Drought Assessment and Forecasting Using a Nonlinear Aggregated Drought Index

By

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Dedicated

To

My Parents, Elder Brother and My Beloved Wife

Who are My Inspirations

ABSTRACT

Drought is a natural phenomenon, and has widespread and significant impacts on the world's economy, environment, industries and the community. Early detection of droughts helps to implement drought mitigation strategies and measures, before they occur. Therefore, drought forecasting plays an important role in the planning and management of water resources systems, especially during dry climatic periods. However, drought assessment and forecasting are not always easy. This study developed an objective drought assessment tool through a Drought Index (DI), and then developed a drought forecasting tool using the developed DI to forecast future drought conditions. These tools can be used to assist water managers to assess droughts effectively and forecast future drought conditions, which will allow them to plan ahead the water management activities during droughts. The Yarra River catchment in Australia was considered as the case study catchment, since the management of water resources in this catchment has great importance to majority of Victorians.

To achieve the objectives of this study, an evaluation of existing DIs was first conducted in terms of their suitability for the assessment of drought conditions in the study catchment. Based on the findings of the evaluation study, a new Nonlinear Aggregated Drought Index (NADI) was developed and evaluated for the Yarra River catchment. The NADI considers five hydro-meteorological variables (i.e., rainfall, potential evapotranspiration, streamflow, storage reservoir volume and soil moisture content) that affect the droughts. It proved to be more useful than the other DIs in this study. It uniquely describes the broad perspective of drought beyond the traditional meteorological, hydrological and agricultural subcategories, and also more representative of the fluctuations in water resources variables within the hydrologic cycle. The Artificial Neural Network (ANN) technique, which has proved to be one of the most successful drought forecasting modeling techniques, was then used to develop and test several drought forecasting models to forecast the NADI values as drought forecasts. The results showed that the best developed drought forecasting models were capable of forecasting drought conditions reasonably well up to 6 months ahead.

The major innovation of this study was the development of the NADI for assessing drought conditions. Moreover, the new drought forecasting model using the NADI, forecasts the overall dry conditions better than the traditional rainfall based DI forecasting models.

DECLARATION

I, Shishutosh Barua, declare that the PhD thesis entitled ‘Drought Assessment and Forecasting Using a Nonlinear Aggregated Drought Index’ is no more than 100,000 words in length including quotes and exclusive of tables, figures, appendices, bibliography, references and footnotes.

This thesis contains no material that has been submitted previously, in whole or in part, for the award of any other academic degree or diploma. Except where otherwise indicated, this thesis is my own work.

Signature ……………….........

Shishutosh Barua

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LIST OF PUBLICATIONS AND AWARDS

Journal Articles:

1. **Barua, S.**, Ng, A.W.M. and Perera, B.J.C. 2010, *Comparative Evaluation of Drought Indices: A Case Study on the Yarra River Catchment in Australia*, ASCE Journal of Water Resources Planning and Management, (In Press).

2. **Barua, S.**, Perera, B.J.C., Ng, A.W.M. and Tran, D. 2010, *Drought Forecasting using an Aggregated Drought Index and Artificial Neural Network*, IWA Journal of Water and Climate Change, (In press).

3. **Barua, S.**, Ng, A.W.M. and Perera, B.J.C. 2010, *Artificial Neural Network Based Drought Forecasting using a Nonlinear Aggregated Drought Index*, Water Resources Research Journal, (Under review).

4. **Barua, S.**, Perera, B.J.C. and Ng, A.W.M. 2010, *Artificial Neural Network Based Drought Forecasting using an Aggregated Drought Index*, Journal of Hydrology, (Under Review).

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6. **Barua, S.**, Muttil, N., Shanmugasundram, S., Ng, A.W.M. and Perera, B.J.C. 2010, *Rainfall Trend Analysis and Implications to Water Resources in the Yarra River catchment, Australia*, Journal of Hydrological Processes, (Under review).

Conference Articles:

7. **Barua, S.**, Perera, B.J.C. and Ng, A.W.M. 2010, *Drought Forecasting using an Aggregated Drought Index and Artificial Neural Networks*, Proceedings of IWA World Water Congress and Exhibition 2010, Montreal, Canada, September 19-24, 2010, (In CD).

8. **Barua, S.**, Perera, B.J.C. and Ng, A.W.M. (2009), *Drought Assessment using Nonlinear Principal Component Analysis*. Proceedings of 32nd Hydrology and Water Resources Symposium, Newcastle, Australia, November 30–December 3, 2009, pp. 1341 – 1352.

9. **Barua, S.**, Ng, A.W.M. and Perera, B.J.C. (2009), *Drought Forecasting: A Case Study within the Yarra River Catchment in Victoria*, *Australia*, Proceedings of IEEE Science and Engineering Graduate Research Expo 2009, The University of Melbourne, Australia, September 25, 2009, pp. 25 – 27.

10. **Barua, S.**, Perera, B.J.C. and Ng, A.W.M. (2009), *A Comparative Drought Assessment of Yarra River Catchment in Victoria*. Proceedings of 18th World IMACS Congress and MODSIM09 International Congress on Modelling and Simulation. Cairns, Australia, July 13-17, 2009, pp. 3245-3251.

11. **Barua, S.**, Ng, A.W.M. and Perera, B.J.C. (2009), *Development of an Aggregated Drought Index using Hydrometeorological Variables*, Proceedings of Australia's National Water Conference and Exhibition, Ozwater'09, Melbourne, Australia, March 16 – 18, 2009, ISBN: 978-1-921335-08-2.

12. Muttil, N., **Barua, S.**, Ng, A.W.M. and Perera, B.J.C. (2009), *Spatio-Temporal Analysis to Detect Climate Changes within the Yarra River Catchment*, Proceedings of Australia's National Water Conference and Exhibition, Ozwater'09, Melbourne, Australia, March 16 – 18, 2009, ISBN: 978-1-921335-08-2, (Partially related to this thesis).

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

LIST OF ABBREVIATIONS

The following list of abbreviations is used throughout this thesis. The other abbreviations, which were used only in particular sections or chapters are defined in the relevant sections or chapters.

ADI	Aggregated Drought Index
ANFIS	Adaptive Neuro-Fuzzy Inference System
ANN	Artificial Neural Network
ARIMA	Autoregressive Integrated Moving Average
ASCE	American Society of Civil Engineers
BOM	Bureau of Meteorology
BP	Back Propagation
DI	Drought Index
DMSNN	Direct Multi-Step Neural Network
EPA	Environmental Protection Authority
NADI	Nonlinear Aggregated Drought Index
NLPCA	Nonlinear Principal Component Analysis
OPR	Office of Post-graduate Research
PC	Principal Component
PC1	First Principal Component
PCA	Principal Component Analysis
PN	Percent of Normal
RMSNN	Recursive Multi-Step Neural Network
SARIMA	Seasonal Autoregressive Integrated Moving Average
SPI	Standardized Precipitation Index
SWSI	Surface Water Supply Index

1. INTRODUCTION

Background; Motivation for this Study; Aims of the Study; Research Methodology in Brief; Research Significance, Outcomes and Innovations; Outline of the Thesis

1.1. Background

Drought is one of the world's costliest natural disasters, causing an average US$6–8 billion in global damages annually, and affecting more people than any other form of natural catastrophe (Keyantash and Dracup, 2002). In Australia, many parts of land suffer from frequent droughts. Australia is often referred to as the driest inhabited continent on earth and this is evident when rainfall and run-off are compared with other countries (Davidson, 1969; Pigram, 2006). It has also been seen that many parts of Australia have experienced their worst single and multi-year droughts on record over the last decade (Tan and Rhodes, 2008). These recent frequent droughts have severely stressed water supply systems and the community that depends on them. Drought management has therefore become an important issue especially in south-eastern Australia. The frequency of drought occurrences is reasonably well studied for the purpose of drought management using the historic time series of hydro-meteorological variables such as rainfall and streamflow. However, the forecasting aspect, which is very important from the point of view of drought preparedness and early warning, is still fraught with great difficulty (Panu and Sharma, 2002). Nevertheless, Beran and Rodier (1985) and Panu and Sharma (2002) suggested that it may be possible to forecast the probable timings of inception and termination of droughts reasonably well over a short period such a month or a season.

Typically, when a drought event and resultant disaster occur, governments and donors follow impact assessments, and response, recovery and reconstruction activities, to return the region or locality to a pre-disaster state. All these activities are generally followed with the assessment of the past drought condition which is often carried out with a drought assessment tool. Although, in general people cope with drought impacts by taking recovery actions after any drought, the society can reduce drought vulnerability and therefore lessen the risks associated with droughts by making a future drought plan. Moreover, the likelihood of increasing frequency, duration and severity of droughts in Australia due to possible climate change impacts reinforces the need for future drought plans (Hennessy *et al.*, 2008). Therefore, it is well recognized that preparedness for drought is the key to the effective mitigation of drought impacts which is becoming more important for water resources managers to handle the challenges in water resources management.

