On the Importance of Input Variables and Climate Variability to the Yield of Urban Water Supply Systems

by

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ABSTRACT

Yield plays a central role in the processes, practices, management and operation of urban water supply systems. In Australia, yield is commonly defined as the maximum average annual volume of water that can be supplied from the water supply system subject to climate variability, operating rules, demand pattern and adopted level of service (or security criteria). For a given water supply system, yield is typically estimated via computational simulation using the entire sequence of available historic climate data. This means that the simulation, and hence the estimation of yield, is subject to a range of extreme climate events consisting of various dry and wet spells with a multitude of severities and durations, present in the historic data. System management policies and rules are optimised to a single climate scenario that may not match the planning length of the studies conducted by the water authority, nor allowing for the effects of future climate variability.

This study is on the importance of input variables and climate variability to the estimation of yield of an urban water supply system. Primarily, the effects of planning period and the climate variability on the yield and on the importance of input variables are assessed.

A preliminary case study on a simple, hypothetical urban water supply system was conducted primarily to assess the applicability and limitations of three sensitivity analysis (SA) techniques, namely the Morris Method, the Fourier Amplitude Sensitivity Test and Sobol's method of SA. These techniques produced mostly reliable results which revealed some limitations of the SA framework adopted. The findings and conclusions of the preliminary study bore important improvements before use on the complex case study of the Barwon Water supply system.

Employing 20 climate scenarios over four simulation lengths, the input variables used in the estimation of yield for the Barwon urban water supply system were subjected to SA using the above-mentioned techniques. Significant findings of the study showed that the estimation of yield is more volatile to changes in the input variables and climate variability for shorter planning periods. This was indicated by the average and the range of the yield estimate decreasing as the planning length increased.

From this study, the main recommendation for water authorities is to consider a number of yield estimates over a simulation planning period the same as the study design period. Consequently, a water supply system should not have a single yield estimate but several; each representative of certain planning period and a possible climate scenario.

DECLARATION

I, David Michael King, declare that the PhD thesis entitled 'On the importance of input variables and climate variability to the yield of urban water supply systems' is no more than 100,000 words in length including quotes and exclusive of tables, figures, appendices, bibliography, references and footnotes.

This thesis contains no material that has been submitted previously, in whole or in part, for the award of any other academic degree or diploma. Except where otherwise indicated, this thesis is my own work.

David King
March 2009

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Chapter 1
Introduction

1.1 Background

Potable water and its supply systems are viewed as increasingly valuable commodities throughout Australia and the rest of the world. Changing climate and the increasing growth in population has put many water supply systems under immense pressure, often being required to supply a demand which is close to or exceeding its sustainable demand limit, or yield. Such pressures have been exerted on most Australian water supply systems, resulting in record restriction periods and in some cases the introduction of permanent water saving measures (DSE, 2004). Demand shortfalls can be alleviated by decreasing the demand via water saving measures and schemes, and education; and/or increasing the yield of the system by optimising system management and/or augmentation with additional water sources. All of these methods, and many operational processes undertaken by water authorities, rely heavily on the yield of the water supply system.

Yield can be thought of as the maximum volume of water that can be sustainably supplied from the system over a given period. It is subject to inflows, outflows and management rules and policies, and therefore it is a direct indicator of the performance of the system and its management. Not only does it define the maximum target demand, it is also an essential part in water supply system management and policy development and enforcement. It is used in augmentation studies, guides water sharing, and assists in decision-making polices. Optimising the management of an existing water supply system is a continual process that is largely the responsibility of water authorities and their processes and practices. The management and operational improvements of a system ultimately aim at maximising the performance of the system, namely the yield of the system. However, the estimation of the yield of a system contains various sources of uncertainty, such as the natural variability inherently implicated in being affected by climatic events, and the lack of knowledge of the optimum set of management policies and rules, which themselves are subject to climatic events.

Typically, the estimation of yield of a specific urban water supply system is performed using a computational model of the physical system simulated over the entire available sequence of historic climate data. This sequence is usually all historic data since recording began until the present day. Using a historic sequence provides a realistic scenario to which the authorities optimise system management operations, policies and rules whilst balancing stakeholder requirements. These operations, policies and rules are input variables in the estimation of yield of the system. This method is based on only one climate scenario and provides no flexibility to assess the impact of different climate realisations or to observe the effects of different planning lengths. Furthermore, it implies that there is a fixed set of optimised system policies and rules for any and all future scenarios.

The use of computational modelling is a critical element in the processes, practices, management and operation of urban water supply systems. However, uncertainty exists throughout all aspects of managing and modelling urban water supply systems, from the collection and handling of data, the interpretation of the physical system into a computational simulation, accuracy of future predictions, value of input variables, operation of the model, etc. This uncertainty propagates through the model to the model output: the yield. Following, this uncertainty in the estimation of yield will be instilled onto any management policies derived from the yield estimate. Although the exact realisation of yield is impossible to obtain (due to the variability that occurs from climate events and lack of knowledge of the optimal position of the system polices, rules and thresholds), certainty in its estimation can be improved by identifying highly influential input variables, and investigating and refining their knowledge. This will improve the confidence in the yield estimate and any management procedures and processes that consider it, leading to optimised system policy development and enforcement, augmentation studies, water sharing strategies and other decision-making practices, as well as an optimal target demand.

The yield of a water supply system is dependant on numerous variables including data (e.g. streamflow and demand), empirical inputs (e.g. operating rules), and model parameters. As these inputs are determined through either measurement, optimisation or modeller experience, they inherently contain unquantified errors which are conveyed through the model structure to the output. Minimising these errors will increase the confidence in the output, or yield. However, input variables may have different significance in terms of their influence on the output. Therefore, it is desirable to identify, investigate and improve the input variables that have considerable effects on the output. The identification of important variables is a primary goal of Sensitivity Analysis (SA).

SA is a set of frameworks and techniques that have been explicitly developed to investigate the effects of input variability on the output of a model. SA is the study of how perturbations to the model inputs propagate through the model causing changes to the output. The greater the output change resulting from a unit perturbation in an input variable, the greater the sensitivity of the model and output to that input variable. Sensitivity of an output to changes in an input variable shows the importance of that variable to the model. In this study, the SA will assess how perturbations to input variables effect the estimation of yield. The application of SA to a given problem is potentially powerful in identifying, assessing and measuring the importance of the input variables on the model and its output. The success depends on the applicability of the SA framework adopted, specifically the aptness of the selected technique(s), design of SA experiments, accuracy required and determined, and the examination of the results.

1.2 Aims of the Study

The aim of this study was to identify the importance of the variables used in the estimation of yield of an urban water supply system. Understanding the importance of the variables used in the estimation of yield provides an indication as to where water authorities should prioritise research and focus their resources to improve the understanding of the input variables. Greater understanding of the input variables used in the estimation of the yield will improve the confidence of its estimation, leading to optimised management procedures and policies, as well as more reliable target demand.

The first case study used in this research employed a simple, hypothetical urban water supply system as a proof-of-concept study to assess the adopted SA techniques and framework, and to provide preliminary results. A number of limitations, findings and conclusions became apparent whilst attempting to achieve the above aim using this hypothetical model. The principal deficiencies were in the adopted definition of yield and the associated handling of time series variables, such as streamflow and rainfall.

Subsequently, a revised aim was realised for use on the second case study considering the more complex Barwon urban water supply system in Victoria, Australia. The modified aim was to identify the importance of input variables, climate variability and planning length (i.e. the simulation length used in the simulation models) on the estimation of yield of an urban water supply system. Findings of this aim will highlight deficiencies (or otherwise) in the approach that is typically used to estimate yield, leading to shortcomings in studies that depend on a yield estimate and similar weaknesses in other studies that use the entire sequence of available historic data. If the estimation of yield is indeed sensitive to climate variability and planning period, it highlights the deficiency in the current approach of estimating the yield of an urban water supply. Ultimately, it highlights the need to consider the planning period and possible climate scenario in the estimation of yield, and for policies, rules and practices that depend on yield.

To achieve the modified aim, a series of SA experiments were used to identify and quantify the sensitivity of the model, and its output(s), to changes in the model inputs. The sensitivity of the model and yield to changes in the model inputs will be observed under different climate realisations, giving an indication of the need for a dynamic set of policies and rules that accommodate possible future climate realisations. Furthermore, by assessing the sensitivity of the model to changes in the input variables over different planning lengths, the necessity of using the same or similar simulation period (in the simulation of the water supply model) as the planning length of the system studies will become apparent.

1.3 Research Methodology

To achieve the above aims, several denotable steps were used:

1. Review of SA theory and SA techniques
2. Design of SA framework for the preliminary case study
3. Conduct SA on preliminary study and review findings
4. Design of SA framework for the Barwon urban water supply system case study
5. Conduct SA on the Barwon system and review findings

Task 1 – Review of SA theory and SA techniques

A review of uncertainty and sensitivity theory highlighted the difference between the two and introduced the significance and purpose of SA. A number of the more modern and commonly used SA techniques were then examined. Each technique was assessed against several ideal characteristics for application to the urban water supply system models, considering the input/output types and structure, the model type and availability, the accuracy and computational requirements. From this review three SA techniques were selected to assess the sensitivity of the estimation of yield to input variable perturbations, namely the Morris method, the Fourier Amplitude Sensitivity Test (FAST) and Sobol's method of sensitivity analysis. The extended Fourier Amplitude Sensitivity Test (eFAST) was also selected as a natural extension to the original FAST.

Task 2 – Design of SA framework for the preliminary case study

Following the review of SA in Task 1 and the requirements of the selected SA techniques, a SA framework for the preliminary case study was developed. The framework consists of SA methodology, input variable handling strategies and design of SA experiments. This case study was a proof-of-concept study primarily to assess the applicability of the SA techniques to an urban water supply system model, and to uncover limitations of the adopted SA framework, if any.

The SA methodology applied was largely based upon measurement and handling errors, where all input variables had an uncertainty margin about their nominal values that defined the range of perturbations. Variable handling strategies attempt to convert input variables so that input variables can be perturbed by a scalar value in SA, if they are not already.

The SA experiments were designed so that progressively accurate, yet computationally expensive, information was obtained. The Morris method was used to screen for variables that have zero or negligible importance to the estimation of yield, with the results confirmed using the FAST/eFAST techniques. SA using the FAST/eFAST and Sobol' methods were then performed on the most important variables identified through the Morris method experiments. Grouping of variables was also completed using the Morris and eFAST methods.

Task 3 – Conduct SA on preliminary study and review findings

Whilst conducting the framework developed in Task 2, important findings and conclusions revealed limitations of the SA techniques and more importantly in the SA framework adopted. Input variable handing strategies were also found to be limited when considering variables with multiple parts, or when perturbing time series variables.

Results of the SA showed mixed success of the three techniques used. The Morris and FAST/eFAST methods performed reliably but the Sobol' method gave erroneous results due to approximations in its algorithm, the model structure and non-independent input variables. The findings of the Morris and FAST/eFAST methods showed domination of results by the streamflow variable. This result caused a review on the handling of streamflow variable, and other time series variables, which highlighted a shortcoming in the SA framework adopted.

Task 4 – Design of SA framework for the Barwon urban water supply system case study

The limitations found in Task 3 resulted in the modified aim discussed in Section 1.2, hence a major alteration in the SA framework applied to the Barwon urban water supply system. The revised SA methodology consists of establishing a number (20) of different climate scenarios over four different planning periods. The climate scenarios consist of the four climate dependant variables (streamflow, rainfall, evaporation and demand) leaving the remaining input variables to be tested in the SA. This method also avoids handling issues with time series variables as experienced in Tasks 2 and 3 and preserves cross correlations between the climate dependant variables.

Task 5 – Conduct SA on the Barwon system and review findings

This task once again showed the success of the Morris and eFAST methods and the deficiency of the Sobol' method. FAST was not used in this study because the accuracy and efficiency of eFAST made it redundant. Few trends were discovered regarding the evolution of the importance of the input variables over the scenarios and planning length, responding directly to the modified aim. Significant findings regarding the average yield estimate and the range of the yield estimate were also made which highlighted the shortcomings to the current approach of how yield is estimated and used throughout water supply planning studies.

1.4 Significance of the Research

The focus of this study is to identify the importance of input variables used in the estimation of yield of an urban water supply system. Knowing the importance of input variables provides insight into where water authorities' resources should be spent and research focussed so that a better understanding of the input variables is gained. This greater knowledge will ultimately lead to improved confidence in the estimation of yield and flow through to other studies, practices and processes of water authorities that depend on yield. By performing the SA on a number of climate scenarios and over a number of planning lengths, the change of the importance of the input variables can be assessed. Also it provides opportunity to observe the impact of the climate variability and the planning length on the estimation of yield.

As discussed in Section 1.1, the estimation of yield is typically performed using the entire available sequence of historic climate data which provides a realistic climate scenario but gives no concern as to the length of time in question in the study. This approach does not provide any flexibility for a different future climate or planning length in the yield estimate and implies that there is a fixed set of optimised system policies and rules for all future scenarios and planning lengths. The findings of this thesis will indicate whether this is an acceptable approach if, and only if, the planning length and climate variability do not have a great effect the estimation of yield. If they do have an effect on the yield estimate, then there is an argument to use an appropriate planning length in the estimation of yield.

Additionally, there is evidence that over the past decade many water supply systems in Australia have experienced a reduced inflow which seems to be permanent. Seen in Figure

1-1 is the total annual inflow into the Barwon water supply system storages, together with the average annual inflow for two periods (1927 to 1996 and 1997 to 2003). There is a clear change in the average annual inflow from 155 Gl in the 1927 to 1996 period to the 76 Gl in the 1997 to 2003 period: a 51% reduction in average inflow. This reduced inflow has continued to 2008. It is not known whether this recent period is simply another dry period, such as the period from 1937 to 1946 that has an average annual inflow of 105Gl, 33% below average, or due to a more permanent feature of the climate. Conversely, it is not known whether the 50-60 years prior to 1997 were exceptionally high inflow as the records do not date back far enough. The worst case scenario is that the lower average inflow is permanent. If this is assumed, it means that only 10 years of correct climate data is available for water supply system studies, including yield studies.

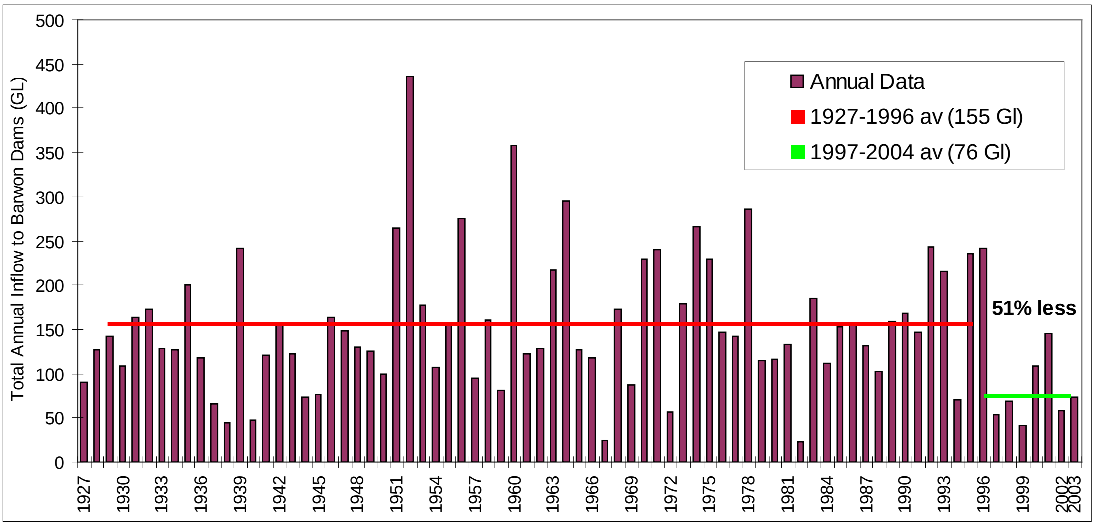


Figure 1-1. Average Annual Inflow to the Barwon Urban Water Supply System.

Potentially 10 years of data is not sufficient to robustly estimate yield and for use in other studies. The SA on the yield estimate of the Barwon urban water supply system in Chapter 5 is performed over various planning lengths (20, 40, 60 and 77 years). This provides an opportunity to assess what length of data is required to provide a robust estimate of yield, which can then be used with confidence. In the case that more than 10 years of data are required (certainly this will be the finding of this study as the minimum planning length used was 20 years), then it is necessary to generate the data for the remaining period prior to these 10 years, which have similar characteristics to the data post-1996. Although this is not part of this study, this can be done by downscaling the data prior to 1997 so that they have similar statistical characteristics as the 10 years after 1997. From these points of view, it is considered that this study is timely to account for ‘climate shift’ that has been experienced in most parts of Australia since 1997.

1.5 Layout of the Thesis

This thesis consists of several components that generally follow the order of the tasks outlined in Section 1.3. Before undertaking Task 1, a discussion of the management practices of urban water supply systems is presented in Chapter 2, including a summary of general system policies and rules used in the simulation of an urban water supply system.

Following this is Chapter 3 which outlines the principles and available techniques that can be used to perform sensitivity analysis are outlined, including a comparative assessment of the techniques in light of the selected models' requirements and limitations. This discussion then leads into a more detailed analysis of the most applicable (and currently available) sensitivity analysis techniques.

The subsequent two chapters (Chapters 4 and 5) individually introduce the two case studies and give an in depth description of the systems, their models and the input variables. A section on the design of experiments relates to how the sensitivity analyses were performed, followed by results and discussion. Different conclusions were drawn from the two case studies relating to the aims of the thesis, on the applicability and limitations of the selected sensitivity analysis framework. These are discussed at the end of Chapters 4 and 5, as well as further conclusions that were revealed whilst undertaking the case studies.

Chapter 6 ends the thesis by presenting a summary of the research work conducted, findings and conclusions drawn from this study, including recommendations to industry, limitations of the study and potential future research.

Chapter 2
Urban Water Supply System Yield

2.1 Introduction

The reliable supply of clean potable water is essential for the well-being and success of communities. Government authorities continually confront various issues, problems and limitations in their attempt to provide the community's needs of clean and reliable water supply. Indeed, water can impose limits on national development by restricting population growth when at limited volumes and impeding national production (agricultural and otherwise) through poor quality (Smith, 1998). Not only is water a political issue, but also social, environmental and economical. Lack of rainfall, water quality, suitability of source, infrastructure and storage, cost, and the community's acceptable security of water supply are factors which need to be addressed in the amelioration of urban water supply. Above all, the management of a system is critical in optimising an existing urban water supply system which aims at maximising the system's yield while balancing stakeholder requirements. Optimisation is largely dependant on the water authority and its processes and practices.

Recently climate change and the increasing growth in population have put many water supply systems under immense pressure, often being required to supply a demand which is close to or exceeding its limit, or yield. Such pressures have been exerted on most Australian water supply systems, resulting in record restriction periods and in some cases the introduction of permanent water saving measures.

Since the early 1990s efforts to slow the growth of demand in Australian cities have had modest results (Dingle, 2008) with urban water authorities implementing education, awareness and conservation measures. The arrival of the current drought at the turn of the century[1] saw a more escalated approach to reducing demand with consumers and developers (with encouragement from the government and water authorities) largely accepting and employing alternative water sources and smart water practices. The use of rainwater tanks, water saving devices and water sensitive designs has not only become fashionable for the domestic consumer but essential for industrial consumers and their products to appear environmentally friendly.

[1] The length of the drought is subjective. MJA (2006) claims that Melbourne moved into drought in 2002 while Melbourne Water recognises the drought beginning in 1997.

To increase the yield of the system, potential exists through obtaining new water sources by building new dams and diversions, constructing desalination plants or augmentation with new ground water sources. These methods are only feasible if supply increase outweighs the economic and environmental costs. A number of major Australian cities have tried to increase supply via this approach. Such as the Melbourne metropolitan area where the implementation and initial construction of major pipelines (to introduce water transfers between previously unconnected water supply systems) and a desalination plant have been met with opposition concerning their environmental costs for only modest improvements to supply (Dingle, 2008).

The estimation of yield of an urban water supply system is a critical process that water authorities use in many important and essential system management practices and processes. The primary aim of this thesis is to identify and quantify the important input variables used in the estimation of yield of an urban water supply system. Doing so indicates where water authorities should concentrate resources and focus research to efficiently reduce uncertainty in the input variables and hence increase confidence in the estimation of yield itself. To do this, Sensitivity Analysis (SA) was performed on two urban water supply system case studies. The first case study used was a preliminary study used as a proof-of-concept to review the application of SA techniques to an urban water supply system model, and to investigate feasibility of input variable handling strategies and the SA framework adopted. An improved SA framework and input variable handling strategies were then be applied in a SA on the much more computationally expensive Barwon urban water supply system.

This chapter continues, in Section 2.2, by providing a discussion of the management of an urban water supply system putting into context the significance of the yield and the importance of its accurate estimation. A brief discussion on water supply system modelling and available computational models is then presented in Section 2.3, including a more focussed description of the REALM (REsource ALlocation Model) software that is used in this study. A discussion on various definitions of yield provided in the water resources literature culminates in the definition adopted in this study, with a general discussion of the input variables required follows in Section 2.4. Section 2.5 provides a review of the procedure used to estimate yield, with Section 2.6 summarising the chapter.

2.2 Urban Water Supply System Management

A major concern of urban water supply management is how to sustain a sufficient supply of water during drought periods or low storage volumes. Drought response plans and water conservation measures have been developed in response to the threat of low storage volumes and are generally only implemented when the storage volume falls below a threshold. However, some water authorities in Australia have implemented permanent water saving measures that attempt to provide the system, water authorities and consumers greater security of supply in the future. It is these sorts of plans and policies – and their associated education schemes, rebates and incentives – that aid in limiting water consumption and reducing demand.

The management of urban water supply systems encompasses a wide range of studies, including drought management, allocation and augmentation based on short- and long-term planning periods. Management of a complex multi-purpose, multi-reservoir water supply system requires the assessment of numerous variables, objectives, risks and uncertainties. Water authorities and their water supply modellers are continually aiming at developing and studying the future plausibility of optimal rules and policies. They try to meet the various, often conflicting, objectives and stakeholders while complying with legal contracts, agreements and traditions affecting water allocation and use.

To meet the objectives and requirements of the stakeholders, water authorities develop alternative operating rules that dictate how the system is managed under different conditions. These alternative operating rules cannot satisfy all objectives of all stakeholders but a reasonable and rational judgment can be made as to which set of operating rules is best for the current and future use and conditions of the system. The operating rules of a system typically balance the needs of the water end users such as: the domestic and industrial water demands; the environmental needs of the natural river systems and other water courses; and the security of continual supply to both.

The most important objective for water authorities is to balance the demand with the available supply of potable water. This, as discussed in Section 1.1, can be done from two opposing directions. The demand can be reduced though education, incentives and regulation through restrictions, whilst the volume of water that can be supplied can be maximised by water source augmentation and by optimizing the management policies, processes and rules related to the supply system. For many water supply system management policies and processes the yield of the system is an essential component. It provides an estimate of the volume of water that can be safely supplied from a system without system failure whilst also satisfying stakeholder objectives. It is also a key indicator of the maximum demand allowable for a sustainable operation of an urban water supply system.

2.3 Water Supply System Modelling

Water authorities and system modellers have used computational modelling for several decades to provide information on water resources systems and as decision making tools. A number of reviews of research in reservoir operation and bulk water harvesting allocation models have been made in the past. Yakowitz (1982) provides an early survey of dynamic programming models for water resource problems and the techniques used to achieve solutions. Yeh (1985) explored reservoir management and operational methods and simulation models including linear programming, dynamic programming and nonlinear programming and simulation models. Similarly, Wurbs (1993) provided an inventory and comparison of reservoir-system analysis models, emphasising their practical applications. Recently Labadie (2004) and Wurbs (2005) provide similar reviews of computational models related to river/reservoir water supply systems and their applications. Wurbs (2005) offers a list of references that provide general reviews of modelling techniques for reservoir/river yield and reliability. These are: McMahon and Mein (1986), Votruba and Broza (1989), Wurbs (1993, 1996), ReVelle (1999) and Nagy et al. (2002).

Numerous water harvesting and distribution models are available. Early developments in modelling water resources include HEC-3 and HEC-5 models (Hydrologic Engineering Center, 1971; 1979). The 1980’s saw an increase in the number of software packages that include MODSIM (Labadie et al., 1986), IRIS (Loucks et al., 1987) and WASP (Kuczera and Diment, 1988). REALM (Diment, 1991 and Perera and James, 2003), WATHNET (Kuczera, 1992), IQQM (Department of Land and Water Conservation, 1999), RiverWare (Zagona et al., 2001) and Aquator (Oxford Scientific Software, 2004) are just some more recent software packages that are available.

The REALM simulation software package is used extensively in the water supply industry in Australia, becoming a standard package for simulation of water supply systems throughout Victoria and much of Australia. Of particular relevance is the use of REALM by Barwon Water Corporation for simulation of the Barwon water supply system, which is a case study considered in this thesis. A description of REALM including the structure and configuration details relating to urban water supply system modelling is presented below. The two models considered in this thesis are described in Sections 4.2 and 5.2.

2.3.1 REALM Simulation Software

REALM (REsource ALlocation Model) is a generalised computational simulation software package that models the harvesting and bulk distribution of water resources within a water supply system. Useful features of REALM include generality in modelling a wide range of water supply systems with diverse forms of operating rules, flexibility in terms of analysing 'what if' scenarios, and high reliability obtained through extensive testing and use in practical applications. It has been developed by Department of Sustainability and Environment (formally the Department of Conservation and Natural Resources) in close conjunction with its major users, with many enhancements made in response to suggestions and feedback from these users. As a result, not only is it now able to meet the needs of a diverse set of users in the water industry, but it has also developed into a comprehensive tool for water supply planning and management. There is now a REALM water resource planning model for all major water supply schemes in Victoria, Australia. Western Australia and South Australia are also major users of REALM. The REALM software and its manuals are freely available for download from the Department of Primary Industries (DPI) website: http://www.dpi.vic.gov.au/dpi/vro/vrosite.nsf/pages/water-surfacemod.

REALM uses a fast network linear programming algorithm to optimise the water allocation within the network during each simulation time step, in accordance with user-defined operating rules (Perera and James, 2003 and Perera et al., 2005). It requires three main inputs that are generally arranged into:

- System description and parameters – including system layout and connections, relevant storage data, operating rules etc. The configuration details are inputted into REALM's graphical interface that records it into system files.

- Streamflow and climate data – such as streamflow data, rainfall and evaporation data, and other climate indices. These system inflows are stored in streamflow files.

- Demand and other consumption data – These are stored in demand text files and include unrestricted demands for each demand centre, which can include, rural and urban demands, environmental flows, hydropower generation demand, etc.

The system file contains information on the nodes (i.e. storages, demand centres, pipe junctions, etc.) and carriers (i.e. rivers and pipes) in the network from which it configures the system, including constraints, priorities for water releases, etc. It also contains information regarding operating rules controlling water transfers and demand restrictions. The streamflow file contains data relating to system inflows and climatic influences on the system. The system inflows are the unregulated streamflow into the storages available for harvesting. Climatic influences can include temperature, evaporation, rainfall and/or climatic indices data, which are used to model the reservoir evaporation losses and seasonally adjust monthly demands from the average annual demand values. The demand file contains unrestricted demands for each demand centre in the system.

During each simulation time step, REALM uses the fast network linear programming algorithm to optimise the allocation of water within the system considering user-defined penalties and operating rules. When allocating the water within the system, the optimisation process attempts to satisfy the following criteria, in order of priority (Perera and James, 2003):

1. Satisfy evaporation losses (and rainfall gains) in the reservoirs.
2. Satisfy transmission losses in carriers.
3. Satisfy all demands (which may be restricted).
4. Minimise spills from the system.
5. Satisfy in-stream requirements defined by minimum capacity of carriers.
6. Attempt to meet the end of season storage target volumes.

2.4 Definition of Yield

There are many definitions and interpretations for the yield of a water supply system. Each is applicable under different circumstances and/or system management operations. Most water resource references provide a discussion on the range of definitions and provide their own, often with varying lexicon.

Linsey et al. (1992) give a general definition of yield as “the volume of water that can be supplied from a reservoir or multi-reservoir system over a given duration”. This is synonymous with McMahon and Mein’s (1986) definition of: “the amount of water that can be supplied from a reservoir or catchment during a specified interval of time”.

In their discussion of yield, McMahon and Mein (1986), cite a more precise definition of yield by Law (1955) as “… the uniform rate at which water can be drawn from the reservoir throughout a dry period of specified severity, without depleting the contents to such an extent that withdrawal at that rate is no longer possible”. This definition is often referred to as safe yield, or sometimes firm yield, which denotes the volume of water that can be supplied over the worst drought in recorded history. For instance, Linsley et al. (1992) state safe, or firm, yield as “the maximum quantity of water that can be guaranteed during a critical dry period” where the critical dry period is regarded as the lowest historic streamflow volume. Twort et al. (2000) similarly offer: “the steady supply that could just be maintained through a repetition of the worst drought on record” but term it historic yield.

McMahon and Adeloye (2005) provide different definitions to safe yield and firm yield, yet they are fundamentally the same. They state firm yield “is a term used mainly in the USA to describe the yield that can be met over a particular planning period with a specified no-failure reliability usually based on the historical record”, whilst expressing that safe yield implies 100% reliability in the supply. They recommend that hydrologists not use the term safe yield but give no such warning to firm yield.

Additional extensions to the above definitions of safe yield include: the secondary yield which defines “the volume of water above safe yield that becomes available during periods of high streamflow” (Linsley et al., 1992) and probability yield which denotes: “the steady supply that could just be maintained through a drought of specified severity and probability” (Twort et al., 2000). Twort et al. (2000) also defines failure yield as “the steady supply that could be maintained for a given percentage of days in a year (as averaged over two decades or more)”.

McMahon and Adeloye (2005) provide a more quantitative definition of yield as the controlled release from a reservoir system, often expressed as a ratio or percentage of the mean annual inflow to the reservoir. However this seems to be more applicable to a single reservoir as they suggest that release, draft and regulation are terms for yield.

These are a few of the commonly referred to definitions which are mostly intended for use for studies concerning a single reservoir but they are easily translated to a multi-reservoir system. However, the above definitions consider only a quantity or uniform flow of water that is supplied from the system. They do not explicitly consider seasonal patterns of demand, nor do they allow for the effects of demand restrictions. These are important considerations as they play an integral part in the behaviour of the system.

McMahon and Adeloye (2005) also give a definition of operational yield that considers seasonal patterns of demand and demand restrictions. They state that operational yield is determined by reducing supply so that reservoirs do not become empty during a prevailing drought and assume no knowledge of future inflows. This definition does not allow for other types of system failure, only the storage drawdown.

Taking this into account, a generalised definition of yield that is commonly used throughout Australia’s water authorities (SKM, 2003), and used in this study, is:

Yield – The maximum average annual volume of water that can be supplied from the water supply system subject to streamflow variability, operating rules, demand pattern

> *and adopted level of service (or security criteria)* (VU and DSE, 2005).

The estimation of yield of an urban water supply system is a fundamental element in the management and operation of an urban water supply system. It is a direct representation of the performance of the physical characteristics of the system and the optimum operational and management of the system. Considering the above definition, it can be reasoned that the yield of an urban water supply system is the upper limit of the demand of the system (i.e. the sustainable volume of water that can be supplied from a system over a given period). Therefore the yield of the system is synonymous to the maximum Average Annual Demand (AAD) that can be supplied over a given period. If the actual operating AAD is greater than the yield, the system will drawdown and water supply will eventually run out, i.e. the system is unsustainable.

Following are short descriptions of each of the components that are included in the above definition. See Sections 4.2.1 and 5.2.1 for system specific description of these components relating to the two case study systems used in this thesis.

2.4.1 Streamflow/Climate Variability and Demand Pattern

Climate variability represents the meteorological changes that affect climate dependant variables such as: streamflow, rainfall, temperature and evaporation. Hourly, daily, seasonal and yearly variability occurs, as well as longer trends and oscillations such as the short period El Nino – Southern Oscillation (ENSO) and much longer Pacific Decadal Oscillation (PDO). All of which can be further modified by other chaotic climate processes and natural forces (such as volcanic activities and solar fluctuations), and by human induced impacts (McKeon, 2006). As these variables change temporally, so they do spatially. This spatial variability is a result of geological characteristics that effect local meteorological conditions.

Temporal climate variability, specifically of the rainfall and temperature patterns, affects water consumption patterns. Water demand generally increases with higher temperatures and decreased rainfall, and it is therefore important to consider demand as a climate dependant variable. Similarly, spatial climate variability can also affect the local demand patterns and system management policies. Other factors such as changes in consumers' attitudes towards water conservation, education and water restriction polices also affect the demand pattern. However, the study on social effects on demand pattern is not within the scope of this study.

These climate patterns are important as they influence the location and magnitude of water inflows at various times, and where and when it will be needed to satisfy demand. This in turn has bearing on numerous management policies and operating rules of a water supply system.

2.4.2 Operating Rules

The operating rules are system specific and ensure that optimal allocation of water is observed whilst a satisfactory performance level according to the stakeholders requirements is satisfied. These cover restriction rule curves, target storage curves and other operating rules.

2.4.2.1 Restriction Rule Curves

Restriction Rule Curves (RRC) are an essential component of the management of an urban water supply system. They are used to determine the required level of restrictions to the ex-house demand to ensure that the system is not too severely depleted and remains able to supply demand in the future. They are essentially a set of curves that are derived through optimisation that provide a balance between system depletion and public's acceptance to the severity of restrictions. Each urban water supply system has a unique set of RRCs optimised to their policies and requirements.

Figure 2-1 depicts a set of 5-stage urban RRCs. The total system storage at the beginning of a given month, expressed as either an absolute value or a percentage of AAD, is used to define the restriction trigger level for that month. Restrictions are imposed when the total system storage drops below the level defined by the upper rule curve for that month. When the total system storage is above the upper rule curve, no restrictions are implemented and when below the lower rule curve, the water demand is restricted to the base demand (i.e. in-house water use only). Between the upper rule curve and lower rule curve, the intermediate curves are associated with various percentages of restrictable demand, increasing in severity as the storage volume decreases. Only the demand above the base demand is restricted, i.e. only outside house demand is restricted.

2.4.2.2 Target Storage Curves

For multiple reservoir systems, target storage curves specify the preferred distribution of individual storage volumes for various total system storage volumes at each time step (Kuczera and Diment, 1988; Perera and Codner, 2006). These curves impose inter-storage transfers to distribute water in the system to ensure water is available to supply demand centres. Target storage curves are widely used in the simulations models developed in Australia such as REALM (Diment, 1991; Perera and James, 2003; Perera et al. 2005), WASP (Kuczera and Diment, 1988) and WATHNET (Kuczera, 1990; Kuczera, 1992). Figure 2-2 shows a typical set of target storage curves for a two-reservoir system. For a given total system storage S_T at a given time-step, the target rule curves specify the storage volumes at reservoirs 1 and 2 as S_1* and S_2* respectively, where the sum of S_1* and S_2* equals to S_T.

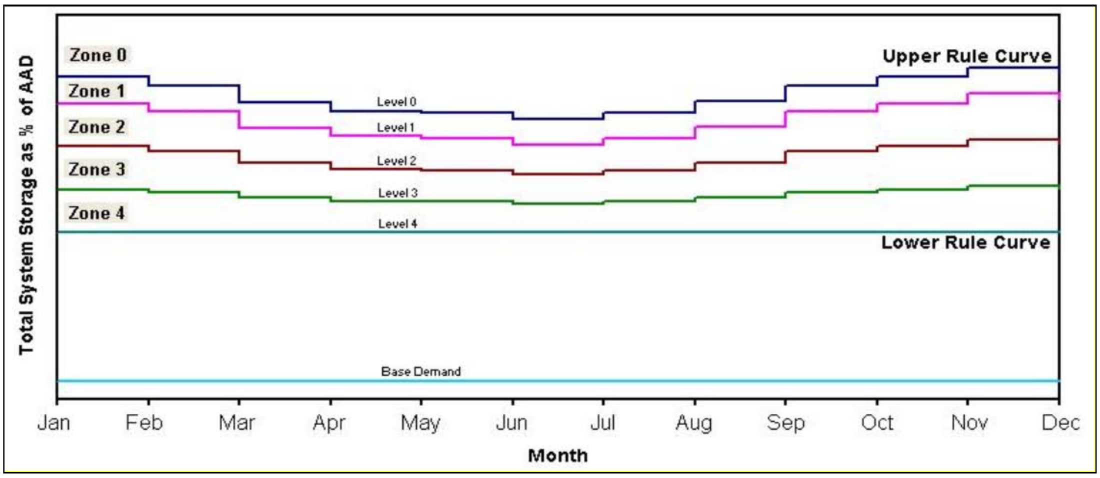


Figure 2-1. Example of a 5-Stage Urban Restriction Rule Curves.
(Source: VU and DSE, 2005)

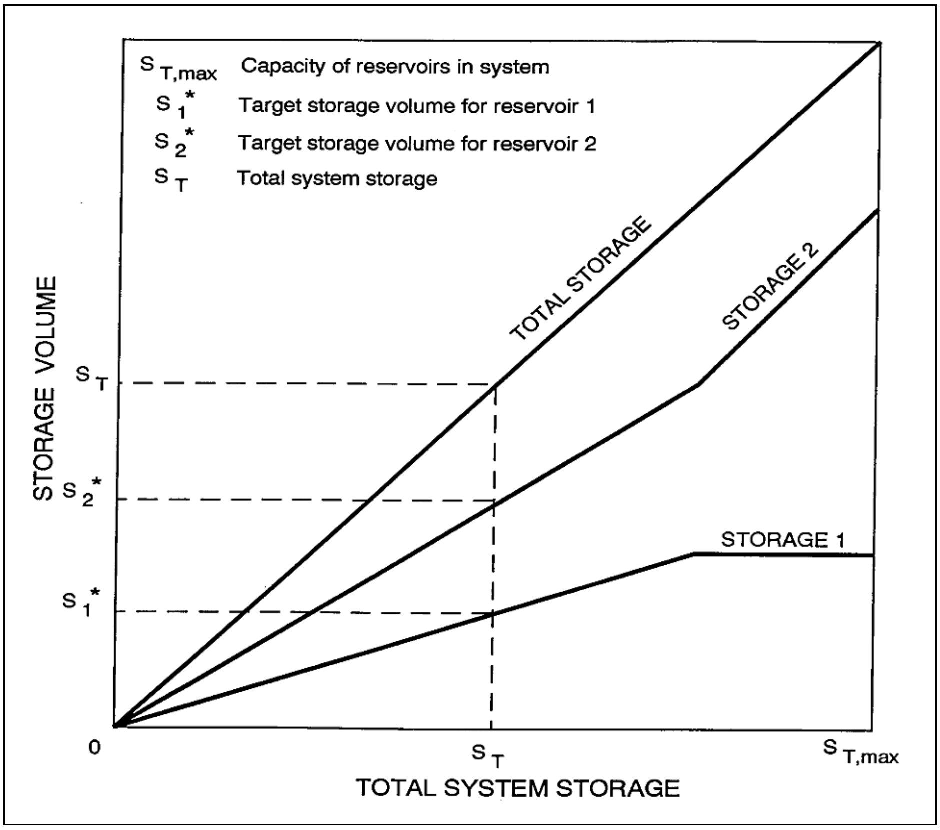


Figure 2-2. Target Storage Curves for a Typical Two-Reservoir System.
(Source: Perera and Codner, 1996)

Different sets of target storage curves can be used for different months of the year. Commonly urban water supply systems with large storage capacity will use a ‘filling’ target rule curves set over the higher streamflow, lower demand months and a ‘drawdown’ set for lower streamflow, higher demand months. This helps avoid spills in the higher streamflow periods and allows water to be stored in smaller storages closer to the demand centres in higher demand periods.

Target storage curves can be determined through optimisation, however they are generally established through modeller experience and/or system limitations and requirements. They are important so that spills are avoided during filling and demand shortfalls minimised by ensuring water is available at the time and location it is required.

2.4.2.3 Other Operating Rules

Further system operating rules could also be defined through other system variables such as environmental flow releases, diversions, hydropower generation etc. These rules are derived from studies relating to river health and hydropower generation requirements undertaken by relevant environmental and power generation authorities. They are incorporated into the models through node and carrier rules and considered permanent rules in this study, hence will not considered in the sensitivity analyses in Chapters 4 and 5.

2.4.3 Required Level of Service

Potentially the most important consideration used in estimating yield is the level of service required from the system. Also called the security of supply, the level of service is measured using one or more security criteria and their thresholds. These system specific security criteria rules can include:

- Reliability of supply – the percentage of time-steps in which restrictions are not implemented.
- Worst severity restriction stage – the worst severity restriction stage permissible.
- Maximum consecutive restriction period – the maximum consecutive number of time steps that restrictions are allowed to be imposed on demand.
- Minimum storage level – the minimum total system storage level at any time during the simulation period.

The tolerance levels of the security criteria or thresholds are determined from the acceptance of the end water users but mostly from the requirements and risks of the system and its management. That is, although the public may accept lenient performance thresholds the water authorities may adopt strict rules to avoid system failures. They are therefore generally determined by the decision makers with respect to the risk of system failure and future supply security, with some consideration given to the public opinion.

2.5 Estimation of Yield

Given the definition of yield in Section 2.4, for this study yield is synonymous with the maximum average annual demand a certain system can supply. Simply, the yield is the largest volume of water that can be supplied, on average, over a given period, without system failure.

Yield is commonly estimated by increasing or decreasing the Average Annual Demand (AAD) until the accepted level of service is almost violated. This is done using a computational water supply system model that simulates the specific water supply system that incorporates streamflow variability, operating rules and demand pattern. Throughout this study, the yield estimate was determined using such a heuristic iterative procedure which is common within the water resources industry (See SKM, 2003 for an application by the Sydney Catchment Authority). REALM is commonly used in yield estimation of urban water supply systems (SKM, 2006; ANRA, 2007; Barwon Water, 2007). Several simulations are required to converge sufficiently to the final yield estimate of the system under a specific system realisation. Within the sensitivity analysis used in this study (See Chapters 4 and 5) each yield estimate is a result of a different system realisation which includes a different combination of randomly selected variable values, positions or states. The computational expense for each estimation of yield depends on the complexity of the system being modelled, the length of the simulation, the number of simulations required to obtain the yield estimate and the power of the computer.

Two water systems are used in this thesis. A simple, hypothetical system (based on Getting Started Example given in VU and DSE, 2005) is used as a preliminary case study in Chapter 4 and the Barwon urban water supply system (SKM, 2006) is used in Chapter 5. Both of these models are simulated using the REALM computational package.

2.6 Summary

Potable water consumption and the awareness that clean water is a commodity, has changed dramatically over the past decade as a result of political, economical and environmental pressures. Climate change and the increasing growth in population has put many water supply systems under immense pressure, often being required to supply a demand which is close to or exceeding its limit. Such pressures have been exerted on most Australian water supply systems, resulting in record restriction periods and in some cases the introduction of permanent water saving measures. Balancing the available supply and demand is the foremost concern for water authorities. To match demand and supply, several possibilities are available, such as; reducing demand through education and water saving measures, and increasing supply through augmentation with new water sources and by optimal management of the system, policies and rules.

The yield, the volume of water that can sustainably be supplied by a system over a given period, is a key component in the management of an urban water supply system. Therefore, its accurate estimation is necessary for correct managerial procedures and practices. Although the estimation of yield consists of a number of input variables that inherently contain uncertainty and/or a range of variability. These uncertainties may be due to lack of precise knowledge of the parameters in the physical system, an unknown optimal position of the variable or combination of the two.

The typical approach used to estimate yield is to consider a computational model of the system and perform simulations with various Average Annual Demands (AAD) until the system is on the verge of failing. That is, the yield is the maximum AAD the system can supply without a level of service criteria (or security criteria) threshold violated.

Chapter 3
Sensitivity Analysis

3.1 Introduction

Urban water supply systems are subject to the three key influences that significantly affect the performance of the system, affecting the yield of the system in particular. These are the inflows and outflows of the system (e.g. rainfall, streamflow and demand), the physical characteristics of the system, and the management and handling of both. To assist in the management of a system, water authorities use computational models that are an abstraction of the physical system; an approximate representation of the actual system. This approximation includes estimations and assumptions that inherently introduce imperfections and errors into the model, leading to a degree of variability that is not present in the physical system. This modelling variability, combined with the above three key influences, influence the performance of the model to correctly match the physical system. All these elements of variability lead to uncertainty regarding the performance of the model and the model output(s); in this study the yield estimate of an urban water supply system.

If the uncertainty in the input variables of a model is reduced, then the confidence in the model performance would improve and the uncertainty in the output will consequently reduce. However, simply improving of knowledge of the input variables with the greatest amount of uncertainty may not be an efficient course in effectively reducing output uncertainty. The influence of a change in an input variable on the output must also be considered. This is known as the sensitivity of a model and its output to changes in input variables. The greater aim of this project is to indicate which input variables water authorities should focus resources and research to improve the accuracy of their values so that the confidence in the yield estimate increases. This is done by identifying and quantifying the sources of variability in the yield estimate by means of sensitivity analysis. Before doing so, uncertainty and sensitivity must be understood and defined in light of water supply modelling and appropriate Sensitivity Analysis (SA) techniques selected.

The current chapter begins with a discussion on the sources and typologies of uncertainty, highlighting the difficulty of developing a single definition and typology of uncertainty. Following on in Section 3.3 is an overview of SA, with definitions, uses, indices and general requirements for successful application. A review of SA techniques is then given in Section 3.4. This review is presented in a classification arrangement with the most significant techniques and some of their applications presented under each classification.

Section 3.5 gives a comparison of each of the reviewed techniques against a number of preferable criteria for their application to an urban water supply system model and its variables. This culminates in the selection of the most appropriate SA techniques (the Morris method, the Fourier Amplitude Sensitivity Test and the method of Sobol'), with further details regarding their algorithms, indices, advantages and disadvantages following.

A brief review of SA in water resources and hydrology is presented in Section 3.6 and finally the chapter summary is given in Section 3.7, providing a discussion of the main findings.

3.2 Sources and Typologies of Uncertainty

Ronon (1988) succinctly expressed the importance of understanding uncertainty in science and engineering with: "It seems that the only certain aspect of science is that it is uncertain".

A degree of uncertainty surrounds everything we do: in every action, choice, decision, within all aspects of everyday life we encounter a certain degree of uncertainty. This uncertainty is assessed almost automatically, somewhat instinctively, generally as a quick qualitative risk assessment that we evaluate by drawing from past experiences. In this case, we are generally equating the uncertainty involved in an action as a lack of confidence or a lack of control over that event, considering the possible variations in the outcome that may result. Similarly, in scientific fields, uncertainty is inherent within all aspects, particularly in the field of computational modelling of a physical system. Here however, modellers and analysts equate uncertainty to a perceived lack of knowledge and/or randomness.

Uncertainty is typically defined as the lack of knowledge of the true state of a phenomenon, process and/or data; where the extreme case of uncertainty is total ignorance (Harwood and Stokes, 2003; Walker et al., 2003). Conversely, having exact knowledge of the system's processes and precise knowledge of all possible variables within a system, the outcomes are then perfectly predictable. That is to say that if the true value of all inputs are realised, then the outcome has no variability. However, this is never the case in reality; there will always be a source of uncertainty and therefore variability in the outcome: it is unavoidable.

A computational model is an abstraction of a physical system that can be represented by Figure 3-1 (Frantz, 1995). As such, it is not only subject to the same sources of uncertainty as the real system but also a range of additional uncertainties arising from assumptions and approximations used in the formulation, parameterisation, calibration, execution and interpretation of the model. In terms of computational modelling, uncertainty can be defined as: “a potential deficiency in any phase or activity of the modelling process that is due to the lack of knowledge" (Oberkampf et al. 1998).

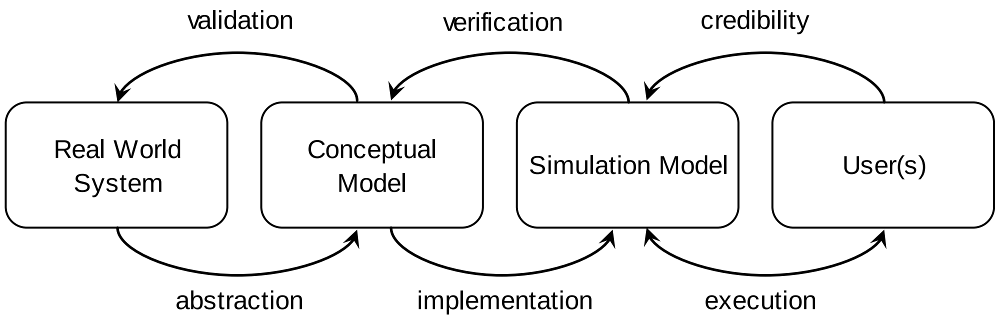


Figure 3-1. Generalised Model Abstraction from Physical System (Frantz, 1995).

Burges and Lettenmaier (1975) suggests that two main sources of uncertainty exist in computational models; i) the selection of the incorrect model with correct (deterministic) parameters, and ii) the choice of correct model with incorrect, or uncertain parameters. These are often referred to as Type I uncertainty and Type II uncertainty, and almost always exist simultaneously. Within the two broad groups that Burges and Lettenmaier (1975) suggest, specific sources of uncertainty are expediently acknowledged. The identification of the sources of uncertainty of a simulation model is particularly subjective to the purpose of the application, field of investigation and the subjectivity of the analyst (Kondolf, 1995; Lewin, 2001; Ascough et al., 2008, Wheaton et al., 2008). Therefore, numerous typologies that attempt to categorise the sources of uncertainty have been developed.

Ascough et al. (2008) provide a discussion on various uncertainty typologies that are present in uncertainty and risk assessment literature from 1990 to present. Table 3-1 lists the typologies presented by Ascough et al. (2008) to which they comment on the divergence and overlap of various sources. The original table given by Ascough et al. (2008) has been extended to include other works directly related to uncertainty in the hydrology and water resources discipline.

Table 3-1: Uncertainty Typologies from the Literature.

Reference From Literature	**Types of Uncertainty Considered**
Burges and Lettenmaier (1975)	Type I : model selection error with correct parameters Type II: incorrect or uncertain parameters, with correct model
Beck (1987)	model uncertainty, parameter (coefficient) uncertainty, future prediction uncertainty, operational uncertainty
Morgan and Henrion (1990); Hofstetter (1998)*	statistical variation, subjective judgment, linguistic imprecision, inherent randomness disagreement, approximation
Funtowicz and Ravetz (1990)*	data uncertainty, model uncertainty, completeness uncertainty
Yeh and Tung (1993) citing Yen et al. (1986)	randomness of natural processes, model uncertainty, parameter uncertainty, operational uncertainty data uncertainty: measurement, inconsistency, handling errors
Lei and Schilling (1994)	input data, model structure, model parameter, undetected numerical error
US EPA (1997*, 2003)	scenario uncertainty, parameter uncertainty, model uncertainty
Bedford and Cooke (2001)*	aleatory uncertainty, epistemic uncertainty, parameter uncertainty, data uncertainty, model uncertainty, ambiguity, volitional uncertainty
Huijbregts et al. (2001)*	parameter uncertainty, model uncertainty, uncertainty due to choices, spatial variability, temporal variability, variability between sources and objects
Bevington and Robinson (2002)*	systematic errors, random errors
Regan et al. (2002)*	epistemic uncertainty, linguistic uncertainty
Walker et al. (2003)*	location: context uncertainty, model uncertainty (input, structure, technical, parameter, outcome) level: statistical uncertainty, scenario uncertainty, recognized ignorance, total ignorance nature: epistemic uncertainty, variability uncertainty
van Asselt and Rotmans (2002)	structural uncertainties: irreducible ignorance, indeterminacy, reducible ignorance, conflicting evidence unreliability uncertainties: practically immeasurable, lack of observations and measurements inexactness
Tung and Yen (2005)	natural variability: climatic, geomorphic, hydrologic, seismic, structural knowledge deficiency: model, operational, data
Maier and Ascough II (2006); Maier et al. (2008)*	data: measurement error (instrument and calibration, reading/logging, transmission/storage), type, length, handling, presentation model: method, data available, calibration, validation, input variability human: experience, knowledge, political

* Denotes table entries originating from Ascough et al. (2008)

From Table 3-1 it becomes clear there are many typologies of uncertainty applicable to different discipline systems and models, and ideology of sources of variability therein. Walker et al. (2003) identify and attribute the lack of a shared generic typology and common terminology of uncertainty to different decision support purposes. While van Asselt and Rotmans (2002) suggest the many classifications of uncertainty that exist are due to the difficulty of defining uncertainty. Wheaton et al. (2008) discuss the extensive range of lexicon that can be used as synonyms of uncertainty. They identify and list 24 potential synonyms for the noun uncertainty and 27 synonyms for the adjective uncertain. They also consider 10 concepts related to and influenced by uncertainty, such as risk, accuracy, precision, repeatability, confidence, etc. Whatever the reasons for the numerous typologies of uncertainty, the classification of the types and sources of uncertainty allows for their identification in a systematic fashion (Wheaton et al., 2008).

It is not the aim of this discussion to suggest a common typology, just to draw attention to the extent of uncertainty sources and the various typologies that exist. For in depth commentary of the definitions and typologies of uncertainty, see the recently published: Norton et al. (2006), Refsgaard et al. (2007), Ascough et al. (2008), Wheaton et al. (2008) and citations therein. Indeed, it is the opinion of the author that there is not, and should not be, a common, shared typology and common terminology. Each field and discipline, each application and stage therein, and each analyst and set of stakeholders will have opinion to where uncertainty originates, which are deemed important and the potential results of these uncertainties.

From the citations given in Table 3-1 and other discussions regarding uncertainty, it is generally recognised that two distinct types of uncertainty exist: objective uncertainty, relating to natural variability or inherent randomness of a system, and subjective uncertainty, relating to the lack of accurate knowledge of the system, its model and variables. Objective uncertainty will always exist, even if or when all subjective uncertainty is eliminated. A number of other terminologies have been given to the same two types of uncertainty, such as those given in Table 3-2.

Understanding and quantifying uncertainty is an important step in the design, development, calibration, validation and use of computational models. It is a quantitative evaluation of the quality of the result and gives an indication on the reliability of the model output via error estimations. Suppose either the structure or the input variables of a model are highly uncertain, the outcomes from the model will also have a high level of uncertainty. This identification allows the modeller, analyst and operator to focus resources and research into areas that cause the highest amount of uncertainty in an attempt to increase the confidence in the model output.

Table 3-2. Alternative Terminology for the Two Distinct Types of Uncertainty.

Natural Variability	**Knowledge Deficiency**	**Reference From Literature**
Objective	Subjective	Yen and Ang, 1971
Aleatory	Epistemic	NRC, 2000; Bedford and Cooke, 2001
Non-Cognitive	Cognitive	Halder and Mahadevan, 2000; Tung and Yen, 2005
Stochastic	Epistemic	Walker et al., 2003
Irreducible	Reducible	Ascough et al., 2008

However, the impact of input variable perturbation on the output of the model is also important to consider before efficient prioritisation of research and resources is possible. That is, a model and its output may be considerably sensitive to a perturbation in an input variable even though there is little uncertainty in its knowledge. The sensitivity of a model and its output to changes in the input model variables is also commonly termed importance of a variable. Synonymously, sensitivity can be deemed as the level of dependency of the model (and output) on an input variable. Knowing the importance of the input variables, in conjunction to their uncertainty, indicates to the analyst how to efficiently prioritise future research and resource expenditure to improve knowledge of the input variables and hence improve confidence in the yield estimate. The assessment of the importance of input variables, the sensitivity or dependence of a model to the variables, is the primary objective of sensitivity analysis (SA).

3.3 Sensitivity Analysis

Sensitivity Analysis (SA), occasionally termed uncertainty propagation analysis (Lei and Schilling, 1994), attempts to provide an understanding of how the model response variables (the outputs, numerical or otherwise) respond to changes in the inputs. Saltelli (2000) defines SA as: “the study of how the variation in the output of a model (numerical or otherwise) can be apportioned, qualitatively or quantitatively, to different sources of variation, and how the given model depends upon the information fed into it”. Sources of variation include input variables or factors, model parameters, model structure, assumptions and specifications.

Traditionally SA has been closely associated with uncertainty analysis, with its application generally being a part of uncertainty analysis. In this setting the strength of SA has been often overlooked, considered as "the easiest analysis" of uncertainty analysis which produces "only rudimentary results" (Zhang and Yu, 2004). Zhang and Yu (2004), paraphrasing the U.S. EPA (1999), claim: "Consequently, it is best suited for making preliminary uncertainty analyses". Uncertainty analysis determines the uncertainty in the model output as a function of the uncertainties in the model itself and the model inputs (Tung and Yen, 2005). It does not provide evidence of the importance of the uncertainty of the inputs on the model, its output and the uncertainty therein.

SA is a useful collection of tools for system and model analysts and decision makers who can use it without the explicit consideration of uncertainty of each variable. For analysts of models that utilise variables with a range of natural variability, not necessarily a range of knowledge deficiency, sensitivity becomes a useful decision and management tool. Indeed, the range of perturbation assigned to each input variable in sensitivity analysis is generally the feasible extent of its realisations, but it can extend beyond this range to analyse system behaviour at extreme values or it can be a sub-range to observe how the system performs in a specific region.

Sensitivity analysis can be used for several purposes. These include:

1. Establishing model dependence on input variables
2. Verify the model structure
3. Identify over-parameterisation of a model
4. Observe model reaction of extreme values/events
5. Identify critical areas of lack of knowledge and data
6. Decision making tool

SA differs greatly from uncertainty analysis whereby the application, outputs and principles encompass much more than evaluating the effects of uncertainty on a model output. Indeed, SA is becoming a significant discipline of its own. The sensitivity of a model output to changes in an input variable can be thought of as the importance of that input variable to the output. Similarly, it shows the dependence of the model structure to that input variable. Sensitivity analysis can be summarised by: "How important are the individual elements of the input with respect to the uncertainty in the output?" (Helton, 2000). Whereas uncertainty analysis can be similarly summarised by: "What is the uncertainty in the output given the uncertainty in the input?” (Helton, 2000). As stated previously, SA apportions output variability to the input variables without necessarily requiring accurate knowledge of the uncertainty of each input variable. Whereas uncertainty analysis requires accurate knowledge of the input variables, including their ranges and probability distributions, in order to determine their contribution to the uncertainty in the model output(s).

The sensitivity of the model output to changes to an input variable is an indication of the effect that a perturbation of that input will have on the output. An input variable associated to a high sensitivity will result in a greater variation of the model output and vice-versa. This sensitivity illustrates the care that modellers must take to obtain and employ an appropriate value for the input variable, but can also signify its importance in relation to its dependency by the model structure (Saltelli et al., 1999).

The successful application of SA largely depends upon the model structure, including the input variable type, possible model linearity and correlations, and the selection of an appropriate SA technique, or techniques, to investigate accurately the nature of the input variables and model output. For example, a purely linear model (i.e. a model where the input-output relationship is linear) can be easily investigated with the use of first-order, differential or one-at-a-time (OAT) techniques[1]. However, for a model that is non-linear or non-monotonic, first-order differential analyses are ineffective as they cannot either identify or handle non-linearity, interactions, or correlations between variables.

3.4 Sensitivity Analysis Techniques

The basis of SA, regardless of the technique selected, is the principle of perturbing the input variables of a model and observing how the output of the model reacts. Most SA techniques assume a scalar input variable; therefore, the required perturbations generated by the techniques’ sampling strategy assume a scalar change. For input variables that are indeed scalar, this perturbation can be a percentage change, an absolute change or a replacement of the nominal value. For other input variable types such as discretely distributed variables and non-scalar variables (i.e. vector, time-series, etc.) variable handling is required. Appropriate handling techniques for such variables are discussed in the relevant sections of Chapters 4 and 5.

[1] One-at-a-time (OAT) techniques test the importance of input variables on a model output by perturbing each input variable in turn and observing the impact on the model output.

A number of taxonomies are available in which to classify the SA techniques. Isukapalli (1999) suggests broadly classifying the SA techniques into (a) sensitivity testing, (b) analytical methods, (c) sampling based methods and (d) computer algebra based methods. Isukapalli's (1999) system of classification is convoluted as some of the groups, namely groups (a) and (d), only contain one technique, which are derivatives of techniques found in groups (c) and (b), respectively. The use of two groups, i.e. analytical methods and sampling based methods, would be an improvement. Also, the sampling based methods group within Isukapalli's (1999) taxonomy contains sampling strategies such as Monte Carlo Sampling (MCS) and Latin Hypercube Sampling (LHS) as well a variety of sensitivity indices, some that use MCS or LHS. It is the view of the author that sampling strategies such as MCS and LHS should not be considered an SA technique as they do not provide a sensitivity measure, but a strategy to ensure that random model responses are simulated from which sensitivity indices are then determined.

Campolongo et al. (2000a) offers a common classification system based largely on the extent of the input variable range that the technique assesses. Here, the techniques are divided into three levels:

1. Factor Screening Methods - Techniques that are typically qualitative, producing only ranked results through computationally efficient sampling strategies. They generally employ some local or global properties and "some can provide univariate assessment (one-at-a-time, 1st order of Morris and Cotter) while others allow for assessment of factor interaction (e.g. factorial designs)" (US EPA, 2003). The primary objective of factor screening experiments is to identify the most important variables of a model that contains a large number of input variables (US EPA, 2003). The variables that have negligible effect on the output can then assume their nominal values and be disregarded from further analysis.

2. Local SA Methods - The emphasis is on the local impact of the input factors on the model by working intensively in a small, local region focused around a specific point of the input parameter space. This point is typically the nominal, mean or the failure point of the space. Local methods typically consider the input to output relationship to be linear and only provide univariate assessment through differentiation based techniques (US EPA, 2003).

3. Global SA Methods - Apportions the variance in the output to the changes in the input over the entire, or relatively large, range of the input parameter space. Saltelli

(2000) highlights two properties global methods possess: i) sensitivity measures incorporate the effects of the range and shape of the variable probability density function, and ii) individual variable measures are estimated while varying all other variables. Generally, they are computationally demanding MCS based techniques that use correlation, regression or variance based principles. They are capable of presenting a multivariate assessment of model sensitivity (US EPA, 2003).

The above system of classification is widely used in SA studies (e.g. Ronen, 1988; Saltelli et al., 2000; US EPA, 2003; Campolongo et al., 2000a; Tung and Yen, 2005). However, this arrangement is ambiguous as the classification of a technique as local or global is subject to whether a range is large enough to be perceived as global, or whether the number of simulations used with a local or global method can be considered as a screening experiment. The nature of some techniques can also cause problems when classifying into such an arrangement: such as the Morris Method (Morris, 1991), which is deemed a screening method, but assesses global sensitivities using locally determined sensitivities.

Another taxonomy that is based on the form of sensitivity assessment is outlined by Frey and Patil (2002). Again, three classes are presented:

1. Graphical Methods - are a simple set of methods that provide a quick overview of the sensitivity of a model by visually representing the input to output relationship using charts, graphs, surfaces, etc. They can be useful as screening techniques or as a complement to mathematical or statistical methods.

2. Mathematical Methods - determine the sensitivity of a model output to variation of an input by assessing the rate of change via differential methods. They typically require only a few values of an input value that represent the possible range of the input and as a result cannot address the variance in the output due to the variance in the input. The model equations are not always required.

3. Statistical Methods - determine sensitivity indices by perturbing the input variables, performing running the required simulations and assessing the resultant output variance. These methods can be one-at-a-time designs or vary multiple input variables simultaneously to allow the identification of the effect of interacting variables.

The ambiguities and problems identified in the classification of Isukapalli (1999) and Campolongo et al. (2000a) are not present in the classification of Frey and Patil (2002). SA techniques are easily identified as graphical, mathematical or statistical by understanding their methods, particularly the indices produced. If the techniques' indices use statistical expressions, such as means, standard deviations or variances, the method is a statistical method. This classification also avoids categorising sampling strategies as SA methods as found in Isukapalli's (1999) taxonomy.

Structured on the classification given by Frey and Patil (2002), the following sections provide brief discussions of commonly used techniques, including examples of their application. This list is not an exhaustive list of SA techniques, just some of the more commonly used and often referred techniques. For other reviews and classifications of SA techniques see: Helton (1993), Hamby (1995), Iskupalli (1999), Saltelli et al. (2000, 2004), Helton and Davis (2003), Oakley and O'Hagan (2004), Frey and Patil (2002), Christiaens and Feyen (2002), US EPA (2003), Patil and Frey (2004) and Tang et al. (2006).

3.4.1 Graphical Methods

Graphical methods have been used throughout most SA studies, for instance Frey and Patil (2002), Saltelli et al. (2000), and Cooke and van Noortwijk (2000). They provide visual representation of input to output relationships that can give the analyst a qualitative insight into trends, non-influential variables, and the effective range of input variables. However, not much literature exists explicitly on graphical methods due to them being viewed as a method of presenting results of statistical and mathematical methods. Cooke and van Noortwijk (2000) note that the main source for graphical methods are software packages. The main difficulty with graphical methods is constructing useful graphical presentations and solutions when analysing complex problems with many input variables (Cooke and van Noortwijk, 2000).

Two commonly used graphical methods, scatter plots and contour plots are briefly described below. However, other graphical methods such as histograms, cobweb plots and radar diagrams have also been used for SA (Cooke and van Noortwijk, 2000; van der Sluijs et al., 2005; Ababei et al., 2007).

3.4.1.1 Scatter Plots

Scatter plots of each variable against a model output presents a graphical representation of trends and importance of each variable to the model and its output. If a scatter plot shows a significant pattern, i.e. a trend, the associated variable can be considered important and the model sensitive to changes in that input variable. If the scatter plot does not show a trend (i.e. the spread of output is relatively uniform across the input range), then the model output is not so dependant on that variable. Hamby (1995) shows that correlations can be qualitatively determined using a scatter plot of the input variable and the model output variable.

Frey and Patil (2002) note that scatter plots can be used as a guide to the selection of appropriate SA techniques as they can help in visualising and identifying complex dependencies between input and output. However, as Vose (2000) observes, the number of points that are plotted must be enough to observe any pattern but not too many as to obscure any variability.

Application of scatter plots for SA covers many fields of research. See Frey and Patil (2002) provides an extensive list, and Tang et al. (2006) and Helton and Davis (2003) for some examples of examination of scatter plots.

The logical extension of scatter plots is the determination of correlation coefficients to assess trends of the input to output relationship (see Section 3.4.3.3).

3.4.1.2 Contour Plots

Contour plots can be used to represent a three-dimensional relationship in two-dimensions. When the contours represent the model output, the regions that contain tighter lines signify regions of greater sensitivity. They can be a simple representation of varying two inputs and the resulting output change, such as those used by Risbey and Entekhabi (1996) to show the simulated streamflow changes to temperature and precipitation changes in the Sacramento Basin in the California, USA. Chu et al. (2006), also use contour plots to present sensitivity measures against regions of parameter space, such as confidence limits.

3.4.2 Mathematical Methods

Mathematical methods discussed here consist of differential sensitivity analysis methods, nominal range sensitivity, and difference in log-odds ratio. Other methods, such as the break-even analysis (von Winterfeldt and Edwards, 1986), the Management Option Rank Equivalence (MORE) method of sensitivity analysis (Ravalico et al., 2007), algebraic sensitivity analysis (Norton, 2008), and difference in log-odds ratio method (Walpole and Myers, 1993) are also categorised under mathematical methods of SA, however they are not discussed here due to their obscurity or relatively new arrival to the SA field.

3.4.2.1 Differential Sensitivity Analysis

Differential analysis is a branch of SA that indicates the sensitivity of a model to variation in each input from partial derivatives of the model equation. The importance of the *i*-th input variable (x_i) is indicated by $\partial y/\partial x_i$, determined via finding the partial derivative of the output (y) with respect to that input (x_i). The greater $\partial y/\partial x_i$, the more sensitive the model is to changes in the *i*-th input variable. If the equation(s) expressing the model's input to output relationship is explicitly known, the direct method (Hamby, 1995) or a Neumann expansion (Isukapalli, 1999) can be used. If the model's equation(s) are not readily available, a Taylor series expansion approximation of the model can be used. From the partial derivatives of the Taylor series, the importance of the input variables can be determined (See Saltelli et al., 2000).

Two commonly used first-order differential SA based methods are the First Order Error Analysis (FOEA) and First Order Reliability Analysis (FORA). The basic principle of these two methods is to construct a truncated expansion of the Taylor series around a pre-determined point of each input variable distribution and determine the mean and variance. The Taylor series is expanded about the failure point is used for FORA (Melching and Yoon, 1996), and about the mean for FOEA (Yen et al., 1986; Zhang and Yu, 2004; Carrasco and Chang, 2005).

