MODELLING THE ASSET ALLOCATION PROCESS AND THE EFFECTIVENESS OF THE MODELS THROUGH TIME

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CONTENTS

Abstract

This thesis considers the predictability of asset prices for financial reserving via a cascade style stochastic investment model for the asset classes of cash, equites and fixed interest. Structural breaks occur in 1947 and 1973 but stability since then means that stochastic investment modelling is a feasible proposition. The final model contains four real variables with inflation as the sole exogenous variable. Inflation modelling is both difficult and not critical in a stochastic investment model. Nominal returns are determined from inflation scenarios applied to the real variables. The equations for fixed interest satisfy appropriate diagnostic criteria and produce the features observed in the data. Those for equities are simple but limited. The model is tested with forecasts and scenarios involving different inflation outlooks.

List of Abbreviations

ACF	Autocorrelation Function
ADF	Augmented Dickey Fuller
ADL	Autoregressive Distributed Lag
AEH	Adaptive Expectations Hypothesis
AIC	Aikake Information Criterion
AR	Autoregressive
ARCH	Autoregressive Conditional Heteroskedastic
ARMA	Autoregressive Moving Average
ARIMA	Autoregressive Integrated Moving Average
CAPM	Capital Asset Pricing Model
CCF	Cross Correlation Function
CIR	Cox Ingersoll Ross
CPI	Consumer Price Index
CRDW	Cointegrating Regression Durbin Watson
DDM	Dividend Discount Model
DF	Dickey Fuller
DGP	Data Generating Process
DW	Durbin Watson
ECM	Error Correction Model
EGARCH	Exponential Generalised Autoregressive Conditional Heteroskedastic
EMH	Efficient Markets Hypothesis
ERCH	Exponential Regressive Conditional Heteroskedastic
ERP	Equity Risk Premium
EWMA	Exponentially Weighted Moving Average
GARCH	Generalised Autoregressive Conditional Heteroskedastic
IID	Independent Identically Distributed
IRR	Internal Rate of Return
KPSS	Kwaitkowski Phillips Schmidt Shin
LM	Lagrange Multiplier
LR	Likelihood Ratio
LP	Linear Programme

MA	Moving Average
NPV	Net Present Value
OLS	Ordinary Least Squares
PACF	Partial Autocorrelation Function
PP	Phillips Perron
RBA	Reserve Bank of Australia
SBC	Schwarz Bayes Criterion
SDE	Stochastic Differential Equation
SFE	Sydney Futures Exchange
SPI	Share Price Index
TAA	Tactical Asset Allocation
VAR	Vector Autoregression
YTM	Yield to Maturity

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Chapter 1

Introduction

1.1 Background

The discipline of financial reserving or the setting aside of reserves to meet future liabilities is central to a wide range of activities. Life insurance, superannuation and long-tailed classes of general insurance are activities which fall into this class and are typically long term in nature. Insurance contracts and the determination of superannuation benefits depend upon having a level of assets available to meet long term contractual liabilities. Since assets and liabilities are stochastic variables when they are matched an appropriate level of risk is required. This is often given as the probability of not meeting some target. In the case of an insurance company a typical target would be the solvency ratio.[1] There is a statutory requirement that the solvency ratio should exceed a certain level which is laid down by regulatory authorities as a prudential standard. Thus to write more business upon which to generate more profit an insurer will normally have to raise more capital. This would normally be in the form of equity.

The financial reserving process is about making financial estimates. Historically this work has been the domain of actuaries. Traditional methods involve discounting future liabilities with appropriate discount factors and matching these against available assets. Hence solvency ratios can be determined. Following Markowitz and the introduction of portfolio theory more detailed analyses of assets utilising these principles have entered business practice. Past data on asset classes is collected and a wide range of statistics calculated. These are then put together to form an *efficient frontier* comprising asset mixes formed in an optimal manner. The efficient frontier is a plot of the set of points yielding the best possible return for a given level of risk with the given asset classes where risk is defined as the standard deviation of returns to the

[1] The solvency ratio can be measured as the ratio of assets to liabilities or net assets to liabilities.

portfolio of assets. Portfolio optimisation packages based upon quadratic programming are now readily available.

Implicit in the production of financial estimates is the assumption of stability in the mean level of asset class returns. The variance of returns is also assumed constant as are the correlations between returns to asset classes. In the Markowitz method there is no attempt to explain, for example, the values of the correlation coefficients between returns to asset classes. If the *random walk hypothesis*[2] with respect to share prices is true then attempts to predict *ex ante* returns to equity by other models will be impossible. Increasingly however evidence has emerged to suggest a degree of predictability in asset prices for both the fixed interest and equity markets. If this is so then the returns to the particular asset class can be found. So with the help of models projections can be made.

The finance literature now contains many studies detailing the predictability of asset prices and in particular the concept of *mean reversion*. Mean reversion is the idea that although returns to asset classes can wander away from a long run mean level, economic forces eventually take over and bring the mean level of returns back to a long run level. There is an extensive literature on mean reversion with no complete agreement either on its existence or extent. Nevertheless many in the finance industry take mean reversion as a given and use valuation models to decide whether or not a particular asset class is over or under valued relative to other asset classes.

The term *stochastic investment model* is extensively detailed in the Wilkie model, see Wilkie (1984, 1987, 1992, 1995a and 1995b), which consisted of a series of stochastic equations. The stochastic investment model is a multi 'asset class' model composed of single asset class models. Each single asset class model is assumed to be described by a number of *factors*. In fixed interest, for example, a "parallel shift" factor is the movement up or down along the whole yield curve. This factor can be captured by a financial *variable* such as the long bond rate.

[2] The random walk hypothesis is that share prices move randomly and therefore the best forecast of the share price one period ahead is the current price. See Brailsford and Heaney (1998) p.389-91 for a discussion.

The financial variables can be modelled by stochastic equations. There are potential connections or links within an asset class between individual stochastic equations. The links may be a variable or coefficient that is shared by two or more stochastic equations. The link may be between the random error terms in two or more stochastic equations. For example, in fixed interest it may be hypothesised that the random error term in any stochastic equation for the short term interest rate is linked to the random error term in any stochastic equation for the long term interest rate. If the hypothesis is correct then simulations should mirror this link.

There are potential links between asset classes through individual stochastic equations. In a similar manner to the potential links within an asset class, links may be made between the elements of the stochastic equations describing the different asset classes. An example could be a potential link between long term bond rates and the dividend yield. Wilkie postulates a number of such links between his stochastic equations.

Individual stochastic equations modelling share or bond prices and respective rates of return have been proposed in the literature. By applying a detailed analysis to a given time series the underlying data generating process (DGP) of that series are modelled. New econometric techniques have been developed which, in conjunction with the increase in computing power, means that complex models can be developed and intensive simulations on these models can be easily conducted. As a result there is a wide range of competing models that are based upon stochastic equations within a single asset class. Further, as the models become more complex in order to fit a given data set, new evidence from a new data set can quickly contradict the more complex model.

There are however few stochastic investment models available in the public domain. The essential 'facts' as to the stochastic equations and links between the equations are different between competing models. There are also differences with the choice and application of exogenous variables. In the Wilkie stochastic investment model the rate of inflation is a driving force. The rate of inflation is modelled as a stochastic process. An alternative is to model real variables, that is to determine stochastic equations with the variables discounted by the CPI. This is the approach followed in the stochastic investment model in this thesis and is a feature differentiating it from competing models.

There are advantages of a stochastic investment system of real rather than nominal variables. Firstly it is not essential to model the rate of inflation. If a satisfactory model is available it can be applied. In practice the rate of inflation has proven difficult to model. Otherwise future annual rates of inflation can be assumed as a scenario. Thus a high or low inflation outlook can be considered. The stochastic investment system can then be applied to generate returns under each different scenario. Any changes in the rate of inflation under each scenario would therefore cascade through the model system. Probabilities could be assigned to various scenarios thereby generating a weighted average set of returns.

Secondly the scenario approach gives the ability to directly determine the impact of changes in the rate of inflation. For superannuation trustees or others responsible for meeting long term liabilities, there is a desire to understand what would happen to their particular portfolio if the financial environment, as defined by the inflation outlook, were substantially different from that originally envisaged.

Another feature of the approach to the stochastic investment system proposed is simplicity. Complex mathematical models may not be readily accepted by many of those in positions of authority. Superannuation trustees cannot be expected to accept something which they do not understand. Actuarial consultants have to communicate what they propose and their reasons. Hence not only can one question the statistical value of more complex models, but also whether they are useful tools for an actuary or adviser.

Finally a brief comment on the econometric methodology is warranted at this stage. The philosophy adopted is a pragmatic one consistent with the approach of Hendry *et al.* at the London School of Economics. They are less concerned with how one finds a model, rather the real test is whether the model stands up to scrutiny against alternative models. This subject is discussed at some length in Chapter 6, when exogenous variables are introduced. Data analysis and univariate modelling, which can be viewed as part of data analysis are conducted in Chapters 3-5.

1.2 Aims of the Document

Funds need to be managed and trustees amongst others need advice as to how to undertake the process of setting asset allocations. Methods to determine reserve estimates and to aid in the decision making process such as a stochastic investment model should therefore be targeted at the needs of the client. The stochastic investment model needs to be easily understandable, flexible and capable of dealing with the kinds of questions that decision makers must answer, such as the solvency ratio for an insurance company or the probability of negative return for an industry superannuation fund.

An aim of the thesis is to devise an appropriate stochastic investment model and apply it. However to do so requires much background work. The design of a model requires a close examination of the base data and a detailed investigation of the structure of the financial series. It is necessary to reconcile existing research on financial markets. Hence a primary aim of the thesis is to place the stochastic investment model in context; what can be achieved is investigated. The predictability of asset prices for financial reserving is examined in the context of stochastic investment model development.

In developing the model potential relationships between asset classes will be examined. The risk premia or extra compensation for being in one particular more risky asset class over another will be investigated; in particular the *equity risk premium*, the premium obtained from investing in equities rather than long term bonds. In order to limit the scope of the thesis only the three asset classes of cash, bonds and equities[3] are considered. Property and international investments are not covered. These latter asset classes can be added at a later stage. Hence an objective is

[3] Any stochastic investment system should be complete. Property presents special difficulties such as data availability. Each country can be separately modelled, as in this thesis, then connected. Limiting

modularity in the stochastic investment model, whereby asset classes may be simply added or subtracted.

It is important that the model is used for projections under various scenarios. To see a model operate under various scenarios shows how useful it can be, for example, in performing asset allocations. The results obtained should be sensible. A range of potential scenarios of the rate of inflation may be considered, then returns can be compared and the level of the risk premium evaluated.

1.3 Outline of the Document

The plan to achieve the above aims is outlined in this section. There are some general comments to be made about the document structure. Inflation, covered in chapter 3, is central to actuarial estimates. If a satisfactory model of inflation can be found then it may be applied and estimates made. However the model is one of real variables and so modelling inflation as a stochastic process is not an essential requirement. Scenarios generate inflation outlooks. Chapters 4-8 develop the model in a sequential fashion. Thus each step refers back to previous results and conclusions. The chapters nevertheless have a degree of independence each dealing with a separate feature of model development. Chapter 9 is not central to the final stochastic investment model. The equity risk premium, covered in chapter 9, is of prime importance to the cost of capital and asset allocation. It is an output of the model and not a key step in model development. Each chapter has a similar structure with an introduction, literature review, exposition of theory, analysis and results followed by a summary and conclusions. An overview of each chapter in the thesis now follows.
Chapter 2 reviews the literature. It summarises the literature and places the study in the context of the overarching finance literature.

Chapter 3 covers inflation modelling. The first section introduces the inflation data series and discusses the sources of information and any shortcomings. The Consumer Price Index (CPI) is reviewed in the second section and measures of the mean level

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the task to an individual country such as Australia is still a substantial requirement. Chapter 11 concludes with a brief discussion of these possible extensions.

and volatility of the CPI are considered. In the third section these are then explored for stationarity via unit root testing. Univariate modelling is applied to the data utilising standard Box-Jenkins methodology. In the fourth section some aspects of potential heteroskedasticity are also investigated with the introduction of a range of non-linear models. Various statistical measures generated from these processes will help to understand the nature of the time series. The final section investigates inflationary expectations and the role of unexpected inflation in financial market action. A model of inflationary expectations is then developed.

Chapter 4 covers data analysis and univariate modelling of equities and fixed interest. The first section details the data sources for the financial series used in the model and any limitations. The second section considers the equity series. Unit root tests are conducted to determine the order of integration of the financial variables. A series of univariate models with stationary data are then developed to describe the DGPs. Then heteroskedasticity and the normality of the residuals are considered with the application of non-linear models. The third section performs the same task using the fixed interest series. In so doing an understanding is gained of the common features in the series.

In chapter 5 the inter-relationships between inflation, equities and fixed interest are investigated within a cointegration framework. Hence long run relationships between the integrated series can be assessed. In the first section concepts and definitions are introduced. The Engle-Granger two-step procedure is used in the data analysis. The Johansen maximum likelihood test is applied as a check on the procedure. In the second section the set of real and nominal bivariate relationships is tested. The link between inflation and nominal share prices is examined in the third section with a long annual data series. In the fourth section the trivariate relationship between the bond yield and the components of the dividend yield is investigated for both nominal and real cases. In the final section all the real variables are considered together.

Chapter 6 covers the modelling of the stationary series with inflation as an exogenous variable. The first section deals with issues in model building, reviewing some of the literature and explaining the chosen methodology. The second section describes the relationship between inflation and each financial series. The chosen model form, from the wide class of models available and the diagnostic tool kit are then reviewed. In the third section this is applied to modelling the financial time series with the introduction of inflation as an explanatory variable. Box-Jenkins transfer functions are used in the fourth section. This provides an alternative approach to the standard linear regression techniques. Unexpected inflation is then introduced as the independent variable in the fifth section. The aim is to determine the role of expectations and to compare the results with those obtained from using observed inflation. If unexpected inflation provides superior results then the results of Chapter 3 can be applied to obtain unexpected inflation from observed inflation. The chapter is concluded with a discussion in the last section of the best equations for the stationary components in the stochastic investment model. A comparison of results with the use of qualitative judgments as well as the available diagnostics is made.

Chapter 7 covers the levels modelling of the integrated series for the fixed interest asset class. The first section reviews some of the literature, the competing models and the progress of research in Australia. In the second section an empirical analysis is conducted which results in a real bond model which satisfies the requirements of non-negativity and mean reversion. The third section covers the relationship between real bonds and real T-notes. The number of factors needed to adequately model interest rates and the yield curve is examined. The cointegrating relationship between real bond rates and real T-note rates is utilised to enable the introduction of error correction models (ECM) involving the levels of the two variables. The results of this analysis can then be compared to the forecasting ability of the long/short ratio, which is another way of viewing the cointegrating relationship. However the ratio does not contain the extra information available in the levels data. Thus the question as to whether a better model can be obtained from this extra data is considered.

Chapter 8 covers the levels modelling of the integrated series for the equities asset class and any potential links between the non-stationary or integrated components of equities and fixed interest. It also covers any potential links between the stationary components of equities and fixed interest. The first section reviews some of the literature and the progress of research in Australia. The second section reviews the long run links between inflation, dividends and share prices, with levels data. An ECM connecting real share prices and real dividends is found and compared to the dividend yield model. The third section reviews connections between the differenced stationary components of the series. Any potential links between the residuals from the final equations are then reviewed to complete the investigation. The final section reviews the stochastic investment model in the light of findings to present the working model.

Chapter 9 introduces the equity risk premium (ERP). In the first section the concepts, some definitions and the literature are reviewed. In the second section a methodology for dealing with *ex ante* values is proposed. The value of the ERP is then adjusted for dividend imputation and an assessment of the reasonableness of the current level is performed. Future trends are then reviewed to see what range of values the ERP could take. In the final section a potential modelling process for the risk premium is outlined. A model of inflation expectations employing the cost of capital is put forward. The model has significant limitations but has the potential for further development. This provides a direct method of obtaining returns to shares via bond returns.

Chapter 10 summarises the stochastic investment model providing out-of-sample forecasts and scenario based simulations. The first section provides a schematic overview of the model and the logic behind scenario building. Then in the second section mechanisms are introduced to convert the stochastic model equations into returns. The third section provides a set of forecasts which can be compared to actual returns. There are available out-of-sample values for the period December 1997 to September 1999. Scenarios are then set and simulations performed yielding sets of asset class returns. The section is rounded off with a discussion of the shortcomings and weaknesses of the model and some potential applications. The chapter concludes with a discussion of the model.

Chapter 11 then concludes the thesis. The first section reviews the main findings. In the second section these findings are then discussed in terms of any unresolved issues and what impact the findings have in the wider arena of asset liability determination.

The final section considers some potential directions that stochastic investment modelling may take.

Finally, there is the caveat that care needs to be taken in moving between real and nominal variables; yields to maturity or discount rates and returns; *ex ante* and *ex post* returns. Much of the disagreement in the literature arises from differing definitions, differing time periods for data analysis or results from different countries with different legal and taxation systems.

Chapter 2

Survey of the Literature

2.1 Introduction

Financial reserving, as discussed in section 1.1, is an essential process for ensuring financial security. The meeting of financial obligations ensures the future welfare of both corporations and individuals. Hence the methodology for estimating financial assets and liabilities is an extremely important business decision. Actuaries employ different techniques to arrive at reserve estimates. These methods suppose a degree of predictability in the assets and liabilities. The methods are often simple involving assumptions about trends in asset returns and liabilities with appropriate asset allocations and discount factors to find a NPV. A more formal method for determining asset allocation is the Markowitz mean-variance technique, a single stage quadratic programme. The Markowitz approach has been extended to liabilities with the introduction of the concept of value at risk. The development of the techniques in the actuarial profession continues. Stochastic investment models as outlined in section 1.1 are in the process of development to accommodate the perceived shortcomings of the Markowitz method.

The aim of the literature review is to provide the scope of research in the arena of stochastic investment modelling. As indicated in section 1.3 each chapter covers literature pertinent to the issues under consideration in each chapter and is a more comprehensive review. The literature review in chapter 2 puts the research in context.

The literature review breaks down into seven sections. In the first three sections 2.2-2.4 a review is conducted of the predictability of asset prices in the finance literature. The review covers a wide range of topics. In section 2.5 multi-asset class models are reviewed. These are termed stochastic investment models in this document, as discussed in section 1.1. Section 2.6 looks at single asset class models. In section 2.7

possible approaches to model building are discussed. Econometric methodology is explored. Section 2.8 considers statistical issues. This section details techniques of data analysis, econometric modelling and statistical inference, stochastic processes. and recent advances in financial economics. It covers more general statistical problems as well as specific issues faced in the analysis of financial data.

2.2 Asset Class Relationships and Predictability: US and International Experience

Research into anomalies in asset returns can be categorised in many different ways. Broad categories could include asset predictors, both short and long run; mean reversion and excess volatility; and a time varying risk premium. In this section studies are categorised in order to capture the main directions of research both internationally and in Australia.

2.2.1 Early Studies, Market Efficiency

The search for predictability in asset returns has a long history. The random walk hypothesis and the efficient markets hypothesis (EMH) goes back to Bachelier (1900). He proposed that the distribution of price changes were independent identically distributed normal variables. Not until 1939 in the aftermath of the Great Depression did the next important review, the Cowles Commission in the US, occur. The intervention of WWII meant that research interest in asset price predictability recommenced in the 1950's.

Kendall (1953) considered the autocorrelation structure of various financial time series and concluded they were random walks. One exception he found to the random walk hypothesis was a cotton price series where he had used averages. This inspired the brief note from Working (1960), who showed that first differences of averages in a random chain exhibit significant autocorrelation, thereby removing the exception.

The collection by Cootner (1964) contains a series of papers whereby various authors[1] conducted a range of tests on series, in an attempt to see whether various trading rules could be applied to "beat the market". Fama (1970) in his review of financial markets, pulled together some of the results of earlier research, to provide his definitions of forms of market efficiency. The net result was that share prices were considered a random walk.

Research since then has challenged the notion that asset prices are a random walk process[2]. A comprehensive review of speculative prices in the finance literature was published by Fama (1991) revisiting his 1970 paper. It shows both the scope of the field and the enormous growth in interest and research conducted since 1970. In this later work Fama is less sure about the efficiency of markets. He cites many examples of the predictability of share prices.

2.2.2 Cross Sectional Studies: Asset Predictors

A variety of papers have investigated potential predictors of asset prices, the list in this sub-section is but a small sample; the results favour predictability. Keim and Stambough (1986) sought *ex ante* variables with which to predict *ex post* risk premia. They concluded that the level of asset prices contained information about expected premiums, especially for the bond portfolios.

Fama and French (1989) investigated the relationship between expected returns and general business conditions. They looked at certain defining characteristics of business conditions and their relationship to the default spread, term spread and dividend yield. Fama and French (1992) considered the predictability of cross sectional returns. They found that average stock returns were not positively related to

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[1] See the papers by Roberts (1959), Larsen (1960) and Alexander (1961) in the Cootner compendium.
[2] Graham, Dodd and Cottle (1962) prior to Fama (1970) had proposed in his text on security analysis and other writings that the price of a stock was driven by its earnings power. The claim is that in the long run share prices follow earnings and are therefore predictable (assuming earnings are predictable). Graham's publications including *The Intelligent Investor* are highly regarded by professional stock and share analysts.

the stock beta, a central tenet of the Sharpe-Lintner-Black model[3]. However they concluded that for the 1963-1990 period, size and book-to-market equity captured the cross sectional variation in average stock returns. These two predictors were sufficient, explaining other predictor such as the earnings yield. Fama (1995) looked at specific practice in stock markets, notably technical and fundamental analysis. This short paper is aimed at the practitioner. Fama finds too little connection between the academic and the practitioner. He feels that much market practice has little merit; share market analysts need to expose their predictions to the ultimate test of demonstrating a track record.

Hawawini and Keim (1995) reviewed the global evidence on the predictability of stock returns. The authors investigated the application of forecasting variables such as the term structure and dividend yield, noting that the recent research supported the use of these and related variables to predict stock returns. The survey of a wide range of empirical studies showed predictabilty in various facets of stock returns.

Zhou (1995, 1996) considered various models of share returns. Zhou (1996) assessed the predictability of returns from the share market based upon the term structure. The author claimed that there was a strong link between *ex ante* interest rates and expected share returns and his single measure of the term structure.

2.2.3 Asset Returns and Inflation

The previous sub-section reviewed a small portion of the literature on general predictabilty. The focus now moves to specific links between asset returns and inflation.

Fama and Schwert (1977) considered the link between asset returns and the rate of inflation. They found returns to bills, bonds and real estate to vary with their definition of expected inflation, but stock returns to respond negatively and therefore

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[3] There are a range of studies in the literature testing the CAPM and the utility of stock beta. A full discussion of related papers for and against the Fama and French findings is given in Chapter 6, section 6.5 on empirical tests of the CAPM in Brailsford and Heaney (1998).

not to be a hedge against inflation. They concluded that the risk premium on stocks, which they defined as the difference between the expected returns on stocks and bills, was negatively correlated with the interest rate.

Lee (1992) found that interest rates explained a substantial fraction of the variation in inflation and that inflation responded negatively to shocks in real interest rates. Lee also found that inflation and stock returns were negatively correlated. He concluded that stock returns explained much of the changes in real activity. However Campbell and Ammer (1993) suggested a positive relationship between inflation and stock returns. They pointed out the difference between a long run and a short run effect. In the long run stocks are a hedge against inflation. This proposition was tested by Boudoukh and Richardson (1993) who conducted an empirical analysis of the relationship between stock returns and inflation, based upon annual US data for 1802-1990 and UK data for 1820-1988. The authors concluded that strong support was provided for a positive relation between nominal stock returns and inflation at long horizons. They pointed out that to the extent that researchers developed theories to explain the negative correlation at short horizons, these models should also be consistent with their evidence.

In terms of the importance of potential predictors Golob and Bishop (1996) concluded that stock prices follow inflation more closely than they followed interest rates. Any link with long term interest rates was minimal, beyond that which was contained through inflation and its effect in turn on interest rates. This potential link was reviewed in Graham (1996) with an investigation into the causality of the negative relationship between share returns and inflation, and the role that monetary policy plays. This investigated Fama's conjecture that the link between inflation and stock returns is in turn due to their common link to real activity. He concluded that the negative relationship appears to arise only in periods when monetary policy is either neutral or counter cyclical and when variability in the inflation rate is associated with variability in the growth of real output.

Rather than inflation driving interest rates Lahiri and Dasgupta (1991) applied interest rates as a predictor of inflation, comparing their results with other composite leading indicators. They concluded by suggesting that the utility of a composite leading indicator of inflation can be greatly enhanced if price forecasts obtained from the bond market are included as one of its components.

2.2.4 Excess Volatility and Mean Reversion

The concepts of excess volatility and mean reversion discussed in this sub-section are introduced. These anomalies in finance have generated much debate in the literature.

Shiller (1979) considered the expectations model of the term structure of interest rates. The expectations model implies that long term rates are a long term average of short term rates plus the liquidity risk premium. Shiller demonstrated that any model based on expectations cannot have the degree of volatility actually observed in long rates, hence *ipso facto* the expectations model is incorrect. Shiller applied the same technique to share prices. Campbell and Shiller (1988), Shiller (1989) found that share prices exhibit 'excess volatility' that is movements in share prices are far greater than that implied by the NPV model of dividends. In Campbell and Shiller (1988) excess volatility is connected to the predictability of share prices via various factors such as the dividend yield. The excess volatility observed is the basis of the predictability of prices.

West (1988) reviewed the excess volatility debate and the associated 'bubbles' literature. There are firstly statistical issues of inference. There is a small sample bias caused by near unit roots. West finds that correcting the situation with tests accounting for unit roots improves the situation but still leaves excess volatility. Explanations for the excess volatility involve "rational bubbles"; alternative models of returns or "fads" models. West reviews bubbles but rejects them as a potential explanation. He looks at variations in expected returns but finds that traditional models do not provide an explanation either. As regards fads he finds that there is little formal positive evidence pertaining to fads models to sway someone unsympathetic to fads models. West feels that non-constant expected returns are a reality. This is the cause of the excess volatility. However he concludes that non-constant expected returns have manifested themselves in a variety of ways, the difficulty is the source of the non-constant expected returns.

The mean reversion hypothesis (see section 1.1 for a discussion) was tested by Poterba and Summers (1988). They applied the variance ratio[4] test to U.S. stock prices and found mean reverting behaviour. They showed that there is initially positive autocorrelation in stock prices followed by negative correlation at longer lags. The degree of this negative autocorrelation is small at any given longer lag but accumulates to a significant total. Fama and French (1988a, 1988b) in two separate papers looked at mean reversion and found similar results; though the extent of mean reversion in the long run was dependent on the inclusion of the 1930's period. Lo and McKinlay (1988) concluded that although stock prices were not a random walk, they were also not described as a random walk plus a transient, mean reverting component. Kim, Nelson and Startz (1998) revisited the Poterba and Summers results by applying a different sampling procedure to generate a sampling distribution for the variance ratio statistic which differed from that of Poterba and Summers. They therefore generated different *p*-values to those obtained by Poterba and Summers and found results which substantially weakened the evidence in favour of mean reversion.

De Bondt and Thaler (1989) reviewed mean reversion, event studies and the predictability of short term price movements via predictors such as P/E ratios. They suggested that many players are not entirely rational. They were pessimistic about the chances for success for traditional models in which all agents are assumed fully rational. They suggested that models in which some agents have non-rational expectations of future cash flows or have faulty risk perceptions may offer greater promise.

2.2.5 Time Varying Risk Premia

A potential explanation of excess volatility is that the risk premium varies over time. Keim and Stambough (1986) sought *ex ante* variables which predict *ex post* risk

[4] For a detailed discussion and application of the variance ratio test to Australian data see Hart (1996, chapter 3).

premia. The third section of the paper considers predicting risk premia. Their regressions indicated better predictability with the lower grade stock and bonds (on the basis the more risky the specific stock or paper the more leveraged it is to the risk premium). They concluded that there is evidence of time varying risk premia and that the levels of asset pices contain information about expected premiums especially for bond portfolios.

Benari (1990) suggested that there are various eras which are more or less favourable to different asset classes. These eras may persist for a long time until eventually the system corrects itself. He outlined three periods 1966-72; 1973-82 and 1983-88. A determining factor of these eras was the nature of price inflation. The first period can be characterised as one of stable but rising inflation. The second as accelerating inflation and the third as declining inflation. Different types of asset did well under each scenario. This view is consistent with Bollerslev and Hodrick (1995). The authors summarised the key issues in finance and the econometric tests and estimation procedures that have been applied to these topics, and concluded in favour of time varying risk premia or eras in financial markets.

A similar conclusion was reached by Kurz and Beltratti (1996) and Kurz (1997), who hypothesised that endogenous uncertainty is dominant in the equity market. There are eras in the stock market when significant amounts of money, in real terms, could have been lost. They extended these ideas to a theory of rational beliefs which they claimed explains many of the anomalies in the finance literature. Chow and Ming (1997) approach the excess volatility debate from the viewpoint of different eras or regimes. The authors took a simple two state regime of dividend growth rates, those of expansion and contraction. Thus hypothesising that the timing activity of investors generated the observed excess volatility. They then defined their expansion/contraction models, conducted simulations and showed 'excess volatility' behaviour.

2.3 Asset Class Relationships and Predictability: Australian Experience

This next section reviews the literature on the issues discussed in section 2.2 in an Australian context. Volker (1982) applied inflationary expectations and other potential explanatory variables to model the Treasury bill rate. He concluded that for the period 1968-1979, liquidity conditions have been the major determinant of short term interest rates in Australia and that inflationary expectations have been reflected to a significant but relatively small degree.

Inder and Silvapulle (1993) discussed the Fisher effect for bank accepted bills. They find that a 1% rise in inflation leads to a 0.5% rise in interest rates and hence a fall in the real rate of 0.5%. Interest rates show stickiness, even in the long run, with nominal interest rates not responding fully to rises and falls in the inflation rate. Mishkin and Simon (1995) examined the Fisher effect for T-notes. They tested the idea that a rise in short term rates implied a rise in expected inflation rather than a tightening of monetary policy. The authors concluded by finding that changes in short term interest rates can reflect the stance of monetary policy. However in the long run it is changes in inflation that affect the level of interest rates. Olekalns (1996) tested a less demanding relationship, that of a partial adjustment of the expected inflation rate, rather than the nominal interest rate fully incorporating anticipated changes in the price level. He concluded that the nominal interest rate only partially adjusted to anticipated inflation. However for the post-deregulation period alone analysis showed that complete adjustment was achieved.

A series of Australian actuarially based papers were published in the 1990's. Carter (1991) found no causal link between inflation and the All Ords index. The index was best modelled as a random walk. He also found that interest rates were not a significant determinant of the All Ords index. Fitzherbert (1992) found against inflation as the long run determinant of the All Ords index. He maintained that retained profits[5] plus a partial adjustment for inflation, determined the long run trend in share prices. This view is consistent with Graham, Dodd and Cottle (1962). Harris

[5] The level of retained profits is dependent upon the leverage and dividend policy of a given company.

(1994) concluded there was a positive relationship between quarterly real GDP growth and quarterly All Ords index returns. He also found that there is a connection between T-notes and All Ords index monthly returns but none between inflation and All Ords index monthly returns.

Groenewald, O'Rourke and Thomas (1997) found a negative relationship between stock returns and inflation. They did not find that monetary policy, either pro or counter cyclical, has an impact. They concluded that the negative relationship between stock returns and inflation was the outcome of interactions in the economy as a whole. Inflation rates affected many macro variables some of which in turn, affected stock returns in addition to any direct effect. Crosby (1998) found that a 1% positive shock to inflation lead to a 0.1% positive increase in the share index. The negative link between stock prices and inflation is a short run and not a long run feature of the data.

2.4 Asset Class Relativities and the Equity Risk Premium

From individual asset predictability, the predictability of asset class relativities follows. This naturally leads to tactical asset allocation (TAA) in funds management, see Hodges (1995). Hence the attempt to build models to time movements between asset classes. This leads to a measure of asset class return relativities and the concept of the equity risk premium.

The *ex post* ERP has been determined by various authors in different markets. It is noteworthy that only a few Anglo-Saxon markets can provide long term studies. In the US Ambatscheer (1989) puts forward a risk premium of 2.5% over bonds and 4% over T-bills. Siegel (1992) conducted a US study over the period 1802-1990 and found a steady level of the premium over short term bonds of 6%. Hathaway (1995) for Australia found a post war arithmetic average equity premium over T-notes of 6.6%. Hibbard (1998) examined the New Zealand experience. He defined the ERP as the excess return on equity over investment in a 90-day New Zealand Treasury bill. He found that for the period September 1982 to December 1997 the value of the average ERP was 6.2%. This is consistent with the Australian result. These results exhibit a consistent excess return from equities over bonds.

The Australian results included no benefit for dividend imputation introduced in Australia in July 1987. Officer (1992) believes that the effect of imputation should be included in the premium either through the calculation of the rate of return, by including the grossed up dividends or by making an adjustment to the normal CAPM equation to include the value of the tax credits. This view is consistent with that of Davis (1999) who argued for the full inclusion of the value of the franking credit.

Obtaining the *ex ante* ERP is harder. Clothier (1989) used an IRR as the expected rate of return. Harris and Marston (1992) used analysts forecasts and the DDM to derive the equity return for the US market over the period 1982-91. The *ex ante* ERP found by Harris and Marston is similar to the long term average *ex post* ERP at approximately 6%. Blanchard (1993) used annuity values to find real returns for the variables under question. He found that the Great Depression of the !930's had distorted the long term results. He concluded that the equity premium had gone down steadily since the early 1950's. Inflation had contributed to the transitory increase above trend in the 1970's and the transitory decrease below trend in the 1980's. Yamaguchi (1994) defined the required risk premium as the excess return for equity over bond yield demanded by an investor tolerant of a specific level of downside probability. He deduced that prices in 1994 argued for a holding of greater than normal weight in bonds.

Mehra and Prescott (1985) found a 'premium puzzle', caused by the size of the *ex post* ERP. They defined the ERP as the return from the market over the return from the riskless asset and found that it was too large relative to the level of the return from the riskless asset. Hibbard (1998) found in favour of a similar 'puzzle' occurring in New Zealand. Kurz and Beltratti (1996) and Kurz (1997) attempted to explain the 'premium puzzle' of Mehra and Prescott. They outlined a theory of Rational Belief Equilibrium. The observations in financial markets are often the result of mistakes or incorrect assessments. For example, making judgements based upon the norms for a previous era when circumstances change. It takes time for participants to catch up with the changes. If beliefs and financial eras dominate then there will be periods of consensus views and non-consensus views. Consequently in a strategic sense the requirement is to find out the nature of the current era and adjust assets accordingly.

Then either trends are followed or upon the realisation that there has been a change of state, assets are adjusted to the new era.

Papers from a meeting of academics and practitioners to discuss the ERP were collected in the compendium by Sharpe and Sherred (1989). A problem with generalisations is the conflicting definitions, data transformations and analytical methods employed. However even allowing for this there would appear to be significant differences of opinion both with respect to the fact of predictability and the potential causes.

French (1989) considered risk premia, defined as the return on the particular portfolio minus the T-bill rate, with respect to various fundamental indicators such as the dividend yield, the default spread and the term spread. He found evidence of the predictability of the premium using these factors. Summers (1989) found that markets consist of much noise but a long run tendency to revert to fundamental value. This predictability of prices based upon fundamental indicators argues for mean reversion strategies at times and momentum based strategies at other times. Sharpe (1989) accepted mean reversion and then aimed to explain variations in the risk premium with a wealth based proxy. On the other hand Nelson (1989) found problems with mean reversion post-war, since the effect of the Great Depression of the !930's overstated the issue. He argued that regression results found to date were due to sample bias, accounting for much of the R^2 found in results. The data to be really conclusive was lacking.

Practitioners were more concerned with TAA and appropriate models. Clothier (1989) described the system in place at Wells Fargo. The IRR is used as a proxy for the expected rate of return and the risk premia between the respective asset markets is used as the indicator in TAA. He claimed excellent results with this approach. Henriksson (1989) (Kidder Peabody) followed a similar approach to Clothier. The gains come from mean reversion and from doing the opposite of what the majority of the market does. Grossman (1989) described a system called informational TAA. The model he described attempted to segment out the risk preference induced moves in price, from changes in price caused by yield, for example, by looking at yield relativities. Brinson (1989) (Brinson & Associates) applied his analysis to the global scene. He gave a table of values for risk premia, ranging from 5.6% for the UK to 4.6% for Japan. Brinson's method of determining risk premia is a process based on both historical data and the understanding of the local market. He then applied TAA, by using deviations from equilibrium at various levels, in a similar way to that described by the other practioners.

A more behavioural approach to market dynamics is provided by Tversky (1989). He outlined how people actually think about risk. He identified examples of 'irrational' behaviour such as loss aversion; the segregation of individual assets (seeing the trees not the wood); and the overconfidence inherent in estimates about the future when actually tested.

Reichenstein and Rich (1993) assessed the predictability of stock returns with predictors such as the earnings yield, dividend yield and the ERP. They found value in the ERP and used it in TAA and indicated where it could add value. Finnerty and Leistikow (1993) considered various risk premia and modelled the processes with a mean reverting model, both with and without a trend component. The authors took the period 1926-89 and found evidence of both mean reversion and a downtrend in the ERP.

2.5 Stochastic Investment Models

A stochastic investment model is a multi-asset class model composed of stochastic equations which represent the DGP of the factors that are sufficient to describe each asset class as described in section 1.1.

There are few stochastic investment models. Stochastic investment models were popularised by Wilkie (1984, 1987, 1992, 1995a and 1995b). The Wilkie approach used inflation as a driver with expected inflation modelled as an exponentially weighted moving average (EWMA) of observed inflation. Wilkie postulated that in the long run inflation will be fully incorporated into the income stream of assets. Hence a 1% change in the rate of inflation will lead to a 1% change in dividends but with lags. Wilkie included many links between predictive variables. For example the random shock term from the dividend yield equation is a component in the equation describing the yield on long term bonds. There is no justification given for the stochastic equations and links between them, and these failings form the basis of certain criticisms of the model.

As a public document Wilkie's model has been extensively reviewed. The paper by Geoghegan *et al.* (1992) resulted from a comprehensive report to the UK Institute of Actuaries, authored by Harvey. The report reviewed the value and application of stochastic models with particular reference to Wilkie's model. The report is in three parts. The first outlined Wilkie's model, alternative models and the use of the Wilkie model. The second part is a series of appendices which included an updated non-linear inflation model and the consideration of ARCH[6] effects in the series. The last three sections of the second part investigated the application of the model in aspects of actuarial work, such as pension funds, life offices and investment management. The third part is a discussion by participants and others of various topics raised by the report as well as more general issues. The discussion included; the inflation model (negative rates of inflation, ARCH effects); the use and modification of the model by life offices; and technical issues such as parameter estimates, the number of simulations and the extent of actuarial judgement.

Other actuaries have seen a role for such modelling[7]. TPF&C[8] (Mulvey and Vladimirou 1992, Mulvey 1994,1996, Mulvey and Ziemba 1995 and Mulvey and Thorlacius 1997) have directly funded research. The CAPLINK model of TPF&C is a cascade structured model, see Mulvey (1994) for a description. Mulvey made certain assumptions such as stock returns were dependent upon a number of factors including the economic conditions of the companies, inflation, interest rates and momentum. Whilst considerable detail is given, the TPF&C model is proprietary. For example, the

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[6] Engle (1982) introduced models with varying volatility or conditional heteroskedasticity as one method of dealing with heteroskedasticity. Engle applied this model to the UK inflation rate.

[7] A recent model by Duval, Teeger and Yakoubov (1999) appeared whilst the model herein was being finalised. Stochastic investment models developed for Japan, South Africa and Finland are also mentioned.

[8] Towers, Perrin, Foster and Crosby, a US based global actuarial consulting firm. They have offices in Australia.

bond model, which lies at the top of the cascade structure, is based upon the two factor model of Brennan and Schwartz (1982). The equations are given in Mulvey (1996); however extra terms appearing in the model are not justified and individual components are only explained in general terms.

In Australia limited work on a potential model has been conducted by Carter (1991). The main focus was the model outlined in section 4. Carter investigated inflation over the period 1970-1990, fitting an ARIMA(1,2,2) to the CPI. The All Ordinaries index was modelled as a random walk. Fitzherbert (1992), as a response to Carter, was a descriptive document which reviewed theoretical aspects of stochastic investment models, suitable transformations and applications. The Fitzherbert approach is to find suitable transformations for the data rather than the application of first differences. Harris (1995) compared various stochastic investment models for long term studies. Harris compared 5 approaches namely; the random walk model; an IAA (Institute of Actuaries of Australia) model; Wilkies model applied to Australia; a VAR model and his own ERCH[9] model. He found that his ERCH model performed the best based upon his ranking criteria. He found that the VAR is over-parameterised. The Wilkie model gave unacceptable results, in particular for the 10-year bond yield. He suggested that annual share price returns are unpredictable. A random walk model performed as well as any other model.

More recent coverage comes from a series in Sherris (1995, 1997a, 1997b) and Sherris, Tedesco and Zehnworth (1996). These papers do not provide a stochastic investment model but investigated the features of the variables which could form such a model. There would not appear to be an Australian stochastic investment model.

An application of stochastic investment models is in asset liability modelling. The advent of increased computing power has led to the introduction of linear programming (LP) techniques. These employ a multi-stage decision process using stochastic or dynamic programming, a technique due to Bellman. Each LP contains a

[9] Harris defined an ERCH (Exponential Regressive Conditional Heteroskedasticity) model, where the conditional variance was expressed as a linear combination of lagged exogenous and dependent variables, see Harris (1994 , p.48).

vast array of equations and identities representing such items as cash flows, fiduciary constraints or models for the DGP of variables. These are then analysed by a network or decision tree. An early example is Bradley and Crane (1972) who employed a multi-stage decision process and applied it to bond portfolio management. More recent papers include the series from Mulvey and Vladimirou (1992), Mulvey (1994) and Mulvey and Ziemba (1995) in co-operation with TPF&C and Carino *et al.* (1994) in co-operation with Frank Russell. Other papers are found in collections by Zenios (1993) and Zenios and D'Ecclesia (1994).

The LP's are on a range of financial planning problems. McKendall, Zenios and Holmer (1994) used stochastic programming to match a given liability under interest rate uncertainty with a portfolio of mortgage backed securities. The equations in the LP's for the asset returns have a range of origins. Judgements or simple extrapolation models have been used, as in Carino *et al.* Another possibility is the Markowitz efficient frontier quadratic programme. A further alternative is a set of stochastic equations for multi-asset classes, representing the DGP of the asset returns in the LP. The stochastic equations come from stochastic investment modelling. Therefore an example of the application of stochastic investment model building is found as a component of the wider set of LP solutions to asset and liability reserving.

2.6 Single Asset Class Models

The next section reviews the literature on single asset class models. These are categorised under the headings of equity, fixed interest and inflation models. The models include those which are part of stochastic investment models as well as ones for specific asset classes. They include both continuous and discrete models. Those covered are not a complete list, in the case of inflation models there are a host available. The emphasis is Australian models.

2.6.1 Equity Models

Equity models appear as part of both the Wilkie and the Mulvey stochastic investment models. In the Wilkie model the logarithm of the dividend yield is modelled as an

AR(1) process plus a component dependent upon current inflation. The dividend in turn depends on the current value of observed inflation plus an EWMA of observed inflation, and on the residual from the dividend yield model. The parameters are such that there is a unit gain. The mean rate of increase of real dividends has a value of zero in the 1984 parameters but is taken as non-zero in 1995. The share price index is found from the product of the mean reverting dividend yield and the nominal dividend.

In the CAPLINK model of Mulvey, stock returns are composed of dividends plus capital gain. Real dividends are constant and stock prices are modelled as a diffusion process. The equations are not given but they include interest rates, inflation and time, see Mulvey (1996).

Carter (1991) models the All Ords index as a random walk. Fizherbert (1992) applied the variance ratio test and concluded that a random walk does not adequately describe the All Ords index. He suggested that there is an equilibrium value about which share prices fluctuate based upon the ratio of share prices to book values. The ratio of price to book suggested by Fitzherbert was 1.5 times.

Harris (1994, 1995) modelled Australian Ords index returns. He considered Wilkie's model, and those of Fitzherbert (1992) and the Maturity Guarantees Working Party. The final section in Harris (1995) conducted an empirical analysis on the various models, including those from the ARCH/GARCH/EGARCH class, as well as his own ERCH class, defined previously. He concluded in favour of a random walk model for annual All Ords index returns.

Gosling (1999) provided a stock valuation model with only three variables required for stock returns: the initial dividend yield, the growth rate of company earnings and the change in the P/E ratio. The earnings growth was predicted from nominal GDP and the change in the P/E ratio by the rate of inflation. The change in the P/E was the most important factor with a negative correlation between inflation and the P/E ratio. This was not justified in the article.

The evidence suggests considerable disagreement about equity models. There are a variety of models on offer[10], from the simple random walk to ones with many components such as that of Mulvey. Fortunately the fixed interest asset class presents more agreement.

2.6.2 Fixed Interest Models

A large area of research in the pricing of fixed interest securities is based upon *continuous* time models of stochastic processes. These can be modelled by stochastic processes such as arithmetic or geometric Brownian motion. These provide the pricing models widely used in the valuation of derivative securities. The theory underlying stochastic processes can be found in Cox and Miller (1965). A review of the some of the common models in use are described in Shimko (1992) and Sawyer (1993). Discrete analogues of these processes form alternative models for lower frequency data. Authors such as Vasicek (1977), Brennan and Schwartz (1982), Cox, Ingersoll and Ross (1985) and Mulvey (1996) have proposed solutions with these forms.

One of the earliest models of the term structure is the mean reverting Ornstein-Uhlenbeck (O-U) model proposed by Vasicek (1977). This can be solved in closed form yielding the price of a zero-coupon bond. General solutions for a wide range of the commonly applied SDEs are given in Sawyer (1993). Musiela and Rutkowski (1997) provide an extensive review of potential models and solutions.

An extension of the single factor model of Vasicek is the two factor model due to Brennan and Schwartz (1982). They defined the stochastic processes for the long or consol rate and the short rate as general O-U processes. To estimate parameters they moved from the continuous model to a discrete approximation and fitted this to Canadian government bond data from December 1958 to December 1979. The Wilkie stochastic equations for long term and short term bonds are consistent with those from the Brennan and Schwartz model.

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[10] See Duval, Teeger and Yakoubov (1999) for another. It is an active research topic.

Cox, Ingersoll and Ross (1981) applied the techniques of diffusion processes to bond prices to evaluate various theories about the term structure of interest rates. The final section of that paper reviewed some widely used stochastic diffusion models and gave functional forms which could be used in model building in empirical studies. They discussed the cases of models only using lagged values of interest rates as predictors and models which are linear in the state variables. Cox, Ingersoll and Ross (1985) presented a single factor model as a stochastic differential equation which they then solved. This is referred to in the literature as the CIR model. They applied the model to evaluating long term bond prices and discussed the implications in detail. An advantage of the CIR model is that it ensures non-negativity in bond prices. Rogers (1995) surveyed stochastic differential equation solutions for bond modelling and the strengths and weaknesses of each of the various formulations, including a discussion of the class of non-negative models.

Empirical studies into the success of the single factor CIR model, have been conducted by Brown and Dybvig (1986) and Brown and Schaefer (1994). Brown and Dybvig found a reasonable fit for the volatility of the model but the model systematically overestimated short term interest rates. Brown and Schaefer found the model needed a good deal of modification if it were to adequately represent the facts. Pagan, Hall and Martin (1996) reviewed models on the term structure. The authors concluded that the predictions from CIR-type models were diametrically opposed to the data.

Brown and Schaefer (1995) applied a two factor model. Brown and Schaefer concluded that only two factors were needed to explain changes in bond yields. The long rate and the spread between the long rate and the short rate were the two factors chosen. Duffie and Kan (1995) supported this view. They found that empirical studies suggested 2 or 3 state variables might suffice for practical purposes. Sherris (1995) conducted a factor analysis of yield curve changes using Australian interest rate data and supported these conclusions.

The usefulness of the spread as a predictive measure was tested by Lahiri and Wang (1996). They considered the utility of three measures of the spread in the term structure of interest rates as predictors of the business cycle. They found the spread between 10-year bonds and 1-year bonds to be the best predictor, signalling all the major turning points with no false signals.

The Towers Perrin two factor model is a based upon the Brennan and Schwartz model, see Mulvey (1994), Mulvey and Ziemba (1995) or Mulvey(1996). The model is a variant of the Brennan and Schwartz approach, which allows for a compete yield curve by curve fitting. The interest rate process involved mean reverting values for both long and short rates. The price inflation equations were modelled as an ARCH process. Price inflation at time t depends upon price inflation in previous time periods and on the current yield curve. This links inflation, in part, as a function of interest rates and the yield curve.

The Wilkie model has the consol equation with a real component modelled as an AR(3) plus a residual from the dividend yield. The 1995 version uses an AR(1) model. The inflation component of the yield is determined from the current value of observed inflation plus an EWMA of observed inflation.

2.6.3 Inflation and Inflationary Expectations Models

The final set of models are those of observed and expected inflation. Eckstein (1981) resulted from a study prepared for the Joint Economic Committee of the US Congress. Eckstein found that shocks to the system tend to persist. He supported the concept of a 'core' rate of inflation.

Wilkie (1984, 1987, 1992, 1995a and 1995b) modelled the annual rate of inflation as an AR(1) with constant mean. The rationale is that annual inflation may depend upon previous annual inflation, without there being much connection between successive quarters, for example features such as indexation in a range of prices. The model suffers from three problems noted in Geoghegan *et al.*; bursts of inflation persisting once established, large irregular shocks and the non-normality of residuals. The prevalence of negative rates of inflation in the inflation model was also commented upon in the discussion in Geoghegan *et al.*. The model presented by Clarkson in

Geoghegan *et al.* (1992), Appendix A, allowed for exogenous shocks such as occurred in the early 1970's with the application of a non-linear process. Wilkie in Geoghegan *et al.*, Appendix B applies ARCH techniques to his model. The investigation is brief. Mulvey (1996) used an autoregressive model with ARCH residuals to model inflation. The model given by Mulvey has two diffusion equations, one for the price level and one for the volatility of the price level.

Carter (1991) for the period 1970-1990, fitted an ARIMA(1,2,2) to the Australian quarterly CPI. Simulations from this model were too volatile and negative inflation too prevalent. Mishkin and Simon[11] (1995) found evidence to suggest ARCH residuals in modelling the rate of inflation. They modelled the process with ARCH terms, though full results are not given. On the other hand Sherris (1997a) in a study on modelling inflation suggested that the volatility in the series could not be successfully modelled using an ARCH process.

Data on inflation expectations is difficult to obtain. An alternative is is to model it with observed inflation data. Eckstein (1981) applied various models of expected inflation including the adaptive expectations hypothesis (AEH) model, which is equivalent to an EWMA of observed inflation. He concluded that the learning process was slow; the strongest statistical results being achieved with a relatively slow learning process. Expected inflation was similarly modelled by both Wilkie and Mulvey as an EWMA of observed inflation.

Data on consumer inflation expectations is available in Australia from 1973. McDonnell (1992) summarised this expectations series, the definitions and any shortcomings. He covered the role of expectations in economic theory and what has been learnt since the series commenced. There are some prior studies using this data. Defris and Williams (1979) applied econometric modelling to consumer expectations of inflation. Three different approaches were used. Firstly lagged values of observed inflation were modelled in an AEH model with extensions/generalisations to other

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[11] This is not a complete list of potential models. The RBA research department has available a number of papers modelling inflation on the RBA website at http//www.rba.gov.au/rdp/index.html which can be downloaded. Mishkin and Simon (1995) is also available at the RBA site.

learning models. Secondly, a range of other economic indicators and thirdly other survey based data such as the index of consumer sentiment were modelled. They concluded that expectations were determined primarily by recent rates of actual inflation. Volker (1982) in modelling the Treasury bill rate used expectations plus other explanatory variables. Lagged values of observed inflation were introduced as an alternative measure of inflationary expectations.

Nerlove and Schuermann (1995) discussed and compared various theories of expectations. The authors formulated a method for joint testing of the rational, adaptive and naive expectations hypotheses. They questioned the appropriateness of the AEH and concluded that all three alternative formulations of the expectations hypotheses were misspecified.

2.7 Model Building Methodology

Sections 2.2-2.4 reviewed the predictability of asset prices and returns. The evidence indicates predictability. Sections 2.5 and 2.6 considered various models that have been proposed. These include multi-asset and single asset models. Given that model building is a worthwhile proposition, a methodology is required. The methodology of econometric model building determines the strategy that is adopted.

The various methodologies of econometrics are discussed in Darnell and Evans (1990). Pagan (1991, 1995) compared the three main approaches to econometric model building, denoting these the 'Hendry'; 'Leamer' and 'Sims' methodologies respectively.

The Hendry methodology is where a set of equations, defined by economic theory is modelled, simplified, then a range of diagnostics and tests are applied. This is the 'general-to-specific' method outlined in Hendry and Mizon (1991). The Leamer methodology is Bayesian. Bayesians add to the usual information sets their own beliefs, knowledge and accumulated experience in a precise way via prior probabilities. Sims (1991) methodology views systems of equations as interdependent, rejecting the use of a series of single equation studies in favour of the vector autoregression (VAR). Sim's believes that the simplifying assumptions built into such large scale models are too strong. In his approach he constructed a VAR with a given lag length and used the resulting model to address questions of interest. Canova (1995) reviewed the VAR methodology. He discusses the background to the VAR approach and the relation of VAR to topics such as Granger causality. An application of the VAR methodology to GNP for the US, Japan and Germany is given.

Other papers in the Granger compendium include those from Spanos and Fair. Spanos (1991) provided an historical context, noting the criticisms of the 'textbook' approach to econometric modelling and the breakdown of even some of the best empirical relations during the early 1970's (caused by the oil shock and the end of the Bretton Woods agreement). Spanos integrated some of the criticisms of various approaches and connected the disparate methodologies into a unified framework. Fair (1991) emphasised the importance of the data and the role of simulation, in proceeding from theory to results. He discussed the role and position of theoretical models and what one may expect in the long run of various models.

2.8 Statistical Issues

This section considers a portion of the literature relevant to some of the technical issues arising in stochastic investment modelling. Eight categories are chosen, they are not independent. The categories provide structure to the literature which represent major themes.

2.8.1 Unit Roots

Economic time series tend to grow together and may exhibit a high degree of correlation even when there is no economic relationship. Hence a linear regression between two such variables will show a high R^2 value. However there will be a tendency in such a case for errors to be positively serially correlated. Hence there is a low value for the Durbin Watson[12] (DW) statistic. This feature of the nature of co-

[12] This is defined and discussed in section 5.2 and equation (5.1).

movements in time series is covered in Hendry (1991). Slutzky (1927) provided one of the earliest investigations of 'spurious regressions'. The last section in Slutzky reviewed correlation theory and is related to the work of Yule. Yule showed that correlation theory is appropriate if the variables are stationary or $I(0)$; then the correlation distribution is normal. If the variables are $I(1)$ and the differences are stationary then the correlation distribution is an inverted semi-ellipse. If they are $I(2)$ and thus second differences stationary, then the distribution is a U-shape. So the most likely correlations for unrelated $I(2)$ series are ± 1. The topic of spurious regressions is well covered in Granger and Newbold (1986). Since then much research has been dedicated to this issue, as it is central to the validity of diagnostic tests in linear regression such as the critical values for the distribution of the coefficients. First differencing for stationarity is equivalent to finding a unit root in an ARIMA model. Hence tests for stationarity are called unit root tests.

The first formal unit root test for a first order autoregresssive model, the Dickey Fuller (DF) test, was proposed in Dickey and Fuller (1979). This test was extended to higher order autoregresssive processes as the augmented Dickey Fuller test (ADF). Said and Dickey (1984) further extended the testing procedure to a more general ARMA(p,q). Phillips and Perron (1988) introduced a test which relaxed some of the assumptions of the ADF test. These tests are widely applied in the finance literature, but there are others. Mayadunne, Evans and Inder (1995) examined a test due to Kwaitkowski-Phillips-Schmidt-Shin (KPSS). The ADF and PP test for the null hypothesis of a unit root whereas the KPSS reverses the hypothesis and tests for a null of no unit root. Extensions of unit root testing further relaxing the assumptions and including a wider array of potential models to which to apply unit root testing, are discusssed in Mills (1993) and Banerjee *et al.* (1993).

The literature on unit root testing is voluminous. The nature of the individual test depends upon the model form under investigation. In the ADF and PP tests these are presumed to be an autoregressive model, which may or may not include a constant and trend term. The test takes the form of a regression using differences and lagged differences. If there is a unit root then the variable is $I(1)$, so normal inference is not valid, as was shown by Slutsky and Yule. Hence different critical values in the regression are needed depending upon the model form. Thus different unit root tests perform better under different circumstances, such as the sample size or nature of the presumed underlying model. Unit root tests may also be conducted by returning to first principles and determining the sampling distribution. Mishkin and Simon (1995) did this with Australian inflation data. They determined the critical values for the PP and DF tests using Monte Carlo simulations based upon a presumed model structure. Bleaney and Mizen (1996) did so in an investigation of exchange rate dynamics. They put forward a polynomial model of degree 3 of mean reversion to accentuate deviations from the mean. They performed simulations to obtain critical values in associated unit root tests.

There are a number of technical difficulties with standardised unit root testing. The order of the underlying process and thus the lag length for the test regression is unknown. Said and Dickey (1984) suggested applying a lag length in the autoregression growing at a controlled rate. There are a number of other suggestions for the lag length to be applied. Information based criteria may be applied or some form of recursive procedure, where a long lag length is initially applied and then reduced until the estimated coefficient of the last included lag is found to be significant. A discussion of some of these technical issues are covered in Harvey (1989), Mills (1993), Perron (1994) and Mayadunne, Evans and Inder (1995). Holden and Perman (1994) reviewed the unit root testing procedure and gave a step-by-step method. Sherris, Tedesco and Zehnworth (1996) provided an application of the Holden and Perman procedure to a range of Australian financial variables.

The basis of unit root tests assume that the required differencing to stationarity is integer valued. This need not necessarily be so. Hosking (1981) introduced the idea of fractional differencing where he extended the ARIMA class to fractional models. He applied a differencing parameter which was a fraction. These models exhibit slowly declining but nevertheless small levels of autocorrelation, which accumulate to a significant size, typical of long memory processes. These ideas are found in Granger and Joyeux (1980) where the characteristics of such models were determined. The use of these models are an alternative or a complement to the use of unit root tests. A discussion of fractional differencing can be found in Hart (1996).

2.8.2 Univariate Modelling

The work of Slutsky (1927) or Kendall (1953) involved time consuming calculations for even the most basic of data analysis. Simple trend extrapolative models of the autoregressive and moving average form had been introduced[13]. The advent of computing made the massive manual calculations underlying these ARIMA style models straightforward. The seminal work on this class of univariate models is the text by Box and Jenkins (1970). Univariate modelling of the mean with the ARIMA class of models and the related diagnostics are covered in this text. There are many additional texts on the subject, see Anderson (1976), Kendall and Ord (1990) or Mills (1990).

In fitting an ARMA model to a given stationary univariate series various diagnostic tests are required. The residuals are assumed independent identically distributed normal variates. Box and Pierce (1970) developed a 'portmanteau' diagnostic test for serial correlation in the residuals. This was modified in Ljung and Box (1978) leading to the Box-Ljung test statistic. The residuals can be checked for normality with the Jarque-Bera test, see Godfrey (1988), Mills (1993) or Sherris (1997a). Criteria for selecting between different competing models each with satisfactory diagnostics have been developed. The Akaike Information Criterion (AIC) and Schwartz Bayes Criterion (SBC) are data based criterion for model selection. However as reported by Harvey (1990) the evidence suggests that there is a tendency for the AIC to choose a model which is overparameterised. There are other criteria available. Hughes and King (1997) discussed an improved version of the AIC based on the Kullback-Leibler information. On the basis of empirical studies, Kendall and Ord (1990) suggested that the different criteria usually provide similar or even identical results.

Financial series exhibit heteroskedasticity or a non-constant level of the variance. Various tests for heteroskedasticity have been proposed as well as a variety of functional forms to model the heteroskedastic variance. There are a many potential non-linear solutions. One solution is the class of ARCH (Autoregressive Conditional

[13] Models such as single exponential smoothing or Holt-Winters double exponential smoothing were introduced in the US in the 1950's for such applications as stock control.

Heteroskedasticity) models introduced by Engle (1982) and related refinements. These models are ARMA style models in the variance. Another set of non-linear models are switching regime models; there are other potential candidates.

Tests for heteroskedasticity have been introduced by White (1980) and Engle (1982). White found a covariance matrix estimator which was consistent in the presence of heteroskedasticity. By comparing the proposed estimator to the usual one obtained he provided a direct test of heteroskedasticity. As the estimator proposed is not dependent on the structure of the heteroskedasticity the test is therefore a general one. The ARCH-LM test due to Engle is a specific test for ARCH errors.

A variety of extensions of the ARCH model have been proposed. Examples include the generalised ARCH or GARCH model and the asymettric EGARCH or exponential GARCH model introduced by Nelson (1991). These models have become popular for modelling the variance in financial series, see Bollerslev, Chou and Kroner (1992) for a review of ARCH modelling in finance.

An example of the application of the non-linear modelling is found in Pagan and Schwert (1990). The authors reviewed monthly US stock returns from 1834-1925 and applied a GARCH(1,2), an EGARCH(1,2), a two state switching regime model, and three non-parametric estimators. They compared the within sample predictive power and out-of-sample forecasts as well as reviewing particular highly volatile episodes. They concluded that the parametric models did not capture the squared returns as well as the non-parametric models but did better in out-of-sample predictions. The authors suggested merging the two approaches.

A similar study to that of Pagan and Schwert was conducted by Kearns and Pagan (1993) to compare US and Australian experience. The authors performed a detailed study with monthly data into the volatility of the Australian stock market over the period 1875-1987. They extended their analysis to model building applying a GARCH(1,2), EGARCH (1,2) and an autoregressive iterative two-step procedure. They concluded that the EGARCH model was superior. They found that whilst the degree of predictability was higher than for the US experience there was still considerable room for improvement. The authors then applied the models and found there was persistence of shocks in volatility and that this persistence was as true of small shocks as it was of large ones. They also concluded that there was no evidence that the persistence was due to structural change; it remained remarkably constant over long periods.

Brailsford and Faff (1993) provided another application modelling daily Australian stock market volatility over the period 1974 to 1993. The authors considered various possible ARCH and GARCH model forms. The evidence found was suggestive of considerable limitations on the use of basic ARCH or GARCH modelling for Australian share prices and that a model that captures asymmetry was required. These results are consistent with those of Sherris (1997a, 1997b) whose results did not suggest that volatility in all the series could be modelled using a simple ARCH process. Sherris also cast doubt on the use of AR models due to the non-normality of the residuals as found from the Jarque-Bera test.

2.8.3 Stochastic Trends

Nelson and Plosser (1982) in a seminal paper on unit roots in macroeconomic time series introduced the concepts of difference and trend stationarity. Analysts had until then considered underlying trends in economic time series to be deterministic. So the cyclic factor was transitory or stationary and the sole cause of the long run variation. Nelson and Plosser demonstrated that the trend component was a stochastic process. The implications are that shocks to the system are permanent in the non-cyclic component and that variations come from both components. Therefore a deviation from a trend caused by a shock means that the system will not necessarily compensate to return to the long run trend path. There may be a permanent change. The conclusion is that most economic series are now considered to show evidence of stochastic trends.

There have been studies conducted into the persistence of shocks in a wide range of macroeconomic aggregates. Cochrane (1988) undertook a study into GNP to determine whether an economy, operating below potential due to a shock, will over-correct to return to the long-run potential path. Thus the question asked was whether or not these aggregates mean reverted. Cochrane found little shock persistence in GNP, suggesting a random walk for that variable. Stock and Watson (1991) investigated the way trends in macroeconomic series vary. The issue was whether cycles are independent of the changes in growth trends. They asked whether or not it is possible to separate out the non-stationary component of a time series from the stationary one. By fitting various models to the trend and stationary components they estimated the contribution to long run growth and variance of the trend component. They concluded that stochastic trends are part of macroeconomic processes and careful thought is needed to analyse such series. They commented that it is possible to apply a variety of tests and some simple rules of thumb to reduce the possibility of making dramatic errors of inference. Mayadunne, Evans and Inder (1995) investigated shock persistence for a wide range of economic time series. They found many series with orders of integration significantly above and below one. They found it difficult to decide whether or not shock persistence existed.

2.8.4 Cointegration and Error Correction Models

If as series is integrated then the normal diagnostics from any regression applying standard OLS are not valid for purposes of inference. But there is information in the 'levels' that should be captured. The key is to find series which trend together. This leads naturally to the concept of cointegration or series that are individually integrated but are such that a linear combination of them is stationary. This has become a topic of significant research effort. Engle and Granger (1991) contains a selection of papers on cointegration; the contribution by Hendry (1991) in the selection provided an overview of developments in cointegration. Banerjee, Dolado, Galbraith and Hendry (1993) presented the econometric analysis of non-stationary data. Standard texts on cointegration and related issues include Stewart (1991), Cuthbertson, Hall and Taylor (1992) and Mills (1993).

Tests are required to determine whether or not two or more integrated series are cointegrated. Two alternative tests are the Engle-Granger two-step procedure and the Johansen maximum likelihood test. The first test requires an investigation of the cointegrated series to see if the linear combination is stationary. The critical values for the test arise from a regression involving integrated variables and are non-standard. The critical values were tabulated by Engle and Yoo (1987). A more comprehensive simulation study was provided by MacKinnon (1991). The Johansen test is a multivariate analogue of the Dickey Fuller test. This test was first proposed in a paper by Johansen (1988). Dickey, Jansen and Thornton (1994) provided a discussion and application of the method of cointegration to money and income. They covered a range of aspects including a geometric interpretation, the various tests for cointegration and a review of the strengths and weaknesses of each test.

There are a number of papers covering empirical work on cointegration and the error correction model (ECM). The ECM arises when two integrated series are cointegrated since the cointegrating relationship treated as a variable allows a regression equation where the variable is stationary. Hence the standard results of statistical inference apply. Mills (1991) provided an example of cointegration techniques applied to finance. He considered the links between some of the fundamental factors in finance; equity prices, dividends and bond yields and their relationship to each other. He firstly found that the logarithms of the series were cointegrated with a single cointegrating vector. Then he considered an ECM leading to formulae which detailed the impulse responses associated with innovations to each of the variables. The final section considered the relationship between the dividend yield and the bond yield. He found that in equilibrium the ratio of the bond yield and the dividend yield were in constant proportion to each other. The log of the ratio was a stationary series and mean reverting, though only very slowly so because of a near unit root. He found evidence of ARCH errors which were then modelled.

Alogoskoufis and Smith (1995) presented an overview and application of ECMs. They surveyed the historic development of ECMs and the alternative formulations by Phillips, Sargan, Hendry and Granger. They considered the long run parameter estimates found from each formulation and provided an economic interpretation for each. They considered ECMs as an optimal adjustment mechanism or a return to a long run mean in an optimal fashion. They introduced an application of an ECM to wage setting in the UK. The authors fitted the various formulations by specifying the structural ECMs, estimated the required parameters and drew conclusions from the alternatives.

2.8.5 Regression with Stationary Series

Given that there are a set of stationary variables standard OLS applies. Thus the introduction of inflation as a driving force may be modelled with other stationary series. Standard texts such as Fomby, Carter and Johnson (1984) or Johnson (1987) discuss modelling with stationary variables. More specific finance texts include Harvey (1990), Cuthbertson, Hall and Taylor (1992), Mills (1993) or Campbell, Lo and MacKinlay (1997). The use of lagged values of the dependent and independent variable introduces a range of statistical issues in diagnostic testing, see Godfrey (1988). A particular problem occurs in diagnostic tests for autocorrelation in the residuals when there are lagged dependent variables. Inder (1984) investigated the effectiveness of the DW test in the presence of lagged independent variables. He found the DW test generally more powerful than Durbin's *h*-test. Godfrey discussed more appropriate tests than the DW including the Breusch-Godfrey LM test as an alternative test for serial correlation in the residuals.

2.8.6 Statistical Issues in Finance

Statistical difficulties limit what can be deduced in finance. The importance of stationarity is discussed in Stock and Watson (1991). They detailed errors made in conclusions from applying techniques as if the regressors were stationary when variables were integrated. They discussed developments in cointegration and applied them to the results. They concluded with a framework for applying regression in levels series and comment that there are no simple 'recipes' for performing time series analysis with integrated variables. Given their conclusion that stochastic trends are part of macroeconomic processes, careful thought is needed to analyse such series. They found that it is possible to apply a variety of tests and some simple rules of thumb to reduce the possibility of making dramatic errors of inference.

However testing for stationarity with unit root tests is not a straightforward task. Gulley (1995) examined the characteristics of inflation and interest rate series for a variety of countries to determine the order of integration of these series. Gulley found much instability in the results for *ex post* rates, which varied widely depending upon the exact time period chosen. Hence conclusions about *ex ante* real rates inferred from inflation rates and expectations is fraught with danger. He concluded that the tests have low power when the true root is close to one or when the sample size is small and/or spans a time period that is not long enough to bring out the true properties of the series. Bollerslev and Hodrick (1995) also discussed the issue of sample size and the low power of various tests such as the variance ratio test. Olekalns (1996) commented on the difficulties with unit root tests. He finds real interest rates to be integrated as was found by many other authors. Yet the stability in rates observed over many centuries implies that a random walk process for nominal interest rates is extremely unlikely, see Homer and Sylla (1996).

Maravell (1995) focused on the statistical problems which arose in the application of a range of filters to a series to extract the various components. Each filter has its own particular characteristic, in that they resolve out features at a particular frequency but in doing so leave behind certain distortions. The author reviewed a wide range of difficulties that arise even when a model form is proposed. He gives the example of US GNP, showing problems particularly with seasonality. Another example was UK money supply, where he showed the kinds of real and practical problems that could arise when adjustments were made without understanding how the original series had been affected. He found that AR or VAR models should not be applied to seasonally adjusted series nor should some of the popular unit root tests.

Kaul (1996) summarised some of the issues involved with the predictability of stock returns, particularly the commonly used statistical procedures. He found that there are problems with both estimation based upon small sample sizes as well as difficulties with the level of R^2 due to 'data mining'. Kaul reviewed the use of overlapping observations, done to maximise the availability of data in long term studies. He concluded that the gains from overlapping data may be severely diluted in certain cases. Kaul remained positive despite the view that commonly used approaches to resolving the small-sample problem inherent in long run studies may be unsatisfactory.

Chapter 3

Inflation Modelling

3.1 Introduction

Stochastic investment models[1] are a recent development. Most financial models for asset liability studies have been based upon the Markowitz mean variance optimisation method. This approach assumes that past correlations and trends can be projected without any consideration of the causes of these relationships. Now recent research such as Poterba and Summers (1988), Fama and French (1988a, 1988b, 1989, 1992), Campbell and Shiller (1988), Fama (1991) suggest that share prices are predictable. Industry practitioners such as Sharpe, Henriksson, Grossman and Brinson in their contributions to Sharpe and Sherrerd (1989) also all agree as to the predictability of asset prices and put forward strategies to exploit it. If so, it follows that share prices can be modelled as can other asset classes, which also show predictability.

Stochastic investment models have been introduced by Wilkie (1984, 1986, 1992, 1995a and 1995b). Other actuaries have seen a role for such modelling[2]. International consultants such as Frank Russell (Carino *et al.* 1994) and Towers, Perrin (Mulvey and Vladimirou 1992, Mulvey 1994,1996, Mulvey and Ziemba 1995 and Mulvey and Thorlacius 1997) have directly funded some of this research. In Australia limited work on aspects of potential models have been conducted, see Carter (1991), Fitzherbert (1992) and Harris (1994 and 1995). More recent coverage comes from a series in Sherris (1995, 1997a,b) and Sherris, Tedesco and Zehnworth (1996).

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[1] A stochatic investment model is a multi-asset class model including links between individual stochastic equations. See the discussion in section 1.1.

[2] See the recent contribution from Duval, Teeger and Yaboukov (1999), presented as this thesis was being finalised.

Stochastic investment models such as those of Wilkie or Mulvey *et al.* model the rate of inflation. Wilkie's model has perceived defects such as negative values and an inability to capture the volatility actually observed in prices (see Geoghegan *et al.* (1992)). However the Wilkie ARCH model in Wilkie (1995b) does capture some of the observed excess volatility in inflation. By considering real variables this can be avoided. Then attention can be focussed on scenarios of inflation or deflation, for example. Nevertheless it is still valuable to review the history of price inflation and attempt to model it. As discussed in section 1.1 if a satisfactory model is available it can be applied. Considerations such as the post war Keynesian revolution in economics, the role of inflationary expectations and non-linearities in modelling requires investigation. The main aim of this chapter is to introduce and explore the observed and expected inflation data. It will also enable the introduction of techniques which can be used in the investigation of various time series.

The chapter is structured as follows. Section 3.2 introduces the base data series and discusses the sources of information and any shortcomings. The next section 3.3 looks at the history of prices as measured by the Consumer Price Index (CPI). In section 3.4 these are then explored for stationarity via unit root testing. Univariate modelling is then applied to the data utilising standard Box-Jenkins methodology. In section 3.5 some aspects of potential heteroskedasticity are investigated with the consideration of possible ARCH effects in the residuals. Various statistical measures generated from these processes will aid in the understanding of the time series. The final section 3.6 considers inflationary expectations and the role of unexpected inflation in financial market action. A model of inflationary expectations is then developed.

3.2 Sources of Information

The consumer price index (CPI) was obtained from the Australian Bureau of Statistics (ABS). Annual data was available from 1855 to 1997 using a long term linked series generated by the ABS. Originally the series was called the Retail Price Index and contained only basic items such as food, clothing and rent. Quarterly data was available from September 1948.

There are available quarterly inflation expectations data for the period March 1973 to September 1997. The data comes from the Melbourne Institute of Applied Economic and Social Research (IAESR) operating in conjunction with the Reserve Bank of Australia. The survey began in 1973 on a quarterly basis but changed to monthly in 1993. The institute conducts surveys of consumers on a range of issues via a telephone survey conducted by Oz Info market research. The coverage of the survey is all persons over 18 years of age and in all states and the ACT. The current sample size is 1200 and the survey is stratified by age and sex. The essential question asked is "By what percentage do you think prices will have gone up/down this time next year?" (which assumes that people reveal their true expectations). The aim of this process as outlined in the overview from McDonnell (1992) is to provide a direct measure of inflationary expectations. The point is made that this measure is important not because it is a good predictor of actual inflation but because it is likely to affect household decisions such as wage demands, spending or saving. McDonnell (1982, p.3) comments that "Our price expectations series has virtually nothing to say in the area of price increases....". The measure is not a good predictor of observed inflation but it is not intended to be. The RBA in its Bulletin[3] comments that "As well as overstating the actual level of inflation, the survey results tend to be unreliable as forecasts of changes in inflation rates.". McDonnell finds it unreasonable to judge the indicator by its value as a predictor, as the RBA does, as that was not the intention.