There are several methods that have been used in the past as the drought assessment tools such as measurement of lack of rainfall, shortage of streamflow, reduced levels of water storage, and Drought Indices (DIs). Of these, DIs are widely used for drought assessment (Heim Jr, 2002; Keyantash and Dracup, 2002; Smakhtin and Hughes, 2004; Morid *et al.*, 2006). DI is a function of a number of hydro-meteorological variables (e.g., rainfall and streamflow), and expresses with a numeric number which is more functional than raw data during decision making (Hayes, 2003). However, defining an appropriate DI is not always an easy task, and researchers and professionals face challenges for developing a suitable DI (Panu and Sharma, 2002). Therefore, the development of an appropriate DI for defining drought conditions is the first task in this thesis.

Historically little attention has been given to drought forecasting aspect which is very important from the point of view of drought preparedness and early warning as mentioned earlier. Because of this emphasis on crisis management, many societies have generally moved from one disaster to another with little, if any, reduction in risk (Wilhite, 2005). In addition, in drought-prone regions, another drought event is likely to occur before the region fully recovers from the last event. However, early indication of drought conditions could reduce future impacts and lessen the need for government intervention in the future (Panu and Sharma, 2002; Wilhite, 2005). Therefore, the development of a drought forecasting tool that can be used for drought preparedness is considered as the second task in this thesis.

1.2. Motivation for this Study

In Australia, drought and drought management have always been an important issue in the context of water resources management. In this regard, drought forecasting activities in Australia had been more intensified in recent years because of the occurrences of the longest dry circumstances in history in the last decade (Neal and Moran, 2009). However, as was mentioned in Section 1.1, drought forecasting is not yet well advanced. Therefore, the lack of an appropriate drought forecasting tool to forecast future drought conditions was the principle motivation of this research project.

This study was also motivated by the fact that there is a lack of an appropriate drought assessment tool (such as a DI) that can be used to define critical drought conditions for providing government support to the drought affected community (Senate Standing Committee on Rural and Regional Affairs, 1992). In 1992, the Commonwealth and State governments in Australia agreed on a National Drought Policy (NDP) (Senate Standing Committee on Rural and Regional Affairs, 1992), which was then reaffirmed in 1994 and revised in 1997 (White and Karssies, 1999). In the NDP, the concept of drought Exceptional Circumstances (EC) was introduced to provide support to farmers and rural communities. To be classified as a drought EC event, the event must be rare, that is, it must not have occurred more than once on average in every 20-25 years. However, it has shown that based on the concept of drought EC, some regions have been continuously drought-declared for 13 of the past 16 years (Hennessy *et al.*, 2008). The Australian primary industries ministers of the Commonwealth and State Governments have now agreed that the current approach of defining drought EC is no longer the most appropriate in the context of a changing climate (Hennessy *et al.*, 2008). Therefore, the development of an appropriate drought assessment tool (i.e., DI) to define drought conditions including drought EC is an important issue.

This study was also motivated by the fact that although there are many DIs developed around the world, the majority of the existing DIs were developed for specific regions. Suitability of these DIs had not been tested for Australia, although few studies have been carried out in other parts of the world (Heim Jr, 2002; Keyantash and Dracup, 2002; Smakhtin and Hughes, 2004; Morid *et al.*, 2006).

1.3. Aims of the Study

The main aim of this research project was to develop a drought forecasting tool to forecast future drought conditions across short to medium term time horizons. In addition, the development of a generic DI was the sub-aim of this study. For the development of generic DI, a DI evaluation study on the existing DIs was first considered. In the development of the generic DI, all important hydro-meteorological variables responsible for droughts were considered so that the generic DI can provide an objective method for defining drought conditions including the drought EC. Once the generic DI namely Nonlinear Aggregated Drought Index (NADI) was developed, the time series of this index was considered for developing drought forecasting models to forecast NADI values to represent the future drought conditions.

1.4. Research Methodology in Brief

In order to achieve the abovementioned aims, the following tasks were conducted in this research project:

1. Review of drought assessment and drought forecasting methods
2. Selection of study area, and data collection and processing
3. Evaluation of selected drought indices
4. Development of a generic Nonlinear Aggregated Drought Index
5. Development of drought forecasting models

Brief descriptions of each of the above tasks are given below.

1.4.1. Review of Drought Assessment and Drought Forecasting Methods

As was mentioned in Section 1.1, there are several methods that have been used in the past as the drought assessment tools. Among those, several DIs have been most commonly used for drought assessment by researchers and professionals around the world (e.g., Gibbs and Maher, 1967; Shafer and Dezman, 1982; McKee *et al.*, 1995; Keyantash and Dracup, 2004). However, most of these DIs were developed for specific regions, and some DIs are better suited than others for specific uses (Redmond, 2002; Hayes, 2003). Therefore, a review of the existing DIs was first conducted in this research to understand the suitability of existing DIs for use in the regions outside of those for which they were originally developed for. Similarly, there are several drought forecasting modeling techniques that have been used for developing drought forecasting models (e.g., Kim and Valdes, 2003; Mishra and Desai, 2005; Barros and Bowden, 2008; Cutore *et al.*, 2009). A review of these techniques was carried out to select the appropriate drought forecasting modeling technique for developing drought forecasting models in this study.

1.4.2. Selection of Study Area, and Data Collection and Processing

The Yarra River catchment in Victoria (Australia) was selected as the case study area in this research to develop and evaluate the DIs and drought forecasting model. This catchment was selected, since the management of water resources in this catchment has great importance to majority of Victorians and one third of Victorian population depends on the water resources of this catchment. Hydro-meteorological data (for several locations in the Yarra River catchment) were collected from several organizations to use in this research. Data processing was then carried out to obtain the catchment representative values which were used for development and evaluation of the DI and the drought forecasting model.

1.4.3. Evaluation of Selected Drought Indices

As was mentioned in Section 1.4.1, existing DIs were developed mostly for use in specific regions, and therefore may not be directly applicable to other regions due to inherent complexity of drought phenomena, different hydro-climatic conditions and catchment characteristics (Redmond, 2002; Smakhtin and Hughes, 2007). There had been few DI evaluation studies around the world (Heim Jr, 2002; Keyantash and Dracup, 2002; Smakhtin and Hughes, 2004; Morid *et al.*, 2006). However, no such study had been conducted in Australia as was mentioned in Section 1.2. Therefore, an evaluation of some selected DIs was carried out to investigate the most appropriate DI for defining drought conditions in the Yarra River catchment. A number of decision criteria were used in this DI evaluation study.

1.4.4. Development of a Generic Nonlinear Aggregated Drought Index

Based on the findings of the DI evaluation study (Section 1.4.3), a generic DI namely, the Nonlinear Aggregated Drought Index (NADI) was developed using five hydro-meteorological variables (i.e., rainfall, potential evapotranspiration, streamflow, storage reservoir volume and soil moisture content) for the case study catchment. The NADI was developed to overcome an important limitation of the Aggregated Drought Index (ADI), which was found to be the best DI for the Yarra River catchment in the DI evaluation study (Section 1.4.3). Nonlinear Principal Component Analysis (NLPCA) was introduced and used to aggregate the above mentioned five hydro-meteorological variables in this study to develop the NADI. The NADI was then evaluated to investigate how well this DI defined drought conditions for the Yarra River catchment.

1.4.5. Development of Drought Forecasting Model

Several drought forecasting models based on the different combinations of potential input variables were developed to forecast the NADI values as future drought forecasts. The Artificial Neural Network (ANN) which was found to be the best suitable technique for developing drought forecasting models (e.g., Kim and Valdes, 2003; Mishra and Desai, 2006; Mishra *et al.*, 2007; Barros and Bowden, 2008; Cutore *et al.*, 2009) was used in this research. Two types of ANN architectures namely, Recursive Multi-Step Neural Network (RMSNN) and Direct Multi-Step Neural Network (DMSNN) were used in the model development. The best drought forecasting model for each type of ANN architectures was then selected by conducting a comparative performance evaluation between the developed models.

1.5. Research Significance, Outcomes and Innovations

In this section, the significance of the research project and the overall outcomes are discussed. A list of innovative ideas which have been evolved from this study is also presented in this section.

1.5.1. Significance

This research project has produced several significant contributions in the field of water resources management, especially in the management of water resources during continuing dry climatic conditions. These contributions are outlined below:

- As was mentioned in Section 1.2, majority of the existing DIs were developed for use in a specific region, and the suitability of these DIs had not been investigated for any Australian catchment, although some suitability studies had been carried out in other parts of the world. Therefore, the evaluation of existing DIs that was carried out in this research was the first study for any Australian catchment. The ADI was found to be the best DI amongst the existing DIs investigated in this study. The overall outcome of this DI evaluation study (Section 1.5.2) was a valuable contribution to the hydrologic and water resource management community throughout the world in general and to Australia in particular.

- The outcome of the quantitative assessment of DIs has provided evidence that by considering all potential hydro-meteorological variables responsible for droughts, the drought conditions including the drought EC can be defined more robustly than other DIs which consider only rainfall as the single variable (Section 1.5.2). This study also led to the development of the generic NADI. The developed NADI in this study is therefore a significant contribution towards developing drought impact assistance plans by providing an objective method for defining drought EC.

- The developed drought forecasting model is a useful tool, which can become part of a drought preparedness and early warning system to provide early indication of future drought conditions.

1.5.2. Outcomes

The outcomes of this study are outlined below:

- Amongst the existing DIs, the ADI was found to be the best DI for defining drought conditions of the Yarra River catchment. However, the use of linear Principal Component Analysis (PCA) in ADI assumed that the hydro-meteorological variables which were used for developing the ADI have linear relationships between them and therefore could not capture the nonlinear relationship between the variables, when they exist.