As first-order differential SA provides a measure of local sensitivity around a nominal point, only a small part of the input variable space is addressed, and linearity assumed. Therefore, non-linearity of input variables, interaction between variables and correlated inputs are not considered: these methods may only be useful for functions that are linear or near-linear.

Automatic approaches based on differential sensitivity analysis are available, most significantly the Automatic Differentiation Method and Green's function (See Isukapalli, 1999). These approaches are fundamentally the same, therefore the details are not provided here.

3.4.2.2 Nominal Range Sensitivity

Also known as local sensitivity analysis or threshold analysis (Cullen and Frey, 1999), the nominal range sensitivity analysis applies a change across the entire range of the plausible values of each input. As one input is perturbed across its plausible range, the remaining inputs are kept at their nominal or base-case values (i.e. an OAT design). The percentage change in the model output as a result of input perturbations indicates the sensitivity (also called swing weight) corresponding to that input variable (Tang et al., 2006). This can be represented as either a positive or a negative percentage difference with respect to the nominal value (Frey and Patil, 2002).

Nominal range sensitivity is a simple and computationally efficient method of providing an approximate estimate of the importance of each input variable, and therefore is used as a screening method. It is generally applicable to deterministic models and not usually used for probabilistic analysis (Frey and Patil, 2002).

Considering its OAT design, this method performs well with linear models but does not allow for non-linearity, interactions or correlations between or among input variables. The ranking indices produced are reliable only if there are no considerable interactions and the plausible ranges are properly specified for each input (Frey and Patil, 2002; Tang et al, 2006).

A variation of the nominal range method is the difference in log-odds ratio (Patil and Frey, 2004), which uses the ratio of probability that an event occurs to the probability that the event does not occur to assess the sensitivity of a model to an input variable. However, the log-odds Ratio has been sparsely used and not discussed further. See Patil and Frey (2004) and references therein for more details.

3.4.3 Statistical Methods

Statistical methods are characterised by the use of mean, variance or standard deviation as the primary source of indicating sensitivity of a model to input variables. Included here are the more traditional regression and correlation based SA measures, ANalysis Of VAriance (ANOVA), response surface methodology, the Morris method, the notably popular and robust variance based methods and the closely related Generalised Likelihood Uncertainty Estimation (GLUE) and the Regionalised Sensitivity Analysis (RSA) methods. Further statistically based SA methods, not discussed below, are available such as Cotter's OAT design (Cotter, 1979), Andres' Iterated Fractional Factorial Design (IFFD) (Andres and Hajas, 1993), the sensitivity analysis method based on regional splits and regression trees (SARS-RT) developed by Pappenberger et al. (2006a) and the fast probability integration technique developed and expanded by a number of contributors (for details see Haskin et al., 1996).

3.4.3.1 Regression Analysis

The basis of regression analysis is to assume that the input to output relationship of a model is characterised by (assuming a linear regression model with one output is required):

$$y = b_0 + b_1 x_1 + b_2 x_2 + ... + b_k x_k + e \tag{3.1}$$

where	y	is the model output variable
	b_i	is the regression coefficient for the i-th (of k) input to be determined
	x_i	is the i-th (of k) input variable. x_i can be an input term (x_i), an interaction term ($x_i \times x_j$), or any higher order term
	e	is the error term

The effect of an individual input variable or the combined effects of multiple inputs on the output variable y is indicated by the magnitude of the regression coefficients, b_i. These b_i coefficients are commonly estimated by least-squares analysis (Campolongo et al., 2000a) and indicate the sensitivity of the model output, y, to the i-th input variable. To standardise each b_i for ease of comparison between variables, Draper and Smith (1981) proposed the Standardised Regression Coefficient (SRC):

$$SRC(y, x_i) = \frac{b_i \hat{s}_i}{\hat{s}_y} \tag{3.2}$$

where	b_i	is the regression coefficient of the random input variable x_i
	$\hat{s}_i$	is the standard deviation of the random input variable x_i
	$\hat{s}_y$	is the standard deviation of the model output

When used for sensitivity analysis, SRC is only as good as the fit of the regression model. The model coefficient of determination, R^2, defined by Equation (3.3), indicates the linearity of the original model. When the associated model coefficient of determination, R^2, is high (i.e. close to 1) the regression model accounts for most of the amount of variability in y, indicating a linear model. If R^2 is low, the model has a non-linear input to output behaviour and the SRC-based SA is of little value as the regression model assumes a linearity (Campolongo et al., 2000a). If the model is found to be highly non-linear, Standardised Rank Regression Coefficients (SRRC) can be used, but only if the model is monotonic. If the model is both non-linear and non-monotonic variance decomposition is recommended (Ekström, 2005).

$$R_y^2 = \frac{\sum_{i=1}^{n}\left(\hat{y}_i - \overline{y}\right)^2}{\sum_{i=1}^{n}\left(y_i - \overline{y}\right)^2} \tag{3.3}$$

where $\hat{y}_i$ is the approximated output of the regression model

y_i is the original output values

$\overline{y}_i$ is the mean of the output

n is the number of values

Use of the Standardised Rank Regression Coefficient (SRRC), a SRC measure with the ranked model output, may improve the R^2 value, but the cost of the transformation alters the model under analysis (Saltelli and Bolado, 1998). SRRCs are calculated using Equation (3.2) using the ranks of the inputs and output instead of the original value of the parameter where ranking 1 corresponds to the smallest original value (Manache and Melching, 2004). The use of rank transformed data results in an analysis based on the monotonic relationship strength rather than a linear relationship strength (Helton and Davis, 2002). Therefore they are more robust, and provide a useful solution when the model has long tailed input to output distributions.

Stepwise regression analysis can be used to automatically exclude statistically insignificant inputs. It produces a regression model by progressively including the next most significant input variable, until no significant input can be identified (Helton and Davis, 2003). The R^2 value represents the significance of each variable; the variable causing the greatest increase in the total R^2 is included in the regression model in progressive analysis steps. The model's coefficient of determination R^2 and SRCs at each step indicate the influence of the selected input variables on the output and the importance of individual input variables, respectively. For uncorrelated input variables, the R^2 and the SRC attributed to a variable are identical. The inclusion of correlated variables cause unrepresentative changes in the results as effects can be added to or deleted from the regression model at each step (Campolongo et al., 2000a).

3.4.3.2 Response Surface Method

A particular case of the regression analysis is the response surface method. The methodology here involves developing a response surface of the first- or higher-order relationship (i.e. input variable interactions) between the model output and one or more input variables. An equation of appropriate order is fitted to data obtained from the original model, typically using a least-squares regression. Once the response surface is developed, the importance of input variables can be determined via inspection of the functional form of the response surface, or appropriate sensitivity analyses, such as nominal range sensitivity, differential analysis, regression analysis, multiple information index method, variance based SA, etc. (Frey and Patil, 2002).

For a computationally intensive model, the RSM approach is advantageous as it can reduce a complex model into a form that is much easier and/or faster to produce model outputs. However, a surface is limited to those variables and their ranges used in its construction and calibration (Frey and Patil, 2002). If another variable is included, or a range extended, the entire surface needs to be re-computed so that input variable interactions are captured, which can be computationally expensive.

3.4.3.3 Correlation Analysis

Correlation analysis can be thought of as an extension to the scatter plots as it attempts to quantify the characteristics that the scatter plots display. In terms of SA, the correlation analysis provides information on the importance of a variable to a model and its output. Two correlation measures are usually recognised; Pearson’s product moment Correlation Coefficient (*CC*) and Partial Correlation Coefficients (*PCC*).

Correlation Coefficient (*CC*):

Otherwise known as Pearson’s sensitivity measure, the *CC* provides the strength of the linear correlation between each input variable and the model output by use of Equation (3.4):

$$CC_{xy} = \frac{\sum_{i=1}^{n}\left(x_i - \overline{x}\right)\left(y_i - \overline{y}\right)}{\sqrt{\sum_{i=1}^{n}\left(x_i - \overline{x}\right)^2 \times \sum_{i=1}^{n}\left(y_i - \overline{y}\right)^2}} \tag{3.4}$$

where CC_{xy} is the correlation between input variable x and output variable y

$\bar{x}$ is the mean of x

$\bar{y}$ is the mean of y

The importance of an input variable is demonstrated by the magnitude of CC_{xy}, a unitless index between -1 and +1. The greater the absolute magnitude of CC_{xy} is the greater the importance of the variable. A flaw that the *CC* measure possesses is that it measures the linear relationship between the input and output with the effects of other variables included, therefore only suitable for linear models with uncorrelated input variables. For a non-linear monotonic model, the Spearman Coefficient, or the Ranked Correlation Coefficient (*RCC*), is used. The *RCC* measure is calculated using the ranks of both y and x_i instead of the original value (Campolongo et al., 2000a).

Partial Correlation Coefficient (*PCC*):

The Partial Correlation Coefficient (*PCC*) provides the linear relationship between an input and output free of any of the linear effects of all other input variables (Campolongo et al., 2000b). The *PCC* between x_i and y ($PCC_{x_i y}$) is determined by first constructing, the following two regression models:

$$\hat{x}_i = c_0 + \sum_{\substack{p=1 \\ p\neq i}}^{n} c_p x_p, \qquad \hat{y} = b_0 + \sum_{\substack{p=1 \\ p\neq i}}^{n} b_p x_p \tag{3.5}$$

where b_i, c_i are the regression coefficients

Two new variables are then defined as $(x_i - \hat{x}_i)$ and $(y - \hat{y})$. $PCC_{x_i y}$ is the *CC* between as $(x_i - \hat{x}_i)$ and $(y - \hat{y})$, i.e. $(x_i - \hat{x}_i)$ replaces x_i and $(y - \hat{y})$ replaces y_i in Equation (3.4) (Campolongo et al., 2000b).

Similar to the *SRC*s and *CC*s, this measure is useful for linear models (i.e. high R^2 value). When dealing with non-linear monotonic models, a rank transformation can be applied in the *PCC* which gives Partial Ranked Correlation Coefficient (*PRCC*).

3.4.3.4 Analysis of Variance

ANalysis Of VAriance (ANOVA) is a probabilistic SA technique that partitions output variance into components due to different input variables (individually or grouped) by determining whether there is a statistical relationship between a model output and one or more inputs (Frey and Patil, 2002). In the ANOVA algorithm, each factor assumes a limited number of distinct points in the variable space (the levels), from which the significance of an input variable is tested individually, or multiple input variables at a time. If all variables and combination of variables are tested for their significance, complete decomposition of the output variance is determined. Ginot et al. (2006) states that for linear variables a two level design is practical while a three or four level design for non-linear variables. If there is no significant association, then the variation in the output is random, and the input variable(s) is not important. Typically, the coefficients of the F-test are used to indicate sensitivity, but the coefficients of the Tukey test or Scheffe test can also be used (See Montgomery, 1997; Hochberg and Tamhane, 1987, as cited in Frey and Patil, 2002).

ANOVA is model independent, therefore does not require knowledge of input to output relationships. It assumes the output is normally distributed and requires accurate knowledge of input variable range. It is difficult to assess individual variables effects if correlations exist (Frey and Patil, 2002). The number of model simulations becomes great when considering a large number of input variables as ANOVA requires p^k model simulations, where k is the number of variables each with p levels. For instance, a 10 variable model where $p = 4$ requires $4^{10} = 1{,}048{,}576$ model simulations. More efficient alternatives are the FAST/eFAST and Sobol' methods which also sample the space from the full range, not the sparse sampling of ANOVA.

3.4.3.5 The Morris Method

The Morris method is a specialised randomised OAT design that proves to be an efficient and reliable technique to identify and rank important variables (Morris, 1991; Campolongo et al., 2000b). The method is based on the OAT assumption that if all variables are changed by the same relative amount, the variable that exhibits the largest variation in the output is the most influential. The efficiency of the Morris method is obtained from the construction of a trajectory (a pathway through the input variable space) so that an Elementary Effect (EE) is calculated for each input variable using requiring $(2k + 1)$ model simulations, where k is the number of input variables. Multiple trajectories are constructed providing a series of EEs for each input variable.

The mean of the set of EEs for each input variable, denoted by μ, assesses the overall influence of the factor on the output. It represents the sensitivity strength between the input variables and the output responses due to all first- and higher-order effects. While the original design does not allow for the separation between the orders of effects, an extension to the original, the New Morris method (Campolongo and Braddock, 1999; Cropp and

Braddock, 2002) allows identification of two-factor interaction effects. The New Morris method is described in Section 3.5.1.2.

The spread, or variance, of the set of EEs represented by σ, provides a measure that indicates a possible interaction of an input variable with other variables and/or the input variable has a non-linear effect on the output (Campolongo and Braddock, 1999). This is a useful advantage of the Morris method, however, distinction of whether the variance is a result of non-linearity or interactions is not possible with the original Morris method design – it is possible with the New Morris method.

The Morris method is primarily a screening technique to provide only ranking of importance of variables (Ratto et al., 2007). It is useful at identifying important variables (and variables with negligible importance) from within a large collection of input variables or those that are associated with a computationally demanding model. Due to its efficient EE sampling strategy the number of model simulations required is proportional to the number of considered input variables, and the number of trajectories, r. Therefore a Morris method SA experiment requires $(k + 1) \times r$, where k is the number of input variables considered. The Morris method employs a linear approximation of the output change across a fundamentally sparse input variable sampling space. The change of input over which an EE is estimated may miss a large input to output non-uniformity, therefore it must be stressed that the results can only be used for qualitative, ranking purposes. These shortcomings are also present in similar one-at-a-time methods, such as nominal range sensitivity. However, the Morris method is preferred over such techniques due to its computational efficiency.

Further details of the Morris method are given provided in Section 3.5.1.

3.4.3.6 Variance Based Techniques

The variance based techniques, namely the Fourier Amplitude Sensitivity Test (FAST) (Cukier et al., 1973), and Sobol’s method of sensitivity analysis (Sobol’, 1993), use the concept of variance as a measure of the importance of an input variable to a model, and its output, by determining the fractional contribution of each input to the variance of the output (Kioutsioukis et al., 2004). These methods can identify and quantify interactions between variables, and can be applied to a single or group of variables. They are model independent so they can be used on a model which algorithms are unknown or complex.

The FAST and Sobol' methods determine the same first-order sensitivity index (S_i), first proposed by Hoffman and Gardner (1983) (Hamby, 1994) that measures the relative contribution of an individual input variable (X_i) to the variance in the models' output (Y):

$$S_i = \frac{V(E(Y \mid X_i))}{V(Y)} \tag{3.6}$$

where S_i is the first-order sensitivity index for i-th input variable, X_i.

$E(Y \mid X_i)$ is the expected value of Y conditional on the value of X_i

$V(Y)$ is the total variance of the model output Y

The numerator of this expression, $V(E(Y \mid X_i))$, is the expected amount of variance that would be removed from the total output variance if the true value of X_i known. It is known as the first-order effect. The first order sensitivity index, S_i, also called *importance measure* or *first order effect*, is simply the ratio of the variance due to the i-th input variable (void of any interaction effectsS) to the variance due to the effects of all variables. Therefore, if the model is purely additive the sum of S_i equals 1, while for non-uniform, non-additive models the sum of S_i is less than 1. The natural progression is then to estimate the conditional variance of the X_i and X_j, $V(E(Y \mid X_i, X_j))$, and therefore S_{ij}, and so on. Eventually, sensitivity estimates of increasingly higher order can be estimated and summed as in Equation (3.7):

$$\sum_{i=1}^{k} S_i + \sum_{1 \le i < j \le k} S_{ij} + \ldots + S_{1,2,\ldots,k} = 1 \tag{3.7}$$

where k is the number of input variables

The second sensitivity measure that can be computed using variance based methods is the total sensitivity index S_{Ti} (where S_{Ti} is the total-order sensitivity index of the i-th input variable). It can be computed using Sobol' and extended FAST (eFAST) – a derivative of the original FAST proposed by Saltelli et al. (1999). This is defined as the sum of all effects involving the i-th input variable. For instance, for a three variable model S_{Ti} is calculated using Equation (3.8):

$$S_{T1} = S_1 + S_{12} + S_{13} + S_{123} \tag{3.8}$$

Sensitivity indices of higher order (i.e. second- and higher-order) can also be estimated using Sobol' method. The eFAST method cannot estimate higher-order measures, only S_i and S_{Ti}. Higher-order sensitivity indices quantify the combined effect of changing two or more variables at the same time. This can provide important information regarding interaction effects that is not possible to identify from the S_i or S_{Ti} indices.

The variance based methods have some important advantages:

- They are model independent in the sense that the model structure and/or characteristic does not effect the accuracy of the method. e.g. will work regardless of the additivity or linearity of the test model (Chan et al., 2000).

- They do not require explicit knowledge of the model algorithms, therefore can be applied to complex computational models.

- These methods can identify and quantify interactions between variables, and can be applied to a single variable or group of variables.

The main drawback of the variance based methods is their computational cost as they require a large number of model simulations in order to estimate a k-dimensional integral (k is the number of input variables considered). Another disadvantage is the errors that occur when applying the variance based techniques to a model that contains discretely distributed input variables (i.e. variables that are not continuous). Discretely distributed variables cause problems with the estimation of the integral due to the non-continuity and the possible lack of relationship between the adjacent discrete points.

3.4.3.7 Regionalised Sensitivity Analysis

Regionalised Sensitivity Analysis (RSA), originally termed Generalised Sensitivity Analysis (GSA) and sometimes referred to as the Hornberger–Spear–Young-method (Sieber and Uhlenbrook, 2005), determines which input variables are most important in the production of the model output(s) by firstly separating the input variable space into the dichotomy of behavioural or non-behavioural. Behavioural is defined as a pattern of model responses that mimicked the qualitative behaviour of the real system. For each input variable, RSA then compares the cumulative distributions of the behavioural and non-behavioural parts of the input space. The greater the difference between the cumulative distributions the greater the importance of the input variable being investigated (Pappenberger et al., 2006a). This can be done by observing the vertical distance between the distributions, as measured by the Kolmogrov-Smirnov test (Spear and Hornberger, 1980).

RSA was initially developed to analyse eutrophication processes of the Peel Inlet in Western Australia but has since been applied to various water quality investigations (See

Hornberger and Spear, 1980; Spear and Hornberger, 1980; Spear et al., 1994; Young, 1999; Ratto et al., 2007 and references therein).

3.4.3.8 GLUE Methodology

Described by Beven and Binley (1992), the Generalized Likelihood Uncertainty Estimation (GLUE) methodology is a Bayesian type methodology for calibration and uncertainty estimation of physically based distributed models based on the RSA methodology (Pappenberger et al., 2006a). It is based upon making a large number of runs of a given model with different sets of parameter values, chosen randomly from specified parameter distributions. On the basis of comparing predicted and observed responses, each set of parameter values is assigned a goodness of fit measure, that Beven and Binley (1992) called a "likelihood measure", which measures how well the model conforms to the observed behaviour of the system. Observing the likelihood measure, the distinction of behavioural and non-behavioural is made and the sensitivities indirectly indentified from the slope of the behavioural cumulative distribution function of each input variable (Pappenberger et al., 2006a).

3.5 Review, Comparison and Selection of Techniques for Use in this Study

The selection of an appropriate set of sensitivity analysis techniques is essential for successful sensitivity analyses. To select the techniques, the characteristics of the model that will be used and its input variables must be considered, as well as a basic appreciation of the design of experiments that will be undertaken.

The two REALM models of urban water supply systems considered in this study (a simple, hypothetical system and the Barwon urban water supply system) are treated as closed, "black box" models. This is to match their use in the water authority's management procedures and because of the vast number of physical characteristics, such as pipe size, capacities and penalties, reservoir sizes, etc. that are deemed to be known precisely and cannot be changed in the physical system. Within the SA ideology, all model parameters, including those that define physical characteristics of the system, should be analysed but this would not be feasible within this study. For instance, the Barwon model contains 511 carriers and 435 nodes that have multiple parameters associated with all of them. This would lead to thousands of potential variables to include in the SA, requiring in an immense number of model simulations. Moreover, knowing the importance of these would be of little benefit to a water authority as they are fixed in the physical system.

The REALM engine is built around a network linear programming algorithm that computes the allocation of water via numerous rules, limitations and criteria. That is, the behaviour of the model can change significantly if a certain rule is changed. Therefore, the model is presumed to be non-linear, non-monotonic and include many input variable interactions.

The characteristics of the input variables themselves are of various forms. Some variables are scalar absolute or percentage variables, some are multi-factored absolute or percentage variables (i.e. restriction rule curves or target storage curves) and some are discrete (or non-continuous) variables, such as maximum consecutive months with restrictions imposed. These variables can also be associated with numerous types of probability distributions, which were not readily available.

A number of ideal selection criteria to select the most appropriate sensitivity analysis technique(s) for use in this study can be identified, viz.:

1. Unknown model function – The model is treated as a “black box”. The technique must not require the model function or equations.

2. Model independent – The technique is free of any assumptions about the model. Linearity or additivity should not influence the analysis accuracy (Saltelli, 2000). The SA techniques should ideally accommodate both non-linear and non-monotonic models.

3. Variable correlations and interactions – The identification of possible variable interaction is desirable, to the second- or higher-order. The SA technique should ideally be able to quantify the interactions.

4. Input/output variable data requirements – The techniques should not require a priori knowledge regarding the characteristics of the input variables, i.e. distributions, likelihood measures etc. The technique must be able to handle a continuous, absolute output.

5. Apportion output variance – Apportioning the output variance is necessary to identify all-order effects.

Computational efficiency is also important as the Barwon REALM model is relatively computationally expensive to run, therefore the technique(s) must be able to compute the sensitivity indices efficiently.

Table 3-3 shows each of the SA techniques considered in Section 3.4 corresponding to their applicability with respect to the above five ideal selection criteria. From this, three main techniques were selected for the sensitivity analyses of the two case study systems. The Morris method was selected, primarily as a screening technique to eliminate non-influential variables (i.e. variables to which the model output are not sensitive).

As the REALM models are assumed to be 'black-box' models with many variable interactions and correlations present, scatter and contour plots, differential analysis, nominal range, correlation, RSA and GLUE were disregarded as appropriate techniques. Furthermore, regression based analysis and response surface method were disregarded due to difficulty handling correlations and non-linearity. The remaining techniques, the FAST, Sobol' and ANOVA, are all capable of handling non-linearity, correlated variables and perform variance decomposition. However, the ANOVA technique can require a greatly number of model simulations compared to the FAST and Sobol' methods. Therefore, the variance based FAST and Sobol' methods were chosen for accurate analysis of the remaining variables.

The FAST and Sobol' methods are model independent and can assess first- and higher-order effects, including input variable non-linearity and variable interactions. One major drawback with the Morris method and FAST is their limitations in handling discrete variables, while the Sobol' method becomes computationally expensive to generate accurate results from a model with a large number of variables or when performing higher-order analysis. These disadvantages will be discussed in the following detailed review of the selected SA techniques.

Following are detailed descriptions of the three selected methods, including discussion of their algorithms, advantages and limitations. Some possible improvements or methods of avoiding limitations are presented where applicable.

Table 3-3. Comparison of the Considered SA Techniques Against Ideal Selection Criteria.

	Unknown Model Function	Model Independent	Variable Correlations and Interaction	Input/Output Variable Data Requirements	Apportion Output Variance
Scatter and Contour Plots	Yes	Yes	No	Input range and distribution	No
Differential Sensitivity Analysis	No	No	No	Functional form of model	No
Nominal Range Sensitivity	Yes	Yes	No	Input range and distribution	No
Regression Analysis	Yes	No	Interactions possible Correlations with rank transformation.	Input range and distribution. Cannot handle discrete variables	Yes
Response Surface Method	Yes	Yes	Depends on how the response surface is used after developed.		
Correlation Analysis	Yes	No	No	Input range and distribution. Cannot handle discrete variables	Yes
ANOVA	Yes	Yes	Interactions only	Input range and distribution. Assumes output is normally distributed.	Possible
Morris Method	Yes	Yes	Possible qualitative identification	Input range and distribution. Issues handling discrete variables	Indirectly, qualitatively
Fourier Amplitude Sensitivity Test	Yes	Yes	Yes, with Extended FAST	Input range and distribution. Issues handling discrete variables	Yes
Method of Sobol'	Yes	Yes	Yes	Input range and distribution.	Yes
Regionalised Sensitivity Analysis	Yes	Yes	Possible, with difficulty	Input range, distribution and likelihood. Requires a binary output dichotomy	No
GLUE Methodology	Yes	Yes	Possible, with difficulty	Input range, distribution and likelihood. Requires a binary output dichotomy	No

3.5.1 Further Details of the Morris Method

The Morris Method is a screening technique useful in identifying variables that have a considerable effect on the model output from within a large collection of input variables or those that are associated with a computationally demanding model. As discussed in Section 3.4.3.5, this design was proposed by Morris (1991) as an efficient screening design. It is based on a special design matrix that results in a more economic design than random sampling. Assuming a computationally expensive model with a large number of input factors, Morris (1991) developed this method to determine which factors have the following effects on the model output:

1. Negligible effects
2. Linear (additive) effects
3. Nonlinear (interaction) effects

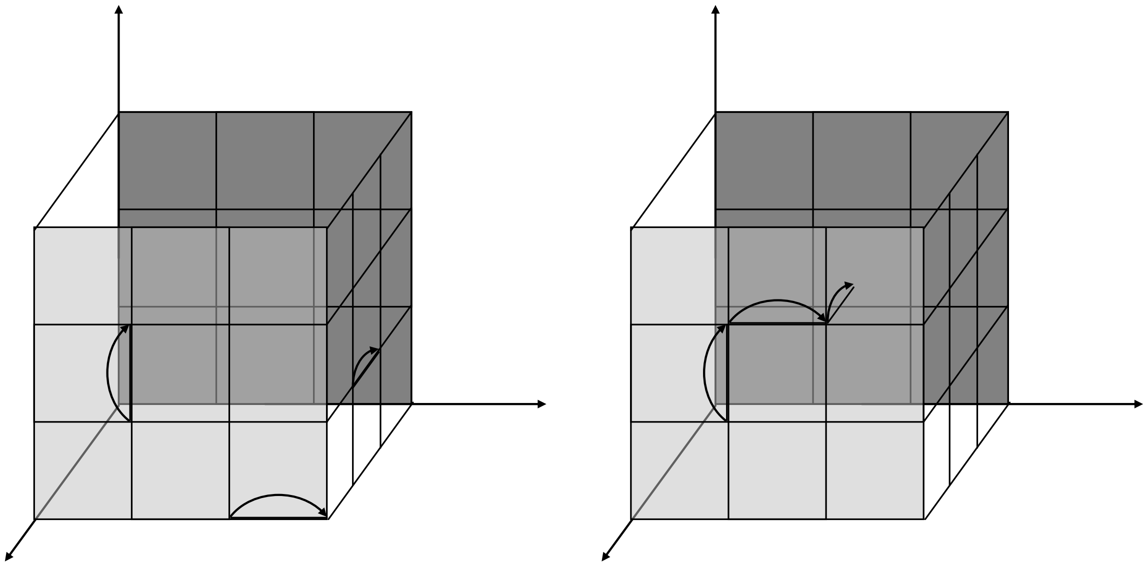
The Morris method (Morris, 1991) evaluates the effect that a change in an input variable has on the model output; termed Elementary Effects (EEs). An EE is simply the ratio of difference in the output (y), before and after a positive Δ change of a single input variable (x_i), to the change in input (Δ), as given in Equation (3.9).

$$EE_i(\mathbf{x}) = [y(x_1, x_2, ..., x_{i-1}, x_i + \Delta, x_{i+1}, ..., x_k) - y(\mathbf{x})] / \Delta \tag{3.9}$$

where	Δ	is a predetermined multiple of $1/(p - 1)$
	p	is the number of ‘levels’, or values, over which the variables can be sampled. Also known as the resolution of sampling.

Assume that k input variables are uniformly distributed over a k-dimensional unit cube and that the EEs are represented by vectors through the unit cube. Conventional OAT designs sample two points per variable to calculate one EE. Seen in Figure 3-2a, the individual EEs (indicated by arrows, one for each variable) would require ($k \times 2$) model simulations to carry out under a conventional OAT design. Morris (1991) proposes that the EEs are positioned in pseudo-random pathways throughout the variable space, a unit cube, so that the tails of each vector shares the same variable space position as the head of the previous vector. The exception is the first and last vectors that do not share simulation results. The EE pathways are termed trajectories with multiple trajectories constructed and simulated to form an SA experiment. The Morris method algorithm connects the individual EEs to create a trajectory through the variable space, as shown in Figure 3-2b. This design requires ($k + 1$) model simulations, producing a ($k - 1$) model simulation saving compared to conventional OAT designs.

Multiple trajectories (where r denotes the number of trajectories) are constructed from which a set of EEs is obtained for each input variable. F_i denotes the finite distribution of EE_i which relate to the r EEs for the i-th input variable. The mean and standard deviation of the set of EE_i (denoted as μ and σ, respectively) are calculated for each input variable. μ indicates the overall influence of a variable on the output and σ indicates a strong variable interaction, a nonlinear variable or both. Further measures are available, which are discussed below.



(a) (b)

Figure 3-2. Region of Experimentation, Ω
(a) Individual EEs for a Three Variable Model. Six Simulations Required. $p = 4$.
(b) Trajectory EEs for a Three Variable Model. Four Simulations Required. $p = 4$.

The Morris design is essentially composed of individual randomized OAT designs, in which the impact of changing the value of each input variable on the model output is evaluated in turn. The region of experimentation Ω is a k-dimensional cube over which the input vector $\mathbf{x}$ is uniformly distributed. Assuming a unit cube, each dimension is resolved into a number of levels, p, resulting in the set $\{0, 1/(p\text{-}1), 2/(p\text{-}1),\ldots,1\}$, from which x_i can be sampled from with equal probability, where p is called the level, or the resolution of sampling. Figure 3-2(a) and (b) shows Ω for a three input variable model in which $p = 4$, which results in three equal spaces between sampling points, and $\Delta = 1/(p\text{-}1) = 1/3$. In this case x_i can assume a value from the set $\{0, 1/3, 2/3, 1\}$.

The number of levels, p, determines the resolution of possible sampling. When p is small, the sampling is sparse and as p increases, the number of possible points increases. The advantage of a low p is that fewer model simulations are required to cover Ω but it can mean that regions of non-uniformity that are between sampling points can be skipped and assumed linear. A high p means that areas of non-uniformity are more likely to be captured but a higher number of model simulations are required to cover Ω sufficiently.

In his original paper, Morris (1991) provided a series of matrices to construct a single trajectory. A detailed description of the Morris method algorithm is provided in Appendix A, which includes a mathematical explanation combined with a simple numerical example of the major stages of the algorithm. Appendix A concludes with a discussion of a shortcoming of the Morris method; which becomes apparent when considering discretely distributed input variables. This shortcoming means that the number of points of a discrete variable must be equal to the number of levels p, or be a multiple of p. This ensures that the two points sampled (over the Δ change) for the calculation of EE of the discrete variable coincide with the possible discrete points that the variable is distributed over. As is discussed in Appendix A, it is possible to avoid this limitation by assigning a different p value to that variable, but this will require further alteration to the algorithm.

3.5.1.1 Elementary Effects and Morris Indices

The finite distribution of the elementary effects due to the i-th input variable is denoted as F_i. Each F_i contains r independent elementary effects (one EE per input variable from each of the r trajectories), from which the sensitivity indices can be computed. Morris (1991) proposed two measures namely the mean (μ) and standard deviation (σ) of the set of EEs for each input variable.

$$\mu_i = \frac{\sum_{n=1}^{r} EE_n}{r} \tag{3.10}$$

$$\sigma_i = \sqrt{\frac{1}{r}\sum_{n=1}^{r}\left(EE_n - \mu_i\right)^2} \tag{3.11}$$

The sensitivity index μ_i, can be used to assess the sensitivity strength between the i-th input variable and the output response due to all first- and higher-order effects that are associated with that variable (Campolongo and Braddock, 1999). When μ_i is high in contrast to other variables, the output is said to be highly sensitive to this input variable as the Δ input variable change causes a large deviation in the output. Conversely, a variable with a low μ_i value has small sensitivity associated to it as the same Δ change causes a relatively low change in output.

Determining the spread (variance) of F_i, denoted by σ_i, indicates possible interactions with other variables and/or that the variable has a non-linear effect on the output (Campolongo and Braddock, 1999). However, the original Morris method does not identify whether the variance is a result of non-linearity or interactions.

A convenient method of presenting the estimated indices is to plot all variables on a μ-σ plane as shown in Figure 3-3. It is then possible to clearly identify the important variables from their position along the μ-axis. When a variable has a high positive μ it signifies that the variable tends to have a strong positive input to output behaviour. A negative μ shows that the variable tends to have an inverse input to output behaviour. Similarly, the σ-axis gives an insight into the strength of interaction and/or non-linearity of an input variable.

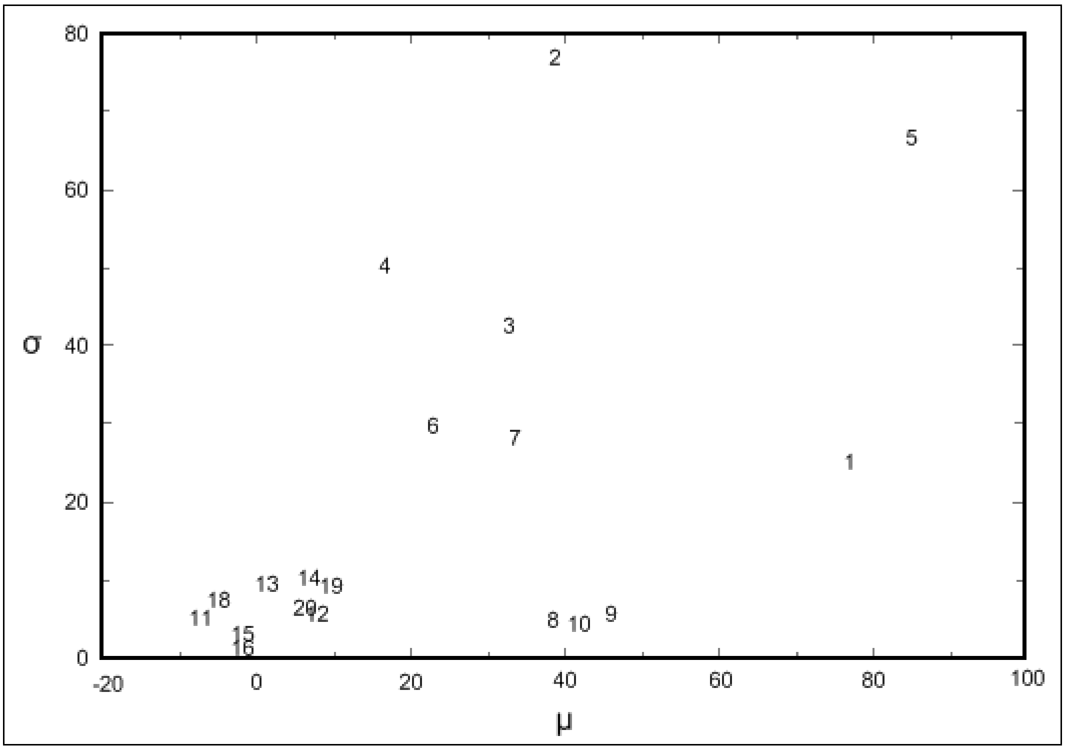


Figure 3-3. Example of μ – σ Plane used to Present Results of a Morris Method Experiment. (Source: Morris, 1991).

One of the weaknesses present in the original work of Morris (1991) is the possible misrepresentation of non-monotonic variables (Campolongo et al., 2007). Such a variable would produce positive and negative elementary effects, from which the mean value, μ, would indicate a lower overall sensitivity measure for a variable that is still highly sensitive. Simply, when calculating μ, effects that have opposite signs cancel out each other (Saltelli et al., 2004); hence using μ can be misleading as to the order of importance of the variables

Initially discussed in Saltelli et al. (2004) and Campolongo et al. (2005), Campolongo et al. (2007) proposed the use an extra sensitivity measure, μ* (See Equation (3.12)); the mean of the finite distribution of absolute values (G_i) of the elementary effects. The benefit of μ*, is that only the magnitudes of the changes are considered, avoiding some effects that may cancel each other out, hence providing a more accurate measure of total influence compared to μ.

$$\mu^* = \frac{\sum_{n=1}^{r} \left| EE_r \right|}{r} \tag{3.12}$$

While μ* provides greater reliability when ranking variables, important information regarding the nature of the effect of the variable on the output can be gained when in combination with μ. For example, if an input variable has different magnitudes for μ and μ*, it suggests that positive Δ changes cause positive and negative EE's over different regions in the variable space. This provides the analyst an insight into the nature of the non-linearity of the model and input variable.

The natural progression of the μ and μ* indices would be to consider σ* as the spread, or standard deviation, of G_i. However, the spread of G_i would be reduced (compared to the spread of F_i) due to the absolute values and not give a true impression of the non-linearity of the model input to output relationship (Campolongo et al., 2007).

The Morris method assumes a linear input to output relationship that is tested over a relatively sparse sampling framework. The measures are also the combined effects of the subject input variable, and the effects of any interaction that may exist between that variable with another input variable. Therefore, the Morris method should only be used as a ranking technique.

3.5.1.2 The New Morris Method

A major variation on the original Morris method was proposed by Campolongo and Braddock (1999), which was later corrected by Cropp and Braddock (2002). Recognising the deficiency of the Morris method to distinguish between variable interaction and non-linear behaviour, Campolongo and Braddock (1999) demonstrated a method of identifying second- and higher-order interaction effects.

To determine the interaction effects of combinations of variables, multiple variables are changed at the same time; for a second-order interaction, two factors are changed, for a third-order interaction, three factors must be changed, and so on. Considering second-order interactions, the measure of a Δ change in both the *i*-th and *j*-th input variables is the Second-order Elementary Effect (SEE_{ij}) and the distribution of multiple SEE_{ij}s denoted as SF_{ij}.

The New Morris method (Cropp and Braddock, 2002) determines one SEE_{ij} for each pair of variables per trajectory. Therefore, $r \times (\frac{k(k-1)}{2})$ model simulations are required to calculate for second-order analysis, in addition to the $r \times (k + 1)$ for first-order analysis.

3.5.1.3 Improved Sampling Strategy

A major drawback of the original Morris method is that the sampling strategy used by Morris (1991) does not guarantee optimum coverage of the sampled points through the variable space, especially when dealing with a large number of input variables. Campolongo et al. (2007) suggest an improved sampling strategy to ameliorate the spread, without additional model simulations required. The aim is to gain a more uniform spread of sampled points.

The improved coverage of the variable space is done by creating a large number of trajectories; say ~500-1000, and selecting trajectories that result in the greatest ‘spread’. Campolongo et al. (2007) show considerable improvements in the distribution of the sampled points is achieved using this strategy. However it was not employed in this study as the distribution of points was not considered an issue due to the small number of variables used in the SA case studies in Chapters 4 and 5. Refer to Campolongo et al. (2007) for details of the implementation of the improved sampling strategy.

3.5.1.4 Grouping of Variables

The original Morris Method (Morris, 1991) has been extended to incorporate grouping of input variables (Campolongo et al., 2007). Adopting grouping with any SA technique is valuable when used to explore the effects of groups of closely related variables, such as the clusters of variables associated with certain processes of a model. For instance, by grouping all variables related to the modelling of evaporation (i.e. empirical factors, volume to surface area relationship etc.) in an urban water supply simulation model, the synergy of their perturbations on the estimation of yield can be assessed. The synergy of any group of input variables is invaluable to modellers as it signifies that even if individual variables cause little sensitivity, together they might be of major importance. Conversely, if individual sensitivities are large, but combined sensitivity is small, it indicates some cancelling out, or lessening effects.

The grouping of variables is performed by changing each variable in a given group at the same time. The EEs now represent a Δ changes of multiple input variables. These Δ changes can be either positive or negative, therefore only the sensitivity indices μ^* and σ are computed when grouping variables. This is because the changes of each variable within the group can be in different directions, i.e. positive or negative. The index μ is not calculated as this index assumes that the Δ changes are all in the same direction.

3.5.2 Further Details of Fourier Amplitude Sensitivity Test (FAST)

The original Fourier Amplitude Sensitivity Test (FAST), developed by Cukier et al. (1973, 1975, 1978), Schaibly and Shuler (1973), Koda et al. (1979) and McRea et al. (1982), provides a means of estimating the first-order sensitivity indices, S_i. Substantial advancements have been made by Saltelli et al. (1999) who presented the extended Fourier Amplitude Sensitivity Test (eFAST), a method of determining the Total Sensitivity indices, S_{Ti}, in addition to S_i, and Fang et al. (2003) who improved accuracy by utilising cumulative probabilities instead of probability density when transforming non-uniform distributions.

The basic tenet that the FAST method is built upon is that a model, or function, can be expanded into a Fourier series. All input variables are simultaneously varied using different frequencies $\{\omega_i\}$ in the required model, and the amplitude of those frequencies are observed in the model output by means of Fourier analysis. From the Fourier coefficients, the mean and variance of the model's output can be determined (Fang et al., 2003), and apportioned, via an ANOVA-like decomposition, to the variance in the input variables (Saltelli and Bolado, 1998). Simply, the importance of each input variable is estimated by observing the amplitudes, $\{\omega_i\}$ in the output. The greater the amplitude of a frequency (as found in the output), the more sensitive the model is to the variable that is assigned that frequency.

Consider the model $Y = f(X)$, where Y is the model output variable vector and X is the random model input variable vector ($x_1, \ldots, x_k$) with a joint probability distribution $p(x_1, \ldots, x_k)$. Assume that Y has a finite mean and variance. As stated in Section 3.4.3.6, the ultimate aim of FAST, and the Sobol' method, is to estimate the sensitivity index, as shown previously as Equation (3.6) and here as Equation (3.13):

$$S_i = \frac{V(E(Y \mid X_i))}{V(Y)} \tag{3.13}$$

As previously stated, the numerator of this expression is the expected amount of variance that would be removed from the total output variance if the true value of X_i known. That is,

$V(E(Y \mid X_i))$ is the conditional variance of input variable X_i. Similarly, the denominator is the total variance of the output variable Y.

The FAST algorithm is centralised around the conversion of the k-dimensional integrals (where k is the number of input variables) in X into a one-dimensional integral in a new variable s by the following function:

$$x_i = G_i\left(\sin \omega_i s\right), \qquad i = 1, \ldots, k \tag{3.14}$$

where s is a scalar variable varying between $-\infty$ and ∞

$\{\omega_i\}$ is a set of incommensurate angular frequencies

G_i is the transform function

For a suitably chosen set of frequencies $\{\omega_1, ..., \omega_k\}$ and G_i, the curve described by X in the k-dimensional space when s varies between $-\infty$ and ∞ completely fills a k-dimensional unit cube Ω^k:

$$\Omega^k = \left(X \mid 0 \le x_i \le 1;\ i = 1, \ldots, k\right) \tag{3.15}$$

The expectation of Y can then be calculated by:

$$E(Y) = f_0 = \int_{\Omega^k} f\left(X\right) dx_1 ... dx_n \tag{3.16}$$

For an incommensurate set of frequencies the integrals in Equation (3.16) is impossible to compute numerically, as it would require calculation of Fourier coefficients over an infinite period. By applying a special case of the ergodic theorem proved by Weyl (1938) the integrals in Equation (3.16) and Equation (3.17) are equal.

$$E(Y) = \hat{f}_0 = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} f(X(s)) ds \tag{3.17}$$

Using Equation (3.17) with an appropriate set of integer frequencies, the curve is now only an approximately space-filling periodic curve with a period of 2π, on which numerical integration can be performed (Saltelli and Bolado, 1998). Applying this to Equation (3.14), s is now varying between $-\pi$ and π.

Incorporating the curve change described above into the computation of variances, $V(Y) = E(X^2) - [E(X)]^2$, Equation (3.18) is obtained:

$$V(Y) = \frac{1}{2\pi} \int_{-\pi}^{\pi} f^2(X(s)) ds - \hat{f}_0^2, \tag{3.18}$$

where:

$$\hat{f}_0^2 = E(Y) = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(X(s))ds \tag{3.19}$$

Applying Parseval's theorem to Equation (3.18) the variance of Y can be expressed as:

$$V(Y) \approx \frac{1}{2}\sum_{j=1}^{\infty}(A_j^2 + B_j^2) \tag{3.20}$$

where A_j and B_j are the cosine and sine Fourier coefficients of $f(s)$, respectively, calculated as follows:

$$\begin{aligned} A_j &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(s)\cos(js)ds \\ B_j &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(s)\sin(js)ds \end{aligned} \tag{3.21}$$

The contribution to the total variance V of the factor X_i is evaluated by:

$$V_i \approx \frac{1}{2}\sum_{p=1}^{\infty}\left(A_{p\omega_i}^2 + B_{p\omega_i}^2\right) \tag{3.22}$$

where $A_{p\omega_i}$ is the p-th harmonic of the frequency ω_i

ω_i is the frequency assigned to the i-th input variable

where the summation extends over all harmonics of ω_i. In practice, of course, only the first several harmonics are summed as the influence caused by higher harmonics are negligible. It also means that the number of simulations required reduces because the angular frequencies can now have common divisors above this harmonic, thus allowing the selection of lower frequencies in the set $\{\omega\}$.

The sensitivity index is then calculated by the usual formula: $S_i = V_i / V$. This quantifies the part of the variance of f that is due to the i-th input variable (Saltelli and Bolado, 1998). The original FAST algorithm cannot calculate the total sensitivity index, S_{Ti}. However both S_i and S_{Ti} can be estimated using the extended FAST (eFAST) which utilises a more efficient sampling strategy than FAST (Saltelli and Bolado, 1998).

Accuracy of the sensitivity indices depends on the selection of space filling curve and the set of angular frequencies. The set of frequencies should be incommensurate (not share a common divisor) and selected so that common Fourier transform issues, such as aliasing and interference, are prevented (Cukier et al. 1973). That is, no ω_i should be obtained by a linear combination of any other frequency in the set. However, if only the first M harmonics are considered in Equation (3.22) then the frequencies only need to be incommensurate up to a common divisor of M. This consequently allows for the use of a lower set of frequencies. However, the lower the frequency, the less the space is filled in the transformation.

Using the highest ω_i and M, the number of simulations (*N*) required by the original FAST algorithm is ($4 \times M \times \max(\omega_i) + 1$).

The selection of the transform function G_i should ideally oscillate uniformly between 0 and 1. Several transformation functions have been suggested by Cukier et al. (1973), Schaibly and Shuler (1973), Koda et al. (1979) and Saltelli et al. (1999). Saltelli et al. (1999) suggested the function given in Equation (3.23) which produces linear curve oscillating between 0 and 1 for all input variables. A two variable example of Equation (3.23) is demonstrated in Figure 3-4 where two variables that are given the commensurable frequencies ω = 11, and ω = 19. The straight lines that it produces mean that the variable space is sampled uniformly.

$$x_i = \frac{1}{2} + \left(\frac{1}{\pi}\right)\arcsin(\sin(\omega_i s + \varphi_i)) \tag{3.23}$$

where $\{\omega_i\}$ is a set of integer angular frequencies

φ_i is a random phase-shift parameter where ($0 \le \varphi_i \le 2\pi$)

s is a scalar variable varying between $-\pi$ and π

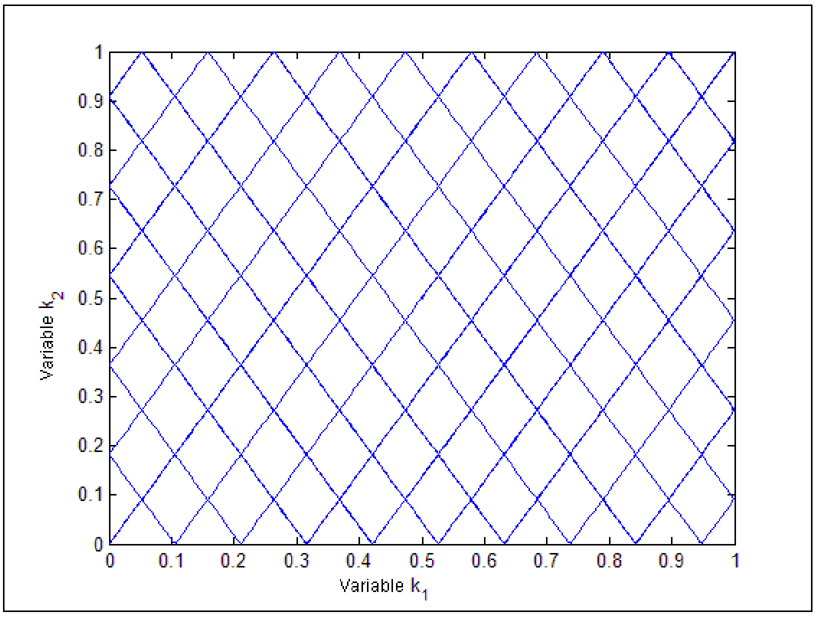


Figure 3-4. Transformation of Two Input Variables using Equation (3.23). $\omega_1 = 11$. $\omega_2 = 19$.

3.5.2.1 Extended FAST

The Extended Fourier Amplitude Sensitivity Test is an extension on the original FAST and was proposed by Saltelli and Bolado (1998). This strategy provides the S_i and S_{Ti} indices in a more economic sampling strategy, which Saltelli and Bolado (1998) claims to provide more accurate estimation of the sensitivity indices than the original FAST design.

The same sampling strategy of Equation (3.23) is utilised, however the frequencies are now selected so that a particular subject input variable takes a high frequency and all other input variables (the complementary set of variables) are assigned low, complementary frequencies. This creates a sample matrix that allows the calculation of V_i and therefore S_i. The partial variance of the complementary set of variables can then be calculated by:

$$V_{\sim i} = \frac{1}{2}\sum_{p=1}^{M}\left(A_{p\omega_{\sim i}}^{2} + B_{pw_{\sim i}}^{2} \right) \tag{3.24}$$

where $\omega_{\sim I}$ is the complementary frequency

Total indices S_{Ti} are calculated by considering the frequencies that are not harmonics of the frequency ω_i, i.e. $\omega_{\sim i}$. These frequencies contain information about the residual variance that is not accounted for by the first-order indices. Hence we can define S_{Ti} as:

$$S_{Ti} = 1 - \frac{V_{\sim i}}{V} \tag{3.25}$$

Two limitations of both FAST algorithms are the aliasing between variables and the interference error due to non-independent variables (Lu and Mohanty, 2001; Xu and Gertner, 2007). Aliasing leads to leaking between frequencies in the output which results in an artificial increase in the sensitivity indices. The interference error relates to the variance that is captured and attributed to a variable but caused by a correlated variable.

3.5.3 Further Details of Sobol’

Sobol’ proposed that his method is an extension to the FAST approach as given in Cukier et al. (1978). Let the region of experimentation Ω is a k-dimensional cube over which the input vector X is uniformly distributed, where k is the number of input variables. The main idea behind Sobol’s (1993) approach is based on the unique decomposition of the model into summands of increasing dimensionality:

$$f(x_1,...,x_k) = f_0 + \sum_{i=1}^{k} f_i(x_i) + \sum\sum_{1\le i\le j\le k} f_{ij}(x_i,x_j) + ...+ f_{1,2,...,k}(x_1,x_2,...,x_k) \tag{3.26}$$

where f_0 is a constant equal to the expectation of the output, $E(Y)$, and the integrals of each summand over any of its own variables is zero as shown in Equation (3.27):

$$\int_0^1 f_{i_1 \ldots i_s}(x_{i_1},\ldots,x_{i_s})dx_{i_k} = 0, \qquad 1 \le k \le s \tag{3.27}$$

The total variance $V(Y)$ is defined as:

$$V = \int_{\Omega^k} f^2(\overline{x})d\overline{x} - f_0^2 \tag{3.28}$$

From the each term in Equation (3.26) the partial variances are computed by:

$$V_{i_1 \ldots i_s} = \int_0^1 \ldots \int_0^1 f^2_{i_1 \ldots i_s}(x_{i_1,\ldots,}x_{i_s})dx_{i_1}\ldots dx_{i_s} \tag{3.29}$$

where $1 \le i_1 < \ldots < i_s \le k$ and $s = 1, 2, \ldots, k$.

The sensitivity measures can then be calculated using:

$$S_{i_1 \ldots i_s} = \frac{V_{i_1 \ldots i_s}}{V(Y)}, \qquad 1 \le i_1 < \ldots < i_s \le k \tag{3.30}$$

Following Homma and Saltelli (1996) and Chan et al. (2000), Monte Carlo methods are used to estimate Equations (3.28) and (3.29), with their respective Equations (3.31) and (3.33):

$$V(Y) = \frac{1}{n}\sum_{m=1}^{n} f^2(X_m) - \hat{f}_0^2 \tag{3.31}$$

where

$$\hat{f}_0 = \frac{1}{n}\sum_{m=1}^{n} f(X_m) \tag{3.32}$$

$$V_i = \frac{1}{n}\sum_{m=1}^{n} f(X^{(1)}_{(\sim i)m}, X^{(1)}_{im})\, f(X^{(2)}_{(\sim i)m}, X^{(1)}_{im}) - \hat{f}_0^2 \tag{3.33}$$

where V_i is the output variance attributed to the i-th input variable

n is the sample size

X_m is the sampled point in Ω^k

$X_{(\sim i)m}$ denotes all sample values of input variables, except variable X_i, e.g. $(x_{1m}, x_{2m}, \ldots, x_{(i-1)m}, x_{(i+1)m}, \ldots, x_{km})$

The superscripts (1) and (2) given in Equation (3.33) indicate two sampling matrices for X, both of dimension $n \times k$. For example:

$$X^{(1)} = \begin{pmatrix} x_{11} & x_{12} & \dots & x_{1k} \\ x_{21} & x_{22} & \dots & x_{2k} \\ \dots & \dots & \dots & \dots \\ x_{n1} & x_{n2} & \dots & x_{nk} \end{pmatrix} \quad \text{and} \quad X^{(2)} = \begin{pmatrix} x'_{11} & x'_{12} & \dots & x'_{1k} \\ x'_{21} & x'_{22} & \dots & x'_{2k} \\ \dots & \dots & \dots & \dots \\ x'_{n1} & x'_{n2} & \dots & x'_{nk} \end{pmatrix} \tag{3.34}$$

Substituting the sampling matrices into Equation (3.33):

$$V_i = \frac{1}{n}\sum f\begin{pmatrix} x_{11} & x_{12} & \dots & x_{1i} & \dots & x_{1k} \\ x_{21} & x_{22} & \dots & x_{2i} & \dots & x_{2k} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ x_{n1} & x_{n2} & \dots & x_{ni} & \dots & x_{nk} \end{pmatrix} f\begin{pmatrix} x'_{11} & x'_{12} & \dots & x_{1i} & \dots & x'_{1k} \\ x'_{21} & x'_{22} & \dots & x_{2i} & \dots & x'_{2k} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ x'_{n1} & x'_{n2} & \dots & x_{ni} & \dots & x'_{nk} \end{pmatrix} - \hat{f}_0^2 \tag{3.35}$$

Equations (3.33) and (3.35) suggest X_i is fixed while the remaining variables, $X_{\sim i}$, are varied. If X_i is an important variable, then the product of $f(X^{(1)}_{(\sim i)m}, X^{(1)}_{im})$ and $f(X^{(2)}_{(\sim i)m}, X^{(1)}_{im})$ will be large producing a large V_i. If X_i is not an important variable $f(X^{(1)}_{(\sim i)m}, X^{(1)}_{im})$ and $f(X^{(2)}_{(\sim i)m}, X^{(1)}_{im})$ will cancel each other out, producing a small V_i (Pastres, et al., 1999).

Higher order partial variances can be determined using:

$$V_{ij} = \frac{1}{n}\sum_{m=1}^{n} f(X^{(1)}_{\sim(ij)m}, X^{(1)}_{ijm}) f(X^{(2)}_{\sim(ij)m}, X^{(1)}_{ijm}) - V_i - V_j - \hat{f}_0^2 \tag{3.36}$$

where V_{ij} is the output variance attributed to the *i*-th and *j*-th input variable

$X^{(1)}_{\sim(ij)m}$ is $X^{(1)}$ with the *i*-th and *j*-th elements swapped with $X^{(2)}$

The sensitivity measures can therefore be calculated using one of the applicable equations below:

$$\begin{aligned} S_i &= \frac{V_i}{V} \\ S_{ij}^c &= \frac{V_{ij}^c}{V} = \frac{V(E(Y \mid X_i, X_j))}{V(Y)} \\ S_{ij} &= \frac{V_{ij}}{V} = S_{ij}^c - S_i - S_j \\ S_{Ti} &= 1 - \frac{V_{\sim i}}{V} \end{aligned} \tag{3.37}$$

where S_i is the importance of the *i*-th variable

S^c_{ij} is the closed effect of all effects involving the i-th and j-th input variable

S_{ij} is the two-factor interaction effect between the *i*-th and the *j*-th input variables

S_{Ti} is the total sensitivity index, a sum of the effects of all orders involving the *i*-th input variable

$V_{\sim i}$ is the total variance excluding the variance due to the *i*-th input variable, estimated using Equation (3.38)

$$V_{\sim i} = \frac{1}{n}\sum_{m=1}^{n} f(X^{(1)}_{(\sim i)m}, X^{(1)}_{im}) f(X^{(1)}_{(\sim i)m}, X^{(2)}_{im}) - \hat{f}_0^2 \tag{3.38}$$

Equation (3.38) suggests X_i is varied while the remaining input variables, $X_{\sim i}$, are fixed. If the $X_{\sim I}$'s are important, then $V_{\sim i}$ will be large and S_{Ti} will be small. If $X_{\sim I}$'s are not important, $V_{\sim i}$ will be small and S_{Ti} will be large (Pastres et al., 1999).

As S_i is a ratio of V and V_i, the sum of all S_i should equate to one if the model is purely additive, and never sum to greater than one. However, due to the Monte Carlo estimates of the integrals, errors can occur. These errors can be reduced by increasing *n*.

The 'closed' effect of the *i*-th and *j*-th input variables, S^c_{ij}, was proposed by Saltelli (2002a). This is a measure of effects of the *i*-th and *j*-th variables, including the individual effects (S_i and S_j), and the interaction effect of the *i*-th and *j*-th variables. Both S^c_{ij} and S_{ij} can only be calculated using the Sobol' method.

An advantage of Sobol' over FAST/eFAST is that Sobol' provides sensitivity information regarding higher-order effects. However, eFAST is more computationally efficient when calculating just the first- and total-order effects. The Sobol' method requires $n(2k + 1)$ model evaluations for calculation of all first- and total-order sensitivity effects, where *n* is the required resolution, i.e. *n* is the number of Monte Carlo samples per sensitivity index.

Saltelli (2002b) provided an enhancement to the original Sobol' (1993) algorithm, so that using $n(2k + 2)$ model simulations, the first-, second- and total-order sensitivity indices can be determined. In this thesis, the Saltelli (2002b) version of Sobol's methodology was used to compute the first-, second- and total-order indices.

3.6 Applications of Sensitivity Analysis in Water Resources and Hydrology

The application of SA in hydrology and water resources has generally been applied as part of uncertainty analysis. Pappenberger and Beven (2006) and Pappenberger et al. (2006b) discuss that uncertainty estimation is a fundamental topic in hydrology and hydraulic modelling. The reason for this is obvious: most of the applications of modelling in hydrology and water resources deal with inputs variables and parameters that contain considerable amounts of uncertainty.

Significant attempts at sensitivity analysis in hydrology and water resources go back to 1972, with the pioneering papers of Freeze (1972) and McCuen (1973; 1974). Freeze presented simulations to examine the effect of variation in certain physical parameters on runoff generation. However, he did not use the word “sensitivity analysis”. Subsequently several authors, McCuen (1974), Burges and Lettenmaier (1975), Coleman and DeCoursey (1976) and Beven (1979) applied an analytical first-order sensitivity analysis to a variety of hydrological models, while Greis (1982) used regression analysis to investigate the variability of water demand for energy production due to climate.

Rogers et al. (1985), Calver (1988) and Binley and Beven (1991) studied the Institute of Hydrology Distributed Model (IHDM) to determine its predictive uncertainty in some of first examples of calibration of physically based models based on sensitivity analysis. They pointed out the importance of calibrating such models against physical measurements and demonstrated the importance of sensitivity analysis in determining which input parameters should be carefully calibrated in view of the sensitivity of the output to their values.

Recognising the importance of SA in investigation and calibration of environmental and hydrologic models, hydrologists have been responsible for the significant RSA/GLUE (Regional Sensitivity Analysis / Generalised Likelihood Uncertainty Estimation) methodology branch of sensitivity analysis. The Generalised Sensitivity Analysis (GSA) method was originally developed by Spear and Hornberger for an analysis of a multi-parameter eutrophication model (Spear and Hornberger, 1980; and Hornberger and Spear, 1980) but soon became known as RSA. RSA was developed further and applied to many different applications by a number of contributors many of whom apply RSA to water quality and rainfall-runoff modelling – see Beck (1987), Jakeman et al. (1990), Spear et al. (1994), Young (1999), and Ratto et al. (2007) and citations therein for discussions and case studies.

The GLUE methodology, proposed by Beven and Binley (1992), is an extension of the RSA method of sensitivity analysis (Ratto et al., 2007). GLUE has been extensively used throughout environmental modelling – see Beven (2006) and Zheng and Keller (2007) for an extensive lists of applications. Hydrology specific applications include Freer et al. (1996) and Montanari (2005) who use GLUE in runoff prediction applications, and Page et al. (2003) who investigated uncertainty surrounding modelling acid deposition in ground water catchments, while Romanowicz and Beven (2003) and Pappenberger et al. (2005) used GLUE in flood inundation applications. The GLUE methodology has also been applied to distributed catchment models (Muleta and Nicklow, 2005; Zheng and Keller, 2007) and groundwater modelling (Christensen, 2003).

More straightforward sensitivity analysis strategies and indices have been used broadly for many years. Burges and Lettenmaier (1975), Chadderton et al. (1982), Tung and Hathhorn (1988), Melching and Anmangandla (1992) and Warwick (1997) have all applied either the FORA or FOEA differential sensitivity analysis techniques (see Section 3.4.2.1 for a brief description of FORA and FOEA) to the well-known Streeter and Phelps (1925) water quality model. FORA and FOEA have also been successfully applied to the QUAL2E model to determine key sources of uncertainty by Brown and Barnwell (1987), Melching and Yoon (1996) and Wagener et al. (1996). Differential approaches have also been used by Yeh and Tung (1993), who applied FOEA to a model simulating the movement of river bed pits that results from sand and gravel mining operations, Sinokrot and Stefan (1994) who performed a differential SA to observe the sensitivity of stream temperature to several input parameters in a dynamic water quality model. Zerihun et al. (1996) used a similar approach to rank 13 input variables to seven output responses in a irrigation model. More recently, Maier et al. (2001) performed an FORA on a water quality model for the Willamette River, Oregon, USA. See Melching and Yoon (1996) and Zhang and Yu (2004) for discussions, applications and references associated to FOEA and FORA.

Besides the FORA and FOEA methods, other mathematical methods of SA (discussed in Section 3.4.2) have been sparsely used. The nominal range SA technique has successfully been applied by Brandt and Elliott (2005), who used it in an agricultural application to determine the effect of input factor perturbations on the phosphorus index score for agricultural biosolids recycling, and Dakins et al. (1994) who used it in a fishing industry assessment of a contaminated harbour. Brandt and Elliott (2005) appreciated the nominal range SA technique as a “straightforward and simple” application however noted that caution must be taken when setting input variable range.

Many of the statistical methods of SA discussed in Section 3.4.3 have also been widely used in environmental modelling, however applications to water resources and hydrology field seem scarce (Sieber and Uhlenbrook, 2005). Regression and correlation based techniques of SA have been used by Pastres et al. (1999) (whom also performed a Sobol' method based SA) on a eutrophication model of Venice lagoon, Italy and by Christiaens and Feyen (2002) for an analysis on important soil hydraulic variables in the integrated surface and ground water model MIKE SHE. Muleta and Nicklow (2004) used stepwise regression analysis on ranked input and output variables to decrease the calibration parameters of a distributed catchment model and Manache and Melching (2004) gave a review of several regression and correlation indices and application using a water quality model. Sieber and Uhlenbrook (2005) used both a regression based SA and RSA to verify the structure of a time-dependent model of the Brugga catchment basin, Germany. In comparison to the RSA results, and to results of a previous study, they judged the regression SA to be successful and proved the model's concept. Sieber and Uhlenbrook (2005) observed the importance of parameters in the model, the dependency of the sensitivity on the initial and boundary conditions and the sensitivity of temporal and spatial variability.

The Morris method, FAST and Sobol' have also been largely overlooked by the water and hydrology community. Recent years has seen an increased adoption of these methods for use in environmental modelling. This is due to their increased viability resulting from increased computer power availability but predominately due to the members of the Econometrics and Applied Statistics Unit (EAS) at the Joint Research Centre (JRC) of the European Commission (Ispra, Varese, Italy). This group has authored or co-authored a significant number of papers that use these three methods, notably are Saltelli et al. (2000) and Saltelli et al. (2004). Indeed one application of Morris and Sobol' was done by Campolongo and Saltelli (1997) in an investigation of sulphur gas production from algal biota.

An early application of the Sobol' method of SA was performed by Pastres et al. (1999) who, as previously mentioned, used linear regression and Sobol' SA on a shallow-water 3D eutrophication model. Since then few hydrological and water resource modellers have used Sobol'. Some notable examples are Hall et al. (2005) who used the Sobol' method to estimate first- and total-order sensitivity indices for six input variables in a flood inundation model, and a string of papers by Tang et al. (2006, 2007a, 2007b) and van Werkhoven et al. (2008) who used Sobol' on various catchment models. Tang et al. (2006) gives a comparison of four SA techniques, concluding that the Sobol' and ANOVA methods were superior to the RSA and differential SA techniques that they also tested.

Using a two-step SA approach, Francos et al. (2003) provided one of the first applications of the Morris method and of FAST to the environmental modelling community. Their study considered the complex SWAT (Soil and Water Assessment Tool) hydrological distribution model using the Ouse catchment in the UK as a case study, with 82 input variables and 22 model output variables. The Morris method was used as a screening pass to determine the qualitative ranking of all 82 input variables over 22 model outputs. This was followed by a deeper analysis of the most relevant input variables for specific sets of model output variables using FAST.

Considering the SWAT model of the Dender catchment in Belgium, van Griensven et al. (2002) combined the Morris method with a Latin Hypercube sampling strategy to screen for the most important out of 129 input variables over five model outputs. Instead of constructing trajectories through the variable space, their methodology randomly selects a number of points in the hypercube ensuring that a uniform spread is generated. From each point, all variables are perturbed, one-at-a-time, resulting in a design of the cost as the Morris method.

Ho et al. (2005) used the Morris and New Morris algorithms to assess two models; a soil erosion and deposition model, and a rainfall runoff model. They commented that no “definitive conclusions” could be drawn relating to the nature of the models.

Particular relevance to this thesis is the use of the Morris method on the REALM Goulburn System Model (GSM) by Schreider et al. (2003) and Braddock and Schreider (2006). In their study, Braddock and Schreider (2006) used the Morris method for first-order analysis and also the rarely implemented New Morris Method for second-order analysis. They considered nine input variables including transmission, operational and evaporation losses, a transfer function, and REALM convergence thresholds, and 16 output variables consisting of water outflows, allocations and diversions. In a series of Morris method experiments they found that the GSM is sensitive to the model convergence thresholds, with second-order effects present, illustrating the need to limit these thresholds more tightly.

3.7 Summary

Computational modelling of water resources systems contains a number of sources of variability and uncertainty. The use of computational models for management of urban water supply systems is a key practice for water authorities. In particular the use of models for estimating the yield of a system is extremely important for water authorities as yield is essential for system performance and many management practices. However, variability surrounds all aspects of the modelling of a physical water system, including the accuracy of climate data measurements and its use for studies of future system use, the accuracy of the model to the physical system, the accuracy of the management policies, etc. These sources of variability, whether originating from knowledge deficiency or from natural variability, propagate through the model and cause output variability. For this study, the output is the yield of urban water supply systems. Since yield is important in management of water supply systems, it is necessary to improve the confidence in its estimate.

Sensitivity analysis is a useful procedure to identify the importance of input variables to a model and its outputs. Often this is referred to as the sensitivity of the model to changes in the input variables or the dependency of the model and outputs to the input variables. This chapter has introduced Sensitivity Analysis (SA) and discussed, under a classification system, a number of commonly used techniques. With consideration given to the most ideal features for the successful application of SA techniques to urban water supply systems, the Morris method, the Fourier Amplitude Sensitivity Test (FAST) and Sobol' method of SA were selected appropriate. The Morris method was chosen as a screening technique to identify and rank the importance of variables, to identify any negligible importance variables and to provide a quick insight into the behaviour of the model. The FAST and Sobol' techniques are variance based methods that can provide first-, higher-, and total-order importance measures of the input variables. The variance based methods are model and variable independent and can handle non-linearity and correlations. The Sobol' method can also quantify interaction effects of input variables.

