The Investigation of Fire Hazards in Buildings using Stochastic Modelling

By

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A thesis submitted in fulfillment of the requirement for the degree of Doctor of Philosophy at Victoria University of Technology

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February 2000

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ABSTRACT

This research investigates the spread of fire and smoke in buildings as well as occupant egress. There are existing deterministic models for these. While deterministic models provide averages of a process, stochastic models give the broad spectrum of all possible scenarios of the process giving the distribution function.

The spread of smoke was first modelled by adding a noise component to the equation of an existing deterministic model. Later a deterministic model was developed and stochastic expressions derived using the Markov chain methodology. Though the Markov chain is a discrete process, it was used in approximating smoke spread which is a continuous process.

The spread of fire was investigated using network analysis. Various methods of modelling the spread of a phenomenon in a network were compared. The Hazard function was shown to be more flexible than the Monte Carlo and the analytical methods. The application and flexibility of the Hazard function methodology was illustrated by simulating the spread of fire in a metropolis and later used in occupant egress from a building.

The research illustrated the importance of stochastic modelling in processes that are uncertain and that are subject to significant variability. Predictions can be made giving the probability of occurrence. Areas for further research are suggested. This research opens up many areas in fire research where stochastic modelling could be better used.

CHAPTER 1

INTRODUCTION

CONTENTS

1.1 Preamble

The initial stage of the research concentrated on the spread of smoke in a building using stochastic modelling. This gradually was extended to cover many of the fire hazards in buildings due to the flexibility of the method used. The method of the hazard function was found to be adaptable and relevant to the spread of hazards in a building. Conceptually it is applicable to many flow phenomena. By reference, the term hazard is either used to connote the risk to life and property or as the specific function used to modulate the spread of any of the hazards. In each case the discussions try to distinguish between the two. The latter is referred to as hazard function. Although the concept of the hazard function is not new (as indicated later in the references) its application in modelling some of the hazard in this research is new, especially the demonstration of its flexibility to accommodate other intervening events shown in chapter 6.

This chapter describes the content of the respective chapters and introduces some of the concepts. Some of the illustrations that demonstrate practical cases might refer to either residential or public buildings, but the approach can be applied to either of them.

1.2 Briefs on the chapters

Chapter two describes the objectives of the research, uncertainties of models, the relevance of stochastic modelling, it's limitation and acknowledges assistance in the research. Chapter three is the literature review that shows the concepts of the methods relevant to the research and previous work already done by others. Chapter four illustrates how to convert an existing deterministic model to a stochastic model. The methods of stochastic modelling are first discussed. The method of adding a noise component, which has been used in other fields of research, is used here to simulate the spread of smoke. There are many deterministic models that need to accommodate the uncertainties in the phenomenon being modelled. Stochastic models give the probability of occurrence of an event and the confidence interval. The chapter opens up the type of modelling that the research addresses. Specifically, a deterministic smoke model was chosen for illustration, a stochastic model is then developed from it. Verifying that its confidence band covers the deterministic model validates the stochastic model. The amplitude of the noise determines the extent of cover.

The Markov chain method of stochastic modelling is discussed and used in chapter 5 to simulate the spread of smoke in a building. A deterministic model is first developed which is later used in developing the stochastic model. The spread of smoke is a continuous process. In practice it is difficult to model hence it was approximated by discretizing the process. The fit and accuracy then depends on the extent of discretization. Results and verification of an illustrative simulation are given.

In chapter 6, a general method is used in simulating the spread of a phenomenon (fire) over a network. The representation of the network and its analysis using the hazard function are given. Discrete approximations of the network are compared with the hazard function approach, the discrete approximations being the analytical and Monte Carlo simulations. A numerical example is given for illustration. The illustration includes an example of incorporating other events using the hazard function, which demonstrates its flexibility.

The stochastic modeling of the spread of fire in a metropolis is presented in chapter 7. The conceptualization of the network involves spread conditions that involve the various conditions of the possible paths of fire spread. The hazard function was used for simulating the spread. An illustration is given that determines the minimum time for fires that start simultaneously in a community to spread to a refuge center.

Chapter 8 presents the stochastic modelling of occupants' evacuation from a building on fire. The occupants are categorized depending on their ability to egress. The simulation considers the respective times. An example of the use of the model is given.

Chapter 9 concludes the research work and gives recommendations of possible related areas for future investigation.

CHAPTER 2

SMOKE SPREAD

CONTENTS

2.1 Definition of the Problem

The spread of smoke in buildings is subject to many variables whose attributes are difficult to concisely define. Many deterministic models have been developed with the aim of describing the movement of smoke arising from fires. Because of the enormous variability involved, these models have the limitation of not incorporating many significant factors. Real life situations indicate that the influence of any of these factors that affect the spread of smoke can be better described by probabilities reflecting the scope of the uncertainties involved.

The aim of this project is to investigate the spread of smoke under uncertainty using a stochastic modelling approach.

2.2 Need for Stochastic Modelling

The need for stochastic modelling of smoke spread is emphasized by the neglect of the effect of kinetic energy and gravitational work on the flow of smoke. It has been observed by the writer in a test of smoke spread in the Environmental Building-Fire facility of Victoria University of Technology, Australia, that the velocity hence kinetic energy of smoke during flashover is a significant factor in the spread. These indeterminate factors can be accounted for by the noise component added to expressions in stochastic modelling.

Another factor not considered in modelling of smoke spread is the initial quantity of air in the compartment before the inflow of smoke. This could affect the initial temperature distribution.

Deterministic models can give averages of a system. Using averages can tell a lot about a system but there are many interesting questions that may not be answered in this way. For instance, enquiries about the probability distribution of certain variables. In order to answer such questions, the evolution of the state of the system with respect to time is necessary. Stochastic process modelling is a relevant methodology in that regard.

The following common life event illustrates the need for stochastic as against deterministic modelling. The average height of female infants will be related to the number of months since birth. To determine the actual height of a female child as a function of age will be difficult. This can only be approximated using a stochastic relationship. The quantity of smoke from a fire will depend on factors like the fuel load, type of fuel, ventilation conditions or availability of oxygen, and the type of fire considered. Though a relationship could be developed between the quantity of smoke generated and these factors, evidently the quantity for a given duration can only be estimated by a stochastic relationship as exact figures are unrealistic.

Apart from the risk associated with fire and smoke there are additional risks arising from using output from models that are themselves fraught with uncertainties. Using stochastic modelling technique allows output from models to be described within some confidence level. Stochastic models can accomplish the following in risk management of fire, smoke, and occupant egress:

1. Assist in developing realistic plans that address loss.

2. Evaluate the levels of risk associated with specific quantities of intoxicants or life or property loss.

Stochastic models allow for both randomness and lack of definition in some of the factors of a relationship. The variability of smoke spread is presented in this write-up as probability distributions of species quantities and of their temperatures. Deterministic models cannot comprehensively accommodate the uncertainties that are inherent in hazards such as fire, the spread of smoke, and occupant egress.

2.3 Boundaries and Constraints

Boundaries and constraints of the models would be defined by the following:

(a) The stipulation of an incorporated stochastic factor reflects the modeller's knowledge of the medium of spread. This depends on the availability of experimental data prior to modelling. Where there is no sufficient data to calibrate the factor, it is assumed. Hence the stipulated value depends on the limited knowledge available at the time of modelling. Data on fire and smoke are relatively scarce because experiments must be full scale.

(b) Reliability and accuracy of the models will depend on the accuracy of the experimental data used for comparison and on the modeller.

2.4 Broad Objectives

The main objective of this program of research is to model the spread of hazards arising from fires in buildings using stochastic techniques which will be validated with results from experiments. The approach is intended to incorporate many of the significant factors that affect the spread of smoke.

2.5 Specific Aims

It is intended in this project to:

1. Establish stochastic model(s) of the spread of fire, smoke and occupant egress and their prediction as an extension of existing deterministic models.
2. Determine the relationship between the models.

3. Compare methods of evaluating the spread of a phenomenon in a network.

2.6 Uncertain Nature of the Spread of Smoke.

Given the same fuel load, compartment, ventilation and environmental conditions, full scale experimental fires at the Fire Facility at Victoria University of Technology, Melbourne, do not produce the same results of the signatures of fire. It is acknowledged that reproductions of fire scenarios are subject to diverse and varying factors that can prove difficult to control. The quantity of intoxicants required to incapacitate or cause death can only be approximated. Estimates of quantities can be stated with a measure of uncertainty. For instance, statements of species concentration could be in the form of a Confidence Interval.

Temperatures and concentrations of species are the main data required from fire. Data of time dependent smoke arising from an experiment used by Hukogo et al [1] also by He and Beck [2] was used as input for the investigation. The main aim of the investigation was to determine the quantities stated above and their probability of occurrence for various locations of interest in a public building.

Two models will be presented:

1. Transformation of deterministic model to stochastic model and
2. A Markov chain model.

Further research will be needed to establish which of these models is better.

Results from the model were compared to those of actual measurements. Investigation also involved comparing different methods of evaluating stochastic spread of a phenomenon such as fire, smoke, or occupants egress in a network. The approaches compared were the analytical, Monte Carlo and the Hazard function. The advantages of the Hazard function over the rest were recognized with appropriate recommendations made. The appendices (A, B, C, D, and E) hold more data and information on the analysis and methodology as indicated in the respective chapter.

The relationship of the respective areas of the investigations to fire safety and their significance were discussed.

2.7 Acknowledgements

The Australian Postgraduate Award provided the fund by way of a scholarship for this investigation.

My supervisors, Professor Emeritus Michael Hasofer and Professor Vaughan Beck provided me with guidance, helped with review, and criticism in this venture. Prof Hasofer was remarkable in streamlining the investigation. Special appreciation is extended to my relations, friends and associates for their encouragement.

CHAPTER 3

CONCEPTS AND LITERATURE

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FIGURES

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3.1 Introduction

Inhalation of smoke has been considered as the main cause of death in building fires [NCFPC 3]. While this may be so, there are discussions [34] about some other important contributory factors to the cause of death. One of such factors is impairment to escape which mostly affects the young and the old. Irritation of sensitive tissues such as the eyes or the lungs is another factor. Smoke is defined by its composition. The American Society for Testing and Materials [ASTM 4] defines smoke to include gases evolved during combustion. It is the total product of combustion. Usually, the composition of smoke includes carbon dioxide, carbon monoxide, hydrogen cyanide (if any) and the incomplete uncombusted fuel particles. The National Fire Protection Association [NFPA 5] defines smoke as consisting of the airborne solid and liquid particules and gases evolved when a material undergoes pyrolysis or combustion, together with the quantity of air that is entrained or otherwise mixed into the gases. This latter definition was adopted in this research. Hartzell [6] enumerated the predominant hazards to humans exposed to products of combustion. These include heat, visual obscurity and narcosis. Narcosis results from the inhalation of asphyxiants and irritation of the upper respiratory tracts. Delays due to visual obscurity, panic and disorientation lead to further inhalation of toxic gases.

3.2 Risk Assessment

Beck et al [7] identified many of the subsystems that react to fire situations. These subsystems would also affect the spread of smoke arising from fires. The subsystems are classified into active and passive. Investigations [7] have been carried out on the active subsystems while much study needs to be done on the passive subsystems. They include: (a) vertical separating elements (b) horizontal separating elements (c) load bearing structure and (d) architectural configuration.

Openings in a building have been identified as important features in determining the effectiveness of external wall subsystems [7]. Windows are the main route of the spread of fire to other buildings, and the ingress of wind can accelerate internal smoke movement and hence it's spread from one compartment to another. The paper [7] also noted that facilities to control the movement of smoke have significant effect on the risk-to-life safety values. These facilities include: stairway doors, smoke exhaust subsystems and stair pressurization.

Factors affecting the spread of smoke include:

1. External wind velocity and relative pressure between the inside and outside of the building.

   The effect of external wind would be important if the openings to the outside are considered opened. External wind causes flow through the external openings and affects the internal spread of smoke to the compartments and to the upper floors. The magnitude of the internal pressure is a function of the wind velocity. Wind parameters are dependent on the season: Spring, Summer, Winter and Autumn. Most models for the spread of smoke do not incorporate this factor. The argument that the effect of the wind can be post-determined after the fire and smoke event does not assist in predicting the effects of future occurrence of fire and the subsequent smoke spread. Completely sealed, partially sealed and completely vented conditions are the three possible states for a building. The effect of wind for the three opening conditions can be of interest. Also of interest is the position of the fire room with respect to the windward wall.

   Pre-existing airflows in a building help to carry toxic gases and smoke from the fire source to other parts. If there are external openings in the building, the air flow will be affected by outside wind. The direction of flow is greatly affected by the positioning of the openings. This could either be in the windward or the leeward side. Wind causes positive and negative wind-induced pressures. It is negative if the fire room has openings on a wall on the leeward side.

   The building conditions for the simulations performed assumed partially sealed conditions with minimal wind effect.

2. External wall openings and internal openings in the partitioning.

3. The type and stage of fire: The emission of smoke will depend on the type of fire. Since fire can change from one stage to another it follows that smoke emission will be dependent on the stage of fire that also depends on time.

4. Horizontal openings: vents, smoke exhaustion systems, etc.

5. Rate of fuel burning.

6. Types of fuel.

Some of the variables that have been investigated which describe some of the factors listed above include:

- Oxygen Concentration / Mass of Oxygen used up
- Mass Burning rate
- Wall Heat Loss
- Air Ventilation Rate.
- Product Gas Concentration.

The rate of generation of smoke depends on the type of fire and fuel. The spread of smoke will initially depend on the rate of combustion which is determined by the availability of fuel and on the type of fire.

3.3 Assumptions in Smoke Spread

Some of the assumptions used by Hagjisophocleous and Yung [8], were assumed in this modelling. The assumptions made for the modelling in this investigation include the following:

1. Spread to floors below that of origin is insignificant as real-world fires show that floors below that of fire are usually fire and smoke free [8, pp 70].

2. For the purpose of this modelling an average spring time condition is assumed as reference time for all fires. This can be adjusted to accomodate other weather conditions.

3. Results of the fire growth model required by the smoke spread model are the mass flow rate of the hot gases leaving the compartment of fire origin, their temperature, the concentrations of the toxic gases and the fire spread source data.

4. The space in a compartment above the door height should be filled with smoke before it spreads to another compartment. Hence the time required to spread from a particular compartment to another is that to fill this volume.

5. There is no looping where flow from a node gets back to itself.

3.4 Smoke Variables Required for Modelling

The list of variables that would be required for modelling the spread of smoke in a building under fire includes:

1. Species concentration (SC): a requirement in the determination of spread.
2. Fractional Incapacitating dosage (FID): required for the investigation into effects

of smoke on humans, loss of life and smoke spread.

3. Gas temperature (T): necessary in determining the effects of smoke on humans, and the loss of life.

Though not exhaustive, the above list indicates the parameters that would be interest in modelling the spread of smoke. The first and the last parameters were of main concern in this investigation. With given Incapacitating Dosage (ID) the effects of smoke on humans and loss of life could be ascertained from the results obtained.

3.5 Models

3.5.1 Deterministic Models

There are very many models available that give estimates without considering the possibility that given the same situation the estimates could change. These models are deterministic. They could be useful in giving guidance and they do approximate reality. The lack of certainty is the major drawback of deterministic models. What is the confidence level of the estimates? Where estimates are used for determining threat to life, accidents and fatalities deterministic models lose credibility. Deterministic models can be of any of the types of models described below.

3.5.1.1 Zone Models

An approach to the spread of fire and smoke is zone modelling. The zone model is based on the observation that smoke, being of lesser density than that of the surrounding air, rises and occupies the upper layer representing the hot layer. The lower zone represents the cold layer. Further more, the model assumes that predicted conditions are uniform. Predicted conditions include the temperature and species concentrations.

Most zone models are fire growth or spread models. Takeda and Yung [10] developed a one-zone fire growth model that predicts fire growth characteristics and species concentrations. Tamura [11, pp102] discussed some of these. The CFAST model developed by the National Institute of Standards and Technology (NIST) is a two-zone model that predicts smoke movement in a multi-storey building. Zone models cannot provide detailed information on fluid flow but their simplicity, ability to run rapidly on computers, ease of transfer from one organisation to another, and low cost makes them attractive. Zone models can be used for multiple compartments.

3.5.1.2 Field Models

These are models that describe phenomena that occur in two or three dimensional spaces. Field modelling involves dividing the enclosures by three (or two) dimensional grids into elements. The physical conditions of each element as a function of time are determined by solving the fundamental mass, momentum, and energy equations. Field models can model the differences in physical parameters throughout the grid. The physical parameters could be temperature, species concentrations, etc. Field models require powerful computers but they provide detailed information.

3.5.1.3 Network Models

Network modelling has been used to solve fire and smoke protection problems [12,13]. The general approach includes representing the network by nodes and links. The building is divided into compartments (nodes). The temperature, pressure and species concentration in each of the nodes is assumed to be uniform. The nodes represent space and also the smoke and/or fire conditions of the space. The nodes are connected by leakage openings (flow paths). The links are the possible movement of fire/smoke from space to space. The mass flow rates and pressure differences are related by the orifice flow equation. The network modelling technique uses the mass balance and flow equations, and expressions for temperature and smoke concentrations. Network and graph theories have been used successfully for studying multi-compartment buildings. The reduction in cost of computers has encouraged the use of network modelling. Network models can predict conditions in many rooms and locations far away from the source of fire. It is most suitable for high-rise buildings.

3.5.2 Non-deterministic Models

Models that give estimates of a phenomenon while considering the uncertainties in the process are said to be non-deterministic. Non-deterministic models present the relative frequency of occurrence of each pattern of spread of smoke in a large number of real fires. Non-deterministic models are of two types; probabilistic and stochastic [9]. The difference between probabilistic and stochastic modelling is that the latter is time dependent while the former is not. Probabilistic modelling is illustrated by Ramachandran [9] as:

"Probability modelling is concerned with final outcomes rather than the detailed knowledge of the processes that make it. For instance, fire damage to properties can be described by a skewed probability distribution (e.g Log-Normal). Hence given an area of property likely to be damaged, the probability can be determined, for a given distribution. This is irrespective of the fact that the area of damage is affected by the type of fire, fuel load and its spatial distribution, ventilation conditions, etc. This non-deterministic approach is used by insurance companies who are least interested in the detailed analysis of the effects of fire."

Ling and Williamson [12, 13] proposed the use of probabilistic networks in analyzing the horizontal spread of smoke and the egress of people in buildings. Occupants' egress was treated as a dynamic network flow problem for a given fire scenario. Calculation of the probability of occurrence for the fire scenarios was based on Mirchandani's algorithm.

3.6 Stochastic versus Deterministic Approach

A process is deterministic if future values (events) can be exactly predicted from past values of the function; otherwise it is random. This definition distinguishes deterministic processes from processes like the Markov processes.

Murthy et al [14, pp28] defined a deterministic system as having variables that assume values that are predictable with certainty. Non-deterministic systems (probabilistic and stochastic) have variables whose values are random and unpredictable. In characterizing a system, it can either be deterministic or non-deterministic based on the significance of the uncertainty therein. Smoke spread is a process fraught with very significant uncertainty that cannot be ignored. It should be considered a non-deterministic process.

Stochastic modelling involves capturing an uncertain process that occurs in time. Deterministic models do not evaluate the uncertainties involved in the spread of smoke. Different patterns of spread can be obtained by varying the input values of the parameters to the models. Non-deterministic models can give the probability with which each pattern of smoke spread can occur in the chosen building under consideration. He and Beck [2] have described a deterministic network approach for modelling smoke spread in multi-storey buildings. A probabilistic (noise) component can be added to this model to obtain a stochastic model as discussed further below.

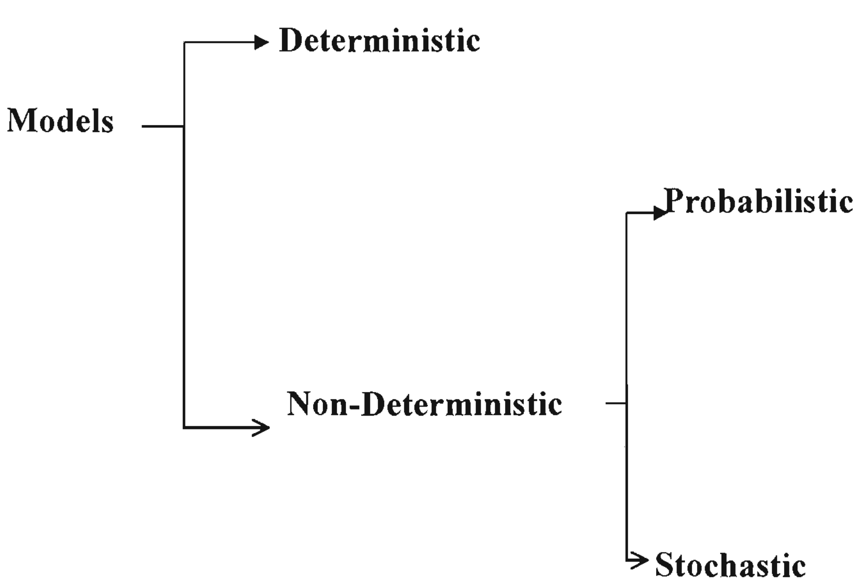


FIG 3.1 CLASSIFICATION OF MODELS

Deterministic models can be converted into stochastic models. Further in this chapter, ways of developing stochastic models is discussed. Hence it is relevant to discuss issues related to the development of a deterministic model. Deterministic models have a very large number of variables to consider as against stochastic models. The number of variables and factors incorporated in a deterministic model depends on availability of information and on the level of detail desired. The more the detail the more accurate the model will be.

3.7 Free-Convectional Flow Systems:

Smoke spread in buildings can be taken as free convectional flow arising from temperature and pressure differences. Some of the parameters that define free convectional flow of fluids will be discussed below.

Free convectional flow is heat transfer from a fluid to a surface, or vice versa, resulting from density or temperature differences arising from the heating process. The buoyancy (or body) force resulting from the change in density exists because of the presence of an external force such as gravity, etc [15].

The dimensionless quantity called the Grashof number, Gr, is important in all free-convectional fluid flow; and is given as:

$$\mathrm{Gr} = \frac{g\beta\ (T_w - T_\infty)\chi^3}{\upsilon^2} \qquad (3.1)$$

where:

g = acceleration due to gravity (m/s^2)

β = volume coefficient of expansion determined from the propeties of the specific fluid. In this instance, for an ideal gas β = 1/T, where T is the gas temperature (K).

T_w = Barrier (wall) temperature (K).

T_∞ = gas temperature at considerable distance from barrier (K).

υ = Kinematic viscosity in m^2/s

χ = Distance along wall, taken as height of wall (m).

The Grashof number is the ratio of the buoyancy forces to the viscous forces in the free convection flow system [15, pp 336]. The role it plays is similar to that played by Reynolds number in forced-convection systems, used as a criterion for transition from laminar to turbulent boundary-layer flow. In free-convection flow it is difficult to predict temperature and velocity profiles analytically hence experimental measurements must be relied upon to obtain relations for heat transfer.

Methods of measuring temperature include Zehneder-Mach and laser interferometers while velocity has been measured by hydrogen-bubble techniques, hot-wire and quartz-fibre anemometers.

Interferometric studies of free convection indicating lines of constant density in fluid flow is one way of measuring temperature. The lines of constant density are assumed to be equivalent to lines of constant temperature. The temperature is obtained, temperature gradient and thermal conductivity calculated hence the heat transfer. This technique can be used in measuring gas temperature without influencing the flow field by the insertion of a measuring device.

Rayleigh number (Ra) is the product of Grashof (Gr) and Prandtl numbers (Pr).

$$Ra = GrPr \qquad (3.2)$$

Prandtl number relates the relative thickness of the hydrodynamic and the thermal boundary layers. It is the link between velocity and temperature fields; and it is given as:

$$Pr = \frac{\upsilon}{\alpha} = \frac{C_p \mu}{k} \qquad (3.3)$$

where

μ	=	Dynamic viscosity (Kg/ms)
C_p	=	Specific heat at constant pressure (KJ/Kg K)
υ	=	kinematic viscosity (m^2/s)
α	=	Thermal diffusivity (m^2/s)
k	=	Thermal conductivity (W/m K)

The average free-convection heat-transfer coefficient is represented by the Nusselt number ($\overline{NU}$). Relations provided by Churchill and Chu (quoted in Holman [15]) for wide ranges of the Rayleigh number are:

$$\overline{NU} = 0.68 + \left[\frac{0.670\, Ra^{1/2}}{(1+(0.492/\mathrm{Pr})^{9/16})^{4/9}}\right]^2 \qquad for \ \ Ra < 10^9 \qquad (3.4)$$

$$\overline{NU} = 0.825 + \left[\frac{0.387\, Ra^{1/6}}{(1+(0.492/\mathrm{Pr})^{9/16})^{8/27}}\right]^2 \qquad for \quad 10^{-1} < Ra < 10^{12} \qquad (3.5)$$

The above equations are valid for walls only. The second equation which covers a wider range of Rayleigh number (Ra) will be used for the modelling in this investigation.

Considering the variations between boundary (walls) and free-stream conditions of gases, in the dimensionless groups, all properties are evaluated at the local film temperature. Film temperature (T_f) is the arithmetic mean between the wall and free-stream temperatures. It is given as [15]:

$$T_f = \frac{T_w + T_\infty}{2} \qquad (3.6)$$

where

T_w	=	Wall temperature (K)
T_∞	=	Gas or free Stream temperature. (K)

When turbulent free convection is encountered, the local heat-transfer coefficient is constant with the height of the wall.

The temperature of the boundary (walls) is T_w. The velocity of gas at wall is zero and the heat transfer to wall is by conduction. By Newton's law of cooling, the heat transfer rate is given as

$$q = hA(T_w - T_g) \tag{3.7}$$

Where

q = Heat transfer rate (KW)

h = Heat transfer coefficient (W/m^2 K)

A = Area (m^2)

T_g = Temperature of gas or free-stream at the wall (K).

The pressure distribution along the stair-shaft is calculated using

$$P(h) = P_b\left(1 - \frac{g}{R}\int_0^h \frac{dh}{T}\right) \tag{3.8}$$

Where

P_b = Reference base pressure (pascals)

h = elevation from a reference floor (m)

g = Acceleration due to gravity (m/s^2)

T = Stair-shaft temperature (K).

R = Gas constant.

The above expression is also used for calculating external pressure distribution for the floor levels.

3.7.1 Mass Flow Rate Through an Orifice

In fire, combustion of flammable materials involves the emission of a great amount of heat that causes the heated gas to expand. This is one of the forces that drives out some of the gas in the compartment through any available vent. Like all substances, gas can only spread if acted upon by external forces such as pressure and or gravity. Flow due to gravity is called buoyant flow. Flow through openings is due to pressure differences across it.

From Bernoulli's equation for flow along a streamline the flow of gas has

velocity, $v = \sqrt{\frac{2\Delta p}{\rho}}$

volume flow $Q = CA\sqrt{\frac{2\Delta p}{\rho}}$ (3.9)

and

mass flow rate $m = CA\sqrt{2\rho\Delta p}$

which can be expressed as

$$m = CA\sqrt{2\rho(P_j - P_k)} \quad (3.10)$$

where

Δp	=	Pressure difference in Pascals (N/m2) = $P_j - P_k$
A	=	Area of opening (m2)
ρ	=	Gas density (Kg/m3)
Q	=	Volume flow rate (m3/s)
m	=	Mass flow rate (Kg/s)
v	=	Flow velocity (m/s).
C	=	Coefficient of Orifice.

j and k are the node subscripts.

The coefficient of orifice C in the equation corrects the effects due to the fluid viscosity, non-uniformity of the velocity through the vent, turbulence, etc... It depends on the Reynolds number, Re. Reynold's number is the dimensionless combination of the variables of flow given as

$$Re = \frac{vD\rho}{\mu} \tag{3.11}$$

Where

v = Velocity of flow (m/s)

D = Diameter of the orifice (m)

ρ = Density of the fluid (Kg/m^3)

μ = Viscosity of fluid. (m^2/s)

Openings in series in the direction of flow have an effective area, A_e, equal to the inverse of the sum of the reciprocals of the openings, A_i.

$$A_e = \left[\frac{1}{A_1^2} + \frac{1}{A_2^2} + \dots + \frac{1}{A_n^2}\right]^{-1/2} \tag{3.12}$$

While openings in parallel have the effective area equal to the sum of all the openings.

$$A_e = \sum_{i=1}^{n} A_i \tag{3.13}$$

3.7.2 Effects of radiation

Effects of radiation in far field modelling can be insignificant especially as the average temperature of gases falls below 500 °C. This observation is made from results of fire experiments conducted at the fire experimental building of the Centre for Environmental Studies and Risk Engineering (CESARE) of the Victoria University of Technology. In fires, the main thermal radiation spectrum lies in the wavelength band between 0.1mm and 100mm. Particles or bodies at a temperature of less than 500 °C do not glow. Emission of light from particles occurs mainly in the burning room(s). It stands to reason that radiation terms may not be very relevant in modelling the spread of smoke in buildings.

3.8 Stochastic Models

Not much has been done on analysing the spread of smoke using a stochastic modelling approach. Hasofer, Beck and Odigie [16] proposed a model using a stochastic approach for spread of fires over a network. The intricate variability of the factors affecting the spread of fire and smoke is indicated in the difficulty of obtaining similar models.

3.8.1 Stochastic Modelling Methodologies

One approach to stochastic modelling, is to superimpose a probabilistic component (noise term) over a deterministic trend of the phenomenon being modelled over space and time.

A stochastic process that consists of a set of independent random variables (r.vs) has the joint probability density function (jpdf) as the product of the marginal density functions. The process can be represented by adding a stochastic variant to the deterministic component. The variant is called a "white noise". It can be used to represent the non-deterministic component in stochastic modelling [18].

Assuming that the spread of smoke can be described by a stochastic function (W) that is made up of a deterministic component (X) and a non-deterministic noise component (Y), then [17, 18, 19]

$$W = X + Y \tag{3.14}$$

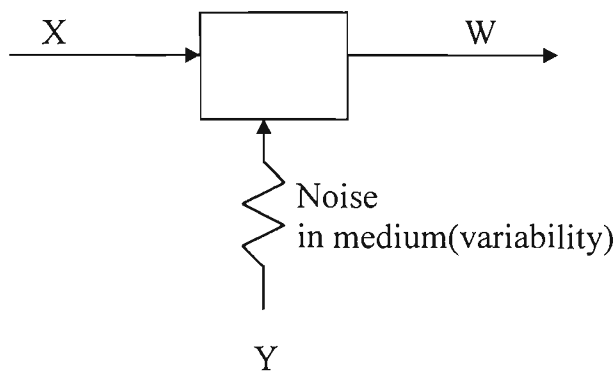


FIG 3.2 SUPERIMPOSING A NOISE COMPONENT ON A DETERMINISTIC TREND.

3.8.2 Steps In Developing a Stochastic Smoke Model

- Determine a deterministic smoke model to use for stochastic modelling.

  An existing deterministic model could be chosen to transform into a stochastic model.

- Identify variables for fire and smoke stochastic modelling

  Stochastic models usually contains fewer variables than a deterministic one. The choice of variables of the deterministic model to use for the stochastic model will depend on the extent of variability in the chosen variables. The variables not included in the model are included in the stochastic variability.

- Determine the effective dosage of smoke to cause incapacitation for a average person.

  Ultimate desire in modelling of smoke spread is to determine the effects on humans. A reasonable and acceptable dosage for modelling would be determined.

- Derive appropriate expressions for the respective variables.

  Expressions for the variables could either be derived, modified or adopted from the chosen deterministic model.

- Determine reasonable assumptions for state of the nodes in the network model.

  The assumptions made in each node will reflect the prevailing conditions and state of the nodes.

- Derive differential expressions.

  Differential expressions for the stochastic modelling incorporating the processes involved in smoke spread would be derived.

- Determine a smoke model or identify data from experiments to use as standard for calibration and verification of model.

A standard measure would be necessary for the calibration of the stochastic model. This can be either experimental data or an acceptable model.

- Determine the dependency relationship of the factors (e.g. correlation).

- Develop the stochastic model.

- Validate the model with experimental data.

These would be the necessary steps in developing stochastic models for smoke generation, spread in one compartment, spread from one compartment to another, and spread in multi-level buildings.

3.9 Stochastic Processes

A stochastic process X(t) is defined by the following quantities:

(a) State space

(b) Index parameter(time)

(c) Statistical dependency between the random variables (r.vs) X(t) for different values of the index parameter t.

The state space is the set of possible values (or states) that X(t) may take. We have a discrete-state process if the values that the process may take are finite or countable. In smoke spread, a particle can occupy any of the infinity of positions about it, hence the process cannot be discrete. The possible positions of the smoke particles, in this case, are over a finite or infinite continuous interval. The spread is thus a continuous-state process.

The index (time) parameter at which changes may occur is anywhere within a set of finite or infinite intervals on the time-axis; hence it is a continuous-parameter process.

Another consideration in classifying smoke spread is stationarity. A stochastic process X(t) is stationary if the joint distribution function, $F_x(x;t)$ is invariant to shifts in time for all values of its arguments [18]. That is, given any constant τ

$$F_x(x; t+\tau) = F_x(x; t) \tag{3.15}$$

Where x and t are vectors of the same length.

Essentially, smoke spread is a non-stationary process.

3.9.1 Markov Processes

The Markov property assumes that the future behaviour of a sequence of events is uniquely determined by a knowledge of the present state only [19,20,22], in which case the knowledge of the present state is sufficient to predict the future behaviour of the process. Markov processes enable us to model uncertainties in some real-life systems that occur dynamically in time. Basically the variables of a Markov process are the state of a system and the state transitions. The Markov process with discrete parameter space is referred to as Markov chain. As indicated above, smoke spread does not occur in discrete time hence can only be modelled approximately as a Markov chain.

A Markov process, $X(t)$, can be illustrated as follows [23]:

If

$t_1 \langle t_2 \langle \ldots\ldots \langle t_n$

then

$$P\{X(t_n) \leq x_n | X(t_{n-1}), \ldots\ldots, X(t_1)\} = P\{X(t_n) \leq x_n | X(t_{n-1})\} \qquad (3.16)$$

where t is the time.

The above is also true for discrete-time processes if $X(t_n)$ is replaced with X_n.

Markov Processes can be classified into four groups [24, pp 14 - 16, 219 -222]:

1. Discrete Markov chain
2. Continuous Markov chain
3. Discrete Markov Process and
4. Continuous Markov Process.

This can be represented as shown in the following table.

TABLE 3.1: TYPES OF MARKOV PROCESSES

	STATE	
Nature of Parameter(e.g time)	Discrete	Continuous
Discrete	Discrete Markov Chain	Continuous Markov Chain
Continuous	Discrete Markov Process	Continuous Markov process

Some of the above will be defined in the following sections.

3.9.1.1 Markov Chain

3.9.1.1.1 Discrete Markov Chain

In a Markov chain we assume that the state transitions can only be at given discrete points in time. A Markov chain is a sequence of random variables in which the dependency of the successive events goes back only one unit in time. That is, that the future probabilistic behaviour of the process depends only on its present state and is not affected by past history [20].

Tijms [19] defined the discrete Markov-chain as follows:

Let $\{\chi_n, \ n = 0,1,.......\}$ be a sequence of random variables having discrete state space I, where χ_n is the state of a system and the set of possible values of the process is finite or countably infinite.

This is a discrete-time Markov chain if it satisfies these conditions:

For each $n = 0,1,.......,$

$$P\{\chi_{n+1} = i_{n+1} | \chi_0 = i_0, \ldots\ldots, \chi_n = i_n\} = P\{\chi_{n+1} = i_{n+1} | \chi_n = i_n\} \qquad (3.17)$$

for all possible values of $i_0, \ldots\ldots\ldots, i_{n+1}$.

If time-homogeneous transition probabilities of the Markov chain only are considered, we can assume that

$$P(\chi_{n+1} = j | \chi_n = i) = P_{i,j} \qquad i, j \in I$$

Where $P_{i,j}$ are the one-step transition probabilities satisfying the conditions

$$P_{i,j} \geq 0, \qquad i, j \in I \;\; and \quad \sum_{j \in I} P_{i,j} = 1, \qquad i \in I$$

The probability distribution of the initial state χ_0 and the one-step transition probability $P_{i,j}$ determine the Markov chain $\{\chi_n, \; n = 0,1,\ldots\ldots\}$.

Smoke spread is not a discrete-time process but it can be approximated by it; in which case the initial smoke concentration in a compartment will be the initial state. Subsequent concentration in time will be determined by the one-step transition probability $P_{i,j}$. The one-step transition probability will be assumed to produce one of two possible outcomes by means of a Bernoulli Trial.

3.9.1.2 Markov Processes

3.9.1.2.1 Discrete Markov Process

The discrete-time Markov chain assumes that the change of state can only occur at fixed times $t = 0,1,\ldots\ldots\ldots$, but in real life situations changes of state could occur at each point in time. The discrete Markov process assumes that the times between successive transitions are exponentially distributed. The succession of states is described by a discrete Markov chain irrespective of how long it takes between transitions. The discrete Markov process can be defined as follows [19]:

Let $\chi(t),\, t\geq 0$ be a stochastic process with discrete state space I with the following properties

1. When the process is in state i, it remains so during an exponentially distributed time with mean $1/v_i$ independently of how the process reached state i and how long it took to get there.

2. When the process leaves state i, it moves to state j with probability $P_{i,j}\,(j\neq i)$ independently of the duration of stay in state i. The transition probabilities $P_{i,j}$ satisfy

$$\sum_{j\neq i} P_{i,j} = 1 \quad \mathit{for\,all}\ \ i\in I$$

3. The rates $v_i,\ i\in I$ are bounded.

The assumption of an exponential distribution which describes the times between successive transitions will not be relevant in this investigation. The input to the model will be the flow rate, concentrations, and temperature of species from a model or real life experiments. These would have been computed or measured at regular intervals. In essence, the times between successive transitions of the input data will not be exponentially distributed. They occur at fixed time intervals.

3.9.1.2.2 Diffusion Process

A diffusion process is a Markov process in which only continuous states occur; the state space is the continuum of real numbers and changes of state are occurring all the time [19,22]. The realizations of this process are expected to be continuous functions.

A diffusion process considers continuous variables in continuous time. In the continuous processes of smoke movement the change in a small unit of time, Δt, will be small. The transition from state to state is relatively very small and may be relatively frequent. This process is used when a very large number of particles as in smoke is involved in a Brownian motion with small, random steps giving a diffusion effect on a large scale.

A diffusion process is defined in terms of its first-order density $p\ (x,\ t)$ [24].
Its transition function is denoted by

$$\pi(x,t;x_0,t_0) \quad = \quad f_{x(t)}(x\,|X(t_0) \quad = \quad x_0) \qquad t \geq t_0$$

$$\pi\ (x,t;x_0,t_0) \quad \to \quad \delta(x - x_0) \quad \text{for} \quad t \to t_0. \qquad 2.18$$

and has the following properties [23]

If $t_0 < t_1 < t$, then

$$\int_{-\infty}^{\infty} p(x,t)\,dx \quad = \quad 1$$

$$p(x,t) \quad = \quad \int_{-\infty}^{\infty} p(x_0,t_0)\pi(x,x_0;t,t_0)dx_0 \qquad \text{and}$$

$$\int_{-\infty}^{\infty} \pi(x,x_0;t,t_0)dx \quad = \quad 1$$

$$\pi(x,x_0;t,t_0) \quad = \quad \int_{-\infty}^{\infty} \pi(x,x_1;t,t_1)\pi(x_1,x_0;t_1,t_0)dx_1$$

An example of diffusion process (both continuous time and continuous state space) is the Brownian motion.

The Brownian process (Wiener process) is governed by the following conditions [24]:

- $\{X(t),\ t \geq 0\}$ has stationary independent increments.

- For every $t > 0$, $X(t)$ is normally distributed.
- For all t 0, $E[X(t)] = 0$.
- $X(0) = 0$.
- $V[X(t) - X(s)] = \sigma^2 |t - s|$.

where

X	=	The process
s, t	=	Time parameters.
$E[X]$	=	Expectation of the process.
$V[X]$	=	Variance of the process.
σ	=	Standard deviation per unit time of the process.

The forward Kolmogorov diffusion equation with infinitesimal mean and variance is given by

$$\partial p/\partial t = -\partial (bp)/ \partial x + 1/2\partial^2 (ap)/ \partial x^2 \quad (3.19)$$

where p is the probability density function of the process, while a and b are deterministic functions of t and x.

A diffusion process can be discretised as a limiting case of a discrete Markov chain. In this way we can have insight into the continuous process and possibly obtain a complete probabilistic description of the process by proceeding to the limit of zero time and displacement intervals.

The concept of diffusion processes as being applicable to smoke spread can be one of the possible stochastic modelling techniques. However a continuous-time process will need to be discretised for computer modelling.

The discrete Markov chain will be assumed to approximately describe the spread of smoke, from an input source, in a building. However it must be remembered that smoke spread is not a stationary process.

3.9.3 Percolation Process

A percolation process is the spread of fluid through a medium under the influence of a random mechanism associated with the medium [21,26]. The medium is made up of an infinite set of atoms and bonds. A bond is a path between two atoms. This can either be an undirected or directed bond. Undirected bonds allow passage from one atom to the other, while directed bonds do the same but do not allow a reverse passage. Two atoms could be linked by several bonds, which can be directed and undirected bonds.

The movement of smoke can be represented on a graph or matrix made up of nodes, that are connected by "directed bonds" which are the only passages in the flow.

The forefront of spread can be any of three states: Stopped or blocked or spread one node forward.