- The NLPCA technique which was introduced in this study in developing the new generic NADI was able to capture the nonlinear relationships between the variables. It also showed that NLPCA was able to explain more variance of the data of the input variables than the traditional linear PCA.

- The NADI was found to be the most suitable DI for defining drought conditions within the Yarra River catchment. A comparative study that was conducted between NADI and ADI showed that the NADI was a better DI than the ADI.

- The ANN modeling technique was found to be the most suitable technique for developing the drought forecasting models to forecast future drought conditions.

- As was mentioned before, two types of ANNs namely - RMSNN and DMSNN were used for developing the drought forecasting models in this study. Forecasting drought conditions in the Yarra River catchment using the RMSNN model were found to be slightly better than the DMSNN model for forecast lead times of 2 to 3 months, and the DMSNN model had given slightly better forecasting results than the RMSNN model for lead times of 4 to 6 months.

1.5.3. Innovations

Several innovative ideas were developed in this study which are outlined below:

- The major innovation of this study was the development of a generic DI, namely NADI for assessing drought conditions. The NADI has also proved that it was more representative of the fluctuations in water resources variables within the hydrologic cycle. The NADI also proved to be more robust than the other DIs.

- The NLPCA technique was widely used in many engineering fields such as in computer science, mainly for data reduction and signal processing purposes. However, this study was the first to apply this technique for data aggregation and has proven success by capturing the nonlinear relationships between the hydro-meteorological variables in the NADI development.
- Drought forecasting models were developed in this study using the time series of NADI to forecast the NADI values as forecasts of future drought conditions. These models forecast the overall dry conditions within the catchment better than the traditional rainfall based DIs which forecast only the meteorological drought conditions.

1.6. Outline of the Thesis

The outline of the thesis is presented in Figure 1.1. This figure shows that the thesis consists of seven chapters. The first chapter describes the background of the research project, the motivation for the study, the aims, a brief methodology, the significance, outcomes and innovations of this project. The second chapter presents a critical review of literature related to the research project. Details on the case study catchment and its importance to the Victorians, drought history in Victoria, data used in this research, and their sources and processing are illustrated in the third chapter. The fourth chapter provides details on the evaluation of selected DIs for the case study catchment. Development of the NADI and its evaluation for drought assessment in the Yarra River catchment are presented in the fifth chapter. The sixth chapter provides the development of drought forecasting models, their performance evaluation, and the selection of the best models in this study. Finally, a summary of the thesis and the main conclusions, and the recommendations for future work are presented in the seventh chapter.

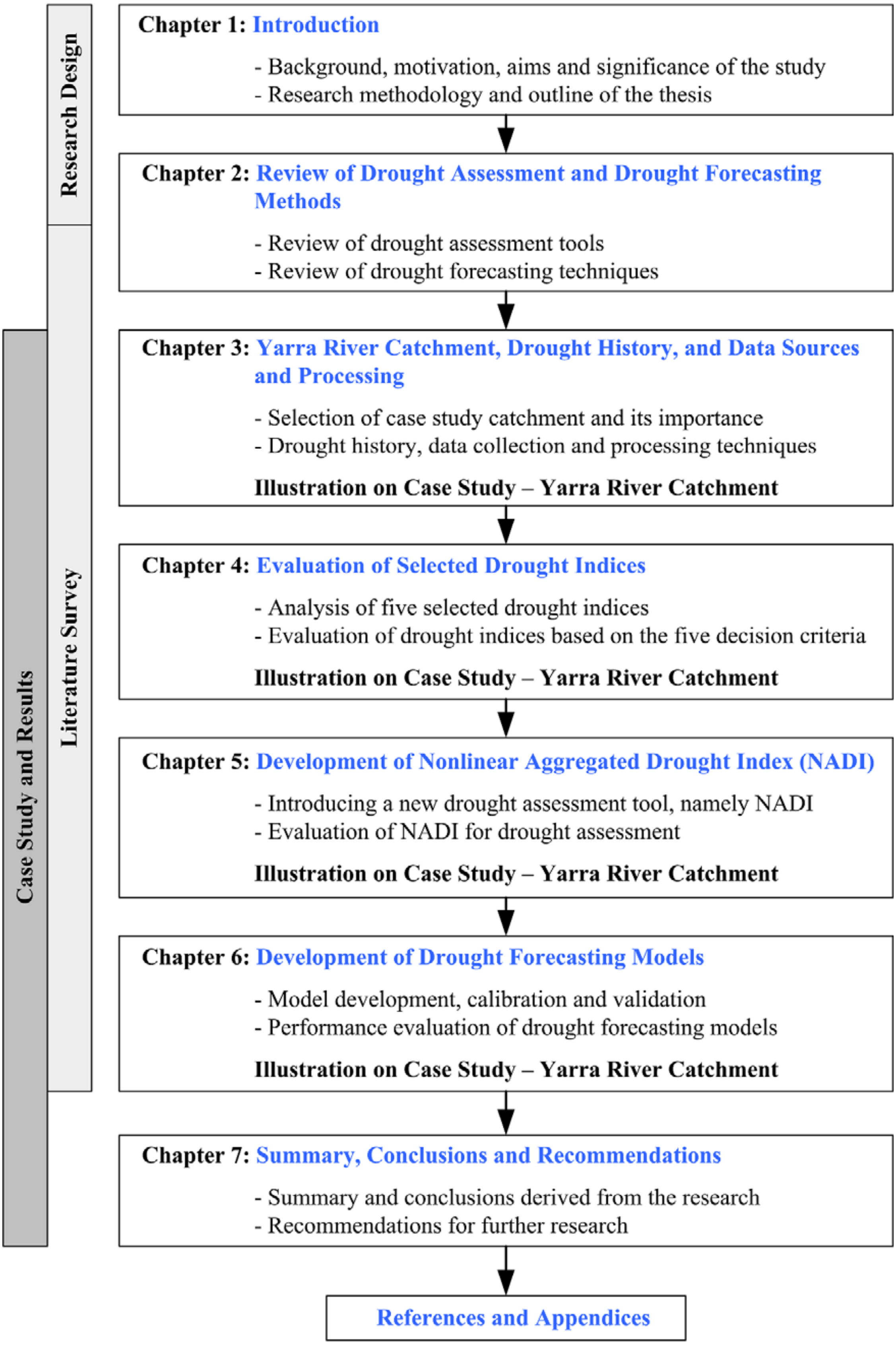


Figure 1.1 Outline of the thesis

2. REVIEW OF DROUGHT ASSESSMENT AND DROUGHT FORECASTING METHODS

Overview; Drought Assessment Tools; Drought Forecasting Techniques; Summary and Conclusions

2.1. Overview

Drought is a complex natural phenomenon and has significant impacts on effective water resources management. In general, drought gives an impression of water scarcity due to insufficient precipitation, high evapotranspiration and over-exploitation of water resources or a combination of all above (Bhuiyan, 2004). There are three main drought categories, i.e., *meteorological*, *hydrological* and *agricultural* droughts. The *meteorological* drought is expressed solely based on the level of dryness measured in terms of rainfall deficiency (Keyantash and Dracup, 2004). The *hydrological* drought, on the other hand, is defined based on deficiency in water availability in terms of streamflow, reservoir storage and groundwater depths (Wilhite, 2000). The a*gricultural* drought is expressed based on soil moisture deficits, and considers rainfall deficits, soil water deficits, variation of evapotranspiration, etc. (Hounam, 1975). In addition, the American Meteorological Society (1997) introduced another drought category called *socio-economic* drought. This category of drought occurs when physical water shortages start to affect the health, well-being and quality of life of people. This drought starts to affect the supply and demand of economic products such as water, fish production, hydroelectric power generation, etc. Drought places enormous demand on rural and urban water resources, and immense burden on agricultural and energy production. Therefore, timely determination of the level of drought will assist the decision making process in reducing the impacts of drought.

As was mentioned in Section 1.1, there are several methods that have been used in the past as drought assessment tools such as measurement of lack of rainfall, shortage of streamflow, Drought Indices (DIs) among others. However, traditionally the estimation of future dry conditions (or drought forecasting) has been conducted using DIs as the most common drought assessment tools. This is because the DI is expressed by a number which is believed to be far more functional than raw data during decision making (Hayes, 2003). The DI in general is a function of several hydro-meteorological variables such as rainfall, temperature, streamflow and storage reservoir volume. In defining DIs, some researchers and professionals argue that drought is just deficiency in rainfall and can be defined with the rainfall as the single variable. Based on this concept, majority of the available DIs including Percent of Normal (PN) (Hayes, 2003), Deciles (Gibbs and Maher, 1967) and many others were developed with rainfall as the only variable. These rainfall based DIs are widely used than other DIs due to their less input data requirements, flexibility and simplicity of calculations (Smakhtin and Hughes, 2004). However, other drought researchers and professionals believe that rainfall based DIs do not encompasses drought conditions of all categories of droughts, since they can be used only for defining meteorological droughts (Keyantash and Dracup, 2004; Smakhtin and Hughes, 2004). Smakhtin and Hughes (2004) also stated that the definition of droughts should consider significant components of the water cycle (such as rainfall, streamflow and storage reservoir volume), because the drought depends on numerous factors, such as water supplies and demands, hydrological and political boundaries, and antecedent conditions (Steinemann, 2003). Byun and Wilhite (1999) had also supported this idea previously, stating that a valid drought index should comprise of a mixture of hydro-meteorological variables. Based on these ideas, two DIs, viz., Surface Water Supply Index (SWSI) (Shafer and Dezman, 1982) and Aggregated Drought Index (Keyantash and Dracup, 2004), had also been developed considering a number of hydro-meteorological variables such as rainfall, streamflow and others. It should also be noted that most of the DIs that had been developed were regionally based and some DIs are better suited than others for specific uses (Redmond, 2002; Hayes, 2003; Mishra and Singh, 2010). Therefore, a review of the existing DIs is necessary before adapting any of the existing DIs, for use in areas/catchments outside those areas for which they were originally developed.