The aggregate measure shows quite wide dispersion within the various demographic categories of income, occupation and education as reported by the survey. For example in the April 1998 survey, individuals with an income less than $20,000 p.a. expect a 4.8% increase in inflation whilst those with an income greater than $70,000 p.a. expect an increase of only 2.3%. The institute therefore constructs from the data two measures, the mean price rise and the median[4] one. The median is a better estimator of consumer inflationary expectations due to the potential bias in averaging given the wide dispersion in values actually reported. An interesting point is that

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[3] 'Measures of Expected Inflation', *RBA Bulletin*, April 1991, 18-22.

[4] "Median price rise is the median of the population. Estimate the median person, the cohort that person is in then calculate the position within this cohort to get the median. Exclude those who said don't know whether up or down." Definition as stated in the IAESR monthly publication *Survey of Consumer Inflationary Expectations*. Further details can be obtained from the monthly document.

consumer responses tend to cluster at points such as 10% or 5% the two modal responses. In the 1990's for the first time the number of respondents at the 5% level outweighed those at the 10% one.

3.3 Consumer Price Index (CPI): Annual Data

Because of the central importance of inflation in asset prices, it is considered worthwhile to discuss this series in some detail. Figures 3.1 to 3.4 below are given; the natural log[5] of the actual index, the percentage change in the index and two estimators of the variance or volatility of the series. The data is calendar year. A few points stand out:

1. The *continuous* inflationary period of the 1970's is an aberration when viewed in the long term context. Prior to 1948 periods of actual deflation were normal.
2. The annual % change in the CPI 1856-1997 series is shown in figure 3.2. The inflation rate has fluctuated substantially over time. Nevertheless the rate has always moderated after short periods of high inflation.
3. The volatility of the series has declined substantially over time.

A measure of this varying level of volatility can be given by a wide range of estimators. Shorter term moving averages would be very volatile with additional smoothing required. Hence consideration is given as to how to best measure the volatility.

A recursive style estimate could be made, corresponding to an increasing sample size as more data points are added and time is allowed for the series to run-in. The estimate after time t would then be:

$$\hat{\sigma}_t^2 = \frac{1}{t}\sum_{j=1}^{t}\left(i_j - \bar{i}\right)^2$$

where i_t is the rate of inflation, the percentage change in the CPI. Given this estimator there will be time needed for it to settle down as individual data points have a much

[5] Hereafter whenever a log is taken it will be a natural logarithm or one to the base e. Note a log will reduce the exponentiation inherent in an index.

greater ability to move the series in the earlier period. The recursive estimator does have the virtue of incorporating all the information in the latest estimates.

There are many other alternative estimators that could be applied, say, a 4-year rolling estimator. These shorter term moving averages would be very volatile. So a compromise is needed between the estimator being of sufficient length to smooth the data but not too long in that it has the potential to filter out the variability in the measure. An appropriate measure for the effectiveness of a moving average is its *error reduction power*. Denoting a series of random observations by z_t, where successive values are independent and identically distributed with variance σ^2 then a moving average with weights α_j may be defined by:

$$\bar{z}_t = \sum_{i=-k}^{k} \alpha_j z_{t-j}$$

then the variance of the smoothed series will be given by:

$$\mathbf{var}(\bar{z}_t) = \sigma^2 \sum_{j=-k}^{k} \alpha_j^2$$

Now when the variance of a moving average is compared to that of the original series then the σ^2 term cancels out. Hence a simple moving average of length 4 will have an error reduction power of 0.25. A 10 period one will have an error reduction power of 0.1. The longer the moving average the greater its error reduction power but also the greater the *end effect*. That is there is a loss of data points due to the length of the moving average; for a 10 point average 10 data values are lost. One way to capture the longer term movements is to determine the variance of a 10-period rolling window, which acts like a moving average as a new data point is added and the oldest dropped. The estimator then becomes:

$$\hat{\sigma}_t^2 = \frac{1}{10} \sum_{j=t-9}^{t} \left(i_j - \bar{i}\right)^2, \; t \geq 10$$

The chart is given in figure 3.3. By way of comparison the 20-period rolling window is given in figure 3.4. The error reduction power is now 0.05. This effect can be seen in the lower chart. The major trends are kept but the peaks and troughs are attenuated.

Figures 3.3 and 3.4 show that the variance has declined quite dramatically. The spikes correspond to economic shocks caused by particular events, for example the Korean War. Nevertheless the downtrend is marked.

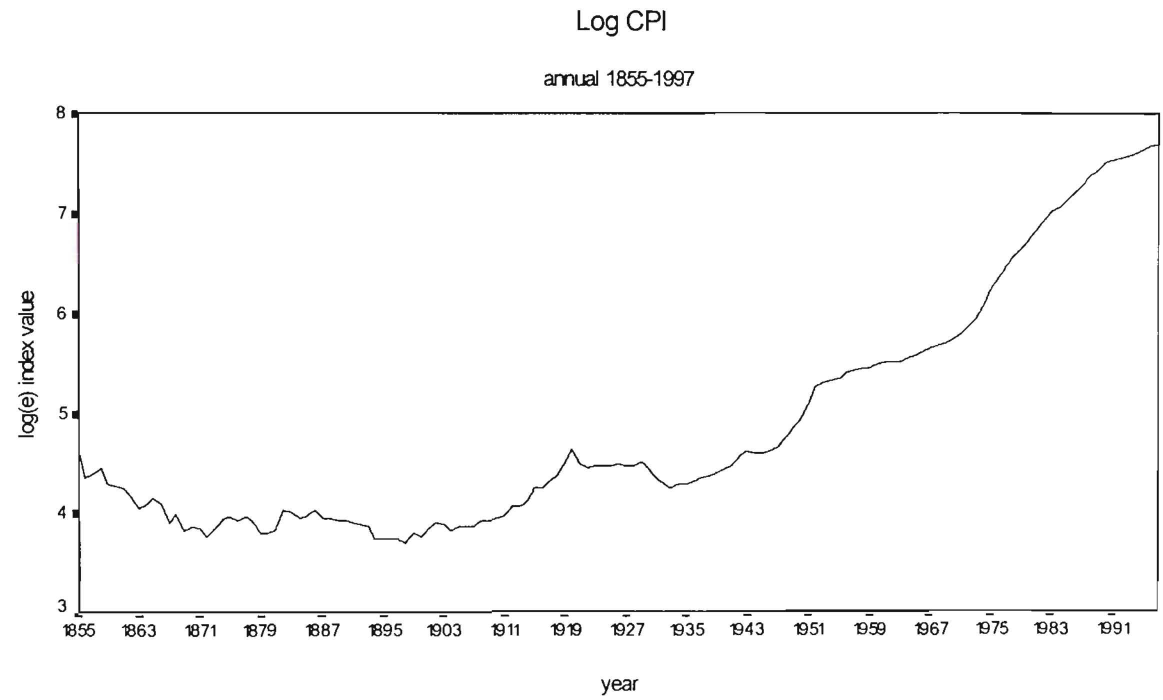


Figure 3.1 Log CPI

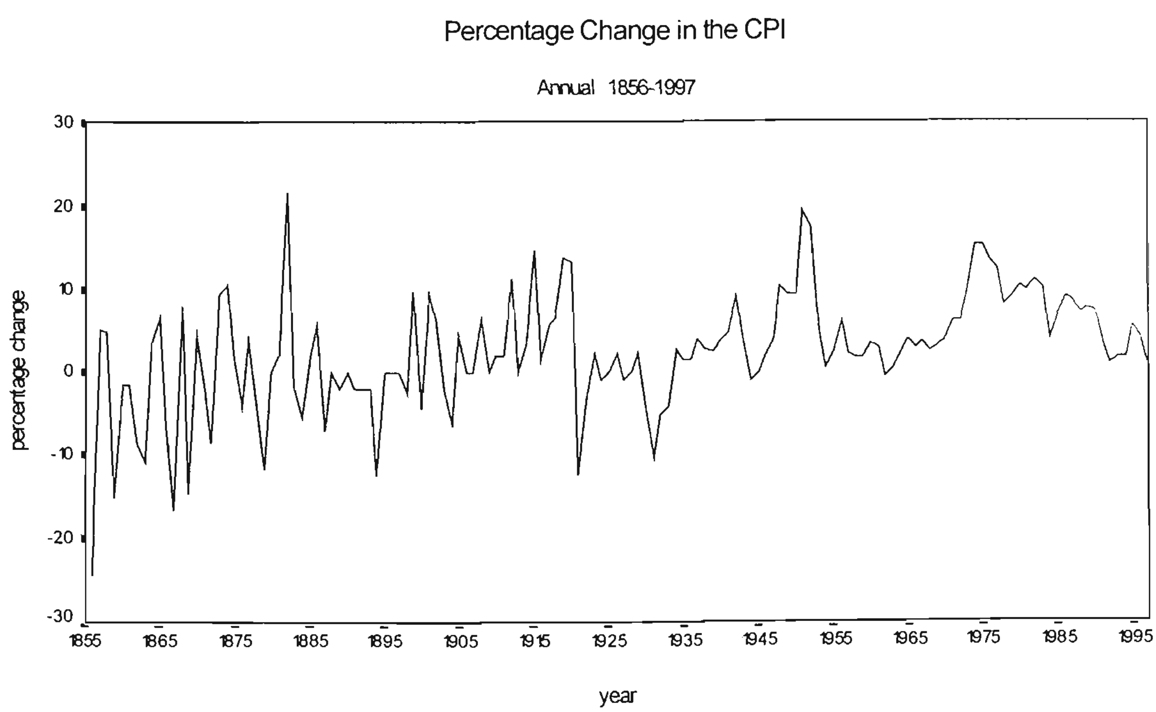


Figure 3.2 % change in the CPI.

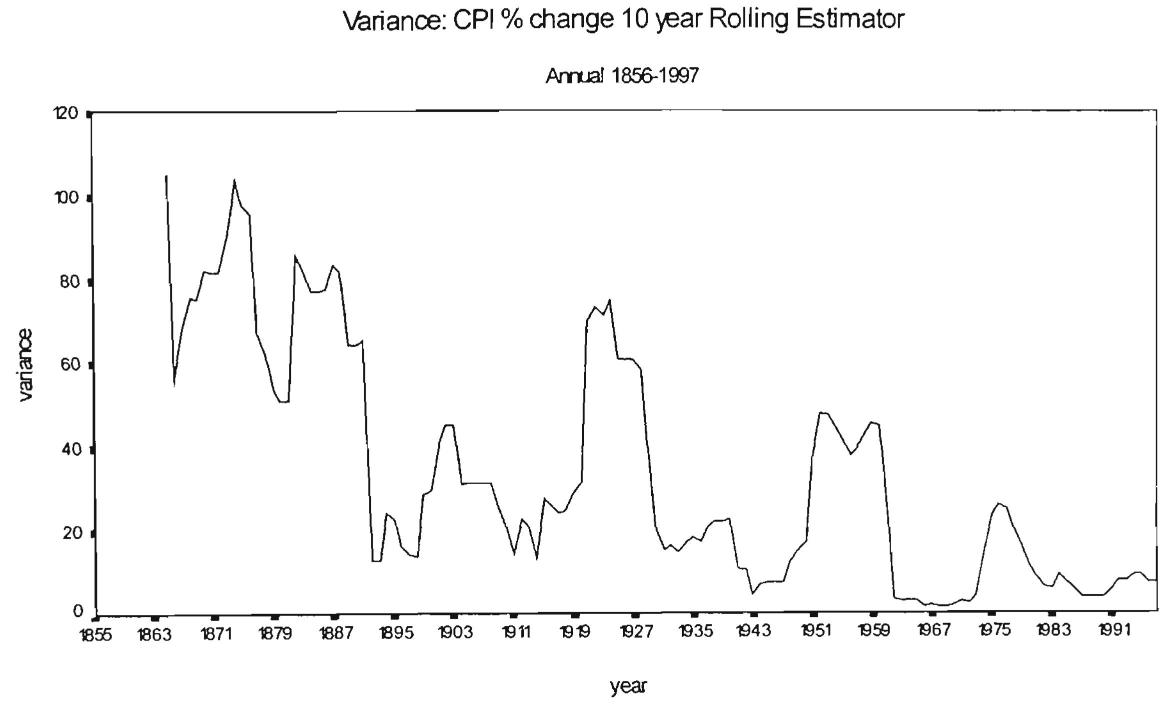


Figure 3.3 10-year Rolling Estimator of the Variance of the % Change in the CPI.

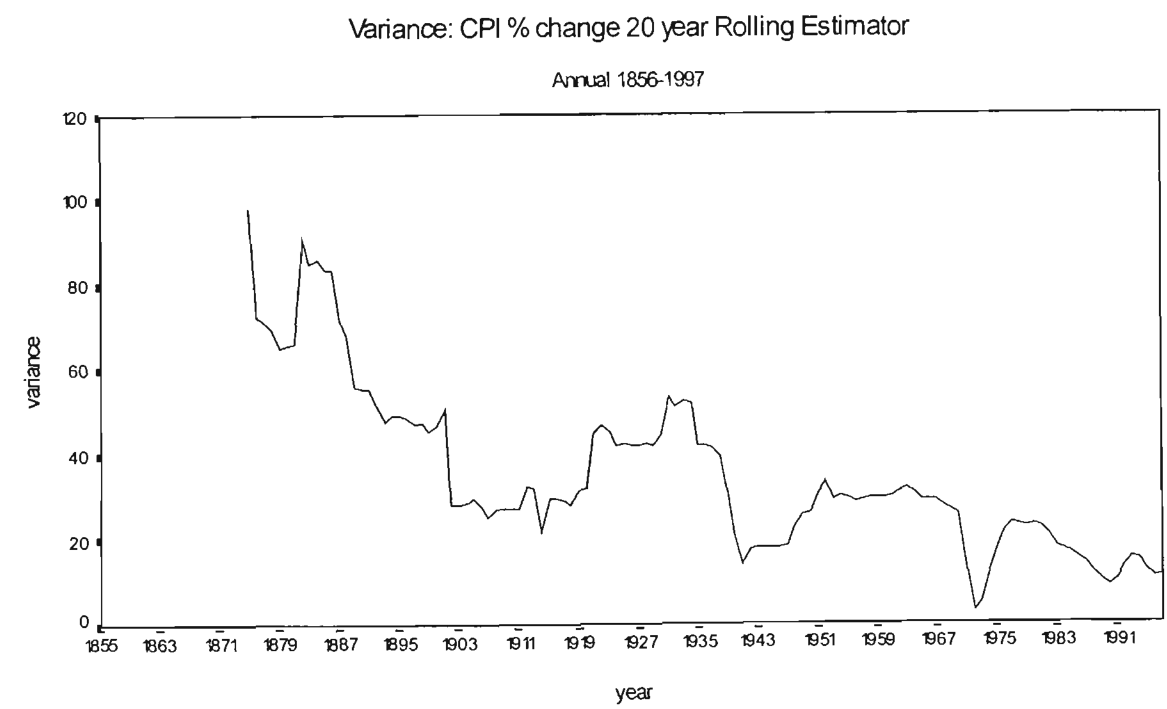


Figure 3.4 20-year Rolling Estimator of the Variance of the % Change in the CPI.

A decision as to the importance of the pre-1948 data, that is prior to the Bretton Woods conference, is required. In order to do so a review of the nature of macroeconomic time series is required. This introduces the work of Nelson and Plosser (1982) who investigated a range of US macroeconomic time series. Analysts until that time had considered underlying trends deterministic. Therefore the cyclic component, which was transitory or stationary in nature, was the sole cause of any long run variation. Nelson and Plosser showed that the characteristics observed in the time series are indicative of difference stationarity rather than trend stationarity. The implications are that shocks to the system are permanent in the non-cyclic component and that variations come from both components. A deviation from a trend caused by a shock means that the system will not necessarily compensate to return to the long run trend path. There may be a permanent change. That is the underlying model is a random walk with or without drift rather than being stationary around a linear trend.

There have been many studies subsequent to the seminal work of Nelson and Plosser. Cochrane (1988) investigated US GNP to determine whether an economy, operating below potential due to a shock, will return to the long-run potential path. Cochrane found little shock persistence in log real per capita GNP suggesting a random walk for that variable. Stock and Watson (1991) investigated how trends in macroeconomic series vary and whether cycles are independent of the changes in growth trends. They ask whether it is possible to separate out the non-stationary component of a time series from the stationary one. By fitting various models to the trend and stationary components they estimate the contribution to long run growth and variance of the trend component. They conclude that stochastic trends are part of macroeconomic processes and careful thought is needed to analyse such series. Mayadunne, Evans and Inder (1995) looked at a wide range of economic time series to investigate such persistence. They considered the order of integration of the series and found many with orders of integration significantly above and below one. They found it difficult to decide whether or not shock persistence existed.

Harvey (1989, p.90-93) in a discussion on structural breaks and switching regimes uses the Nelson and Plosser example of US real GNP demonstrating a structural break in 1947. The facts post 1947 are quite different from those from 1909-1947. Harvey (1989, p.347) comments "This change is attributable to the success of stabilisation policies in the post war period brought about, in large part, by the Keynesian revolution in macroeconomics.". The post 1947 change reflects central bank action, in the post-Bretton Woods context. Central banks and the IMF are a significant part of economic management. A continuation of the efforts of bankers to avoid deflation and the policy mistakes of the Great Depression of the 1930's should be expected, even at the cost of periods of high inflation.

The log of the CPI, $\mathbf{ln}(CPI)$ shown in figure 3.1 is a series with no trend in the first part up to 1945 but with a linear trend thereafter, certainly in the series post-1948. There appears to be a structural break corresponding to the time of the introduction of the Bretton Woods agreement for the Australian data. This is consistent with the US results discussed by Harvey. To investigate this potential structural break the nature of the series pre and post 1947 must be considered. Following Nelson and Plosser the series was differenced to eliminate any trend. Hence the series $\mathbf{ln}(I_t) - \mathbf{ln}(I_{t-1}) = \Delta\,\mathbf{ln}(I_t)$, where I_t is the value of the index at time t, was investigated (see figure 3.5). Note this looks very like figure 3.2, see section 3.4.

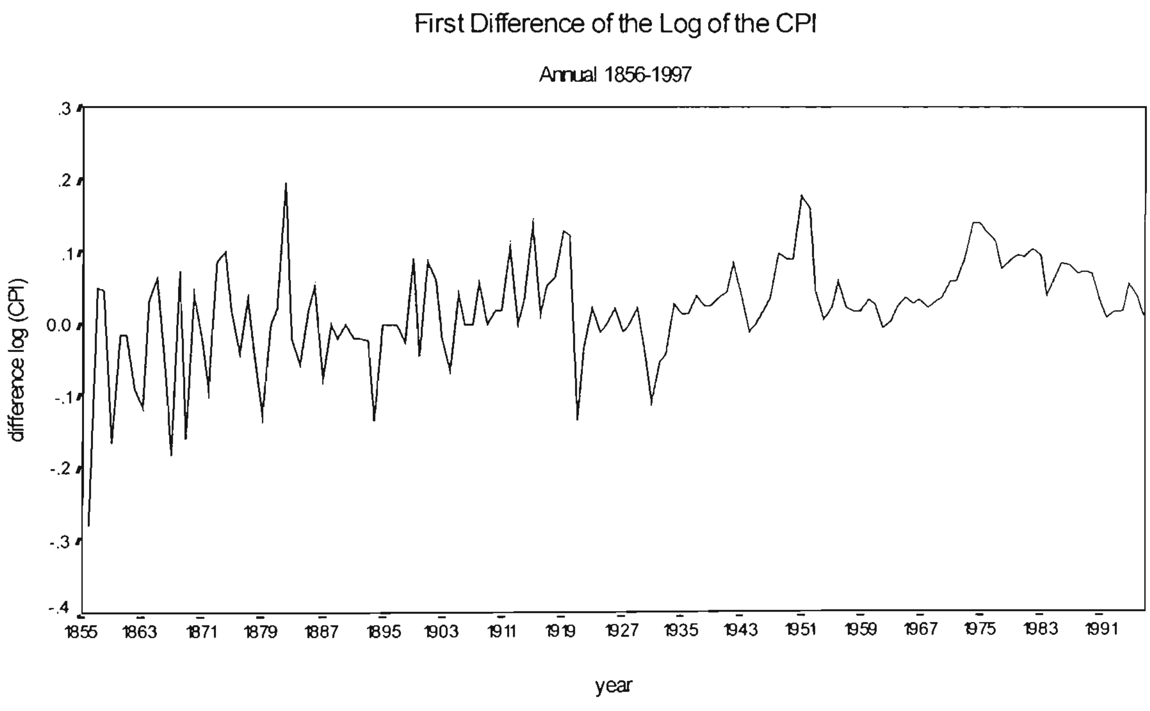


Figure 3.5 First Differences of the Log of the CPI.

An analysis of the data shown in figure 3.5 showed that from 1856-1947 $\Delta\,\mathbf{ln}(I_t)$ was negative 34 times out of a total of 92 observations. Between 1948-1997 it was negative only once out of 50 observations, in 1962 with a value of -0.004. The range for the period 1856-1947 was -0.28 to 0.20 and for the period 1948-1997, 0.00 to 0.18. This indicates that periods of price deflation were common in the 19th century and until 1947. From 1948-1997 prices have fallen once; then only by a small value. This suggests that there is a bias towards positive inflation outcomes since 1947.

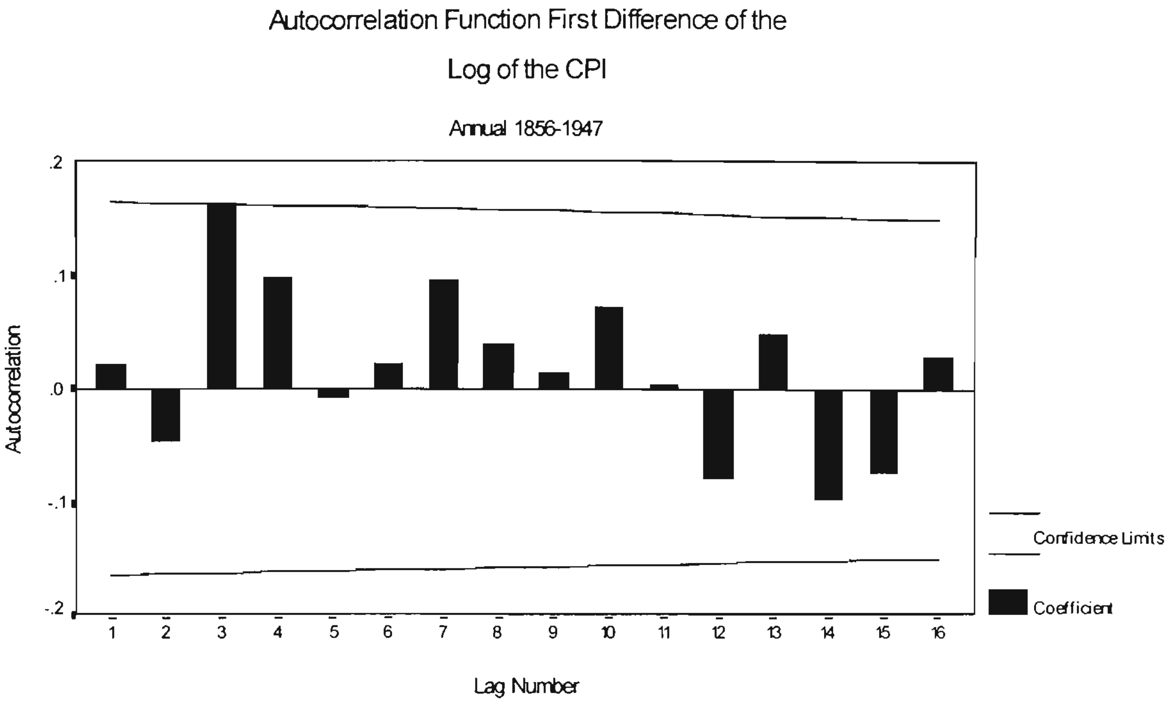
To perform a more rigorous analysis, tools are required to analyse the struture of the two portions of this series. The autocorrelation function (acf) and partial autocorrelation function (pacf) provide a way of looking at the correlation structure of the series. The acf and pacf for each sub-period were then determined as shown in figure 3.6 where the *autocorrelation* at lag k is defined by:

$$\rho_k = \frac{\mathbf{cov}(z_t, z_{t-k})}{\{\mathbf{var}(z_t).\mathbf{var}(z_{t-k})\}^{1/2}}, \quad k = 0,1,\ldots.., \quad \text{hence } \rho_0 = 1.$$

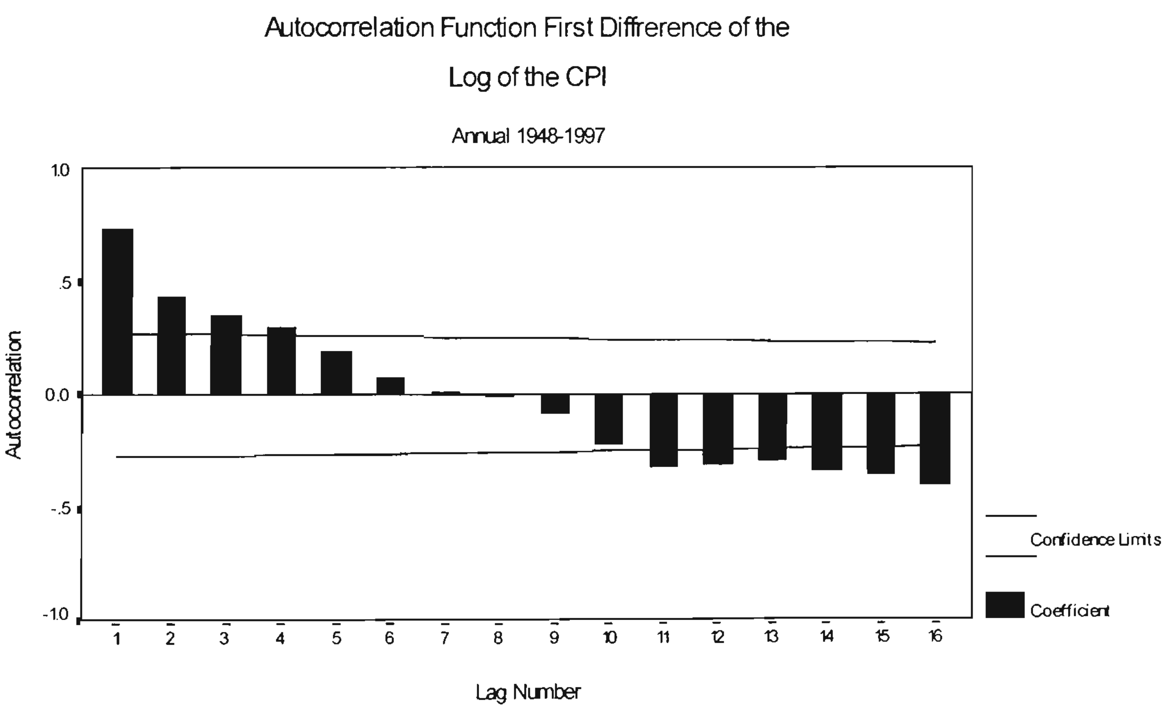
and the *partial autocorrelation* at lag k is defined by:

$$\phi_{kk} = \frac{|P_k^*|}{|P_k|}, \quad \text{where } P_k \text{ is the } (k \times k) \text{ autocorrelation matrix}$$

and P_k^* is P_k with the last column replaced by $(\rho_1, \rho_2, \ldots\ldots. \rho_k)'$.



(a)



(b)

Figure 3.6 (a) and (b) Acf for the First Differences of the Log of the CPI with a Structural Break at 1947.

The acf for the two sub-periods are very different. For the period 1856-1947 the series shows no significant autocorrelation with the value at lag 3 just on the confidence limit. For the period 1948-1997 there is evidence of high first order autocorrelation.

This suggests a connection between successive annual values consistent with the US data and the effects of the success of the stabilisation policies referred to by Harvey.

More recently Perron (1994) investigated real GDP data for structural breaks, for 11 countries including Australia, using annual data from 1870-1986. He used an additive outlier model (see Perron (1994, (3), p.118)). He then employed various methods to find the break date and found dates clustered in the 1940's. Perron (1994, p.146) comments "For Australia, some kind of major "take-off" occurred in the early 1940's.". This result, albeit for real GDP rather than inflation, is consistent with the difference between the acf's in Figures 3.6 (a) and (b) indicating a structural break at this time. The increase in the slope of the GDP growth has implications for returns to financial assets. This confirms the value of employing 1947 as a break point; however it is noted that the exact choice is based upon economic theory; acf's with a cut-off date of 1946 or 1948 would have shown a similar picture.

3.4 A Quarterly CPI Model

The quarterly series from September 1948 is the data set used for the stochastic investment model. The advantages are:

1. There will be a continuation of the post war macroeconomic stabilisation policies. There is a strong bias for positive, even if moderate, inflation.
2. Data problems of availability, definitional changes and so on, plague earlier data for the range of financial variables that are required.
3. There is a span of 49 years of quarterly data or 196 observations. This represents a long time series. Annual data provides only one quarter of the data points.

There is also a decision as to the use of continuous versus discrete returns on which there is an extensive literature. In many circumstances stochastic differential equations (SDE) have been used to model continuous processes. Empirical model fitting in those cases has taken place with a discrete approximation to the continuous model (see Brennan and Schwartz (1982), Cox, Ingersoll and Ross (1985)[6]). A review of continuous versus discrete models is given in Sawyer (1993)[7]. As the inflation data is quarterly, as indeed are the financial variables introduced later, discrete rather than continuous returns are assumed. For high frequency data continuous models may be more appropriate.

The period post 1948 can be modelled with the use of many different estimators. There is a choice of:

1. $i_t = \dfrac{(I_t - I_{t-1})}{I_{t-1}} * 100$ which is in terms of percentage changes per quarter,
2. $i_t' = \ln(I_t) - \ln(I_{t-1}) = \Delta \ln(I_t)$, a log form,
3. $i_t'' = \dfrac{(I_t - I_{t-4})}{I_{t-4}} * 100$ which is in annualised terms (given a quarterly series),

where I_t is the value of the index at time t.

The first is more natural because returns and asset allocations, in the market place, are quoted in percentage changes rather than logs. Also given that $\ln(1+x) \cong x$, for small values of x then:

$$\ln(I_t) - \ln(I_{t-1}) = \ln\left(\frac{I_t}{I_{t-1}}\right) = \ln\left(1 + \frac{(I_t - I_{t-1})}{I_{t-1}}\right) \cong \frac{(I_t - I_{t-1})}{I_{t-1}}$$

By way of numerical example the range of quarterly percentage changes for the rate of inflation is from -0.9% to +6.5%. Now a 6.5% change or 0.065 translates to a difference in logs of 0.063. The mean quarterly change is 1.5%. The difference of the logs in this case is 0.0149. These two different transformations produce series that are nearly identical. The structure of the two series will also be nearly identical.

The third is the form used to deflate the bonds and T-notes, which are also given in annualised terms. A univariate model of the general form $z_t = \alpha + \text{ARMA}(p,q) + \varepsilon_t$

[6] See chapter 6 which covers the application of SDEs to fixed interest models.

where z_t is a given time series, α is a constant and $\varepsilon_t \sim N(0, \sigma^2)$ may then by applied.

3.4.1 Unit Root Testing: Theory

The nature of each of the series must be reviewed, in particular the *stationarity* of the series. A stochastic process is called strictly stationary if its properties are unaffected by a change of time origin. Hence the process has a constant mean and variance and so does not 'explode'. In this case shocks to the series will die out. For a more formal definition and discussion see Box and Jenkins (1970) p.7-12. Note also that a process is called covariance stationary if the covariance of the series with itself is constant; for a normal distribution strict stationarity implies covariance stationarity.

For stationary series standard regression results and conventional procedures of inference are valid. However the condition of stationarity places restrictions on possible parameters for a model. Now for an autoregressive series the condition for stationarity is that the AR parameters should be within the region of stationarity[8] (see Box and Jenkins (1976, p.53-4)). The nature of the stationary region can be found by considering the roots of the general *p*th order process. The roots should lie outside the unit circle. The critical boundary value is therefore when one or more of the roots lie on the unit circle and so tests for stationarity are called unit root tests. Suitable differencing can normally be applied to bring an otherwise non-stationary series to stationarity. Now if a series X_t is non-stationary but is such that the differenced series $\Delta X_t = X_t - X_{t-1}$ is stationary then the series X_t is said to be *integrated of order one* denoted $I(1)$.

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[7] Sawyer reviewed continuous time stochastic models and in addition considered discrete approximations to the SDEs and the relation of ARCH processes to continuous models. See section 3.5 for a discussion of ARCH processes.

[8] Writing an AR(p) process as $\phi(B)z_t = \varepsilon_i$, where B is the backshift operator and $\varepsilon_i \sim N(0, \sigma_\varepsilon^{\ 2})$ then in general the necessary and sufficient conditions for stationarity for the process is that the zeroes of $\phi(B)$ should all lie outside the unit circle.

There are alternative unit root tests available. The original test is the Dickey Fuller test with the extension of this test, the *augmented* Dickey Fuller test (ADF). A later test relaxing some of the assumptions of the original formulation has been introduced by Phillips and Perron (1988), the PP test. Other possibilities include one from Kwaitkowski-Phillips-Schmidt-Shin (KPSS) examined in a paper by Mayadune, Evans and Inder (1995). The ADF and PP test for the null hypothesis of a unit root; the KPSS reverses the hypothesis. Extensions of unit root testing further relaxing the assumptions and including a wider array of potential models to which to apply unit root testing, are discusssed in Mills (1993, p.59-60) and Banerjee *et al.* (1993, p.119-135). The previous tests all rely on classical methods of statistical inference. Mills comments that recently Bayesian methods for unit root testing have also been developed.

The nature of the Dickey Fuller test depends upon the model form under investigation. This is presumed to be an autoregressive model which may or may not include a constant and trend term. The nature of the test takes the form of a regression using differences and lagged differences. This test assumes the residuals from the regression are independent and identically distributed (i.i.d.). An AR(1) with constant term can be written:

$$z_t = \alpha + \phi_1 z_{t-1} + \varepsilon_t$$

subtracting z_{t-1} from both sides yields the reparameterisation:

$$\Delta z_t = \alpha + (\phi_1 - 1) z_{t-1} + \varepsilon_t$$

Therefore to test for a single unit root for a simple AR(1) process with drift and deterministic trend term the following regression is used:

$$\Delta z_t = \alpha + (\beta - 1) z_{t-1} + \gamma t + \varepsilon_t \qquad (3.1)$$

and test for the null hypothesis of a unit root corresponding to $\beta = 1$.The test statistic is therefore the ratio of the coefficient of z_{t-1} to its standard error. The test is thus that $(b-1)/se(b) = 0$, where b is the OLS estimate of β and $se(b)$ is the standard error.

This has a distribution different from the standard t-distribution and is tabulated by Dickey and Fuller.

The cases when either or both of α and γ are zero must also be covered. The existence of a drift and/or a trend term will change the underlying distribution for the t-statistic. In the case $\gamma = 0$ the unit root null hypothesis corresponds to a random walk with or without drift depending as to whether $\alpha = 0$ or not. Otherwise the underlying model being tested corresponds to one with a linear trend with or without drift, depending again as to whether $\alpha = 0$ or not. The strategy outlined in the example given in Mills[9] (1993, p.62) may be applied. This can be extended to the case of two unit roots or more by suitable differencing.

The test can be extended to an AR(p) process. A reparameterisation is applied to an AR(p) in the same fashion as equation (3.1). An AR(p) with constant term can be written:

$$z_t = \alpha + \phi_1 z_{t-1} + \phi_2 z_{t-2} + \ldots\ldots + \phi_p z_{t-p} + \varepsilon_t$$

and this transforms to:

$$\Delta z_t = \alpha + \left(\sum_{i=1}^{p} \phi_i - 1 \right) z_{t-1} - \sum_{i=1}^{p-1} \left(\sum_{j=i+1}^{p} \phi_j \right) \Delta z_{t-i} + \varepsilon_t$$

Letting $\beta = \sum_{i=1}^{p} \phi_i$, $\delta_i = -\sum_{j=i+1}^{p} \phi_j$ and adding the trend term the test regression for this, the augmented Dickey Fuller test, is:

$$\Delta z_t = \alpha + (\beta - 1) z_{t-1} + \gamma t + \sum_{i=1}^{p-1} \delta_i \Delta z_{t-i} + \varepsilon_t \qquad (3.2)$$

[9] This raises issues of trend stationarity versus difference stationarity. Nelson and Plosser (1982) demonstrated the importance of these ideas (see also Mills (1993, p.57)).

Said and Dickey (1984) extend the testing procedure to a more general ARMA(p,q). They show the procedure remains valid asymptotically when the orders p and q are unknown, provided the lag length in the autoregression increases with sample size. For a detailed discussion refer to Mills (1993, p.126-128) or Stewart (1991, p.200-203). Holden and Perman (1994, p.64-65) give a good overview of unit root testing including a step-by-step procedure.

The Phillips Perron test (PP) relaxes the i.i.d. assumption in the errors from the regression (see Phillips and Perron (1988)). Their test statistic can deal with serial correlation or heteroskedasticity in the errors. Hence the PP test can account for a shift in the mean and/or trend in a time series. The PP test regression is of the form equation (3.1) and unlike the ADF test does not include any lagged differences. The distribution of the test statistic requires 'correcting' and this involves an adjustment to the variance. The lag length for an estimate of the variance as an input to finding the test statistic is needed (see Phillips and Perron (1988, formula (16) , p.340)). Therefore the required truncation lag length of the PP test is unknown. Phillips and Perron evaluate their test against the ADF test in section 7 of their paper. The results of their simulations with respect to the size and power of the respective tests indicate that each test performs well under different circumstances.

In a similar vein to that of the PP test one difficulty with the ADF test is the lag length to include in the test regression in equation (3.2). The order of the underlying process and thus the lag length is unknown prior to reviewing the correlogram and model fitting. Said and Dickey (1984) suggest a lag length in the autoregression growing at a controlled rate less than $N^{1/3}$ where N is the number of observations. Mills (1993, p.54) suggests the number of lagged differences is given by the integer part of $N^{0.25}$. Given N varying between around 100 up to 196 implies up to six lagged differences with the Said and Dickey suggestion, and three lagged differences with Mills. There are other suggestions. Another way of dealing with the lag length avoiding finding the number of lagged differences based upon sample size is suggested by Mayadunne, Evans and Inder (1995, p.148). They suggest that one starts with a long lag length and they choose 10, then this is "...reduced systematically, until the estimated coefficient of the last included lag was found to be significant.". They take a similar approach to the truncation lag length for the PP test. Their treatment of the ADF lag length is consistent with that of Harvey (1989, p.293) in his example of US GNP. It is also consistent with the approach advocated by Perron (1994). Referring specifically to the truncation lag parameter Perron (1994, p.138) advocates a recursive *t*-statistic procedure commenting "...the procedure selects that value of k, say k^*, such that the coefficient on the last lag in an autoregression of order less than k^* is significant and that the coefficient on the last lag in an autoregression of order greater than k^* is insignificant up to some maximum order *kmax* selected *a priori*.". Perron recommends the use of the recursive procedure over information based criteria such as the Akaike Information Criterion (AIC) or Schwartz Bayes Criterion (SBC) (see section 3.4.3 formulae (3.3) and (3.4) for definitions of these terms). Sherris, Tedesco and Zehnworth (1996) applied the Holden and Perman procedure. The lag length that is chosen is such that the value of p in equation (3.2) is selected to ensure the errors are uncorrelated. Sherris, Tedesco and Zehnworth (1996, Table 2, p.10) give the lag lengths they use for various variables and their first differences.

Finally the unit root tests may be conducted by returning to first principles and determining the sampling distribution. Mishkin and Simon (1995) take this approach to unit root testing with Australian inflation data, since they find that test results can be misleading in either small samples or if the DGP includes moving average terms. Then the power of the tests is extremely low. They determine the critical values for the unit root tests (PP and DF) using Monte Carlo simulations. They assume that the data is difference stationary so that it can be described by ARIMA models with ARCH residuals. But this presupposes that the data has a unit root and a model form to test for unit roots, as against given forms with critical values already found in the standard ADF or PP test values.

3.4.2 Unit Root Testing: Application to Inflation Measures

This sub-section applies the theory outlined in section 3.4.1. Throughout this document the ADF or DF test is applied in conjunction with the PP test as a check on the ADF results. The reason that these are chosen is that these two tests are the most widely applied in the literature on stochastic investment modelling. This enables a comparison of the results of this document with those of others using similar series.

The ADF test for a unit root was applied to the rate of inflation series i_t. Utilising the reduction approach of Mayadunne *et al.* 4 lagged differences were found appropriate with the test regression as in equation (3.2). The test equation then became (with t-values below):

$$\Delta i_t = 0.288 - 0.000t - 0.182 i_{t-1} + \sum_{i=1}^{4} \delta_i \, \Delta i_{t-i} + \varepsilon_t$$

(1.685) (-0.276) (-2.941)

The ADF test statistic is -2.941 and this compares to the MacKinnon (1991) critical value at the 5% level of significance of -2.877 and -2.5748 at the 10% level of significance. The null hypothesis of a unit root is thus rejected at the 5% level. The test was then applied with an intercept term only and 4 lagged differences. The test equation then became (with t-values below):

$$\Delta i_t = 0.254 - 0.182 i_{t-1} + \sum_{i=1}^{4} \delta_i \, \Delta i_{t-i} + \varepsilon_t$$

(2.182) (-2.951)

The ADF test statistic is not significant (see table 3.1). If the underlying model is an AR(2) then only one lagged difference is required in the regression. In a regression with intercept and 1 lagged difference the ADF test value was -3.665 and the unit root null hypothesis was again rejected at the 5% level. The test was then applied with no intercept or trend term (see table 3.1). The unit root null hypothesis was again rejected at the 5% level. The series is therefore considered to be stationary. The rate of inflation series i_t is therefore described as an $I(0)$.

This procedure was carried out for inflation measures i_t, i'_t and i''_t. In addition the order of integration of the CPI, I_t was also found. The results are summarised in table 3.1. The coefficients of the regression are given with t-values in brackets underneath the coefficient.

Table 3.1 ADF Regression CPI series September 1948 to September 1997: Various Estimators

ADF 5% level critical value below		trend and intercept (-3.434)			intercept (-2.878)		none (-1.942)
variable	lag length	α	$(\beta-1)$	γ	α	$(\beta-1)$	$(\beta-1)$
i_t ($\%\Delta I_t$)	4	0.288 (1.685)	-0.182 (-2.941)	-0.000 (-0.276)	0.254 (2.1830	-0.182* (-2.951)	-0.073* (-1.994)
i'_t ($\Delta \ln I_t$)	4	0.003 (1.674)	-0.179 (-2.912)	-0.000 (-0.274)	0.002 (2.161)	-0.179* (-2.923)	-0.072* (-1.980)
i''_t	8	0.109 (0.538)	-0.525 (-2.718)	0.001 (.0921)	0.241 (1.687)	-0.051 (-2.655)	-0.024* (-2.273)
$\Delta\ i''_t$	7	-0.187 (-1.073)	-0.937* (-7.260)	0.001 (0.700)	-0.078 (-0.992)	-0.934* (-7.252)	-0.924* (-7.198)
$\ln I_t$	6	0.015 (2.159)	-0.007 (-1.786)	0.000 (1.672)	0.004 (1.536)	-0.001 (-0.721)	0.001 (1.704)
ΔI_t	7	0.007 (0.126)	-0.189 (-2.581)	0.001 (1.337)	0.0634 (1.593)	-0.117 (-2.361)	-0.058 (-1.750)
$\Delta^2 I_t$	3	0.040 (0.690)	-2.289* (-5.777)	-0.000 (-0.856)	-0.004 (-0.148)	-2.263* (-5.734)	-2.265* (-5.755)

* Significant at the 5% level.

From table 3.1 in each line consider the first regression with both α and γ; if the value of γ is small with a low T-ratio, as it is in every case, we move on to the next regression which omits γ; then if the value of α is small with a low T-ratio, as again it is in every case, we move on to the final regression which omits both α and γ; then we look at the T-ratio of $(\beta-1)$; if this is significantly large we reject the hypothesis that the series is $I(1)$. This is quite certainly the case for $\Delta\ i''_t$ and $\Delta^2\ I_t$. It is probably the case for i_t, i_t' and i_t'' those these are marginal in one direction, and for **ln** I_t and ΔI_t, though these are marginal the other way.

These results may be compared with those of Sherris, Tedesco and Zehnworth (1996). They carried out extensive unit root testing with the use of quarterly data for a range of relevant financial series. Their results indicated that $i_t' = \Delta \ln(I_t)$ is a stationary series or $I(0)$. Mishkin and Simon (1995) find using quarterly data from September 1962 to December 1993 that the Australian inflation rate, by which they mean the annualised quarterly value or i_t'', is an $I(1)$. Crosby (1998) applying the ADF test with a constant but no trend and up to 3 lags to Australian annual data from 1875 to 1976, found that the rate of inflation is stationary. However table 2, p.8, makes it clear that Crosby means $i_t' = \Delta \ln(I_t)$, which is consistent with the results in table 3.1. Crosby finds the same result if the GDP deflator is used as a measure of inflation instead of the CPI. Gulley (1995) conducted unit root tests on the US inflation rate for various time periods. He finds that the inflation rate is $I(1)$ for some sample periods but $I(0)$ for others. Further he found qualitatively similar results with both the ADF and PP tests. He extended the analysis to the G-7 countries finding that the rate of inflation is an $I(1)$ for the period March 1970 to December 1991. Gulley (1995, p.205) concluded that "...inflation rates are more likely to be non-stationary in sample periods limited to the 1970's and 1980's.".

To test this further ADF tests were performed with different sample periods using the above method. This time only the rate of inflation ($\%\Delta I_t$) estimator was applied. The results are given in the table below.

Table 3.2 ADF Regression for the Rate of Inflation: Various Time Periods

ADF 5% level critical value below		trend and intercept (-3.455)			intercept (-2.891)		none (-1.943)
period	lag length	α	$(\beta-1)$	γ	α	$(\beta-1)$	$(\beta-1)$
Sep48 Sep73	1	0.266 (0.980)	-0.206 (-2.377)	-0.000 (-0.015)	0.263 (1.867)	-0.206 (-2.483)	-0.099 (-1.630)
Sep48 Sep70	1	0.545 (1.790)	-0.263 (-2.821)	-0.006 (-1.246)	0.207 (1.496)	-0.205 (-2.529)	-0.126* (-2.034)
Dec73 Sep97	6	2.731 (4.187)	-0.825* (-4.357)	-0.026 (-4.055)	0.195 (0.969)	-0.143 (-1.516)	-0.061 (-1.483)

* Significant at the 5% level.

On the basis of the above the period September 1948 to September 1973 is an $I(1)$. The periods December 1973 to September 1997 and September 1948 to September 1970 are $I(0)$. Subtracting just 3 years significantly changes the result.

The Phillips-Perron test was applied to the above. Truncation lags 1, 5 and 10 were applied to test for sensitivity. Only the PP test statistic is given rather than the coefficients for the regression equation as in table 3.1 for the ADF test. The relevant critical value is given for comparison with the test values.

Table 3.3 PP Test CPI series September 1948 - September 1997: Various Estimators

crit.val. below	trend and intercept (-3.434)			intercept (-2.876)			none (-1.941)		
variable	1	5	10	1	5	10	1	5	10
i_t	-6.228	-7.143	-8.049	-6.237	-7.148	-8.043	-3.498	-3.772	-4.313
i_t'	-6.201	-7.116	-8.025	-6.211	-7.123	-8.021	-3.498	-3.772	-4.272
i_t''	-2.103	-2.661	-2.401	-2.104	-2.663	-2.397	-1.536	-1.818	-1.660
$\Delta\, i_t''$	-10.13	-10.49	-10.02	-10.16	-10.51	-10.05	-10.17	-10.53	-10.07
$\mathbf{ln}\, I_t$	-0.576	-0.956	-1.192	-0.357	-0.349	-0.369	11.09	7.022	5.651
$\Delta\, I_t$	-5.586	-6.409	-7.228	-4.418	-4.827	-5.516	-2.981	-3.002	-3.387
$\Delta^2\, I_t$	-24.02	-26.98	-29.31	-24.03	-26.88	-29.11	-24.10	-26.96	-29.21

There is agreement that the rate of inflation series i_t is an $I(0)$, $\mathbf{ln}\, I_t$ is an $I(1)$ and i_t' $= \Delta\, \mathbf{ln}\, I_t$ is an $I(0)$. The annualised series i_t'' is an $I(1)$, but only just failing to reject the null indicating a near unit root. The 10% critical value for the PP test with no trend or intercept is -1.617. So the test values at truncation lags 5 and 10 would reject the unit root null. This is consistent with the ADF test result. The view of $i_t'' \sim I(1)$ is consistent with Mishkin and Simon who used the PP and DF tests but with adjusted critical values based upon simulated sampling distributions, and a different time period. The major area of difference is $\Delta\, I_t$ which the PP finds an $I(0)$ and hence it follows that the CPI (I_t) is an $I(1)$. This inconsistency is examined by the acf and univariate modelling (see the discussion at the end of section 3.4.3).

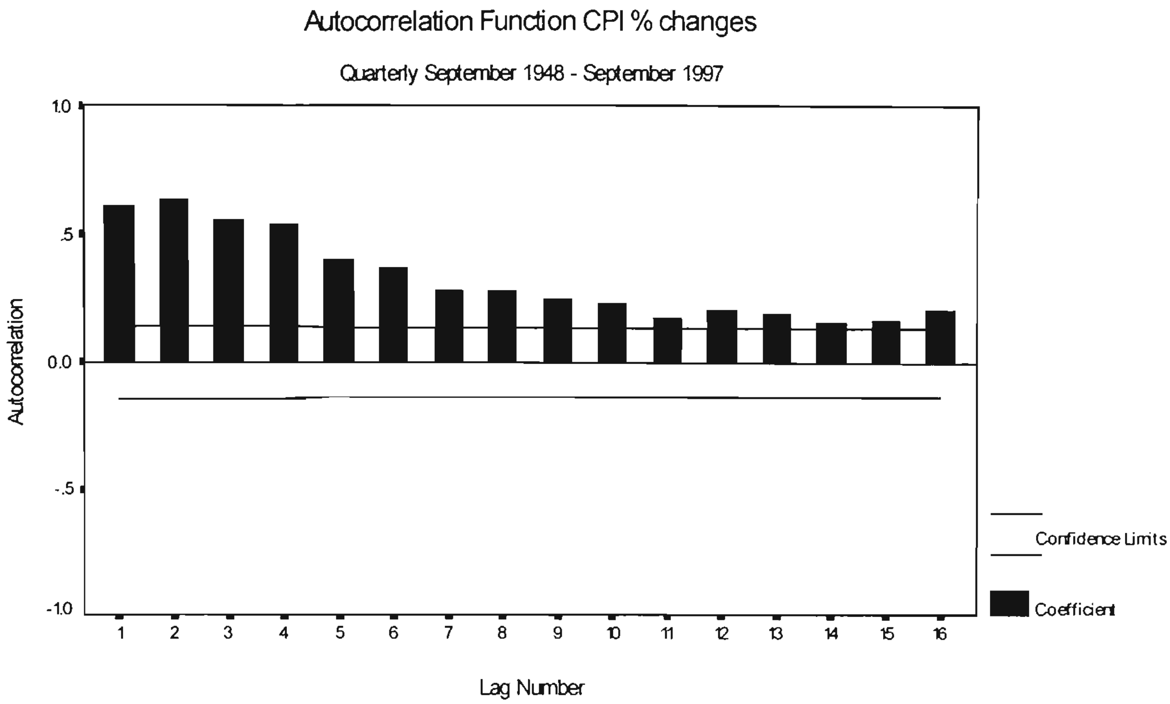
Table 3.4 PP Test for the Rate of Inflation: Various Time Periods

crit.val. below	trend and intercept (-3.455)			intercept (-2.891)			none (-1.943)		
period	1	5	10	1	5	10	1	5	10
Sep48 Sep73	-4.607	-5.346	-5.788	-4.438	-5.140	-5.605	-2.708	-3.144	-3.571
Sep48 Sep70	-4.770	-5.321	-5.525	-3.921	-4.455	-4.807	-2.758	-3.063	-3.365
Dec73 Sep97	-9.130	-9.353	-9.392	-4.539	-5.126	-5.892	-2.353	-2.303	-2.535

This time when the PP test was applied to different sub-periods for i_t, the test firmly rejects the unit root null. Thus the rate of inflation series i_t is an $I(0)$ for all three sub-periods.

3.4.3 Univariate Modelling

In the preceding section the ADF and PP tests for i_t indicated that this is a stationary series. The mean value of the series $\bar{i}=1.503$ which translates to an annualised value for the inflation rate of 6.15% per annum. Now $S.E.(\bar{i}) = 0.097$ so the mean value is significantly different from zero as is expected. So the transformation $z_t = i_t - \bar{i}$ is applied to the data. The standard Box-Jenkins approach, as described in Anderson (1976) or Mills (1993) was then employed. Note that in this section and whenever a similar analysis occurs possible outliers are examined by 'eye', via a histogram. For any potential non-normality refer to section 3.5 for an extensive discussion. The autocorrelation function (acf) and partial autocorrelation function (pacf) were firstly determined with the following results:



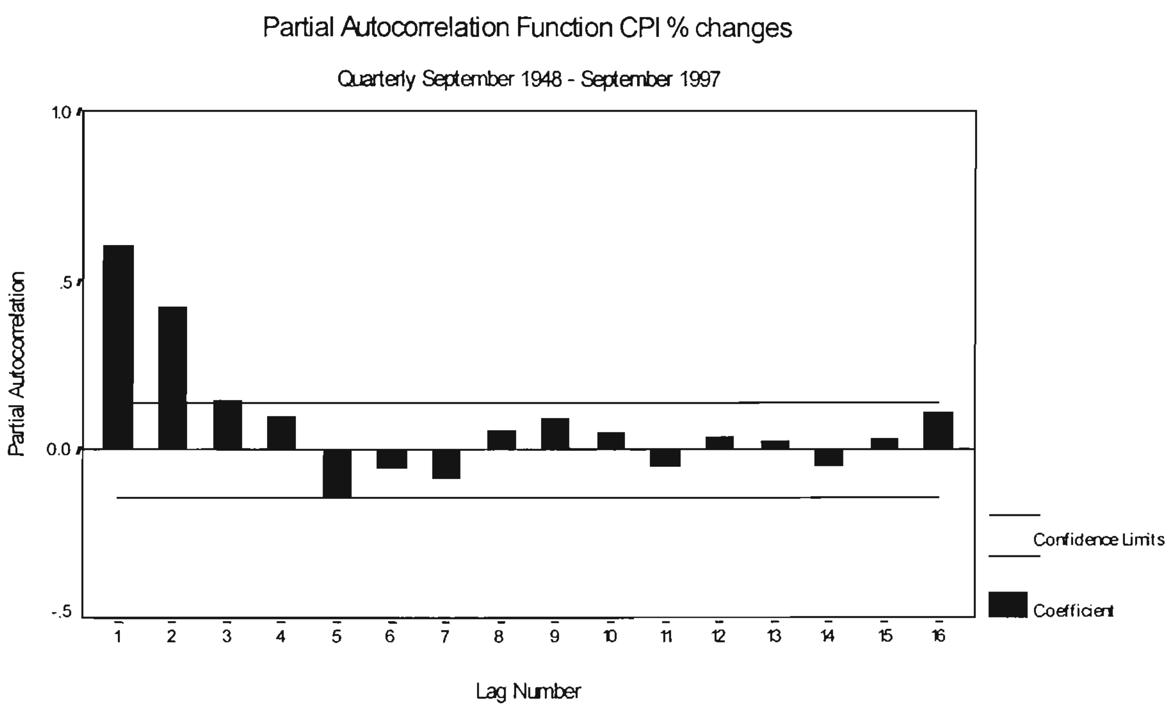


Figure 3.7 Acf and pacf of the % change in the CPI.

There is no evidence of seasonality as the acf does not exhibit highly significant autocorrelations at the seasonal frequencies nor a clear cut off in the pacf at lag 4. This makes sense as there is no reason to expect seasonality in an all encompassing index like the CPI. The slowly declining acf, still significant at lag 12 plus the cut off of the pacf at lag 2, though nearly significant at lag 3, suggests an AR(2) or possibly an

AR(3) as an 'overfit'. Some other possibilities were also considered. This was done with the results shown in table 3.5.

Table 3.5 ARMA Models, Fitting and Diagnostic Checking: Series i_t

Model	Parameter Estimates		t-values	$\hat{\sigma}_\varepsilon$	AIC	$Q(12)$
AR(2)	ϕ_1	0.343	5.35	0.970	547.0	15.46(0.217)
	ϕ_2	0.433	6.68			
AR(3)	ϕ_1	0.280	3.95	0.962	544. 8	11.33(0.501)
	ϕ_2	0.380	5.51			
	ϕ_3	0.149	2.08			
ARMA(1,1)	ϕ_1	0.920	24.46	0.975	549.0	17.65(0.127)
	θ_1	0.545	6.75			
ARMA(1,2)	ϕ_1	0.885	19.52	0.955	542.2	8.81(0.719)
	θ_1	0.648	8.16			
	θ_2	-0.244	-3.28			

Here $\hat{\sigma}_\varepsilon$ is the estimated standard deviation of the residual, $Q(12)$ is the Box-Ljung test statistic for serial correlation in the residuals and AIC represents the Akaike Information Criterion.

The Box-Ljung test is a modification of the 'portmanteau' diagnostic test of Box and Pierce (1970), which tests the joint null hypothesis:

$$H_0 : \rho_1 = \rho_2 = \ldots\ldots = \rho_k$$

where ρ_k is the autocorrelation at lag k, whereby Ljung and Box (1978) show that the statistic:

$$Q(k) = n(n+2)\sum_{i=1}^{k}\frac{\hat{\rho}_i^{\,2}}{(n-i)}$$

where n is the number of observations approximately follows the distribution $\chi_{(k-m)}^{2}$, where m is the number of parameters in the model. In table 3.5 this particular diagnostic, with associated p-values shown in brackets, was satisfactory in all cases.

The AIC is a selection criterion which can be used to determine an appropriate model. It is defined as:

AIC = -2 x (maximised log likelihood) + 2 x (no. of parameters)

or $$\text{AIC}(p,q) = n\ln\hat{\sigma}^2 + 2(p+q) \qquad (3.3)$$

for the ARMA models, where n is the number of observations and p and q are the orders of the AR and MA model fits respectively. As such it compares the improvement in residual variances adjusted for the number of parameters as a penalty term. An alternative to the AIC is the Schwartz Bayes Criterion (SBC). This criterion has been introduced to overcome the difficulty that the AIC is not consistent in that it does not select the true model with probability approaching 1 as $n \to \infty$. The SBC is defined as:

$$\text{SBC}(p,q) = n\ln\hat{\sigma}^2 + (p+q)\ln(n) \qquad (3.4)$$

Other criteria have been developed. Hughes and King (1997) discuss an improved version of the AIC based on the Kullback-Leibler information denoted AICc. Monte Carlo simulation indicated that the performance of the AICc relative to the AIC and SBC was, in most instances, very good. There are certain qualifications on the use of this citeria as Hughes and King (1997, p.9) note, "Under certain circumstances, AIC and SBC perform better than AICc even in exceptionally small samples.". In summary, as concluded by Kendall and Ord (1990, p.117), "Several other criteria have been proposed....In practice they usually provide similar, or even identical results.".

The AIC or SBC is only one of many diagnostic tests for potential models. They will help in selection between two competing models provided other criteria such as significant t-values for the parameters are satisfied. To test the efficacy of the two criteria the following table summarises the results obtained when applying these criteria to the inflation series.

Table 3.6 Values of the AIC and SBC criteria for Different ARMA(p,q) Models for the Rate of Inflation series: September 1948-September 1997

		AR				
AIC		0	1	2	3	4
MA	0		584.4	547.0	544.8	544.6
	1	629.5	549.0	544.3	546.1	541.0
	2	596.7	542.2	546.2	542.6	546.0
	3	585.6	542.4	540.8	539.6	539.2*
	4	559.7	541.9	541.7	543.7	540.4

		AR				
SBC		0	1	2	3	4
MA	0		587.6	553.6	554.6	557.8
	1	632.8	555.5	554.2	559.2	557.4
	2	603.2	552.1*	559.3	559.0	565.7
	3	595.5	555.5	557.2	559.3	562.2
	4	572.8	558.3	561.3	566.7	566.6

The * indicates a minimum. The AIC is indicating an ARMA(4,3) as the minimum. The coefficients ϕ_1, ϕ_4 and θ_1 are not significant for this model. The next best choice is the ARMA(3,3). The SBC chooses the ARMA(1,2) followed by the ARMA(2,0). This result is consistent with Harvey (1990, p.80) who reports evidence which "...suggests that there is a tendency for the AIC to choose a model which is overparameterised.".

The results in table 3.6 are given on the assumption that all the lags are in the equation. If the various models are fitted with only those lags which are significant then different results arise. For example, if the ARMA(4,3) model is used where the AIC is at a minimum and only the significant lags are used a different result is obtained. The log likelihood function changes as does the number of parameters which is now reduced to 4 from 7; AR lags 2 and 3 and MA lags 2 and 3 only remain. The AIC = 537.5 and the SBC = 550.6. These are below the values in the above table for the corresponding AR and MA terms. The SBC is now at a new minimum below the ARMA(1,2) value. When tested further the model diagnostics were inferior. The value of $Q(12)$ rose to 18.82, above that reported for the 4 models in table 3.5. The ARMA(1,2) which had the lowest SBC value had a test statistic $Q(12)$ value of 8.81 (p = 0.719). This suggests that the SBC is a more reliable information criterion and that model comparison using either criteria needs to be treated cautiously when lags are dropped in higher order ARMA specifications.

The R^2 statistic, measuring the goodness of fit in a similar fashion to that of multiple regression analysis, may also be determined. The recursive formula given in Mills (1990) and due to Nelson for an AR(k) process can be used, where:

$$R_k^{\ 2} = \phi_{kk}^{\ 2}\left(1 - R_{k-1}^{\ 2}\right) + R_{k-1}^{\ 2} \tag{3.5}$$

and ϕ_{kk} is the kth order partial autocorrelation. In table 3.5 the ARMA(1,1) can be eliminated as it is not as good a fit as the AR(2). In the case of the AR(3) model, $\hat{\phi}_3$ is just significant and there is a increase in the SBC. The test statistic $Q(12)$ is acceptable in all cases. The AR(3) has a lower value for $Q(12)$. However, a review of the resulting acf for the residuals of the AR(3) indicated a significant value at lag 4. The only other candidate is the ARMA(1,2). There are better diagnostics for this model but at the expense of an extra parameter and MA terms. As a further comparison the stability of the coefficients was analysed. In the earlier consideration of unit roots the time period was divided into various sub-periods. It was found that using a breakpoint of September 1973 the ADF test gave conflicting results for sub-periods (see table 3.2). The same breakpoint was therefore taken. The results are encapsulated in table 3.7, with p-values in brackets:

Table 3.7 ARMA Models, Fitting and Diagnostic Checking: Series i_t with a Break Point of September 1973

Time Period	Model	Parameter Estimates		t-values	SBC	$Q(12)$
Sep 1948 to Sep 1973	AR(2)	ϕ_1	0.349	3.892	296.4	5.60(0.935)
		ϕ_2	0.435	4.762		
	ARMA(1,2)	ϕ_1	0.831	10.340	299.6	3.95(0.984)
		θ_1	0.533	4.538		
		θ_2	-0.301	-2.789		
Dec 1973 to Sep 1997	AR(2)	ϕ_1	0.341	3.748	266.6	21.66(0.042)
		θ_1	0.433	4.650		
	ARMA(1,2)	ϕ_1	0.975	40.688	258.1	15.55(0.213)
		θ_1	0.845	8.035		
		θ_2	-0.169	-1.625		

The ARMA(1,2) exhibits unstable coefficients. For the latter period θ_2 is not significant. It was concluded that the AR(2) would be preferred as it offered almost as good a fit, with one less parameter and coefficients which exhibited stability. The acf of the residuals from the AR(2) is shown in figure 3.8.

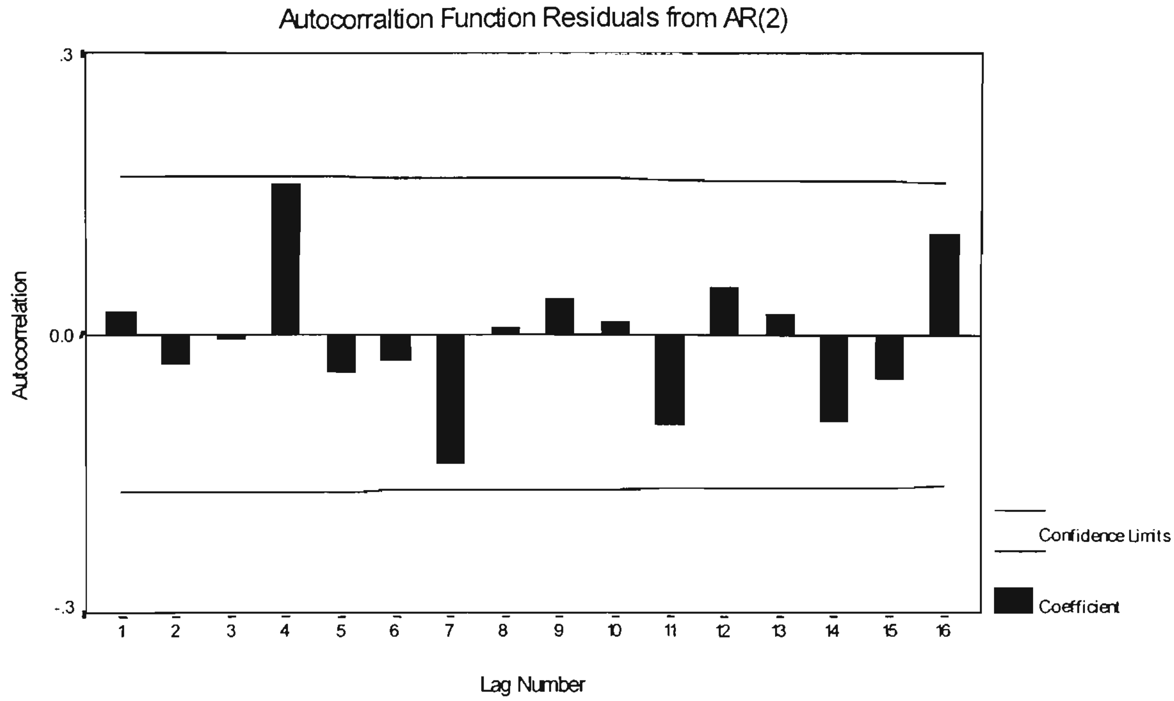


Figure 3.8 Acf of the Residuals from Fitting an AR(2) Model to the % change in CPI.

This then yielded the model:

$$z_t = 0.343 z_{t-1} + 0.433 z_{t-2} + \varepsilon_t$$

where $\varepsilon_t \sim N(0,0.94)$ and $z_t = i_t - \bar{i}$ which when re-transformed lead to:

$$i_t = 0.352 + 0.343 i_{t-1} + 0.433 i_{t-2} + \varepsilon_t \tag{3.6}$$

where the constant term is given by 1.503(1-0.343-0.433). For this model using the recursive formula (3.5) the R^2 statistic is 0.638. The error term is assumed normal and with a constant variance. This will be tested in section 3.5.

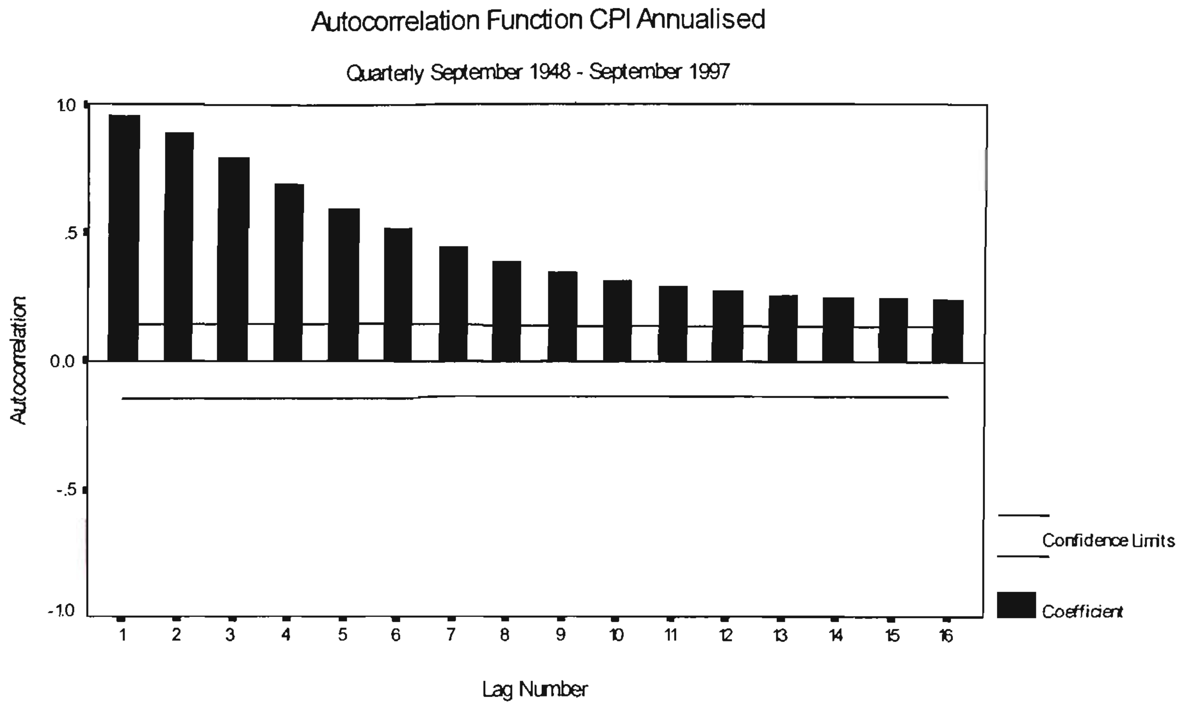
For $i_t' = \Delta \ln I_t$, the difference of the log form, then $\bar{i}'$=0.015 and $S.E.(\bar{i}') = 0.001$. Hence the mean value is significantly different from zero; so the mean adjusting transformation was applied. The result was as follows but with the AR(3) omitted as $\hat{\phi}_3$ was not significant.

Table 3.8 ARMA Models, Fitting and Diagnostic Checking: Series $i_t' = \Delta \ln I_t$

Model	Parameter Estimates		t-values	$\hat{\sigma}_\varepsilon$	SBC	$Q(12)$
AR(2)	ϕ_1	0.406	6.58	0.0097	-1251.5	20.01(0.067)
	ϕ_2	0.502	8.16			
ARMA(1,1)	ϕ_1	0.972	56.02	0.0096	-1252.8	18.63(0.098)
	θ_1	0.575	8.85			

The parameters of the models for i_t (see table 3.5) and i_t' are similar in magnitude. The more natural percentage changes approach will be utilised.

In the annualised form i_t'', the mean value is $\bar{i}'' = 6.27\%$ with $S.E.(\bar{i}'') = 0.349$. Hence the mean is significantly different from zero. The data was mean adjusted by the transformation $z_t = i_t'' - \bar{i}''$ The acf shows a classic AR representation with a high first order correlation coefficient $\rho_1 = 0.956$. This near unit root was observed in the unit root tests. The pacf however does not die down quickly with ϕ_5 still significant at 0.315. The ADF test regression had 7 lagged differences suggesting a high order AR model. Given the pacf an AR(10) was fitted, then the non-significant coefficients eliminated one by one. This left an AR(5).



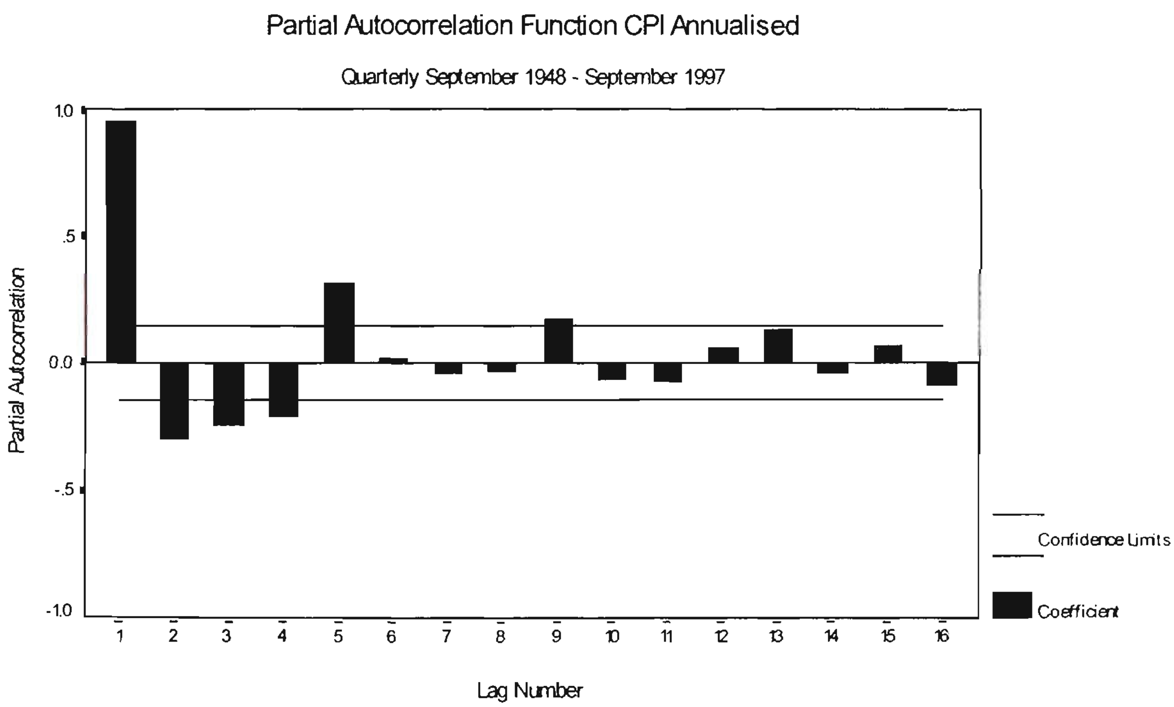


Figure 3.9 Acf and pacf for the Annualised CPI.

Then since a high order AR may be simplified with MA terms, MA parameters were introduced. A process of diagnostic checking was pursued similar to those described for i_t and i_t'. This led to the model:

$$z_t = 1.270 z_{t-1} + 0.278 z_{t-4} + \varepsilon_t - 0.932 \varepsilon_{t-4}$$
$$(65.595) \quad (-14.346) \qquad (-40.096)$$

which yields on re-transformation:

$$i_t'' = -3.436 + 1.270 i_{t-1}'' + 0.278 i_{t-4}'' + \varepsilon_t - 0.932 \varepsilon_{t-4} \qquad (3.7)$$

with the value of the test statistic $Q(12)$ being 10.943 with an associated p-value of 0.280. Time aggregation creates a potential difficulty in that the annualised CPI is the product of four quarters, the last three of which will be carried forward to the subsequent annualised value. Bollerslev and Hodrick (1995) make reference to a similar problem with annualised time series when finding a suitable dividend series for the NYSE. They used annualised monthly data, defined as the current value of the monthly dividends over the previous year, rather than quarterly data. They then took the difference of the log of the dividend as a measure of growth in real dividends.