Considering a two-dimensional matrix of nodes, a node already invaded by smoke can only contaminate the immediate neighbouring nodes. In such a situation a node can contaminate one or more of its immediate neighbouring nodes with a probability of

$$P = 1 - (1 - P_1)(1 - P_2)\ldots(1 - P_8) \qquad (3.20)$$

Where P is the probability of a node being contaminated by a neighbour.

In his discussion on the application of the percolation process to fire spread Ramachandran [9] assumed that these eight spread probabilities are not necessarily symmetrical due to factors such as wind and topography. This can be taken to be reasonable as the medium of spread is fraught with randomness. For instance the probability of a node being contaminated can increase if the link between it and the contaminating node has a higher proportion of openings.

A node can be in any of three states at any time: uncontaminated (U), or contaminated (C) and contaminating (CT). A node is contaminating if the smoke concentration has reached its threshold and there is a link and passage to the other node. The threshold is reached when the smoke fills up the space between the ceiling and the lintel soffit of the opening. A node can be contaminated but not contaminating. A contaminating node has been contaminated.

The medium has the following random mechanism: each bond has an independent probability p of being undirected and q = 1 - p of being directed; a directed bond does allows passage in one direction; and fluids supplied at a set of source nodes spread only along undirected bonds contaminating or wetting other nodes they reach.

Tat and Hasofer [25] represented the percolation process on a two dimensional lattice as shown in figure 2.3 where o represents nodes of the medium with directed bonds and the • represent nodes in the medium with undirected bonds. An analogy between a percolation process defined by the spread of a fire on a level of a building and the spread of a fluid in a medium was given. Percolation processes can be used for modelling the spread of smoke in a level of a building.

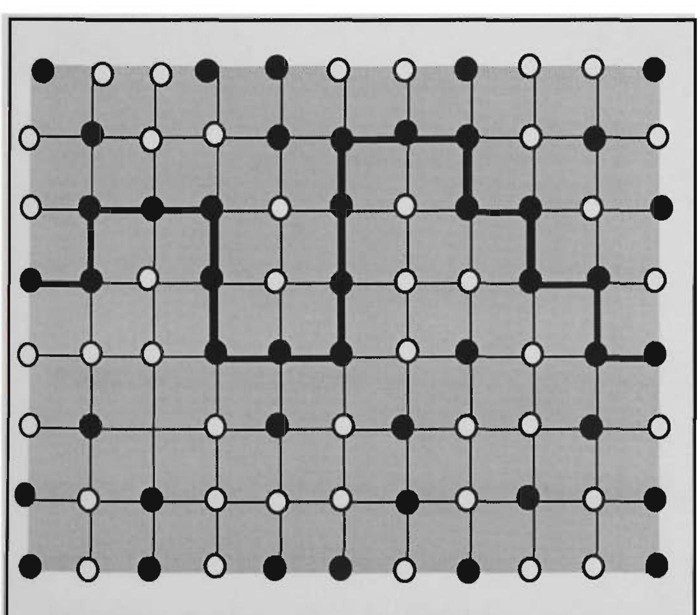


FIG 3.3. TWO DIMENSIONAL REPRESENTATION OF A PERCOLATION PROCESS.

3.9.4 Birth-Death Processes

This is a special class of discrete Markov processes. The definition of a birth-death process requires that the state transitions are only between neighbouring states. Being a discrete Markov process, it will only approximately describe smoke spread.

3.9.5 Epidemic Model

The epidemic theory is a simple class of discrete Markov processes. Different processes are obtained for different problems by varying the assumptions without disturbing the Markov property [21,27]. The main characteristic of the epidemic process is the transfer of infection.

The simple epidemic model is the spread of a relatively mild infection through a finite population in which none of the infected individuals is removed from the population by isolation, recovery or death.

Transition probabilities for the general epidemic model are given by Becker [26] as below:

TRANSITION	PROBABILITY
$(S, I, R) \rightarrow (S - 1, I+1, R)$	$\beta SI + o(h)$
$\rightarrow (S, I-1, R + 1)$	$\gamma hI + o(h)$
$\rightarrow (S, I, R)$ no change	$1 - \gamma hI - \beta SI + o(h)$

where S(t) = Susceptibles, I(t) = Infectives and R(t) = removals in the population.

The stochastic Epidemic Threshold theorem states that [26]:

$$\Pr(\text{minor epidemic}) = 1 - \Pr(\text{major epidemic})$$

$$= \min\{1, (\gamma / \beta k)^{\chi_o}\}$$

where initial conditions $S(0) = k$, $I(0) = \chi_{o,}$ and $R(0) = 0$. And the probability of a major outbreak is determined by the initial infection potential $k\beta / \gamma$.

The birth-death process and the epidemic model can be useful in modelling fire spread.

3.9.6 Random Walk

The random walk is a stochastic process in discrete time [19,20,22]. A particle can undergo a step or jump X, where X is a random variable having a given distribution. The next position to be occupied is equal to the previous position plus the random variable. The value of the r.v. is independently drawn from an arbitrary distribution which does not change with the state of the process. The sequence of r.vs is called a random walk:

$$S_n = X_1 + X_2 + \ldots\ldots\ldots + X_n \qquad n = 1, 2, \ldots\ldots. \quad (3.21)$$

Where X_1 , X_2 ,.... is a sequence of independent random variables (r.v) from a common distribution, and S_n is the position of the particle at time n.

If the state space is continuous then the steps X_i will be continuous. Steps that are restricted to integral values are discrete. The presence of barriers in the particle motion forces some restriction. The effects of the restriction and or absorption at the barriers can be determined.

Though smoke particles can be considered as a random walk process, it is certainly not a simple or linear one. It is a multidimensional random walk on a plane or three-dimensional space.

3.9.7 Discretization in Stochastic Modelling

Natural phenomena like the spread of smoke, do not change their characteristics at specific points in time but on a continuous basis. They are continuous processes.

Time dependent modelling involves the follow-up of events from start to end points. The result at the end point can only be approximated by discretization of the duration of interest. The discretization approach involves dividing the range of time interest into discrete units such that the cumulation of the results gives the end event value. The unit of time depends on the desired fineness and the known range from start to end points. For a segment of a sample function X(t) between the times t and t + dt, the discretization approaches the desired continuous effect as time unit dt -> 0. Hasofer et al [16] used this approach in discretising the hazard function as a tool in the network representation of fire spread.

3.10 Smoke Spread

3.10.1 Smoke Spread Within A Compartment

Spread of smoke in a compartment is related to that of fire although they may not be the same. It is therefore relevant to consider the fire spread while assessing that of smoke.

A fire in a compartment releases radiant and convectional energy to all objects about it, increasing their temperature. These objects rapidly lose their external then internal moisture content into the air. In course of doing this an object's point of ignition is delayed due to the resulting cooling effect of evaporation. As the temperature increases so does the dryness. The next object on fire can be ignited by transported combusting fragments carried by the forces in the medium or by those of combustion reactions. The rate and duration of combustion of the previous object determine the readiness of the current object hence its speed of ignition.

The eddies and difference in concentrations between the upper and lower sections of a compartment result in the layering effect. He and Beck [2] noted that the layering effect in a stairshaft and the corridors on floors above that of fire origin was not obvious. As the force creating the eddies and stratification dissipates and the extent of mixing increases the smoke loses its layering effect as it propagates from the fire source. Smoke spread within a compartment is also affected as described above. The force creating eddies may be caused by pressure differences; which could be due to temperature differences or by mechanical devices. Pressure differences between the inlets and outlets of the compartment create the general flow trend. Within the compartment the flow medium is subject to significant variability.

3.10.2 Spread From One Compartment To Another

Steps in developing a model for the spread of smoke from one compartment to another on a floor will include the following:

1. Obtain concentrations of combustion products from the source of fire.
2. Determine fire and smoke spread.
3. For locations of interest determine concentrations of products with time from the source. To determine these concentrations, we move from one instant of time to the next after each iteration round.
4. Determine the same as in (3) above but also for additional flow from other nodes on fire arising from the spread.

Due to conservation of mass, fluid (air and smoke mixture) in a compartment flows to adjacent compartments through openings as fluid flows in. The rate of out flow is dependent on the sizes of the outflow openings and the inflow rate.

Below is a flow chart of a possible simplified algorithm for the spread of smoke compartments on a floor of a building.

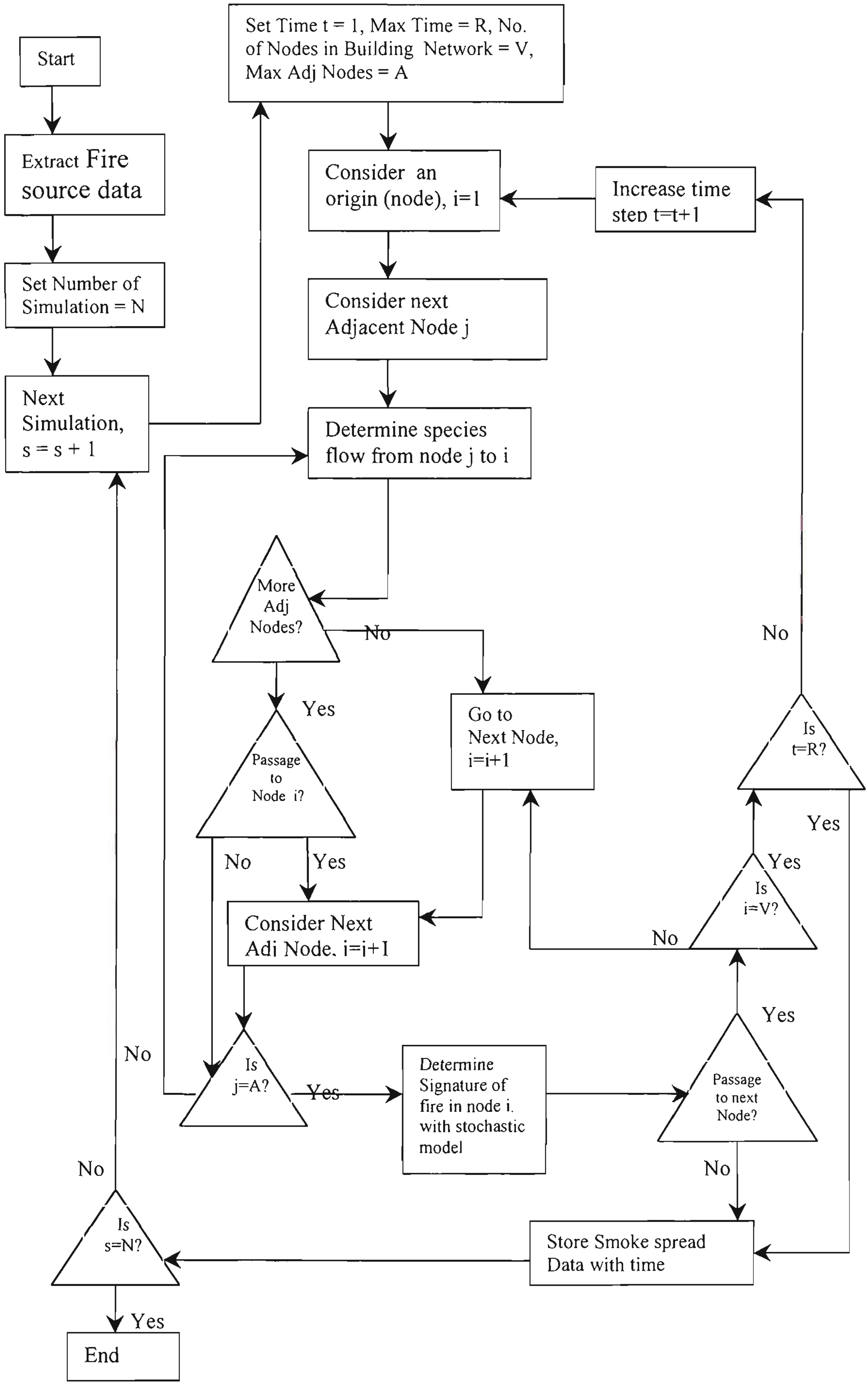


FIG 3.4 ILLUSTRATION OF ALGORITHM OF SMOKE SPREAD BETWEEN COMPARTMENTS ON A FLOOR

CHAPTER 4

TRANSFORMATION OF A DETERMINISTIC MODEL TO A STOCHASTIC MODEL

CONTENTS

FIGURES

TABLES

4.1 Introduction

Much has been done on deterministic models. Deterministic models avail us the opportunity to predict the signatures of fire. Their limitation lies in the lack of certainty. Part of the investigation done was establishing a procedure for the converting a typical deterministic model to a stochastic model. The advantage will be the ability to estimate the signatures within certain probability.

Presented below is the theory, analysis and results of converting a deterministic model to a stochastic model. It is assumed that the signatures of fire from the burning room (source) are the input to the model. Validation of the resulting stochastic model was done by comparing with results of the chosen deterministic model and real life data from experiments.

The following are required as input from a source into the smoke spread model.

1. Carbon monoxide (CO) concentration
2. Carbon dioxide (CO_2) concentration
3. Hydrogen cyanide (HCN) concentration
4. Oxygen (O_2) concentration
5. Mass flow rate (m).

At the time of the simulation only the mass flow rate of species and carbon dioxide concentration was available as input to the model. This was sufficient for illustration.

4.2 Methods of Stochastic Modelling:

The model chosen for this illustration is that by He and Beck [2]. Theirs is a modification of the NRCC smoke spread model. It was derived from physical laws. Three equations were derived. This deterministic model incorporates many considerations of the factors affecting the spread of smoke in a building.

Three of the methods available in transforming a deterministic model to a stochastic model are: (1) infusing stochasticity in inputs of high uncertainty, (2) adding stochastic components to equations, and (3) evaluating the process as experiencing jumps (Markov chain). The first method is a common approach in stochastic modelling. The second method has been used in random signals [17]. The last method will be used to develop a stochastic model and is described in the next chapter.

4.3 Choice of Stochastic Method:

(a) Infusion of Stochasticity:

Infusing stochasticity to expressions that describe a phenomenon is one of the methods used in converting a deterministic growth model to a stochastic model. This involves identification of inputs of high uncertainty and modelling the variability with a known or empirical distribution function. If the assumptions are realistic, this method would approximate the actual situation. These assumptions include among others the following:

- That the uncertainty can be described by the chosen distribution.
- That the uncertain inputs identified are the only significant ones that affect the uncertainties.
- That the expressions containing the uncertain inputs fully describe the phenomenon.

While the above could be a useful method, in this research other methods will be considered.

(b) Brownian motion method (noise addition):

The other method of turning the deterministic model to a stochastic one is by adding a stochastic component to the equations. It involves:

- Determining or choosing equations that describe the phenomenon.

  The equations derived form the deterministic model. Usually, these do not fully describe the phenomenon.

- Choosing appropriate stochastic noise components to add to the equations.

  The noise component includes the most uncertain of the components that make up the spread expressions. This induces a spread of noise about the expectations.

The basic assumption made in this method is that the noise component represents all the uncertainties affecting the phenomenon. In the case of smoke, this will be the velocity of spread while in fire spread, it will represent among other things the uncertain spatial distribution of fuel, combustibility of fuel, sudden changes of pressures that affect direction of spread, etc. As expected some of these uncertainties are common to both fire and smoke spread.

The problem is in the choice of an appropriate noise component to add. For instance, the choice of the parameter that defines the degree of stochasticity is at present arbitrary. In essence, the modeller tries many values before making a choice. It is suggested herein that this parameter can be classified through extensive simulations that could be verified by experimental data. This will require much data from experiments of different variability (fluctuations). For smoke spread, the variability will be a function of changes in temperature, pressure, weather and time of the year, elevation from base floor, etc. Values of the degree of stochasticity will depend on the type of medium which is a function of many other variables. Each phenomenon can have its own classification. That for fire spread would be different from that of smoke spread. Though this is a possible area to study, it will require much experimental data and time for simulations.

The significant advantage of this method is its simplicity and ease of modelling. In view of the difficulty of identifying appropriate distributions that describe the inputs in the flow equations, the first method will not be used in modelling. The stochastic model derived in this investigation consists of a deterministic model and stochastic noise component that describes smoke spread. The noise component comprises the selected variables that most induce the lack of certainty. These variables strongly influence the medium of spread of smoke. This will be the case when modelling species and temperature variations in the building.

4.4. Differential Expressions.

The general form of differential equation for the analysis of stochastic dynamic systems is [29]

$$\dot{X}_t = f(t, x_t) + G(t, x_t)\xi_t \tag{4.1}$$

Where ξ_t = m-dimensional white noise

$G(t, x_t)$ = d x m matrix, and

X_t and f are R^d (R dimension) valued functions.

The corresponding integral equation is given as [29, pp. 57]:

$$X_t = c + \int_0^t f(s, x_s)ds + \int_0^t G(s, x_s)\xi_s ds \tag{4.2}$$

where:

c = arbitrary random variable.

For an m-dimensional Wiener process W_t:

$$dW_t = \xi_t dt$$

Hence:

$$X_t = c + \int_{t_0}^{t} f(s, x_s)ds + \int_{t_0}^{t} G(s, x_s)dW_s \tag{4.3}$$

This can be rewritten by *Itô*'s theorem (transforming stochastic integrals to stochastic differential equations) in the following differential form [29, pp 88 - 92]:

$$dX_t = f(t, x_t)dt + G(t, x_t)dW_t \tag{4.4}$$

Choice of the Most Variable Parameters

The list of variables that affect the spread of smoke can be long but some of the commonly used include the following [1,2,7]:

- Temperature
- Pressure
- Mass flow rate
- Heat loss rate
- Species mass fraction

The last three of the above all depend on the first two. In essence, the first two variables are fundamental. Hence, changes in them induce significant changes in the others. It is evident that these two variables (temperature and pressure) will be input in defining the noise component to add to the model.

He and Beck [2] derived the following set of differential equations for the mass flow rate, the rate of variations in temperature and species concentration.

$$m_o = \left(m_i T_i - Q / C_p\right) / T \tag{4.5}$$

$$\frac{dT}{dt} = \frac{RT}{PV}\left[m_i(T_i - T) - Q / C_p\right] \quad (4.6)$$

$$\frac{dY}{dt} = \frac{RT}{PV} m_i(Y_i - Y) \quad (4.7)$$

Where:

m = mass flow rate.

T = temperature (K).

Q = heat loss rate (W).

C_p= specific heat of constant pressure (J/Kg K).

P = pressure (Pa)

R = gas constant = 8.32×10^3 (J/Kg K).

V = volume (m^3).

Y = species mass fraction.

t = time (sec).

Subscript:

i = in-flow

o = out-flow

The above equations give the deterministic model to be converted to a stochastic model.

4.5. Deriving set of stochastic noise component for smoke spread.

A stochastic process with all frequencies participating with the same intensity has a "white" spectrum as with white light in optics that contains all frequencies of visible light uniformly. It is not an ordinary process but a "generalized" process. The advantage of such a generalised process is that its derivative always exists and is itself a generalised stochastic process. The concept can be a useful mathematical tool. Such processes can serve as models of "noise", that is, of stationary and rapidly fluctuating phenomena because of the independence of values at every point.

White noise ξ_t is the derivative of the Wiener process W_t when we consider both processes as generalized stochastic processes [29, pp. 57].

$\xi_t = \dot{W}_t$ *and*

$W_t = \int_0^t \xi_s ds$

Generally, d-dimensional Gaussian white noise is the derivative of the d-dimensional Wiener process.

The mass flow rate in and out of a compartment is the same. There is no loss or gain in mass. We can express the stochastic equations as follows:

$$m_o \; - \; (m_i T_i - Q/c_p)/T)dt \; = \; f1(T)dW_0 \tag{4.8}$$

$$dT \; - \; \frac{RT}{PV}\left[m_i(T_i - T) - Q/C_p\right]dt \; = \; f2(T,P)dW_1 \tag{4.9}$$

$$dY \; - \; \frac{RT}{PV} m_i(Y_i - Y)\,dt \; = \; f3(T,P)dW_2 \tag{4.10}$$

where

m = Mass flow rate (Kg/m^3)

We can change the amount of stochasticity by multiplying the right hand side of the above equations by a coefficient and obtain different realisations.

The choice of the noise component, in this instance, depends on the factors that most influence variability in the spread namely changes in temperature and pressures. The amplitude of the niose component should be chosen in such a way that the amplitude of the band of the noise around the theoretical results will cover the experimental results.

Definition of the noise component involves carrying out series of simulations on trial basis. The amplitude of the noise component should be chosen in such a way that the band of the noise around the theoretical results that will cover the experimental results. Work need to be done in calibrating the amplitude against the variable factors to facilitate ease of choice.

Then f1, f2 and f3 were chosen as

$$f1(T) = \phi T/10 \quad (4.11)$$

$$f2(T,P) = 50\phi TP/10000000 \quad (4.12)$$

$$f3(T,P) = 12\phi TP/1000000 \quad (4.13)$$

where *T and P* are the temperature and pressure respectively.

ϕ was taken as 200.

Addition of a noise component to deterministic models has been used in other areas of engineering [17]. Similar equations were used by Hasofer and Beck [16a] in their simulation of the spread of fire. As the spread of fire is quite different from that of smoke the considerations for choosing the variables are different. Using the above three expressions a series of simulations were performed adjusting the value of ϕ and the coefficients of the equations until the amplitude of the noise was sufficient to cover all fluctuations in the deterministic model. This is evident in the plots or in the computation of the standard deviation of the stochastic model, ensuring that the deterministic model is always within the desired confidence interval. It is important to point out here that a lot needs to be done in calibrating the value of ϕ and the coefficients of the equations to real life experimental results to enable easy choice of such expressions as the above. The calibration has been left out of this research to focus more on the methodology with the hope that later work will address that. The above expressions define the stochastic equations used for the simulations of the spread of smoke.

4.6. Determining reasonable assumptions for state of nodes.

The assumptions used are as stated in He and Beck [2]. More of the assumptions had earlier been presented in section 3.3 of chapter 3.

4.7. Results and Analysis

The plots of the results of the simulations show jagged behavior of the process of smoke spread. They exhibits the Markov property [29, pp. 27] which states that with a known present it is not possible to transmit information from the past into the future.

The distribution of the temperature of species for the stochastic model for the stairway on the second floor is shown in Fig 4.1 below. The input data is same as that used by He and Beck [2], which was earlier used by Hukogo [1]. The simulation is for the stairway of the NRCC building. Detail of the building configuration is given in the figure below.

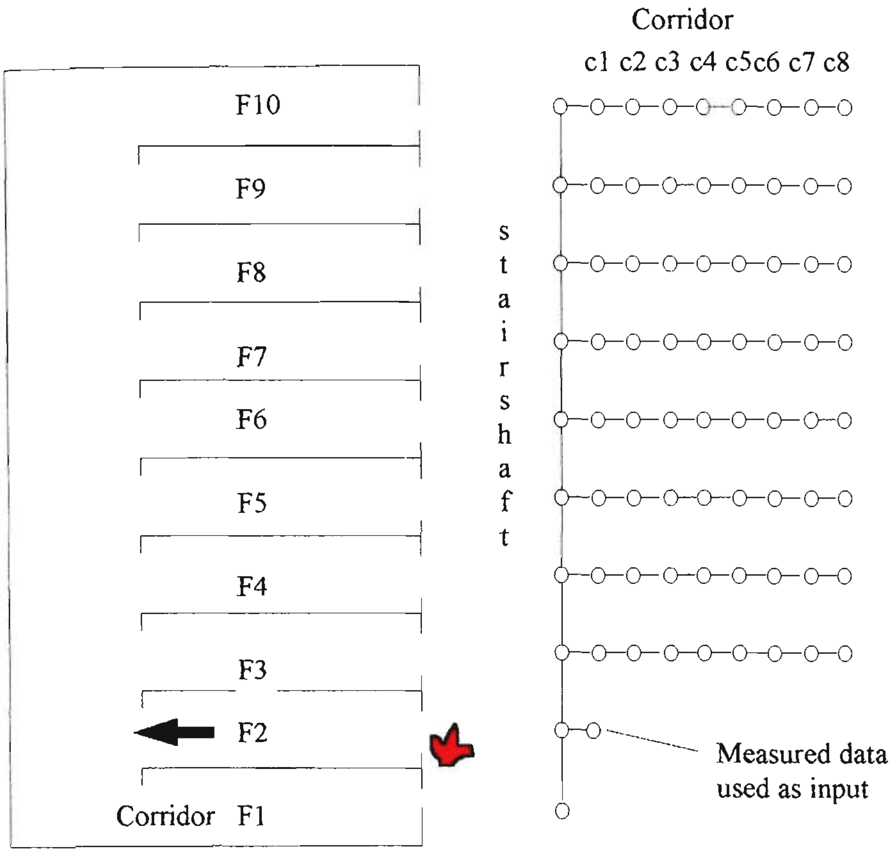


Fig 4.1. Simplified sketch of the NRCC smoke tower (left) used and a network representation for modelling (right).

The figure below shows the temperature distribution for the stairway on the second level (NRCC Building) using the stochastic model. It shows the deterministic and stochastic model results.

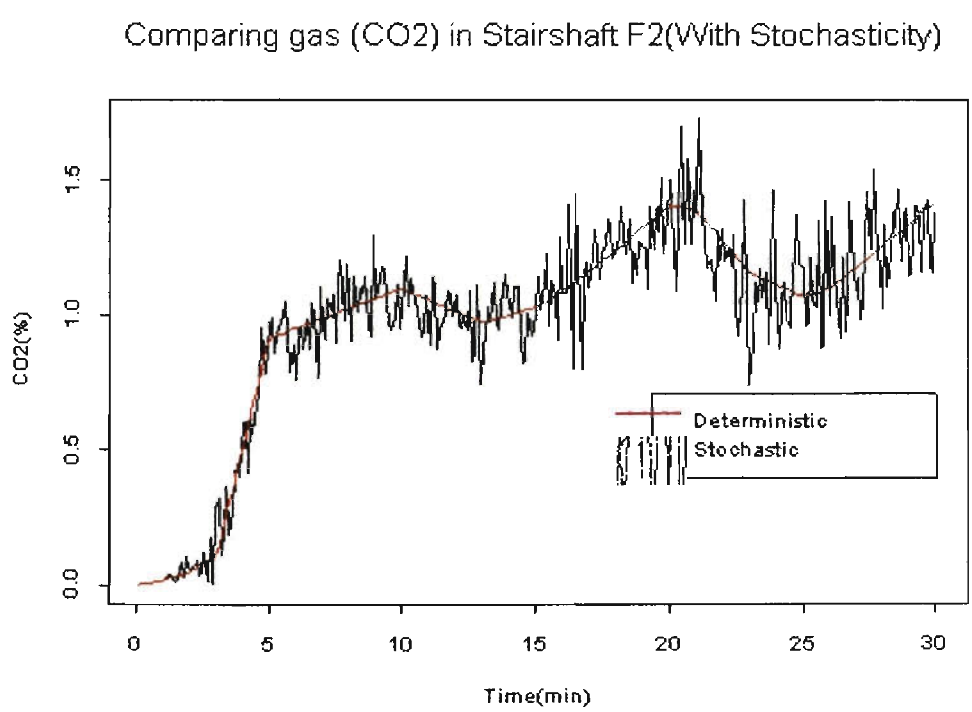


FIG 4.2 RESULTS FOR THE STAIRWAY ON THE SECOND LEVEL (NRCC BUILDING)

4.8. Validating the model with experimental data.

The results obtained from the above model will be compared to that from experimental data.

Hukugo et al [1] results for real experiment for carbon dioxide concentrations using the same building for some of the levels is presented below. Results of the stochastic model developed will later be compared with these results.

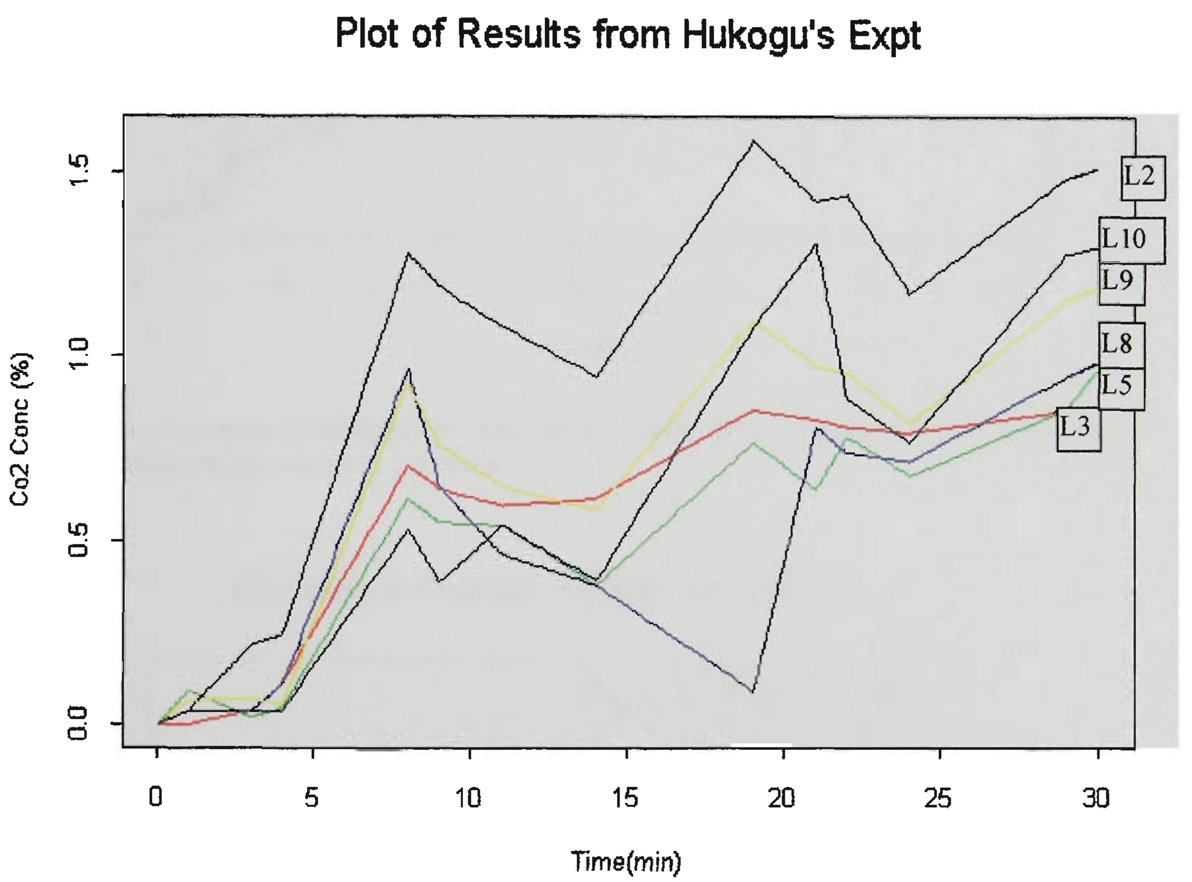


FIG 4.3 HUKUGO ET AL'S RESULTS FOR THE VARIOUS LEVELS OF THE STAIRCASE (NRCC BUILDING) FOR A REAL EXPERIMENT.

The smoke spread on the second floor was modelled with increased noise amplitude; increasing the amount of stochasticity hence obtaining a different realization. Result of such is shown in Fig 4.4 below. The noise component of the stochastic model can be increased or decreased to reflect the variability of the medium of spread. The second level result for carbon dioxide from the above figure is compared with that for the stochastic model.

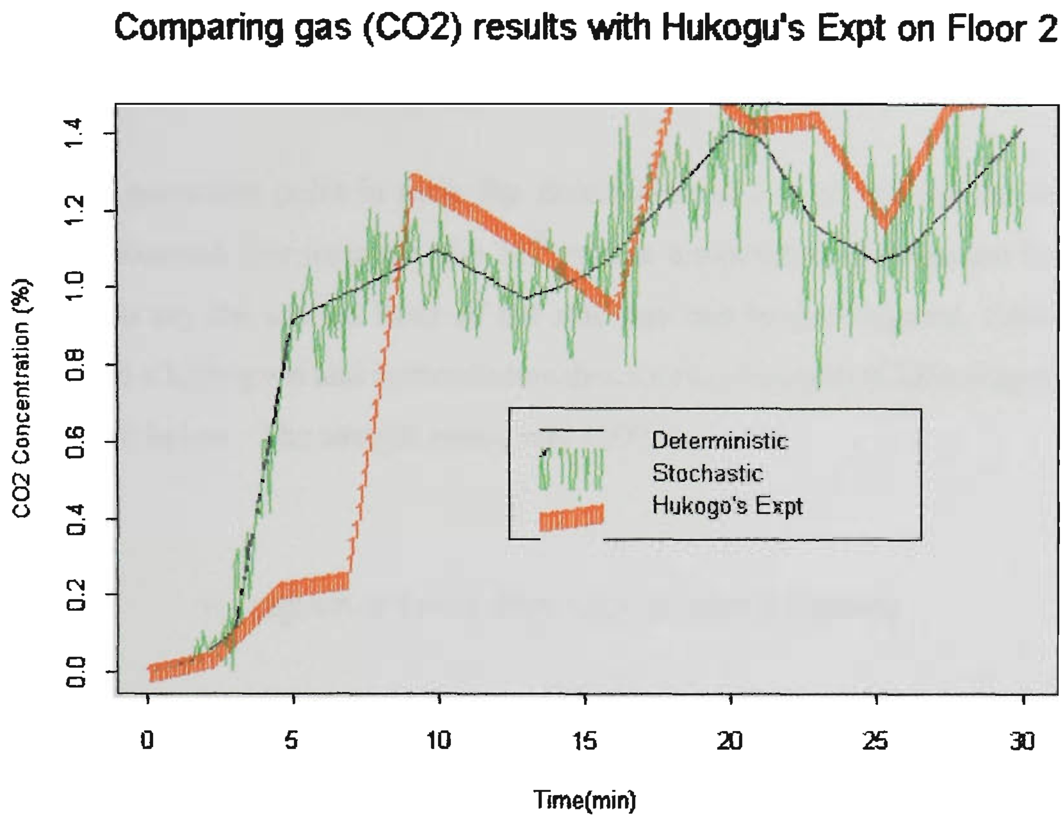


FIG 4.4 COMPARISON OF STOCHASTIC AND DETERMINISTIC MODELS (CARBON DIOXIDE CONCENTRATIONS ON LEVEL 2 STAIRWAY)

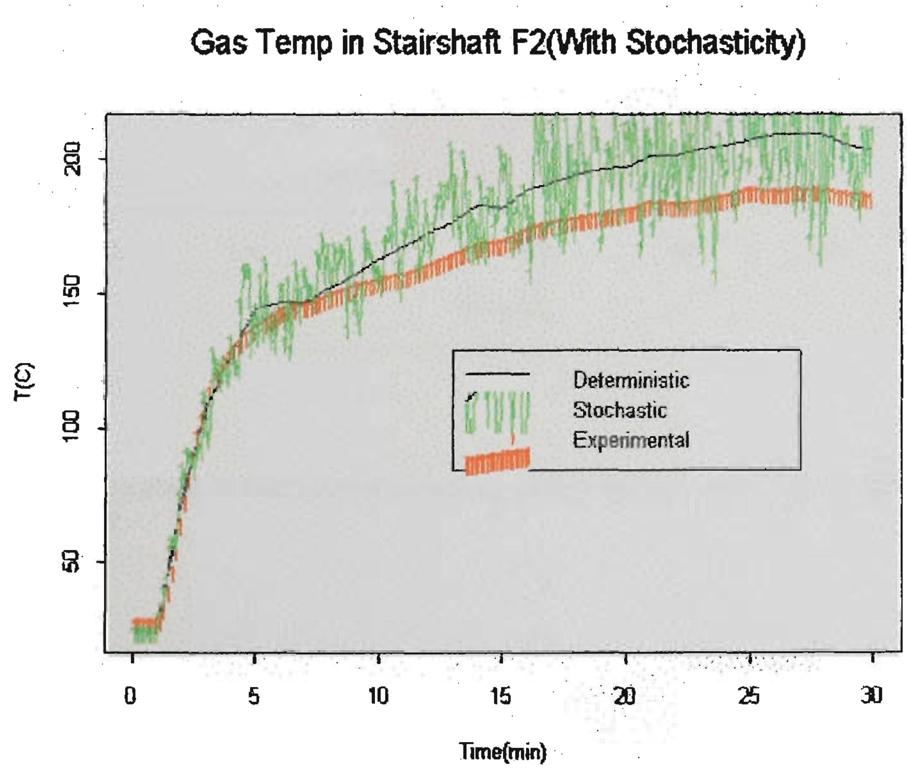


FIG 4.5 COMPARISON OF THE STOCHASTIC AND DETERMINISTIC MODELS (TEMPERATURE ON LEVEL 2 STAIRWAY)

The figures above compares the stochastic model to experimental results [1] and He and Becks' [2] deterministic result for level 2. Expectation of a stochastic model is the deterministic model, which is evident in the figure.

Given a particular point in time, the distribution of any of the signatures of the fire can be obtained. For instance, the 30th minute temperature distribution from the start of fire on say the second floor of the stairway can be investigated. Such simulation produced a histogram and cummulative distribution function (CDF) diagram shown in the figure below. The sample space was 1000.

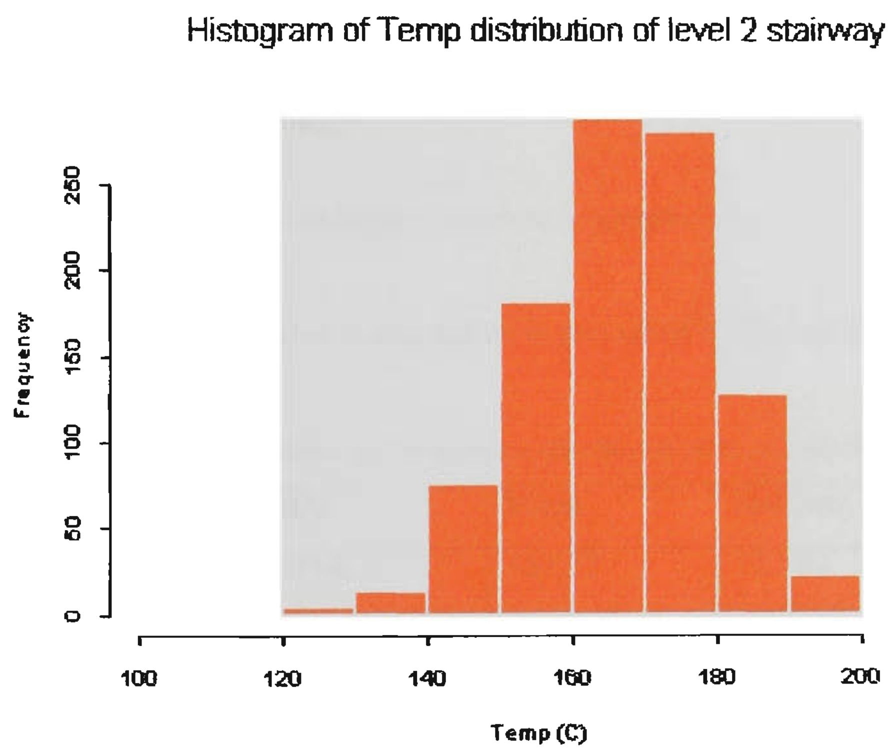


FIG 4.6: HISTOGRAM OF THE TEMPERATURE DISTRIBUTION FOR THE 30TH MINUTE FOR LEVEL 2 OF THE STAIRWAY.

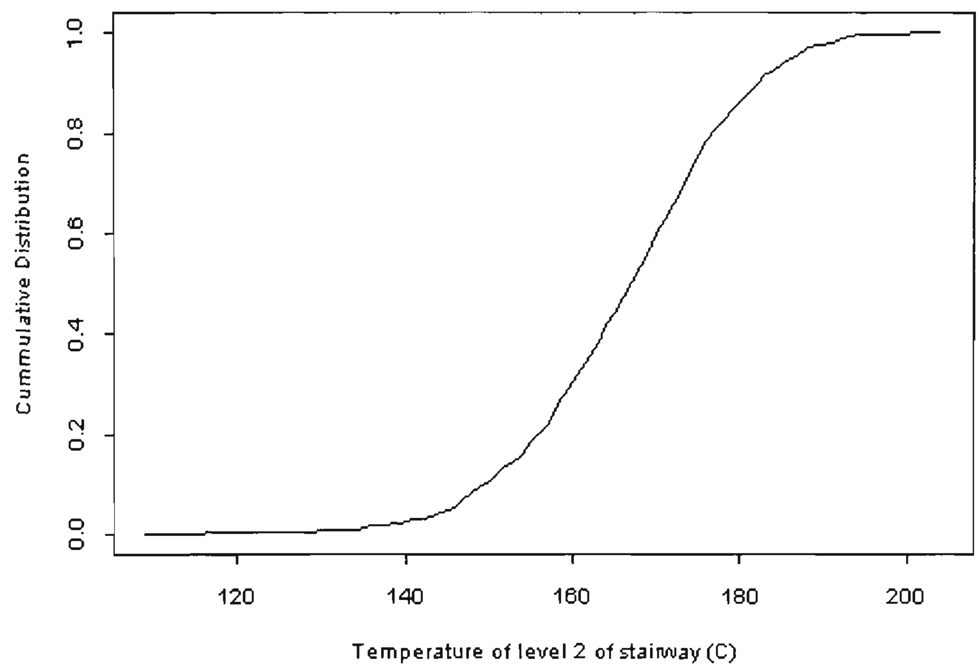


FIG 4.7: CUMMULATIVE DISTRIBUTION OF TEMPERATURE FOR THE 30TH MINUTE ON LEVEL 2 OF THE STAIRWAY.