Similarly, there are several techniques that have been used for developing drought forecasting models including Autoregressive Integrated Moving Average (ARIMA) (Mishra and Desai, 2005), Markov Chains (Paulo and Pereira, 2007) and

Artificial Neural Network (ANN) (Kim and Valdes, 2003; Mishra and Desai, 2006; Mishra *et al.*, 2007; Barros and Bowden, 2008; Cutore *et al.*, 2009). Understanding these techniques will help in selecting the appropriate method in developing a drought forecasting model.

There are two aims of the current chapter: (1) to review the existing drought assessment tools (e.g., DIs) which have been used to define the drought conditions, and (2) to review the existing drought forecasting modeling techniques which have been used for developing drought forecasting models. The outcome of this chapter will be the identification of the most suitable DI and drought forecasting technique for use in this research.

The chapter first reviews the existing drought assessment tools, followed by the review of the existing drought forecasting techniques. A summary of the review are presented at the end of the chapter.

2.2. Drought Assessment Tools

As stated earlier, there are several drought assessment tools that have been used in the past, and of these, Drought Indices (DIs) have been most commonly used to assess drought conditions around the world, since it is more functional than raw data in decision making. These DIs were used to trigger drought relief programs and to quantify deficits in water resources to assess the drought severity. Also, they were used as drought monitoring tools.

In the mid-twentieth century, Palmer (1965) first introduced a DI called Palmer Drought Severity Index (PDSI) in the U.S. to define meteorological droughts using a water balance model. The application of PDSI became popular immediately after its development and was the most prominent DI used in the U.S. until its limitations were recognized by Alley (1984). There were also other DIs developed around the world at different times including the widely used Percent of Normal (PN) (Willeke *et al.*, 1994), Deciles (Gibbs and Maher, 1967), Standardized Precipitation Index (SPI) (McKee *et al.*, 1993) and Surface Water Supply Index (SWSI) (Shafer and Dezman, 1982), and the theoretically sound Aggregated Drought Index (ADI) (Keyantash and Dracup, 2004). There are many other DIs that have been developed around the world which have limited use or which are regionally based. Details of some of the well known DIs and their usefulness are presented below.

It should be noted that mathematical details on any of the DIs are not provided in this section, as all of them were not used in this research. Mathematical details will be provided only for the DIs that were used in this study in the various sections of Chapters 4 and 5. Moreover, the threshold ranges of these DIs which are used to classify droughts will also be provided in these chapters. The reader is referred to the original references for details of the other DIs.

2.2.1. Palmer Drought Severity Index

Palmer (1965) developed a meteorological drought index (widely known as Palmer Drought Severity Index (PDSI)) considering criteria for determining when a drought or a wet spell begins and ends. These criteria considered the following two conditions: (1) an abnormally wet month in the middle of a long-term drought should not have a major impact on the index, and (2) a series of months with near-normal rainfall following a serious drought does not mean that the drought is over. The PDSI is also described in detail by Alley (1984).

The PDSI uses a simple monthly water budget model, with inputs of rainfall, temperature and available catchment soil moisture content. It does not consider streamflow, lake and reservoir levels or other hydro-meteorological variables that affect droughts (Karl and Knight, 1985). Human impacts on the water balance, such as irrigation, are also not considered. The PDSI is also known as the Palmer Hydrological Drought Index (PHDI). This is because the concept used in the model is based on moisture inflow (i.e. rainfall), outflow as moisture loss due to temperature effect and storage as soil moisture content (Karl and Knight, 1985).

The PDSI has been widely used for a variety of applications across the United States including the States of New York, Colorado, Idaho and Utah as part of their drought monitoring systems and also to trigger drought relief programmes (Loucks and van Beek, 2005). It was most effective at measuring impacts that were sensitive to soil moisture conditions (Willeke *et al.*, 1994). It has also been useful as a drought monitoring tool, and had been used to trigger actions associated with drought contingency plans (Willeke *et al.*, 1994). Alley (1984) identified three positive characteristics of the PDSI that contributed to its popularity:

1) It provided decision-makers with a measurement of the abnormality of recent weather for a region,
2) It provided an opportunity to place current conditions in historical perspectives, and
3) It provided spatial and temporal representations of historical droughts.

Although the PDSI has been widely used within the U.S., it has little acceptance elsewhere (Smith *et al.*, 1993; Kogan, 1995). It does not do well in regions where there are extremes in variability of rainfall or runoff, such as in Australia and South Africa (Hayes, 2003). Nevertheless, the usefulness of the PDSI had been tested outside the U.S. by several researchers including Bruwer (1990) and du Pisani (1990) in South Africa. They found that the PDSI was a poor indicator of short-term (i.e., periods of several weeks) changes in moisture status affecting crops and farming operations. The limitations of the application of PDSI had also been highlighted by Alley (1984), Karl and Knight (1985) and Karl (1986).

2.2.2. Percent of Normal

Percent of Normal (PN) is one of the simplest drought monitoring tools which is commonly used by the TV weathercasters and general audiences (Hayes, 2003). The U.S. Army Corps of Engineers have used the PN across the U.S. for setting policy, making short and long term plans for water use, and also for making operational decisions. The PN was also used in New South Wales, Australia by Osti *et al.* (2008) for classifying drought conditions. It is expressed as the actual rainfall in percentage compared to the normal rainfall. Usually either the long-term mean or median rainfall value at a location was used as the normal rainfall and was considered as 100% (Hayes, 2003; Morid *et al.*, 2006). One of the disadvantages of using the PN is that the mean rainfall is often not the same as the median rainfall Hayes (2003), and therefore there is a confusion in selecting the mean or the median for defining the PN. The PN can be calculated for a variety of time scales including monthly, multiple monthly (e.g., 1-, 3-, 6-, 12-, 24- and 48- month), seasonal and annual or water year. However, PN in multiple monthly time scales is still calculated for each month except accumulating only the specified numbers of present and past month rainfall values. For example, 3- month April PN value means that the rainfall from February to April month were accumulated and used to calculate the PN 3- month April value. There are no threshold ranges published for this DI; usually, lower PN (PN<100%) values indicate dry conditions. Further details on PN are given in Section 4.3.1.

2.2.3. Deciles

Deciles were developed by Gibbs and Maher (1967) in Australia. In Deciles, the long-term monthly rainfall record is first ranked from highest to lowest to construct a cumulative frequency distribution. The distribution is then divided into ten parts, which are called "deciles". The first "decile" is the rainfall amount not exceeded by the lowest 10% of the rainfall occurrences. The second "decile" is the rainfall amount between the lowest 10 and 20% of occurrences. The definitions of these deciles continue until the rainfall amount identified by the tenth "decile" which is the largest "decile" within the long-term record.

The Deciles approach has been used as the meteorological measurement of droughts within the Australian Drought Watch System, as it was relatively simple to calculate, and required less data and fewer assumptions than more comprehensive DIs such as the PDSI (Smith *et al.*, 1993). In this system, farmers and ranchers could request government assistance only if the drought was considered to have a return period of more than 20-25 years (i.e., approximately in deciles 1 and 2) and has lasted for longer than 12 months (White and O'Meagher, 1995). One disadvantage of the Deciles approach is that a reasonable long rainfall record (i.e., generally 30-50 years) is required to calculate the deciles accurately (Hayes, 2003). However, Smakhtin and Hughes (2004) noted that this was not a shortcoming of the approach, but rather a requirement of the statistical analysis. Further details on Deciles are given in Section 4.3.2.

2.2.4. Surface Water Supply Index

The Surface Water Supply Index (SWSI) was developed by Shafer and Dezman (1982) for Colorado, U.S.A., as an indicator of the surface water conditions. The objective of the development of the SWSI was to incorporate both hydrological and climatological features into a single index (Shafer and Dezman, 1982; Doesken *et al.*, 1991). Four input variables are required to calculate the SWSI such as snow water content, streamflow, rainfall and storage reservoir volume. The snow water content, rainfall and storage reservoir volume are used to calculate the SWSI values for the winter months. During the summer months, streamflow replaces the snow water content.

To determine the SWSI for a particular river basin, the monthly catchment average values are calculated first at all rainfall, reservoirs and snow water content/streamflow measuring stations over the basin. These monthly data are then fitted to individual probability distributions for each month and for each input variable. Each variable has a weight assigned individually for each of the 12 months depending on its typical contribution to the surface water within that basin. The weighted variables are summed to determine the SWSI value representing the entire basin for each month. In determining variable weights, Shafer and Dezman (1982) hypothesized that the additive nature of the variables causes the SWSI to be normally distributed and they have used the Chi-square statistic (Cochran, 1952) to optimize the goodness of fit.