In a discussion of the results of tables 3.1 and 3.3, the order of integration of I_t was uncertain. The ADF test found ΔI_t an $I(1)$ and the PP test indicated it as an $I(0)$. So the series ΔI_t was investigated. The acf revealed a first order correlation coefficient $\rho_1 = 0.746$, well below 1. A process of diagnostic checking as done previously led to the pure AR model:

$$z_t = 0.336 z_{t-1} + 0.270 z_{t-2} + 0.202 z_{t-3} + 0.248 z_{t-4} - 0.174 z_{t-7} + \varepsilon_t$$

(4.693) (3.598) (2.555) (3.267) (-2.635)

with the value of the test statistic $Q(12)$ being 6.473 ($p = 0.486$). In this case the sum of the AR coefficients, $\sum_{j=1}^{7} \phi_j = 0.882$. This compares to values of 0.776 in table 3.5 and 0.908 in table 3.8 for the i_t and i_t' models respectively. Thus the rejection of the unit root null depends on the power of the relevant test. This result suggests that the rejection of the unit root null for ΔI_t by the PP test is soundly based. This would imply I_t is an $I(1)$.

These results can be compared with those of Carter (1991). Carter applied the difference of the logarithm of the CPI, corresponding to $i_t' = \Delta \ln I_t$, to quarterly data from December 1971 to June 1990. He found no ARMA model adequately fitted the data. So assuming non-stationarity of the force of inflation series as the cause of this failure, he differenced the series a second time. He then fitted an ARMA(2,2) to $\Delta^2 \ln(I_t)$. The results in this chapter suggest a potential over differencing. The period Carter examined contains a period of high inflation which covers a large portion of the total; that may have influenced the result.

Only a few of the possible alternative estimators of the inflation rate have been considered. The choice reflects a compromise between what is available and the use made of it. Percentage changes satisfies naturalness and is log equivalent. Unfortunately to find real interest rates an annualised series is required since rates are given on a yield to maturity (YTM) basis. That introduces the problems of aggregation outlined above. The elements of the interest rate series are assumed to be independent. Therefore the deflated series will further dilute any aggregation difficulties. This has the advantage of preserving real rates as they are quoted.

3.5 Normality and Heteroskedasticity

So far the error terms are assumed to be identical independently distributed normal variates. Serial correlation is tested by the acf of the residuals and the Box-Ljung statistic $Q(12)$. However neither the normality nor the homoskedasticity assumptions have been investigated.

A test for normality is the Jarque-Bera test. This test measures departures from normality in terms of the skewness and excess kurtosis of the distribution (see Mills (1993, p.143-144), Sherris(1997a)). If μ_k is the kth central moment, $\mu_k = \frac{1}{n}\sum_{t=1}^{n}\left(\varepsilon_t - \bar{\varepsilon}\right)^k$ with n observations and defining $m_3 = \frac{\mu_3}{\mu_2^{3/2}}$ and $m_4 = \frac{\mu_4}{\mu_2^{2}} - 3$ then the Jarque-Bera statistic is:

$$n\left(\frac{m_3^2}{6} + \frac{m_4^2}{24}\right) \sim \chi_2^2$$

That is, it has a χ^2 distribution with 2 degrees of freedom. The critical value is 5.991 at the 5% level of significance. Godfrey (1988, p.145) comments that this test is asymptotically valid and that "Jarque and Bera carry out simulation experiments and find that asymptotic theory provides a poor approximation to the actual finite sample behaviour of their test statistic.". Godfrey notes that there needs to be simulations to obtain critical values for finite samples and this tends to be ignored in empirical work.

The heteroskedasticity observed[10] in financial series may take many functional forms, (see Godfrey (1988, p.123-136), Mills (1993, p.101-105)). There are a wide variety of non-linear solutions. One set is the class of ARCH (Autoregressive Conditional Heteroskedasticity) models introduced by Engle (1982) and related refinements. Another set of non-linear models are switching regime models; there are other potential candidates (see Mills (1993, p.113-126). The ARCH class has been widely applied in the stochastic investment model literature (see Geoghegan *et al.* (1992), Harris (1994), Mulvey (1996) or Sherris (1997a)). These models have also been widely and successfully applied in the general finance literature. A comprehensive review of ARCH modelling in finance is given in Bollerslev, Chou and Kroner (1992). The application of non-linear univariate models will therefore be restricted to this class.

One possible general test for heteroskedasticity is that due to White (1980). A more specific test for non-linear dependence in the residuals for the ARCH class is given by the ARCH-LM test due to Engle (1982). This gives a specific functional form to the heteroskedasticity to be able to test for. The ARCH-LM test is based upon a regression of ε_t^2 on ε_{t-1}^2 (see Engle (1982)). The resulting R^2 from the linear regression (which includes a constant term) times the number of observations n has an asymptotic χ^2 distribution with 1 degree of freedom. That is $nR^2 \sim \chi_{(1)}^2$, the value of which is 3.841 at the 5% level of significance.

[10] The distributions of many economic series show fatter tails than expected on the basis of a normal distribution. One potential explanation of this non-normality is varying levels of volatility over time. This is not the only possible explanation. A discussion of this is in Hart (1996, p.86-90).

Now the ARCH(p) model allows for time varying conditional volatility by modelling the variance as:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2$$

where ε_t are the residuals from fitting an equation. An extension of the above is to the GARCH(p,q) which models the variance as:

$$\sigma_t^2 = \alpha_0 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 + \sum_{k=1}^{q} \alpha_k \varepsilon_{t-k}^2 \qquad (3.8)$$

There have been many further developments of these two basic models. One variant is to consider the relation between the variance and error terms, hence the IGARCH(1,1) sets $\alpha_1 + \beta_1 = 1$ and $\alpha_0 = 0$. Another set sees the distribution of ε_t which is assumed normal in the ARCH model, modelled with different distributions such as a normal-Poisson or power exponential. A further possibility is to allow the variance to be a function of explanatory variables. Therefore the GARCH-M extends the GARCH model to a regression framework and brings the conditional variance into the equation for the mean level (see Mills (1993, p.137)).

Another important set is the EGARCH or exponential GARCH introduced by Nelson (1991). Nelson's paper summarises model development to that date and the shortcomings of the various models. Nelson then introduced a log term to ensure non-negativity in the variance. He also introduced a leverage effect by creating an asymmetric response to positive and negative shocks. The model is set out in Nelson (1991, p.353-355) and applied to modelling the US risk premium. The EGARCH(1,1), with logical extensions to higher orders, is of the form:

$$\ln\left(\sigma_t^2\right) = \alpha_0 + \beta_1 \ln\left(\sigma_{t-1}^2\right) + \alpha_1 \left|\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right| + \gamma_1 \frac{\varepsilon_{t-1}}{\sigma_{t-1}}$$

In the stochastic investment modelling literature various non-linear models have been proposed for inflation, Clarkson in Appendix A in Geoghegan *et al.* (1992) puts forward a possible model. His model is of the form:

$$i_{t+1} = i_t - A_t(i_t - B) + D_t + E_t$$

where i_t is the rate of inflation in year t and $A_t > 0$, B is the mean rate of inflation, D_t is a random error term and E_t is a random shock term. A_t is the rate of mean reversion, which will vary if there is an upward trend in inflation. Clarkson in model fitting makes the variable A_t constant. E_t allows for a jump process which differs from the usual random error term D_t. This shock can take the process to a new level before slowly returning to the long run mean. E_t then allows for exogenous shocks such as occurred in the early 1970's. The jump process could be modelled as a Poisson distribution.

Wilkie also in Geoghegan *et al.*, Appendix B applies ARCH techniques to his model. The investigation is brief, with the added comment that "...ARCH models are worth investigating further.". Mulvey (1996) uses an autoregressive model[11] with ARCH residuals to model inflation. The model given by Mulvey (1996, p.13) is represented by two diffusion equations, one for the price level and one for the volatility of the price level. The equations are:

$$dp_t = ndr_t + g(p_0 - p_t)dt + h(v_{pt})dZ_{3t}$$

and $$dv_{pt} = k(v_{p0} - v_{pt})dt + m\sqrt{v_{pt}}dZ_{4t}$$

where p_t is inflation at time t, v_{pt} is volatility at time t, n is a constant, r_t is the short term rate, g and k are functions which are not given, and h and m are parameters representing the contributions of the white noise processes dZ_{3t} and dZ_{4t}.

[11] A model developed by Mulvey and others for the consulting actuarial firm Towers, Perrin, Foster and Crosby (TPFC).

Interestingly, this models inflation as a process dependent on price inflation in previous periods and on the current yield curve via the short rate r_t, given in the first equation above. The short rate itself is modelled as a stochastic process.

Sherris (1997a) reviewed the topic using Australian data, with a detailed consideration of ARCH effects in models for inflation, equity returns and interest rates. Applying simple ARCH models Sherris tests whether better fits to the data could be obtained with ARCH models rather than by using simple AR models (he adheres to AR(1) representations). He does not find in favour of these particular ARCH models. Some of these issues will be tested using the quarterly inflation data and with the application of a range of non-linear models as described above.

In table 3.10 the AR(2) equation (3.6) shows evidence of non-normality, with significant excess kurtosis. It also fails the ARCH test, indicating non-linearity, at least in some form. The next step is to determine the orders p and q for modelling the variance with the GARCH(p,q). The SBC is a criterion for model selection used previously and found preferable to the AIC. There are other possible criteria. One such is the R^2 test applied in Pagan and Schwert (1990). Pagan and Schwert reviewed monthly US stock returns from 1834-1925 and applied various alternative models of conditional heteroskedasticity and compared them. The comparison is made on the basis of an R^2 test. Another possibility is a series of likelihood ratio tests based upon $\ln L$.

The mean was modelled with an AR(2) process and mean adjusted. The order of the conditional heteroskedastic ARCH/GARCH model are as in the following table. A joint estimation procedure for the conditional mean and variance equations was used. The period of the fit is September1948 to September 1997.

Table 3.9 Values of the SBC criterion for Different GARCH(p,q) Models for the Variance with the Mean Modelled as an AR(2) for the Rate of Inflation series: September 1948-September 1997

SBC		ARCH 1	2	3
GARCH	0	536.0	530.2	534.7
	1	524.2	524.2	529.4
	2	528.8	529.4	530.3
	3	511.6	NA*	523.6

* No value determined as a near singular matrix occurred during optimisation.

As a check a GARCH(1,4) was fitted with an SBC of 527.4. This confirms the preferred model on this criterion as the AR(2)-GARCH(1,3). Higher order ARCH models exhibited higher values for the SBC. With the SBC as a criterion of selection there has been an improvement over the AR(2). An SBC value of 553.6 was recorded[12] (see table 3.6).

Three models were chosen and diagnostics compared. The AR(2) discussed previously, an AR(2)-ARCH(1) by way of comparison and the preferred AR(2)-GARCH(1,3). Note that the diagnostics for the original equation are for the residuals. Those for the ARCH models are for standardised residuals. Table 3.10 below outlines the situation (p-values in brackets where appropriate):

[12] Different software for the ARCH/GARCH modelling meant that the SBC was slightly different. The log likelihood function differs. This may be due to different optimisation algorithms.

Table 3.10 AR(2)-ARCH/GARCH Models Diagnostic Testing

Model	ARCH-LM test	skewness	excess kurtosis	Jarque-Bera statistic	$Q(12)$
AR(2)	6.522	0.952	2.794	92.41	15.458
	(0.011)				(0.217)
AR(2)-ARCH(1)	0.015	0.826	3.226	106.21	23.338
	(0.902)				(0.010)
AR(2)-GARCH(1,3)	0.275	0.626	0.961	17.15	17.150
	(0.608)				(0.071)

Introducing ARCH or GARCH terms improves the standardised residuals so that they pass the ARCH test. The standardised residuals for the AR(2)-GARCH(1,3) model have a significantly lower Jarque-Bera test statistic. It still remains outside the critical value of 5.991, thus rejecting the assumption of the normality of the standardised residuals. The other diagnostics are acceptable with the p-value for $Q(12)$ at 0.071.

The stability of the model coefficients must next be checked. The model AR(2)-GARCH(1,3) for the period September 1948 to September 1997 is given by (with t-values in brackets beneath):

$$z_t = \underset{(36.179)}{0.464} z_{t-1} + \underset{(9.155)}{0.348} z_{t-2} + \varepsilon_t \quad \text{where } z_t = i_t - \bar{i}$$

and

$$\hat{\sigma}_t^{\,2} = \underset{(1.483)}{0.034} + \underset{(3.025)}{0.275}\hat{\varepsilon}_{t-1}^{\,2} + \underset{(11.701)}{0.686}\hat{\sigma}_{t-1}^{\,2} - \underset{(-8.530)}{0.646}\hat{\sigma}_{t-2}^{\,2} + \underset{(6.960)}{0.649}\hat{\sigma}_{t-3}^{\,2}$$

The time period was divided into various sub-periods. A suitable breakpoint is 1973 when inflation rose rapidly (see figure 3.2). The use of this breakpoint is consistent with that in table 3.4. The September 1973 breakpoint was used and the preferred model fitted to the two sub-periods. The results of this analysis is given in table 3.11.

Table 3.11 AR(2)-GARCH(1,3) Parameter Values for GARCH terms for the Whole Period and Applying a Break Point at September 1973

period	α_0	α_1	β_1	β_2	β_3
Sep 1948 to	0.504	0.244	0.528	0.084	-0.034
Sep 1973	(2.561)	(1.814)	(1.429)	(0.203)	(-1.454)
Dec 1973 to	0.256	0.590	-0.149	0.044	0.182
Sep 1997	(1.626)	(2.239)	(-2.367)	(0.358)	(0.808)
Sep 1948 to	0.034	0.275	0.686	-0.646	0.649
Sep 1997	(1.483)	(3.025)	(11.701)	(-8.530)	(6.960)

The equations are unstable across time periods. The standardised residuals were unsatisfactory for the sub-period December 1973 to September 1997. The value of $Q(12)$ for this sub-period was 30.816 with a p-value of 0.001.

Therefore on the basis of the analysis conducted above the use of ARCH/GARCH modelling has not proved successful. The approach has not provided an improvement over the simpler AR(2) representation. These results are consistent with those of Sherris (1997a, p.23), who concluded that "The results do not suggest that volatility in the series can be successfully modelled using an ARCH process.". Mishkin and Simon (1995) however found evidence to suggest ARCH residuals in modelling the rate of inflation. They then modelled the process with ARCH terms though full results are not given. Hence in determining their sampling distributions for the unit root tests they then applied ARCH error terms in the DGP rather than normal ones (see Mishkin and Simon (1995, p.219-221)).

Brailsford and Faff (1993) modelled Australian stock market volatility using *daily* data over the period 1974 to 1993. Their best or preferred model for the variance component was a GARCH (3,1) of the general form given in (3.8), where $p = 3, q = 1$. However Brailsford and Faff (1993, p.129) concluded that, " But diagnostic tests ....suggested the model was unable to fully capture asymmetric responses to past innovations.". The authors then employed an approach which modified the GARCH model to incorporate the asymmetric effects. Whilst this improved the model, the authors found that the model was unstable across periods.

On the basis of the results of Brailsford and Faff an asymmetric[13] model was fitted to the quarterly data. The AR(2)-EGARCH (p,q) model of Nelson was applied for the range of values $p = 1$ to 2, $q = 1$ to 2. The best model on the SBC criterion was an EGARCH (1,2) but with inferior diagnostics to the GARCH (1,3) above. The value of $Q(12)$ was for this EGARCH was 24.462 with a p-value of 0.006. The other EGARCH models also had many non-significant coefficients.

There is evidence of non-normality in the residuals from table 3.10. On the assumption of heteroskedasticity there are a wide range of possibilities for the functional form of the variance relationship. Hence either the assumption of normality in the errors can be maintained as a suitable approximation or else some other functional form for the residuals can be applied. By way of example a mixture of normal distributions could be applied. The normality assumption will be maintained. This heteroskedasticity is common to most financial series.

3.6 Inflation Expectations: the *ex ante* Values

So far only *ex post* inflation values have been considered. This is not the same as expected inflation. The behaviour of the financial markets needs to be thoroughly understood if a successful model is to be built. This issue is most clear when the risk premium is considered, since arguably what matters most is not what happens to *ex post* returns but *ex ante* returns. The latter is a determined by behaviour which can aid in an explanation of some of the anomalies (see Tversky (1989)).

There are available two series of monthly inflation expectations data, the mean and median for the period March 1973 to September 1997, as explained in section 3.2. The median is a better estimator of consumer inflationary expectations due to the potential bias in averaging given the wide dispersion in values actually reported. A chart of the median value of inflation expectations versus actual inflation[14] is given in figure 3.10.

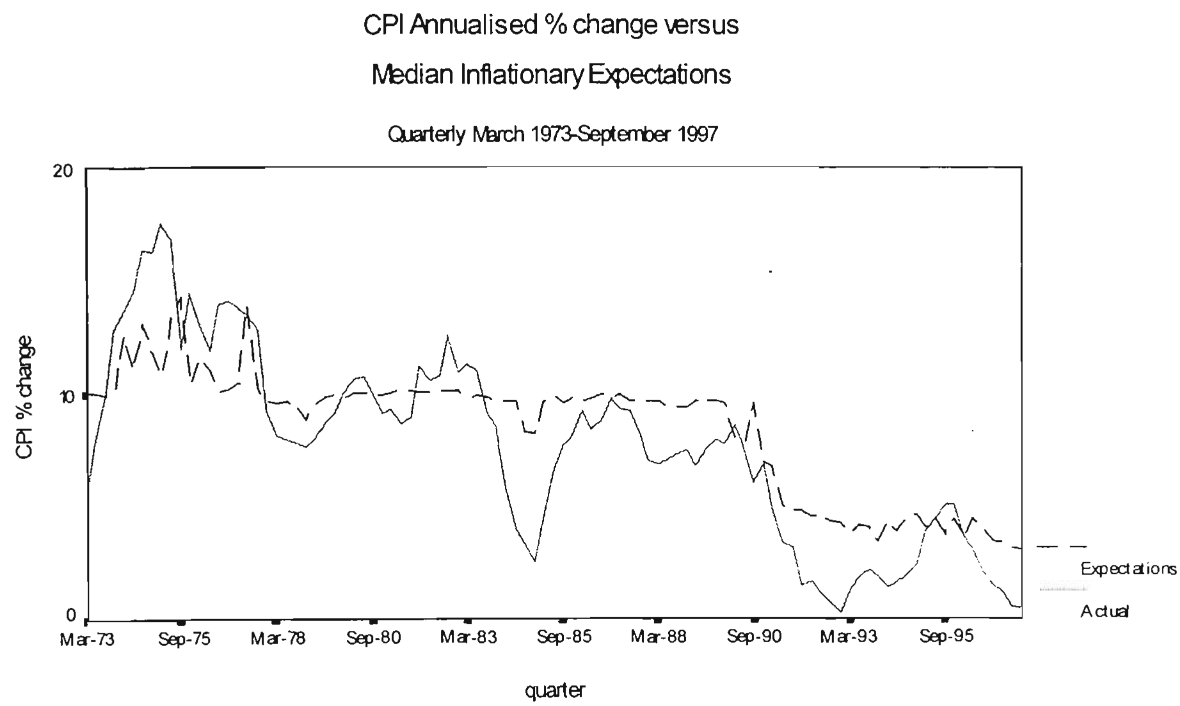


Figure 3.10 Median Value of Inflationary Expectations versus the Annualised % change in the CPI.

This demonstrates that the public's view of inflation is remarkably smooth over the short term. There was an element of uncertainty in the 1970's perhaps as a result of the inflation shocks occurring then. It has proved difficult for expectations to rise and stay above 10%. From 1978 until 1989/1990 the consensus view was that 10% was correct. It took a lot of convincing before the general public came to the view that there was a downtrend in place. Expectations rapidly fell to the 5% region where they have stayed. There has been a period of very high real interest rates but one which the general public *did not expect.* An assessment of prospective inflation has been made based upon a long period of actual values of inflation. To take the analysis further the relationship of expectations to actual or observed inflation needs to be investigated. Economic theory would suggest that the unobserved or unexpected component of inflation should be white noise. This can be tested and potential models related by using the identity:

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[13] The asymmetric model would suggest a levels effect or the magnitude of the volatility depends upon the overall level of inflation. Chapter 7 reviews this proposition for real bond rates.

observed inflation = expected inflation + unexpected inflation

Unexpected inflation was determined by the difference between actual and expected inflation. That is, if i_t'' is observed inflation, $e_t = E(i_{t+1}''|I_t)$ is the expected value of inflation conditional on the information set available to agents from the current period, and u_t is the unexpected inflation all at time t, then $u_t = i_t'' - e_t$. Note that discounting the actual or observed inflation by inflationary expectations $u_t = \left\{ \frac{\left(1 + \frac{i_t''}{100}\right)}{\left(1 + \frac{e_t}{100}\right)} - 1 \right\} * 100$ could also be used. For small values, and this is the case with the data, there is little difference[15]. The method used here is therefore consistent with the way, for example, nominal bond yields are discounted. The chart is given in figure 3.11.

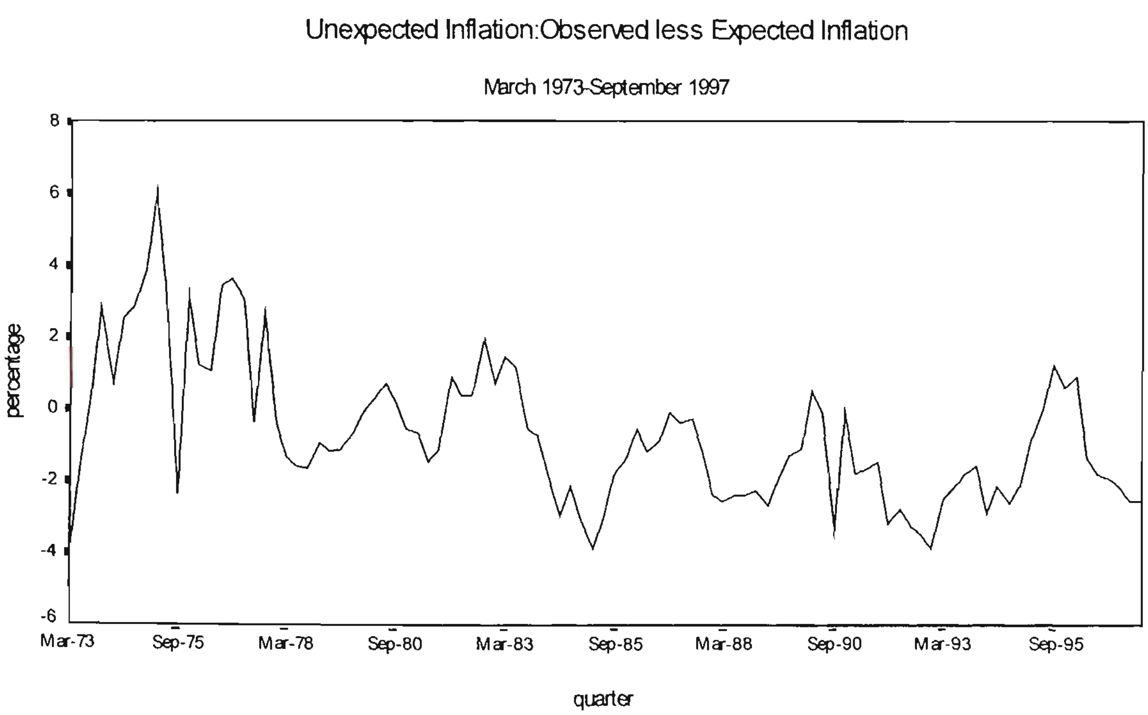

Figure 3.11 Unexpected Inflation: Actual Inflation less Inflationary Expectations.

[15] This can easily be seen by expanding the discounting expression and dropping higher powers of e_t.

3.6.1 Unit Root Testing

An ADF test was applied to test for the stationarity of each of the series. The method used was as in section 3.4.1.

Table 3.12 ADF Regression: Actual, Unexpected and Expected Inflation series: March 1973 to September 1997

ADF: 5% level critical value below		trend and intercept (-3.455)			intercept (-2.891)		none (-1.943)
variable	lag length	α	$(\beta-1)$	γ	α	$(\beta-1)$	$(\beta-1)$
i_t''	4	3.553 (5.447)	-0.247* (-5.301)	-0.033 (-5.301)	0.273 (1.075)	-0.038 (-1.307)	-0.010 (-0.785)
u_t	3	0.485 (1.420)	-0.391* (-3.783)	-0.016 (-2.331)	0.229 (-1.486)	-0.232* (-2.916)	-0.191* (-2.546)
e_t	2	2.180 (2.422)	-0.162 (-2.366)	-0.018 (-2.725)	-0.133 (-0.432)	0.001 (0.039)	-0.013 (-1.257)
Δe_t	1	0.095 (0.475)	-1.734* (-10.40)	-0.004 (-1.319)	-0.122 (-1.330)	-1.706* (-10.28)	-1.678* (-10.15)

* Significant at the 5% level.

The observed inflation series i_t'' is an $I(0)$. The trend and intercept terms are significant suggesting trend stationarity. This may be compared to the earlier results in section 3.4.1, tables 3.1 and 3.3. For u_t a value of -3.783 below the critical 5% level of -3.457 is found and thus the null hypothesis of a unit root is rejected. The test regression was re-run without the trend term and the null was again rejected. Finally the test was applied without intercept or trend terms. The null was again rejected. Hence it follows that u_t is an $I(0)$. Expected inflation e_t is an $I(1)$. The PP test was performed in the same fashion as in section 3.4.1 with the results shown in table 3.13.

Table 3.13 PP Test Actual, Unexpected and Expected Inflation series: March 1973 to September 1997

crit.val.	trend and intercept (-3.455)			intercept (-2.891)			none (-1.943)		
variable	1	5	10	1	5	10	1	5	10
i_t''	-4.156	-4.135	-4.227	-1.111	-1.474	-1.375	-0.816	-0.928	-0.876
u_t	-5.500	-5.748	-5.743	-4.005	-4.366	-4.623	-3.804	-4.131	-4.409
e_t	-3.385	-3.503	-3.851	-0.781	-0.548	-0.646	-1.098	-1.179	-1.199
Δe_t	-14.64	-15.91	-16.19	-14.57	-15.60	-15.72	-14.50	-15.21	-14.97

There is agreement that i_t'' and u_t are both $I(0)$. In table 3.3 $i_t'' \sim I(1)$, with this shorter series the unit root null is easily rejected at the 5% level. However for e_t the PP test rejects the unit root null at truncation lags 5 and 10 when an intercept and trend are applied. To check the test was re-run with truncation lags 1 to 4. Resulting test values were -3.385, -3.285, -3.403 and -3.405 for lags 1 to 4 respectively. This suggests a near unit root but an acceptance of the null. Hence this suggests that e_t is an $I(1)$. First differencing was therefore necessary for stationarity of this series. When this was done the ADF test was applied with an intercept term and trend term giving a value of -10.401 well above the critical value. Dropping the intercept term in our regression equation gave a value of -10.148 still well outside the critical region (see tables 3.12 and 3.13).

The observed inflation series i_t'' is an $I(0)$, as is the unexpected inflation series u_t. The expected inflation series is an $I(1)$. Now the sum of an $I(1)$ and an $I(0)$ is an $I(1)$, since the order of integration of a linear combination of two series is the maximum of the order of the components. This is not expected as both i_t'' and u_t are $I(0)$. The result for e_t arises from the smoothness of expectations. This result is consistent with those of Gulley (1995, p.205) who concluded that "The tests are known to have low power when the true root is close to one or when the sample size is small and/or spans a time period that is not long enough to bring out the true properties of the series.".

There are several possibilities to explain the above result for e_t. Firstly there may simply be a Type II error. The data set has lead to the failure to reject the null of a unit root which is false. The PP test suggests this has happened. Additional observations could see a correction where respondents underestimate future inflation. This seems plausible. Secondly in the analysis an $I(1)$ null is tested against an $I(0)$ alternative. The order of differencing is assumed integer. It could be fractional. This opens a wide avenue of possibilities. Granger and Joyeux (1980) or Hosking (1981) provide an introduction to the topic of fractional integration. These issues will not be pursued since it will detract from the main objectives. It does serve to underline the statistical issues of estimation and hypothesis testing with unit root methodology.

3.6.2 Modelling Expected and Unexpected Inflation

The mean value of the unexpected inflation series u_t was -0.701 with a standard error of 0.199. Hence the mean value is significantly different from zero, so the data was transformed by mean adjustment. The acf and pacf for the resulting series was then reviewed. There was no evidence of seasonality from the acf and pacf. The acf showed a classic AR(1) representation[16], so a model was fitted as before with the result shown in table 3.14.

[16] In figure 3.14 there appears to be a possible downtrend, but given the nature of the series, there is no reason to suppose that the surprises will continue to be negative. If inflation rises errors could be the other way. Furthermore the acf for the unexpected inflation series was very well behaved, in marked contrast to that of observed inflation, which had significant values at lag 5 and above.

Table 3.14 ARMA Models, Fitting and Diagnostic Checking: Series u_t

Model	Parameter Estimates		t-values	$\hat{\sigma}_\varepsilon$	SBC	$Q(12)$
AR(1)	ϕ_1	0.726	10.51	1.370	347.6	10.37(0.583)
AR(2)	ϕ_1	0.605	6.09	1.358	349.5	8.11(0.776)
	ϕ_2	0.167	1.67			

The AR(1) was accepted as ϕ_2 for the AR(2) is not significant. Thus the unexpected inflation series is not white noise. If there were a downtrend, which *a priori* one would discount, then white noise would still not represent the series. Figure 3.11 demonstrates the point showing consistent over or under-estimation of inflation.

The model may be determined by re-transforming the mean, where u_t is the unexpected inflation at time t as:

$$u_t = -0.19 + 0.73u_{t-1} + \varepsilon_t \tag{3.9}$$

where $\varepsilon_t \sim N(0,1.877)$.

The picture is completed by considering the data generating process (DGP) of the expectations series e_t, and modelling it. Some form of linear filter may be appropriate as a fit. For example a weighting system which weights the more recent data is suggested. An exponentially weighted moving average (EWMA) where the weights form a geometrically declining series, was applied (see Box and Jenkins (1976, p.105-108)). This was fitted using actual inflation, so as to fit as closely as possible that expected. The public's view of expected inflation is being determined. The value of the weight θ is found such that:

$$\min_{0\le\theta\le1}\left\{\sum_{i=1}^{\infty}\left[g(i_t'') - e_t\right]^2\right\}$$

where $g(i_t'') = (1-\theta)i_t'' + \theta g(i_{t-1}'')$. The optimal value was $\theta = 0.874$, so that:

$$e_t = 0.126\sum_{j=0}^{\infty}(0.874)^j i''_{t-j} \qquad (3.10)$$

The "average age" of the data is therefore 7.94 quarters. This gives a low weight of 0.110 to the last data point, and a value of 0.033 to the 10th value. Indeed it takes 19 periods or 5 years before the contribution falls below 1%. The fit obtained is reasonable though expectations were held at the 10% level for longer than would be predicted by the EWMA approach.

An alternative way of expressing the EWMA is with the Adaptive Expectations Hypothesis (AEH), for a discussion and comparison of various theories of expectations see Nerlove and Schuermann (1995). The AEH postulates that:

$$e_{t+1} - e_t = \gamma\,(i''_t - e_t) + \varepsilon_t \Rightarrow e_{t+1} = \gamma\, i''_t + (1-\gamma)e_t + \varepsilon_t \qquad (3.11)$$

This may be expanded recursively with the coefficient of i''_{t-n} being $\gamma(1-\gamma)^n$. Now $\Delta e_t = (e_t - e_{t-1}) \sim I(0)$ and $(i''_t - e_t) = u_t \sim I(0)$ so standard OLS applies. The regression was conducted with a resulting value of $\gamma = 0.140$, where t = 3.411, $R^2 = 0.102$, $LM(4)$ = 10.567[17] (p = 0.032) and $LM(12)$ = 18.274 (p = 0.108). This value is consistent with the value $\theta = (1-\gamma) = 0.874$ in equation (3.10). It is also consistent with Eckstein (1981) who found a value of 0.84 for the US price expectations factor. This suggests similar behaviour operating in both countries. Learning would not appear to have speeded up based upon these facts. Defris and Williams (1979) conducted econometric modelling of consumer expectations of inflation. The authors used the AEH plus a range of explanatory variables including consumer sentiment. Defris and Williams (1979, p.147) concluded that expectations are "...determined primarily by recent rates of actual inflation.". The only economic variables that exerted an impact on expectations were wage indexation and changes in money supply. This result supports the view that observed inflation is the best predictor of expected inflation. There were two areas of concern in the Defris and

[17] See section 5.4 for a discussion of the various diagnostic tests used in OLS regression.

Williams study. Firstly the study was carried out with only 20 observations. Secondly the inference for the regression was based on OLS. Given the conclusion that expected inflation is an integrated variable this would question the validity of the results. Any future study would therefore need to investigate the nature of the explanatory variables. The actual model formulation and for example the extent of the lags involved remains undecided. Nerlove and Schuermann question the merits of the AEH and indeed other common models which attempt to model expectations. These aspects will not be pursued as they are not central to the aims of the thesis.

The DGP may also be modelled by standard univariate methods. The mean value of the differenced series Δe_t was $\overline{\Delta e_t} = -0.071$ with a corresponding standard error, $S.E.\left(\overline{\Delta e_t}\right) = 0.098$. Hence the mean is not significantly different from zero. The acf was produced and showed a cut off at lag 1, with the pacf showing significant values at lags 1 and 2 but no evidence of seasonality. An MA(1) is strongly suggested by these characteristics, which turned out to be the case as shown in table 3.15.

Table 3.15 ARMA Models, Fitting and Diagnostic Checking: Series Δe_t

Model	Parameter Estimates		t-values	$\hat{\sigma}_\varepsilon$	SBC	$Q(12)$
MA(1)	θ_1	0.432	4.70	0.886	258.2	7.75(0.804)
MA(2)	θ_1	0.448	4.39	0.890	262.7	7.87(0.795)
	θ_2	-0.024	-0.23			

This leads to the model; $\Delta e_t = \varepsilon_t - 0.43\varepsilon_{t-1}$ or $e_t = e_{t-1} + \varepsilon_t - 0.43\varepsilon_{t-1}$ an IMA(1,1) model in the Box-Jenkins terminology or more commonly known as an exponentially weighted moving average. Following the approach of equating coefficients as in Box and Jenkins (1976, p.106) and using the same format as the EWMA used for modelling the expected inflation rate from the observed one, a value of $\theta = 0.432$ was determined. This compares with the value of 0.874 for the previously fitted series. The heavier discount or filter is needed to follow the data more closely, remembering that expectations were held at the 10% level for longer than would be predicted by the EWMA approach. What is important is that the process is best modelled as an EWMA. This provides support to the hypothesis that expectations are the result of a process which can be defined by an EWMA of past inflation. In conclusion there are the following results:

1. Unexpected inflation is not a random series but follows an AR(1) process.
2. Expected inflation is determined primarily by recent rates of actual inflation. An appropriate description of expected inflation is as an EWMA of past inflation.

Now $i_t'' = e_t - u_t$, hence following from this the observed inflation model for i_t'' may be deduced from (3.9) and (3.10) or equivalently (3.11). Alternatively u_t may be found by substituting (3.10) into the identity. The model that was fitted to i_t'' in section 3.4.3 equation (3.7) was an ARMA(4,4). The inconsistency is caused by a question as to the order of integration of the series, and therefore whether an ARMA or ARIMA model is the appropriate one to use. This inconsistency is not pursued here but observed and expected inflation should 'trend together' in the long run. Figure 3.13 shows a consistent over estimation of inflation. This is because the data is for a period exhibiting a long downtrend in inflation. A solution would be for $e_t \sim I(0)$ and hence both observed and expected inflation would be stationary. This would then create difficulties for the AEH as an EWMA is an integrated or ARIMA model.

3.7 Conclusions

This chapter has reviewed the data sources for the inflation indices that form the basis of the proposed stochastic investment model. The nature and characteristics of inflation have been reviewed with various estimators of the mean level of inflation proposed. Estimates of the volatility of the rate of inflation has also been considered.

A structural break in 1947 has been determined. There is evidence to suggest that both the rate of inflation and real GDP growth have a different DGP post war. Thus the data series commence in 1947. The results with respect to inflation suggest that Keynesian post war macroeconomic stabilisation policies have been successful.

The augmented Dickey-Fuller and Phillips Perron tests have been used to determine the existence of unit roots. Determination of the stationarity or otherwise of the series is valuable in evaluating whether shocks to the system are permanent, as argued in Nelson and Plosser (1982). It also has important technical implications for time series modelling. The order of integration of various estimators of inflation and expected/unexpected inflation has been tested. Where series were not stationary, suitable differencing has been applied to achieve stationarity. Henceforth the order of integration of each of the series are determined. In particular the rate of inflation which is the percentage change in the CPI is a stationary or an $I(0)$ series.

The autocorrelation function and the partial autocorrelation function have been detailed for each series. The acf's indicate that there is no evidence of seasonality in any of the series. The acf and pacf have been used to fit univariate Box-Jenkins models to each series. Selection criteria have been considered with the SBC the chosen criterion. Various diagnostic criteria have been used to assess the validity of the resulting models. An AR(2) model has been fitted to inflation.

Normality and heteroskedasticity of the residuals from modelling have been investigated. Some aspects of potential heteroskedasticity have been considered. The best model fit to the AR(2), equation (3.6) for inflation, using the SBC as a selection criterion, was a GARCH(1,3). Further investigation showed this model was not stable. The conclusion reached is that whilst models have residuals which are leptokurtokic simple ARCH/GARCH/EGARCH models have not provided a satisfactory solution.

The links between inflation and expected inflation have been investigated. Expected inflation is best modelled as an exponentially weighted moving average of observed inflation. The results are consistent with the view that observed inflation is the best predictor of expected inflation. Unexpected inflation does not appear as a random series as one would expect but as an AR(1). There is a predictability in expectations of inflation. Periods of consistent over or under-estimation of future inflation suggest a slow learning process. This results in expected inflation being found as a non-stationary series.

The resulting model of inflation (3.6) is of limited value. It cannot capture the volatility actually observed in prices. Shocks to the system will die out with this model. Large shocks such as occurred in the 1972 will not be adequately modelled. Further this shock persisted so as to raise the overall level of inflation above the general level in the 1960's. This can be dealt with in a system of real variables where levels of inflation can be considered exogenously via scenarios.

Chapter 4

Data Analysis and Univariate Models

4.1 Introduction

The seminal paper of Nelson and Plosser (1982) showed that many economic series show evidence of stochastic trends[1] (see Cochrane (1988), Stock and Watson (1991) for definitions and a further discussion of stochastic trends). Economic time series may tend to grow together and often show a high degree of correlation even when there is no actual economic relationship. Hence a linear regression between two such variables will show a high R^2 value. However as the series are only trending roughly in the same direction, there will be a tendency in such a case for errors to be positively serially correlated. Hence there is a low value for the DW[2] statistic. This feature of the nature of co-movements in time series has a long history (see Hendry (1991)). The paper by Slutzky (1927) is one of the earliest works on this topic of 'spurious regressions'. Slutzky considered correlation theory, which is related to the work of Yule. Yule showed that correlation theory is appropriate if the variables are $I(0)$, in which case the null distribution for the correlation coefficient is normal. If the variables are $I(1)$ the distribution is a semi-ellipse and if $I(2)$ then it is a U-shape. So the most likely correlations for unrelated $I(2)$ series are ± 1. A more recent revisitation of the topic is given by Granger and Newbold (1986). Since then much research has been dedicated to this issue as it is central to the validity of diagnostic tests in linear regression, such as the critical values for the distribution of the coefficients.

Therefore a straightforward approach to stochastic model building is to determine the order of integration of the series, difference appropriately to stationarity and then proceed with modelling. However this method loses information contained in the

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[1] Nelson and Plosser (1982) use the term difference stationarity (DS) for this class of non-stationary processes, for which the first or higher order differences is a stationary and invertible ARMA process. The trend process in this case is not a deterministic function of time. See the discussion in Nelson and Plosser (1982) p.141-143.

[2] See equation (5.1) and section 5.2 for a discussion of the Durbin-Watson statistic.

'levels' data. Further if there is a long run economic relationship between two or more variables then these results can be exploited. The modelling process can be simplified. If no such relationship exists then pretending there is will lead to poor models. This will result in erroneous forecasts.

The design of any stochastic investment model requires decisions as to the financial variables to be included. A set of variables based on economic considerations and the results of previous research are put forward. Thus the aims of this chapter are to introduce and investigate the order of integration of the variables. Only then can formal modelling proceed. This is a necessary step in developing the long run economic relationships in a stochastic investment system. Chapter 3 discussed inflation, which is a key driver of asset prices. Various methods are used in Chapter 3 to describe and understand the relevant DGP. The same approach is applied to the financial variables in the proposed model, which are adjusted for inflation to produce real variables.

The chapter is structured as follows. The first section 4.2, details the data sources and any limitations inherent in the data. The next section 4.3, considers the equity series. A brief introduction and discussion of the literature follows. Unit root testing with the ADF and PP tests are conducted to determine the order of integration of the financial variables. A series of univariate models are then developed to describe the DGPs. Then heteroskedasticity and the normality of the residuals are considered. The next section 4.4, performs the same tasks, this time using the fixed interest series. In so doing an understanding is gained of the common features in the series and how they relate to each other.

4.2 Sources of Information

The Australian Stock Exchange (ASX) provides monthly data on the All Ordinaries Index (AOI) as far back as January 1875. From 1875 the index was known as the Commercial and Industrial Index and it continued until June 1936. From July 1936 until December 1979 the index was known as the Sydney All Ordinaries Index; then when the ASX became fully national, the current AOI was created. These earlier indices are comparable to the current All Ordinaries Index. SBC Warburg[3] provided an AOI series back to 1900. The SBC Warburg series being re-based was validated against the Commercial and Industrial Index.

A dividend yield series was available from the ASX for the period 31 October 1882 to 30 September 1983 which was, however, an *unweighted* quarterly series. Weighted dividend yield series for the All Ordinaries index and the All Industrials index were made available from the J.B.Were Research Department, covering the period from January 1973 to June 1997 and March 1961 to June 1997 respectively. The J.B.Were series are regarded as being the appropriate ones. Thus the task is to reconcile the unweighted and weighted All Ords dividend yields. The ratio of the two series was calculated for the overlapping period, figure 4.1 detailing this ratio.

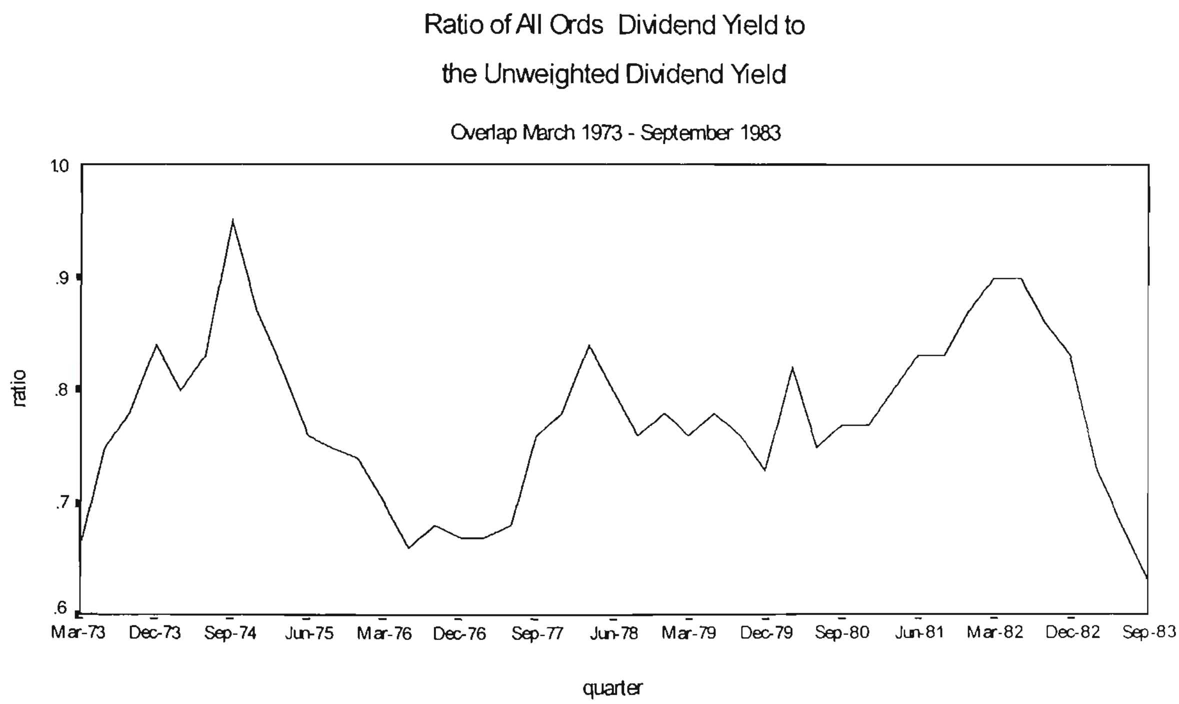


Figure 4.1 Ratio of All Ords Dividend Yield to the Unweighted Dividend Yield.

As can be seen from figure 4.1 there is no trend in this ratio. It reached a peak in September 1974 and a trough in September 1983. Given that some of the larger companies on the Australian stock market are mining companies with low dividend yields it explains why the unweighted yield is consistently higher than that applied to

[3] Where available the indices that were used were validated against each other. There is available data

the All Ordinaries as a whole which is weighted by market capitalisation. This proposition is checked in figure 4.2 the chart of the ratio of the All Ordinaries dividend yield to that of the All Industrials. This shows that the dividend yield on the All Resources index is lower than that of the All Ords.

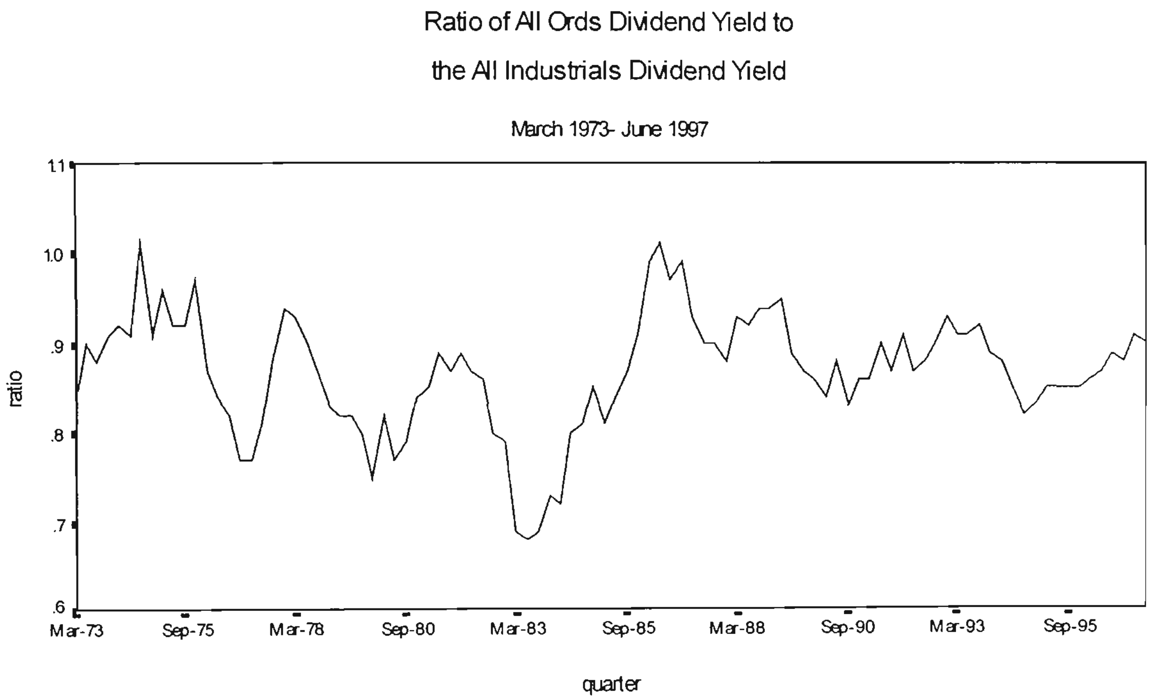


Figure 4.2 Ratio of All Ords Dividend Yield to the All Industrials Dividend Yield.

Finally the chart for the ratio of the All Industrials dividend yield to that of the unweighted All Ords index is given in figure 4.3. The advantage here is that the data can be extended back to March 1961.

from Warburg Dillon Reed, J.B.Were and ANZ Funds Management.

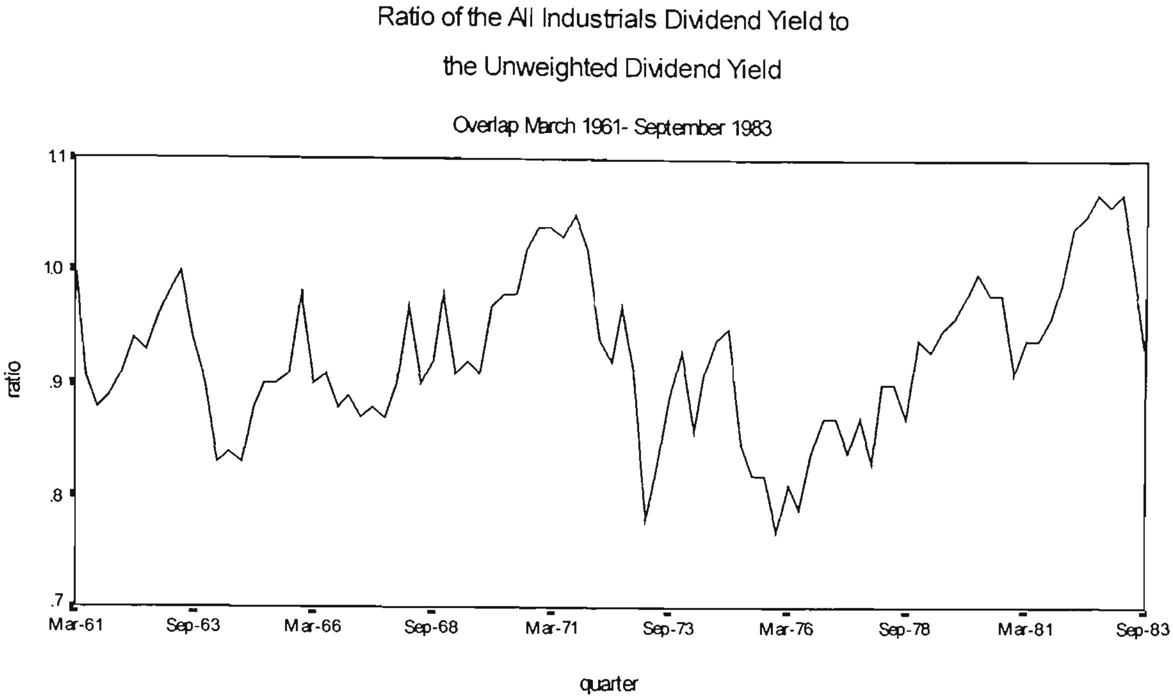


Figure 4.3 Ratio of All Industrials Dividend Yield to the Unweighted Dividend Yield.

Going back to 1948 is more difficult. Data for this earlier period is not consistent due to the changes that have taken place. Table 4.1 gives an indication of trends drawn from official records of securities listed on the Melbourne Stock Exchange. The percentage of mining stocks is given in brackets. The number of mining stocks declined over the period 1948-1965 but then rebounded in the boom year of 1969. This gives no indication of the value of the relative sectors nor the dividend yield.

Table 4.1 Relative Importance of Industrial and Mining Shares 1948-1969

Number of Shares Listed on the Melbourne Stock Exchange[4]

Year	1948	1950	1955	1960	1965	1969
Industrial	657	788	1086	1096	1112	1025
Mining	257(28.1)	248(23.9)	181(14.3)	131(10.7)	130(10.5)	261(20.3)
Total	914	1036	1267	1227	1242	1286

[4] Sourced from Bruns (1970), Table I, p.47; the original data is obtained from the Stock Exchange of Melbourne Committee Reports.

Nevertheless for the period from March 1973 to September 1983 figure 4.1 shows that the ratio of the weighted and unweighted All Ords dividend yields has remained stable and with no discernible trend. The two other charts in figures 4.2 and 4.3 which extend the time period also exhibit consistency with no observable trend. The number of listed mining stocks has varied but would not appear to have affected the ratio of the All Industrials dividend yield to the unweighted dividend yield in any significant way. From the ratio of the All Ords dividend yield to the unweighted dividend yield an average ratio was determined. Then that average was applied to the unweighted series to extend the dividend yield series back to September 1948. Having both an All Ordinaries index series and the corresponding dividend yield means that the nominal dividend index may be calculated by multiplying the two together. There are therefore three series starting from September 1948; the same period as for the CPI. In the analysis of the equity series the difficulties with the early data will be taken into account. Therefore more emphasis will be placed on the period for which there is a consistent series, that is from March 1973 onwards.

In the case of Treasury notes (abbreviated to T-notes) the RBA has available monthly T-note prices as far back as December 1959. Because there is no really suitable proxy prior to 1959 it was considered better to start the series at that point rather than try to take it back to September 1948, the start of the inflation and equity price series. This was also the case with the bond yield series. Here the data collected by the RBA has changed over time. There have been two substantive changes. From March 1960 to September 1964 the series was for a 15-year bond[5], the 10-year series starting in June 1963. Prior to December 1968 the 10-year series was for rebateable bonds[6], where there was a tax concession. In both cases there was a series overlap. There was an average premium of 29 basis points[7] (b.p.) for non-rebateable 10-year stock over the rebateable 10-year stock for the period December 1968 to June 1970. Applying this adjustment took the time series back to December 1964.

[5] Defined as '15-year theoretical average for the week centred on the last Wednesday of the month'.
[6] For rebateable bonds 'income is subject to income tax at current rates less a rebate of 10c in the $1 on loan interest included in taxable income'.
[7] A term used in the bond market, where 1 basis point (b.p.) = 0.01%.

The adjustment for the series to March 1960 is more difficult. There is available a series for 2-year bonds. Given a potentially changing yield curve, the yield on the 10-year bond was obtained by a linear interpolation between the 15-year and 2-year series. Then the premium of 29 b.p. was added to the portion of the series generated by the interpolation. For example, in March 1960 the yield on the 15-year bond was 4.93% and that of the 2-year bond 4.14%, then the yield on the 10-year bond is given by:

$$4.14 + (4.93-4.14)*(8/13) + 0.29 = 4.92\%$$

Table 4.2 lays out the original data and adjustments made. As is the case with the equity series these approximations will be taken into account[8]. More importance will be placed on the more recent and consistent data. Structural breaks and the stability of model coefficients are also relevant considerations.

[8] In fact the final model ignores this data altogether. See Chapter 7.

Table 4.2 Adjustments to Extend the 10-year Treasury Bond Series to March 1960.

quarter	15-year T-bond	2-year T-bond	interpolation	10-year T-bond
Mar 1960	4.93	4.14	4.63	4.92
June 1960	4.94	4.19	4.65	4.94
Sept 1960	4.97	4.30	4.71	5.00
Dec 1960	5.30	5.60	5.42	5.71
Mar 1961	5.35	5.40	5.37	5.66
June 1961	5.36	5.23	5.31	5.60
Sept 1961	5.33	4.59	5.05	5.34
Dec 1961	4.98	4.36	4.74	5.03
Mar 1962	4.95	4.39	4.73	5.02
June 1962	4.93	4.30	4.69	4.98
Sept 1962	4.94	4.25	4.67	4.96
Dec 1962	4.88	4.13	4.59	4.88
Mar 1963	4.84	4.04	4.53	4.82
June 1963	4.50	3.75	4.21	4.50
Sept 1963	4.46	3.68	4.16	4.45
Dec 1963	4.44	3.64	4.13	4.42
Mar 1964	4.46	3.63	4.14	4.43
June 1964	4.70	4.36	4.57	4.86
Sept 1964	4.89	4.48	4.73	5.02

4.3 Equity Models

To establish a long term stochastic investment model appropriate sub-models which form the various elements in the asset picture are needed. The theoretical basis for this rests upon the predictability of stock and bond returns (see Keim and Stambaugh (1986), Fama and French (1988a, 1988b, 1989, 1992), Campbell and Shiller (1988) and Summers (1989)). This is closely related to the issue of 'excess volatility' and 'bubbles' (see West (1988) or Shiller (1989)). Campbell and Shiller (1988, p.663) argue that "... excess volatility and predictability of multi-period returns are not two phenomena but one.".

The methodology applied to inflation in section 3.4, is applied to the key variables in the equity area; real stock prices, real dividends and dividend yields. The real dividend index, that is the nominal dividend index discounted by inflation and real stock prices as represented by the All Ordinaries index discounted by inflation are used. The earnings yield will not be used. The data is not available, even though there is a good case for the theory that stock prices follow earnings[9]. Campbell and Shiller found that the best predictor of share prices was a long moving average of earnings.

Then the identity $y_t = \frac{D_t}{P_t}$ is applied, where y_t is the dividend yield, D_t[10] is the nominal dividend index and P_t is the nominal stock price index. Real dividends and the dividend yield are then modelled. Nominal dividends can be determined by applying an inflation projection to real dividends. Projections of the All Ordinaries index are then obtained by applying the dividend yield to the nominal dividend index.

One point to note is that the monthly All Ordinaries index is a weighted average of prices over the month not the month end closing value. The daily All Ords index is composed of individual shares whose price movement in the index is weighted by market capitalisation. Now a result due to Working (1960) showed that the expected first-order serial correlation of first differences between averages of terms in a random chain with n items in each successive segment is given by:

$$corr\left(\bar{x}_i - \bar{x}_{i-n}, \bar{x}_{i-n-1} - \bar{x}_{i-2n}\right) = \frac{n^2 - 1}{2\left(2n^2 - 1\right)}$$

[9] Campbell and Shiller (1988, p.664) refer to the view long held by Benjamin Graham of using an average of earnings in computing the earnings price ratio.

[10] The convention is that upper case symbols will be used to indicate nominal variables and lower case symbols real variables.

where each average is of length n, and $\bar{x}_i = \frac{1}{n}\sum_{n=0}^{n-1} x_{i+n}$. Hence there is a first order serial correlation converging on 0.25. By using quarters with data points spaced three months apart there is now no induced autocorrelation between the elements, as serial correlation coefficients of higher order than the first remain zero.

A final comment is on structural breaks. Section 3.3 discussed the major structural break at the time of the Bretton Woods Conference in 1947. Logically a second similar break may be expected on the breakdown of the system when the US$ was no longer convertible into gold at the fixed rate of 32US$ per ounce. A result of this change was the rapid and unexpected increase in the rate of inflation, which caused the economic shock in 1973. This would be convenient since the data is consistent only from 1973 onwards, as outlined in section 4.2 above. Other breaks such as floating the A$ and the introduction of dividend imputation may be postulated. These are important 'local' changes but the two breakpoints 1947 and 1972/3 are global in context. Therefore rather than assert *a priori* the existence of such local breaks, data and econometric analysis can be used to investigate whether or not these time points are significant. Looking ahead, based on the analysis conducted herein, there is no strong evidence to suggest that these local factors do provide structural breaks.

4.3.1 A Quarterly Dividend Yield Model

The chart in figure 4.4 plots the dividend yield.

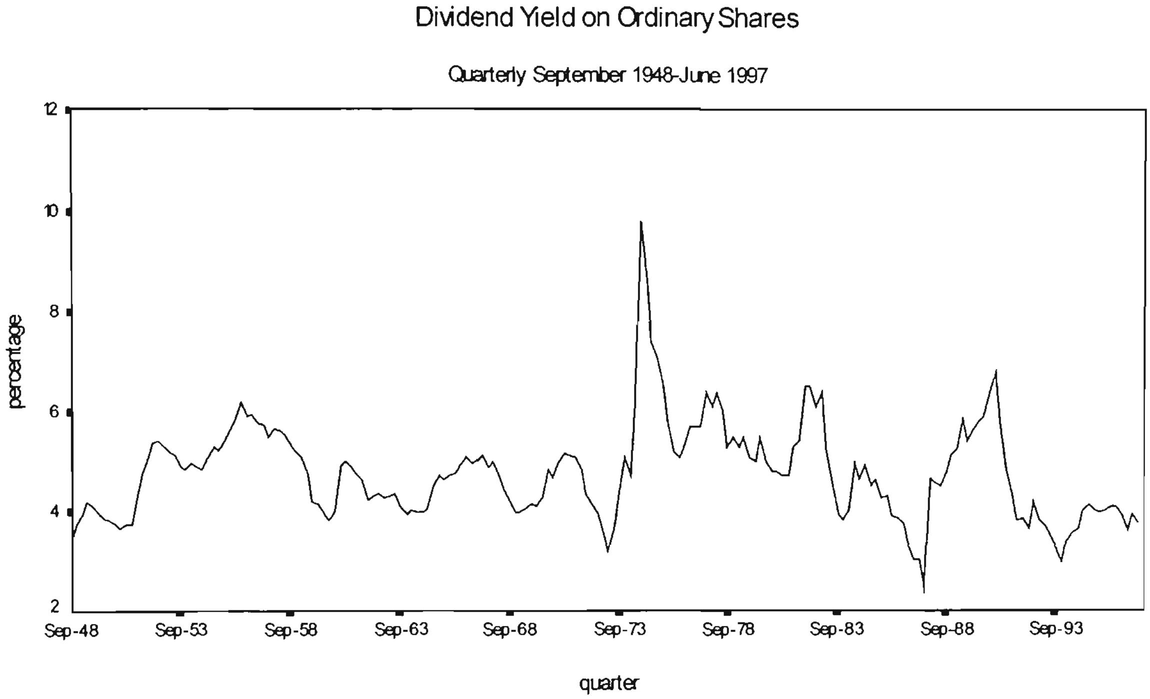


Figure 4.4 The Dividend Yield on Ordinary Shares.

The dividend yield series in figure 4.4 appears to have a constant mean level. The spike in September 1974 is consistent with a structural break at that time. In more recent times it does disguise a marked increase in the payout ratio or the ratio of dividends to earnings. Nevertheless dividends are much more stable than earnings. In practice a period of reduced earnings often implies a rise in the payout ratio. Company boards then let the ratio remain high until improved earnings restore the ratio to acceptable levels, before again increasing dividends.

ADF and PP tests were carried out and the results are given in the tables 4.3 and 4.4. The procedures for unit root testing throughout this chapter and subsequent ones are as per section 3.4.1 with the lag length chosen by the reduction method of Mayadunne *et al.* (1995). Note that the discussion of the results for d_t and p_t follow in sections 4.3.2 and 4.3.3 respectively.

Table 4.3 ADF Regression Equity series September 1948 to September 1997

ADF 5% level critical value below		trend and intercept (-3.434)			intercept (-2.878)		none (-1.942)
variable	lag length	α	$(\beta-1)$	γ	α	$(\beta-1)$	$(\beta-1)$
y_t	3	0.921 (4.508)	-0.182* (-4.672)	-0.001 (-0.858)	0.853 (4.531)	-0.179* (-4.613)	-0.006 (-0.822)
$\ln y_t$	7	0.247 (3.301)	-0.148 (-3.215)	-0.000 (-1.404)	0.211 (2.994)	-0.136* (-3.005)	-0.001 (-0.251)
d_t	2	0.032 (2.332)	-0.051 (-2.487)	0.000 (0.460)	0.033 (2.483)	-0.049 (-2.449)	-0.000 (-0.114)
Δd_t	10	0.001 (0.185)	-0.938* (-4.654)	-0.000 (-0.011)	0.001 (0.389)	-0.938* (-4.672)	-0.933* (-4.669)
p_t	3	0.909 (2.186)	-0.076 (-2.706)	0.002 (1.111)	1.025 (2.545)	-0.068 (-2.510)	-0.002 (-0.217)
Δp_t	0	-0.082 (-0.368)	-1.031* (-14.30)	0.001 (0.653)	0.044 (0.403)	-1.029* (-14.31)	-1.029* (-14.33)
y_t**	3	1.179 (3.616)	-0.223* (-3.806)	-0.002 (-0.967)	1.032 (3.581)	-0.212* (-3.691)	-0.010 (-0.870)

* Significant at the 5% level.

** Using the shorter time period Sep 1967 to Sep 1994.

Table 4.4 PP Test Equity series September 1948 - September 1997

crit.val. below	trend and intercept (-3.434)			intercept (-2.878)			none (-1.941)		
variable	1	5	10	1	5	10	1	5	10
y_t	-3.869	-4.083	-3.561	-3.817	-4.041	-3.359	-0.721	-0.722	-0.559
$\ln y_t$	-3.721	-4.077	-3.538	-3.626	-3.927	-3.481	-0.426	-0.439	-0.346
d_t	-1.978	-2.359	-2.503	-1.941	-2.312	-2.450	0.027	-0.114	-0.152
Δd_t	-13.49	-13.77	-13.90	-13.53	-13.98	-13.93	-13.55	-13.82	-13.95
p_t	-2.413	-2.608	-2.569	-2.174	-2.370	-2.330	-0.222	-0.253	-0.176
Δp_t	-14.30	-14.30	-14.33	-14.31	-14.30	-14.33	-14.33	-14.33	-14.35
y_t**	-3.091	-3.267	-2.822	-2.982	-3.168	-2.758	-0.853	-0.848	-0.755

** Using the shorter time period Sep 1967 to Sep 1994.

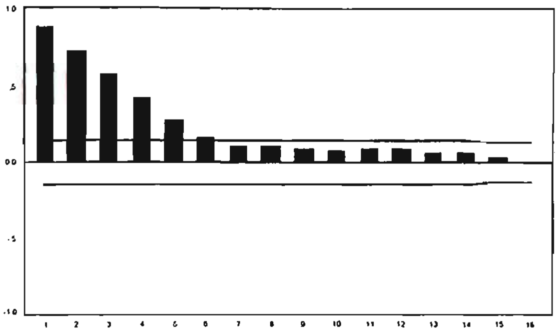
The ADF test value for the intercept only model for y_t was -4.613 well below the MacKinnon critical value at the 5% level of significance of -2.878. Therefore y_t and also $\ln y_t$ from table 4.3 are $I(0)$. This is consistent with Wilkie (1995, p.291) who takes the dividend yield series to be stationary commenting that "It is clear that share prices and share dividends are what statisticians would call cointegrated series....This connection is represented by modelling the dividend yield as a stationary series.". Mills (1993, p.56) finds the log of the dividend yield to be stationary which agrees with tables 4.3 and 4.4 and is consistent with a stationary dividend yield. By contrast Sherris, Tedesco and Zehnworth (1996) find the dividend yield series to be an $I(1)$. To check this the unit root tests were performed with the time period shortened to that in Sherris *et al.*, September 1967 to December 1994. The ADF test statistic (table 4.3, row 7) from the regression with both trend and intercept terms dropped to -3.806. The 5% level critical value increased marginally to -3.451 due to the smaller sample size. But still the unit root null is rejected. Even when the trend term is dropped as the coefficient is not significant and employing an intercept only the unit root null is still rejected. However the PP test (table 4.4, row 7) is not fully supportive of this result.

Nevertheless employing the intercept only model leads to a rejection of the null at the 5% level for truncation lags 1 and 5 but not 10 given a critical value of -2.888. The recursive *t*-statistic procedure for lag length suggests 3 lags. The test would then reject the unit root null. Employing the Sherris *et al.* 10% level of significance for the one-sided test would lead to a stronger rejection of the unit root null. The overall conclusion is that for this shorter time period there is a near unit root in the series. This is supported by the first order serial autocorrelation, $\rho_1 = 0.851$. The net result is that the evidence in tables 4.3 and 4.4 is supportive of the conclusion that the full series y_t is an $I(0)$.

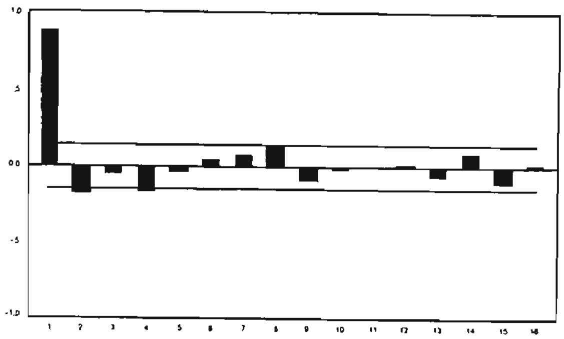
The data for the period September 1967 to September 1997 was mostly covered by the J.B.Were data, which is for the period January 1973 to September 1997. It therefore includes only a small portion of the adjusted data; section 4.2 gives a description of the information. By an inspection of the plots for the two data sets it is apparent that the data is different from that of Sherris *et al.* Analysts at J.B.Were may adjust the data, particularly since the RBA data is based upon averages. The sources of disagreement amongst other authors are varied. Some use log transformations, different time periods or adjust the data in some way thereby generating results which are not comparable. The stationarity or otherwise of this key series is important. The results are on balance in favour of stationarity. If the dividend yield series is a random walk then shocks will be permanent and the long horizon predictability of Fama and French and others will be suspect. This aspect will be further reviewed in the discussion on cointegration (see chapter 5). Indeed Holden and Perman (1994, p.71) comment that "In fact it has become increasingly common not to rely too heavily on pre-testing variables for their orders of integration before considering the relationship between them in a cointegration framework.".

The effect of a log transformation or its inverse the exponential function on the stationarity of a given series may also be considered. The definition of stationarity for a series involving a constant mean and finite variance, independent of time, implies that such a transformation should not alter the essential characteristics of the series. If x_t is stationary then $\ln x_t$ should be stationary and vice versa. Therefore in order to cover the two alternatives, modelling of both y_t, the dividend yield and $\ln y_t$ will be reviewed and compared for any differences.

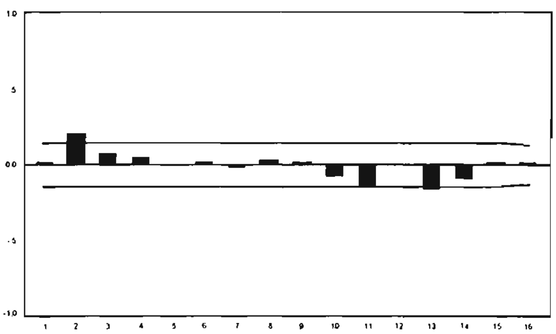
Univariate modelling will now be applied to the dividend yield y_t following the procedures outlined in detail in section 3.4.3. Results in chapter 4 are given only in summary form. Now $\bar{y} = 4.763$ and $S.E.(\bar{y}) = 0.069$. The mean is therefore significantly different from zero. So the mean adjusting transformation $z_t = y_t - \bar{y}$ is made. The acf and pacf were then viewed, again with no evidence of seasonality (see figure 4.5 (a) and (b)).



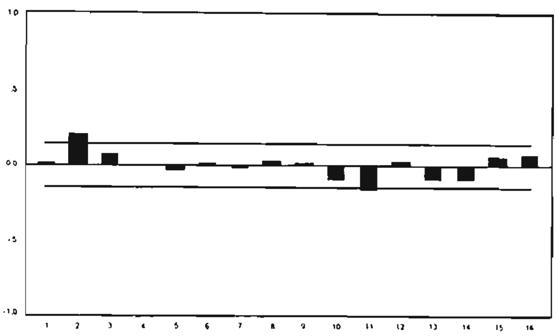
(a)



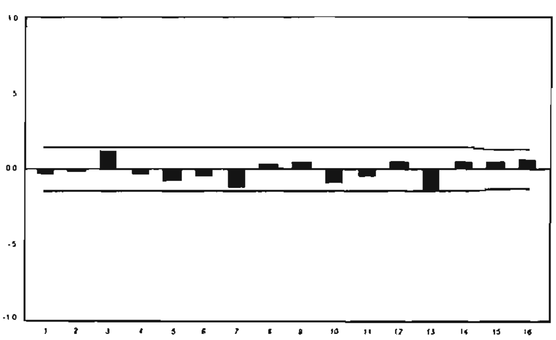
(b)



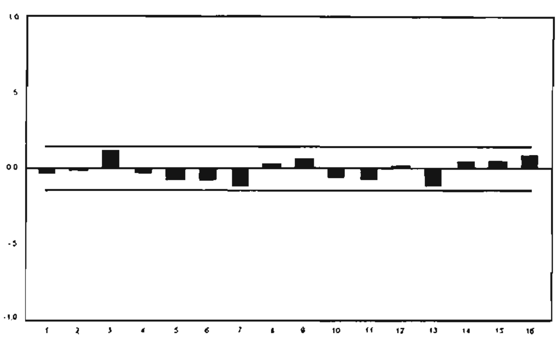
(c)



(d)



(e)



(f)

Figure 4.5 Acf and Pacf for equity series (a), (b) y_t ; (c), (d) Δd_t and (e), (f) Δp_t . Confidence limits given by the lines parallel to the x-axis.

The acf suggested an AR representation, with the significant value of ϕ_2 suggesting an AR(2), as in figure 4.5 (a) and (b). The table 4.5 details, with the various model fits.

Table 4.5 ARMA Models, Fitting and Diagnostic Checking: Series y_t

Model	Parameter Estimates	t-values	$\hat{\sigma}_{\varepsilon}$	SBC	$Q(12)$
AR(1)	ϕ_1 0.879	25.98	0.466	259.0	26.36(0.006)
AR(2)	ϕ_1 1.045	14.87	0.459	256.9	14.93(0.135)
	ϕ_2 -0.190	-2.70			
ARMA(1,1)	ϕ_1 0.832	18.64	0.459	257.4	15.68(0.109)
	θ_1 0.212	-2.69			

The AR(1) model is inadequate due to the significant $Q(12)$ value. The critical value is 19.675 and hence there is a reported p-value of 0.006. Both two-parameter models have satisfactory diagnostics, but the AR(2) is superior, and is the chosen model. Re-transforming yields the model:

$$y_t = 0.691 + 1.045y_{t-1} - 0.19y_{t-2} + \varepsilon_t \tag{4.1}$$

where $\varepsilon_t \sim N(0,0.217)$. Note that this satisfies the AR(2) invertibility condition $r_1^{\,2} < \frac{1}{2}(r_2 + 1)$ (see Box and Jenkins (1976, p.61)), here $0.871^2 = 0.759 < \frac{1}{2}(1.716) = 0.858$. For this model $R^2 = 0.766$, using Nelson's formula (3.5). Now $\phi_1 + \phi_2 = 0.855$ for this model supporting the unit root test results for this variable as an $I(0)$.

The model (4.1) is close to an AR(1) which would perhaps be considered more 'natural'; indeed any realisations of the either AR(1) or AR(2) would be difficult to distinguish. In practice the only time there would be a major difference in the DGP would be after a large shock. The AR(2) would tend to cause greater persistence. A large negative shock causing a price fall would imply that the next value of the dividend yield would be higher than for the AR(1) and vice versa. The model (4.1) can be considered as an AR(1) with a parameter change on a large shock. A plausible explanation could be the fact that dividends are a trailing entity. Since dividends are paid semi-annually about half are paid each quarter[11]. It takes time for the dividend to catch up.