Chapter 4
Preliminary Sensitivity Analysis Using a Hypothetical Urban Water Supply System

4.1 Introduction

Chapter 2 discussed the definition, importance and method of the estimation of yield of an urban water supply system. Yield is central to many management policies and practices of an urban water supply system, and therefore important to the performance and management of the system. However the estimation of yield is subject to many sources of input uncertainty which propagate through the model. Reducing the uncertainty in the input variables would reduce the uncertainty in the yield estimate, and therefore improve the confidence on the management policies and practices which depend on it. As discussed in Sections 3.2 and 3.3, quantifying the sensitivity of the model and the output to changes in the input leads to an indication as to the most important variables to the model and the output. Knowing this gives an insight into which variables research should be focussed and resources spent so as to reduce the input uncertainty and hence the output uncertainty. To do this Sensitivity Analysis (SA) can used to identify and quantify the sensitivity of a model to an input variable. Sections 3.4 and 3.5 presented common SA techniques and culminates in the selection of three appropriate techniques for the sensitivity analysis of the yield estimate of an urban water supply system.

This chapter discusses the use of a simple, hypothetical case study as a ‘proof of concept’ for the application of the Morris method, the Fourier Amplitude Sensitivity Test (FAST) and Sobol’ method of sensitivity analysis. This case study is a proof-of-concept study to assess the applicability of these techniques and to identify their limitations and shortcomings before applying them to a more computationally expensive urban water supply system. Also to be assessed is the SA framework and the variable handling strategies used for the many input variables used in the estimation of yield. Where limitations are found, improvements or alternatives will be applied to the case study of the Barwon urban water supply system, which is described in Chapter 5.

The case study used in this chapter is the Getting Started Example model found in VU and DSE (2005). A description of this case study is provided in Section 4.2, including details of the model inputs used. Section 4.3 describes the SA framework, including the input variable handling strategies (Section 4.3.1) and the design of experiments (Section 4.3.2). Sections 4.4 and 4.5 present the sensitivity analysis results of using the Morris method and the variance based techniques, respectively. Following this in Section 4.6 is a discussion on issues and limitations of the framework used in this case study and recommendations for improvements for use in the Barwon urban water supply system case study.

4.2 System Description

The Getting Started Example model (VU and DSE, 2005), a hypothetical two-reservoir system, was considered as the case study. The schematic diagram of this system is shown in Figure 4-1, while basic system, streamflow and demand data, given in VU and DSE (2005), were used for the case study. The only modification to the VU and DSE (2005) system is the inclusion of evaporation modelling for Reservoir B; in VU and DSE (2005) the evaporation is modelled only in Reservoir A.

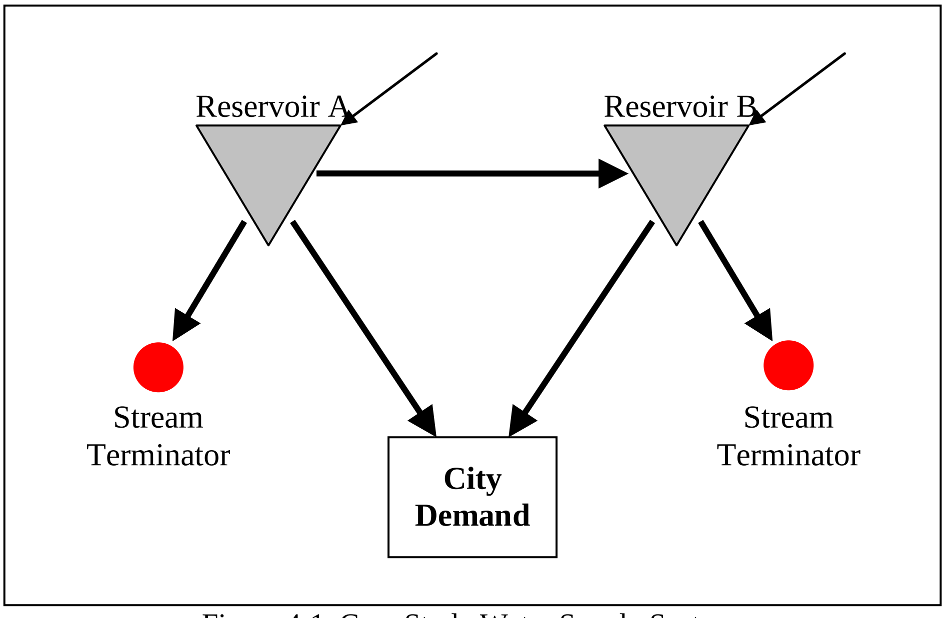


Figure 4-1. Case Study Water Supply System.

Reservoirs A and B receive a combined mean annual streamflow of 104,000 Ml from their own catchments and supply water to a City. Reservoir A, which has a capacity of 100,000 Ml, can transfer water to the 60,000 Ml capacity Reservoir B. Both reservoirs have a minimum capacity of zero Ml. The reservoirs are also subject to rainfall gains and evaporation losses.

Monthly demand disaggregation factors, which reflect typical high demands during summer months and low demands during winter months, are used to disaggregate annual demand (which is used in yield estimation) into monthly demands. These monthly demands are further adjusted by a climate index variable to account for the effects of climate variability over the simulation period. The streamflow data at the reservoirs, climate data for modelling reservoir losses and gains (i.e. rainfall and evaporation data), and climatic index variable data for disaggregating annual demand data into monthly data were available for a period of 28 years.

The current method of estimating yield involves using an entire available historic data sequence. In effect, the dates of the historic sequence have no consequence on the estimation of yield. They are only used as a requirement of REALM and for planning purposes where it provides an identification of the sequence used. For this study, the historic time-series data (streamflow, evaporation and rainfall) were used, but the simulation period was considered from January 1996 to December 2023 as obtained from, and to be consistent with, the REALM Getting Started Example.

4.2.1 Model Input Variables Used in this Study

Following is a brief discussion regarding the input variables that were considered in the study, with the nominal values presented. These nominal values are considered as the base case of the study, as given in VU and DSE (2005), and are used as a basis for the variable perturbation ranges required in the following SA.

4.2.1.1 Streamflow Data

Twenty eight years of unregulated monthly historical streamflow data is available for this case study and is shown in Figure 4-2. Reservoir A has a mean annual streamflow of approximately 68,000 Ml with a minimum monthly flow of 42,550 Ml and maximum monthly flow of 96,000 Ml. Similarly Reservoir B receives approximately 36,000 Ml average annual streamflow with a minimum monthly flow of 5,900 Ml and maximum of 79,280 Ml.

Additionally shown in Figure 4-2 is the monthly combined streamflow of Reservoir A and Reservoir B, the combined average (~8650 Ml) and the 12 month rolling average. The 12 month rolling average curve shows the average combined streamflow of the 12 months prior to the marked point in Figure 4-2. Most noticeable from the rolling average is the low streamflows that occur in 2006. In the period 1996 to 2005, the rolling average is mostly above the combined streamflow average, while the rolling average for the period 2006 to 2023 show several low 12 monthly combined streamflow minima.

Table 4-1 shows the mean, standard deviation and coefficient of variance for the monthly and annual streamflow data for Reservoir A. The same statistical quantities for Reservoir B are given in Table 4-2.

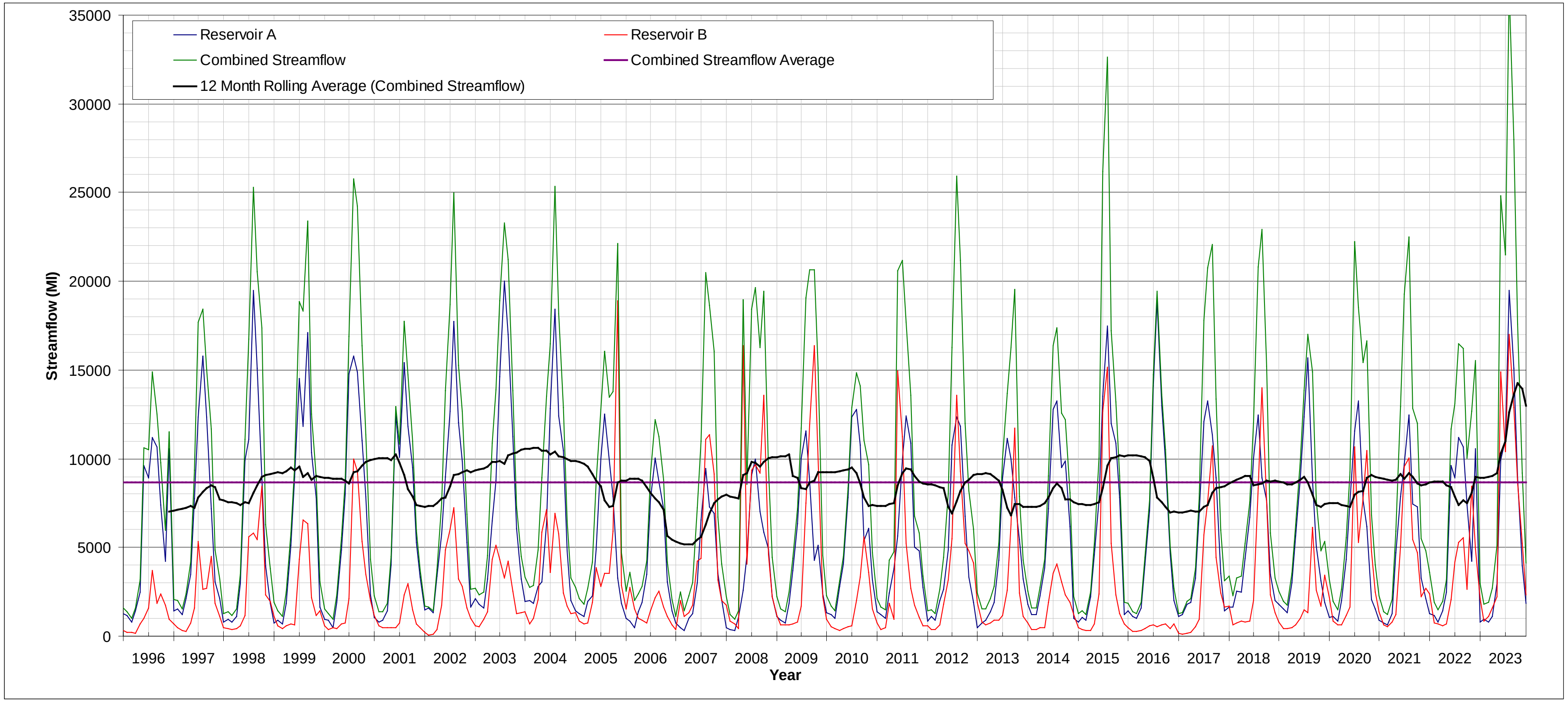


Figure 4-2. Monthly Streamflow Data for Reservoir A, Reservoir B and Combined Streamflow. Also Shown is the Trend and 12 Month Moving Average of the Total Streamflow.

Table 4-1. Statistical Properties of Streamflow into Reservoir A.

		Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Annual
Reservoir A	Mean (Ml)	1179	1125	1007	1932	3662	7323	11191	14115	11171	8033	4647	2682	68067
	Standard Deviation (Ml)	426	374	491	676	1278	2190	2150	3220	2791	1984	1527	2308	12411
	Coefficient of Variance (C_v)	0.361	0.332	0.487	0.350	0.349	0.299	0.192	0.228	0.250	0.247	0.329	0.861	0.182

Table 4-2. Statistical Properties of Streamflow into Reservoir B.

		Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Annual
Reservoir B	Mean (Ml)	1042	650	575	715	1796	3117	4736	6439	6189	5335	3429	1785	35808
	Standard Deviation (Ml)	636	545	331	493	3102	3782	3585	4230	3626	4022	3758	1085	17048
	Coefficient of Variance (C_v)	0.611	0.838	0.576	0.690	1.727	1.213	0.757	0.657	0.586	0.754	1.096	0.608	0.476

4.2.1.2 Evaporation Data

The evaporation losses and rainfall gains in both reservoirs are modelled through climatic data, and the evaporation empirical factors ‘A’ and ‘B’ (VU and DSE, 2005), as defined by Equations (4.1) and (4.2):

$$\text{Evaporation}(mm) = B \times [\text{Evaporation Data}] + A - [\text{Rainfall Data}] \tag{4.1}$$

$$\text{Net Evaporation }(Ml) = \text{Evaporation}(mm) \times \text{Surface Area}(Ha)/100 \tag{4.2}$$

where A and B are empirical factors.

The evaporation data time-series and the rainfall data time-series are historic monthly measurement records and are assumed to be valid for both reservoirs. For each reservoir, evaporation (in millimetres) is determined using Equation (4.1). The evaporation and rainfall data are adjusted by the A and B parameters, individually set for each reservoir. The final loss/gain volume (net evaporation) is determined during model simulation by multiplying the evaporation in millimetres by the reservoirs’ surface area, as shown in Equation (4.2). The volume to surface area relationships for Reservoirs A and B are defined in Table 4-3 and Table 4-4, respectively.

Table 4-3. Volume to Surface Area Relationship of Reservoir A.

Volume (Ml)	**Surface Area (Ha)**
0	0
10,000	176
50,000	700
100,000	1,000

Table 4-4. Volume to Surface Area Relationship of Reservoir B.

Volume (Ml)	**Surface Area (Ha)**
0	0
10,000	176
50,000	700
60,000	1,000

4.2.1.3 Demand

In this study the yield estimate is synonymous to the Average Annual Demand (AAD) when the system is performing at a level of service threshold (see Section 2.4 for further discussion). This means that the demand itself is not an input variable, but its value in this study – termed yield – is the output of the model. However, the two variables, temporal disaggregation factors and the climate index variable are used to modify the demand before model simulation.

4.2.1.4 Temporal Disaggregation Factors

Temporal Disaggregation Factors (TDFs) disaggregate the AAD into monthly demands. As the monthly values (nominal values used in this study and in the REALM Getting Started Example are shown in Figure 4-3) are percentages of the annual demand, the sum of the 12 individual factors is required to sum to 100%, therefore TDFs are a multi-factored variable. TDFs are generated using historic water use data to determine the typical breakdown of annual to monthly demands.

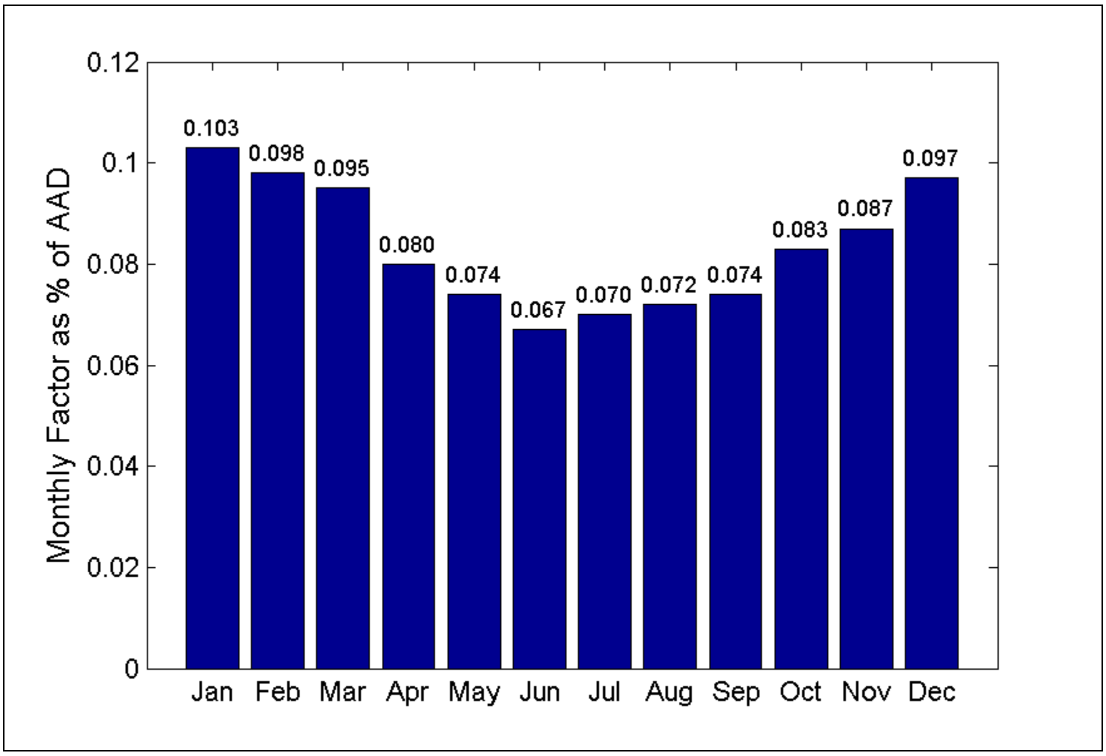


Figure 4-3. Nominal Temporal Disaggregation Factors used in this Study.

4.2.1.5 Climate Index Data

The so called "climate index" variable time series consists of 28 years of monthly data which characterises ex-house water use. The data is an empirical representation of demand behaviour that results from climatic fluctuations over the 28 year period. The climate index variable is used after the application of the TDFs to seasonally adjust the monthly demand as shown in Equation (4.3).

$$Monthly\ Demand = \frac{CI_j}{100} Y_j + B_j \tag{4.3}$$

where *Monthly Demand* is the seasonally adjusted monthly demand

CI_j is the climatic index for the j-th month

Y_j is the ex-house water use (i.e. the difference between the j-th month demand and the j-th month base demand, B_j)

B_j is the base demand for the j-th month

The climate index variable is generally inversely proportional to the streamflow and correlated to rainfall data to a certain extent. It can also depend on a combination of rainfall, temperature and time of the year (VU and DSE, 2005). The climate index variable can be determined by hindcasting procedures which produces monthly data. The climate index variable must average to 100 for the entire historic period.

4.2.1.6 Restriction Rule Curves

A five-stage demand restriction policy (shown graphically in Figure 4-4 and numerically in Tables 4-5 and 4-6) is adopted for the system to restrict the demand during low system storage volume periods. It consists of upper and lower rule curves, including four intermediate restriction zones (with definitions of relative positions and percentage restrictable levels), and a base demand curve. The upper, lower and intermediate Restriction Rule Curves (RRCs) denote the restriction stage triggered when the total system storage, expressed as percentages of AAD in Figure 4.4, drops below a certain level. The base curve denotes the unrestrictable demand, generally the in-house water demand. Restrictions are only applied to ex-house water demand, which is the difference between the monthly (unrestricted) demand and the base demand curve.

The intermediate zone curves are defined by a relative position (Table 4-6) between the upper and lower curves, measured from the upper RRC. If the total system storage volume is in an intermediate zone, the ex-house water demand is restricted by the appropriate percentage restrictable. More severe restrictions are progressively imposed as the total system storage volume continues to drop, until it falls below the lower RRC. In this zone, zone 5, 100% ex-house water demand restriction applies, i.e. only the base demand is supplied (Perera and James, 2003). The nominal trigger levels of the upper and lower curves and base demand are given in Table 4-5 and values of the relative position and percentage restrictable for the intermediate curves are given in Table 4-6.

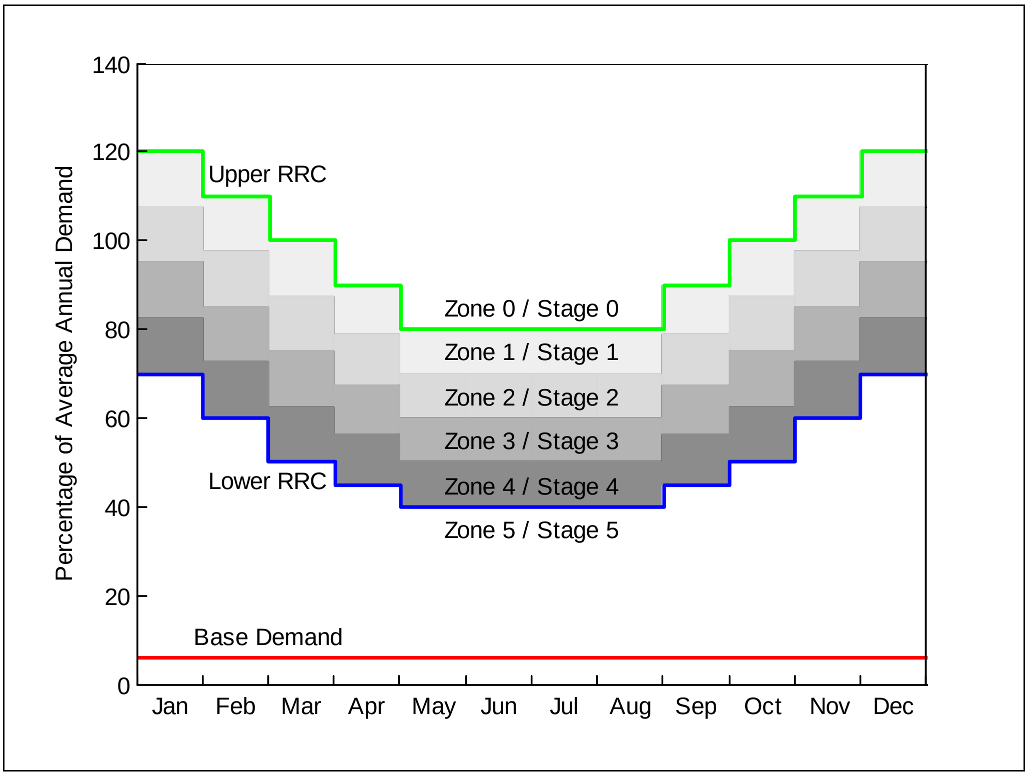


Figure 4-4. Set of 5-Stage Urban Restriction Rule Curves.

Table 4-5. Restriction Rule Curve Values.

	Base Demand (% AAD)	**Lower RRC (% AAD)**	**Upper RRC (% AAD)**
January	6	70	120
February	6	60	110
March	6	50	100
April	6	45	90
May	6	40	80
June	6	40	80
July	6	40	80
August	6	40	80
September	6	45	90
October	6	50	100
November	6	60	110
December	6	70	120

Table 4-6. Percentage Restrictable and Relative Position of the Intermediate Curves.

	Relative Position as % from Upper RRC		**Percentage Restrictable**
	Upper Bound	**Lower Bound**	
Zone 0	–	0	0
Zone 1	0	25	20
Zone 2	25	50	40
Zone 3	50	75	60
Zone 4	75	100	80
Zone 5	100	–	100

The set of RRCs (including positions and percentage restrictable) are developed through optimisation, experience and stakeholder requirements. There are obvious interactions and correlations within the RRC set and also interactions to other REALM input variables.

4.2.1.7 Target Storage Curves

The reservoir target storage curves specify the preferred storage volumes of individual reservoirs for a given total system storage. In this case study they are defined by a single set of five-point target curves for all months of the year. In practice, they are generally produced from optimisation, and are designed to force inter-reservoir transfers to ensure that demands can be supplied at various demand centres.

Given in Table 4-7 and shown graphically in Figure 4-5 are the nominal values used in this study. For a given total system storage at a given month, say 65,000 Ml, the target storage curves specify the storage volumes of Reservoirs A and B to be 40,000 Ml and 25,000 Ml respectively. Linear interpolation is used for total system storage volumes between the points provided in Table 4-7 during REALM simulation.

Table 4-7. Target Storage Curve Values for Simple Case Study.

Total System Storage (Ml)	**0**	**65,000**	**125,000**	**140,000**	**160,000**
Reservoir A Storage (Ml)	0	40,000	65,000	80,000	100,000
Reservoir B Storage (Ml)	0	25,000	60,000	60,000	60,000
Point	1	2	3	4	5

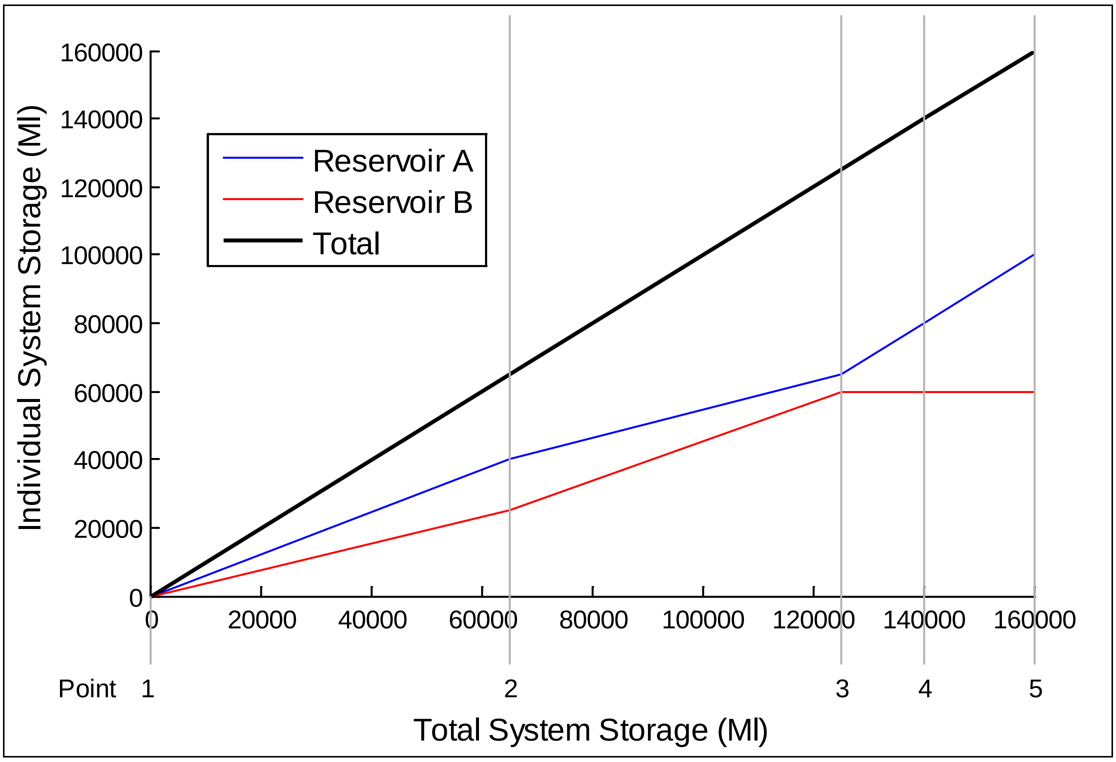


Figure 4-5. Target Storage Curves for the Hypothetical Case Study System.

4.2.1.8 Security of Supply

Security of supply thresholds ensure that the system is able to supply the demand over the planning period while meeting stakeholder requirements. In this study, three thresholds are employed:

1. Reliability of supply – The percentage of simulation time-steps in which restrictions are not imposed is nominally considered as 95%. For planning period of 28 years, 95% reliability means a maximum of 15 months in restriction.

2. Maximum allowable consecutive months in restriction – Nominally the maximum allowable consecutive months in which restrictions are imposed is set to 12.

3. Worst severity restriction stage – The worst severity restriction stage is nominally considered to be stage 3.

The system is considered to have failed if one or more of these thresholds are violated.

4.2.1.9 Initial Storage Volumes

Initial storage volumes set the storage of each reservoir at the beginning of the simulation period. Much discussion surrounds the values of the initial storage volumes. Theoretically they can be set anywhere between 0% and 100% of capacity. If the initial volumes are below the upper RRC then restrictions are immediately imposed in the simulation (i.e. at the start of the model simulation) and if low enough the maximum severity threshold violated and the system already failed.

In practice, the initial storage volume(s) depend of the purpose of the study and can be set to the current storage volume, to an ‘online’ volume for augmentation studies (level which is reached before an additional storage begins to supply water) or to an arbitrary percentage such as 80%.

4.3 Sensitivity Analysis Framework

The aim of this preliminary case study on a hypothetical urban water supply system is to evaluate the applicability of the selected sensitivity analysis techniques on an urban water supply allocation model, specifically used to determine the sensitivity of the yield estimate to its input variables. Intended as a proof-of-concept study – an exploratory process – the accuracy of the sensitivity framework, the variable attributes (ranges, distributions) and the variable handling strategies used in this study were not designed nor expected to be perfect. Instead, the limitations, conclusions and recommendations from this proof-of-concept study will be considered for use in the estimation of yield of the Barwon urban water supply system (Chapter 5).

The definition of yield used in this study is: *the maximum average annual volume of water that can be supplied from the water supply system subject to streamflow variability, operating rules, demand pattern and adopted level of service (or security criteria), which are defined by supply reliability, worst restriction level and consecutive number of months of restrictions* (VU and DSE, 2005). The estimation of yield is determined using the process described in Section 2.5.

As discussed in Section 3.3, Sensitivity Analysis (SA) is: “the study of how the variation in the output of a model (numerical or otherwise) can be apportioned, qualitatively or quantitatively, to different sources of variation (e.g. input variables, model parameters, structure etc.), and how the given model depends upon the information fed into it” (Saltelli, 2000). SA assesses the effect of input variation on the model output, indicating the importance of each input variable to the processes of the model. The greater the output change that is incurred by a unit change in an input variable, the more sensitive the model is to changes in that input variable. The sensitivity of the model to an input variable illustrates the care that modellers must take to obtain and employ an appropriate value for the variable, but also signifies its importance in relation to its dependency by the model structure (Saltelli et al., 1999).

The basis of SA is to perturb the input variables, within a predetermined range, and observe the changes they have on the model output. The pattern of perturbations depends on the selected SA technique. For this study and the study on the Barwon urban water supply system (Chapter 5), three SA techniques were selected (Section 3.5). The Morris method was used primarily as a screening method to identify input variables that the yield estimate has a negligible sensitivity to and sot they can be eliminated from further, more detailed SA. The more accurate but computationally expensive Fourier Amplitude Sensitivity Test (FAST) and method of Sobol' were then used on the remaining input variables.

The methodology applied in this case study is a basic application of sensitivity analysis based around uncertainty and errors in the values of the input variables. All input variables are considered to have data error resulting from instrument errors, reading and handling errors, etc. The error range for all the input variables are assigned using common error margins considered standard within the water resources industry or where limitations due to variable characteristics exists. The distributions of the input variables are considered to be uniform so that the SA explores the range evenly, and should preferably be continuous for accurate SA. These were the general handling guidelines used in this preliminary study with further discussion on the handling of individual variables presented in the following paragraphs. Other sources of uncertainty, such as physical system characteristics (e.g. reservoir and carrier capacities) and model operation uncertainty (e.g. REALM's internal operations, such as the hierarchy of optimisation) were considered fixed (or no uncertainty) in this study.

In SA, the input factors or variables need to be sampled over a reasonable range of absolute values, or a percentage change from their nominal values. This sampling range is up to the analysts' discretion; generally it is considered to be a feasible range in which a variable can exist in the physical system. It can, however, exceed the feasible range in some cases; such as when the feasible range is determined by stakeholders' choice, or when observing a model's behaviour at extreme parts of the variable space.

The random samples that are generated in accordance to the selected SA technique represent the perturbation of each input variable as a scalar value. Variables must therefore have the ability to be perturbed by a single, scalar sample. Where the model contains only scalar variables perturbing the parameters is simple. For cases where the input variables are time-series or have multiple factors within the variable (e.g. the 12 monthly values constituting the TDFs in Section 4.2.1.4) special handling strategies are required. This is because the random scalar samples cannot simply perturb variables that have more than one factor associated with them.

The types of data relevant to this case study are as follows:

1. Time series data variables – The time series data variables used in this case study are: 28 years of monthly streamflow, rainfall, evaporation and climate index data. These data variables are based on historic measurements and therefore subject to data collection and handling errors: the industry standard is to assume a ±5% error margin on individual datum. A special handling technique is required to perturb this type of variables.

2. Percentage scalar variables – This group consists of scalar variables that nominally assume a percentage value. A single percentage randomly selected from the variable's range is used to perturb the nominal value of the variable. The variables contained in this group are: initial storage volumes as percentage of capacity, reliability of supply threshold, upper RRC and lower RRC, base demand, and relative position and percentage restrictable demand for various intermediate restriction stages.

3. Absolute scalar variables – Consisting of the consecutive number of restriction months threshold, worst restriction stage threshold, and evaporation modelling empirical factors *A* and *B*, the variables within this group are characterised by an absolute scalar value. The range for these input variables is defined by absolute values from which a randomly value is selected.

4. Multi-factored variables – This group contains variables that have multiple factors attributed to them, such as the TDFs which have 12 monthly values, but should be considered as a single input variable. The individual factors within these variables could be tested individually in the sensitivity analysis but from a system management position they should be considered as a single variable. Also, some multi-factored variables have intricate relationships within them that should remain intact when performing SA. Temporal disaggregation factors, volume to surface area relationship of reservoirs and the target storage curves are the multi-factored variables in this study.

There are several variables that are required to sum or average to a certain value, therefore require a special handling strategy. They are the TDFs, climate index time series and target storage curves. The TDFs and climate index variables are handled using the algorithm presented in Section 4.3.1.3, which perturbs individual factors so that their sum maintains the required property, while the handling technique used for the target storage curves is discussed in Section 4.3.1.6.

A list of variables and their assigned ranges used in this case study is shown in Table 4-8. Also presented in Table 4-8 are reference numbers of the variables (variable numbers) and groups they are assigned to. Their grouping is used in the grouping SA experiments in Sections 4.4 and 4.5. The fourth column provides the sampling ranges of each variable and the fifth column displays the variable type as listed above.

4.3.1 Input Variable Handling

The following is a discussion on the handling strategies for each of the identified input variables arranged in the groupings given in Table 4-8. In the SA, the variables are required to be perturbed in accordance to the SA techniques' requirements. A range must therefore be defined for each input variable from which a sample that represents the perturbation can be randomly selected.

4.3.1.1 Streamflow

The streamflow data is based on historic measurements and therefore subject to data collection and handling errors. Therefore, a sampling range of -5% to +5% for streamflow is chosen to reflect the accepted water resources industry error margin. Each streamflow data point is changed uniformly in this study by the same randomly selected number from the above range.

4.3.1.2 Evaporation

The rainfall and evaporation time series consist of 28 years of monthly historic data. They are subject to similar data collection and handling errors as the streamflow variable, therefore a range of -5% to +5% of the recorded data is suitably chosen. A single percentage randomly selected from this range is used to change all the data points in the time series using a uniform change method as in Section 4.3.1.1.

The two evaporation empirical factors associated with the modelling of evaporation of each reservoir are sampled individually. Factor *A* has a nominal value of 0 and is sampled between -5 and +5. Factor *A* is simply added to the evaporation height in Equation (4.1).

Table 4-8. Description of Input Variables Used in this Study.

Group Name	Variable Number	Variable	Range	Remarks
Streamflow	1	Streamflow	-5% - +5% of historic data	Time-series variable
Evaporation	2	Rainfall	-5% - +5% of historic data	Time-series variable
	3	Evaporation	-5% - +5% of historic data	Time-series variable
	4	Evaporation Factor A for Reservoir A	0 – 5	Absolute scalar variable
	5	Evaporation Factor A for Reservoir B	0 – 5	Absolute scalar variable
	6	Evaporation Factor B for Reservoir A	0.95 – 1.05	Absolute scalar variable
	7	Evaporation Factor B for Reservoir B	0.95 – 1.05	Absolute scalar variable
	8	Volume to Surface Area Relationship	-5% - +5% of nominal volumes	Multi-factored variable
Demand Pattern	9	Temporal Disaggregation Factors	-5% - +5% of nominal position	Multi-factored variable
	10	Climate Index	-5% - +5% of nominal data	Time-series variable
Restriction Rule Curves	11	Upper Restriction Rule Curve Position	-5% - +5% of nominal position	Percentage scalar variable
	12	Lower Restriction Rule Curve Position	-5% - +5% of nominal position	Percentage scalar variable
	13	Base Demand Position	-5% - +5% of nominal position	Percentage scalar variable
	14	Stage 1 Percentage Restrictable	-5% - +5% of nominal position	Percentage scalar variable
	15	Stage 2 Percentage Restrictable	-5% - +5% of nominal position	Percentage scalar variable
	16	Stage 3 Percentage Restrictable	-5% - +5% of nominal position	Percentage scalar variable
	17	Stage 4 Percentage Restrictable	-5% - +5% of nominal position	Percentage scalar variable
	18	Stage 1 Relative Position	-5% - +5% of nominal position	Percentage scalar variable
	19	Stage 2 Relative Position	-5% - +5% of nominal position	Percentage scalar variable
	20	Stage 3 Relative Position	-5% - +5% of nominal position	Percentage scalar variable
Security of Supply	21	Consecutive Month in Restriction	6 – 18 months	Absolute scalar variable
	22	Worst Severity Restriction Stage	3 – 4	Absolute scalar variable
	23	Supply Reliability	80% – 98%	Percentage scalar variable
Target Storage Curves	24	Target Storage Curves – Point 2	-5% - +5% of nominal position	Percentage scalar variable
	25	Target Storage Curves – Point 3	-5% - +5% of nominal position	Percentage scalar variable
	26	Target Storage Curves – Point 4	-5% - +5% of nominal position	Percentage scalar variable
Initial Storage Volumes	27	Initial Volume of Reservoir A	25-100% of capacity	Percentage scalar variable
	28	Initial Volume of Reservoir B	25-100% of capacity	Percentage scalar variable

This means that the -5 to +5 range causes up to a ±5 mm change in the evaporation. Factor *B* is sampled from 0.95 to 1.05, with its nominal value of 1. The range for Factor *B* was selected as it results in approximately a ±5% change to the evaporation via Equation (4.1).

The volume to surface area relationship, given in Tables 4-3 and 4-4, is assigned a continuous distributed sampling range of -5% to +5%. This range was considered as a representation of the measurement errors of the reservoir profile which will subsequently error the evaporation modelling for a given storage volume. In this study the random sample changes all surface area values simultaneously, leaving the storage volume unchanged. i.e. a +3% random sample changes all intermediate surface areas given in Tables 4-3 and 4-4 by +3% simultaneously.

4.3.1.3 Demand Pattern

Monthly demand pattern is affected by two variables; the TDFs and climate index variables. Both of these variables are multi-factor variables and require special handling strategies to ensure that they maintain the requirement that they sum to, or their average equals, a required value. The TDFs, shown in Figure 4-3, require the 12 monthly values to sum to 100% and climate index time series must average 100 over the simulation period of 28 years.

Both variables are handled using the same algorithm which changes each factor (approximately) by the randomly selected percentage change. The algorithm ensures that all individual factors within a multi-factored variable are perturbed, requiring only one randomly selected sample. The algorithm is given on the next page.

Figure 4-6 provides an example of the following algorithm applied to the nominal TDFs (given in Figure 4-4) using a random number: ρ = +0.035. The rows in Figure 4-6 show notable values determined in each step in the above algorithm. As it can be seen, the initial calculation of X_i^* results in a total sum error of 0.002, therefore the 12 monthly are scaled back to sum to unity. The algorithm perturbs each monthly factor by either -3.3%, or +3.7%; i.e. approximately the expected 3.5%. This is a satisfactory result for perturbation algorithm.

The sampling range of -5% to +5% was considered for perturbing the TDFs as it produces adequate perturbation of the individual factors while still ensuring a realistic correlation between monthly figures. Using the above algorithm, only one random sample is now required to perturb the TDFs.

The perturbations of climate index variable are sampled over a range of -5% to +5%. This was considered a reasonable range for similar reasons to the streamflow variable. One random sample is used to perturb the climate index variable.

Let multi-factor variables: $X_1, \ldots, X_k > 0$ be the set of inputs such that: $\sum_{i=1}^{k} X_i = 1$.

Let $\varepsilon_1, \ldots, \varepsilon_k$ be a random permutation of +1's and -1's (normally as near as possible to $\frac{k}{2}$ for each +1's and -1's), so as not to introduce bias.

$$Y_i = \varepsilon_i X_i, \qquad i = 1, \ldots, k$$
$$\bar{Y} = \tfrac{1}{k} \sum_{i=1}^{k} Y_i$$

Let ρ be a random variable uniformly distributed between, say $-a$ and $+a$ (e.g.± 0.05).

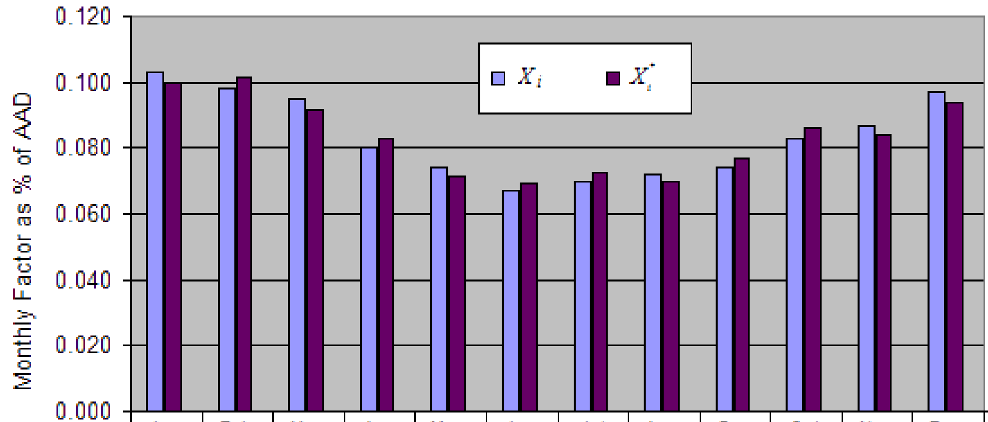
$$Z_i = \rho(Y_i - \bar{Y})$$
$$\text{Then } \sum_{i=1}^{k} Z_i = \rho\left(\sum_{i=1}^{k} Y_i - k\bar{Y}\right) = 0$$

Let the modified value of X_i be $X_i^* = X_i + Z_i$, ; $i = 1, \ldots, k$.

Check that all X_i^* are positive for $\rho = \pm a$. Otherwise, the range $-a$ to $+a$ of ρ must be suitably reduced.

Finally check that $\sum X_i^* = \sum X_i + \sum Z_i = 1$.

If $\sum X_i^* \neq 1$, standardise X_i^* to sum to unity.



	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	SUM
■ X_i	0.103	0.098	0.095	0.080	0.074	0.067	0.070	0.072	0.074	0.083	0.087	0.097	1
ε_i	-1	1	-1	1	-1	1	1	-1	1	1	-1	-1	0
Y_i	-0.103	0.098	-0.095	0.080	-0.074	0.067	0.070	-0.072	0.074	0.083	-0.087	-0.097	0
Z_i	-0.004	0.003	-0.003	0.003	-0.003	0.002	0.002	-0.003	0.003	0.003	-0.003	-0.003	-0.002
X_i^*	0.099	0.101	0.092	0.083	0.071	0.069	0.072	0.069	0.077	0.086	0.084	0.094	0.998
■ X_i^*	0.100	0.102	0.092	0.083	0.072	0.069	0.073	0.070	0.077	0.086	0.084	0.094	1
% change	-0.033	0.037	-0.033	0.037	-0.033	0.037	0.037	-0.033	0.037	0.037	-0.033	-0.033	

■ X_i^* - is standardised to unity

Figure 4-6. Example of Perturbation Algorithm on Temporal Disaggregation Factors.

4.3.1.4 Restriction Rule Curves

Referring to Figure 4-4, and Tables 4-5 and 4-6, three sub groups of variables in the RRCs are identifiable: primary curves (i.e. upper and lower RRCs, and base demand), intermediate curves and percentage restrictable. All RRC variables are assigned a ±5% error margin around the nominal value, therefore all are assigned a continuous distribution ranging from -5% to +5%. Each sub group is handled as follows:

<u>Primary Curves</u>

Primary curves define the positions of the upper and lower RRCs and the base demand. A single random percentage value is used to perturb the twelve monthly values of the upper RRC. For example, if a +4.5% is random selected, each monthly value of the upper RRC is changed by +4.5%. Similarly, another random percentage is used to perturb the lower RRC and yet another to change the base demand. Therefore, three random percentage values are required, one for each of the primary curves.

<u>Percentage Restrictable</u>

There are four percentage restrictable variables, one for each of the four restriction zones. In this study, each percentage restrictable value is perturbed individually, requiring four random numbers. For example, a single percentage value (that is randomly selected in accordance to the SA techniques requirement) is used to change the stage 1 percentage restrictable value. Another randomly selected percentage value is used to perturb the stage 2 percentage restrictable value, and so on.

Intermediate Curves

The intermediate curves define the trigger levels between restriction zones 1, 2, 3 and 4. The relative positions of the three intermediate curves (between zones 1-2, 2-3 and 3-4) was perturbed by changing the nominal scalar values. Each relative position variable is perturbed individually, therefore requiring three randomly selected numbers (one for each relative position). This effectively changes their relative position between the upper and lower RRCs either positively or negatively.

4.3.1.5 Security of Supply

A continuous distribution range of 80% to 98% was assigned to the reliability of supply threshold variable. The lower limit was considered to be representative of what the water users could consider a reasonable minimum reliability of supply. An upper limit was initially considered to be 100%, however no restriction periods leads to very low yield estimates, therefore was set to 98%.

The maximum number of consecutive months in restrictions threshold was assigned a range of 6 to 18 months. Restrictions are implemented on a monthly basis; therefore the distribution of the sampling range for this variable must be discrete. It was discussed in Chapter 3 that discrete distributions should be avoided as some SA techniques are not able to handle them correctly. For instance, the Morris method observes the difference between two consecutive sample points to determine the elementary effect of such a change when are then standardised between all variables. If these consecutive sample points do not have a relative association between points (as is possible with discrete variables) the elementary effect produces a sensitivity measure that is not representative to the perturbation. Discrete distributions also create problems in the estimation of the integral used in the variance based methods. However, in this case a discrete distribution is suitable and was adopted as there is a direct and relative relationship between each of the discrete points (i.e. the discrete points are representative of the 6 to 18 month range, which are ordered and evenly spaced), and the consecutive months variable cannot be reduced to fractions of months.

A discretely distributed range of stage 2 to stage 4 was assigned to the worst severity restriction stage allowable threshold variable. Again, a discrete distribution is suitable due to the direct representation of the variable, and that the stages are themselves integer values. Stage 1 was excluded from this range as it is generally accepted by stakeholders of the water system to be too severe a threshold as it does not allow stage 2 restrictions to be imposed on the ex-house demand.

4.3.1.6 Target Storage Curves

The target storage curves (see Figure 4-5 and Table 4-7 for nominal values) could be considered as a multi-factored variable (i.e. should be considered as a single variable in the SA). However, they are not handled as such in the SA of this case study as an appropriate handling strategy could not be established. Target storage curves possess a requirement that individual storage volumes must sum to a given total system storage, therefore this must be considered in developing the handling strategy. Also, the storage volume of a reservoir should not decrease as the total system storage increases.

When setting the values of the reservoirs the lower limits and upper limits of both must be considered so that they sum to the nominal total system storage for each point. Table 4-9 shows, in order of descending rows, the nominal total system storage (as given by VU and DSE, 2005), the nominal individual storage volumes, the lower bound of Reservoir B (with the associated Reservoir A volume given in parenthesis), the upper bound of Reservoir B and lastly the curve reference points. At point 1 both storages are at zero, and at point 5 they are both at capacity. The intermediate points 2, 3 and 4 are set so their sum equals the total system storage for that point. Reservoir B is considered as the controlling reservoir as it is the smallest, hence used for setting the lower and upper limits. Reservoir A then makes up the remaining total system storage volume, with the associate volume given in parenthesis.

Table 4-9. Reservoir B Sampling Limits for Nominal Total Storage Volumes. The Required Reservoir A Volumes are Given in Parenthesis.

Nominal Total System Storage (Ml)	**0**	**65,000**	**125,000**	**140,000**	**160,000**
Reservoir B Nominal Values (Ml) (Reservoir A Nominal Values, Ml)	0 (0)	25,000 (40,000)	60,000 (65,000)	60,000 (80,000)	60,000 (100,000)
Reservoir B Lower Limit (Ml) (Reservoir A Storage, Ml)	–	0 (65,000)	25,000 (100,000)	40,000 (100,000)	–
Reservoir B Upper Limit (Ml) (Reservoir A Storage, Ml)	–	60,000 (5,000)	60,000 (65,000)	60,000 (80,000)	–
Point	1	2	3	4	5

To handle the target storage curves, several methods were considered:

1. Change points 2, 3 and 4 for Reservoir B independently by sampling three random numbers. The range of sampling of point 2 is defined by the lower and upper limits in of column 3 in Table 4-9. The range for point 3 is given by the lower and upper limits shown in column 4 in Table 4-9, and so on. Once the storage of Reservoir B is set, Reservoir A takes up the difference to the total system storage with the corresponding volumes shown in parenthesis.

2. Change the total system storage volumes for points 2, 3 and 4 either independently (using three random numbers) or simultaneously (using one random number and a predefined direction pattern). Changing the total system storage also requires the individual storages to be adjusted. The individual storages are altered so that their percentage of total storage remains the same, effectively changing the individual storages by the same percentage as the total system storage.

3. Generate multiple sets of target curves before the SA experiments and assigned a discrete number to each. The curves were generated so that the minimum and maximum storage limitations were observed and they summed to the correct total system storage.

The first method listed above was used in the SA for this case study as it provides perturbations without using discrete distributions. The second method was disregarded as it required an extra, superfluous change to the individual volumes as well as the total system storage. The third method was not considered as it requires a discrete distribution which would not have a relationship between the sampling points, hence causing issues with the SA techniques.

4.3.1.7 Initial Volume of Reservoirs

The storage capacity of Reservoirs A and B are 100,000 Ml and 60,000 Ml, respectively. The initial volume of the reservoirs can theoretically be sampled from 0% to 100% storage capacity.

Figure 4-7 shows the yield estimate considering the initial storage volume as a variable between 0% to 100% storage capacity for both reservoirs, keeping the other input variables at their nominal values as given in Section 4.2 and VU and DSE (2005). When the initial volumes are low the model produced small yield estimates. When both of the initial storage volumes were increased to sum to approximately 35% to 40% of the total storage capacity, the yield increases to ~60,000 Ml (indicated by the yellow dashed line in Figure 4-7), and the rate of change of the yield estimate decreases as can be seen from the wider contours.

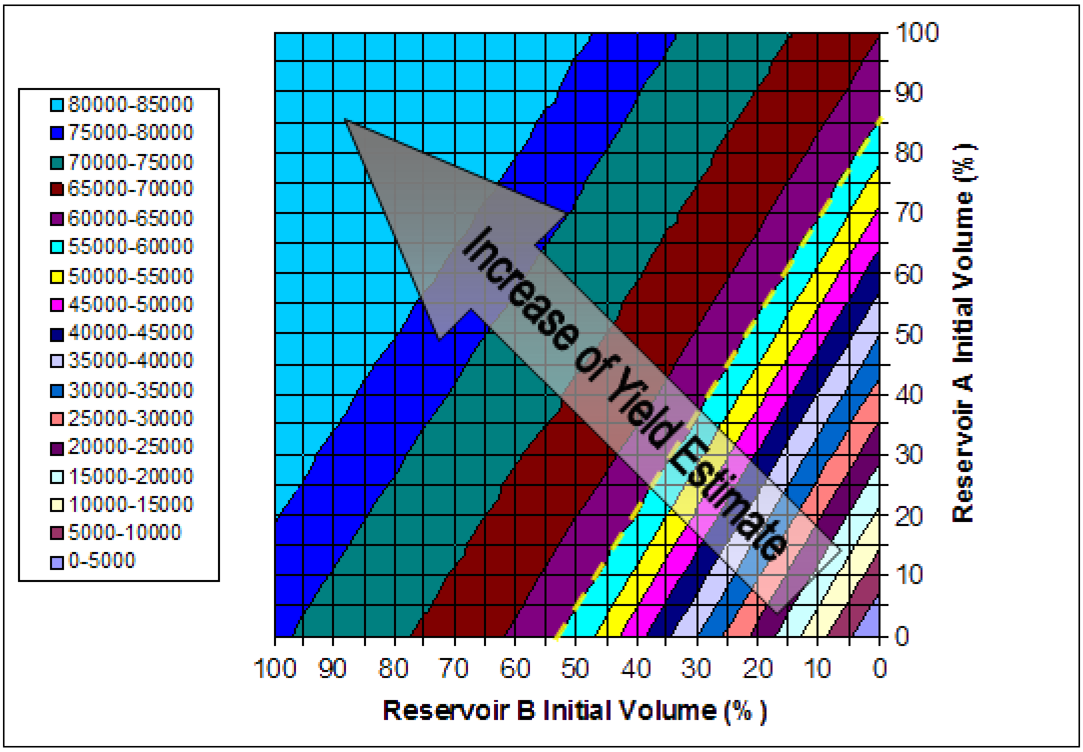


Figure 4-7. Yield Estimate Over 0-100% Initial Storage Volumes.

At the yield estimate of approximately 60,000 Ml (highlighted by a yellow dashed line in Figure 4-7), a change of the critical security of supply threshold (that which is violated) occurs. For yield estimates lower than 60,000 Ml the worst severity restriction stage (set at stage three) becomes the critical threshold. This is due to the reservoirs filling from a small initial volume, causing the maximum severity threshold to be violated at early part of the simulation period, giving low yield volume. When the yield estimate is greater than 60,000 Ml, the supply reliability (set at 95%) becomes the critical threshold. The streamflow (Figure 4-2) at the beginning of the historic sequence provides high volumes of inflow in the reservoir. As a result, storage filling is quite rapid. Within only a few years the reservoirs come close to or do spill, depending on the initial storage volumes. After this point, the reservoirs never drawdown further than the stage one restriction level, or never long enough to violate the consecutive number of months of restrictions (nominal value is 12 months).

Considering the unrealistically low yield that is produced from a low initial storage volume, the sampling range of the initial storage volume parameter for Reservoirs A and B were considered as uniformly distributed from 25% to 100% storage capacity in initial investigations. However, since the definition of yield is not dependent on the initial storage volumes and considering that yield is a planning attribute which depends on system characteristics, a methodology was developed to determine yield estimate without the influence of initial storage volume.

An iterative procedure is adopted in which the simulation is repeated several times with end storage becoming the initial storage of the subsequent iteration until the end storages converge. Studies conducted on this case study system showed that within 2-3 iterations the initial storage volume of the final iteration converges to the same value and hence, the yield converges, regardless of the starting initial storage volume. This approach was used in this case study as well as the Barwon urban water supply system study (Chapter 5).

The initial storage volumes contain modelling uncertainty unlike all other input variables which were assumed to contain measurement and handling error uncertainty. The initial storage volumes are included in the SA of this case study to observe if there remains any effect on the estimation of yield while employing this procedure.

4.3.2 Design of Sensitivity Analysis Experiments

A tiered approach is the most widely used methodology for SA (e.g. US EPA, 2003; Saltelli et al., 2000, 2004); starting with simple and computationally inexpensive experiments, and progressively increasing the computational cost and the accuracy of the experiments. Often preliminary random experiments using graphical techniques, such as scatter plots, are performed as a screening experiment to gain a basic understanding of each input variable to model output behaviour. The purpose of simple random experiments as screening experiments is to identify trends, non-influential variables and possible limitations (e.g. non-influential regions of input space) of each input variable. Then more complex, comprehensive and accurate SA techniques are used on input variables that show importance in the screening experiments.

The Morris method, Fourier Amplitude Sensitivity Test (FAST) and the Sobol' method of sensitivity analysis were selected as the most appropriate SA techniques for use in estimating the sensitivity of the yield estimate of urban water supply systems. See Section 3.5 for a review, comparison and selection of these techniques. The Morris method was selected as the screening technique to provide identification and ranking of the most important variables. It also indicates the variables that have possible non-linearity or interaction effects. The results of the Morris method are then confirmed by the FAST and extended FAST (eFAST) methods.

The variance based methods – the FAST/eFAST and Sobol' methods – were selected for detailed analyses of the importance of variables used in the estimation of yield of urban water system. FAST provides first-order importance measures, with eFAST providing first- and total-order sensitivity effects. Comparing the first-order and total-order sensitivity indices gives further indication of the importance of variables and identifies variables with higher-order effects. The method of Sobol' is selected to quantify higher-order interaction effects between input variables.

SA on groups of variables (groupings of variables indicated in Table 4-8) will also be performed using the Morris method and eFAST. This is done to gain information regarding the effect of a set of linked variables, showing possible synergy in the estimation of yield amongst the variables. Sobol' does not allow selection of groups of variables, but determines effects of multiple variables by calculating second- and higher-order effect of all combinations of input variables of a particular order. For instance, a second-order analysis will quantify the effects all pair-wise combinations of input variables, a third-order will quantify the combined effects of all three variable combinations, etc.

4.4 Sensitivity Analysis: The Morris Method

All input variables listed in Table 4-8 were tested for their importance in the estimation of yield of the hypothetical urban water supply system using the Morris method. The Morris method algorithm provides three indices:

μ - The overall sensitivity effect due to all first- and higher-order effects.

μ^* - The 'true' importance measure, free of any non-monotonic input to output behaviour that could be present in μ.

σ - The possible non-linearity of an input variable or interactions of an input variable with other variables. The Morris method does not distinguish between these two effects.

The Morris algorithm requires the selection of the number of levels p, Δ as a multiple of $1/(p - 1)$, the number of trajectories to perform and a random seed.

The experiments in this study were performed using a variety of algorithm settings and random seeds as noted in Table 4-10. The level p determines the number of equally spaced sampling points in the variables' range (i.e. the sampling resolution) from which two are sampled with a Δ change between. The higher p is, the higher the number of possible points that can be sampled from the variable space. Different number of levels, p, were used so that the sampling can be over a sparse and a fine resolution. Different Δ were also used so that small and broad perturbations were produced. The number of trajectories required, r, and the number of input variables, k, determine the number of required model simulations: $r(k + 1)$. The random seed ensures that different sets of trajectories are constructed for each p and Δ setting.

The results from all experiments are combined for simplicity and shown in Table 4-11. The reason for the various settings and the combined results is so that any bias that may occur from a particular setting is avoided and that all effects from small and large Δ changes are captured.

Table 4-10. Algorithm Settings for the Morris Method Sensitivity Analysis Experiment.

Experiment	Number of Trajectories	Level	Δ	Seed
1	10	4	1	18936437
2	10	4	1	874366872
3	10	6	2	18936437
4	10	6	2	874366872
5	10	6	3	18936437
6	10	6	3	874366872
7	10	6	4	18936437
8	10	6	4	874366872
9	10	8	3	18936437
10	10	8	3	874366872
11	10	8	4	18936437
12	10	8	4	874366872
13	10	8	5	18936437
14	10	8	5	874366872
15	20	4	2	18936437
16	20	4	2	874366872

Individual results of the each experiment listed in Table 4-10 are given in Appendix B. The results in Appendix B show that the μ, μ* and σ measures for most input variables are relatively stable for the various setting used. Of particular significance is the target storage curve point 3 variable which results in zero μ, μ* and σ measures for all experiments except experiment 16. Similarly the relative position point 1, stage 2 percentage restrictable and lower RRC position variables show that in some experiments they return zero μ, μ* and σ measures, while in other experiments they have a low importance measure. The combined results given in Table 4-11 reflect a more reliable estimation of the importance of the input variables as the effects of the various Morris algorithm settings are capture.

Included in Table 4-11 is the μ, μ^* and σ measures and the μ^* rankings. μ^* is used for ranking as it is a better measure of the total sensitivity of an input variable because it considers only the magnitude of the change, whereas μ also considers the direction of the change and can therefore include cancelling out of effects. The μ, μ^* and σ results are also presented graphically on a μ-σ axis in Figure 4-8. Noticeably there are only a few input variables that show any importance to the estimation of yield. These variables are labelled in Figure 4-8, whereas the remaining variables that have negligible μ, μ^* and σ results in comparison are not labelled for clarity.

The streamflow variable is clearly the most important input variable, with the reliability, consecutive months and the upper RRC showing noteworthy effects on the yield estimate, as indicated from the position along the μ/μ^*-axis. The consecutive months threshold, reliability threshold and streamflow show large interaction or non-linearity behaviours as indicated by the large σ indices.

Interestingly, most of the input variables show a negative input to output behaviour, this is revealed by the difference between μ and μ^*. When these indices are equal but opposite (note that μ^* will always be positive) it shows that the variable have a monotonically negative input to output behaviour. When they are not equal but μ is still negative it shows that the input to output behaviour is non-monotonic but tends to be negative. The streamflow, consecutive months threshold and the rainfall variables all produce equal μ and μ^* indices. This indicates that when they have been perturbed they produce an output change in the same direction, i.e. when they are increased, the yield estimate also increases, when they are decreased, the yield decreases.

There are a number of variables that show a zero influence on the output. These include the initial storage volume variables, the target curve points 3 and 4, stage 3 and stage 4 percentage restrictable, stage 2 and stage 3 relative position, and the worst restriction stage threshold. The initial storage volume variables are zero due to the iterative handling procedure used. The zero measures for the stage 2 (and 3) relative position and stage 3 percentage restrictable suggests that the worst severity restriction stage that is triggered is stage 2 restrictions, i.e. the system never triggers a stage 3 restriction. This is also indicated by the zero results for the worst restriction stage, which itself suggests that total system storage does not drawdown enough for the worst restriction stage (either stage 3 or 4) to be the critical threshold. The zero effects of the target curves points shows that changing these points does not effect the yield estimate.

Table 4-11. Combined Results of the Morris Method Experiments.

Factor	**μ**	**μ***	**σ**	**μ* Ranking**
Streamflow	6151	6151	996	1
Rainfall	852	852	189	5
Evaporation	-629	640	234	9
Evaporation Factor A for Reservoir A	-658	713	391	8
Evaporation Factor A for Reservoir B	-728	728	192	7
Evaporation Factor B for Reservoir A	-294	295	244	12
Evaporation Factor B for Reservoir B	-315	315	211	11
Volume to Surface Area Relationship	200	205	209	14
Temporal Disaggregation Factors	-433	444	264	10
Climate Index	-737	737	270	6
Upper RRC Position	-1169	1195	692	3
Lower RRC Position	44	44	145	16
Base Demand Position	-204	210	241	13
Stage 1 Percentage Restrictable	108	129	204	15
Stage 2 Percentage Restrictable	23	31	93	18
Stage 3 Percentage Restrictable	0	0	0	20
Stage 4 Percentage Restrictable	0	0	0	20
Stage 1 Relative Position	-31	31	103	18
Stage 2 Relative Position	0	0	0	20
Stage 3 Relative Position	0	0	0	20
Consecutive Months in Restriction	1190	1190	2063	4
Worst Severity Restriction Stage	0	0	0	20
Supply Reliability	-3891	3891	1438	2
Target Storage Curves – Point 2	-31	31	102	17
Target Storage Curves – Point 3	0	0	0	20
Target Storage Curves – Point 4	0	0	0	20
Initial Volume of Reservoir A	0	0	0	20
Initial Volume of Reservoir B	0	0	0	20

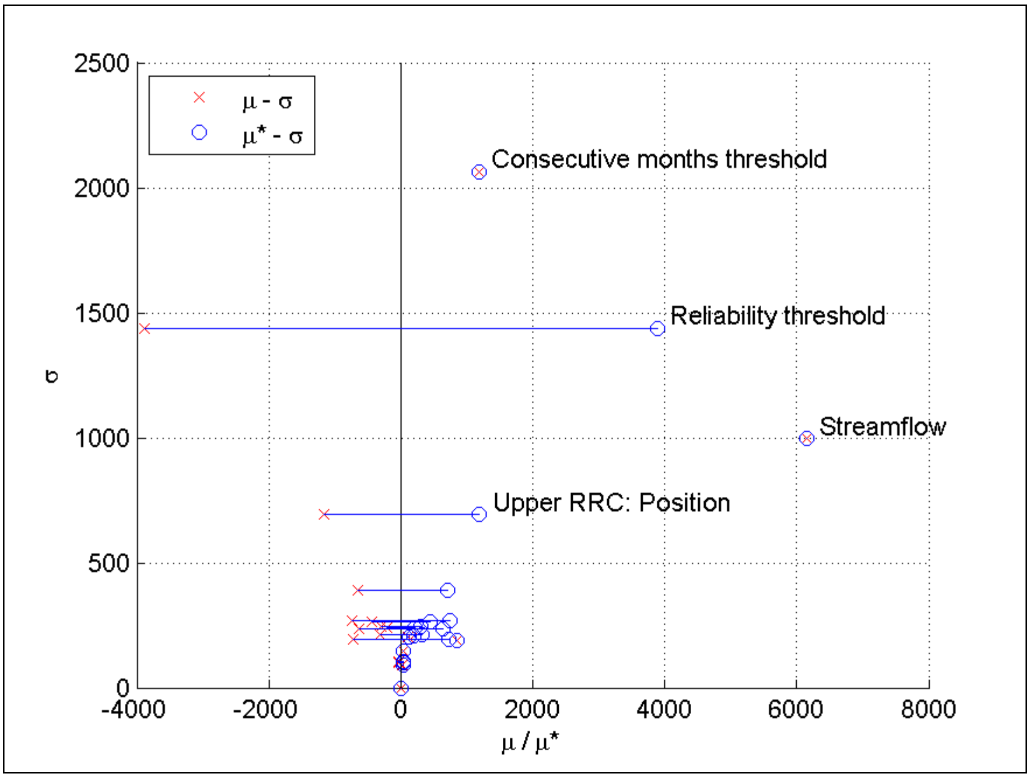


Figure 4-8. Combined Results of the Morris Method Experiments.

The following points summarise the findings from the Morris method experiments:

1. The most influential variables in the estimation of yield for the hypothetical urban water supply system are the streamflow, reliability of supply threshold, the upper RRC position and the consecutive months in restriction.

2. Interactions and/or non-linearity behaviour exists, primarily in the consecutive months, reliability of supply and streamflow variables.

3. The most severe restriction stage imposed on this system is stage 2. This effectively negates the use of several variables, namely the worst restriction stage (which had a range of 3-4), stage 3 and stage 4 percentage restrictable, and stage 2 and stage 3 relative position.

4. The target curve points 2 and 3 are not influential, either suggesting that the storages never fill past curve point 2 (65,000 Ml total storage) for any Morris method experiment, or that the yield is not sensitive to changes of target curve point 3 and point 4.

5. The Morris method has successfully been applied to test the input variables used in the estimation of yield of an urban water supply system. It has efficiently identified

variables with negligible influential to the estimation of yield and also revealed system behaviour, such as highlighted in the points above.

The Morris method identified several variables that have zero influence on the estimation of yield and should therefore be neglected from further SA experiments to decrease the number of simulations required. Nevertheless, all variables as listed in Table 4-8 will be used in preliminary experiments using the variance based FAST and eFAST methods to confirm the results of the Morris method. Once these results are confirmed, the FAST, eFAST and Sobol' methods will be used a reduced number of variables.

4.5 Sensitivity Analysis: Variance Based Methods

Two variance based methods are employed in this study: the Fourier Amplitude Sensitivity Test (FAST) and Sobol' method of sensitivity. The following four sensitivity indices are used in this section to assess the sensitivity of the estimation of yield to changes in the input variables:

S_i - The first-order sensitivity effects of the *i*-th input variable, free of any higher-order or interaction effects. S_i can be calculated via the classic FAST, eFAST and the Sobol' SA techniques. These indices should be positive and $\Sigma S_i \leq 1$.

S_{Ti} - The total-order sensitivity effects of the *i*-th input variable which includes the effects of all possible combinations that the *i*-th input variable is included in. S_{Ti} can be calculated via the eFAST and the Sobol' SA techniques. $S_{Ti} \geq S_i$ for the same variable.

S_{ij} - A measure of the first-order interaction effects of the *i*-th and *j*-th input variables, free of the effects of all other interactions and individual effects of the *i*-th and *j*-th input variables. S_{ij} can only be calculated using the Sobol' method and is determined by: $S_{ij} = S_{ij}^c - S_i - S_j$. S_{ij} should always be positive.

S_{ij}^c - The 'closed' (Saltelli, 2002a) effect of the *i*-th and *j*-th input variables. This is a measure of effects of the *i*-th and *j*-th input variables, including the individual effects (S_i and S_j), and the interaction effect of the *i*-th and *j*-th input variables. S_{ij}^c can only be calculated using the Sobol' method and should always be positive.

Note that the Sobol' technique has two commonly used algorithms, Sobol's (1993) own algorithm and a more accurate and computationally efficient algorithm developed by Saltelli

(2002b). The calculation of the sensitivity indices are done using the same equations, as presented in Section 3.5.3, but the sampling design differ in the two methods. For this study the Saltelli algorithm is used as it provides the same S_i and S_{Ti} indices as the original Sobol' algorithm but at a lower computational cost, and can also calculate higher-order sensitivity indices which the original Sobol' method cannot.