The SWSI has been used, to trigger the activation and deactivation of the Colorado Drought Plan. It has also been applied in other western states in the U.S. including Oregon, Montana, Idaho and Utah. One of the advantages of SWSI is that it gives a representative measurement of surface water supplies across the basin (Shafer and Dezman, 1982). However, the SWSI had not considered some of the other important hydro-meteorological variables such as soil moisture content and potential evapotranspiration that affect droughts. The SWSI also suffers from another limitation, which is the subjectivity involved in determining weights that are used in SWSI calculations. Furthermore, the additional changes in the water management within the basin, such as flow diversions or new reservoirs, mean that the entire SWSI algorithm for that basin needs to be redeveloped to account for these changes. Thus, it is difficult to maintain a homogeneous time series of the index (Heddinghaus and Sabol, 1991). Further details on SWSI are given in Section 4.3.4.

2.2.5. Standardized Precipitation Index

McKee et al. (1993) developed the Standardized Precipitation Index (SPI) as an alternative to the PDSI for Colorado, U.S.A. The SPI was designed to quantify the rainfall deficit as a drought monitoring tool, and has been used to monitor drought conditions across Colorado since 1994 (McKee *et al.*, 1995). Monthly maps of the SPI for Colorado can be found on the Colorado State University home page (http://ulysses.atmos.colostate.edu/SPI.html). It is also currently being used by the National Drought Mitigation Center and the Western Regional Climate Center in the U.S.

To calculate SPI, the long-term historical rainfall record is fitted to a probability distribution (generally the gamma distribution), which is then transformed into a normal distribution (McKee and Edwards, 1997).

The rainfall deficits over different time scales have different impacts on different water resources components such as groundwater, soil moisture content and streamflow. As an example, soil moisture conditions respond to rainfall anomalies relatively quickly (e.g., days/weeks to a month), while groundwater, streamflow and reservoir storage reflect the longer-term rainfall anomalies (e.g., months to seasons). Because of this reason, the SPI was originally calculated for a monthly or multiple monthly time scales (i.e. 1-, 3-, 6-, 12-, 24- and 48- month) as in the PN (McKee *et al.*, 1993).

To date, SPI finds more applications around the world than other DIs due to its less input data requirements and flexibility in the SPI calculations (Hughes and Saunders, 2002; Hayes, 2003; Bhuiyan, 2004; Smakhtin and Hughes, 2004; Mishra and Desai, 2005; Morid *et al.*, 2006; Bacanli *et al.*, 2008). Osti *et al.* (2008) used the SPI to identify and characterize droughts in New South Wales (NSW), Australia. Barros and

Bowden (2008), used the SPI to forecast drought conditions within the Murray-Darling River basin in Australia. Although, the SPI has more popularity than any other DI, it is not strong enough to define the wider drought conditions since many other important hydro-meteorological variables (e.g., streamflow, soil moisture condition, evapotranspiration and reservoir storage volume) that affect droughts were not considered in SPI (Keyantash and Dracup, 2004; Smakhtin and Hughes, 2004). Further details on SPI are given in Section 4.3.3.

2.2.6. Aggregated Drought Index

The Aggregated Drought Index (ADI) is a multivariate DI developed by Keyantash and Dracup (2004) in California, U.S. It comprehensively considers all categories of drought (i.e., meteorological, hydrological and agricultural) through selection of input variables that are related to each drought type. The ADI input variables represent volumes of water fluctuating within the catchment. The six hydro-meteorological variables (i.e., rainfall, streamflow, potential evapotranspiration, soil moisture content, snow water content and reservoir storage volume) were used as the input variables to calculate the ADI (Keyantash and Dracup, 2004). These variables may be used selectively, depending on the characteristics of the catchment of interest. For example, if a region does not have snow, then snow water content can be omitted in the ADI calculation. Similar to the SWSI, the ADI has been used to assess the surface water conditions within the basin/catchment. However, the SWSI does not consider potential evapotranspiration and soil moisture content which have important roles in drought occurrence.

The Principal Component Analysis (PCA) was used as the numerical approach to extract the essential hydrologic information from the input data set to develop the ADI. The PCA has been widely used in the atmospheric and hydrologic sciences to describe dominant patterns appearing in observational data (Barnston and Livezey, 1987; Lins, 1997; Hidalgo *et al.*, 2000). The PCA was carried out on the hydro-meteorological monthly data for three selected catchments in California, U.S in the study carried out by Keyantash and Dracup (2004). The first Principal Component (PC) was considered as the ADI value for each month (Keyantash and Dracup, 2004), since it explains the largest fraction of the variance described by the full *p*-member standardized data set, where *p* is the number of input variables.

The ADI thresholds values were calculated probabilistically for the selected catchment using the empirical cumulative distribution function of ADI values. These thresholds are used to classify the drought conditions. Keyantash and Dracup (2004) used the SPI thresholds to generate the ADI thresholds. The SPI dryness thresholds are the Gaussian variates of -2, -1.5, -1 and 1 standard deviations, which correspond to 2.3^{th}, 6.7^{th}, 16.0^{th} and 84.0^{th} percentiles in SPI distributions. The ADI values corresponding to these percentiles were used as the ADI thresholds for the catchment for which ADI values were developed.

The ADI methodology provides an objective approach for describing wider drought conditions beyond the traditional meteorological droughts. However, the use of PCA in ADI assumes that the variables have linear relationships between them in formulating Principal Components (PCs) (Monahan, 2000, 2001; Linting *et al.*, 2007). Therefore, if nonlinear relationships exists between the variables which are used in the ADI, then the PCs generated through PCA will represent less variance in the data than expected (Linting *et al.*, 2007). Moreover, difficulties may arise in data poor regions in getting all required data to develop the ADI. Further details on ADI are given in Section 4.3.5.

2.2.7. Other Drought Indices

There are many other DIs that were cited in the literature which had limited applications. A list of some of these DIs and their brief descriptions are presented in Table 2.1. It can be seen from this table that majority of the DIs were developed using the rainfall as the single variable. Also it can be seen that most of these DIs have limited use, mainly in the U.S.A. Further details on the applicability of these DIs and their limitations can be found in Alley (1984), Keyantash and Dracup (2002), Heim Jr (2002), Tsakiris *et al.* (2002), Morid, *et al.* (2006), Hayes (2003), Smakhtin and Hughes (2004), and Loucks and van Beek (2005).

Table 2.1 Other Drought Indices

Drought Index	**Drought Definition**	**Application**
Munger Index (Munger, 1916)	Length of period in days with daily rainfall less than 1.27 mm.	Daily measure of comparative forest fire risk in the Pacific Northwest, U.S.A.
Kince Index (Kincer, 1919)	30 or more consecutive days with daily rainfall less than 6.35 mm.	Producing seasonal rainfall distribution maps in the U.S.A.
Marcovitch Index (Marcovitch, 1930)	Drought Index = $\frac{1}{2}(N/R)^2$; where *N* is the total number of two or more consecutive days above 32.2 ^{0}C in a month and *R* is the total rainfall for the month.	Alarming bean beetle in the eastern United States of America.
Blumenstock Index (Blumenstock Jr, 1942)	Length of drought in days, where drought is terminated by occurrence of 2.54 mm of rainfall in 2 days.	Short term drought management in the U.S.A.
Keetch - Byram Index (KBDI) (Keetch and Byram, 1968)	Rainfall and soil moisture analyzed in a water budget model with a daily time step.	Used by fire control managers for wildfire monitoring and prediction in the U.S.A.

Crop Moisture Index (CMI) (Palmer, 1968)	The CMI was developed from procedures within the calculation of the PDSI. The PDSI was developed to monitor long-term meteorologicall wet and dry spells, however the CMI was designed to evaluate short-term moisture conditions across major crop producing regions. The CMI is computed using the mean temperature and total rainfall for each week within the catchment, as well as the CMI value of the previous week.	Used in the U.S. to monitor week to week changes in moisture conditions affecting crops.
Reclamation Drought Index (RDI) (Weghorst, 1996)	RDI is calculated at the river basin (or the catchment) level using a monthly time step, and incorporates temperature, rainfall, snow water content, streamflow and reservoir levels.	Used as a tool for defining drought severity and duration, which assisted the Bureau of Reclamation in the U.S.A. in providing drought mitigation measures.
Effective Drought Index (EDI) (Byun and Wilhite, 1999)	The EDI is the rainfall amount needed return to normal condition (or to recover from the accumulated deficit since the beginning of a drought).	Used to monitor day to day drought conditions in the U.S.A. It was also tested in Iran (Morid *et al.*, 2006).