Another explanation is that it simply an artifice of the data. A different result could be obtained over a different time period. Anticipating somewhat this will be reviewed; the current results are preliminary in nature. However there is a case for taking the result as it is. Picking another period because it gives a more 'acceptable' conclusion is data mining. The AR(1) is statistically inadequate.

An identical process for $\ln y_t$ was undertaken. Now $\overline{\ln y} = 1.543$ and $S.E.\left(\overline{\ln y}\right) = 0.014$. The mean is therefore significantly different from zero and so the mean adjusting transformation $z_t = \ln y_t - \overline{\ln y}$ is made. The result, with p-values for $Q(12)$ in brackets, was as follows:

Table 4.6 ARMA Models, Fitting and Diagnostic Checking: Series $\ln y_t$

Model	Parameter Estimates		t-values	$\hat{\sigma}_\varepsilon$	SBC	$Q(12)$
AR(1)	ϕ_1	0.885	26.45	0.090	-377.5	24.76(0.010)
AR(2)	ϕ_1	1.019	14.23	0.090	-376.7	16.99(0.075)
	ϕ_2	-0.151	-2.11			
ARMA(1,1)	ϕ_1	0.856	20.51	0.090	-375.8	18.65(0.045)
	θ_1	0.138	1.72			

When re-transformed this lead to the equation:

$$\ln y_t = 0.132 + 1.019 \ln y_{t-1} - 0.151 \ln y_{t-2} + \varepsilon_t \qquad (4.2)$$

[11] Dividends do not have a quarterly seasonal pattern, see the discussion in section 4.3.2.

where $\varepsilon_t \sim N(0,0.0081)$. The parameter coefficients and the various diagnostics for $\ln y_t$ are virtually identical to that applying to y_t. In this case the ARMA(1,1) shows evidence of serial correlation in the residuals with a p-value of 0.045 for $Q(12)$.

4.3.2 A Quarterly Real Dividend Model

The stochastic investment model is a system of real variables. Hence the next step is to generate a real dividend index $d_t = \frac{D_t}{I_t}$. This was done and the following chart figure 4.6 was plotted.

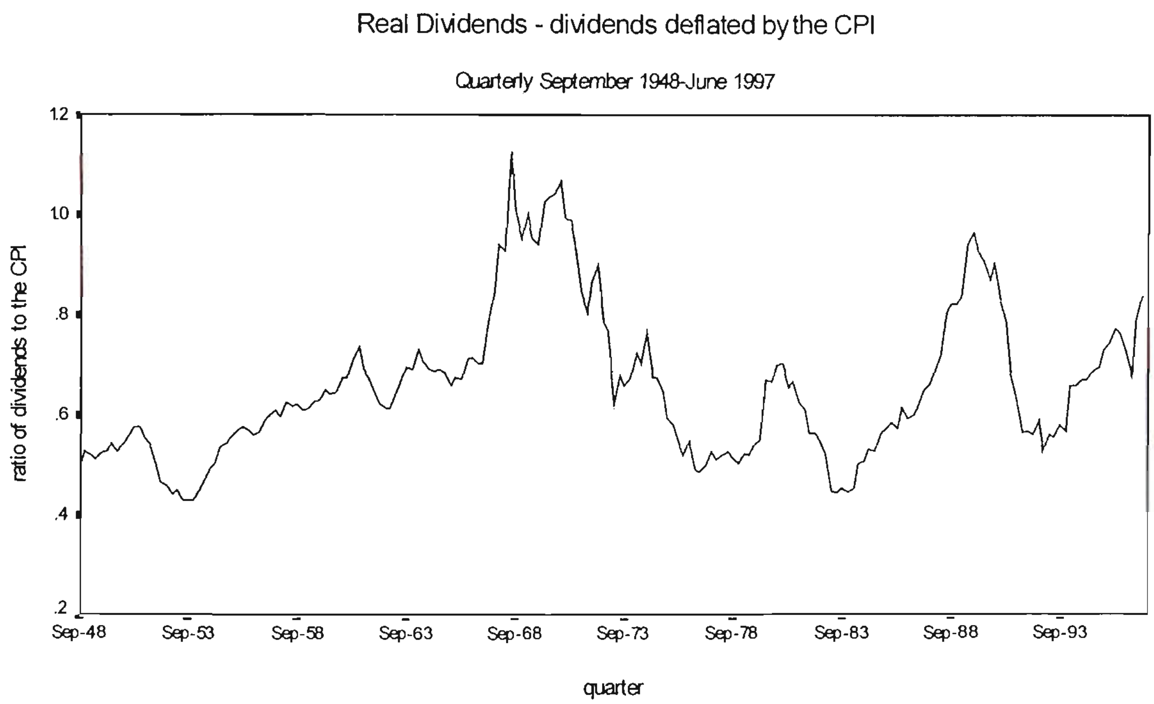


Figure 4.6 Real Dividends on Ordinary Shares, the Nominal Dividend Index Deflated by the CPI.

This has a wandering characteristic more like a random walk. The ADF and PP tests were again applied with the results shown in tables 4.1 and 4.2 above. Both tests cannot reject a unit root null for d_t. This is not surprising given a high first order autocorrelation for the series of 0.955 and a slowly declining acf. First differencing led to a stationary series Δd_t with the null of a unit root being easily rejected under various alternatives. Hence d_t is an $I(1)$.

The series d_t is a differenced stationary series with $d_t = d_{t-1} + \Delta d_t$, where d_t is non-stationary and Δd_t is stationary. The two variables form components of the series; the non-stationary or 'levels' component describes the long run dynamics and the stationary component the short run dynamics. The dividend yield y_t is a stationary variable, requires no differencing and so has no such components[12].

Real dividends have shown little change. The mean value of the percentage change in real dividends was 0.429. The standard error was 0.407 and therefore not statistically different from zero. This agrees with Wilkie's model for the UK where he finds no increase in real dividends, though this may well be affected by the pre-Depression starting date of 1919. Wilkie (1995, p.292) comments "This is the sort of area where judgement is needed ...and assume zero real dividend increase for the future.". It is also consistent with Mulvey (1996, p.13) which shows a graph of the real dividend on the S&P 500. As Mulvey comments "It has been relatively stable over the last several decades. This makes forecasting this component of stock returns fairly straightforward...". From an economic perspective it might be anticipated that earnings and therefore dividends would rise at nominal GDP and so real dividends would rise at real GDP. One may speculate as to the causes of the above result. The stockmarket may be over represented in mature industries. For example, currently the banking sector forms a much larger part of the AOI than it does for the economy as a whole, as all major banks and regional ones are listed. Whilst this is undoubtedly an important subject, the result stands. Growth in real dividends is not statistically different from zero.

Returning to the real dividend model it can now be taken a step further by taking first differences and modelling the growth rate in dividends[13] Δd_t. The series is stationary with a mean of $\overline{\Delta d} = 0.002$ and $S.E.\left(\overline{\Delta d}\right) = 0.003$. The mean value is not significantly different from zero, hence a mean adjusting transformation is not

[12] The stochastic investment model introduced in this chapter is then seen to contain two stationary and four differenced stationary variables. Only the latter have non-stationary components to be investigated in chapter 5. These non-stationary components are stochastic trends.

necessary. Once again there was no evidence of seasonality from a consideration of the acf and pacf shown in figure 4.5 (c) and (d). A random walk model was initially discounted because of the significant value of r_2, from the acf at lag 2, though the value of $Q(12)$ was only 14.92. Ignoring r_2 would then imply a random walk model for real dividends:

$$d_t = d_{t-1} + \varepsilon_t \quad \text{or } \Delta d_t = \varepsilon_t \tag{4.3}$$

where $\varepsilon_t \sim N(0,0.0017)$.

Alternatively, proceeding as before table 4.7 results:

Table 4.7 ARMA Models, Fitting and Diagnostic Checking: Series Δd_t

Model	Parameter Estimates	t-values	$\hat{\sigma}_\varepsilon$	SBC	$Q(12)$
AR(1)	ϕ_1 0.001	0.01	0.041	-679.3	15.82(0.148)
AR(2)	ϕ_1 -0.003	-0.52	0.040	-669.6	7.60(0.668)
	ϕ_2 0.219	3.02			

Hence the difference in real dividends follows an AR(2) process. The model can be written as:

$$\Delta d_t = 0.219 \Delta d_{t-2} + \varepsilon_t \tag{4.4}$$

where $\varepsilon_t \sim N(0,0.0016)$. The value of $R^2 = 0.045$, using (3.5) which is a very low level of explanatory power, despite the significant t-statistic. This then implies that equation (4.3) remains a viable alternative.

[13] Percentage changes could have been used. However for consistency between equity and fixed interest differences are used. When bonds and notes are analysed small values or negative real rates lead to very erratic percentage changes.

4.3.3 The All Ordinaries Index

Even though the All Ords index is to be calculated by dividing the nominal dividend index by the dividend yield, it can be directly modelled. This enables both a degree of completeness and a comparison with other authors. Carter (1991), Fitzherbert (1992) and Harris (1995) all model the All Ords directly. The real All Ords is obtained by deflating the All Ords by the CPI, which is denoted $p_t = \frac{P_t}{I_t}$. The chart is given in figure 4.7.

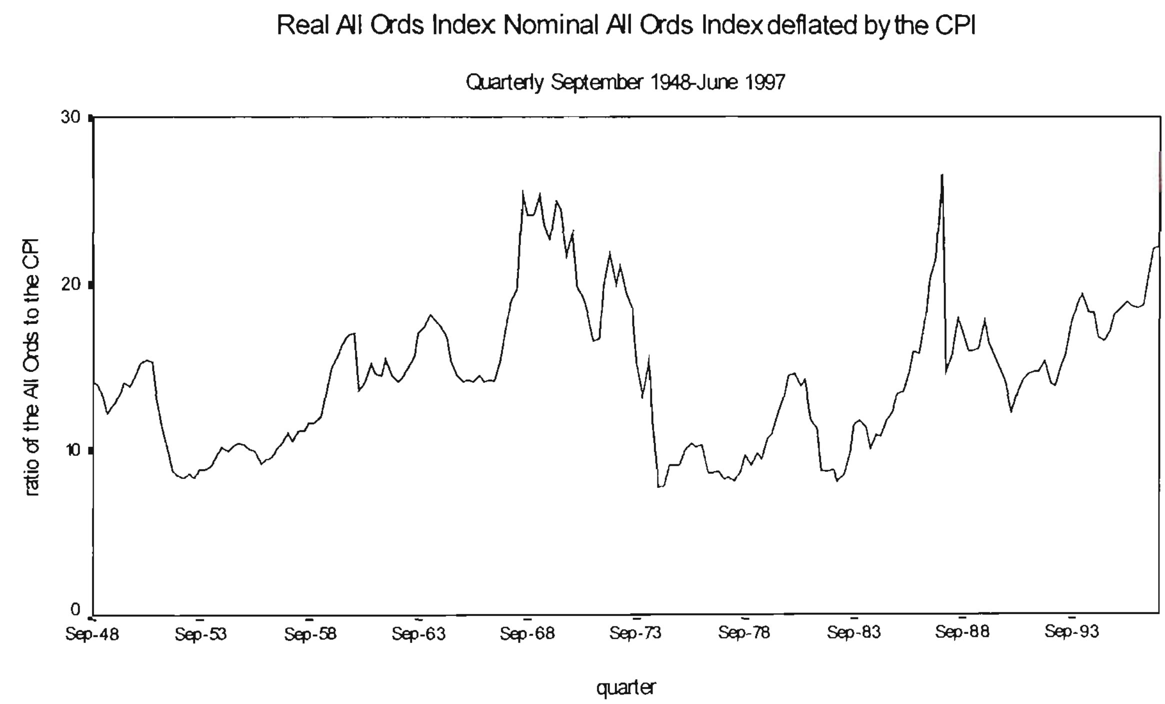


Figure 4.7 Real All Ordinaries Index: the Nominal All Ordinaries Index Deflated by the CPI.

The annualised return from the All Ords over this period is 7.1% p.a.; this compares with a rate of inflation of 6.1% p.a. This is dependent on the time period under consideration, the extra real growth is a result of the latest bull market. The nature of the series was tested for unit roots with the ADF and PP tests as shown in tables 4.3 and 4.4. The ADF and PP test statistics for p_t were not significant for any model formulation. So the null hypothesis of a unit root cannot be rejected. Differencing, yielding Δp_t led to a very strong rejection of the unit root null. Hence it may be concluded that $p_t \sim I(1)$. This is consistent with Crosby (1998, p.8) who finds the log of the nominal stock price index or $\ln P_t \sim I(1)$.

A review of the acf for Δp_t in figure 4.5 (e) showed no significant autocorrelation coefficients and a value for Box Ljung statistic $Q(12)$ of 10.114 (p = 0.606). The best model fit is then:

$$\Delta p_t = \varepsilon_t \tag{4.5}$$

where $\varepsilon_t \sim N(0,2.320)$, implying that the real All Ords is best modelled as a random walk. This is consistent with the results of Carter and Harris. Carter models $\Delta \ln P_t$ as a random walk[14]. Harris uses a continuously compounded share price index return series and finds a random walk model to outperform a range of alternatives (see Harris (1995, p.55)).

4.3.4 Heteroskedasticity and ARCH Effects

As in section 3.5 the univariate equity models need to be tested for normality and heteroskedasticity. Again the application of non-linear stochastic models is restricted to the ARCH/GARCH/EGARCH class. Other possible univariate models such as the bilinear process or asymmetric models are in the literature (see Mills (1993, p.113-126)).

The process outlined in section 3.5 was pursued. The residuals from the preferred equity models were tested for ARCH effects. Then ARCH/GARCH models were fitted using the SBC criterion for goodness of fit. Values of p from 0 to 3 and q from 1 to 3 for the GARCH(p,q), equation (3.7), were used; a total of 12 models. The log

[14] Brailsford and Faff (1993) use $\Delta \ln P_t$ and apply a MA(1) model to account for non-trading effects. With daily data this is approximately the continuously compounded return, Their data is daily, hence there are likely to be non-trading effects, as has been found by others. With monthly data this is unlikely to be the case, see the footnote to Kearns and Pagan (1993) in section 4.3.4. With quarterly data non-trading effects are virtually impossible. Any stock that traded that infrequently would be de-listed.

likelihood function **ln** L was also compared. The results with the best ARCH/GARCH model chosen as per the SBC criterion are in table 4.8. The diagnostics for the equation of the mean are for the residuals. Note that the diagnostics for the ARCH models are for the standardised residuals. P-values are given in brackets where appropriate.

Table 4.8 Preferred ARCH/GARCH Equity Models Diagnostic Testing

Model	ARCH-LM test	skewness	excess kurtosis	Jarque-Bera statistic	$Q(12)$
y_t: Eqn. (4.1)	10.00	3.097	20.629	3730.50	14.93
	(0.002)				(0.135)
GARCH(1,1)	0.801	1.015	1.759	58.00	18.89
	(0.371)				(0.042)
Δd_t: Eqn. (4.3)	8.919	0.249	3.049	75.95	7.59
	(0.003)				(0.749)
ARCH(1)	1.243	0.506	2.890	74.62	9.01
	(0.265)				(0.621)
Δp_t: Eqn. (4.5)	4.725	-2.287	18.901	3088.42	10.114
	(0.030)				(0.606)
ARCH(1)	0.040	0.171	3.385	95.548	19.844
	(0.842)				(0.070)

For Δd_t and Δp_t the minimum for **ln** L corresponded to the minimum on the SBC criterion. For y_t the minimum for **ln** L was a GARCH(2,3). The ARCH parameters were not significant for that model. The introduction of the non-linear process means that the models all pass the ARCH-LM test and the standardised residuals are more normal. The $Q(12)$ statistic is higher in all cases and just outside the 5% critical level for y_t. For each of the ARCH models the coefficients are significant. The following table 4.9 gives the AR coefficients ϕ_1 and ϕ_2 first and then the ARCH/GARCH coefficients next. T-values are given in brackets below.

Table 4.9 Preferred ARCH/GARCH Equity Models Parameter Values

model variable	ϕ_1	ϕ_2	α_0	α_1	β_1
y_t	1.160	-0.272	0.011	0.443	0.584
	(13.810)	(-2.870)	(2.217)	(6.126)	(10.233)
Δd_t		0.335	0.001	0.445	
		(2.654)	(5.494)	(2.687)	
Δp_t			1.085	0.566	
			(10.484)	(4.856)	

As in section 3.5 the EGARCH (p,q) model of Nelson was applied to the conditional variance for the range of values $p = 1$ to 2, $q = 1$ to 2. For y_t the $Q(12)$ statistic was in all cases outside the 1% critical value. For Δp_t this was more strongly so with p-values of $Q(12)$ at the 0.000 level. For Δd_t the best model on the SBC criterion was an EGARCH (2,2) with appropriate diagnostics. However when the model was applied to the period December 1973 to September 1997 to test for stability, the coefficients were completely different. The EGARCH coefficients for example changing from -1.17 and -0.75 for the period September 1948 to September 1997 to 0.08 and 0.93 for December 1973 to September 1997.

The ARCH models whilst representing an improvement, still have non-normal residuals. Extra parameters are also introduced. This adds significantly to the computational burden imposed in any stochastic programme. These results are consistent with Pagan and Schwert (1990). Pagan and Schwert analysed monthly US stock returns from 1834-1925. The authors applied a GARCH(1,2), an EGARCH(1,2), a two-state switching regime model, and three non-parametric estimators. They then compared the predictive ability of both in and out-of-sample forecasts as well as predictions for highly volatile episodes. They found that the parametric models do not capture the squared returns as well as the non-parametric models but do better in out-of-sample predictions. The authors suggest merging the two approaches. Pagan and Schwert (1990, p.289) concluded "Our results imply that standard parametric models are not sufficiently extensive.".

A similar study to that of Pagan and Schwert was conducted by Kearns and Pagan[15] (1993) to compare US and Australian experience. The authors performed a detailed study with monthly data into the volatility of the Australian stock market over the period 1875-1987. They extended their considerations to model building with a GARCH(1,2), EGARCH (1,2) and an autoregressive iterative two-step procedure (see Kearns and Pagan (1993, p.170-4) for further details). They found the EGARCH model to be superior with an R^2 of 0.196. They found a satisfactory value for $Q(12)$ for the EGARCH(1,2) of 8.5, in contrast to the results found here. Kearns and Pagan used monthly data over 1875-1987, a quite different data set and frequency. They also appear to have used average prices (see footnote). Parameter stability and additional residual diagnostics were not given. The evidence does not appear to suggest that the models are an improvement on the results in table 4.9 and associated discussion. Indeed Kearns and Pagan (1993, p.172) found that whilst the degree of predictability is higher than the US experience "...there is no doubt still room for improvement.". Applying the models they found "...that there is persistence of shocks in volatility and that this persistence is as true of small shocks as it is of large ones. Moreover, there is no evidence that the persistence is due to structural change; over long periods it has remained remarkably constant." (Kearns and Pagan (1993, p.177)).

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[15] There are some points arising that need to be considered. Firstly they appear to have used average prices for the month, that is the average of the daily closing prices. For the All Ords for the period Sept-Oct-Nov 1987, the *average* prices went from 2238.7 to 1885.1 to 1280.0, whereas the *month end* closing prices went from 2249.2 to 1294.5 to 1329.5. Hence they attribute the large changes which occur in successive months in the average series to non-trading effects. Given that the 50 leaders represents 75% of the index and these trade in large volume all the time, one would expect the index to react very rapidly indeed to any price changes, as indeed was the case. Further, using averages means that the variance is approximately 2/3 of the month end series, using a result due to Working (1960), see section 4.3. Secondly, the AOI does not contain all stocks. Hence those that do not trade generally are not in the index and the weighting of certain stocks with overseas parentage are altered to reflect the fact that the stock is not available to trade plus the lack of control premium and so on, which may affect fundamental value. Thus some care is necessary in assuming any non-trading effects or indeed calendar ones. As Kearns and Pagan point out, there are substantial differences between the US and Australian experiences.

4.4 Fixed Interest Models

So far models have been developed for inflation and equities. Next the fixed interest asset class and the risk free asset as represented by the 90-day T-note will be reviewed. The structure of the yield curve and the relationship between long term (longs) and short term (shorts) interest rates will also be investigated. Previous models such as that of Brennan and Schwartz (1982) were based upon a two factor model. That is two financial variables are sufficient to describe the fixed interest asset class. Brennan and Schwartz used the spread between longs and shorts and a long term bond series as the two factors. They, as does Wilkie, use consols or perpetual bonds to represent the long term bond series. In Australia the Reserve Bank 10-year Treasury bond series, as given in the RBA Bulletin, represents the appropriate consol proxy. The reasons for this are:

1. The Commonwealth of Australia does not issue consols. The normal long term instrument is of a term to maturity of 10 years.
2. The 10-year bond is a standard. In the derivatives market futures and options exist for the 10 year Treasury bond. The security is therefore highly liquid and there are reliable secondary market prices.
3. Because it is a standard there is a good data series available.

The data is as described in the section on sources of information, section 4.2. The definitional limitations are as outlined there. An issue is the different periods covered by the respective data sets. To the extent that different financial eras show different behaviour the fact that the bond and bill series are of shorter period (1960-97) than the equity series (1948-97) is important. This can be evaluated by comparing the results of various periods for features such as stability in coefficients.

The ADF and PP tests are used to determine the order of integration of the series as before. The results are tabulated in tables 4.10 and 4.11. There is agreement in Lahiri and Dasgupta (1991), Inder and Silvapulle (1993), Sherris *et al.* (1996) or Mills (1993), that the nominal and real interest rate series are $I(1)$. Inder and Silvapulle

(1993, p.842) apply the ADF and the KPSS tests to Australian inflation, nominal and real interest rates and US real interest rates with ambiguous results finding "...there may well be a unit root in all the series; we are quite confident of this for the nominal interest rate, and less so for the other series.". Mishkin and Simon (1995) determine the critical values for the unit root tests (PP and DF) using simulations based upon a model structure assumed ARMA in differenced form and with ARCH residuals. They report that in no case do they find a rejection of the unit root null for the nominal note rate. However they comment "... any reasonable model of the macro economy would surely suggest that real interest rates have mean reverting tendencies which make them stationary..." (Mishkin and Simon (1995 p.23)). This implies that they expect real T-note rates to be $I(0)$. Mayadunne, Evans and Inder (1995, Table 1b, p.149), apply three alternative unit root tests to a range of Australian economic time series. The results are consistent with the Australian bond yield being an $I(1)$. Olekalns (1996) comments on some of the conflicting results by noting the low power of the unit root tests. He comments wryly[16] that the interest rate should be stationary in levels, thus making it an $I(0)$.

Keim and Stambough (1986) apply a von Neumann ratio test to the difference between long term corporate bonds and T-bills as a measure of the spread. They find this measure to reject the unit root null but conclude that they cannot make reliable inferences about stationarity. Lahiri and Wang (1996) define three different measures of the spread. They use the spread between 10-year bonds and the bill rate, 10-year bonds and the 1-year rate and the spread between the bill rate and the commercial paper rate at six months maturity. They find the ADF test strongly rejects the unit root null at the 1% level for all three measures. Pagan, Hall and Martin (1996, table 2, p.93) come to similar conclusions with their definitions of spread and applying the DF and ADF tests. That the spread or ratio of longs to shorts is a stationary series or $I(0)$ therefore finds support. This would not appear a contentious conclusion.

[16] Olekalns gives the following quote "any test tells you that interest rates have unit roots, and lag selection procedures indicate a near random walk structure...Yet interest rates are almost certainly stationary in levels. Interest rates were about 6% in ancient Babylon: they are about 6% now. The chances of a process with a random walk component displaying this behaviour are infinitessimal."

4.4.1 A Quarterly Liquidity Premium Model

The 10-year nominal bond yield (longs) is denoted by B_t and the nominal 90-day Treasury note yield (shorts) by N_t, then the ratio of longs to shorts is defined by $m_t = \frac{B_t}{N_t}$. The ratio of longs to shorts is the liquidity premium or compensation for the added risk of being in longer term bonds in comparison to the risk free rate. There are alternative definitions of the liquidity premium[17]. Real bond yields, following the convention of lower case for real variables, are denoted by b_t and real T-notes by n_t. Formal definitions are in sections 4.4.2 and 4.4.3 respectively. Figure 4.8 is a chart of this ratio.

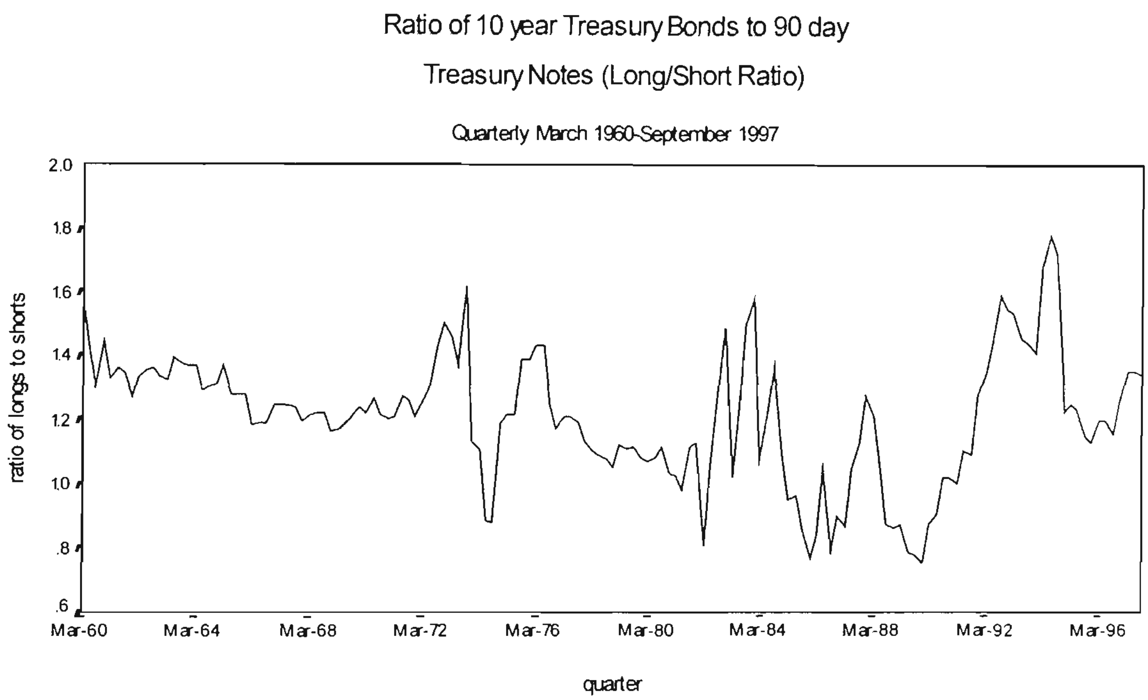


Figure 4.8 Long /Short Ratio a Measure of the Spread in the Yield Curve Between 10-year Treasury Bonds and Treasury Notes.

As was the case with the dividend yield series this appears to be stationary. This was confirmed as in tables 4.10 and 4.11. Both the ADF and PP test statistics with an intercept and trend terms reject the unit root null. Therefore it follows that $m_t \sim I(0)$.

[17] Different authors use different definitions. They may use the T-note rate or 'short term' bonds, *ex post* or *ex ante* and so on. The Sharpe and Sherred (1989) compendium is a good source of coverage of this topic.

Table 4.10 ADF Regression Fixed Interest series March 1960 to September 1997

ADF 5% level critical value below		trend and intercept (-3.434)			intercept (-2.878)		none (-1.942)
variable	lag length	α	$(\beta-1)$	γ	α	$(\beta-1)$	$(\beta-1)$
B_t	3	0.205	-0.017	-0.000	0.186	-0.020	-0.002
		(1.333)	(-0.854)	(-0.290)	(1.346)	(-1.434)	(-0.510)
b_t	4	-0.139	-0.061	0.003	0.150	-0.042	-0.018
		(-0.512)	(-1.904)	(1.177)	(1.295)	(-1.518)	(-0.875)
Δb_t	3	-0.036	-1.225*	0.001	0.032	-1.223*	-1.220*
		(-0.133)	(-7.829)	(0.268)	(0.373)	(-7.851)	(-7.866)
N_t	4	0.345	-0.081	0.003	0.522	-0.064	-0.012
		(1.007)	(-2.274)	(0.771)	(2.056)	(-2.281)	(-0.978)
n_t	2	-0.219	-0.081	0.003	0.151	-0.062	-0.045
		(-0.578)	(-2.085)	(1.042)	(1.113)	(-1.807)	(-1.458)
n_t**	2	-0.408	-0.123	0.012	0.133	-0.057	-0.044
		(-1.213)	(-2.344)	(1.860)	(0.779)	(-1.456)	(-1.245)
Δn_t	1	0.013	-1.400*	0.013	0.039	-1.396*	-1.395*
		(0.035)	(-11.50)	(0.035)	(0.323)	(11.55)	(-11.58)
m_t	4	0.288	-0.224*	-0.000	0.260	-0.215*	-0.004
		(3.316)	(-3.682)	(-0.572)	(3.637)	(-3.667)	(-0.451)

* Significant at the 5% level.

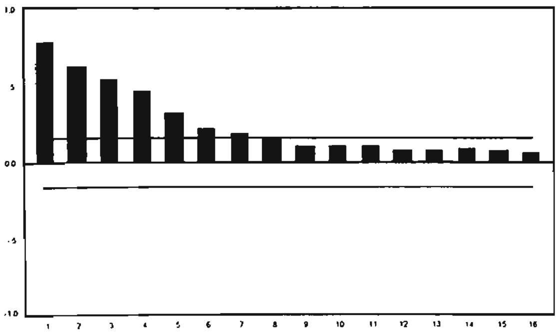
** Using the shorter time period Sep 1969 to Sep 1997.

Table 4.11 PP Test Fixed Interest series March 1960 to September 1997

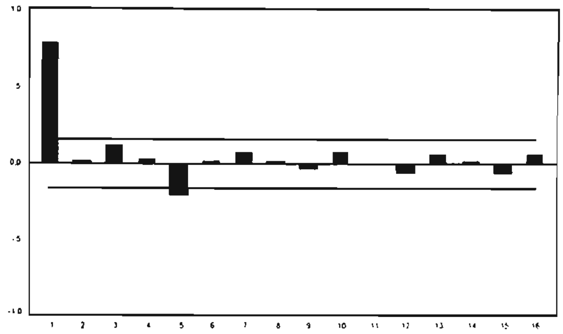
critical value	trend and intercept (-3.434)			intercept (-2.878)			none (-1.941)		
variable	1	5	10	1	5	10	1	5	10
B_t	-0.282	-0.445	-0.444	-1.308	-1.397	-1.430	-0.296	-0.336	-0.349
b_t	-2.522	-2.533	-2.420	-2.004	-1.985	-1.854	-1.386	-1.333	-1.174
Δb_t	-13.29	-13.37	-13.79	-13.34	-13.42	-13.85	-13.38	-13.45	-13.87
N_t	-2.053	-2.069	-1.890	-2.184	-2.173	-2.036	-0.885	-0.839	-0.716
n_t	-2.864	-2.823	-2.820	-2.483	-2.401	-2.370	-2.119	-2.008	-1.945
n_t**	-3.109	-3.096	-3.080	-2.080	-1.991	-1.965	-1.838	-1.728	-1.679
Δn_t	-13.98	-14.26	-15.08	-14.02	-14.31	-15.13	-14.07	-14.36	-15.18
m_t	-4.305	-4.374	-4.330	-4.330	-4.385	-4.344	-0.793	-0.734	-0.690

** Using the shorter time period Sep 1969 to Sep 1997.

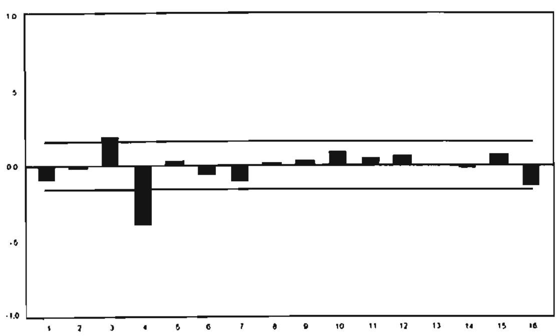
The mean ratio was 1.211 or an average premium of 21.1% over the risk free rate. Now $\overline{m} = 1.211$ and $S.E.(\overline{m}) = 0.016$, hence the mean is significantly different from zero. The mean adjusting transformation, $z_t = m_t - \overline{m}$ was therefore made. The acf and pacf indicated a classic AR(1) pattern, with no evidence of seasonality as exhibited in figure 4.9 (a) and (b). An AR(2) overfit was fitted as a check with the results shown in table 4.12.



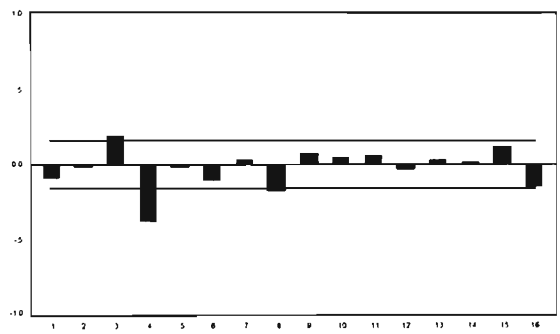
(a)



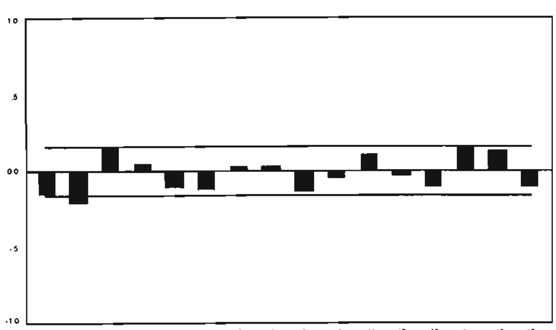
(b)



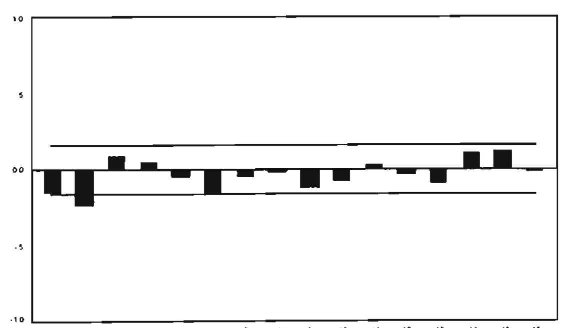
(c)



(d)



(e)



(f)

Figure 4.9 Acf and Pacf for fixed interest series (a), (b) m_t ; (c), (d) Δb_t and (e), (f) Δn_t . Confidence limits given by the lines parallel to the x-axis.

Table 4.12 ARMA Models, Fitting and Diagnostic Checking: Series m_t

Model	Parameter Estimates	t-values	$\hat{\sigma}_{\varepsilon}$	SBC	$Q(12)$
AR(1)	ϕ_1 0.785	15.87	0.123	-202.1	18.22(0.077)
AR(2)	ϕ_1 0.786	9.54	0.122	-194.9	18.41(0.049)
	ϕ_2 0.005	0.06			

As ϕ_2 was not significant and the AR(1) satisfied the diagnostics it was accepted and hence re-transforming leads to:

$$m_t = 0.260 + 0.785 m_{t-1} + \varepsilon_t \tag{4.6}$$

where $\varepsilon_t \sim N(0,0.0151)$ with an R^2 value of 0.628 from formula (3.5).

4.4.2 A Quarterly Treasury Bond Model

As in the case of real dividends, nominal bond prices are deflated, that is the inflation component is removed. The deflator used is the annualised CPI $i_t'' = \frac{(I_t - I_{t-4})}{I_{t-4}} * 100$, as outlined in section 3.4.1. This then yields $b_t = \left\{ \frac{\left(1 + \frac{B_t}{100}\right)}{\left(1 + \frac{i_t''}{100}\right)} - 1 \right\} * 100$, in the same fashion observed inflation was discounted by expected inflation in section 3.6. Hence a real YTM series is generated, as in figure 4.10. The steep rise from the trough in March 1975 is consistent with a structural break at that time. This trough is 2 quarters after the peak in the dividend yield and the bottom in the real All Ordinaries Index.

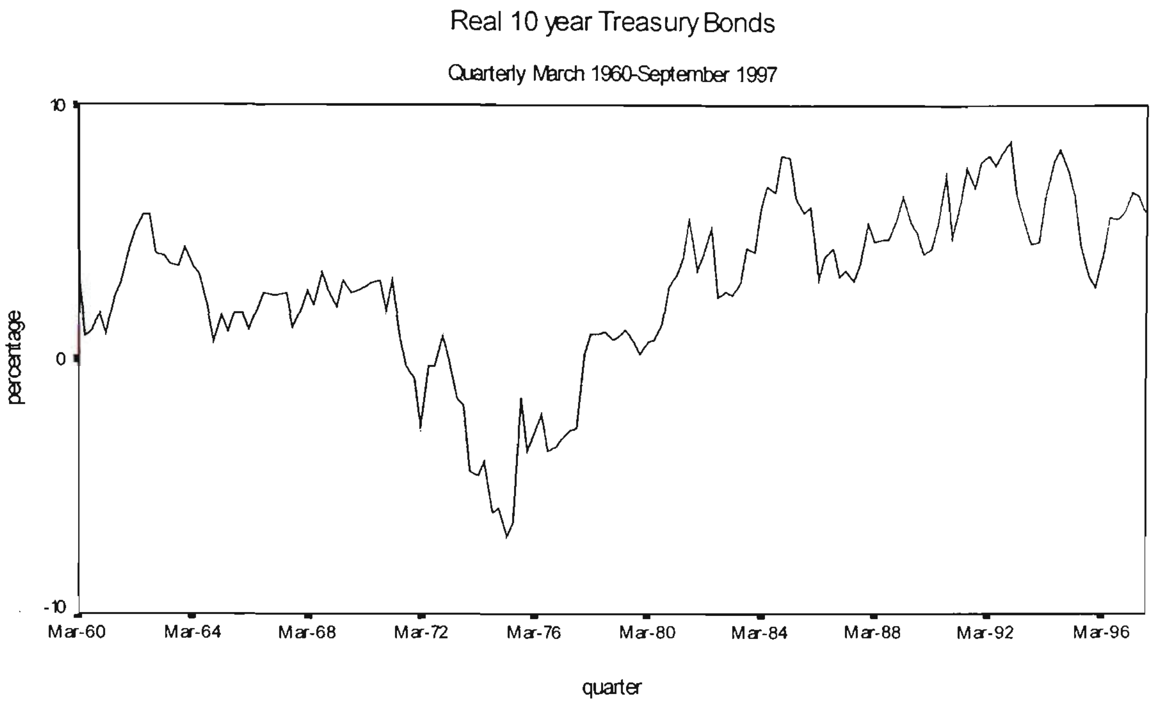


Figure 4.10 Real 10-year Treasury Bond Rates, Nominal YTM of Treasury Bond Deflated by Annualised Rate of Inflation.

This does not appear stationary. The ADF and PP tests were carried out as before and the results are in tables 4.10 and 4.11. Nominal bond yields B_t and real bond yields b_t are both $I(1)$ series, as $\Delta\, b_t$ strongly rejects the unit root null. These results are consistent with Lahiri and Dasgupta (1991), Sherris *et al.* (1996), Mills (1993) and Mayadunne, Inder and Evans (1995).

Now that $\Delta\, b_t$ has been found to be $I(0)$ univariate modelling was applied to this stationary series. The mean value is $\overline{\Delta b} = 0.016$ and $S.E.\left(\overline{\Delta b}\right) = 0.093$ and hence the mean is not significantly different from zero. This is reasonable *a priori*, as growth in real interest rates would not necessarily be expected. There was an average positive real interest rate for bonds of 2.84% with a standard error of 0.27%. There was a current value of some 6%. The level of growth will depend upon the starting point and will be heavily influenced by the trough of negative real rates in the 1970's and the higher average level up to current times. The acf and pacf as shown in figure 4.9 (c) and (d) indicated significant values at lags 3 and 4 of 0.187 and -0.390 respectively compared to a standard error of 0.080. The sign at lag 4 is negative indicating that a change in real rates leads to an *opposite* change with a delay of 4 quarters. A seasonal would be indicated by positive autocorrelation. Proceeding as before with the acf, pacf yielded table 4.13:

Table 4.13 ARMA Models, Fitting and Diagnostic Checking: Series Δb_t

Model	Parameter Estimates	t-values	$\hat{\sigma}_\varepsilon$	SBC	$Q(12)$
AR(3)	ϕ_1 -0.082	-1.01	1.109	462.6	25.40(0.003)
	ϕ_2 0.010	0.12			
	ϕ_3 0.190	2.34			
AR(4)	ϕ_1 -0.007	-0.09	1.028	442.8	6.18(0.519)
	ϕ_2 0.012	0.16			
	ϕ_3 0.153	2.00			
	ϕ_4 -0.378	-4.93			

Hence the AR(4) model:

$$\Delta b_t = 0.153\Delta b_{t-3} - 0.378\Delta b_{t-4} + \varepsilon_t \qquad (4.7)$$

where $\varepsilon_t \sim N(0,1.057)$ with an R^2 value of 0.180 using formula (3.5).

4.4.3 A Quarterly Treasury Note Model

The real T-note rate, denoted by n_t, is deflated in the same manner as for the Treasury bond series. Therefore $n_t = \left\{ \frac{\left(1+\frac{N_t}{100}\right)}{\left(1+\frac{i''_t}{100}\right)} - 1 \right\} * 100$ and the chart of this variable is given in figure 4.11.

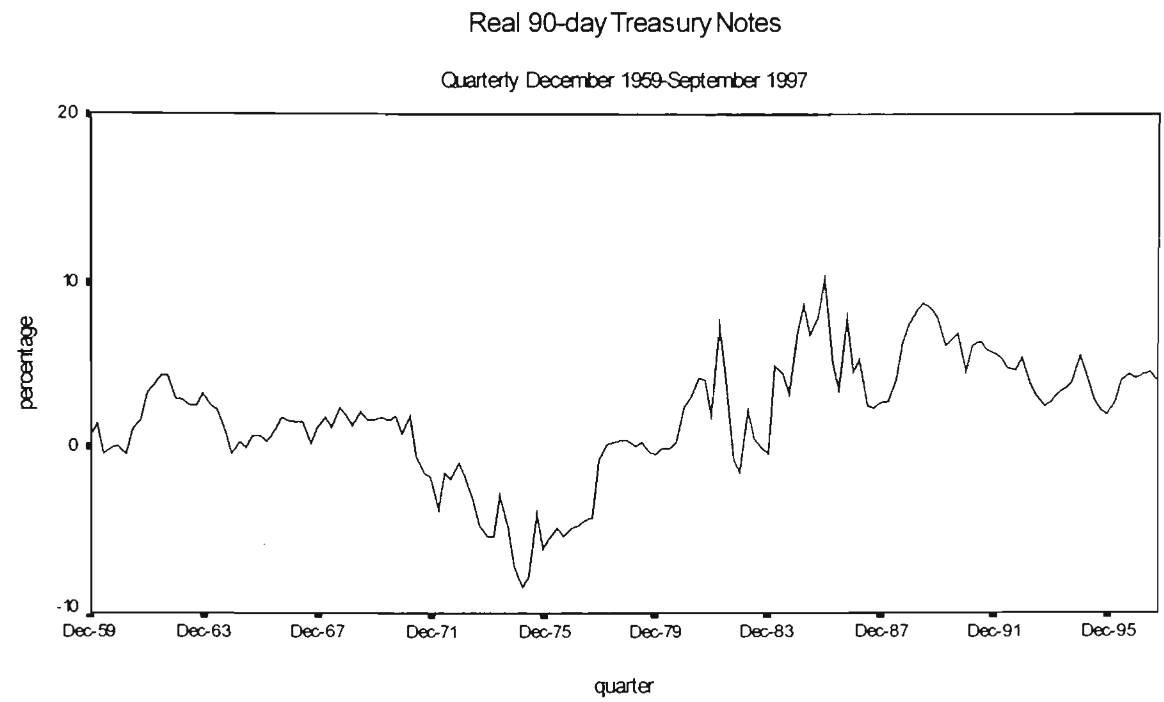


Figure 4.11 Real Treasury Note Rates, Nominal Yield of Treasury Note Deflated by Annualised Rate of Inflation.

It exhibits similar features to those observed in figure 4.10, the chart of real bond rates. The ADF and PP tests are shown in tables 4.10 and 4.11. The ADF test statistics cannot reject the unit root null. Denoting the first differences in real T-notes by Δn_t, the ADF test statistics now strongly reject the null of a unit root. This implies that $n_t \sim I(1)$. The PP test however does not support this result. In the case of no trend or intercept (table 4.11, row 6, columns 7-9) the unit root null is rejected at the 5% level for truncation lag lengths 1, 5 and 10. This suggests a near unit root. The first order autocorrelation ρ_1 for the series n_t was 0.910. Now during the early 1960's short term rates were very stable given the degree of government control over the rate and a stable inflationary environment. Therefore it was decided to review the unit root tests over the shorter period September 1969 to September 1997. The last rows in tables 4.10 and 4.11 refer. Now the unit root null is no longer rejected at the 5% level. For the nominal interest rate series N_t the unit root null is not rejected using either test over the longer period (row 4 of the Tables). Therefore it follows that $N_t \sim I(1)$. From section 3.4.1 the T-note deflator, the annualised series i_t'', is an $I(1)$. The quotient is not necessarily an $I(1)$. The result is not clear. However for consistency the underlying logic of the series is accepted. It is the same as for the bonds; nominal T-notes are an $I(1)$ as are real T-notes. Mishkin and Simon (1995) using data from September 1963 to December 1993 find in favour of the stationarity of real T-note rates or $n_t \sim I(0)$ where the real T-note rate is defined by $N_t - i_t''$. This fact is deduced by the authors as a consequence of i_t'' and $N_t \sim I(1)$ (see Mishkin and Simon (1995, equation (7), p.222)) and related discussion.

The mean value is $\overline{\Delta n} = 0.022$ and $S.E.\left(\overline{\Delta n}\right) = 0.125$ and hence the mean is not significantly different from zero. This result may be compared with that for Δb_t. There is a greater mean and standard error or a greater change in short rates on average, than for long rates. This reflects the fact that YTM is under consideration not price changes. Given the long duration of the 10-year bond far greater price changes for the bond series would be expected. Hence, and in contrast with the average real bond rate the average real T-note rate is 1.82% and the standard deviation of real T-note rates is 3.64%. The acf shown in figure 4.9 (e) is almost white noise with no evidence of seasonality but with a value at lag 2 which is just significant. The Box-Ljung statistic $Q(12)$ was 22.46 above the 5% critical level, so the simple random walk model was not considered appropriate. The results are given in table 4.14:

Table 4.14 ARMA Models, Fitting and Diagnostic Checking: Series Δn_t

Model	Parameter Estimates		t-values	$\hat{\sigma}_\varepsilon$	SBC	$Q(12)$
AR(1)	ϕ_1	-0.138	-1.71	1.521	555.7	20.83(0.035)
AR(2)	ϕ_1	-0.169	-2.11	1.471	548.8	8.69(0.562)
	ϕ_2	-0.229	-2.87			
MA(1)	θ_1	-0.211	-2.64	1.508	556.9	17.74(0.088)
MA(2)	θ_1	-0.131	-1.63	1.486	557.8	11.06(0.524)
	θ_2	-0.192	-2.39			
ARMA(1,1)	ϕ_1	0.629	3.44	1.500	557.0	15.05(0.130)
	θ_1	-0.799	-5.64			

The AR(2) candidate is preferred and is consistent with the other results, giving the model:

$$\Delta n_t = -0.169\Delta n_{t-1} - 0.229\Delta n_{t-2} + \varepsilon_t \qquad (4.8)$$

where $\varepsilon_t \sim N(0,2.195)$ and an R^2 value of 0.071 from formula (3.5). The MA(1) is included for comparison with a result given in Mills (1993) where UK real Treasury bill rates were modelled over the period 1952 to 1982. First differences were used to obtain $\Delta x_t = \varepsilon_t - 0.678\varepsilon_{t-1}$ where x_t is the real Treasury bill rate and $\varepsilon_t \sim N(0,14.5)$. This model is significantly different. It is reflective of the complete absence of the period 1982 -1997 when the value and volatility of the series took a very steep jump. It has remained at a high, albeit slowly declining, level. This underscores the vital importance of the time period under investigation. One can reach very different conclusions with an assessment of the data utilising different sub-periods. A second order MA was tried but θ_1 was not significant (see table 4.14).

4.4.4 Heteroskedasticity and ARCH Effects

As in section 4.3.4 the normality and homoskedasticity assumptions are tested. The results of the ARCH-LM test on the univariate models indicates that there is not a strong ARCH effect. There are other tests for non-linearity but they require a specification of the type of non-linearity. From table 4.15 Δb_t does not have ARCH residuals; m_t and Δn_t only just.

Table 4.15 Preferred ARCH/GARCH Fixed Interest Models Diagnostic Testing

Model	ARCH-LM test	skewness	excess kurtosis	Jarque-Bera statistic	$Q(12)$
m_t : Eqn. (4.6)	4.087	-0.815	2.519	56.27	18.223
	(0.043)				(0.077)
ARCH(1)	0.665	-0.406	2.781	52.97	12.877
	(0.415)				(0.301)
Δb_t : Eqn. (4.7)	1.498	0.114	1.213	9.27	6.300
	(0.221)				(0.789)
ARCH(2)	1.271	0.193	0.859	5.39	7.874
					(0.641)
Δn_t : Eqn. (4.8)	4.076	0.286	1.745	29.93	8.688
	(0.043)				(0.562)
ARCH(1)	0.440	0.682	2.039	37.37	7.436
	(0.507)				(0.684)

The non-linear models resulting from the above are given in table 4.16. The table has the same format and style as table 4.9.

Table 4.16 Preferred ARCH/GARCH Fixed Interest Models Parameter Values

model variable	ϕ_1	ϕ_2	ϕ_3	ϕ_3	α_0	α_1	α_2
m_t	0.761				0.008	0.589	
	(8.001)				(3.608)	(2.743)	
Δb_t			0.121	-0.416	0.872	0.364	-0.127
			(4.913)	(-7.999)	(6.553)	(5.447)	(-6.273)
Δn_t	-0.151	-0.197			1.388	0.435	
	(-1.980)	(-2.567)			(4.364)	(4.304)	

The EGARCH (p,q) model of Nelson was applied to the conditional variance for the range of values p = 1 to 2, q = 1 to 2. For m_t the best model on the SBC criterion was an EGARCH (2,2) with appropriate diagnostics. However when the model was applied to the period December 1973 to September1997 to test for stability, the coefficients changed substantially. The EGARCH coefficients for example changing from 1.45 and -0.49 for the period September 1960 to September 1997 to 1.05 and -0.17 for December 1973 to September 1997. For Δb_t the best model on the SBC criterion was an EGARCH (2,1) but the SBC value was below that of the AR(4)-ARCH(2) above with $\ln L$ increasing from -206.6 to -204.6 with 3 extra parameters. For Δn_t all models had better Jarque-Bera statistics but still none were satisfactory. The EGARCH models also require many extra parameters.

The models do not suggest that there are significant gains to be made with the application of non-linear stochastic models of the ARCH/GARCH form. For the fixed interest series the normality and homoskedasticity assumptions are more reasonable than for the equity series.

4.5 Conclusions

This chapter has reviewed the financial time series that form the basis of the proposed stochastic investment model. The data sources have been detailed as a precursor to investigating each of the financial time series in detail. The ADF and PP tests have been used to determine the order of integration of each series. Where series were not stationary, suitable differencing has been applied to achieve stationarity.

The real interest rate and real price indices series are found to be $I(1)$. The evidence is that all these 'levels' series are in fact near integrated. Economic theory would suggest that the level of real interest rates is constrained within bounds and that these should be stationary series. These points are made by several authors (see Mishkin and Simon (1995), Olekalns (1996)). That is they cannot wander as would a random walk. Nevertheless for statistical purposes the levels series are regarded as representing a stochastic trend component. Thus the differenced stationary components of the time series can be analysed with OLS regressions, where from section 3.7 it was concluded that quarterly rate of inflation was also stationary. The conflicting nature of real and nominal interest rates will be revisited in Chapter 7. The dividend yield and long/short ratio are $I(0)$. The latter two results are not generally contentious conclusions.

The autocorrelation function and the partial autocorrelation function has been detailed for each stationary series. The acf's indicate that there is no evidence of seasonality in any of the series. The acf and pacf have been used to fit univariate Box-Jenkins models to each series. Various diagnostic criteria have been used to assess the validity of the resulting models. Autoregressive series of low order have been found suitable for modelling each of the variables. The dividend yield and long/short ratio are stationary series modelled as an AR(2) and AR(1), equations (4.1) and (4.6) respectively. Both models possess good explanatory power. Real dividends are close to a random walk. The model given by equation (4.4) has a very low R^2. The AOI is best modelled as a random walk, given by equation (4.5). This latter result is consistent with the real dividend model and the results of other Australian authors. Univariate models for real bonds and real T-notes, given by equations (4.7) and (4.8) respectively, have a very low level of explanatory power.

Heteroskedasticity has been investigated. For equity variables ARCH modelling represented an improvement but the residuals were still non-normal. For the fixed interest variables ARCH effects were not strong. The results did not suggest significant gains from this form of modelling. The conclusion reached is that whilst models have residuals which are leptokurtokic simple ARCH models do not provide a satisfactory solution. Provided serial correlation is not present using our diagnostic tests, the residuals will be regarded as independent identically distributed (i.i.d.) normal variates.

Chapter 5

Levels Relationships and Cointegration

5.1 Introduction

Stochastic investment models in the literature such as Wilkie (1984, 1987, 1992, 1995a and 1995b) or Mulvey and Thorlacius (1997) employ a set of interconnecting equations. The linkage between the equations for equity prices on the one hand and fixed interest prices on the other for each model are different (see the preliminary discussion in section 1.1). The determination and evaluation of models requires an analysis of the long run relationships between the time series. Section 4.1 discussed some of the issues of spurious regressions. This leads to the concept of cointegration or series that are 'integrated' in some special way.

Extensive research has been devoted to the study of cointegrated processes. Engle and Granger (1991) contains a selection of papers covering some of the salient features of this research. Banerjee, Dolado, Galbraith and Hendry (1993) overview the econometric analysis of non-stationary data. These show that the use of standard regression techniques depends upon the stationarity of the series being modelled. The use of integrated series is much more difficult unless there is a long run relationship between the variables. Standard regression techniques can be applied using "....dynamic specifications which take into account any cointegrating relationships among the variables." (Banerjee *et al.* (1993, p.162)).

In this chapter the interrelationships between the financial variables introduced in chapters 3 and 4 are investigated within a cointegration framework. Therefore any long run relationships between the integrated series can be assessed.

The chapter is structured as follows. Section 5.2 introduces definitions and testing. The Engle-Granger two-step procedure is used in the data analysis. The Johansen maximum likelihood test is applied as a check on the procedure. In section 5.3 the set of real and nominal bivariate relationships is tested. In section 5.4 a special case, the link between inflation and share prices, is examined with a long annual data series past the structural break in 1947. Then in section 5.5 the trivariate relationship between the bond yield and the components of the dividend yield is also investigated for both nominal and real cases. Finally in section 5.6 all real variables are considered together.

5.2 Cointegration: Definitions and Testing

So far a range of series has been tested for the existence of unit roots and differenced appropriately for stationarity and then modelled. It is of importance for financial modelling to see whether or not there is any relationship between the integrated series. If there is, in that the series trend together, then there may a linear combination of them that is stationary. Suppose there are two time series $x_t \sim I(1)$ and $y_t \sim I(1)$, then it is generally expected that any linear combination will also be $I(1)$. However it may be the case that $u_t = y_t - ax_t$ is an $I(0)$ for some value a. If so, the series are then said to be *cointegrated.* In the general case Engle and Granger (1987) provide the following definition of cointegration. The components of the vector $\mathbf{x}$ = $(x_{1t}, x_{2t}, \ldots x_{nt})$ are said to be *cointegrated of order d, b* denoted $\mathbf{x} \sim CI$(d,b) if:

1. All components of $\mathbf{x} \sim I$(d).
2. There exists a vector $\mathbf{a} = (a_1, a_2, \ldots a_n)$ such that the linear combination

   $\mathbf{a}'\mathbf{x} = a_1 x_{1t} + a_2 x_{2t}, \ldots . a_n x_{nt} \sim I$(d-b) where b$> 0$.

Then **a** is the called the *cointegrating vector*. Mills (1993, p.176-182) or Cuthbertson, Hall and Taylor (1992, p.135-148) outline the general theory behind cointegration testing.

A test is required for cointegration. Now if the series u_t derived from the linear combination is to be stationary then the cointegrating vector must be found and then an ADF or PP test applied to the resulting series u_t. The critical values for the ADF and PP tests from the cointegrating regression are different from those for a unit root test on a single variable. These critical values are tabulated by Engle and Yoo (1987). A more comprehensive tabulation is available as the result of a simulation study in MacKinnon (1991). Alternatively the cointegrating regression Durbin Watson statistic (CRDW) derived from u_t may be applied. The Durbin Watson (DW) test for serial autocorrelation in the residuals is defined as:

$$d = \frac{\sum_{t=2}^{n} \left(e_t - e_{t-1}\right)^2}{\sum_{t=1}^{n} e_t^{\,2}} \tag{5.1}$$

where e_t are the residuals. If there is no autocorrelation then the expected value of d is 2. With perfect positive correlation $\rho_1 = 1$ the expected value of d is 0. Hence a test that $u_t \sim I(1)$ is given by testing if the DW statistic d is significantly greater than zero. This two-step procedure of employing a cointegrating regression plus a test for the residuals being $I(0)$ is called the Engle-Granger method.

Engle and Yoo (1987) examined the behaviour of the Durbin-Watson statistic from the cointegrating regression. They concluded, on the basis of simulations to generate critical values for the Engle-Granger cointegration test, that "...the discrepancy between the critical values for different systems remains significant even for the sample of two hundred.....Hence this statistic does not appear to be too useful for testing cointegration." (Engle and Yoo (1987, p.157)). Holden and Perman (1994) discussed the power of the various tests and suggested that the PP test has a higher power than the DF or ADF test.

Each of the above tests is applicable to a bivariate situation. If there are only two variables the cointegrating vector found using OLS is unique. For more than two variables there is no guarantee that this is so. In this case, with n variables, one could find up to $(n-1)$ cointegrating vectors. An alternative method for cointegration testing is a vector autoregressive approach (VAR) due to Johansen (1988). This method has the advantage of dealing with the circumstance of more than two variables in the cointegrating equation. The Johansen approach is a maximum likelihood method of estimating all the distinct cointegrating vectors which may exist between a set of variables. It also shows how to test which of the cointegrating vectors are statistically significant (see Cuthbertson, Hall and Taylor (1992, p.144-154)). Since the method is applicable to the bivariate case the Johansen test procedure will be applied to check the Engle-Granger results. An unknown is the lag length of the VAR. Holden and Perman (1994, p.106-109) use a procedure employing a likelihood ratio test applied to the log likelihood functions. These are obtained from running the VAR with different lag lengths commencing with a "large" lag length plus satisfactory diagnostic tests for white noise residuals. They then select the lag length as the shortest lag length which is consistent with the restrictions imposed in moving to that reduced lag length being statistically acceptable. Banerjee, Dolado, Galbraith and Hendry (1993, p.286) report on studies into the optimum lag length commenting that "... the loss of efficiency from choosing too long a lag length is small...even if a lag of four periods is used for the short run dynamics instead of the correct value of zero.". They also suggest the SBC criterion (see formula (3.6)) does well in a data based lag length selection exercise.

5.3 Bivariate Relationships

In this next section cointegration between elements in both the set of real variables and the set of nominal variables are considered. The set of real variables are the principal focus of attention. The addition of the nominal set is both for completeness and comparison with other research.

5.3.1 Nominal Variables

Let us now consider the set of nominal $I(1)$ variables as determined by the ADF and PP tests and then look for any cointegration amongst them. The dividend yield

$y_t = \frac{D_t}{P_t}$ is an $I(0)$. Thus the nominal All Ords index and the nominal dividend index may be cointegrated series. Similarly cointegration between the nominal bond rate and the nominal T-note rate may be anticipated as they are connected via the stationary long short ratio.

Of interest are the long run relationships between the various integrated measures of inflation and all of our series. Now $\mathbf{ln}(I_t)$ and ΔI_t are $I(1)$, though tables 3.1 and 3.3 yielded conflicting evidence about ΔI_t. In a similar context Pagan, Hall and Martin (1996, p.92) discuss the 'near integrated' nature of interest rates commenting that "It may be that the autoregressive root is close to unity, rather than identical to it, but such "near integrated" processes are best handled with integrated process technology rather than that for stationary processes.". The near integrated series are analysed on the assumption of being integrated series rather than stationary ones. Hence for the cointegration analysis it is assumed that $\Delta I_t \sim I(1)$, as are nominal share prices, nominal dividends, nominal bonds and nominal T-notes. Therefore any potential bivariate relationships can be tested. An underlying trend in the cointegrating series is assumed. Thus the cointegrating equation between the difference in the CPI and the All Ords is $P_t = \alpha_0 + \alpha_1 \Delta I_t$.

The cointegrating regressions were run for each pair of variables. The number of lagged differences for the ADF test and the truncation lag length were obtained by the recursive *t*-statistic procedure of Mayadunne *et al.* (1995) (see section 3.4.1). Note that since there is a constant term in the cointegrating regression it need not appear in the ADF test. The residuals will have zero mean. The following table 5.1 gives the results. The first variable given in the table is the independent variable. Note that in the tables that follow upper case symbols represent nominal variables and lower case symbols represent real variables.

Table 5.1 Cointegration between Nominal Series: Tests Applied to the Residuals from the Cointegrating Regression. Various Lengths of Time Period

Variable ind:dep	N	DW	lag length	ADF** (1)	ADF** (2)	PP** (1)	PP** (2)
$\Delta I_t : P_t$	196	0.124	3	0.393	1.841	-1.206	0.644
$\Delta I_t : N_t$	152	1.305*	3	-3.052	-2.898	-8.987*	-8.606*
$\Delta I_t : D_t$	194	0.234	3	-0.395	0.860	-3.022	-1.162
$\Delta I_t : B_t$	151	1.064*	3	-2.989	-2.603	-8.354*	-7.355*
$B_t : P_t$	151	0.038	7	0.367	2.056	-0.113	1.778
$B_t : N_t$	151	0.624*	4	-4.591*	-4.548*	-5.330*	-5.297*
$B_t : D_t$	149	0.019	0	-0.186	1.435	-0.186	1.435
$D_t : P_t$	194	0.304*	6	-4.481*	-4.463*	-4.180*	-4.162*
$D_t : N_t$	150	0.153	9	-0.269	-0.808	-1.610	-1.866
$N_t : P_t$	152	0.045	7	0.201	2.069	-0.591	1.437

* Significant at the 5% level.

**ADF(1) is the ADF test with trend and intercept terms and number of lagged differences given by lag length, column (4). ADF(2) is the ADF as in (1) but with intercept term only. PP(1) is the Phillips Perron test with trend and intercept terms and truncation lag as per lag length column (4). PP(2) is as per ADF(2).

The 5% level critical value for the CRDW test is given in Banerjee, Dolado, Galbraith and Hendry (1993, table 7.1, p.209) as 0.38 for $N = 100$ and 0.20 for $N = 200$ observations with two regressors. The CRDW test results suggest a cointegrating relationship between inflation and nominal bonds, between inflation and nominal T-notes, between nominal bonds and nominal T-notes and between the All Ords index and nominal dividends.

The asymptotic critical values for the DF and PP tests are given in MacKinnon (1991, table 1, p.275). This table is used in conjunction with the interpolation formula

MacKinnon (1991, formula (8), p.273) for smaller sample sizes. So the 5% critical value for ADF(2); that is, with a constant but no trend term and with $N = 150$ observations is given by:

$$-3.3377-(4.039/150)-(17.83/(150)(150)) = -3.378.$$

and in a similar fashion for ADF(1). That is, with a constant and trend term the critical value is -3.844. The ADF tests for $\Delta I_t : N_t$ and $\Delta I_t : B_t$ lie inside the critical values, though they are close to them. The PP tests strongly reject the unit root null. Hence accepting the Holden and Perman suggestion that the PP test is more powerful than the ADF test along with the CRDW results, implies support for a cointegrating relationship between inflation and bonds and between inflation and T-notes. Otherwise the above results confirm the CRDW test and suggest a cointegrating relationship between bonds and T-notes and between the All Ords index and dividends.

The Johansen procedure was applied as a check on the results given in table 5.1. The appropriate lag length to be applied requires consideration. To investigate this the variables B_t and N_t were chosen and the lag length p was analysed using the SBC criterion. The following table 5.2 gives the results of the log likelihood function ($\ln L$) and the Johansen likelihood ratio statistic (LR).

Table 5.2 Log Likelihood Function and LR Statistic for Various Lag Lengths in the Johansen Cointegration Test VAR. Nominal Bonds and Nominal T-notes.

lag length(p)	**ln** L	LR statistic
1	-359.50	23.106*
2	-356.03	15.823*
3	-343.34	15.176
4	-332.36	23.483*
5	-329.28	20.778*

* Significant at the 5% level.

Now the SBC criterion penalises extra parameters at the rate of $\mathbf{ln}(151) = 5.02$ for $N = 151$ observations times the number of extra parameters. The number of parameters in the VAR is given by $k^2 p + k = k(kp + 1)$ where k is the number of variables (and here k = 2), p is the lag length in the VAR, and there is assumed a constant term in each equation. For a VAR therefore the parameters increase by $k^2 p$ where k is the number of variables in a k-dimensional VAR. Since $-2\,\mathbf{ln}\,L$, the first term in the SBC formula (3.6) is increasing more slowly than the penalty which is increasing by $k^2\,\mathbf{ln}(N) = 4\,\mathbf{ln}(151) = 20.08$. it follows that the optimum lag length on this data based criterion is p =1. The appropriate critical value at the 5% level of significance is 15.41. Thus the null of no cointegrating equations is rejected at this level. Now the LR statistic is significant for lag lengths 1, 2, 4 and 5 but not 3. In this case the LR statistic is only just inside the critical value for 3 lags. The null of no cointegrating equations is therefore rejected over the range if the lag 3 value is discounted.

Table 5.2 suggests a degree of consistency in the significance of the LR statistic for various lag lengths. Also Banerjee *et al.* suggest that the loss of efficiency from choosing too long a lag length is small. Hence a complete tabulation is not conducted as per table 5.2 for all the combinations, 10 in all, in table 5.3. Significance at lag 4 confirmed by significance at lag 2 is regarded as supporting evidence for cointegration, particularly in the context of the results in table 5.1. The Johansen test is used as a check. When a conflict arises a more detailed investigation will be conducted. The likelihood ratio values for the other bivariate relationships are given in table 5.3. The table gives the VAR = 4 result in the top row of each line and VAR = 2 in the bottom row. The sample is over the period September 1948 to September 1997 with the length of the bivariate series as in table 5.1.

Table 5.3 Johansen Cointegration Test: That the Hypothesised Number of Cointegration Equations is None. Time Periods as in Table 5.1

variable		P_t	N_t	D_t	B_t
ΔI_t	VAR=4	8.625	20.357*	9.842	19.567*
	VAR=2	7.918	30.863*	11.478	26.050*
B_t	VAR=4	6.717	23.483*	7.633	
	VAR=2	5.199	15.823*	6.354	
D_t	VAR=4	31.005*	8.750		
	VAR=2	34.121*	7.234		
N_t	VAR=4	7.598			
	VAR=2	5.231			

* Significant at the 5% level.

Table 5.3 confirms the results of table 5.1 that there is a cointegrating relationship between the difference in the CPI and the two interest rates. Mishkin and Simon (1995) examined the Fisher hypothesis that changes in the expected inflation rate will be fully incorporated in nominal interest rates and that therefore real interest rates will remain constant over time. They apply the Engle-Granger method and ECM's, as an alternative due to the low power of the two-step method. They find that the data is generally supportive of the long run Fisher effect or that nominal T-note rates and inflation are cointegrated. There is also cointegration between the two interest rate series on the one hand and between the All Ords and dividend on the other. These results are consistent with Sherris *et al.* (1996). These relationships are captured by the dividend yield and the ratio of longs to shorts. Some authors have used the spread or the difference between the long rate and the short rate rather than the ratio as a measure of the liquidity premium.

There is no cointegrating relationship between difference in the CPI and nominal dividends. This result is consistent with those reported by Carter (1991), Fitzherbert (1992), Harris (1994,1995) and Sherris *et al.*(1996). This is also consistent with Crosby (1998) who used annual data from 1875-1976 to test the link between nominal stock returns and inflation using regression techniques. He finds no long run regression between these two variables. This differs from Wilkie, who presumes that in the long run inflation will find its way fully into the income stream of assets and hence their value. A more detailed discussion of the literature on the relationship between inflation, dividends and stock returns is presented in sections 8.2 and 8.3.1.

5.3.2 Real Variables

The test is next applied to real variables. An identical approach is taken to that applied to the nominal variables. The Johansen test follows in table 5.5.

Table 5.4 Cointegration between Real Series: Tests Applied to the Residuals from the Cointegrating Regression. Various Length of Time Period

Variable ind:dep	N	DW	lag length	ADF** (1)	ADF** (2)	PP** (1)	PP** (2)
$\Delta I_t : p_t$	196	0.130	3	-2.891	-2.598	-2.679	-2.358
$\Delta I_t : n_t$	152	0.234	2	-2.065	-1.796	-2.875	-2.525
$\Delta I_t : d_t$	194	0.076	2	-2.491	-2.452	-2.124	-2.082
$\Delta I_t : b_t$	151	0.123	4	-1.926	-1.540	-2.582	-2.045
$b_t : p_t$	151	0.181	3	-2.339	-2.395	-2.345	-2.424
$b_t : n_t$	151	0.564	4	-4.430*	-4.448*	-4.983*	-5.001*
$b_t : d_t$	149	0.094	2	-2.246	-2.255	-1.983	-1.973
$d_t : p_t$	194	0.277	3	-4.712*	-4.624*	-4.202*	-4.126*
$d_t : n_t$	150	0.183	2	-2.095	-1.763	-2.748	-2.293
$n_t : p_t$	152	0.185	3	-2.344	-2.462	-2.357	-2.437

* Significant at the 5% level.

**ADF(1) is the ADF test with trend and intercept terms and number of lagged differences given by lag length, column (4). ADF(2) is the ADF as in (1) but with intercept term only. PP(1) is the Phillips Perron test with trend and intercept terms and truncation lag as per lag length column (4). PP(2) is as per ADF(2).

Table 5.5 Johansen Cointegration Test: That the Hypothesised Number of Cointegration Equations is None. Time Periods as in Table 5.4

variable		p_t	n_t	d_t	b_t
ΔI_t	VAR=4	11.624	11.103	12.907	12.255
	VAR=2	10.390	8.237	12.652	8.300
b_t	VAR=4	16.013*	19.364*	12.282	
	VAR=2	12.893	18.333*	10.303	
d_t	VAR=4	31.125*	15.959*		
	VAR=2	28.074*	11.231		
n_t	VAR=4	14.273			
	VAR=2	11.331			

* Significant at the 5% level.

Tables 5.4 and 5.5 support cointegrating relationships between real bonds and real T-notes and between real dividends and real share prices. This makes sense *a priori*. However from table 5.4, rows 1-4 or table 5.5, rows 1 and 2 there is no support for such long term relationships between the level of inflation and real interest rates, real share prices or real dividends. The results of table 5.4 suggest no other cointegrating relationships. However in table 5.5 there are two significant values of the LR statistic at lag 4 not confirmed at lag 2 or in table 5.4. This is considered in more detail in table 5.6.

Table 5.6 LR Statistic for Various Lag Lengths in the Johansen Cointegration Test VAR for $b_t : p_t$ and $d_t : n_t$.

lag length (p)	LR stat. $b_t : p_t$	LR stat. $d_t : n_t$
1	12.370	10.396
2	12.893	11.231
3	20.640*	15.218
4	16.013*	15.959*
5	15.074	13.718

* Significant at the 5% level.

In both cases the LR statistic is not significant at the shorter lags. A review of **ln** L for $b_t : p_t$ gave a value of -502.2 for lag 2 and -477.4 at lag 4. Therefore the SBC would choose the shorter lag. In fact lag 1 would be chosen with the SBC. The significant value at lag 4 was therefore discounted. Moreover, if there was cointegration between p_t and b_t, then given cointegration between p_t and d_t by definition it follows that there exists λ and μ such that:

$$u_t = p_t - \lambda d_t \sim I(0) \text{ and } v_t = p_t - \mu b_t \sim I(0)$$

Thus $v_t = u_t + \lambda d_t - \mu b_t$ and hence $v_t - u_t = \lambda d_t - \pi b_t \sim I(0)$ and so it follows that b_t and d_t are cointegrated. This is not supported by the evidence. A similar set of arguments follows for d_t and n_t. Hence only the two cointegrating relationships $b_t : n_t$ and $d_t : p_t$ remain.

5.4 Special case: Nominal Inflation and Nominal Share Prices

The lack of cointegration between nominal share prices and inflation is an important special case arising from the results of sections 5.3.1 and 5.3.2. This is *a priori* not expected and warrants further investigation.

Now since P_t and D_t are cointegrated as are p_t and d_t then only cointegration between the level of inflation and share prices needs checking. To investigate this, long term annual series for both the CPI and the All Ords index were obtained. The CPI series started in 1855 and the All Ords in 1875 with the commencement of the Australian Stock Exchange (see section 4.2). A chart of the logarithm of these two long term indices is shown in figure 5.1.