Detail data for the above figure is presented in appendix A.

Statistics for the above simulation is presented in the table below.

Table: 4.1 Statistic of Temperature distribution on level 2 of stairway.

Max	Min	Mean	Std.Dev	Var
204.75	109.62	167.22	12.782	163.379

Comparing the result in the table above with the real life experiment, Hukugo et al's result at the 30th minute on the level 2 of the stairway was 173.45 ^{o}. Hence the experimental result lies within one standard deviation of the mean of the simulation.

A deterministic model can be used to simulate the spread of smoke. The spread process is subject to so much variability that it is not satisfactory to state the signatures of fire as being definite. The stochastic model as illustrated above provides the opportunity to estimate the signatures of fire with some probability based on the amount of added noise component; which represents the variability in the medium. Usually the broad spectrum of the results of the stochastic model covers the results of the deterministic model as shown in Fig 4.4 and 4.5 above. The deterministic model being the expectation of the process. By choosing appropriate amplitude of the noise component, the band of the results of the stochastic model gives realistic results that is more reliable than that that the a deterministic model will provide. For instance by increasing the noise component as in Figs 4.4 and 4.5 the results of the stochastic model covers what is expected in real life experiments.

4.9. Summary

In this chapter, a deterministic model has been converted to a stochastic model. With the cumulative distribution function (CDF) of the model, temperature and species concentration can be estimated with given probability. The converse is true. The computer program is listed in Appendix A.

The simulation was done using the most relevant of the factors affecting flow. The basis for the choice of the factors used was presented. Equations of flow were developed. A deterministic model was first developed with a stochastic model given and used for the simulations. The stochastic model attempts to represent the variation in the medium of spread. The extent to which adding noise to a deterministic model represents the effects of uncertainty in data and modelling is demonstrated by the coverage of the experimental data by the stochastic model. This is reflected by the amplitude of the noise. Fig 4.5 show the coverage of the experimental and deterministic models by the stochastic model. It rests with the modeller to use such level of noise to enable this. Input data from the fire was from that used by previous modellers. The building considered for the simulation is a ten storey (NRCC building) with a stairwell, corridors and compartments. The simulation was limited to spread in the stairwell, corridors on the floors and between levels.

The limitation of this methodology is when the noise added cannot cover the band of the deterministic model. Where there are experimental data, the model can be verified by determining if the experimental result all fall within the stochastic model as demonstrated in section 4.8 above. The experimental result of Hukogo plotted in Fig 4.4 for Carbon dioxide is not entirely covered while it is in Fig 4.5 for temperature.

Increasing the noise component for the carbon dioxide simulation is then required. Another limitation is not knowing if the chosen variables that cause the significant variability in the process used for the noise are exhaustive. Where there are other factors not considered that introduce variability in the process the noise becomes too loud with increased variance. As stated before there is need for further work to calibrate the required noise for easy choice. The stochastic model attempts to represent the variation in the medium of spread. The extent to which adding noise to a deterministic model, represent the effects of uncertainty in data and modelling, is demonstrated by the coverage of the experimental data by the stochastic model. This is reflected by the amplitude of the noise.

With this type of modelling, estimates can be stated within some confidence interval. This is most useful in the management of hazards due to fire and smoke.

The usefulness of adding the noise component lies in the fact that with the noise component the theoretical results do cover the experimental results showing that the deviation of the experimental result from the theory is due to hidden uncertainties in the experimental data.

CHAPTER 5

Markov Chain Model for Smoke Spread

CONTENTS

FIGURES

TABLES

5.2 Introduction

In this chapter, the spread of smoke in a building is simulated using a stochastic process, the Markov chain. The spread of smoke is approximated by a discrete Markov chain with very small time unit step. The simulation was done using the most relevant of the factors affecting flow. The basis for the choice of the factors used is presented. Equations of flow are developed. The effect of buoyancy pressure on the spread of smoke was first investigated with the assistance of Professor W. K. Chow. The result indicated that buoyancy pressure does not play significant part in the spread of smoke beyond the fire room. Hence a deterministic model that excludes effect of buoyancy pressure and a stochastic model were then developed and used for the simulations. The stochastic model attempts to represent the variation in the medium of spread. Input data from the fire is from that used by previous modellers. The building considered for the simulation is a ten storey (NRCC building) with a stairwell, corridors and compartments. The simulation is limited to spread in the stairwell, corridors on the floors and between levels.

Various averages can be used to describe a system but there are many interesting questions that may not be answered in this way. Such include questions about the distribution of certain variables. Stochastic process modelling is a convenient tool in such matter. The last chapter illustrated the transformation of a deterministic model to a stochastic model by adding a noise component. In this chapter a deterministic model was first developed then converted to a stochastic model by approximating the process by a Markov chain.

A stochastic process X(t) is defined by the following quantities [22]:

(a) State space

(b) Index parameter (time)

(c) Statistical dependencies between the random variables (r.vs) X(t) for different values of the index parameter t.

The state space is the set of possible values (or states) that X(t) may take. We have a discrete-state process if the values that the process may take are finite or countable. In smoke spread, a particle can occupy any of the infinity of positions about it, hence the process cannot be discrete. The possible positions of the smoke particles, in this case, are over a finite or infinite continuous interval. The spread is thus a continuous-state process. The index (time) parameter at which changes may occur is anywhere within a set of finite or infinite intervals on the time-axis; hence it is a continuous-parameter process.

The flow of smoke is considered as a stochastic process. This is due to the instability and variability in the process. One of the stochastic processes that can describe this is the Markov process. The Markov property assumes that the future behaviour of a sequence of events is uniquely decided by a knowledge of the present state only. In essence, given the state of the process at any time, the Markov process assumes that its subsequent behaviour is independent of its past history. The Markov process with discrete parameter space is referred to as a Markov chain. Smoke spread does not occur in discrete time hence it can only be modelled approximately as a Markov chain.

The definition of smoke determines its composition. The American Society for Testing and Materials [4] defines smoke to include gases evolved during combustion. It is the total product of combustion. Usually, the composition of smoke includes carbon dioxide, carbon monoxide, hydrogen cyanide (if any) and the incomplete combusted fuel particles. The National Fire Protection Association [NFPA 5] defines smoke as consisting of the airborne solid and liquid particulate and gases evolved when a material undergoes pyrolysis or combustion, together with the quantity of air that is entrained or otherwise mixed into the gases. Hartzell [6] enumerated the predominant hazards to humans exposed to products of combustion. These include heat, visual obscurity and narcosis. Narcosis results from the inhalation of asphyxiants and irritation of the upper respiratory tracts. Delays due to visual obscurity, panic and disorientation results in further inhalation of toxic gases. Consideration will be given to the concentration and temperature of species. A deterministic model will be converted to a non-deterministic model as a

Markov chain process. Data from a chosen model will input into the model.

5.3 Flow between nodes on a level

Consider a network of nodes. At a given time t, node A is in some initial state with species concentration x of mass m_1 and of temperature t_1 and node B with species concentration y of mass m_2 and of temperature t_2. Assume there is a link (opening) between the two nodes such that smoke flows from node A to B. For small infinitesimal time dt there is some additional quantity of smoke flowing from node A to B resulting in increase in temperature and species concentration in node B.

At time t

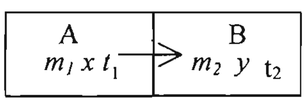


Fig 5.1 Flow from node A to B.

5.4 Modelling Assumptions

1. There is no accumulation of mass in a node. In which case the total mass of gases in a node at any time does not change. The mass flow of gases from a compartment is assumed to be equal to that leaving it. This is a necessary assumption to ensure that the total mass flow rate, ω , of smoke through the nodes is the same.

The air in the zone above the doors' lintels is first displaced before there is flow to the next compartment. Gases in fire plume rise directly and impinge on the ceiling. With time there is an accumulation of stagnant smoke layer in the upper portion of the compartment. This continues until the entire portion above the lintel level of available openings is filled. Evans [30] illustrated the ceiling jet flow beneath an unconfined ceiling.

2. The model focuses on smoke spread outside the combustion zone, locations remote from the fire (far field effects). Occupants in distant locations from the source of fire are most unlikely to be aware of it. These areas could then steadily become hazardous especially if the occupants are asleep or drunk.

3. The boundaries of the compartments attain the temperature of the compartment within one unit time step. So the walls are assumed to have the temperature of the compartment after the unit time step (of say 10 seconds). Heat transfers from the incoming gas to the walls and also to the assumed uniform space in the compartment within one unit time step.

5. As hot gases enter a node, they occupy the upper section while cooler gases flow out at the bottom of the opening. The flow through an opening may be in both directions especially if the node has only one opening. Most of the nodes used in the validation and simulation have more than one opening. One opening lets in smoke and another lets out smoke. The model developed does not consider flow in both directions. Gases can only flow in one direction dictated by the pressure difference across the opening.

5.5 Deterministic Model

The first step in developing a stochastic model is to develop a deterministic process that describes the spread of smoke. This can then be converted to a stochastic process. Flow in deterministic processes is defined mainly by known input parameters. For instance, the spread of smoke is mainly due to differential pressures. Difference in pressure could be due to many factors.

5.5.1. Pressure Differences

The pressure difference Δp , causing the flow can be due to many factors. These include stack, buoyancy, wind and mechanical devices. The relevance of any or a combination of these will depend on the location of the fire, height of building or number of levels, and the building configuration. Each of these is discussed below.

5.5.1.1. Buoyancy:

Buoyancy is the effect of the pressure difference between hot gases from a fire compartment and its surroundings. The effect of buoyancy pressure will depend largely on the gas temperature distribution along the levels of the building. For situations where the fire is located such that the gas temperatures are significantly less at distances from it, buoyancy will not be relevant in the spread of smoke. This assumption was first investigated as illustrated below. Gases usually mix with the cold air as they flow away from the fire source. A particular case will be where the fire is located on the second floor and by the door to the stairway of a ten-storey building. It is expected that the temperature distribution reduces rapidly as it goes up the levels in the stairway.

The pressure difference due to buoyancy is given as [Klote 31]

$$\Delta p_b \quad = \quad kb\ (1/TTs - 1/TTc)\ H \qquad (5.1)$$

Where

kb	=	Pressure Constant, 3460 (NKm^{-3})
TTc	=	Temp of gases into stairshaft (K): a variable
TTs	=	Assumed average initial temp in stairshaft (K)
H	=	Height above Neutral Plane in m.

Buoyancy pressure is only significant on the floor of the fire but does not have much effect for locations far from the fire source. Hence it can be assumed that the effect due to buoyancy for flows in a building is not significant in the flow of gases. That is the buoyancy of fire gases is not the major driving force in the flow of smoke in a building. Tamura [11, pp 66] also illustrated this with examples.

5.5.1.2. Stack Pressure

Stack pressure is caused by the difference in temperature between the outside environment of the building and the inside. When the outside is colder there is an upward lift of flow (normal stack effect), while the reverse is the case when the outside is warmer (reverse stack) [Federic et al, 33]. Klote [31] states that the significance of normal stack effect is greater for low outside temperatures and for tall buildings.

Tamura [11] gave an expression for the total stack pressure for multistorey building as

$$dP_{TS} = dP_{ewT} + dP_{ewB} + \sum_{i=1}^{n-1} dP_{fi} \qquad (5.2)$$

Where:

dP_{TS} = Total stack pressure difference (Pa),

dP_{ewT} = pressure difference across the exterior wall at the top,

dP_{ewB} = pressure difference across the exterior wall at the bottom,

dP_{fi} = pressure difference across the floor construction on the i - th floor,

n = total number of floors.

Stack action can be due to the temperature of the fire and to the difference in temperature between the inside and outside of the building. While the first acts only on the fire floor, the later acts over the entire height of the building [Tamura 11].

The expression for stack pressure differences at standard atmospheric pressure is given as

$$\Delta p_s = ks\ (1/TT - 1/TTs)\ H \qquad (5.3)$$

Where

p_s = pressure difference from shaft to outside (Pascal)

TTs = Temp of inside air in stair shaft (K)

TT = Temp of outside air (K)

H = Height above neutral plane in m

ks = 3460 $N.K.m^{-3}$ Coefficient.

The stack pressure depends largely on the height of the neutral plane from the base of the building. Hence the elevation of the building is the determining factor in deciding whether to incorporate stack pressure in the flow computation. Evidently it will not be relevant for low-rise buildings. The location of the neutral plane also depends on the location of the fire in the building. Klote et al [32] illustrated this. Apart from the location of the neutral plane, the leakage factor of the building also determines the flow. The leakage factor being the extent or proportion of openings at the boundary of the building. See Tamura [11, pp 35 - 45]. For a building with uniform air leakage to the outside the neutral plane will be located fairly close to the mid-height of the building.

Below the neutral plane, air moves into the building while the opposite is the case for part of the building above the neutral plane (NP). Hence, air will move from all the corridors below the NP into the stairway. The concentration of species in the corridors below the NP will be insignificant. This is illustrated by Klote et al [32, pp 22]. Corridors above the NP are expected to have significant species concentration.

5.5.1.3 Wind

Wind effect is a major effect especially in high rise buildings. The pressure due to wind on smoke movement is given as [Klote 31]:

$$P_w = \frac{1}{2} C_w \rho_0 V^2 \tag{5.4}$$

Where

C_w = Dimensionless pressure coefficient (-0.8 to 0.8),

ρ_o = Outside air density,

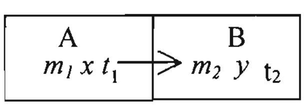
V = Wind velocity.

The effect of wind on smoke flow will not be included in this simulation. However it should be noted that it is of great importance; see for example BSI DD240 Part 2 from 1997. The effect of wind is not considered here for simplicity and to correspond with the conditions of the experiment and data available for verification [2]; see section 5.5.8 ahead. The building is made of concrete and external openings are closed. Only the openings in the corridor in the outside wall are opened. The input data does not include the situation where a window is broken. It is taken that the effect is not consequential in this case.

5.5.2. Species Concentration

Considering figure 5.1 again, the following could be derived.

At time t



The quantity of species flowing from node A to B in $(t, t+dt)$ =

$$\left\lfloor CA\sqrt{\rho\Delta p}\right\rfloor x\,dt \quad = \quad \left\lfloor CA\sqrt{\rho\Delta(p_b + p_s)}\right\rfloor x\,dt \qquad (5.5)$$

where

ρ	=	Gas density
A	=	Area of orifice (e.g. door)
C	=	Coefficient of orifice
Δp	=	Pressure difference between A and B
x	=	Concentration in previous node (A)
y	=	Concentration in current node (B)
p_b	=	Pressure due to buoyancy effect.
P_s	=	Pressure due to stack effect.

Similarly, the quantity of species flowing out of node B in $(t,\ t+dt)$

$$= \quad \left[CA\sqrt{\rho\ \Delta p}\right]ydt \quad = \quad \left[CA\sqrt{\rho\ \Delta(p_b + p_s)}\right]ydt$$

The quantity of species in node B at time t = $m_2 y$

and the quantity of species in node B in time $t+dt$ =

$$m_2 y + \left[CA\sqrt{\rho\ \Delta(p_b + p_s)}\right]xdt - \left[CA\sqrt{\rho\ \Delta(p_b + p_s)}\right]ydt$$

The concentration of species in node B at time $t+dt$ is given by

$$y + dy = \frac{m_2 y + \left[CA\sqrt{\rho\ \Delta(p_b + p_s)}\right]xdt - \left[CA\sqrt{\rho \Delta(p_b + p_s)}\right]ydt}{m_2} \tag{5.6}$$

$$= y + \frac{(x - y)}{m_2} CA\sqrt{\rho \Delta(p_b + p_s)}\ dt$$

$$dy = \frac{(x - y)}{m_2} CA\sqrt{\rho \Delta(p_b + p_s)}\ dt \tag{5.7}$$

$$\frac{dy}{dt} = \frac{(x - y)}{m_2} CA\sqrt{\rho \Delta(p_b + p_s)} \tag{5.8}$$

For horizontal flow, effect due to buoyancy can be assumed to be insignificant. Going by the assumption that the openings are of the same size, equation (4.8) above becomes:

$$\frac{dy}{dt} = \frac{(x - y)}{m_2} CA\sqrt{\rho \Delta p_s} \tag{5.9}$$

By the above expression, the rate of increase in concentration with time in node B depends on the difference in concentration between the two nodes, the pressure differences due to buoyancy and stack, and the mass of gas in the node. Pressures due to buoyancy and stack are functions of temperature. The area of openings is assumed to be the same. Otherwise the expression needs to be modified.

5.5.6. Discretising the Model

The spread of smoke is a continuous process. This must be discretised in order to be able

$$y_{n+1} - y_n = \frac{CAh\sqrt{\rho\, \Delta p}(x_n - y_n)}{m_2} \qquad (5.10)$$

to simulate the process. Let the unit step of time in discretising be h.

Equation (4.9) above can also be discretised as follows

where $\Delta p = \Delta(p_b + p_s)$

Equation (5.10) will be used in computing species concentration at different locations in the building.

In computing the species concentration in the corridor, it is taken as one compartment. The corridor can be subdivided into smaller units and the equation modified. If the corridor is subdivided then the flow will be horizontal and equation (5.8) developed earlier will be applicable.

5.5.7. Species Temperature

Consider a control volume that could represent a compartment in the simulation.

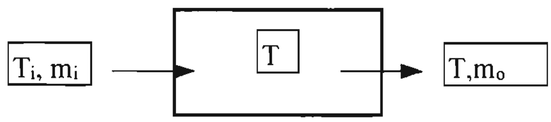


Fig 5.3 A control volume

The differential equations for the temperature of gas could be derived as in the above section for species concentration or the concept of a control volume. Using the latter, the specific heat capacities of gas are defined as

$$C_v = \left(\frac{\partial h}{\partial T}\right)_v \quad \text{and} \quad C_p = \left(\frac{\partial h}{\partial T}\right)_p \qquad (5.11)$$

Where h = Enthalpy per unit volume.

T = Temperature of gas in unit volume.

C_p = Specific heat capacity of gas at constant pressure.

C_v = Specific heat capacity of gas at constant volume.

Since smoke is considered here as a perfect gas, C_v *and* C_p will be constants at all temperatures and pressures. Although C_v *and* C_p vary with temperature their averages are assumed to be constant for practical purposes.

For a perfect gas, enthalpy $Q = M C_p T$ (5.12)

Where M = Mass of gas (Kg)

T = Temperature of gas (^{o}K)

$$\frac{dQ}{dt} = C_p\frac{d(MT)}{dt} = C_p\left[M\frac{dT}{dt} + T\frac{dM}{dt}\right] \tag{5.13}$$

Where the quantity of heat entering the control volume

$$Q_i = C_p\sum_{i=1}^{n} m_i(T_i - T) \tag{5.14}$$

The total enthalpy in the volume is assumed to be constant, so that $\frac{d(MT)}{dt} = 0$. We assume that is no accumulation of gas in the control volume.

$$M\frac{dT}{dt} = \sum_{i=1}^{n}\left[m_i(T_i - T)\right] - \frac{Q_0}{C_p} \tag{5.15}$$

From gas law $P = \rho R_g T$

$$M = \rho V = \frac{\mu P V}{R T}$$

Where

R_g = Gas Constant

R = Universal gas Constant (8320 m^2/s^2. K)

ρ = Density of gas

μ = Molecular weight of gas

Hence substituting in equation (5.15) above gives

$$\frac{dT}{dt} = \frac{RT}{\mu PV}\left[\sum_{i=1}^{m} m_i (T_i - T) - \frac{Q_0}{C_p}\right] \qquad (5.16)$$

The total mass flow rate into the volume is equal to the total mass flow rate out of the volume. Also the temperature of gas flowing out of the volume is equal to the temperature of gas in the volume.

Hence equation (5.16) becomes

$$\frac{dT}{dt} = \frac{RT}{PV}\left[m_i (T_i - T) - \frac{Q}{C_p}\right] \qquad (5.17)$$

Where

Q = Enthalp

m_i = mass transfer rate (inflow)

The above differential equation accounts for the elemental increase in temperature in unit time. He and Beck [2] in their deterministic model used a similar expression to the one above (Equation 5.17).

5.5.8. Consideration of Input Data for Simulations:

In large room fires the temperature of hot gases coming out of the room as the fire approaches flashover is known to exceed 700 °C [Frederic et al, 34]. The concentration of CO can be as high as 10%. Concentration of CO is significant in considering smoke toxicity. Clarke [35] stated that the CO fraction in fires under oxygen-deficient conditions approaches a common limit of about 0.2 Kg Co/Kg of burned. Babrauskas et al [35] has this to say of flashover fires:

- Smoke toxicity is dominated by carbon monoxide.
- All organic materials produce roughly the same fraction of carbon monoxide.
- For large fires where carbon monoxide is the dominant toxicant, there is no significant difference in the smoke toxicity of materials.

The concentration of carbon monoxide is a considerable factor in smoke toxicity. Presently, there is no ready data of carbon monoxide emission from fires available as input to the model. For illustration purpose, input smoke data will be that used by He and Beck. The carbon dioxide data used by Hukogu and He and Beck [2] will be used as input to the model. The spread of other signatures can be considered later.

The building used for this simulation is a ten-storey high rise building. Stack pressure will play a major role in the flow since the height of the neutral plane will be significant. The doors from the stairway to the corridors are assumed to be ten percent opened (10%). The staircase is assumed to give forty- percent obstruction to the flow. The walls of the building are made of concrete. The external openings are closed.

Since the openings to the corridor for the levels is fairly uniform it will be assumed that the neutral plane is about half way the building height. A time unit step of 1.5 minutes was used in the simulation.

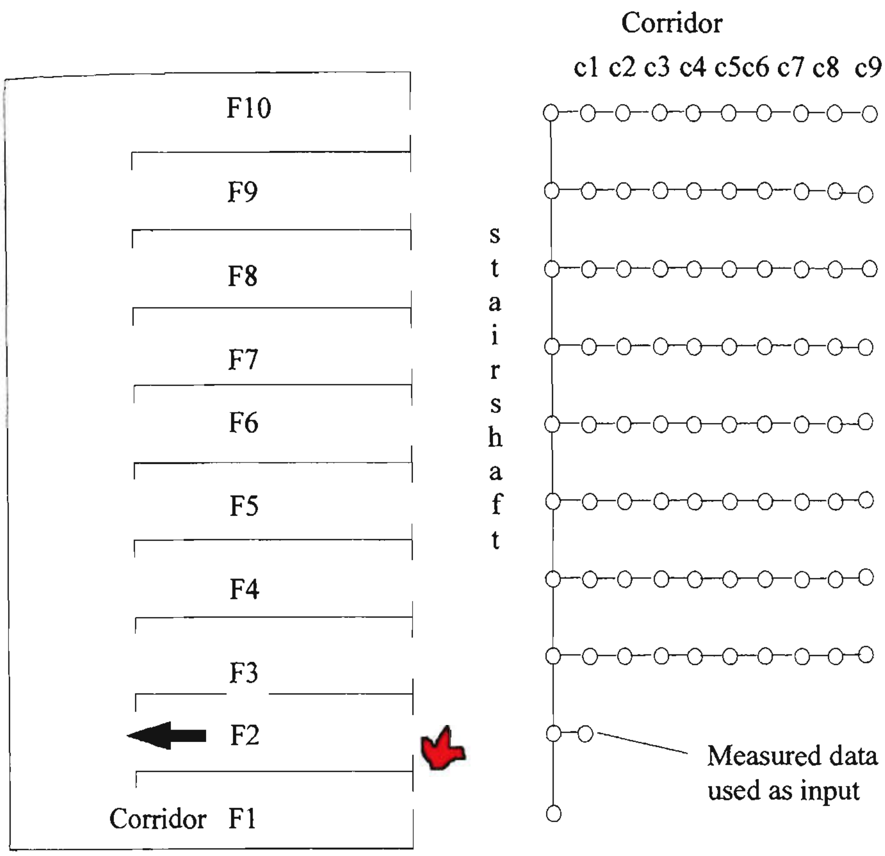


Fig 5.4. Simplified sketch of the NRCC smoke tower (left) used and a network representation for modelling (right).

Deterministic modelling gives estimates of species concentrations at various location of the building. These estimates may not take into account events that will induce fluctuations. Such events would include when doors are opened and closed and when windows are opened, closed, or broken. The precise estimates arising from these actions will entail rigorous computations that are themselves only estimates.

5.6 Source of Data

Data used for validation of the model is from Hokugo and Hadjisophocleous [8]. The experiments were conducted in the NRCC ten-storey experimental smoke spread tower. The fire is by the door to the stairwell on the second floor. Hence there is flow of smoke into the stairwell on the second floor. This is used as input to the model. The data has been used in validating the NRCC smoke spread model. It has also been used by He et al [2]. Details of the input data is presented in appendix B.

The data is for the condition of flashover fire with the door opened or closed. Wind velocity was less than 5 km/hr and atmospheric pressure about 24 °C. Hokugo [1, pp 810-811] shows graphs of velocity and temperature distribution during the test and pressure differences across points of interest. The data will be input into this model.

5.7 Results of Simulations for Deterministic model

The following figures are the results of the simulations of the spread of smoke in the NRCC building for the deterministic model carried out by He and Beck [2]. The deterministic model does not incorporate uncertainties. Results for the stochastic models are presented later in the chapter.

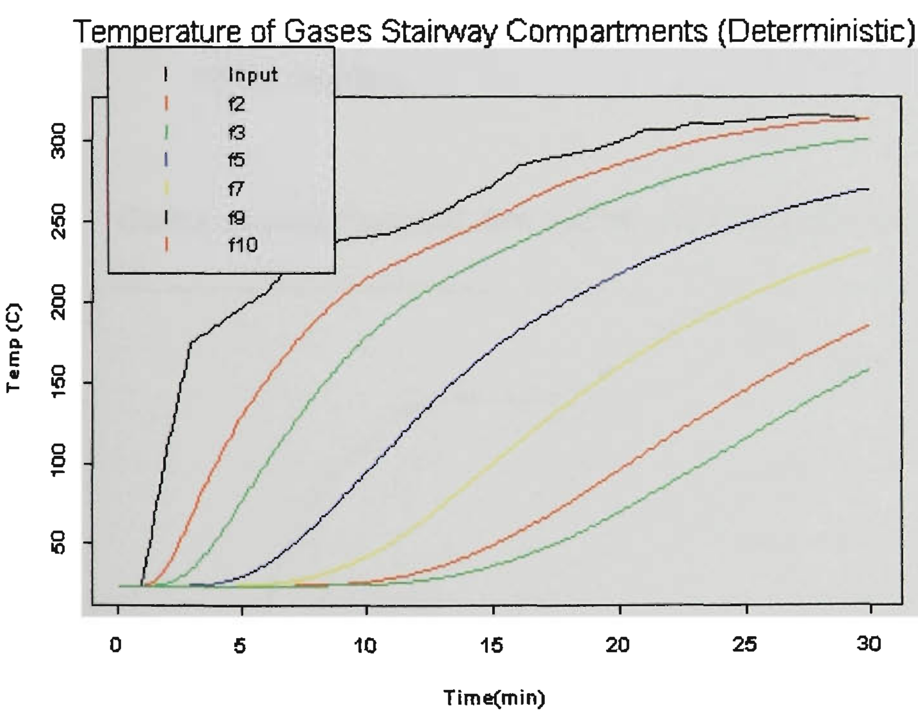
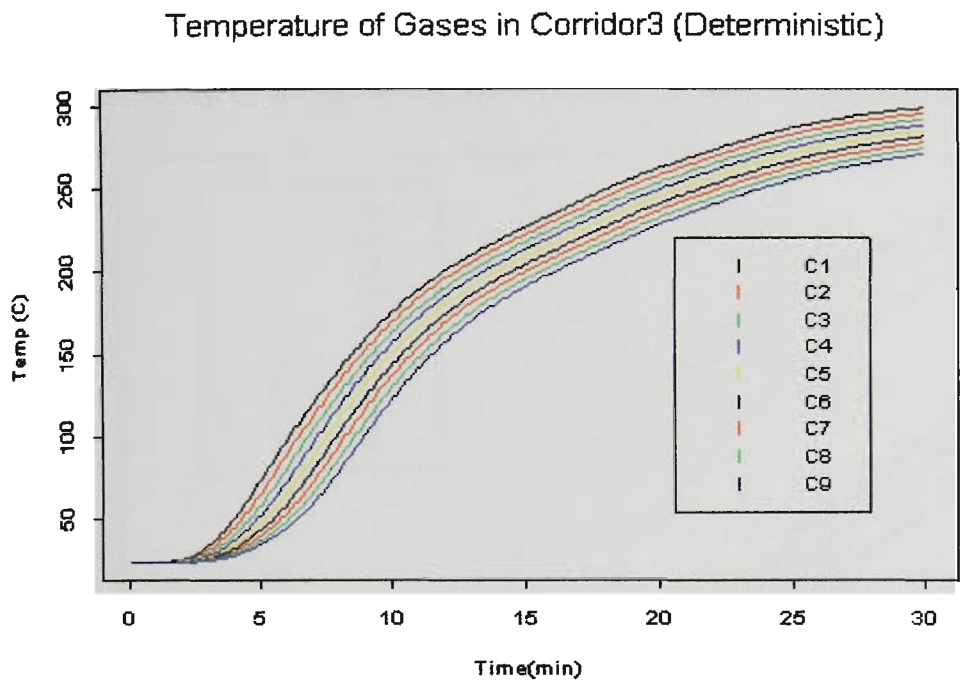


Fig 5.6 Temperature distribution for some levels of stairway compartments of the NRCC building (Deterministic). The fire was located by the door to the stairway on the second floor [2].



Time(min)

Fig 5.7 Temperature distribution for network nodes for the corridor on level 3 of NRCC building.

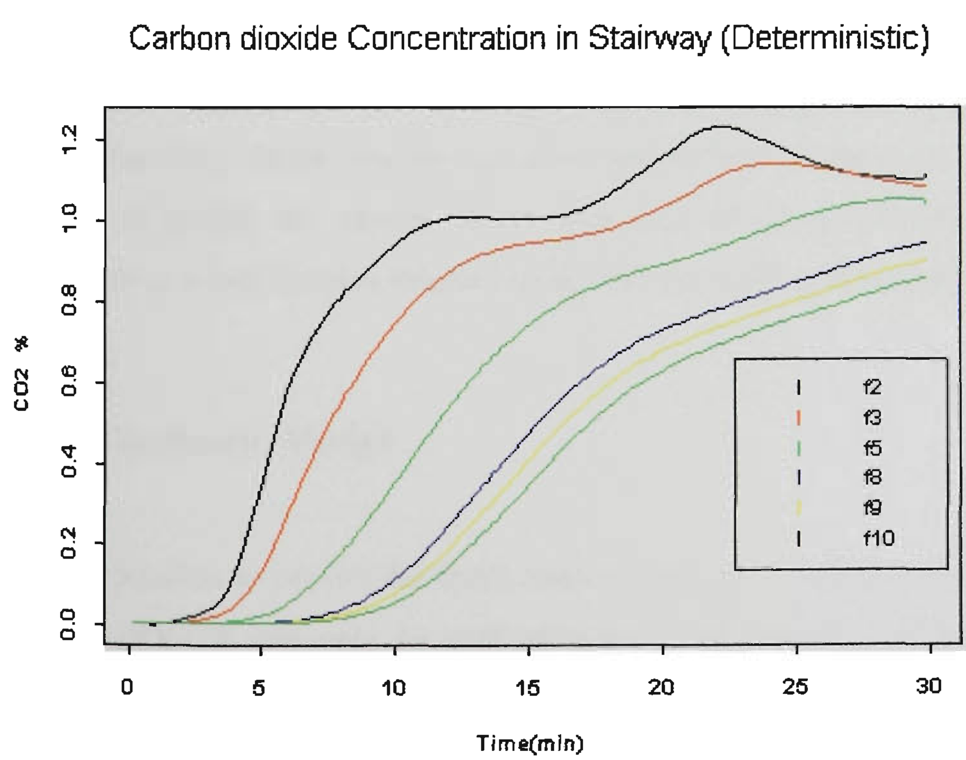


Fig 5.8 Carbon dioxide distribution for the stairway of the NRCC building.

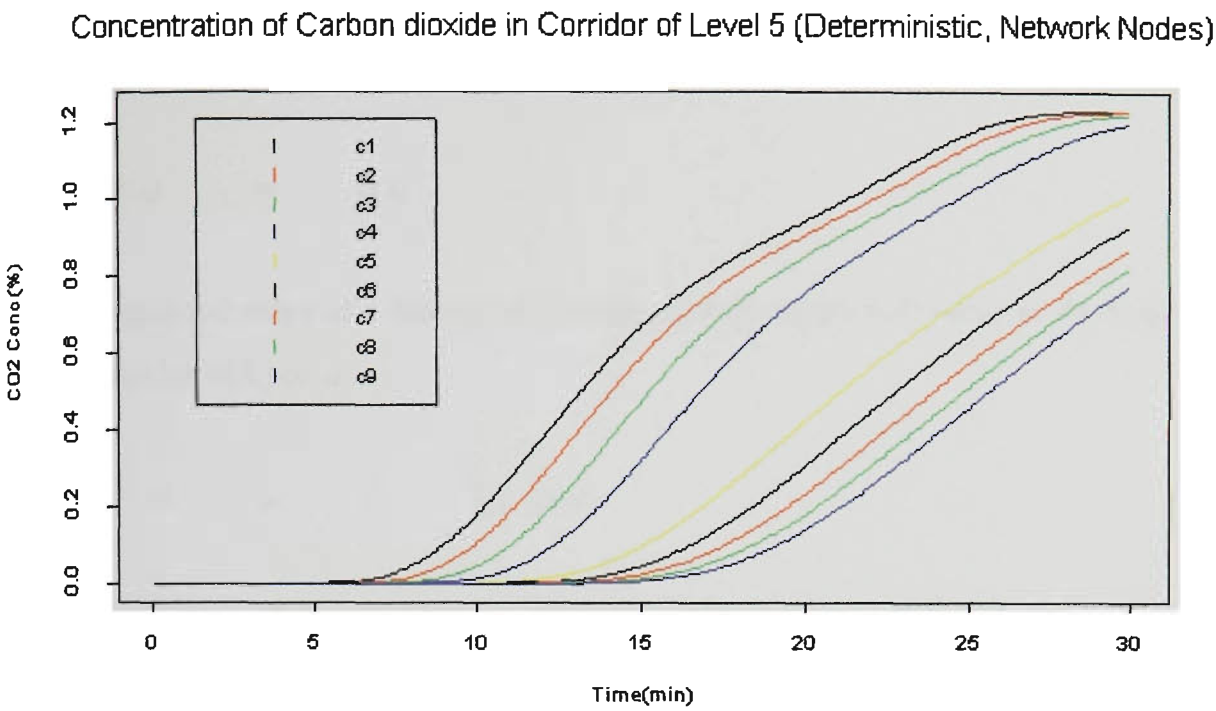


Fig 5.9 Carbon dioxide distribution for corridor on level 5 of NRCC building

The above model can be turned into a stochastic model that induces variability with the objective of incorporating some fluctuations in the process due to some unpredictable events. The amount of variation will depend on the magnitude of stochasticity induced. The stochasticity factor can be classified depending on the type of fire, stability of medium of spread, the season of the year, and on the proportion of openings. Such classification would involve detailed study and simulations not contemplated here.

5.9. Stochastic Model

It is not possible to predict the exact outcome of an event that is described by a random variable (RV). It can only be estimated by a probability. Smoke spread is a highly variable stochastic process. Though a continuous process, smoke spread can be approximated by a discrete process. Continuous processes are difficult to simulate in practice.

Probability distributions can be represented by a graph. A discrete probability distribution gives the probability of each discrete outcome occurring. The probability that a discrete RV X will take the value x is the probability of x written as

$$P(X = x) \quad = \quad p(x)$$

The expected value of a discrete RV is the average (expected) value in the long run. It is denoted by E(X) or μ .

$$E(X) \quad = \quad \mu \quad = \quad \sum x \; p(x)$$

that is the sum of the x - values multiplied by their corresponding probabilities.

The variance is given as

$$Var(x) \quad = \quad \sigma^2 \quad = \quad E(X^2) \; - \; (E(X))^2$$

$$= \quad \sum x^2 \; p(x) \; - \; (\sum x \, p(x))^2$$

5.9.1 Species Concentration

Let α be the discretising parameter that is the unit step of the stochastic jumps. If the probability of the change in the phenomenon being investigated (in this case, concentration of smoke) is β, then the probability of no increment in the concentration will be 1 - β. This can be expressed as follows:

Let

$$\frac{CA\ \alpha\sqrt{\rho\Delta p}}{m_2}(x_n - y_n) = E(\Delta y) = k$$

(The above follows from equation 5.9 of the deterministic model in section 5.5.2).

$$P_\alpha = P\{\Delta y = \alpha\} = \beta \qquad (5.18)$$

$$P_0 = P\{\Delta y = 0\} = 1 - \beta$$

$$E(\Delta y) = \alpha P_\alpha + 0P_0 = \alpha P_\alpha = \alpha\beta = k$$

Hence

$$\beta = \frac{k}{\alpha} \qquad (5.19)$$

The variance of the process is given as

$$\text{Var}(\Delta y) = E[\Delta y^2] - [E(\Delta y)]^2$$

$$= \alpha^2\beta + 0.(1-\beta) - \alpha^2\beta^2 = \alpha^2\beta(1-\beta)$$

The coefficient of variation of the process is given by

$$CV = \frac{Std.Dev(\Delta y)}{mean(\Delta y)} = \frac{\alpha\sqrt{\beta(1-\beta)}}{\alpha\beta} = \frac{\sqrt{\frac{k}{\alpha}(1-\frac{k}{\alpha})}}{\frac{k}{\alpha}}$$

$$= \sqrt{\frac{1-\frac{k}{\alpha}}{\frac{k}{\alpha}}} = \sqrt{\frac{\alpha}{k} - 1} \qquad (5.20)$$

The above expression implies that for a constant mean increment (k) the variation in the process increases with increasing values of the discretising parameter (α). The magnitude of α indicates the extent of variability in the process.

5.9.2. Temperature

Similarly the stochastic model for temperature distribution will follow the same analysis as for that for the species concentration above.

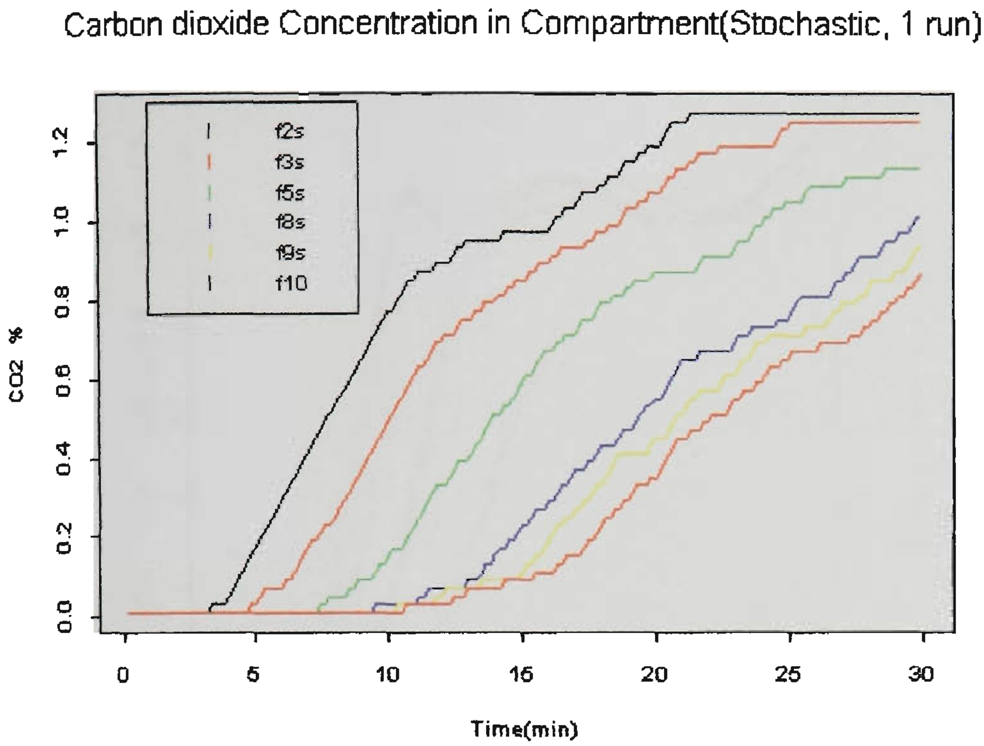
The expectation of the distribution can be derived from equation 5.32 as follows:

$$E(\Delta T) = \alpha P_\beta + 0P_0 = \alpha P_\beta = \alpha\beta = \frac{RTh\sqrt{\rho}}{PV\alpha}\left\{m_i(T_i - T) - \frac{Q}{C_p}\right\} = k$$

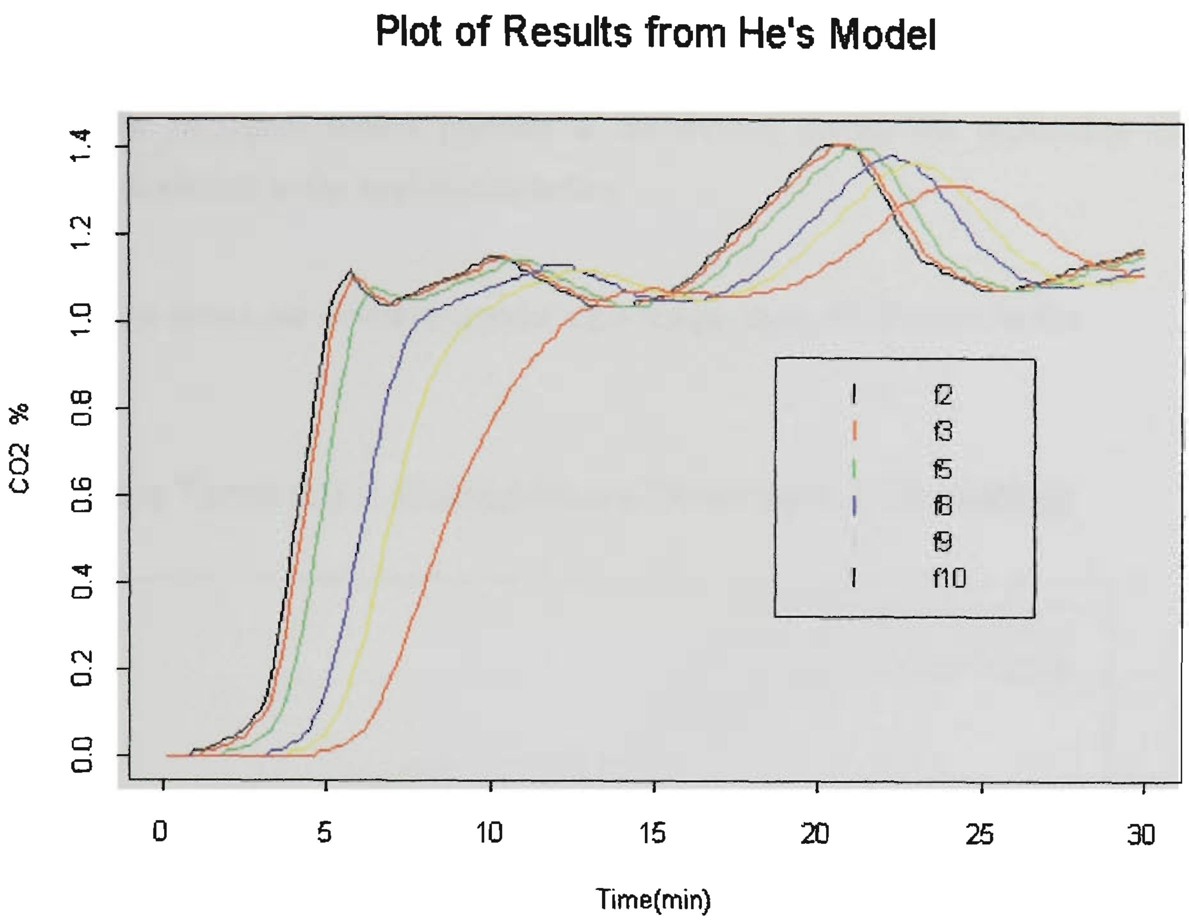
And the variance ,

$$\text{Var}(\Delta T) = \alpha^2\beta(1-\beta)$$

5.9.3 Results of Simulations for Stochastic model



(a)



(b)

Fig 5.10

(a) Plot of Carbon dioxide concentration variation with time for the respective stairway floor levels (NRCC building) for a stochastic simulation.