2.3. Drought Forecasting Techniques

There are several modeling techniques that have been used to develop drought forecasting models including Autoregressive Integrated Moving Average (ARIMA) and Seasonal Autoregressive Integrated Moving Average (SARIMA) (Mishra and Desai, 2005; Durdu, 2010), Markov Chain (Paulo and Pereira, 2007), Loglinear (Moreira *et al.*, 2008), Artificial Neural Network (ANN) (Kim and Valdes, 2003; Mishra and Desai, 2006; Mishra *et al.*, 2007; Barros and Bowden, 2008; Cutore *et al.*, 2009) and Adaptive Neuro-Fuzzy Inference System (ANFIS) (Bacanli *et al.*, 2008). These techniques have been used to forecast the DI values to represent future drought conditions for future planning of water resources management activities. In the review, it was found that two types of drought forecasting models have been used around the world to provide: (1) deterministic forecasts where the models were used to forecast DI values for future time steps from the current time step; ARIMA, SARIMA, ANN and ANFIS models come under this type, and (2) probability based drought class transition forecasts where the models were used to estimate the probability drought class transition from one stage at current time step to another for future time steps; Markov Chain and Loglinear models fall under this type of models. Details of these drought forecasting modeling techniques are discussed below.

It should be noted that in-depth mathematical details and calibration procedures for any of the drought forecasting modeling techniques are not provided in this section, as all of them were not used in this research. Mathematical details and calibration procedures will be provided only for the technique that was used in this study; these will be given in various sections in Chapter 6. The reader is referred to the original references for details of the other techniques.

2.3.1. Autoregressive Integrated Moving Average Model

A useful class of models for time series forecasting called Autoregressive Moving Average (ARMA (p, q)) models is formed by combining an Autoregressive (AR) model of order *p* and a Moving Average (MA) model of order *q*. This ARMA (p, q) model is defined by Equation (2.1).

$$\begin{aligned} x_t &= \phi_1 x_{t-1} + \ldots + \phi_p x_{t-p} + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \ldots - \theta_q \varepsilon_{t-q} \\ &= \sum_{i=1}^{p} \phi_i x_{t-i} + \varepsilon_t - \sum_{j=1}^{q} \theta_j \varepsilon_{t-j} \end{aligned} \tag{2.1}$$

where, x_t	is the random variable at discrete time (usually $t = 0, \pm1, \pm2, ...$)
ε_t	independent random variables with mean μ and variance σ_ε^2
$\phi_1, \ldots, \phi_p$	are coefficients for AR model
$\theta_1, \ldots, \theta_q$	are coefficients for MA model

Also known as Box-Jenkins type models (Thyer, 2001), this class of models has been widely used by many researchers and professionals for hydrological time series simulation. With ARMA (p, q) models, it is assumed that the time series to be modeled is stationary and approximately normal (Salas *et al.*, 1980; Chatfield, 2003). However, in practice most time series are non-stationary (Chatfield, 2003). Therefore, the ARMA (p, q) model has been extended to Autoregressive Integrated Moving Average (ARIMA) model (also known as ARIMA (p, d, q) model) in order to fit the non-stationary time series by allowing differencing of the time series (Salas *et al.*, 1980; Chatfield, 2003). Differencing of the time series removes its non-stationarity and it can be done for first, second or in general the d^{th} times. As an example, the first and second order differences are defined by Equations (2.2) and (2.3) respectively. Salas *et al.* (1980) noted that in practice only one or two difference operations are used, and Chatfield (2003) mentioned that the first difference is often found to be adequate in many cases.

$$u_t = x_t - x_{t-1} \tag{2.2}$$

$$w_t = u_t - u_{t-1} = (x_t - x_{t-1}) - (x_{t-1} - x_{t-2}) = x_t - 2x_{t-1} + x_{t-2} \tag{2.3}$$

The behavior of the differenced series u_t or w_t can be then represented in ARIMA (p, d, q) model by Equation (2.4).

$$u_t = \sum_{i=1}^{p} \phi_i u_{t-i} + \varepsilon_t - \sum_{j=1}^{q} \theta_j \varepsilon_{t-j}$$
$$\text{or} \qquad (2.4)$$
$$w_t = \sum_{i=1}^{p} \phi_i w_{t-i} + \varepsilon_t - \sum_{j=1}^{q} \theta_j \varepsilon_{t-j}$$

Box and Jenkins (1976) popularized the ARIMA (p, d, q) model for forecasting hydrologic variables including rainfall and streamflow. Mishra and Desai (2005) introduced the ARIMA model for drought forecasting in the Kansabati river basin in India, and this approach was recently adopted by Durdu (2010) in the Buyuk Menderes river basin, Western Turkey. Mishra and Desai (2005) developed the drought forecasting model to forecast SPI values for multiple time scales (i.e. 3-, 6-, 9-, 12- and 24- months). They found that their model is only able to forecast 1- month ahead SPI values, and was not able to forecast longer lead time values due to the seasonality effects in the data used in the model. Therefore, Mishra and Desai (2005) further improved the model by removing the seasonality effects in the data, which is discussed in Section 2.3.2.

2.3.2. Seasonal Autoregressive Integrated Moving Average (SARIMA) Model

In practice, most time series contain a seasonal periodic component, which repeats at every ω observations (Chatfield, 2003). With monthly observations, $\omega = 12$ and typically expect x_t to depend on values at annual lags, such as x_{t-12}, and perhaps x_{t-24}, as well as on more recent non-seasonal values such as x_{t-1} and x_{t-2}. Box and Jenkins (1976) generalized the ARIMA (p, d, q) model to deal with seasonality, and defined a general multiplicative seasonal ARIMA model using seasonal differencing in Equations (2.2) and (2.3) as defined by Equations (2.5) and (2.6).

$$u_t = x_t - x_{t-\omega} \qquad (2.5)$$

where, ω is the period. As stated earlier, typically ω is equal to 12 for monthly series. If necessary, to achieve stationarity the seasonal differencing could be repeated D times. For example, for $D = 2$ and $\omega = 12$

$$w_t = u_t - u_{t-12} = (x_t - x_{t-12}) - (x_{t-12} - x_{t-24}) = x_t - 2x_{t-12} + x_{t-24} \tag{2.6}$$

This model is commonly known as the Seasonal Autoregressive Integrated Moving Average (SARIMA) model. In short notation, the SARIMA model described as ARIMA (p, d, q) (P, D, Q)s, where (p, d, q) is the non-seasonal part of the model and (P, D, Q)s is the seasonal part of the model.

The SARIMA model was used by Mishra and Desai (2005) to develop a SPI-based drought forecasting model by removing seasonality. The technique was also used by Durdu (2010) in the Buyuk Menderes river basin, Western Turkey to forecast drought conditions using several time scales SPI time series. Both Mishra and Desai (2005) and Durdu (2010) found that their SARIMA models were able to give reasonably good results up to 2 month ahead drought forecasts. The results were better for higher time scale SPI series (i.e. 9-, 12-, and 24- month), and with these higher time scale SPI forecasting models, forecasting had produced good results up to 3 month ahead forecasts (Mishra and Desai, 2005). Both studies conducted by Mishra and Desai (2005) and Durdu (2010) demonstrated that the improved results might be due to the increase in rainfall accumulation in higher SPI time series which reduced the noise more effectively. Mishra and Desai (2005) also recommended that the SARIMA models can be used in other river basins for forecasting SPI series of multiple time scales.

2.3.3. Markov Chain Model

A Markov chain (Cinlar, 1975) is a stochastic process having the property that the value of the process at time t, X_t, depends only on its value at time $t - 1$, X_{t-1}, and not on the sequence of values X_{t-2}, X_{t-3}, . . ., X_0 that the process had passed through in arriving at X_{t-1}. This can be written as (Mishra *et al.*, 2009):

$$\begin{aligned}&\text{Prob }\left(X_t = a_j \middle| X_{t-1} = a_i, X_{t-2} = a_k, X_{t-3} = a_1, \ldots, X_0 = a_q\right)\\&\quad = \text{Prob }\left(X_t = a_j \middle| X_{t-1} = a_i\right)\end{aligned} \tag{2.7}$$

The conditional probability, Prob $\left(X_t = a_j \middle| X_{t-1} = a_i\right)$, gives the probability that the process at time t will be in ‘‘state j’’ given that at time t – 1 the process was in ‘‘state i’’. The Prob $\left(X_t = a_j \middle| X_{t-1} = a_i\right)$ is commonly called first-order transition probability and usually denoted by p_{ij}.