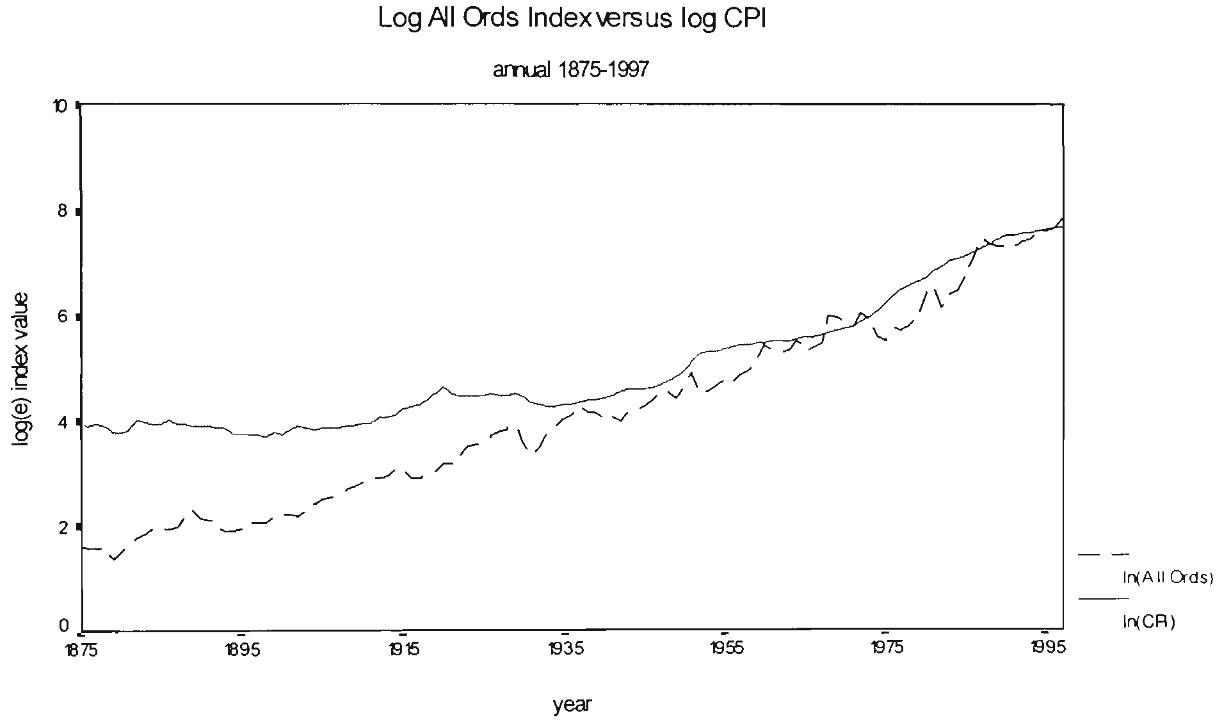


Figure 5.1 Log of the All Ordinaries Index versus the log of the CPI from 1875-1975.

The full period 1875-1997 was then sub-divided by the structural break in 1947 yielding two periods 1875-1947 and 1948-1997 (see section 3.3). The results are summarised in tables 5.7 and 5.8. The methods applied are as in section 5.3.

Table 5.7 Unit Root Tests for Inflation and Share Prices 1875-1947 (Rows 2-5) and 1948-1997 (Rows 6-9): ADF Test (Columns 2-5) and PP Test (Columns 6-8).

Variable	Lag Length	Trend + intercept	Intercept	None	Trend + intercept	Intercept	None
I_t	3	-2.660	-0.828	0.253	-3.485*	-1.319	-0.295
ΔI_t	0	-10.734*	-10.333*	-10.366*	-10.734*	-10.333*	-10.366*
P_t	7	-0.043	3.553	5.070	-0.058	2.908	4.576
ΔP_t	6	-6.604*	-4.701*	-3.1108	-6.681*	-6.301*	-6.020*
I_t	5	-2.599	-1.608	-1.192	-0.705	2.821	5.450
ΔI_t	10	-2.866	-3.202*	-2.874*	-2.743	-2.260	-1.417
P_t	0	0.202	2.448	3.686	0.202	2.448	3.686
ΔP_t	10	-1.071	0.398	1.189	-5.709*	-5.240*	-5.163*

* Significant at the 5% level.

In section 5.3.1 it was assumed that $\Delta I_t \sim I(1)$. For the period 1875-1947 it is an $I(0)$ as is ΔP_t . The PP test with trend and intercept (row 2, column 6) rejects the unit root null but only just, the 5% critical value being -3.459. This would make $I_t \sim I(0)$. The second period is less clear cut. The near unit root suggested in ΔI_t means that the order of integration is unclear. The strong rejection of the unit root null by the PP test for ΔP_t implies $P_t \sim I(1)$. Determining cointegration is difficult when the order of integration of the series shows such variation. The differences in the structure of the series for the two periods confirms the breakpoint at 1947[1] found in section 3.3. Also given that the long run trend in **ln** P_t is steady showing no breakpoints (see figure 5.1), suggests that inflation is *not a determinant of long run nominal share prices.*

This proposition of the lack of long run connection between the CPI and nominal share prices is checked in table 5.8.

[1] No other structural breaks were found in the analysis in section 3.3. This does not preclude the possible existence of other breakpoints.

Table 5.8 Summary Cointegration Statistics between Inflation and Share Prices 1875-1947 and 1948-1997

Vars	No. Obs	α	β	CR DW	No. lags	ADF trend	ADF none	PP trend	PP none	Joh. LR
$P_t : I_t$	73	-37.0	0.953	0.16	1	-2.01	-1.70	-1.73	-1.43	8.06
$P_t : I_t$	50	-92.6	0.966	0.53	0	-1.42	-1.75	-1.42	-1.74	26.05
$P_t : \Delta I_t$	50	277.0	9.004	0.24	7	-0.19	-0.57	0.57	1.12	11.19

The table 5.8 is interpreted with columns 3 and 4 being the cointegrating regression coefficients with the related DW statistic in column 5. Columns 7-10 are the results of the unit root test on the residuals with lag length/truncation lag in column 6 and determined by the recursive *t*-statistic method. The last column is the result of the Johansen test with 2 lags in the VAR. Cointegration is rejected in both periods. The result for $P_t : I_t$ over 1948-97 is less clear with conflicting evidence. The Johansen LR test statistic indicated a cointegrating vector. However the cointegrating vector obtained from the two methods was different. This result is consistent with Dickey, Jansen and Thornton (1994) who find in an empirical study on money and income, that evidence concerning cointegration is sensitive to the method used. They also find that the cointegrating vector is nearly identical when the Johansen and Engle-Granger methods indicate cointegration. The vectors are different when they do not agree.

5.5 Share Prices, Dividends and Bond Yields: 'Confidence'

With the use of UK data Mills (1991) finds in favour of cointegration between the log of the dividend, log share price and the gilt yield. An identical procedure to that performed by Mills was carried out using Australian data. In the first instance, instead of logs, real variables were used. The real All Ords was regressed against real bonds and real dividends. The cointegrating regression equation was:

$$p_t = 0.021 + 0.330 b_t + 20.598 d_t + \varepsilon_t \qquad (5.2)$$

$$(0.019) \quad (4.807) \quad (13.717)$$

for $N = 149$ observations with an R^2 of 0.592 and a DW of 0.374.

The 5% level critical value for the CRDW test is given in Banerjee, Dolado, Galbraith and Hendry (1993, table 7.1, p.209) as 0.48 for $N = 100$ and 0.25 for $N = 200$ observations with 3 regressors. The CRDW test therefore suggests a borderline rejection of the unit root null implying that $\varepsilon_t \sim I(0)$ and the series are cointegrated. This was confirmed by the ADF and PP tests on the residuals. For the ADF test applying the recursive *t*-statistic procedure the lag length was set at 3. The resulting test values were -4.369 without trend and -4.359 with trend included. The critical values at the 5% level for $N = 149$ observations and 3 variables given in MacKinnon (1991) are -3.800 without trend and -4.201 with trend. The PP test with 3 truncation lags confirmed this with values of -4.136 without trend and -4.112 with trend.

The Johansen cointegration test procedure was then applied to validate the equation and determine if there are in fact two cointegrating vectors. The test was run with the VAR lag lengths at 4 and 2 to test for sensitivity to this factor consistent with the approach taken in section 5.3. The results are presented in table 5.9.

Table 5.9 Johansen Cointegration Test: Trace test LR Statistics for p_t, b_t and d_t.

March 1960-September 1997.

Hypothesised Number of Cointegrating Equations	None	At most 1	At most 2
Likelihood Ratio (VAR=4)	39.76*	13.56	4.49*
Likelihood Ratio (VAR=2)	34.08*	11.05	4.57*
5% critical value	29.68	15.41	3.76

* Significant at the 5% level.

The test result indicates only one cointegrating equation at the 5% level. The cointegrating vector is therefore unique. The normalised cointegrating vector at VAR length 4 was (1,-0.275,-23.095) and that at VAR length 2, (1,-0.242,-25.921) close to the cointegrating regression equation (5.2). This is consistent with Dickey, Jansen and Thornton (1994, p.36) who comment "It is interesting to note, however, that the estimated cointegrating vectors obtained from the Johansen and Engle-Granger approaches are nearly identical when both indicate cointegration.".

Now tables 5.4 and 5.5 showed there was cointegration between p_t and d_t but not between b_t and these two. However the above suggests cointegration between all three. By definition it follows that there exists λ, μ and π such that:

$$u_t = p_t - \lambda d_t \sim I(0) \text{ and } v_t = p_t - \mu d_t - \pi b_t \sim I(0)$$

Thus $v_t = u_t + \lambda d_t - \mu d_t - \pi b_t$ and hence $v_t - u_t = (\lambda - \mu)d_t - \pi b_t \sim I(0)$ and so it follows that b_t and d_t are cointegrated. This contradicts the previous result. As in section 5.4.1 the result of cointegration was therefore checked for the period June 1975 to September 1997 to see if the result for September 1960 to September 1997 remained valid or was simply a Type II error.

Table 5.10 Summary Cointegration Statistics between b_t, p_t and d_t: June 1975 to September 1997

No.Obs	const. coeff.	b_t coeff.	d_t coeff.	CRDW	No. Lags	DF Trend	DF None	Joh. LR
88	2.934	0.433	14.645	0.425	0	-3.394	-3.129	32.536

The residual CRDW from the cointegration does not reject a unit root null nor does the DF test on the residuals. The PP test is the same when the truncation lag is zero. The Johansen test however does suggest cointegration. The cointegrating vector found was (1,-0.345,-25.214) different from that for the cointegrating regression given in table 5.10 of (1,-0.433,-14.645). This contradictory result suggests that the null of no cointegration cannot be rejected.

Mills (1991) extends his analysis to a consideration of the link between the bond rate and the dividend yield. He finds that there is a constant relationship between the ratio of the two, that is, in terms of the levels of the two series (the nominal values):

$$\frac{B_t}{y_t} = \frac{B_t}{D_t / P_t} = C \text{, a constant labelled by Mills ‘confidence’.}$$

This implies that $\mathbf{ln}\, C = \mathbf{ln}\, B_t - \mathbf{ln}\, D_t + \mathbf{ln}\, P_t$ or $\mathbf{ln}\, P_t = \alpha + \mathbf{ln}\, D_t - \mathbf{ln}\, B_t$ is a cointegrating regression if the variables are $I(1)$ and then the cointegrating vector is (1,-1,1). The ADF and PP tests on the variables in tables 5.11 and 5.12 confirmed that the series are $I(1)$.

Table 5.11 ADF Regression ‘Confidence’ series: March 1960-September 1997.

ADF 5% level critical value below		trend and intercept (-3.434)			intercept (-2.878)		none (-1.942)
variable	lag length	α	$(\beta-1)$	γ	α	$(\beta-1)$	$(\beta-1)$
$\mathbf{ln}\, P_t$	0	0.275 (2.535)	-0.062 (-2.465)	0.001 (2.556)	0.017 (0.420)	0.000 (0.004)	0.003 (2.566)
$\Delta\, \mathbf{ln}\, P_t$	0	0.009 (0.707)	-0.947* (-13.14)	0.000 (0.589)	0.016 (2.426)	-0.945* (-13.15)	-0.913* (-12.76)
$\mathbf{ln}\, D_t$	2	0.098 (3.332)	-0.067 (-3.022)	0.001 (3.332)	0.020 (1.490)	-0.002 (-0.575)	0.004 (2.551)
$\Delta\, \mathbf{ln}\, D_t$	10	0.013 (1.379)	-1.010* (-5.061)	0.000 (0.370)	0.016 (3.105)	-1.009* (-5.070)	-0.625* (-3.911)
$\mathbf{ln}\, B_t$	3	0.033 (1.151)	-0.012 (-0.666)	-0.000 (-0.332)	0.036 (1.316)	-0.017 (-1.347)	-0.001 (-0.294)
$\Delta\, \mathbf{ln}\, B_t$	10	0.040 (2.342)	-1.198* (-4.744)	-0.000 (-2.286)	0.003 (0.518)	-1.007* (-4.156)	-0.985* (-4.141)

* Significant at the 5% level.

Table 5.12 PP Test 'Confidence' series: March 1960-September 1997.

critical value	trend and intercept (-3.434)			intercept (-2.878)			none (-1.941)		
variable	1	5	10	1	5	10	1	5	10
$\ln P_t$	-2.562	-2.737	-2.568	-0.031	-0.066	0.966	2.494	2.428	2.796
$\Delta \ln P_t$	-13.14	-13.14	-13.14	-13.15	-13.15	-13.15	-12.76	-12.83	-12.78
$\ln D_t$	-2.233	-2.727	-2.824	-0.416	-0.503	-0.510	3.818	3.096	3.015
$\Delta \ln D_t$	-13.33	-13.67	-13.72	-13.36	-13.71	-13.75	-12.28	-12.97	-13.49
$\ln B_t$	-0.078	-0.262	-0.104	-1.390	-1.469	-1.462	0.022	-0.015	-0.011
$\Delta \ln B_t$	-9.612	-9.571	-9.437	-9.402	-9.400	-9.334	-9.433	-9.430	-9.371

Next $\ln P_t$ was regressed against $\ln D_t$ and $\ln B_t$. The resulting cointegrating regression equation was:

$$\ln P_t = 3.325 + 1.157 \ln D_t - 0.357 \ln B_t + \varepsilon_t \quad (5.3)$$
$$(39.70)\ (44.25)\qquad (-6.752)$$

for $N = 149$ observations with an R^2 of 0.954 and a DW of 0.330.

The CRDW test statistic suggests a failure to reject the null hypothesis of no cointegration. The ADF and PP tests were applied to the residuals. The resulting ADF test with no lagged differences in the regression yielded a test value for the without trend case of -3.668 and -3.670 for the trend case. Therefore the null of a unit root cannot be rejected. In this case with no truncation lags the PP test would give the same result as the DF test.

The Johansen cointegration test was then applied as before.

Table 5.13 Johansen Cointegration Test: Trace test LR Statistics for $\ln P_t$, $\ln B_t$ and $\ln D_t$. March 1960-September 1997.

Hypothesised Number of Cointegrating Equations	None	At most 1	At most 2
Likelihood Ratio (VAR=4)	26.30	4.60	1.22
Likelihood Ratio (VAR=2)	24.14	3.47	0.39
5% critical value	29.68	15.41	3.76

The likelihood ratio test therefore accepts the null of no cointegrating equations. This then confirms the results from the regression residuals two-step procedure. So for Australia, unlike the result of Mills for the U.K., there is no cointegration between these variables. Long run parameter estimates are therefore unlikely to be of value. There is a high R^2 but low DW from the regression (5.3). The series trend together but the regression is potentially spurious. A chart of 'confidence' is given in figure 5.2.

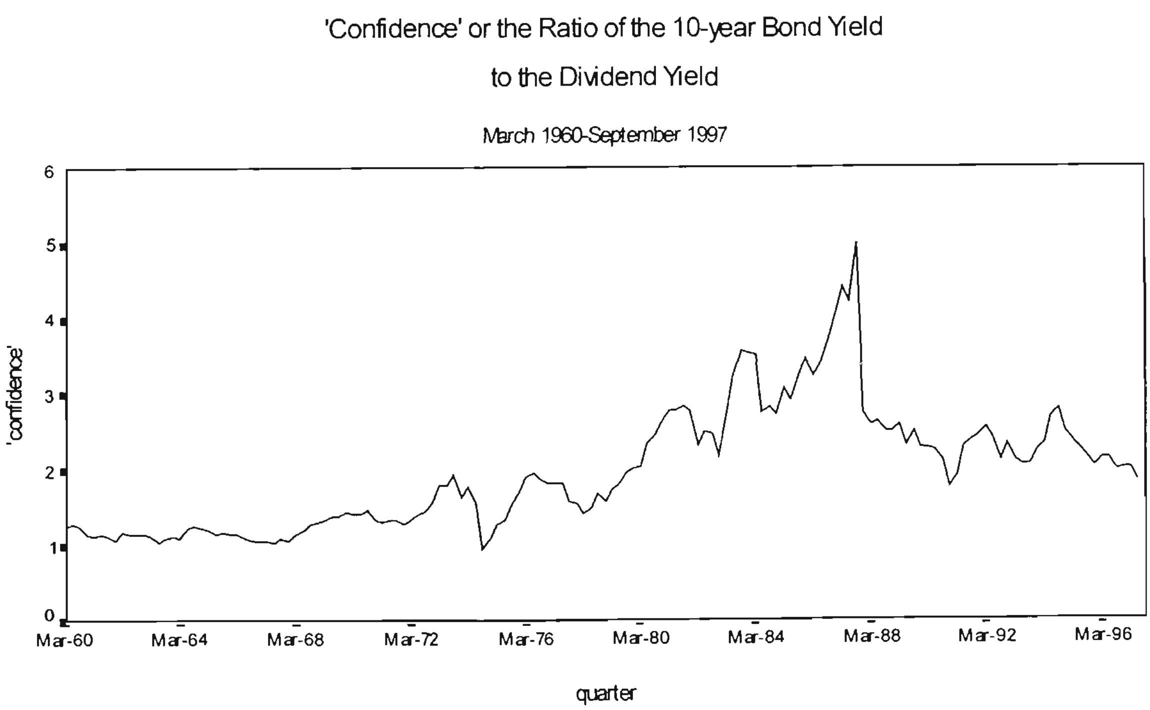


Figure 5.2 Ratio of the Bond Yield to the Dividend Yield or 'Confidence'.

The cause of the very different results can be speculated upon. Mills (1993, p.251) quotes practitioners suggesting why such a relationship *should* exist, "Many investors were accustomed to noting.....what was the difference between the yield on the industrial ordinary share index and that on consols.". Investors switch between shares and bonds according as to yield. In Australia taxation may have played a part. The introduction of the capital gains tax in 1985 and dividend imputation in 1988 shifted the relative attractiveness of bonds and shares. Imputation has made dividends more attractive and led to an increase in the proportion of total return from income.

If there is an equilibrium relationship then the past has not revealed it. Such a fact would give a strong timing signal and make the task of asset allocation and stochastic investment modelling much easier.

5.6 Mulivariate Relationships

A final assessment is to consider all the variables jointly: $\Delta I_t, p_t, b_t, n_t$ and d_t. There were 145 observations, the length of the shortest series. This was done and the cointegrating regression equation was:

$$p_t = 1.333 - 0.789\Delta I_t - 0.335n_t + 0.676b_t + 20.783d_t + \varepsilon_t \qquad (5.4)$$

(0.072) (-2.053) (-2.020) (3.771) (13.070)

for $N = 149$ observations with an R^2 of 0.631 and a DW of 0.453.

The 5% level critical value for the CRDW test is given in Banerjee, Dolado, Galbraith and Hendry (1993, table 7.1, p.209) as 0.68 for $N = 100$ and 0.35 for $N = 200$ observations with 5 regressors. The CRDW test therefore suggests that the null of no cointegration cannot be rejected, so $\varepsilon_t \sim I(1)$ and the series are not cointegrated. For the ADF test applying the recursive *t*-statistic procedure from Mayadunne *et al.* the lag length was set at 0. The resulting test values were -4.326 without trend and -4.329 with trend included. Now the critical values for the ADF test are different for the residuals in a cointegrating regression than those required when testing for a unit root in an individual series. The critical values at the 5% level for $N = 149$ observations and 5 variables given in MacKinnon (1991) are -4.511 without trend and -4.833 with trend. This confirms that the series are not cointegrated. In this case with no truncation lags the PP test would give the same result as the DF test.

The Johansen test results, which considers if there is more than one cointegrating vector, is shown in table 5.14.

Table 5.14 Johansen Cointegration Test: Trace test LR Statistics for $\Delta I_t, p_t, b_t, n_t$ and d_t . March 1960-September 1997.

Hypothesised Number of Cointegrating Equations	None	At most 1	At most 2	At most 3	At most 4
Likelihood Ratio VAR=4	83.81	54.50	31.05	14.96	2.84
VAR=2	70.70	38.64	21.38	10.08	3.92
VAR=1	81.32	40.81	19.86	8.34	2.99
5% critical value	68.52	47.21	29.68	15.41	3.76

Even allowing for the fact that under the SBC criterion the lag length p is small the test still suggests one cointegrating equation when $p = 1$ or 2. The normalised cointegrating vector for $p = 2$ is given by (1,-2.162,2.160,-2.471.-31.472) with the variables in the same order as in the regression (5.4). The vector is different from the vector in (5.4). For $p = 1$ the vector is (1,-10.522,7.037,-7.120,-52.802). This instability suggests that not withstanding the test result the hypothesis of cointegration should be treated cautiously. This is consistent with the vector being different when the two methods disagree as was found in section 5.4. Pagan (1995) brings out one of the difficulties of the cointegrating vector in that any linear combination of the basic vector is also a feasible vector. He gives a simple example of when this can occur and the false conclusions that can be drawn from it. One may therefore obtain some strange looking cointegrating vectors. The hypothesis of cointegration at the 5% level therefore cannot be rejected. The disagreement is noted.

The results of sections 5.3.2 and 5.5 suggested cointegration only between b_t and n_t and between p_t and d_t. This is consistent with no cointegration between all the variables jointly. Otherwise finding a cointegrating vector for the other four variables which would follow as the variables are pairwise cointegrated would mean $\Delta I_t \sim I(0)$. This is not the case though section 5.3.1 discussed the conflicting evidence on the order of integration of ΔI_t. This conflict may be reflected in the results. A result in favour of cointegration would indicate that there is a direction in 5-dimensional space where there is a stationary equilibrium. There exists a direction where a meaningful long run relationship exists between the non-stationary variables. Dickey *et al.* (1994, p.17) in discussing the economic interpretation of cointegration comment " In general cointegrating vectors are obtained from a reduced form of a system where all variables are assumed to be jointly endogenous. Consequently, they cannot be interpreted as representing structural equations...". Pagan (1995, p.34) views cointegration analysis as "...best thought of as another form of data analysis.".

5.7 Conclusions

Cointegration of the financial series has been investigated. There is a cointegrating relationship between the nominal All Ords and the nominal dividend as well as between the nominal bond and T-note rates. These features are captured in the dividend yield and long short ratio. These are both stationary series. There is cointegration between the difference of the CPI, nominal bond and T-note rates, but none between this measure of inflation and nominal dividends. With real rates as defined herein the only bivariate cointegrating relationships remaining are those between real bonds and real T-notes and between the real All Ords and real dividends.

No long run relationship between inflation and nominal or real share prices or nominal or real dividends was found. A test employing 122 years of annual data confirmed this result. This contradicts Wilkie's assertion that inflation is fully incorporated into dividends and hence share prices via the dividend yield. This result in section 5.4 is consistent with that of other authors using Australian data, albeit for different sample periods (see Carter (1991), Harris (1994, 1995) and Crosby (1998)).

Initial results were in favour of cointegration between real share prices, real dividends and real bonds. There was one unique cointegrating vector. The vector found with both methods was similar. However contradictory results from the bivariate analysis and taking a time period defined by a structural break suggests that the null of no cointegration cannot be rejected. Unlike the UK no long run relationship between the dividend yield and the nominal bond rate was found. Such a relationship would give a strong timing signal and make the task of asset allocation and stochastic investment modelling much easier.

There is a complete set of financial series to describe the asset classes of equities, fixed interest and cash. Due to the cointegrating relationships only the dividend yield y_t and real dividend d_t are necessary for equities, and the long/short ratio m_t and real bond yield b_t for fixed interest.

When all real variables were considered together the cointegration result was not clear. The Engle-Granger method accepted the null of no cointegrating relationship. The Johansen method rejected the null but suggested only one cointegrating vector. This instability suggests that not withstanding the test result the hypothesis of cointegration should be treated cautiously. The economic significance of the result even if cointegration were to be accepted is not clear.

Chapter 6

Stationary Series Modelling

6.1 Introduction

A variety of modelling methods have been applied in stochastic investment modelling. Wilkie (1984, 1986, 1992 and 1995) and Carter (1991) used a time-series oriented strategy applying univariate Box Jenkins models. Transfer functions were applied to determine causal links between variables. Harris (1994, 1995) considered and compared a range of methods including vector autoregressive (VAR) models. Sherris, Tedesco and Zehnworth (1996) introduced state space models based upon the Kalman filter. Mills (1991) applied a cointegration methodology to consider the links between equity prices, dividends and bond yields.

Vasicek (1977), Brennan and Schwartz (1982) and Cox, Ingersoll and Ross (1985) have applied stochastic differential equations (SDE), based upon the Ornstein-Uhlenbeck process, to modelling bond prices. The two factor bond model which formed part of the stochastic investment model due to Mulvey and Thorlacius (1997) was a variant of the Brennan and Schwartz model. A description of the stochastic equations was given in Mulvey (1996). Links between individual equations were given by common parameters or variables in separate equations.

The aim of this chapter is to model the equations that describe the short run economic relationships, that is, modelling the six stationary variables. Since the series are stationary the standard techniques of inference in econometric analysis may be applied. The rationale for the chosen methodology is to allow flexibility within a sparse coefficient model structure. This will enable a concentration on the driving forces of financial market returns via scenarios without imposing computational burdens.

The chapter is structured as follows. Section 6.2 deals with general issues in model building reviewing some of the literature and explaining the methodology that has been chosen. It gives context to the endeavour and helps to compare and contrast previous research. The next section 6.3 describes the relationship between inflation and the financial series. The chosen model form, from the wide class of models available within the overall methodological approach and the diagnostic tool kit are then reviewed. The financial time series are modelled with the introduction of inflation as an explanatory variable in section 6.4. A comparison of the OLS regression method with Box-Jenkins transfer functions is made in section 6.5. Transfer functions provide an alternative approach to the standard linear regression techniques and enables a comparison of results with those of other authors. In section 6.6 unexpected inflation is introduced as the independent variable replacing observed inflation. The aim is to determine the role of expectations and to compare the results with those obtained from using observed inflation.

The chapter is concluded in section 6.7 with a discussion of the best equations for the stationary variables in the stochastic investment model, involving the use of qualitative judgments as well as various diagnostics. The system of equations as developed by this stage is then presented.

6.2 Structural Model Building: Methodology and Literature

Reviews of the various methodologies of econometrics may be found in Darnell and Evans (1990), Granger (1991) and Pagan (1991, 1995). Darnell and Evans (1990) provided a comprehensive guide to the schools of thought on methodology in modern econometrics. The compendium edited by Granger (1991) included articles by a range of authors, some of which are the source articles outlining the differing methodologies. At the risk of some simplification there are three major approaches. These three differing methodologies are reviewed and contrasted by Pagan (1991, 1995).

Firstly, the traditional or Cowles Commission approach is to model a set of equations, defined by economic theory, simplify it as much as possible then use a range of diagnostics and tests. This is the 'general-to-specific' method of Hendry. This view, representative of the London School of Economics (LSE), is expounded in Hendry and Mizon (1991). They distinguish between discovery or how one finds a model, and justification or how one validates it. They are less concerned with how one finds a model; it doesn't matter if its luck or intuition that gets you there. The real test is whether the model stands up to scrutiny vis-à-vis alternative models. They see the critical process of testing as being essentially destructive in that it aims to eliminate unsatisfactory models. This process then defines a necessary but not sufficient set of conditions for model acceptance. The authors then go on to expand these ideas under four headings: 'fragile' models and their value, the nature of discovery, the nature of critical evaluation and the nature of the research process. The essence of their approach is a pragmatic one. It does not matter how a model is found; it is its usefulness that counts. It must then be rigorously tested, as in the physical sciences, by within sample tests. This is the nature of research progress.

Pagan contrasted the general-to-specific method with the Bayesian method, the chief proponent of which is Leamer. Granger defined Bayesians as those who "... add to the usual information sets by using their own beliefs, knowledge and accumulated experience in a precise way via prior probabilities." (Granger (1991, p.193)). He added that all modellers are informal Bayesians and it is personal choice whether or not to become a formal one. Leamer (1991) argued that one needed prior views. He claimed that "A fact is merely an opinion held by all, or at least held by a set of people you regard to be a close approximation to all." (Leamer (1991, p.38)). Hence one must study the nature of these priors in a systematic way.

The third methodology is that of Sims. Sims (1991) takes the view that equations within a system are in fact interdependent, rejecting the use of a series of single equation studies in favour of the vector autoregressive or VAR methodology. The essence of Sims' criticism of the standard approach to macroecomomic model building is that the simplifying assumptions built into such large scale models are 'incredible' to use Sims' expression. Sims feels that such assumptions are both distorting and unnecessary at the same time. They are distorting because there are elements of feedback and response involved in all macro aggregates, with various leads and lags involved, and he argued this has important impacts on actual policy formulation. He constructed the VAR with a given lag length, using transformed data if necessary, then simplified the VAR and used the model to address questions of interest. An overview of the VAR methodology with an application to GNP for the US, Japan and Germany was provided by Canova (1995).

Pagan notes that a major difficulty of the VAR method is that it uses many degrees of freedom because of the number of parameters that need to be estimated and thus restrictions are placed on the lag length. These restrictions placed upon the system make Sims' claim of being prior free somewhat illusory. Pagan finds that none of the methodologies is necessarily the best, each delivering something and revealing insights into the system under study. He concludes that "Our data are such that we cannot ignore the fact that information therein may need to be extracted by a wide range of techniques borrowed from many different approaches."(Pagan (1991, p.118)).

Other papers in the Granger compendium include that by Spanos (1991), covering the history and development of the subject, and Fair (1991). Fair outlines his methodology under five headings: data collection, unobserved variables, specification, estimation and testing. The importance of the data and the role of testing via simulation, in the overall back and forth movement from theory to results are emphasised. He concluded with a general discussion on the role and position of theoretical models ("...theories being 'useful' or 'not useful' ") and what one may expect in the long run of various models.

An attempt could be made to build sets of equations which are all encompassing and inter-linked in a strong way. Some of the large models of the Australian economy attempt to do this, by using interacting simultaneous equations. Methods such as the VAR method of Sims, impose a similar structure on sets of equations. There lag structures of low order are imposed and vectors of variables are used to define relationships.

Harris (1994, 1995) reviewed the performance of various stochastic investment models for long term studies. Harris defined his own Exponential Regressive Conditional Heteroskedasticity (ERCH) class whereby the conditional variance was expressed as a linear combination of lagged exogenous and dependent variables (Harris (1994 , p.48)). A VAR model was included. In Harris (1995) he ranked them using various criteria. The measures he reviewed for the ranking were those of goodness of fit, parameter stability and predictive ability for returns and volatilities. He found that using his ranking criteria his ERCH model was the best. The Wilkie model was unacceptable, showing poor results, for example, for the 10-year bond. Harris found that the performance of the VAR model was poor. He suggested that this was due to over-parameterisation.

Complex models such as given by the VAR methodology run the risk of not providing the degree of stability required for long term forecasting. Over-parametrisation as found by Harris is a potential cause of instability. The aim is to be consistent and focus on ensuring a sound process rather than assuming the models reflect the 'true' DGP. The method pursued in this document is therefore in line with the pragmatic general-to-specific method of Hendry and Mizon, the LSE tradition in model building.

As outlined in Chapter 1 in most practical applications, for example asset and liability studies for pension funds or insurance companies in their claims reserving processes, inflation plays a key role. The basic assumption in an asset and liability study is that inflation is the driving force. Inflation drives the value of assets through nominal returns as it does liabilities through, for example, average weekly earnings. Inflation is therefore a potential exogenous variable in the proposed stochastic investment model equations. It need not necessarily enter the equations. It will only do so if it is demonstrably significant; it 'cascades' through the model. It may be postulated, as does Wilkie, that in the long run inflation will find its way fully into the income stream of assets and hence their value. This is equivalent to saying that a 1% change in the rate of inflation will lead to a 1% change in dividends but with time leads or lags. However, in section 5.3.1 the cointegration tests showed that at the 5% level of significance there was no long run connection between the difference in the CPI and nominal dividends. The short run dependence of dividends and the All Ords on the rate of inflation will also require investigation.

The univariate models outlined in chapter 4 will therefore be augmented to include the effect of inflation. There are various modelling methods which can be applied. All forms of model building can help understand the data. Some are better for certain sets of circumstances than others, but no one approach is necessarily the 'correct' one. This is consistent with Pagan's view. There are a number of criteria to be applied in assessing any potential model.

1. *Parsimony.* A situation of fewer parameters rather than more is preferred. The noise in the data makes more complex model building difficult. Parameter instability or unreliable in-sample forecasts can arise from over-specified models.
2. *Stability.* The coefficients in any model should be stable over time. The coefficients should also not be substantially affected by individual observations or outliers.
3. *Economically sensible.* The model must be plausible and fit the empirical evidence. It must appear reasonable in the context of the financial markets. Clients of any stochastic investment model such as trustees need to believe that the system makes sense.

In addition one might include other desirable features. For example, does the model have sensible long and short run features; can it be made consistent with a continuous model; are the right series stationary in the long run.

The goal is models which are parsimonious and stable. A virtue is not made of sparseness. Only strong relationships can be take into account, defined by such characteristics as goodness of fit or degree of significance. A set of simple equations which can describe the financial process is sought. By constructing the model in this way it is easier to alter one of the 'modules'. If differing relationships are found the particular part of the model may be changed without the need to re-calibrate the whole. This approach provides the flexibility to add or subtract asset classes.

The model structure presumes a good approximation to the DGP. In practice this runs up against a number of problems.

1. The Slutzky-Yule effect or the realisation that apparently systematic fluctuations can be generated merely as the average of random events. Slutzky[1] (1927) in a classic paper showed that what appear to be cycles are merely the result of averages of random events. Slutzky was able in his paper to mimic an actual trade cycle of the 19th century by the use of a moving average process. Many features that are observed and recorded including financial time series are the result of the sum of many events which are themselves random. Kendall and Ord (1990) provide a discussion of this effect and its consequences.
2. Given a random series a pattern can often be found after extensive searching. This is termed data mining in the literature.
3. Structural breaks in the time series. For example in a high inflation period, certain patterns or trends could be deduced which may not be appropriate at all in a low inflation period. Evidence on this may be found in Stock and Watson (1991) and Perron (1994). Stock and Watson reviewed variable trends in US real GNP, discussing whether cycles are independent of the changes in growth trends. Perron's paper looked directly at the issue of structural change in macroeconomic time series. He investigated a range of 11 countries finding structural breaks in real GNP for all of them. This raises questions as to the linkeage between structural breaks and economic events.

6.3 Modelling with Inflation: Data Analysis and Model Choice

In chapter 4 univariate models of the stationary variables were developed; causality was not discussed. As an initial step in data analysis, correlations between the exogenous and endogenous variables are required to gain an appreciation of any lag

[1] Slutsky (1927, p.123) summarises his results thus "The summation of random causes generates a cyclic series which tends to imitate for a number of cycles a harmonic series of a relatively small number of sine curves. After a more or less considerable number of periods every regime becomes disarranged, the transition to another regime occuring sometimes rather gradually, sometimes more or less abruptly, around certain critical points.".

structures. This can be done with the use of the *cross correlation function* (ccf). The cross correlation function at lag k is defined by:

$$\rho_{u,z}(k) = \frac{\mathbf{cov}(u_t, z_{t-k})}{\{\mathbf{var}(u_t).\mathbf{var}(z_{t-k})\}^{1/2}}, \quad k = 0, \pm 1, \pm 2, \ldots\ldots,$$

This is a logical extension of the autocorrelation function. In general it is not symmetric, that is $\rho_{u,z}(k) \neq \rho_{u,z}(-k)$.

Firstly the correlation matrix between inflation and the variables was found. This is the ccf at lag 0 and is shown in table 6.1 with p-values in brackets.

Table 6.1 Correlations coefficients between i_t and Financial Variables: Time Period Maximum of the Available Data[2]

y_t	Δp_t	Δd_t	m_t	Δb_t	Δn_t
0.279	-0.203	-0.112	-0.323	-0.364	-0.256
(0.000)	(0.004)	(0.120)	(0.000)	(0.000)	(0.001)

Secondly the full ccfs were found. Initially the ccf for each variable versus i_t for lags of $-12,\ldots\ldots,12$, that is 24 quarters or 6 years, was determined. This was considered sufficient for the impact of inflation to have worked its way through the system. It was found that values at longer lags and leads were not significant, hence only values up to lags of ± 7 are presented in figure 6.1. There are complex lag structures evident. The table is to be interpreted where i_t is leading or lagging, so for example figure 6.1 (e) is interpreted as saying that $corr(\Delta b_t, i_{t-4}) = 0.390$.

[2] The equity data is quarterly 1948-1997; the fixed interest data quarterly 1960-97. Hence there are more data points for the correlations for the equity series; the maximum available has been used.

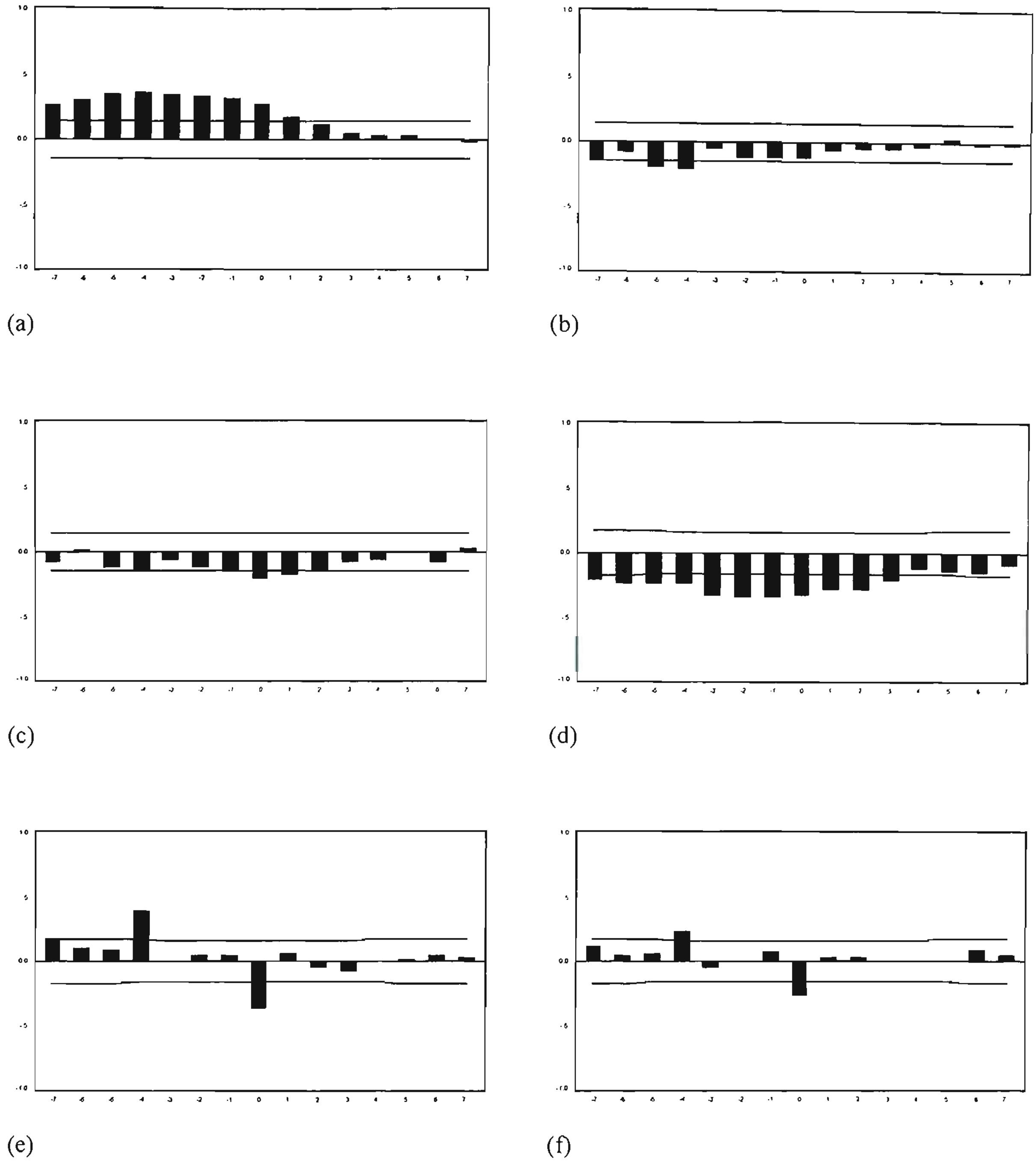


Figure 6.1 Ccf for equity series: (a) y_t; (b) Δd_t and (c) Δp_t and for fixed interest series: (d) m_t; (e) Δb_t and (f) Δn_t. Values from -1.0 to +1.0 and lags -7 to +7. Confidence limits given by the lines parallel to the x-axis.

For Δd_t, figure 6.1(b) the negative impact of a rise in inflation is delayed, lags 4 and 5 are significant, and lags 0, 1, 2 and 7 have values close to significance. Now nominal dividend payments are a trailing entity since any change will not be observed until the next dividend is declared. They are also slow to change due to the action of company boards smoothing dividend payments to shareholders. This is a feasible explanation as to the non-significant result for Δd_t at lag 0 in table 6.1. It may also be speculated that the adverse impact of inflation on the earnings power of a given ordinary share, and hence dividend, is delayed as the impact of inflation works its way through company operations. The impact is felt through aspects of working capital management such as receivables and pricing delays. Industrial companies typically have limited ability to rapidly pass through price increases. Other factors may be at work and more complex interactions make it difficult to delineate the inflation impact.

The correlation at lag 0 in table 6.1 for Δp_t, is significant at the 1% level. For Δp_t in figure 6.1(c) there are significant values at lag 0 and at lead 1. Values otherwise die away after 4 quarters. The cause of the lead value is a matter of speculation. The market may anticipate a poor inflation outcome. This is hard to reconcile with figure 6.1(e) and (f) where the bond market shows no significant leads over inflation.

The dividend yield y_t in table 6.1 has a significant *positive* correlation with the rate of inflation. Both Δd_t and Δp_t respond negatively and in the case of Δp_t significantly so. Nominal dividends are slower to change as discussed above. Figure 6.1(a) shows the ccf for y_t. The strong positive autocorrelation comes out in the ccf. The strongest effect is with inflation lagging but there are also significant terms when inflation is leading. This slow adjustment to changes in inflation is a reflection of the autocorrelation.

The correlations in table 6.1 for Δb_t and Δn_t are as expected, the correct sign and significant. In figures 6.1(e) and (f) are the ccfs of Δb_t and Δn_t. For Δb_t in figure 6.1(e) significant values are visible at lags for quarters 0 and 4. Now inflation causes changes in real interest rates but nominal interest rates are also a component of the CPI calculation, for example directly via mortgage interest rates. There would not appear to be any element of feedback as there are no significant leads. The ccf in figure 6.1 (e) is interpreted as saying that $corr(\Delta b_t, i_{t-4}) = 0.390$. Hence there is an initial negative impact of an increase in the rate of inflation followed by a later positive change. The economic justification for this is that after the initial negative impact real rates fall, then the market adjusts the real interest rate to compensate for the negative surprise and builds in extra compensation for the added risk. It follows from this that positive surprises would be expected to reduce the level of the real interest rate and is a plausible explanation as to why low inflation is seen as such an important goal of economic policy. This makes sound economic sense. Figure 6.1(f) for Δn_t shows an identical pattern to the ccf for Δb_t, with the same economic logic applying.

For the ratio of longs to shorts m_t in table 6.1 the correlation is negative. Whilst both long and short real interest rates respond negatively to inflation at lag 0 the ratio of the nominal values goes down. A tightening of short term interest rates as a response to a rise in inflation leads to a more inverse yield curve. This would seem *a priori* a reasonable response. The ccf for m_t exhibits similar characteristics to that of y_t. The ratio has a strong positive autocorrelation and this is apparent in figure 6.1(d).

From a modelling viewpoint the autocorrelation effect needs to be separated from the effect of the rate of inflation. The dividend yield and long/short ratio are derived from the $I(1)$ series that form the ratios. These $I(1)$ series are cointegrated and so there is a long run relationship between them. There are also short run relationships. Δb_t and Δn_t are correlated as are the levels[3] series b_t and n_t.

The modelling needs to take into account the potentially complex responses of the variables to changes in inflation. There are a wide range of techniques that may be applied.

1. Standard linear regression may be applied under the assumption that only a few lags are significant. For Δb_t and Δn_t only lags 0 and 4 are significant. In the case of Δd_t only lags 4 and 5 need be included, though it is less clear cut than in the other two cases.

[3] As these series are being treated as $I(1)$ the usual correlation theory is not appropriate. These series are cointegrated. See the discussion in Chapter 5.

2. Standard linear regression could be applied utilising the general-to-specific methodology. The models would be reduced down to significant lags satisfying the necessary diagnostics. 1 is a special nested case of 2.
3. The lag structure could be specified *a priori*. There are many possibilities in the literature. A Koyck lag structure could be employed with an exponentially declining set of weights (see Johnston (1987, p.346-352)). Alternatively some other polynomial representation for the lag coefficients which sidesteps the multi-collinearity issue could be utilised, such as Almon variables. The ccfs indicate that such a set of exponentially declining weights is unlikely to fit the data. These methods are unnecessarily restrictive.
4. The series could be modelled employing a time series approach. Transfer function modelling provides an alternative method of estimating the parameters. The methodology for this is outlined in detail in Box and Jenkins (1976, Part III). This is attractive since in terms of their terminology the model is of the form $y_t = \frac{\omega(B)}{\delta(B)} x_{t-b} + N_t$, where the orders of $\omega(B)$ and $\delta(B)$ are r and s respectively and the delay parameter is b. In the case of Δb_t for example, a value of $b = 0$ and r and s with low values would appear a sensible solution.
5. The Kalman filter as an engineering style adaptive response solution could be introduced. Harvey (1993) or Cuthbertson *et al.* (1992) discuss this approach to econometrics in some detail. In the Australian context Sherris, Tedesco and Zehnwirth (1996) employed the Kalman filter and *state space* models. The state space model consists of measurement and transition equations which allow for time varying parameters. The recursive equations can project forward one step ahead. Beyond that an idea of how the parameters are changing in the long run is needed. The Kalman filter will not be pursued.

Modelling methods 2 and 4 are the preferred options. At a later stage long run effects or levels will also be investigated as well as the links between the series of single equations.

6.4 Linear Regression Modelling

The first of the two chosen approaches is standard linear regression. In this section there are six sub-sections each devoted to modelling the dependence of each of the six variables on the rate of inflation. These sub-sections are prefaced with a discussion of methodology and diagnostic testing.

The models considered in this section are of the general linear form[4]:

$$z_t = \alpha_0 + \sum_{i=1}^{n} \alpha_i z_{t-i} + \sum_{i=0}^{m} \beta_i i_{t-i} + \varepsilon_t \qquad (6.1)$$

for a given time series z_t and where $\varepsilon_t \sim N\left(0, \sigma^2\right)$. The method could be extended to include extra explanatory variables. Here two exogenous variables[5], expected and unexpected inflation are considered, since as discussed in Chapter 1 the rate of inflation is central to the financial reserving process. All the preferred univariate models in chapter4 involve only AR parameters. The model (6.1) is called an *autoregressive distributed lag* model denoted ADL(n,m). The ADL general model form (6.1) is employed and a Hendry style general-to-specific methodology applied. The univariate models of chapter 4 yield a benchmark in terms of autoregressive lags and model fit. The methodology is the same for all the variables, in that a backward approach of gradual elimination of non-significant variables is taken. The time periods for which data is available vary. Rather than lose a lot of data points by shortening all series to the shortest, models have been fitted to the available data. The value and appropriateness of a particular model is judged by the diagnostics and parameter stability. Structural breaks can then be determined by a combination of economic and econometric analysis. There are 196 observations for the equity series with the loss of 1 observation on differencing. For the bond and T-note series there are 153 and 151

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[4] This can be thought of as a linear transfer function model in the Box-Jenkins terminology, of the form $\Phi(B)z_t = \Omega(B)i_{t-1} + N_t$, where $\Phi(B)$ and $\Omega(B)$ are lag polynomials and N_t is a disturbance or noise term.

[5] Other exogenous variables are possible, Harris (1994) discusses this. The range of potential variables is wide. Inflation is however central to financial reserving, which is recognised in the stochastic investment model of Wilkie for example, where inflation is treated as an endogenous variable.

observations respectively with 152 for the long short ratio, with differencing again causing the discrepancies.

Standard diagnostics were reviewed including the R^2. The Durbin Watson (DW) statistic to test for serial autocorrelation in the residuals, defined by equation (4.9) is given. Since there are lagged values of the dependent variable in the regression the standard DW test requires modification. To deal with lagged values of the dependent variable in the regression Durbin modified the test statistic to provide Durbin's *h*-test (see Fomby *et al.*(1984, p.244-245)). Inder (1982) investigated the effectiveness of the DW test in the presence of lagged dependent variables with the use of simulation techniques. In particular Inder was concerned with the power of the test with respect to Durbins *h*-test. He concluded that "...the DW test is generally more powerful than the *h* test..... The only problem with the use of the DW test is in obtaining critical values for the statistic." (Inder (1982, p.184)). The Inder findings suggest that the DW test can still be used provided that the appropriate critical value is chosen. Johnson (1987, p.314-317) discusses this issue suggesting that the upper bound d_U be simply used as the critical value and the null hypothesis is rejected if the calculated value $d < d_U$. The DW test statistic may then be compared with the top end of the range[6].

The Breusch-Godfrey LM test provides an alternative test for serial correlation (see Stewart (1991, p.168-170) or Cuthbertson *et al.* (1992, p.108)). This is valid in the presence of lagged dependent variables. The test is applicable whether the residuals follow an AR(p) or MA(q) model. If the error process follows an AR(p) model where the residuals are from fitting (6.1) so that $\varepsilon_t = \rho_1\varepsilon_{t-1} + \rho_2\varepsilon_{t-2} + \ldots. + \varepsilon'_t$ then the null is $\rho_i = 0$ for all i. The following auxiliary regression is run:

$$\hat{\varepsilon}_t = r_1\hat{\varepsilon}_{t-1} + r_2\hat{\varepsilon}_{t-2} + \ldots.. + \alpha_0 + \sum_{i=1}^{n} \alpha_i z_{t-i} + \sum_{i=0}^{m} \beta_i i_{t-i}$$

[6] For the sample size varying between 100 and 200 observations the critical value of d_U at the 5% level of significance lies between 1.69 and 1.78, when there are two regressors and between 1.74 and 1.80 when there are four regressors.

where $\hat{\varepsilon}_t$ is the residual without serial correlation. Then the test statistic is $LM(p) = NR^2 \sim \chi_{(p)}{}^2$, where N is the number of observations. Identical logic applies for an MA(q) error term.

In addition the acf of the residuals was checked for significant values. For the univariate models the Box-Ljung statistic $Q(12)$ was found. However Godfrey (1988) points out that the LM test is valid when the regressors include both exogenous and lagged dependent variables. The Q test is not. The Q test is valid if the regressors are either all exogenous variables or all lagged values of the dependent variable. In comparing the two diagnostic tests Godfrey (1988, p.122) comments "However, even if the null specification corresponds to one of these two special cases, the Q test may well be markedly inferior to LM/score tests.". These comments suggest the need to apply the LM test with different lags. As the data is quarterly the test with 4 lags is needed. As well the test with 12 lags is required to consider a potential build up in serial correlation. Hence $LM(4)$ and $LM(12)$ values are found. For the purely autoregressive models in chapters 3 and 4 the residuals showed no significant serial correlation.

The possibility of multicollinearity must also be investigated. In the case of estimating a linear equation involving lagged independent variables there are diagnostic tools that are available. This topic is investigated at length in Fomby *et al.* (1984, Chapter 13, p.283-306). The nature of multicollinearity is that one or more of the regressors are linearly related. This means that the coefficient estimates are likely to have less precision, since the coefficients will have a relatively large standard error and hence wide confidence limits. The method of determining multicollinearity is to derive the characteristic roots or eigenvalues of the matrix X of observations and then perform a variance decomposition across the estimates associated with each eigenvalue. Fomby *et al.* (1984, p.293-296) give a detailed explanation and derivation of the formulae. The relative size of the eigenvalue is then compared with the proportion of the variance associated with each estimate for that particular eigenvalue. Letting $v_i = \left(v_{i,1}, \ldots . v_{i,p}\right)$ be the eigenvector associated with eigenvalue λ_i with p

explanatory variables and $\Phi_{ij} = v_{ij}^{2} / \lambda_i$ and $\Phi_j = \sum_{i=1}^{p} \Phi_{ij}$, then the variance decomposition proportion for the j th regression coefficient associated with the i th component is defined as $\pi_{ij} = \Phi_{ij} / \Phi_j$.

One way of dealing with multicollinearity is to add more data. The equations can also be estimated over different periods to see how stable the coefficients are. Structural breaks, if there are any, can provide such varying periods. The study by Stock and Watson (1991) considered the way trends in macroeconomic series vary and identified changes in growth trends. Their work on US GNP suggests there are changes in long run trends and these are difficult to distinguish from business cycles.

Next potential breakpoints require consideration. Chapter 3 identified a structural break in 1947 (see section 3.3). The breakdown of the Bretton Woods system in 1972 may be postulated as another breakpoint (see section 4.3 for a preliminary discussion of this structural break)[7]. The change in 1972 to the convertibility of the US$ into gold at a fixed rate of 32US$ per ounce devalued the US$. One result was to increase the world price of oil, and eventually the gold price rose more than tenfold. The resulting price changes caused by the increase in the price of oil boosted the rate of inflation. The Australian economy, like most of the rest of the developed world, entered a sharp recession combined with steep rises in interest rates. The Australian equity market plunged and real interest rates became negative as yields struggled to rise as fast as inflation. This external shock forced permanent changes to the operation of the Australian economy as agents came to terms with the much changed pricing structure. The new pricing regime potentially altered many of the factors driving asset prices. Hence it is reasonable to expect that a new regime was created.

For the dividend yield figure 4.4 suggests a breakpoint of March 1973. This breakpoint which is consistent with the preceding discussion is then applied to the

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[7] Spanos (1991) gives an overview of the development of econometrics over the last 50 years. He notes the breakdown of even some of the best empirical relations during the early 1970's caused by the oil shock and the end of Bretton Woods.

other equity series. This divides the data set approximately in half. The second portion includes the more reliable and consistent data set.

For the fixed interest series a breakpoint of March 1975 was chosen as this was when real interest rates reached a low point. The stability of the 1960's was replaced with rapid increases in inflation (see figure 3.2). Real rates climbed steeply from that time before settling back in the 1990's as shown in figures 4.5 and 4.6. This breakpoint is again consistent with the preceding discussion of the underlying economics. The second portion also includes the majority of the more reliable and consistent data set (see section 4.2).

Models for each variable were fitted to the full period for which data was available. Then the period was divided by the respective breakpoints and models fitted to the resulting portions. This was done to test for the stability of the coefficients. By fitting models to various periods better judgements may be applied as to the structural equations. It also gives a range of other diagnostics to consider the merits of the model as a whole. Recursive least squares tests are available and are an important tool for testing for structural change in the model. The most useful of the recursive least squares tests is the recursive coefficient estimates which yields a plot of the selected coefficients. This traces the evolution of the coefficients as more of the sample data is used in the estimation.

Stability tests are available in the econometric literature. The Chow Breakpoint test, a test of parameter stability between different periods or categories and its associated test, the Chow Forecast test are employed. The Chow Breakpoint test statistic is given by the formula:

$$F = \left[\left(S_R - \sum S_i\right) / (m-1)k\right] / \left[\sum S_i / (n - mk)\right] \qquad (6.2)$$

where there are k regressors, m categories and n observations in total. S_R is the sum of squares of the residuals for the entire period and S_i is the sum of squares of the residuals for the regression on data in category i. This test statistic follows an F-

distribution with $(m-1)k$ and $(n-mk)$ degrees of freedom. Stewart (1991, p.79-88 and p.101-102) provides a detailed discussion of both the forecast and breakpoint tests. The forecast test formula is almost identical to (6.2). In using the forecast test the last 2 years or 8 quarters data were kept for forecasting purposes, that is the period September 1995 to September 1997. Here the test statistic follows an F-distribution with g and $(n-k)$ degrees of freedom, where g is the number of periods ahead for forecasting.

For completeness it is useful to mention an alternative approach to modelling structural breaks that has been developed; the use of regime switching models. This topic is well covered in texts such as Tong (1990), Granger and Terasvirta (1993) or Hamilton (1994). The approach is that the different mean levels may be caused by a time dependent process. That is, the interaction effects of economic variables in different financial eras produce different results in the time series. Economic agents learn; thus responses to the same event may be significantly different in different times; thus interest rate rises may potentially be more effective in a subdued or pessimistic time than in a more buoyant period. Therefore the model parameters will be different during the successive eras. Further the transition between these eras is likely to be over a reasonable time period. That is, some form of relatively smooth transition is in order.

Hence a further refinement is to allow a smooth transition from one state to another via the use of the exponential or logistic functions. These models are highly complex and researchers have simplified their assumptions by restricting their attention to, for example, two-state models. This gives us the smooth transition autoregressive model (STAR) of the form:

$$z_t = \alpha f(z_{t-2}) z_{t-1} + \varepsilon_t \ ,$$

where $f(y)$ is a smooth non-decreasing function, such as the logistic function.

Another alternative is the switching model with a sharp or sudden change. This then embeds the process in a Markov chain of transition probabilities (rather than some other function). An example is Tyssedal and Tjostheim (1988) where they introduce a chain of transition probabilities between various states described individually by AR(1) processes with different parameters. That is:

$$z_t = \theta_t z_{t-1} + \varepsilon_t \quad , \qquad t = 0, \pm 1, \pm 2, \ldots..$$

and the $\{\theta_t\}$ are a Markov chain. They use this approach on the IBM stock price data from Box and Jenkins and use it to identify change points in the stock data which correspond to economic events.

Another example of the switching regime approach is given by Lahiri and Wang (1996). The authors consider the utility of three measures of the spread in the term structure of interest rates as predictors of the business cycle. They use the spread between 10-year bonds and the bill rate, 10-year bonds and the 1-year rate and the spread between the bill rate and the commercial paper rate at six months maturity. They use a two state Markov switching regime which differentiates between the expansion side of the cycle and the contraction side. They solve for the parameters and then test the predictive power of the model by reviewing various turning points in the business cycle.

A discussion of the issues and some available tests for heteroskedasticity and normality of the residuals were covered in section 3.5. The results in sections 4.3.4 and 4.4.4 for univariate models did not show encouraging results using a range of ARCH, GARCH and EGARCH models as an explanation of heteroskedasticity. The strategy applied in this chapter is to find suitable models, which are then compared on the criteria outlined earlier in this section, then test the preferred model for normality and heteroskedasticity. To the extent that the models of chapter 4 are preferred models, as indeed proves the case for some series, the results of sections 4.3.4 and 4.4.4 apply. This assumes the OLS estimators are not unduly affected by these factors.

The OLS estimates were performed with the heteroskedasticity consistent covariance method due to White (1980).

The maximum lag in the ADL equation (6.1) was set at 5. This yields a model of the form:

$$z_t = \alpha_0 + \sum_{i=1}^{5} \alpha_i z_{t-i} + \sum_{i=0}^{5} \beta_i i_{t-i} + \varepsilon_t \tag{6.3}$$

where $\varepsilon_t \sim N(0, \sigma^2)$. Each table for the results of each variable has a column with the lag above it for the significant autoregressive components followed by significant lags for the independent variable. In some cases the column below appears empty. This is because it is not in the regression. This signals some instability in the coefficients. The time period of the estimation is given at the start of the respective row in the form; year then quarter. *T*-values or *p*-values where appropriate are given under the respective coefficients or test statistics in italics. As in the univariate case both the dividend yield and long/short ratio were mean adjusted prior to the regression as was inflation. A circumflex over the variable indicates mean adjustment. The ultimate equations can be found by a suitable re-transformation.

6.4.1 The Dividend Yield

Table 6.2 ADL(5,5):Regression of the Dividend Yield on Inflation*

qtr:yr	$\hat{y}_{t-1}$	$\hat{y}_{t-2}$	$\hat{y}_{t-4}$	$\hat{y}_{t-5}$	$\hat{i}_t$	$\hat{i}_{t-2}$	R^2	DW	$LM(4)$	$LM(12)$
Sep48	.922		-.119		.081		0.790	1.887	4.769	13.41
Sep97	*21.97*		*-2.88*		*3.29*				*.312*	*.340*
Sep48	.853				.085		0.783	1.704	10.19	20.14
Sep97	*25.04*				*3.49*				*.037*	*.064*
Sep48	1.303	-.370				.032	0.893	1.993	5.88	15.57
Ma73	*13.28*	*-3.74*				*2.06*			*.208*	*.212*
Jun73	.811			-.127	.209		0.790	1.927	0.711	6.493
Sep97	*14.71*			*-2.47*	*4.19*				*.950*	*.889*

* A circumflex over the variable indicates mean adjustment.

The regressions results are in table 6.2. The first order AR coefficient dominates. The model forms are otherwise unstable. The sum of the AR coefficients is 0.803 for the full period (row 1), and for the first and second portions 0.933 (row 5) and 0.684 (row 7) respectively. The model for the whole period was re-run without the lag 4 dependent variable as in table 6.2, row 3 to see if a simpler form was acceptable. The *LM*(4) test statistic for this model was 10.19 with a *p*-value of 0.037 thus indicating serial correlation in the residuals. The value of $LM(12)$ was 20.14 with an associated *p*-value of 0.064 just inside the 5% limit. The lagged inflation component is questionable. It is small and unstable between time periods. The coefficients vary significantly in terms of order of magnitude. This compares to the earlier univariate result equation (4.1) of an AR(2).

This is a process dominated by the AR terms. The high *t*-values for the first order AR coefficient and high R^2 in the univariate case all point towards this result. Using the recursive formula (3.5) for R^2 for an AR process, $R_k^{\ 2} = \phi_{kk}^{\ 2}\left(1 - R_{k-1}^{\ 2}\right) + R_{k-1}^{\ 2}$

where ϕ_{kk} is the kth order partial autocorrelation and with $\phi_{11} = 0.871$ and $\phi_{22} = -0.177$, yields $R_1^{\ 2} = 0.759$ and $R_2^{\ 2} = 0.766$ (see section 4.3.1). This suggests a contribution to R^2 of 0.031 for the combined second AR and inflation terms.

A check of the multicollinearity diagnostics for the full period model is shown in table 6.3.

Table 6.3 Multicollinearity Diagnostics: Regression Table 6.2, rows 1 and 2.

		Variance Proportions		
Variable	Eigenvalue	$\hat{y}_{t-1}$	$\hat{y}_{t-4}$	$\hat{i}_t$
$\hat{y}_{t-1}$	1.614	0.195	0.183	0.034
$\hat{y}_{t-4}$	0.986	0.001	0.064	0.873
$\hat{i}_t$	0.399	0.805	0.752	0.093

There is a small eigenvalue associated with the inflation term with large variance proportions for the lagged dependent variable, indicating problems with the inflation term. This is consistent with the instability in inflation terms observed between the first and second time periods, rows 5 and 6, and rows 7 and 8 respectively. The Chow test for the full model gave an F- value of 5.261 (p = 0.002) using March 1973 as a breakpoint. This shows that the coefficients are unstable. This compares to the result with a March 1973 breakpoint for equation (4.1) with a Chow breakpoint test value of 1.067 (p = 0.346). This latter result was confirmed when the forecast test was applied with an F- value of 0.234 (p = 0.984). This problem combined with evidence of multicollinearity suggests keeping to the AR(2) form, equation (4.1). This does indicate some of the practical issues involved in analysing noisy series. All the analyses will be reviewed in the conclusion.

6.4.2 Real Dividends

It was much more difficult to obtain a satisfactory model for Δd_t. The results are shown in table 6.4.

Table 6.4 ADL(5,5):Regression of the Real Dividend on Inflation

qtr:yr	Δd_{t-2}	$\hat{i}_{t-3}$	$\hat{i}_{t-4}$	$\hat{i}_{t-5}$	R^2	DW	$LM(4)$	$LM(12)$
Sep48	.208	.007	-.006	-.007	0.113	1.978	4.131	8.461
Sep97	*2.917*	*2.424*	*-2.086*	*-2.303*			*.389*	*.748*
Sep48				-.007	0.032	1.871	4.387	10.217
Mar73				*-2.448*			*.356*	*.597*
Jun73	.278				0.105	1.765	4.207	19.433
Sep97	*2.981*						*.379*	*.079*

For the full period the inflation parameters are small but significant. The inflation parameters are however not stable and those lags which are significant vary. For the first portion only inflation at lag 5 is in the equation. The R^2 value is very low. For the second portion there are no inflation terms. A single AR term remains the value of which is similar to the value of 0.219 found for equation (4.4) for the full period univariate equation. The multicollinearity diagnostics indicate problems with inflation at lag 5 with a small eigenvalue 0.327 and large variance proportions, 0.840 being associated with $\hat{i}_{t-3}$. The Chow test applied with a March 1973 breakpoint yielded an F-value of 6.535 ($p = 0.011$) thus demonstrating the instability of the coefficients over the two periods. The forecast test gave similar results with a p-value of 0.085. The R^2 has shown only a modest improvement from the 0.039 value for an AR(2). These results are indicative of the stationary component of real dividends being a random walk. This yields equation (4.3).

6.4.3 Real All Ordinaries

Table 6.5 ADL(5,5):Regression of the Real All Ordinaries on Inflation

qtr:yr	Δp_{t-3}	Δp_{t-5}	$\hat{i}_{t-5}$	R^2	DW	$LM(4)$	$LM(12)$
Sep48			-.232	0.041	2.137	4.044	11.764
Sep97			*-2.890*			*.400*	*.465*
Sep48	.293	-.307	-.206	0.193	1.806	3.467	18.765
Mar73	*3.069*	*-3.060*	*-2.565*			*.483*	*.097*
Jun73			-.259	0.034	2.232	1.512	4.763
Sep97			*-1.868*			*.829*	*.965*

Table 6.5 completes the equity regressions. The inflation impact is significant as shown in figure 6.1(c). Model (4.5) in section 4.3.3 suggested a random walk. The AR coefficients are not significant for the full period nor the second portion. They are for the first portion. The data for the second portion is more reliable so the result for the first portion is discounted. The Chow test was applied to the inflation only model with the breakpoint at March 1973. The resulting F-value was 0.159 ($p = 0.690$) indicating coefficient stability. The R^2 from this model is very low. This result is consistent with the short run negative correlation between inflation and share prices generally reported (see Crosby (1998)).

6.4.4 Long/Short Ratio

Table 6.6 ADL(5,5):Regression of the Long/Short Ratio on Inflation

qtr:yr	$\hat{m}_{t-1}$	$\hat{m}_{t-4}$	$\hat{m}_{t-5}$	$\hat{i}_t$	$\hat{i}_{t-3}$	$\hat{i}_{t-4}$	R^2	DW	$LM(4)$	$LM(12)$
Ma60	.746	.287	-.243	-.031		.022	0.683	1.937	3.382	14.070
Sep97	*12.54*	*3.46*	*-3.12*	*-2.99*		*2.07*			*.496*	*.296*
Ma60	.772	.250	-.238				0.647	1.925	1.089	7.268
Sep97	*13.13*	*2.97*	*-2.98*						*.896*	*.839*
Ma60	.590				-.028		0.464	2.044	5.677	15.075
Ma75	*5.71*				*-2.37*				*.225*	*.205*
Jun75	.792	.366	-.364				0.686	1.805	3.179	9.683
Sep97	*11.14*	*3.67*	*-3.77*						*.528*	*.644*

The regression results are in table 6.6. The results are similar to that for the dividend yield. The inflation component is relatively minor with different lags being significant in the full period and first portion. It does not enter at all in the second portion (rows 7 and 8). The full period regression was then run with no inflation terms, that is as a univariate equation, as in table 6.6 rows 3 and 4. This then compares to equation (4.6) in section 4.3.1. The sum of the two coefficients of the AR terms at lags 4 and 5 is 0.012 (0.250-0.238). The result of adding these extra lags does not significantly improve the equation. The R^2 has increased to 0.649 from the value of 0.628 given for equation (4.6). The penalty from extra two parameters would mean the AR(1) is a better model fit based upon the SBC criterion. It was not necessary to review the multicollinearity diagnostics as there are no inflation terms.

The Chow test for the full period for the AR(1) version of the model with a March 1975 breakpoint were satisfactory with a test value of 0.863 (p = 0.354). The forecast test confirmed this with a test statistic of 0.234 (p = 0.989). This suggests that the simpler AR(1) model given by equation (4.6) should be retained. This is consistent with the findings of Wilkie (1995b) for the UK who has an AR(1) for this ratio.

6.4.5 Real Bond Yields

In line with the ccfs in figure 6.1(d) and (e) significant values at lags 0 and 4 but of opposite sign may be expected. The results in table 6.7 were obtained, showing the different reaction of the bond market to the rate of inflation as compared with the stock market.

Table 6.7 ADL(5,5):Regression of the Real Bond Yield on Inflation

qtr:yr	Δb_{t-2}	$\hat{i}_t$	$\hat{i}_{t-4}$	$\hat{i}_{t-5}$	R^2	DW	$LM(4)$	$LM(12)$
Mar60	-.090	-.933	.970		0.770	1.862	9.347	15.393
Sep97	*-2.23*	*-19.36*	*19.90*				*.053*	*.221*
Mar60		-.863	.974	-.099	0.912	1.903	4.245	19.184
Mar75		*-19.78*	*18.68*	*-2.04*			*.374*	*.084*
Jun75	-.122	-.948	.986		0.712	1.866	4.655	10.177
Sep97	*-2.13*	*-12.30*	*13.98*				*.325*	*.600*
Mar60		-.927	.962		0.769	1.900	14.206	19.497
Sep97		*-19.65*	*20.30*				*.007*	*.077*
Mar60		-.900	.945		0.905	1.769	3.967	21.118
Mar75		*-22.25*	*20.21*				*.411*	*.049*
Jun75		-.951	.975		0.697	1.915	8.427	13.119
Sep97		*-12.11*	*13.59*				*.077*	*.360*

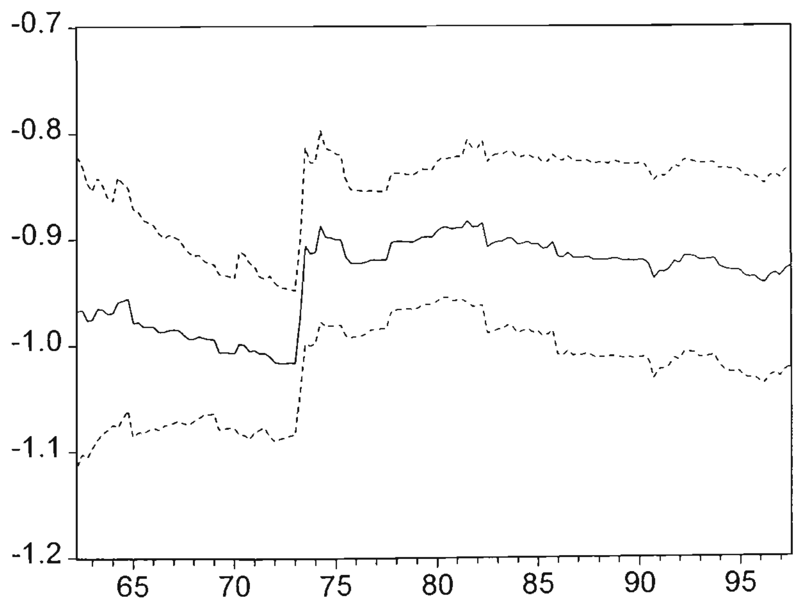
Inflation is an important causal factor in the bond market. Prices react strongly to moves in reported inflation. The values at lag 0 and 4 are consistently highly significant, contributing greatly to the explanatory power of the model. The coefficients exhibit stability over the three time periods. There is a small second order AR component, negative in value. This is not significant in the first time period.

Table 6.8 presents the multicollinearity diagnostics indicating problems with the Δb_{t-2} component with large variance proportions associated with the lagged inflation parameters. These results suggest dropping the AR term and keeping inflation.

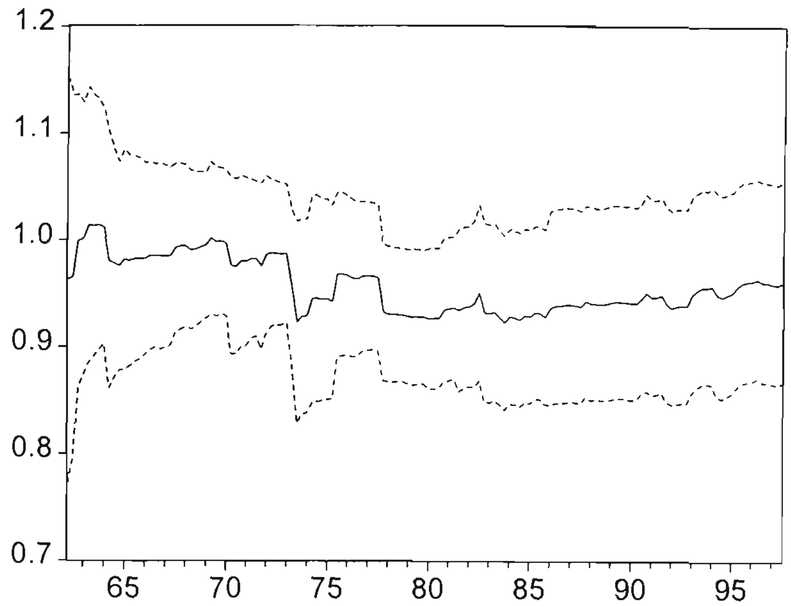
Table 6.8 Multicollinearity Diagnostics: Regression Table 6.7, rows 1 and 2.

		Variance Proportions		
Variable	Eigenvalue	Δb_{t-2}	$\hat{i}_t$	$\hat{i}_{t-4}$
Δb_{t-2}	1.620	0.000	0.188	0.188
$\hat{i}_t$	1.006	0.974	0.004	0.002
$\hat{i}_{t-4}$	0.374	0.025	0.808	0.809

The next step therefore is to examine what happens when the autoregressive components are dropped and the rate of inflation is kept as the explanatory variable. The results of this are in table 6.7, rows 6 to 12 for the respective periods. There are diagnostic problems for the full and first portions. This indicates serial correlation in the residuals. The evolution of the coefficients over time can be seen using recursive least squares. The plots are given in figure 6.2.



(a)



(b)

Figure 6.2 Recursive least squares coefficients of inflation (a) at lag 0 and (b) at lag 4.

The coefficient at lag 0 in figure 6.2(a) in 1973 moved from of order -1.0 to closer to -0.9. The values have remained stable since then. The breakpoint suggested is March 1975. The coefficient at lag 4 has remained close to 0.95 throughout as in figure 6.2(b) confirming the results in table 6.7. This confirms the earlier choice of this breakpoint discussed in section 6.4. There is no indication in figure 6.2 of a structural break when the A$ was floated in 1983. For the model over the full period with inflation components at lags 0 and 4 and no AR terms and with a March 1975 breakpoint, the Chow breakpoint and forecast test values were 0.728 ($p = 0.485$) and 1.447 ($p = 0.174$) respectively. This would suggest no structural break occurred at that time.

There is a strong case for only taking the results from after the adjustment in 1975. Firstly this is the period for which the data is more reliable. The bond data is only consistent post December 1968 (see section 3.2 where this was covered in detail). Secondly this model is also the only one which does not indicate evidence of serial correlation in the residuals. The final set of tests are for normality and heteroskedasticity. The ARCH-LM test value was 0.387 ($p = 0.534$) and Jarque-Bera test value 2.871 ($p = 0.238$). Hence the residuals are normal with no significant

ARCH effects. On mean re-adjustment given the fact that the coefficients are equal in magnitude but opposite sign means that the constant term is minor and therefore dropped. This will then be the preferred model.

$$\Delta b_t = -0.951 i_t + 0.975 i_{t-4} + \varepsilon_t \qquad (6.4)$$

6.4.6 Real Treasury Note Yields

The final regression equations are for T-note yields as in table 6.9.