Using all variables given in Table 4-8, ten preliminary FAST/eFAST experiments were used to confirm the results of the Morris method experiments given in Section 4.4. Table 4-12 details the settings of these experiments. Both the classic FAST and the eFAST algorithms are used to confirm the Morris method results, as well as a grouped experiment using the eFAST algorithm. The accuracy of the FAST and eFAST techniques increases as the number of simulations increases, therefore increasing accuracy experiments were considered until sufficient convergence was reached. Different seeds are used to provide two experiments for each of the same resolution experiments (i.e. same or similar number of model simulations) which are then averaged. Due to the eFAST grouping algorithm the different random seeds produce slightly different number of required model simulations. This can be seen in different number of simulations between experiments 7 and 8 and between experiments 9 and 10. Only 40,000 run classic FAST experiments were performed as this was expected to be sufficiently accurate. The results of these experiments (experiments 1 and 2) are confirmed with the eFAST experiments (experiments 3 to 6). For experiments that have the resolution of sampling (i.e. the same or similar number of model simulations), the results are averaged and presented.

Table 4-12. Settings of the Preliminary FAST Experiments.

Experiment Number	**FAST Algorithm**	**Number of Variables/Groups**	**Number of Simulations**	**Random Seed**
1	Classic	28 Variables	40000	9825169
2	Classic	28 Variables	40000	3584381
3	Extended	28 Variables	9884	9825169
4	Extended	28 Variables	9884	3584381
5	Extended	28 Variables	19964	9825169
6	Extended	28 Variables	19964	3584381
7	Grouped	7 Groups	9615	8974561
8	Grouped	7 Groups	9559	3584381
9	Grouped	7 Groups	19627	8974561
10	Grouped	7 Groups	19571	3584381

Table 4-13 presents the averaged first-order sensitivity indices (S_i) for the classic FAST experiments 1 and 2. Acceptable results are gained from these experiments as the sum of S_i is not greater than one. The sum of S_i indicates the degree of additivity of the model; the closer S_i is to unity the more additive the model, where the sum of S_i is exactly 1 for a completely additive model. It is clear that the first-order measure is dominated by the streamflow, followed by the reliability threshold. The upper RRC position, consecutive months threshold and the rainfall variables are then the most important, with the remaining variables showing negligible difference in their importance. These results of experiments 1 and 2 confirm the Morris method ranking for the 12 most important input variables in the estimation of yield. Also corresponding with the Morris method are most of the zero importance variables. However, three variables, the lower RRC position, the relative position of intermediate curve 1 and target curve point 2, show a zero S_i results whereas their μ values were not.

Four experiments using eFAST (experiments 3, 4, 5 and 6) were also performed on the individual variables shown in Table 4-8 to confirm the accuracy and the results of the Morris method experiments. The averaged first-order and total-order sensitivity indices of experiments 3 and 4 are presented in Table 4-14 and the averaged results of experiments 5 and 6 are presented in Table 4-15. The rankings of the 10 most important variable in experiment 5 and 6 correspond to the ranking of the Morris method experiments, confirming the accuracy of the Morris method at screening for important variables. However, errors are present. The results of experiments 3 and 4 sum to less than one, however the averaged results of experiments 5 and 6 sum to greater than one. This is counter to the theory that an increased number of model simulations should provide a more accurate estimation of the sensitivity indices. The source of this error is unknown, it could be due to aliasing or interference between frequencies or due to the issues handling discretely distributed variables. Nevertheless, the results will be used with caution. Between the two sets of experiments, the streamflow, reliability threshold, upper RRC position, consecutive months threshold and the rainfall variables have the same S_i ranking and the sensitivity indices have same order of magnitude. The magnitude of S_{Ti} and the difference between S_i and S_{Ti} from Table 4-14 and Table 4-15 indicate that there are high-order effects in all input variables, specifically in the streamflow, reliability threshold, volume to surface area and the evaporation factor A of Reservoir B.

Table 4-13. First-Order Indices (S_i) for FAST Experiments 1 and 2 (Averaged).

Variable	S_i	**Ranking**
Streamflow	0.6286	1
Rainfall	0.0112	5
Evaporation	0.0058	9
Evaporation Factor A for Reservoir A	0.0074	8
Evaporation Factor A for Reservoir B	0.0077	7
Evaporation Factor B for Reservoir A	0.0015	12
Evaporation Factor B for Reservoir B	0.0017	11
Volume to Surface Area Relationship	0.0008	13
Temporal Disaggregation Factors	0.0035	10
Climate Index	0.0091	6
Upper RRC Position	0.0284	3
Lower RRC Position	0	17
Base Demand Position	0.0007	14
Stage 1 Percentage Restrictable	0.0003	15
Stage 2 Percentage Restrictable	0.0001	16
Stage 3 Percentage Restrictable	0	17
Stage 4 Percentage Restrictable	0	17
Stage 1 Relative Position	0	17
Stage 2 Relative Position	0	17
Stage 3 Relative Position	0	17
Consecutive Months in Restriction	0.0164	4
Worst Restriction Stage	0	17
Supply Reliability	0.2273	2
Target Storage Curves – Point 2	0	17
Target Storage Curves – Point 3	0	17
Target Storage Curves – Point 4	0	17
Initial Volume of Reservoir A	0	17
Initial Volume of Reservoir B	0	17

Some irregularities and limitations of the eFAST technique become apparent between the two sets of experiments (experiments 3 and 4 and experiments 5 and 6). The most obvious, and previously discussed, is the sum of S_i for experiments 5 and 6 is greater than one. There are also numerous variables that have a zero S_i in Table 4-15 whereas they are non-zero in Table 4-14. Another observation is that the variables with high S_i (streamflow and reliability threshold) in experiments 3 and 4 increase in experiment 5 and 6, while most other variables decreases. A significant finding from the point of the performance of the eFAST method is the non-zero S_i results for the initial storage volumes.

Table 4-14. First-Order Indices (S_i) and Total-Order (S_{Ti}) for eFAST Experiments 3 and 4 (Averaged).

Variable	S_i	**Ranking**	S_{Ti}	**Ranking**	$S_{Ti} - S_i$
Streamflow	0.6298	1	0.6504	1	0.0207
Rainfall	0.0112	5	0.0166	8	0.0055
Evaporation	0.0050	9	0.0104	16	0.0054
Evaporation Factor A for Reservoir A	0.0066	8	0.0122	10	0.0056
Evaporation Factor A for Reservoir B	0.0103	6	0.0320	5	0.0218
Evaporation Factor B for Reservoir A	0.0016	11	0.0112	14	0.0096
Evaporation Factor B for Reservoir B	0.0013	13	0.0074	23	0.0061
Volume to Surface Area Relationship	0.0015	12	0.0286	6	0.0271
Temporal Disaggregation Factors	0.0041	10	0.0119	12	0.0078
Climate Index	0.0069	7	0.0115	13	0.0046
Upper RRC Position	0.0315	3	0.0440	4	0.0126
Lower RRC Position	0.0002	20	0.0084	19	0.0082
Base Demand Position	0.0010	14	0.0080	21	0.0070
Stage 1 Percentage Restrictable	0.0004	15	0.0105	15	0.0101
Stage 2 Percentage Restrictable	0.0002	20	0.0069	27	0.0067
Stage 3 Percentage Restrictable	0.0002	20	0.0081	20	0.0079
Stage 4 Percentage Restrictable	0.0003	17	0.0090	17	0.0087
Stage 1 Relative Position	0.0003	17	0.0072	24	0.0069
Stage 2 Relative Position	0.0004	15	0.0158	9	0.0154
Stage 3 Relative Position	0.0002	20	0.0077	22	0.0075
Consecutive Months in Restriction	0.0229	4	0.0582	3	0.0354
Worst Restriction Stage	0.0001	26	0.0120	11	0.0119
Supply Reliability	0.2096	2	0.2392	2	0.0296
Target Storage Curves – Point 2	0.0002	20	0.0055	28	0.0053
Target Storage Curves – Point 3	0.0003	17	0.0089	18	0.0086
Target Storage Curves – Point 4	0.0002	20	0.0187	7	0.0185
Initial Volume of Reservoir A	0.0001	26	0.0070	26	0.0069
Initial Volume of Reservoir B	0.0001	26	0.0071	25	0.0070
SUM	**0.9461**		**1.2745**		

Table 4-15. First-Order Indices (S_i) and Total-Order (S_{Ti}) for eFAST Experiments 5 and 6 (Averaged).

Variable	$\boldsymbol{S_i}$	**Ranking**	$\boldsymbol{S_{Ti}}$	**Ranking**	$\boldsymbol{S_{Ti} - S_i}$
Streamflow	0.6841	1	0.7135	1	0.0294
Rainfall	0.0102	5	0.0210	7	0.0108
Evaporation	0.0055	9	0.0157	10	0.0102
Evaporation Factor A for Reservoir A	0.0074	8	0.0197	9	0.0123
Evaporation Factor A for Reservoir B	0.0084	7	0.0214	6	0.0130
Evaporation Factor B for Reservoir A	0.0016	11	0.0120	12	0.0104
Evaporation Factor B for Reservoir B	0.0014	12	0.0129	11	0.0115
Volume to Surface Area Relationship	0.0013	13	0.0201	8	0.0188
Temporal Disaggregation Factors	0.0038	10	0.0073	15	0.0035
Climate Index	0.0097	6	0.0217	5	0.0121
Upper RRC Position	0.0289	3	0.0357	4	0.0068
Lower RRC Position	0.0001	16	0.0035	21	0.0034
Base Demand Position	0.0011	14	0.0053	17	0.0042
Stage 1 Percentage Restrictable	0.0003	15	0.0035	21	0.0032
Stage 2 Percentage Restrictable	0.0000	20	0.0047	18	0.0047
Stage 3 Percentage Restrictable	0.0000	20	0.0042	20	0.0042
Stage 4 Percentage Restrictable	0.0000	20	0.0028	28	0.0028
Stage 1 Relative Position	0.0001	16	0.0046	19	0.0046
Stage 2 Relative Position	0.0000	20	0.0033	24	0.0033
Stage 3 Relative Position	0.0001	16	0.0069	16	0.0069
Consecutive Months in Restriction	0.0219	4	0.0555	3	0.0336
Worst Restriction Stage	0.0000	20	0.0033	24	0.0032
Supply Reliability	0.2418	2	0.2706	2	0.0289
Target Storage Curves – Point 2	0.0000	20	0.0031	27	0.0031
Target Storage Curves – Point 3	0.0000	20	0.0035	21	0.0034
Target Storage Curves – Point 4	0.0000	20	0.0032	26	0.0032
Initial Volume of Reservoir A	0.0000	20	0.0113	14	0.0112
Initial Volume of Reservoir B	0.0001	16	0.0120	12	0.0119
SUM	**1.0276**		**1.3023**		

It was shown earlier in the Morris method (Section 4.4) that the initial storage volumes have zero influence on the estimation of yield due to the iterative procedure used. However the Morris method only changes one input variable at a time, so when the initial storage volumes were changed no other variables were perturbed, meaning that the initial storages would converge to the same volume causing no effect on the estimation of yield, hence the zero μ, μ* and σ results. On the other hand, the FAST and eFAST perturb variables at the same time and although the S_i estimates the effect of only the *i*-th input variable, it is prone to interference from non-independent variables and aliasing from the Fourier transform. As briefly discussed in Section 3.5.2, aliasing error is due to variance leaking from other frequencies during the Fourier transform that is central to the FAST technique and the interference errors is due to correlations in the input variables. Both of these errors lead to an artificial increase in the attributed S_i and S_{Ti} of an input variable. On the other hand, these errors may not be present and the the initial storage volumes may indeed be important in the estimation, hence the non-zero S_i and S_{Ti}. Whatever the reason for these results, the results show the importance of these variables are small enough to not be of any concern.

The next four experiments (experiments 7, 8, 9 and 10) were performed on groups of variables as defined in the first column of Table 4-8. Experiments 7 and 8 were performed with the same number accuracy experiment but different random seeds which causes the slightly different number of model simulations. The S_i and S_{Ti} sensitivity indices are averaged and presented in Table 4-16. The averaged S_i and S_{Ti} sensitivity indices for experiments 9 and 10 are shown in Table 4-17. Again the streamflow dominates both S_i and S_{Ti} with the security criteria group second important. The evaporation group and restriction rule curves group show similar S_i and S_{Ti} results. The same groups also show a large difference between S_i and S_{Ti}, indicating that they are involved in interactions. The remaining groups have low first-order sensitivity yet show high higher-order effects.

Table 4-16. First-Order Indices (S_i) and Total-Order (S_{Ti}) for Grouped eFAST Experiments 7 and 8 (Averaged).

Variable	$\boldsymbol{S_i}$	**Ranking**	$\boldsymbol{S_{Ti}}$	**Ranking**	$\boldsymbol{S_{Ti} - S_i}$
Streamflow	0.6311	1	0.6528	1	0.0217
Initial Storage Volumes	0.0010	7	0.0297	7	0.0287
Security Criteria	0.2708	2	0.3082	2	0.0374
Restriction Rule Curves	0.0338	4	0.0689	4	0.0351
Target Curves	0.0011	6	0.0320	6	0.0309
Evaporation	0.0393	3	0.0738	3	0.0345
Demand	0.0152	5	0.0431	5	0.0279
SUM	**0.9923**		**1.2085**		

Table 4-17. First-Order Indices (S_i) and Total-Order (S_{Ti}) for Grouped eFAST Experiments 9 and 10 (Averaged).

Variable	S_i	Ranking	S_{Ti}	Ranking	$S_{Ti} - S_i$
Streamflow	0.6276	1	0.6459	1	0.0183
Initial Storage Volumes	0.0010	7	0.0294	7	0.0285
Security Criteria	0.2810	2	0.3192	2	0.0383
Restriction Rule Curves	0.0323	4	0.0666	3	0.0343
Target Curves	0.0013	6	0.0318	6	0.0305
Evaporation	0.0367	3	0.0647	4	0.0280
Demand	0.0164	5	0.0477	5	0.0313
SUM	**0.9962**		**1.2053**		

There is good agreement between the results shown in Table 4-16 and Table 4-17 with only one change of ranking existing in the S_{Ti} measures. For both sets of experiments the sum of S_i are relatively high. According to the variance based principles of the FAST method, this indicates a high additivity of the input variables, i.e. there exist only weak interactions between variables. Higher-order effects for all groups are present as indicated by the difference between S_{Ti} and S_i.

The results of the experiments 7 and 8 (not provided here) demonstrate similar S_i and S_{Ti} results with only some minor discrepancies. Similarly, the experiments 9 and 10 results have similar S_i and S_{Ti} results. The results of experiments 9 and 10 are generally closer than the S_i and S_{Ti} results of experiments 7 and 8, suggesting that convergence of S_i and S_{Ti} results occurs with greater number of model simulations used.

Following the common procedure for SA, the variables with negligible importance on the estimation of yield are set at their nominal values and further experiments performed on the important variables. The 10 variables with the highest S_i and S_{Ti} rankings were used in the following SA experiments. These 10 most important variables are the same as found in the Morris method experiments. These variables are given in Table 4-18. The remaining variables are set at there nominal values as given in various sections of Section 4.2.1.

Table 4-18. Top 10 Important Variables used in Detailed SA Experiments.

Variable	Variable
Streamflow	Climate Index
Rainfall	Evaporation
Evaporation Factor A for Reservoir A	Evaporation Factor A for Reservoir B
Consecutive Months Threshold	Reliability Threshold
Upper RRC Position	Temporal Disaggregation Factors

The purpose for these experiments is to gain more detailed sensitivity measures for the most important variables, without the effects of the less important variables. For these experiments, the FAST, eFAST and Sobol' methods were used, where the Sobol' method was used to determine higher-order sensitivity indices. Details of the experiments are given in Table 4-19; note that the method of Sobol' uses a quasi-random sample generation, therefore does not require a random seed. The Sobol' method was not employed in the previous preliminary experiments as the high number of simulations required would not be practical and the preliminary experiments were used mainly to confirm the results of the Morris method.

Table 4-19. Settings for the 10 Variable SA Experiments.

Experiment Number	**SA Technique**	**Algorithm**	**Number of Simulations**	**Random Seed**
11	FAST	Classic	5000	75132541
12	FAST	Classic	10000	75132541
13	FAST	Extended	4970	75132541
14	FAST	Extended	9930	75132541
15	FAST	Extended	14970	75132541
16	FAST	Extended	19930	75132541
17	Sobol'	Up to 2nd Order	58368	NA*
18	Sobol'	Up to 2nd Order	116736	NA*

*NA: that Sobol' method is a pseudo-random design that does not require a random seed

The results of these experiments are shown below in following five tables. Table 4-20 shows the S_i results of the classic FAST experiments 11 and 12. These results show excellent parity, showing the accuracy of the classic FAST algorithm at a relatively low number of simulations. The first eight importance rankings for the classic FAST experiments shown in Table 20 and Table 4-13 are the same. The first five ranks also matches with the S_i rankings of the combined eFAST experiments shown in Table 4-15.

Figure 4-9 shows the S_i indices of eFAST experiments 13, 14, 15 and 16 which are performed over 5000, 10000, 15000 and 20000 model simulations, respectively. Similarity exists over these experiments, with only the experiment 13 (5000 model simulations) providing unsatisfactory results due to the sum of S_i greater than one. As the results seem to be stable, only experiment 16 will be considered from this set of experiments for the following analysis of the importance of input variables used in the estimation of yield of the hypothetical urban water supply system.

Table 4-20. First-Order Indices (S_i) for FAST Experiments 11 and 12.

Variable	**S_i (Exp 11)**	**Ranking**	**S_i (Exp 12)**	**Ranking**
Streamflow	0.6243	1	0.6243	1
Climate Index	0.0096	6	0.0097	6
Rainfall	0.0124	5	0.0123	5
Evaporation	0.0016	10	0.0016	10
Evaporation Factor A For Reservoir A	0.0078	8	0.0076	8
Evaporation Factor A For Reservoir B	0.0080	7	0.0080	7
Consecutive Months Threshold	0.0202	4	0.0213	4
Reliability Threshold	0.2360	2	0.2397	2
Upper RRC Position	0.0307	3	0.0309	3
Temporal Disaggregation Factors	0.0040	9	0.0042	9
SUM	**0.9546**		**0.9596**	

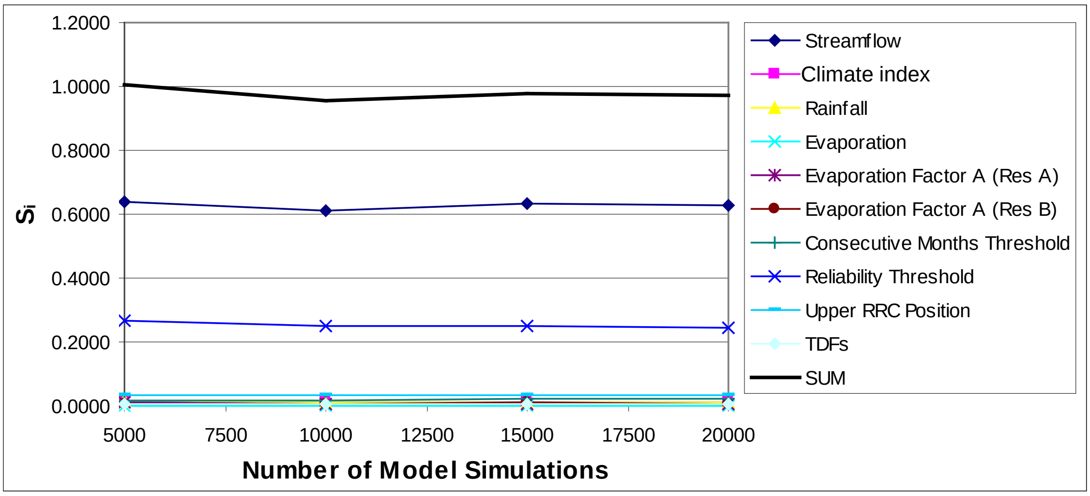


Figure 4-9. First-Order Indices (S_i) for Experiments 13, 14, 15 and 16 Indicating Parity of Results Across all Experiments.

The results in Table 4-21 (for experiment 16) show similar results as the FAST experiment in Table 4-20 with the only difference being the swap of the evaporation Factor A variables ranks. The S_{Ti} and the ($S_{Ti} - S_i$) results in Table 4-21 show that some higher-order effects exist, mostly in the consecutive months and reliability threshold. The streamflow variable also shows possible higher-order effect, while the upper RRC position variable shows a large change between S_i and S_{Ti} relative to the S_i measure.

Table 4-22 presents the results for the Sobol' Experiment 18. Experiment 17 is not shown here due to unsatisfactory results produced, where $S_{Ti} < S_i$ for some variables. Table 4-22 shows that the first four ranked variables, the streamflow, reliability threshold, upper

RRC position and consecutive months threshold, have satisfactory results where the S_i > 0.02 for these variables. The S_i order of magnitude and their rankings shown in Table 4-22 show excellent comparison with the FAST and eFAST results in Table 4-20 and Table 4-21. Interestingly, the erroneous results in Table 4-22 occur for most variables that have S_i < 0.02. To improve the accuracy of the Sobol' experiment, a higher number of model simulations is required; however, the next more accurate Sobol' experiment requires 233,472 model simulations and was deemed computationally infeasible. Errors in the S_{Ti} measures makes comparison of the results difficult between experiments 18 and the eFAST experiments.

Table 4-21. First-Order Indices (S_i) and Total-Order (S_{Ti}) for eFAST Experiment 16.

Variable	S_i	**Ranking**	S_{Ti}	**Ranking**	$S_{Ti} - S_i$
Streamflow	0.6288	1	0.6451	1	0.0163
Climate Index	0.0100	6	0.0162	7	0.0062
Rainfall	0.0118	5	0.0178	5	0.0060
Evaporation	0.0016	10	0.0070	10	0.0054
Evaporation Factor A For Reservoir A	0.0083	7	0.0165	6	0.0082
Evaporation Factor A For Reservoir B	0.0079	8	0.0139	8	0.0060
Consecutive Months Threshold	0.0228	4	0.0560	3	0.0332
Reliability Threshold	0.2472	2	0.2780	2	0.0308
Upper RRC Position	0.0307	3	0.0414	4	0.0107
Temporal Disaggregation Factors	0.0047	9	0.0113	9	0.0066
SUM	**0.9782**				

Table 4-22. First-Order Indices (S_i) and Total-Order (S_{Ti}) for Sobol' Experiment 18

Variable	S_i	**Ranking**	S_{Ti}	**Ranking**	$S_{Ti} - S_i$
Streamflow	0.6329	1	0.6377	1	0.0048
Climate Index	0.0121	6	0.0083	7	-0.0038
Rainfall	0.0162	5	0.0125	5	-0.0036
Evaporation	0.0013	10	0.0003	10	-0.0010
Evaporation Factor A for Reservoir A	0.0099	7	0.0092	6	-0.0007
Evaporation Factor A for Reservoir B	0.0089	8	0.0081	8	-0.0008
Consecutive Months Threshold	0.0219	4	0.0462	3	0.0244
Reliability Threshold	0.2508	2	0.2734	2	0.0226
Upper RRC Position	0.0325	3	0.0330	4	0.0005
Temporal Disaggregation Factors	0.0042	9	0.0062	9	0.0020
SUM	**0.9907**				

Tables 4-23 and 4-24 present the pair-wise first-order indices (S_{ij}) and closed pair-wise first-order indices (S_{ij}^c) of the Sobol' experiment 18, respectively. Notably, the S_{ij}^c indices show all positive results, whilst the S_{ij}, which must be positive for satisfactory results, show many negative sensitivities. A principal objective of this preliminary case study is to assess the applicability of SA techniques to an urban water supply system. Therefore, the Sobol' results shown in Tables 4-23 and 4-24 are presented here only to show a shortcoming of the Sobol' method. The closed indices reveal little extra information than what gained in the previous experiments. The S_{ij}^c results (Table 4-24) for most pairs of variables are close to the addition of the S_i for the same two variables shown in Table 4-22. For instance, $S_{ij}^c = 0.0387$ for the rainfall and consecutive months threshold variables, where as their individual S_i sum to 0.0381 (from Table 4-22). This suggests negligible pair-wise effects, but this cannot be relied upon due to the errors present.

From the above FAST/eFAST and Sobol' method experiments, the following points can be summarised:

1. Results from FAST/eFAST technique experiments proved the ranking results of the Morris method experiments, highlighting the reliability of the Morris method as a screening technique.

2. The streamflow, reliability of supply threshold, the upper RRC position and the consecutive months in restriction are the four most influential variables, with the streamflow being the greatest source of sensitivity for the estimation of yield.

3. The FAST and eFAST were used successfully with only some minor errors produced when performed over limited model simulations. The results above lead to the eFAST being the preferred technique as it calculates S_i at a similar accuracy as FAST using less computational cost (with the addition of S_{Ti}), making FAST obsolete. The Sobol' method was not successful at estimating the importance measures with errors present, such as the $S_{ij} < 0$, and $S_{Ti} < S_i$.

4. Similar to the results of the Morris method experiments, the results of the FAST experiments show that there are higher-order effects between input variables. Most likely this is pair-wise interactions, however the Sobol' method results were contradicted this finding, suggesting that only a small pair-wise interaction effect

remained in the S_{ij}^{c} once S_i and S_j were removed. However the Sobol' results were erroneous, therefore could not quantify or confirm this satisfactorily.

Table 4-23. Pair-Wise Interaction Indices (S_{ij}) for Sobol' Experiment 18

	Streamflow	**Climate Index**	**Rainfall**	**Evaporation**	**Evaporation Factor A for Reservoir A**	**Evaporation Factor A for Reservoir A**	**Consecutive Months Threshold**	**Reliability Threshold**	**Upper RRC Position**
Climate Index	-0.0023								
Rainfall	-0.0039	-0.0014							
Evaporation	0.0002	-0.0003	-0.0015						
Evaporation Factor A for Reservoir A	-0.0016	-0.0014	-0.0017	0.0005					
Evaporation Factor A for Reservoir B	-0.0028	-0.0033	-0.0022	-0.0003	-0.0022				
Consecutive Months Threshold	0.0030	0.0017	-0.0008	-0.0004	-0.0009	-0.0006			
Reliability Threshold	0.0011	-0.0022	-0.0021	0.0009	-0.0016	-0.0019	0.0155		
Upper RRC Position	0.0001	-0.0018	-0.0015	0.0008	-0.0004	-0.0016	0.0001	-0.0032	
Temporal Disaggregation Factors	-0.0007	-0.0006	-0.0017	0.0007	-0.0010	-0.0003	0.0001	-0.0008	-0.0259

Table 4-24. Closed Pair-Wise Interaction Indices (S_{ij}^{c}) for Sobol' Experiment 18

	Streamflow	**Climate Index**	**Rainfall**	**Evaporation**	**Evaporation Factor A for Reservoir A**	**Evaporation Factor A for Reservoir A**	**Consecutive Months Threshold**	**Reliability Threshold**	**Upper RRC Position**
Climate Index	0.6395								
Rainfall	0.6410	0.0238							
Evaporation	0.6318	0.0118	0.0136						
Evaporation Factor A for Reservoir A	0.6386	0.0192	0.0219	0.0109					
Evaporation Factor A for Reservoir B	0.6387	0.0186	0.0227	0.0115	0.0181				
Consecutive Months Threshold	0.6591	0.0382	0.0387	0.0260	0.0340	0.0356			
Reliability Threshold	0.8801	0.2572	0.2603	0.2501	0.2561	0.2572	0.2893		
Upper RRC Position	0.6648	0.0432	0.0466	0.0357	0.0431	0.0431	0.0595	0.2791	
Temporal Disaggregation Factors	0.6343	0.0148	0.0168	0.0060	0.0128	0.0148	0.0298	0.2518	0.0124

4.6 Issues, Limitations and Recommendations

The aim of the hypothetical urban water supply system case study was to evaluate the applicability of the selected sensitivity analysis techniques on a water supply allocation model, specifically used to determine the sensitivity to input variables used in the estimation of yield. The methodology applied to the simple, hypothetical case study is a basic application of sensitivity analysis based on an uncertainty error framework.

The range of the perturbations for all input variables in the hypothetical case study were individually assigned using common error margins considered standard within the water resources industry or where feasible limitations of the variables exist. All input variables were considered to only have data error (including instrument errors, reading and handling errors, etc.) and were implemented as such in the sensitivity analysis. The distributions of the ranges were mostly considered uniform and continuous, but occasionally discrete distributions were required as some input variable structures could not be handled otherwise.

Throughout the SA of this case study several disadvantages, limitations and improvement associated with the techniques were identified. Some of these have already been indicated in the review of the SA techniques (Section 3.5) and the discussion on the input variable handling strategies (Section 4.3.1). For completeness, these are listed below, along with a number of weaknesses in the adopted SA framework, and other observations, conclusions and recommendations:

1. Discretely distributed variables – The main issue with discrete variables for the three SA techniques relates to the possible lack of relationship between the discrete points which would give misrepresentative sensitivity measures. Limitations of discrete variables in SA also exist in the sampling and calculation of EEs in the Morris method, and in the approximation of the integrals for both variance based methods. These can be avoided easily by ensuring that all variables have a continuous distribution. If this is not possible, a relationship between discrete points should be sought at the least.

2. Integral estimation and approximation – The variance based methods have shown mixed results in their estimation of the sensitivity indices. The FAST and eFAST methods have produced acceptable results, however the method of Sobol’ suffered from errors resulting in negative sensitivity indices. Both methods are still to be used in the Barwon case study in the optimism that the Sobol’ method will perform

acceptably in light of the following limitation regarding the handling of time-series variables (Point 4 below).

3. Historic data use - When a single set of historic data is used for future planning in the approach that is commonly used, the management variables, such as the restriction rule curves, target rule curves and security of supply thresholds, will always have the same importance. Therefore, a yield estimate and an optimum set of operating rules that is established from this historic sequence may not be suitable for another possible climate realisation or for a different planning period.

4. Time-series variables – It was noted that in all SA experiments in Sections 4.4 and 4.5 that the streamflow variable and streamflow group dominate the sensitivity measures. This is not surprising considering the handling of the streamflow variable consisted of increasing or decreasing each streamflow time-series data point, that in effect changes the total streamflow entering the system. This handling strategy breaks cross correlations that inherently exist between the streamflow and the other climate dependant variables (i.e. rainfall, evaporation and climate index variables). Appendix C provides a discussion on a number of perturbation strategies that could be applied to the time-series variables. However, they all still break cross correlations between climate variables and only provide perturbations on a single time-step basis (i.e. a week or month change), rather than long term variability that is present in climate events.

5. Importance of climate dependant variables – knowing the effect of the streamflow, evaporation, rainfall and demand is of little use to water authorities as they are uncontrollable.

6. Definition of yield – The handling strategy employed for a time series variable in this case study changed all data points by the same amount, causing the total volume to change with little change the variability. Other perturbation strategy that are possible to perturb time-series (see Appendix C) are also flawed as the variability changes are on short term basis (such as week or month), not long term as found in climate events (such as several months, years or decades). Additionally, and in light of the points 3 and 4 above, the definition of yield should be changed so that yield is dependent on *climate variability* rather than just *streamflow variability*. The use of *climate* implies that the yield is dependent not only on *streamflow*, but also *rainfall*, *evaporation* and *demand*.

7. Additional sensitivity measures – The above SA experiments show that the streamflow variable dominates the standardised S_i and S_{Ti} indices. Due to this dominance, the effects of other input variables are lost due to their very small S_i and S_{Ti} indices. Therefore, additional sensitivity measures that relay non-standardised sensitivity measures are required to observe the influence of an input variable on the estimation of yield without the effect of the dominating variable(s). In the Barwon case study, the partial variance due to each variable (V_i) will be used to reflect the effect on the estimation of yield in terms of non-standardised measure. In addition, the total output variance, $V(Y)$, is used so that the sensitivity of the estimation of yield on the climate scenario can be observed.

To improve the limitations of the adopted methodology as described in points 3, 4 and 5 above, and also considering point 6, it is proposed that the framework for SA on the input variables used in the estimation of yield of the Barwon urban water supply system uses a climate scenario approach, which will be discussed in Chapter 5. In this methodology, the time-series variables, all of which are climate dependant, are broken into several scenarios over a variety of time periods. A sensitivity analysis is then performed on each scenarios using the same random samples (as generated according to the SA techniques' requirements). The use of climate scenarios ensures that cross correlations between climate dependant time series are maintained and also provides a more rigorous analysis of the effects of climate variability on the estimation of yield.

4.7 Summary

This chapter describes in detail the sensitivity analysis of the input variables used in the estimation of yield of a hypothetical urban water supply system. This case study was used as a preliminary study to assess the applicability of selected sensitivity analysis techniques, review input variable handling strategies and aid in developing a sensitivity analysis framework for the application on the complex Barwon urban water supply system case study given in Chapter 5.

Sensitivity analysis of the hypothetical case study system showed that the streamflow variable was the most important to yield of the system. However, the type of perturbations that were applied to the streamflow variable, and the other climate dependent variables, caused a change to the total volume of the series with little change to the variability.

Sensitivity analysis using the Morris method, the Fourier Amplitude Sensitivity Test (FAST) and extended FAST (eFAST) provided reasonably acceptable and reliable results, with only some minor errors produced when performed with a small number of model simulations. The results indicated that streamflow is clearly the most important variable in the estimation of yield, followed by the reliability of supply threshold. Only a few of the remaining variables show any substantial importance. The Sobol’ method of sensitivity analysis was also used to provide first- and total-order importance measures, and pair-wise interaction effects on the estimation of yield. However, the results were mostly erroneous. Grouping the variables showed the streamflow dominating followed by security criteria group. The evaporation group and Restriction Rule Curve (RRC) group show synergistic effects when grouped whereas their individual variables have indistinguishable sensitivities.

The hypothetical urban water supply system case study was adopted as a proof-of-concept for the application of sensitivity analysis on the estimation of yield of complex urban water supply systems. From this case study, a number of areas were identified as needing improvement before application to the more complex, and computationally demanding, Barwon urban water supply system. The most important outcomes of this case study relate to the recommended changes to the SA framework. Drawn from the variable handling strategies used, the recommendation of using climate scenarios in the case study using the Barwon urban water supply system in particular avoids a number of issues that were present in this case study.

Chapter 5
Sensitivity Analysis Using the Barwon Urban Water Supply System

5.1 Introduction

Chapter 4 described the identification and quantification of the importance of input variables used in the estimation of yield of a hypothetical water supply system detailed in REALM Getting Started Manual (VU and DSE, 2005). This preliminary case study was conducted as a proof-of-concept study for the assessment of the applicability of three techniques for Sensitivity Analysis (SA) on the yield of an urban water supply system before use on a much more computationally expensive case study. A number of conclusions on the performance of the SA techniques and on variable handling strategies were identified (See Section 4.6).

The SA framework of the hypothetical urban water supply system case study was based around a simple error uncertainty methodology where all input variables (those that are considered in the definition of yield) were subject to only measurement and handling error uncertainty ranges. The findings of this study showed that streamflow, reliability threshold and upper restriction rule curve position were the most important variables used in the estimation of yield. However, several limitations were identified in adopted SA framework:

1. The streamflow variable was perturbed in a way that not only was the variability changed but also the total volume. The effect of the change of variability was minor compared to the change of total streamflow volume. This did not comply with the definition of yield that was dependant on the variability of streamflow. Other perturbation strategies (Appendix C) avoid changing the streamflow volume whilst changing the variability, yet these strategies result in a short-term variability, such as those attributed to instrument error (i.e. over a single time-step), not the desirable long-term variability (occurring over multiple months, or years) that results from climatic events.

2. The climate dependant variables (streamflow, evaporation, rainfall and demand) are uncontrollable to water authorities and therefore their importance to the estimation of yield is of little use, as research cannot improve their accuracy. Furthermore, yield is ultimately used for studies of future purposes; therefore, the climate dependant variables are implying a future climate scenario, which cannot be known precisely in any case.

3. The estimation of yield and other system management practices generally use a single sequence of all available historic data. Although this is a plausible climate, polices that are derived or optimised from this sequence will not be suitable under all climate scenarios.

The current chapter begins by describing the Barwon water supply system (Section 5.2), with an explanation of all considered input variables in light of the SA framework adopted. The input variables used in this case study and their nominal values are discussed.

Presented in Section 5.3 is the SA framework adopted for the Barwon urban water supply system case study. A discussion on the justification of this adopted framework is accompanied by a typology of the sources of variation in the estimation of yield of an urban water supply system. Particular reference is made to the difference between uncertainty and variability. This section includes a discussion on the scenario selection and the various handling procedures of the controllable variables and concludes in the design of SA experiments for this study.

The results of the SA on the estimation of yield of the Barwon urban water supply system using the Morris method, eFAST and the method of Sobol' are desribed in Section 5.4. The conclusions, recommendations and further improvements are then presented in Section 5.5, with the chapter summarised in Section 5.6.

5.1.1 Hypothesis for Barwon Water Supply System Case Study

The hypothesis to be tested in this case study using the Barwon water supply system is that the estimation of yield of an urban water supply system and the importance of the input variables used in its estimation change under different climate scenarios and over different planning periods.

If this hypothesis is correct then it places doubt on the use of historic data and/or a single length of climate data for system planning purposes. The use of a single set of historic data for future planning provides a plausible realisation climate conditions. However, since no other possible climate scenario is considered the controllable variables, such as the restriction rule curves, target rule curves and security of supply thresholds, will always have the same importance. Therefore, a yield estimate and an optimum set of operating rules that are established from this historic sequence may not be suitable for another possible climate realisation or for a different planning period length. Consequently, a system should not be considered to have a single yield value, but a variety of estimates for different planning periods and for different climate conditions. The yield of a system can be estimated under various plausible conditions and over various planning periods to gain a greater understanding of system behaviour. A suitable yield estimate can then be adopted for planning according to the planning length and predicted future climate events.

To substantiate the hypothesis that climate variability and planning length are important in the estimation of yield, several lengths of planning period (20, 40, 60 and 77 years) are used with a number of scenarios selected for each period. Seven scenarios are selected with a 20 year planning period, five scenario are selected with 40 year length, five with 60 year length and three with a 77 year length. Each of these climate scenarios contain the four climate dependant variables (streamflow, rainfall, evaporation and demand) selected to represent the same period to ensure that cross correlations between the climate variables are maintained. These climate variables are considered as being uncontrollable variables with the remaining, controllable variables subject to sensitivity analysis.

5.2 Barwon Water Supply System Description

The Barwon urban water system is operated and maintained by the Barwon Region Water Corporation (hereafter referred to as Barwon Water) which was formed in 1994. Barwon Water is now the largest regional water corporation in Victoria (Australia) supplying water and sewerage services to 275,000 permanent residents over 8,100 square kilometres (www.barwonwater.vic.gov.au). Highlighted in Figure 5-1 is the region of operation that covers a regional and coastal area in south-west Victoria. This area encompasses the City of Greater Geelong, the Borough of Queenscliffe, the Surf Coast and Colac Otway Shires and part of Golden Plains Shire. The headworks and region of service under the management of the Barwon Water is shown in Figure 5-2.

The Barwon headworks consist of over 5,000 kilometres of pipes, six major reservoirs, six water treatment plants and nine water reclamation plants. Water is sourced from the Barwon River, the East Moorabool River, the West Moorabool River and pumped from a number of groundwater sources. Approximately 70 percent of potable water supply for Greater Geelong and surrounding coastal region (consisting of the Bellarine Peninsula and Surf Coast) is supplied from the Barwon River via the Wurdee Boluc Reservoir. The remaining water is obtained from catchments on the Moorabool River system, which also provides water to the inland demand centres, including; Anakie, Staughton Vale, Bannockburn, Gheringhap, Teesdale, Shelford and Inverleigh.

The REALM (REsource ALlocation Model) model of the Barwon Water system was supplied by Barwon Water for use in this study. This model was developed by SKM and is described in detail in the Central Region Sustainable Water Strategy (SKM, 2006). The REALM model comprises of four sub-models; the Upper Barwon model, the Lower Barwon model, the Central Highlands Water (CHW) model and the Moorabool River Catchment model, which supply Geelong and Ballarat urban demands, and many rural demands. The service area of the CHW model is not shown in Figure 5-2 but covers the region north of Lal Lal Reservoir servicing the Ballarat demand centre encompassing the upper reaches of the Moorabool River, upstream of Lal Lal reservoir.

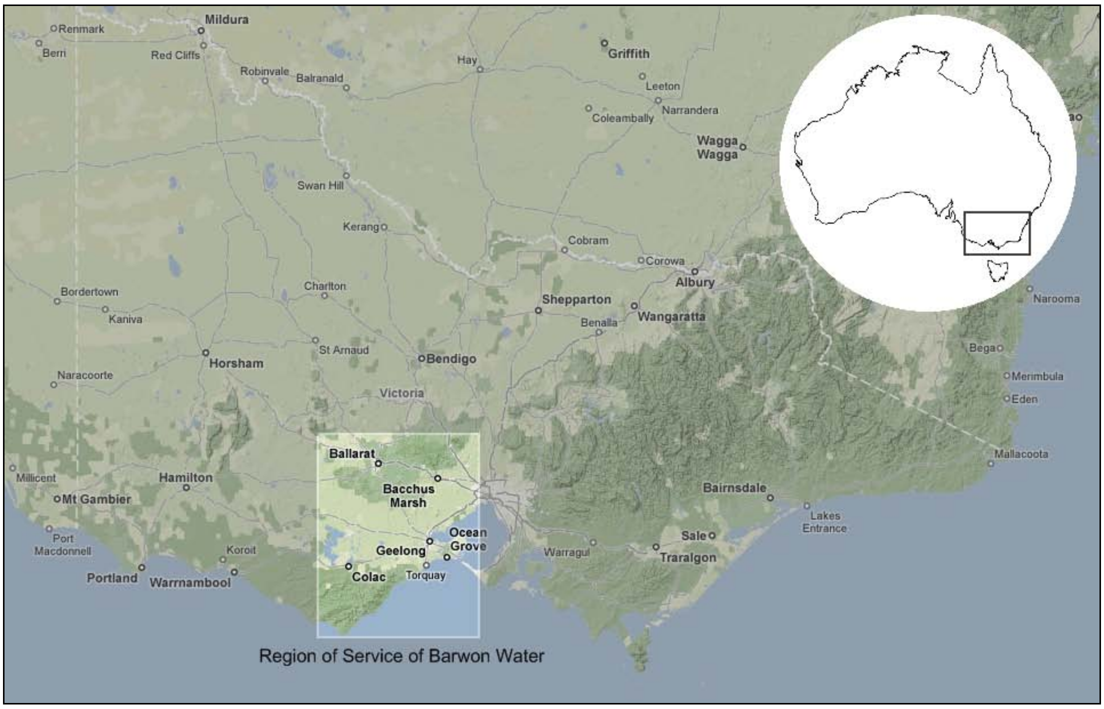


Figure 5-1. Region of Service of the Barwon Water. Inset Shows Location of Victoria within Australia. (Source: www.maps.google.com)

The subject of this study is the Barwon headworks consisting of the Upper Barwon model, the Lower Barwon model and the Moorabool catchment. The Barwon headworks consists of the main system is shown as Figure 5-2, without the coastal Apollo Bay, Lorne and Aireys Inlet systems, and the Colac sub-system from Gellibrand Reservoir in the south to Cressy in the north. From now on, the Barwon headworks is referred to as the Barwon urban water supply system, or simply the Barwon system.

Table 5-1 presents the list of the six main reservoirs of the Barwon urban water system. Note that the Barwon system has a one-third share of the Lal Lal Reservoir and CHW has a two-thirds share. The individual storage capacities of the six major storages range from 2,091 Ml to 38,056 Ml, with the total system storage capacity of 98,285 Ml: approximately two and a half times the annual consumption in 2006 (SKM, 2006). The REALM model consolidates all urban demand centres into two demand centres: Geelong North and Geelong South. More details of the Barwon REALM can be found in SKM (2006).

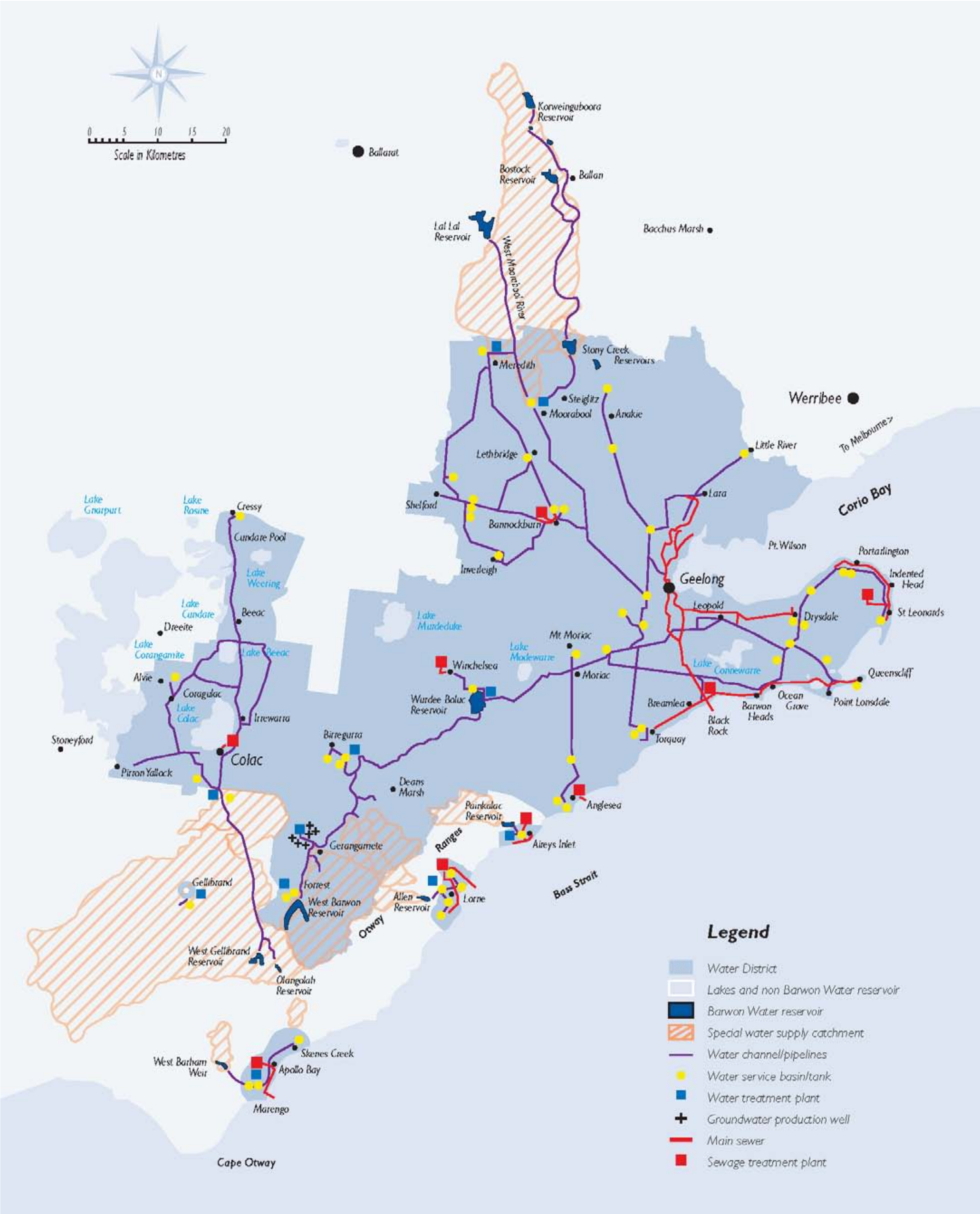


Figure 5-2. Headworks of the Barwon Water Region of Service.
(Source: http://www.barwonwater.vic.gov.au/emplibrary/Service Area map1.pdf)

Table 5-1. Major Storages of the Barwon Urban Water Supply System.

Location	Nominal Capacity (Ml)
Wurdee Boluc Reservoir	38,056
West Barwon Reservoir	21,504
Korweinguboora Reservoir	2,091
Bostock Reservoir	7,455
Stony Creek Reservoir	9,494
Lal Lal Reservoir	19,685*
Total Storage	98,285

* Barwon Share

The Barwon model that was provided for use in this study (SKM, 2006) has the Barwon system connected to the CHW system via Lal Lal Reservoir and some spills from minor reservoirs. The CHW system, along with rural demands and environmental flows are ignored from the sensitivity analysis of input variables related to the estimation of yield of the Barwon system and considered as compulsory rules and policies.

The REALM model of the Barwon system considers a number of input variables which include physical system attributes (network configuration, storage capacity, pipe capacities and penalties, etc.), operational policies and rules (restriction rules, storage target rules, etc.), system climate data inputs (streamflow, rainfall and evaporation) and system output (urban and rural demand, environmental flows, etc.). This study is only concerned with the operational policies and rules, and the system inputs and outputs of the REALM model, which are described below in Section 5.2.1. The physical system attributes (e.g. storage volumes, pipe penalties, etc.) are fixed and not considered in the SA as they cannot be easily changed and the definition of yield does not depend on them.

5.2.1 Input Variables Used in this Study

The following sections discuss the input variables, including the climate dependant variables and the remaining management controllable variables. Reference is made to their nominal values which refer to the 'base case' values of the variables that were provided by Barwon Water and are present in SKM (2006).

5.2.1.1 Climate Dependant Variables

Seventy-seven years of weekly historic data beginning 1st January, 1927 and ending 31st December, 2003 was supplied by Barwon Water for the four climate dependant variables (streamflow, evaporation, rainfall and demand).

The streamflow records include inflow into the reservoirs from rainfall-runoff, river diversions and groundwater pumping across the Barwon system. This inflow data is derived from a combination of historic measurements, hindcasting and calibration. Evaporation and rainfall data for the various reservoirs are also historic data derived through similar measurements, hindcasting and calibration.

Evaporation and rainfall time series data are used in the calculation of the gains and losses from the reservoir using the default functions of REALM evaporation modelling as given in Equations (4.1) and (4.2). They are repeated here as Equations (5.1) and (5.2):

$$\text{Evaporation } (mm) = B \times [\text{Evaporation Data}] + A - [\text{Rainfall Data}] \tag{5.1}$$

$$\text{Net Evaporation } (Ml) = \text{Evaporation } (mm) \times \text{Surface Area (Ha)}/100 \tag{5.2}$$

where A and B are empirical factors

The empirical parameters A and B are not tested in the SA of the Barwon case study as they ultimately adjust the evaporation time-series which is handled using the scenario approach. If the empirical factors A and B were adjusted it would be akin to the measurement error perturbations used in the hypothetical case study in Chapter 4. Since the measurement and handling error uncertainty of climate variables is disregarded from this case study, so will the empirical evaporation factors A and B.

The demand data for the Geelong North and Geelong South demand centres is unrestricted weekly demand. Other demands, such as rural and irrigation, are outside the scope of this study which is to consider only urban water supply. As such they are assumed to be compulsory outflows similar to environmental flows and therefore they are not considered in the sensitivity analysis.

5.2.1.2 Security of Supply

Security of supply criteria, also referred to security criteria in this thesis, provide performance targets for the system ensuring that a supply of demand is reached without total drawdown whilst meeting stakeholders' requirements. At a given Average Annual Demand (AAD) a system is deemed to have failed when at least one security of supply criteria is violated. For this study, the maximum AAD (the greatest demand before system failure) is considered as the yield of the system (see Section 2.5 for further description of the procedure for the estimation of yield). The Barwon system is subject to two security of supply criteria and their thresholds:

1. Reliability of supply – The percentage of months in which demand restrictions are not imposed on the Geelong demand centres, with respect to the total number of months in the simulation. A commonly used value for supply reliability used in water supply management is 95% (SKM, 2003; Barwon Water, 2007), which allows the system to have water restrictions imposed 5% of the planning period.

2. Minimum total system storage level – The minimum total system storage of the six main storages in the Barwon system (as listed in Table 5-1) at any point in the model simulation period. In correspondence with modellers at Barwon Water, they expressed that a minimum total system storage volume threshold is somewhat unknown. The water authority has not considered it a critical threshold in the past, but was informally considered to be between 10-15% of the total capacity.

The base case simulation (given in SKM, 2006) of the Barwon system considers a supply reliability of 92% and a minimum storage level of approximately 20% storage capacity. Further discussion regarding the range and handling of these variables is in Section 5.3.1.2.

5.2.1.3 Restriction Rule Curves

The REALM Barwon system model nominally has a four-stage restriction policy to restrict demand during low storage volume periods. The restriction rule curves are defined by upper and lower rule curves, including three intermediate restriction zones (with their definitions of relative positions and percentage restrictable levels), and a base demand. The values of the restriction rule curves (RRC) currently used and provided by Barwon Water in the REALM simulation model are presented in Figure 5-3. The nominal trigger levels of the upper and lower RRCs, base demand, and the intermediate curves are given in Table 5-2. The upper, lower and intermediate RRCs are shown as an absolute value of storage, while the base demand is given as a percentage of annual demand. The nominal values of the relative position and percentage restrictable for the intermediate curves are given in Table 5-3. The three intermediate curves are defined by a relative position between the upper and lower curves, measured as a percentage from the lower RRC.

When the total system storage is drawn down to below the upper RRC (Table 5-2), demand restrictions are imposed at the percentage restrictable associated whichever zone the storage falls in (nominal values given in Table 5-3). If further drawdown occurs and an intermediate trigger level reached, a more severe restriction stage is enforced. Restrictions are imposed monthly and are introduced at the start of a month in the REALM model. Restrictions are only applied to ex-house water demand which is the difference between the monthly (unrestricted) demand and the base demand.

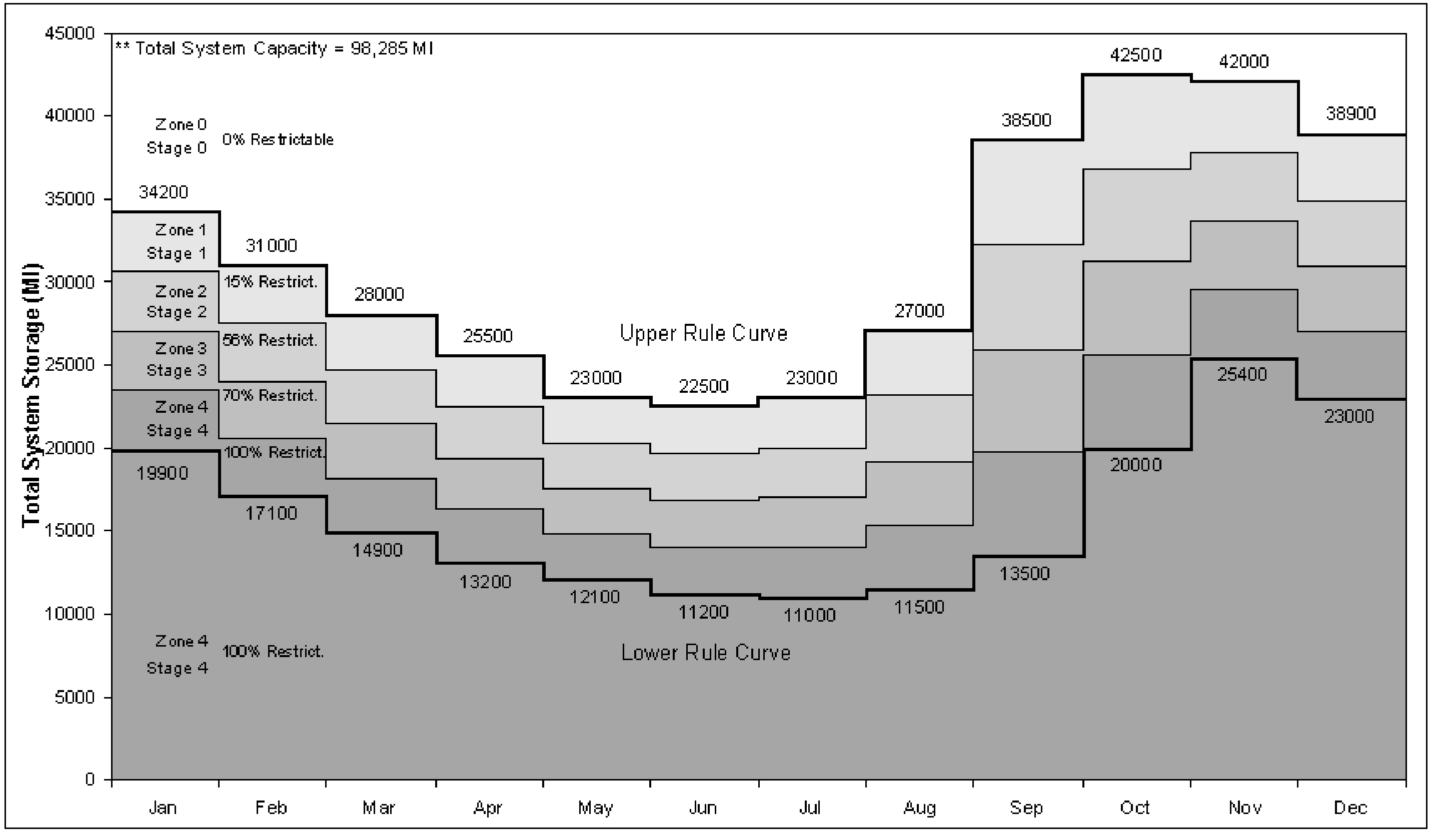


Figure 5-3. Nominal Restriction Rule Curves for Barwon Urban Water Supply System. Values given in Table 5-2.

Table 5-2. Nominal Values of the Upper and Lower Rule Curves, Base Demand and the Intermediate Curves 1, 2 and 3. Relative Positions of the Intermediate Curves given in Table 5-3.

	Jan	**Feb**	**Mar**	**Apr**	**May**	**Jun**	**Jul**	**Aug**	**Sep**	**Oct**	**Nov**	**Dec**
Upper Rule Curve	34,200	31,000	28,000	25,500	23,000	22,500	23,000	27,000	38,500	42,500	42,000	38,900
Intermediate Curve 1	30,625	27,525	24,725	22,425	20,275	19,675	20,000	23,125	32,250	36,875	37,850	34,925
Intermediate Curve 2	27,050	24,050	21,450	19,350	17,550	16,850	17,000	19,250	26,000	31,250	33,700	30,950
Intermediate Curve 3	23,475	20,575	18,175	16,275	14,825	14,025	14,000	15,375	19,750	25,625	29,550	26,975
Lower Rule Curve	19,900	17,100	14,900	13,200	12,100	11,200	11,000	11,500	13,500	20,000	25,400	23,000
Base Demand (% AAD)	6.1%	6.1%	6.1%	6.1%	6.1%	6.1%	6.1%	6.1%	6.1%	6.1%	6.1%	6.1%

All values given in Ml unless otherwise shown

Table 5-3. Percentage Restrictable and Relative Position of the Intermediate Curves.

	Relative Position as % from Lower RRC		**Percentage Restrictable**
	Lower Bound	**Upper Bound**	
Zone 0	100	–	0
Zone 1	75	100	15
Zone 2	50	75	56
Zone 3	25	50	70
Zone 4	0	25	100
Zone 5	–	0	100

The lower RRC in the REALM model of the Barwon system provides a basis for positioning intermediate curves. It does not separate restriction zones of different percentage restrictable. That is, zone four (100% restrictable demand) is below intermediate curve three down to zero total system storage (see Figure 5-3). This means that the RRC curves are set up as a five-stage demand restriction policy (as in the hypothetical case study – see Section 4.2.1.6), but act as a four-stage policy. The intermediate curve separating zone three and zone four is operationally the lower RRC; it separates the intermediate zone 3 from the 100% restrictable demand zone (zone 4).

5.2.1.4 Target Storage Curves

In the Barwon system REALM model, the target rule curves are defined by a single set of five-point curves for all months of the year, indicating the preferred distribution of individual storage volumes for various total system storage volumes. These curves impose inter-storage transfers to distribute water in the system so as to ensure the required demand at various demand centres can be supplied. Figure 5-4 shows the five point target rule curves used to model the water sharing between each of the major storages of the Barwon system with the values supplied by Barwon Water given in Table 5-4. For total system storages between points, the Barwon Water REALM model uses linear interpolation to compute individual reservoir targets.

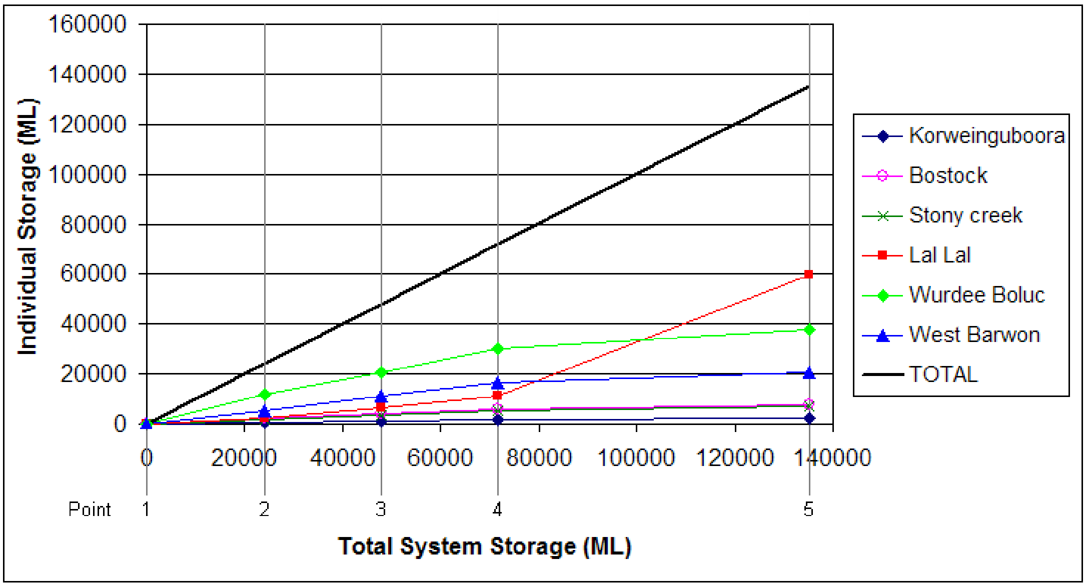


Figure 5-4. Five-Point Target Rule Curves for the Barwon Urban Water Supply System.

Lal Lal Reservoir is modelled at full capacity in the REALM model, including the Central Highlands Water's two-third share. This explains the ~40,000 Ml difference in the total system storage seen in Table 5-1 and Figure 5-4 (134,996 Ml) and the total system storage given in Table 5-4 (98,285 Ml).

Table 5-4. Nominal Values of the 5-point Target Storage Curves of the Barwon Water Supply System.

Total System Storage	**0**	**23,824**	**47,648**	**71,471**	**134,996**
Korweinguboora	0	586	1,133	1,666	2,091
Bostock	0	2,089	4,041	5,942	7,455
Stony Creek	0	1,961	3,794	5,578	7,000
Lal Lal Barwon	0	2,363	6,756	11,345	59,550
Wurdee Boluc	0	11,605	20,596	30,284	38,000
West Barwon	0	5,220	11,328	16,656	20,900
Point	1	2	3	4	5

* All values given in Ml unless otherwise shown

5.3 Sensitivity Analysis Framework

The adapted definition of yield of an urban water supply used in this case study is *the maximum average annual volume of water that can be supplied from the system over a given planning period subject to climate variability, demand pattern and operating rules, without violating the adopted level of service*. By definition, the yield of an urban water supply system experiences variability as a response to changes in the input variables used in its estimation. These input variables include two distinct groups: climate dependent variables which cannot be controlled or accurately predicted by the water authority, and management variables, which include system policies and rules controlled by the water authority. These fit neatly into the main two types of computational model uncertainty identified in Section 3.2:

1. Natural variability (or inherent randomness) – Also termed objective, non-cognitive, irreducible, stochastic and aleatory uncertainty, this is uncertainty that random by nature and is unavoidable.

2. Knowledge deficiency – Also called subjective, cognitive, irreducible or epistemic uncertainty, knowledge deficiency can be reduced through research, improved techniques, modelling and experience and better understanding of the physical system, the processes and data used.

Considering these two types of uncertainty, the hierarchy of the variability of the estimation of yield of the Barwon urban water supply system can be developed as shown in Figure 5-5. At this point, a move away from the term *uncertainty* to the term *variability* is made to better represent the framework of SA used in this case study. Briefly, this framework considers the effects of possible variability of the input variables, regardless of whether the source of this variability is from measurement and data handling uncertainty, optimisation and calibration errors or simply perturbing the input variables to various positions to observe the model and yield behaviour to such a change. For instance, climate dependant variables have uncertainty regarding the accuracy of the historic data, however this accuracy is not relevant as future climate will certainly be different, hence voiding the importance of the data errors. Furthermore, the security of supply thresholds are model input variables which the water authority set; therefore contain no uncertainty regarding the accuracy of the values. However, they have a range of positions that they can be set, depending on numerous stakeholder requirements, performance targets, optimisation, etc.

The natural variability in the estimation of yield relates to inherent fluctuations that can occur within the urban water supply system. These fluctuations are not due to uncertainty of the state or value, but are due to the natural characteristics of the variable. In this case study, natural variability of the climate dependent variables – streamflow, rainfall, evaporation and demand – cause variability in the yield estimate. Following the natural variability branch in Figure 5-5, the inherent fluctuation in these variables is primarily due to spatial and temporal trends and patterns, with some randomly occurring events such as extreme weather events causing further fluctuations (i.e. individual heterogeneity).

The second type of variability in the estimation of yield of an urban water supply system is caused by knowledge deficiency. This, as the name suggests, is caused by an uncertainty of the true value of a variable or not knowing the variable’s optimal value. Knowledge deficiency exists in the operation of the system and model, including interpretation; model uncertainty, consisting of formulation, numerical, parameter and execution errors; and data uncertainty, which consists of measurement errors, handling errors and inconsistent sampling. Indicated in Figure 5-5 are the input variables that contribute to the knowledge deficiency, including: RRCs, security of supply thresholds, target rule curves, evaporation empirical factors, etc. These variables are management policies, rules and empirical parameters that are derived primarily through optimisation, calibration and modeller experience. With exception of the evaporation factors, knowledge deficiency of the input variables is not related to measurement error and handling uncertainty but regarding the position of their optimum position. The operational positions (i.e. the values used in the management and studies of the system) of these variables are known to the water authority but their optimum positions are relatively unknown as they are subject to the climate sequence over which variable optimisation is performed.

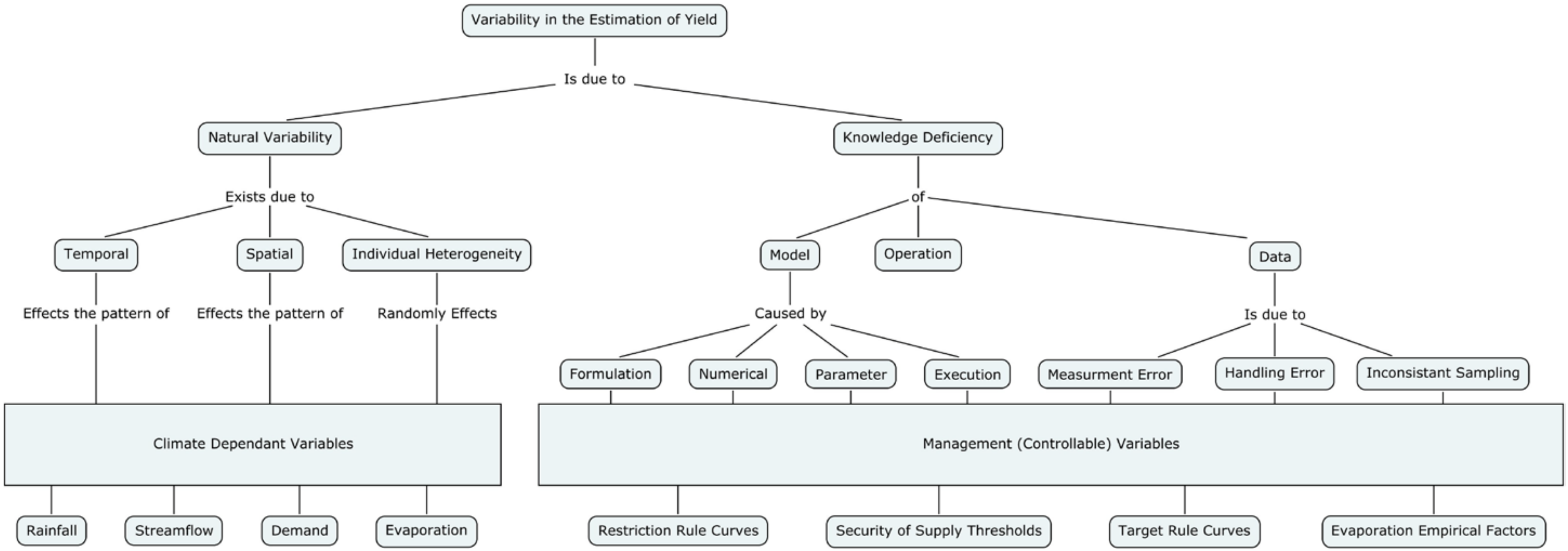


Figure 5-5. Hierarchy of the Sources of Variability of the Estimation of Yield of the Barwon Urban Water Supply System

Variables that have knowledge deficiency can, and do, have a natural variability associated with their values as a product of their dependence on the climate dependent variables. It can also be suggested that the climate dependent variables have some knowledge deficiency associated with them due to measurement and handling errors. However, it is meaningless assessing the impact of measurement and handling errors of historical data of climate dependant variables (which was done in Chapter 4) as the estimation of yield is used for future prediction of system performance and future planning purposes for which the climate will be different and will encompass much more variability.