If a process is divided into m states, then m^2 transition probabilities must be defined. The m^2 transition probabilities can be represented by an (m x m) matrix P given as

$$P = \left[p_{ij}\right] = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1m} \\ p_{21} & p_{22} & \cdots & p_{2m} \\ p_{31} & p_{32} & \cdots & p_{3m} \\ \cdots & \cdots & \cdots & \cdots \\ p_{m1} & p_{m2} & \cdots & p_{mm} \end{bmatrix} \tag{2.8}$$

This is called the transition probability matrix of the Markov chain where sum of row elements is always 1 and the size of the matrix is equal to the number of states to be considered (Ochola and Kerkides, 2003). The transition probability matrix can be estimated from the observed data by tabulating the number of times the observed data had gone from state i to state j, n_{ij}. Then, an estimate of p_{ij} would be

$$p_{ij} = \frac{n_{ij}}{\sum_{j=1}^{m} n_{ij}} \tag{2.9}$$

Since the Markov chain approach was introduced by A. A. Markov in 1906, it has been widely applied in the disciplines of natural science, engineering, economics and management (Liu *et al.*, 2009). This approach has also been widely used in drought forecasting (Lohani and Loganathan, 1997; Lohani *et al.*, 1998; Paulo and

Pereira, 2007; Liu *et al.*, 2009). Paulo and Pereira (2007) stated that the Markov chain modeling approach is useful in understanding the stochastic characteristics of droughts through the analysis of probabilities for each severity class, times for reaching the non-drought class from any drought severity state, and residence times in each drought class. Lohani and Loganathan (1997) and Lohani *et al.* (1998) developed an early warning system for drought management using the PDSI and the Markov chain, in two climatic areas of Virginia (U.S.A.). The same approach was also adopted for developing a meteorological drought forecasting model by Liu *et al.* (2009) in Laohahe catchment in northern China. In their study, spatio-temporal distributions of PDSI were analyzed and forecasted by Markov chain. Steinemann (2003) adopted six classes of severity, from wet to dry conditions, similar to those in PDSI, and used the Markov chain to characterize probabilities for drought class and duration in a class. The results obtained were used to propose triggers for early-activating of the drought preparedness plans at the basin scale. Paulo *et al.* (2005), and Paulo and Pereira (2007) used the Markov chain approach to characterize drought conditions and forecast the SPI drought classes. They found that the approach can be satisfactorily used as a predictive tool for forecasting transitions among drought severity classes up to 3 months ahead. Liu *et al.* (2009) demonstrated two shortcomings of the Markov chain technique for forecasting drought conditions. They were: (1) the predictive performance decreased greatly as the severity of drought increased, and (2) the predictive performance was not always satisfactory for drought state transitions, and the prediction performance was only acceptable for the successive and smooth states.

2.3.4. Log linear Model

The log linear model is one of the special cases of generalized linear models which can be used with Poisson-distributed data. It is an extension of the linear modeling process that allows models to fit data that follow probability distributions other than the normal distribution, such as the Poisson distribution. It is also known as an extension of the two-way or two-dimensional contingency tables where the relationships between two or more discrete, categorical variables are analyzed by taking the natural logarithm of the cell frequencies within a contingency table. The variables used in the loglinear model are called as “response variables” (Agresti, 1990).

In a recent study, Moreira *et al.* (2008) used the three-dimensional loglinear model for drought forecasting in Alentejo and Algarve regions in Portugal using the 12-month SPI. With the three-dimensional loglinear model, it is possible to compute how most probable is the transition from class i to class j compared with the transition from class i to class k by analyzing the value of the odds. The odd is defined here by the ratio E_{ij} / E_{ik}, where E_{ij} and E_{ik} are the expected frequencies of transitions from class i to class j and from class i to class k respectively. Modeling is performed for the cell counts in contingency tables (Haberman, 1977). The Poisson sampling model is usually used for the counts in the contingency tables and assumes that the counts are independent Poisson random variables. The use of three-dimensional loglinear model aims at fitting the observed frequencies of transitions between each drought class, denoted as O_{ijk}, and to model the corresponding expected frequencies, denoted as E_{ijk}, which are the estimates of the observed frequencies for each cell of a three-dimensional contingency table (Table 2.2). In this table, four drought classes are considered. In general, several models are fitted to the response variables. Moreira *et al.* (2008) found that the quasi-association model is proved to be most adequate model with the drought class transitions. Further mathematical details on quasi-association model for the three-dimensional contingency table (i.e., Table 2.2) are given in Moreira *et al.* (2008).

Table 2.2 Three-dimensional contingency table for two consecutive transitions between drought classes (Moreira *et al.*, 2008)

Drought class at month $t-2$	Drought class month t															
	1				2				3				4			
	Drought class month $t-1$				Drought class month $t-1$				Drought class month $t-1$				Drought class month $t-1$			
	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
1	O_{111}	O_{121}	O_{131}	O_{141}	O_{112}	O_{122}	O_{132}	O_{142}	O_{113}	O_{123}	O_{133}	O_{143}	O_{114}	O_{124}	O_{134}	O_{144}
2	O_{211}	O_{221}	O_{231}	O_{241}	O_{212}	O_{222}	O_{232}	O_{242}	O_{213}	O_{223}	O_{233}	O_{243}	O_{214}	O_{224}	O_{234}	O_{244}
3	O_{311}	O_{321}	O_{331}	O_{341}	O_{312}	O_{322}	O_{332}	O_{342}	O_{313}	O_{323}	O_{333}	O_{343}	O_{314}	O_{324}	O_{334}	O_{344}
4	O_{411}	O_{421}	O_{431}	O_{441}	O_{412}	O_{422}	O_{432}	O_{442}	O_{413}	O_{423}	O_{433}	O_{443}	O_{414}	O_{424}	O_{434}	O_{444}

The above contingency table (i.e., Table 2.2), has three classification criteria, i.e. drought classes at months t - 2, t - 1 and t respectively with four levels from 1 to 4. The levels 1, . . ., 4 are associated with the drought classes: 1 - non-drought class, 2 - near normal drought class, 3 - moderate drought class, and 4 - severe/extreme drought class. The observed frequencies (O_{ijk}) are the response variables for the three-dimensional loglinear model and refer to the observed number of transitions between the drought class i at month $t – 2$, drought class j at month t - 1 and drought class k at month t. For example, the observation O_{111} is the number of times that a given study area stays for three consecutive months in drought class 1 (Non-drought). Therefore, the three-dimensional loglinear model allows modeling of the expected frequencies of drought class transitions corresponding to a 2 months step transition from drought class i to class j (t - 2 to t - 1) and from class j to class k (t - 1 to t).

The drought forecasting study using the three-dimensional loglinear model conducted by Moreira *et al.* (2008) showed that the technique was a useful tool for short-term drought warning systems, i.e., knowing the drought class values for two previous months it is possible to make reliable predictions for drought class in the two following months. However, they found that the forecast and the actual drought classes did not match in few cases. This happened when the SPI value was near the upper or lower boundary of the class, which can easily change to the nearby class when the rainfall deficit increases or higher rainfall occurs. Results showed that forecasting of drought class transitions was able to describe the trend of drought initiation and development as well as the trend for drought dissipation. It should be noted that the loglinear modeling for predicting more than two consecutive drought classes require a very large number of model parameters, and then the respective contingency tables become complex, and the interpretation of the model results is difficult (Moreira *et al.*, 2008). Hence, it is not foreseen to use the loglinear models to increase the lead time of forecasts. The approach is therefore appropriate only for short-term drought monitoring and mitigation measures, e.g. 2 months ahead.

2.3.5. Artificial Neural Network Model

An Artificial Neural Network (ANN) is an information processing system that resembles the structure and operation of the brain (Hassoun, 1995; ASCE Task Committee on Application of Artificial Neural Networks in Hydrology, 2000; Maier *et al.*, 2010). The ANN modeling approach was developed in the 1940s by McCulloch and Pitts (1943) and gradually progressed with advances in calibration methodologies

(Rumelhart *et al.*, 1986). Given sufficient data and complexity, ANN can be designed to model any relationship between a series of independent and dependent variables – inputs and outputs to the network respectively (Hornik *et al.*, 1990; ASCE Task Committee on Application of Artificial Neural Networks in Hydrology, 2000; Luk *et al.*, 2000). One of the advantages of the ANN technique is that there is no need for the modeler to fully define the intermediate relationships (i.e., physical processes) between inputs and outputs (Morid *et al.*, 2002; Anctil *et al.*, 2004; Tran *et al.*, 2009). This feature makes ANNs particularly suitable for the analysis of complex processes, like drought forecasting, where the relationships of a large number of input variables with the output need to be explored (Morid *et al.*, 2007). Because of this advantage, in recent years, the ANN modeling approach has been used in many research fields including drought forecasting (Sajikumar and Thandaveswara, 1999; Kim and Valdes, 2003; Mishra and Desai, 2006; Mishra *et al.*, 2007; Morid *et al.*, 2007; Barros and Bowden, 2008; Maity and Kumar, 2008; Ochoa-Rivera, 2008; Cutore *et al.*, 2009). A brief description of the ANN modeling approach is given below.

The ANN modeling approach mimics the human brain in processing information through a network of neurons which are connected together. In this sense, a biological neuron acts as a function which receives a set of input signals and produces an output. The biological nervous systems of humans come in various architecture; some are simple, while others are complex. But all these different types are composed of the same type of building blocks, called the neural cells or neurons. Figure 2.1 shows the basic structure of a biological neuron. A neuron receives signals or inputs, and produces a response or an output, after processing information through cell of the body of the neuron. In the biological neuron, the inputs and the response are electrical pulses. The input pulses are passed to the neuron through multiple channels (i.e. dendrites) and the output is passed forward via its only one output channel (i.e. axons). Each dendrite has a contact point (called synapses) which acts as a gate to open or close, and thus allows some input signals to flow in or stops some others depending on the mode of operation.