Table 6.9 ADL(5,5):Regression of the Real Treasury Note Yield on Inflation

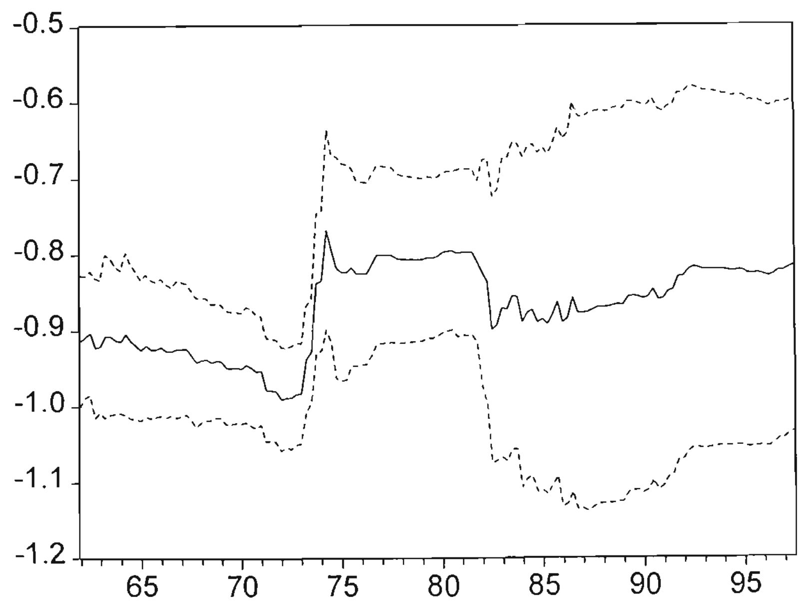
qtr:yr	Δn_{t-1}	Δn_{t-2}	Δn_{t-3}	Δn_{t-4}	$\hat{i}_t$	$\hat{i}_{t-4}$	R^2	DW	$LM(4)$	$LM(12)$
Ma60	-.134	-.166		.212	-.861	.902	0.394	1.923	1.076	15.365
Se97	*-2.01*	*-2.45*		*2.98*	*-7.87*	*7.87*			*.898*	*.222*
Ma60					-.821	.834	0.710	1.848	8.433	27.811
Ma75					*-10.7*	*9.14*			*.077*	*.006*
Ju75			.186	.300	-.931	.934	0.323	2.173	3.368	10.077
Se97			*2.11*	*3.31*	*-5.13*	*5.52*			*.498*	*.610*
Ma60					-.817	.809	0.310	2.187	19.79	32.215
Se97					*-7.50*	*7.36*			*.001*	*.001*
Ju75					-.813	.798	0.214	2.216	14.62	21.847
Se97					*-4.30*	*4.63*			*.006*	*.039*

As with bonds the rate of inflation at lags 0 and 4 is significant but this time there are extra AR terms. These are not stable across the structural breakpoint of March 1975. During the first period, incorporating the surge in inflation in the early 1970's, the model shows that only inflation is needed. There is evidence of serial correlation in the residuals. The Breusch Godfrey *LM*(4) test *p*-value was 0.077 and the *LM*(12) *p*-value was 0.006. The multicollinearity diagnostics for the full period regression rows 1 and 2 are shown in table 6.10. There is only a small proportion of the variance associated with the AR lag 1 and lag 2 terms.

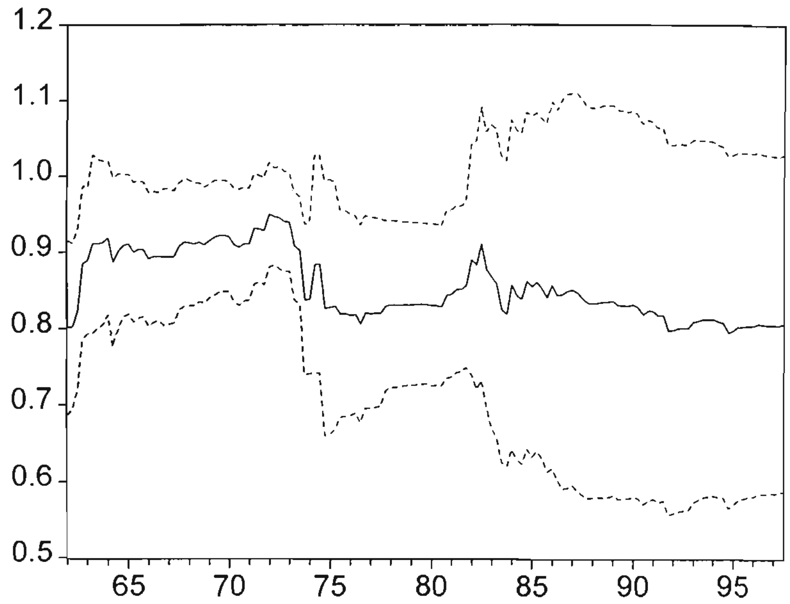
Table 6.10 Multicollinearity Diagnostics: Regression Table 6.9, rows 1 and 2.

		Variance Proportions				
Variable	Eigenvalue	Δn_{t-1}	Δn_{t-2}	Δn_{t-4}	$\hat{i}_t$	$\hat{i}_{t-4}$
Δn_{t-1}	1.689	0.008	0.013	0.054	0.132	0.152
Δn_{t-2}	1.294	0.202	0.234	0.172	0.051	0.011
Δn_{t-4}	0.878	0.747	0.277	0.053	0.001	0.002
$\hat{i}_t$	0.818	0.038	0.435	0.493	0.068	0.008
$\hat{i}_{t-4}$	0.321	0.004	0.042	0.228	0.748	0.827

The lagged values of the dependent variable were then dropped and the regression re-run with lagged values of inflation as regressors. Table 6.9, rows 8 to 11 shows the results. As was the case for the first period March 1960 to March 1975 there were problems of serial correlation. The p-values in rows 8 and 10 show this. Applying recursive least squares the evolution of the coefficients over time may be observed. There are no AR terms in the regression. Figure 6.3 details the results.



(a)



(b)

Figure 6.3 Recursive least squares coefficients of inflation (a) at lag 0 and (b) at lag 4.

The inflation surge in 1974 as discussed previously has caused a shock to the system particularly for the coefficient of $\hat{i}_t$. The Chow breakpoint test statistic for this model with a March 1975 breakpoint was 0.011 (p = 0.989). This suggests no structural break occurred at that time. The coefficients since then exhibit stability. This is consistent with figure 6.2. There is a good case as with Δb_t for only considering the data post 1975. Because of serial correlation AR terms are required. This yields the model in rows 5 and 6 table 5.9 for the period June 1975 to September 1997. The final set of tests for this preferred model are for normality and heteroskedasticity. The ARCH-LM test value was 11.166 (p = 0.001) and Jarque-Bera test value 11.206 (p = 0.004). Hence the residuals are not normal with significant ARCH effects. As was the case with bonds the mean re-adjusted model remains the same.

$$\Delta n_t = 0.186\Delta n_{t-3} + 0.300\Delta n_{t-4} - 0.931 i_t + 0.934 i_{t-4} + \varepsilon_t \qquad (6.5)$$

The R^2 for this model is 0.323 a marked improvement on the value of 0.071 for the univariate equation (4.8). It is still well below the R^2 of 0.697 for the preferred Δb_t model. The Chow forecast test statistic for this version was 0.213 with a p-value of 0.992. The non-normality and heteroskedasticity of the residuals remains an issue.

However, anticipating the summary in section 6.7, cointegration between real bonds and real T-notes means that this equation is unnecessary.

6.5 Box Jenkins Transfer Models

In section 6.3 various methods for modelling the financial series with inflation incorporated were discussed. The second of the two approaches chosen, the transfer function modelling of Box and Jenkins is a method to be investigated. This time-series oriented strategy is discussed in Box and Jenkins (1976, Part III) and the development outlined there will be followed. The strategy involves a five step process which can be applied to Δb_t as the variable with the clearest and strongest response to inflation. Now the function is of the form:

$$y_t = \frac{\omega(B)}{\delta(B)} x_{t-b} + N_t$$

where $\omega(B)$ and $\delta(B)$ are lag polynomials and N_t is a noise term. It is shown by Box and Jenkins that estimation is difficult unless the input series is transformed to be a random series or white noise[8]. It is then easier to see the effect of any transformation. The same transformation that is applied to the input series is then applied to the output series. Estimation can then proceed on the basis of the ccf between the two transformed series. The ccf is used to estimate the value of the '*dead time*', that is the time until the lag cut in b. It is also used to find the orders r and s of the lag polynomials $\omega(B)$ and $\delta(B)$ and the corresponding polynomial coefficients.

The stochastic investment model is one of real variables, hence inflation can be entered separately and a model is not required to generate inflation. However the transfer modelling process requires the input series to be pre-whitened. Hence the input series is modelled in order to provide the required transformation. The input inflation series is best modelled as an AR(2) equation (3.6). The 10-year bond data was not available until 1960 so it was necessary to re-calibrate the inflation model

[8] This process is called *pre-whitening* by Box and Jenkins.

over the shorter time period. The inflation model was estimated with t-values below in brackets to yield the equation:

$$\hat{i}_t = 0.347\hat{i}_{t-2} + 0.253\hat{i}_{t-3} + 0.251\hat{i}_{t-4} + \varepsilon_t$$
$$(4.411) \quad (3.472) \quad (3.187)$$

where $\hat{i}_t = i_t - \bar{i}$, R^2=0.523 and $Q(12) = 9.083$ (p = 0.430). This can then be expressed as $\left(1 - 0.347B^2 - 0.253B^3 - 0.251B^4\right)\hat{i}_t = \varepsilon_t$, where B is the backshift operator. Thus letting $\phi(B)z_t = \left(1 - 0.347B^2 - 0.253B^3 - 0.251B^4\right)\hat{i}_t = \alpha_t$ yields a white noise input series α_t.

The same transformation is now applied to the output series. There are no MA terms in the transformation hence the series $\beta_t = \phi(B)\Delta b_t$ is generated. This was done and the resulting ccf between the transformed input and output series α_t and β_t is shown in figure 6.4.

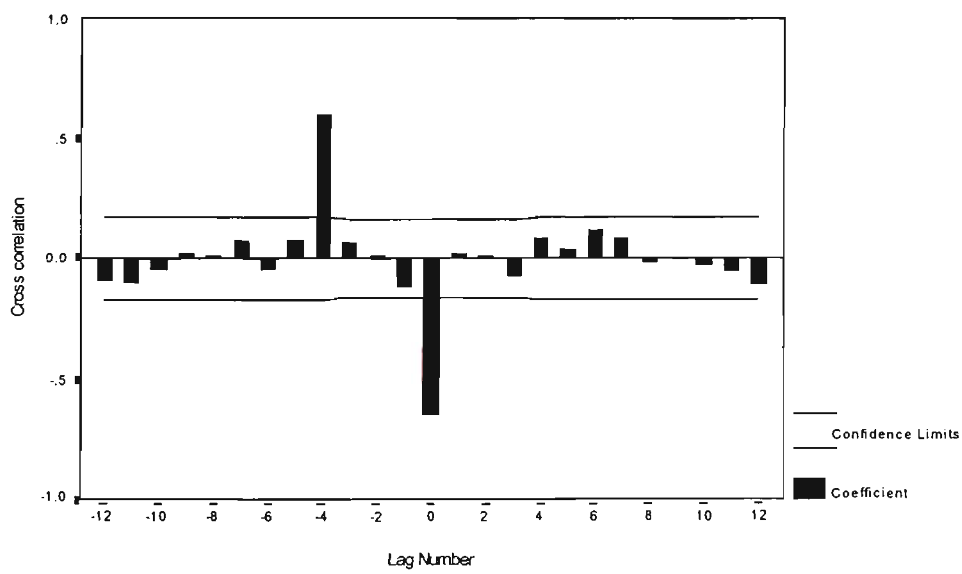


Figure 6.4 Transfer Function Model for Δb_t: Ccf between the transformed input and output series α_t and β_t.

There are highly significant values at leads 0 and 4. Letting $v(B)=\frac{\omega(B)}{\delta(B)}$, then the coefficients of $v(B)$ are given by $v_k=\frac{r_{\alpha\beta}(k)s_\beta}{s_\alpha}$, where s_α and s_β are the standard deviations of α_t and β_t, the input and output series respectively (see Box and Jenkins (1976, p.380)). In this case s_α= 0.832 and s_β= 1.311 and so $\frac{s_\beta}{s_\alpha}=1.576$.

From the ccf the lag cut off $b=0$. Hence the transfer function takes the form (6.6) where the component for Δb_t is assumed of low order.

$$\left(1-\delta_1 B-\delta_2 B^2\right)\Delta b_t=\left(\omega_0+\omega_4 B^4\right)\hat{i}_t+N_t \tag{6.6}$$

Equating coefficients in $v(B)\delta(B)=\omega(B)$ yields:

$$\left(1-\delta_1 B-\delta_2 B^2\right)\left(v_0+v_1 B+v_2 B^2+\ldots\right)=\left(\omega_0+\omega_4 B^4\right)$$

$$\left(v_0+\left(v_1-\delta_1 v_0\right)B+\left(v_2-\delta_1 v_1-\delta_2 v_0\right)B^2+\ldots\right)=\left(\omega_0+\omega_4 B^4\right)$$

Now $r_{\alpha\beta}(0)=-0.643$, $r_{\alpha\beta}(4)=0.598$ and $r_{\alpha\beta}(k)\cong 0 \Rightarrow v_k=0$ otherwise, therefore

$$\omega_0=v_0=(1.576)(-0.643)=-1.013,$$

$$\omega_4=v_4=(1.576)(0.598)=0.942$$

and $\delta_1=\delta_2=0$. This yields the transfer function model:

$$\Delta b_t=\left(-1.013+0.942B^4\right)\hat{i}_t+N_t \tag{6.7}$$

A model is fitted to the error term N_t, by finding $N_t=\Delta b_t-\hat{v}(B)$ using the estimates found above. Doing this and applying the standard ARMA model fitting procedure yielded:

$$N_t=0.297N_{t-3}+\varepsilon_t$$

(3.671)

or $N_t = \dfrac{\varepsilon_t}{\left(1 - 0.297B^3\right)}$ with *t*-values in brackets and $Q(12) = 12.666$ (p = 0.316).

This can be combined into the complete transfer function model:

$$\Delta b_t = \left(-1.013 + 0.942B^4\right)\hat{i}_t + \frac{1}{\left(1 - 0.297B^3\right)}\varepsilon_t \qquad (6.8)$$

and then finally substitute $i_t = \hat{i}_t + 1.470$ to re-transform the mean.

There are several points to be made:

1. In equation (6.6) introducing more lag terms on the right hand side, that is inflation terms, will add complexity with relatively small coefficients. The ccf shows no other terms are significant.
2. Equation (6.8) is not a particularly attractive expression. A larger parameter at lag 4 for the inflation input function would help clean up the residual term.
3. The above process forms the starting point for an iterative estimation procedure. These are best dealt with using some of the advanced Box Jenkins packages available. The aim would be for the ARMA error term N_t to be white noise.

   Most importantly the model is not significantly different from the linear regression approach given in section 6.5.4, even allowing for the fact that the estimates are initial ones. For the period June 1975 to September 1997 in table 6.7, rows 11 and 12 the coefficients of $\hat{i}_t$ and $\hat{i}_{t-4}$ were -0.951 and 0.975 respectively. This compares to values of -1.013 and 0.942 for the equivalent coefficients in (6.8).
4. Judgements are required for the choice of lag cut off and model form in (6.6). The ADL general-to-specific method is a more systematic reduction process.

The equivalence of these modelling methods is expected. The linear regression does the estimation simultaneously whereas the transfer function does it in a step by step procedure, using an iterative process to find the best estimates. The importance of inflation comes through in both methods. This gives added support to the results in table 6.7. By way of comparison with figure 6.1 the ccfs of the pre-whitened series are shown in figure 6.5. As per figure 6.1 only lags of ±7 are presented. Figure 6.5(e) is therefore identical to figure 6.4 but with fewer lags shown. Note that the data is quarterly identical to that in figure 6.1 and as described in the footnote to Table 6.1.

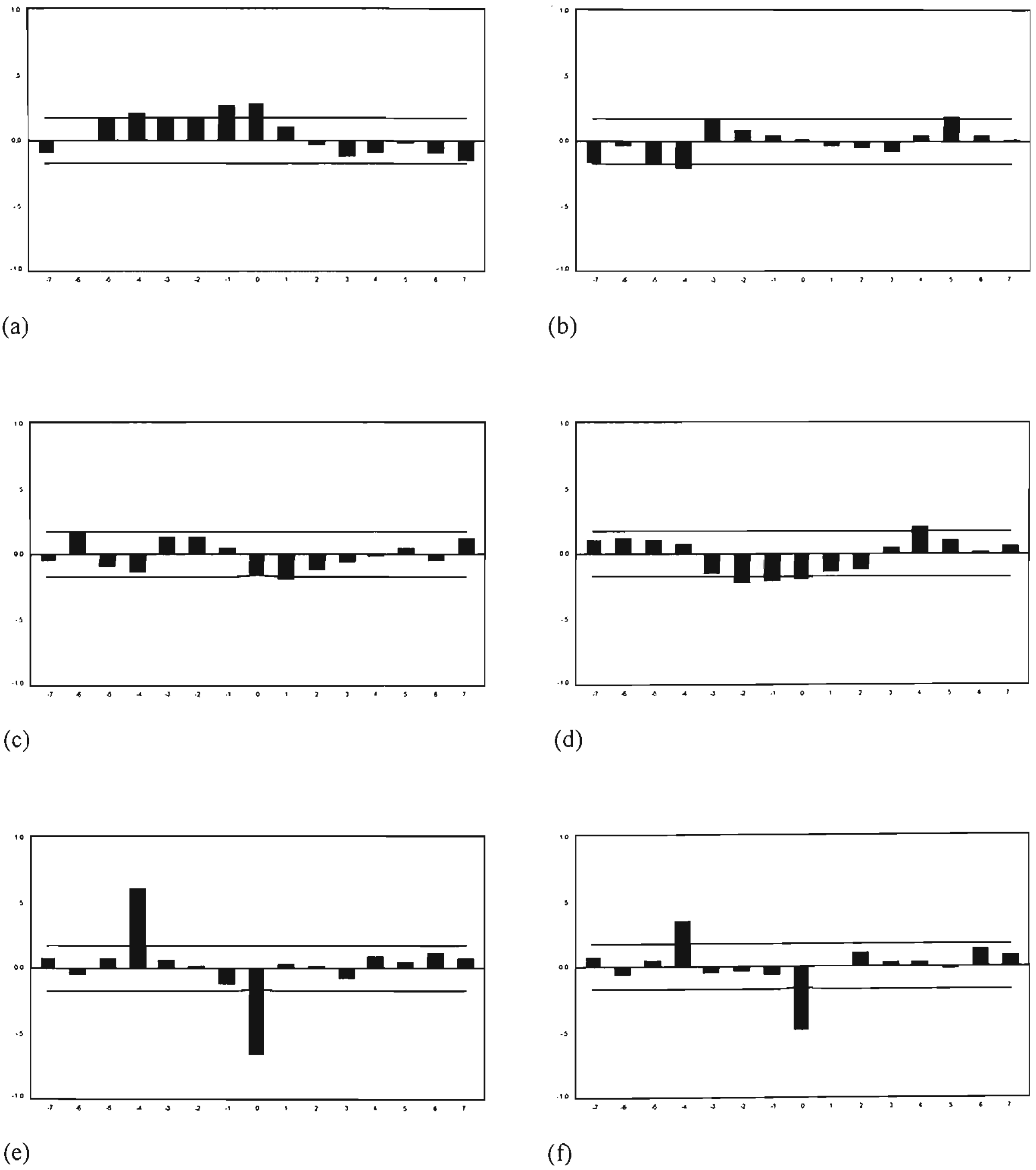


Figure 6.5 Ccf for pre-whitened equity series: (a) y_t^T; (b) Δd_t^T and (c) Δp_t^T, and for pre-whitened fixed interest series: (d) m_t^T; (e) Δb_t^T and (f) Δn_t^T, where T represents the pre-whitening transform. Values from -1.0 to +1.0 and lags -7 to +7. Confidence limits given by the lines parallel to the x-axis.

The ccf figure 6.5(b) for Δd_t^T indicates a minor inflation effect. For Δp_t^T figure 6.5(c) there is no significant cross correlation coefficients. The ratio quantities y_t^T and m_t^T in figure 6.5(a) and (d) respectively show small significant values at lag 0 but a dominance by the AR terms. This is consistent with the results in tables 6.2 and 6.6. Finally Δn_t^T in figure 6.5(f) exhibits the same dynamics as figure 6.1(e). There are significant values at lags 0 and 4 but smaller in magnitude. The particular comments made in points 1-5 above have general validity for the application of transfer modelling to each of the other 5 variables. Therefore transfer function models will not be formally applied to these series.

6.6 The Roles Of Expected and Unexpected Inflation

The stock and bond markets are anticipatory in nature, so it would not be surprising if the stock and bond markets can be used as leading indicators of the economy. Financial variables should therefore have an *ex ante* content. However expected inflation has been shown to be an $I(1)$ needing differencing for stationarity. Therefore unexpected inflation u_t is taken as the independent variable in the linear regressions, since $u_t \sim I(0)$ and hence standard inference applies. As was the case for observed inflation, up to 5 lags were used in the ADL model of the form (6.1), and the dividend yield, long/short ratio and unexpected inflation were mean adjusted prior to the regression. A circumflex over the variable indicates mean adjustment. The period for which expectations data is available is March 1973 to September 1997. The regressions were performed and the results placed in table 6.11. The layout is identical to that in table 6.2 and the other result tables in section 6.4. Here z_t represents the respective variable in the first column lagged once, twice and so on, hence the coefficient of $\Delta d_{t-2} = 0.384$ in the z_{t-2} column.

Table 6.11 ADL(5,5):Regression of the Real Variables on Unexpected Inflation

	z_{t-1}	z_{t-2}	z_{t-3}	z_{t-4}	$\hat{u}_t$	$\hat{u}_{t-1}$	$\hat{u}_{t-3}$	$\hat{u}_{t-4}$	R^2	*LM*4	*LM*12
$\hat{y}_t$	.928			-.121					.751	2.50	14.47
	15.3			*-1.98*						*.645*	*.272*
Δd_t		.384							.146	3.23	12.36
		4.05								*.520*	*.418*
$\hat{m}_t$	.779								.601	7.77	10.38
	12.5									*.100*	*.583*
Δb_t		-.242	.200	-.162	-.571	.430	.188		.641	3.68	12.87
		-3.37	*3.03*	*-2.48*	*-10.1*	*7.20*	*3.50*			*.451*	*.379*
Δn_t	-.290	-.315		.192	-.392			.374	.349	0.58	6.83
	-3.26	*-3.52*		*2.09*	*-4.89*			*4.64*		*.968*	*.868*

The equity equations and $\hat{m}_t$ have no significant lagged values of unexpected inflation. For Δp_t no lags in the equation (6.1) were significant. Hence no model appears. For $\hat{y}_t$ there is a second AR term at lag 4. The *t*-statistic is just significant. The only models that include the independent variable are Δb_t and Δn_t. The consistency between these two models in terms of lags in the model when actual inflation is the independent variable is not reproduced when unexpected inflation is the independent variable. There is no evidence of serial correlation in any of the residuals.

The R^2 values for Δb_t and for Δn_t are comparable to those models for the similar time period June 1975 to September 1997 in tables 6.7 and 6.9 which include AR terms. The Chow breakpoint test was applied to Δb_t with a breakpoint of March 1985. This was when real rates had peaked. Figure 4.10 for real bonds exhibits a plateau after that time and figure 4.11 exhibits a slow decline from then. For Δb_t the breakpoint test statistic was 1.167 (p = 0.332) and for Δn_t 0.395 (p = 0.823). The forecast test produced similar results. Test values of 0.824 (p = 0.595) for Δb_t and 0.295 ($p = 0.974$) for Δn_t were obtained.

Table 6.12 Multicollinearity Diagnostics: Regression Table 6.11, rows 7 and 8.

		Variance Proportions					
Variable	Eigenvalue	Δb_{t-2}	Δb_{t-3}	Δb_{t-4}	$\hat{u}_t$	$\hat{u}_{t-1}$	$\hat{u}_{t-3}$
Δb_{t-2}	2.434	0.000	0.026	0.007	0.050	0.048	0.051
Δb_{t-3}	1.187	0.468	0.064	0.001	0.020	0.008	0.034
Δb_{t-4}	1.076	0.011	0.216	0.579	0.005	0.006	0.000
$\hat{u}_t$	0.772	0.077	0.671	0.293	0.019	0.005	0.026
$\hat{u}_{t-1}$	0.293	0.423	0.014	0.008	0.422	0.014	0.767
$\hat{u}_{t-3}$	0.240	0.021	0.009	0.111	0.485	0.918	0.122

Table 6.13 Multicollinearity Diagnostics: Regression Table 6.11, rows 9 and 10.

		Variance Proportions				
Variable	Eigenvalue	Δn_{t-1}	Δn_{t-2}	Δn_{t-4}	$\hat{u}_t$	$\hat{u}_{t-4}$
Δn_{t-1}	1.531	0.068	0.068	0.145	0.105	0.162
Δn_{t-2}	1.272	0.042	0.236	0.097	0.204	0.091
Δn_{t-4}	0.978	0.662	0.017	0.075	0.057	0.104
$\hat{u}_t$	0.721	0.042	0.679	0.421	0.083	0.000
$\hat{u}_{t-4}$	0.498	0.187	0.001	0.261	0.552	0.643

The multicollinearity diagnostics for Δb_t and Δn_t are shown in tables 6.12 and 6.13 respectively. Table 6.12 shows that Δb_{t-2} contributes little. Most of the variance comes from the lagged $\hat{u}_t$ terms. The same is true for Δn_t where table 6.13 shows only Δn_{t-4} has a large contribution to the variance. This is consistent with the multicollinearity results in tables 6.8 and 6.10 for actual inflation.

6.7 Summary of Model Equations

The next step is to integrate the results to produce the next stage in the stochastic investment system. Equations have been developed:

1. In Chapter 4 utilising a univariate approach.
2. In the first part of this chapter with inflation as the independent variable.
3. In section 6.6 with unexpected inflation as the independent variable.

Table 6.14 summarises the results. Mean re-adjustments are made where appropriate. The time period for the regressions vary. The results for each variable in (1) are as per those discussed in sections 4.3 and 4.4. In (2) they are for the model produced with the full data set, the same as in the univariate case, section 6.4 refers. In (3) the time period is March 1973 to September 1997. Variables with individual significant coefficients on the lags are given in braces for the lagged dependent variable, the independent variable is given after the braces. For comparative purposes the model R^2 is given in brackets with the particular model form.

Table 6.14 Summary of Modelling Results with Three Alternative Approaches

Financial variable	univariate (1)	inflation (2)	unexpected inflation (3)
y_t	AR(2) $\{\phi_1,\phi_2\}$ [0.766]	AR(2)$\{\phi_1,\phi_2\}$,i(0) [0.790]	AR(2) $\{\phi_1,\phi_2\}$ [0.751]
Δd_t	AR(2) $\{\phi_2\}$ [0.045]	AR(2)$\{\phi_2\}$ i(3),i(4),i(5) [0.113]	AR(2) $\{\phi_2\}$ [0.146]
Δp_t	random walk	random walk	random walk
m_t	AR(1) [0.628]	AR(5) $\{\phi_1,\phi_4,\phi_5\}$ i(0),i(4)[0.683]	AR(1) [0.601]
Δb_t	AR(4) $\{\phi_3,\phi_4\}$ [0.180]	AR(2)$\{\phi_2\}$i(0),i(4) [0.770]	AR(4)$\{\phi_1,\phi_2,\phi_4\}$ u(0),u(4) [0.655]
Δn_t	AR(2) [0.071]	AR(4)$\{\phi_1,\phi_2,\phi_4\}$ i(0),i(4)[0.394]	AR(4) $\{\phi_2,\phi_3,\phi_4\}$ u(0),u(1),u(3)[0.341]

The dividend yield[9] y_t in row 1 shows a sum of coefficients in the univariate case given by equation (4.1) of 0.855 and a relatively small and unstable inflation component when regressed against actual inflation (table 6.2). The unexpected inflation regression is a straightforward AR(2); no coefficients on inflation terms are significant. Wilkie's result for the UK was an AR(1) representation, though with a small but significant influence from the current rate of inflation, as is found with the linear regression relationship in row 1, column 2. Carter (1991, equation 4.6.3(1), p.359), applying transfer functions found a significant link between inflation and the force[10] of the dividend yield. The force of inflation at lags 1 and 5 enter the model, which has a complex representation. If in row 1 the inflation element is rejected as it contributes little, then a similar argument may be applied to the second AR component, thus yielding a simple AR(1) representation. For the AR(1) model

[9] Note that all references in this discussion are to table 6.14 unless otherwise stated.
[10] The force of a variable is an actuarial term (see the discussion in section 3.4.3).

the $Q(12)$ Box-Ljung statistic for the residuals was 26.36 (p = 0.006) as in table 4.5. The extra parameter is therefore required which leaves the AR(2) model.

For Δd_t, row 2, an AR(2) where only the lag 2 parameter is significant has very little explanatory power. Whilst adding inflation as an explanatory variable resulted a modest improvement to the explanatory power of the model the R^2 values were still low . This relationship also proved to be unstable. This suggests a random walk model with zero drift for Δd_t given by equation (4.3).

For Δp_t, row 3, the result is a random walk in all cases. This result is consistent with the literature.

The long/short ratio m_t, row 4, is an AR(1). There is a minor inflation impact when actual inflation is the regressor in the ADL model (6.1). The inflation component adds only a small amount to the explanatory power of the resulting model. The simpler AR(1) representation is therefore preferred. This yields the univariate equation (4.6).

The univariate bond model for Δb_t in row 5, given by equation (4.7) is an AR(4) with a low R^2 value of 0.184 found using either Nelson's formula or by conducting a regression with only AR terms. This compares to an R^2 value of 0.770 for the full period inflation series regression and 0.655 for the shorter unexpected inflation regression. The equations with i_t as regressor will therefore be applied rather than the ones with u_t. For the inflation model given in row 5, column 2 there is an AR(2) component plus the inflation values at lags 0 and 4. Including the AR(2) component only takes the overall R^2 from 0.769 to 0.770 and the autoregressive components are unstable. As discussed in section 6.4.5, equation (6.4) is preferred.

There is stability in the coefficients after 1975 for the Δb_t model, as shown in figure 6.3. However the small difference in magnitude between the inflation coefficients at lags 0 and 4 would lead to a continually rising real yield under constant inflation. For example in the preferred inflation only model equation (6.4)

$\Delta b_t = -0.951 i_t + 0.975 i_{t-4} + \varepsilon_t$, there is a difference in values between the coefficients at lags 0 and 4 of +0.024. There is a positive drift of 0.024, small but significant over time. So for a constant 5% inflation rate over 10 years the expected drift would be $(40)(5)(0.024) = 4.8\,\%$. Higher rates of inflation would lead to larger drifts and vice versa. This is a logical consequence of the actual positive drift observed over the last 37 years. The value of $\overline{\Delta b} = 0.016$ from section 4.3.2 and this is not statistically significant from zero, since $S.E.\left(\overline{\Delta b}\right) = 0.093$. The model reproduces this drift. Nevertheless small though this factor is, cumulatively there has been a steep increase in real rates. There is information in the levels of the series which has not been incorporated. The model is for the stationary component of the time series. The stochastic trend component must be investigated.

A similar situation to Δb_t prevails for Δn_t and the univariate and unexpected inflation models are rejected. For Δn_t, row 6, column 2 the inflation model there are significant inflation values at lags 0 and 4 plus an AR(4) with 3 significant terms. For Δn_t, in table 6.9 the addition of the AR terms added significantly to the R^2 in the sub periods but much less so taking the period as a whole. The much lower R^2 for the T-note model in the most recent period, June 1975 to September 1997, is a concern. Given the instability in the AR parameters there is a good case for rejection of these parameters. However because of serial correlation AR terms are required. So this leads to equation (6.5). The non-normality and heteroskedasticity of the residuals qualifies the utility of this particular model.

In this case the parameter estimates at lags 0 and 4 are almost equal. There is a positive drift of 0.003 in magnitude. So over 10 years with inflation constant at 5% the total drift would be +0.15%. With only the exogenous variable present the evolution of the coefficients can be seen as in figure 6.4. The coefficients are stable and this demonstrates that collinearity in the variables is not a concern. The coefficient estimates have not moved around rapidly as more data is added, as would be expected if this were so.

Carter (1991, equation 4.3.4(1)) finds a link between the change in the force of T-notes[11] and inflation. His model incorporates an effect of inflation over the previous two quarters plus an MA term at lag 4. The shock term has an immediate effect plus a delayed effect in one years time. He rejects past inflation as a direct determinant of the change in the force of bond rates. However he finds a direct link between changes in the force of bond yields in the current quarter and changes in the force of T-note yields in the current quarter. There is then an indirect link between bond rates and inflation through T-notes. Carter has a T-note equation plus a link between long rates and short ones.

Volker (1982) attempted to model the Treasury bill rate over the period 1968-1979 with the application of the then available expectations data. He used expectations plus other explanatory variables such as GDP and real growth in M1. The best equation was found when a liquidity factor was introduced (excess LGS assets). The OLS system used by Volker was one of integrated variables, though many of the techniques of cointegration had not been developed at that time. The study was pre-Campbell Inquiry so there are issues with respect to the changes brought about by that inquiry, as well as the fairly short period for which expectations were available.

The above summary then moves the development of the stochastic investment model forward. All six equations are given for completeness. Cointegration between the variables implies that two of the equations may be dropped at a later stage (see section 5.3.2). The equations now may be summarised (applying the same symbols as used throughout).

[11] Defined as $\ln\left(1 + \frac{1}{4} N_t\right)$ where N_t is the effective T-note rate per annum (see Carter (1991, equation 4.3.1(1))).

$$y_t = \frac{D_t}{P_t} \qquad \text{(cointegrated series)}$$

$$y_t = 0.691 + 1.045 y_{t-1} - 0.190 y_{t-2} + \varepsilon_t \quad \varepsilon_t \sim N(0, 0.217) \ \sigma_\varepsilon = 0.466 \qquad (4.1)$$

$$\Delta d_t = \varepsilon_t \qquad \varepsilon_t \sim N(0, 0.0017) \ \sigma_\varepsilon = 0.041 \qquad (4.3)$$

$$\Delta p_t = \varepsilon_t \qquad \varepsilon_t \sim N(0, 2.320) \ \sigma_\varepsilon = 1.523 \qquad (4.5)$$

$$m_t = \frac{B_t}{N_t} \qquad \text{(cointegrated series)}$$

$$m_t = 0.260 + 0.785 m_{t-1} + \varepsilon_t \quad \varepsilon_t \sim N(0, 0.0151) \ \sigma_\varepsilon = 0.123 \qquad (4.6)$$

$$\Delta b_t = -0.951 i_t + 0.975 i_{t-4} + \varepsilon_t, \quad \varepsilon_t \sim N(0, 0.429) \ \sigma_\varepsilon = 0.655 \qquad (6.4)$$

$$B_t = \left\{ \left(1 + \frac{b_t}{100}\right)\left(1 + \frac{i_t''}{100}\right) - 1 \right\} * 100 \qquad \text{(definition, see section 4.3.2)}$$

$$\Delta n_t = 0.186 \Delta n_{t-3} + 0.300 \Delta n_{t-4} - 0.931 i_t + 0.934 i_{t-4} + \varepsilon_t, \qquad (6.5)$$

$$\varepsilon_t \sim N(0, 2.140) \ \sigma_\varepsilon = 1.463$$

$$N_t = \left\{ \left(1 + \frac{n_t}{100}\right)\left(1 + \frac{i_t''}{100}\right) - 1 \right\} * 100 \qquad \text{(definition, see section 4.3.3)}$$

For the correlation matrix of the residuals refer to Table 8.1. A discussion of this and further analysis is given in section 8.4.2 at a later stage in model development.

6.8 Conclusions

In this chapter further steps in the development of the stochastic investment model have been made. The approach chosen combines flexibility with a set of sparse coefficients.

The first section investigated the links between inflation and the endogenous variables via product moment correlations and the cross correlation function. There is a strong relationship between the difference in real bond rates or the difference in real T-note rates and the rate of inflation, with an initial negative impact of inflation at lag 0 followed by a strong positive response at lag 4. For the difference in real dividends the situation is less clear cut. The effect of inflation is delayed and appears at lags 4 and 5 and is of the expected sign. However the impact does not die out so quickly, reflecting the different dynamics of dividends. The difference in the real All ordinaries index is negatively correlated to the rate of inflation; a significant value at lag 0 is observed.

The resulting equations describe the dividend yield as a simple AR(2) process and the liquidity risk premium, defined as the ratio of longs to shorts, as an AR(1). The difference in real dividends is best modelled as random walk with zero growth. The difference in the real All Ordinaries index is modelled as a random walk. In the fixed interest market both the difference in real bond rates and the difference in real T-notes are found to be affected mainly by the rate of inflation with an initial negative impact of inflation followed by a positive impact with a lag of 4 quarters. The autoregressive component is small in both cases.

Alternative approaches such as transfer function modelling do not produce statistically superior models but do increase greatly computational time and complexity. The results of transfer function modelling are consistent with those found from linear regression models with the ADL form. Utilising unexpected inflation as the independent variable in lieu of observed inflation also did not generate models with superior statistical properties, so the equations use observed inflation as the exogenous variable.

Chapter 7

Fixed Interest Series Levels Modelling

7.1 Introduction

The fixed interest and equity asset classes are different in nature. For a fixed interest security both the coupon and redemption value are known. The only unknown is the yield to maturity. For equities the dividend, long run share price and the discount rate are all unknown. Factors such as the supply/demand balance or general confidence are extremely hard to determine but are important elements in setting price levels.

The fixed interest asset class and the shape of the yield curve are the subject of this chapter. The model (6.4) is one of differences. It deals with the stationary component of real interest rates but not the stochastic trend component (see Nelson and Plosser (1982) or Stock and Watson (1991) for a discussion of stochastic trends in macroeconomic series). This component of the real interest rate must be considered. There are certain empirical issues which need attention. The nominal bond rate cannot be negative. The real bond model should possess other characteristics observed in the data. Hence a near integrated bond series should be created by the model exhibiting a tendency to wander but to do so within reasonable bounds.

High frequency data means that continuous models, given by stochastic processes such as Brownian motion, may be applied to tasks such as derivative pricing. Discrete analogues of these processes form viable alternative models for lower frequency data, such as the quarterly data used in the stochastic investment model. Authors such as Vasicek (1977), Brennan and Schwartz (1982), Cox, Ingersoll and Ross (1985) and Mulvey (1996) have proposed solutions in these forms. So a review of competing models for the fixed interest class is required.

The chapter is organised as follows. Section 7.2 reviews some of the literature and alternative models in existence. The real bond rate is considered in some depth and the mean reverting nature of real interest rates investigated. In section 7.3 an empirical analysis is conducted using Australian data. This results in a real bond model which satisfies the requirements of mean reversion for real bond rates and the non-negativity of nominal bond rates. In the next section 7.4 links within the fixed interest asset classes are reviewed. The number of factors needed to adequately model interest rates and the yield curve are examined. The cointegrating relationship between real bond rates and real T-note rates enables the introduction of error correction models (ECM) involving the levels of the two variables. The results of this modelling can then be compared to that of the long/short ratio, which is another way of considering the cointegrating relationship between nominal bond and nominal T-note rates. However this measure does not contain the extra information available in the levels data. Therefore the question as to whether a better model can be obtained from this extra data is considered.

7.2 Non-Negativity of Nominal Bond Rates and Mean Reversion in Real Bond Rates

The modelling needs to satisfy various criteria in order to be acceptable. Such features as mean reversion in real bond rates and ensuring the non-negativity of nominal bond rates will be investigated. These features are not independent of each other.

7.2.1 Diffusion Equations: Some Solutions

In the literature there are various solutions to the twin difficulties of mean reversion in real bond rates and the non-negativity of nominal bond rates which involve more complex functions to capture the effects. A large area of research is based upon *continuous* time models of stochastic processes. These provide the pricing models widely used in the valuation of derivative securities. There are several standard models used to describe continuous processes. Arithmetic and geometric Brownian motion and the mean reverting Ornstein-Uhlenbeck (O-U) process are examples. These are described in Shimko (1992).

Sawyer (1993) reviewed continuous time stochastic models. A variety of common processes were investigated with solutions provided where they can be found and conditions for finding them. Sawyer related the stochastic differential equations (SDE) from continuous models to their discrete counterparts. The relation of ARCH processes to continuous models was also discussed. Sawyer (1993, p.761) referred to recent work which, "...showed that properly chosen sequences of EGARCH and GARCH models will converge weakly to bivariate diffusion processes...". Sawyer (1993, p.740) commented that "These contributions strengthen the contention that continuous time financial models are indeed proper approximations to a wide class of discrete time formulations.".

The general diffusion process is defined by the SDE:

$$dX = \kappa(\mu - X)dt + \sigma X^{\gamma} dz \tag{7.1}$$

where κ, μ and γ are positive. The parameter κ determines the speed of adjustment or rate of mean reversion, μ is the long term mean level of the process and σ^2 is the variance of the process. The O-U process is given by letting $\gamma = 1$. These can then be solved by Ito's lemma and the use of, for example, Laplace transforms applied to the resulting partial differential equation. Note an SDE can have higher order erivative terms, though they are difficult to relate to discrete time series models.

One of the earliest SDE models is the mean reverting O-U model originally proposed by Vasicek (1977). This is of the form:

$$dr = (a - br)dt + \sigma dz \tag{7.2}$$

where a, b and σ are strictly positive constants. This can be solved in closed form yielding the price of a zero-coupon bond (see Musiela and Rutkowski (1997, p.288-290)).

A logical extension of the single equation or single factor model of Vasicek is the two factor model due to Brennan and Schwartz (1982). They started by defining two differential equations which represented the stochastic processes that the long or consol rate[1] and the short rate should follow, which were given as general O-U processes. The SDE's were described by the following:

$$dr = \left(a_1 + b_1\left(l - r\right)\right)dt + \sigma_1 r dz_1 \tag{7.3}$$

$$dl = l\left(a_2 + b_2 r + c_2 l\right)dt + \sigma_2 l dz_2$$

where r is the short term interest rate, l is the consol rate, $\sigma_1{}^2$ and $\sigma_2{}^2$ are the variance of the short rate and consol rate respectively. Brennan and Schwartz then applied a discrete approximation to the SDE's. They proceeded to an empirical investigation employing data on Canadian government bonds.

A further extension of the use of SDE's was given by Cox, Ingersoll and Ross (1985). The resulting model is referred to generally as the CIR model. The single factor CIR model is described by the SDE:

$$dr = \kappa\left(\theta - r\right)dt + \sigma\sqrt{r}dz \tag{7.4}$$

where κ, θ and σ^2 are constants. The parameter κ determines the speed of adjustment or the risk adjusted rate of mean reversion, θ is the long term mean level of the process and σ^2 is a parameter of the process. Cox *et al.* then extended and generalised these ideas to multi-factor models involving two or more state variables. The introduction of the $\sqrt{r}$ term ensured non-negativity (see Rogers (1995, p.99-103) for a discussion of the class of squared Gaussian models).

[1] Brennan and Schwartz use consols or irredeemable government bonds in their analysis, so does Wilkie (1985). As discussed in section 4.3 consols are not available in Australia.

Empirical studies into the success of the single factor CIR model, have been conducted by Brown and Dybvig (1986) and Brown and Schaefer (1994). Brown and Dybvig found a reasonable fit for the volatility of the model by comparison with the time series values under study but they found the model systematically overestimated short term interest rates. Further they found the model fitted short rates better than long ones, that is errors in pricing were not identically distributed across Treasury issues. They postulated a possible tax effect for some of the observed biases. Brown and Schaefer (1994, p.19) compared the observed term structure with that predicted by the CIR model and concluded "..for the UK at least the CIR model has sufficient flexibility to closely match the term structure of real interest rates.". They then considered various parameter estimates finding instability in the parameters. Brown and Schaefer further found three independent but strongly inconsistent estimates of the degree of mean reversion in real interest rates. They concluded that the model needed a good deal of modification if the issues that they had found were to be dealt with satisfactorily. Pagan, Hall and Martin (1996) undertook a review of models on the term structure. They reviewed the CIR model testing as to whether the model can produce the features observed in the data. Hence they wished the model to have, for example, yields that were near integrated. Pagan, Hall and Martin (1996, p.111) concluded that "The predictions from CIR type models are therefore diametrically opposed to the data.".

More recent papers have further generalised the SDE models. Brown and Schaefer (1995) applied a two factor model using the long rate and the spread between the long rate and the short rate. Describing the process via two SDE's, in the same format as the CIR single variable case, they conducted an empirical investigation with bond data involving six major currencies. They found reasonable and consistent values for the mean reversion parameter for the spread but the mean reversion parameter for the long rate in all cases was close to zero. The US data spanned 1930 to 1979 and the data for the other five major currencies was from 1984 onwards. Brown and Schaefer (1994, p.119) came to the view that only two factors are needed to explain changes in bond yields, "...we do not need to think in terms of five or ten factors but rather two or possibly three.". Duffie and Kan (1995, p.131) come to similar conclusions noting that "Some of the empirical studies mentioned above suggest that 2 or 3 state variables might suffice for practical purposes.". Sherris (1995), using Australian interest rate data, supported these conclusions. He conducted a factor analysis of yield curve changes finding that two factors explained some 96% of the variance in yield curve changes. The third factor contributed an extra 3%. He described the factors as representing a "parallel shift" factor (83%), a "slope" factor (13%) and "curvature" factor (3%). This is consistent with Pagan *et al.* (1996, p.98-104) in their discussion of the number of factors.

The Towers Perrin two factor model is a variant of the Brennan and Schwartz model (see Mulvey (1994), Mulvey and Ziemba (1995) or Mulvey(1996)). The success of this model would not appear to have been tested in the public domain. The description of the stochastic equations is given in Mulvey (1996) and are for the short rate, r_t:

$$dr_t = a(r_0 - r_t)dt + b\sqrt{r_t}\,dz_1 \tag{7.5}$$

and for the long rate, l_t:

$$dl_t = c(l_0 - l_t)dt + e\sqrt{l_t}\,dz_2$$

However, there were inter-linkages between the variables which were not detailed. Mulvey (1996, p.13) indicated that a and c were "...functions which depend upon the spread between long and short rates.". Further whilst b and e were specified as constants, the random coefficients dz_1 and dz_2 were *correlated* Wiener terms. These links were the critical relationships that integrated the planning model.

A survey of SDE solutions was given by Rogers (1995). Rogers investigated the range of mathematical solutions and considered the strengths and weaknesses of each of the various formulations. Rogers (1995, p.111) concluded that "..what is needed now is not *more* (more complicated) mathematical models, but rather a serious attempt to combine practitioner input with (probably extremely simple) mathematical models...".

A set of equations were given by Wilkie in his series of papers. His consol equation had an inflation component plus a real yield. The real yield is modelled with a mean reverting component using the long term average real yield (μ) of 3.1%. The model form for the real component was:

$$\mu \exp(N_t)$$

where $N_t = \alpha N_{t-1} + \eta E_y(t) + E(t)$, $E_y(t) = N(0, \sigma_y)$ from the dividend yield equation and $E(t) = N(0, \sigma)$. By linking the bond rate to the dividend yield Wilkie was postulating the non-independence of shocks to the system as well as allowing the real rate to slowly revert to its long term value via the exponential function. Carter (1991) rejected the Wilkie model for long term bonds as he found the consol yield to be a non-stationary series. His stochastic investment model had a two factor model for interest rates. He had an equation for T-notes and one connecting T-notes and bonds.

In a much earlier paper Volker (1982) attempted to model the nominal T-note rate with the use of inflationary expectations. He used a variety of integrated economic series finding the best explanatory variable was when excess LGS assets were introduced as a liquidity factor. Volker (1982, p.19) concluded that for the pre-Campbell Inquiry era 1968-1979, "To sum up, it appears that liquidity conditions have been the major determinant of short term interest rates in Australia over the last 10 years, and that inflationary expectations have been reflected to a significant but relatively small degree.". Volker's paper hinted that the use of the levels series of inflationary expectations was unlikely to produce a statistically better model for the level of T-notes.

There are other alternative functions that might be considered such as the cubic mean reversion model applied in a paper considering exchange rate dynamics by Bleaney and Mizen (1996). The models of Bleaney and Mizen do not include exogenous variables. All of these functions require either parameter estimation or some model of interest rates with a long term level to which these rates ultimately revert.

7.2.2 Real Bond Model

In the discussion of the model equation (6.4) in section 6.7, reference was made to the positive drift apparent in real bond rates and the carry over of this to the model for Δb_t. The standard mean variance optimisation process for asset allocation takes the history of returns, variances and covariances and projects these forward. The stochastic modelling method explicitly recognises the underlying factors that are creating the observed correlations between asset classes. Hence the cause of the drift must be investigated. The paper by Benari (1990) suggested there are a number of factors driving returns, one of which is inflation. He broke up the period 1966-1988 into four sub-periods defined in part by changes in both the overall level of the inflation rate and trends in it. Therefore particular financial eras are defined in part by both the level of, and trends in, real interest rates. For example high inflation followed by declining rates as in the 1980's. Asset allocation can then be viewed as defining the current era and anticipating future ones.

There are alternative strategies that amy be employed.

1. The positive or upward drift observed can be assumed to continue. However there is no reason to expect a continuation of this upward drift in real interest rates.
2. An *ad hoc* adjustment could be made, such as to hold real rates constant under a constant inflation scenario. It is by no means clear that this is what would happen if investors were confident that low to moderate stable inflation rates were to prevail.
3. An equilibrium value for the level of real interest rates could be found. This presupposes that real rates mean revert and that this level is a constant.

This does not detract from a potential explanation as to the causes of this positive drift in real rates. It can be postulated that the market under-estimated inflation in the 1970's. The chart of inflation expectations in figure 3.10 lends credence to this view. The liquidity risk premium for bonds was adjusted to compensate. With falling inflation the market has remained skeptical that low levels of inflation can be maintained leaving a high real rate of interest. Explanatory variables for this phenomenon will not be sought; the use of unexpected inflation did not solve this problem. An equilibrium value may be found but any choice made requires justification.

The nominal bond rate must remain positive. Therefore the model formulation must ensure that negative bond rates do not occur; this can happen if the real rate of interest is sufficiently large and negative. To investigate this and the success of the model (6.4) the difference equation was converted to real bond rates using actual inflation as the generating mechanism. This can then be compared to actual real bond rates to see how well the model (6.4) performed. The initial value of the real bond rate was set at the actual value in March 1960 and then the relation $b_{t+1} = b_t + \Delta b_{t+1}$ yielded progressive values of the real rate. The chart is given in figure 7.1.

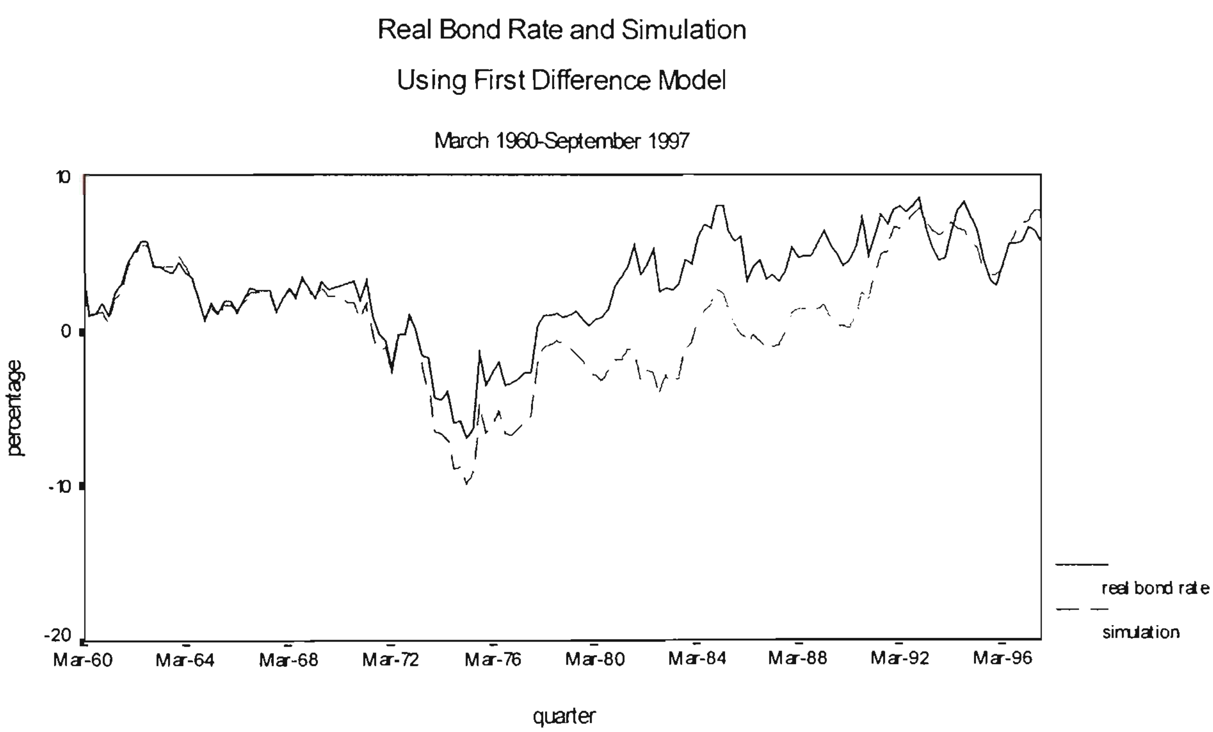


Figure 7.1 Effectiveness of the Real Bond Model (6.4).

The stochastic simulation tracks the changes in the difference in real rates. It fits the real bond series b_t closely up to the mid-1970's when real rates became highly negative due to the steep and unexpected rise in the rate of inflation. These very large negative real rates of interest were not sustainable. The speed of adjustment is much faster than the model allows and it has taken until the early 1990's for the model to catch up with this effect. It now seems to be performing well again.

The model (6.4) was calibrated over the period June 1975 to September 1997. The simulation is over the longer period September 1960 to September 1997. If the model was only fitted to the period post June 1975 then figure 7.1 shows that not only would the model have still been too slow to correct the large negative real rate but it would also have over-shot. That is, there would be a current real rate far higher than that observed. The model given by equation (6.4) needs to be able to take account of this mean reverting[2] characteristic.

7.2.3 Real Interest Rate Level

The long term real rate given by Wilkie of 3.1% for the UK, 2.65% for the US and 2.5% for France, may be contrasted with the value of 2.82% found for Australia. There is a degree of consistency though the periods are different. The magnitude of the real rate was investigated further. The best indicator that is available is the yield on indexed bonds, since the market is giving a direct judgement about real rates. There is available from the RBA a series of indexed bonds commencing in September 1986, and a chart of these is shown in figure 7.2. Real rates on indexed bonds during this period have ranged from 3.64% to 5.80% with an average of 4.89% and standard deviation of 0.58%. By comparison real bond rates over the same period have an average value of 5.64% and standard deviation of 1.56%. This is logical as the inflation risk has been taken out of the indexed bonds so there is lower volatility as well as a lower return. Nevertheless these values are, in both cases, well above the long term average real bond rate of 2.82%. This suggests that the level of the real rate is not constant. Expanding on the suggestions inherent in Benari (1990) a speculative hypothesis is that the level remains stable for extended periods then shocks to the system drive the rate to a new equilibrium which then persists. Figure 7.2 indicates

[2] Mishkin and Simon (1995, p.223) in an investigation of the relationship between real T-notes and inflation comment "... any reasonable model of the macro economy would surely suggest that real interest rates have mean reverting tendencies which make them stationary thus yielding a long run Fisher relationship.". Their comments would presumably be valid for real bonds given the nature of their analysis.

considerable stability in yields to indexed bonds and that a mean level of real interest rates of around 5% is applicable to the current 'era'.

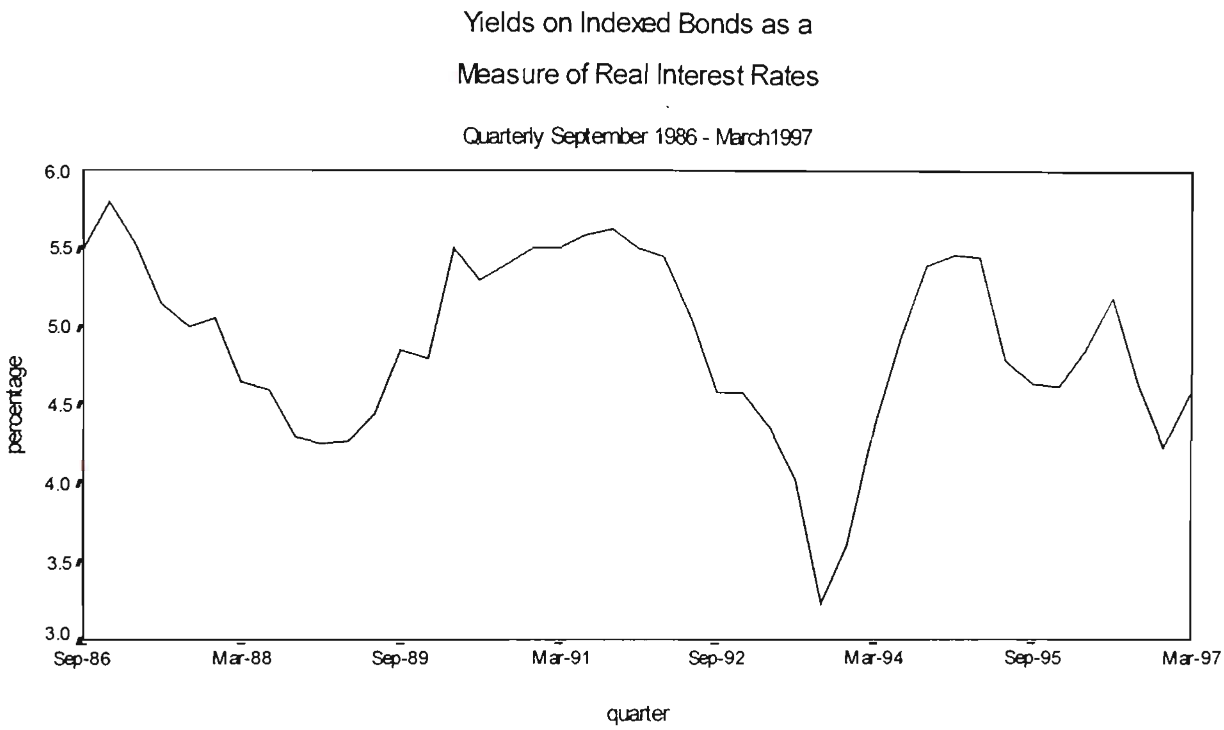


Figure 7.2 Yields on Indexed Bonds.

7.2.4 Real Interest Rate Volatility

The first decision as to how to modify the basic model to include levels information is to investigate the volatility of the interest rate process. The SDE equations assume a relationship exists between the absolute level of interest rates (b_t) and the volatility of the change in interest rates (Δb_t). The CIR model, equation (7.4), assumes the absolute variance of the difference in interest rates increases when the interest rate increases, whereas the Brennan and Schwartz model assumes that relationship is with the standard deviation rather than the variance. That is, it does not have the square root term. This may be tested by considering the volatility of Δb_t. This was done using a 5-year or 20-quarter rolling estimator of the variance as a measure of volatility. This is the same estimator as was used in section 3.3. Based upon the series

Δb_t the following estimator, $\hat{\sigma}_t^2 = \frac{1}{20}\sum_{i=t-19}^{t}\left(\Delta b_i - \overline{\Delta b}\right)^2$ was applied. A plot of the series is given in figure 7.3.

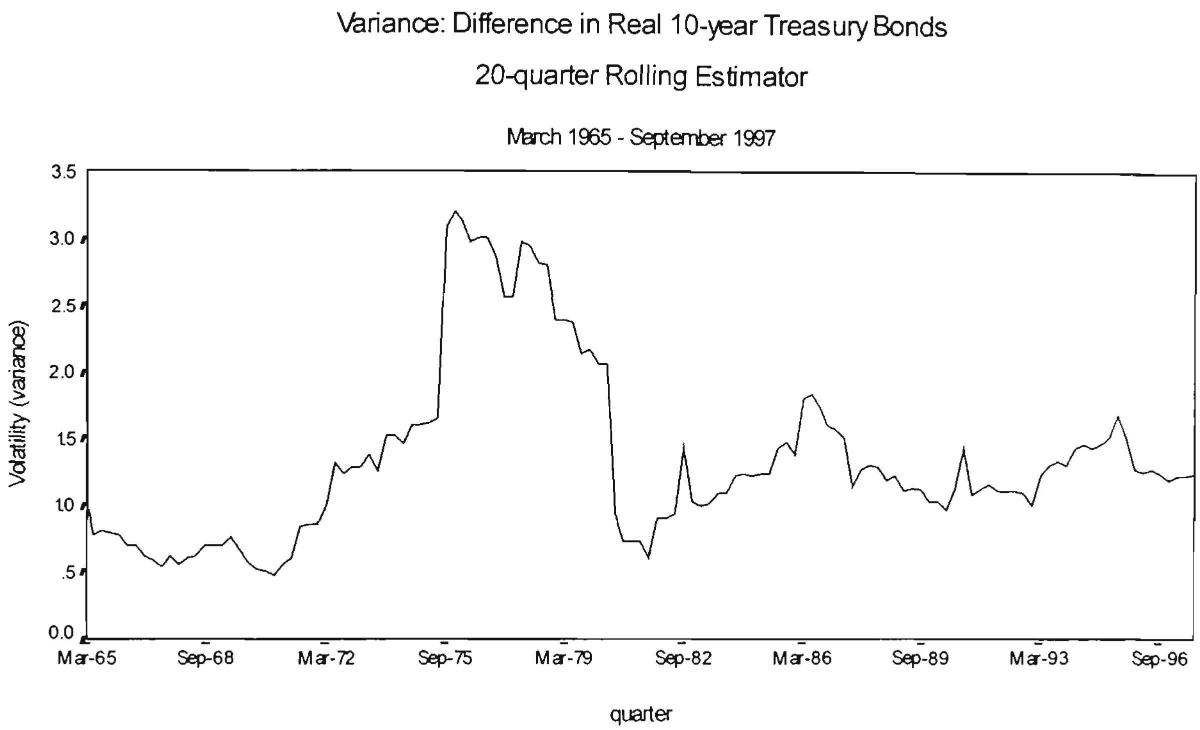


Figure 7.3 Volatility of the Difference in Real 10-year Treasury bonds using a 20-quarter Rolling Estimator of the Variance.

There was a sharp rise in volatility associated with the inflation increase in the early 1970's; see figure 4.10 showing the occurrence of large negative real rates and section 6.4.5 for a discussion of the underlying economics. Since then the volatility has declined as the effect has passed through. This is consistent with the application of the June 1975 breakpoint and related discussion in section 6.4.5. The series appears to be homoskedastic since 1980 when the volatility returned to the lower levels prevailing prior to 1972. A scatter plot of the relationship between b_t and volatility of Δb_t is shown in figure 7.4.

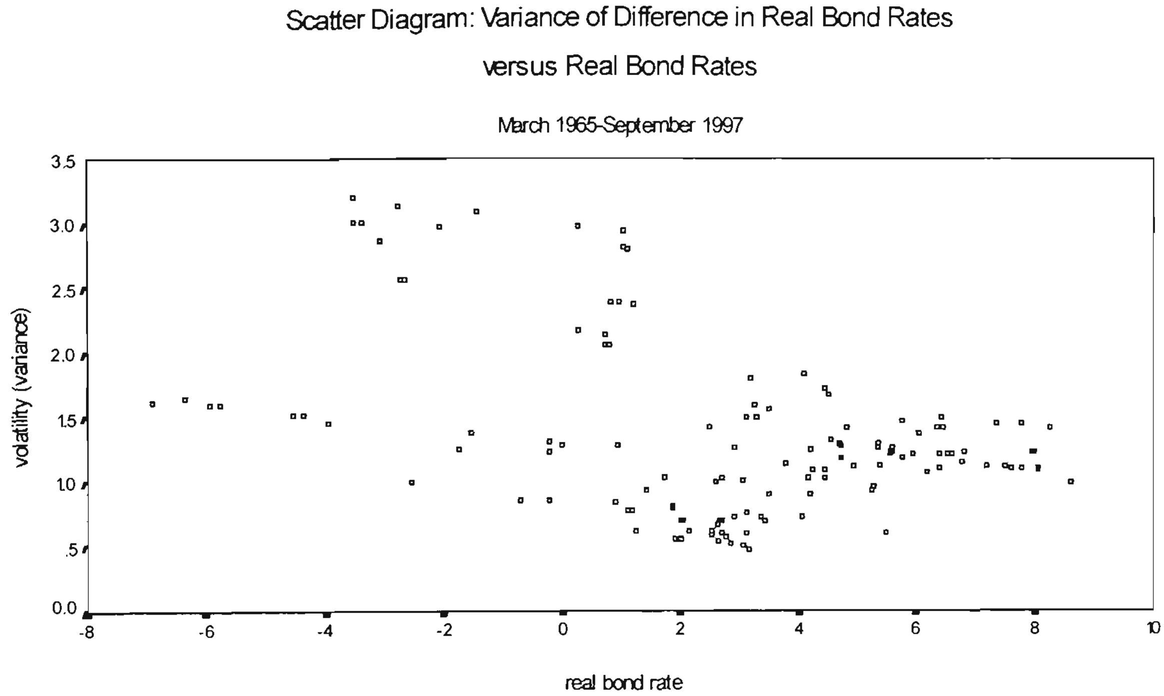


Figure 7.4 Scatter diagram of the 20-quarter Rolling Estimator of the Variance of the Difference in Real 10-year Treasury Bonds versus the Real 10-year Treasury Bond Rate.

The correlation between the two at -0.419 ($p = 0.000$) is highly significant. The plot in figure 7.4 shows a cluster of points with volatility values between 2.0 and 3.5 on the y-axis. These points represent the approximate period 1975-1980 in figure 7.3 when the volatility jumped. The balance of the data points in figure 7.4 exhibit no clear relationship. The data was re-examined for the period September 1980 to September 1997 when the volatility dropped. This time the correlation was 0.027 ($p = 0.829$).

Pagan, Hall and Martin (1996) examined this 'levels effect' on volatility. They used a different estimator; the nominal yield on a zero coupon bond with one month to maturity. The authors in the first section of their paper took the general model:

$$dr = (\alpha - \beta r)dt + \sigma r^{\gamma} dz \tag{7.6}$$

which is equivalent to (7.1) and converted it into discrete form. They then investigated the discrete form by regression techniques. They found in favour of the 'levels effect' on volatility and obtained estimates of the parameters α, β and γ. They concluded that "Based on the evidence from the indirect estimators, $\gamma = \frac{1}{2}$ seems a reasonable choice for the shortest maturity, which would correspond to the diffusion process used by Cox *et al.* (1995)." (Pagan, Hall and Martin (1996, p.97)).

The evidence outlined above with the use of quarterly real bond rate data does not suggest a 'levels effect'. This implies a simplification of the second volatility term in (7.6) and retaining the mean reverting component. This leads to $db = (a - br)dt + \sigma dz$ where b is the real bond rate. This is the model (7.2) originally proposed by Vasicek (1977). However this model allows for negative interest rates.

In the Australian case exhibited in figure 7.1, there appears to be a "snap-back" effect in the real interest rate trend, rather than the smoother relationship indicated by the solutions to the models (7.2) to (7.6). The modelling process could be extended to include a 'jump process'. A Poisson process could be used to model the "snap-back" feature with a model of the form $dX = \alpha(t)dt + \sigma(t)dz + \xi\, dw$. This model is such that the increment of X is given by the sum of a normally distributed random variable with mean $\alpha(t)dt$ and variance $\sigma^2(t)dt$ but with an occasional shock of magnitude ξ. Shimko (1992) discussed this possibility but bearing in mind Rogers' comment on the need for simpler models and the stated aim of a simple model structure, such an extension will not be pursued.

In summary the models examined in this section do not satisfy the criteria of non-negativity of nominal interest rates, mean reversion in real interest rates and simplicity at the same time. An alternative approach is suggested. This is now pursued.

7.3 An Empirical Investigation: The Real Bond Model

The consideration of more general discrete mean reverting models reflecting the facts is necessary. The use of reflecting barriers was considered. This model has a long history going back to Cootner (1964) who outlined a model of stock behaviour with prices being a random walk within reflecting barriers. These are formed by prices deviating too far from the mean (see Hart (1996, p.62-65) for a further discussion of a model utilising these principles). One limitation of this augmentation for the regression model (6.4) is that once real rates reach this barrier unless inflation falls, yields will not revert to the long run mean. Real rates may stay negative for a long time, since any increment in Δb_t is entirely dependent on the random term[3]. This would not be acceptable from an economic perspective as the elimination of negative real rates over a reasonably short time frame is to be expected. Real interest rates became negative in June 1973 and stayed so until September 1977 or for 17 quarters (this ignores some negative rates in an earlier period but these were interspersed with positive ones).

A alternative is to re-model our original equations to include the information on levels. A discrete form of the mean reverting function is indicated. This yields a regression of the form:

$$\Delta b_t = \alpha_1 i_t + \alpha_2 i_{t-4} + \alpha_3 f(\theta - b_t) + \varepsilon_t \qquad (7.7)$$

where $f(\theta - b_t)$ may take a variety of functional forms such as $(\theta - b_t)$ or $(\theta - b_t)^3$ and the impact is assumed small such as not to alter the significant lags in equation (6.4). The difficulty here is that $b_t \sim I(1)$ as is $(\theta - b_t)$. Thus it follows that the real interest rate series is difference stationary. This implies that there is no mean to which the series reverts as shocks are permanent. Now from table 4.11 row 2 the PP test showed b_t to have a unit root with a decisive rejection of the unit root null for Δb_t. The suggestion is then that there is a near unit root for this series. A possible interpretation of this is that the there is a mean level which is changing. To check the order of integration the ADF test was applied to the shorter time period June 1975 to September 1997 over which model (6.4) was calibrated. The test equation (3.2) was applied with 4 lagged differences using the recursive t-statistic procedure and with an intercept but no trend term. The ADF test statistic was -3.084 (p = 0.003) thus rejecting the unit root null. The PP test confirmed this applying a truncation lag length of 4 and with an intercept but no trend term the test value was -3.202 (p = 0.002). Thus $b_t \sim I(0)$ for this period. This does provide support to the hypothesis of a mean

[3] A test using simulations was carried out with different inflation scenarios but with a plateauing of the

level which is constant over various periods (see section 7.2.3). However the conflicting evidence is such that this series is best handled as a "near integrated" series which implies using an econometric approach for integrated series rather than that for stationary series. This also confirms the importance of the structural break in 1975 in the modelling exercise.

Bleaney and Mizen (1996) in their exchange rate analysis did not use an independent variable. The comparison that they made was between a linear and a cubic (polynomial of degree 3) mean reverting model with the level of the exchange rate plus lagged values of the difference. The linear model form in Bleaney and Mizen (1996, equation (12), p.40) is the ADF regression equation (3.2) with an intercept but no trend term. Critical values for this are available from MacKinnon (1991). The cubic function equation (11) in Bleaney and Mizen requires a Monte Carlo simulation to find the critical values in the regression. The null for the simulation for the test was a random walk. Now in general with integrated processes the test statistics are different. As Hendry (1991, p.56) commented "A major complication relative to conventional testing theory is that the distribution of tests tend to be influenced in important ways by unknown features of the data process.". Hence in the ADF test equation (3.2) different tables are required for the cases of a trend, intercept or trend and intercept. In the case here for equation (7.7) there is a lagged independent variable, so (7.7) needs modification. Now:

$$\Delta b_t = b_t - b_{t-1} = (\theta - b_{t-1}) - (\theta - b_t) \cong \alpha(\theta - b_{t-1}) - (\theta - b_t) \text{ if } \alpha \cong 1$$

hence a variation of (7.7) is obtained:

$$(\theta - b_t) = \alpha \ (\theta - b_{t-1}) + \alpha_2 i_t + \alpha_3 i_{t-4} + \varepsilon_t \qquad (7.8)$$

The model is of the form $(\theta - b_t) = \alpha(\theta - b_{t-1}) - \Delta b_t$ which is a simple variant on the model form (6.4). Now if $\Delta b_t \sim I(0)$ then so is $((\theta - b_t) - \alpha(\theta - b_{t-1}))$ for $\alpha \cong 1$,

---

inflation rate. The real bond rate in all cases converged on a constant value.

particularly given the near integrated nature of b_t. Therefore a grid of values of α over the region $0.9 \leq \alpha \leq 1$ will yield a set of stationary series which can then be regressed with the the rate of inflation and the rate of inflation lagged four periods as independent variables for the period June 1975 to September 1997. The mean real rate of interest was θ = 3.805%. This is consistent with the rates in section 3.2.2 and those given for indexed bonds which cover a time period that is more closely aligned with this. This was done in table 7.1. The resulting regression can then be transformed to yield a set of predictions of deviations from the mean. Then the fit can be optimised using $\mathbf{min}\left((\theta - b_t) - (\theta - b_t)'\right)^2$, where the predicted deviation is given by $(\theta - b_t)'$.

This procedure relates to section 6.4.5 table 6.7, the result when $\alpha = 1$ is the same as the last rows in that table. The data was mean adjusted to maintain consistency.

Table 7.1 Model Fit for Various Values of α in Regression (7.8)

α	0.90	0.92	0.94	0.95	0.96	0.98	1.00
coeff. i_t	-.998	-.989	-.980	-.975	-.970	-.961	-.951
coeff. i_{t-4}	.902	.917	.931	.939	.946	.961	.975
obj.func.	40.02	38.22	37.20	36.97	36.94	37.46	38.77

The objective function is smooth and flat bottomed with a mininum between 0.95 and 0.96. Now the above has been performed with mean adjustment and on the assumption of $b_t \sim I(1)$. These assumptions are changed, firstly by assuming $b_t \sim I(0)$ then OLS may be applied. This yielded the following regression:

$$(\theta - b_t) = 0.952(\theta - b_{t-1}) + 0.971 i_t - 0.907 i_{t-4} + \varepsilon_t$$

(39.188) (12.451) (-11.498)

There is a sign change on the inflation terms. This is because there is a $(\theta - b_t)$ term, changing the sign returns the expected relationships. The result is the same as that obtained in table 7.1, with a minor difference in the i_{t-4} coefficient. Re-inserting a constant with no mean adjustment to inflation yielded the following regression:

$$(\theta - b_t) = 0.093 + 0.940(\theta - b_{t-1}) + 0.949 i_t - 0.878 i_{t-4} + \varepsilon_t \quad (7.9)$$
$$(0.508)\ (30.435) \qquad (11.934)\ (-9.398)$$

where $\varepsilon_t \sim N(0, 0.433)$ and the various diagnostics were $LM(4) = 9.866$ ($p = 0.043$), $LM(12) = 14.595$ ($p = 0.251$), the Jarque-Bera statistic $= 1.437$ ($p = 0.487$) and the ARCH-LM test value $= 1.658$ ($p = 0.198$).