(b) Plot of Carbon dioxide concentration variation with time for the respective stairway floor levels (NRCC building) for a deterministic model by He and Beck [2].

Figure 5.10 (a) above is comparable to that of the deterministic in Fig 5.8. The stochastic model is able to estimate the probability of occurrence of the results by running the model a number of times. The value of α used in the above simulations was 0.03. The probability of the change, β, in the phenomenon was determined by Bernoulli trial, giving either a 0 or 1. For each trial the change may or may (1) not occur (0). A value of 1 gives a jump while a 0 does not. The simulation was performed 1000 times. The Markov chain modelling methodology is able to estimate the spread of smoke.

The figures above are the results of the stochastic model for species concentration (a) and that of the deterministic model by He and Beck [2]. Although the results are again comparable the stochastic model presents a distribution giving the probability of occurrence as illustrated in the next section below.

The figure below shows the stochastic model's gas temperature distribution for the stairway.

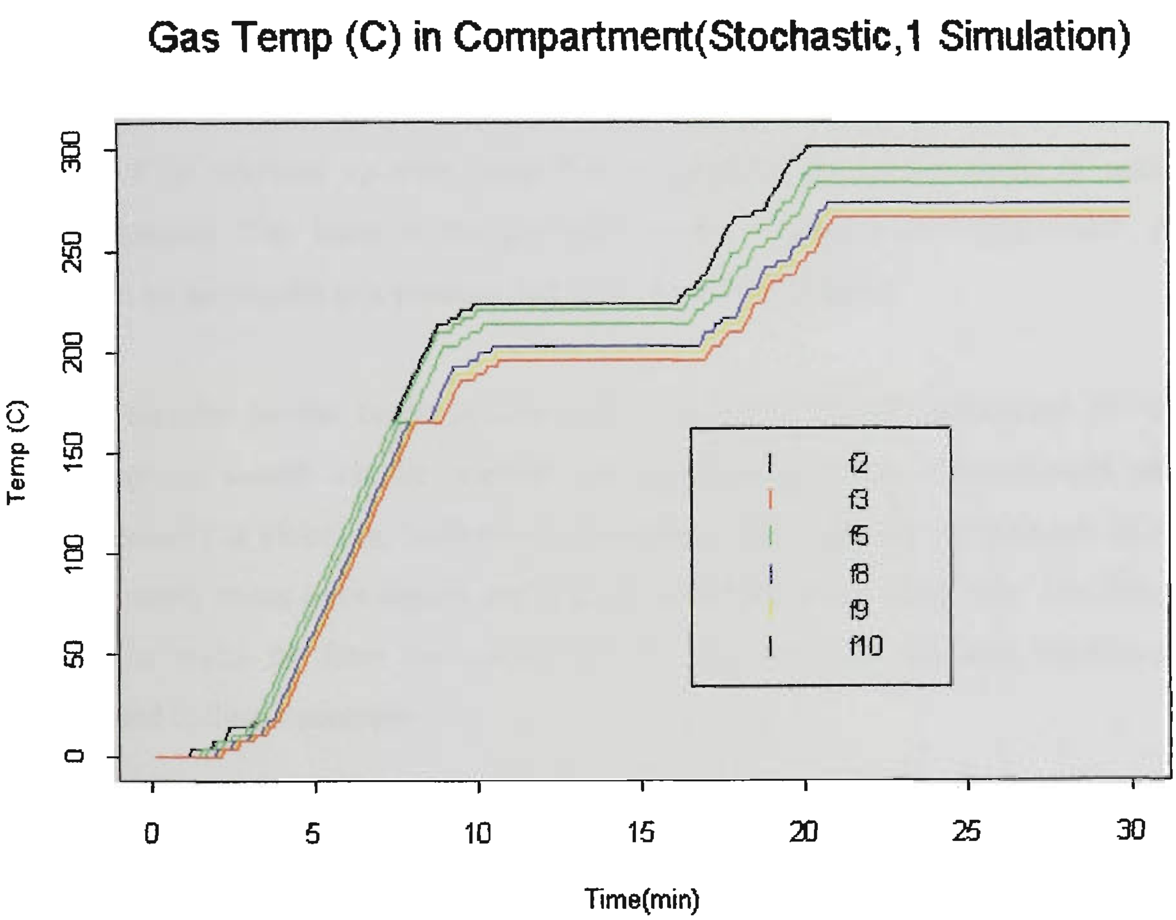


Fig 5.11 Plot of Gas Temperature variation with time for the respective stairway floor levels (NRCC) for a simulation.

5.10. Observations

The spread assumes a constant value for the viscosity of smoke. The viscosity is a function of the temperature. This variation was initially incorporated by using data input from Table A-5 at page 659 of Holman [15] for properties of air. The table shows the variation of viscosity with temperature. Simulations were performed using data from the table then using a constant value. There was no significant change in the results. Hence in the remaining simulations the viscosity was taken as constant.

The above was also the case for the Prandt number. This was assumed to be constant at 0.77. It is relevant to state here that judgement has to be made in making these assumptions. The issue is the precision versus relevance or significance. Again, we cannot be so precise in a process that lacks concise definition.

Heat transfer to the walls is assumed to a depth of 10 centimeters (4 inch). This assumption would not be realistic for the burning room. Observations at real fire experiments at Fiskville, Melbourne, shows that the boundary walls (made of cement) of the burning room have significant high temperature at the other side. But this is not the case for walls far from the source of fire. The depth of the heat transfer evaluation assumed is thus reasonable.

Below is the result of a run of the stochastic model for the species concentration in the stairway. The flow is assumed to be incompressible; the density is taken as being constant. This run can be compared later with that for a varying density; that is that for compressible flow.

The above figures represent the result from a single simulation. It is now possible to determine the condition of an occupant moving from one location to another in a bid to escape the effects of smoke.

1000 runs were made and results for a particular time of interest was obtained and analysed. The 29th minute of the process was taken and results from the runs were analysed. Analytical results for chosen floors are as presented in the table below.

Table 5.3 : Descriptive analytical result for the 29th minute for the 8^{th} floor(Carbon dioxide concentration)

Staircase level 8				
Max	**Min**	**Mean**	**Std Dev**	**Var**
1.09	0.85	0.97632	0.037447	0.001402

The above result is for the stochastic run of the model. A time unit step of 10 seconds was used. The values obtained for each staircase is representative of all possible scenario for the assumed conditions. Histograms showing the distribution of the species concentration for some of the floors in the stairway are shown in the appendix B. The choice of the number of classes ensures that not too much detail is lost and the shape of the distribution is easily seen. Although nothing could be said about the individual values the histogram does give the necessary probabilities for a range of values necessary for future estimates.

The above results in table 5.3 can be compared with that of the real life fire experiment conducted by Hokugo et al for the 29^{th} minute presented below.

	F8s
Stochastic mean	0.97632
Hukugo	0.945

The experimental mean lies within one standard deviation of the mean of the simulation.

Cumulative Relative Frequency Distribution:

Apart from knowing the minimum and maximum possible concentration at a location at a specific time after the start of a fire, it will be interesting to know the distribution between these values. The proportion of all possible observations that lie within various intervals between these extreme values could be determined.

The cumulative relative frequency of a particular class is the proportion of concentrations that fall below the upper limit of that class. From it the probability at or to the left of each point χ can be specified. For a random variable, say the concentration of smoke (C), with a given distribution, this probability is a function of χ such that

$$F(\chi) = P(C \leq \chi)$$

This is the cumulative distribution function (c.d.f) of smoke concentration C. If the distribution function of smoke concentration at a location is (say) normal then the Standard Normal c.d.f is

$$F(C) = \Phi\left[\frac{x - \mu}{\sigma}\right]$$

Where μ is the mean and σ is the standard deviation of C.

The probability that C falls in an interval can be obtained from the graph, that is

$$P(a < C \leq b) = F(b) - F(a) \qquad (5.48)$$

The cumulative relative frequency distribution graph (ogive) of the smoke concentration for the 29th minute after the start of fire for the staircase on the third floor of the building concerned is presented below.

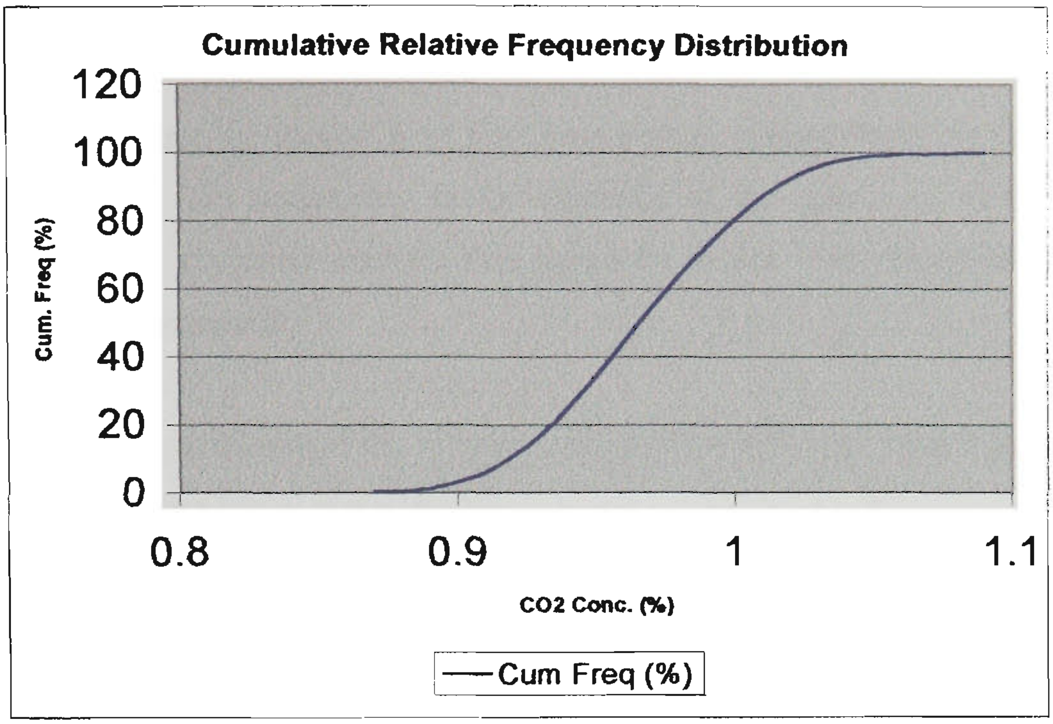


Fig 5.12 Cumulative Relative Frequency Distribution for the Carbon dioxide Concentrations for the 29th minute after the start of Fire on the eighth level of the Stairway.

The same result can be done for any other signature of the fire that can be approximated by similar mode of spread, Markov chain. Gases like hydrogen cyanide (HCN) and carbon monoxide (CO) could be of interest. The combined effect of the gases can then be evaluated. The distribution of the time to effective incapacitating dosage (EID) can be obtained for an occupant moving from say the tenth to the third floor.

5.11 Conclusion

In this chapter, the spread of smoke in a building was simulated using a stochastic process, the Markov chain. The spread of smoke was approximated by a discrete Markov chain with very small time step unit. The extent of accuracy depends on the fineness of the time unit step that is stipulated by the modeller.

The above illustrates the feasibility of the Markov chain methodology in modelling the spread of smoke. The uncertainty in the estimates of the signatures of fire can be addressed by this stochastic method. The signatures of fire were presented with their probabilities of occurrence.

The extent of the usefulness of the Markov chain method will need further investigation to determine its exact useful predictability.

CHAPTER 6

STOCHASTIC SPREAD OVER A NETWORK

CONTENTS

FIGURES

TABLES

6.1 Introduction

The previous chapters have been concerned with modelling the spread of smoke within and between compartments in a building. From the cumulative probability function (ogive) diagram obtained from the simulations concentrations at various locations in a building can now be determined for given probabilities. Most experimental work on fire has been for compartment fires. The cost factor does not allow experiments involving the spread of fire from one compartment to another in a building. But with enough information about the times for fire to break boundary elements it is possible to investigate the spread of fire without experimentation. Boundary elements that could be broken down in fire include doors, windows, walls and openings. It is possible to obtain data of the time at which such elements collapse in say flashover fires. For a given compartment with different boundary elements, the possible times for fire to spread to the neighboring compartment will be a collection of fire resistance times for these elements. This chapter investigates the spread of a phenomenon from one vertex to the other in a network. Although the methodology is extensible to other phenomena it is relevant to the spread of fire and smoke. The presentation herein is first given in general terms before specific illustrations are given.

The investigation involves comparing different approaches to calculating the time of spread of a phenomenon over a network [16]. A network consists of inter-linked vertices. To an edge between vertices corresponds a random variable that describes the time of spread. The vertices can be compartments or areas of defined boundaries. These definitions depend on the type of network considered. For the purpose of this investigation vertices are assumed to be rooms or compartments in a building. References to nodes, vertices, rooms or compartments identify bounded confinements.

Duration of time of spread will be the time taken for the phenomenon to occur in the vertex and to break through the confinement or boundaries in spreading to the next vertex. In essence, for a network of rooms in a building, it would be the duration in time for the phenomenon (say fire) to reach a sufficient severity to break through the boundary that requires the least effort. In spread of fire, the time to spread is a random variable whose distribution will depend among others on the compartment internal fuel load and the state of the boundary.

The stochastic method of modelling a network is applicable to most phenomena that spread. This method can be extended to the spread of smoke. The time allocations for the edges will be that required for the smoke to enter, fill up to the soffit of the door before entering the next and nearest available vertex if there is an opening. Alternatively it could be the time taken to attain a particular dosage (eg. Incapacitating Dosage). Definitions of the above terms are given in the respective paragraphs where they are used.

6.2 Network Representation

It is well known that, for the purpose of investigating fire spread, a building can be represented by a graph. See e.g. Ling and Williamson [12] and the full discussion and literature review in Ramachandran [51]. A vertex represents each compartment, and vertices are connected by an edge if there is between the two corresponding compartments a direct path through which the fire can spread, such as a fire-rated barrier, a door or a floor, or if the fire can spread through windows. Since there are situations where the fire can spread only in one direction, (e.g. if it is assumed that the fire will not spread from a level to the level below) each edge is given a direction, so that it has a beginning and an end. Thus the type of graph to be considered is a directed graph or "digraph". If the fire can actually spread in both directions (albeit not necessarily at the same speed) we introduce two directed edges. For an elementary discussion of graph theory see Wilson [52]. We denote the number of vertices by V and number them from 1 to V.

Suppose there are K edges and that we number them from 1 to K. To describe the time-dependent fire spread we assign to each directed edge a random variable representing the time taken by the fire to spread from the compartment at the start of the edge to the compartment at the other end from the time the fire started in the compartment represented by the start of the edge. Let us denote the random variable assigned to edge number r, which links vertex i to vertex j, by T_r. It is important to note that there is usually a non-zero probability p_r that the fire will not spread at all e.g. if it is contained by fire resistant barriers until it burns out. This can however be dealt with by allowing the random variable T_r to be "defective" See section 6.4.

It should be noted that the random variable T_r is a global expression of a number of successive phenomena:

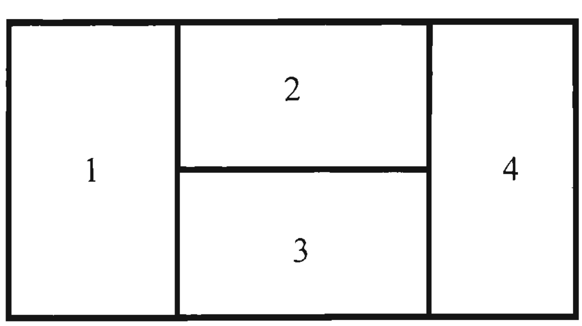
1. The growth of the fire in compartment i.

2. The transmission of heat from compartment i to compartment j, including the thermal and structural damage to the barrier between compartments i and j.

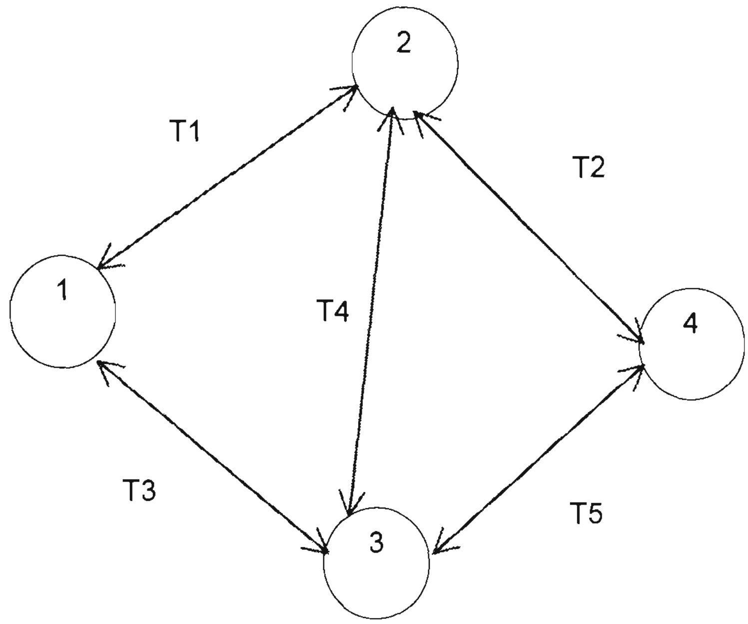
3. The eventual spread of fire from compartment i to compartment j.

4. The cessation of the fire in compartment i.

As an example, we shall consider a floor of a building consisting of four compartments numbered one to four. The fire is assumed to spread in the same way in both directions of the edges. So the five edges are numbered from one to five and are shown in figure 6.1.



(a)



(b)

FIG 6.1 (A) PLAN OF FLOOR WITH ROOM NUMBERS
(B) NETWORK OF FLOOR WITH ROOM NUMBERS AND EDGE VARIABLES.

This floor configuration was studied as early as 1981 by Elms and Buchanan [52] and more recently by Pratt, Elms and Buchanan [55].

The information provided by the network can be encoded in a computer program in the form of an n x n matrix as follows: The diagonal elements are set to zero and the (i, j)th entry is the number of the directed edge starting at vertex i and ending at vertex j. For example, the matrix corresponding to the network in Figure 6.1 is

$$\begin{pmatrix} 0 & 1 & 3 & 0 \\ 1 & 0 & 4 & 2 \\ 3 & 4 & 0 & 5 \\ 0 & 2 & 5 & 0 \end{pmatrix}$$

6.3. THE SPEED OF FIRE SPREAD

We are interested in the time taken by the fire to spread from a "source", i.e. the compartment of fire origin, to some target compartment. This time, which we will denote by U, is itself a (possibly defective) random variable. We can write down a formula for U in terms of the component variables T_r as follows: We first need to define a "directed path". This is simply a sequence of edges such that the beginning of the first edge is the source, the end of the last edge is the target, and the end of each edge is on the same vertex as the beginning of the next edge. Suppose that there are K directed paths in the digraph from the source to the target. Let P_i be the set of indices of the edges constituting the i-th path. Then the time V_i taken by the fire to spread from the source to the target along the i-th path will be given by

$$V_i = \sum_{j \in P_i} T_j \tag{6.1}$$

and the random variable U will be given by

$$U = \min_i V_i \tag{6.2}$$

In this general formulation the random variables T_r need not be independent. However, for simplicity, we shall assume in the rest of this thesis that they are independent.

Even with the independence assumption, there is no simple method for calculating the probability distribution of U in terms of the probability distribution of the T_r unless there is only one path or all paths contain only one edge. This is because it will be necessary to obtain the joint distribution function of the V_i, which are not independent in general since the different paths may have common edges.

6.4 WAITING TIMES

In dealing with time-dependent phenomena it is often required to introduce a random waiting time T between the occurrence of one event and the occurrence of a second event. Two aspects of waiting times that distinguish them from the usual random variables are:

1. they are non-negative,

2. the second event may not occur at all. It is possible to deal with such a circumstance by introducing the concept of 'defective' random variable (See Feller [52]). It is defined as a random variable which, with probability 1 – p > 0, say, does not assume a finite numerical value. For our purpose, we can assign to T the symbol ∞. For such a random variable, we have

$$\lim_{x \to \infty} P(W \leq x) \quad = \qquad p \tag{6.3}$$

with $p < 1$.

6.5 THE DISCRETE APPROXIMATION

One method that will considerably reduce the computational load is to assume that the time is discrete, taking values at discrete time points, and that they only take a small number of values each. The case when the variable T_r is defective with probability p_r can be taken care of within the framework just outlined by attributing to T_r with probability p_r an extra value M larger than the sum of all the other actual values taken by the spread times. The paths which include the variable T_r will be automatically rejected by formula (6.2) when T_r takes the value M as long as there are shorter paths available.

The choice of the increment size on which the discrete time is based will of course depend on the amount of fine detail required. Clearly the smaller the increment the more computing will be required and the more unstable the calculations may turn out to be, because at each step very small probabilities will be involved. Thus, as with all methods involving discretization, a balance must be struck between fine detail, cost and stability of the computation.

Assuming that T_r takes n_r different values, the total number of points in the sample space of the random variables T_r (r = 1,...,n) will be equal to

$$\prod_{r=1}^{n} n_r \qquad (6.4)$$

6.5.1 The Analytical Approach

With the discretization assumption in place, it is possible to calculate for each point in the sample space its probability and the corresponding value of U, using formula (6.1) and (6.2). The discrete probability function of U can then be obtained by adding the probabilities of all the sample space points corresponding to each value taken by U.

Some shortcuts are available to reduce somewhat the computational burden. One such shortcut is described in Ling and Williamson [12]. It consists of two steps. Suppose as above that the variable T_r is defective and takes n_r finite values. The first step in the shortcut calculation is to replace the edge r by n_r parallel edges. On each of these edges the spread time takes only two values: one of the finite values and infinity. The spread times of the parallel edges are assumed independent. The resulting network is known as "Mirchandani's equivalent network". The second step consists in enumerating all paths in the modified network and then calculating the probability distribution of U through the use of some recursive relations.

6.5.2 Monte Carlo simulation of the fire spread.

Unfortunately, as soon as the number of values taken by the edge variables and the number of edges increase beyond small numbers, the full analytical calculation of the probability function of the random variable U, even using shortcuts, becomes unmanageable. Recourse must then be had to approximate methods. The simplest, but still very effective, method is a Monte Carlo simulation. First we generate a sample of suitable size N from the random vector $T = (T_1, ..., T_k)$. Then for each element of the sample we calculate the corresponding value of U, using equations (6.1) and (6.2). Finally we calculate the probability function of the sample of U-values. The values $p_u = P(U = u)$ obtained are unbiased estimators of the true values and standard Monte Carlo simulation theory will provide confidence intervals for the true values of the p_u.

6.6 Hazard Function.

6.6 THE HAZARD FUNCTION APPROACH

Even though the approaches described in the previous sections do take account explicitly of the time, they nevertheless suffer from a severe defect: the random variable variables T_r ; ($r = 1,...,k$), or at least their probability functions, must be given in advance. However, when fire spread is studied, we are usually interested in the interaction between the fire spread and the fire fighting, which can include such items as operation of sprinklers, closing of fire doors, exhaust fans activation and stair pressurization as well as the actions of the fire brigade. All these events affect the probability distributions of T_r . Moreover, we are also interested in the occupant egress, for which we need to know the status of the fire and smoke spread at each instant of time. The above approaches do not readily lend themselves to such uses.

For these reasons, a new approach is hereby proposed to calculate the spread of fire in a network, based on the use of discrete Hazard Functions. A comparison of the three methods will be given in section 6.7.2.

6.6.1 Continuous version

The Hazard Function is a standard tool of probabilistic reliability assessment. It is mostly used in its continuous form: Let X be a continuous, non-negative random variable with cumulative distribution function $F(x)$ and density function $f(x)$. Then its Hazard Function $h(x)$ is defined [38, 39] by

$$h(x) = \frac{f(x)}{1-F(x)} = \frac{\text{Probability density function}}{\text{Survivor function}} \qquad (6.5)$$

as long as $1 - F(x) > 0$ and is equal to 0 otherwise. $F(x)$ is the Cumulative Distribution Function (CDF) of x. The hazard function can be regarded as the instantaneous failure rate or the conditional density of failure at time x, given that the unit has survived until time x. The denominator of the equation is known as the survivor function, used mainly in biomedical applications. The hazard function can either be said to be increasing or decreasing depending on whether the unit is deteriorating or improving with age. For units that do not depend on age the exponential distribution is used as it gives a constant hazard function (no-memory property): the reciprocal of the mean time to failure.

The Cumulative Hazard Function $H(x)$ is defined by

$$H(x) = \int_0^x h(y)dy \qquad (6.6)$$

Conversely

$$F(x) = 1 - e^{-H(x)} \qquad (6.7)$$

and

$$f(x) = h(x)e^{-H(x)} \qquad (6.8)$$

For a function $h(x)$ to qualify as a hazard function it is necessary and sufficient that $h(x) \geq 0$ for all x. For the random variable X not to be defective, it is necessary and sufficient to have

$$\int_0^\infty h(x)\, dx = +\infty \qquad (6.9)$$

The hazard function can be used in simulating the spread of a phenomenon in a network. Rather than considering the failure rate, the hazard function can be used to evaluate the instantaneous success rate of using the edge between vertices at the time of consideration.

Given the hazard function of a process the cumulative density function (CDF) is defined and vice-versa.

6.6.2 *Discrete version*

The system operation can either be intermittent or continuous. The discrete Hazard function will be applicable to an operation that is intermittent or approximates a continuous operation.

Let X be discrete with probability function $P(X = n) = p_n$, (n = 0,1,2,...). Let $q_0 = 1$ and for n > 0.

$$q_n = P(X \geq n) = 1 - \sum_{r=0}^{n-1} p_r \qquad (6.10)$$

Then the discrete hazard function h_n, is defined by

$$h_n = \frac{p_n}{q_n}, \quad (n = 0,1,2,\ldots) \qquad (6.11)$$

as long as $q_n > 0$ and is equal to 1 otherwise. Conversely, q_n and p_n can be recovered from h_n by means of the equation $q_0 = 1$ and, for $n > 0$,

$$p_n = P(X = n) = h_n \left[\prod_{r=0}^{n-1} (1 - h_r) \right] \qquad (6.12)$$

$$q_n = P(X \geq n) = \prod_{r=0}^{n-1} (1 - h_r) \qquad (6.13)$$

For a sequence h_n, ($n = 0,1,2,\ldots$) to qualify as a hazard function it is necessary and sufficient that $0 \leq h_n \leq 1$ for all n.

For the random variable X not to be defective, it is necessary and sufficient to have

$$\sum_{0}^{\infty} h_n = \infty \qquad (6.14)$$

In some parts of this chapter, h_n will be written for convenience $h(n)$.

6.6.3 Modifying the hazard function.

In view of the fact that the only restriction on the h_n is

$$0 \leq h_n \leq 1 \qquad (6.15)$$

it is clear that we can modify any term of the hazard function, say h_n as long as the above condition remains satisfied.

6.6.4 Examples of discrete Hazard functions

Example 1. Let $p_n = (1 - a)a^n$, $n = 0,1,2,...$ (the "geometric" distribution). Then $q_n = a^n$ so that $h_n = 1 - a$, i.e. the hazard is constant for all n. This is the discrete version of the "lack of memory" property of the exponential distribution.

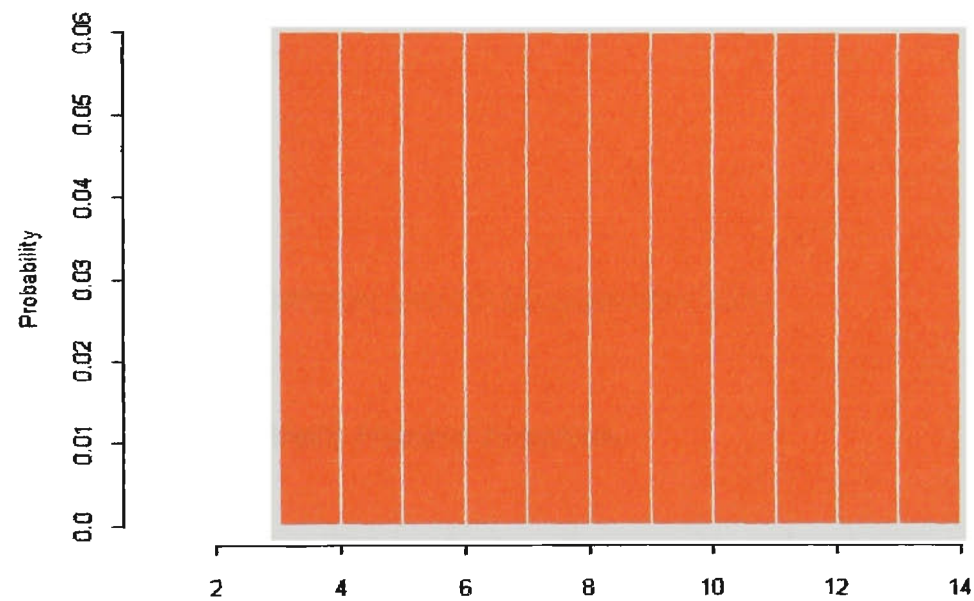
Example 2. Let a, b be two positive integers and let x be a positive number such that $(b + 1)\,x \leq 1$. Let as before X be a discrete random variable with probability function $P(X = n) = p_n$, $(n = 0, 1, 2, ...)$. Let $p_n = x$ for $a \leq n \leq a + b$ and zero elsewhere. In other words, we take X to be uniformly distributed over the integers from a to $a + b$. It is then easy to check that $h_n = 0$ for $n < a$ and that

$$h_n = \frac{x}{1 - (n - a)x} \qquad (6.16)$$

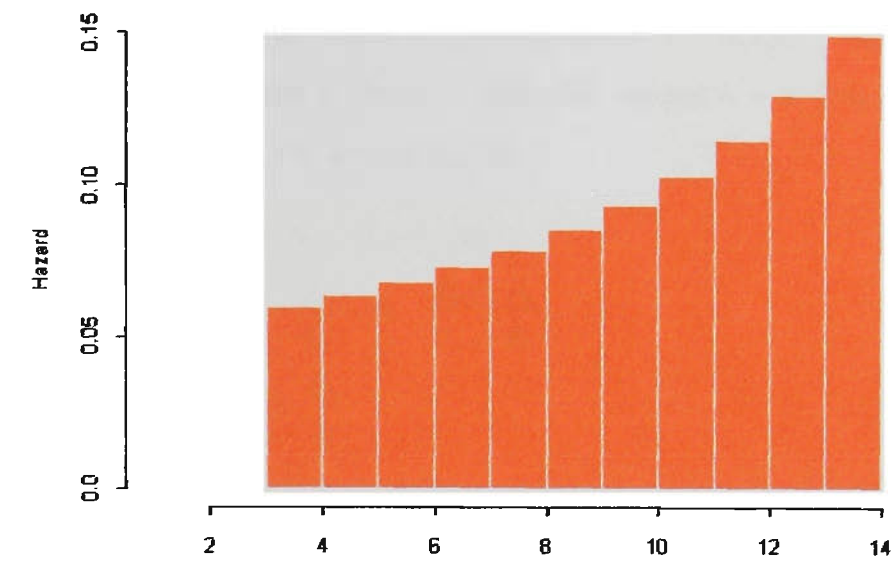
For $a \leq n \leq a + b$.

For X not to be defective we must have $x = 1/(b + 1)$. In that case, $h_n = 1$, for $n > a + b$. Otherwise, $h_n = 0$ for $n > a + b$.

Figure 6.2 below shows the shapes of the probability function and the hazard function for $a = 3$, $b = 10$ and $x = 0.06$. Here the probability of the finite values of X is 11 x 0.06 = 0.66 and the probability that X is equal to infinity, i.e. that the event never occurs, is 0.34.



(a)



(b)

FIGURE 6.2: (A) PROBABILITY FUNCTION AND (B) HAZARD FUNCTION.

6.6.5 Simulating with a discrete Hazard function.

Suppose that we want to simulate a discrete waiting time X with probability function p_n, $n = 0,1,2,\ldots$. The standard method is to draw a random number x between 0 and 1 and to find n such that $q_n \geq x > q_{n+1}$, where, as before, $q_n = P(X \geq n)$. Then $X = n$. If $\lim_{n \to \infty} q_n = q_\infty > 0$ then if $x \leq q_\infty$ we put $X = \infty$, i.e. the second event never occurs.

This approach, however, is correct only if there have been no disturbances to the process that generates the waiting time X during the n intervals of time. But if we want to be able to modify the properties of the remaining waiting time at any point of time n_1, say, the method just presented becomes useless.

To deal with such situations, we propose the following algorithm: At time 0, we perform a Bernoulli trial with probability of success h_0. In case of success, we put $X =$

0. Otherwise, we perform a second Bernoulli trial, this time with probability of success h_I and in case of success we put $X = 1$. In general, we continue to perform successive Bernoulli trials with probabilities of success h_i, $I = 0,1,2,\ldots$ until the first success. If that first success occurs at the $(n + 1)th$ trial, we put $X = n$. It is easy to see that the probability function of X is indeed p_n, for

$$P(X=n) = (1-h_0)(1-h_1)\ldots(1-h_{n-1})h_n \quad (6.17)$$

$$= (1-\frac{p_0}{q_0})(1-\frac{p_1}{q_1})\ldots(1-\frac{p_{n-1}}{q_{n-1}})\frac{p_n}{q_n} \quad (6.18)$$

$$= \frac{q_1}{q_0}.\frac{q_2}{q_1}\ldots\frac{q_n}{q_{n-1}}.\frac{p_n}{q_n} \quad (6.19)$$

$$= p_n \quad (6.20)$$

While the proposed algorithm is being performed, we have the capability of appropriately modifying the values of the subsequent h_i at any time, as long as the first Bernoulli success has not yet occurred, to take into account any disturbances affecting the process which generates the waiting time, as discussed in Section 6.4.

Of course, if after some number N the hazard function vanishes, indicating that X is defective, and if until N no success has been encountered, we put $X = \infty$, i.e. the second event will never occur, unless the subsequent terms of the hazard function are modified.

Assuming that the total time taken to get to a compartment i is t_o. To obtain the time of the next compartment j, the transit time T_r for the edge r is added to t_o where r is the edge under consideration. The transit time T_r is a random variable that can assume a varying number of values. To determine the value to be assumed by T_r in a particular simulation, the hazard function is used. This is illustrated diagrammatically below.

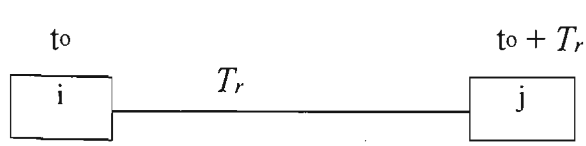


FIG 6.3 VERTEX-EDGE REPRESENTATION

The duration of the phenomenon is determined arbitrarily but greater or equal to the latest time for which at least one of the hazard functions of the edges adjacent to the compartment is still positive.

Given the hazard function, the probabilities of taking a particular value in a set of possible values that can be taken by T_r between two vertices can be recovered.

6.7 A NUMERICAL EXAMPLE

As a numerical example consider the floor with four compartments shown in Figure 6.1. The five random variables T_1, T_2, T_3, T_4, T_5, assumed independent, each take four values with equal probabilities. The values are shown in Table 6.1.

Table 6.1 Values of variables T_1 to T_5

T_1	5	7	9	10
T_2	11	12	15	17
T_3	18	19	20	22
T_4	6	7	8	9
T_5	9	10	11	12

6.7.1 Estimating Values of T_r.

One of the main issues in the network analysis is the determination of the possible values of the random variable (T_r). This is a requirement in the analysis. The duration of the phenomenon in the compartments is foreknowledge or information input in the analysis. The duration of fire in a compartment before it spreads to a neighbouring one is dependent on many factors. There are many models that can predict the fire growth in compartments. Also available from these models are the temperatures and times for fire to break through boundary elements. In building design and construction, it is usual to have the boundary elements in compartments specified, such as one-hour fire resistant door. Other sources of information in estimating the values of T_r include results of real full-scale experimental data.

As there is not sufficient real life data available some values will be assumed for T_r for the analysis. It is expected that with the availability of enough real experimental data, the same procedure can be used to evaluate the spread of fire in a building.

6.7.2 Comparing the Three Methods

In this illustration we focus on calculating the distribution of time of spread of fire from compartment 1 to compartment 4, denoted by U.

There are $4^5 = 1024$ points (i.e realisations) in the sample space, so that the full analytical calculation is not unmanageable. The calculation was carried out and a Monte Carlo simulation as well as a Hazard Function simulation were carried out as well. The simulation was performed 1000 times.

It turned out that the only values taken by U with non-zero probability were {16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, and 27}. A comparison of the probability functions as well as the mean and the standard deviation for U yielded by the Analytical, the Monte Carlo and the Hazard function methods is given in Table 6.2.

Table 6.2 Comparison of the three methods

t	P(U = t)		
	Analytical	Monte Carlo	Hazard Functions
16	0.063	0.035	0.060
17	0.063	0.061	0.054
18	0.063	0.053	0.051
19	0.063	0.098	0.067
20	0.128	0.101	0.139
21	0.133	0.169	0.140
22	0.179	0.195	0.181
23	0.008	0.006	0.009
24	0.116	0.145	0.129
25	0.074	0.056	0.069

26	0.059	0.058	0.062
27	0.051	0.023	0.039
Mean	21.43	21.42	21.46
S.D.	3.01	2.71	2.91

Here, since there are no defective random variables, the probabilities add up to one. Finally, the full information about the spread of the fire in one realization is given in Figure 6.4. The fire is assumed to have already started in vertex one. Subsequent times for fire to spread to the other vertices are shown by the beginning of the blocks; the size of which indicates the duration of the fire.

The advantage of the Hazard function over the Monte Carlo lies in its flexibility. The Hazard function can be modified to reflect any other external influence such as wind, fire brigade, etc. See the conclusion in section 6.8.