Table 5-5 presents a summary of the variables used in the study of the importance of input variables and climate variability to the estimation of yield of the Barwon urban water supply system. Each variable that is perturbed in the SA is listed in Table 5-5 including their range, and the two groupings they are assigned. Following the table, in Section 5.3.1, is a description of the variable handling techniques and the perturbation methods used to change the input variables listed in Table 5-5.

5.3.1 Scenario Selection and Input Variable Handling

The following section describes the selection of the climate scenarios using the 77 year historic climate data sequence, followed by variable handling of the remaining controllable variables.

5.3.1.1 Scenario Selection

Seventy-seven years of weekly historic data is available for the climatic dependent variables, i.e. streamflow, rainfall, evaporation and demand. Four planning lengths were selected, consisting of 20 years, 40 years, 60 years and 77 years as an even spread between minimum planning period considered (20 years) and the maximum possible (77 years). Planning lengths less than 20 years were not considered practical for industry and for properly capturing climate events such as drought and wet periods which the system must provide a buffer against. For each of these planning period lengths several scenarios were selected.

Scenarios were identified by ranking the moving total streamflow volume of specific planning length through each of the weekly time steps in the 77 year sequence. The scenarios were then selected from equal spacing of the rankings. Streamflow was used to select the scenarios as it provides a robust representation of climate behaviour. For every streamflow sequence that was identified as suitable, the same time period of the remaining climate dependant variables (rainfall, evaporation and demand) were also selected to complete each scenario and so as to maintain cross correlations. For the 20 year scenarios, a 20 year moving total streamflow was used, for the 40 year scenarios a 40 year moving average was used, and so on. The procedure of scenario selection is further explained using the 20 year planning length as an example.

Table 5-5. Description of Input Variables Used in this Study

Grouping 1	Grouping 2	Variable	Range
Restriction Rule Curves	Upper Curve	Upper Restriction Rule Curve Curvature	Up to -10% - +10% of nominal position
		Upper Restriction Rule Curve Position	-5% - +5% of nominal position
	Lower Curve	Lower Restriction Rule Curve Curvature	Up to -10% - +10% of nominal position
		Lower Restriction Rule Curve Position	-5% - +5% of historic data
	Base Demand	Base Demand Position	70% – 76%
	Percentage Restrictable	Stage 1 Percentage Restrictable	0.10% – 0.20%
		Stage 2 Percentage Restrictable	0.50% – 0.60%
		Stage 3 Percentage Restrictable	0.70% – 0.80%
	Relative Positions	Stage 1 Relative Position	0.20% – 0.30%
		Stage 2 Relative Position	0.45% – 0.55%
		Stage 3 Relative Position	0.70% – 0.80%
Security of Supply	Security Criteria	Supply Reliability	80% – 98%
		Minimum Storage Level	4% – 20%
Target Storage Curves	Target Curve	Target Storage Curves	Discrete distribution 0-10,000

Seven 20 year scenarios were selected from a 20 year total moving streamflow volume. Shown in Figure 5-6 are the weekly streamflow volumes, 20 year total streamflow volume and the seven selected scenarios. The blue line represents the 20 year streamflow totals (for the following the 20 years) with the red circles indicating selected scenarios. The year and season are given in the format YYYY.SS, where YYYY is the year and SS is the week in that year.

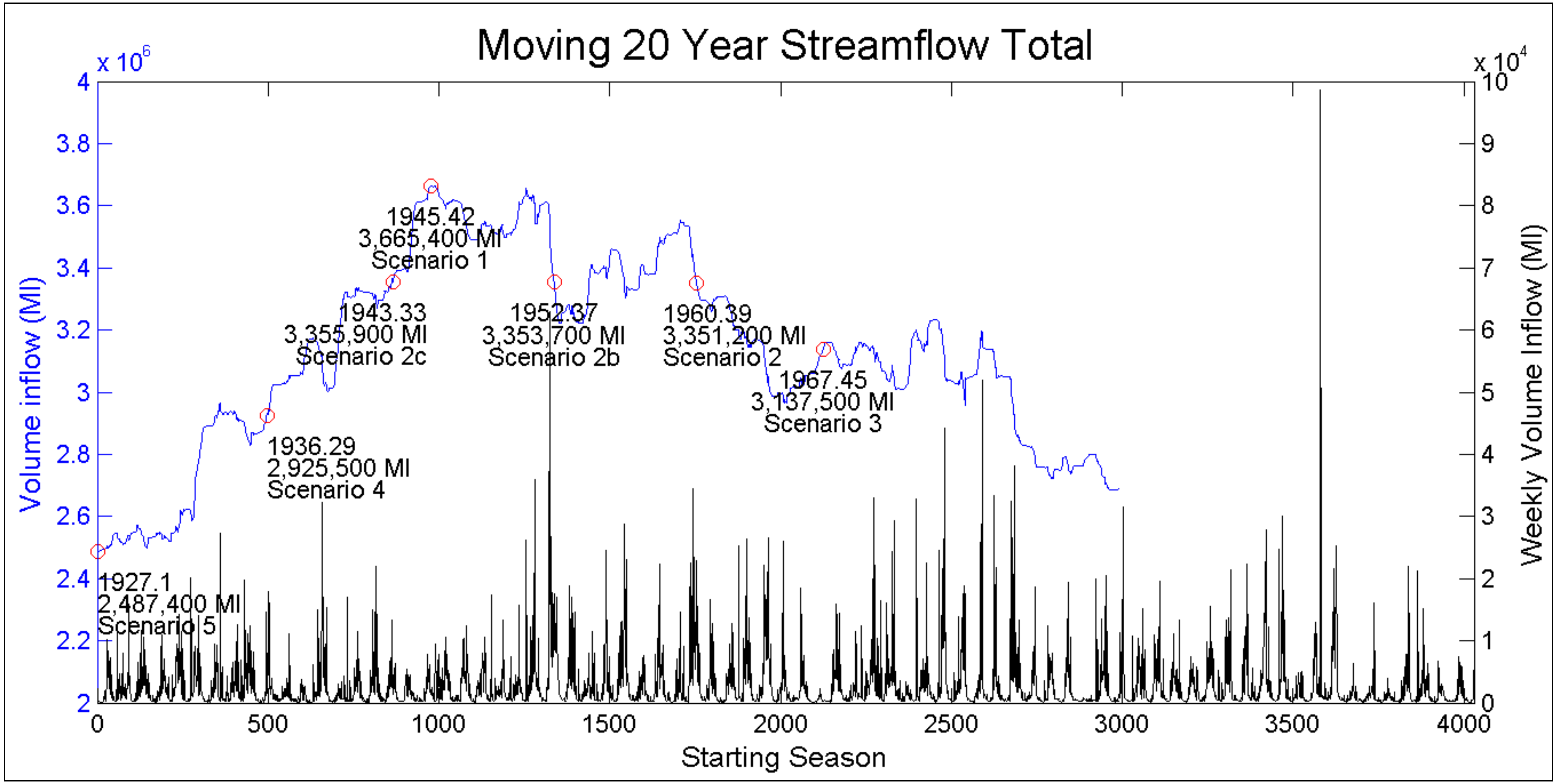


Figure 5-6. Weekly Streamflow Totals and 20 Year Moving Streamflow Total.
Starting Time Step Indicated by Year.Week. i.e. 1945.42 Represents the 42nd Week of 1945.

Scenarios were selected by ranking the 20 year streamflow totals and choosing the maximum and minimum streamflow volumes (scenario 1 and scenario 5, respectively). Three intermediate scenarios were selected using evenly spaced ranking intervals. The scenario ranks (rank 1 has the lowest streamflow total), the starting time step and 20 year total streamflow volume are shown in Table 5-6. Two more scenarios were selected that have a similar total streamflow volume as scenario 2, but at significantly different positions in the historic sequence. As the two extra scenarios have almost the same streamflow volume as scenario 2, and are labelled scenario 2b and 2c. The additional scenarios (scenarios 2b and 2c) were selected so that the effect of climate variability can be assessed without the effect of the total streamflow volume.

Table 5-6. 20 Year Planning Length Scenario Selection Data.

Scenario	**Rank**	**Starting Year.Week**	**20 Year Total Streamflow (Ml)**
1	2990	1945.42	3,665,400
2	2242	1960.39	3,351,200
3	1495	1967.45	3,137,500
4	747	1936.29	2,925,500
5	1	1927.01	2,487,400
2b	2244	1952.37	3,355,900
2c	2243	1943.33	3,353,700

The same method of scenario selection was performed for the 40 and 60 year scenarios producing the ranking, starting week and total streamflows shown in Table 5-7 and Table 5-8, respectively. In both of these cases, five scenarios each were selected, while the three 77 year scenarios required a different selection approach (as described below). Extra scenarios, such as scenario 2b and 2c of 20 year simulation length, are not selected for the 40 and 60 year scenario sets as no significantly different sequences of 40 and 60 years were found.

Table 5-7. 40 Year Planning Length Scenario Selection Data.

Scenario	**Rank**	**Year.Week**	**40 Year Total Streamflow (Ml)**
1	1951	1951.07	6,730,400
2	1464	1949.17	6,600,700
3	976	1957.50	6,430,200
4	488	1931.35	6,160,200
5	1	1964.26	5,839,500

Table 5-8. 60 Year Planning Length Scenario Selection Data.

Scenario	**Rank**	**Year.Week**	**60 Year Total Streamflow (Ml)**
1	911	1939.08	9,509,100
2	684	1941.22	9,384,300
3	456	1940.03	9,306,700
4	228	1930.32	9,228,000
5	1	1928.25	9,152,000

A shuffling (or recycled) method is used for the 77 year planning period scenarios. The 77 year historic sequence is divided into a number of blocks and reordered to produce replicate climate sequences. Blocks of a whole years must be used to ensure that the correct weekly climate pattern within the year is observed in the replicated climate sequence. In this study 11 blocks of seven years is used. The advantage of this method is that a number of new scenarios of maximum length are easily generated, without generating data using stochastic data generation methods (See Srikanthan and McMahon, 1985; Srikanthan and McMahon, 2001; McMahon and Adeloye, 2005, for discussions and reviews of stochastic data generation methods). It is possible that this approach can break severe droughts or create worse droughts, providing new climate event sequences. The disadvantage is that it breaks serial correlations between six pairs of years at the end and beginning of the blocks.

As there is little total streamflow volume change between scenarios 2, 2b and 2c (Table 5-6) and no volume change in the three 77 years scenarios, these scenarios provide an opportunity to test the importance of input variables due to changes in climate variability without the effects of a volume change of streamflow, rainfall and evaporation. All other scenarios test the effects of different streamflow, rainfall and evaporation volumes and their climate variability.

5.3.1.2 Security of Supply

As stated in Section 5.2.1.2, the two security of supply criteria used in the management of the Barwon urban water supply system are the reliability of supply threshold and the minimum storage level threshold. The reliability threshold sets the limit on the number of restriction periods that are imposed in the ex-house demand as a percentage of total number of time steps. The minimum storage level threshold is the minimum volume of water stored in the system at any time step during the simulation.

The reliability of supply threshold was nominally set at the generally accepted industry standard of 95% (Barwon Water, 2007). This variable was been assigned a continuous, uniformly distributed range of 80% to 98% for the random sampling in the SA in this study. These limits were considered for the same reasons as the hypothetical water supply model case study (Section 4.3.1.5); the lower limit set at a reasonable minimum reliability expected by water users and the upper set to 98%, as 100% would produce very low yield estimates.

As stated in Section 5.2.1.2, Barwon Water does not have a typical minimum total system storage volume for use in their yield studies and operational planning. However, modellers from Barwon Water casually mentioned a range of 10% to 15% capacity. The lower bound for the random sampling of the minimum storage volume can be as low as 0%, while the upper bound can be limited by the RRCs. It is reasonable to consider the lowest value of intermediate RRC three as the upper bound of the minimum storage volume threshold as it defines the lowest point of the boundary between zone three (75% restrictable demand) and zone four (100% restrictable). The use of this value allows all three restriction zones to be introduced throughout all months of the year. From the nominal position of the RRCs as given in Table 5-2, the minimum level of intermediate curve three is 14,000 Ml, occurring in July, which equates to 14.2% of total system storage (capacity = 98,285 Ml). This storage level is illustrated on Figure 5-7. However, the position of intermediate RRC three will change due to the random sampling in the SA, therefore the above rationalisation becomes invalid.

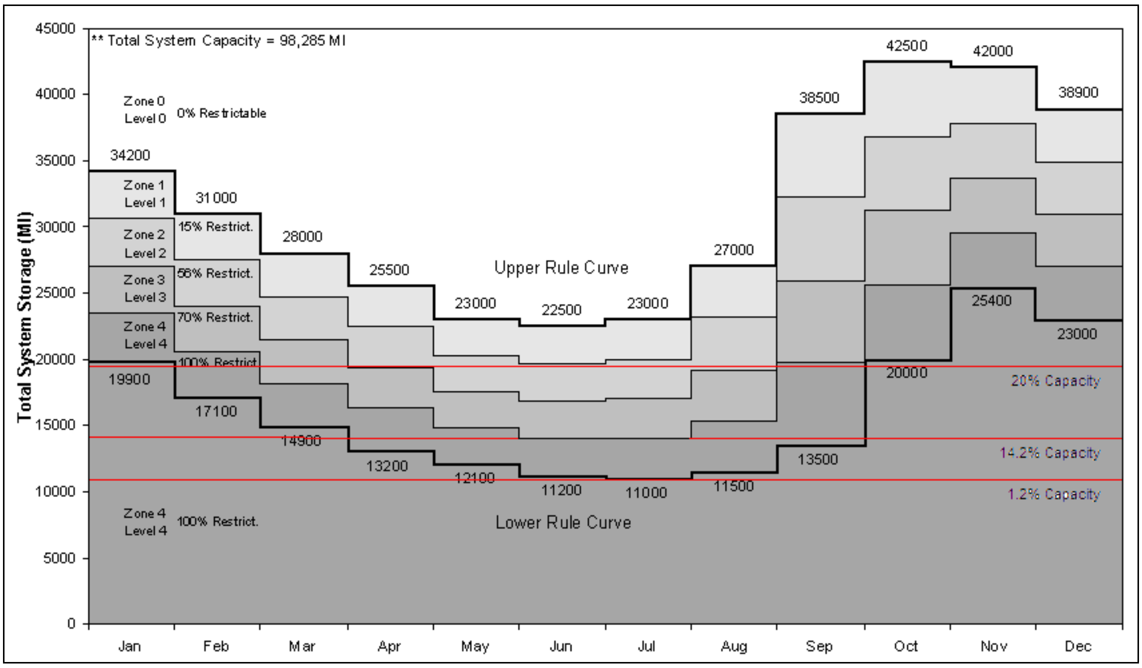


Figure 5-7. Nominal Restriction Rule Curves Showing Percentage of Total System Capacity.

To determine a suitable range for the minimum storage level threshold and confirm the 80-98% reliability range selected above, a preliminary SA observing the behaviour of the estimation of yield to changes in the security criteria thresholds was performed. The test consisted of estimating the yield of the Barwon system considering various combinations of the two security criteria thresholds, while keeping all other REALM input variables constant. The reliability of supply threshold ranged between 80% to 100% at steps of 2%, while the minimum storage threshold was sampled at 1% steps between 1% and 20% of total system storage. The ranges were extended to beyond those provided above (i.e. minimum storage threshold 10% to 15% total storage volume and 80% to 98% supply reliability) to assess the behaviour at extreme values. The tests were done for each scenario. Figure 5-8 and Figure 5-9 show some of the more interesting results of the security of supply criteria range tests. Figure 5-8 shows the results of scenarios 1, 2, 3 and 5 for the 20 year planning length, while Figure 5-9 shows the results for scenarios 1, 2, 3 and 4 for the 40 year planning length. They show the minimum storage threshold versus reliability with the yield estimate shown in contours. The dashed line represents the separation between the critical security of supply threshold. In the areas on the left and upper left of the dashed line the reliability threshold is the critical threshold, i.e. the system has failed due to violating the reliability threshold. In the area to the right and lower right of the dashed line, the minimum storage level is critical. All other scenarios (20 year scenarios 4, 2b and 2c, 40 year scenario 5, the five 60 year scenarios and the three 77 year scenarios) show similar results.

From Figure 5-8, it can be seen in the 20 year planning period scenarios the yield estimate is highly sensitive to changes in the reliability, particularly between 94% and 98% (shown by the tightness of the contour lines). When the reliability decreases towards the lower bound (80%) the yield estimate is not as sensitive. The exception is scenario 3, which is largely dominated by the minimum storage threshold. This indicates that importance of the severe drought that occurs in this scenario, as it causes the system to drawdown greatly, violating the minimum storage threshold at a lower AAD (average annual demand) than would be needed to violate the reliability threshold.

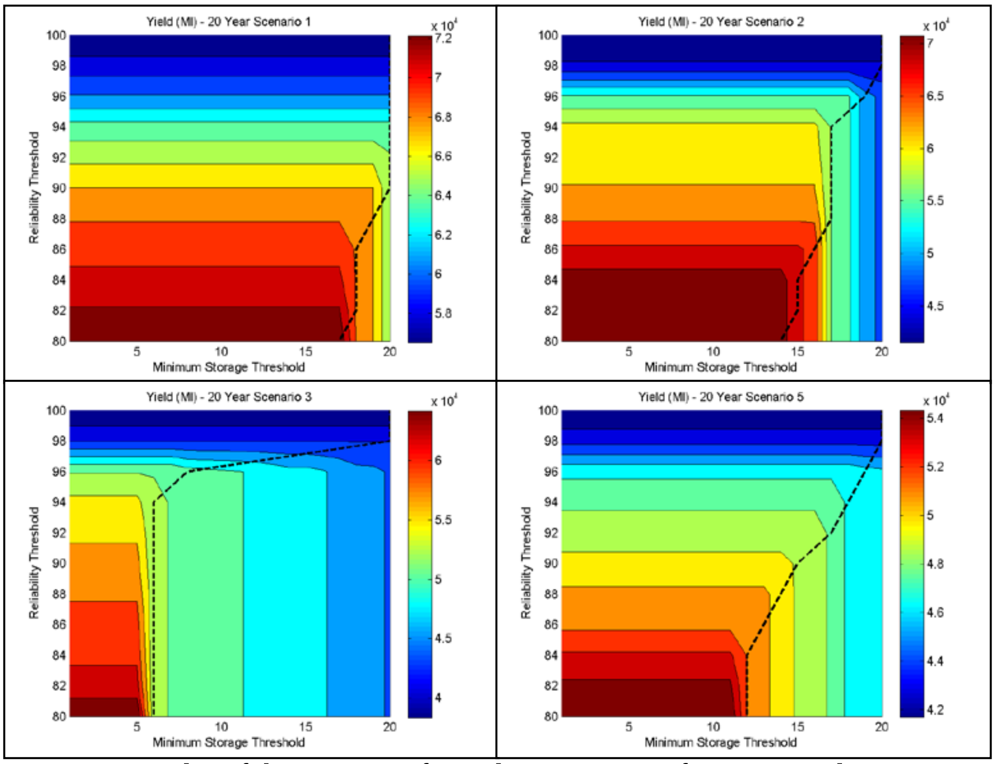


Figure 5-8. Samples of the Security of Supply Range Tests for 20 Year Planning Period. Showing Yield Versus Reliability/Minimum Storage Volume Thresholds.

Comparing the 20 year scenarios (Figure 5-8) to the 40 year scenarios (Figure 5-9), it is clear that there is a difference in the behaviour of yield with respect to the security thresholds. The 40 year scenarios show a greater dependency on the minimum storage threshold with an effective range from as low as 6-7% up to 20%, whereas the 20 year scenarios it has an effective range from 12% upwards (with the exception of 20 year scenario 3).

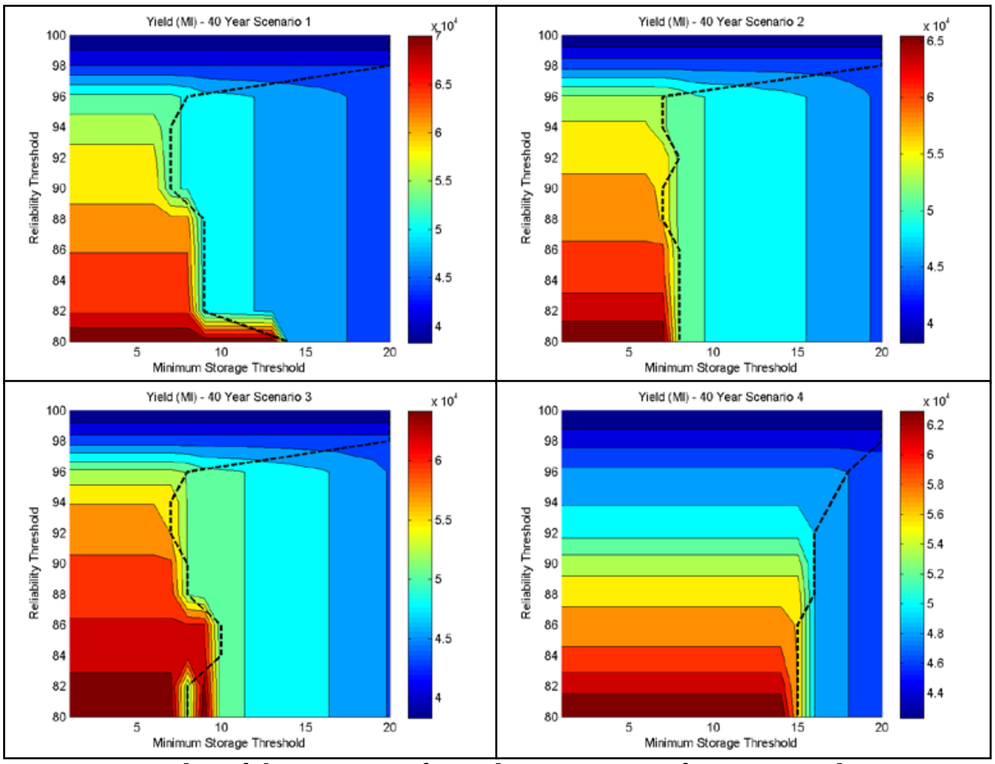


Figure 5-9. Samples of the Security of Supply Range Tests for 40 Year Planning Period. Showing Yield Versus Reliability/Minimum Storage Volume Thresholds.

The test described above highlights the regions of variable space that the two security criteria thresholds are effective. Therefore, the range selected for the minimum storage threshold is 4% to 20% of total system storage to capture the effective range with an extreme lower minimum storage threshold still possible. The reliability threshold has a uniform distributed range of 80% to 98%. The implications of testing the security criteria within SA provide an indication of the importance of correctly setting the thresholds.

To reiterate, the reliability threshold has a range of 80% to 98% and the minimum storage volume threshold has a perturbation range of 4% to 20%. The upper limit of 20% for the minimum storage volume consequently sets the minimum storage volume threshold to 19,657 Ml. Shown in Figure 5-7 is a line highlighting the position of 20% capacity. The implication of an upper limit of 20% is that in the months from April to August, inclusive, the imposed demand restrictions are limited to zone one and zone two. For most of the other months the restrictions can enter zones three and four.

5.3.1.3 Restriction Rule Curves

Three groups of factors are within the set of restriction rule curves. These are the upper and lower curves and base demand, the percentage restrictable, and the relative positions of the intermediate curves.

<u>Upper and Lower Restriction Curves and Base Demand Perturbations</u>

Two types of perturbations were performed on the upper and lower RRCs and a single perturbation is applied to the base demand. A curvature change is applied to the upper and the lower RRCs which adjusts the slope of the curves shown in Figure 5-3. A position change is applied to the upper and lower RRCs and the base demand. The base demand is assumed to be constant throughout the year so it is not subject to a curvature perturbation.

The curvatures of the upper curve and lower curve are changed separately and done so that the slopes of the curves become flatter or steeper. Changing the curvature alters the trigger volumes of the restriction zones; a steeper curve means that the difference of the trigger volumes between the filling and summer months (September to March), and the other months is increased. When a flatter curve is generated, the difference between the trigger volumes decreases. The purpose is to generate a variation in the curvature of the RRCs so that the importance of the slope can be assessed.

To change the curvature, a percentage value, randomly selected from between a range of -10% to +10% in accordance with the SA technique, is used to generate a new curve by interpolating between predefined bounds. An example of these bounds is shown in Figure 5-10 surrounding the nominal position of the upper and lower curves (i.e. the 0% curvature change) used in REALM model of the Barwon system as provided by Barwon Water. The curves shown in Figure 5-10 have a maximum of 10% deviation from the nominal positions at the peaks (reservoir filling and summer months) and troughs of the nominal curve. The remaining monthly values are generated so that a smooth curve was maintained.

The positions of the upper curve, lower curve and base demand are perturbed with respect to the total system storage (Figure 5-3), i.e. they are raised or lowered against the total system storage. Each curve is perturbed separately using a single randomly generated percentage value: i.e. three random numbers are required; one for each curve. Each random percentage changes the 12 monthly values associated to a curve by that random percentage; either increasing or decreasing the monthly values from the curves nominal position. The range of the position perturbations for both the upper curve and the lower curve is ±5%. This is applied to the curve after the curvature perturbation is performed, or to the nominal position of the curves (given in Table 5-2) if the curvature perturbation is not required in the SA. The nominal value of the base demand curve is 73% of AAD (combined for all months of the year) which is replaced by a randomly selected value from a range of 70% to 76% (approximately ±5% from the nominal value). Note that the base demand values given in Table 5-2 are monthly values with respect to the 73% AAD, i.e. (73% / 12 months) = 6.1% AAD per month; the base demand is the same for each month.

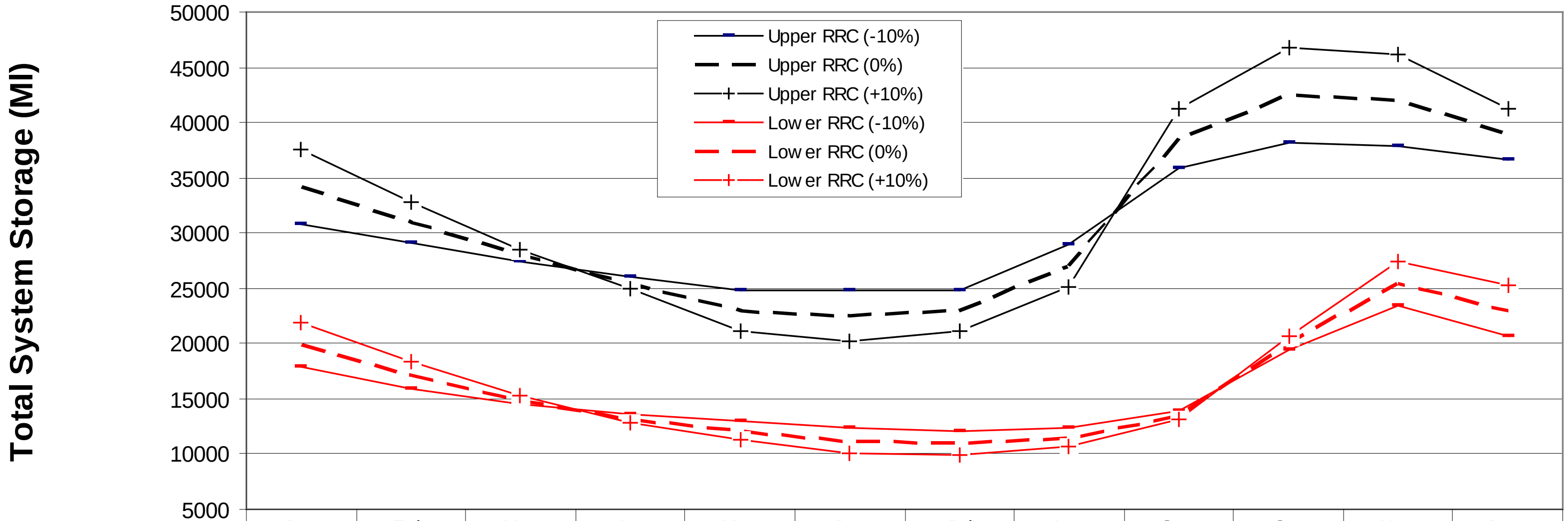
When the upper and lower curves are perturbed, the trigger level for each intermediate zone will also change. This means that restrictions are triggered at a different total system storage volume. When the base demand curve is increased, the in-house water demand is increased, effectively decreasing the volume of water that can be restricted since only the ex-house demand is restricted. The ex-house demand is the difference between the total demand and the in-house demand, i.e. the base demand. By perturbing the base demand in the SA, the importance of accurately determining and modelling the in-house water demand is measured.

Relative Position Perturbations

The relative positions of the intermediate curves are set nominally to the values shown in Table 5-3. Each position is individually perturbed, each using a random value selected from a ±5% uniformly distributed range. Therefore, a total three random numbers are required to perturb the three intermediate curves.

Percentage Restrictable Perturbations

The values of the percentage of demand restrictable for each restriction zone is perturbed in a ±5% uniformly distributed range. The nominal values are in Table 5-3. Note that the percentage restrictable in zone four is not perturbed and is set at 100%. Each percentage restrictable value uses a single random value for this perturbation; therefore, three randomly selected values are required.



	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Upper RRC (-10%)	30780	29140	27440	26010	24840	24750	24840	28890	35805	38250	37800	36566
Upper RRC (0%)	34200	31000	28000	25500	23000	22500	23000	27000	38500	42500	42000	38900
Upper RRC (+10%)	37620	32860	28560	24990	21160	20250	21160	25110	41195	46750	46200	41234
Lower RRC (-10%)	17910	15903	14453	13596	12947	12320	12100	12305	13905	19400	23368	20700
Lower RRC (0%)	19900	17100	14900	13200	12100	11200	11000	11500	13500	20000	25400	23000
Lower RRC (+10%)	21890	18297	15347	12804	11253	10080	9900	10695	13095	20600	27432	25300

Figure 5-10. Upper and Lower Interpolation Limit Bounds for Perturbation of Curvature.

5.3.1.4 Target Storage Curves

In the hypothetical water supply model case study (Chapter 4), the target storage curves were perturbed in the SA by changing the intermediate points of one of the two reservoirs. The intermediate points of the second reservoir were then assigned volumes so that the associated total system storage was met (see Section 4.3.1.6).

This strategy is simple to implement for a two reservoir system as one of the two reservoirs is controlled by the required perturbation, while the other reservoir takes up the remaining total system storage. This method only requires one random number to perturb two reservoirs, two random numbers to perturb three reservoirs, and so on. The inclusion of additional reservoirs means that the required properties of the target curves become increasingly complex to maintain. The properties are the sum of the individual storages sum to the total system storage at each point and the reservoir storage only increases as the total system storage increases.

The Barwon system has six reservoirs, and therefore this handling strategy is not suitable as it would require five random numbers per each intermediate point (i.e. points 2, 3 and 4 in Figure 5-4), totalling 15 random numbers and would be extremely complex to ensure that the target storage curves maintain to the correct properties. Instead, 10,000 sets of target storage curves were randomly generated and assigned a 1 to 10,000 discrete sampling distribution from which the SA technique selects. The sets were generated ensuring that individual storage volumes summed to the required total system storage and that the individual volumes of reservoirs do not decrease at higher total system storage. As the sets of curves were generated randomly, similarity to the nominal curves, given in Figure 5-4, was not guaranteed. Although it has been stressed in this thesis that a discrete distribution should be avoided in SA (due to problems with integral estimation and index calculations), a suitable handling strategy to perturb the target storage curves could not be found during the SA experiments of this study.

5.3.2 Design of Sensitivity Analysis Experiments

The three SA techniques that have been identified as suitable (Section 3.5) for analysis of the input variables used in the estimation of yield of the Barwon urban water supply system are the Morris method, the extended Fourier Amplitude Sensitivity Test (eFAST) and the method of Sobol'. The same tiered approach used in the SA of the yield of the hypothetical water supply case study (Section 4.3.2) is applied to the Barwon urban water supply system case study. Increasingly more accurate but computationally expensive experiments were performed so that more information was progressively gained to provide a better understanding of the individual input variables used in the estimation of yield. The effects of groups of associated variables were also tested so that any synergism, or cancelling out, of perturbing a group of individual variables could be identified. The grouping of variables was done so that the closely related variables are allocated into the same group, or groups.

The Morris method was used as a screening technique to identify input variables, or groups of input variables, that have no or very little importance on the estimation of yield and to give an initial indication of the behaviour of the yield estimate to perturbations of input variables. As the indices are not reliable quantitative estimates of input variable behaviour they can only be used for ranking input variables; therefore limited experiments using the Morris method were used in this study. If, using the Morris method, an input variable was identified as having little effect on the yield estimate over all scenarios, it would be kept at its nominal value and neglected from the subsequent experiments. If an input variable shows importance in just a single experiment then it should remain in all other experiments so that the evolution of importance can be assessed completely.

The eFAST and Sobol' methods were then used in an attempt to accurately quantify the importance of the input variables used in the estimation of yield. Note the FAST method was not used as the eFAST technique provides more importance measures and better accuracy at a lower computational cost and was found to perform adequately in Chapter 4. The eFAST and Sobol' methods produce the same first- and total-order sensitivity indices, but use different techniques. The first-order importance measure (S_i) provides information on the sensitivity of the yield estimate to variations in the i-th input variable, free of the effects of all other variables. The total-order measure (S_{Ti}) provides the overall importance measure of an input variable which includes all combinations of interactions with the remaining input variables. eFAST is also used to determine the importance of groupings of input variables to identify possible synergy of groups of related variables, or lack there of. The Sobol' method was also used to determine two-factor interaction effects, S_{ij}. Negative importance indices of the Sobol' method experiments were estimated in the pilot study in Chapter 4, which voids the accuracy and success of the technique. With this in mind, the Sobol' method is still used with caution.

Each experiment has a unique set of randomly selected input variable samples, or perturbations, in accordance to the requirements of the technique used. Each set of samples was applied to all scenarios for each experiment. This ensures the importance of the input variables could be compared across the scenarios without the introduction of additional variability. By comparing the S_i and the S_{Ti} indices of an input variable across scenarios, the change of importance of that variable can be observed. If a large change in the importance of input variables exists over different planning periods, it would indicate how the importance of the management of the controllable variables would change under different planning lengths. A change of importance of input variables over different climate scenarios of the same length signifies the effect of climate variability on the behaviour of the model and, thus, on the yield estimate.

Findings of the hypothetical case study led to the recommendation of using the total output variance, $V(Y)$, and the partial output variance (V_i) due to each input variable so that non-standardised effects of input variables can be observed. $V(Y)$ provides insight into the sensitivity of the estimation of yield to the climate sequence. A high $V(Y)$ indicates a large range of yield estimates, indicating sensitivity to the climate sequence of that scenario. V_i provides similar information as S_i, however it can be compared across scenarios and planning lengths to give understanding of the effects of the i-th variable.

As the 77 year scenarios are generated by shuffling blocks of years, they can be used to show the influence of the climate variability, free of the influence of the volume of streamflow entering the system. The same can be done with 20 year scenarios 2, 2b and 2c. These are significant in terms of the hypothesis, which brings into question the use of a single climate sequence for a all planning purposes. If these sets of scenarios (20 year scenarios 2, 2b and 2c, and the three 77 year scenarios), show a change in importance of input variables, it highlights the need to consider a number of climate scenarios.

5.4 Sensitivity Analysis Results

5.4.1 Morris Method Results

The Morris method is commonly used as a screening technique to provide information regarding the sensitivity of yield estimate to input variable perturbations and information on possible interactions or non-linear behaviour of input variables. The Morris method involves a randomly generated One-At-a-Time (OAT) sampling procedure that provides ranking estimates of the importance of a model's input variables (to the output) with a low computational cost. The algorithm generates a trajectory through the input variable space which links model realisations together to estimate an Elementary Effect (EE_i) for each of the k variables ($1 \leq i \leq k$) at a cost of k+1 model simulations. Multiple trajectories, r, are constructed to provide r EEs for each input variable. Further details of the Morris method are given in Section 3.5.1. The following Morris method sensitivity indices are used in this discussion:

μ_i - the mean of the EE_is – provides overall sensitivity of changes in the *i*-th input variable. This includes all interaction effects and all-order effects.

μ_i* - the mean of the absolute EE_is is – provides the overall sensitivity void of any cancelling out effects that can result from μ_i.

σ_i - the standard deviation of the EE_is – the spread of the EE_is indicates possible non-linearity or interactions.

In the individual input variable experiments following, further information regarding the monotonicity, or non- monotonicity, of the input variable to model output relationship can be identified by observing the difference between μ and μ*. If a difference exists, it shows non-monotonicity of an input variable, i.e. a change in the input in one direction (positive or negative) causes the yield estimate to both increase and decrease. This could be due to a non-uniform input to output behaviour or due to interactions with other input variables.

Three sets of Morris method experiments were performed, consisting of individual input variable experiments and two sets grouped input variable experiments. The individual input variables are shown in the third column of Table 5-5, whilst the groups that they are assigned to are given in columns 1 and 2 of the same table. All Morris method experiments were performed using an 8-level sampling resolution and used 50 trajectories to ensure that results sufficiently converged.

Shown in Figure 5-11 is the cumulative μ, μ* and σ indices of all 20 year simulation length scenarios. Shown here is the evolution of the indices for the restriction rule curve (RRC) group showing that 50 trajectories are sufficient to reach convergence. Other variables show similar results.

The Morris method experiments were performed on 18 climate scenarios over four simulation time lengths. The μ - σ plane typically used to present the Morris method indices will allow only one scenario to be presented and viewed with ease. Therefore, μ, μ* and σ are shown in table and bar chart formats so that they can be compared over all scenarios. In the two grouping experiments, only μ* and σ are considered. The μ index considers the direction of model output change due to either a positive or a negative input variable change. As variables within a group can have a positive and a negative change at the same time, μ is not used when grouping variables. The μ* index avoids this issue by considering only the magnitude of the output change and not the direction (See Campolongo et al., 2007 for further details).

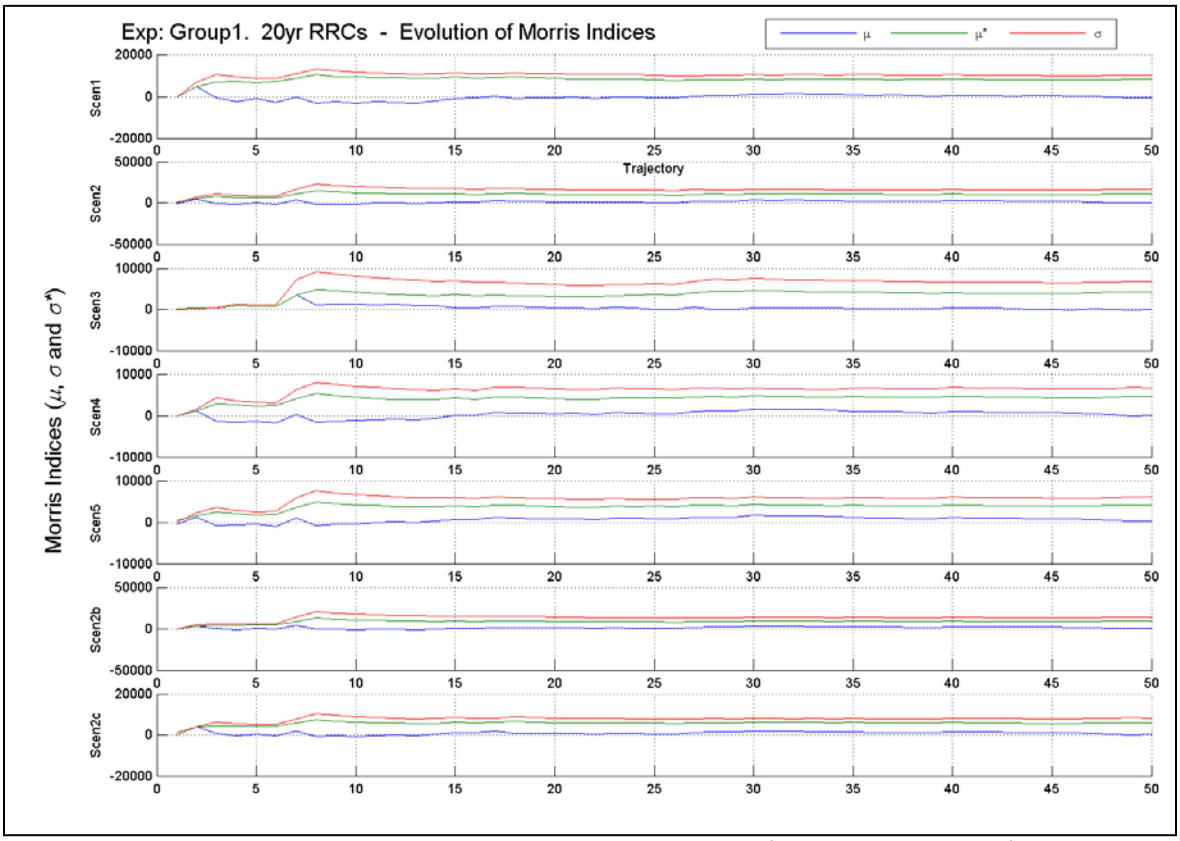


Figure 5-11. Grouping Experiment 1. Showing the Evolution of the Morris Indices for the Restriction Rule Curves Group over 20 Year Planning Period.

5.4.1.1 Individual Input Variable Experiments

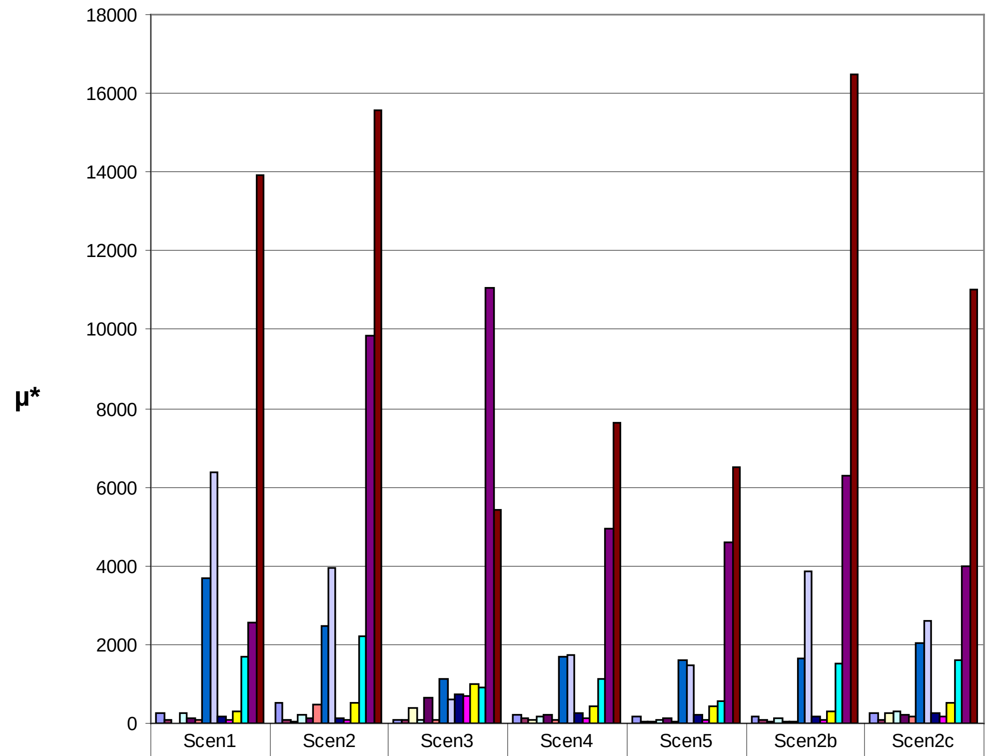
The individual input variable Morris method experiment includes all 14 variables shown in Table 5-5. Individual experiments were performed using i) a 50 trajectory, 8-level, $\Delta = 2$ design, ii) a 50 trajectory, 8-level, $\Delta = 4$ design, and iii) a 50 trajectory, 4-level, $\Delta = 2$ design (See Section 3.5.1 for details regarding the Morris method algorithm). These experiments showed similar results, therefore only the 50 trajectory, 4-level, $\Delta = 2$ design results are shown below and discussed for brevity.

The 20 year simulation period results are shown in Figure 5-12 and Table 5-9. Figure 5-12 shows the μ* results that indicate four input variables are notably important for all scenarios, except scenario 3. These are reliability of supply threshold, the minimum storage threshold, the upper RRC curvature, the upper RRC position and the target curves. Interestingly, when the reliability of supply threshold is the most important variable (for all scenarios except scenario 3), the upper RRC curvature and upper RRC position are clearly defined as being the next most important, while the remaining variables showing inconclusive difference. For scenario 3, the most important variable is the minimum storage threshold, then the reliability of supply threshold, with all other variables showing minor sensitivity effects. The reason for the apparent correlation between the reliability threshold variable and upper RRC curvature and position variables is related to which security criteria threshold is critical (i.e. which threshold will cause the system to fail at a lowest average annual demand - AAD) and the climate variability. The reliability threshold has a reliance on the upper RRC as the upper RRC determines when restrictions are triggered. When the upper RRC is increased with respect to the total system storage, restrictions will be imposed at a higher total storage. The reliability threshold will then be violated at a lower AAD and is more likely to be the critical threshold. Conversely, the minimum storage threshold does not rely on the upper RRC and will be violated regardless of the position of the upper RRC. Therefore, changing the upper RRC does not effect the likelihood of the minimum storage threshold being the critical threshold, hence the lack of correlation.

The results of the 20 year individual variable scenario (shown in Table 5-9) suggest considerable non-linearity or interaction behaviour as indicated by high σ values in comparison to the μ and μ* results. Furthermore, almost all input variables over all scenarios show non-monotonicity as indicated by the difference between μ and μ*. The exceptions are highlighted in Table 5-9. The yellow highlights the input variables that have inverse monotonic input to output relationship and the green highlights positive monotonic input to output relationships. The minimum level and reliability thresholds both have an inverse monotonic input to output relationship, signifying that when they are increased (i.e. they become more strict thresholds) the yield estimate decreases. Alternatively, the yield estimate increases when the thresholds are lessened, i.e. become less strict.

The Morris results of the 40 year simulation period experiment (presented in Table 5-10) shows that the two security of supply threshold are the most important in the estimation of yield for this time period. The next important are again the upper RRC position and curvature variables, with the base demand and target curve variables showing some importance. The remaining variables do not show a notable trend across the scenarios. Many variables show non-monotonicity while the highlighted cells show the monotonic variables that have an inverse input to output relationship.

The results for the 60 year simulation period experiment are shown in Table 5-11 and the 77 year simulation period experiment is shown in Table 5-12. For both simulation lengths, the reliability of supply and minimum storage level threshold variables are the most important and the base demand, target curves, upper RRC position and upper RRC curvature show some significance across all scenarios. Again, non-monotonic input to output relationships exist for most variables with the highlighted cells of Table 5-11 and Table 5-12 showing the exceptions for the 60 year and 77 year experiments.



	Scen1	Scen2	Scen3	Scen4	Scen5	Scen2b	Scen2c
Relative Position 1	251	507	108	216	152	160	269
Relative Position 2	82	98	67	147	50	71	90
Relative Position 3	1	55	400	86	47	25	258
Perc. Restrictable 1	257	226	91	168	82	134	299
Perc. Restrictable 2	116	114	667	213	123	53	232
Perc. Restrictable 3	108	472	84	84	48	63	159
Upper RRC - Curvature	3679	2460	1129	1712	1621	1658	2023
Upper RRC - Position	6367	3941	599	1729	1489	3858	2581
Low er RRC - Curvature	157	129	754	268	209	154	279
Low er RRC - Position	100	98	682	110	104	91	191
Base Demand	284	518	1000	449	416	287	503
Target Curves	1699	2193	930	1116	575	1526	1610
Minimum Storage Threshold	2552	9862	11063	4943	4610	6297	3983
Reliability Threshold	13944	15585	5401	7620	6488	16491	11002

Figure 5-12. μ* Results of the Individual Input Variable Morris Method Experiment – 20 Year Planning Period.

Table 5-9. Results of the Individual Input Variable Morris Method Experiment – 20 Year Planning Period.

	Factor	Scen1	Scen2	Scen3	Scen4	Scen5	Scen2b	Scen2c
µ	Relative Position 1	-208	158	-108	-159	-141	-50	-94
	Relative Position 2	-63	-98	-66	-78	-45	-7	-10
	Relative Position 3	-1	-27	-389	-53	-28	-25	-219
	Percentage Restrictable 1	150	115	58	162	71	104	143
	Percentage Restrictable 2	39	10	661	182	25	42	157
	Percentage Restrictable 3	41	401	84	20	13	63	18
	Upper RRC Curvature	-2954	-2221	-555	-1299	-1073	-797	-1374
	Upper RRC Position	-6177	-3306	-355	-1541	-1176	-3744	-2265
	Lower RRC Curvature	140	75	-487	112	154	106	110
	Lower RRC Position	53	98	682	83	82	42	148
	Base Demand	-240	-240	-956	-398	-388	-150	-469
	Target Curves	294	-312	-143	-37	-132	-226	790
	Minimum Level Threshold	-2552	-9862	-11063	-4943	-4610	-6297	-3983
	Reliability Threshold	-13944	-15585	-5401	-7620	-6488	-16491	-11002
µ*	Relative Position 1	251	507	108	216	152	160	269
	Relative Position 2	82	98	67	147	50	71	90
	Relative Position 3	1	55	400	86	47	25	258
	Percentage Restrictable 1	257	226	91	168	82	134	299
	Percentage Restrictable 2	116	114	667	213	123	53	232
	Percentage Restrictable 3	108	472	84	84	48	63	159
	Upper RRC Curvature	3679	2460	1129	1712	1621	1658	2023
	Upper RRC Position	6367	3941	599	1729	1489	3858	2581
	Lower RRC Curvature	157	129	754	268	209	154	279
	Lower RRC Position	100	98	682	110	104	91	191
	Base Demand	284	518	1000	449	416	287	503
	Target Curves	1699	2193	930	1116	575	1526	1610
	Minimum Level Threshold	2552	9862	11063	4943	4610	6297	3983
	Reliability Threshold	13944	15585	5401	7620	6488	16491	11002
σ	Relative Position 1	539	2385	269	380	294	418	623
	Relative Position 2	393	343	192	311	164	236	248
	Relative Position 3	10	273	2601	236	129	155	843
	Percentage Restrictable 1	482	853	227	248	205	264	670
	Percentage Restrictable 2	306	339	4211	392	262	156	443
	Percentage Restrictable 3	262	2220	186	235	140	172	422
	Upper RRC Curvature	3166	2510	1922	1621	1502	2070	2203
	Upper RRC Position	3810	3875	1071	1518	1439	2889	2938
	Lower RRC Curvature	477	320	4244	419	373	361	503
	Lower RRC Position	276	364	4209	240	218	281	503
	Base Demand	407	972	4302	591	372	613	761
	Target Curves	2332	3310	1254	1541	709	1903	2046
	Minimum Storage Threshold	5865	15007	9044	6097	5561	11751	7337
	Reliability Threshold	4953	11233	8380	4474	4628	8804	5767

Table 5-10. Results of the Individual Input Variable Morris Method Experiment – 40 Year Planning Period.

	Factor	Scen1	Scen2	Scen3	Scen4	Scen5
μ	Relative Position 1	-50	-73	-26	-70	-137
	Relative Position 2	-62	-65	-65	-70	-66
	Relative Position 3	-41	-45	-39	-46	-67
	Percentage Restrictable 1	45	74	98	36	107
	Percentage Restrictable 2	89	20	67	23	74
	Percentage Restrictable 3	-667	149	878	78	90
	Upper RRC Curvature	-547	-185	-423	-1232	-48
	Upper RRC Position	-1092	-327	-966	-2203	-238
	Lower RRC Curvature	139	111	48	32	114
	Lower RRC Position	62	124	84	187	224
	Base Demand	-328	-740	-915	-340	-1000
	Target Curves	202	-221	-19	351	-827
	Minimum Level Threshold	-17777	-14610	-16306	-6941	-9183
	Reliability Threshold	-5951	-4885	-5560	-12927	-4919
μ*	Relative Position 1	220	166	97	103	179
	Relative Position 2	73	67	65	124	66
	Relative Position 3	58	45	82	52	90
	Percentage Restrictable 1	118	166	357	146	116
	Percentage Restrictable 2	176	110	111	114	163
	Percentage Restrictable 3	879	179	889	102	92
	Upper RRC Curvature	1025	644	838	1559	688
	Upper RRC Position	1337	1118	1204	2354	1080
	Lower RRC Curvature	206	154	205	137	200
	Lower RRC Position	87	135	118	249	224
	Base Demand	550	768	1316	413	1000
	Target Curves	880	799	865	1258	915
	Minimum Level Threshold	17777	14610	16306	6941	9183
	Reliability Threshold	5951	4885	5560	12927	4919
σ	Relative Position 1	610	515	239	253	367
	Relative Position 2	190	169	148	453	206
	Relative Position 3	168	136	228	154	257
	Percentage Restrictable 1	283	520	1324	345	228
	Percentage Restrictable 2	533	334	344	208	306
	Percentage Restrictable 3	5364	414	4780	230	222
	Upper RRC Curvature	1638	908	1211	1491	910
	Upper RRC Position	2510	2368	2271	2130	1437
	Lower RRC Curvature	343	324	384	377	362
	Lower RRC Position	200	454	276	523	351
	Base Demand	632	2159	5034	586	805
	Target Curves	1780	1504	1868	1656	750
	Minimum Storage Threshold	14247	10542	12442	10691	5130
	Reliability Threshold	9237	7303	8308	8009	6373

Table 5-11. Results of the Individual Input Variable Morris Method Experiment – 60 Year Planning Period.

	Factor	Scen1	Scen2	Scen3	Scen4	Scen5
μ	Relative Position 1	-86	-208	-179	-111	-116
	Relative Position 2	-141	-50	-84	-67	-13
	Relative Position 3	-46	-92	-82	-35	-14
	Percentage Restrictable 1	37	145	134	97	67
	Percentage Restrictable 2	76	99	100	-37	-9
	Percentage Restrictable 3	65	111	112	311	72
	Upper RRC Curvature	-230	-189	123	-221	-438
	Upper RRC Position	-1020	333	134	-505	-895
	Lower RRC Curvature	128	121	104	119	134
	Lower RRC Position	75	256	165	92	53
	Base Demand	-844	-1192	-929	-646	-344
	Target Curves	-61	-748	-477	41	-108
	Minimum Level Threshold	-12187	-11312	-9836	-11457	-12180
	Reliability Threshold	-5351	-2621	-3141	-5748	-6021
μ*	Relative Position 1	115	208	201	121	116
	Relative Position 2	181	87	90	70	62
	Relative Position 3	62	116	82	92	83
	Percentage Restrictable 1	86	153	161	103	124
	Percentage Restrictable 2	130	170	133	156	121
	Percentage Restrictable 3	101	149	118	345	100
	Upper RRC Curvature	649	768	451	616	848
	Upper RRC Position	1286	931	687	756	1199
	Lower RRC Curvature	172	132	151	142	185
	Lower RRC Position	103	256	186	115	95
	Base Demand	855	1192	929	737	464
	Target Curves	784	855	689	687	781
	Minimum Level Threshold	12187	11312	9836	11457	12180
	Reliability Threshold	5351	2621	3141	5748	6021
σ	Relative Position 1	224	357	325	235	257
	Relative Position 2	583	232	179	194	167
	Relative Position 3	157	287	261	262	231
	Percentage Restrictable 1	231	272	304	234	269
	Percentage Restrictable 2	263	278	250	457	256
	Percentage Restrictable 3	201	291	212	1665	203
	Upper RRC Curvature	954	1183	639	879	1135
	Upper RRC Position	2346	1127	985	1019	1789
	Lower RRC Curvature	314	308	301	253	315
	Lower RRC Position	208	344	315	245	219
	Base Demand	2515	867	711	1714	496
	Target Curves	1385	766	765	1138	1387
	Minimum Storage Threshold	8801	4163	4394	8472	9352
	Reliability Threshold	7713	4898	5455	7826	8203

Table 5-12. Results of the Individual Input Variable Morris Method Experiment – 77 Year Planning Period.

	Factor	Scen1	Scen2	Scen3
μ	Relative Position 1	-153	-144	-136
	Relative Position 2	-82	-111	-239
	Relative Position 3	-96	-104	-136
	Percentage Restrictable 1	124	55	42
	Percentage Restrictable 2	76	125	198
	Percentage Restrictable 3	61	143	41
	Upper RRC Curvature	-153	-282	-236
	Upper RRC Position	166	-173	-113
	Lower RRC Curvature	155	310	242
	Lower RRC Position	251	276	204
	Base Demand	-1184	-1143	-1114
	Target Curves	-734	-343	-541
	Minimum Level Threshold	-11132	-13697	-13299
	Reliability Threshold	-2816	-5147	-4992
μ*	Relative Position 1	176	163	172
	Relative Position 2	82	133	263
	Relative Position 3	96	164	136
	Percentage Restrictable 1	132	130	166
	Percentage Restrictable 2	123	243	220
	Percentage Restrictable 3	112	229	129
	Upper RRC Curvature	722	947	956
	Upper RRC Position	963	1080	1129
	Lower RRC Curvature	156	403	378
	Lower RRC Position	257	276	206
	Base Demand	1184	1143	1114
	Target Curves	837	827	993
	Minimum Level Threshold	11132	13697	13299
	Reliability Threshold	2816	5147	4992
σ	Relative Position 1	372	270	327
	Relative Position 2	215	268	954
	Relative Position 3	279	363	299
	Percentage Restrictable 1	232	246	383
	Percentage Restrictable 2	219	571	532
	Percentage Restrictable 3	237	395	288
	Upper RRC Curvature	1193	1359	1377
	Upper RRC Position	1294	1359	1534
	Lower RRC Curvature	318	645	654
	Lower RRC Position	407	463	329
	Base Demand	809	1009	1041
	Target Curves	777	1294	1715
	Minimum Storage Threshold	4077	6389	6692
	Reliability Threshold	4920	7372	7495

The main findings of the individual variable experiments using the Morris method are:

1. The most important input variables in all scenarios over all simulation lengths are the reliability of supply and the minimum storage level thresholds. The base demand, target curves, upper RRC position and upper RRC curvature variables also show considerable importance across all scenarios.

2. A number of input variables show an inverse input to output relationship (i.e. a positive input change causes a negative output change and vice versa) as indicated by the large negative μ results.

3. The above results suggest a correlation between the importance indices of input variables exists. When the reliability threshold is the critical security of supply criteria, the upper RRC curvature and position variables are also important. When the minimum storage level threshold is critical, the remaining variables show non-conclusive importance. This correlation is further investigated and discussed in the results of the variance based methods.

4. Most input variables show a non-monotonic relationship with the yield estimate. This means that a positive change to the input variable can results in a positive or a negative change in the yield estimate. The notable exceptions to this generality is the minimum storage and reliability thresholds which show a negatively inverse monotonic relationship with the yield estimate. i.e. a positive change to the threshold(s) causes a decrease in the estimation of yield.

5.4.1.2 Grouping 1 Experiments

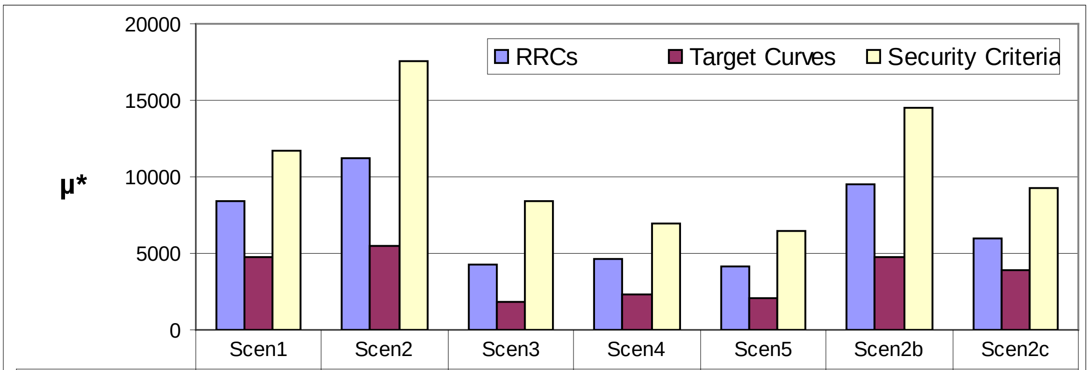
In grouping 1 experiments, the individual input variables were assigned into three groups (11 variables in the restriction rule curves group, two variables in the security of supply group and one in the target storage curves group) as shown in Table 5-13, with the ranges as indicated in Table 5-5. The Morris method indices for the grouping 1 experiment for simulation length 20, 40, 60 and 77 years are presented in Tables 5-14, 5-15, 5-16 and 5-17, respectively. The μ* indices for all scenarios are shown graphically in Figures 5-13, 5-14, 5-15 and 5-16. The grouping 1 experiments were performed using a 50 trajectory, eight level, $\Delta = 4$ Morris design so that the same input variable sampling points as the individual input variable experiment could be selected.

Table 5-13. Assignment of Input Variables for the Grouping 1 Experiments.

Group Name	Individual Variable	Grouping Name	Individual Variable
Restriction Rule Curves (RRCs)	Upper RRC Curvature	Target Storage Curves (Target Curves)	Target Storage Curves
	Upper RRC Position		
	Lower RRC Curvature	Security of Supply (Security Criteria)	Reliability Threshold
	Lower RRC Position		Minimum Storage Threshold
	Base Demand		
	Percentage Restrictable 1		
	Percentage Restrictable 2		
	Percentage Restrictable 3		
	Relative Position 1		
	Relative Position 2		
	Relative Position 3		

Table 5-14. Results of the Grouping 1 Morris Method Experiment – 20 Year Planning Period.

Morris Index	Group	Scen1	Scen2	Scen3	Scen4	Scen5	Scen2b	Scen2c
μ*	RRCs	8364	11195	4253	4641	4163	9537	5961
	Target Curves	4714	5497	1815	2365	2072	4698	3877
	Security Criteria	11747	17605	8440	7002	6442	14573	9314
σ	RRCs	10315	16731	6698	6700	6031	14253	8218
	Target Curves	7690	11316	3592	4234	3881	9213	7059
	Security Criteria	14101	22940	12012	8933	8511	18143	11625



	Scen1	Scen2	Scen3	Scen4	Scen5	Scen2b	Scen2c
RRCs	8364	11195	4253	4641	4163	9537	5961
Target Curves	4714	5497	1815	2365	2072	4698	3877
Security Criteria	11747	17605	8440	7002	6442	14573	9314

Figure 5-13. μ* results of the Grouping 1 Morris Method Experiment – 20 Year Planning Period.

Table 5-15. Results of the Grouping 1 Morris Method Experiment – 40 Year Planning Period.

	Factor	Scen1	Scen2	Scen3	Scen4	Scen5
μ*	RRCs	5508	4350	5057	7312	4017
	Target Curves	2427	2125	2488	3129	1852
	Security Criteria	11951	9679	10769	11915	7636
σ	RRCs	8468	6826	7865	10692	6654
	Target Curves	6213	5282	5808	7201	3491
	Security Criteria	18736	14458	16154	15261	9683

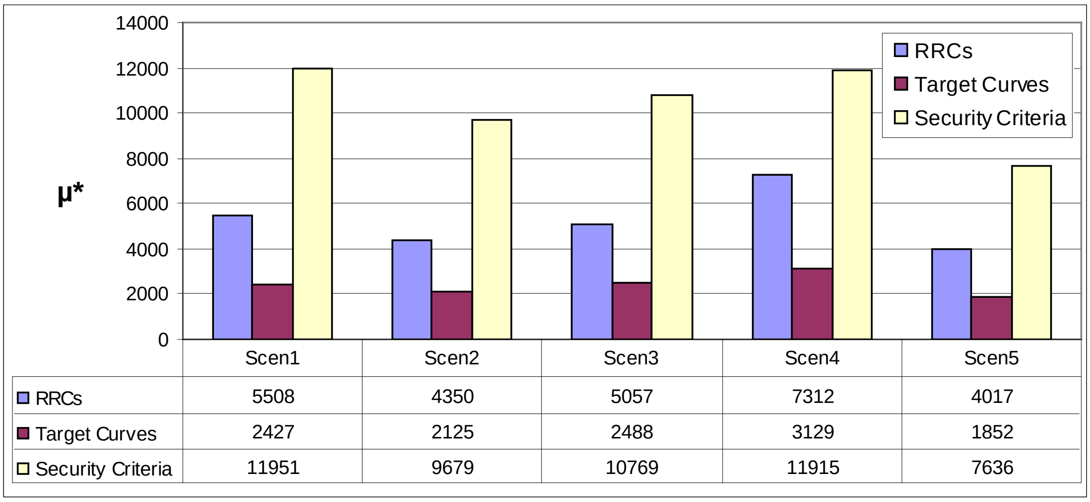
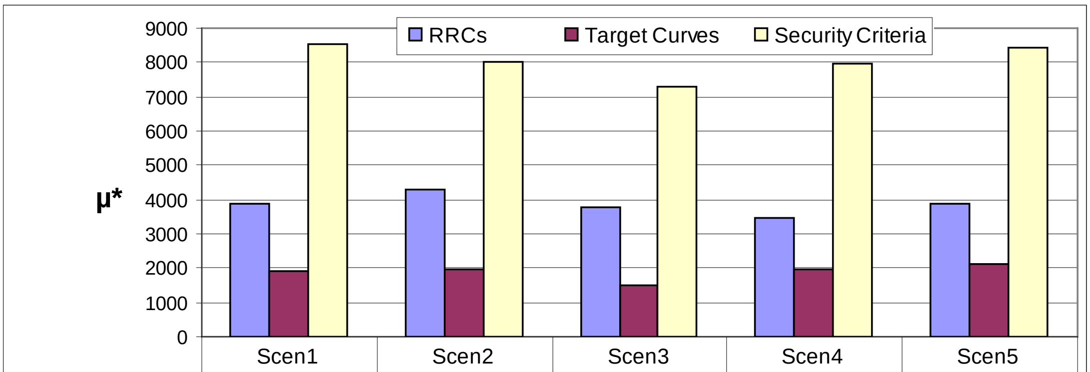


Figure 5-14. μ* results of the Grouping 1 Morris Method Experiment – 40 Year Planning Period.

Table 5-16. Results of the Grouping 1 Morris Method Experiment – 60 Year Planning Period.

	Factor	**Scen1**	**Scen2**	**Scen3**	**Scen4**	**Scen5**
μ*	RRCs	3873	4294	3753	3488	3867
	Target Curves	1931	1943	1484	1983	2125
	Security Criteria	8538	7997	7317	7940	8455
σ	RRCs	6189	6881	6138	5610	6150
	Target Curves	4695	3660	2959	4478	4783
	Security Criteria	12625	9855	9174	11218	12210



	Scen1	Scen2	Scen3	Scen4	Scen5
RRCs	3873	4294	3753	3488	3867
Target Curves	1931	1943	1484	1983	2125
Security Criteria	8538	7997	7317	7940	8455

Figure 5-15. μ* results of the Grouping 1 Morris Method Experiment – 60 Year Planning Period.

Table 5-17. Results of the Grouping 1 Morris Method Experiment – 77 Year Planning Period.

	Factor	Scen1	Scen2	Scen3
μ*	RRCs	4102	4812	4128
	Target Curves	1992	2732	2600
	Security Criteria	7876	10734	9406
σ	RRCs	6606	8186	6922
	Target Curves	3747	5359	5159
	Security Criteria	9724	13706	12201

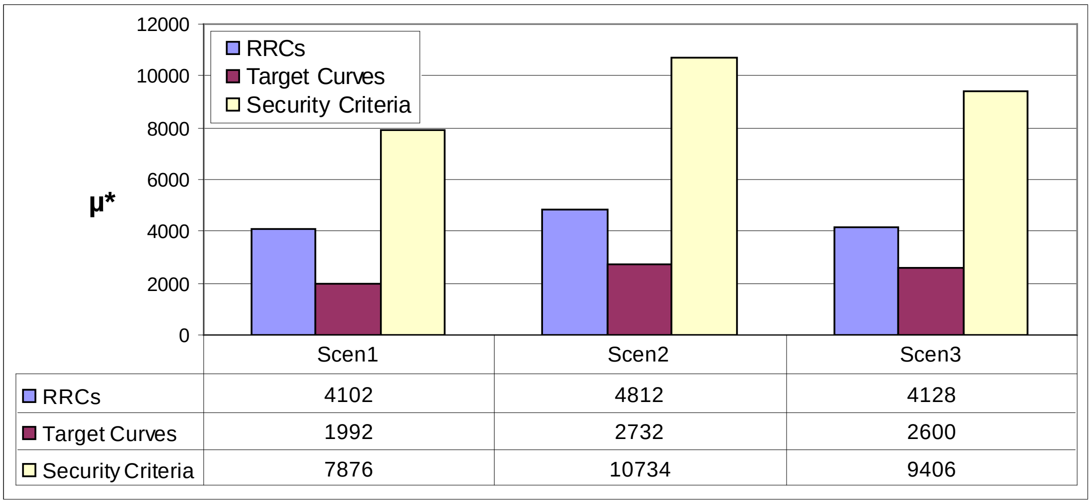


Figure 5-16. μ* results of the Grouping 1 Morris Method Experiment – 77 Year Planning Period.