The equation (7.9) can be expanded by dropping the constant term which is not significant and substituting in the mean value of $\theta = 3.805$. This gave:

$$b_t = 0.228 + 0.940 b_{t-1} - 0.949 i_t + 0.878 i_{t-4} + \varepsilon_t \qquad (7.10)$$

For comparison a regression was applied applying the same method for the period June 1960 to September 1997 to yield the following equation:

$$(\theta - b_t) = 0.050 + 0.956(\theta - b_{t-1}) + 0.942 i_t - 0.912 i_{t-4} + \varepsilon_t$$
$$(0.050)\ (58.192) \qquad (19.642)\ (-18.068)$$

with an R^2 of 0.974. Substituting the value for the mean of $\theta = 2.828$ will yield the required equation.

There is consistency in each result with (7.10) the equation chosen. The rate of adjustment coefficient is given by (1-0.940) = 0.06 and the drift due to the difference in inflation terms is +0.071. The rate of adjustment parameter is small and of the same order as reported in Brown and Schaefer (1995) in their six country sample.

The non-negativity of nominal bond rates (B_t) derived from the real bond rate equation (7.10) needs investigation. Now between September 1948 and September

1997, $i_t \geq 0$ on 188 occasions and $i_t < 0$ on 8 occasions. The range of values was $-0.9 \leq i_t \leq 6.5$. The effectiveness of (7.10) then depends upon the rate at which a surge in inflation will impact on b_t and hence B_t. This is best tested via simulations of extreme but plausible ranges. The historic range suggests values on a annual basis between -5% and +30%. Simulations were run using (7.10) for a range -10% to +40% as in table 7.2.

Table 7.2 Non-Negativity Test: Annual Inflation Rates used in Model (7.10) Simulation

Year	1	2	3	4	5	6	7	8	9	10
Inflation	0	20	40	10	-10	0	30	0	-10	40

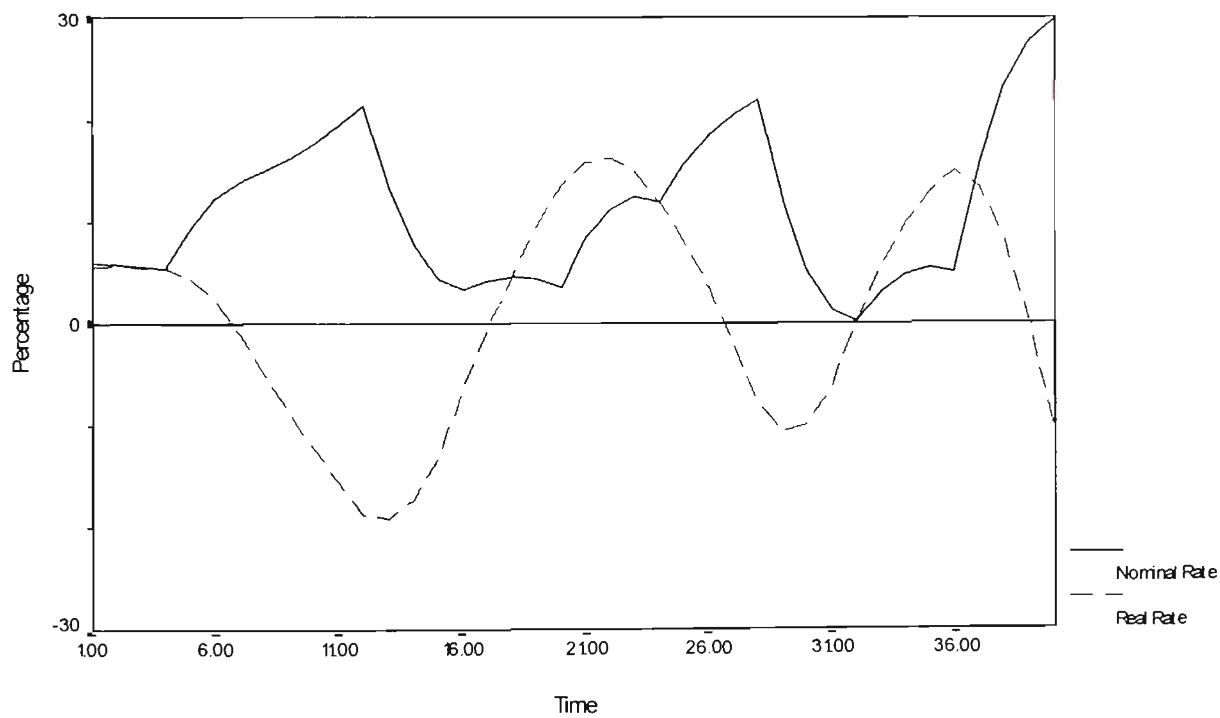


Figure 7.5 Real and Nominal Bond Rates from a Simulation of the Real Bond Model (7.10) using Inflation Rates from Table 7.2.

The rates of inflation in table 7.2 represent an extreme set of outcomes, particularly against the experience of the last 50 years. Despite this and real bond rates fluctuating between a band of approximately ±20% the nominal bond rate remained non-negative. The serial autocorrelation in the quarterly rate of inflation would guarantee a smoother outcome and the nominal bond rate which came close to zero in the simulation would be significantly above zero. The figure 7.5 also suggests a wandering characteristic for the real and nominal rate indicative of a near integrated series.

The real bond model provided by equation (7.10) satisfies the criterion of mean reversion in real interest rates. The scenario in table 7.2 showed that the criterion of the non-negativity of nominal interest rates was satisfied for even an extreme set of outcomes. The model is also mathematically simple, consistent with Rogers' conclusion (see the end of section 7.2.1)

7.4 Real Bond and Real Note Yields

Real T-notes are cointegrated with real bond rates. Hence predictions of nominal T-note rates may be obtained from the levels relationship for the real variables with inflation incorporated. Alternatively nominal T-note rates may be obtained from nominal bond rates and the long/short ratio which captures cointegration of nominal T-note and bond rates.

7.4.1 The Real T-note Model

The research evidence in section 7.2.1 is that only two factors are required to define the term structure. Evidence in support of this hypothesis comes from Duffie and Kan (1995); Brown and Schaefer (1994) and in an Australian context by Carter (1991) or Sherris (1995). Of the factors mentioned by Sherris, the "parallel shift" factor is given by the long bond rate and the "slope" factor by the long short ratio. This therefore implies that a separate equation for real T-notes is unnecessary. The yield curve can be determined from the long rate and the long short ratio or alternatively the spread between long and short rates. This yields a modelling approach in the style of that pursued by both Mulvey *et al.* (1994, 1995, 1996) and Wilkie (1984, 1987, 1992, 1995a and 1995b). Wilkie (1995a, p.299) described the interest rate on short term bonds by the stochastic equation:

$$\ln B(t) = \ln C(t) - N_B(t) - \mu_B$$

where $B(t)$ is the yield on short term bonds, $C(t)$ is the yield on an irredeemable bond (such as consols), $N_B(t) = \alpha_B\, N_B(t-1) + E_B(t)$ and $E_B(t) = \sigma_B\, N(0,1)$. The 'typical parameter value' given for α_B was 0.75 very close to the value of 0.785 that has been determined for the AR(1) coefficient in the long/short ratio model equation (4.6).

The choice of the second equation comes down to the use of either the long/short ratio which encapsulates cointegration at the nominal level between N_t and B_t or a direct relationship between n_t and b_t.

Now tables 4.10 and 4.11 showed that b_t and $n_t \sim I(1)$ for the period September 1960 to September 1997 and in section 5.3 cointegration tests on a range of bivariate relationships were carried out. Cointegration was found between real bonds (b_t) and real T-notes (n_t) (see tables 5.4 and 5.5). The structural break indicated at June 1975 and data problems with earlier data implies that the cointegrating relationship between n_t and b_t requires investigation over the shorter time period. The chart in figure 7.6 exhibits the connection for the shorter period June 1975 to September 1997.

An immediate difficulty is that $b_t \sim I(0)$ over this period. Now section 4.4.3 discussed the order of integration of n_t finding that $n_t \sim I(0)$ for September 1969 to September 1997. Subtracting the data points from the beginning of the data set has changed the series from $I(1)$ to $I(0)$. For n_t over the period September 1960 to September 1997, $\rho_1 = 0.910$ and over June 1975 to September 1997 this fell to $\rho_1 = 0.848$. Interestingly since the A$ float, over the period September 1983 to September 1997, ρ_1 has fallen further to 0.739. A similar situation prevails for b_t the value of ρ_1 falling from 0.938 to 0.876 to 0.659 over the same periods. This suggests that the shock to the system in 1973 has induced a unit root in the full series and the series September 1975 to September 1997 is no longer near integrated. However there is no reason to suppose that shocks will not occur in future[4]. This lends support to a near integrated characteristic for n_t as well as b_t. As in section 7.3 given the near integrated nature of these series following Pagan *et al.* they will be treated as integrated.

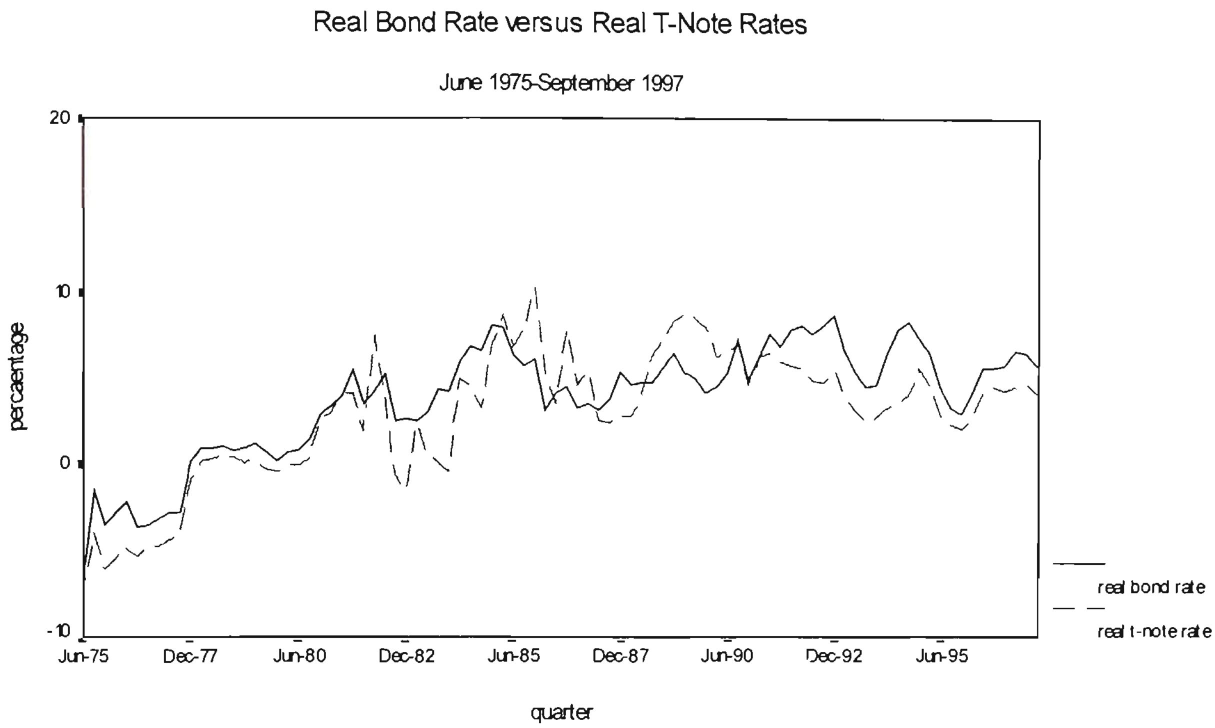


Figure 7.6 Real Bonds Versus Real T-Notes Showing Cointegration.

The first step is to check for cointegration between n_t and b_t. The cointegrating regression is given by:

$$n_t = -1.017 + 1.023 b_t + \varepsilon_t \qquad (7.11)$$

$$(-3.267) \quad (16.435)$$

for N = 90 observations, with an R^2 of 0.754 and a DW statistic of 0.547. The 5% level critical value for the CRDW test is given in Banerjee *et al.* (1993, Table 7.1, p.209) as 0.38 for N = 100 observations. This suggests a rejection of the unit root null

[4] If the series is assumed $I(0)$ then this would suppose an ability to predict a structural break, when presumably the series would return to a near integrated one. A speculative hypothesis would suggest that after such a shock an investor should understand the new financial era that has arisen and adjust assets accordingly.

and hence an acceptance of cointegration between n_t and b_t [5]. The ADF and PP tests were then applied to the residuals. For the ADF test applying the recursive t-statistic procedure the lag length was set at 4. The resulting test values were -3.427 without trend and -3.453 with trend included. The critical values at the 5% level for $N = 90$ observations and 2 variables given in MacKinnon (1991) are -3.405 without trend and -3.888 with trend. The PP test with 3 truncation lags confirmed this with values of -3.759 without trend and -3.726 with trend. This then confirms the rejection of the unit root null for the residuals, thus finding evidence in favour of cointegration.

The Johansen procedure was applied as a check on the results. The test allowed for a linear deterministic trend in the data. The lag length p was investigated using the SBC criterion (see the discussion in section 5.3). The penalty term, from equation (3.4) in multivariate form, is increasing by $k^2 \ln(N) = 4\ln(90) = 18.00$. The SBC suggests $p = 1$ for the lag length. In any case the results of table 7.3 confirmed cointegration as the LR statistic is significant at all lags.

Table 7.3 SBC Criterion and LR Statistic for Various Lag Lengths in the Johansen Cointegration Test VAR. Real Bonds and Real T-notes.

lag length(p)	SBC	LR statistic
1	611.38	22.853*
2	621.62	17.841*
3	627.44	15.485*
4	618.56	18.813*
5	635.64	19.368*

* Significant at the 5% level.

The cointegrating vector from (7.11) is (1,-1.023) not significantly different from the (1,-1) expected. For the optimal lag the cointegrating vector was (1,-1.127) which

[5] Since there is still autocorrelation in ε_t , OLS is not the most efficient technique. The cointegration result is checked with the Johansen procedure.

given a standard error of 0.146 for the second term in the normalised equation means it is not significantly different from the (1,-1) expected. Given quarterly data and the way the SBC performed, the value of the vector at $p = 4$ was found. The cointegrating vector was (1,-0.850) again not significantly different from (1,-1).

Now ε_t the residual series from the cointegrating regression (7.11) is an $I(0)$. So the *error correction mechanism* (ECM) may be used. This is a pair of equations of the form:

$$\Delta n_t = \alpha_{n0} + \alpha_n\left(n_{t-1} - \beta\, b_{t-1}\right) + \sum_{i=1}^{k} \alpha_i\, \Delta n_{t-i} + \sum_{i=1}^{k} \beta_i\, \Delta b_{t-i} + \varepsilon_{nt} \qquad (7.12)$$

$$\Delta b_t = \alpha_{b0} + \alpha_b\left(n_{t-1} - \beta\, b_{t-1}\right) + \sum_{i=1}^{k} \gamma_i\, \Delta n_{t-i} + \sum_{i=1}^{k} \delta_i\, \Delta b_{t-i} + \varepsilon_{bt} \qquad (7.13)$$

where now each term is an $I(0)$ and β is the parameter of the cointegrating vector. The error terms may be correlated but are individually i.i.d. normal variates or white noise. These equations can be considered as reparameterisation of the dynamic linear regression model in terms of differences and levels. It is similar to equation (6.1) without the inflation variable. It may be expected as is the case that here $\beta = 1$

A discussion of the ECM, the historic development and alternative formulations of Phillips, Sargan, Hendry and Granger was given by Alogoskoufis and Smith (1995). They considered the long run parameter estimates given from each formulation and interpreted them in an economic way. An example of an ECM for wage setting in the UK was presented, using the different formulations, with a critical discussion of the ECM approach including various restrictions not always observed which should be.

Now the equations (7.12) and (7.13) are a simple VAR in two variables. There are two alternative methods to solving these equations. The Engle Granger method is to take the results of the cointegrating regression (7.11) and substitute the lagged residual ε_{t-1} as an instrument for the error correction term. Then the VAR can be solved using

OLS. Otherwise the ECM can be solved by obtaining the cointegrating vector from the Johansen procedure and then the VAR applied. The Engle Granger method was applied using a VAR with lag length k in (7.12) and (7.13) of 5. This is consistent with the ADL equation (6.3). The following result was obtained where insignificant parameters were dropped and t-values are below.

$$\Delta n_t = -0.281(n_{t-1} - 1.023b_{t-1}) + 0.339\Delta n_{t-4} - 0.494\Delta b_{t-4} + \varepsilon_{nt} \qquad (7.14)$$

$$\quad (-2.414) \qquad\qquad\qquad (2.500) \qquad (-2.506)$$

$$\Delta b_t = -0.347\Delta b_{t-4} + \varepsilon_{bt} \qquad (7.15)$$

$$(-2.457)$$

The Johansen method yielded:

$$\Delta n_t = -0.297(n_{t-1} - 0.986b_{t-1}) + 0.345\Delta n_{t-4} - 0.498\Delta b_{t-4} + \varepsilon_{nt} \qquad (7.16)$$

$$\quad (-2.584) \qquad\qquad\qquad (2.570) \qquad (-2.559)$$

$$\Delta b_t = -0.354\Delta b_{t-4} + \varepsilon_{bt} \qquad (7.17)$$

$$(-2.531)$$

and these are almost identical. The residuals were tested for normality and serial correlation. For ε_{nt} the Jarque-Bera test statistic was 1.945 (p = 0.378) and the value of $Q(12)$ was 3.551 (p = 0.990) thus indicating there was no serial correlation in the residuals. The result for ε_{bt} was the Jarque-Bera test statistic at 5.028 (p = 0.081) and $Q(12)$ = 12.306 (p = 0.421). The error terms are significantly correlated with $corr(\varepsilon_{nt}, \varepsilon_{bt}) = 0.639$.

Comparing (7.13) with the results in (7.17) it follows that $\alpha_b = 0$ and $\gamma_i = 0$ for all i. Then it can be said that Δn_t does not *Granger cause* Δb_t. The long rate is setting the overall level of the short rate and not vice versa. Short term interest rates cannot deviate too far from the long rate. The speed of adjustment parameter is approximately 0.3. It is negative since long rates are normally above short rates. This is a rapid rate of adjustment and is consistent with figure 4.8 showing the change in the shape of the yield curve. From (7.11) where $\beta = 1$ then in the long run $n_t = n_{t-1} = n$ and $b_t = b_{t-1} = b$ hence $b - n = 1.017$, the constant in (7.12). By way of contrast the Johansen cointegrating equation used in finding (7.16) and (7.17) on the same assumption that $\beta = 1$ had a value for the spread of 0.874%. The actual spread between June 1975 and September 1997 is 0.928%.

The response to a one standard deviation innovation in Δn_t and Δb_t can be measured. This can be seen for the case of Δn_t as in figure 7.7. This shows that the impulse quickly dies down and by lag 6 the effect has reduced substantially; by lag 10 the response has died away.

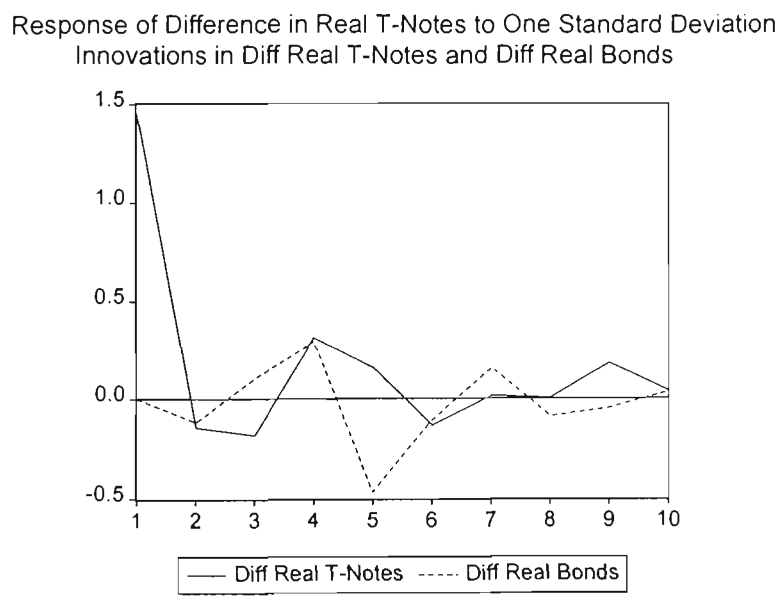


Figure 7.7 Impulse Response in Δn_t Due to One Standard Deviation Innovations in Δn_t and Δb_t.

The equations (7.15) and (7.17) are consistent with the univariate result, equation (4.7) in section 4.4.2, though the period differs. The results (7.14) and (7.16) may be compared to equations (4.8) and (6.5); the significant value at lag 4 for Δn_t is seen in both cases.

Mishkin and Simon (1995) found $n_t \sim I(0)$, hence b_t and n_t cannot be cointegrated. This they claim would invalidate the use of any analysis dependent upon this connection. The evidence here suggests a near integrated characteristic for real T-notes which acts like a stochastic trend process but does so within constraints. That is there are limits to the wandering of the real T-note, tied as it is to real bond rates. These are in turn tied to inflation so that real bond rates cannot become too negative or positive for a sustained period. This gives real bond rates their slowly mean reverting characteristic. This is consistent with Mishkin and Simon (1995, p.218) who concluded that changes in short term interest rates can reflect the stance of monetary policy however "...the evidence does not support the existence of a short run Fisher relationship in which a change in expected inflation is associated with a change in interest rates.". They also found for cointegration between the annualised rate of inflation and nominal interest rates and thus a long run effect. In the long run it is changes in inflation that affect the level of interest rates.

Inder and Silvapulle (1993, p.842) investigated bank accepted bills and concluded that interest rates show stickiness, "Nominal interest rates do not respond fully to rises and falls in the inflation rate, even in the long run.". Olekalns finds that after deregulation the Fisher effect can no longer be rejected. Olekalns (1996, p.855) concluded "For a pooled sample ...the nominal interest rate only partially adjusts to anticipated inflation. However analysis conducted on post-deregulation data alone shows that complete adjustment is achieved.".

The suggestions from these articles is that it is the speed and extent of the response of short term interest rates to inflation that is at issue. The results are consistent with cointegration between inflation and interest rates in the long run but with short run effects being the result, at least in part, of monetary policy.

7.4.2 Long Short Ratio Model

The long short ratio is an AR(1) model. In mean adjusted form it is given by $z_t = 0.785 z_{t-1} + \varepsilon_t$ where $z_t = m_t - \overline{m}$ (see equation (4.6) in section 4.4.1). Denoting by z_{t+l} the forecast made at time t for *lead* l, the forecasts can be determined recursively using, $z_{t+l} = 0.785 z_{t+l-1}$. Since only values up to $l = 0$ are known, from $l = 2$ successive values are deterministic as $E(\varepsilon_{t+l}) = 0$, $l > 0$. The forecast function is therefore precisely determined after z_{t+1}. The value z_{t+1} is called the *pivotal value*, see Anderson (1976, p.90-98) or Box and Jenkins(1976, p.126-170).

The forecast function generates an exponential path towards the long term average long short ratio. The rate of convergence slows as the deviations of the forecast values from the mean reduce. The rate of decline is a function of the parameters of the process and the pivotal values. In the case of an AR(1) the recursive equation is of simple form, the rate of convergence to the mean being given by $\phi_1{}^n$, where ϕ_1 is the AR(1) parameter. Now $m_t = 1.338$ at September 1997, so $m_{t+1} = 0.260 + 0.785(1.338) = 1.310$ and $m_{t+10} = 1.221$ and this compares to the mean long short ratio of 1.211. Hence the convergence is rapid and this is consistent with figure 4.8 showing rapid movements in the yield curve. It is also in accord with the speed of adjustment parameter in (7.14) and (7.16) and the impulse responses in figure 7.7 which are seen to be dying out after 10 quarters. This is consistent with a period of short term business cycles of order 4 years or 16 quarters.

7.4.3 Summary Discussion

Now equation (7.10) yielded an augmented bond model which included mean reversion. Equation (7.16) yielded an ECM for T-notes. The speed of adjustment parameter in (7.16) is such as to give a degree of equivalence between it and the exponential decline observed in the forecast function for the long short ratio model (4.6). The ECM does not capture essential features that are not found in (4.6). Further the ECM relies upon cointegration between n_t and b_t which is questionable *a priori* if the Mishkin and Simon view of no possible cointegration as $n_t \sim I(0)$ is accepted. The variable m_t, which relies upon cointegration between the nominal variables N_t and

B_t, is stationary. No difficulties with the order of integration arises. Therefore (4.6) will be used to generate T-note yields in preference to (7.16).

This is consistent with Wilkie but different from Mulvey *et al.* (see equations (7.5)). They used a variant of the Brennan and Schwartz model (see equations (7.3)). However the model of Mulvey (1996, p.12) was constructed in such a way as to have constraints between the long and short rate SDE's, "At its simplest, we assume that long and short interest rates are linked in their movements through a correlated white noise term and are further linked by means of a stabilising term that keeps the difference between the short and long rates under control.".

By applying the long short ratio to the nominal bond rate the nominal T-note rate is found. The implication of the modelling is that the yield curve moves to a fixed shape after 8 to 10 quarters. The model does not attempt to predict business cycles and there is no cyclic effect in the modelling procedure. However inverse yield curves or when the ratio of long rates to short rates falls below unity, have heralded economic slowdowns or recession in the past. Lahiri and Wang (1996) tested this proposition for the US via the utility of three measures of the spread in the term structure of interest rates as predictors of the business cycle[6]. They found the spread between 10-year bonds and 1-year bonds to be the best predictor signalling all the major turning points with no false signals.

7.5 Conclusions

A variety of solutions to finding a bond equation involving levels has been reviewed. The solutions from continuous time models involving SDE's have added complexities to models without necessarily a better representation of the observed facts.

Applying the yields on indexed bonds as a measure of real yields suggests that the mean level of real interest rates has not remained constant over time. Further an

[6] Lahiri and Dasgupta (1991, p.351) used interest rates as a predictor of inflation, comparing their results with other composite leading indicators by commenting that "This suggests that the usefulness of a composite leading indicator of inflation can be greatly enhanced if price forecasts extracted from the bond market are carefully included as one of its components.".

investigation did not find in favour of a levels effect on volatility for real bond yields. This confirmed the suggestion that modelling via SDE's using the standard equations in the literature such as that of CIR was not likely to produce a real bond model satisfying the observed facts.

Results from unit root testing of the real bond rate suggests that the series is a near integrated one. For the period June 1975 to September 1997 $b_t \sim I(0)$ whereas for the longer period June 1960 to September 1997, tables 4.10 and 4.11 showed that $b_t \sim I(1)$. This provides support to the hypothesis of a mean level which is constant over various periods. The real bond model fitted using levels data over the period September 1975 to September 1997 yielded equation (7.10). This equation finds that the real bond rate is best described by a mean reverting term plus the impact of the current rate of inflation and the inflation rate lagged by 4 quarters. The rate of adjustment parameter is small, hence mean reversion is slow. This is consistent with the empirical evidence in the finance literature.

The yield curve is found to be adequately defined by two factors. The long bond rate and either the long short ratio or the T-note rate are the only two factors required. Cointegration between real bond rates and real T-note rates allowed the derivation of an ECM linking real bonds and T-notes. A consideration of the ECM and the long/short ratio model (4.6) indicated that the more complex representation does not capture essential features that are not found in (4.6). So (4.6) will be used to generate T-note yields. This is consistent with the evidence in the literature outlined in section 7.2.1.

Chapter 8

Equity Series Levels Modelling and Connections

8.1 Introduction

The yield to maturity (YTM) is the principal factor when a fixed interest security is valued (see Sherris (1995)). Chapter 6 showed that the YTM responds significantly to a factor such as inflation and chapter 7 directly modelled the relationship. However the quarterly All Ords index does not exhibit such a relationship with inflation, as was seen in chapters 5 and 6. Indeed Australian authors such as Carter (1991) and Harris (1994, 1995) have found that the best model for the quarterly All Ords index is the random walk.

The models of Wilkie (1985) and Mulvey (1996) assumed connections between the fixed interest and equity markets. Mulvey (1996, p.14) assumed that "Stock returns are tied to a number of factors, including economic conditions of the companies (profits, taxes), inflation, interest rates and momentum.". Harris (1995, Appendix) covered a range of models showing significant differences in model structure. Amongst these alternative models there appears to be minimal agreement as to either the range of connections or the degree of connectivity between the fixed interest and equity markets.

The first objective of this chapter is therefore to investigate the equity equations. The second objective is to analyse the potential inter-relationships between the set of non-stationary variables, and then do likewise for the set of stationary variables of the proposed stochastic investment model.

The chapter is organised as follows. Section 8.2 reviews some of the literature, the competing models and the progress of research in Australia. The next section 8.3 analyses the long run connection between inflation, dividends and share prices. An

ECM connecting real share prices and real dividends is found and compared to the dividend yield equation (4.1). Section 8.4 investigates the dependence of each endogenous stationary variable on lagged values of the other stationary endogenous variables. A range of methods is used for data analysis employing ccfs, and both VAR and ADL models. Any potential connections between the residuals from the final equations are then reviewed to complete the investigation. The final section 8.5 reviews the stochastic investment model in the light of findings to present the working model.

8.2 Bonds and Equity Relationships

The extent and nature of any relationships requires investigation. Wilkie in his model included the random shock term from the dividend yield equation in the equation describing the yield on long term bonds (see section 7.2.1). Wilkie's approach would suggest that shocks to the system are mutually correlated in some way[1]. This possible link needs to be investigated.

The literature is extensive on the general topic of the predictability of stock returns and the connection between inflation, long term interest rates or other potentially predictive variables and stock returns. Chapter 2 reviewed the literature. The focus of the majority of the research is the US market. A sample would include DeBondt and Thaler (1989), Campbell and Ammer (1993) and Bollerslev and Hodrick (1995). Shiller (1989) and Mills (1991) analysed the predictability of the UK market. Hawawini and Keim (1995) addressed the over-concentration on the US market with a review of the evidence on the predictability of stock returns world wide. In this chapter specific studies on the relationship between inflation, interest rates and equity returns are the main focus.

[1] Economic theory would suggest that interest rates and monetary policy are factors in the variation of the dividend yield. Inverse yield curves, when the ratio of long rates to short rates falls below unity, have heralded economic slowdowns or recession in the past (see Lahiri and Wang (1996)). The business cycle mechanism would potentially transmit this slowdown into equity returns thereby affecting the dividend yield.

Fama and Schwert (1977) considered the link between asset returns and inflation. Given the time the paper was written the data is dated. They found that returns to bills, bonds and real estate to vary with their definition of expected inflation, but stocks returns to be negatively correlated with inflation and hence not a hedge. They concluded that "... the risk premium on stocks, the difference between the expected returns on stocks and bills, varies inversely with the interest rate." (Fama and Schwert (1977, p.519)). The idea that the discount rate explains the variation in returns was suggested by Fama and French (1988a)[2]. They used the dividend yield as a predictor of stock prices. Based upon a regression analysis with US monthly data over the period 1927-86, they found that the predictive power of the dividend yield improved as the period lengthened, and that this was caused by the discount rate effect. Hence a rise in the discount rate will cause stock prices to fall, thereby raising the dividend yield and therefore leading to future higher expected returns.

Mills (1991), using monthly UK data from the period January 1969 to May 1989, linked causality via cointegrating equations from bond yields to stock returns, with an error correction model for prices, dividends and gilts in section III of his paper. In his concluding remarks Mills states that "...gilt yields are strongly exogenous, that changes in gilt yields have a strong short-run effect on equity prices, and that the influence of dividends on prices is initially small and ambiguous, only becoming discernible after almost a year." (Mills (1991, p.254)).

Lee (1992) applied a vector autoregression (VAR) to monthly US data, for the sample period January 1947 to December 1987, for real stock returns, real interest rates, growth in industrial production and the rate of inflation. He found that inflation and stock returns are negatively correlated and concluded that share returns helped explain real activity, rather than vice versa. He found the direction of causality between interest rates and inflation to be from interest rates to inflation, rather than vice versa. Lee (1992, p.1602) commented that ".. interest rates explain a substantial fraction of

[2] There are other Fama and French papers covering the predictability of stock returns. Fama and French (1989) looked at the relationship between expected returns and general business conditions. The relationships found were mixed. The general message imparted is that expected returns are low when times are good and higher when they are poor. Fama and French (1988b) focused upon mean reversion

the variation in inflation, with inflation responding negatively to shocks in real interest rates.".

Campbell and Ammer (1993) used a VAR framework with excess stock returns, real interest rates, the change in nominal interest rate and the long/short spread plus two other variables. They applied their methodology to post war US monthly data, with a full sample period of January 1952 to February 1987. They suggested a positive relationship between inflation and stock returns, which differs from the usual interpretation of a negative one. They pointed out the difference between a long run versus a short run effect. In the long run stocks are a hedge against inflation. Earnings adjust to inflation via price rises for goods sold. In the short run there is an adverse effect on both stocks and bonds.

Boudoukh and Richardson (1993) conducted an analysis of the relationship between stock returns and inflation, based upon annual US (1802-1990) and UK (1820-1988) data. They used 1-year and non-overlapping 5-year returns. They regressed the returns on 1-year and 5-year inflation using instrumental variables to find an *ex ante* relation. They found small values for 1-year returns but larger and often significantly positive values at longer time horizons. The authors concluded that "...this paper provides strong support for a positive relation between nominal stock returns and inflation at long horizons. To the extent that researchers develop theories to explain the negative correlation at short horizons, these models should also be consistent with the evidence presented here." (Boudoukh and Richardson (1993, p.1354)).

Golob and Bishop (1996) attempted to settle the issue as to whether stock prices are driven by interest rates or inflation. They test the hypothesis via regressions with the earnings yield as the dependent variable and inflation, deviation of GDP from trend, and various measures of interest rates (30-year Treasuries to corporate paper) as independent variables. They concluded that stock prices follow inflation more closely and that any link with long term interest rates is minimal beyond that which is contained through inflation and its effect in turn on interest rates.

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and the extent to which shocks or unexpected changes to the discount rate are absorbed. Fama and French (1992) considered the predictability of cross sectional returns.

Graham (1996) applied regression analysis to quarterly US data for the period March 1953 to December 1990 to determine the relationship between real stock returns and inflation. He was interested in the causality of the often observed negative relationship and the role that monetary policy plays in Fama's conjecture that the link between inflation and stock returns was in turn due to their common link to real activity. On the basis of his analysis he found in favour of the Fama interpretation of the negative link between inflation and stock returns, concluding that "The negative relation appears to arise only in periods when monetary policy is either neutral or counter cyclical and when variability in the inflation rate is associated with variability in the growth of real output." (Graham (1996, p.35)).

Zhou (1995, 1996) investigated the predictability of returns from the stock market. Zhou (1995) applied the Kalman filter technique to evaluating various models of stock returns based upon the perceived requirement for any model to show the characteristics of both short run positive autocorrelation and long run negative autocorrelation. His preferred model only used past returns; there are no explanatory variables such as dividend yield. Zhou (1996) attempted to assess the predictability of returns from the stock market based upon the term structure, using regression methods and monthly US data from 1952-90. He claimed that there was a strong link between *ex ante* interest rates and expected stock returns and that his single measure of the term structure explained a significant part of the variation in dividend price ratios. He also found that his results suggested that the high volatility of the stock market was related to the high volatility of long term bonds.

Campbell and Shiller (1988) used a VAR framework and annual data from 1871-1987 to relate excess volatility to the predictability of stock prices via various factors (in particular, dividend yield; dividend growth rates and the earnings yield along with two long term moving averages of the earnings yield). They found the best predictability from a long term moving average of the real earnings yield, and that the dividend yield has predictive power for dividend growth. Shiller (1989) suggested that stock prices are predictable but not by a specific causative factor. Shiller compared the historic P/E ratio with the value based ratio or one based upon a dividend discount model of actual dividends. Shiller (1989, p.11) then postulated that markets have an irrational component causing this excess volatility, there was "...a tendency for stock prices to react to self generated fashions or fads.". This excess volatility makes stock prices predictable but not by a specific causative factor. A summary of the 'bubbles' literature[3] and the excess volatility debate is given by West (1988).

In the Australian context, Carter (1991) used Box Jenkins transfer functions applied to quarterly data for various periods in the broad span of 1970-1990. Carter found no significant correlation between share returns and inflation. These results are consistent with the later findings of Harris (1994). Carter (1991, p.354) investigated the possible link between interest rates and share prices concluding "...that interest rates are not a significant determinant of share price yields[4].". Harris developed his own ERCH (Exponential Regressive Conditional Heteroskedasticity) model class. He modelled annual (1949-92) share price index returns using T-notes as one of the lagged exogenous variables (see Harris (1994, p.50)). He found in favour of a link between 13-week T-notes and SPI returns but no link between CPI inflation and SPI returns. He found for a link between quarterly real GDP growth and quarterly SPI returns[5].

Groenewold, O'Rourke and Thomas (1997) investigated the relationship between nominal stock returns and inflation with quarterly Australian data from September 1960 to September 1991. They found a negative relationship between stock returns and inflation. They did not find that monetary policy, either pro or counter cyclical, had an impact. They set up a simple macroeconomic model and simulated their model in a variety of ways to pinpoint the source of the negative correlation between stock returns and inflation. They concluded that the negative relationship was "... the outcome of interactions in the economy as a whole. Inflation rates affect many macro

[3] "Bubbles" occur because investors seeing others profit will act in an irrational manner. West rejects bubbles as a potential explanation for the excess volatility. West feels that non-constant expected returns are a reality. This is the cause of the excess volatility. However he comments "...non-constant expected returns have manifested themselves in a variety of ways. What is left...is the source of the non-constant expected returns." West (1988, p.659).

[4] Carter uses the actuarial term *force of share price yields* defined as the logarithmic differenced All Ordinaries Share Price Index.

[5] The success of this approach is based upon the presumption of the ARCH nature of stock returns. Harris (1994, p.37) observed that "The heteroskedasticity extends for at most a few years, since the variance of annual SPI returns can be assumed to be constant from decade to decade.". Sherris (1997a) has tested these assumptions using inflation, equity and interest rate returns, as outlined in section 3.5. The results in Sherris did not find in favour of modelling the volatility in these series using an ARCH process.

variables some of which in turn, affect stock returns in addition to any direct effect." (Groenewold *et al.* (1997, p.134)).

This conclusion of Groenewold *et al.* is not at variance with Crosby (1998), as the results reflect short run features of the data. The negative link between stock returns and inflation is a short run and not a long run feature of the data. Crosby employed annual data from 1875-1996 and found share price returns and the rate of inflation stationary, hence there can be no cointegration and inflation can have no permanent impact on returns. This would appear to contradict Boudoukh and Richardson (1993), discussed earlier in this section, who find in favour of a long run positive connection between nominal share returns and inflation. However Boudoukh and Richardson's conclusions were based upon data from different countries, used a different period and regressions employing non-overlapping 5-year returns.

All the papers reviewed in this sample from literature on the relationship between inflation, interest rates and equity returns present the same problems, in that the choice of the data set and the econometric techniques employed allow of conflicting conclusions. Notwithstanding these caveats a distillation of the suggestions from the literature are that:

1. There is evidence in favour of predictability in stock returns. A range of factors have been put forward as predictors, such as the dividend yield.
2. There is evidence from some results to suggest that the level of long term interest rates is a potential factor in stock returns.
3. There is no clear support for monetary policy as a factor in nominal short term stock returns.
4. The exact relationship between inflation and nominal share returns is not clear; nor is the direction of any causality. The suggestion in the literature is that there is a short run negative relationship between share returns and inflation. There would appear to be no agreement on a positive long run relationship, which is taken for granted by Wilkie.

8.3 Equity Price Model: Non-Stationary Variables

In chapter 4 univariate linear and non-linear models for the stationary variables were considered. The models were extended in chapter 6 to include inflation as an independent variable. Then chapter 7 introduced the modelling of non-stationary fixed interest variables. In this way a process has been followed of developing separate single equations then giving them more rigour by considering any potential connections. The next aim is therefore to consider the connections between the non-stationary variables.

8.3.1 Non-Stationary Variables: Inflation, Share Prices and Dividends

A distinction is drawn between short run and long run experience. Fama and French (1988a) found that the predictability obtained from the dividend yield is long run in nature; the series is assumed to be mean reverting. The suggestion of an equilibrium value is consistent with the mean reversion hypothesis of Poterba and Summers (1998) and the results of Hart (1996, p.39-47). Fitzherbert (1992) suggested that there was an equilibrium value, given by the ratio of price to book value, about which stock prices fluctuate. The ratio of price to book suggested by Fitzherbert is 1.5 times implying that there is a liquidity premium[6]. The existence of such an equilibrium value forms the basis for tactical asset allocation (TAA) models.

Wilkie's model has a connection between nominal dividends, and therefore nominal share prices, and inflation[7]. Wilkie (1987, p.67) contended that "The parameters are such that a given percentage increase in the Retail Price Index ultimately results in the same percentage increase in the dividend index, so the model is said to have unit gain.". This assertion is questioned by other authors. Carter found no link between inflation and real share prices, suggesting various reasons for the observed decline in real share prices such as declining real growth rates. Crosby (1998) found that a 1%

[6] There is a premium for the advantage of public listing. This is reasonable *a priori* since public listing gives access to a wide group of potential investors.
[7] Note the discussion here is with respect to levels. Share returns are regarded as stationary (see Groenewold *et al.* (1997)).

positive shock to the price *level* leads to a 0.1% positive increase in the share price index.

Graham, Dodd and Cottle (1962) and Fitzherbert suggested that in the long run growth in ordinary share earnings comes from retained earnings rather than being the result of inflation. Retained earnings in an individual company is generally used for what may be called "stay in business" capital expenditure. For an industrial company this might be plant and equipment upgrades or for a service company brand development. This therefore maintains the earnings of an individual corporation. There is an argument that even though there is no cointegrating relationship between difference in the CPI and nominal dividends the use of retained earnings does provide a long run connection (see Campbell and Ammer (1988)). If the rate of inflation increases then, in the short run, more of the current earnings will need to be retained to maintain the level of future earnings. It may simply mean that the connection is one with long and variable lags which is consistent with the above argument.

The real All Ords series is not stationary, though table 4.4, row 5 showed that the ADF test statistic was not far outside the 10% level of significance. The chart in figure 4.7 shows no evidence of a trend in the real All Ords. This gives support to Wilkie's contention. The stochastic investment model developed herein uses the real All Ords p_t and real dividends d_t. Hence to translate these into nominal returns means that the inflation component is compounded as an accounting identity.

For the non-stationary or levels variables sections 5.3 and 5.4.2 showed that ΔI_t was not cointegrated with P_t, p_t, D_t or d_t. Sections 6.4.1-6.4.3 showed that for the stationary equity variables the impact of inflation was not significant. This does not mean that the price level is irrelevant (see Crosby (1998)) as listed companies produce goods and services that are part of the CPI. However for equities it may be concluded that since no significant and stable inflation relationships have been found, then there are none in the equity equations of the stochastic investment model.

Next the real levels series b_t, p_t and d_t were reviewed together. Now sections 5.3.2 and 5.5 investigated the cointegration characteristics of these series. The results indicated cointegration only between p_t and d_t. The 'confidence' result from the cointegrating regression (5.3) and as shown in figure 5.2, indicated no long run relationship between nominal 10-year bonds and the dividend yield. Indeed since these variables are $I(1)$ and $I(0)$ respectively, cointegration is impossible. Nevertheless this ratio cannot wander without bounds, otherwise if the yields were too far out of line investors would move between the share and bond markets. As was the case with nominal interest rates the tests show a unit root but this ratio must have 'stationary characteristics'. The near unit root for the nominal bond rate is the cause of the difficulty between the econometric result and the logic of financial arbitrage. Mishkin and Simon (1995) and Olekalns (1996) both comment on this feature of the nominal bond rate. The lack of a link between real bond rates and the real All Ords index or the real dividend is consistent with much of the literature. Poterba and Summers (1988) when looking for the economic justification for the transitory component they believe is present in share *returns*, find difficulty with the view of the discount rate effect as the cause due to the lack of success of interest rates as a predictor of stock returns.

However cointegration between p_t and d_t does allow an ECM to be constructed. The approach follows section 7.4.1. Again both the Engle-Granger and Johansen methods of obtaining the ECM were employed. Insignificant parameters were dropped and t-values are given below the coefficients. The Engle-Granger method yielded:

$$\Delta p_t = \underset{(-2.594)}{-0.139}\left(p_{t-1} - 21.511 d_{t-1}\right) + \varepsilon_{pt} \qquad (8.1)$$

$$\Delta d_t = \underset{(2.088)}{0.003}\left(p_{t-1} - 21.511 d_{t-1}\right) + \underset{(3.263)}{0.267}\Delta d_{t-2} + \varepsilon_{dt} \qquad (8.2)$$

The Johansen method yielded:

$$\Delta p_t = -0.101(p_{t-1} - 27.552 d_{t-1}) + \varepsilon_{pt} \qquad (8.3)$$
$$(-1.933)$$

$$\Delta d_t = 0.004(p_{t-1} - 27.552 d_{t-1}) + 0.279 \Delta d_{t-2} + \varepsilon_{dt} \qquad (8.4)$$
$$(3.005) \qquad (3.441)$$

The results from the two methods are similar. In equation (8.3) the cointegrating term is just outside the 5% confidence level. It is included for comparative purposes. The R^2 for (8.1) was 0.070 and that for (8.2) was 0.102. The residuals were almost identical hence the notation for the error terms is the same. The residuals were tested for normality and serial correlation. The diagnostics given are from the VAR method. For ε_{pt} the Jarque-Bera test statistic was 2542.7 (p = 0.000), indicating non-normality. The value of $Q(12)$ was 5.576 (p = 0.936) thus indicating no serial correlation in the residuals. For ε_{dt} the Jarque-Bera test statistic = 116.7 (p = 0.000) and $Q(12) = 7.050$ (p = 0.854). The error terms are significantly correlated and the value of $corr(\varepsilon_{pt}, \varepsilon_{dt}) = 0.435$.

The results in equations (8.1) and (8.2), and (8.3) and (8.4) may be compared with equations (7.12) and (7.13) by changing the variables. From (8.1) and (8.3) long run deviations from equilibrium have an impact on Δp_t via the speed of adjustment parameter which is approximately equal to -0.1. There are no other terms. This random walk process is consistent with section 4.3.3 and the literature. Similarly for Δd_t the speed of adjustment parameter is of order 0.004 and the only significant term is Δd_{t-2}. This is consistent with the AR(2) equation (4.4) found for Δd_t in section 4.3.2. The coefficient in equation (4.4) for Δd_{t-2} was 0.219, close to the estimates in (8.2) and (8.4). The R^2 for (4.4) was 0.045 by comparison with the 0.102 found for (8.2). This latter value is much smaller than the R^2 of 0.766 found for the dividend yield model, which reflects cointegration of the nominal variables.

In the long run using expected values $E(p_t) = E(p_{t-1}) = p$ and $E(d_t) = E(d_{t-1}) = d$. The constant term in the cointegrating regression applied in the VAR is not significant therefore in equilibrium $p - 21.511d = 0$ or $\frac{d}{p} = 0.0465$. Now the dividend yield is $\frac{D_t}{P_t} \cong \frac{d_t}{p_t}$ since the real variables are deflated by the same deflator, hence the long run average dividend yield is approximately equal to 4.65%. This compares to the value of 4.76% found in section 4.3.1. The ECM results are consistent with the previous conclusions.

The effect of an impulse response to unit innovations can be measured. This can be seen for the case of Δp_t as in figure 8.1. This shows that the impulse very quickly dies down and by lag 2 the effect has reduced substantially; by lag 6 the response has died away.

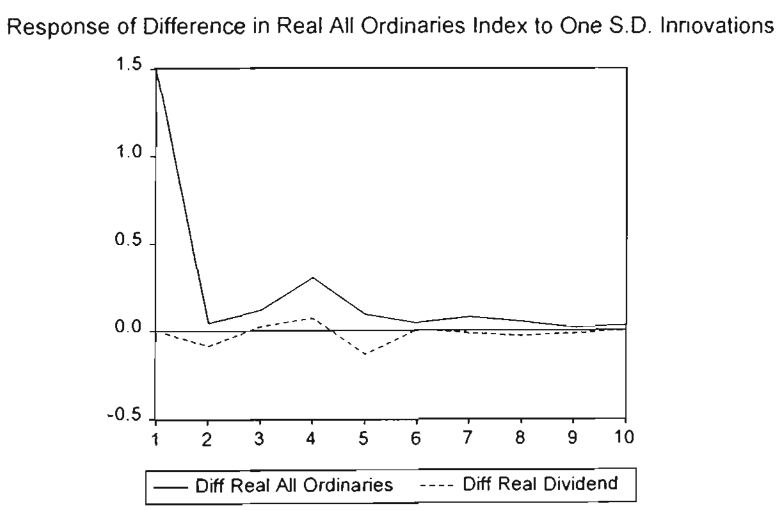


Figure 8.1 Impulse Response of Δp_t Due to One standard Deviation Innovations in Δp_t and Δd_t.

8.3.2 Dividend Yield Model

The quarterly dividend yield model is an AR(2). In mean adjusted form it is given by equation (4.1), $z_t = 1.045z_{t-1} - 0.19z_{t-2} + \varepsilon_t$ where $z_t = y_t - \bar{y}$. As in section 6.4.2 the forecasts can be determined recursively using $z_{t+l} = 1.045z_{t+l-1} - 0.19z_{t+l-2}$. Since only values up to $l = 0$ are known, from $l = 3$ successive values are deterministic as

$E(\varepsilon_{t+l}) = 0$, $l > 0$. The forecast function is therefore precisely determined after z_{t+2}, the pivotal value. The forecast function generates an exponential path towards the long term average dividend yield. The rate of convergence slows as the deviations of the forecast values from the mean reduce. The rate of decline is a function of the parameters of the process and the pivotal values. Now $y_t = 3.79$ and $y_{t-1} = 3.93$, the December and September 1997 values respectively, which yields a forecast of:

$$y_{t+1} = 0.69 + 1.045(3.79) - 0.19(3.93) = 3.90$$

Likewise $y_{t+2} = 4.05$ and so on, converging on the mean value 4.76. $y_{t+10} = 4.63$, so the convergence is rapid.

8.3.3 Summary Discussion

As was the case with real T-notes in section 7.4.1, the ECM in section 8.3.1 confirms earlier results, both of univariate modelling of Δp_t and Δd_t, and of the dividend yield. The rapid decline of unit impulses shown in figure 8.1 suggests rapid mean reversion. This is consistent with the dividend yield parameters in equation (4.1) which sum to 0.86. The more complex ECM representation does not capture essential features that are not found in (4.1). This may be compared to the ECM connecting real bond and real T-note rates in section 7.4.1. There the smaller parameter in the long/short ratio equation (4.6) of 0.785 was consistent with the slower mean reversion shown in the impulse response diagram in figure 7.7. The preferred model is then the dividend yield model.

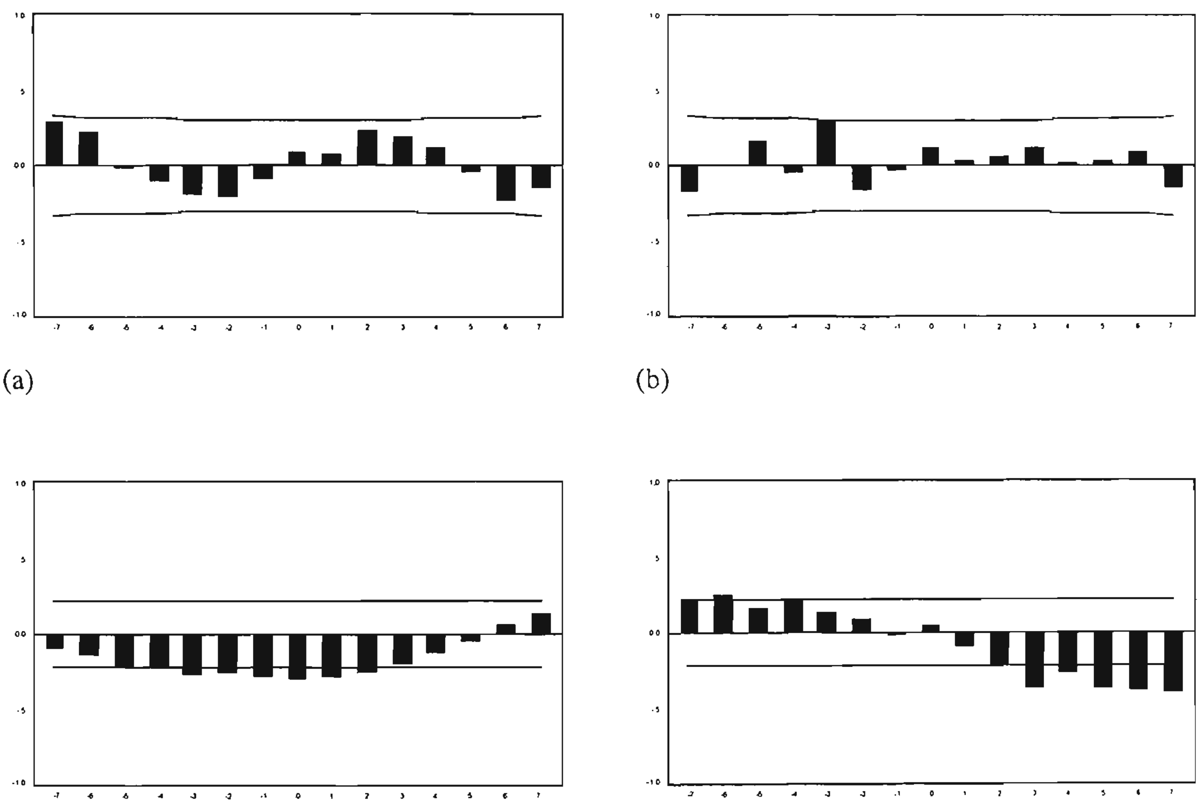
8.4 Equity Price Model: Stationary Variables

The Australian data is now investigated for potential dependence of one endogenous stationary variable on lagged values of the other stationary endogenous variable. In Chapter 6 the connections between inflation and the stationary variables was covered. Chapter 7 found that for fixed interest the required equations were those for b_t and

m_t, so Δn_t can be eliminated. In equities cointegration between real dividends and real share prices implies Δp_t can be eliminated. Hence only connections between the two stationary equity variables and two stationary fixed interest variables are required, reducing the permutations to four; Δb_t with Δd_t and y_t, and m_t with Δd_t and y_t.

8.4.1 Connections between Stationary Variables

The ccf between each pair of these variables was determined. As the bond equation (7.10) has been calibrated past the structural break at 1975 the data for the ccfs was from June 1975 to September 1997.



(a) (b) (c) (d)

Figure 8.2 Ccfs for the quarterly period June 1975 to September 1997 for the Response to Δb_t of (a) y_t and (b) Δd_t and to m_t of (c) y_t and (d) Δd_t. Values from -1.0 to +1.0 and lags -7 to +7. Confidence limits given by the lines parallel to the x-axis.

The ccfs of Δb_t with Δd_t and y_t shown in figure 8.2 (a) and (b) did not reveal any significant relationships. As a check an unrestricted VAR was run. The VAR may also yield some cross connections not observed when the variables are viewed two at a time. The VAR lag length p was set at 5, consistent with the ADL models in chapter 6, with $k = 3$ variables, therefore requiring 48 parameters including constants for estimation with 88 observations available. The resulting VAR with only significant coefficients given and t-statistics below was:

$$\Delta b_t = 0.381\Delta b_{t-4} + \varepsilon_t$$
$$(-3.774)$$
$$\Delta d_t = 0.046 - 0.022y_{t-4} + 0.017y_{t-5} + 0.323\Delta d_{t-4} + \varepsilon_t$$
$$(2.014)\ (-2.162)\quad (2.256)\quad (2.679)$$
$$y_t = 0.618 + 0.906y_{t-1} + \varepsilon_t$$
$$(2.100)\ (7.829)$$

No connections from Δb_t are observed. Δb_t is an AR(4) similar to equation (4.7). In Δd_t the lagged y_t coefficients are small and equal in magnitude but opposite in sign. y_t is an AR(1). These confirm the ccfs. The results may be also compared to those in chapters 4 and 6.

The ccfs of m_t with Δd_t and y_t shown in figure 8.2 (c) and (d) do appear to have a connection. There are both individually significant values and a collective significance observed in the patterns. A VAR was run as before with the following result:

$$m_t = 0.431 + 0.790m_{t-1} + 0.285m_{t-4} - 0.263m_{t-5} - 0.995\Delta d_{t-3} + \varepsilon_t$$
$$(3.219)\ (7.217)\quad (2.180)\quad (-2.606)\quad (-2.289)$$
$$\Delta d_t = -0.020y_{t-4} + 0.014y_{t-5} + 0.274\Delta d_{t-4} + \varepsilon_t$$
$$(-2.016)\quad (2.015)\quad (2.115)$$
$$y_t = 0.955y_{t-1} + \varepsilon_t$$
$$(8.187)$$

m_t has values at lags 4 and 5 equal in magnitude but opposite in sign. The equation for m_t may be compared to equation (4.6) which is an AR(1) with parameter 0.785.

The Δd_{t-3} term is discounted. The yield curve factor m_t does not enter the equation for Δd_t, nor for y_t which is an AR(1).

Given the difference between the ccfs and VARs, the series were modelled using ADL format equation (6.1). An ADL(5,5) was run using lagged values of Δd_t and y_t with m_t as the independent variable. A backwards reduction procedure was applied with a gradual elimination of non-significant terms. The result for Δd_t was:

$$\Delta d_t = -0.041 + 0.228\Delta d_{t-2} + 0.217\Delta d_{t-4} + 0.036 m_{t-4} + \varepsilon_t$$
$$(-2.068)\ (2.080) \qquad (2.015) \qquad (2.175)$$

where $R^2 = 0.186$, $LM(4) = 2.978$ ($p = 0.562$), $LM(12) = 7.753$ ($p = 0.804$), ARCH-LM = 0.037 ($p = 0.847$) and the Jarque-Bera Statistic = 8.951 ($p = 0.011$). Stability was tested by fitting the model over the shorter period June 1985 to September 1997. This breakpoint divides the time period approximately in two. The latter half is post the A$ float in 1983, the start date coinciding with the introduction of the capital gains tax in 1985. The results indicated considerable instability, with the coefficient of Δd_{t-4} nearly doubling to 0.417. In summary this does not suggest that the yield curve factor is a significant determinant of Δd_t.

There exists a complex inverse relationship between m_t and y_t with both lags and leads. The regression result was:

$$y_t = 0.834 y_{t-1} + 0.275 y_{t-3} - 0.105 y_{t-4} + 0.629 m_{t-2} - 0.636 m_{t-3} + \varepsilon_t \qquad (8.5)$$
$$(14.769) \quad (6.037) \quad (-2.382) \quad (2.196) \quad (-2.202)$$

where $R^2 = 0.868$, $LM(4) = 28.398$ ($p = 0.000$), $LM(12) = 34.216$ ($p = 0.000$), ARCH-LM = 4.504 ($p = 0.034$) and the Jarque-Bera Statistic = 27.442 ($p = 0.000$). These diagnostics for the residuals are not satisfactory. The coefficients of m_{t-2} and m_{t-3} are almost equal in magnitude and opposite in sign, so any potential yield curve effect is almost immediately reversed. This compares to the univariate dividend yield equation

(4.1) of $y_t = 0.691 + 1.045y_{t-1} - 0.190y_{t-2} + \varepsilon_t$ with an R^2 of 0.766. There is a gain in explanatory power by adding in the long short ratio and a link across to the term structure. The sum of the coefficients in (4.1) is 0.855 by comparison with 1.004 in (8.5). With a breakpoint of June 1985 the Chow test value was 2.775 (p = 0.735) suggesting stability in the coefficients. However the failure of the model to pass the diagnostic tests implies that this model cannot be profitably applied.

The evidence suggests a potential relationship between the dividend yield and the long short ratio. But the investigation did not reveal a satisfactory model linking the two. This suggestion is consistent with the results of Graham (1996) and Groenewold *et al.* (1997), albeit for stock returns. Graham found that the impact of monetary policy on stock returns and inflation varies, and Groenewold *et al.* (1997) found no impact of monetary policy on stock returns and inflation.

8.4.2 Correlations Between Residual Terms

Shocks to the financial system affect all variables. Hence it may be conjectured that the error terms are correlated. This was the case for the error terms in the ECM equations (7.14) and (7.15), and (8.1) and (8.2). Hence the independence of the error terms requires checking. The error terms from the equations (7.10), (4.1), (4.6) and (4.3) were investigated. The terms are labelled $\varepsilon_{ht}, \varepsilon_{yt}, \varepsilon_{mt}$ and ε_{dt}. The results are tabulated in the correlation matrix, table 8.1 with p-values in brackets.

Table 8.1 Correlation Matrix Between Stochastic Equation Error Terms for the Period June 1975 to September 1997

error term	ε_{ht}	ε_{yt}	ε_{mt}	ε_{dt}
ε_{ht}	1			
ε_{yt}	0.221(*.039*)	1		
ε_{mt}	-0.056(*.602*)	-0.040(*.710*)	1	
ε_{dt}	0.167(*.119*)	0.247(*.019*)	0.074(*.491*)	1

This suggests a possible connection between the real bond and dividend yield residuals, and those from the dividend yield and real dividends. These connections were tested further by dividing the period with the breakpoint March 1985, as well as including the period from September 1948 to March 1975 for those series for which the data was available. The results are shown in table 8.2.

Table 8.2 Correlations Between Selected Error Terms for Different Time Periods

Period	Sep48 to Mar75	Jun75 to Mar85	Jun85 to Sep97
$\rho\ (\varepsilon_{bt}, \varepsilon_{yt})$	NA	0.080(*.628*)	0.299 (*.037*)
$\rho\ (\varepsilon_{yt}, \varepsilon_{dt})$	0.168(*.089*)	0.226(*.161*)	0.299 (*.037*)

The results in table 8.1 masks the variation over time. This instability leads to the conclusion that the correlations are not significantly different from zero and that the results in table 8.1 are Type I errors. Hence the error terms are independent.

8.5 Stochastic Investment Model: Review and Summary

Both the final bond model equation (7.10) which incorporates a modification of equation (6.4) and the original equation (6.4) were calibrated over June 1975 to September 1997. Logically it is appropriate to recalibrate the equations (4.1) and (4.6) over this shorter period. Doing so for m_t yielded:

$$m_t = 0.241 + 0.801 m_{t-1} + \varepsilon_t$$

This is almost identical to (4.6) indicating the stability in this equation. Likewise when y_t was re-run the best equation was an AR(1) but with a parameter value of 0.869 by comparison with the sum of the AR terms in (4.1) of 0.855. The coefficients of these rapidly mean reverting autoregressive equations are stable. This is a reflection of cointegration where shocks are absorbed by the component series so as to maintain a stable relationship in the quotient. The structural break in the bond market is not reflected in movements in the shape of the yield curve. So the original equations (4.1) and (4.6) are kept.

The dividend yield model (4.1) is a simple autoregression. There needs to be a reconciliation between the models for the dividend yield and the real dividend, with that for the real All Ords index. There is no evidence supporting the connection of real dividends or real share prices to inflation or interest rates. No other variables have been found to help predict the level of p_t. The model for Δd_t is a random walk with zero drift. The dividend yield is modelled as an AR(2); the dividend yield reverts over a relatively short time frame, as seen in section 8.2.2. When a forecast of the dividend yield is applied to nominal dividends to obtain a forecast of the All Ords, an AR(2) model for the All Ords is implied. This appears to be at variance with the results in sections 4.3.3 and 5.4.3, suggesting that a random walk model best fitted p_t. Further by modelling real dividends as a random walk implies a nominal dividend stream which is correlated with the level of inflation. But this contradicts the findings of section 5.4.2 that inflation is not a determinant of long run share prices[8].

In section 7.3 a mean reverting element was introduced into the real bond to generate equation (7.10). There were sound economic arguments for this, such as the long run requirement for a non-negative real rate. Because of the significance of the inflation terms the mean reverting component only contributes significantly when deviations are large. The economic arguments for a constant level of the real dividend are more difficult. The key question is "Is $\alpha = 1$?", in the model, $d_t = \alpha d_{t-1} + \Delta d_t$, where the stationary component $\Delta d_t = \varepsilon_t$. The unit root tests given in tables 4.3 and 4.4. showed that the value of α is not significantly different from 1. This leaves a random walk model for real dividends and by a similar argument for the real All Ords index.

[8] See the earlier discussion in section 8.3.1. There the relationship of the level of long run share prices with retained earnings provided justification for a potential connection of long run share prices to inflation. Sections 11.3 and 11.4 also provides a discussion of this issue.

The dividend yield prediction is 'modifying' the All Ords random walk model to force the All Ords index back towards the long term trend line[9]. The net result of the forecast function and the indexation of dividends means that there are models for share prices, dividends and hence returns which are simple but limited. This is a logical consequence of the random walk nature of real dividends.

The decisions by trustees or consultants on asset allocation are best made by reviewing alternative scenarios and performing a sensitivity analysis based upon those scenarios. Point estimates based upon relationships that are unstable or not proven is inappropriate. Nor can allocations be made on the basis of historic correlations without understanding the reasons for those facts.

The stochastic investment model can now be put in its final form.

$$m_t = 0.260 + 0.785 m_{t-1} + \varepsilon_{mt} \quad \varepsilon_{mt} \sim N(0, 0.0151) \quad \sigma(\varepsilon_{mt}) = 0.123 \qquad (4.6)$$

$$b_t = 0.228 + 0.940 b_{t-1} - 0.949 i_t + 0.878 i_{t-4} + \varepsilon_{bt} \qquad (7.10)$$

$$\varepsilon_{bt} \sim N(0, 0.433) \quad \sigma(\varepsilon_{bt}) = 0.658$$

$$B_t = \left\{\left(1 + \frac{b_t}{100}\right)\left(1 + \frac{i_t''}{100}\right) - 1\right\} * 100 \qquad \text{(definition, see 4.3.2)}$$

$$N_t = \frac{B_t}{m_t} \qquad \text{(cointegrated series)}$$

$$y_t = 0.691 + 1.045 y_{t-1} - 0.190 y_{t-2} + \varepsilon_{yt} \qquad (4.1)$$

$$\varepsilon_{yt} \sim N(0, 0.217) \quad \sigma(\varepsilon_{yt}) = 0.466$$

$$d_t = d_{t-1} + \varepsilon_{dt} \qquad \varepsilon_{dt} \sim N(0, 0.0017) \quad \sigma(\varepsilon_{dt}) = 0.041 \qquad (4.3)$$

$$D_t = d_t I_t \qquad \text{(definition, see 4.2.2)}$$

$$y_t = \frac{D_t}{P_t} \qquad \text{(cointegrated series)}$$

[9] It may be speculated that the predictability of the dividend yield is the result of the majority of dividends arising from industrial shares. The more volatile and lower dividend paying resource shares contribute more to the price index (see figure 4.2). As the resource sector of the Australian market continues to decline in its weighting and importance then there is the potential for the index to become more predictable.

B_t is on a YTM basis. This necessitates a conversion from a bond yield into a bond return. The T-note return can be found from the T-note yield directly . The modelling procedure generates nominal dividends and the nominal All Ords index for equities. These can then be converted into equity returns.

8.6 Conclusions

The exact relationship between inflation, interest rates and equity yields is not clear; nor is the direction of causality. Any potential negative link between share returns and inflation is a short run rather than long run effect. The evidence favours the proposition that inflation is not a determinant of long run nominal share prices. The suggestion is that long run growth in share earnings, and therefore prices, is the result of retained earnings. There is evidence from some results to suggest that the level of long term interest rates is a potential factor in stock returns. However the results of section 5.5 suggested that the connection between the dividend yield and gilt yields observed in the UK is not observed in Australia.

Cointegration between the real All Ords and real dividends allowed the derivation of an ECM linking the real All Ords and real dividends. A consideration of the ECM and the dividend yield equation (4.1) indicated that the more complex representation does not capture essential features that are not found in equation (4.1). The dividend yield equation has greater explanatory power. Therefore equation (4.1) will be used to generate share prices.

The AR(2) dividend yield model mean reverts rapidly. The real dividend is best modelled as a random walk, implying that nominal dividends are indexed to the rate of inflation. The All Ords index resulting from the product of the dividend yield and the nominal dividend mean reverts over 2-3 years. This is at variance with both the random walk nature of the short term All Ords and the lack of long term relationship between inflation and share prices.

An analysis of the of the stationary variables was conducted. The evidence suggests a potential relationship between the dividend yield and the long short ratio. However a satisfactory model linking the two was not obtained. There is no significant or stable link between the residuals from each of the series in the stochastic investment model. In summary no stable relationships were found between the stochastic trend components of the differenced stationary variables or the stationary variables.

The evidence from the literature exhibits substantial areas of conflict. This is consistent with the results in this chapter to the degree that differing results, with different definitions of variables can be obtained over different periods. This may help to explain why such often contradictory results have been obtained by various authors. The proposed stochastic investment model, based upon Australian data, is therefore different from the stochastic investment models by either Wilkie or Mulvey.

Chapter 9

Equity Risk Premium

9.1 Introduction

Chapter 8 discussed the connection between equity price indices and yields on fixed interest. A more traditional measure of asset class relativities is defined by returns. The equity risk premium (ERP) is defined as the difference between returns to the equity and fixed interest asset classes, typically the risk free asset. These may be on an *ex ante* and *ex post* basis. The *ex ante* or *ex post* ERP are therefore new variables. Explanations as to the magnitude and direction of the ERP are dependent on explanations as to movements in and between, its component parts. For example the cost of capital is an *ex ante* return which is given by the expected return from the All Ords accumulation index.

Whilst not central to the modelling process *per se*, the ERP will be an output from the stochastic investment model. It can therefore provide a useful benchmark for comparing the results of any simulations. Movements in the ERP are central to tactical asset allocation (TAA). Models used in funds management apply measures of valuation to decide on movements between cash, equities and fixed interest. These TAA methodologies normally assume mean reversion. Sharpe and Sherred (1989) contains a range of papers from Sharpe, Clothier (Wells Fargo), Henriksson (Kidder Peabody), Brinson (Brinson & Associates) and Grossman. These strategic and tactical issues are the *raison d'être* of stochastic investment modelling.

This chapter is structured as follows. Section 9.2 introduces the concepts and some definitions and then follows with a literature review. The next section 9.3 details a methodology for dealing with *ex ante* values. The value of the ERP is then adjusted for dividend imputation and an assessment of the reasonableness of the current level is performed. Future trends are then discussed to see what range of values the ERP could take. In section 9.4 a potential modelling process for the risk premium is outlined. A model utilising expectations and employing the cost of capital is put forward. The model provides a direct method of obtaining returns to shares via bond returns. This would then allow a change in the equity equations in the stochastic investment model presented in section 8.5.

9.2 The Equity Risk Premium: Definitions and Discussion

In this section the observed difference between bond and equity returns, termed the risk premium is introduced and discussed. Both the *ex post* and, by employing various simplifying assumptions, the *ex ante* values of the risk premium may be found. The nature and extent of any simplifying assumptions are central to the discussion of a number of studies into the *ex ante* risk premium.

9.2.1 A Discussion of the Risk Premia

A risk premium is the extra return obtained from taking on the additional risk in investing in one particular asset class over another. The major risk premia are defined by the following. Note that the three measures are not independent; any one of the three is deducible from the other two (see table 9.1):

1. The liquidity risk premium, this is the premium of the returns from 10-year Treasury bonds over the returns from the riskless 13-week Treasury note (T-note).
2. The equity risk premium (ERP), this is the premium of the returns from the All Ordinaries index over the returns from 10-year Treasury bonds.
3. The short term equity risk premium, this is the premium of the returns from the All Ordinaries index over the returns from the riskless T- note.

The liquidity risk premium has a degree of equivalence with the long/short ratio[1] but the premium is based upon returns rather than YTMs. A ratio would be inappropriate

[1] The long short ratio is the ratio of the *ex ante* returns, if the YTM of the 10-year bonds is taken as the expected return to bonds. A stationary positive ratio is obtained. The same would not be true of *ex post* returns.

as negative or zero return values are possible in the denominator. Therefore these three measures, the liquidity risk premium, the ERP and the short term ERP, are differences. In modelling the yield curve the long/short ratio[2] was used.

There is a difference between the *ex post* and *ex ante* risk premia. The *ex post* risk premium is that obtained after the event and represents the difference in returns actually found from the two asset classes under consideration. The *ex ante* risk premium represents the risk premium based upon the expectations of market participants and is the difference in expected returns. The determination of the *ex post* risk premia is straightforward. However the volatility of such a measure makes conclusions about long term trends difficult (see figure 9.1). The *ex ante* value implies a need to consider expectations of such variables as dividends, earnings and the rate of inflation. Such statistics are difficult to find, particularly series of sufficient length. Hence certain simplifying assumptions have to be made.

9.2.2 Background: Studies on the Risk Premia

The academic literature contains a number of studies detailing aspects of the risk premium. Many of these studies, particularly those from practitioners, discuss at the same time the closely related topic of TAA. The overwhelming majority of such studies use US data.

Siegel (1992) conducted a long term US study over the period 1802-1990, finding a steady level of the average US *ex post* premium over short term bonds of 6%. Ambatscheer (1989) posits a much lower average US *ex post* risk premium of 2.5% over bonds and 4% over T-bills. Hathaway (1995) for the post war period found an average Australian *ex post* premium over T-notes of 6.6%. Hibbard (1998) examined the New Zealand experience. He defined the ERP as the excess return on equity over investment in a 90-day New Zealand Treasury bill. He found the value of the average New Zealand *ex post* premium for the period September 1982 to December 1997 to be

[2] By using a ratio of interest rates the measure becomes scale independent. Given a high interest rate structure a larger spread would arise for a given relative degree of tightness in the yield curve.

6.2%. This is consistent with the Australian result. It does mask a high degree of variability in all the *ex post* results.

A number of authors grapple with the more difficult task of obtaining the *ex ante* risk premium. Harris and Marston (1992) used short term share analysts forecasts and the dividend discount model (DDM) to derive the equity return for the US market over the period 1982-91. This was a short term study, with few data points and hence of limited value.

Yamaguchi (1994, p.18) worked backwards from a risk profile. He defined the risk premium as "...the excess return for equity over bond yield demanded by an investor tolerant of a specific level of downside probability.". He estimated a downside probability by counting the number of months that the equity total return failed to outperform the bond income return. Then he applied a normal distribution for expected returns with the volatility deduced from past data. He thereby generated an equity risk premium

Blanchard (1993) tackled the issue by using annuity values to find real returns for the variables under question. He then defined the *ex ante* equity risk premium as the difference between the expected return to stocks and either the medium or long term bond rate. Blanchard (1993, p.113) concluded that "...the equity premium has gone down steadily since the early 1950's and that inflation contributed to the transitory increase above trend in the 1970's and the transitory decrease below trend in the 1980's.". He suggested that the Great Depression has distorted the long term results and was atypical, therefore it was unlikely to be repeated in the short term. The current value put forward by Blanchard for the *ex ante* risk premium is of order 2-3%.