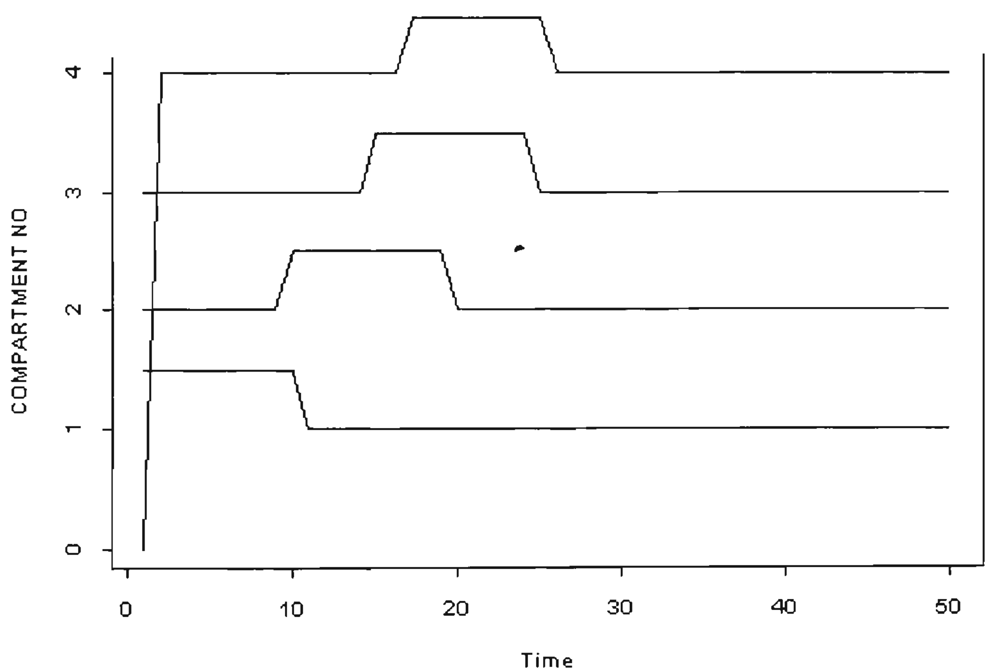


FIG 6.4 GRAPHICAL REPRESENTATION OF A REALIZATION OF SPREAD

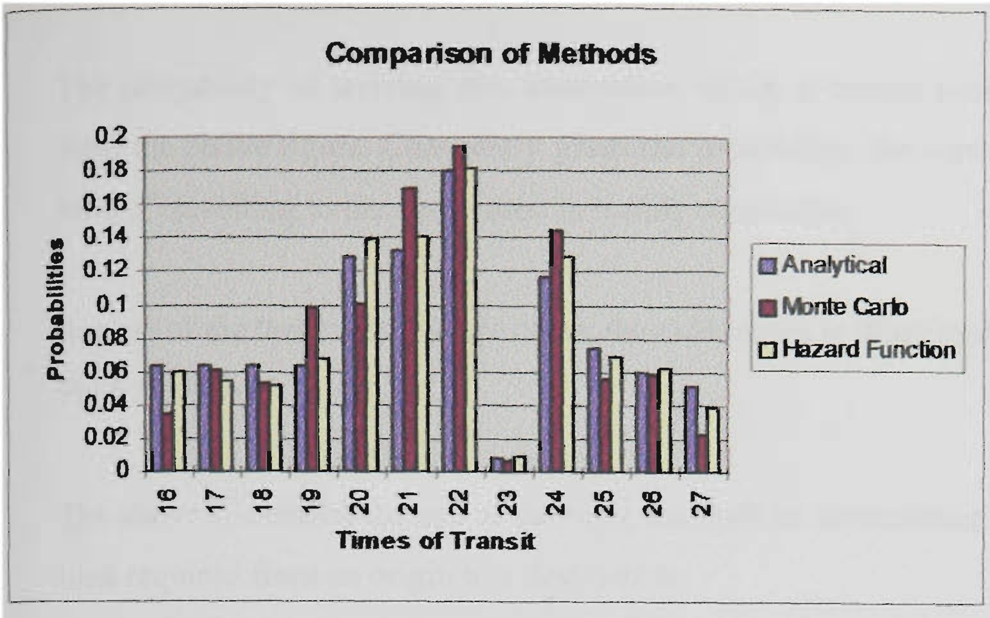


FIG 6.5 HISTOGRAM OF TIMES OF TRANSITING THE NETWORK FOR THE METHODS.

The above shows that the spread using the Hazard function is a possible means of simulating the phenomenon.

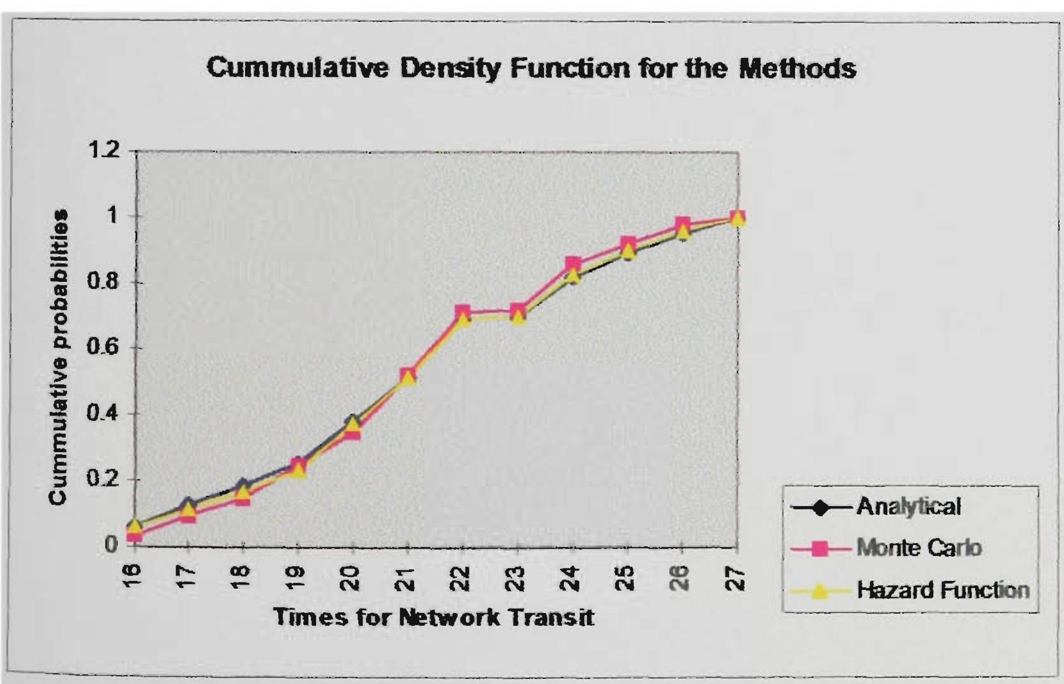


FIG 6.6 THE OGIVE DIAGRAM FOR THE METHODS.

The probability of arriving at a destination within a certain time is easily available from the above figure. Conversely given the probability, the corresponding minimum time of travelling to the destination is readily obtainable.

Results of the three methods are close; the differences is illustrated in the histogram of Fig 6.5 above.

The above illustrated the use of the three methods in determining the least or shortest time required from an origin to a destination.

The Figure 6.7 below is a flow chart using the hazard function in analyzing the network. The source of fire for the respective vertex is obtainable.

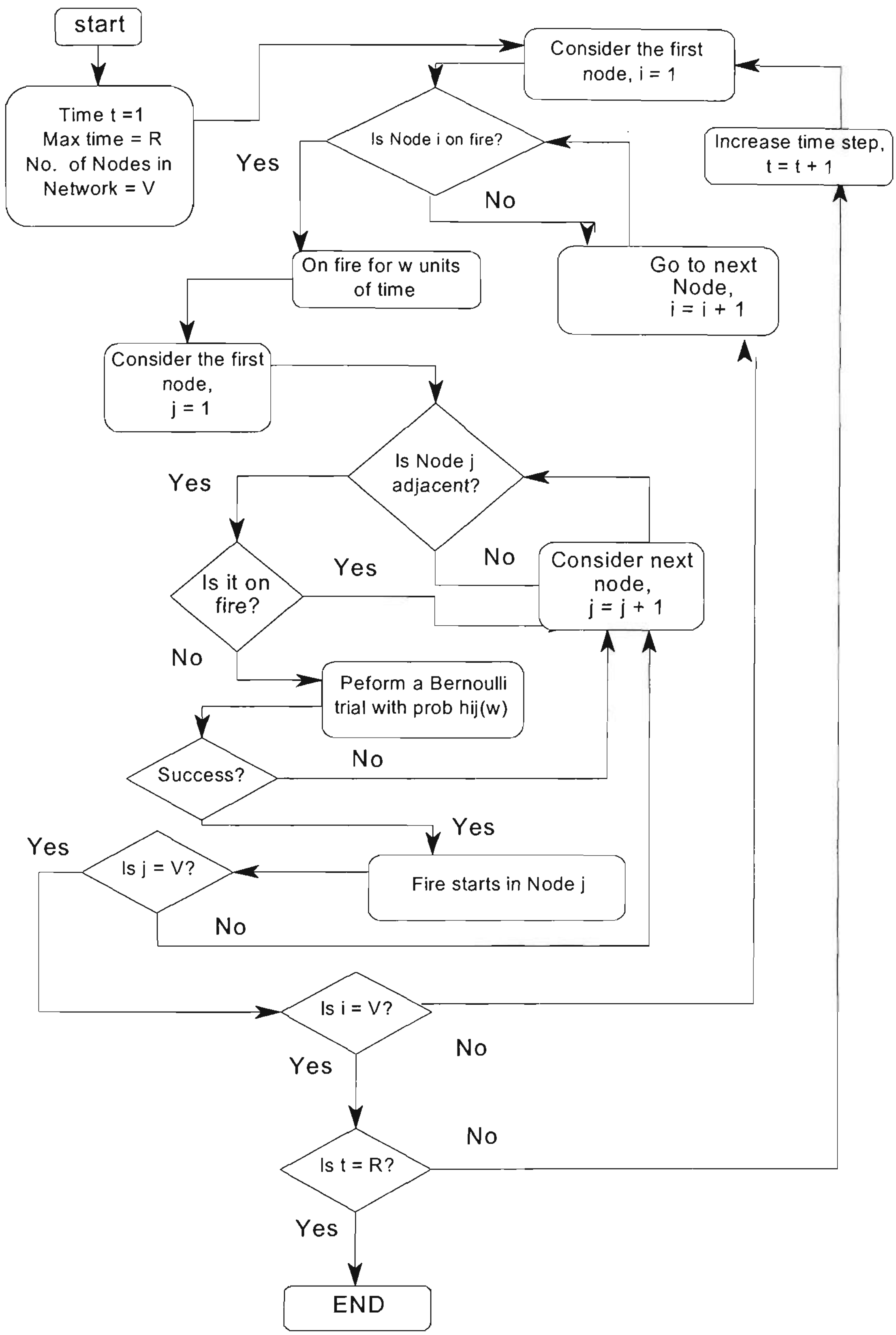


FIG 6.7 HAZARD FUNCTION NETWORK SIMULATION

The illustration is for one realization. To obtain the probability distribution function of the times to transit from an origin to a destination the process is simulated a large number of times.

A typical source matrix of a realization is shown below. It is indicated that vertex two caught fire from vertex one, vertex three from two, and vertex four from two.

Source Matrix:

	VERTICES
	1 2 3 4
SOURCE	1 2 2

In the next section one of the advantages of using the hazard function over Monte Carlo is illustrated. The convenience of being able to incorporate the effects of other events is shown. Strategically, the hazard function is modified to accommodate the effects of other events. The hazard function approach is shown to possess a flexibility that the other methods do not have.

The pseudo code and SPLUS program for the simulation are presented in Appendix C.

6.7.3 Example of Incorporating Other Events

The ultimate desire in analyzing the spread in a network of a phenomenon is to be able to evaluate the effect of external events on the spread. For fire and smoke spread, the external events may include the following:

- Fire Brigade Intervention
- Wind Velocity
- Smoke Exhaust Systems
- Pressurization and HVAC Systems
- Sprinklers

These systems could be activated during smoke spread so that the expected paths taken in spreading is altered. Some paths could have any or a combination of these systems influencing the time taken to spread from vertex to vertex. To incorporate these effects the hazard function expression is adjusted by a modifier. For example, hazard function given on equation (6.5) can be written as

$$h(n)=\frac{xf}{1-(n-a)x} \quad for \ a \leq n \leq a+b \tag{6.21}$$

where f is the hazard adjustment factor, all other parameters remain as previously defined.

An adjustment factor f is used to modify the function such that different result are obtained for various weights of the factor. f takes various values commensurate with the anticipated influence of the event(s) being incorporated. Consider an edge in the network. Suppose that the random variable T_r can take up any of four possible values for the edge. If some (say two) of these values are results of some external influence that can accelerate or decelerate the spread of fire on the edge. Assuming that, for one possibility, the spread of fire is impeded by a barrier hence reducing the spread rate to half ($f = 0.5$). The other possible values of T_r are assumed to be unaffected. Then the adjustment required for simulating the spread is as shown in the third column of Table 6.3 below.

The probabilities p_n can be recovered using equation 6.11 as previously discussed.

Table 6.3 Recovering Probabilities from the Hazard Function on the same Edge (using equation 6.11 above) with other events incorporated.

$b = 3,\ a = 1,\ n = 1,..,4,\ x = 0.25$							
	Event	*f*	*h(n)*	*h(r)*	*1-h(r)*	$\prod_{r=0}^{n-1}[1-h(r)]$	P_n
1	*Barrier*	*0.50*	0.266667	0	1	1	0.266667
2	*Nil*	*1.00*	0.333333	0.266667	0.733333	0.733333	0.244444
3	*Nil*	*1.00*	0.5	0.333333	0.666667	0.488889	0.244444
4	*Nil*	*1.00*	1	0.5	0.5	0.244444	0.244444

where

h(n) = Hazard value of current vertex.
h(r) = Hazard value of previous vertex ($r = n - 1$).

In the above Table 6.3, column three is calculated using equation 6.20 (the adjusted hazard function); column four is defined above, column six from equation 6.11, and column seven from equation 6.12.

A similar graph representation as in Fig 6.4 is presented below. The graph shows the spread of fire after incorporating other events as explained above. The fire is assumed to have already started in vertex one. Subsequent times for fire to spread to the other vertices are shown by the beginning of the blocks; the size of which indicates the duration of the fire

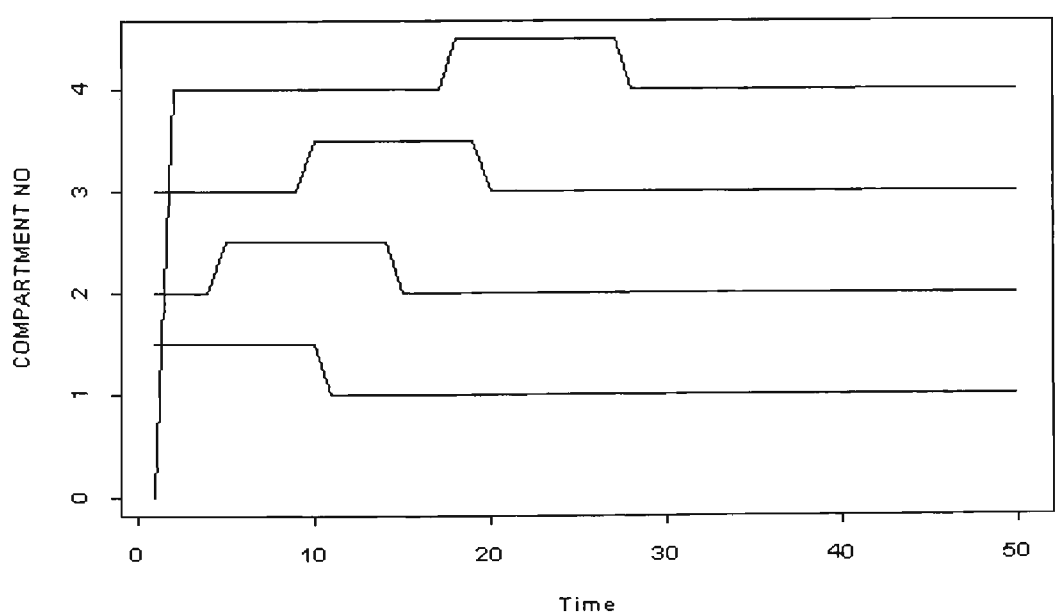


FIG 6.8 A REALIZATION WITH OTHER EVENTS INCORPORATED

Source Matrix:

VERTICES

1 2 3 4

SOURCE 1 2 3

The source matrix above indicates that vertex one is the origin of fire, vertex two caught fire from vertex one, vertex three from vertex two and vertex four from vertex three. This result is different from the earlier case where influence of external events on the spread was not considered. The difference is shown in the two figures.

Another interesting aspect is the time taken to simulate the spread for both cases. For the latter case, the influence of the barrier increased the time. This increased the time of simulation significantly.

6.7.4 Relevance of the Hazard Function Approach

The hazard function has been used as a means of simulating the spread of a phenomenon in a network. Some of the advantages have been highlighted in some sections above. The hazard function can be used for many other types of analysis, for instance, in production lines, the minimum-cost flow algorithms in financial analysis, etc [39, pp30-45]. The relevance of this approach should be assessed in its possible applications.

Some of the possible areas of application of this approach in fire safety will include the following:

1 Recommending the desired performance of building designs for approval.
2 Determining the fastest strategy of evacuating occupants in a building.
3 Fire management and damage control.
4 Risk undertaking (Insurance Policies).
5 Sensitivity analysis of the effects of external events on the spread phenomenon.
6 Management of danger and determination of its effect.

The approach is a versatile tool for analyzing flow in a network.

6.8 CONCLUSION

The numerical example just given highlights the superiority of the Hazard Function method over the other two methods presented. The analytic method is only feasible when the number of compartments is small. The Monte Carlo method is efficient but will not yield a step by step history of the fire spread that can be used for interaction with fire fighting and evacuation. Stochastic models for the latter are being developed at present and when integrated with the fire spread model presented in this paper will provide a means for calculating

(a) the probability distribution of the number of deaths in the fire,

(b) the probability distribution of the building damage.

These two measures will form the foundation for probability-based design of the fire safety and protection in the building.

6.9 Summary

In this chapter, three approaches to the calculation of the time of fire spread over the network were discussed and compared: the Analytical approach, the (direct) Monte Carlo approach and the Hazard Function approach.

Hazard function will be more adequate when other events that affect the spread are to be incorporated. It can predict the following:

1. History of phenomenon (fire spread)
2. Probability of any particular scenario or group of scenarios, a priori probabilities are not required.
3. Effect of other events (e.g. fire fighting measures) if their effect on the hazard function can be prescribed.

CHAPTER 7

METROPOLIS FIRE SPREAD FOR MULTIPLE HIT SCENARIO

CONTENTS

FIGURES

TABLES

7.1 INTRODUCTION

The objective of this investigation is to evaluate the time available from time of ignition to when the destination becomes inaccessible. This requirement is crucial in planning evacuations and other hazard management.

Fire could result from one or a combination of the following:

1. Natural events(e.g. Lightning)
2. Arson or
3. Other Human Activities (Conflicts, etc).

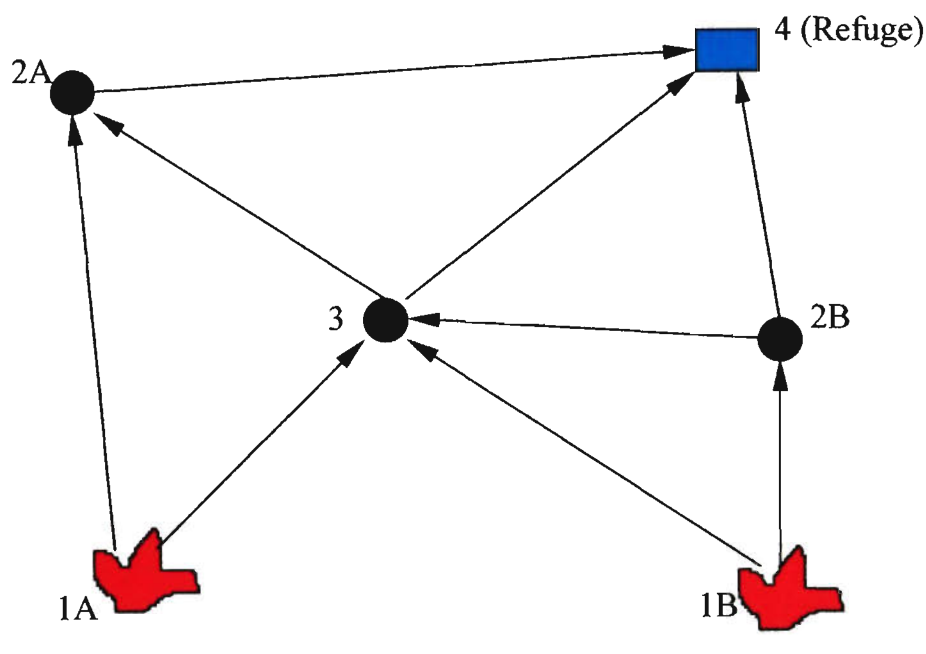
Strategic planning will include area mapping and classification of the above causes. Usually the levels of classification are Low, Low Medium, High Medium, High and Very High. Areas classified as having High and Very High probabilities of being hit can be identified as origins. Classification is a requirement in identifying possible origins of fire. A typical process of classification of an area (city) is well presented in the document of Fire Hazard Mapping: Shire of Eltham, Victoria, Australia [40]. The document illustrates classification of vegetable or grasslands. The same methodology can be adopted in classifying building lines or Central Business Districts (CBD). More than one area can be the origin for multiple and or simultaneous hits or ignition. Destinations can be any of the following:

1. Refuge Centers
2. Shelters
3. Locations of interest.

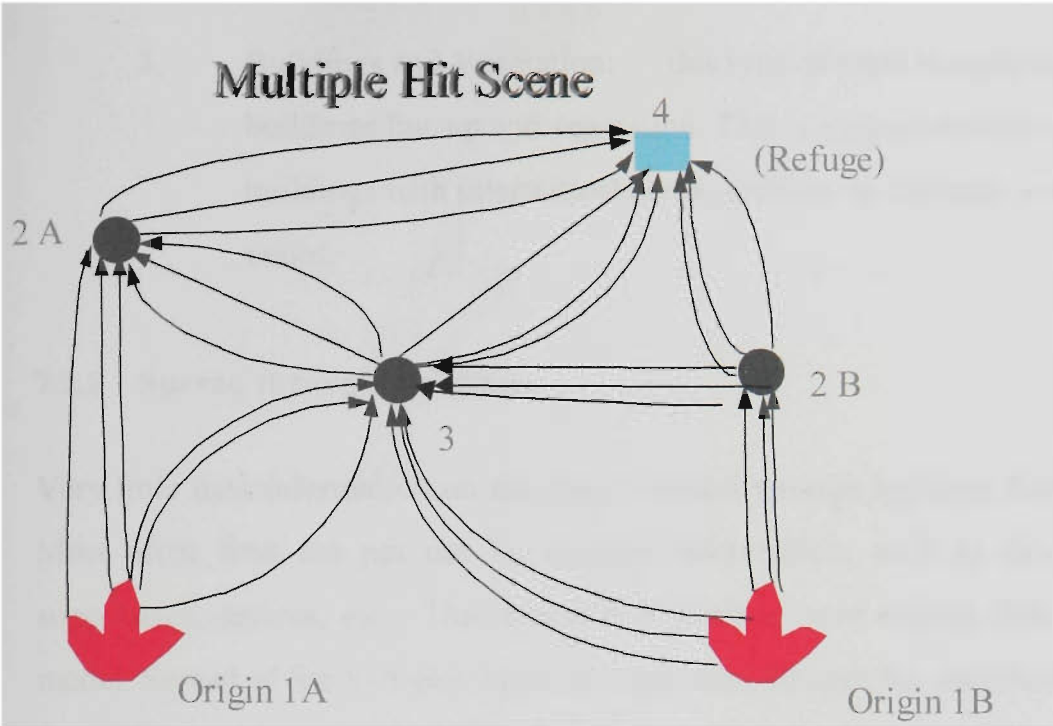
The State of Victoria, Australia, has a fire hazard rating scale of five levels ranging from Low to Very High [40].

7.2 NETWORK CONCEPTUALISATION

A metropolitan city is considered to be made up of a network of nodes (locations of interest) and edges (spread passages). The nodes are connected by edges. An edge starts from a node and ends in the other node. Edges can be directed or undirected. A directed edge has its tail on the originating node and the head on the designating node; or it can have both ends as heads. The first is uni-directional while the latter is dual-directional. For simplicity, the edges will be considered in this investigation as uni-directional. Then flow can only be in one direction. An edge has a random variable representing the time of spreading from a node to another. Graphically, each edge can have a number of links representing the possible values of the random variable. A path is a sequence of edges from the origin node (the beginning of the spread) to some other node. The above represents the graphical conception of the network. This is illustrated in the figures below.



(a)



(b)

Fig 7.1 Graphical illustration of network: (a) Directed Edges (b) Directed Edges (made up of links).

Each link represents one value of the random variable.

Although graphs can be used to represent networks, it is difficult to analyze them using the computer. It is more efficient to represent a network by a matrix. Nodes are numbered from 1 to V. In the $V \times V$ network matrix, entry (i,j) is zero if nodes i and j are not connected. If they are connected, the entry is a number that refers to the random variable which represents the time for the fire to spread from node i to node j. The network matrix can be used as an input to the stochastic analysis described and illustrated later in this chapter.

7.3 APPLICATION TO METROPOLIS FIRE SPREAD

Route conditions include the following:

1. Line of Buildings: this route consists of continuous blocks of buildings from one node to the next.
2. Vegetation: route referred to as vegetation are parks, gardens, orchards, farms, etc, in between nodes.

3. Buildings and Vegetation: this type of route is made up of part buildings line up and vegetation. This is a representation of blocks of buildings with intermittent parks, gardens, or orchards in between nodes.

7.3.1 Spread through Buildings:

Very little data information on the rate of spread through building lines is available. Most often fires are put out by external intervention, such as fire brigade, fire suppression devices, etc... This makes it difficult to have enough data to satisfy the model. Spread of fire to higher floors is most often through the outside of the building [41]. The fire spreads from the incident floor through a window to higher floors due to effects of buoyancy. Spread from a compartment to another occurs when a boundary collapses. Instances of such failures can be where windows are broken, doors collapse, walls fail, etc... Times of fire to break through windows in the incident floor and to break through the window of upper floors, are available. These are experimental data for fires in compartments conducted at the Victoria University of Technology fire experimental facility [40]. Observation of the data indicates that the time to spread is approximately equal to the fire resistance of the boundary element with the least resistance. Compartment fires that spread tend to reach flashover before this time [40]. Using the fire resistance of the element with the least resistance is reasonable in approximating time to spread. These were used to estimate the time to spread from compartment to compartment. Times to failure of these boundary elements are obtainable from compartment fires [42, 43, 44]. Time to spread through a building will depend on the paths available for spread. For a path, it is the summation of the times of fire resistance of the least resistant boundary elements. The more information available about the configuration of the buildings and the boundary elements the better the estimates with other factors considered. Fire resistance ratings for most boundary elements are obtainable from literature [43, 44]. Table 7.5 in section 7.6 shows the calculation of the time to spread through a building given the layout, usage, and type of construction. Though this is a stochastic phenomenon we will assume the average for simplicity.

7.3.1.1 Spread between Buildings:

Large scale spread of fire in a community is not common due to the general adoption of regulations and strategies to contain fire within a building. Hence there is not much data of real life fire spread in a community.

The most common methods of containment of fire within a building are:

- fire-resistant building components
- spatial separation, and
- firefighting

Fire spread from one building to another separated from it by a vacant space may be due to one or more of the following:

- flying brands,
- convective heat transfer, and
- radiative heat transfer.

Flying brand may initiate secondary fire at substantial distances from the primary fire, e.g., 0.5 km. Convective heat transfer will cause ignition only if the temperature of the gas stream is several hundred degrees Celsius [40a]. The radiation levels of an exposed building must be kept below a hazardous level (generally taken as 12.5 kW/m^2) [40a]. This is achieved by spatial separation and by minimizing the total area of windows that could emit radiation from a burning building. McGuire J.H., Williams-Leir G. [40a] states that "Once a fire had reached very large proportions involving say, a whole city block, the level of radiation issuing from it was so great that materials 100 m (300 ft) away could be ignited. At this stage, containment of the fire might no longer be possible". Spread between buildings depends largely on the spatial separation of the buildings, type of materials used for the exteriors, wind, and the extent of external intervention like fire brigade.

7.3.2 Spread Through Vegetation

The spread of fire through land is affected by any or a combination of the following [40]:

1. Length of fire season
2. Predominant aspect of Slope
3. Steepness of Slope
4. Vegetation-Ground cover
5. Vegetation-Annual driest state.

The Fire Danger Index (FDI) is the classification of fire based on the factors that influence its rate of spread [40, 45]. These factors are the maximum temperature, humidity, wind direction and wind speed. The more the number of factors incorporated the better the FDI.

McArthur [46] gave data for the fire forward speed for different fire danger index for various fuel loads. The following figure illustrates the forward speed of fire for these conditions. From this type of graph, the forward speed of fire through vegetation can be obtained.

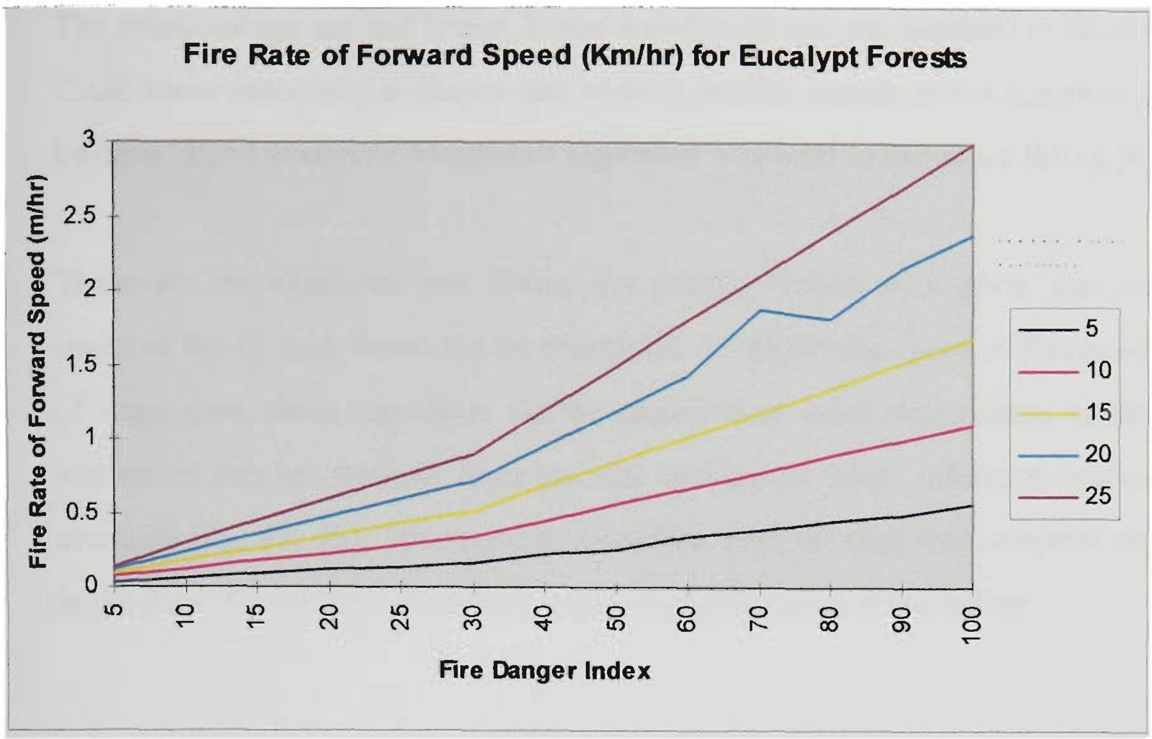


Fig 7.2 Fire Rate of Forward Speed (Km/hr) for Eucalypt Forests for different fuel quantities.

The above figure is the plot of data given by McArthur [46]. The relationships between the variables that affect the speed of fire as derived from the above figure are as follows:

Table 7.1 Expressions for Fire Forward Speed for various Fuel Loads

Fuel Load(t/ha)	Speed Equations
5	$Y = (2.1078 \times 10^{-6})x^2 + 0.00531x + 0.00736$
10	$Y = (2.3818 \times 10^{-6})x^2 + 0.01068x + 0.01914$
15	$Y = (-1.7469 \times 10^{-6})x^2 + 0.01692x + 0.00798$
20	$Y = (-0.00002 \times 10^{-6})x^2 + 0.02612x + 0.02945$
25	$Y = (-1.5616 \times 10^{-6})x^2 + 0.03019x + 0.00489$

LEGEND:

Y = Forward Speed

x = Fire Danger Index

The relationships are not linear, hence suitable curves are required to describe them.

The relationships are not linear, hence suitable curves are required to describe them. Close observance of the shapes can identify known graphs and a transformation can be done. The Levenberg-Marquardt algorithm was used in the curve fitting [47].

These are the equations that define the graphs. Hence, with given parameters, the speed of fire in such forest can be calculated. To obtain the speed of fire in other types of vegetation, these equations can be modified or more data related to the type of vegetation can be obtained from real fire incidences. More information about this is available [40, 45, 46]. The above presentation does not deal with inherent uncertainty in the data. This will be dealt with in the application described below.

7.3.3 Spread Through Vegetation and Buildings:

The spread of fire through edges that are made up of both building lines and vegetation will be affected by factors determining the spread for the other two previously discussed above (See [40, 45, 46] for details of the effect of these factors). Fires through vegetation can spread to buildings by one of four ways: (1) burning debris, (2) radiant heat, (3) direct flame contact, and (4) wind. Burning debris is acknowledged as the most common [48]. This is more so if it is windy.

7.4 ANALYSIS

A network can be made of one or more of the categories identified above. The categories have been listed in paragraph one of section 7.3. For instance, it can represent the spread through contiguous building lines. In such instance, possible values assumed by the random variable can be affected by any of the conditions for spread through buildings discussed in section 7.3.1. For each of the categories, the time taken from an origin to a destination along the i – th path is represented as in equations 6.1 and 6.2 in chapter 6.

The two equations are used to compute the probability and the value of *U*.

Starting from a map of a city the procedure required to start the fire spread analysis is shown in the figure below.

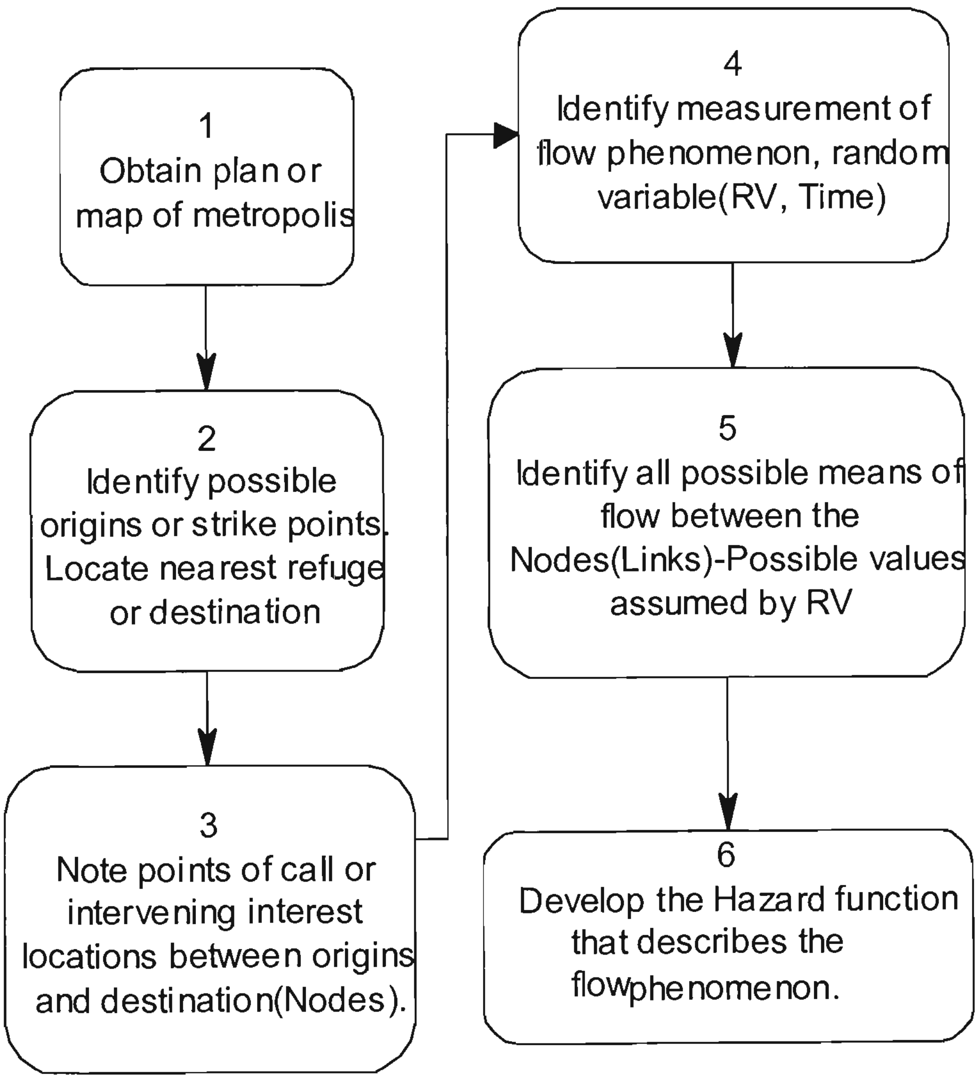


Fig 7.3 Analytical Chart

7.4.1 Hazard Function.

The hazard function used for the simulation is as presented in the previous chapter. The duration of the fire is determined arbitrarily but greater or equal to the latest time for which at least one of the hazard functions of the edges adjacent to the node is still positive. Given the hazard function, the probabilities of taking a route in a group of routes between two nodes can be recovered as before.

7.5 DATA SOURCING

Real life fire input data is scarce, especially information on the rate of spread in various conditions.

Data for fire experiments on entire buildings are not readily available. In order to make reasonable estimates of the times of spread of fire from one building to another, available data for compartment fires were used. These were extrapolated based on stated assumptions. Data for compartmental fire experiments for various conditions, that indicate the times for the glazed windows to break and spread to adjacent compartments were used based on the following conditions [44]:

The fuel loads in the other compartments in the building are assumed to have the same spatial distribution as that of the origin. This assumption is reasonable if all the sections in the building are subjected to the same usage with same furnishing. Estimation of fuel load for the other compartments is assumed to be proportional to their size (area). For a typical building, the time for fire to spread from one end to another can then be estimated; that being the accumulation of times along a route through the building. To obtain the time to spread through a line of buildings, it is assumed that the proximity of buildings to one another is within the spotting distance of the fire given the fire conditions. Spotting distance is that for wind or other mechanism to carry ignited fragments.

The CSIRO Australia has compiled data on fire incidences in Australia [50]. The percentage of fires that spread beyond the structure of origin for various property uses can be computed as presented below (See Table 12 of the document).

Table 7.2 Percent of fires that spread beyond the structure of origin for various property use in Australia.

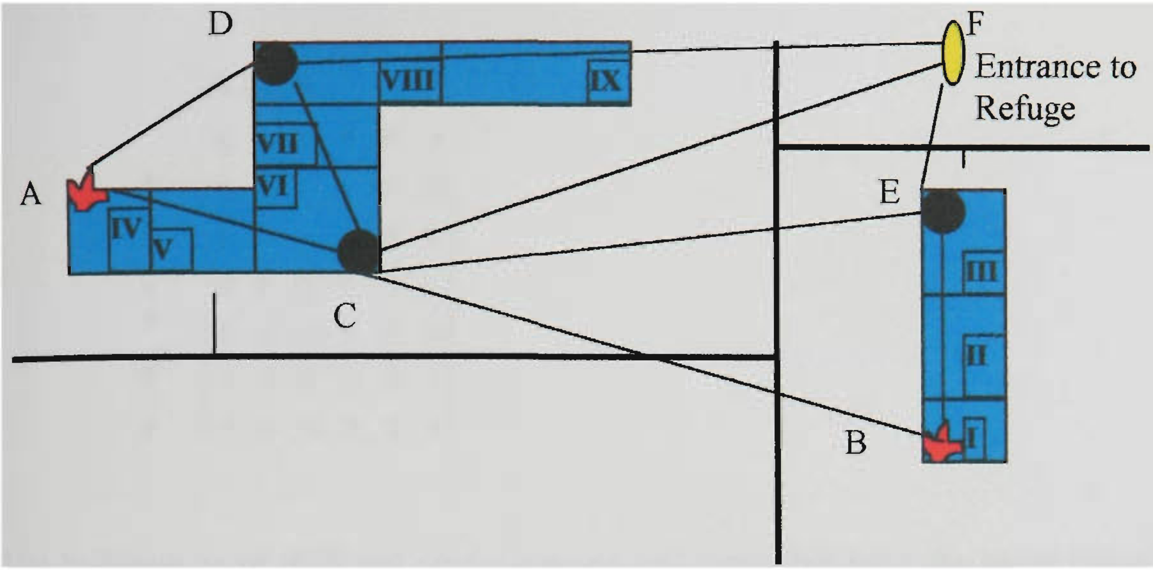
EXTENDED FIRE DAMAGE BEYOND STRUCTURE OF ORIGIN

	Public Assem Prop	Educat-ional Prop	Instit-utional Prop	Resid-ential Prop	Shop, Office Prop	Basic Indust Prop	Manu-facturing prop	Storage Prop	Special Prop	Unclass-ified
No.	11	7	0	146	27	5	17	55	34	1
Total	647	306	571	8007	1465	149	833	912	793	103
%age	1.7	2.3	0	1.8	1.8	3.4	2.0	6.0	4.3	1.0

The above table shows the proportion of fires that extended beyond the structure of origin. For most of the fires there were external interventions, such as fire brigade. Hence the percentages are low. Although this indicates low probabilities it is known that fire spread can be accelerated by chemicals and wind. The probabilities are expected to be higher during events like war; when all fire management facilities can be exhausted. The most important consideration in metropolitan fire spread is the potential of spread in such crises and the effect that that will have on access to refuge centers.