From the above tables and figures a number findings for the variable grouping 1 experiments can be extracted:

1. All groups of input variables show some influence on the yield estimate (indicated by the non-zero μ* indices) and show considerable interaction or non-linearity (indicated by the high σ indices).

2. The μ* indices shown in Figures 5-13, 5-14, 5-15 and 5-16 clearly indicate that the input variable groups maintain the same rankings for every scenario in every simulation length. The most important group of variables in the estimation of yield is the security criteria group, followed by the restriction rule curves and the target curves. These findings are expected as the security criteria thresholds essentially drive the estimation of yield with dependency on the restriction rule curves.

3. A large range of magnitudes of the μ* and σ indices exists across scenarios of the same length and across different planning lengths. No obvious trends exist based on the total streamflow volume (scenarios are selected using the total streamflow volume, see Section 5.3.1.1) or the planning lengths.

5.4.1.3 Grouping 2 Experiments

The grouping 2 experiments consist of a larger number of groups compared to the grouping 1 experiments, with the restriction rule curves group of the grouping 1 experiments separated into smaller groups of related variables. These experiments were designed to test the importance of the components of the restriction rule curves. They consist of groups of variables as indicated in Table 5-18, with their ranges shown in Table 5-5. An eight level, $\Delta = 4$ Morris design over 50 trajectories was used.

The Morris method indices for the grouping 2 experiment for the 20 year planning length are presented in Table 5-19 with the μ* indices shown graphically in Figure 5-17. The μ* and σ results for the 40 year planning length are given in Table 5-20, with μ* presented visually in Figure 5-18. Similarly, the 60 year indices are presented in Table 5-21 and in Figure 5-19. Finally, the μ* and σ results of the 77 year grouping 2 experiment are shown in Table 5-22 and Figure 5-20. The grouping 2 experiments were performed using a 50 trajectory, eight level, $\Delta = 4$ Morris design so that the same input variable sampling points as the individual input variable experiment could be used.

Table 5-18. Assignment of Input Variables for the Grouping 2 Experiments.

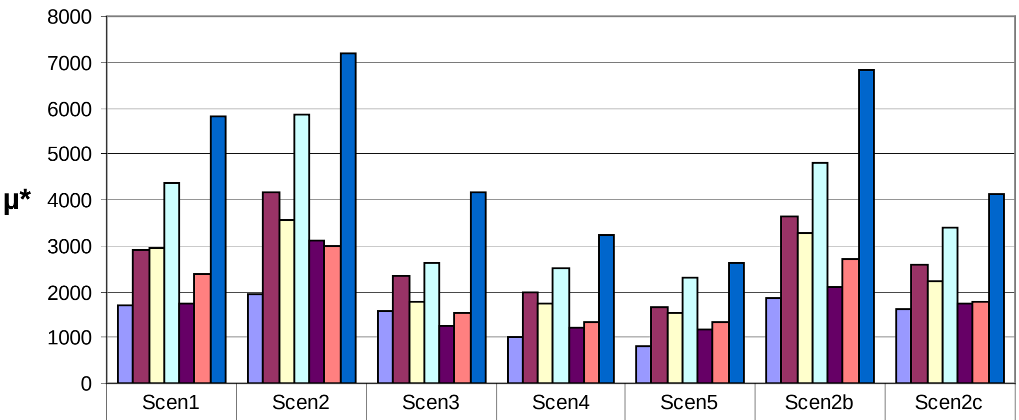
Group Name	**Individual Variable**	**Group Name**	**Individual Variable**
Upper Restriction Rule Curve (Upper RRC)	Upper RRC – Curvature	Base Demand	Base Demand
	Upper RRC – Position	Relative Positions	Relative Position 1
Lower Restriction Rule Curve (Lower RRC)	Lower RRC – Curvature		Relative Position 2
	Lower RRC – Position		Relative Position 3
Percentage Restrictable	Percentage Restrictable 1	Security of Supply	Reliability Threshold
	Percentage Restrictable 2		Minimum Storage Threshold
	Percentage Restrictable 3	Target Storage Curves	Target Storage Curves

Table 5-19 and Figure 5-17 show the μ* index of the 20 year simulation period scenarios. These results show that the most important group of input variables for each scenario is the security criteria, followed by the lower RRC group. The ranking of the remaining groups of variables show some stability over the scenarios with the percentage restrictable, the upper RRC and the target curves groups ranked three, four and five over most of the scenarios. The base curves and relative position groups are ranked six and seven for most of the scenarios.

The security criteria group and the lower RRC group are the most important variables for all scenarios in the 40 year scenarios (shown in Table 5-20 and Figure 5-18), the 60 year scenarios, (Table 5-21 and Figure 5-19) and the 77 year scenarios (Table 5-22 and Figure 5-20). The rankings of the remaining variables change providing little conclusions with these results.

Table 5-19. Results of the Grouping 2 Morris Method Experiment - 20 Year Planning Period.

Morris Index	Group	Scen1	Scen2	Scen3	Scen4	Scen5	Scen2b	Scen2c
μ*	Relative Position	1714	1949	1568	1000	790	1851	1610
	Percentage Restrictable	2913	4165	2334	1967	1674	3656	2577
	Upper RRC	2940	3540	1790	1725	1539	3263	2233
	Lower RRC	4347	5867	2616	2510	2294	4823	3412
	Base Demand	1741	3111	1238	1232	1158	2104	1736
	Target Curves	2381	3009	1544	1316	1348	2724	1758
	Security Criteria	5828	7184	4168	3225	2623	6845	4102
σ	Relative Position	4461	6020	4258	2584	2329	5393	3847
	Percentage Restrictable	6438	10575	5477	4120	4194	9017	6016
	Upper Curve	5452	8897	5213	3718	3652	8004	4487
	Lower Curve	7897	12380	6026	4847	4427	9926	6728
	Base Demand	4410	7465	2919	3076	2923	5602	4205
	Target Curves	5110	8664	3508	3110	3323	7585	4300
	Security Criteria	9020	12357	8094	5256	4736	11566	6675

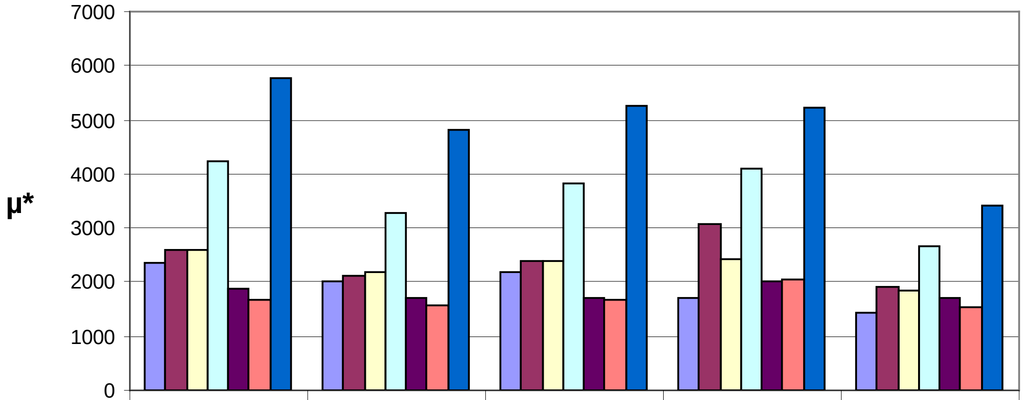


	Scen1	Scen2	Scen3	Scen4	Scen5	Scen2b	Scen2c
Relative Position	1714	1949	1568	1000	790	1851	1610
Perc. Restrictable	2913	4165	2334	1967	1674	3656	2577
Upper Curve	2940	3540	1790	1725	1539	3263	2233
Low er Curve	4347	5867	2616	2510	2294	4823	3412
Base Demand	1741	3111	1238	1232	1158	2104	1736
Target Curves	2381	3009	1544	1316	1348	2724	1758
Security Criteria	5828	7184	4168	3225	2623	6845	4102

Figure 5-17. μ* Results of the Grouping 2 Morris Method Experiment - 20 Year Planning Period.

Table 5-20. Results of the Grouping 2 Morris Method Experiment – 40 Year Planning Period.

Morris Index	Group	Scen1	Scen2	Scen3	Scen4	Scen5
μ*	Relative Position	2344	2017	2176	1720	1432
	Percentage Restrictable	2591	2122	2403	3089	1927
	Upper Curve	2588	2176	2380	2441	1831
	Lower Curve	4220	3280	3816	4084	2668
	Base Demand	1887	1723	1699	2016	1704
	Target Curves	1684	1567	1669	2044	1537
	Security Criteria	5755	4799	5247	5210	3401
σ	Relative Position	7170	5763	6661	4969	4039
	Percentage Restrictable	6175	4833	5561	7448	4657
	Upper RRC	8066	6552	7319	5783	4008
	Lower RRC	9995	7992	9118	8083	5467
	Base Demand	4921	4364	4470	5162	3871
	Target Curves	4049	3607	3913	5480	3692
	Security Criteria	11784	9311	10250	9302	6211

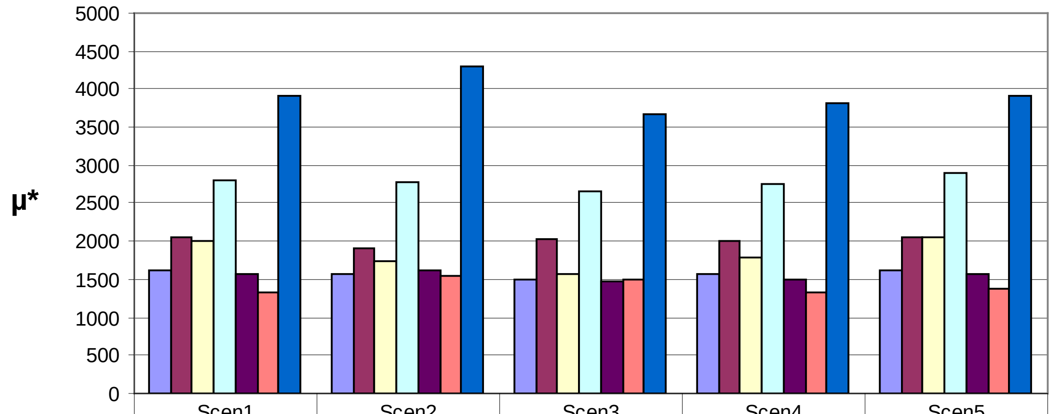


	Scen1	Scen2	Scen3	Scen4	Scen5
Relative Position	2344	2017	2176	1720	1432
Perc. Restrictable	2591	2122	2403	3089	1927
Upper Curve	2588	2176	2380	2441	1831
Low er Curve	4220	3280	3816	4084	2668
Base Demand	1887	1723	1699	2016	1704
Target Curves	1684	1567	1669	2044	1537
Security Criteria	5755	4799	5247	5210	3401

Figure 5-18. μ* Results of the Grouping 2 Morris Method Experiment - 40 Year Planning Period.

Table 5-21. Results of the Grouping 2 Morris Method Experiment – 60 Year Planning Period.

Morris Index	Group	Scen1	Scen2	Scen3	Scen4	Scen5
μ*	Relative Position	1622	1571	1489	1568	1629
	Percentage Restrictable	2061	1914	2033	1996	2054
	Upper RRC	1995	1734	1562	1789	2052
	Lower RRC	2790	2788	2667	2764	2895
	Base Demand	1579	1614	1472	1503	1571
	Target Curves	1333	1550	1488	1338	1381
	Security Criteria	3912	4310	3679	3818	3923
σ	Relative Position	4937	4064	3940	4802	4946
	Percentage Restrictable	4421	4394	4662	4591	4569
	Upper RRC	5496	3711	3459	4750	5553
	Lower RRC	6717	5314	5299	6706	7124
	Base Demand	3981	3555	3237	3664	4018
	Target Curves	2905	3739	3490	2807	2960
	Security Criteria	7730	7259	6259	7610	7658



	Scen1	Scen2	Scen3	Scen4	Scen5
Relative Position	1622	1571	1489	1568	1629
Perc. Restrictable	2061	1914	2033	1996	2054
Upper Curve	1995	1734	1562	1789	2052
Low er Curve	2790	2788	2667	2764	2895
Base Demand	1579	1614	1472	1503	1571
Target Curves	1333	1550	1488	1338	1381
Security Criteria	3912	4310	3679	3818	3923

Figure 5-19. μ* Results of the Grouping 2 Morris Method Experiment - 60 Year Planning Period.

Table 5-22. Results of the Grouping 2 Morris Method Experiment – 77 Year Planning Period.

Morris Index	Group	Scen1	Scen2	Scen3
μ*	Relative Position	1463	1611	1670
	Percentage Restrictable	1820	2509	2259
	Upper RRC	1693	2186	2246
	Lower RRC	2647	3551	2903
	Base Demand	1600	1933	2119
	Target Curves	1534	1899	1435
	Security Criteria	4000	5035	4393
σ	Relative Position	3850	4466	4859
	Percentage Restrictable	4359	6185	5198
	Upper RRC	3709	5382	5248
	Lower RRC	5248	7321	6101
	Base Demand	3555	5123	4974
	Target Curves	3743	4799	3603
	Security Criteria	6858	9107	7753

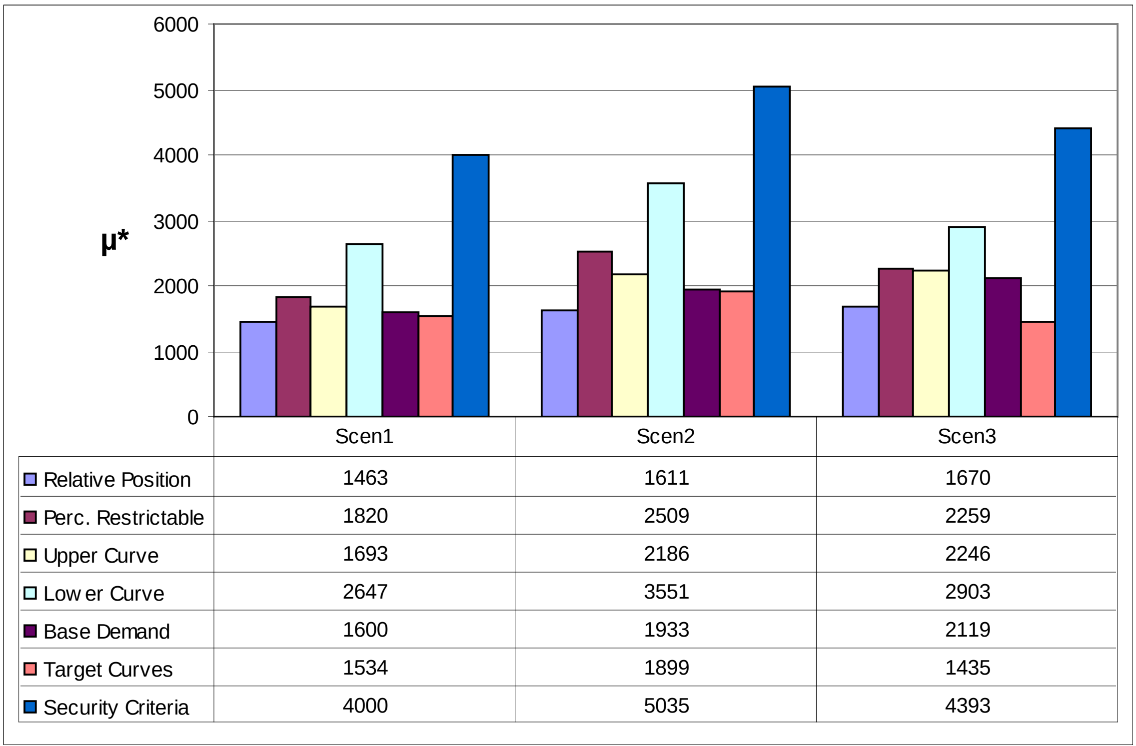


Figure 5-20. μ* Results of the Grouping 2 Morris Method Experiment - 77 Year Planning Period.

Based on Tables 5-19, 5-20, 5-21 and 5-22 and Figures 5-17, 5-18, 5-19 and 5-20 given above, the following conclusions can be drawn from the grouping 2 Morris method experiments:

1. All experiments indicate that the security criteria group and the lower RRC group are the most important. The rankings of the remaining groups show consistency across scenarios in the 20 year planning length (Figure 5-17) but inconsistent results for the 40, 60 and 77 year planning length scenarios (Figures 5-18, 5-19 and 5-20, respectively).

2. All groups show influence on the estimation of yield (indicated by the non-zero μ^* indices) and show considerable interaction or non-linear effects (indicated by non-zero σ).

3. A large range of magnitudes of the μ^* and σ indices exists across scenarios of the same planning length and also across different planning lengths. No obvious trends exist regarding the total streamflow volume or the planning lengths.

4. This experiment showed that the lower RRC group was important, which is contradictory to the individual experiment results that shows the upper RRC curvature and position as more important. This could be a result of a synergy of the lower RRC when grouped, or by some cancelling out of the upper RRC variables when grouped.

5.4.1.4 Summary of Morris Experiments

The following conclusions can be deduced from comparison of the individual, grouping 1 and grouping 2 experiments:

1. The μ^* and σ indices across the grouping 1 and grouping 2 experiments have somewhat similar magnitude, while most μ^* and σ indices in the individual experiments are considerably different. Consider the relative position 1, 2 and 3 variables. The μ^* indices of these variables in the individual experiments are considerably less than the μ^* for the relative position group in grouping 2 experiment. This shows that the yield estimate is effected more when they are changed at the same time, indicating a synergistic effect. On the other hand, the two security of supply thresholds seem to have 'cancelling out' effect when changed at the same time.

2. No trends are evident across planning periods or between scenarios within a single simulation length. In other words, neither the streamflow volume nor the planning length have significant influence on the importance of input variables used in the estimation of yield. Climate variability, however, does effect the importance of input

variables, and groups of, as shown by the significant differences in the μ, μ* and σ indices across scenario 2, 2b and 2c of the 20 year planning period and between the 77 year planning period scenarios. (If these indices showed little change amongst scenarios 2, 2b and 2c it would suggest that climate variability is not influential.) These findings are explored further in the results of the variance based methods (Section 5.4.2).

The Morris method was used successfully to identify important input variables and groups of input variables in the estimation of yield of an urban water supply system. The primary aim of the Morris method was to screen out input variables that show no importance over all scenarios for all simulation periods. All variables and groups of input variables show some importance with a number of input variables that have a cause a high sensitivity, therefore none will be omitted from the following SA using the variance based techniques.

5.4.2 Variance Based Method Results

The extended Fourier Amplitude Sensitivity Test (eFAST) was used here to estimate the first- and total-order importance measures of input variables used in the estimation of yield of the Barwon urban water supply system. Only eFAST (see Section 3.5.2) was used here as it accurately estimates the same first-order importance index as the classic FAST in less model simulations with the addition of estimating the total-order importance index (See Section 4.5 for comparison). The same grouping experiments as the Morris method experiments were performed using the eFAST technique. Experiments using the method of Sobol’ (see Section 3.5.3 for details on the Sobol’ method) were also performed in which first-, second- and total-order importance indices are estimated.

As explained in Section 3.4.3.6, the first- (S_i) and total-order (S_{Ti}) indices determined using the eFAST and Sobol’ methods are theoretically the same. However, these methods use different numerical approximations that lead to slight discrepancies. As seen in Section 4.5, the Sobol’ method can provide erroneous results such as negative importance indices and $S_i > S_{Ti}$. If these are encountered, increasing accuracy Sobol’ experiments should be performed until satisfactory results are met, i.e. $S_i \geq 0$ and $S_i < S_{Ti}$. However this may not be possible as the next accurate Sobol’ experiment would require twice the number of model simulations. If indeed the number of model simulations becomes infeasible to compute, the erroneous results can be analysed in a qualitative manner, disregarding any quantitative inconsistencies. By design, the eFAST algorithm never produces negative indices but can produce $\Sigma S_i > 1$ or $S_i > S_{Ti}$ for a given experiment. If this occurs, the number of simulations that eFAST is performed over should be increased.

Details of the FAST and Sobol' techniques are in Sections 3.5.2 and 3.5.3, respectively. The following importance indices are used in the experiments using the variance based techniques:

S_i – first-order importance index which measures the effect of the *i*-th input variable, free of interaction and higher-order effects, where $S_i = V_i/V(Y)$. V_i and $V(Y)$ are explained below.

S_{Ti} – total-order sensitivity index that measures the combined first- and higher-order effect of the *i*-th input variable. This includes all interaction effects involving the *i*-th input variable, i.e. for a three variable model: $S_{Ti} = S_i + S_{ij} + S_{ik} + S_{ijk}$. See below for definition of S_{ij} and S_{ijk}.

S_{ij}^c – 'closed' second-order importance index that measures the effect of the *i*-th and *j*-th input variables, individually and combined. S_{ij}^c is only calculated using the method of Sobol'.

S_{ij} – second-order importance index quantifying the combined effect of the *i*-th and *j*-th input variables only. It is calculated using the closed index, i.e. $S_{ij} = S_{ij}^c - S_i - S_j$, therefore can only be determined using the Sobol' method. This index can be expanded to even higher-order indices such as S_{ijk}, which is the combined effect of the *i*-th, *j*-th, and *k*-th input variables.

The above four indices are standardised indices within an SA experiment. Because of this, comparison between experiments can only be qualitative comments on the ranking of importance of the input variables. To observe the absolute effect of the input variables on the estimation of yield in a method that allows a better comparison between scenarios and experiments, the following measures are used:

$V(Y)$- the total variance of the yield estimate (Y) due to all input variables, including individual and combined effects. A single $V(Y)$ value is computed for each scenario. The value of $V(Y)$ indicates volatility of the estimation of yield to changes of all input variables for each scenario. Scenarios with a high yield estimate variance are more sensitive to the changes to the input variables, suggesting that the positions, states or values of the input variables are more important for that climate sequence than scenarios with a low yield variance. Conversely, a scenario with a low output variance is robust against changes in the input variables and the input variables are then less important for that scenario.

V_i – is a non-standardised measure of the partial output variance due to the *i*-th input variable only. A V_i measure is computed for each variable in every scenario. It demonstrates the effect of the *i*-th variable on the estimation of yield, comparable between scenarios and experiments. A changing V_i across scenarios and planning periods reflects the change of importance of the input variable due to the climate sequence or planning length used. This change of importance may not become clear using the standardised S_i index.

5.4.2.1 Individual Experiments

This section presents the results of the individual input variable experiments similar to Section 5.4.1.1. The S_i and S_{Ti} indices for the eFAST SA experiments (using 1918 model simulations) are presented in Tables 5-23 and 5-24. The graphs of the eFAST experiments showing the S_i and S_{Ti} results for the individual experiments are given in Appendix D. However, the results for the reliability threshold and minimum storage level threshold are shown in Figures 5-21 and 5-22 respectively, as examples since they are the most important input variables identified by the SA study.

Table 5-23 shows that the most important input variables in all scenarios of all experiments are the security criteria thresholds: the minimum storage threshold and the reliability threshold. For all scenarios, either the minimum storage threshold or the reliability threshold is the most important input variable. This corresponds to the most critical threshold within each scenario (the threshold that will causes system failure at the lowest AAD - Average Annual Demand): an indication of the dependence of these variables on the climate sequence. For instance, the reliability threshold is the most important variable for all 20 year scenarios, except scenario 3, which can be seen from the S_i results in Figures 5-21 and 5-22 for both variables. The 20 year scenario 3 streamflow sequence has a large drought that causes a severe drawdown; violating the minimum storage criteria. Whereas the minimum storage threshold is the most important for all 40 year scenarios, except scenario 4 which has a relatively constant climate variability with no severe dry periods. This constant climate results in the reliability threshold is violated at a lower AAD than required to violate the minimum storage threshold, therefore making the reliability threshold critical and important in scenario 4. Interestingly, the reliability threshold has little importance in the 60 and 77 year scenarios. This is due to these scenarios containing a severe drought that causes large system drawdown, hence violating the minimum storage threshold.

Table 5-23. S_i Results of the Individual eFAST Experiment using 1918 Simulations

		First-Order Sensitivity Index (S_i)														
Simulation Period	Scenario	Relative Position 1	Relative Position 2	Relative Position 3	Percent. Restrict. 1	Percent. Restrict. 2	Percent. Restrict. 3	Upper RRC Curvature	Upper RRC Position	Lower RRC Curvature	Lower RRC Position	Base Demand	Target Curves	Minimum Storage Threshold	Reliability Threshold	SUM
20 Year	1	0.0031	0.0065	0.0068	0.0009	0.0077	0.0007	0.0417	0.0935	0.0018	0.0053	0.0064	0.0045	0.0229	0.7519	0.9537
	2	0.0021	0.0019	0.0045	0.0019	0.0025	0.0003	0.0021	0.0300	0.0028	0.0051	0.0024	0.0084	0.3075	0.4515	0.8230
	3	0.0012	0.0022	0.0045	0.0176	0.0011	0.0022	0.0066	0.0057	0.0026	0.0029	0.0043	0.0050	0.5918	0.0901	0.7378
	4	0.0016	0.0119	0.0083	0.0033	0.0057	0.0023	0.0119	0.0234	0.0034	0.0022	0.0026	0.0022	0.1393	0.6088	0.8269
	5	0.0035	0.0076	0.0040	0.0036	0.0078	0.0013	0.0166	0.0185	0.0066	0.0029	0.0056	0.0026	0.2384	0.5200	0.8390
	2b	0.0036	0.0062	0.0095	0.0021	0.0018	0.0019	0.0040	0.0369	0.0047	0.0040	0.0042	0.0147	0.2675	0.4837	0.8448
	2c	0.0087	0.0112	0.0059	0.0029	0.0169	0.0047	0.0074	0.0383	0.0005	0.0067	0.0130	0.0045	0.1448	0.6626	0.9281
40 Year	1	0.0055	0.0031	0.0075	0.0094	0.0041	0.0089	0.0049	0.0059	0.0079	0.0036	0.0050	0.0023	0.6632	0.0719	0.8032
	2	0.0057	0.0017	0.0037	0.0165	0.0043	0.0069	0.0042	0.0091	0.0040	0.0025	0.0035	0.0033	0.6924	0.0499	0.8077
	3	0.0086	0.0018	0.0072	0.0110	0.0028	0.0031	0.0048	0.0079	0.0086	0.0023	0.0044	0.0023	0.6588	0.0671	0.7907
	4	0.0038	0.0041	0.0065	0.0016	0.0031	0.0030	0.0081	0.0219	0.0012	0.0025	0.0031	0.0043	0.2761	0.5096	0.8489
	5	0.0027	0.0021	0.0012	0.0060	0.0021	0.0007	0.0017	0.0029	0.0012	0.0014	0.0117	0.0006	0.5805	0.1294	0.7443
60 Year	1	0.0081	0.0026	0.0040	0.0124	0.0039	0.0064	0.0053	0.0093	0.0066	0.0029	0.0028	0.0035	0.6047	0.1184	0.7909
	2	0.0022	0.0035	0.0006	0.0042	0.0031	0.0007	0.0021	0.0022	0.0007	0.0014	0.0232	0.0027	0.8303	0.0230	0.9000
	3	0.0004	0.0006	0.0002	0.0044	0.0015	0.0021	0.0002	0.0004	0.0013	0.0007	0.0121	0.0010	0.7073	0.0545	0.7869
	4	0.0011	0.0049	0.0024	0.0030	0.0041	0.0014	0.0047	0.0029	0.0039	0.0026	0.0016	0.0031	0.5740	0.1569	0.7666
	5	0.0084	0.0035	0.0041	0.0075	0.0037	0.0035	0.0065	0.0095	0.0082	0.0027	0.0029	0.0022	0.5653	0.1462	0.7742
77 Year	1	0.0025	0.0029	0.0004	0.0044	0.0028	0.0005	0.0019	0.0014	0.0011	0.0012	0.0170	0.0031	0.8069	0.0313	0.8774
	2	0.0027	0.0058	0.0012	0.0027	0.0041	0.0037	0.0020	0.0037	0.0030	0.0013	0.0077	0.0010	0.7079	0.1021	0.8489
	3	0.0048	0.0028	0.0019	0.0035	0.0073	0.0028	0.0034	0.0020	0.0016	0.0020	0.0143	0.0029	0.7029	0.0640	0.8162

Table 5-24. S_{Ti} Results of the Individual eFAST Experiment using 1918 Simulations

		Total-Order Sensitivity Index (S_{Ti})														
Simulation Period	Scenario	Relative Position 1	Relative Position 2	Relative Position 3	Percent. Restrict. 1	Percent. Restrict. 2	Percent. Restrict. 3	Upper RRC Curvature	Upper RRC Position	Lower RRC Curvature	Lower RRC Position	Base Demand	Target Curves	Minimum Storage Threshold	Reliability Threshold	SUM
20 Year	1	0.0578	0.0997	0.0986	0.0483	0.0967	0.0376	0.1335	0.1657	0.0922	0.1041	0.0833	0.1135	0.0770	0.8448	2.0528
	2	0.0451	0.0632	0.0645	0.0438	0.0728	0.0360	0.0668	0.0836	0.0454	0.0843	0.0426	0.0944	0.4496	0.5846	1.7767
	3	0.1739	0.1177	0.1185	0.1536	0.1173	0.1238	0.0845	0.1311	0.1193	0.0879	0.1315	0.0806	0.7984	0.2425	2.4806
	4	0.0435	0.0836	0.1458	0.0718	0.1090	0.0420	0.1060	0.0804	0.0799	0.1077	0.0353	0.1347	0.2920	0.8000	2.1316
	5	0.0557	0.0768	0.1048	0.0830	0.1252	0.0465	0.0971	0.0715	0.1001	0.0901	0.0416	0.1064	0.4033	0.7141	2.1161
	2b	0.0585	0.0767	0.1003	0.0691	0.0759	0.0530	0.0734	0.1021	0.0657	0.0777	0.0609	0.1258	0.4257	0.6422	2.0070
	2c	0.1043	0.1402	0.0861	0.0719	0.1928	0.0813	0.0960	0.1299	0.1332	0.0952	0.0973	0.1554	0.2799	0.8493	2.5129
40 Year	1	0.1042	0.0824	0.1003	0.0928	0.0941	0.1306	0.0890	0.1169	0.1027	0.0951	0.0867	0.0833	0.8802	0.2697	2.3282
	2	0.0983	0.0685	0.0758	0.1269	0.0802	0.1245	0.0900	0.1288	0.0699	0.0812	0.0952	0.0740	0.8865	0.2125	2.2124
	3	0.1149	0.0794	0.0887	0.1027	0.0954	0.1211	0.0819	0.1236	0.1114	0.0912	0.0889	0.0760	0.8660	0.2397	2.2809
	4	0.0508	0.0478	0.0773	0.0494	0.0586	0.0523	0.0624	0.0861	0.0393	0.0605	0.0550	0.0800	0.4505	0.6838	1.8539
	5	0.0490	0.0433	0.0222	0.0926	0.0529	0.0611	0.0378	0.0485	0.0494	0.0251	0.0444	0.0266	0.7544	0.3309	1.6383
60 Year	1	0.1086	0.0760	0.0634	0.1531	0.0749	0.1286	0.0862	0.1040	0.0694	0.0722	0.0631	0.0984	0.8411	0.3410	2.2798
	2	0.0461	0.0489	0.0597	0.0649	0.0681	0.0583	0.0384	0.0499	0.0583	0.0395	0.0602	0.0452	0.9341	0.1295	1.7011
	3	0.0260	0.0336	0.0250	0.0870	0.0355	0.0485	0.0313	0.0374	0.0338	0.0236	0.0393	0.0255	0.8277	0.1327	1.4068
	4	0.0545	0.0748	0.0617	0.0700	0.0737	0.0626	0.0684	0.0597	0.0620	0.0719	0.0377	0.0708	0.7968	0.3804	1.9448
	5	0.1042	0.0751	0.0666	0.1172	0.0808	0.1146	0.0818	0.0974	0.0697	0.0739	0.0593	0.0652	0.8083	0.3800	2.1942
77 Year	1	0.0466	0.0426	0.0440	0.0724	0.0658	0.0589	0.0387	0.0468	0.0603	0.0357	0.0538	0.0405	0.9246	0.1491	1.6798
	2	0.0647	0.0548	0.0283	0.0687	0.0408	0.0620	0.0538	0.0410	0.0582	0.0370	0.0478	0.0403	0.8815	0.2893	1.7683
	3	0.0956	0.0664	0.0408	0.0925	0.0688	0.1033	0.0571	0.0422	0.0784	0.0543	0.0828	0.0352	0.8870	0.2665	1.9710

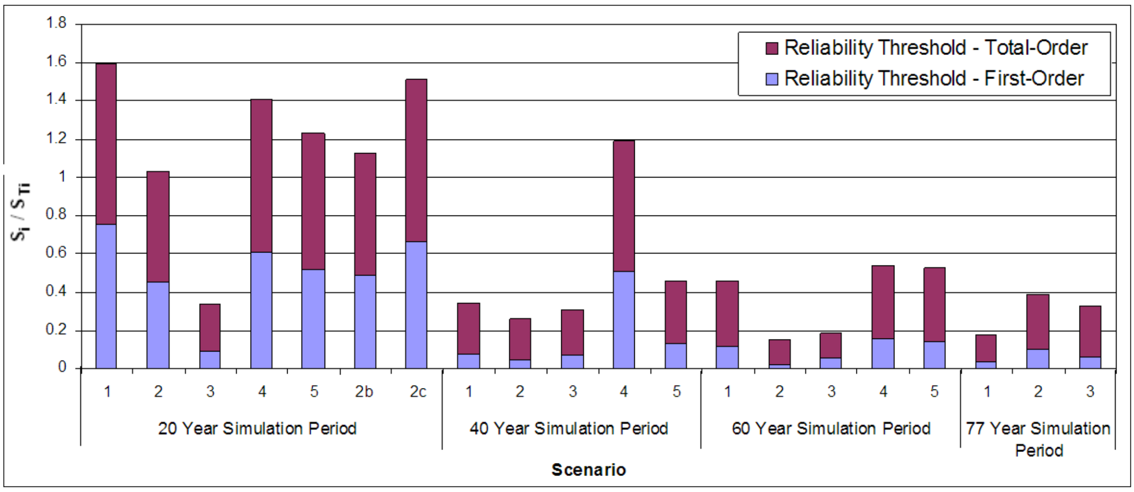


Figure 5-21. eFAST Individual Experiment. Reliability of Supply Threshold. S_i and S_{Ti} Results for all Scenarios.

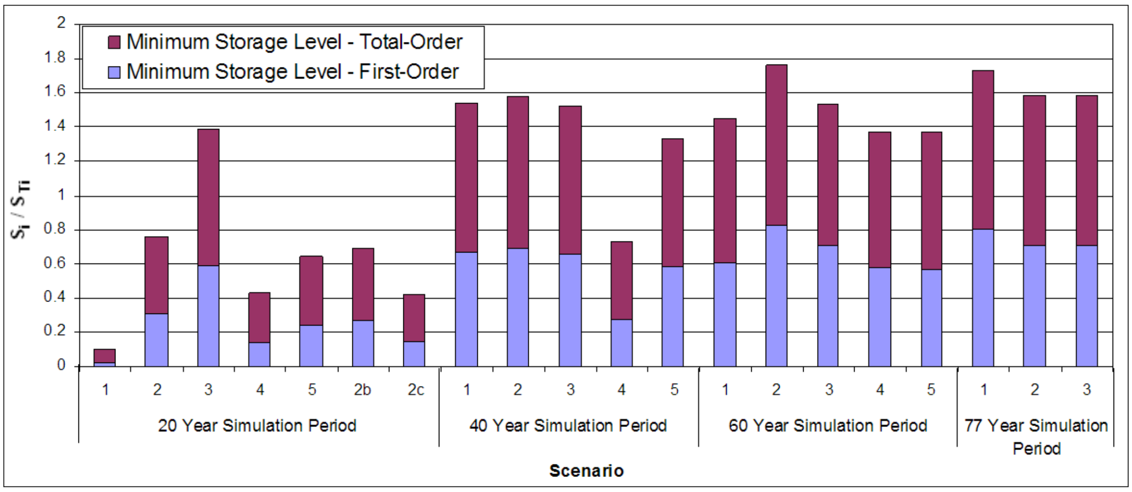


Figure 5-22. eFAST Individual Experiment. Minimum Storage Level Threshold. S_i and S_{Ti} Results for all Scenarios.

Results of the Morris method experiments (Section 5.4.1.1) indicated that correlation of the importance indices of input variables existed. Section 5.4.1.1 described that when the reliability of supply threshold is the most important, the upper RRC curvature and upper RRC position variables are important. Similarly, when the minimum storage threshold is the most important the other variables have somewhat indistinguishable μ* measures. This is also identifiable in S_i and S_{Ti} results shown in Figures 5-21 and 5-22, and also from some of the charts of the S_i and S_{Ti} indices of the eFAST individual experiments presented in Appendix D. Progressing with this investigation, Table 5-25 presents a correlation of the S_i importance measures of all input variables.

Table 5-25. Correlation Matrix of the First-Order Indices (S_i) for the eFAST Individual Experiment.

	Relative Position 1	Relative Position 2	Relative Position 3	Percent. Restrict. 1	Percent. Restrict. 2	Percent. Restrict. 3	Upper RRC Curvature	Upper RRC Position	Lower RRC Curvature	Lower RRC Position	Base Demand	Target Curves	Minimum Storage Threshold	Reliability Threshold
Relative Position 1	1													
Relative Position 2	0.043	1												
Relative Position 3	0.325	0.450	1											
Percent. Restrict. 1	0.309	-0.459	0.007	1										
Percent. Restrict. 2	0.417	0.693	0.184	-0.281	1									
Percent. Restrict. 3	0.599	-0.057	0.274	0.529	0.182	1								
Upper RRC Curvature	-0.008	0.414	0.394	-0.226	0.366	-0.171	1							
Upper RRC Position	0.046	0.469	0.553	-0.386	0.419	-0.195	0.852	1						
Lower RRC Curvature	0.536	-0.103	0.412	0.398	-0.165	0.472	-0.003	-0.142	1					
Lower RRC Position	0.352	0.439	0.602	-0.153	0.612	0.135	0.458	0.729	0.043	1				
Base Demand	-0.155	-0.078	-0.593	-0.240	0.157	-0.292	-0.203	-0.206	-0.586	-0.294	1			
Target Curves	-0.036	0.121	0.528	-0.170	-0.078	-0.164	0.043	0.428	-0.006	0.525	-0.248	1		
Minimum Storage Threshold	-0.016	-0.708	-0.671	0.443	-0.492	0.195	-0.666	-0.819	0.070	-0.712	0.408	-0.418	1	
Reliability Threshold	-0.010	0.748	0.613	-0.541	0.542	-0.236	0.662	0.833	-0.164	0.699	-0.284	0.411	-0.984	1

The correlations are calculated using all scenarios over all planning lengths. From this table it can be seen that a number of strong correlations between the importance indices of variables exist. A positive correlation demonstrates that when the importance of an input variable increases, the importance of the correlated variable also increases. When there is a negative correlation, the importance of the other variable decreases. The strong negative correlation between the reliability threshold and minimum storage threshold confirms the suspected correlations of the important of variables discussed above. The positive correlations between the reliability threshold variable and both the upper RRC position and curvature variables are also understandable as the upper RRC directly affects the reliability threshold. The remaining correlations are of little interest.

The difference between the first- and total-order importance measures can be gained from a comparison of Tables 5-23 and 5-24. The highlighted cells in Tables 5-23 and 5-24 designate the variables that increase by greater than 0.1 between S_i and S_{Ti}. The minimum storage threshold, the reliability threshold and a few other input variables in the 20 year scenarios show a large increase between S_i and S_{Ti}, implying interactions and/or higher-order effects. No other results are clearly obvious from Tables 5-23 and 5-24.

The S_i and S_{Ti} indices for the Sobol' SA experiments using 6848 model simulations are given in Tables 5-26 and 5-27, respectively. However, these results are unsatisfactory due to the large number of negative results. The Sobol' experiments also determined second-order importance indices, S_{ij}^c and S_{ij}, but these are also unsatisfactory (note that indices from the variance based methods should be always be positive). This is a limitation of the Sobol' method, as the algorithm produces negative sensitivities when a model has a number of variables with relatively negligible importance (Saltelli et al., 2004). The S_{ij}^c and S_{ij} results of 20 year scenarios can be seen in Appendix D from which the negative results can be seen. All other scenarios had similar errors.

Table 5-26. S_i Results of the Individual Sobol' Second-Order Experiment Using 6848 Model Simulations.

		First-Order Sensitivity Index (S_i)														
Simulation Period	Scenario	Relative Position 1	Relative Position 2	Relative Position 3	Percent. Restrict. 1	Percent. Restrict. 2	Percent. Restrict. 3	Upper RRC Curvature	Upper RRC Position	Lower RRC Curvature	Lower RRC Position	Base Demand	Target Curves	Minimum Storage Threshold	Reliability Threshold	SUM
20 Year	1	-0.0061	0.0010	0.0013	0.0063	0.0043	0.0002	0.0446	0.0335	0.0018	-0.0011	-0.0053	0.0474	0.0608	0.7260	0.9148
	2	-0.0066	0.0011	-0.0007	-0.0066	0.0079	-0.0012	0.0183	0.0110	0.0027	0.0005	-0.0311	-0.0127	0.4404	0.4563	0.8793
	3	0.0013	-0.0014	0.0019	0.0002	0.0000	-0.0061	0.0081	0.0200	-0.0008	-0.0010	-0.0020	0.0357	0.7262	0.2161	0.9981
	4	-0.0028	0.0005	0.0013	0.0020	-0.0012	0.0031	-0.0188	0.0029	-0.0008	0.0035	-0.0026	-0.1150	0.1696	0.5385	0.5802
	5	0.0020	0.0009	0.0009	-0.0033	-0.0073	0.0068	-0.0171	0.0412	0.0048	0.0178	-0.0121	0.0296	0.2782	0.4710	0.8131
	2b	0.0045	0.0620	0.0028	0.0025	0.0031	0.0005	0.0573	0.0055	0.0442	0.0505	0.0521	-0.0252	0.3091	0.5437	1.1126
	2c	0.0011	-0.0022	-0.0004	-0.0045	-0.0009	-0.0002	0.0279	-0.0188	0.0027	0.0021	-0.0044	-0.0664	0.1384	0.5808	0.6551
40 Year	1	0.0564	0.0060	0.0084	-0.0022	-0.0007	0.0074	0.0797	0.0072	0.0046	-0.0008	0.0041	0.0411	0.7915	0.1640	1.1665
	2	0.0017	-0.0005	-0.0023	-0.0026	-0.0152	-0.0093	-0.0137	0.0035	0.0020	0.0010	0.0089	0.0488	0.9063	0.1390	1.0677
	3	0.0064	0.0018	-0.0016	-0.0001	0.0019	-0.0011	0.0225	0.0147	0.0045	0.0025	-0.0028	0.0579	0.8335	0.1845	1.1247
	4	0.0015	0.0016	0.0059	0.0036	0.0025	-0.0010	-0.0073	-0.0024	0.0004	-0.0011	-0.0007	-0.0032	0.2998	0.3371	0.6367
	5	-0.0003	-0.0052	0.0013	-0.0030	-0.0046	-0.0051	-0.0311	0.0156	0.0030	0.0000	0.0018	0.0257	0.7587	0.1107	0.8676
60 Year	1	0.0042	-0.0007	-0.0046	-0.0039	0.0053	-0.0031	-0.0162	0.0104	0.0014	0.0032	0.0005	0.0145	0.7704	0.2145	0.9959
	2	-0.0065	-0.0002	0.0020	0.0002	0.0035	-0.0002	-0.0036	0.0259	-0.0005	0.0019	-0.0050	0.0529	0.9718	0.0400	1.0823
	3	-0.0037	-0.0058	-0.0016	-0.0047	-0.0056	-0.0009	0.0033	0.0171	-0.0062	-0.0021	-0.0083	-0.0054	0.9100	0.1107	0.9969
	4	0.0106	0.0009	-0.0039	0.0015	0.0071	0.0008	-0.0144	0.0124	0.0014	-0.0009	0.0104	0.0320	0.6962	0.2257	0.9798
	5	0.0020	-0.0002	-0.0061	-0.0088	0.0020	-0.0040	-0.0258	0.0161	0.0008	0.0001	-0.0021	0.0194	0.7439	0.2513	0.9886
77 Year	1	-0.0086	-0.0031	0.0015	0.0021	0.0036	-0.0011	0.0006	0.0259	0.0000	0.0017	-0.0098	0.0679	0.9275	0.0520	1.0602
	2	-0.0064	0.0000	0.0044	-0.0021	-0.0032	0.0055	-0.0187	0.0152	-0.0043	0.0000	-0.0057	0.0905	0.7368	0.1294	0.9415
	3	0.0009	0.0019	-0.0035	0.0115	0.0040	0.0003	-0.0026	-0.0040	-0.0038	0.0029	0.0044	0.0730	0.7830	0.1247	0.9927

Table 5-27. S_{Ti} Results of the Individual Sobol' Second-Order Experiment using 6848 Simulations.

		Total-Order Sensitivity Index (S_{Ti})														
Simulation Period	Scenario	Relative Position 1	Relative Position 2	Relative Position 3	Percent. Restrict. 1	Percent. Restrict. 2	Percent. Restrict. 3	Upper RRC Curvature	Upper RRC Position	Lower RRC Curvature	Lower RRC Position	Base Demand	Target Curves	Minimum Storage Threshold	Reliability Threshold	SUM
20 Year	1	0.0013	0.0014	0.0019	0.0015	0.0049	0.0017	0.0828	0.0609	-0.0059	0.0093	0.0025	0.0349	0.0550	0.7446	0.9969
	2	-0.0032	0.0003	-0.0006	-0.0025	-0.0013	-0.0006	0.0272	0.0323	-0.0005	0.0044	-0.0002	0.0026	0.5702	0.5871	1.2154
	3	0.0023	-0.0016	-0.0008	0.0027	-0.0055	-0.0017	-0.0151	-0.0087	0.0033	-0.0007	0.0016	0.0469	0.8911	0.2521	1.1660
	4	0.0073	-0.0012	-0.0028	0.0123	0.0118	0.0082	0.0448	0.0470	0.0130	0.0042	0.0040	0.1309	0.3538	0.7619	1.3952
	5	-0.0048	0.0001	-0.0052	0.0091	0.0026	0.0004	0.0299	0.0152	0.0027	0.0036	-0.0122	0.1676	0.4554	0.7010	1.3655
	2b	0.0017	0.0182	-0.0049	-0.0032	-0.0018	-0.0020	0.0544	0.0401	0.0427	0.0418	0.0467	0.0493	0.4164	0.6371	1.3365
	2c	-0.0065	-0.0004	-0.0065	0.0006	-0.0073	-0.0071	0.0284	-0.0225	-0.0083	-0.0072	-0.0060	0.0838	0.2039	0.7080	0.9529
40 Year	1	0.1268	0.1277	0.1221	0.1242	0.1273	0.1276	0.1284	0.1218	0.1375	0.1428	0.1149	0.1878	0.9791	0.3280	2.8961
	2	0.0020	0.0003	0.0007	0.0012	-0.0044	-0.0006	-0.0002	-0.0047	0.0033	0.0038	0.0074	0.0810	1.0525	0.1213	1.2636
	3	-0.0049	-0.0011	-0.0001	-0.0003	-0.0011	-0.0029	0.0178	0.0099	-0.0011	0.0052	0.0023	0.0802	1.0089	0.1725	1.2854
	4	-0.0015	0.0024	-0.0032	0.0008	0.0043	0.0045	0.0242	0.0195	0.0054	0.0096	-0.0039	0.0687	0.4610	0.6250	1.2168
	5	0.0020	0.0013	-0.0027	-0.0015	-0.0099	-0.0078	-0.0030	-0.0033	-0.0002	-0.0118	-0.0128	0.0355	1.0098	0.2091	1.2048
60 Year	1	0.0207	0.0181	0.0002	0.0170	0.0237	0.0165	0.0353	0.0196	0.0148	0.0240	0.0169	0.0727	1.0434	0.2659	1.5888
	2	0.0008	0.0043	-0.0022	-0.0110	-0.0040	0.0000	-0.0094	0.0095	0.0020	-0.0033	0.0134	0.0650	1.0599	0.0347	1.1595
	3	-0.0025	0.0014	-0.0018	0.0004	-0.0101	-0.0005	0.0030	0.0173	0.0024	0.0018	0.0136	0.0258	0.9548	0.1410	1.1466
	4	0.0147	0.0034	-0.0025	0.0005	0.0044	-0.0049	0.0222	0.0084	-0.0086	0.0063	0.0128	0.0828	1.0020	0.2665	1.4080
	5	-0.0020	-0.0005	-0.0016	-0.0050	-0.0012	-0.0047	0.0030	0.0025	-0.0023	0.0045	-0.0156	0.0458	1.0446	0.2371	1.3046
77 Year	1	0.0110	0.0034	0.0012	-0.0010	0.0017	0.0041	-0.0012	0.0040	0.0034	0.0008	0.0128	0.0875	1.0936	0.0407	1.2619
	2	-0.0013	-0.0042	0.0058	0.0052	0.0067	-0.0024	-0.0063	0.0020	0.0024	-0.0012	0.0083	0.1097	0.9606	0.2038	1.2892
	3	0.0071	0.0004	-0.0025	0.0228	0.0069	0.0087	0.0113	-0.0009	0.0029	-0.0057	0.0048	0.0318	1.0545	0.1067	1.2489

To improve the accuracy of the Sobol' experiment in an attempt to avoid erroneous results, experiments of greater sampling resolution (and therefore greater number of model simulations) should be used. However, the next Sobol’ experiment would require 13,696 model simulations, then 27,392 model simulations, and so on, i.e. the number of required model simulations doubles for each progressively greater sampling resolution experiment. Experiments using 27,392 model simulations were begun but initial results still gave erroneous S_i and S_{Ti}, and therefore were not completed. The next more accurate experiments would require 54,784 model simulations, which was deemed impracticable to calculate due to the amount of computational time required. Still some qualitative comments can be given based on the 6848 model simulation Sobol’ experiment. The S_i and S_{Ti} results this experiment show that the reliability threshold and the minimum storage threshold are the most important for all scenarios, with their ranking in each scenario matching the ranking of the FAST experiments. No other reliable comparisons can be made between the Sobol’ and the FAST experiments.

Table 5-28 shows the partial output variance due to each input variable (V_i) corresponding to each scenario, and the average V_i for each input variable under scenarios of the same length. The partial variances provide an alternative view of the effects of each input variable on the estimation of yield. Comparing each input variable across scenarios (of the same length and of different planning length) it can be seen that there are no discernable trends, demonstrating the variability in the yield estimate due to each input variable is not dependant on the planning length or the streamflow volume. From the partial variances it can again be seen that the reliability threshold and the minimum storage threshold cause the greatest amount of output variance, hence their high S_i results. A number of variables (relative position 2, percentage restrictable 2, upper RRC position and curvature, target curves and reliability threshold) exhibit a reduction of variance as the planning length increases from 20 to 60 years. This indicates that the yield estimate becomes less sensitive to changes in these variables as the planning length increases.

The 77 year scenarios and the 20 year scenarios 2, 2b and 2c show large changes in the V_i of many input variables. The percentage restrictable 1 and 2, lower RRC curvature, target curves, reliability threshold and minimum storage threshold show considerable range in the 20 year scenarios 2, 2b and 2c. The same do not show such a range in the 77 year scenarios. Indeed, the V_i range of all input variables in the 77 year scenarios are moderate. This could suggest that the variability of climate is significant for a shorter planning length than a long planning length.

Table 5-28. V_i Results of the Individual eFAST Experiment using 1918 Simulations.

		Partial Variance (V_i)													
Simulation Period	Scenario	Relative Position 1	Relative Position 2	Relative Position 3	Percent. Restrict. 1	Percent. Restrict. 2	Percent. Restrict. 3	Upper RRC Curvature	Upper RRC Position	Lower RRC Curvature	Lower RRC Position	Base Demand	Target Curves	Minimum Storage Threshold	Reliability Threshold
20 Year	Scen1	291	422	432	159	459	140	1069	1600	222	381	419	351	792	4538
	Scen2	374	356	547	356	408	138	374	1414	432	583	400	748	4525	5484
	Scen3	148	201	287	567	142	201	347	323	218	230	280	302	3289	1283
	Scen4	131	356	298	188	247	157	356	500	190	153	167	153	1219	2548
	Scen5	172	253	183	174	256	105	374	395	236	156	217	148	1416	2092
	Scen2b	406	533	660	310	287	295	428	1301	464	428	439	821	3502	4709
	Scen2c	438	497	360	253	610	322	404	918	107	384	535	315	1786	3820
	AVERAGE	280	374	395	287	344	194	479	921	267	331	351	405	2,361	3,496
40 Year	Scen1	497	373	580	649	429	632	469	514	595	402	474	321	5453	1796
	Scen2	399	218	322	679	347	439	343	505	335	264	313	304	4402	1182
	Scen3	554	254	507	627	316	333	414	531	554	287	397	287	4853	1549
	Scen4	353	366	461	229	319	313	515	847	198	286	319	375	3006	4084
	Scen5	181	160	121	270	160	94	144	188	121	131	377	89	2658	1255
	AVERAGE	397	274	398	491	314	362	377	517	361	274	376	275	4,074	1,973
60 Year	Scen1	408	231	287	505	283	362	330	437	368	244	240	268	3524	1559
	Scen2	177	223	95	244	210	100	173	177	99	141	574	196	3433	571
	Scen3	67	83	50	217	127	150	47	69	118	88	359	103	2748	763
	Scen4	132	279	195	219	255	149	274	215	249	203	160	222	3023	1580
	Scen5	419	270	293	396	278	270	369	446	414	238	246	214	3437	1748
	AVERAGE	241	217	184	316	231	206	238	269	250	183	316	201	3233	1244
77 Year	Scen1	183	198	73	243	194	85	160	137	122	127	478	204	3295	649
	Scen2	246	361	164	246	303	288	212	288	259	171	416	150	3984	1513
	Scen3	308	235	194	263	379	235	259	199	178	199	531	239	3724	1124
	AVERAGE	246	264	144	251	292	203	210	208	186	165	475	198	3668	1095

The average yield estimates for each scenario from these experiments are shown in Table 5-29. The average yield estimate for each scenario in Table 5-29 was calculated using the 1918 yield estimates from the individual input variable eFAST experiments for that scenario. Also given in Table 5-29 is the range of the average yield estimates of scenario of the same length. As can be seen from this table, the average yield estimate decreases as the simulation length increases. There are no observable trends within simulations of the same simulation length, signifying that the greater total streamflow volume (where scenario 1 has the greatest to scenario 5 at the least) does not necessarily result in increased yield. It can be seen from the range of the average yield estimates from scenarios of the same length, i.e. the scenarios 1 to 7 of 20 year length, the five 40 year scenarios, five 60 year scenarios and the three 77 year scenarios, that the range of the average yield estimate reduces as the planning length increases. The decreasing average yield and the reducing range of the average yield over increasing planning period indicates that the yield estimate stabilises at a generally lower value and becomes less sensitive to changes in the climate variability and changes to the input variables.

Table 5-29. Average Yield Estimates for Each Scenario with Individual eFAST Experiment.

Yield (Ml)	**20 Years**	**40 Years**	**60 Years**	**77 Years**
Scenario 1	67,490	51,061	49,986	42,932
Scenario 2	61,173	50,595	43,041	44,500
Scenario 3	49,595	50,771	41,461	44,292
Scenario 4	50,887	53,320	49,647	
Scenario 5	49,475	42,279	49,884	
Scenario 2b	60,028			
Scenario 2c	60,031			
Average	**56,954**	**49,605**	**46,804**	**43,908**
Range	**18,015**	**11,041**	**8,525**	**1,568**

Table 5-30 presents the standard deviation of the yield estimates of each scenario with the last row showing the range of the standard deviations, i.e. the difference between the largest standard deviation and the smallest standard deviation. The standard deviation is simply the square root of $V(Y)$ and is used instead of $V(Y)$ for simplicity. As the simulation length increases, the range of standard deviation of the yield estimate decreases, showing that the estimation of yield becomes more stable with respect to the input variable changes and less sensitive. That is, the 20 year scenarios show a 5,260 difference in the standard deviation of the yield estimate, the 40 year scenarios has 3,208 difference, 60 year scenarios contains a 1,303 range and the 77 year scenarios only show approximately a 1,066 difference. This particularly suggests that the selection of a climate scenario for a short planning period is more important than the selection for a longer period. The estimation of yield for longer periods seems to be more robust in this case, which is due to them having the same extreme climate events, i.e. the shuffling approach used to generate the 77 year scenarios does not break the 'critical' climate event.

Table 5-30. Standard Deviation of Yield Estimates for Each Scenario with Individual eFAST Experiment.

	20 Years	**40 Years**	**60 Years**	**77 Years**
Scenario 1	5,233	6,696	4,531	3,669
Scenario 2	8,161	5,290	3,767	4,735
Scenario 3	4,276	5,979	3,268	4,442
Scenario 4	3,266	5,721	3,990	
Scenario 5	2,901	3,488	4,571	
Scenario 2b	6,771			
Scenario 2c	4,693			
Range	**5,260**	**3,208**	**1,303**	**1,066**

The 20 year scenarios 2, 2b and 2c show a wide range of standard deviation (Table 5-30) yet similar average estimation of yield (Table 5-29), explicitly demonstrating that the yield estimation is sensitive to climate variability. Similar conclusions can be drawn from the 77 year scenarios.

The implications of the yield estimate becoming more robust as the planning length increases are interesting with respect to the approach of estimation, handling and use of yield of an urban water supply system. On one hand it can be argued that a simulation length used for the yield estimate should be the same or similar length as the planning period. In this way a greater maximal yield is estimated and adopted. That is, if the study is for a 20 year period, then the estimation of yield should only consider 20 years, giving rise to a greater yield estimate. On the other hand, using the average yield estimate resulting from the 77 years planning length provides a conservative estimate which will lead to conservative planning measures. These findings and conclusions are of course only relevant to this study and the data used.

The individual eFAST experiments provide the following findings:

1. The minimum storage threshold and the reliability threshold dominate the S_i measures. For the 20 year scenarios, the reliability threshold is the most important with the minimum storage threshold the second most important. This order is swapped for the longer planning periods. The upper RRC position shows some significance in the 20 year scenarios while the base demand becomes more influential in the 60 and 77 year scenarios. The remaining variables show mostly inconclusive results.

2. The S_{Ti} are also dominated by the minimum storage threshold and the reliability threshold. The target rule curves become less important as the planning length increases while the upper RRC position is significant in the 40 year scenarios within the total-order sensitivity measures. The remaining variables show mostly inconclusive results.

3. Significant increases from S_i to S_{Ti} exist for most input variables indicating higher-order effects, i.e. interaction effects.

4. The average yield estimate reduces as the planning length increases. Also, as the planning length increases, the estimate becomes more stable as shown by the reduced range of variance of the yield estimate.

5. The climate variability has a considerable effect on the sensitivity of the yield estimate, on the importance of individual input variables and on the partial variance of each variable. There are no significant trends that can be identified for the importance measures of input variables over scenarios of the same planning length. Similarly, the partial variances show no significant trend, over scenarios of the same length or over different lengths.

6. Sobol' method experiments produced unacceptable results, therefore no first-, higher- or total-order effects were largely disregarded.

5.4.2.2 Grouping 1 Experiments

The grouping 1 experiments were performed using the variable groupings as shown in Table 5-13. The eFAST experiments were performed over 979 randomly selected model simulation samples with acceptable results. The first-order sensitivity measures (S_i) and the total-order sensitivity measures (S_{Ti}) are shown in Table 5-31.

Noticeably from Table 5-31, the security criteria group of variables is dominant, providing the greatest S_i and S_{Ti} for all experiments. The RRCs are then the next dominant for both measures over most scenarios with the target curves generally the least important. All groups show higher-order effects as indicated by the difference between S_i and S_{Ti}.

Table 5-31. S_i and S_{Ti} Results of the Grouping 1 eFAST Experiment using 979 Simulations.

		First-Order Sensitivity Index (S_i)				Total-Order Sensitivity Index (S_{Ti})		
	Scenario	**RRCs**	**Target Curves**	**Security Criteria**	**SUM**	**RRCs**	**Target Curves**	**Security Criteria**
20 Year Simulation Period	1	0.1612	0.0023	0.7025	0.8660	0.2600	0.1397	0.8167
	2	0.0299	0.0017	0.8442	0.8758	0.1220	0.0641	0.9555
	3	0.0216	0.0034	0.8108	0.8358	0.1945	0.0816	0.9788
	4	0.0617	0.0018	0.7854	0.8489	0.1712	0.0888	0.9329
	5	0.0497	0.0007	0.7987	0.8491	0.1719	0.0925	0.9525
	2b	0.0317	0.0033	0.8612	0.8962	0.1408	0.1057	0.9698
	2c	0.0487	0.0017	0.7638	0.8142	0.1706	0.1712	0.9163
40 Year Simulation Period	1	0.0314	0.0015	0.8229	0.8558	0.1756	0.0598	0.9753
	2	0.0263	0.0012	0.8551	0.8826	0.1565	0.0562	0.9775
	3	0.0274	0.0012	0.8444	0.8730	0.1652	0.0620	0.9761
	4	0.0278	0.0023	0.8896	0.9197	0.0964	0.0635	0.9640
	5	0.0236	0.0010	0.9111	0.9357	0.0945	0.0334	0.9841
60 Year Simulation Period	1	0.0225	0.0007	0.8808	0.9040	0.1200	0.0399	0.9841
	2	0.0175	0.0006	0.9198	0.9380	0.0842	0.0363	0.9812
	3	0.0192	0.0002	0.9268	0.9462	0.0819	0.0239	0.9865
	4	0.0196	0.0017	0.8970	0.9183	0.1059	0.0484	0.9868
	5	0.0240	0.0012	0.8833	0.9085	0.1255	0.0481	0.9845
77 Year Simulation Period	1	0.0169	0.0005	0.9228	0.9402	0.0732	0.0288	0.9838
	2	0.0087	0.0015	0.9173	0.9275	0.0680	0.0619	0.9908
	3	0.0117	0.0010	0.9047	0.9174	0.0839	0.0586	0.9854

The S_i and S_{Ti} indices for the three groups of variables (RRCs, target curves and security criteria) for all scenarios are represented in Figures 5-23, 5-24 and 5-25, respectively. It can be seen from Figure 5-23 and Figure 5-24 that the importance of the RRCs and the target curves tends to decrease as the simulation period increases. However, generalising a trend within scenarios of the same planning period is not possible. The lack of trend indicates that the planning period is significant in terms of the importance of the groups, more so than the total streamflow volume entering the system. It also suggests that the variability of streamflow is significant. This is most clear when observing the 20 year period Scenarios 2, 2b and 2c and the 77 year simulation periods of the target curves (Figure 5-24). Figure 5-25 shows the S_i and the S_{Ti} results for the security criteria group in which it is clear of the dominance of the group for all scenarios.

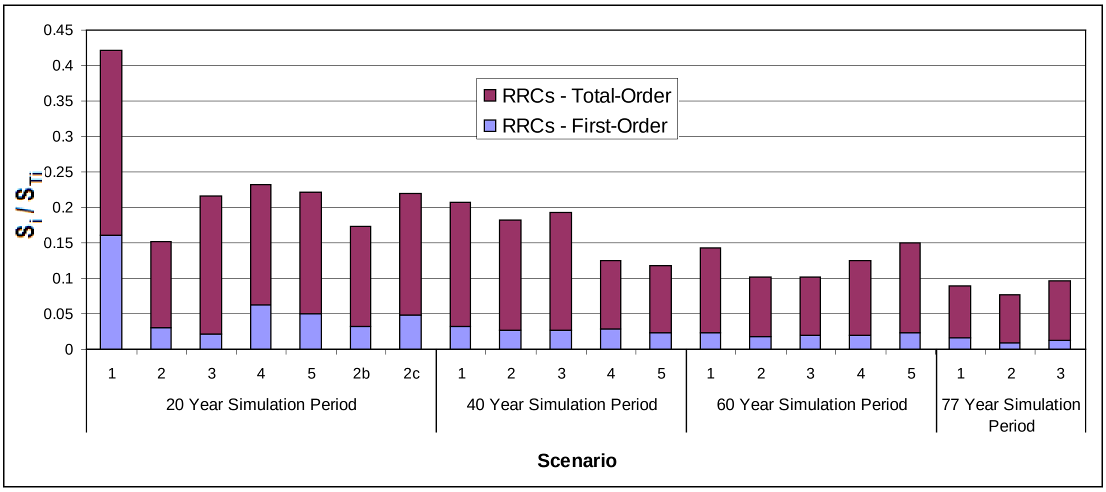


Figure 5-23. RRCs S_i and S_{Ti} Results for all Scenarios in the eFAST Grouping 1 Experiments.

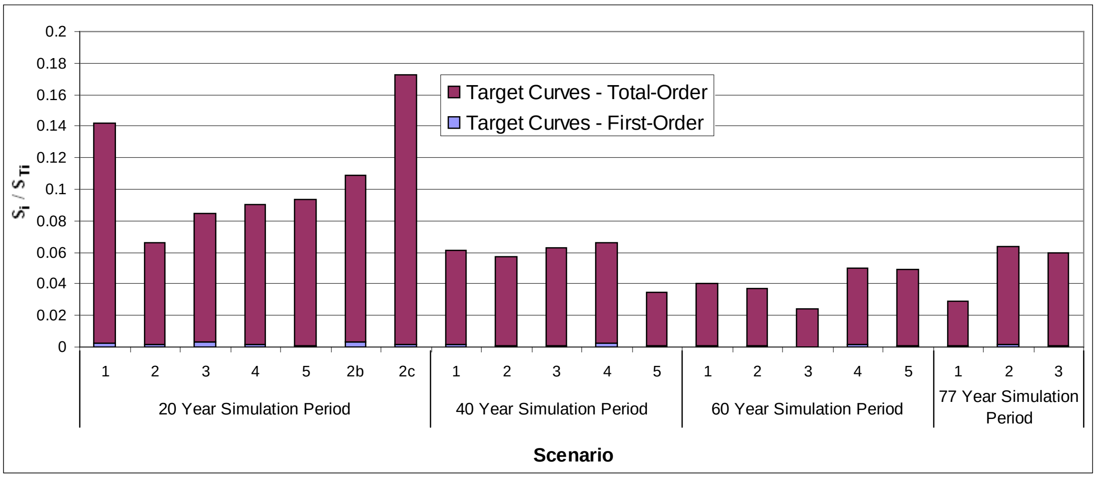


Figure 5-24. Target Curves S_i and S_{Ti} Results for all Scenarios in the eFAST Grouping 1 Experiments.

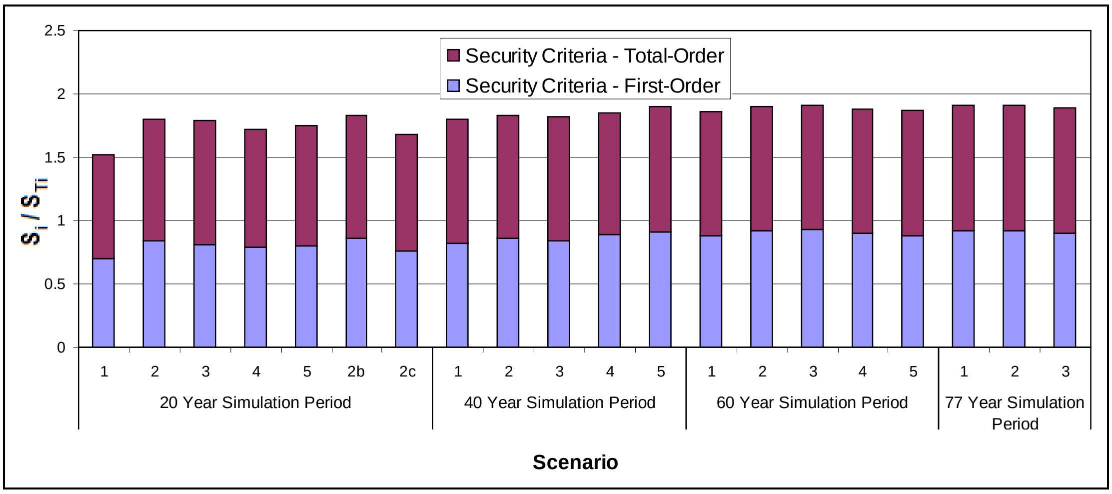


Figure 5-25. Security Criteria S_i and S_{Ti} Results for all Scenarios in the eFAST Grouping 1 Experiments.