The compendium Sharpe and Sherred (1991), contains many pertinent papers on the topic. Summers examined variance ratios and found short term positive autocorrelation, and long term negative autocorrelation or mean reversion[3]. Markets therefore consist of much noise but a long run tendency to revert to fundamental

[3] This would appear to imply a constant mean to which prices revert. Hence the risk premium would itself be constant if similar behaviour is assumed for bonds or bills.

value. This predictability of prices based upon fundamental indicators argues for mean reversion strategies at times and momentum based strategies at other times. French considered risk premia (the return on the particular portfolio minus the T-bill rate) with respect to various fundamental indicators (dividend yield, the default spread and term spread). He found evidence of the predictability of the premium using these factors.

Nelson suggested that the pre and post war situations were different. He found that the effect of the Great Depression overstated the case for mean reversion. He suggested that post war mean aversion, or a tendency for price weakness to persist and take prices even further away from prior averages, was the rule. Taking the two halves he argued that there was no reason to prefer either model for the future, and hence not to bet on what appear to be 'trends' which may in fact be no such thing.

The remaining papers divide into TAA methodologies. These normally assume mean reversion. The compendium contains contributions in this style from Sharpe, Clothier (Wells Fargo), Henriksson (Kidder Peabody), Brinson (Brinson & Associates), and Grossman.

Sharpe accepted mean reversion and aimed to explain variations in the risk premium with a wealth based proxy (real security value per capita). He tested this measure, found it significant then applied it to TAA type strategies. He evaluated various TAA strategies and looked at the risk return trade-off for these strategies, finding benefits for counter cyclical investors who can truly take the long view.

Clothier described the system in place at Wells Fargo. He defined the risk premium as ".. the difference between the internal rate of return on the stock market and the internal rate of return on the bond market..". This premium is assumed constant in the long term and mean reverting (by 'eye' around about 3.8%). The IRR is used as a proxy for the expected rate of return and the risk premia between the respective asset markets is used as the indicator in TAA. Henriksson used a similar approach to Clothier. He propounds that for liquidity, wealth or other reasons the market changes its risk premium. The gains come from mean reversion and holding your risk level or function (however defined) constant. The benefit comes from doing the opposite of what the majority of the market does.

Grossman described a system called informational TAA, which he claimed has produced significant cumulative outperformance. The model he described attempted to segment out the risk preference induced moves in price from those changes in price caused by yield (by looking at yield relativities). Having found expected returns to each asset he then added in prospective returns to find efficient frontiers. He outlayed his claim to outperformance with some results tables.

Brinson expanded the horizon to international risk premia. He reviewed the differences between risk premia in various markets, postulating a further difference between segmented markets (where a national market operates independently), and integrated ones (where there is a lower risk premia due to diversification between markets). He gives a table of values for risk premia (ranging from 5.6% for the UK to 4.6% for Japan) and makes the comment of risk premia "....it is a process based on both historical data and one's understanding of the local market; there is no scientific way of determining these values." (Sharpe and Sherred (1989, p.69)). He then applied TAA in much the same way as previous authors have described, by using deviations from equilibrium.

Tversky provided an alternative psychological approach considering market activity from the viewpoint of human behaviour. He outlined how people think about risk and identified examples of 'irrational' behaviour. Individuals when facing losses may take large risks to recoup those losses. Individual assets are viewed in isolation from the portfolio (seeing the trees not the wood). There is also often overconfidence inherent in estimates about the future when actually tested. Individuals are more confident about the future than they should be.

Mehra and Prescott (1985) defined a 'premium puzzle', arguing that the size of the *ex post* equity risk premium was too large relative to the level of the return from the riskless asset. Hibbard (1998) found a similar 'puzzle' in New Zealand. Kurz and Beltratti (1996) explained the Mehra and Prescott 'puzzle' by claiming that endogenous uncertainty, that not due to fundamentals, was dominant in the equity market[4]. In effect there are eras in the stock market, when significant amounts in real terms and over a prolonged period could have been lost. Individuals or funds are sensible to hold, for example bonds, which underperform over the long term. Hence diversification into these assets is sensible because the periods of underperformance can have such an impact on the welfare of a retiree who expects a reasonable income in old age. The retiree may be wrong but wishes to generate income however 'cheap' equities may be. The different beliefs held by investors cause this risk premium. Under rational expectations it is assumed all hold the same beliefs. Consequently in a strategic sense one needs to find out the era one is in and adjust assets accordingly. Then either existing trends would be followed, or realising that there has been a change of state, strategic asset allocations would be adjusted accordingly.

Attempts to build suitable TAA models have been conducted by Reichenstein and Rich (1993) and Finnerty and Leistikow (1993). Reichenstein and Rich found value in the *ex post* risk premium and used it in TAA and indicated where they claim it can add value. Finnerty and Leistikow considered various risk premia and modelled the processes by the use of mean reverting models, both with and without a trend component. They took the period 1926-89 and found evidence of both mean reversion and a downtrend in the equity risk premium. The R^2 values were extremely low so the usefulness of this particular model would appear to be questionable.

9.3 Equity Risk Premium: *Ex post* and *Ex ante* values

The *ex post* values for the premia as defined in section 9.2.1 were found and are shown in table 9.1. These provide a benchmark for the *ex ante* values.

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[4] This is only one explanation. There are a wide variety of potential explanations for the 'puzzle'. These range from the universe of shareholders being different from the non-shareholders to one of statistical significance. No hypotheses appear to satisfactorily explain the puzzle.

Table 9.1 *Ex post* Risk Premia December 1977 - April 1997

liquidity risk premium	1.87%
equity risk premium	5.98%
short term equity risk premium	7.85%

The chart of the *ex post* ERP is given in figure 9.1.

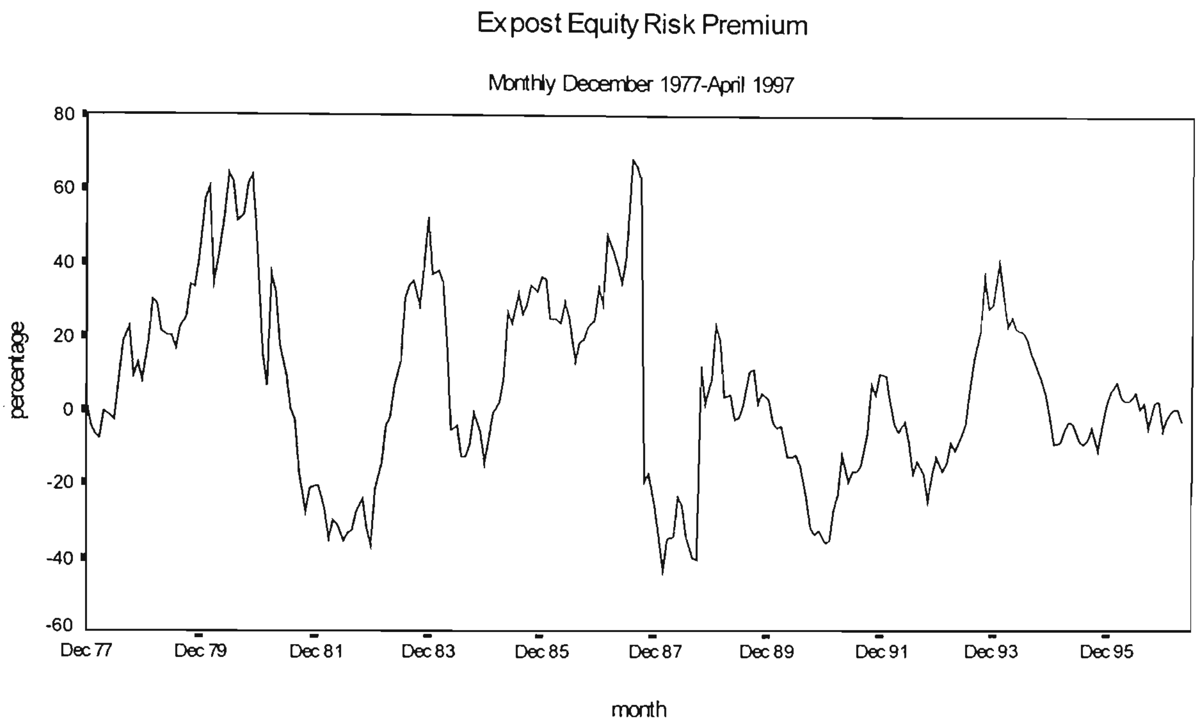

Figure 9.1 Equity Risk Premium: *Ex post* Value of Equity Returns less 10-year Treasury Bonds on a Rolling 12-month Returns Basis.

In the case of the liquidity risk premium the YTM of the 10-year bond is taken as the expected return to bonds. The expected real return to bonds at a given time is thus the YTM less the expected annualised rate of inflation at that time. It is noted that strictly speaking the NPV of cash flows discounted at the rate of inflation expected to prevail at the time of the particular cash flow should be considered. The best information that is available is the median expected rate of inflation for a particular period. Inflation expectations look 12 months ahead hence the expected discount factor for the cash flows past this period are unknown (see section 3.2 for definitions). Another limitation is that due to the variation of the coupon on issuance of the benchmark stock at each particular period of time, the duration of the 10-year bonds will be different during different periods.

The discount rate for ordinary shares is even harder to determine. Estimates as to the expected dividend return plus any expected capital gains or losses are required. The method applied is similar to that used by Blanchard (1993), which used the traditional Gordon growth model. For a series of dividends expected to grow at a constant rate the model is given by:

$$P_t = \frac{D_{t+1}}{(K_t - G_t)} \Rightarrow K_t = \left(\frac{D_{t+1}}{D_t}\right) y_t + G_t \qquad (9.1)$$

where at time t, D_t, P_t and $y_t = \frac{D_t}{P_t}$ are as previously defined, G_t is the long run growth rate in nominal dividends and K_t is the nominal discount rate for equity or the nominal cost of capital. Share prices will then vary with changes in expected dividends, dividend growth rates and the expected cost of capital. The expected future growth in dividends is given by inflation expectations plus an anticipated real growth in dividends. If the real growth rate in dividends is assumed to be constant, say g then:

$$G_t = e_t + g$$

where $e_t = E(i''_{t+1}|I_t)$ is given by the inflation expectations series (see section 3.6). The constant term g is given by the historic real dividend growth rate one measure of which is Δd_t. In section 4.3.2 it was found that there was no significant growth in real dividends, which suggests that $g = \Delta d_t = 0$. However the mean value of the growth in dividends on an annualised basis over the period March 1973 to September 1997 for which expectations data is available is 1.73%; low but non-zero. Any change to this would just move the plot of the *ex ante* ERP shown in figure 9.2 up or down.

A central assumption is that the inflation component of dividends is anticipated to grow at a constant rate equal to the current assessment of inflation. In practice share analysts tend to forecast not much further than the next 12-18 months. Information and plans are available from the individual companies that analysts cover to provide a basis for these forecasts. Beyond that, if forecasts are given, assumptions are made about long term earnings and dividends which largely consist of a projection of existing trends. Thus the model of a constant inflation rate plus a factor for real growth is representative of much of current practice.

Officer (1992) believed that the effect of dividend imputation, introduced on 1st July 1987, should be included in the premium. This view was consistent with that of Davis (1999) who argued for the full inclusion of the value of the franking credit[5]. Following this principle an allowance is required for franking credits captured in the dividend and hence dividend yield. This can be done either through the calculation of the rate of return, by including the grossed up value of the dividends or by making an adjustment to the normal CAPM equation to include the value of the tax credits. There are two inputs required in valuing the franking credit, the rate of company tax (t_c) and the average percentage franking for the All Ordinaries dividend index (f). The dividend yield may therefore be adjusted by a factor given by the formula:

$$\left(1-\frac{f}{100}\right)+\left(\frac{f}{100}\right)\left(\frac{1}{\left(1-\frac{t_c}{100}\right)}\right)$$

Given the changes in the company tax rate the adjustment factor can be found from the following table 9.2. The average franking level is for a range of 50%-100%. This

[5] In practice the full value may not be incorporated due to foreign ownership of stocks and the inability of these investors to arbitrage the franking credit by selling the franking credit to domestic investors and capitalising the gain, e.g. the new 45-day tax ruling on short term holdings. Officer argued that given that the pool of franking credits is far less than the potential tax payable prices would be bid up to incorporate the full value. Even if some of the tax benefit were wasted the price should reflect the marginal benefit to an Australian taxpayer. Davis commented that regulators were using a much smaller value in their cost of capital calculations, of order 50%. This would be consistent with a high percentage value but yet below 100%. In summary both Officer and Davis argued in favour of full inclusion.

spans the current estimate in 1997 of the average level of franking of 80% based upon data from the J.B.Were[6] database.

Table 9.2 Dividend yield Grossing Up Factors for the Franking Benefit by Tax Rate and Average Franking Level

date of change	company tax rate	average franking for dividend(%) 50	60	70	80	90	100
1/7/1987	49	1.4804	1.5765	1.6725	1.7686	1.8647	1.9608
1/7/1988	39	1.3197	1.3836	1.4475	1.5115	1.5754	1.6393
1/7/1993	33	1.2463	1.2955	1.3448	1.3940	1.4433	1.4925
1/7/1995	36	1.2813	1.3375	1.3938	1.4500	1.5063	1.5625

For example y_t at March 1997 was 3.9% so on the assumption that dividends are on average 80% franked then the gross dividend yield is $3.9 * 1.45 = 5.655\%$. The post July 1987 adjustment is included in the dividend yield calculation. The 50% and 100% franking levels provide upper and lower bounds for the *ex ante* ERP, $K_t - B_t$ which is shown in figure 9.2.

[6] J.B.Were Research Department, 101 Collins St., Melbourne. Coverage is those companies analysed by the research team, which is the majority of companies with a significant level of market capitalisation listed on the ASX.

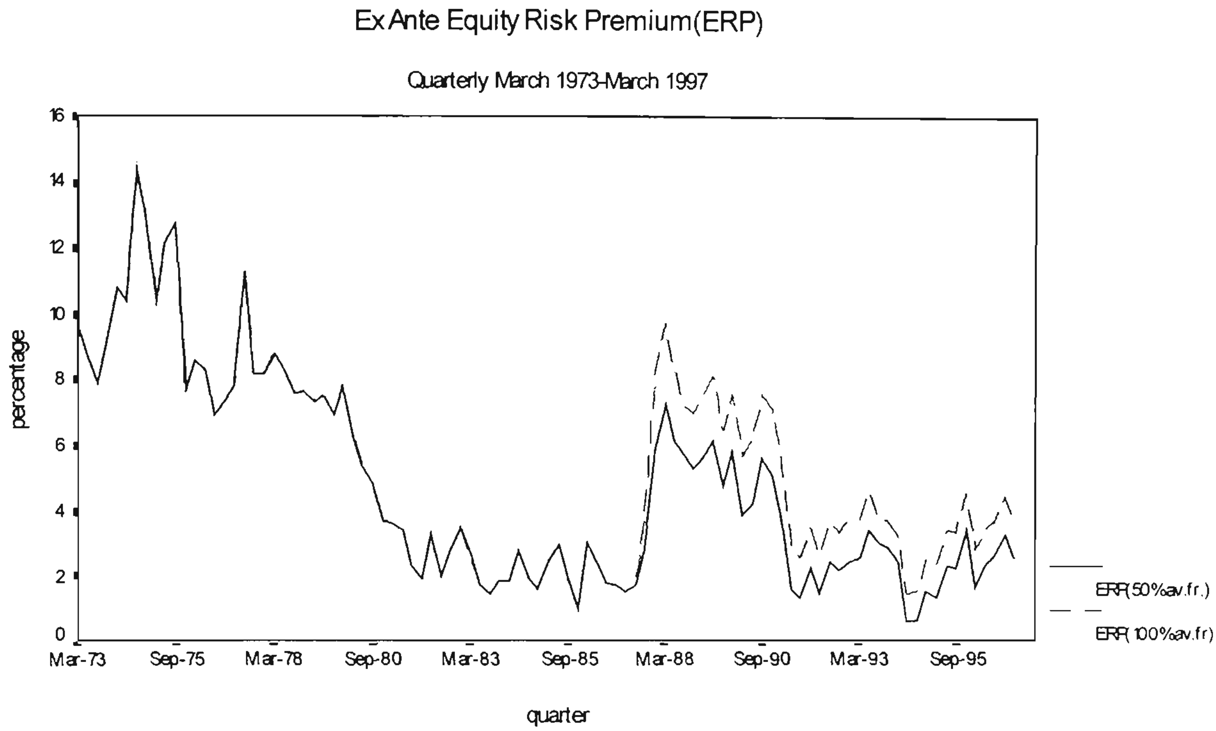

Figure 9.2 Equity Risk Premium (ERP): *Ex ante* Value of Equity Returns less 10-year Treasury Bond Returns Adjusted at the 50% and 100% Average Level of Franking for the Imputation Benefit Post July 1987.

The *ex post* data exhibited in figure 9.1 moves around in an erratic pattern due to the variation in returns. Figure 9.2 shows a smoother trend due to the method of calculating the *ex ante* value[7]. The premium exhibits a lower level of volatility. There is a rapid decline in the 1970's, dropping to the 2-3% zone during the 1980's. There was a spike upwards due to the October 1987 market correction which was close to the introduction of imputation in July 1987. Most recently the imputation adjusted series would appear to have fallen back into the 2-3% zone; the exact value dependent upon the average level of franking. The imputation benefit has shifted the ERP upwards. This tangible benefit to shareholders has taken time to be appreciated. After 10 years of operation the benefit should be fully priced in. That implies that the ERP is between 2 and 3%. This is consistent with the values given by Ambatscheer (1989) and Blanchard (1993) for the US.

[7] This comparison is not unlike the excess volatility argument where a DDM model smoothes the actual data which is far more volatile than it 'should reasonably be'. *Ex ante* values in the long run should be smoothed *ex post* values.

A consequence of this method of calculation are the orders of integration of the derived series. In equation (9.1) $y_t \sim I(0)$ and the nominal dividend factor $\left(\frac{D_{t+1}}{D_t}\right)$ will not alter this as the ratio hovers around unity. Hence $(K_t - G_t) \sim I(0)$. Now G_t the growth rate in nominal dividends is given by $G_t = e_t + g$, where $e_t \sim I(1)$ from section 3.6.1. Thus G_t and $K_t \sim I(1)$.

Now K_t and G_t are deflated by observed inflation (i_t'') to yield k_t the real cost of capital and g_t, where $(k_t - g_t) \sim I(0)$. By deflating both sides of the equation $G_t = e_t + g$ by observed inflation (i_t''), then $g_t = u_t + const.$ is obtained (from the definition of unexpected inflation in section 3.6). Now unexpected inflation $u_t \sim I(0)$ (see tables 3.12 and 3.13), hence $g_t, k_t \sim I(0)$.

The mean level of the real cost of capital found from equation (9.1) was 7.7% for the adjusted series and 6.9% for the unadjusted series. The nominal adjusted cost of capital given an average inflation rate of 7.7% is thus approximately 15.4%. The growth in earnings over January 1974 to February 1997 using monthly data from the J.B.Were Research Department[8] was 6.9% p.a. This translates to an average P/E ratio over the period of 1/(.154-.069) = 11.8X. This is consistent with further J.B.Were data showing an average P/E for the All Ords excluding loss making companies of 11.9X. The data set is internally consistent and the conclusions appear reasonable.

Accepting that this is an accurate representation of the equity risk premium then future trends are the next issue. Firstly, the lower value has only been prevalent since the 1991 recession. This is the result of long term downtrend, as confidence in equities has grown. Whilst a sudden reversal would appear unlikely, it is valuable to gain an historical perspective. Secondly the components that make up the premium require consideration to see where likely trends may take the ERP and to provide an assessment of the risks. Now:

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[8] J.B.Were Research Department, 101 Collins St., Melbourne. The presentation is courtesy of Michael Fitzsimmons and Albert Peker, Jardine Fleming Capital Partners, 600 Bourke St., Melbourne.

$$\text{ERP} = K_t - B_t \cong y_t + e_t + g - b_t - i_t'' = y_t + g - b_t - u_t$$

using (9.1) and where $\bar{u} = -0.7$ but $E(u_t) = 0$ (see section 3.6). The real bond rate in March 1997 was 6.7% and the gross dividend yield was 5.7%, hence the risk premium = 5.7 + g - 6.7. Thus a 2% risk premium yields a real growth rate in dividends of 3.0%. Such a high value has not been previously observed. The real growth rate in dividends was 2.19% for the much better dividend growth period March 1974 to September 1997.

Real growth in dividends of 0.43% p.a., the long term average value found in section 4.3.2, would yield a risk premium of -0.6% (= 5.7+0.4-6.7). Alternatively a value for the real bond rate of 2.8%, the long term average, would yield a premium of 3.3% (=5.7+0.4-2.8) . A real bond rate of 4% is consistent with a premium of around 2% (= 5.7+0.4-4).

A sample of the factors that could affect the likely future direction of the premium would include:

1. The premium reflects the preferences of the public[9]. The continuing decline of the premium is a factor reflected in outperformance by the equities asset class.
2. Whilst the series is of relatively short duration it is difficult to see any particular level to which the *ex ante* premium should tend. It should remain positive.
3. Real interest rates are a significant factor. If markets are convinced that low inflation is here to stay, then a reduction in real rates of interest should follow.
4. Any increase in the level of the premium may require a shock to the system such as it received in the early 1970's. The possibility is that such an event may then herald a long period of underperformance by equities.

[9] A note in the publication *Finance and Development* (December 1997) by Charles F. Kramer, an economist with the IMF research Department, canvases this public tendency via US mutuals. In the note he attributes 74% of the decline in the dividend yield to mutual fund flows.

9.4 Modelling the Equity Risk Premium

The stochastic investment model generates a set of interest rates, share prices and dividend indices. These could then be converted into returns to generate a value for the risk premium. Alternatively a model for the *ex ante* ERP, $K_t - B_t$ can be found, by using the constructed series for the real cost of capital $k_t \sim I(0)$ which can be modelled as a univariate series. Hence K_t can be generated via scenarios by adding an inflation component to forecasts of k_t. Equation (7.10) yields b_t and thus B_t . Alternatively this representation would see projections of the *ex ante* ERP as $(k_t - b_t)$, since the inflation component for the nominal values are the same.

Equation (9.1) may be used to generate equity returns. Now:

$$P_t = \frac{D_{t+1}}{(K_t - G_t)} \cong \frac{y_{t+1} P_{t+1}}{(k_t + i_t'' - (e_t + g))} \tag{9.2}$$

where i_t'' is the annualised rate of inflation and this value is determined from the scenarios. Forecast values for e_t can be obtained from the EWMA equation (3.10) or the Adaptive Expectations Hypothesis (AEH) model (3.11). This would be consistent with Wilkie (1995, p.294) who modelled expected inflation with a set of exponential weights applied to past inflation with a long average period. Therefore (9.2) can be re-arranged to give:

$$\frac{P_{t+1}}{P_t} = \frac{(k_t + i_t'' - e_t - g)}{y_{t+1}} \tag{9.3}$$

Now y_t will rapidly mean revert to a constant value (see section 8.3.2). The growth rate g is constant so that price changes are in part given by the spread between observed (i_t'') and expected (e_t) inflation. Three cases can be observed.

1. Rising inflation or $i_t'' > e_t$. Then in equation (9.3) any price appreciation will be reduced by the difference; the sharper the inflation rate increases, the sharper the impact. The early to mid-1970's are representative of this situation.
2. Falling inflation $i_t'' < e_t$. The reverse of 1 and consistent with the evidence of the 1980's.
3. Steady inflation or $i_t'' = e_t$. The model suggests that prices would increase steadily, irrespective of the level of inflation.

The model suggests that it is transition periods that cause the most difficulties for investors. Another way of viewing this model is putting it into the context of Benari's paper. He defined 5 factors which create a financial era and hence determine asset class preferences. The factors were productivity growth, business cycle risk, the volatility of interest rates, GDP potential gap and inflation. Eras are defined not just by the rate of inflation but whether it is rising, falling or stable, and the overall level from which it is changing. In addition there are other general characteristics which can influence real growth rates.

Now the equity return $r_t = \dfrac{(P_t - P_{t-1} + D_t)}{P_{t-1}} * 100\%$, then using equation (9.3):

$$r_t = \frac{P_t(1 + y_t) - P_{t-1}}{P_{t-1}} = \{(k_{t-1} + i''_{t-1} - e_{t-1} - g)(1 + y_t)\} - 1 \qquad (9.4)$$

The expression for $\left(\dfrac{P_{t+1}}{P_t}\right)$ in equation (9.3) will smooth the returns; translated to (9.4) the equation will yield a very smooth set of forecasts from each scenario.

9.5 Conclusions

The future direction of the risk premium is of importance both for the cost of capital and the relative performance of bonds and equities. The Australian *ex post* ERP over the period 1977-1997 was 6%. The Australian *ex ante* equity risk premium, based upon the Gordon growth model with the simplifying assumptions made in section 9.3, has declined substantially from a peak in 1974 of nearly 15%. Incorporating the benefit of dividend imputation, with changing company tax rates and a range of values for the average level of franking of the All Ordinaries dividend index, suggests that the current level of the *ex ante* ERP is between 2 and 3%. A real bond rate of 4% is consistent with a premium of around 2%, the current level of the *ex ante* ERP.

The decline in the *ex ante* ERP is an important factor reflected in the outperformance of equities against bonds over the period 1977-1997. There would appear to be no particular value to which the premium might attain but the decline to the current level cannot be repeated. Any significant increase in the level of the premium may well need a shock to the system such as it received in the early 1970's. This would undermine the current level of confidence in equities as an asset class. The possibility is that such an event may then herald a long period of underperformance by equities.

The *ex ante* ERP is modelled as a function of the dividend yield plus the nominal growth in dividends less the YTM of the long bond. This model can be reformed to generate returns to equities as in equation (9.4). The model will both smooth the returns and be highly sensitive to small changes in the denominator. This means that the model has limited use as a stochastic equation replacing some of the existing equity equations.

Chapter 10

Overview: Forecasting and Simulation

10.1 Introduction

The stochastic investment models covered in this document are those due to Wilkie (1984, 1987, 1992, 1995a and 1995b) in the UK and Mulvey and Thorlacius (1997) in the US. With the exception of Carter (1991) there would appear to be no specifically Australian stochastic investment model.

The approach due to Carino *et al.* (1994)[1] was to develop an asset liability model using multistage stochastic programming. The basic model is a series of linear equations which describe the cash flows in and out of accounts and returns as random coefficients. The basic risk measure is one of shortfalls, rather than the traditional variance. The application to Yasuda Insurance given in the paper has an interactive component with asset returns based upon forecasting models and judgements. The returns are not formally modelled as a set of stochastic equations. Hence this portion of the LP is not a stochastic investment model[2], as defined in Chapter 1. The result is a massive LP with 318,121 non-zero coefficients.

Wilkie's model has been the most studied. It has been in the public domain since the first paper in 1984. Geoghegan *et al.* (1992) entitled 'Report on the Wilkie Stochastic Investment Model' was a comprehensive review of that model by the Institute of Actuaries in London. Some of the model limitations and other cautionary notes were detailed. A range of practical applications is given in Appendix C. The Mulvey model is only partly in the public domain. The connection to Towers, Perrin, Foster and Crosby (TPF&C) gives it a proprietary nature. Carter (1991, part 2) applied his model

[1] The study was conducted and funded by Frank Russell a US based asset consulting group. The consultants worked for Yasuda a Japanese insurance company,

[2] Mulvey *et al.* also apply multi-stage stochastic programming. The difference in approaches is that Carino *et al.* eschew the use of a stochastic investment model. They make no comment on this difference.

to the pricing of investment account products. Harris (1994) compared the performance of various stochastic investment models for long term studies (see section 6.2.1).

The Working Party in Geoghegan *et al.* (1992, p.184) suggested that "...one can get quite good estimates of means and variances with only 100 simulations.". Hence only a small number of simulations are required. Indeed as Harvey, the author of the Report from the Working Party commented, "...the aim of stochastic investment modelling is to investigate the medium and long run behaviour of the relevant economic and financial variables. The aim is not simply to predict these variables. In fact, in the long run, the predictions....will converge to the underlying mean level implied by the model" (Geoghegan *et al.* (1992, p.183)). Hence the aim in this chapter is to conduct simulations of the stochastic investment model, presented at the end of chapter 8, and then translate the stochastic equations outputs into returns. Results will then be reviewed.

This chapter is structured as follows. Section 10.2 provides a schematic overview of the model and discusses the logic behind scenario building. The next section 10.3 introduces the mechanisms to convert the stochastic model equations into asset class returns. The bond YTM provides most difficulty in conversion. The final section 10.4 provides a set of forecasts which can be compared with actuals. The model is calibrated up to September 1997. There are available out of sample values since then. Scenarios are then set and simulations performed generating sets of values for asset returns. The section concludes with a discussion of the model and its shortcomings.

10.2 Model Structure and Scenario Generation

The model structure is outlined in figure 10.1 with arrows indicating directions of causality. Inflation is the only exogenous variable with the dividend yield, real dividends, the long/short ratio, and real bond rates as endogenous variables. A summary of the equations in the model is given in section 8.5.

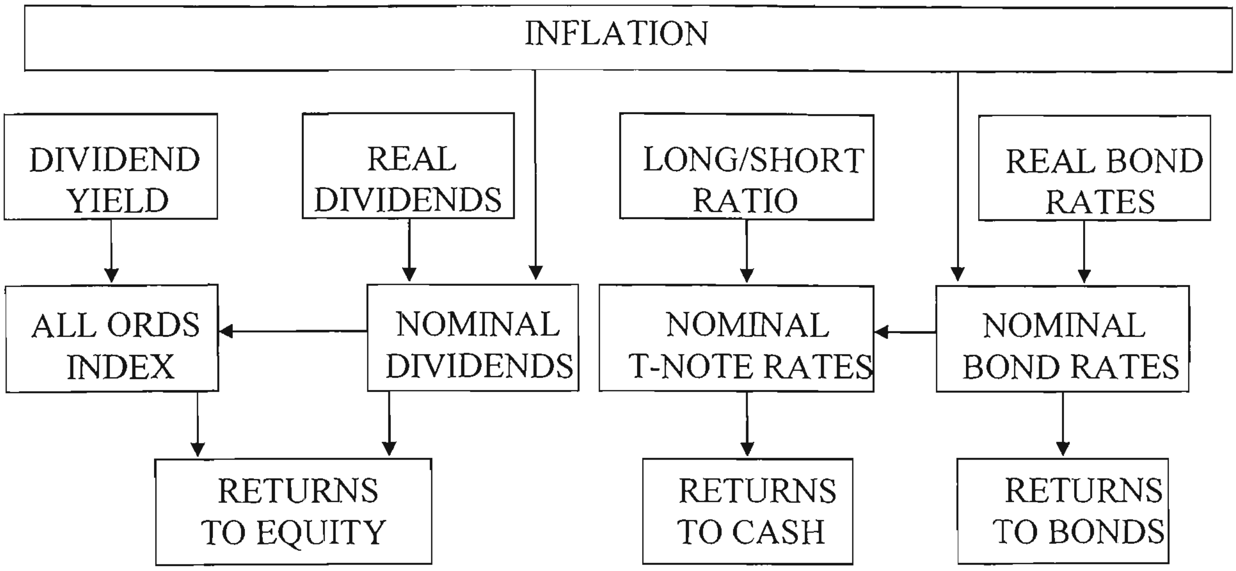

Figure 10.1 Schematic Diagram of the Proposed Model Structure.

Inflation models of the simple autoregressive type have difficulty in emulating both the changing level and volatility observed in the rate of inflation. In section 3.4.3 inflation was as modelled an AR(2) with a non-zero mean value. The inflation equation (3.6) is $i_t = 0.352 + 0.343 i_{t-1} + 0.433 i_{t-2} + \varepsilon_t$ so taking expectations $E(i_t | I_{t-1}) = 1.57\,\%$ to which the quarterly rate of inflation converges with this model. Now the closer the AR parameter is to unity the closer the model is to a random walk and the longer any shock to the system will take to die out. Since the inflation model AR parameters sum to 0.776 the model mean reverts, albeit more slowly than for the AR(2) dividend yield model where the AR parameters sum to 0.855 (see sections 7.4.2 and 8.3.2 for a discussion of the forecast function). Applying inflation equation (3.6) with values for the quarterly percentage change in the CPI for December and March 1997 of 0.2%, yields $i_{t+10} = 1.31\%$. Initial values of 3.0% (12.6% annualised) leads to a value of $i_{t+10} = 1.84\,\%$. Hence directly modelling inflation in this way will not deal with the desire of consultants or trustees to gain an impression of the impact of high, low or changing inflation scenarios.

This leads to models with more explanatory terms such as that of Clarkson discussed in section 3.5, where a range of models and their weaknesses were reviewed. Carter (1991, p.371) in the discussion of his simulation results commented, "The model of inflation appears to be too volatile. In particular negative inflation is too prevalent....".

Considering the history of inflation shocks to the system tend to persist. A potential hypothesis is that there is a 'core' rate of inflation. This concept is discussed in depth in Eckstein (1981) which originated from a study for the Joint Economic Committee of the US Congress. High inflation persisted through the 1970's. Now in the 1990's rates of inflation are lower. It is therefore much more realistic to view scenarios whereby inflation moves to a high level, subject to appropriate initial conditions[3], comparable with the 1970's and see what happens if this persists. Because the modelling is of the real YTM and real dividends, high or low inflation scenarios can be constructed to see what happens to returns for various asset classes. Therefore the result of a certain inflation outcome can be seen directly, rather than attempting to predict the rate of inflation. It simply removes one set of forecasts. A scenario of continuing low inflation may also be considered, or one where it rises and falls. Different asset allocations may well be appropriate under each alternative scenario. An assessment of the outcomes can then indicate asset mixes either to avoid or that may be favourable, in many alternative scenarios. This method can then dimension the asset allocation process rather than seek point estimates. This approach is a distinguishing feature of the method employed.

10.3 Conversion of the Stochastic Equations into Returns

The stochastic investment model of section 8.5 has equations for nominal bond and nominal T-note yields, as well as values for the nominal All Ords index and nominal dividends. These must be converted into returns[4].

10.3.1 Conversion to Equity Returns

For the equity returns there are the two equations:

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[3] The initial conditions considered here are market ones; the current values. There are various other possibilities such as long term means or some form of random sampling.

[4] The model as proposed has the ability to be cast in terms of the 'benchmark' for the respective asset classes. The All Ordinaries index is a performance measure for Australian equities. A possible performance measure for the fixed interest asset class is the Commonwealth Bank Bond Index (All Maturities). The duration of the CBBI can be matched by a suitable combination of 10-year bonds and T-notes. The convexity of the CBBI may not exactly be matched but this is a second order effect.

$$y_t = 0.691 + 1.045 y_{t-1} - 0.190 y_{t-2} + \varepsilon_{yt} \text{ and}$$

$D_t = d_t I_t$ where $d_t = d_{t-1} + \varepsilon_{dt}$, along with the identity $y_t = \dfrac{D_t}{P_t}$.

Hence D_t and P_t may be found and therefore returns to equity by:

$$r_t = \frac{(P_t - P_{t-1} + D_t)}{P_{t-1}} * 100\%$$

10.3.2 Conversion of YTM of a Bond to Returns

The real bond model equation (7.10) allows scenario generation of real bond yields which can then be translated into nominal yields to maturity. However the returns to a holder of a 10-year bond include both coupon income and any capital gains or losses over the holding period. The change in capital value will depend upon the duration of the particular bond, itself a function of the coupon. A general method is required to determine the coupon of a bond with a given nominal YTM.

For statistical purposes the RBA determines the YTM of the *benchmark* 10-year bond as the estimated closing figure for the last business day of the month. In the notes to the RBA Bulletin[5] the RBA states that "Interest rates and yields in the table are representative. They are the mid-point of predominant bid and offer quotations in each market as identified by the RBA.". Now until the 1980's the RBA had many bonds on issue. If extra 10-year stock were needed it would issue the stock with a coupon close to the current YTM in the market place, thus issuing the bond with a capital value close to par or "par yield". Since the mid-1980's the RBA has changed its policy and radically reduced the number of bonds on issue. In 1985 there were 153 different series on issue and as at May 1996 this has been reduced to 51. Furthermore it has concentrated these issues into so called benchmark stocks, of which the 10-year bond is one, and these account for 97% of stock on issue. Concentrating the stock on issue into a smaller range of securities increases the depth or liquidity in the marketplace.

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[5] See the commentary in the May 1996 issue of the *RBA Bulletin* for the full details of RBA policy.

The RBA via its open market operations carries out its charter by buying or selling securities. When securities are sold the RBA asks the market what securities they want and then it issues the appropriate series into the professional market, so that they have various terms to maturity to 'fill in'. They do this to increase liquidity and to allow for arbitrage overseas. Issuance is done with the coupon that is on the stock that is being added to. The coupon of the stock being added to may be at any level depending on rates prevailing at the time it was first issued.

The benchmark 10-year stock is set by the market itself. Suppose that the RBA issues an 11-year stock with a coupon approximating the YTM at the time of issue. After 1 year the stock, because of liquidity or other considerations, becomes the 'hot' or benchmark 10-year stock. It will still have the original coupon and if the RBA issues more, then the extra will also have the same coupon. Now given a normal yield curve and the desire of institutions to seek extra yield for longer term maturities, the RBA will push out the term to maturity to 15 years. Therefore after 5 years, if the series becomes the 10-year benchmark, it is almost certain to have a coupon different from the current YTM. Therefore in general one cannot assume that the coupon on the 10-year stock is the current YTM nor can one assume that the coupon is at any particular level.

Nevertheless a methodology is required to convert a YTM into returns. To do so two alternatives will be considered and then the results compared in terms of returns. The first alternative is where the coupon is equal to the YTM. The second is for a fixed coupon of 12%. This choice is driven by the recognition that the futures contract has been, since its inception, priced with a 12% semi-annual coupon[6]. The standard contract unit is a 10-year government bond offering a coupon rate of 12% per annum, semi-annual payments and with a face value of $100,000. This contract is quoted as 'one hundred minus'. This means that as interest rates rise and the value of the future falls, the quotation will also fall, and vice versa.

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[6] See Chapter 17 in Brailsford, T. and Heaney, R. (1998), *Investments: Concepts and Applications in Australia*, Harcourt Brace, Sydney. An extensive discussion of the Sydney Futures Exchange (SFE) futures contracts is given.

The formulae for price, duration and thus percentage change in price may be found. The price is given by the normal annuity formula:

$$P = C\frac{\left[1-(1+i)^{-n}\right]}{i} + V(1+i)^{-n}$$

where there are cashflows of C per period, over n periods with a discount rate per period of i and the bond has face value V. If the coupon rate equals the YTM then the bond will trade at par or $P = V$.

The *duration* D is defined as the discounted weighted average term to maturity or 'average time to get your money back' on a NPV basis. It can also be thought of as the 'centre of gravity' of the bond. Hence, by definition:

$$D = \left[\frac{C}{(1+i)} + \frac{2C}{(1+i)^2} + \ldots\ldots + \frac{nC}{(1+i)^n} + \frac{nV}{(1+i)^n}\right] \cdot \frac{1}{P} \tag{10.1}$$

$$= \frac{C}{(1+i)P}\left[1 + 2(1+i)^{-1} + 3(1+i)^{-2} + \ldots + n(1+i)^{-n+1}\right] + \frac{nV(1+i)^{-n}}{P}$$

where the term in brackets is an arithmetic geometric progression with a sum to n terms S_n given by:

$$S_n = \frac{\left(1-(n+1)r^n + nr^{n+1}\right)}{(1-r)^2}$$

where $r = (1+i)$. Hence the duration may be found.

The approximation from Taylor's theorem can then be used to find the percentage change in price for a small change in interest rates that is:

$$f(a+x)=f(a)+\frac{x}{1!}f'(a)+\frac{x^2}{2!}f''(a)+..........+\frac{x^n}{n!}f^{(n)}(a)+\varepsilon$$

Letting $a=i$, $x=\Delta i$ we have:

$$f(i+\Delta i)=f(i)+f'(i)\Delta i+f''(i)\frac{(\Delta i)^2}{2}+....$$ or new price = old price + changes.

In practice the second term in the expansion is a sufficient approximation for moderate changes in interest rate. This introduces the concept of *convexity* or degree of non-linearity in the change, given by the term involving the second derivative. However to a first approximation it follows that:

$$f(i+\Delta i)=f(i)+\frac{dP}{di}\Delta i \quad \text{or} \quad f(i+\Delta i)-f(i)=\Delta P=\frac{dP}{di}\Delta i$$

$$\text{then} \quad \frac{\Delta P}{P}=\left(\frac{dP}{di}\right).\left(\frac{1}{P}\right).\Delta i=-\frac{D\Delta i}{(1+i)} \quad \text{from (10.1).}$$

Then substituting in $n=20$ periods, a face value V say of 1000 and values for C of $\frac{i}{2}$ or 6% respectively gives the required returns.

In order to test the two formulae for the returns generated, a comparison was made using the actual 10-year bond YTM over the period March 1960 to September 1997. Figure 10.2 is a chart of the cumulative difference between the YTM version and the fixed 12% version. It is noted that the higher the coupon the shorter the duration and the smaller the price change for a given change in interest rates, either up or down. Therefore since the bond rate has mostly been below 12%, larger changes result from the YTM version. Hence there is an upward drift. There is a difference in total returns of 9.81%. This compares to the total return of 2191.2% obtained using the YTM version. This translates to an annualised return of 8.58% over the 37.5 years with a difference of 0.25% per annum from the two methods.

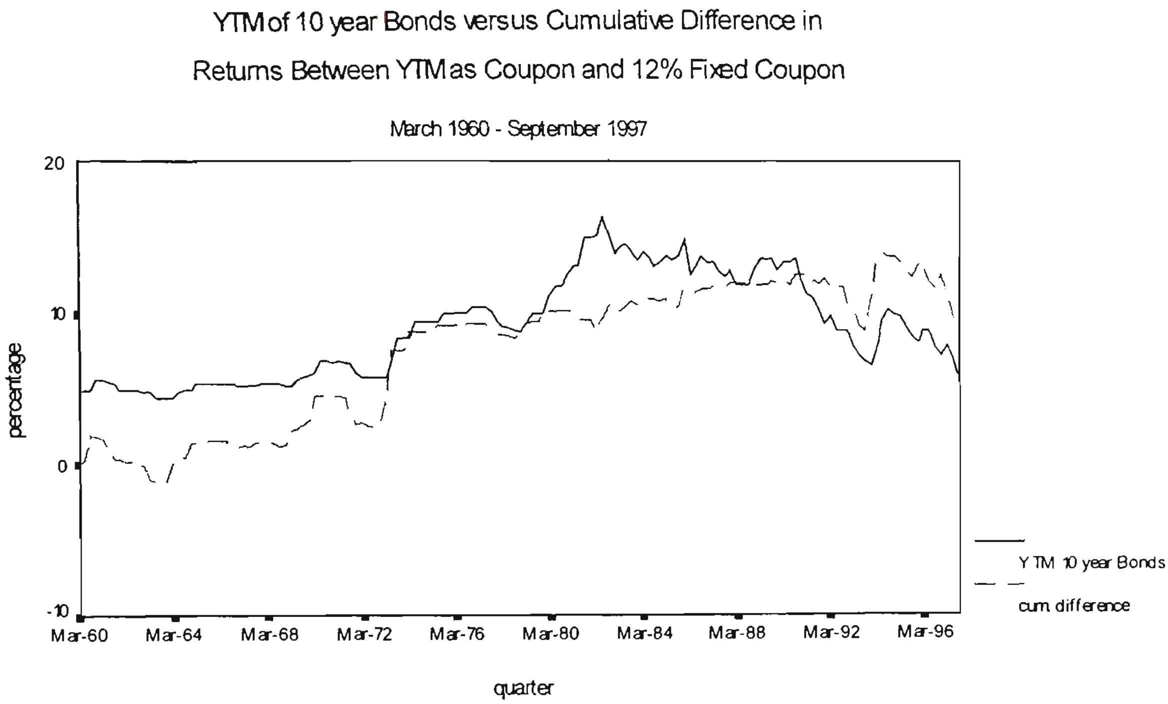

Figure 10.2 Comparison of Cumulative Difference in Returns between Using YTM as the Coupon Versus a Fixed 12% Coupon.

The choice between the two methods is not critical; particularly given the long term nature of the stochastic investment model. Since scenarios of high or low inflation are considered, it is more likely that bias will arise from assuming a constant 12% coupon than a coupon that will change if rates move significantly higher or lower. Thus the YTM version is the one that will be used to generate the required returns.

10.4 Model Performance and Interpretation

A satisfactory stochastic investment model in terms of diagnostics, such as parameter stability and an explanation of the observed features of the relevant series has been shown in section 8.5. The forecasting ability of the model and its operation under various scenarios is now reviewed to evaluate the plausibility and consistency of results. Note that the principal focus of the model is scenario based forecasts, not short term forecasts based upon inflation modelling.

10.4.1 Forecasts

The stochastic investment model has been calibrated with data up to September 1997. Information is now available up to September 1999. There are 8 quarters to forecast for each stochastic equation. Actual inflation data has been used to generate the forecasts, since inflation is not directly modelled in the stochastic investment model. The tables 10.1 and 10.2 describe the results, firstly for equities then for fixed interest. The forecast is for the whole period, denoted $|F_0$. The 1-step ahead value is based upon a forecast made when actuals become available as each quarter rolls by, denoted $|F_t$.

Table 10.1 Forecasts of Equity Indicators

Qtr.	Dividend yield			Nominal dividends			All Ords index		
	actual	$\|F_0$	$\|F_t$	actual	$\|F_0$	$\|F_t$	actual	$\|F_0$	$\|F_t$
Dec97	3.9	4.09	3.43	99.4	94.98	89.00	2549	2321	2592
Mar98	3.6	4.24	4.16	98.4	95.27	99.70	2734	2246	2398
Jun98	3.6	4.37	3.73	93.9	95.84	99.01	2608	2195	2655
Sep98	3.9	4.47	3.79	99.2	96.03	94.08	2543	2149	2485
Dec98	3.5	4.55	4.10	96.1	96.51	99.67	2745	2120	2430
Mar99	3.2	4.62	3.62	94.5	96.42	95.99	2954	2087	2649
Jun99	3.3	4.68	3.39	97.8	96.80	94.91	2963	2070	2804
Sep99	3.3	4.72	3.55	97.2	97.67	98.66	2945	2068	2782

Nominal dividends are indexed to the rate of inflation. The mean reverting characteristic of the dividend yield asserts itself, producing an All Ords forecast which declines towards the long term trend. However the time horizon for the stochastic investment model is long term. The best short term forecast is a random walk, yet mean reversion is presumed to operate over a long term horizon.

Table 10.2 Forecasts of Fixed Interest Indicators

Qtr.	10-year bonds			long/short ratio			T-notes		
	actual	$\|F_0$	$\|F_t$	actual	$\|F_0$	$\|F_t$	actual	$\|F_0$	$\|F_t$
Dec97	6.05	6.48	5.97	1.22	1.30	1.28	4.96	5.00	4.66
Mar98	5.75	6.19	5.79	1.18	1.28	1.22	4.88	4.85	4.76
Jun98	5.58	6.24	5.82	1.12	1.26	1.18	4.98	4.94	4.91
Sep98	5.08	6.22	5.60	1.05	1.25	1.14	4.85	4.97	4.91
Dec98	5.01	6.26	5.18	1.10	1.24	1.08	4.57	5.04	4.79
Mar99	5.49	6.16	4.99	1.18	1.24	1.12	4.67	4.98	4.45
Jun99	6.27	6.13	5.51	1.34	1.23	1.18	4.69	4.99	4.66
Sep99	6.30	6.00	6.13	1.32	1.23	1.31	4.77	4.90	4.68

The bond yield forecast is adjusted by the actual inflation values over the period. The long short ratio mean reverts. The downward movement in bond yields has initially been far more rapid than predicted by the model but has now moved back in line with the forecast. The actual long short ratio has moved around far more than the smooth prediction. The yield curve has now steepened after flattening out in September 1998 with the ratio of longs to shorts well above the long term average. The net effect is that T-notes are close to that predicted. Short rates have now been raised by the RBA reflecting the trend shown by the model. It is nevertheless difficult to draw significant conclusions from short term forecasts.

10.4.2 Scenario Simulations

The final stage in model development is to evaluate the model under different inflation outcomes. The table 10.3 gives 5 possible scenarios of annual rates of inflation for the next 10 years. In simulations these are converted into annual rates of inflation for each quarter by linear interpolation of the annual rates, then finding the quarterly rates that generate the particular annual rates. The initial values chosen are the current values; the current rate of inflation is 2%.

Table 10.3 Inflation Scenarios: Annual Rate of Inflation for the Next 10 Years

year	scenario 1	scenario 2	scenario 3	scenario 4	scenario 5
1	2	2	2	2	2
2	3	5	1	4	4
3	4	8	1	6	6
4	5	11	1	8	4
5	6	13	1	10	2
6	7	15	1	10	2
7	8	15	1	8	4
8	9	15	1	6	6
9	10	15	1	4	4
10	10	15	1	2	2

These scenarios are a representative set with a range of states. There was a steady rising rate; a rapid rise to a high peak; constant inflation; a rise and fall and an oscillating experience. There are many other possibilities but this set should yield an indication of model performance. The simulation was conducted over 40 quarters with 100 simulations. The average annual nominal returns from equities, bonds and T-notes for the full 10-years, first 5 years and second 5 years were found. The resulting ERP on an annual basis is also given. The results are shown in table 10.4.

Table 10.4 Simulation Scenarios 1 to 5: Average Annual Nominal Returns and ERP

scenario	period	equity returns	bond returns	T-note returns	ERP
1	full 10 yrs.	3.84	5.32	6.22	-1.48
	1st 5 yrs.	-1.39	5.33	5.33	-6.72
	2nd 5 yrs.	9.06	5.33	7.12	3.76
2	full 10 yrs.	8.83	5.24	7.93	3.59
	1st 5 yrs.	1.51	3.45	6.24	-1.94
	2nd 5 yrs.	16.14	7.03	9.62	9.11
3	full 10 yrs.	-1.39	6.31	4.26	-7.70
	1st 5 yrs.	-4.10	7.73	4.49	-11.83
	2nd 5 yrs.	1.32	4.88	4.03	-3.56
4	full 10 yrs.	4.24	7.15	6.26	-2.91
	1st 5 yrs.	0.67	3.96	5.87	-3.28
	2nd 5 yrs.	7.80	10.34	6.65	-2.54
5	full 10 yrs.	0.97	6.82	5.49	-5.84
	1st 5 yrs.	-0.48	7.06	5.47	-7.54
	2nd 5 yrs.	2.43	6.58	5.52	-4.15

There are some points arising from each of the scenarios. Firstly the current dividend yield lies above the long term average. As a consequence of the stationarity of the dividend yield and the mean reverting nature of the AR(2), the equity return is low over the first 5 years. It is below that of the bond return in all cases. Secondly, since the real bond rate is mean reverting, albeit slowly as a consequence of its near stationarity, then the real bond rate should fall. This is because it is currently above its long term average. The model suggestion is that equities are over-valued relative to bonds.

The model suggests outperformance from equities during periods of higher inflation. This is due to the linking of equity returns to inflation from the model. The evidence here is conflicting. The cointegration tests in section 5.4.2 did not reject the null hypothesis of no cointegration between inflation and share returns over the period 1875-1997. The post war situation is less clear (see section 5.4.2 and figures 4.6 and 5.2). The bond model suggests that high rates of inflation create significant under-performance from bonds. This is consistent with the evidence. T-note rates perform better than bonds during these high inflation periods indicating that the fixed interest portfolio should be positioned with a short duration.

Scenarios 3, 4 and 5 are generally poorer for equities than bonds. The model indicates that a continuation of the low inflation conditions prevailing in the late 1990's are not conducive to equity performance. The evidence in chapters 5 and 6 does not suggest a link between either inflation and the dividend yield or the dividend and bond yields via the 'confidence' ratio; though this is observed in the UK (see Mills (1991)). The keys are the relative valuation of each asset class and the potential sources of earnings growth for shares. With a low level of pricing power, growth cannot come from nominal price increases in the cost of goods or services sold. Productivity increases and a continuation of the recent high profit share of gross operating surplus as against the wage share would seem potential important factors. A detailed investigation of these factors lies outside the scope of this thesis.

A more recent stock valuation model applying similar logic to the above was described by Gosling (1999), providing results consistent with those in table 10.4. In the model by Gosling only three variables were required for stock returns; the initial dividend yield, the growth rate of company earnings and the change in the P/E ratio. Of these three, the initial dividend yield was known, the earnings growth was predicted from nominal GDP and the change in the P/E ratio by the rate of inflation. This latter was the determining factor and no justification was given for the negative relationship between inflation and the P/E ratio. Six scenarios were set up. Two deflation scenarios, benign and destructive deflation, and four inflation scenarios, ranging from zero price change to high inflation. Returns were then forecast based upon whether or not a 'valuation shift' has occurred under the current disinflation regime (see Gosling (1999, table 6, p.4). Gosling then considers the possibility of a regime shift from disinflation to either deflation, stable inflation or rising inflation. None of these Gosling argues is good for equity market performance. Gosling (1999, p.5), when discussing the current high P/Es, commented that " What is more likely is that the market has built in expectations of an ongoing disinflationary environment, failing to appreciate the imminence of a regime shift which is likely to be far less supportive.".

The scenario outcomes in table 10.4 show that the ERP is negative except in the high inflation situation. It was suggested in the discussion in chapter 9 that the continuing decline of the *ex ante* premium is reflected in outperformance by the equities asset class. Based upon the model in 1974, the time of the high *ex ante* ERP, equities were under-valued and bonds over-valued (see figure 9.1). In September 1974 the dividend yield stood at 9.8% and the real bond yield -5.93%. This proved to be a time to borrow to buy shares *if there is confidence that inflation will eventually decline*. The inflation surge created the under-valuation. It was the cause of excess volatility (see West (1988), Shiller (1989)) and is consistent with the 'rational beliefs' view of Kurz (see Kurz and Beltratti (1996), Kurz (1997)). Investors cannot be certain that inflation will fall. As discussed in section 9.2.2, there are eras in the stock market when one could have lost significant amounts in real terms over a prolonged period. Individuals or funds are sensible to hold bonds, which underperform over the long term, because of the impact of these periods.

10.4.3 Model Review: Shortcomings and Potential Applications

The model as given thus far is not without significant shortcomings. The model is one of real variables. Nominal returns are generated from scenarios on the assumption that an inflation model is not of itself important. This is not the case; many examples occur where the modelling of inflation is a critical issue, for example in asset allocation. The model as it stands would benefit from a suitable inflation model, though it is fair to say that those actuarial factors that depend upon inflation, such as forecasts of average weekly earnings, could be found using a scenario approach. Indeed an advantage of taking the scenario route is it enables a matching of assets and liabilities under many different scenarios. Hence it is possible to determine sets of scenarios which are more or less favourable to a particular set of liabilities. It can be determined which scenarios deliver shortfalls and to what extent. This leads to ideas such as mini-max strategies or asset allocations which seek to minimise maximum losses. Indeed there may be asset allocations which are optimal over a range of inflation scenarios.

The model does not cover international assets; nor for that matter the property asset class. As indicated in section 1.2 covering the major Australian asset classes of equities, fixed interest and cash, in a rigorous fashion, reviewing the literature and comprehensively examining various econometric techniques, has led to a long and detailed document. Any operational model should include international assets. The model framework and methods of analysis proposed herein represents a blueprint for any necessary extension. The addition of property is much more difficult due to the lack of a suitable index.

An important issue which has been revealed by the document are structural breaks. In model building the approach has been to discount information relating to periods prior to the two breaks, that of the late 1940's (the commencement of Bretton Woods) and that of the early 1970's (the breakdown of Bretton Woods) respectively. Regime switching models could have been applied, as described in section 6.4. They have not for several reasons. Firstly the present regime has been in operation for nearly 30 years; if another regime arises it is unlikely to be predictable. Implicitly an assumption is made of a central and continuing role for the reserve banks of the major OECD[7] countries. Secondly knowing that the generating process is different between structural breaks with no ability to know if or when a change might occur, nor the nature of any regime change will not help at all. Thirdly it presents yet another econometric technique. Whilst this is valuable perhaps of more importance is what the differences were and what different effects they have caused. Clearly a lot more could be said about such fundamental issues as structural breaks, but this lies beyond the scope of the thesis.

The model is based on quarterly data. A fundamental reason for this is that the CPI is provided quarterly. It represents a balance between the long term nature of stochastic investment models and a need to relate any such model to shorter term models used

[7] Organisation of Economic Co-operation and Development. It represents all the major economies of Europe, North America, Asia and Australasia.

for other purposes such as valuations. This a limitation but a clearly expressed one. Equity data for example is available at a wide range of intervals to virtually a continuous basis. There are many models in the literature that have been developed and many potential ones. It is recognised that any practical model should not offer any arbitrage opportunities with any shorter term model, yet the models may emphasise different features. Models that suit one frequency may or may not be consistent with another frequency. Again this lies beyond the scope of this thesis. A deal of time is spent considering for example the nature of nominal and real interest rates or the connection of inflation to share returns; and this is necessary.

In section 10.4.1 a brief test of the limited model was made. The forecasting ability of the model was reviewed using the extra data that has become available over the years 1997-99. The next section 10.4.2 considered at performance over a 10 year horizon under various scenarios and with the current values of inflation, interest rates and so on as initial conditions. It offers a limited test drive and no more. At periods of say 20 years or greater, forecasts can only be considered speculative; it can be argued that if a projection is so off course after 5 years then a correction will be needed such as to render any long term projections of limited value. Of course any viable stochastic investment model has great value over shorter ranges as a tactical asset allocation model. A central role can be given to any model in deciding 'fair value' for an asset class; hence based upon the discovery of over or under valuation asset allocators can make decisions. Of course this is tacitly a presumption of mean reversion, and this process can be very slow.

Finally there are potential applications of any stochastic investment model. Examples of applications in the general area of finance or particular area of actuarial studies could have been given. Areas would include asset allocation and valuation. There are two key reasons why this is not done. Firstly this is not the major focus of the thesis. The final model results but it is the discussion and analysis to get there that is the central theme of the thesis. Secondly the limitations of the model are recognised. An attempt has not been made to rush it into action; the suggestions of the model in a valuation context are commented upon in section 10.4.2. A reference to the role of such modelling and a brief discussion of some of the relevant literature was given in section 2.5. This places the endeavour in the appropriate context. The application to stochastic asset liability modelling unfortunately lies outside the scope of the thesis.

10.5 Conclusions

This chapter has demonstrated the operation of the stochastic investment model. Procedures for converting the stochastic equation outputs into returns for each asset class have been determined. Conversion into equity returns is straightforward. For bond returns assumptions were required with respect to the coupon. A comparison was made between the YTM of the bond as the coupon and a fixed 12% coupon. The 12% value represents the notional coupon on the 10-year bond futures contract on the Sydney Futures Exchange (SFE). The choice between the two methods is not critical. It was considered that greater bias would arise from assuming a constant 12% coupon, than a coupon that will change if rates move significantly higher or lower. Therefore the YTM version is applied to generate the required returns.

Forecasts of the model were made to compare with actuals over the period September 1997 to September 1999. The model suggests current over-valuation in share prices. The downward movement in bond yields has been far more rapid than predicted by the model. The yield curve has steepened after flattening out with the ratio of longs to shorts at September 1999 well above the long term average. The net effect is that T-notes are close to that predicted.

Simulations were conducted with a set of inflation scenarios. In all scenarios, the model suggestion is that equities are over-valued relative to bonds. The model suggests outperformance from equities during periods of higher inflation. The model indicates that a continuation of the low inflation conditions prevailing in the late 1990's are not conducive to equity performance. A consequence of the modelling is that the ERP is negative, except in the high inflation scenarios. Caution is required with the model, particularly the link of equity returns to inflation. The evidence is conflicting but supportive of the hypothesis of no long run link between the two. The final section reviewed model shortcomings and potential applications.

Chapter 11

Conclusions and Final Remarks on Stochastic Investment Modelling

11.1 Introduction

In section 1.2 an aim of the thesis was stated as '... to devise an appropriate stochastic investment model and apply it.' The preceding chapters have developed and implemented such a stochastic investment model. In so doing they have also provided the justification for the modelling in terms of the background discussion in section 1.1. The results of the document places stochastic investment modelling in the general context of asset/liability modelling; what can be achieved has been investigated and therefore the primary aim stated in section 1.2 is satisfied. Hence the nature and predictability of the financial series, the topic of structural breaks, competing stochastic investment models and the utility of inflation equations in the final model has been reviewed. Therefore in this chapter an overview of where the research fits into financial and actuarial practice in Australia and internationally is presented. The question is asked as to what has been achieved and what light does it shed upon the future direction of industry.

This final chapter is structured as follows. The first section 11.2 summarises and discusses the main conclusions that have been reached in the thesis. This section is a distillation of the conclusions rather than a recapitulation of the conclusions at the end of each chapter. The primary outcome places the stochastic modelling in context. It determines what can be achieved in modelling. Each chapter contributes to this decision making process. Hence each conclusion is not 'secondary' as it creates the bounds within which stochastic investment modelling can be placed. The second section 11.3 then considers the conclusions in the context of applied financial research. It extends the commentary given in section 11.2. The third section 11.4 then considers the future direction and scope of research in stochastic investment modelling and any assistance this research may offer.

11.2 Summary and Discussion of the Main Findings

A structural break in 1947 has been determined (see section 3.3). There is evidence that both the rate of inflation and real GDP growth have a different DGP post war. The results in chapter 3 showed that Keynesian post war macroeconomic stabilisation policies have been successful.

A second structural break was indicated in modelling the real bond series in 1973 (see section 6.4 and chapter 7). This break was not reflected in the inflation or equity series nor the long/short ratio. No break was found in 1983 when the A$ was floated. The conclusion is that for the period of 22 years, from 1975 to 1997, there has been a stable environment. Given that the stochastic investment model is designed for long term forecasts, of an order of 10 years or greater, the existence of frequent structural breaks would invalidate the process. The results therefore indicate that stochastic investment modelling is a feasible undertaking.

The quarterly rate of inflation, the percentage change in the CPI, is a stationary series. There is no evidence of seasonality. Univariate linear and non-linear models were fitted to the rate of inflation in chapter 3. However the simple non-linear models discussed there did not provide a satisfactory solution. The resulting models of inflation cannot capture the volatility actually observed in prices. Shocks to the system will die out with these models, so large shocks will not be adequately modelled. This is consistent with other evidence in the literature where satisfactory models of inflation were not found. A tentative hypothesis is that stochastic investment models which rely upon stochastic inflation equations will therefore not prove satisfactory in asset liability modelling.

Results from unit root testing in chapter 4 of the real bond rate and real T-note rate led to the conclusion that the series are near integrated ones. This provides support to the hypothesis of a mean real bond level which is constant over various periods. In chapter 6 applying the yields on indexed bonds as a measure of real yields supported the hypothesis that the mean level of real interest rates has not remained constant over time. Now economic theory would suggest that the level of real interest rates is constrained within bounds and therefore these should be stationary series. More recent data is consistent with these series being stationary, that is they cannot wander as would a random walk. Nevertheless for statistical purposes the levels series, the real bond rate and real T-note rate, are regarded as representing a stochastic trend component. There is no evidence of seasonality in the derived stationary series.

There is a cointegrating relationship between the nominal bond and nominal T-note rates. This feature is captured in the long short ratio which is $I(0)$ with no evident seasonality. Cointegration was found between real bonds and real T-notes and between the difference of the CPI and nominal bond rates, and nominal T-note rates.

In chapters 6 and 7 a real bond model was developed. There was a strong relationship between the difference in real bond rates or the difference in real T-note rates and the rate of inflation, with an initial negative impact of inflation at lag 0 followed by a strong positive response at lag 4. This was modelled using linear regression with inflation as the independent variable. The real bond model chosen described the real bond rate by a mean reverting term plus the impact of the current rate of inflation and the inflation rate lagged by 4 quarters. The rate of adjustment parameter is small, hence mean reversion is slow. The real bond model chosen showed no significant ARCH diagnostics. An investigation indicated that the levels effect observed by other authors in nominal bond yields did not carry over to real bond yields. This confirmed that modelling with the application of SDE's using the standard equations in the literature, such as the CIR model, was not likely to produce a real bond model satisfying the observed facts.

In chapter 3 expected inflation was found as a non-stationary series and was best modelled as an exponentially weighted moving average of observed inflation. The results are consistent with the view that observed inflation is the best predictor of expected inflation. Unexpected inflation is not a random series as one would expect but instead follows an AR(1) model. Utilising unexpected inflation in chapter 6 as the independent variable in lieu of observed inflation, did not generate models with superior statistical properties.

The yield curve is found to be adequately defined by two factors, though this does not define the whole yield curve. The nominal long bond rate and, either the long short ratio or the nominal T-note rate, are the only two factors required. The long/short ratio is modelled as a univariate autoregressive series. A consideration of the ECM between real bond rates and real T-note rates with the univariate long/short ratio model indicated that the more complex representation does not capture essential features that are not found in the autoregressive model.

An identical approach was taken to modelling the equity series as to that for fixed interest. The real price and dividend indices series were found to be $I(1)$. There was no evidence of seasonality in the derived difference stationary series. There was a cointegrating relationship between the nominal All Ords and the nominal dividend, captured in the dividend yield which is a stationary series with no evident seasonality. There was cointegration between the real All Ords and real dividends but not between the difference of the CPI and nominal dividends, and real dividends. The latter result questions Wilkie's assertion that inflation is fully incorporated into dividends and hence share prices.

The real dividend index was best modelled as random walk with zero growth, implying that nominal dividends are indexed to the rate of inflation. The AOI was best modelled as a random walk. This latter result is consistent with the real dividend model and the results of other Australian authors. The dividend yield is modelled as a univariate autoregressive series. A comparison of the ECM between the real dividend and real price indices with the dividend yield equation showed, as was the case with the long/short ratio equation, that the ECM did not improve upon the autoregressive model. However the autoregressive dividend yield model mean reverts rapidly. The All Ords index resulting from the product of the dividend yield and the nominal dividend therefore mean reverts over 2-3 years. This is at variance with both the random walk nature of the short term All Ords and the lack of long term relationship between inflation and share prices. For equity variables univariate non-linear modelling represented an improvement but the residuals were still non-normal and did not suggest significant gains from this form of modelling.

The exact relationship between inflation, interest rates and equity yields is not clear; nor is the direction of causality. Any potential negative link between share returns and inflation is a short run rather than long run effect. A proposed hypothesis is that in the long run growth in share earnings, and therefore prices, is the result of retained earnings. Unlike the UK, no long run relationship between the dividend yield and the nominal bond rate was found. Such a relationship would give a strong timing signal and make the task of asset allocation and stochastic investment modelling easier. No stable relationships were found between the stochastic trend components of the differenced stationary variables or the stationary variables.

A review of the equity risk premium was conducted in chapter 9. The Australian *ex post* ERP over the period 1977-1997 was 6%. Applying a method incorporating the benefit of dividend imputation showed that Australian *ex ante* equity risk premium has declined from a peak in 1974 of approximately 15% to a current value of between 2 and 3%. The current level of the *ex ante* ERP is consistent with a real bond rate of 4%. The decline in the *ex ante* ERP, reflecting a change in public attitudes towards the risks[1] of equity investing, has been an important factor in the outperformance of equities against bonds over the period 1977-1997. There would appear to be no particular value to which the premium may reach but the decline to the current level cannot be repeated. If the premium is to remain positive then it cannot fall far.

Procedures for converting the stochastic equation outputs into returns for each asset class have been determined. Forecasts of the model were made to compare with actual values over the period September 1997 to September 1999. The model yielded share price forecasts below current values.

Simulations were conducted with a set of inflation scenarios. In all scenarios, the model indicated that equities are over-valued relative to bonds. The model yielded

[1] See chapter 9 for a discussion of 'risk'.

greater returns from equities than from bonds during periods of higher inflation. The model indicates that a continuation of the low inflation conditions prevailing in the late 1990's are not conducive to equity performance. A consequence of the modelling is that the ERP is negative, except in the high inflation scenarios. Caution is required with the model, particularly the link of equity returns to inflation.

11.3 Concluding Remarks on the Findings

The Markowitz mean variance optimisation method is based upon the assumption of stability of the DGP underlying asset returns. The shortcomings of this method were outlined in chapter 1. So, do stochastic investment models present a way forward? Is the more heuristic model as proposed by Carino *et al.* (1994) a better approach? Can fund managers, or those who determine and implement investment strategy, afford to underweight equities for the length of time it takes for mean reversion to operate (which presumes that it does)[2]?

A central factor in long term modelling is the existence of structural breaks. Structural breaks are critical to the development of any methodology, whether mean variance optimisation, stochastic investment model or simple trend extrapolation, for if the underlying DGP is changing then any modelling is suspect. This feature does not appear to have been directly considered in the Mulvey and Wilkie models. The results here have concluded that in Australia there has been stability since 1975, after the breakdown of the Bretton Woods system, which implies that stochastic investment modelling is feasible. However further consideration of potential structural breaks is merited. This raises wider questions as to the connection between structural breaks and economic events commented upon in chapter 6.

[2] A near unit root, as is found in the real interest rate series, implies that shocks to the system are not permanent but will only slowly correct. This means that markets can remain significantly under or over valued for long periods of time, which is consistent with the excess volatility argument. If managers act on 'fundamental valuations' then there is a risk of the loss of funds under management if, over a reasonable period of time, markets do not respond to such assumed fundamentals. The business risk of the manager needs to be considered separately from the investment risk of the investor.

The stochastic investment model given in this thesis suggests that there is predictability and a real bond model which passes a range of statistical diagnostic criteria (see section 6.2 for a discussion of econometric methodology).

The results for the equity side are less satisfactory. No cointegrating variable has been suggested in the literature for the real AOI. The evidence is that inflation and the nominal AOI are not cointegrated. If the long run growth in share prices is due to retained earnings, then this is consistent with the hypothesis of earnings per share as the best predictor of share prices. Now given that the dividend yield is a stationary series and the payout ratio is assumed to lie between 0 and 1, then the earnings yield is also stationary. Hence the earnings index and the share price index are cointegrated. If this is the case than there should be real growth in share prices consistent with growth in real GDP, (unless the publicly listed portion of the economy represents the 'no growth' sector of the economy). Post war share prices as shown in chapter 4, exhibited no real growth. However shareholders have enjoyed a dividend income stream which has also matched inflation. So the accumulation share price index return has been well above the CPI, and far better than that from the comparable bond index as evidenced by the *ex post* ERP. The conclusion from this line of argument is that the earnings index should be compared to total returns and that the earnings index should be cointegrated with the accumulation return.

Non-linear models for the equity series could be further investigated. To the extent to which the quarterly data has been tested, any potential success observed with higher frequency data was not mirrored when quarterly data was used. Nevertheless it is the mean level of the variable which determines the long run returns, not the short term volatility. Hence, even were such a model to be successful in terms of statistical validity the assistance that could be provided in long term scenario generation is questionable.

The above discussion of the equity series implies that, rather than assume relationships or equations without a sound basis, only that which is well founded should form part of a stochastic investment model. This means that there will be a wider band of uncertainty or larger confidence limits around projections from a given scenario than would be the case if there were extra equations. There will be fewer equations available in any stochastic linear programme and potentially more subjective input (a la Carino *et al.*(1994)). In effect the process would lie somewhere between the models of Mulvey and Wilkie with detailed stochastic investment equations (which are not justified) and the heuristic approach of Carino *et al.*, for that portion of the LP which is formed by the stochastic investment model (see section 2.5 for a discussion). Extra equations could be inserted or deleted as justification to add or delete them arises.

This aspect of the actuarial and financial profession would potentially stand on firmer ground. This will not eliminate the subjective component in consulting but does enable confidence to be built in areas where such confidence is well founded. This allows those in the financial and actuarial profession such as fund managers and consultants to be able to more effectively market their work.

It may also assist in matching theory and practice more closely. For the academic either econometrician, actuary or financial economist it provides a link across to important practical considerations. It places research where it can add value.

11.4 Some Final Remarks on Stochastic Investment Modelling

There are a number of issues covered in this thesis. Statistical and econometric differences have created a confusing picture (see the brief discussion in section 2.8). The use of a wide range of estimators, over different time periods with different techniques has produced a situation where various authors have come to conclusions which are potentially contradictory. A desirable objective for Australia would be to have an agreed database on the lines of that from the Centre for Research in Security Prices (CRSP) in the US[3]. Then agreement on basic facts would be more easily obtainable.

[3] This already exists to a certain extent in the form of the SIRCA and AGSM databases.

Unit root testing provides a number of problems. Different results were obtained over different time periods. A working hypothesis is that the inflation shocks of the early 1970's have induced unit roots in the fixed interest series. This hypothesis has parallels in the mean reversion literature, wherein the importance of the Great Depression is emphasized. Removing these periods, such as the 1930's and 1970's, changes the results. However shocks to the system whilst infrequent are to be expected.

The equity side of the stochastic investment model presents significant challenges. It is difficult to find reliable long term predictors. A connection between bond rates and the dividend yield would be valuable. A preferred alternative is a connection between bond yields and earnings yields which then avoids potential distortions introduced by changes in the payout ratio.

The stochastic investment model is a set of equations involving yields, which are then converted into returns. Now the returns may be separately generated or certain of the returns may be determined through the correlations between returns to asset classes. With this format the real bond model could be placed at the 'top of the tree' and nominal equity returns would then be generated from the correlation with nominal bond returns. However the results of chapter 8 did not indicate such connections were statistically reliable[4]. Mulvey *et al.* (1996) does mention the use of such correlations, but the results were not available for public comment, hence no assessment can be made of their reliability.

The implementation and use of the stochastic investment model requires investigation in the stochastic programming context. This would enable an understanding of the relative importance of the stochastic investment model in the overall asset/liability framework. The research in this thesis did not cover this topic.

Finally there is the potential extension of the modelling approach to both other domestic asset classes such as property or international assets. This was briefly

[4] The conclusions from chapter 8 would tend to invalidate the use of mean variance optimisation which assumes the predictability and stability of the correlations between asset class returns.

mentioned in section 1.2 and note 3 in that section. The magnitude of the study conducted here implies a necessary restriction to the three asset classes covered. There is interest in the development of such models as shown by a most recent paper presented by Duval, Teeger and Yakoub (1999)[5]. The value of such models is twofold. Firstly they provide stochastic equations for international markets. These models also provide a consistency check; results in different markets may have varying parameters but equations which are widely different may lead to a conclusion of data mining. At this stage these international developments are noted. It may well need parties with global interests to provide the necessary synthesis of results from diverse international capital markets.

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[5] A presentation to the Staple Inn Actuarial Society in November 1999. The comments made of the paper in *The Actuary* of November1999 were that it was designed for practical asset/liability management for financial institutions in the UK and it is one of the few fully published models in this area.

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