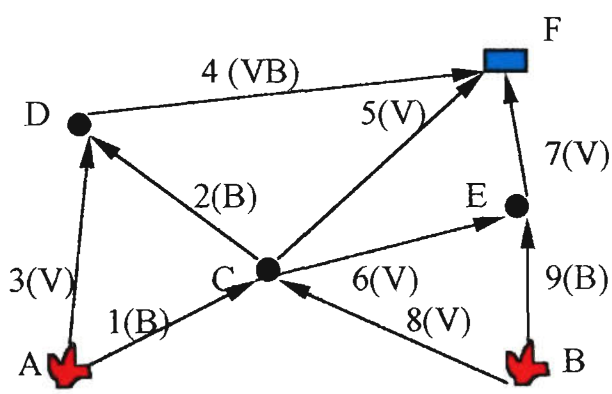
7.6 AN APPLICATION

We will consider part of a community for illustration. A plan of the area is shown in the figure below. All area apart from the buildings and refuge center are vegetation (Eucalypt trees). The widths of the roads are far less than the fire spotting distance, hence roads will not impede spread in this instance. The building lines are made up of contiguous buildings. Locations of interest are points A to F. A and B are fire origins. F is a refuge center. Other locations are indicated by black circles.



LEGEND: I-IX = Building blocks

Fig 7.5 Multiple Hit Scene: Contiguous building lines and vegetation



LEGEND:

B = Edge made up of building line

V = Edge made up of vegetation

VB = Edge made up of building line and vegetation.

1-9 are the edges in the network.

Fig 7.6 Network representation of the scene under consideration.

The network diagram in fig 7.6 is derived from the plan. The network can be represented by an $n \times n$ matrix with the diagonal elements set to zero. The (i,j)th entry is the number of the directed edge starting at vertex i and ending at vertex j. The matrix corresponding to the network is as follows:

NODES

	A	B	C	D	E	F
A	0	0	1	3	0	0
B	0	0	8	0	9	0
C	0	0	0	2	6	5
D	0	0	0	0	0	4
E	0	0	0	0	0	7
F	0	0	0	0	0	0

The buildings have different configurations and sizes, but have the same usage, all being residential. In this illustration, the buildings are all bungalows. The same fuel load is assumed for all since they have the same usage. Below is a demonstration of how the time to spread through one of the buildings (I) was obtained. The layout is given. The respective compartments are designated with lowercase alphabets (a to i). All doors are opened and windows are closed. Doors are made of 1/4-inch plywood while the windows are glazed. Times to spread through them are obtainable [44, 46]. Time to spread through doors is assumed to be 80 percent of the fire resistance.

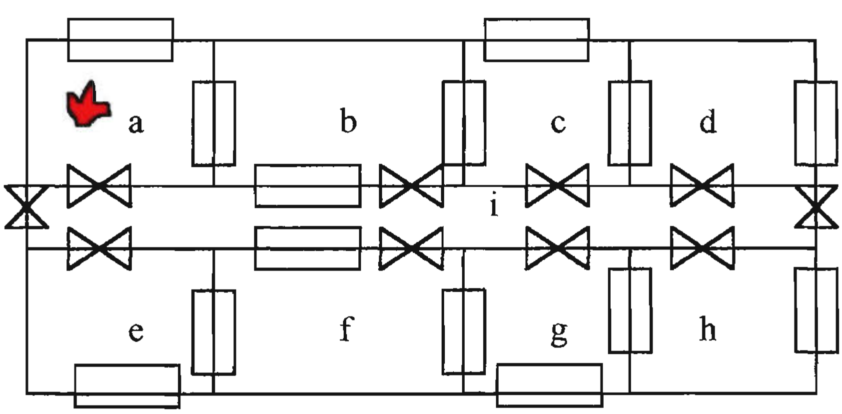


Fig 7.7 Layout of building (I) in the network.

Table 7.3 Spread times for the building (I).

	a	b	c	d	e	f	g	h
Area	13.36	19.88	13.26	12.05	13.36	19.88	13.26	12.05
Time(min)	7.92	11.79	7.86	7.14	7.92	11.79	7.86	7.14

Spread through the corridor (i) is evaluated in segments corresponding to the length of the adjacent rooms; ranging from (ia) to (id). Fire can spread from the segments into the corresponding rooms by the carpets through the usual slight opening under the doors. Fuel load in the corridor is only the carpet flooring. Observation of experimental fires by the author at Fiskville shows that the possibility of spread of fire by only the carpet is small. This may be due to inadequate fuel load, fire resistance of carpets, and for many other reasons. Only the first two carpet segments close to the source of fire (ia and ib) will be considered as capable of spreading fire. Their proximity enables exposure to higher heat flux and flammability. Using the above table the possible paths in the building network and the corresponding time of transit is given in the table below.

Table 7.4 Time to transit paths in building (I).

Paths	Time, T_b, to Transit (min)
a-b-c-d	34.71
a-ia-e-f-g-h	44.63
a-ia-ib-b-c-d	48.71
a-b-ib-f-g-h	48.45
Mean Time to Transit Building (I).	44.14

Forward speed through the carpet ≅ 0.5m/min.

Above table shows all possible values that can be assumed by the random variable T_b for fire to transverse the building. Having determined the time for building I, those for other buildings can be approximated by extrapolation; that is by assuming that the time of transit is proportional to the area. Extrapolation is the proportionate extension of a value based on precedence.

Table 7.5 Times to Transit Buildings in the Network.

Building	Area (m2) (Length x breadth)	Times to transit (min)
I	9x15 = 135	35,45,48,49
II	9x25 = 225	58,75,80,82
III	9x24 = 216	56,72,77,78
IV	10x12 = 120	31,40,43,44
V	10x30 = 300	78,100,107,109
VI	15x35 = 525	136,175,187,191
VII	10x20 = 200	52,67,71,73
VIII	7x50 = 350	91,117,124,127
IX	7x55 = 385	100,128,137,140

The above table presents the computation of the times of transit for the respective buildings based on the assumption of the same usage as for building I whose computation has been illustrated above.

The table below presents the computed values of the times to transit the route condition of vegetation only. This is further illustrated in the next paragraph.

Table 7.6 Possible times to transit edges with vegetation component.

Edge	Distance of vegetation	Times to transit
3	68.0	23,35,120
4	98.0	17,33,50

5	180.0	31,61,92
6	158.0	27,54,81
7	45.0	8,15,23
8	203.0	35,69,104

Times to transverse vegetation are given by dividing the distance between nodes by the fire speed, which depends on the fuel load. Fuel load of the areas covered by vegetation in this case ranges from 5 to 15 ton/hectare; the FDI for the period being evaluated is 30. The corresponding forward speeds are 0.17, 0.34 and 0.51 m/hr (See fig 7.2 above). Times to transverse an edge that has both building and vegetation will be the addition of the respective components.

$$T_r = T_b + T_v \tag{7.1}$$

where

T_r = Time to transverse the edge *r*.

T_b = Time to transverse the building component of the edge.

T_v = Time to transverse the vegetation component of the edge.

Using tables 7.5 and 7.6 above the times to transit the edges are presented below. It represents the possible times for the respective edges in the network.

Table 7.7 Possible times to transit edges in the network (min).

Edges	Building Line	Vegetation	Times to transit
1	245,278,311,344	N/A	245,278,311,344
2	279,316,354,391	N/A	279,316,354,391
3	N/A	23,72,120	23,35,120,500
4	191,216,242,267	17,34,50	208,250,292,500
5	N/A	31,62,92	31,62,92,500

6	N/A	27,54,81	27,54,81,500
7	N/A	8,16,23	8,16,23,500
8	N/A	35,70,104	35,70,104,500
9	149,192,205,209	N/A	149,192,205,209

LEGEND: N/A - Not Applicable. Impossible value of 500 given to complete values of edges that are incomplete.

Consider the first row of the table above. Column one is the number of the edge. Column two gives the transit times for the four possible routes through the building line. Edge one (1) in Fig 6.6 is from vertex A to C in Fig 6.5. This edge is made up of only a building line consisting of buildings IV, V and VI in Fig 6.5. The times to transit these buildings have been presented in table 6.5. The total time to transit these buildings (line of buildings) is the summation of the respective times.

In this illustration, each building has four possible routes through it. The number of routes can vary for the respective buildings. As the fire spreads through a building line the number of choices as route increases exponentially; from say 4, to 4^2, to 4^3, etc.... this can be analyzed with the aid of a computer. To reduce the computational complexity we adopted the following. The minimum and maximum times to transverse a building are taken. For the remaining number of possible routes, we interpolate uniformly between these values. The possible times to transverse a building were the accumulation of the times for the corresponding routes. The effect of this procedure can be further investigated to determine the computational advantage if need be. The above table is reconstituted below as input into the simulation.

Table 7.8 Values of the variable T_1 to T_9 for the respective edges.

Edge (Time, T_r)	Times to transit (min)			
T_1	245	278	311	344
T_2	279	316	354	391

T_3	23	72	120	500
T_4	208	250	292	500
T_5	31	62	92	500
T_6	27	54	81	500
T_7	8	16	23	500
T_8	35	70	104	500
T_9	149	192	205	209

where T_9 is the time to transit edge 9 in the network. Impossible value of 500 were given to the edges that are incomplete.

The hazard values calculated using the hazard function described above are given below. It is at this point that uncertainties are introduced through the use of the Hazard function methodology.

```
      1        2       3   4
[1,] 0.25 0.3333333  0.5   1
[2,] 0.25 0.3333333  0.5   1
[3,] 0.25 0.3333333  0.5   1
[4,] 0.25 0.3333333  0.5   1
[5,] 0.25 0.3333333  0.5   1
[6,] 0.25 0.3333333  0.5   1
[7,] 0.25 0.3333333  0.5   1
[8,] 0.25 0.3333333  0.5   1
[9,] 0.25 0.3333333  0.5   1
```

Each row of the above matrix corresponds to each of the transit times, each element is the hazard function.

6.7 Simulation of the fire spread

We are concerned in this illustration with the distribution of time of spread of fire from the origin of fire in building (I) to the entrance to the refuge center (F). For a single realization of the spread of fire in the network, we carry out a stochastic net work analysis using the hazard function earlier described. The figure below illustrates the flow chart using the hazard function

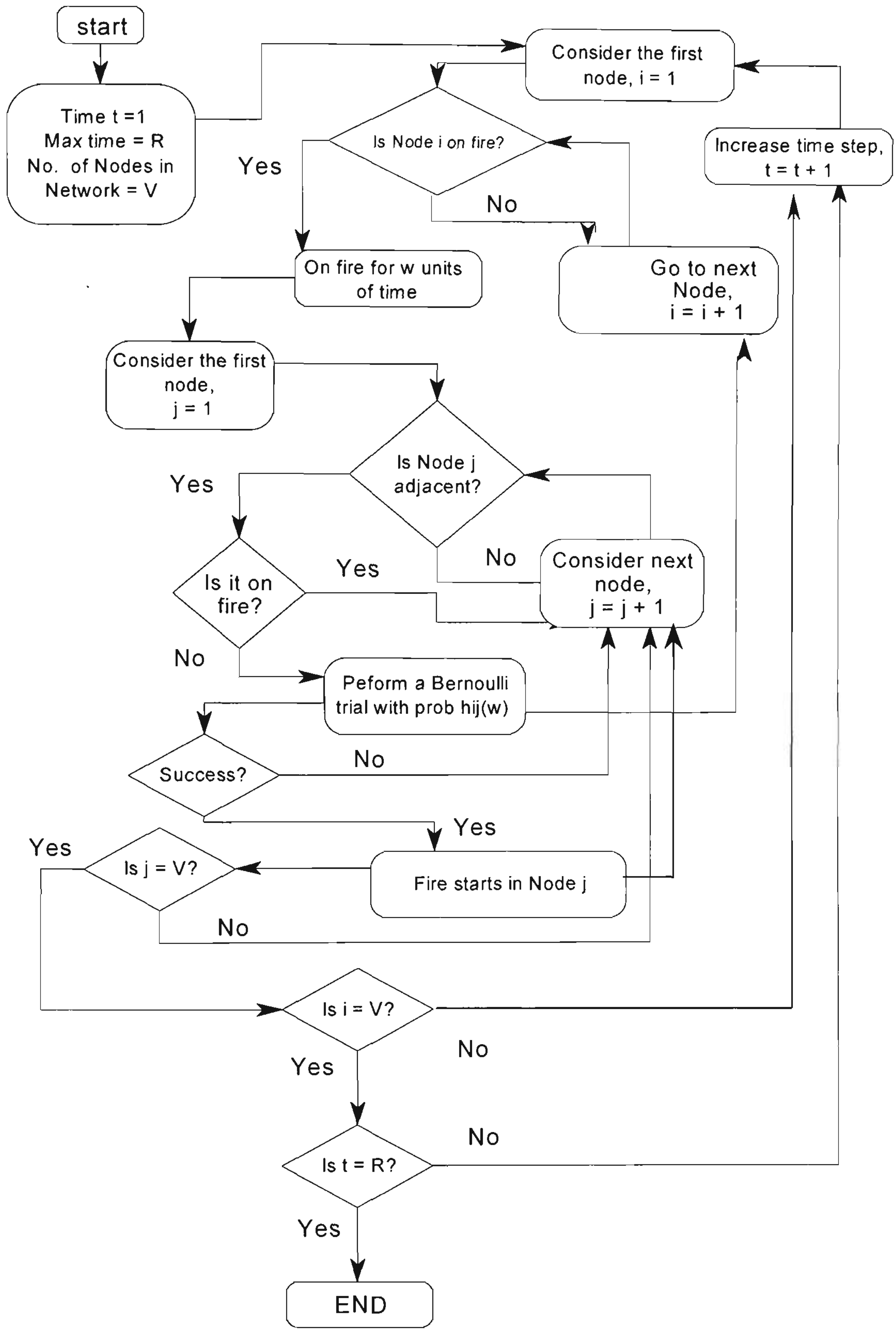
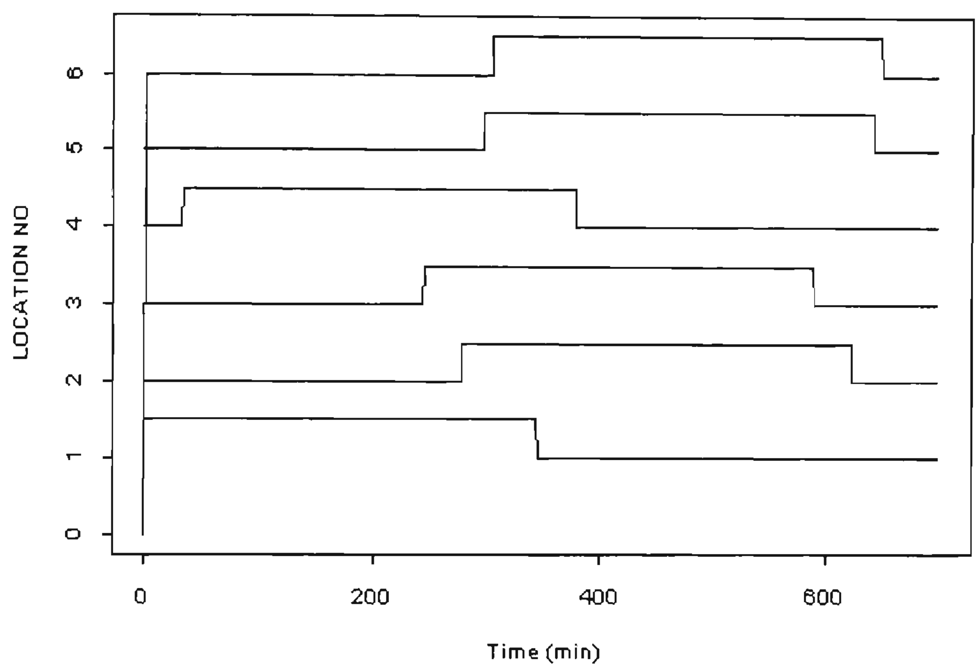


Fig 7.8 Flow Chart for One Realization of Network hazard Analysis

The illustration is for one realization. This evaluation did not consider external interventions, such as the fire brigade. To obtain the probability distribution function of the times to transit from an origin to a destination the flow process is simulated many times. The outcomes are influenced by the hazard values.

7.8 RESULTS

A typical graphical output of the simulation is shown below.



LEGEND: Location 1 = Node A, 2 = Node B,…,6 = Node F

Fig 7.9 Fire spread in the network (One realization).

Fire will get to the refuge center (F) in the 300th minute (5 hours) after ignition in nodes A and B. For this community, all effort to evacuate should be accomplished before then. Times of arrival of fire for the other locations of interest are also indicated. Below is the source matrix from the simulation. Node A is given a source value of (-1) being the origin of fire.

SOURCE :

A B C D E F

-1 3 1 1 3 3

A CA A C C

From the above, node A is the source of fire, B caught fire from node C, C from A, D from A, E from C and the refuge center from node C. Nodes A and C happen to be the sources of fire for the other nodes in this instance.

To obtain the time distribution the analysis is repeated a large number of times. Table below shows the time of spread of the fire from nodes A and B to the refuge center. The cumulative density function (CDF) is also given.

Table 7.9: Probability function and Cumulative Distribution Function (CDF) of the time of spread of fire from nodes A and B to the refuge center (F).

Time (min)	P(U=t)	CDF
298	0.197	0.197
341	0.325	0.522
354	0.22	0.742
358	0.135	0.877
384	0.039	0.916
397	0.055	0.971
401	0.02	0.991
410	0.007	0.998
414	0.002	1

Apart from knowing the minimum and maximum possible time for fire to get to a location after the start of a fire, it will be interesting to know the distribution between these values. The proportion of all possible observations that lie within various intervals between these extreme values could be determined.

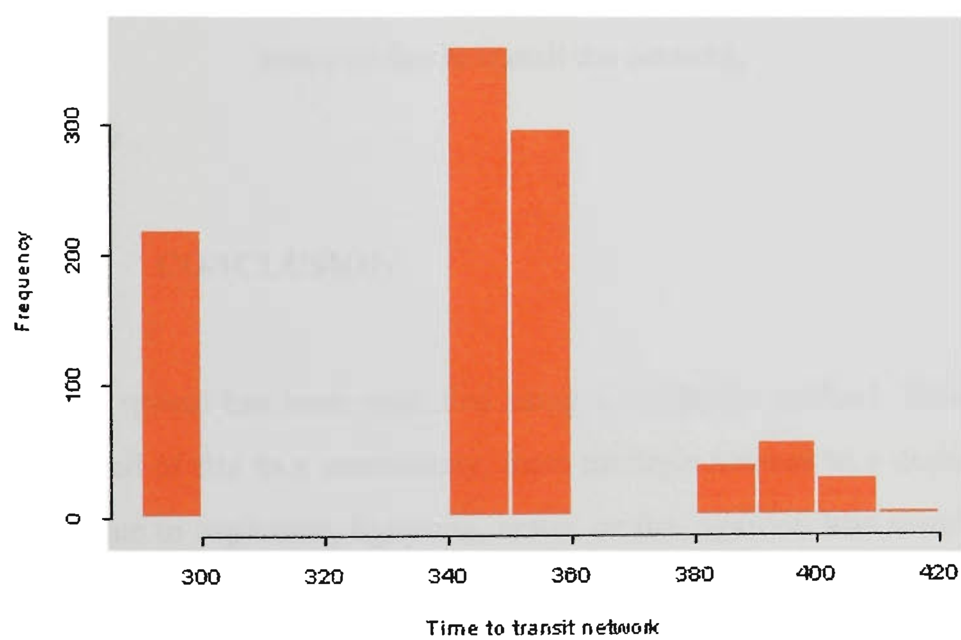
transit the network that fall below the upper limit of that class. From it the probability at or to the left of each point χ can be specified. For a random variable, say the time of the fire to transit the network (T), with a given distribution, this probability is a function of χ such that

$$F(\chi) = P(T \leq \chi) \tag{7.2}$$

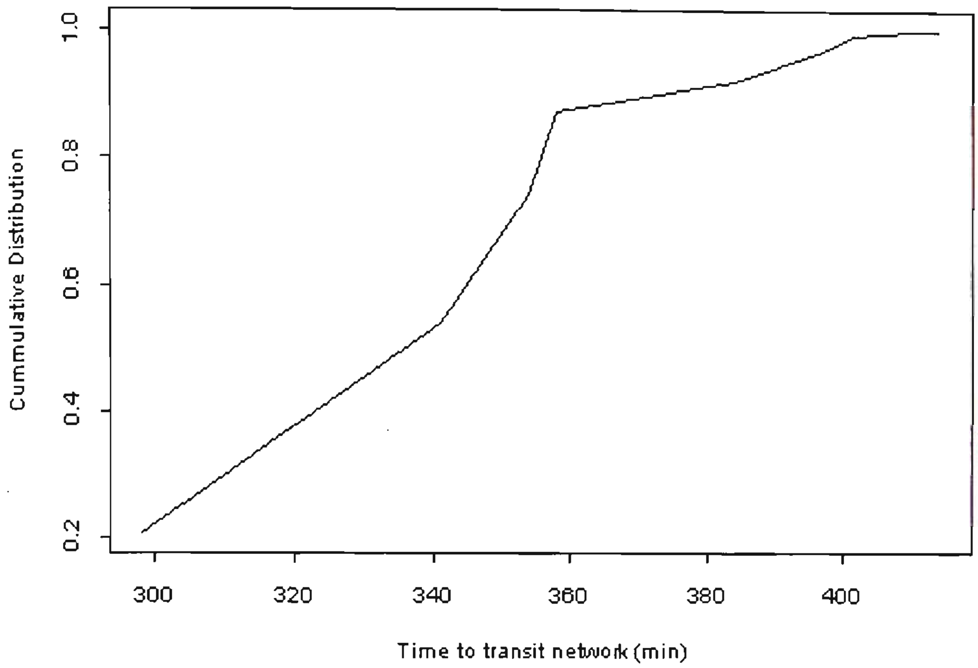
This is the cumulative distribution function (CDF) of the time of the fire to transit the network (T). The probability that T falls in an interval or that it is an integer can be obtained from the graph, that is

$$P(a < T \leq b) = F(b) - F(a) \tag{7.3}$$

The corresponding histogram and cumulative distribution function for the above are shown below.



(a)



(b)

Fig 7.10 (a) Histogram (b) Cumulative Distribution Function (CDF) of the times of fire to transit the network.

7.9 CONCLUSION

Fire spread has been modelled using a stochastic method. This chapter analysed the spread of fire in a community, from multiple sources to a destination. Sources could be due to explosion, lightning, arson, or fire; ignition was simultaneous. Destinations could be refuges, shelters, or vital locations. A metropolitan city was considered to be made up of a network of nodes (locations of interest) and edges (spread passages). An edge has a random variable representing the time of spreading from a node to another. Each edge can have a number of links representing the possible values of the random variable. The safety analysis involves human and property critical hazard management with the attendant risk. The proposed methodology determines the probability of occurrence of the events. A real life scenario was used as a case point. Practical relevance of the methodology developed in the previous chapter was discussed.

The stochastic methodology previously developed [30] has been applied here to investigate hazard from fire and safety issues in a metropolis. The analysis used many previous experimental data, and illustrated their use in hazard management.

This investigation has highlighted the areas where relevant data are not available; also areas for further investigation. The objective of evaluating the time available from time of ignition to when the destination becomes inaccessible was met. This is provided by the distribution and the CDF obtained. The information provided by the result of the methodology presented here can be used in planning evacuations and other hazard management. This is a relevant tool for disaster management, the opportunity exist to predict issues and establish preventive measures well before crisis.

CHAPTER 8

STOCHASTIC MODELLING OF OCCUPANTS EGRESS FOR SAFETY IN A BUILDING

CONTENTS

FIGURES

TABLES

8.1. INTRODUCTION

Designing an efficient building configuration to ensure the safety of a mixed ability population in the event of a fire is a challenging task. Occupant egress models currently in use for studying evacuation times in buildings are essentially deterministic and assume exact knowledge of the input parameters.

Given a typical configuration and fuel load in a building, it is possible in principle to obtain the time after ignition for signatures of fire to attain incapacitating dosage. With known intake rate of intoxicants by occupants, the period available for egress after the fire cue has been received can be computed. But, in general, the values of all these variables cannot be known in advance. Finally it must be pointed out that most current models consider evacuation only and disregard interaction with the fire.

An alternative approach is to model the uncertainty explicitly by introducing random variables in the model. The outcome of the evacuation is then obtained as a probability distribution. Usually, interest will be focussed on the distribution of the number of fatalities. In particular, two measures of the efficiency of the design have been proposed: the expected number of deaths in a fire, and the probability of no deaths. In most cases, analytical solutions will not be feasible, but Monte Carlo simulations will be comparatively easy to perform. With almost unlimited computing power being readily available to engineers, the Monte Carlo simulations provide a quick way of obtaining reliable estimates of the two above measures.

A major difficulty in introducing random variables in occupant egress models is that there is continuous interaction between the spread of fire cues and incapacitating conditions, occupant response, and passive and active fire-fighting systems.

The situation is further complicated by the fact that the fire spread as well as the egress routes usually form a network with multiple available paths. Deterministic network analyses cannot be readily generalized to deal with stochastic parameters.

Developed herein is a novel approach to the simulation of such stochastic network models, based on the concept of the discrete hazard function. The hazard function approach is an elegant and flexible way of dealing with the simulation of waiting times between one event and the next one. It is equal to the probability of a particular time being just exceeded, assuming that it has been reached. Instead of simulating the whole random variable, one performs a Bernoulli trial at each successive step, using the value of the hazard function.

The use of hazard functions makes it possible to follow the time evolution of the global fire scene step by step and to modify the probabilities which drive the model at any step in accordance with the interactions that eventuate. Models based on hazard functions are flexible enough to deal with a wide range of situations, from apartment blocks to large office buildings.

The main obstacle to the use of stochastic models in fire engineering remains the dearth of observational data to support the choice of appropriate probability distributions for the random variables of the models. But this should not delay the development of the theoretical tools required. In the meantime, the appropriateness of the probability distributions can be tested by calibrating the designs derived from the simulations against designs which are known to be safe.

In this chapter, a discussion of the network representation will be given. The building will be considered to be made up of nodes (compartments or apartments) and edges (passages, stairs). To each edge will correspond a random variable representing the time of moving from one node to another. An unusual characteristic of such variables is that they can take the value "infinity" with non-zero probability. Such variables are called "defective".

The basic properties of discrete hazard functions have already been given in section 6.4 of chapter 6. It was shown that hazard functions can deal naturally with "defective" random variables. Moreover, the value of the hazard can be readily modified at any point of time to accommodate changes in the environment.

A stochastic representation of the abilities and locations of the occupants will be introduced. The points of time at which the alerting cues reach the occupants as well as the nature of their response to the cues will also be modelled. Occupant response in the room of fire origin must be treated separately. This is a particularly acute problem as it is known that a large proportion of fatalities in fires occurs in the room of fire origin. Congestion of evacuation routes can be modelled by appropriately modifying the hazard functions. Flowcharts of the Monte Carlo simulations will be presented and finally an example will be worked out. The risk to life is incapacitation and death. The risk factor is smoke or fire.

The analysis starts from when the cue to evacuate is given, which could be indicated by fire, smoke alarm, or any other known cue.

Large buildings with many compartments are relatively difficult to evacuate because of the numerous options available to the occupant. In an emergency panic and other disorientating factors disable reasoning. Occupants are then subject to inappropriate means of escape. Recalling the functional routes and exits is one of the easily forgotten tasks in such crisis.

In this investigation, egress routes are represented by a graph, made up of nodes and edges. Nodes between an origin and a destination can be points of interest or of change of direction. Times to evacuate are assumed to be the time taken by an occupant to reach an exit from his or her compartment (origin). Total time to evacuate (T_r) will include time of recognition (T_B), time to respond (T_D) and time of egress (T_X) [50]. Usually time of recognition (T_B), for public buildings, is small as most public buildings have fire and or smoke warning systems. Response time (T_D) can reasonably be estimated except for physical and or psychologically handicapped occupants. The total time (T_r) to evacuate is then a summation of these times given as

$$T_r = T_D + T_B + T_X \tag{8.1}$$

where

T_r = Total evacuation time, without external intervention (min).

T_B	=	Recognition time (min).
T_D	=	Response time (min).
T_X	=	Egress time (min).

All the above times have distributions defined by hazard functions.

8.2 Occupants Categorization

The first issue to resolve is the determination of all possible categories of occupants that can be in the building. In this regard occupants can be grouped into categories depending on their mobility and response to cues.

Table 8.1 Classification of Possible Occupants:

Type	Description
1	Very Mobile: e.g. Physically and Psychologically fit to Evacuate
2	Mobile: As above but with reduced mobility.
3	Reduced Mobility: Children and Aged.
4	Assisted Mobility: Mobile only if assisted e.g. Drunks & Wheelchairs
5	Reduced Mobility and unassisted.

Occupancy in the originating compartments can be grouped into the above classes. For instance, if there are 10 occupants, classifying entails identifying and allocating them to groups as shown in the above table.

8.3 Recognition Time

The time of recognition of cue (T_B) is that between when the cue is given and the occupant becomes aware of it. Its probability distribution can be influenced by the condition of the occupant that is indicated by the classification into the respective categories.

8.4 Time to Respond

The time to respond to cue is defined as from when there is recognition to when actual egress begins. This is a variable time that depends on the occupant's thoughts before deciding to start evacuation, and would increase with the occupant's concerns, assumed to be proportional to age and inversely so with agility. We can assume that the older the occupant the larger the response time.

8.5 Egress Time

The time to egress (T_X) is that between when the egress starts to the time to reach an exit or the location of death. T_X can be obtained from historical data of egress from buildings, where available, or from other egress models.

8.6. Network Representation

It is well known that, for the purpose of investigating building occupants egress, a building can be represented by a graph. See e.g. Ling and Williamson [13] and the full discussion and literature review in Ramachandran [51]. The graph representation is particularly useful in modelling flows.

Each compartment is represented by a vertex, and vertices are connected by an edge if there is between the two corresponding compartments a direct path through which the occupants can move, such as a door. Since there are situations where movement is possible only in one direction, (e.g. if it is assumed that an occupant will not move back to a previous node) each edge is given a direction, so that it has a beginning and an end. Thus the type of graph to be considered is a directed graph or "digraph". If the occupants can actually move in both directions (albeit not necessarily at the same speed) we introduce two directed edges. For an elementary discussion of graph theory see Wilson [52].

8.6.1 Notation

Let us denote the number of vertices by v and number them from 1 to v. Furthermore, suppose there are K edges and that we number them from 1 to K. To describe the time-dependent occupant egress we assign to each directed edge a random variable representing the time taken by the occupant to move from the compartment at the start of the edge to the compartment at the other end from the time the occupant started in the compartment representing by the start of the edge. Let us denote the random variable assigned to edge number r, which links vertex i to vertex j, by T_r. It is important to note that there is usually a non-zero probability p_r that the occupant will not move at all e.g. if he is totally incapacitated until he dies. This can however be dealt with by allowing the random variable T_r to be "defective" i.e. to take the value "infinity" with probability p_r.

8.6.2. Matrix representation

It is easy to encode all the information provided by the network in a form which is eminently suitable for computer work: in the form of a v x v matrix. This is done as follows: The ith row and ith column of the matrix correspond to the ith node. The diagonal elements are set to zero and the (i,j)th entry identifies the random variable attached to the directed edge starting at node i and ending at node j. It should be noted that the (i,j)th entry is not in general identical to the (j,i)th entry. The (i,j)th entry refers to the waiting time connected to the flow from node i to node j while the (j,i)th entry refers to the waiting time connected with the flow from node j to node i. If there is no directed arc between the nodes i and j, the (i,j)th entry is set to zero.

Steps taken in simulating occupant's egress are illustrated in the figure below.

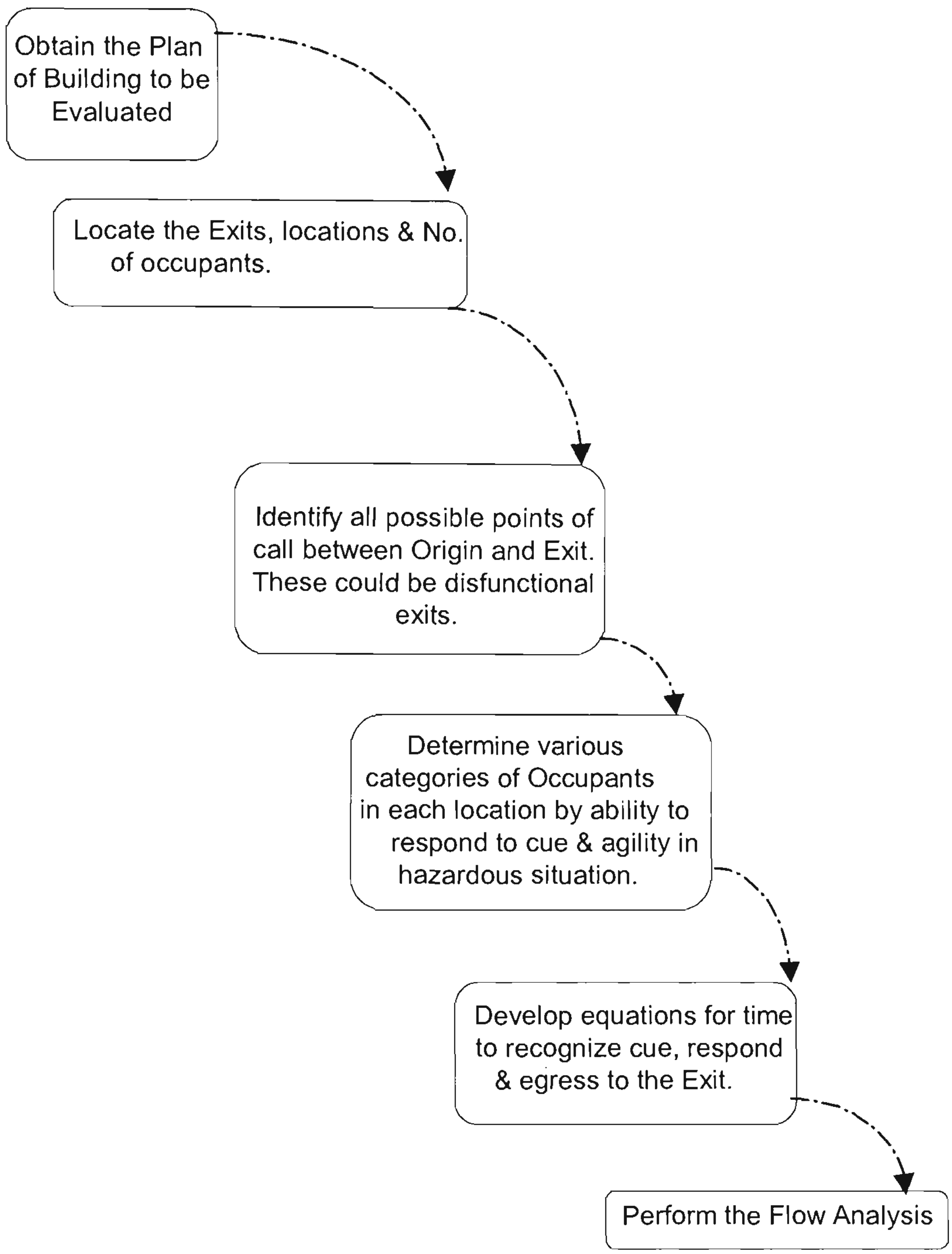


Fig 8.1 Steps in Building Evacuation Analysis

8.7 SIMULATION

8.7.1 Input data

To implement a stochastic model for occupant egress the following input data are required:

1. The configuration of the building, represented by a network, and the evacuation routes from each compartment (node).

2. The numbers of occupants in various categories of speed of response to alerting cues and mobility. The network node at which each one is located.

3. The time when alerting cues reach the various compartments (nodes).

4. For each category of occupant, the time to recognize the cue, respond to the cue, and egress from the building.

5. The time-dependent spread of untenable conditions over the network.

8.7.2 Methodology

The proposed simulation algorithm determines the time, location and disability level of each occupant at successive instants of time. For the purpose of this illustration we shall assume that the random variables T_r (eqn 8.1) are independent. The instant at which the alerting cue reaches the occupant is recorded. From then on a hazard function simulation (as described in section 6.6 of chapter 6) is carried out until the instant of beginning of evacuation is reached. The next step is to determine the adjacent edge available for evacuation. Where there are more than one possible edge to take, the probability of choosing each edge must be given, and the simulation chooses the edge to be followed at random according to the given probability distribution. We shall subsume the information attached to the edge from i to j in the form of two discrete hazard functions: $h_{ij}(w)$ for the egress time from i to j and $h_{ji}(w)$ for the egress time from j to i. Suppose that at time t_o the occupant was in compartment i. Suppose that at some subsequent random time $t_o + W$ the occupant moves to the next adjacent compartment j. The hazard function $h_{ij}(w)$ corresponds to the random variable W. The process is repeated for each node reached. At each instant of the simulation the number of evacuees of each category located at each node is recorded as well as the disability level of each one. Eventually, either the occupant reaches an exit or reaches the lethal disability level. The lethal disability level is the point of incapacitation, when an occupant is unable to exit due to the accumulation of the total effects of the fire. The simulation ends when the final outcome is determined for each occupant. Suppose we are given, for a particular building, the graph that represents the building together with the set of hazard functions corresponding to the edges of the graph and the time-dependent spread of untenable conditions. We can then construct the matrix A corresponding to the graph. Suppose there are v compartments, so that the matrix A is v x v.

8.8 A ILLUSTRATIVE EXAMPLE

8.8.1 Building plan and corresponding network

As a numerical example consider the layout of a building. The building considered consists of two floors, labeled floor 1 and floor 2. On each floor there are three apartments, a corridor and access to two stairways. Each apartment has a balcony. But they will not be included in this analysis. A full egress model would also require smoke and fire spread, in this simulation it is assumed that untenable condition is when the total effects of fire would not enable egress. The total effects of fire include smoke and fire. The building plan and the corresponding network are shown in Figure 8.2 .

The nodes are labeled in the following ways:

Node type (1) level number (2) location number (3).

The location number of corridors is 1.

Thus a2.3 represents the third apartment on level 2.

We now number the network as described in Section 8.6.1. The numbered network is shown in the figure below.

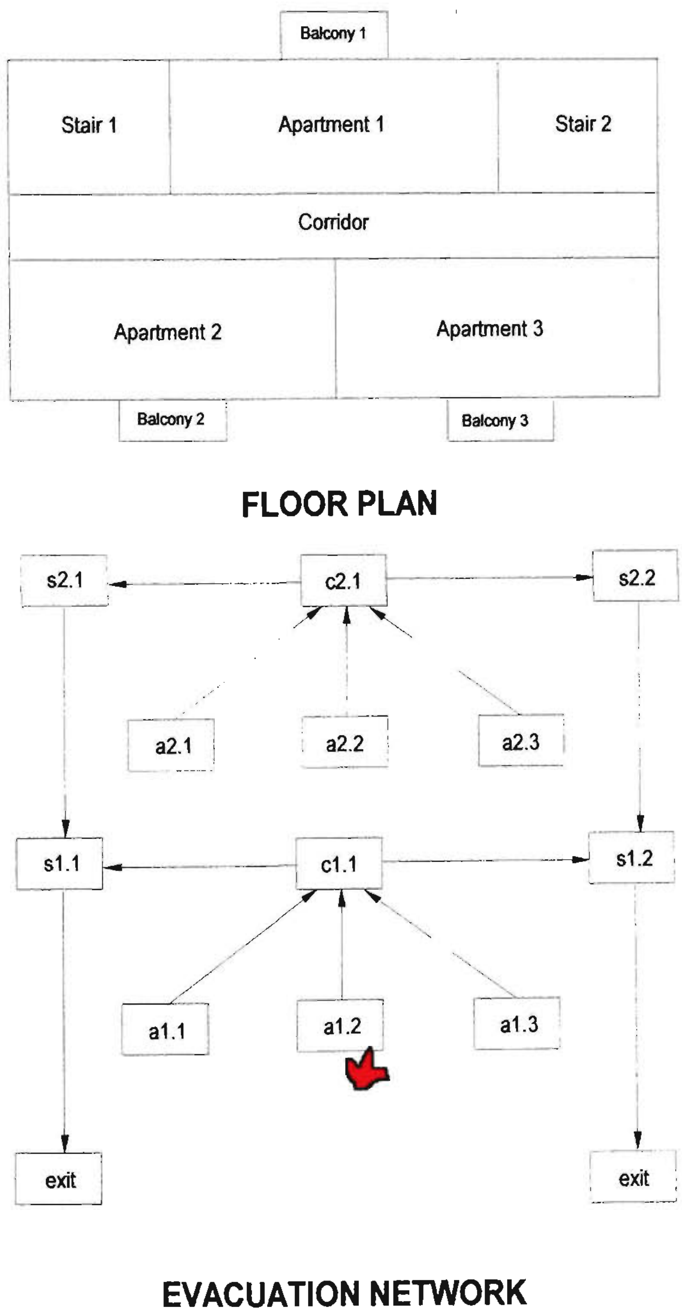


Fig 8.2 Floor plan and evacuation network

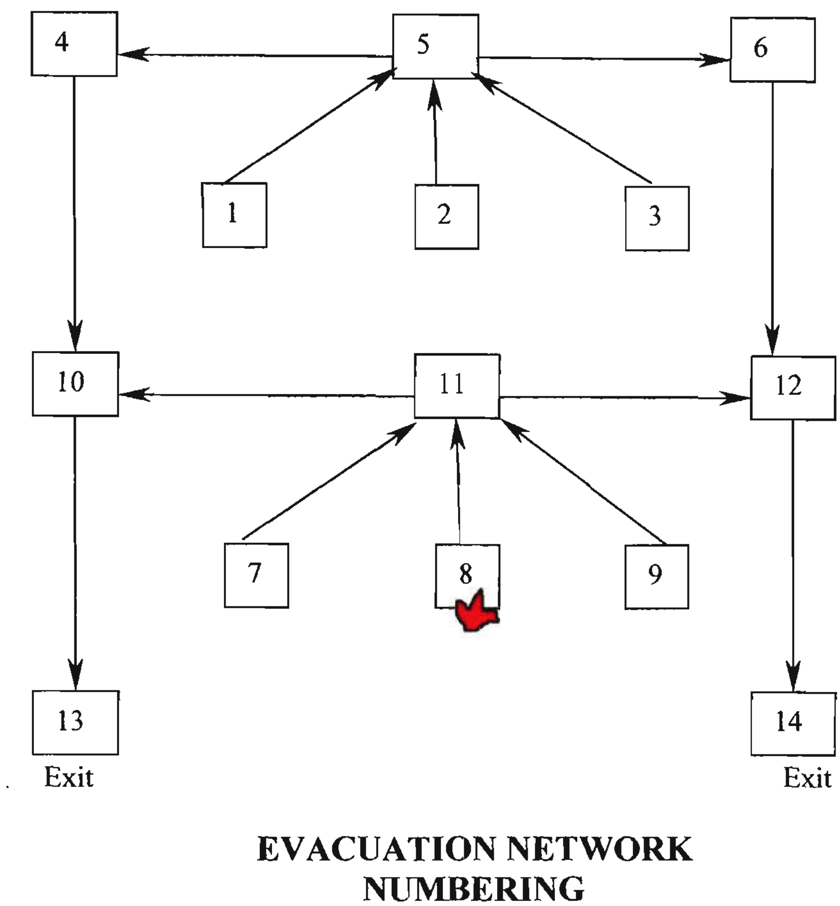


Fig 8.3 Numbering network

8.8.2. ASSUMPTIONS:

For the purpose of this illustration we shall assume the following:

1. The time is measured in minutes.
2. The fire starts in apartment a1.2 at time zero (0).
3. There is just one occupant in apartment a2.2 of Category type 3.
4. If the occupant opens the door to stair 1 or 2 and untenable conditions have already reached the stair s1.1 or s1.2, this will result in the death of the occupant.