Tables 5-32 shows the average yield estimate for each scenario. Each scenario average yield was calculated from the 979 yield estimates used in the eFAST grouping 1 experiment. Also given in Table 5-32 is the average and range of the average yield estimates. From Table 5-32 it can be seen that as the planning length increases the average yield estimate decreases and the range of the average yield estimate decreases. For this case study, this indicates that, regardless of the climate variability and input variable variability, the yield estimate and the spread of the yield estimates will generally decrease as the planning length increases.

Table 5-32. Average Yield Estimates for Each Scenario in the Grouping 1 eFAST Experiments.

Yield (Ml)	**20 Years**	**40 Years**	**60 Years**	**77 Years**
Scenario 1	67,378	51,114	49,950	43,077
Scenario 2	60,972	50,518	43,179	44,561
Scenario 3	49,644	50,781	41,611	44,413
Scenario 4	50,864	53,194	49,674	
Scenario 5	49,455	42,346	49,828	
Scenario 2b	59,921			
Scenario 2c	59,907			
Average	**56,877**	**49,591**	**46,849**	**44,017**
Range	**17,923**	**10,848**	**8,339**	**1,484**

Table 5-33 shows the standard deviation of the yield estimates for each scenario, with the range (difference between maximum and minimum standard deviations) given in the last row. The standard deviations shown are the square root of the total variance of each scenario, $V(Y)$. The total variance is shown here as the standard deviation for ease of reading. From Table 5-33 it is clear that the range of the standard deviations decreases as the planning length increases, indicating that the variability of the yield estimate caused by climate variability decreases as the planning length increases. That is, the yield estimate becomes more robust against the climate variability and changes in the input variables as the planning length increases. Once again there are no obvious trends between scenarios in the same planning length that can be observed from Tables 5-32 and 5-33.

The average yield estimates for scenarios of the same streamflow volume (20 year Scenarios 2, 2b and 2c) show that the yield estimate seems to be reasonably similar with regards to the average yield estimate. However, the range of the average yield (Table 5-32) and the total variance (shown as the standard deviation in Table 5-33) of the yield estimates for the 20 year scenario 2, 2b and 2c are significantly different. Similarly, the three 77 year scenarios have similar average yield estimates but has relatively similar standard deviation of the yield estimate.

Table 5-33. Standard Deviation of the Yield Estimates for Each Scenario in the Grouping 1 eFAST Experiments.

	20 Years	40 Years	60 Years	77 Years
Scenario 1	4,609	6,281	4,112	3,792
Scenario 2	7,488	4,822	3,900	4,655
Scenario 3	4,145	5,515	3,315	4,420
Scenario 4	2,929	5,178	3,772	
Scenario 5	2,603	3,386	4,068	
Scenario 2b	6,309			
Scenario 2c	4,214			
Range	**4,885**	**2,895**	**797**	**863**

The following findings can be drawn from this grouping experiment:

1. All groups of variables show importance (indicated by S_{Ti}) and show significant interaction between groups (indicated by the difference between S_i and S_{Ti}).

2. Table 5-31 clearly shows the importance of the groups for every scenario over every simulation length. The most important group of variables in the estimation of yield is the security criteria group, followed by the RRCs and then the target curve groups.

3. The importance of the RRCs group and the target curves group decrease as the simulation length increases. The security criteria group does not show a notable trend.

5.4.2.3 Grouping 2 Experiments

The first-order (S_i) and the total-order indices (S_{Ti}) of the grouping 2 SA experiments using the eFAST technique are shown in Table 5-34. The variables are grouped as shown in Table 5-18. The experiment was performed over 1862 model simulations, which produced acceptable results, i.e. ΣS_i not greater than 1 and the $S_i < S_{Ti}$.

Table 5-34. S_i and S_{Ti} Results of the eFAST Grouping 2 Experiments using 1862 Simulations.

		First-Order Sensitivity Index (Si)								**Total-Order Sensitivity Index (S_{Ti})**						
Simulation Period	Scenario	Relative Position	Percent. Restrict.	Upper Curve	Lower Curve	Base Curve	Target Curves	Security Criteria	Sum	Relative Position	Percent. Restrict.	Upper Curve	Lower Curve	Base Curve	Target Curves	Security Criteria
20 Year	1	0.0110	0.0087	0.1664	0.0175	0.0007	0.0013	0.7235	0.9291	0.0883	0.0965	0.2705	0.1337	0.0801	0.0489	0.8376
	2	0.0066	0.0057	0.0483	0.0081	0.0010	0.0005	0.8646	0.9348	0.0811	0.0963	0.1451	0.0991	0.0528	0.0450	0.9549
	3	0.0209	0.0095	0.0129	0.0148	0.0051	0.0004	0.8569	0.9205	0.1672	0.1400	0.1654	0.1483	0.0775	0.1174	0.9918
	4	0.0143	0.0101	0.0636	0.0101	0.0036	0.0013	0.8189	0.9219	0.0901	0.0867	0.1620	0.1033	0.0796	0.0513	0.9275
	5	0.0103	0.0123	0.0491	0.0181	0.0028	0.0020	0.8055	0.9001	0.1117	0.0998	0.1607	0.1301	0.0721	0.0460	0.9298
	2b	0.0125	0.0074	0.0389	0.0137	0.0027	0.0020	0.8373	0.9145	0.1037	0.1153	0.1453	0.1149	0.0826	0.0586	0.9456
	2c	0.0074	0.0141	0.0632	0.0265	0.0015	0.0026	0.8122	0.9275	0.0922	0.1168	0.1887	0.1914	0.0826	0.0701	0.9275
40 Year	1	0.0261	0.0209	0.0199	0.0201	0.0027	0.0004	0.8700	0.9601	0.1773	0.1315	0.1493	0.1433	0.0843	0.0741	0.9843
	2	0.0240	0.0201	0.0211	0.0153	0.0020	0.0015	0.8896	0.9736	0.1582	0.1151	0.1420	0.1117	0.0694	0.0759	0.9880
	3	0.0246	0.0175	0.0219	0.0193	0.0017	0.0010	0.8737	0.9597	0.1788	0.1091	0.1520	0.1318	0.0732	0.0687	0.9843
	4	0.0099	0.0037	0.0383	0.0101	0.0005	0.0009	0.8981	0.9615	0.0760	0.0491	0.1084	0.0824	0.0588	0.0390	0.9685
	5	0.0079	0.0051	0.0099	0.0086	0.0096	0.0005	0.9123	0.9539	0.0582	0.0513	0.0890	0.0566	0.0433	0.0276	0.9782
60 Year	1	0.0235	0.0153	0.0134	0.013	0.0027	0.0015	0.9110	0.9804	0.1259	0.0878	0.1045	0.0930	0.0735	0.0921	0.9901
	2	0.0061	0.0025	0.0059	0.0082	0.0112	0.0013	0.9172	0.9524	0.0684	0.0330	0.0775	0.0667	0.0474	0.0393	0.9816
	3	0.0065	0.0066	0.0091	0.0084	0.0091	0.0007	0.9134	0.9538	0.0601	0.0528	0.0825	0.0604	0.0286	0.0232	0.9793
	4	0.0158	0.0135	0.0074	0.0117	0.0018	0.0005	0.9195	0.9702	0.1001	0.0813	0.0885	0.0762	0.0610	0.0439	0.9886
	5	0.0212	0.0146	0.0192	0.0118	0.0020	0.0013	0.9022	0.9723	0.1253	0.0891	0.1315	0.0905	0.0716	0.0731	0.9885
77 Year	1	0.0052	0.0020	0.0064	0.0087	0.0115	0.0008	0.9195	0.9541	0.0549	0.0302	0.0732	0.0645	0.0469	0.0336	0.9807
	2	0.0070	0.0053	0.0066	0.0087	0.0043	0.0010	0.9202	0.9530	0.0643	0.0541	0.0748	0.0661	0.0398	0.0408	0.9852
	3	0.0048	0.0070	0.0102	0.0082	0.0057	0.0011	0.9044	0.9414	0.0756	0.0621	0.1038	0.0667	0.0622	0.0572	0.9800

The security criteria group dominates the S_i and S_{Ti} indices for all scenarios as shown in Table 5-34. Of the remaining variables, the upper RRC curve group shows some importance in the 20 year scenarios but becomes less important as the planning length increases. There is significant increase between S_i and S_{Ti} for all groups (except the security criteria group), indicating that higher-order effects are attributed to these groups.

The partial variances and total contribution variances are shown in Table 5-35. This explicitly reveals the domination of the security criteria on the total output variance; note: $S_i = V_i / V(Y)$. The higher-order effects of most groups can also be seen in Table 5-35 by comparing the V_i and V_{Ti} results. The V_i due to the upper RRC curve group tends to decrease as the planning length increases, showing that yield estimate becomes more robust to changes in upper RRC curve as the planning length increases. This is related to which security criteria is critical, if the reliability of supply is critical (violates a lower average annual demand) then the upper RRC is important (see Section 5.4.2.1). It was found that as the planning length increases the minimum storage level threshold becomes critical and the upper RRC becomes less significant in the estimation of yield. There are no other obvious trends of V_i and V_{Ti} across different planning lengths.

Table 5-36 presents the average yield estimate for each scenario, the average yield estimate for scenarios of the same length and the range of the average yield estimates. The results presented in Table 5-36 show excellent similarity to the results given in Tables 5-29 and 5-32, which once again show that average yield estimate decreases as the length of simulation increases. There are no discernable trends of the average yield estimates within the same planning length. It also shows that the range of the average yield estimates (i.e. difference between the minimum and maximum average yield estimate) decreases as the simulation length increases. This is the range of the average yield estimates across scenarios of the different length, therefore indicating how the yield estimate generally behaves with respect to the planning period.

Table 5-35. Partial Variance and Total Contribution Variance Results of the eFAST Grouping 2 Experiment using 1862 Simulations.

		Partial Variance (Vi)							**Total Contribution Variance (VTi)**						
Simulation Period	Scenario	Relative Position	Percent. Restrict.	Upper Curve	Lower Curve	Base Curve	Target Curves	Security Criteria	Relative Position	Percent. Restrict.	Upper Curve	Lower Curve	Base Curve	Target Curves	Security Criteria
20 Year	1	527	468	2,048	664	133	181	4,271	1,492	1,560	2,611	1,836	1,421	1,110	4,595
	2	643	597	1,739	712	245	182	7,357	2,253	2,455	3,014	2,490	1,819	1,678	7,731
	3	611	412	480	514	302	85	3,914	1,729	1,582	1,720	1,628	1,177	1,449	4,211
	4	363	305	765	305	182	109	2,745	911	893	1,221	975	856	687	2,921
	5	277	303	605	368	145	122	2,452	913	863	1,095	985	734	586	2,634
	2b	744	572	1,312	779	346	297	6,087	2,142	2,258	2,536	2,255	1,912	1,611	6,467
	2c	385	532	1,126	729	174	228	4,038	1,360	1,531	1,946	1,960	1,287	1,186	4,315
	Average	507	456	1,154	582	218	172	4,409	1,543	1,592	2,020	1,733	1,315	1,187	4,696
40 Year	1	1,055	944	921	926	339	130	6,093	2,750	2,368	2,524	2,473	1,897	1,778	6,480
	2	800	732	750	638	231	200	4,868	2,053	1,751	1,945	1,725	1,360	1,422	5,130
	3	911	768	859	807	239	183	5,428	2,455	1,918	2,264	2,109	1,571	1,522	5,761
	4	546	334	1,074	552	119	165	5,203	1,513	1,217	1,808	1,576	1,331	1,084	5,403
	5	311	250	348	324	343	75	3,341	844	792	1,044	832	728	581	3,459
	Average	725	606	790	649	254	151	4,987	1,923	1,609	1,917	1,743	1,377	1,277	5,247
60 Year	1	670	541	506	498	227	169	4,172	1,551	1,295	1,413	1,333	1,185	1,327	4,349
	2	301	193	296	350	408	139	3,696	1,009	701	1,074	997	840	765	3,824
	3	268	270	317	304	317	90	3,175	815	763	954	816	561	506	3,288
	4	499	461	341	429	168	91	3,806	1,256	1,132	1,181	1,096	980	831	3,947
	5	637	529	607	476	196	158	4,158	1,550	1,307	1,587	1,317	1,171	1,184	4,352
	Average	475	399	413	411	263	129	3,801	1,236	1,040	1,242	1,112	947	923	3,952
77 Year	1	271	168	300	350	402	103	3,598	879	653	1,015	953	812	688	3,716
	2	399	347	388	445	313	148	4,576	1,210	1,109	1,305	1,227	952	964	4,735
	3	312	377	455	408	340	149	4,282	1,238	1,122	1,451	1,162	1,122	1,077	4,457
	Average	327	297	381	401	352	133	4,152	1,109	961	1,257	1,114	962	910	4,303

Table 5-36. Average Yield Estimates for Each Scenario in the eFAST Grouping 2 Experiments.

Yield (Ml)	20 Years	40 Years	60 Years	77 Years
Scenario 1	67,492	51,019	49,987	43,036
Scenario 2	61,077	50,534	43,142	44,600
Scenario 3	49,625	50,730	41,584	44,424
Scenario 4	50,827	53,289	49,695	
Scenario 5	49,424	42,401	49,863	
Scenario 2b	60,182			
Scenario 2c	59,971			
Average	**56,943**	**49,595**	**46,854**	**44,020**
Range	**18,068**	**10,888**	**8,403**	**1,564**

Table 5-37 shows the standard deviation of the yield estimates for each scenario (i.e. the square root of the total yield variance $V(Y)$ for each scenario), with the last row showing the range of these standard deviations (difference between the minimum and maximum deviations). The standard deviations indicate the spread of the possible yield estimates that are possible in each scenario, due to changes in the input groups. The results in Table 5-37 show very similar results to Tables 5-30 and 5-33, where the range of the variance in the yield estimates reduces as the simulation length increases. This, once again, shows that the estimation of yield becomes less sensitive to changes in the input variables as the planning length increases.

Table 5-37. Standard Deviation of Yield Estimates for Each Scenario in the eFAST Grouping 2 Experiments.

	20 Years	40 Years	60 Years	77 Years
Scenario 1	5,021	6,532	4,371	3,752
Scenario 2	7,912	5,161	3,860	4,771
Scenario 3	4,228	5,807	3,322	4,502
Scenario 4	3,033	5,490	3,969	
Scenario 5	2,732	3,498	4,378	
Scenario 2b	6,652			
Scenario 2c	4,480			
Range	**5,180**	**3,034**	**1,055**	**1,018**

The following conclusions can be drawn from the grouping 2 FAST experiment:

1. The security criteria group is the most influential variable in the estimation of yield. The S_i and S_{Ti} indices for the other groups have inconclusive results.

2. Large differences between the S_i and S_{Ti} indices for all groups over all scenarios show that high-order effects of input variables are present.

3. The partial variances given in Table 5-35 show that the sensitivity of the yield estimate to variation in all groups of variables decreases as the planning length increases, indicating that the yield estimate becomes more robust as the planning length increases.

4. The lack of trends within all indices (S_i and S_{Ti}) and measures (V_i, V_{Ti} and $\sqrt{V(Y)}$ - standard deviation) indicates that the importance of these groups on the estimation of yield is not directly driven by the streamflow volume. More so this suggests that the climate variability effects the yield estimate, as is explicitly shown by the difference in the variances of the 20 year 2, 2b and 2c scenarios and the 77 year scenarios.

The following Table 5-38 shows the approximate average yield estimate of the Barwon urban water supply system for each considered planning length. Also shown is the range of the average yield estimate for scenarios of the same planning period. As has been highlighted a number of times, there is a clear reduction of the average yield and the range of the average yield as the planning period increases. The significance of the range of the average yield estimate gives an indication on the minimum length of data required to produce a robust yield estimate and for use in other water supply planning studies conducted by the water authorities. Although there are limited values given in Table 5-38, it can be argued that a minimum of 40 years of data is required so that the range of the average yield estimate is not greater than approximately 10,000 Ml. However, it must be stressed that this claim is valid for only this case study.

Table 5-38. Average Yield Estimate for the Barwon Urban Water Supply System.

	20 Years	**40 Years**	**60 Years**	**77 Years**
Average Yield Estimate (Ml)	57,000	50,000	47,000	44,000
Range of Average Yield Estimates (Ml)	18,000	11,000	8,000	2,000

5.5 Issues, Limitations and Recommendations

The following are summaries of the major findings, including the issues, limitations and recommendations, from all SA experiments on the Barwon urban water supply system detailed in Section 5.4:

1. Application of selected SA techniques – Successful application of the Morris method and eFAST were given in Section 5.4. Sobol' experiments provided erroneous results with negative first- and second-order importance indices, and ΣS_i greater than 1. The Morris method showed excellent accuracy in identifying the important variables in the estimation of yield of the Barwon urban water supply system, and the eFAST technique gave good quantification.

2. Importance of variables – The importance of variables were consistant throughout all experiments over all scenarios with the security criteria variables (reliability of supply and minimum storage threshold) proving to be the most important in the estimation of yield of the Barwon urban water supply system.

3. Importance correlations - When the reliability threshold is the most important, the upper RRC position and curvature variables have a clear defined importance. When the minimum storage threshold is the most important variable, all remaining variables have inconclusive Si indices and partial variances.

4. Grouping – The security criteria group is the most important in both grouping experiments. The only other group of variables that show any significant importance is the upper RRC group, containing the upper RRC position and curvature groups. Significant high-order effects are present within groups, especially within the Grouping 2 experiments.

5. Integral estimation and approximation – Again, the variance based methods have shown mixed results in their estimation of the sensitivity indices. The eFAST methods have reliably given acceptable results at relatively low model simulations, however the method of Sobol' suffered from errors causing negative sensitivity indices. The solution to this is to increase the number of model simulations, however this becomes infeasible due to excessive computational time required for the Barwon system simulation model.

6. Further sensitivity measures – The total output variance, $V(Y)$ (presented as a standard deviation in this study for simplicity), provided an excellent measure to

observe the sensitivity of the yield estimate on the climate scenarios and changes to the input variables. The range of the total output variance for a planning period shows the volatility or stability of the yield estimate. The results given in Section 5.4.2 show that the variance of the yield estimate and the range of the variance decrease as the planning period increases.

7. Average yield estimate – The average yield estimate due to different climate scenarios decreases as the planning period increases. This, combined with the findings considered in point 6 above, suggests that the estimation of yield converges as the planning length increases. Perhaps the estimation of yield will converge to a single value if a planning period of sufficient length is used.

8. Historic data use – In Section 4.6 the use of historic data use for planning purposes was discussed. Doing so provides a plausible set of climate data. However without the consideration of alternative plausible climate sets, the optimal position and the importance of the management variables will not vary. It has been shown in the current chapter that the yield and the importance of variables changes significantly when considering different climate scenarios and planning lengths.

5.6 Summary

This study discussed the estimation of yield of an urban water supply system, considering the Barwon Region Water Corporation water supply system as the case study. The definition of yield adopted was "*the maximum average annual volume of water that can be supplied from the system over a given planning period subject to climate variability, demand pattern and operating rules, without violating the adopted level of service*". For the Barwon water system, the level of service includes a reliability of supply threshold (i.e. the number of time periods without demand restrictions imposed to the total number of time periods) and a minimum total system storage volume threshold. If either threshold is violated, the system is deemed to have failed. The yield of a system can be simply explained as the maximum annual volume of water that can be supplied by system sustainably over a number of years. This volume is synonymous to the maximum allowable annual demand, or the target demand for supply and demand balancing.

The sensitivity analysis (SA) experiments in this chapter indicate the importance of input variables used in the estimation of yield of an urban water supply system considering climate variability and planning length. The security of supply thresholds that are applied to the Barwon system are the most important, which is unsurprising as they directly influence water consumption. Following these only the upper restriction rule curve variables, the curvature and position, show any other discernable importance. The implication of these findings is that these four important variables need to be accurately estimated and set first. If further improvements are then required, another SA will identify which of the remaining variables will provide the next best improvement in the estimation of yield.

The SA methodology used in this study differs from most other SA studies due to the use of the climate scenarios. The use of the total variance and the partial variance allowed for comparison between scenarios and provided extra information that would not have been available from the standard Fourier Amplitude Sensitivity Test (FAST) sensitivity indices.

Most significantly these findings prove the hypothesis that the planning length and the climate variability are influential in the estimation of yield. This questions the use of a single climate sequence in the use of the estimation of yield and other water resources planning studies, and the use of a single set of management polices and rules for all possible future climates.

It has been shown throughout this chapter that the average yield estimate decreases as the planning length increases. Also the range of the yield estimate decreases as the planning length increases, which results in a low and robust average yield estimate for 77 years, whereas the estimation of yield for under a 20 year planning length is high and highly fluctuating. The implication of this to industry is that the use of an entire available historic climate data sequence will generally provide a conservative estimation of yield (as found in this study) and therefore conservative planning designs. This chapter also found that the use of a long climate sequence means that the estimate is more robust to changes in the input variables, i.e. the spread of the yield estimate decreases as the planning length increases. This means that accurate knowledge and estimation of input variables will not significantly improve on the estimation of yield for long planning periods. Conversely, a short planning period results in a generally high estimation of yield that is sensitive to changes in the input variables.

An alternative approach to the estimation, handling and use of yield of an urban water supply system is required. This approach should consider a simulation length appropriate to the water authorities planning period and different climate scenarios. Doing so will provide a better insight into the possible range of behaviour of input variables and the estimation of yield of the urban water supply system in question. The planning length used in the simulation of the system should be the same or similar to the length of the study that it will be used for. For instance, if a planning period is 20 years, the length in the simulation should also be 20 years. Then, various climate scenarios of 20 years can be used to assess the range of possible behaviour of the system, giving rise to a number of yield estimates from which one can be appropriately selected. These issues are further discussed in Chapter 6.

Chapter 6
Summary, Conclusions and Recommendations

6.1 Summary

In concluding this study, this chapter briefly provides a summary of the work undertaken to fulfil the aims of the study. It provides the major results of the Sensitivity Analysis (SA) on the two case studies, commenting on the success of the SA techniques applied to water supply planning models and reiterates the major findings, conclusions and recommendations from the research undertaken. Recommendations are thereafter provided for future research.

Balancing demand and available supply is the foremost issue that water authorities face. Water supply management is primarily concerned with how to sustain a reasonable supply of water during drought periods which cause low storage volumes. It is due to these low storage volumes that drought response plans and water conservation measures have been developed and generally implemented only when the storage volume falls below a threshold. Safeguards and policies, such as consumption restrictions, have been in place for some time to protect water supply systems from low system storage volumes. However the increasing population growth and the recent drought that much of Australia is experiencing have forced many water authorities to impose permanent water saving measures and mandatory water consumption restrictions to reduce urban demand. Still, many water supply systems are required to supply a demand that exceeds a sustainable volume. This shortfall can be reduced by: decreasing the demand via water saving measures and schemes, and education; and/or increasing the yield of the system by optimising system management, or augmentation with additional water sources.

This research initially aimed at finding the most important input variables used in the estimation of yield of an urban water supply system. As well as being used as a sustainable demand (i.e. a target demand), the estimation of the yield of a water supply system is also an essential part in water resources management, and policy development and enforcement, as it is used in processes such as augmentation studies, water sharing and decision-making polices. It is therefore important that an accurate estimation of yield is established and used in these studies. The yield of a water supply system is typically estimated by simulating a computational model of the physical system using the entire length of available climate data. Both the model and its required input variables are subject to inherent uncertainty which propagates through the model to the yield, inducing uncertainty and decreasing confidence in the yield estimate. By identifying the most important input variables used in the estimation of yield via a sensitivity analysis, resources can be allocated and research prioritised so that water authorities can improve their knowledge, hence decreasing their uncertainty and increasing the confidence in the yield estimate.

Sensitivity Analysis (SA) using three techniques on a hypothetical urban water supply system case study showed that the estimation of yield is most sensitive to variations in the streamflow input variable. Through this case study it was found that the selected SA techniques – the Morris method, the Fourier Amplitude Sensitivity Test (FAST) and the Sobol' method of SA – had mixed success. The extended Fourier Amplitude Sensitivity Test (eFAST) was also used as an extension of FAST. The Morris method and FAST/eFAST gave satisfactory results while Sobol' gave erroneous results, as a consequence of many input variables that had negligible importance on the yield. Through this case study it was found that the SA framework that the study was built upon could be improved considerably by considering a different uncertainty/variability methodology, alternative variable handling strategies and different sensitivity indices.

The second significant downfall in the approach used to estimate the yield of an urban water supply system (which is a typical yield study) was thereby realised: using a single climate sequence provides a plausible realisation to perform simulations but without the consideration of alternative future climate sequences, it implies that future climate is the same as the historic climate, including wet and dry event patterns. When a single climate sequence is used for studies pertaining to or using the yield estimate, the results, information, calibration and optimisation are only truly valid for that climate sequence. Any policies, rules or other system management studies that are derived or optimised from this sequence may not be appropriate for another realisation of climate.

Lastly, the third weakness was the use of a single planning period in the estimation of yield. In the hypothetic urban water supply system case study, only a single 28 years of historic climate data was available and used to test the importance of input variables on the estimation of yield. Doing so allowed for the identification of the important variables for only that length of simulation. This is the method that is typically used for yield studies but it does not allow for observation of the effect of the different planning lengths on the yield estimate.

The second case study, using the Barwon Water urban water supply system explicitly addressed the issue of use of a single climate sequence by considering multiple climate scenarios selected from the historic climate data sequence. The scenarios were chosen so that different climate patterns were present over four different planning periods of 20, 40, 60 and

77 years. SA was applied to each scenario with the aim of observing the evolution of the importance of the management variables (i.e. the input variables that the water authority can set) over the different climate scenarios and different planning lengths. The two level of service thresholds, the reliability of supply and the minimum storage level, were found to be the most important in all scenarios. Some correlation effects between the importance indices of the security of supply criteria and the upper restriction rule curve variables. Besides this, few other trends and results were explicitly assessable, which in itself is a significant finding in terms of the handling and application of the estimation of yield of an urban water supply system.

6.2 Findings and Conclusions of the Study

In the following three sections (i.e. Sections 6.2.1 – 6.2.3), the main conclusions related to various aspects of the study are presented.

6.2.1 Sensitivity Analysis in Water Supply System Modelling

Traditionally SA has been viewed as a facet of uncertainty analysis. Indeed they share many common elements, however SA can be regarded as a distinct set of principles and tools that offer an analyst more than a branch of uncertainty analysis. SA is increasingly being appreciated as a major statistical tool for use in the development, operation, calibration, optimisation and application of computational modelling. Given correct selection of technique(s) and planning of SA experiments, a SA can provide information regarding the model structure, the dependence on input variables, the behaviour of the model at extreme values/events, areas of lack of knowledge and data, and can be used as a decision making tool.

6.2.1.1 Sensitivity Analysis Techniques

The available SA techniques discussed in this thesis differ with each other in the quality and quantity of input information required, the methodology and sampling strategy used, the sensitivity indices produced, accuracy, and computational expense. None of the techniques discussed in Chapter 3 are a ‘solve all’ technique that can be easily applied to any SA problem. Rather a technique or techniques should be selected for applicability to the problem and model in subject. In this thesis a number of ideal SA technique characteristics were sought, viz.;

- Does not require the knowledge of the model or its algorithm(s) – The REALM software package and the two water supply planning models used were assumed as ‘black-box’ models which the internal parameters, setting and configuration cannot be changed.

- The techniques are model independent – The techniques should be free of assumptions regarding linearity, additivity (lack of interactions between variables) or monotonicity of the model.

- The ability to handle input variable correlations and interactions – The level of correlation or interaction should not detract from the accuracy of the technique.

- Does not require intricate model input and output characteristics – A priori knowledge of input variable characteristics such as distributions and likelihood measures should ideally not be required. A continuous model output must be required.

- Apportion output variance into different order levels – To assess the first- and higher-order effects without the influence of other order effects.

Computational expense was also considered but was not ultimately a deciding selection criteria.

Using the above five criteria, the three SA techniques selected for use in this study were the Morris method, FAST/eFAST and Sobol’ method of SA. The Morris method and FAST/eFAST were used as screening techniques and the FAST/eFAST and Sobol’ methods used for more detailed and higher-order analyses.

The Morris method and FAST/eFAST proved to be successful in their application of screening variables for negligible importance, while the FAST/eFAST technique was also able to provide non-erroneous first- and total-order indices in the detailed analyses of both case study systems. The Sobol’ method gave erroneous measures illustrating a limitation with its algorithm and application to such an analysis. Early results of an increased accuracy Sobol’ experiment in the preliminary case study and the Barwon system case study showed erroneous results and were therefore not completed. The next iteration of the Sobol’ method required impracticable computational expense and hence were not undertaken. Considering the results of this study, the Morris method and FAST/eFAST are suitable methods for identifying and quantifying the importance of input variables used in the estimation of yield of an urban water supply system, and can therefore be extended to be applicable techniques to apply to other problems related to water supply planning modelling.

Several limitations exists, specifically the lack of accurate estimation of higher-order effects of input variables and application to a model that considers a time series input variable, such as streamflow, evaporation, rainfall and demand used in water supply planning models, and presumably other environmentally dependant models. The limitation of handling time series is due to the requirement that the input variables can be perturbed by a scalar value and that the typical SA techniques indices do not allow comparison across experiments, such as different climate scenarios in this study.

6.2.1.2 Variable Handling

There is a lack of appreciation of variable handling strategies in SA application literature. All SA techniques reviewed in this thesis randomly select a single scalar value from within a predefined range to perturb an input variable. Typically, analysts use discrete distributions for variables that cannot be easily handled using a single scalar value, such as variables that contain a number of interrelated factors, like the target storage curves used in this study. However, discretely distributed variables cause issues for many SA techniques, such as the FAST and Sobol' approximate an integral in which the relationship between sample points is required to be continuous.

Variable handling strategies were therefore a major obstacle that needed to be overcome in this study. Several strategies were established to perturb an input variable that has a number of factors (such as the temporal distribution factors and time-series variables) by a scalar value. An algorithm was developed that approximately perturbs individual factors within a multi-factored input variable by the randomly selected percentage value. Other input variable specific strategies were also developed as discussed in this thesis. However, some limitations relating to the handling of multi-factored input variables and handling of discretely distributed input variables were not overcome as suitable strategies were not available at the time.

6.2.1.3 Additional Sensitivity Analysis Measures

The sensitivity measures, or indices, that are determined via the FAST and Sobol' techniques provide an estimation of the importance of the input variables standardised within each experiment. The quantitative effect of an input variable on the model output is lost through this standardisation; therefore, comparison across scenarios is limited to the qualitative ranking of importance of the input variables. This limitation was identified in the case study on the hypothetical water supply system presented in Chapter 4. In the Barwon urban water supply system case study in Chapter 5, the partial variances due to each input variable, V_i, and the total yield variance, $V(Y)$, were determined for each climate scenario considered in this study. V_i provided a measure of the individual effect of each variable on the estimation of yield, comparable across scenarios and planning periods. $V(Y)$ quantifies the amount of variance in the estimation of yield due to all variables, which indicates the sensitivity of the estimation of yield to the climate scenarios. The findings that these additional sensitivity measures are discussed in Section 6.2.2.

6.2.2 Sensitivity of Yield Estimate to Input Variables

The sensitivity of yield estimate to changes in the input variables was the primary aim of this study. In the preliminary case study using the hypothetical urban water supply system, it was found that the streamflow was the most important input variable, followed by the reliability of supply threshold. The upper Restriction Rule Curve (RRC) and the maximum number of consecutive months in restriction threshold also showed considerable sensitivity effect. The remaining variables had negligible importance to the yield estimate.

The SA framework applied to the Barwon urban water supply system case study included the dichotomy of input variables consisting of: i) climate dependant variables and, ii) management controllable variables. The climate dependant variables consisted of streamflow, evaporation, rainfall and demand. The second source of variability in the estimation of yield, the management controllable variables, consisted of the system management polices and rules such as the target storage curves, restriction rule curves, etc. SA was performed on the management variables while considering various climate scenarios.

Using this climate scenario based approach, the most important input variables in estimation of yield of the Barwon urban water supply system over nearly all scenarios were the security criteria: the reliability of supply threshold and the minimum storage level threshold. It was also shown that when the reliability threshold variable was more important than the minimum storage threshold variable, the upper RRC curvature and upper RRC position are also important. However, when the minimum storage threshold is important, then the remaining variables have mostly ambiguous importance. Furthermore, it was shown that the reliability threshold is the most important security criteria for the 20 year scenarios and the minimum storage level threshold was most important for the 40, 60 and 77 year scenarios. The presence of interaction effects was identified by the Morris method and FAST/eFAST but they cannot provide quantitative estimates. The Sobol' method was used to quantify the second order effects but gave unsatisfactory, erroneous results.

These results highlight the areas and order that research should be focussed and resources spent so a better understanding of these input variables is gained, resulting in a smaller variability of yield and a greater confidence in its estimation. For all scenarios, the minimum storage level threshold and the reliability of supply need to be accurately set, which will remove a significant amount of variability in the estimation of yield. After this is done, if it is found that the reliability threshold is critical, then the upper RRC curvature and the upper RRC position should subsequently be researched and set. The study done on the Barwon system could not provide a definitive preference to the order of research if the minimum storage level threshold is critical as the importance indices did not show a trend. This lack of trend indicates that the input variables, i.e. the system policy and rules, are sensitive to the climate scenarios, including a change of total streamflow volume, change of planning length and climate variability.

The system policy and rules provide a set of operational and management guidelines that are general enough to provide system security and adequate system performance for the unpredictable climatic future. It has been shown in this study that the controllable input variables are sensitive to changes in the climate variability and planning length. Therefore, several climate scenarios should be considered when generating these general set of rules and policies, rather than the current approach that uses a single climate sequence using the entire available historic climate data. Alternative climate scenarios can be achieved using stochastic generation methods or the methods described in this thesis, i.e. selecting a sub-sequence from a longer sequence or using the shuffling block approach used to generate the 77 year scenarios.

6.2.3 Sensitivity of the Yield Estimate to Planning Length and Climate Variability

To assess the sensitivity of the yield estimate to the climate scenarios, the $V(Y)$ for each scenario was computed and compared. For simplicity, the square root of $V(Y)$ (i.e. the standard deviation) as used instead. Comparing $V(Y)$ across the planning length indicated that as the simulation length increases, the variance decreases, i.e. the range of the yield estimates decreases. It was also found that the average of the yield estimates deceases as the planning length increases. As the random samples of the SA are the same across the all scenarios and planning lengths, these findings indicate that the estimation of yield becomes more robust against changes in the input variables and climate variability as the planning length increases.

This highlights the importance of the length of simulation when estimating yield and yield related studies. It is especially important for short planning periods. For a planning period of 20 years, it was found that the estimation of yield for the Barwon system is highly variable compared to the estimates resulting from the 60 and 77 year planning periods. The longer planning periods contain a number of extreme climate events such as drought periods and high streamflow periods. The high streamflow periods that are contained within the longer planning period fill the storages allows them to buffer against the dryer, drought periods, therefore leading to a lower but more stable estimation of yield. Conversely, a short planning period will capture fewer opposing climate events, therefore the range of the yield estimate is higher. This conclusion is only valid for the Barwon urban water supply system under the historic climate sequences used.

The implications of these findings are quite significant with respect to the typical method that yield is estimated and handled, which considers the entire sequence of historic climate data. In the case of the Barwon system, the use of the 77 years of historic data, without the consideration of alternative climate scenarios, suggests that the future climate will be the same as the climate represented in the historic data. Therefore this method will produce an inappropriate yield estimate and set of optimal system polices and rules for any future climate. However, the future climate is unpredictable, so a representative scenario can never be determined. It is recommended here that multiple climate scenarios of appropriate length are used in the estimation of yield and subsequent studies. Table 6-1 shows the average and range of the average yield estimates of the Barwon urban water supply system that are experienced amongst each considered planning period. These were based on the results given as Table 5-38 in Chapter 5.

Table 6-1. Average Yield Estimate for the Barwon Urban Water Supply System.

	20 Years	**40 Years**	**60 Years**	**77 Years**
Average Yield Estimate (Ml)	57,000	50,000	47,000	44,000
Range of Average Yield Estimates (Ml)	18,000	11,000	8,000	2,000

The results of the Barwon case study show that the entire available historic climate data sequence (77 years in length) produces a lower and more robust average estimation of yield compared to the average estimate resulting from a shorter planning length. The average yield result for the 77 year planning length is approximately 44,000 Ml. This could be considered as a safe yield, quoted to the public as a target demand in the attempt to reduce water consumption, which will mean a better security of supply, greater water for environmental flows, etc.

In the operation of the water supply system however, yield should be estimated using an appropriate planning period for simulation (i.e. simulation length) so that the appropriate yield is realised for that planning period. This simulation length should ideally be the same or similar to the study period (i.e. the period considered in the water authorities' studies). In the Barwon case study it was found that a shorter planning period of 20 years results in a greater average yield estimate. For Barwon Water this yield estimate can be used as a guide to system augmentation or other development purposes. For example, based on the 20 year average yield results given in Chapter 5, the Barwon system has a 20 year yield of 57,000 Ml. If this operational yield is to be maintained up to 40 year planning length, then system augmentation resulting in a yield increase of 7,000 Ml is required after 20 years.

Using historic sequences of the appropriate length to a study purpose, and considering multiple climate variability scenarios, water authorities can gain an appreciation of a range of possible yields and the importance of the input variables. However, these results are clearly dependant on the behaviour of the system under the adopted climate sequences, and the appropriateness of these sequences as a representative climate.

Section 1.4 presented the significance of the study, making reference to Figure 1-1 which highlights the change of average annual inflow to the Barwon system over the 77 year historic sequence. It is clearly seen that a 51% decrease in average annual streamflow from an average annual streamflow of 155 Gl from 1927 to 1996, to 76 Gl for the period from 1997 to 2003. The reduced inflow into the Barwon system has continued to 2008. It is likely that the climate sequence from 1997 to present is indicative of what to expect for the future. In that case, the use of the entire sequence prior to 1997, or part thereof, is erroneous, only the last 10 years or so (1997 – 2008) will be relevant to the present day climate.

Based on the average yield estimate and standard deviation of yield estimate results given in Table 6-1, it can be seen that the yield estimate is very sensitive to changes in the climate variability and input variables for short planning lengths. As the planning length increases the average yield estimate becomes more robust (i.e. the standard deviation of the estimation of yield decreases) to climate and input variable changes.

Based on the results of this study, it is the recommendation that climate data of at least 40 years in length is required for a robust estimation of yield, and for that matter, any other water supply planning studies using climatic data. This length will provide a reliable yield estimate and will be long enough to capture an appropriate sample of natural climate variability. The gains in terms of robustness between the 40 year and 60 year planning lengths are marginal. The 77 year length shows considerable robustness, but this could be due to the method of generating the data. However, this claim is only valid for the Barwon system as it is the only urban water supply system considered in this study. There is opportunity here for future research into the determining a more accurate required planning length and to apply the hypothesis on other urban waters supply systems.

6.3 Limitations of the Study and Recommendations for Further Research

The climate scenario approach used to test the sensitivity of the yield estimate to changes in the input variables, the climate variability and the planning length, only used a limited number of climates and planning lengths. As mentioned in the section above, there is opportunity for future research to be carried out using more climate scenarios and planning lengths generated from the historic climate data, using historic data that has been adjusted to the required characteristics and/or using stochastically generated climate sequences. This will confirm the required planning period for a robust representation of climate and will also give a better understanding of the system behaviour, yield estimate and importance of input variables. There are also possibilities to extend the methods used in this study to other urban water supply systems and other investigation techniques, i.e. other sensitivity analysis techniques or to decision making tools.

The sensitivity analysis techniques used in this study were limited by their inability to handle time series variables and discretely distributed variables, and to provide a comparative measure across various sensitivity analysis experiments. The Morris method and the use of the partial- and total-output variances did somewhat provide a quantitative measure to allow comparison across various experiments. However, the comparisons in this study were mostly qualitative since the climate sequences were not characterised robustly in terms of dry/wet periods, critical periods etc. Here also lays scope for a sensitivity analysis technique that is specifically designed to consider time-series variables without the use of discrete sets of sequences.

Only the effect of the changes of input variables over the total planning period were considered in the sensitivity measures determined. Further information regarding the behaviour of the system can be gained if importance of input variables on the model were determined throughout the planning period, i.e. a sensitivity measure determined at each time simulation step. This would then highlight which variables are important for various system conditions and climate events. Combining this information with predictions of future climate can be made based on current conditions, ultimately leads to a dynamic approach to the research and optimisation of management processes and practices.

The performance of an urban water supply system can be measured from the yield of the system. If the physical system and its management are at an optimal state the yield is maximised. In this study, this was the only output of the model considered. Significant improvements in the understanding of the behaviour of the system could be gained by performing sensitivity analysis on other model outputs and/or by changing the purpose of the sensitivity analysis; such as testing the sensitivity of the security criteria to climate variability, urban demand and to changes in the remaining system policies and rules.

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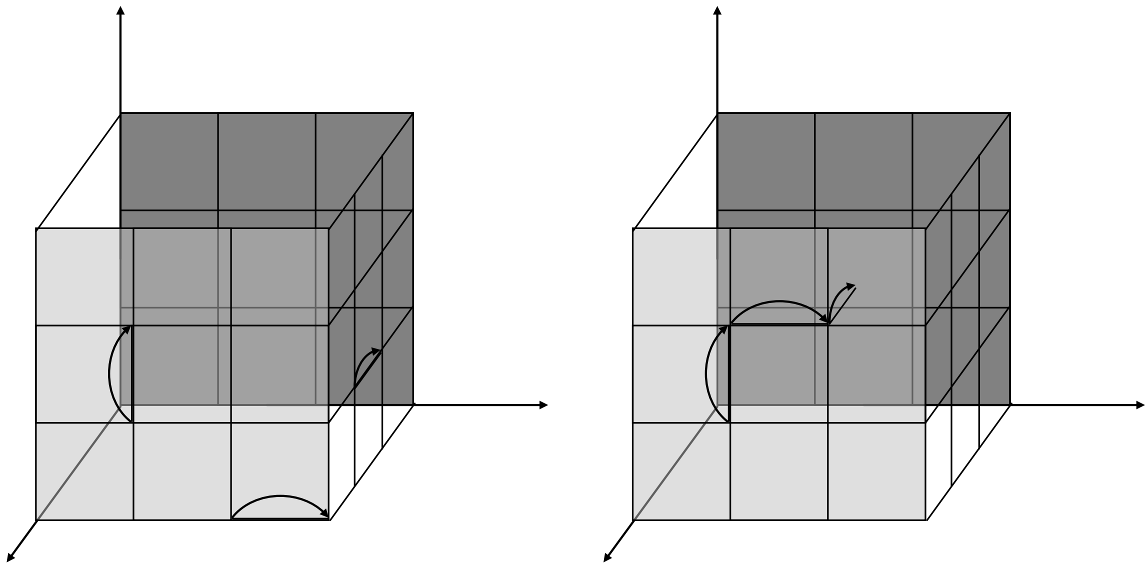
APPENDIX A

Morris Method Algorithm

The following describes the algorithm that Morris (1991) proposed for the construction of trajectory pathways through the region of experimentation, Ω, as shown in Figure A-1(b).



(a) (b)

Figure A-1. The Region of Experimentation, Ω.
(a) Individual EEs for a Three Variable Model. Six Simulations Required. P = 4.
(b) Trajectory EEs for a Three Variable Model. Four Simulations Required. p = 4.

The trajectories are used to denote the variable perturbation from which an Elementary Effect (*EE*) can be calculated for each input variable. An *EE* is determined using Equation (A.1):

$$EE_i(\mathbf{x}) = [y(x_1, x_2, ..., x_{i-1}, x_i + \Delta, x_{i+1}, ..., x_k) - y(\mathbf{x})] / \Delta \qquad \text{(A.1)}$$

where Δ is a predetermined multiple of 1/(p - 1).

p is the number of 'levels', or values, over which the variables can be sampled. Also known as the resolution of sampling.

To define the pathway of each trajectory, Morris (1991) used a series of matrices to construct the final trajectory matrix, **B***, which is defined by Equation (A.2) and explained following. A number of these final trajectory matrices, r, ultimately determine the design input matrix **X**, as shown in Equation (A.3). **X** is an ($n \times k$) matrix where n signifies a the number of model simulations.

$$\mathbf{B}^* = (\mathbf{J}_{m,1}\mathbf{x}^* + (\Delta/2)[(2\mathbf{B} - \mathbf{J}_{m,k})\mathbf{D}^* + \mathbf{J}_{m,k}])\mathbf{P}^* \tag{A.2}$$

$$\mathbf{X} = \begin{bmatrix} \mathbf{B}_1^* \\ \mathbf{B}_2^* \\ \cdots \\ \mathbf{B}_r^* \end{bmatrix} \tag{A.3}$$

Letting $m = k + 1$, the initial step is to create a ($m \times k$) sampling matrix, **B**, which contains elements of 0's and 1's, and has the key property that for each column, $i = 1, 2, 3, \ldots, k$, there are two rows of **B** that differ only in their i-th entries (Morris 1991). A lower left triangle unit matrix for example:

$$\mathbf{B} = \begin{bmatrix} 0 & 0 & 0 & 0 & . & .0 \\ 1 & 0 & 0 & 0 & \ldots & 0 \\ 1 & 1 & 0 & 0 & \ldots & 0 \\ 1 & 1 & 1 & 0 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 1 & 1 & 1 & 1 & . & .1 \end{bmatrix} \tag{A.4}$$

The sampling matrix, **B**, is then modified so that within randomly selected columns 0's become 1's and 1's become 0's. Denoted as **B'** this modified sampling matrix can be produced using Equation (A.5):

$$\mathbf{B}' = (\tfrac{1}{2})[(2\mathbf{B} - \mathbf{J}_{m,k})\mathbf{D}^* + \mathbf{J}_{m,k}] \tag{A.5}$$

where $\mathbf{J}_{m,k}$ is a ($m \times k$) matrix of 1's (unit matrix)

$\mathbf{D}^*$ is a k-dimensional diagonal matrix which the diagonal elements have an equal probability of taking a value of +1 of -1

By introducing -1s, the **D*** matrix allows a negative Δ change to occur. An example of **D*** for k = 6 is shown below:

$$\mathbf{D^*} = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \tag{A.6}$$

Using Equation (A.6) in Equation (A.5), **B'** then becomes:

$$\mathbf{B}' = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix} \tag{A.7}$$

B* could then be constructed as Equation (A.8):

$$\mathbf{B}^* = \mathbf{J}_{m,1}\mathbf{x}^* + (\Delta\mathbf{B}') \tag{A.8}$$

where $\mathbf{J}_{m,1}$ is a (m × 1) matrix of 1's

$\mathbf{x}^*$ **x*** is a set of randomly selected base values from the set of selectable **x** values ranging from 0 to 1 – Δ

The base values, **x***, is only selectable from 0 to 1–Δ so that when Δ**B'** is added to **x***, **B*** does not exceed the upper bounds of Ω. It is interesting to note that **x*** is therefore never used as a sample point in **B***.

Letting p = 6 and Δ = 2/5, the available set of selectable variable points becomes {0, 1/5, 2/5, 3/5}. Assuming the randomly selected base value **x*** = {3/5, 0, 2/5, 1/5, 3/5, 2/5}, Equation (A.8) then resolves to:

$$\mathbf{B^*} = \begin{bmatrix} 1 & 0 & 0.4 & 0.6 & 0.6 & 0.8 \\ 0.6 & 0 & 0.4 & 0.6 & 0.6 & 0.8 \\ 0.6 & 0.4 & 0.4 & 0.6 & 0.6 & 0.8 \\ 0.6 & 0.4 & 0.8 & 0.6 & 0.6 & 0.8 \\ 0.6 & 0.4 & 0.8 & 0.2 & 0.6 & 0.8 \\ 0.6 & 0.4 & 0.8 & 0.2 & 1 & 0.8 \\ 0.6 & 0.4 & 0.8 & 0.2 & 1 & 0.4 \end{bmatrix} \tag{A.9}$$

The final permutation matrix (**P***) is a k-dimensional matrix where each column and row contains only single element equal to 1 and the rest 0's, as demonstrated in Equation (A.10) for a k = 6 matrix.

$$\mathbf{P}^* = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \tag{A.10}$$

P* is not an essential element of **B***, but as the location of the 1's is random, it provides a random change to the order that the variables are perturbed, and increases the number of trajectories possible. Combining Equations (A.5), (A.8) and (A.10) the final trajectory matrix, Equation (A.2), is realised:

$$\mathbf{B^*} = \begin{bmatrix} 0.6 & 1 & 0.8 & 0 & 0.6 & 0.4 \\ 0.6 & 0.6 & 0.8 & 0 & 0.6 & 0.4 \\ 0.6 & 0.6 & 0.8 & 0.4 & 0.6 & 0.4 \\ 0.6 & 0.6 & 0.8 & 0.4 & 0.6 & 0.8 \\ 0.2 & 0.6 & 0.8 & 0.4 & 0.6 & 0.8 \\ 0.2 & 0.6 & 0.8 & 0.4 & 1 & 0.8 \\ 0.2 & 0.6 & 0.4 & 0.4 & 1 & 0.8 \end{bmatrix} \tag{A.11}$$

Here it can be seen that each column, which represents a variable, is changed one-at-a-time by a negative or positive Δ, where $\Delta = 2/5$. From these change, an EE can then be calculated for each input variable using Equation (A.1).

A number r **B*** matrices are constructed and assembled to finally make up the design matrix **X**, given in Equation (A.3). The columns of **X** are then scaled to appropriate range of the input variables.

Here a shortcoming of the Morris method becomes clear. When applying the Morris method to discrete variables, the number of points of a discrete variable must be the same as, or a multiple of, the number of levels, p, used in the above algorithm. This ensures that two discrete points are chosen for the Δ change and the EE is correctly calculated. Alternatively, it is possible to assign different p values (p_i), and hence different Δ values (Δ_i), to each input variable by allowing the choice of **x*** from the appropriate distributions. However, as **x*** may be selected from sets that contain different number of values, resulting from different number of levels, **P*** will need to be modified to ensure that the columns with equal p_i values are permuted together. Although as discussed earlier, **P*** is not essential and if omitted this issue will be avoided.

APPENDIX B
Results of Individual Morris Method Experiments of the Preliminary Case Study

The following are the results of the Morris method experiments performed on the preliminary case study. The individual experiments were performed using the algorithm settings as shown in Table B-1.

Table B-1. Algorithm Settings for the Morris Method Sensitivity Analysis Experiment.

Experiment	Number of Trajectories	Level	Δ	Seed
1	10	4	1	18936437
2	10	4	1	874366872
3	10	6	2	18936437
4	10	6	2	874366872
5	10	6	3	18936437
6	10	6	3	874366872
7	10	6	4	18936437
8	10	6	4	874366872
9	10	8	3	18936437
10	10	8	3	874366872
11	10	8	4	18936437
12	10	8	4	874366872
13	10	8	5	18936437
14	10	8	5	874366872
15	20	4	2	18936437
16	20	4	2	874366872

Table B-2. Results of the Morris Method Experiment 1.

Factor	**μ**	**μ***	**σ**	**μ* Ranking**
Streamflow	6085	6085	796	1
Rainfall	919	919	334	5
Evaporation	-560	560	350	9
Evaporation Factor A for Reservoir A	-742	742	292	8
Evaporation Factor A for Reservoir B	-818	818	412	7
Evaporation Factor B for Reservoir A	-433	433	352	11
Evaporation Factor B for Reservoir B	-163	163	217	15
Volume to Surface Area Relationship	182	182	296	14
Temporal Disaggregation Factors	-480	480	413	10
Climate Index	-827	827	266	6
Upper RRC – Position	-1216	1216	782	4
Lower RRC – Position	76	76	241	16
Base Demand – Position	-261	261	368	12
Stage 1 Percentage Restrictable	254	254	352	13
Stage 2 Percentage Restrictable	13	13	37	18
Stage 3 Percentage Restrictable	0	0	0	19
Stage 4 Percentage Restrictable	0	0	0	19
Stage 1 Relative Position	-15	15	37	17
Stage 2 Relative Position	0	0	0	19
Stage 3 Relative Position	0	0	0	19
Consecutive Months in Restriction	1534	1534	3674	3
Worst Restriction Level	0	0	0	19
Supply Reliability	-4313	4313	1341	2
Target Storage Curves – Point 2	0	0	0	19
Target Storage Curves – Point 3	0	0	0	19
Target Storage Curves – Point 4	0	0	0	19
Initial Volume of Reservoir A	0	0	0	19
Initial Volume of Reservoir B	0	0	0	19

Table B-3. Results of the Morris Method Experiment 2.

Factor	**μ**	**μ***	**σ**	**μ* Ranking**
Streamflow	5906	5906	969	1
Rainfall	920	920	40	5
Evaporation	-674	674	350	7
Evaporation Factor A for Reservoir A	-652	652	332	8
Evaporation Factor A for Reservoir B	-595	595	358	9
Evaporation Factor B for Reservoir A	-396	396	422	11
Evaporation Factor B for Reservoir B	-239	239	284	14
Volume to Surface Area Relationship	279	420	464	10
Temporal Disaggregation Factors	-346	346	367	12
Climate Index	-771	771	323	6
Upper RRC Position	-1172	1172	587	4
Lower RRC Position	76	76	241	16
Base Demand Position	-243	243	368	13
Stage 1 Percentage Restrictable	62	86	230	15
Stage 2 Percentage Restrictable	0	0	0	17
Stage 3 Percentage Restrictable	0	0	0	17
Stage 4 Percentage Restrictable	0	0	0	17
Stage 1 Relative Position	0	0	0	17
Stage 2 Relative Position	0	0	0	17
Stage 3 Relative Position	0	0	0	17
Consecutive Months in Restriction	1652	1652	3125	3
Worst Restriction Level	0	0	0	17
Supply Reliability	-4749	4749	1549	2
Target Storage Curves – Point 2	0	0	0	17
Target Storage Curves – Point 3	0	0	0	17
Target Storage Curves – Point 4	0	0	0	17
Initial Volume of Reservoir A	0	0	0	17
Initial Volume of Reservoir B	0	0	0	17

Table B-4. Results of the Morris Method Experiment 3.

Factor	**μ**	**μ***	**σ**	**μ* Ranking**
Streamflow	6812	6812	924	1
Rainfall	826	826	49	5
Evaporation	-682	682	185	8
Evaporation Factor A for Reservoir A	-776	776	285	6
Evaporation Factor A for Reservoir B	-704	704	219	7
Evaporation Factor B for Reservoir A	-352	372	360	11
Evaporation Factor B for Reservoir B	-306	306	303	12
Volume to Surface Area Relationship	225	225	298	14
Temporal Disaggregation Factors	-498	498	329	10
Climate Index	-640	640	322	9
Upper RRC Position	-1136	1136	717	3
Lower RRC Position	141	141	275	15
Base Demand Position	-305	305	305	13
Stage 1 Percentage Restrictable	91	110	200	16
Stage 2 Percentage Restrictable	-4	6	15	18
Stage 3 Percentage Restrictable	0	0	0	19
Stage 4 Percentage Restrictable	0	0	0	19
Stage 1 Relative Position	-19	19	39	17
Stage 2 Relative Position	0	0	0	19
Stage 3 Relative Position	0	0	0	19
Consecutive Months in Restriction	986	986	2673	4
Worst Restriction Level	0	0	0	19
Supply Reliability	-3953	3953	1133	2
Target Storage Curves – Point 2	0	0	0	19
Target Storage Curves – Point 3	0	0	0	19
Target Storage Curves – Point 4	0	0	0	19
Initial Volume of Reservoir A	0	0	0	19
Initial Volume of Reservoir B	0	0	0	19

Table B-5. Results of the Morris Method Experiment 4.

Factor	**μ**	**μ***	**σ**	**μ* Ranking**
Streamflow	6170	6170	1058	1
Rainfall	867	867	196	4
Evaporation	-753	753	42	8
Evaporation Factor A for Reservoir A	-700	700	189	9
Evaporation Factor A for Reservoir B	-788	788	40	7
Evaporation Factor B for Reservoir A	-173	173	240	13
Evaporation Factor B for Reservoir B	-370	370	284	10
Volume to Surface Area Relationship	197	197	256	12
Temporal Disaggregation Factors	-337	337	302	11
Climate Index	-828	828	210	6
Upper RRC Position	-861	861	618	5
Lower RRC Position	0	0	0	16
Base Demand Position	-97	97	193	14
Stage 1 Percentage Restrictable	54	54	48	15
Stage 2 Percentage Restrictable	0	0	0	16
Stage 3 Percentage Restrictable	0	0	0	16
Stage 4 Percentage Restrictable	0	0	0	16
Stage 1 Relative Position	0	0	0	16
Stage 2 Relative Position	0	0	0	16
Stage 3 Relative Position	0	0	0	16
Consecutive Months in Restriction	943	943	2014	3
Worst Restriction Level	0	0	0	16
Supply Reliability	-4498	4498	1684	2
Target Storage Curves – Point 2	0	0	0	16
Target Storage Curves – Point 3	0	0	0	16
Target Storage Curves – Point 4	0	0	0	16
Initial Volume of Reservoir A	0	0	0	16
Initial Volume of Reservoir B	0	0	0	16

Table B-6. Results of the Morris Method Experiment 5.

Factor	**μ**	**μ***	**σ**	**μ* Ranking**
Streamflow	6215	6215	951	1
Rainfall	882	882	186	5
Evaporation	-700	700	331	8
Evaporation Factor A for Reservoir A	-677	677	211	9
Evaporation Factor A for Reservoir B	-818	818	201	6
Evaporation Factor B for Reservoir A	-234	234	204	13
Evaporation Factor B for Reservoir B	-314	314	191	11
Volume to Surface Area Relationship	211	211	197	14
Temporal Disaggregation Factors	-375	375	312	10
Climate Index	-759	759	252	7
Upper RRC Position	-1391	1391	602	3
Lower RRC Position	0	0	0	17
Base Demand Position	-284	284	305	12
Stage 1 Percentage Restrictable	104	104	203	15
Stage 2 Percentage Restrictable	0	91	193	16
Stage 3 Percentage Restrictable	0	0	0	17
Stage 4 Percentage Restrictable	0	0	0	17
Stage 1 Relative Position	0	0	0	17
Stage 2 Relative Position	0	0	0	17
Stage 3 Relative Position	0	0	0	17
Consecutive Months in Restriction	1195	1195	2226	4
Worst Restriction Level	0	0	0	17
Supply Reliability	-4060	4060	741	2
Target Storage Curves – Point 2	0	0	0	17
Target Storage Curves – Point 3	0	0	0	17
Target Storage Curves – Point 4	0	0	0	17
Initial Volume of Reservoir A	0	0	0	17
Initial Volume of Reservoir B	0	0	0	17

Table B-7. Results of the Morris Method Experiment 6.

Factor	μ	μ*	σ	μ* Ranking
Streamflow	5784	5784	935	1
Rainfall	947	947	231	5
Evaporation	-593	593	147	9
Evaporation Factor A for Reservoir A	-898	898	309	6
Evaporation Factor A for Reservoir B	-668	668	190	8
Evaporation Factor B for Reservoir A	-271	271	211	12
Evaporation Factor B for Reservoir B	-314	314	186	11
Volume to Surface Area Relationship	168	168	191	14
Temporal Disaggregation Factors	-373	373	218	10
Climate Index	-749	749	224	7
Upper RRC Position	-1082	1082	657	4
Lower RRC Position	159	159	320	15
Base Demand Position	-244	244	234	13
Stage 1 Percentage Restrictable	155	155	215	16
Stage 2 Percentage Restrictable	21	21	31	17
Stage 3 Percentage Restrictable	0	0	0	19
Stage 4 Percentage Restrictable	0	0	0	19
Stage 1 Relative Position	-10	10	31	18
Stage 2 Relative Position	0	0	0	19
Stage 3 Relative Position	0	0	0	19
Consecutive Months in Restriction	1110	1110	1767	3
Worst Restriction Level	0	0	0	19
Supply Reliability	-4128	4128	1229	2
Target Storage Curves – Point 2	0	0	0	19
Target Storage Curves – Point 3	0	0	0	19
Target Storage Curves – Point 4	0	0	0	19
Initial Volume of Reservoir A	0	0	0	19
Initial Volume of Reservoir B	0	0	0	19

Table B-8. Results of the Morris Method Experiment 7.

Factor	μ	μ*	σ	μ* Ranking
Streamflow	6099	6099	950	1
Rainfall	791	791	145	5
Evaporation	-663	663	154	8
Evaporation Factor A for Reservoir A	-608	608	153	9
Evaporation Factor A for Reservoir B	-684	684	147	6
Evaporation Factor B for Reservoir A	-408	408	108	11
Evaporation Factor B for Reservoir B	-358	358	23	12
Volume to Surface Area Relationship	175	175	152	13
Temporal Disaggregation Factors	-451	451	113	10
Climate Index	-668	668	235	7
Upper RRC Position	-1390	1390	594	4
Lower RRC Position	0	0	0	18
Base Demand Position	-152	152	170	14
Stage 1 Percentage Restrictable	137	137	166	15
Stage 2 Percentage Restrictable	32	32	100	16
Stage 3 Percentage Restrictable	0	0	0	18
Stage 4 Percentage Restrictable	0	0	0	18
Stage 1 Relative Position	-5	5	15	17
Stage 2 Relative Position	0	0	0	18
Stage 3 Relative Position	0	0	0	18
Consecutive Months in Restriction	1604	1604	2096	3
Worst Restriction Level	0	0	0	18
Supply Reliability	-2866	2866	1674	2
Target Storage Curves – Point 2	0	0	0	18
Target Storage Curves – Point 3	0	0	0	18
Target Storage Curves – Point 4	0	0	0	18
Initial Volume of Reservoir A	0	0	0	18
Initial Volume of Reservoir B	0	0	0	18

Table B-9. Results of the Morris Method Experiment 8.

Factor	μ	μ*	σ	μ* Ranking
Streamflow	5760	5760	939	1
Rainfall	826	826	229	5
Evaporation	-595	595	146	9
Evaporation Factor A for Reservoir A	-674	674	231	7
Evaporation Factor A for Reservoir B	-769	769	153	6
Evaporation Factor B for Reservoir A	-199	199	164	13
Evaporation Factor B for Reservoir B	-308	308	121	11
Volume to Surface Area Relationship	257	257	139	12
Temporal Disaggregation Factors	-394	394	216	10
Climate Index	-605	605	252	8
Upper RRC Position	-1035	1035	597	4
Lower RRC Position	45	45	107	18
Base Demand Position	-166	166	162	14
Stage 1 Percentage Restrictable	54	54	90	17
Stage 2 Percentage Restrictable	74	74	133	16
Stage 3 Percentage Restrictable	0	0	0	19
Stage 4 Percentage Restrictable	0	0	0	19
Stage 1 Relative Position	-87	87	169	15
Stage 2 Relative Position	0	0	0	19
Stage 3 Relative Position	0	0	0	19
Consecutive Months in Restriction	1461	1461	1929	3
Worst Restriction Level	0	0	0	19
Supply Reliability	-3466	3466	1581	2
Target Storage Curves – Point 2	0	0	0	19
Target Storage Curves – Point 3	0	0	0	19
Target Storage Curves – Point 4	0	0	0	19
Initial Volume of Reservoir A	0	0	0	19
Initial Volume of Reservoir B	0	0	0	19

Table B-10. Results of the Morris Method Experiment 9.

Factor	**μ**	**μ***	**σ**	**μ* Ranking**
Streamflow	6913	6913	1045	1
Rainfall	871	871	252	4
Evaporation	-606	606	362	9
Evaporation Factor A for Reservoir A	-648	648	203	8
Evaporation Factor A for Reservoir B	-673	673	177	7
Evaporation Factor B for Reservoir A	-305	305	250	11
Evaporation Factor B for Reservoir B	-283	283	281	12
Volume to Surface Area Relationship	211	211	268	15
Temporal Disaggregation Factors	-458	458	295	10
Climate Index	-798	798	181	5
Upper RRC Position	-1201	1201	773	3
Lower RRC Position	11	11	29	17
Base Demand Position	-267	267	288	13
Stage 1 Percentage Restrictable	138	252	392	14
Stage 2 Percentage Restrictable	-14	14	31	16
Stage 3 Percentage Restrictable	0	0	0	18
Stage 4 Percentage Restrictable	0	0	0	18
Stage 1 Relative Position	0	0	0	18
Stage 2 Relative Position	0	0	0	18
Stage 3 Relative Position	0	0	0	18
Consecutive Months in Restriction	743	743	2349	6
Worst Restriction Level	0	0	0	18
Supply Reliability	-4231	4231	1187	2
Target Storage Curves – Point 2	0	0	0	18
Target Storage Curves – Point 3	0	0	0	18
Target Storage Curves – Point 4	0	0	0	18
Initial Volume of Reservoir A	0	0	0	18
Initial Volume of Reservoir B	0	0	0	18

Table B-11. Results of the Morris Method Experiment 10.

Factor	μ	μ*	σ	μ* Ranking
Streamflow	6320	6320	1024	1
Rainfall	874	874	225	3
Evaporation	-695	695	42	8
Evaporation Factor A for Reservoir A	-665	665	174	9
Evaporation Factor A for Reservoir B	-770	770	182	7
Evaporation Factor B for Reservoir A	-175	175	235	13
Evaporation Factor B for Reservoir B	-347	347	275	11
Volume to Surface Area Relationship	139	139	161	14
Temporal Disaggregation Factors	-448	448	306	10
Climate Index	-796	796	196	6
Upper RRC Position	-840	840	700	4
Lower RRC Position	9	9	29	17
Base Demand Position	-91	202	312	12
Stage 1 Percentage Restrictable	-93	129	246	15
Stage 2 Percentage Restrictable	0	0	0	18
Stage 3 Percentage Restrictable	0	0	0	18
Stage 4 Percentage Restrictable	0	0	0	18
Stage 1 Relative Position	-73	73	188	16
Stage 2 Relative Position	0	0	0	18
Stage 3 Relative Position	0	0	0	18
Consecutive Months in Restriction	802	802	1897	5
Worst Restriction Level	0	0	0	18
Supply Reliability	-4601	4601	1368	2
Target Storage Curves – Point 2	0	0	0	18
Target Storage Curves – Point 3	0	0	0	18
Target Storage Curves – Point 4	0	0	0	18
Initial volume of Reservoir A	0	0	0	18
Initial Volume of Reservoir B	0	0	0	18

Table B-12. Results of the Morris Method Experiment 11.

Factor	**μ**	**μ***	**σ**	**μ* Ranking**
Streamflow	6609	6609	636	1
Rainfall	733	733	192	7
Evaporation	-631	631	173	9
Evaporation Factor A for Reservoir A	-708	708	203	8
Evaporation Factor A for Reservoir B	-775	775	217	5
Evaporation Factor B for Reservoir A	-273	273	218	11
Evaporation Factor B for Reservoir B	-253	253	200	13
Volume to Surface Area Relationship	95	95	139	15
Temporal Disaggregation Factors	-492	492	245	10
Climate Index	-836	836	332	4
Upper RRC Position	-1304	1304	476	3
Lower RRC Position	0	0	0	18
Base Demand Position	-263	263	239	12
Stage 1 Percentage Restrictable	146	146	193	14
Stage 2 Percentage Restrictable	9	9	22	17
Stage 3 Percentage Restrictable	0	0	0	18
Stage 4 Percentage Restrictable	0	0	0	18
Stage 1 Relative Position	-51	51	140	16
Stage 2 Relative Position	0	0	0	18
Stage 3 Relative Position	0	0	0	18
Consecutive Months in Restriction	759	759	2065	6
Worst Restriction Level	0	0	0	18
Supply Reliability	-3676	3676	743	2
Target Storage Curves – Point 2	0	0	0	18
Target Storage Curves – Point 3	0	0	0	18
Target Storage Curves – Point 4	0	0	0	18
Initial Volume of Reservoir A	0	0	0	18
Initial Volume of Reservoir B	0	0	0	18

Table B-13. Results of the Morris Method Experiment 12.