A full egress model should include a detailed fire and smoke spread model as discussed in previous chapters. Here we shall be content with assumption 4 above. In this simulation it is assumed that untenable conditions occur when the total effect of fire would not enable egress. The total effect of fire includes smoke and fire.

The time to transit is described by probability functions. The probability functions of the times to transit the respective edges is assumed to be an input obtained from any existing egress model and are as follows:

a. Probability function of time until the effective cue reaches the occupant, denoted by S82. This include time taken since start of ignition, up until an effective cue reaches the occupants, enabling them to discover or detect the existence of a fire.

S82	15	60
P	0.2	0.8

b. Probability function of time until untenable conditions reach s1.1, denoted by U810

U810	50	70
P	0.5	0.5

c. Probability function of time until untenable conditions reach s1.2, denoted by U812

U812	60	80
P	0.5	0.5

d. Probability function of time until evacuation of a.2.2 starts, denoted by T25

T25	2	3	4
P	0.3	0.4	0.3

e. At c2.1 the occupant chooses to go either right or left with probability 0.5.

f. Probability function of time of egress time from c2.1 to s2.1, denoted by T54

T54	2	3	4
P	0.3	0.4	0.3

g. Probability function of time of egress time from c2.1 to s2.2, denoted by T56

T56	2	3	4
P	0.3	0.4	0.3

h. Probability function of time of egress time from s2.1 to s1.1, denoted by T410

T410	1	2	3
P	0.3	0.4	0.3

i. Probability function of time of egress time from s2.2 to s1.2, denoted by T612

T612	2	3	4
P	0.3	0.4	0.3

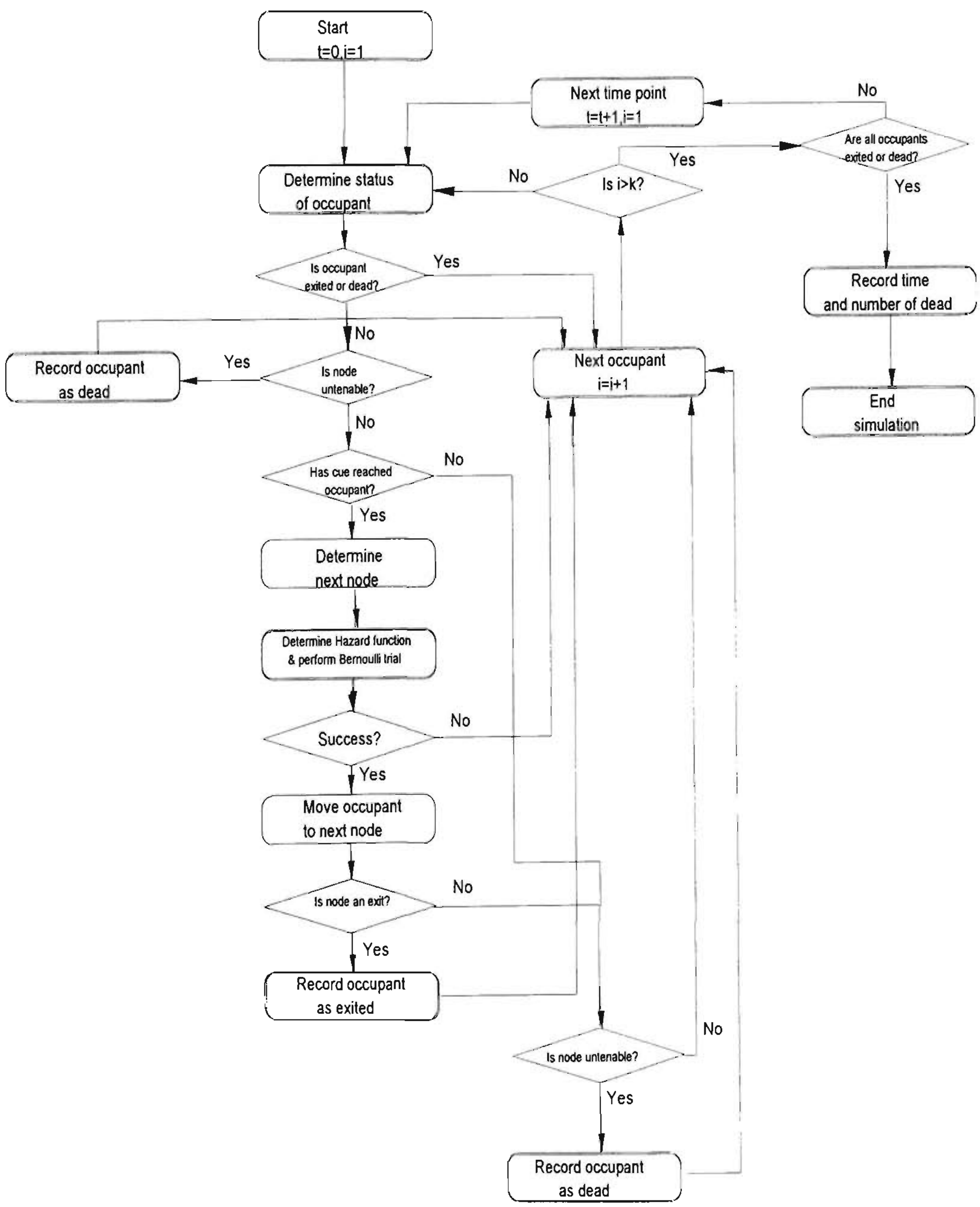
Consider again the network in figure 8.3 above . Let the sequence of consecutive edges from an occupied compartment to an exit in the building be a path. We can trace all possible paths an occupant in the two occupied locations can take to the two exits in the building. Below is a table identifying such possible paths.

Table 8.2 Possible paths in the building network.

Possible Paths in the network
a2.2 - c2.1 - s2.1 - s1.1 - Exit1
a2.2 - c2.1 - s2.2 - s1.2 - Exit2

From the above table of the possible paths and the given distributions of times of transit, the choice matrix showing the probabilities of times of transit is obtained.

A typical simulation program in SPLUS is presented in Appendix E. The flow chart for the simulation process is presented below.



LEGEND:

t = time. Initially t = 0

i = Occupant number, Initially i = 1.

- There are k occupants
- For each occupant at each node the next node is determined at the beginning of the simulation by lottery and is not subsequently changed.

- At each point of time the arrival of cues at each node and the onset of untenable conditions at each node (whether yes or no) are recorded from the fire spread program.
- At each point of time the status of each occupant is recorded, namely whether the cue has been received or not, whether the occupant has exited or not, and whether the occupant has encountered untenable conditions or not.

Fig 8.4 Flow chart of the occupant egress simulation process.

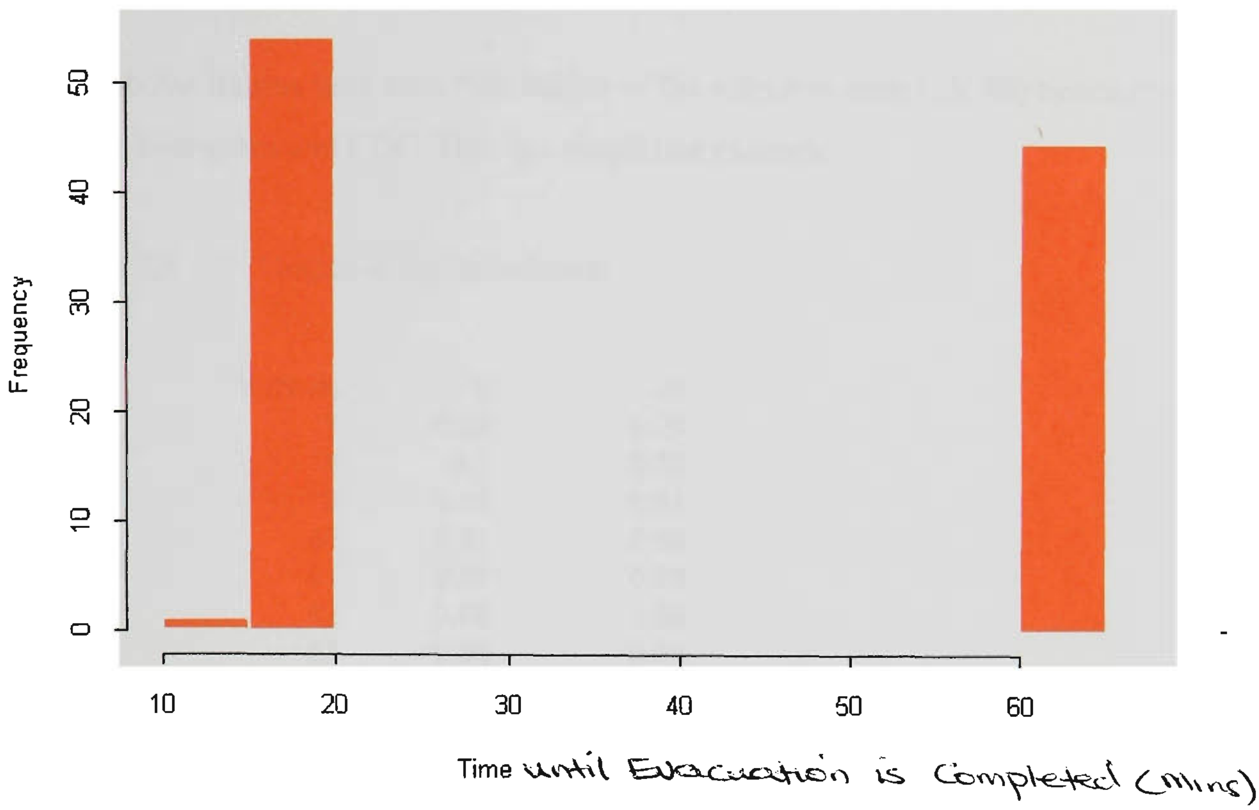
8.8.3 Results of a Simulation.

The simulation was performed 1000 times. In order to obtain the probability of death of the occupants, counters were placed, counting the number of times death occurred (if any). Having the number of times an occupant died, the probability of death is then obtainable. As stated in Assumption 4 (p.162) the death of an occupant occurs if untenable conditions reach the stair s.11 or s.12 and the occupant opens the door to that stair. For every simulation a record is kept if the condition has occurred. The probability of death of one occupant is then the ratio of the number of simulations resulting in death to the total number.

If there are k occupants the simplest assumption is that they will behave independently of each other. Then if p is the probability of death, the number of deaths among the k occupants will follow a binomial distribution with probability p. If, however, we wish to assume some interaction between the occupants, then this can be included in the model. In that case, for each simulation, we will obtain some number r of deaths (out of k). The histogram of the r's will estimate the distribution of the number of deaths.

1000 runs of the simulation gave the following:

Probability of death for occupant in a2.2 is 0.02


Frequency
0
10
20
30
40
50
10
20
30
40
50
60
Time until Evacuation is Completed (mins)

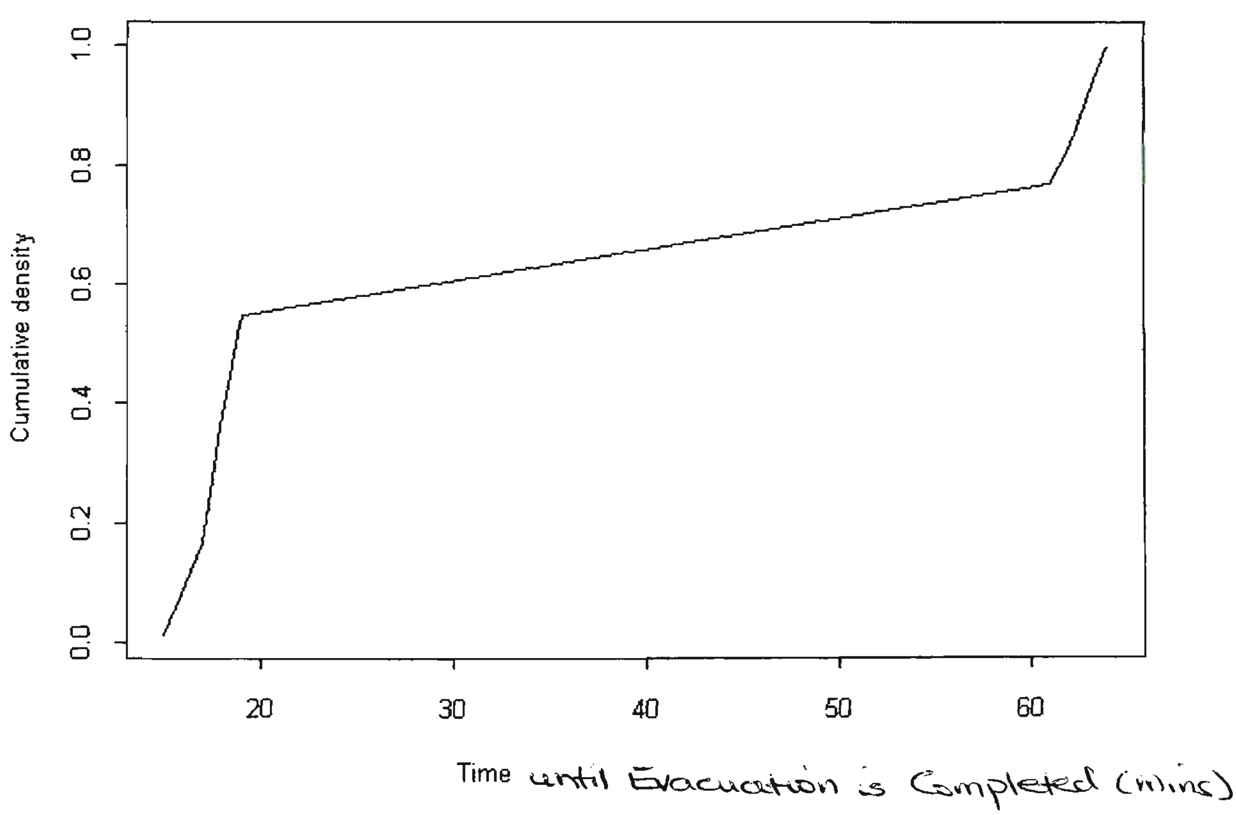

Cumulative density
0.0
0.2
0.4
0.6
0.8
1.0
20
30
40
50
60
Time until Evacuation is Completed (mins)

Fig 8.8 (a) Histogram (b) Cummulative Distribution Function (CDF) of Exit time for the building until evacuation is completed.

The above figures uses only two values of the effective cues (15, 60) hence the shape of the histogram and CDF. This is a simplified example.

Table 8.5 Output of the Simulation

	Y (time)	z	cdf
[1,]	17	0.22	0.22
[2,]	18	0.1	0.32
[3,]	19	0.19	0.51
[4,]	60	0.01	0.52
[5,]	61	0.27	0.79
[6,]	62	0.06	0.85
[7,]	63	0.09	0.94
[8,]	64	0.06	1

Legend:

Y = Time to evacuate building

Z = Probability function

Cdf = Cumulative distributive function of time to evacuate building.

From the above the probability of evacuating the building for any given time is obtainable. This result does not incorporate the restriction to flow due to building structural factors. Further more because we have only two values of the effective cues (15, 60) that is an unrealistic assumption. We dealt with a simplified illustrative example. Similar simulations can be carried out for other exit locations during building design and a comparison made to determine the most effective type that ensures the required safety level.

The simulation also provides the probabilities for the case when an occupant is confronted with the decision of making a choice of taking an edge from more than one edge. A typical simulation indicated if an occupant died, giving the compartment it occurred and the time.

8.9. RELEVANCE:

Similar simulations to the above can be carried out for other combinations of exit locations during building design and a comparison made to determine the most effective combination that gives least risk to life. Also, this simulation process can be used to evaluate the effect of evacuation training programs. The hazard function is then modified accordingly.

The quest for optimal building configuration is the task of a regulatory body. The number of exits required will determine the safe evacuation of occupants. Regulatory bodies need a form of guidance in specifying the performance required for a building in terms of evacuation in emergency. The procedure given in this paper can be used to evaluate the adequacy of hazard management in public places such as old peoples' home.

For a given building it is now possible to estimate the time for evacuating all categories of occupants within certain probabilities. Application of this methodology includes the ability to recommend reasonable terms for building safety regulations.

An example of a regulatory exit provision clause for a residential two floor building, for both prescriptive and performance based code is presented below; for a given built up area of each floor (say, not more than 500 $m^{2)}$.

Table 8.4 Typical Regulatory Clauses.

Prescriptive Based Code	Performance Based Code
Number of terminal exits = 2	Provide number of terminal exits to keep the probability of death for each occupant below 0.02.

8.10. RECOMMENDATION

The above analysis enables the consideration of most possible options for the occupant in evacuating a building. An alternative approach to the analysis will be to consider various transportation algorithms. For instance, the greedy algorithm assumes that all occupants will leave the building through the nearest exit. But could this be the quickest way of evacuating all occupants? Other algorithms can be used and a comparative analysis done. Of course these algorithms are conceptually deterministic hence encumbered with the limitations of not evaluating uncertainties.

The simulation gets more complex as the number and types of occupant increases; also as more factors that influence egress are considered. As the process becomes more complex, the hazard function method of simulation used above becomes more relevant.

CHAPTER 9

CONCLUSION AND RECOMMENDATION

CONTENTS

9.1 CONCLUSIONS

The investigation done has covered the objective and specific aims as listed in the introductory chapter. Chapter 4 illustrated a methodology for the conversion of a deterministic model to a stochastic model. The most variable components of the deterministic model were used in developing a noise component. The stochastic model became the summation of this and the deterministic model. In chapter 5, a complete stochastic model was developed using the discrete Markov chain. A continuous process, smoke spread, was approximated by a discrete process. The fineness of the approximation being determined by the magnitude of the unit time step chosen. The stochastic model compared well with the models used for verification. Different methods of evaluating the spread of a phenomenon (fire) in a network were investigated in chapter 6. The Analytical, Monte Carlo and the Hazard function methodologies were compared. The Hazard function methodology was found to be more flexible especially as there are external events that can modify the spread of these phenomena. Chapter 7 illustrated the use of the Hazard function method in solving a real life problem. The spread of fire in a metropolis for a multiple hit scenario was investigated. Further more the chapter also gave insights into the use of existing data and identified such data that need to be kept for such analysis. The ultimate desire is to be able to predict the probability of death for occupants in a building. Chapter 8 showed the stochastic modelling of occupants' egress for safety from a building. Input data of the migration time distribution was from existing models and the simulation gave the prediction of the probability of death. Each simulation predicts if an occupant exited safe and if not the location of death.

The following have been accomplished

1. Stochastic models of fire spread, smoke spread and occupants egress have been developed using available data from compartmental fires. Smoke spread prediction as an extension of existing deterministic models with probabilities of occurrence of events is now possible. Hence uncertainties not incorporated in the chosen deterministic models has been addressed.

2. Starting with deterministic models stochastic models were developed and the relationship between the models established.

3. Comparisons were made, as occasioned, with other risk-cost assessment models to ascertain the advantages of the methodologies derived in this investigation.

4. Different methods of evaluating the spread of a phenomenon in a network were compared. The phenomena investigated were the spread of fire, spread of smoke and occupants egress.

The objectives of this investigation have been achieved. The spread of fire and smoke arising from fires in buildings using stochastic techniques has been established. The models have been validated using results from experiments and existing models.

Specifically, stochastic models that can predict the spread of fire, smoke and occupants egress have been originated. Some of these models are extensions of existing deterministic models. The relationship between these types of stochastic models and the deterministic models has been identified as the noise component. Comparison has been made between the stochastic models and the deterministic models and the advantages highlighted. Spread of a flow phenomenon in a network was also investigated; various methods were used in the evaluation. Advantages of using the hazard function over and above the other methods were shown.

9.2 RECOMMENDATIONS

9.2.1 POSSIBLE AREAS FOR FUTURE INVESTIGATION

9.2.1.1 Dependency and Correlation

The gas temperature and toxicity determine the effect of smoke. These two variables are correlated. In which case, higher burning rate ensures higher temperatures which implies increasing toxicity. As the temperature increases, the distress condition increases and resistance to toxicity reduces. In essence, the limit states of the variables in the independent situation may not be reached before incapacitation is attained. Independent situation is when the variables are considered to be independent and not correlated and their effects are analysed as such. The combinational effect of temperature and toxicity could reduce the incapacitation dosage. There is a correlation between the two in attaining incapacitation. They cannot be taken to be independent.

This fact of life in the hazard posed by smoke in fires is an interesting aspect to investigate.

9.2.1.2 Other Areas

1. Use of the strategy in chapter 6 above to analyze the time taken for smoke to spread from an origin to a destination in a network, hence the

2. Determination of the time to reach an event at a location.

3. Consideration of inhibitors to smoke spread eg Exhaust system, pressurized staircase, etc. Inhibitors being activities that would reduce or prevent the spread of smoke.

4. Determination of the level of stochasticity factor to be used for different types of fire and smoke.

APPENDIX A

The plot of figure 3.2 is by the program "hes" executable in SPLUS. The program calls the files holding the results of the stochastic simulation of He and Beck's model. The stochastic model was obtained by the transformation explained in the chapter (3). The simulation was for the stairway of the NRCC building.

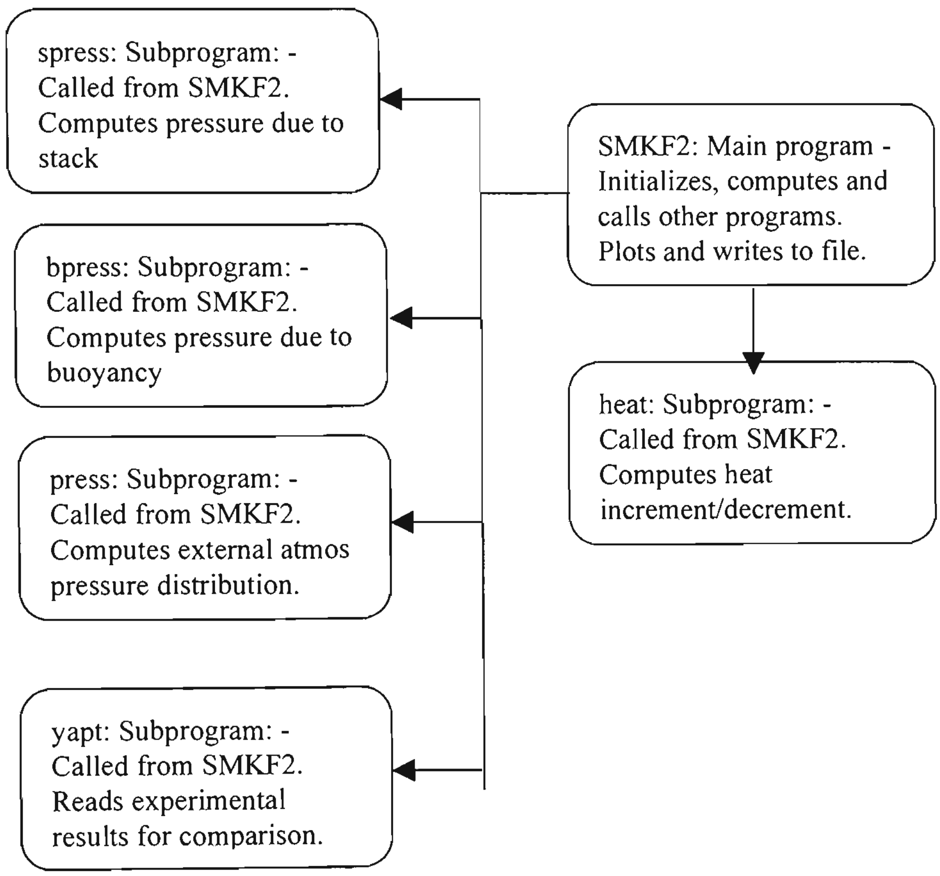
The following is the code for hes:

```
function()
{
        y <- yapt()
        tm <- y[, 1]
        graphics.off()
        win.graph()
        matplot(tm, y[, 2], type = "l", xlab = "Time(min)", ylab =
                "Temp (C)", main =
                "Temperature of species in Stairway")
        matlines(tm, y[, 3], type = "l", col = 2)
        matlines(tm, y[, 4], type = "l", col = 3)
        matlines(tm, y[, 5], type = "l", col = 3)
        matlines(tm, y[, 5], type = "l", col = 5)
        matlines(tm, y[, 6], type = "l", col = 4)
        matlines(tm, y[, 7], type = "l", col = 4)
        matlines(tm, y[, 8], type = "l", col = 5)
        matlines(tm, y[, 9], type = "l", col = 2)
        matlines(tm, y[, 10], type = "l", col = 3)
        colour <- c(1, 2, 3, 4, 5)
        ltype <- c("l", "p", "p")
        pchar <- c("l", "l")
        leg.names <- c("f2", "f3", "f5", "f8", "f9", "f10")
        legend(locator(1), leg.names, col = colour, pch = pchar)
}

function()
{
#yapt: This file: extracts the result of the stochastic simulation for the transformation of
#       He and Beck's model.
#matrix DATAS contains data for other models (Yaping)
        DATAS <- matrix(scan(
                "c:\\data\\prop\\modat\\results\\stair2.dat", sep =
                "\t"), byrow = T, ncol = 10)
        dimnames(DATAS) <- list(1:length(DATAS[, 1]), c("tm", "f2s",
                "f3s", "f4s", "f5s", "f6s", "f7s", "f8s", "f9s",
                "f10s"))
        DATAS
}
```

The SPLUS code that simulates species distribution on the second level for the stairway for the NRCC building and the comparison with real experiment and the deterministic model is presented below. This generates figure 3.3 in chapter 3. The program calls a few others.

The flow chart for the programs call for the simulation for fig 3.3 is shown below.




```
function()
{
#smkf2: This file(main file) computes species conc for second floor stairshaft
        DATA <- data()      #extracts input data from file
        Mi2 <- DATA[, "M"]
        TT <- DATA[, "TT"]
        tm <- DATA[, "tm"]
        CO2 <- DATA[, "CO2"]
        DT <- TT - 297
        frac <- 200
        dt <- 0.1
        SD <- sqrt(dt)
        set.seed(101)
        T0 <- 297 #initial stairshaft temp in K(24 C)
        R <- 8320
        Cp <- 1.06#specific heat
        C <- 0.65  # orifice coefficient
        Ac <- 0.16
        #A <- 5.05 * 2.48 * 0.5 + 0.16   #effective orifice area
        CO20 <- 0.00019      #initial CO2 concentration
        V <- 3.6 * 5.05 * 2.48              #volume of node
        n <- length(TT)
        bp <- bpress(TT)      #pressure from bouyancy
        sp <- spress()        #pressure from stack
        dp <- bp + sp         #pressure difference
        p0 <- press()         #External atmos pressure distr.
        p0 <- p0[2]           #Level external atmos pressure
        p <- p0 + dp
        rho <- c(rep(0, n))    #gas density container
        TTf2s <- c(rep(0, n))
```

```
	CO2b <- c(rep(0, n))
	TTf2sb <- c(rep(0, n))
	dWGT <- c(rep(0, n))
	dWGCO2 <- c(rep(0, n))
	Mi3 <- c(rep(0, n))
	Mc <- c(rep(0, 300))
	for(i in 1:n) {
		Tf <- (TT[i] + T0)/2	#film temperature
		rho[i] <- p[i]/(R * Tf)
	}
	Q <- heat(DATA)
	for(j in 1:n) {
		dWGT[j] <- (frac * DT[j] * rnorm(1, 0, SD))/5000
	#Noise for gas temp
		dWGCO2[j] <- (20 * frac * DT[j] * rnorm(1, 0, SD))/
			100000000		#Noise for CO2
		TTf2s[j] <- ((R * T0)/(p[j] * V)) * (Mi2[j] * (TT[j] -
			T0) - Q[j]/Cp)
		TTf2s[j] <- TTf2s[j] + (T0 - 273)
		if(TTf2s[j] < 24)
			TTf2s[j] <- 24
		TTf2sb[j] <- TTf2s[j] + dWGT[j]
		CO2[j] <- CO2[j] - ((R * TT[j])/(p[j] * V)) * (Mi2[j] * (
			CO2[j] - CO20)) * dt
		CO2b[j] <- CO2[j] + dWGCO2[j]
		Mi3[j] <- (Mi2[j] * (TT[j] - T0) - Q[j]/Cp)/TT[j]
	}
#	TTf2s
#	DATA <- cbind(tm, Mi3, TTf2s, CO2, Mc)
#	dimnames(DATA) <- list(1:length(DATA[, 1]), c(
#		"tm", "M", "TT", "CO2", "Mc"))
#	write.table(DATA, file = 
#		"c:\\data\\prop\\modat\\results\\dataf2s.dat",
#		sep = "\t", append = F, quote.string = 
#		F, dimnames.write = F, na = NA,
#		end.of.row = "\n")
#	print("Output data for level 2 of stairshaft in c:\\data\\prop\\modat\\results\\dataf3s.dat"
#		)
	y <- yapt()
	graphics.off()
	win.graph()
	matplot(tm, TTf2sb, type = "l", xlab = "Time(min)", ylab = 
		"Temp(C)", main = 
		"Gas Temp in Stairshaft F2(With Stochasticity) ")
	matlines(tm, TTf2s, type = "l", col = 3)
	matlines(tm, y[, 2], type = "p", col = 2)	#	DATA
}
```

```
function()
{
#data: This file (called from smkf2): creates matrix DATA
#matrix DATA contains data for previous node
	DATA <- matrix(scan(
		"c:\\data\\prop\\modat\\results\\tdata.dat", sep = 
		"\t"), byrow = T, ncol = 4)
	dimnames(DATA) <- list(1:length(DATA[, 1]), c("tm", "M", "TT",
		"CO2"))
	DATA
}
```

```
function(TT)
{
#bpress:This file: computes pressure difference due to bouyancy
	kb <- 3460#N.K.m-3
#	DATA2 <- data2()
#	TTc <- DATA2[, "f2"] + 273
#temp of gases into stairshaft(K)
	TTs <- 297 #assumed average initial temp in stairshaft(K)
	Ht <- 12.5		n <- length(TT)
	bp <- c(rep(0, n))
	for(i in 1:n)
```

```
                    bp[i] <- (kb * (1/TTs - 1/TT[i]) * Ht)/2
          bp
}
```

```
function()
{
#spress: This file(called from smkf2): computes the pressure difference due to stack effect
#Ht is an array of heights of floors above neutral plane
          kb <- 3460#N.K.m-3
          TTs <- 297          #assumed average temp in stairshaft(K)
          TT <- 278 #temp of outside air(K)
          nt <- height()
          Ht <- nt[length(nt)]/2
          for(i in 1:length(nt)) {
                    nt[i] <- Ht - nt[i]       #height above neutral plane in m
                    if(nt[i] < 0)
                              nt[i] <-  - nt[i]
          }
          ds <- c(rep(0, 10))
          for(i in 1:length(nt)) {
                    ds[i] <- kb * (1/TT - 1/TTs) * nt[i]
          }
          ds
}
```

```
function()
{
#presst: This file(called from smkf2): calculates current
#Pressure w.r.t height and temp
#TT is the current temp in compartment
#h is the height of compartment above base
          int <- init()
          Pb <- int["P0"]
          TT <- int["T0"]
          h <- height()
          n <- length(h)
          #extracts initial external ground level pressure
#extracts initial staircase ground level temp
          R <- 8320 #gas constant
          g <- 9.81  #acceleration due to gravity
          P <- c(rep(0, n))
          dh <- 0    #Base heigth
          for(i in 1:n) {
                    dh <- dh + h[i]
                    P[i] <- Pb * (1 - ((g/R) * (dh/TT)))
          }
          P
}
```

```
function(DATA)
{
#heat: This file(called from smkf2): computes the heat transfer rate Q
          a <- c(rep(1.6, 9))    #Areas of opennings in m
          b <- 5.699e-011 * 60 #Stefan-Boltzmann constant KJ/min m2
          e <- 0.9    #assumed gas emissivity:pp665 Holman J.P
          A <- a[2]
          As <- 2 * 3.6 * (5.05 + 2.48) + (5.05 * 2.48)
          #DATA <- indata()
          Tw0 <- 297
          TT <- DATA[, "TT"]
          tm <- DATA[, "tm"]
          n <- length(TT)
          Qc <- c(rep(0, n))
          Qr <- c(rep(0, n))
          Q <- c(rep(0, n))
          h <- c(rep(0, n))
          h <- heatco(DATA)
          for(i in 1:n) {
                    Qc[i] <- (A * h[i] * (TT[i] - Tw0))/20
          #convection heat transfer
                    Qr[i] <- (As * e * b * (TT[i]^4 - Tw0^4))/840
                    Q[i] <- (Qc[i] + Qr[i])
          }
```

```
        Q
}
--------------------------------------------------------------------------------
```

The above is a typical program that simulates the spread of smoke in a building for the investigation done in chapter 3. The program is run several times to obtain distribution function for the respective parameters of interest. The deterministic model was first developed then transformed to a stochastic model. Similar codes to the above were developed for the other levels for the stairway, compartments and corridors in the building. Hence a program that calls the respective functions was used to simulate flow of smoke through out the building.

Data generated for the cummulative distribution function for 1000 sample space is as follows:

	y	z	cdf
[1,]	109	0.001	0.001
[2,]	123	0.002	0.003
[3,]	124	0.001	0.004
[4,]	126	0.001	0.005
[5,]	129	0.001	0.006
[6,]	130	0.001	0.007
[7,]	134	0.002	0.009
[8,]	135	0.002	0.011
[9,]	136	0.004	0.015
[10,]	137	0.002	0.017
[11,]	139	0.003	0.02
[12,]	140	0.006	0.026
[13,]	141	0.004	0.03
[14,]	142	0.002	0.032
[15,]	143	0.004	0.036
	y	z	cdf
[16,]	144	0.008	0.044
[17,]	145	0.006	0.05
[18,]	146	0.009	0.059
[19,]	147	0.01	0.069
[20,]	148	0.016	0.085
[21,]	149	0.011	0.096
[22,]	150	0.009	0.105
[23,]	151	0.019	0.124
[24,]	152	0.013	0.137
[25,]	153	0.01	0.147
[26,]	154	0.013	0.16
[27,]	155	0.023	0.183
[28,]	156	0.021	0.204
[29,]	157	0.017	0.221
[30,]	158	0.037	0.258
	y	z	cdf
[31,]	159	0.02	0.278
[32,]	160	0.027	0.305
[33,]	161	0.026	0.331
[34,]	162	0.026	0.357
[35,]	163	0.028	0.385
[36,]	164	0.034	0.419
[37,]	165	0.021	0.44
[38,]	166	0.03	0.47
[39,]	167	0.034	0.504
[40,]	168	0.031	0.535
[41,]	169	0.031	0.566
[42,]	170	0.033	0.599
[43,]	171	0.026	0.625
[44,]	172	0.031	0.656
[45,]	173	0.032	0.688
	y	z	cdf
[46,]	174	0.034	0.722
[47,]	175	0.033	0.755
[48,]	176	0.031	0.786
[49,]	177	0.023	0.809
[50,]	178	0.014	0.823
[51,]	179	0.024	0.847

```
[52,]              180      0.011     0.858
[53,]              181      0.019     0.877
[54,]              182      0.019     0.896
[55,]              183      0.019     0.915
[56,]              184       0.01     0.925
[57,]              185      0.013     0.938
[58,]              186      0.012      0.95
[59,]              187      0.006     0.956
[60,]              188      0.014      0.97
          y           z        cdf
[61,]              189      0.005     0.975
[62,]              191      0.005      0.98
[63,]              192      0.008     0.988
[64,]              193      0.003     0.991
[65,]              194      0.004     0.995
[66,]              198      0.001     0.996
[67,]              199      0.002     0.998
[68,]              201      0.001     0.999
[69,]              204      0.001         1
>
```

The SPLUS program that produced the histogram, cummulative density function (CDF) and the above data is presented below:

```
function()
{
        y <- c(rep(0, 5))
        graphics.off()
        y[1] <- max(dts)
        y[2] <- min(dts)
        y[3] <- mean(dts)
        y[4] <- sqrt(var(dts))
        y[5] <- var(dts)
        win.graph()
        hist(dts, nclass = 7, xlab = "Temp (C)",
                ylab = "Frequency", main =
                "Histogram of Temp distribution of level 2 stairway"
                )
        dt <- matrix(rep(0, length(dts) * 2), nrow
                 = length(dts), ncol = 2)
        dt[, 1] <- floor(dts)
        dt[, 2] <- 1/length(dts)
        fm <- freqm(dt)
        p <- dim(fm)[1]
        cdf <- matrix(0, nrow = p, ncol = 1)
        cdf <- fm[, 2]
        for(s in 2:p)
                cdf[s] <- cdf[s - 1] + cdf[s]
        win.graph()
        plot(fm[, 1], cdf, type = "l", xlab =
                "Temperature of level 2 of stairway (C)",
                ylab = "Cummulative Distribution")
        z <- cbind(fm, cdf)
        z
}
```

The program that generated the data for the above for the 1000 sample space is shown below:

```
function()
{
#smkf2: This file computes species conc for second floor stairshaft
        DATA <- data()
        #extracts input data from file
        Mi2 <- DATA[, "M"]
        TT <- DATA[, "TT"]
        tm <- DATA[, "tm"]
        CO2 <- DATA[, "CO2"]
        z <- 1000  #No. of simulations
        DT <- TT - 297
        frac <- 200
```

```
dt <- 0.1
SD <- sqrt(dt)
set.seed(101)
T0 <- 297
#initial stairshaft temp in K(24 C)
R <- 8320
Cp <- 1.06#specific heat
C <- 0.65  # orifice coefficient
Ac <- 0.16
#A <- 5.05 * 2.48 * 0.5 + 0.16    #effective orifice area
CO20 <- 0.00019      #initial CO2 concentration
V <- 3.6 * 5.05 * 2.48                #volume of node
n <- length(TT)
bp <- bpress(TT)      #pressure from bouyancy
sp <- spress()        #pressure from stack
dp <- bp + sp         #pressure difference
p0 <- press()
#External atmos pressure distr.
p0 <- p0[2]           #Level external atmos pressure
p <- p0 + dp
rho <- c(rep(0, n))    #gas density container
TTf2s <- TTf2sb <- dWGT <- dWGCO2 <- CO2b <-
        c(rep(0, n))
TTs <- c(rep(0, z))
Mi3 <- c(rep(0, n))
Mc <- c(rep(0, 300))
y <- c(rep(0, 5))
for(i in 1:n) {
        Tf <- (TT[i] + T0)/2
#film temperature
        rho[i] <- p[i]/(R * Tf)
}
Qs <- Q <- heat(DATA)
for(k in 1:z) {
        for(j in 1:n) {
                dWGT[j] <- (frac * DT[j] *
                        rnorm(1, 0, SD))/
                        5000
#Noise for gas temp
                dWGCO2[j] <- (20 * frac *
                        DT[j] * rnorm(1, 0,
                        SD))/100000000
                Qs[j] <- rlnorm(1, log(Q[j]
                        ), 0.1)
#Noise for CO2
                TTf2s[j] <- ((R * T0)/(p[j] *
                        V)) * (Mi2[j] * (TT[
                        j] - T0) - Qs[j]/Cp
                        )
                TTf2s[j] <- TTf2s[j] + (T0 -
                        273)
                if(TTf2s[j] < 24)
                        TTf2s[j] <- 24
                TTf2sb[j] <- TTf2s[j] +
                        dWGT[j]
                CO2[j] <- CO2[j] - ((R * TT[
                        j])/(p[j] * V)) * (
                        Mi2[j] * (CO2[j] -
                        CO20)) * dt
                CO2b[j] <- CO2[j] + dWGCO2[
                        j]
                Mi3[j] <- (Mi2[j] * (TT[j] -
                        T0) - Qs[j]/Cp)/TT[
                        j]
        }
        TTs[k] <- TTf2sb[n]
}
write.table(TTs, file =
        "c:\\data\\prop\\modat\\results\\dts.dat",
        sep = "\t", append = F,
        quote.string = F, dimnames.write =
        F, na = NA, end.of.row = "\n")
```

APPENDIX B

The simulations in chapter 5 used the experimental result conducted by Hokugo and Hadjisophocleous [8] in the NRCC ten-storey experimental smoke spread tower. The data used for the simulation are for carbon-dioxide and temperature. These are presented below.