Factor	μ	μ*	σ	μ* Ranking
Streamflow	6138	6138	742	1
Rainfall	781	781	226	4
Evaporation	-642	642	259	8
Evaporation Factor A for Reservoir A	-502	590	360	9
Evaporation Factor A for Reservoir B	-706	706	205	6
Evaporation Factor B for Reservoir A	-198	198	200	13
Evaporation Factor B for Reservoir B	-365	365	187	11
Volume to Surface Area Relationship	222	222	198	12
Temporal Disaggregation Factors	-370	479	358	10
Climate Index	-677	677	346	7
Upper RRC Position	-1074	1074	671	3
Lower RRC Position	135	135	218	15
Base Demand Position	-164	164	193	14
Stage 1 Percentage Restrictable	118	118	166	16
Stage 2 Percentage Restrictable	0	7	16	18
Stage 3 Percentage Restrictable	0	0	0	19
Stage 4 Percentage Restrictable	0	0	0	19
Stage 1 Relative Position	-58	58	172	17
Stage 2 Relative Position	0	0	0	19
Stage 3 Relative Position	0	0	0	19
Consecutive Months in Restriction	728	728	1239	5
Worst Restriction Level	0	0	0	19
Supply Reliability	-4170	4170	1413	2
Target Storage Curves – Point 2	0	0	0	19
Target Storage Curves – Point 3	0	0	0	19
Target Storage Curves – Point 4	0	0	0	19
Initial Volume of Reservoir A	0	0	0	19
Initial Volume of Reservoir B	0	0	0	19

Table B-14. Results of the Morris Method Experiment 13.

Factor	**μ**	**μ***	**σ**	**μ* Ranking**
Streamflow	6270	6270	961	1
Rainfall	778	778	325	7
Evaporation	-596	673	386	8
Evaporation Factor A for Reservoir A	-786	786	105	6
Evaporation Factor A for Reservoir B	-819	819	35	5
Evaporation Factor B for Reservoir A	-424	424	233	11
Evaporation Factor B for Reservoir B	-241	241	167	12
Volume to Surface Area Relationship	187	187	164	13
Temporal Disaggregation Factors	-479	479	197	10
Climate Index	-669	669	281	9
Upper RRC Position	-1382	1382	521	3
Lower RRC Position	11	11	23	18
Base Demand Position	-173	173	182	14
Stage 1 Percentage Restrictable	57	57	104	17
Stage 2 Percentage Restrictable	114	114	166	16
Stage 3 Percentage Restrictable	0	0	0	19
Stage 4 Percentage Restrictable	0	0	0	19
Stage 1 Relative Position	-116	116	269	15
Stage 2 Relative Position	0	0	0	19
Stage 3 Relative Position	0	0	0	19
Consecutive Months in Restriction	1061	1061	1955	4
Worst Restriction Level	0	0	0	19
Supply Reliability	-3595	3595	618	2
Target Storage Curves – Point 2	0	0	0	19
Target Storage Curves – Point 3	0	0	0	19
Target Storage Curves – Point 4	0	0	0	19
Initial Volume of Reservoir A	0	0	0	19
Initial Volume of Reservoir B	0	0	0	19

Table B-15. Results of the Morris Method Experiment 14.

Factor	**μ**	**μ***	**σ**	**μ* Ranking**
Streamflow	5744	5744	876	1
Rainfall	798	798	142	5
Evaporation	-663	663	155	8
Evaporation Factor A for Reservoir A	-640	640	161	9
Evaporation Factor A for Reservoir B	-675	675	178	7
Evaporation Factor B for Reservoir A	-379	379	205	12
Evaporation Factor B for Reservoir B	-382	382	328	11
Volume to Surface Area Relationship	223	223	177	13
Temporal Disaggregation Factors	-316	403	286	10
Climate Index	-713	713	272	6
Upper RRC Position	-1069	1069	574	4
Lower RRC Position	100	100	184	15
Base Demand Position	-211	211	277	14
Stage 1 Percentage Restrictable	100	100	133	15
Stage 2 Percentage Restrictable	34	34	104	17
Stage 3 Percentage Restrictable	0	0	0	18
Stage 4 Percentage Restrictable	0	0	0	18
Stage 1 Relative Position	0	0	0	18
Stage 2 Relative Position	0	0	0	18
Stage 3 Relative Position	0	0	0	18
Consecutive Months in Restriction	1170	1170	1637	3
Worst Restriction Level	0	0	0	18
Supply Reliability	-3869	3869	1022	2
Target Storage Curves – Point 2	0	0	0	18
Target Storage Curves – Point 3	0	0	0	18
Target Storage Curves – Point 4	0	0	0	18
Initial Volume of Reservoir A	0	0	0	18
Initial Volume of Reservoir B	0	0	0	18

Table B-16. Results of the Morris Method Experiment 15.

Factor	**μ**	**μ***	**σ**	**μ* Ranking**
Streamflow	5958	5958	1211	1
Rainfall	923	923	139	5
Evaporation	-604	604	169	9
Evaporation Factor A for Reservoir A	-686	686	190	8
Evaporation Factor A for Reservoir B	-699	699	182	7
Evaporation Factor B for Reservoir A	-242	242	185	12
Evaporation Factor B for Reservoir B	-305	305	165	11
Volume to Surface Area Relationship	241	241	184	13
Temporal Disaggregation Factors	-464	464	235	10
Climate Index	-761	761	236	6
Upper RRC Position	-1474	1474	542	3
Lower RRC Position	1	1	6	18
Base Demand Position	-211	211	202	14
Stage 1 Percentage Restrictable	158	158	220	15
Stage 2 Percentage Restrictable	2	2	7	17
Stage 3 Percentage Restrictable	0	0	0	19
Stage 4 Percentage Restrictable	0	0	0	19
Stage 1 Relative Position	-19	19	85	16
Stage 2 Relative Position	0	0	0	19
Stage 3 Relative Position	0	0	0	19
Consecutive Months in Restriction	1110	1110	1935	4
Worst Restriction Level	0	0	0	19
Supply Reliability	-3538	3538	2191	2
Target Storage Curves – Point 2	0	0	0	19
Target Storage Curves – Point 3	0	0	0	19
Target Storage Curves – Point 4	0	0	0	19
Initial Volume of Reservoir A	0	0	0	19
Initial Volume of Reservoir B	0	0	0	19

Table B-17. Results of the Morris Method Experiment 16.

Factor	μ	μ*	σ	μ* Ranking
Streamflow	5833	5833	1181	1
Rainfall	863	863	142	6
Evaporation	-519	613	382	9
Evaporation Factor A for Reservoir A	-275	984	1743	5
Evaporation Factor A for Reservoir B	-700	700	168	8
Evaporation Factor B for Reservoir A	-349	349	230	12
Evaporation Factor B for Reservoir B	-342	342	173	13
Volume to Surface Area Relationship	206	206	186	14
Temporal Disaggregation Factors	-463	463	209	10
Climate Index	-753	753	343	7
Upper RRC Position	-1046	1415	1291	4
Lower RRC Position	25	25	92	19
Base Demand Position	-145	145	178	16
Stage 1 Percentage Restrictable	157	163	194	15
Stage 2 Percentage Restrictable	36	36	110	17
Stage 3 Percentage Restrictable	0	0	0	20
Stage 4 Percentage Restrictable	0	0	0	20
Stage 1 Relative Position	-28	28	92	18
Stage 2 Relative Position	0	0	0	20
Stage 3 Relative Position	0	0	0	20
Consecutive Months in Restriction	1636	1636	2083	3
Worst Restriction Level	0	0	0	20
Supply Reliability	-3628	3628	1905	2
Target Storage Curves – Point 2	-445	445	1992	11
Target Storage Curves – Point 3	0	0	0	20
Target Storage Curves – Point 4	0	0	0	20
Initial Volume of Reservoir A	0	0	0	20
Initial Volume of Reservoir B	0	0	0	20

APPENDIX C
Time-Series Perturbation and Correlation Issues

In the sensitivity analysis of the input variables used in the estimation of yield of the hypothetical urban water supply system, the handling strategy used for perturbing the streamflow time-series variable consisted of changing all datum in the sequence by a randomly selected percentage. This was used as a simple method of perturbation that changes all data points by the same measurement error margin. This in effect changes not only the variability of the streamflow but also the total volume of water entering the system. Below is a discussion on uniform change perturbation method used in the Chapter 4 SA and two more perturbation methods that were consequently considered:

1. Uniform change – A percentage change randomly selected from the variable range to change all the data in the time series. The change can be a positive or negative percentage change applied to all data simultaneously. This is the method that was used in the above SA.

2. Varying change – Like the uniform change, a single percentage change is randomly selected from the sampling range and is used to change all data points in the time series. However, approximately half the data is assigned a positive change and the other half a negative change. The pattern of these positive or negative changes is randomly predefined and used in all experiments to avoid introducing unnecessary uncertainty or variability that would not be accounted for in the sensitivity analysis.

3. Random change – This method of perturbation introduces an extra level of randomness to the magnitude of each datum perturbation. Each datum is randomly perturbed between predefined margins in a way that the total streamflow perturbation is equal to the selected percentage change required by the SA technique. Say the selected required percentage change is +3.5% and a predefined margin of +1% to -1% is used. Each datum in then randomly assigned a perturbed between +2.5% and +4.5%, so that the average change is +3.5%.

Table C-1 shows the effects of the time-series perturbation methods on the streamflow sequence for Reservoir A of the hypothetical urban water supply system case study of Chapter 4. Given are the monthly mean, standard deviation, variance and coefficient of variation (C_v) of the original streamflow and also after the three perturbation methods were performed. A noteworthy point is the standard deviation of the uniform change, which shows a change in variance but an unchanged C_v. This is due to the changed mean and the uniform perturbation dispersing further the data from it. The varying and random perturbation show changes to both the standard deviation and C_v, where the varying method retains a similar mean as the original streamflow.

Table C-1. Analysis of Uniform, Varying and Random Perturbation Methods for a 5% Change.

	Original	**Uniform Change**	**Varying Change**	**Random Change (4% Margin)**
Mean	5672	5955	5669	5955
Standard Deviation	4762	5000	4780	4990
Coefficient of Variation (C_v)	0.8395	0.8395	0.8430	0.8378

In the SA presented in Chapter 4, the uniform perturbation method was used; however, the varying perturbation method would have better corresponded with the definition of yield which states that it is dependent on the variability of the streamflow, not the volume. However this variability is only on a short-term measurement error basis, not on a long-term climate event basis.

A 5000 model simulation eFAST SA experiment was performed with the variables shown in Table C-2 using the three perturbation methods described above. The S_i results are shown in Table C-3 and the S_{Ti} results are given in Table C-4. Immediately the difference in importance is clear with the streamflow variable becoming the least important when the varying change and the random change perturbation strategies are applied. Note that the climate index, rainfall and evaporation time series were all perturbed using the uniform change strategy. Due to the streamflow decreasing its dominance on the total output variance, the remaining variables increase, however not at the same rate. There are some changing of ranks between the variables that can be rationalised by considering an altering of interactions with the streamflow variable and ultimately the volume of streamflow entering the system.

Table C-2. Top 10 Important Variables used in Detailed SA Experiments.

Variable Name
Streamflow
Climate index
Rainfall
Evaporation
Evaporation Factor A (Reservoir A)
Evaporation Factor A (Reservoir B)
Consecutive Months Threshold
Reliability Threshold
Upper RRC Position
Temporal Disaggregation Factors

Table C-3. First-Order Indices (S_i) for eFAST Perturbation Strategies Experiment.

Variable	Uniform Change	Varying Change	Random Change (4% margin)
Streamflow	0.6398	0.0011	0.0002
Climate Index	0.0120	0.0285	0.0267
Rainfall	0.0150	0.0312	0.0316
Evaporation	0.0012	0.0046	0.0044
Evaporation Factor A (Reservoir A)	0.0100	0.0194	0.0216
Evaporation Factor A (Reservoir B)	0.0075	0.0290	0.0277
Consecutive Months Threshold	0.0190	0.0541	0.0515
Reliability Threshold	0.2650	0.6607	0.6681
Upper Restriction Rule Curve Position	0.0333	0.0930	0.0941
Temporal Disaggregation Factors	0.0049	0.0086	0.0094

Table C-4. Total-Order Indices (S_{Ti}) for eFAST Perturbation Strategies Experiment.

Variable	**Uniform Change**	**Varying Change**	**Random Change (4% margin)**
Streamflow	0.6639	0.0272	0.0271
Climate Index	0.0242	0.0574	0.0535
Rainfall	0.0260	0.0601	0.0591
Evaporation	0.0097	0.0264	0.0257
Evaporation Factor A (Reservoir A)	0.0222	0.0435	0.0451
Evaporation Factor A (Reservoir B)	0.0167	0.0502	0.0487
Consecutive Months Threshold	0.0494	0.1307	0.1200
Reliability Threshold	0.3001	0.7388	0.7453
Upper Restriction Rule Curve Position	0.0502	0.1316	0.1305
Temporal Disaggregation Factors	0.0162	0.0315	0.0314

APPENDIX D
SA Results of eFAST Individual Experiments for the Barwon Water Supply System Case Study

The following (Figure D-1 to D-14) show the first- and total-order results of the eFAST individual experiments on the Barwon urban water supply system. The experiments were performed using 1918 model simulations.

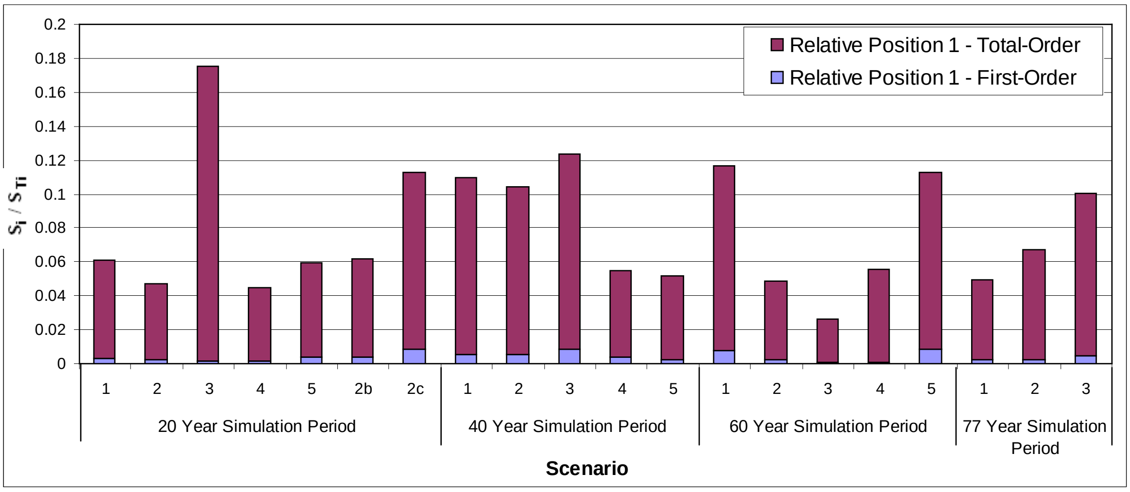


Figure D-1. eFAST Individual Experiment. Relative Position Intermediate Curve 1. S_i and S_{Ti} Results for all Scenarios.

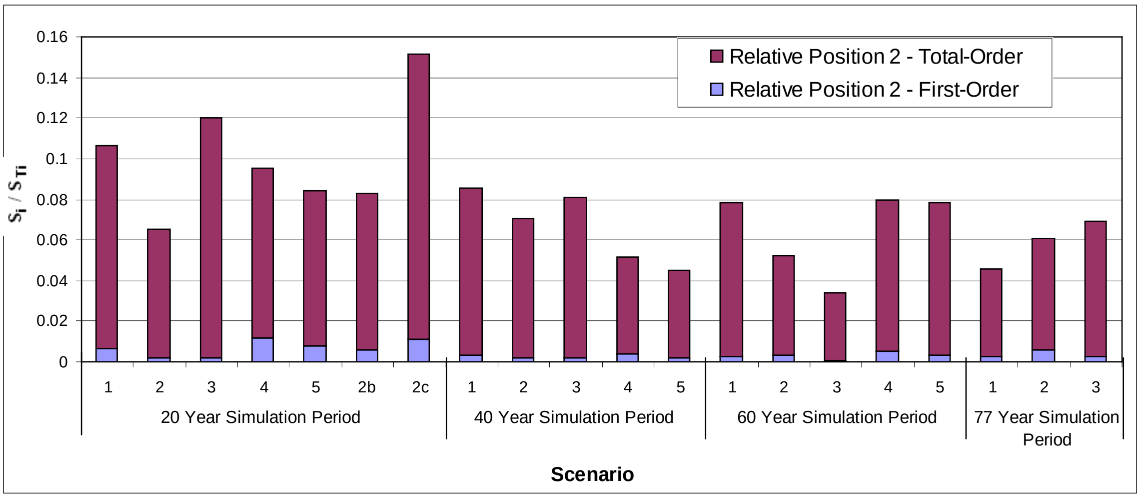


Figure D-2. eFAST Individual Experiment. Relative Position Intermediate Curve 2. S_i and S_{Ti} Results for all Scenarios.

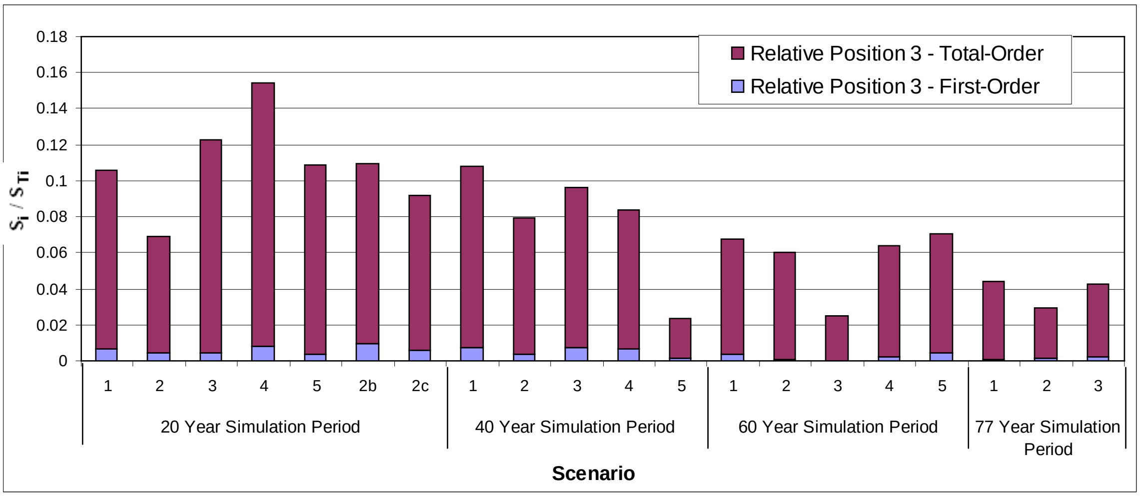


Figure D-3. eFAST Individual Experiment. Relative Position Intermediate Curve 3. S_i and S_{Ti} Results for all Scenarios.

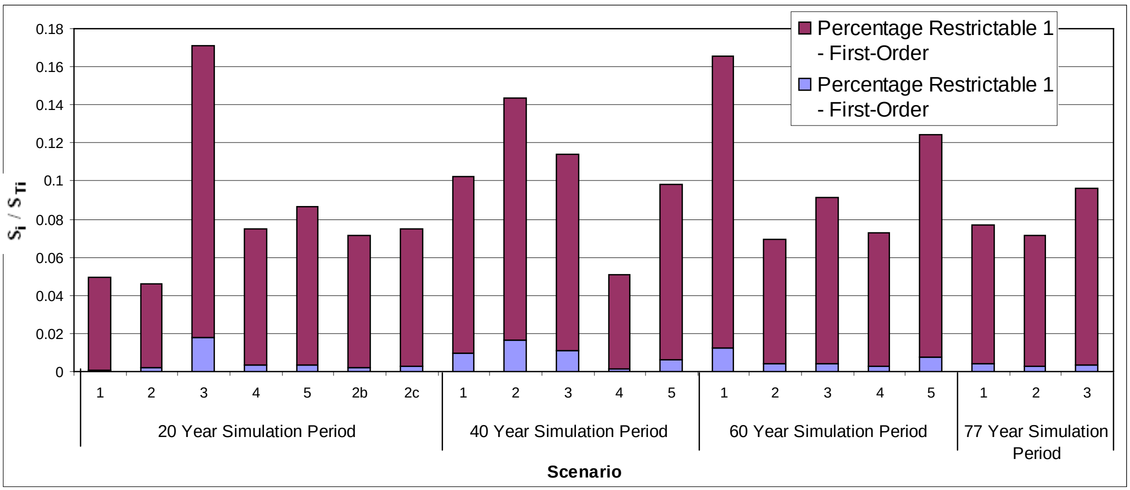


Figure D-4. eFAST Individual Experiment. Percentage Restricatable Zone 1. S_i and S_{Ti} Results for all Scenarios.

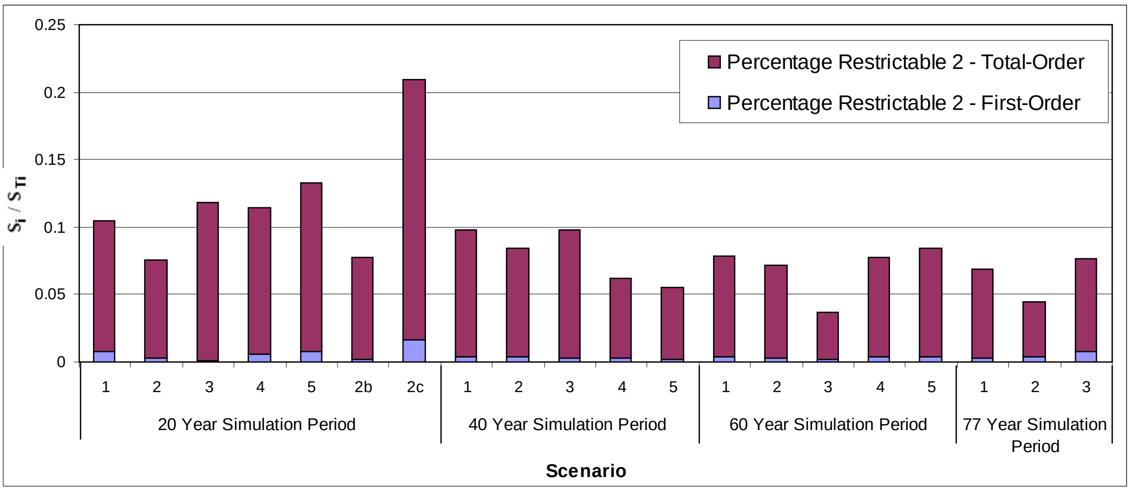


Figure D-5. eFAST Individual Experiment. Percentage Restricatable Zone 2. S_i and S_{Ti} Results for all Scenarios.

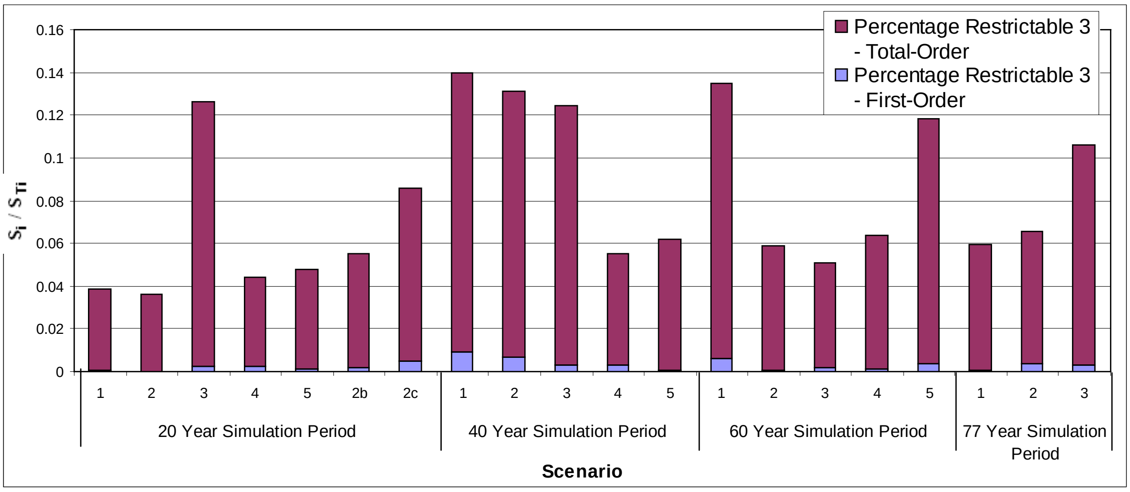


Figure D-6. eFAST Individual Experiment. Percentage Restricatable Zone 3. S_i and S_{Ti} Results for all Scenarios.

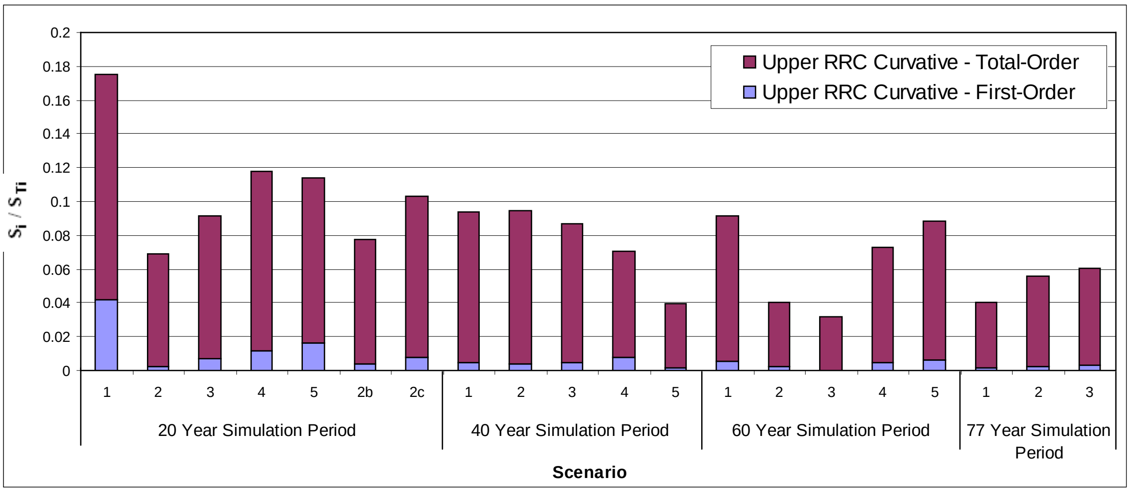


Figure D-7. eFAST Individual Experiment. Upper RRC Curvature. S_i and S_{Ti} Results for all Scenarios.

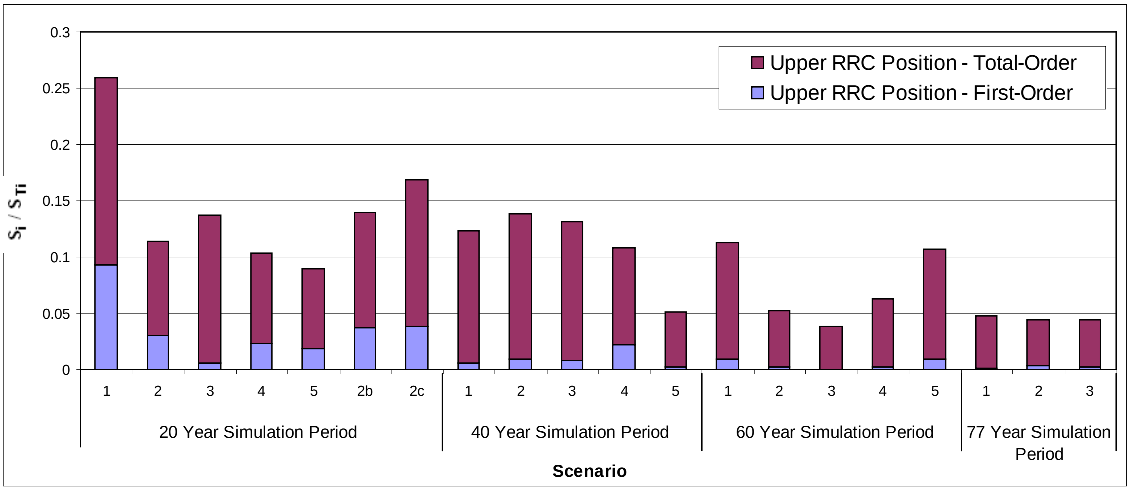


Figure D-8. eFAST Individual Experiment. Upper RRC Position. S_i and S_{Ti} Results for all Scenarios.

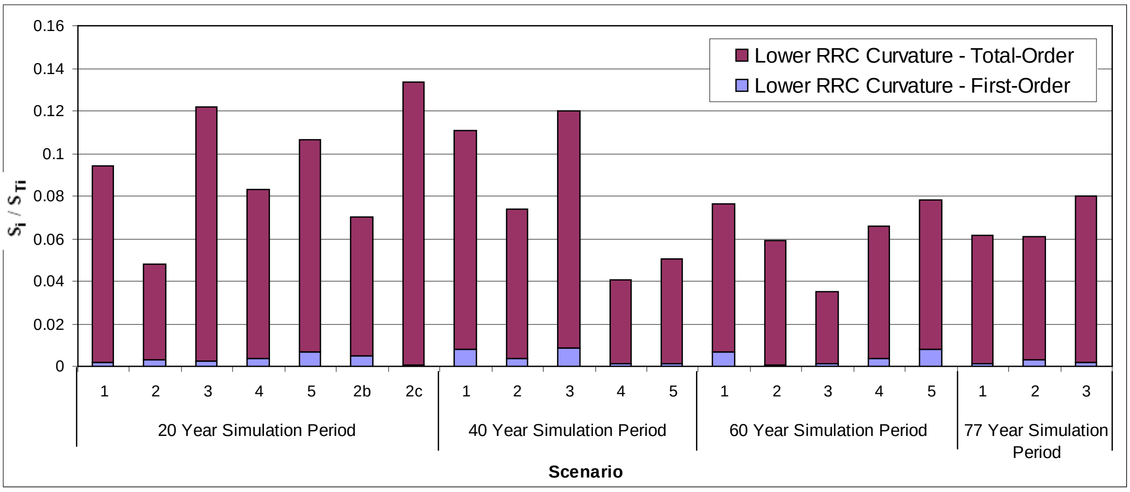


Figure D-9. eFAST Individual Experiment. Lower RRC Curvature. S_i and S_{Ti} Results for all Scenarios.

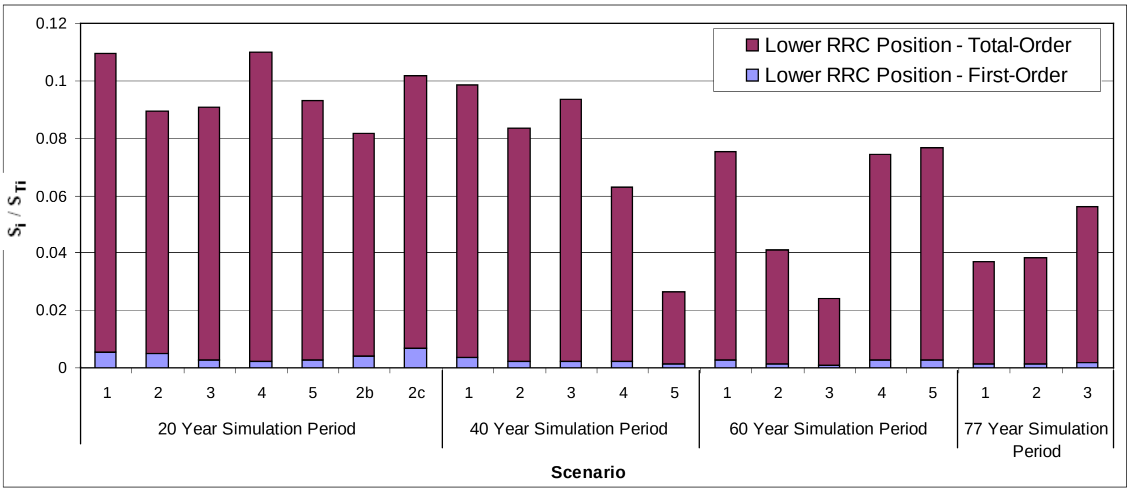


Figure D-10. eFAST Individual Experiment. Lower RRC Position. S_i and S_{Ti} Results for all Scenarios.

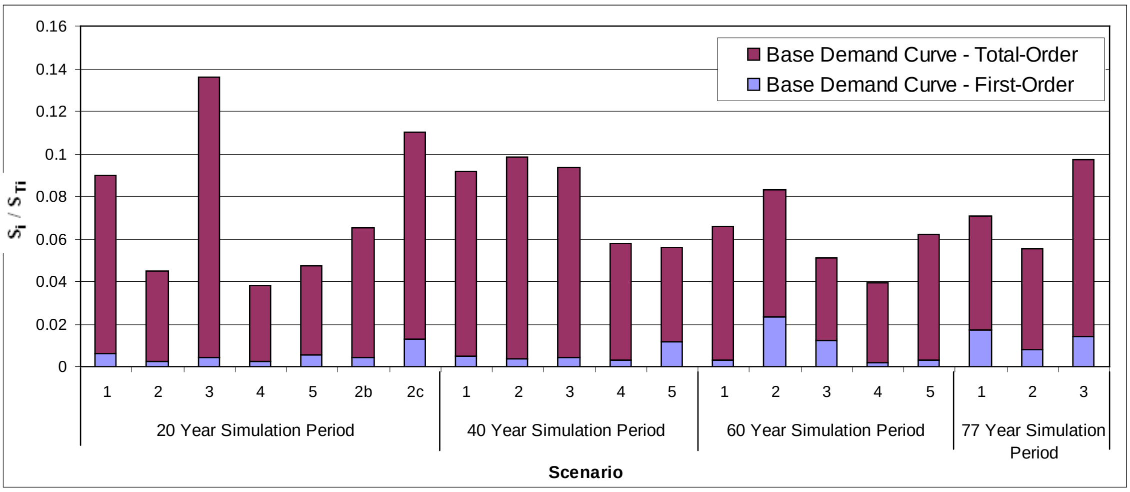


Figure D-11. eFAST Individual Experiment. Base Demand Curve Position. S_i and S_{Ti} Results for all Scenarios.

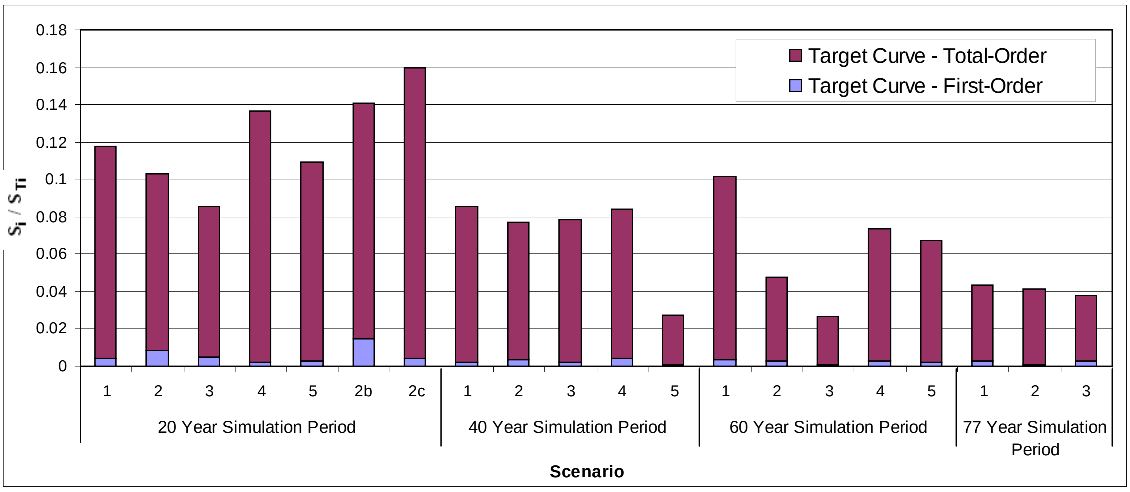


Figure D-12. eFAST Individual Experiment. Target Storage Curves. S_i and S_{Ti} Results for all Scenarios.

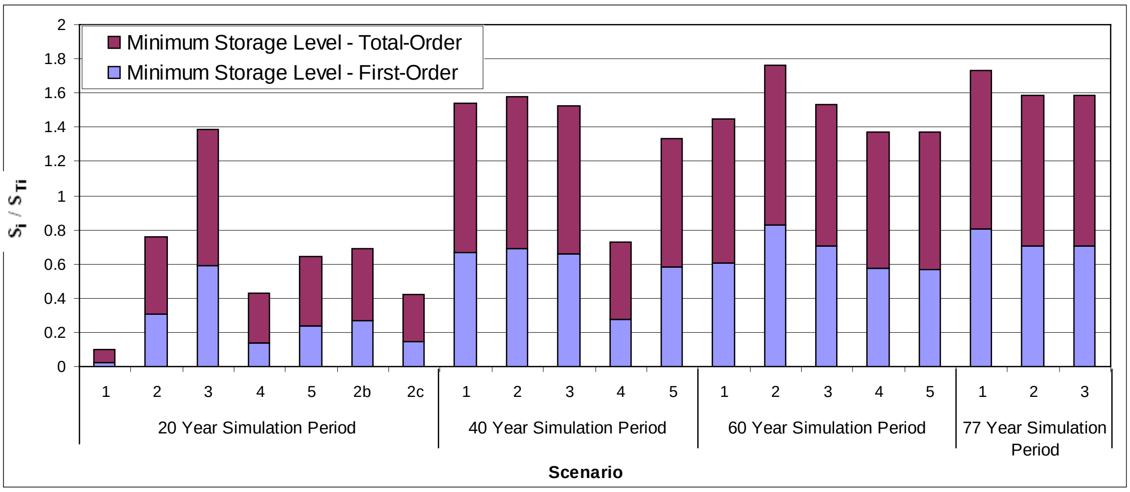


Figure D-13. eFAST Individual Experiment. Minimum Storage Level Threshold. S_i and S_{Ti} Results for all Scenarios.

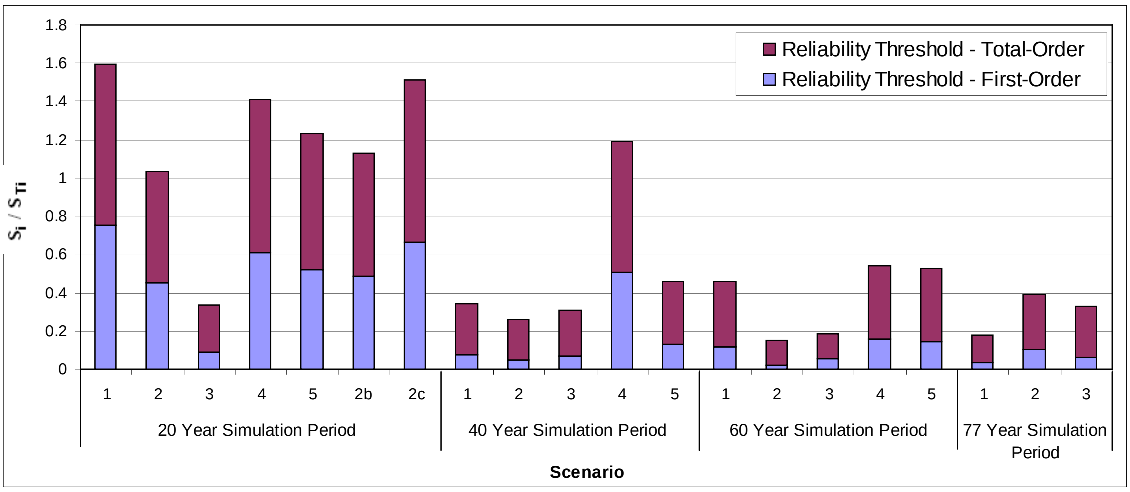


Figure D-14. eFAST Individual Experiment. Reliability of Supply Threshold. S_i and S_{Ti} Results for all Scenarios.

APPENDIX E
Sobol' Second-Order Sensitivity Indices of Individual Experiments for Barwon Urban Water Supply System Case Study

The following tables display the S_{ij}^{c} and S_{ij} results of the 6848 simulation Sobol' experiments performed on the Barwon urban water supply system. They are included her to show the erroneous $S_{ij}^{c} < 0$ and $S_{ij} < 0$ results. Only the 20 year scenarios are shown here for brevity but the other results of other planning lengths show similar errors.

Table E-1. ‘Closed’ Second-Order Importance Measures (S_{ij}^{c}) of the Sobol' Experiment for the Barwon Urban Water Supply System Case Study – 20 year Scenario 1.

	Relative Position 1	**Relative Position 2**	**Relative Position 3**	**Percent. Restrict. 1**	**Percent. Restrict. 2**	**Percent. Restrict. 3**	**Upper RRC Curvature**	**Upper RRC Position**	**Lower RRC Curvature**	**Lower RRC Position**	**Base Demand**	**Target Curves**	**Minimum Storage Threshold**
Relative Position 2	-0.0081	0.0011	0.0064	0.0043	0.0031	0.0465	0.0930	0.0376	-0.0019	-0.0100	0.0343	0.1146	0.7598
Relative Position 3	-0.0058	0.0062	0.0052	0.0060	0.0466	0.0349	0.0443	0.0364	-0.0047	0.0499	0.0634	0.8245	
Percent. Restrict. 1	0.0009	0.0031	0.0002	0.0481	0.0457	0.0016	0.0463	0.0318	0.0476	0.0599	0.7179		
Percent. Restrict. 2	-0.0026	-0.0003	0.0446	0.0386	0.0070	-0.0015	0.0382	0.0694	0.0627	0.7256			
Percent. Restrict. 3	-0.0058	0.0481	0.0339	0.0080	0.0011	0.0034	0.0893	0.0826	0.7238				
Upper RRC Curvature	0.0400	0.0356	0.0032	0.0028	-0.0044	0.0474	0.0854	0.7126					
Upper RRC Position	0.0403	0.0018	-0.0011	-0.0021	0.0420	0.0607	0.7554						
Lower RRC Curvature	-0.0066	-0.0017	-0.0052	0.0473	0.0640	0.7260							
Lower RRC Position	-0.0073	-0.0089	0.0439	0.0632	0.7282								
Base Demand	-0.0225	0.0472	0.0612	0.7270									
Target Curves	0.0511	0.0635	0.7259										
Minimum Storage Threshold	0.0571	0.7275											
Reliability Threshold	0.7325												

Table E-2. 'Closed' Second-Order Importance Measures (S_{ij}^{c}) of the Sobol' Experiment for the Barwon Urban Water Supply System Case Study – 20 year Scenario 2.

	Relative Position 1	**Relative Position 2**	**Relative Position 3**	**Percent. Restrict. 1**	**Percent. Restrict. 2**	**Percent. Restrict. 3**	**Upper RRC Curvature**	**Upper RRC Position**	**Lower RRC Curvature**	**Lower RRC Position**	**Base Demand**	**Target Curves**	**Minimum Storage Threshold**
Relative Position 2	-0.0060	0.0004	-0.0066	-0.0070	0.0071	0.0178	0.0072	0.0076	-0.0024	-0.0314	-0.0016	0.4692	0.9848
Relative Position 3	-0.0073	-0.0068	0.0073	-0.0228	0.0218	0.0094	0.0184	0.0069	-0.0303	-0.0095	0.4333	0.5115	
Percent. Restrict. 1	-0.0093	0.0076	-0.0018	0.0218	0.0151	0.0030	0.0166	0.0069	-0.0112	0.4410	0.4546		
Percent. Restrict. 2	0.0010	0.0024	0.0183	0.0055	0.0087	0.0012	-0.0073	-0.0015	0.4432	0.4583			
Percent. Restrict. 3	-0.0073	0.0187	0.0101	-0.0040	0.0073	-0.0278	-0.0079	0.4514	0.4582				
Upper RRC Curvature	0.0130	0.0078	0.0013	-0.0062	-0.0274	-0.0112	0.4439	0.4529					
Upper RRC Position	0.0093	0.0040	-0.0004	-0.0261	-0.0077	0.4368	0.4732						
Lower RRC Curvature	-0.0050	0.0012	-0.0320	-0.0089	0.4354	0.4505							
Lower RRC Position	-0.0058	-0.0327	-0.0127	0.4412	0.4555								
Base Demand	-0.0294	-0.0104	0.4404	0.4548									
Target Curves	-0.0108	0.4394	0.4563										
Minimum Storage Threshold	0.4334	0.4562											
Reliability Threshold	0.4567												

Table E-3. 'Closed' Second-Order Importance Measures (S_{ij}^{c}) of the Sobol' Experiment for the Barwon Urban Water Supply System Case Study – 20 year Scenario 3.

	Relative Position 1	**Relative Position 2**	**Relative Position 3**	**Percent. Restrict. 1**	**Percent. Restrict. 2**	**Percent. Restrict. 3**	**Upper RRC Curvature**	**Upper RRC Position**	**Lower RRC Curvature**	**Lower RRC Position**	**Base Demand**	**Target Curves**	**Minimum Storage Threshold**
Relative Position 2	-0.0016	-0.0001	0.0016	0.0016	-0.0058	0.0064	0.0305	0.0234	0.0041	-0.0028	0.0321	0.7310	1.0322
Relative Position 3	0.0037	-0.0006	0.0017	-0.0035	0.0100	0.0217	0.0138	0.0220	0.0033	0.0366	0.7014	0.2323	
Percent. Restrict. 1	0.0012	-0.0043	-0.0028	0.0135	0.0186	-0.0020	0.0084	0.0190	0.0400	0.7269	0.2205		
Percent. Restrict. 2	0.0039	-0.0071	0.0048	0.0226	0.0015	-0.0047	0.0070	0.0514	0.7125	0.2148			
Percent. Restrict. 3	-0.0048	0.0079	0.0216	-0.0005	-0.0005	-0.0018	0.0549	0.7539	0.2129				
Upper RRC Curvature	0.0135	0.0202	0.0022	-0.0010	-0.0003	0.0343	0.7429	0.1901					
Upper RRC Position	0.0197	0.0009	0.0029	-0.0012	0.0373	0.7254	0.2070						
Lower RRC Curvature	0.0000	0.0004	0.0009	0.0353	0.7280	0.1998							
Lower RRC Position	0.0000	-0.0099	0.0340	0.7336	0.2150								
Base Demand	-0.0017	0.0310	0.7223	0.2163									
Target Curves	0.0415	0.7241	0.2175										
Minimum Storage Threshold	0.7062	0.2011											
Reliability Threshold	0.2158												

Table E-4. 'Closed' Second-Order Importance Measures (S_{ij}^{c}) of the Sobol' Experiment for the Barwon Urban Water Supply System Case Study – 20 year Scenario 4.

	Relative Position 1	**Relative Position 2**	**Relative Position 3**	**Percent. Restrict. 1**	**Percent. Restrict. 2**	**Percent. Restrict. 3**	**Upper RRC Curvature**	**Upper RRC Position**	**Lower RRC Curvature**	**Lower RRC Position**	**Base Demand**	**Target Curves**	**Minimum Storage Threshold**
Relative Position 2	-0.0042	0.0028	0.0038	-0.0053	-0.0006	-0.0133	0.0058	-0.0098	0.0002	0.0013	-0.1210	0.1167	0.9006
Relative Position 3	-0.0003	0.0011	-0.0007	0.0030	-0.0160	-0.0080	-0.0157	-0.0014	-0.0022	-0.1130	0.1802	0.5738	
Percent. Restrict. 1	-0.0017	0.0020	0.0034	-0.0211	-0.0029	0.0016	-0.0113	-0.0147	-0.1120	0.1690	0.5360		
Percent. Restrict. 2	-0.0009	0.0037	-0.0219	-0.0020	0.0019	0.0047	-0.0276	-0.0605	0.1718	0.5400			
Percent. Restrict. 3	-0.0015	-0.0184	-0.0018	-0.0019	0.0062	-0.0022	-0.1000	0.1857	0.5348				
Upper RRC Curvature	-0.0161	0.0012	0.0021	0.0063	-0.0002	-0.1130	0.1849	0.5309					
Upper RRC Position	0.0018	-0.0012	-0.0004	-0.0031	-0.1260	0.1730	0.5675						
Lower RRC Curvature	-0.0013	0.0003	-0.0038	-0.1200	0.1731	0.5408							
Lower RRC Position	0.0026	0.0006	-0.1160	0.1708	0.5360								
Base Demand	-0.0062	-0.1050	0.1708	0.5317									
Target Curves	-0.1140	0.1657	0.5418										
Minimum Storage Threshold	0.1684	0.5378											
Reliability Threshold	0.5373												

Table E-5. ‘Closed’ Second-Order Importance Measures (S_{ij}^{c}) of the Sobol' Experiment for the Barwon Urban Water Supply System Case Study – 20 year Scenario 5.

	Relative Position 1	**Relative Position 2**	**Relative Position 3**	**Percent. Restrict. 1**	**Percent. Restrict. 2**	**Percent. Restrict. 3**	**Upper RRC Curvature**	**Upper RRC Position**	**Lower RRC Curvature**	**Lower RRC Position**	**Base Demand**	**Target Curves**	**Minimum Storage Threshold**
Relative Position 2	0.0002	0.0001	0.0092	-0.0002	0.0022	-0.0112	0.0141	0.0380	0.0147	-0.0032	0.0216	0.2734	0.9406
Relative Position 3	0.0040	0.0015	-0.0029	-0.0051	-0.0145	0.0394	-0.0075	0.0415	-0.0114	0.0397	0.2859	0.5348	
Percent. Restrict. 1	-0.0049	-0.0040	0.0051	-0.0153	0.0360	0.0121	-0.0047	0.0442	0.0330	0.2939	0.4633		
Percent. Restrict. 2	-0.0054	0.0046	-0.0221	0.0469	-0.0031	0.0153	-0.0210	0.0799	0.2797	0.4759			
Percent. Restrict. 3	0.0055	-0.0174	0.0387	0.0061	0.0102	-0.0125	0.0304	0.3303	0.4680				
Upper RRC Curvature	-0.0179	0.0440	0.0103	0.0154	-0.0180	0.0335	0.2959	0.4756					
Upper RRC Position	0.0412	0.0060	0.0054	-0.0036	0.0253	0.2945	0.4702						
Lower RRC Curvature	0.0021	0.0074	-0.0121	0.0299	0.2741	0.4712							
Lower RRC Position	0.0112	-0.0171	0.0336	0.2752	0.4695								
Base Demand	-0.0163	0.0290	0.2791	0.4782									
Target Curves	0.0311	0.2793	0.4726										
Minimum Storage Threshold	0.2800	0.4649											
Reliability Threshold	0.4682												

Table E-6. ‘Closed’ Second-Order Importance Measures (S_{ij}^{c}) of the Sobol' Experiment for the Barwon Urban Water Supply System Case Study – 20 year Scenario 2b.

	Relative Position 1	**Relative Position 2**	**Relative Position 3**	**Percent. Restrict. 1**	**Percent. Restrict. 2**	**Percent. Restrict. 3**	**Upper RRC Curvature**	**Upper RRC Position**	**Lower RRC Curvature**	**Lower RRC Position**	**Base Demand**	**Target Curves**	**Minimum Storage Threshold**
Relative Position 2	-0.0100	0.0484	0.0061	0.0537	0.0603	0.0593	0.0221	-0.0409	0.0497	0.0070	-0.0125	0.3703	0.9831
Relative Position 3	0.0575	0.0495	0.0076	0.0009	0.0731	-0.0425	0.0574	0.0054	0.0481	-0.0114	0.3198	0.5153	
Percent. Restrict. 1	0.0575	0.0510	0.0629	0.0206	0.0117	0.0013	0.0761	0.0130	-0.0145	0.3101	0.4788		
Percent. Restrict. 2	0.0528	0.0483	0.0739	-0.0398	0.0500	0.0028	0.0226	-0.0630	0.3598	0.4671			
Percent. Restrict. 3	-0.0032	0.0690	-0.0422	-0.0001	0.0046	0.0533	0.0274	0.3534	0.5402				
Upper RRC Curvature	0.0724	0.0196	-0.0013	0.0043	0.0500	-0.0238	0.3272	0.4487					
Upper RRC Position	-0.0331	0.0603	0.0487	0.0520	-0.0115	0.3077	0.5404						
Lower RRC Curvature	0.0015	0.0631	0.0571	0.0215	0.3643	0.5447							
Lower RRC Position	0.0041	0.0035	-0.0123	0.3123	0.4667								
Base Demand	0.0038	-0.0138	0.3092	0.5434									
Target Curves	-0.0089	0.3628	0.5479										
Minimum Storage Threshold	0.3621	0.5413											
Reliability Threshold	0.5435												

Table E-7. ‘Closed’ Second-Order Importance Measures (S_{ij}^{c}) of the Sobol' Experiment for the Barwon Urban Water Supply System Case Study – 20 year Scenario 2c.

	Relative Position 1	**Relative Position 2**	**Relative Position 3**	**Percent. Restrict. 1**	**Percent. Restrict. 2**	**Percent. Restrict. 3**	**Upper RRC Curvature**	**Upper RRC Position**	**Lower RRC Curvature**	**Lower RRC Position**	**Base Demand**	**Target Curves**	**Minimum Storage Threshold**
Relative Position 2	0.0021	-0.0024	-0.0030	0.0032	0.0038	0.0146	0.0198	-0.0226	0.0012	-0.0022	-0.0853	0.1590	0.8681
Relative Position 3	0.0007	-0.0059	0.0001	0.0064	0.0200	-0.0199	0.0197	-0.0182	-0.0080	-0.0709	0.1381	0.5618	
Percent. Restrict. 1	-0.0026	-0.0010	0.0007	0.0244	-0.0248	0.0018	0.0239	-0.0225	-0.0689	0.1487	0.5861		
Percent. Restrict. 2	-0.0043	0.0070	0.0248	-0.0242	-0.0005	0.0030	0.0145	-0.0555	0.1451	0.5843			
Percent. Restrict. 3	-0.0009	0.0291	-0.0236	-0.0081	-0.0030	-0.0033	-0.0610	0.1751	0.5789				
Upper RRC Curvature	0.0223	-0.0209	0.0027	-0.0062	-0.0010	-0.0649	0.1207	0.5333					
Upper RRC Position	-0.0162	-0.0026	-0.0040	-0.0016	-0.0761	0.1364	0.5934						
Lower RRC Curvature	0.0021	0.0003	-0.0056	-0.0674	0.1355	0.5853							
Lower RRC Position	0.0034	-0.0043	-0.0662	0.1374	0.5848								
Base Demand	-0.0116	-0.0726	0.1347	0.5834									
Target Curves	-0.0741	0.1367	0.5816										
Minimum Storage Threshold	0.1386	0.5809											
Reliability Threshold	0.5837												

Table E-8. Second-Order Importance Measures (S_{ij}) of the Sobol' Experiment for the Barwon Urban Water Supply System Case Study – 20 year Scenario 1.

	Relative Position 1	**Relative Position 2**	**Relative Position 3**	**Percent. Restrict. 1**	**Percent. Restrict. 2**	**Percent. Restrict. 3**	**Upper RRC Curvature**	**Upper RRC Position**	**Lower RRC Curvature**	**Lower RRC Position**	**Base Demand**	**Target Curves**	**Minimum Storage Threshold**
Relative Position 2	-0.0029	-0.0013	-0.0013	-0.0063	-0.0014	0.0017	0.0148	0.0023	-0.0027	-0.0036	-0.0078	0.0064	-0.0269
Relative Position 3	-0.0010	-0.0011	-0.0004	-0.0006	-0.0022	0.0012	-0.0021	0.0040	-0.0013	0.0036	0.0078	0.0512	
Percent. Restrict. 1	0.0007	-0.0022	-0.0013	-0.0028	0.0078	-0.0004	0.0028	0.0036	-0.0016	0.0001	-0.0028		
Percent. Restrict. 2	-0.0007	-0.0016	-0.0013	-0.0013	0.0009	-0.0007	-0.0011	-0.0115	0.0001	0.0007			
Percent. Restrict. 3	0.0001	0.0025	-0.0010	-0.0002	-0.0021	0.0084	-0.0027	-0.0118	-0.0040				
Upper RRC Curvature	0.0015	0.0011	0.0000	-0.0025	-0.0034	-0.0002	-0.0200	-0.0469					
Upper RRC Position	0.0129	-0.0010	-0.0013	-0.0032	-0.0096	-0.0003	-0.0152						
Lower RRC Curvature	-0.0023	-0.0016	-0.0012	-0.0064	-0.0011	-0.0002							
Lower RRC Position	-0.0001	-0.0046	-0.0048	-0.0040	-0.0021								
Base Demand	-0.0111	-0.0011	-0.0009	-0.0053									
Target Curves	0.0098	0.0017	-0.0014										
Minimum Storage Threshold	0.0024	0.0005											
Reliability Threshold	0.0127												

Table E-9. Second-Order Importance Measures (S_{ij}) of the Sobol' Experiment for the Barwon Urban Water Supply System Case Study – 20 year Scenario 2.

	Relative Position 1	**Relative Position 2**	**Relative Position 3**	**Percent. Restrict. 1**	**Percent. Restrict. 2**	**Percent. Restrict. 3**	**Upper RRC Curvature**	**Upper RRC Position**	**Lower RRC Curvature**	**Lower RRC Position**	**Base Demand**	**Target Curves**	**Minimum Storage Threshold**
Relative Position 2	-0.0006	0.0000	0.0007	-0.0083	0.0005	0.0008	-0.0220	-0.0061	-0.0056	-0.0008	0.0423	0.0416	0.0881
Relative Position 3	0.0000	-0.0014	0.0002	-0.0150	-0.0043	-0.0004	-0.0025	-0.0045	-0.0018	0.0027	0.0240	0.0679	
Percent. Restrict. 1	0.0038	-0.0014	0.0001	0.0101	-0.0037	0.0015	-0.0022	0.0271	-0.0012	0.0001	0.0294		
Percent. Restrict. 2	-0.0003	0.0024	0.0007	0.0011	-0.0018	0.0018	0.0055	0.0003	0.0001	0.0015			
Percent. Restrict. 3	0.0004	-0.0007	-0.0001	-0.0001	-0.0011	0.0045	-0.0134	0.0000	-0.0008				
Upper RRC Curvature	0.0013	-0.0043	-0.0007	-0.0001	-0.0041	0.0027	-0.0147	-0.0144					
Upper RRC Position	0.0049	0.0001	-0.0003	0.0116	-0.0028	-0.0024	-0.0014						
Lower RRC Curvature	-0.0012	-0.0005	-0.0002	0.0104	-0.0128	-0.0046							
Lower RRC Position	0.0002	-0.0027	0.0007	0.0074	-0.0086								
Base Demand	0.0083	0.0012	0.0007	0.0050									
Target Curves	0.0084	-0.0022	0.0007										
Minimum Storage Threshold	-0.0005	-0.0012											
Reliability Threshold	0.0069												

Table E-10. Second-Order Importance Measures (S_{ij}) of the Sobol' Experiment for the Barwon Urban Water Supply System Case Study – 20 year Scenario 3.

	Relative Position 1	**Relative Position 2**	**Relative Position 3**	**Percent. Restrict. 1**	**Percent. Restrict. 2**	**Percent. Restrict. 3**	**Upper RRC Curvature**	**Upper RRC Position**	**Lower RRC Curvature**	**Lower RRC Position**	**Base Demand**	**Target Curves**	**Minimum Storage Threshold**
Relative Position 2	-0.0015	-0.0006	-0.0006	0.0014	0.0003	0.0044	0.0024	0.0042	0.0059	0.0002	-0.0016	-0.0308	0.0900
Relative Position 3	0.0005	0.0006	-0.0002	0.0023	0.0019	0.0077	0.0066	0.0029	0.0061	0.0019	-0.0228	-0.0194	
Percent. Restrict. 1	-0.0003	-0.0029	0.0014	0.0052	-0.0014	0.0049	0.0013	0.0010	0.0051	0.0018	0.0065		
Percent. Restrict. 2	0.0026	0.0005	-0.0052	0.0023	0.0023	0.0024	0.0009	-0.0043	-0.0129	-0.0003			
Percent. Restrict. 3	0.0000	0.0013	-0.0004	0.0001	0.0004	0.0063	0.0111	0.0077	-0.0024				
Upper RRC Curvature	0.0042	0.0016	0.0011	-0.0002	0.0016	0.0047	0.0087	-0.0460					
Upper RRC Position	-0.0016	0.0031	0.0020	0.0005	0.0015	0.0053	-0.0171						
Lower RRC Curvature	-0.0005	0.0028	0.0009	-0.0006	0.0018	-0.0102							
Lower RRC Position	-0.0003	-0.0065	-0.0036	0.0072	-0.0011								
Base Demand	-0.0010	-0.0032	-0.0057										
Target Curves	0.0045	-0.0006	-0.0005										
Minimum Storage Threshold	-0.0213	-0.0135											
Reliability Threshold	-0.0016												

Table E-11. Second-Order Importance Measures (S_{ij}) of the Sobol' Experiment for the Barwon Urban Water Supply System Case Study – 20 year Scenario 4.

	Relative Position 1	**Relative Position 2**	**Relative Position 3**	**Percent. Restrict. 1**	**Percent. Restrict. 2**	**Percent. Restrict. 3**	**Upper RRC Curvature**	**Upper RRC Position**	**Lower RRC Curvature**	**Lower RRC Position**	**Base Demand**	**Target Curves**	**Minimum Storage Threshold**
Relative Position 2	-0.0019	0.0010	0.0006	-0.0061	-0.0025	0.0025	0.0216	-0.0119	-0.0024	0.0004	-0.0036	0.0619	0.1925
Relative Position 3	0.0012	-0.0014	-0.0008	-0.0021	0.0040	-0.0140	0.0039	-0.0078	0.0012	-0.0021	0.0133	0.1500	
Percent. Restrict. 1	-0.0009	0.0027	-0.0010	-0.0043	-0.0047	-0.0006	0.0040	-0.0150	0.0037	-0.0041	0.0001		
Percent. Restrict. 2	0.0031	0.0001	-0.0043	-0.0069	0.0039	-0.0018	-0.0062	0.0513	0.0030	-0.0020			
Percent. Restrict. 3	-0.0018	-0.0001	-0.0060	-0.0031	0.0039	-0.0027	0.0331	0.0131	-0.0029				
Upper RRC Curvature	0.0054	-0.0022	0.0017	0.0008	0.0036	-0.0016	0.0341	-0.0105					
Upper RRC Position	0.0016	-0.0009	-0.0051	-0.0026	-0.0098	0.0003	0.0478						
Lower RRC Curvature	0.0023	-0.0037	-0.0025	-0.0070	0.0047	-0.0008							
Lower RRC Position	0.0018	0.0027	-0.0025	-0.0008	-0.0013								
Base Demand	-0.0008	0.0091	0.0000	-0.0088									
Target Curves	0.0030	-0.0044	0.0021										
Minimum Storage Threshold	0.0016	-0.0012											
Reliability Threshold	0.0016												

Table E-12. Second-Order Importance Measures (S_{ij}) of the Sobol' Experiment for the Barwon Urban Water Supply System Case Study – 20 year Scenario 5.

	Relative Position 1	**Relative Position 2**	**Relative Position 3**	**Percent. Restrict. 1**	**Percent. Restrict. 2**	**Percent. Restrict. 3**	**Upper RRC Curvature**	**Upper RRC Position**	**Lower RRC Curvature**	**Lower RRC Position**	**Base Demand**	**Target Curves**	**Minimum Storage Threshold**
Relative Position 2	-0.0027	-0.0017	0.0116	0.0105	0.0028	-0.0009	-0.0100	-0.0080	-0.0078	-0.0089	0.0041	-0.0344	0.1915
Relative Position 3	0.0011	0.0039	0.0035	-0.0085	0.0100	-0.0086	0.0048	-0.0175	-0.0041	-0.0077	0.0198	0.0342	
Percent. Restrict. 1	-0.0036	0.0025	-0.0025	0.0051	0.0021	0.0006	-0.0054	0.0150	-0.0014	-0.0020	0.0044		
Percent. Restrict. 2	-0.0001	-0.0031	-0.0059	0.0090	-0.0006	-0.0092	0.0082	0.0091	-0.0033	-0.0128			
Percent. Restrict. 3	-0.0033	-0.0012	-0.0034	0.0046	-0.0003	-0.0071	0.0179	0.0109	-0.0077				
Upper RRC Curvature	-0.0027	0.0019	0.0046	0.0010	0.0014	-0.0028	0.0349	-0.0366					
Upper RRC Position	-0.0021	0.0003	-0.0133	0.0119	0.0031	0.0096	0.0164						
Lower RRC Curvature	-0.0047	-0.0113	-0.0009	0.0036	0.0033	-0.0065							
Lower RRC Position	-0.0086	-0.0059	0.0031	0.0004	0.0059								
Base Demand	-0.0062	-0.0014	0.0001	0.0106									
Target Curves	-0.0005	0.0003	0.0008										
Minimum Storage Threshold	-0.0002	-0.0069											
Reliability Threshold	-0.0048												

Table E-13. Second-Order Importance Measures (S_{ij}) of the Sobol' Experiment for the Barwon Urban Water Supply System Case Study – 20 year Scenario 2b.

	Relative Position 1	**Relative Position 2**	**Relative Position 3**	**Percent. Restrict. 1**	**Percent. Restrict. 2**	**Percent. Restrict. 3**	**Upper RRC Curvature**	**Upper RRC Position**	**Lower RRC Curvature**	**Lower RRC Position**	**Base Demand**	**Target Curves**	**Minimum Storage Threshold**
Relative Position 2	-0.0766	-0.0164	0.0008	0.0480	0.0566	0.0015	-0.0407	-0.0906	-0.0450	-0.0955	-0.0393	0.0865	0.1302
Relative Position 3	0.0502	-0.0150	0.0017	-0.0022	0.0128	-0.0486	-0.0440	-0.0506	-0.0482	-0.0366	-0.0414	-0.0032	
Percent. Restrict. 1	0.0505	-0.0141	0.0596	-0.0392	0.0031	-0.0434	-0.0317	-0.0446	-0.0335	-0.0495	-0.1170		
Percent. Restrict. 2	0.0452	-0.0142	0.0138	-0.0479	0.0027	-0.0482	-0.0867	-0.0433	0.0065	-0.1270			
Percent. Restrict. 3	-0.0083	-0.0503	-0.0505	-0.0469	-0.0489	0.0007	-0.0046	0.0388	-0.0477				
Upper RRC Curvature	0.0106	-0.0480	-0.0483	-0.0487	-0.0052	0.0009	-0.0392	-0.1010					
Upper RRC Position	-0.0431	-0.0459	-0.0045	-0.0027	0.0106	-0.0019	-0.0606						
Lower RRC Curvature	-0.0472	-0.0494	0.0022	0.0442	0.0520	0.0005							
Lower RRC Position	-0.0509	-0.1110	0.0102	0.0007	-0.0801								
Base Demand	-0.0528	-0.0505	-0.0027	-0.0029									
Target Curves	0.0118	-0.0084	0.0015										
Minimum Storage Threshold	0.0484	-0.0644											
Reliability Threshold	-0.0047												

Table E-14. Second-Order Importance Measures (S_{ij}) of the Sobol' Experiment for the Barwon Urban Water Supply System Case Study – 20 year Scenario 2c.

	Relative Position 1	**Relative Position 2**	**Relative Position 3**	**Percent. Restrict. 1**	**Percent. Restrict. 2**	**Percent. Restrict. 3**	**Upper RRC Curvature**	**Upper RRC Position**	**Lower RRC Curvature**	**Lower RRC Position**	**Base Demand**	**Target Curves**	**Minimum Storage Threshold**
Relative Position 2	0.0032	0.0001	0.0020	0.0087	0.0049	-0.0131	0.0107	-0.0065	-0.0036	0.0001	-0.0145	0.0870	0.1489
Relative Position 3	0.0000	0.0008	0.0015	0.0112	-0.0070	-0.0008	-0.0109	-0.0015	-0.0062	-0.0065	0.0042	0.0474	
Percent. Restrict. 1	0.0008	0.0021	0.0014	0.0010	-0.0051	-0.0007	-0.0061	0.0008	-0.0052	0.0082	0.0097		
Percent. Restrict. 2	-0.0045	0.0094	-0.0027	-0.0009	-0.0022	0.0011	-0.0090	0.0298	0.0040	0.0014			
Percent. Restrict. 3	-0.0018	0.0034	-0.0044	-0.0063	-0.0041	0.0014	-0.0225	0.0555	-0.0046				
Upper RRC Curvature	-0.0067	0.0000	0.0004	-0.0038	0.0044	0.0018	-0.0456	-0.0287					
Upper RRC Position	0.0015	-0.0031	-0.0057	0.0074	-0.0087	-0.0018	-0.0153						
Lower RRC Curvature	-0.0017	0.0004	-0.0007	0.0036	-0.0020	0.0047							
Lower RRC Position	0.0002	0.0023	0.0006	0.0036	0.0049								
Base Demand	-0.0083	-0.0041	-0.0033	0.0071									
Target Curves	-0.0089	0.0005	0.0012										
Minimum Storage Threshold	-0.0010	0.0023											
Reliability Threshold	0.0018												