The measured time (min), mass flow rate of gases (Kg/s^2), temperature (OC), and carbon-dioxde (%) through the door into the second floor is as shown below.

Time (min)	Mass flow rate (KG/s2)	Temp (°C)	Co2 (%)
0.00E+00	0.00E+00	0	0
1.00E+00	1.65E+00	0	0.018
2.00E+00	1.80E+00	84	0.054
3.00E+00	1.82E+00	150	0.126
4.00E+00	1.88E+00	161	0.612
5.00E+00	1.96E+00	172	1.098
6.00E+00	1.93E+00	182	1.145
7.00E+00	1.89E+00	199.5	1.192
8.00E+00	1.89E+00	205.5	1.239
9.00E+00	1.87E+00	214.5	1.286
1.00E+01	1.90E+00	216	1.333
1.10E+01	1.91E+00	219	1.285
1.20E+01	1.91E+00	225	1.236
1.30E+01	1.90E+00	231	1.188
1.40E+01	1.89E+00	241	1.224
1.50E+01	1.84E+00	247.5	1.26
1.60E+01	1.83E+00	259.5	1.332
1.70E+01	1.82E+00	264	1.431
1.80E+01	1.82E+00	267	1.53
1.90E+01	1.81E+00	270	1.629
2.00E+01	1.79E+00	274.5	1.728
2.10E+01	1.78E+00	282	1.71
2.20E+01	1.77E+00	282	1.566
2.30E+01	1.76E+00	286.5	1.422
2.40E+01	1.76E+00	286.5	1.368
2.50E+01	1.76E+00	288	1.314
2.60E+01	1.76E+00	289.5	1.35
2.70E+01	1.75E+00	291	1.445
2.80E+01	1.74E+00	291	1.541
2.90E+01	1.71E+00	289.8	1.634
3.00E+01	1.70E+00	288	1.731

The measured CO_2 (%) for the respective floors of the building follows by Hukugo [8].

	f2	f3	f5	f8	f9	f10
0	0	0	0	0	0	0
1	0.036	0	0.09	0.036	0.063	0.036
3	0.216	0.036	0.018	0.036	0.0684	0.036
4	0.243	0.108	0.045	0.108	0.054	0.036
8	1.278	0.702	0.612	0.963	0.927	0.531
9	1.188	0.639	0.549	0.648	0.756	0.3852
11	1.08	0.594	0.54	0.459	0.6444	0.54
14	0.945	0.612	0.378	0.378	0.5832	0.3906
19	1.5876	0.855	0.765	0.09072	1.098	1.08
21	1.422	0.828	0.639	0.81	0.972	1.3086
22	1.44	0.81	0.783	0.738	0.954	0.8874
24	1.17	0.792	0.675	0.7146	0.819	0.765
29	1.485	0.855	0.855	0.945	1.152	1.278
30	1.512	0.965	0.965	0.984	1.186	1.298

The measured interpolated data for temperature distribution in the building is presented below.

	Temp °C					
Time (min)	f2	f3	f5	f7	f9	f10
0.13333	26.8	26.8	26.8	26.8	26.8	26.8
0.3	26.8	26.8	26.8	26.8	26.8	26.8
0.46667	26.8	26.8	26.8	26.8	26.8	26.8
0.63333	26.8	26.8	26.8	26.8	26.8	26.8
0.8	26.8	26.8	26.8	26.8	26.8	26.8
0.96667	26.8	26.8	26.8	26.8	26.8	26.8
1.1333	28.38	26.98	26.8	26.8	26.8	26.8
1.3	32.62	28.05	26.82	26.8	26.8	26.8
1.4667	38.73	30.4	26.93	26.8	26.8	26.8
1.6333	46.15	34.06	27.27	26.81	26.8	26.8
1.8	54.48	38.9	27.97	26.83	26.8	26.8
1.9667	63.43	44.69	29.17	26.9	26.8	26.8
2.1333	72.29	51.08	30.95	27.05	26.8	26.8
2.3	80.7	57.63	33.29	27.31	26.81	26.8
2.4667	88.87	64.2	36.16	27.76	26.82	26.8
2.6333	97.13	71.08	39.61	28.45	26.84	26.8
2.8	103.22	78.02	43.8	29.51	26.9	26.8
2.9667	108.75	83.68	48.17	30.91	26.99	26.81
3.1333	113.16	88.35	52.34	32.6	27.14	26.82
3.3	116.33	91.94	56.09	34.51	27.37	26.85
3.4667	119.04	94.76	59.34	36.53	27.68	26.89
3.6333	121.58	97.14	62.05	38.57	28.09	26.94
3.8	124.09	99.31	64.32	40.53	28.58	27.03
3.9667	126.6	101.38	66.24	42.35	29.15	27.14
4.1333	128.72	103.32	67.91	43.99	29.77	27.27
4.3	130.39	104.96	69.39	45.46	30.44	27.44
4.4667	131.89	106.39	70.71	46.76	31.13	27.64
4.6333	133.33	107.69	71.89	47.91	31.83	27.86
4.8	134.74	108.93	72.95	48.93	32.51	28.1
4.9667	136.14	110.13	73.93	49.83	33.16	28.36
5.1333	137.22	111.18	74.81	50.63	33.78	28.63
5.3	138.01	111.97	75.56	51.33	34.35	28.9
5.4667	138.73	112.61	76.19	51.93	34.87	29.16
5.6333	139.45	113.18	76.71	52.44	35.34	29.42
5.8	140.48	113.77	77.13	52.87	35.75	29.66
5.9667	142.05	114.46	77.44	53.2	36.1	29.89
6.1333	142.94	115.17	77.74	53.48	36.42	30.11
6.3	143.43	115.67	78.06	53.73	36.7	30.32
6.4667	143.76	116.02	78.37	53.95	36.95	30.53
6.6333	144.02	116.28	78.63	54.15	37.18	30.71
6.8	144.26	116.48	78.86	54.34	37.37	30.88

6.9667	144.49	116.67	79.04	54.51	37.54	31.04
7.1333	144.84	116.89	79.22	54.67	37.69	31.18
7.3	145.33	117.22	79.4	54.82	37.83	31.31
7.4667	145.87	117.62	79.6	54.96	37.95	31.42
7.6333	146.41	118.04	79.84	55.11	38.06	31.53
7.8	146.96	118.48	80.09	55.25	38.17	31.62
7.9667	147.51	118.92	80.36	55.4	38.26	31.7
8.1333	148.1	119.36	80.64	55.56	38.35	31.78
8.3	148.74	119.83	80.92	55.72	38.44	31.85
8.4667	149.4	120.31	81.21	55.88	38.53	31.92
8.6333	150.05	120.79	81.49	56.05	38.61	31.98
8.8	150.7	121.28	81.78	56.21	38.69	32.03
8.9667	151.35	121.77	82.07	56.38	38.77	32.09
9.1333	151.87	122.24	82.37	56.55	38.85	32.14
9.3	152.26	122.65	82.68	56.74	38.94	32.19
9.4667	152.61	123.01	82.99	56.92	39.03	32.24
9.6333	152.94	123.34	83.28	57.12	39.12	32.3
9.8	153.28	123.66	83.56	57.31	39.21	32.35
9.9667	153.6	123.98	83.83	57.51	39.31	32.4
10.133	153.93	124.28	84.09	57.7	39.4	32.46
10.3	154.26	124.58	84.33	57.88	39.5	32.51
10.467	154.59	124.87	84.56	58.06	39.6	32.56
10.633	154.92	125.16	84.78	58.23	39.69	32.62
10.8	155.25	125.45	85.01	58.39	39.79	32.67
10.967	155.58	125.73	85.22	58.55	39.88	32.72
11.133	156	126.04	85.44	58.7	39.97	32.77
11.3	156.5	126.41	85.66	58.85	40.06	32.82
11.467	157.02	126.8	85.89	59	40.15	32.87
11.633	157.56	127.22	86.14	59.15	40.23	32.92
11.8	158.09	127.64	86.4	59.31	40.31	32.96
11.967	158.62	128.06	86.67	59.46	40.39	33.01
12.133	159.22	128.52	86.95	59.62	40.47	33.05
12.3	159.88	129.03	87.25	59.8	40.56	33.1
12.467	160.56	129.57	87.57	59.98	40.64	33.14
12.633	161.24	130.12	87.92	60.17	40.73	33.18
12.8	161.92	130.68	88.28	60.37	40.82	33.23
12.967	162.6	131.24	88.64	60.59	40.91	33.27
13.133	163.25	131.75	89	60.8	41	33.32
13.3	163.85	132.2	89.31	61	41.1	33.37
13.467	164.44	132.62	89.6	61.19	41.19	33.41
13.633	165.02	133.03	89.85	61.37	41.29	33.46
13.8	165.6	133.42	90.09	61.54	41.38	33.51
13.967	166.17	133.81	90.31	61.69	41.46	33.56
14.133	166.6	134.16	90.52	61.83	41.55	33.61
14.3	166.87	134.4	90.71	61.96	41.63	33.66
14.467	167.1	134.58	90.87	62.08	41.7	33.7
14.633	167.32	134.73	91	62.19	41.78	33.75
14.8	167.53	134.87	91.11	62.28	41.84	33.8
14.967	167.74	135	91.2	62.37	41.9	33.84
15.133	168.24	135.23	91.3	62.44	41.96	33.88
15.3	169.04	135.67	91.43	62.52	42.01	33.91
15.467	169.92	136.25	91.63	62.6	42.06	33.94
15.633	170.81	136.87	91.88	62.69	42.1	33.96
15.8	171.71	137.51	92.19	62.8	42.15	33.98
15.967	172.59	138.16	92.52	62.93	42.19	34.01
16.133	173.24	138.74	92.86	63.08	42.24	34.03
16.3	173.64	139.15	93.19	63.25	42.3	34.06
16.467	173.98	139.47	93.49	63.42	42.36	34.09
16.633	174.3	139.74	93.75	63.6	42.43	34.13
16.8	174.61	140	93.97	63.77	42.5	34.16
16.967	174.93	140.26	94.17	63.93	42.58	34.2
17.133	175.23	140.51	94.36	64.08	42.65	34.24
17.3	175.51	140.75	94.54	64.22	42.73	34.28
17.467	175.78	140.99	94.72	64.35	42.81	34.32
17.633	176.06	141.22	94.89	64.48	42.88	34.36
17.8	176.33	141.45	95.06	64.61	42.96	34.4
17.967	176.61	141.68	95.23	64.73	43.03	34.44
18.133	176.85	141.9	95.4	64.84	43.1	34.48
18.3	177.07	142.09	95.56	64.96	43.17	34.52
18.467	177.28	142.27	95.7	65.07	43.24	34.56
18.633	177.49	142.45	95.84	65.17	43.3	34.6
18.8	177.7	142.62	95.97	65.27	43.37	34.64
18.967	177.91	142.79	96.1	65.37	43.43	34.67
19.133	178.14	142.97	96.23	65.46	43.48	34.71
19.3	178.38	143.14	96.35	65.55	43.54	34.74
19.467	178.63	143.33	96.46	65.63	43.59	34.78
19.633	178.89	143.51	96.58	65.71	43.64	34.81
19.8	179.14	143.7	96.7	65.79	43.69	34.84
19.967	179.39	143.88	96.82	65.87	43.74	34.87
20.133	179.76	144.11	96.94	65.94	43.78	34.9

20.3	180.26	144.41	97.08	66.02	43.82	34.92
20.467	180.78	144.77	97.24	66.1	43.87	34.94
20.633	181.3	145.15	97.43	66.19	43.91	34.97
20.8	181.83	145.53	97.63	66.29	43.95	34.99
20.967	182.36	145.92	97.84	66.39	43.99	35.01
21.133	182.59	146.21	98.05	66.5	44.04	35.03
21.3	182.52	146.29	98.23	66.61	44.09	35.06
21.467	182.39	146.26	98.34	66.72	44.14	35.09
21.633	182.25	146.18	98.41	66.82	44.19	35.13
21.8	182.1	146.08	98.43	66.89	44.25	35.16
21.967	181.95	145.99	98.43	66.95	44.3	35.19
22.133	181.92	145.91	98.41	67	44.34	35.22
22.3	181.99	145.89	98.38	67.02	44.38	35.25
22.467	182.1	145.92	98.36	67.03	44.41	35.28
22.633	182.2	145.96	98.36	67.04	44.44	35.3
22.8	182.31	146.01	98.36	67.04	44.46	35.32
22.967	182.41	146.06	98.37	67.04	44.48	35.33
23.133	182.64	146.19	98.41	67.05	44.49	35.35
23.3	182.99	146.47	98.51	67.08	44.51	35.36
23.467	183.38	146.8	98.67	67.13	44.52	35.37
23.633	183.78	147.17	98.87	67.21	44.54	35.38
23.8	184.17	147.54	99.11	67.31	44.57	35.39
23.967	184.55	147.91	99.37	67.43	44.61	35.4
24.133	184.95	148.27	99.63	67.58	44.65	35.41
24.3	185.37	148.65	99.9	67.73	44.7	35.43
24.467	185.78	149.02	100.18	67.9	44.76	35.45
24.633	186.2	149.4	100.45	68.07	44.83	35.48
24.8	186.61	149.78	100.73	68.25	44.9	35.51
24.967	187.03	150.15	101.01	68.43	44.98	35.54
25.133	187.21	150.41	101.26	68.6	45.06	35.57
25.3	187.15	150.48	101.44	68.76	45.15	35.62
25.467	187.04	150.43	101.56	68.9	45.23	35.66
25.633	186.92	150.35	101.61	69.02	45.31	35.71
25.8	186.81	150.26	101.63	69.11	45.39	35.76
25.967	186.7	150.17	101.61	69.17	45.46	35.8
26.133	186.68	150.12	101.59	69.21	45.52	35.84
26.3	186.78	150.16	101.59	69.24	45.57	35.88
26.467	186.89	150.24	101.61	69.26	45.62	35.92
26.633	187.01	150.34	101.65	69.29	45.66	35.95
26.8	187.12	150.44	101.72	69.32	45.69	35.98
26.967	187.24	150.54	101.79	69.36	45.72	36
27.133	187.28	150.62	101.86	69.4	45.75	36.03
27.3	187.26	150.65	101.93	69.44	45.78	36.05
27.467	187.22	150.65	101.98	69.49	45.81	36.07
27.633	187.18	150.64	102.02	69.53	45.84	36.09
27.8	187.14	150.62	102.04	69.57	45.86	36.12
27.967	187.09	150.61	102.07	69.61	45.89	36.14
28.133	186.96	150.56	102.08	69.64	45.92	36.16
28.3	186.74	150.44	102.07	69.66	45.94	36.17
28.467	186.51	150.29	102.04	69.68	45.96	36.19
28.633	186.27	150.12	102	69.69	45.98	36.21
28.8	186.04	149.96	101.93	69.69	46	36.23
28.967	185.8	149.79	101.86	69.68	46.02	36.25
29.133	185.61	149.64	101.79	69.66	46.03	36.26
29.3	185.46	149.53	101.73	69.65	46.04	36.27
29.467	185.31	149.44	101.68	69.63	46.04	36.28
29.633	185.18	149.35	101.64	69.61	46.05	36.29
29.8	185.04	149.27	101.61	69.6	46.05	36.3
29.967	184.89	149.19	101.58	69.59	46.05	36.3

See section 5.5.8 and 5.6 of this thesis for more on the data source.

The computer programs that do the simulation in chapter four are presented below. The program for the spread of smoke using Markov chain stochastic simulation is as follows:

```
function()
{
#stok3: This file:computes concentration of smoke using the Markov chain
#Model for storey building
```

```
#It considers effect of pressure differences
#Initial CO fraction          =          1%               =          0.01
#mass of air                  =          0.70 Kg/m3
#Constant gas density is assumed.
#yn+1 = yn + C*h*sqrt(rho)*(A*x*sqrt(Psa)/mb - Ac*yn*sqrt(Psb)/mc - A*yn*sqrt(Ps3)/Mb)
        alpha <- 0.03
        C <- 0.65          #orifice flow constant
        Ac <- 2 * 0.8 * 0.1
        # Area of opening to corridor
        A <- 5.05 * 2.48 * 0.6
        #stairwell opening from level to level
        DATA <- others2()
        xn <- DATA[, "f2"]        #conc at source
        dct <- 0.8
        h <- 0.05
        np <- height()
        np <- np[length(np)]/2     #nplane halfway up
        n <- length(DATA[, 1])
        m <- 0.7 # Kg/m3, density of air
        v <- c(2.48 * 5.05 * 3.6, 2.48 * 5.05 * 3.6,
                rep(2.48 * 5.05 * 2.6, 8))
        hc <- c(3.6, 3.6, rep(2.6, 8))          #floor heights
        hf <- height()
        f <- length(v)
        #Number of stair compartments(m3)
        M <- c(rep(0, f))  #mass of air
        mc <- c(rep(0, f)) #3.6 * 14 * m
        CO <- 0.01        #initial concentration
        y <- matrix(rep(CO, n * f), nrow = n, ncol = f)
        yc <- matrix(rep(CO, n * f), nrow = n, ncol = f
                )
        cor <- matrix(rep(CO, n * f), nrow = n, ncol =
                f)
        sp <- c(rep(0, f))
        sp <- spress()
        sp2 <- sp
        for(i in 2:length(sp)) {
 #total stack pressure on floors
                sp2[i] <- sp[2] + sp[10] + sp2[i - 1] +
                        sp[i]
        }
        tm <- DATA[, "tm"]        #time
        for(g in 1:f) {
                M[g] <- m * v[g]
                mc[g] <- hc[g] * 14 * m * 1.5
        }
        for(i in 2:n) {
 #              if(hf[2] < np) {
 #floor height below nplane?
                rho <- ((C * h * sqrt(dct) * (A * xn[i -
                        1] * sqrt(sp2[2]) - (Ac * sqrt(
                        sp2[2]) + A * sqrt(sp2[2])) * y[
                        i - 1, 2]))/M[2])/alpha
                y[i, 2] <- y[i - 1, 2] + alpha *
                        bernoull(rho)      #              }
        }
        for(j in 3:f) {
                for(i in 2:n) {
```

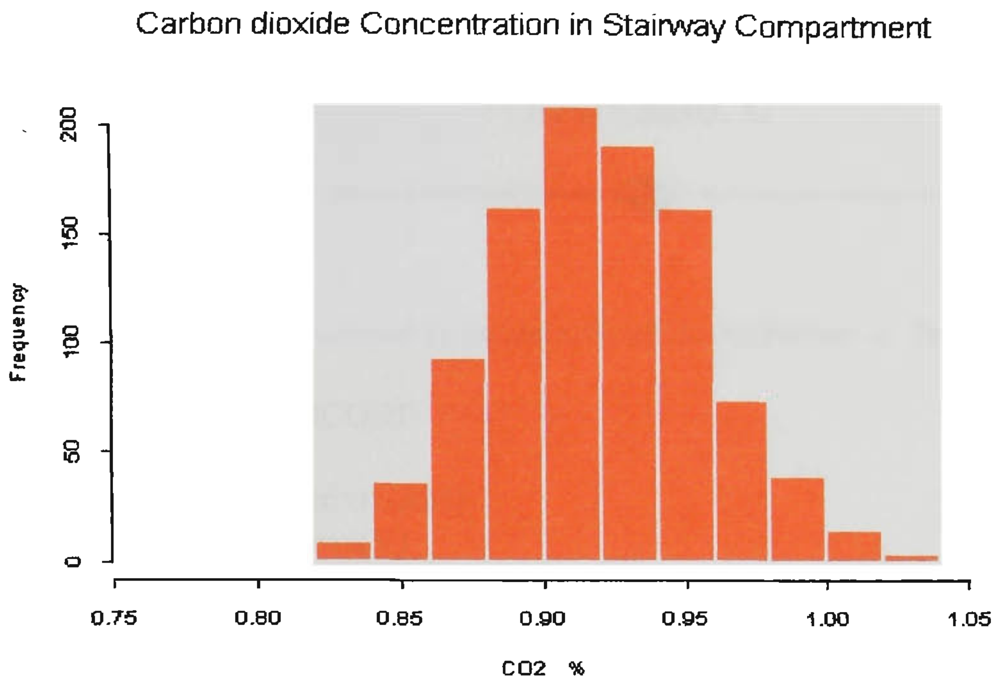
```
                                rho <- ((C * h * sqrt(dct) * (A *
                                          y[i - 1, j - 1] * sqrt(
                                          sp2[j]) - (Ac * sqrt(
                                          sp2[j]) + A * sqrt(sp2[
                                          j])) * y[i - 1, j]))/M[
                                          2])/alpha
                                y[i, j] <- y[i - 1, j] + alpha *
                                          bernoull(rho)
                                rho2 <- ((C * h * sqrt(dct) *
                                          Ac * y[i - 1, j] * sqrt(
                                          sp2[j]))/mc[j])/alpha
                                yc[i, 2] <- yc[i - 1, 2] +
                                          alpha * bernoull(rho2)
        #                                 }
                }
        }
        graphics.off()
        win.graph()
        matplot(tm, y[, 2], type = "l", xlab =
                "Time(min)", ylab = "CO2  %", main =
                "Carbon dioxide Concentration in Compartment(Stochastic,1 Simulation)"
                )
        matlines(tm, y[, 3], type = "l", col = 2)
        #       matlines(tm, y[, 4], type = "l", col = 3)
        matlines(tm, y[, 5], type = "l", col = 3)
        #       matlines(tm, y[, 6], type = "l", col = 5)
#       matlines(tm, y[, 7], type = "l", col = 4)
        matlines(tm, y[, 8], type = "l", col = 4)
        matlines(tm, y[, 9], type = "l", col = 5)
        matlines(tm, y[, 10], type = "l", col = 2)
        colour <- c(1, 2, 3, 4, 5)
        ltype <- c("l", "p", "p")
        pchar <- c("l", "l")
        leg.names <- c("f2", "f3", "f5", "f8", "f9",
                "f10")
        legend(locator(1), leg.names, col = colour, pch
                = pchar)
}
```

The following program simulates the spread of smoke for both temparature and species concentration distributions using Markov chain stochastic simulation:

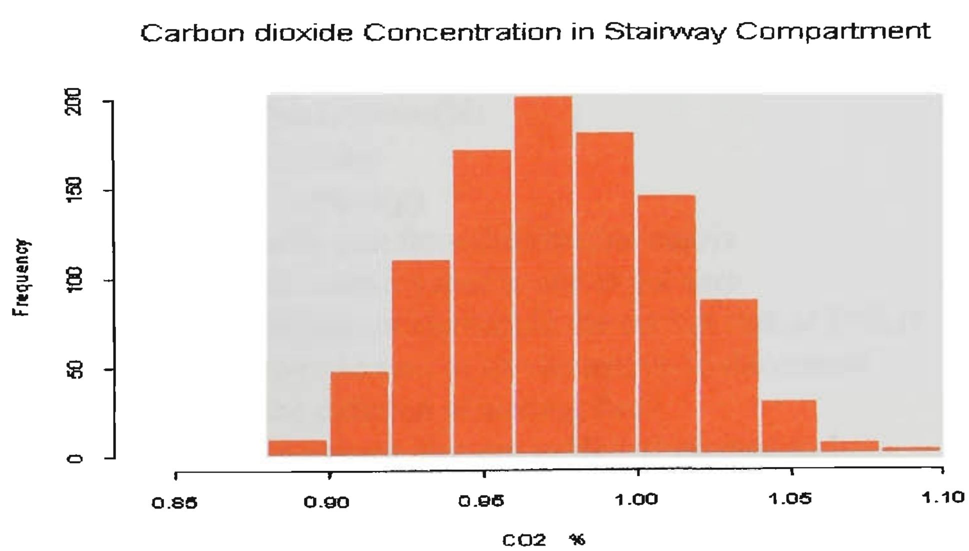
```
function()
{
#stoka: This file:computes concentration of smoke using the Markov chain
#Deterministic Model for storey building
#It considers effect of pressure differences
#Initial CO fraction        =       1%              =       0.01
#mass of air                =       0.70 Kg/m3
#Constant gas density is assumed.
#yn+1 = yn + C*h*sqrt(rho)*(A*x*sqrt(Psa)/mb - Ac*yn*sqrt(Psb)/mc - A*yn*sqrt(Ps3)/Mb)
        alpha <- 0.02
        alpha2 <- 1.8
        z <- 50
        C <- 0.65           #orifice flow constant
        Ac <- 2 * 0.8 * 0.1         # Area of opening to corridor
```

```
        A <- 5.05 * 2.48 * 0.6      #stairwell opening from level to level
#       DATA <- others2()
        DATA <- data()  #temp extracts input data from file
        Mi2 <- DATA[, "M"]
        TTi2 <- DATA[, "TT"]
        xn <- DATA[, "CO2"]
        tm <- DATA[, "tm"]
        T0 <- 297           #initial stairshaft temp in K(24 C)
        R <- 8320
        Cp <- 1.06          #specific heat
#       xn <- DATA[, "f2"]          #conc at source
        dct <- 0.8
        h <- 0.05
        h2 <- h * 1000
        np <- height()
        np <- np[length(np)]/2      #nplane halfway up
        n <- length(DATA[, 1])
        m <- 0.7 # Kg/m3, density of air
        v <- c(2.48 * 5.05 * 3.6, 2.48 * 5.05 * 3.6, rep(2.48 * 5.05 *
                2.6, 8))
        hc <- c(3.6, 3.6, rep(2.6, 8))          #floor heights
        hf <- height()
        f <- length(v)      #Number of stair compartments(m3)
        M <- c(rep(0, f))  #mass of air
        mc <- c(rep(0, f)) #3.6 * 14 * m
        CO <- 0.01          #initial concentration
        y <- matrix(rep(CO, n * f), nrow = n, ncol = f)
        TT <- matrix(rep(CO, n * f), nrow = n, ncol = f)
        ym <- matrix(rep(CO, z * f), nrow = z, ncol = f)
        M0 <- Mc <- yc <- cor <- y
        TTm <- TT
        dp <- spress()      #pressure from stack
        p0 <- press()       #External atmos pressure distr.
        p0 <- p0[2]         #Level external atmos pressure
        p <- p0 + dp
        sp <- c(rep(0, f))
        sp <- spress()
        y[, 1] <- xn
        M0[, 1] <- Mi2
        TT[, 1] <- TTi2
        Q <- heat(DATA)
        for(i in 2:n) {
                rho <- ((C * h * sqrt(dct) * (A * y[i - 1, 1] * sqrt(
                        sp2[2]) - (Ac * sqrt(sp2[2]) + A * sqrt(sp2[2]
                        )) * y[i - 1, 2]))/M0[i - 1, 1])/alpha
                rho2 <- (((R * TT[i - 1, 1] * h)/(p[i] * v[2])) * (abs(
                        M0[i - 1, 1]) * (TT[i - 1, 1] - T0) - Q[i]/Cp)
                        )/alpha2
                y[i, 2] <- y[i - 1, 2] + alpha * bernoull(rho)
                TT[i, 2] <- TT[i - 1, 2] + alpha2 * bernoull(rho)
#                       print(TT[i, 2])
        }
        for(j in 3:f) {
                for(i in 2:n) {
                        rho <- ((C * h * sqrt(dct) * (A * y[i - 1, j -
                                1] * sqrt(sp2[j]) - (Ac * sqrt(sp2[j]) +
                                A * sqrt(sp2[j])) * y[i - 1, j]))/M[j]
                                )/alpha
```

```
                    rho2 <- (( - 1^i) * (((R * TT[i - 1, j - 1] *
                              h)/(p[i] * v[j])) * (M0[i - 1, 1] * (
                              TT[i - 1, j - 1] - T0) - Q[i]/Cp)))/
                              alpha2
                    y[i, j] <- y[i - 1, j] + alpha * bernoull(rho)
                    TT[i, j] <- TT[i - 1, j] + alpha2 * bernoull(
                              rho)
                    M0[i, j] <- (M0[i - 1, j] * (TT[i, j] - T0) -
                              Q[i]/Cp)/TT[i, j]
          }
     }
     graphics.off()
     win.graph()
     matplot(tm, TT[, 2], type = "l", xlab = "Time(min)", ylab =
          "Temp (C)", main =
          "Gas Temp (C) in Compartment(Stochastic,1 Simulation)"
          )     #     matlines(tm, TT[, 2], type = "l", col = 2)
     matlines(tm, TT[, 3], type = "l", col = 2)
     #     matlines(tm, TT[, 4], type = "l", col = 3)
     matlines(tm, TT[, 5], type = "l", col = 3)
     #     matlines(tm, TT[, 6], type = "l", col = 5)
#    matlines(tm, TT[, 7], type = "l", col = 4)
     matlines(tm, TT[, 8], type = "l", col = 4)
     matlines(tm, TT[, 9], type = "l", col = 5)
     matlines(tm, TT[, 10], type = "l", col = 2)
     colour <- c(1, 2, 3, 4, 5)
     ltype <- c("l", "p", "p")
     pchar <- c("l", "l")
     leg.names <- c("f2", "f3", "f5", "f8", "f9", "f10")
     legend(locator(1), leg.names, col = colour, pch = pchar)
}
```



Histogram of Carbon dioxide concentration for the 29th minute on level 8 of the stairway



Histogram of Carbon dioxide concentration for the 29th minute on level 9 of the stairway

APPENDIX C

The following pseudocode is for the simulation performed in chapter 6.

SPREAD PSEUDOCODE

```
Import position matrix(GAM)
Import variable matrix(V)
Import HAZARD matrix(H)

#GAM is the position matrix
#V is a matrix of the possible values of the random variable for all the edges in the
network
#H is a matrix of the hazard function values derived from the hazard function

Ensure same result for all trials(Seed)
Obtain duration of starting fire(d)
Create HOLD container for delays (dim(No of Sim, No of Vertices)
Create SOURCE container that will indicate source of fire

Initialise first element of SOURCE as Origin of fire(e.g -1)
Initialise first elements of HOLD, for the duartion of fire, to be 1

Iterate for the No of Simulations(N)
Iterate for the No of Links(r)
Iterate for the No of Vertices(p)
        Obtain a variable unit from the variable matrix
        Ensure that the current variable unit is not zero
        Ensure that the current element in the current row of HOLD
          is zero and that the sum of its element up to the current
          is less than the duration of starting fire
        Ensure that the current element in HOLD is equal to 1
        Sum up the first elements of HOLD equal to the current
          No of iterations(y)
        Compute the HAZARD on current variable for the delay of y(x)
        Perform a Bernoulli trial x
        Ensure that the trial is 1
        Assign 1 to the elements of HOLD in first column for the
        duration of fire from the current simulation
        Assign element of current SOURCE to be current vertex
End Iterations

Plot graph for the simulations
```

Below is a computer programme that analysis a network giving the SOURCE matrix for each of the vertex. Other functions are called, which are not included here.

```
function()
{
#hazd1.4:This file computes the spread in a network using the
#hazard function
        H <- hf()       #computes hazard function
        V <- p <- matrix(c(5, 11, 18, 6, 9, 7, 12, 19,
                7, 10, 9, 15, 20, 8, 11, 10, 17, 22, 9,
                12), nrow = 5, ncol = 4)
        #variable input
#       p <- pb()
        links <- vertexs <- length(V[1, ])
        #No. of possible values of variable (links) on an edges
        edges <- length(V[, 1])         #No. of edges
        hold <- c(rep(0, edges))
        source <- temp <- c(rep(0, vertexs))
        #source[1] <- -1
        for(j in 1:edges) {
                tm <- max(V[j, ])
                for(i in 1:links) {
                        trial <- bernoull(H[i, j])
                        if((trial == 1) && (V[j, i] <=
                                tm))
                                tm <- V[j, i]
                }
                hold[j] <- tm
        }
        temp[2] <- min(hold[1], (hold[3] + hold[4]))
        temp[3] <- min(hold[4], (hold[1] + hold[3]))
        temp[4] <- min(temp[2], temp[3])
        if(temp[2] == hold[1])
                source[2] <- 1
        else source[2] <- 3
        if(temp[3] == hold[4])
                source[3] <- 1
        else source[3] <- 2
        if(temp[4] == temp[2])
                source[4] <- 2
        else source[4] <- 3
        tm <- times(hold)
        cat(" SOURCE\n", "VERTEXS\n", 1:vertexs, "\n",
                source, "\n")
}
```

```
function()
{
#This file: Computes the probability function and the hazard function graphs for
#Figure 6.2 (Chapter 6 )
        x <- 0.06
        a <- 3
        b <- 10
        cl <- a + b
        rw <- 1:cl
        n <- a
        h <- p <- hr <- c(rep(0, cl))
        for(j in a:cl) {
                h[j] <- x/(1 - (n - a) * x)
                hr[j] <- h[j - 1]
                p[j] <- x
                n <- n + 1
        }
        graphics.off()
        win.graph()
        barplot(p, histo = TRUE, besides = TRUE, ylab = "Probability",
                xlab = "")
        win.graph()
        barplot(h, histo = TRUE, besides = TRUE, ylab = "Hazard", xlab
                 = "")
}
```

APPENDIX D

The following SPLUS code is for the simulation performed in chapter 7.

```
function(V, H, r, u)
{
#HAZARD:yields the hazard at delay u for
#the (discrete)RV TT[r] representing the time of spread.
#V[r,] is the set of values of TT[r] and H[r,] the corresponding hazards.
        n <- dim(V)[2]
        y <- 0
        if(r != 0) {
                for(k in 1:n) {
                        if(V[r, k] == u) {
                                y <- H[r, k]
                                break
                        }
                }
        }
        y
}
```

```
function(NAM, VAR, H2)
{
#PBSCENE: simulates fire spread in a metropolitan network with
#adjacency matrix NAM, ascending values of times in VAR and hazard values in H2
#sctimes: determines the minimum time for the paths
#freqm: determines the probability function of times to transit network
#       set.seed(101)
        N <- 1000       #number of simulations
        p <- dim(NAM)[1]        #number of nodes
        n <- dim(VAR)[2]        #number of links
        m <- dim(VAR)[1]        #number of edges
        RES <- matrix(0, nrow = N, ncol = 2)
        HAT <- c(rep(1, n))
        TT <- c(rep(0, m))
        for(i in 1:N) {
                for(l in 1:m) {
                        for(j in 1:n) {
                                HAT[j] <- bernoulli(H2[m, j])
                                if(HAT[j] != 0)
                                  HAT[j] <- VAR[m, j]
                                else HAT[j] <- NA
                        }
                        TT[l] <- min(HAT, na.rm = T)
                }
                RES[i, 1] <- sctimes(TT)
```

```
            RES[i, 2] <- 1/N
        }
        z <- length(RES[1,  ])
        graphics.off()
        win.graph()
        hist(RES[, 1], xlab = "Time to transit network", ylab =
            "Frequency")
        fm <- freqm(RES)
        p <- dim(fm)[1]
        cdf <- matrix(0, nrow = p, ncol = 1)
        cdf <- fm[, 2]
        for(s in 2:p)
            cdf[s] <- cdf[s - 1] + cdf[s]
        win.graph()
        plot(fm[, 1], cdf, type = "l", xlab =
            "Time to transit network (min)", ylab =
            "Cummulative Distribution")
        z <- cbind(fm, cdf)
        z
}
```

APPENDIX E

The SPLUS main program that simulates the egress of an occupant from a building is presented below. The program calls others.

```
function()
{
# evac: This file simulation egress from a building given the
#matrix containing the respective time distribution for the compartments
# (nodes).
#S82 = distribution of time of cue to reach occupant,
#U810 = distribution of Untenable time to s1.1
#U812 = distribution of Untenable time to s1.2
#T25 = distribution of Time to start of evacuation of a2.2
#T54 = distribution of time from c2.1 to s2.1
#T56 = distribution of time from c2.1 to s2.2
#T410 = distribution of time from s2.1 to s1.1
#T612 = distribution of time from s2.2 to s1.2
#node is an nxn signed node matrix of the probability of paths
#adjacency matrix is EE, indicates node numbers
        N <- 1000       #No. of simulations
        node <- node()
        nodeprb <- nodeprb()
        EE <- EE()
        eg <- dim(node)[1]      #No. of Nodes
        HOLD <- matrix(0, nrow = N, ncol = 3)
        RES <- matrix(0, nrow = N, ncol = eg)
        S82 <- U10 <- U12 <- C21 <- matrix(0, 2, 2)
        HOLD[, 2] <- 1
        LL <- "s1.1"
        LR <- "s1.2"
        orig <- "a2.2"
        U10[1, ] <- c(50, 70)
        U10[2, ] <- U12[2, ] <- c(0.5, 0.5)
        U12[1, ] <- c(60, 80)
        C21[1, ] <- c(1, 2)
        C21[2, ] <- c(0.5, 0.5)
        S82[1, ] <- c(15, 60)
        S82[2, ] <- c(0.2, 0.8)
        prbC21 <- 1
        Tunt <- cnt <- dcnt <- ecnt <- 0
        for(k in 1:N) {
```

```
        prbU10 <- prbU12 <- 1
        Tcue <- lottery(S82[1,  ])
        prbc <- prob(Tcue, S82)
        Tcum <- Tcue
#Time of cue to occupant
        for(i in 1:eg) {
                for(j in 1:eg) {
                        if(node[i, j] > 0)
                         {
                         if(nodeprb[i, j] > 0) {
                           if(EE[i, j] > 0 ) {
                             if(EE[i, j] == 5) {
                               x <- lottery(C21[2, 1])
                               if(x == 1)
                                 {#Left

                                node[5,  6] <- node[6, 12] <- node[12,14] <- -1
                                prbC21 <- C21[2, 1]
                                  HOLD[k, 2] <- HOLD[k, 2] * prbC21
                                Tunt <- lottery(U10[1,  ])
                                prbU10 <- prob(Tunt, U10)
                                  HOLD[k, 2] <- HOLD[k, 2] * prbU10
                               }
                               else {#Right
                                node[5, 4] <- node[4, 10] <- node[10,3] <- -1
                                prbC21 <- C21[2, 2]
                                  HOLD[k, 2] <- HOLD[k, 2] * prbC21
                                Tunt <- lottery(U12[1, ])
                                prbU12 <- prob(Tunt,U12)
                               }
                             }
                             dis <- distr(j)
                             if(dis) {
                               tt <- lottery(dis[1,  ])
                               prb <- prob(tt, dis)
                               HOLD[k, 2] <- HOLD[k, 2] * prb * prbU10 *
                                prbU12
                               HOLD[k, 1] <- Tcum
                               for(m in 1: tt) {
                                if(Tcum > Tunt)
                                  {
                                  HOLD[k,1] <- Tcum
                                  dieshow2(EE[i, j], Tcum)
                                  dcnt <- dcnt + 1
                                  break
                                }
```

```
                                        Tcum <- Tcum + 1
                                      }
                                    }
                                  }
                                }
                              }
                      }
              }
      }
      cat("Probability of death is = ")
      cat(sum(HOLD[, 2]/N))
      print(" ")
      z <- length(HOLD[1, ])
      graphics.off()
      win.graph()
      hist(HOLD[, 1], xlab =
              "Time to Evacuate Building (min)",
              ylab = "Frequency")
      fm <- freqm(HOLD[, 1:2])
      p <- dim(fm)[1]
      cdf <- matrix(0, nrow = p, ncol = 1)
      cdf <- fm[, 2]
      for(s in 2:p)
              cdf[s] <- cdf[s - 1] + cdf[s]
      win.graph()
      plot(fm[, 1], cdf, type = "l", xlab =
              "Time to Evacuate Building (min)",
              ylab = "Cumulative density")
      z <- cbind(fm, cdf)
      print(z)
}
```

The computer output for the simulation is shown below.

```
> evac()
Probability of death for occupant in a2.2 is 0.02
y     z     cdf
[1,] 17 0.03000 0.03000
[2,] 18 0.02000 0.05000
[3,] 19 0.01125 0.06125
[4,] 60 0.00900 0.07025
[5,] 61 0.03750 0.10775
[6,] 62 0.02625 0.13400
     y     z     cdf
[1,] 17 0.03000 0.03000
```

```
[2,] 18 0.02000 0.05000
[3,] 19 0.01125 0.06125
[4,] 60 0.00900 0.07025
[5,] 61 0.03750 0.10775
[6,] 62 0.02625 0.13400
```

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