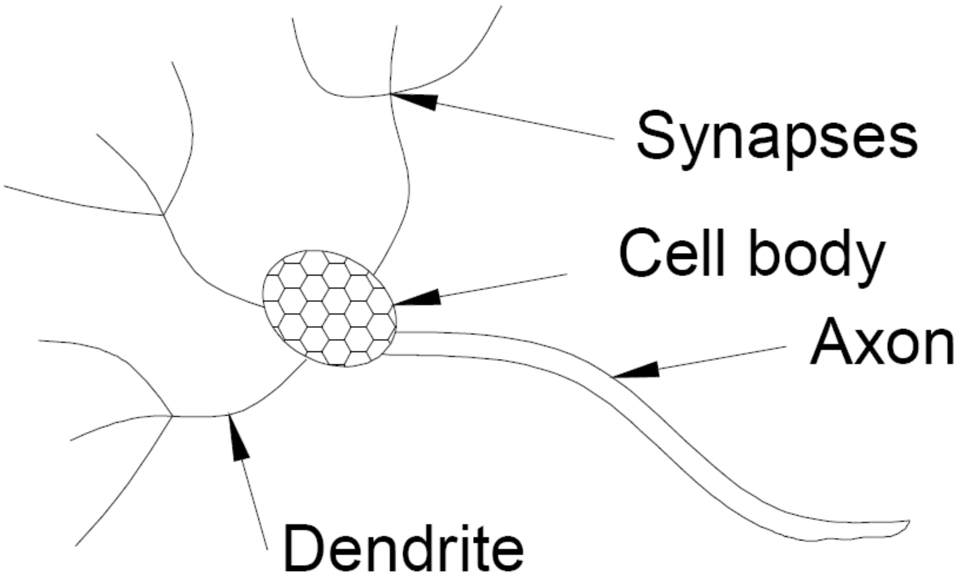


Figure 2.1 Basic structure of a biological neuron

Analogous to the biological neuron, an ANN has multiple-input channels (i.e. dendrites), a cell body and a output channel (i.e. axon), as shown in Figure 2.2. Like input pulses in the biological neuron, input signals (X_1, X_2, …, X_p) are passed to the neuron through multiple input channels. Each channel has an associated weight called connection weight. These weights (w_1, w_2, …, w_p) allow choosing the important signals among all input signals by their large weight values. The neuron has a special input signal to the cell of the body called bias weight (i.e. b as shown in Figure 2.2). The bias weight simulates the function of synapse that can allow (being non-zero value) and stop (being zero-value) the input signals going through cell body. The transmitted signals are integrated usually just by adding up all input signals. A mathematical function called the activation function (which will be discussed shortly) in the cell body is used to produce an output signal. The mathematical relationship between the input signals and the output in an ANN therefore can be formulated as:

$$Y = f\left(I\right) = f\left(\sum_{i}^{p} w_i X_i + b\right) \tag{2.10}$$

where: X_i is the input signal i

w_i is the weight attached to the input signal i

p	is the number of input signals
b	is the bias at the cell of the body
Y	is the output signal
f	is the activation function

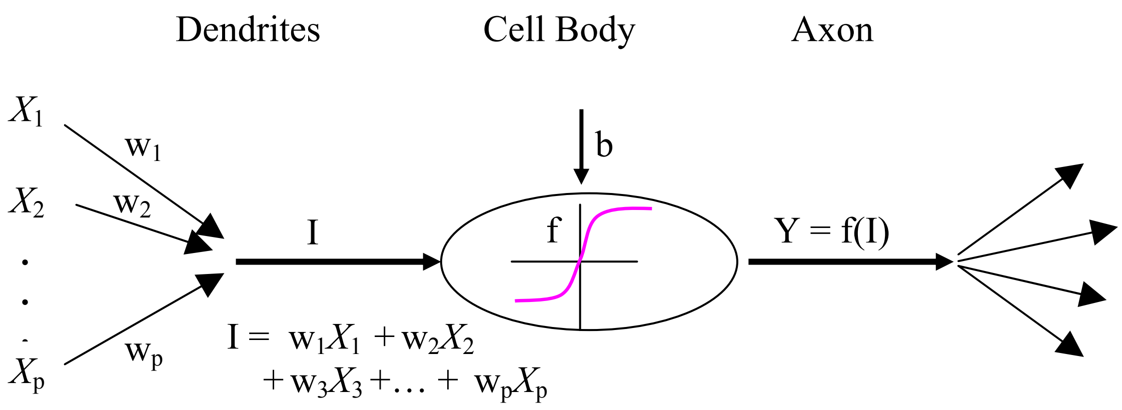


Figure 2.2 A typical artificial neural network

The activation function is a mathematical formula that gives the output of a processing element (i.e. cell body) considering its input signals (Maier and Dandy, 1998). It allows mappings between input and output signals (Maier and Dandy, 2000; Tran, 2007). Several activation functions can be used for the neurons. The non-linear sigmoidal, hyperbolic tangent and linear activation functions as shown in Figure 2.3 have often been used in ANN models as they proved to be successful in most cases (Karunanithi *et al.*, 1994; Maier and Dandy, 1998, 2000; Mishra and Desai, 2006). Furthermore, according to Bishop (1995), the use of these functions with a single hidden layer can approximate any non-linear relationship in real world problems.

There are various types of ANN model architectures that have been proposed and used. Generally, ANN architectures have been divided into two types, namely feed-forward and recurrent networks (Figure 2.4). In feed-forward networks, the information propagation is only in one direction, i.e., from input layer to the output layer. In recurrent networks, on the other hand, information may propagate not only in the forward direction but also in the backward direction through feedback loops. The output layer neurons may feed back the output to input and/or hidden layer neurons.

There is another type of ANN model architecture called hybrid architecture as shown in Figure 2.4 that uses hybrid modeling approaches to exploit the advantages of the available modeling paradigms in order to capture the complexities involved in environmental and hydrological systems. In this approach, several combinations of available modeling approaches are used to model the complexity involved in the problem to be studied. Details on different types of ANN model architecture are available in Fausett (1994) and Samarasinghe (2006). However, more details on some of the ANN model architectures used in this study will be presented in Chapter 6.

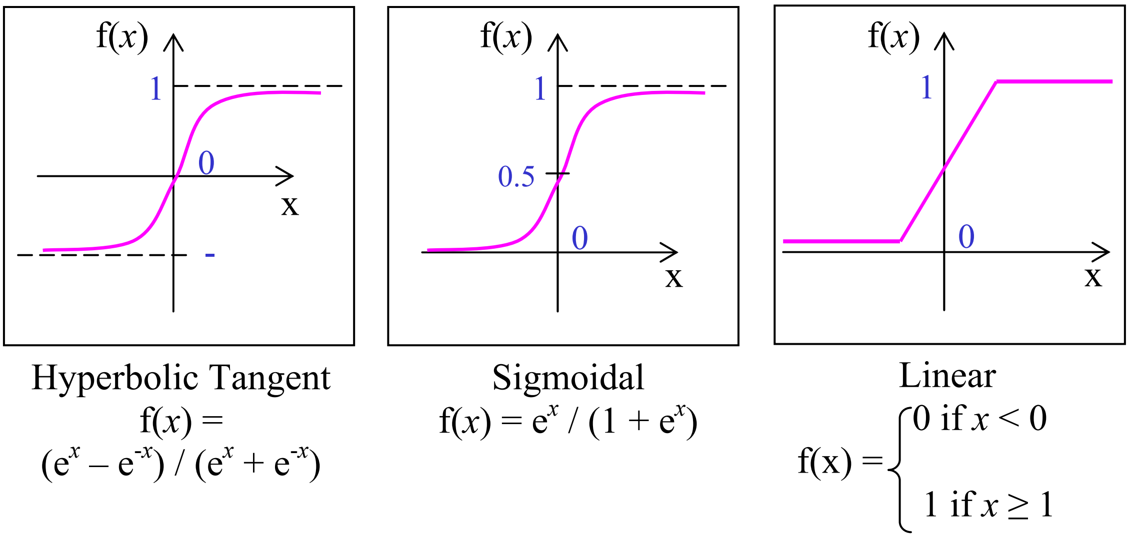


Figure 2.3 Typical activation functions

As outlined earlier, there are several ANN model applications of drought forecasting and to date it was found to be the most widely used modeling approach in drought forecasting around the world (e.g. Kim and Valdes, 2003; Mishra and Desai, 2006; Mishra *et al.*, 2007; Morid *et al.*, 2007; Barros and Bowden, 2008; Ochoa-Rivera, 2008; Cutore *et al.*, 2009). Mishra and Desai (2006) successfully demonstrated ANN models on drought forecasting in Kansabati River basin, India. They tested two types of ANN approaches, namely Recursive Multi-Step Neural Network (RMSNN) and Direct Multi-Step Neural Network (DMSNN) to forecast SPI values. The results obtained from their study showed that the RMSNN approach was best suited for 1-month ahead forecasting, while the DMSNN approach outperformed the RMSNN approach for longer forecasting lead time up to 4 months. These two approaches were used by Cutore *et al.* (2009) in Sicily (Italy), where the Palmer Hydrological Drought

Index (PHDI) was used to forecast future drought conditions. However, they concluded that there were no significant differences in the results obtained from these two approaches. Morid *et al.* (2007) developed several drought forecasting models using Effective Drought Index (EDI) as the DI and different combinations of the past EDI values, rainfall and others variables such as North Atlantic Oscillation (NAO) and Southern Oscillation Index (SOI) as predictors. Similar models were also developed using SPI as the DI. More than 25 different ANN models were tested for each DI at six rainfall stations in the Tehran Province of Iran with forecast lead times from 1 to 12 months. They found that the best model developed in their study was able to forecast values for lead times up to 6 months with high forecasting accuracy, particularly in the case of the EDI. Similar forecasting capabilities were found by Kim and Valdes (2003) who had used ANN to forecast Palmer Drought Severity Index (PDSI) in Conchos River basin, Mexico.

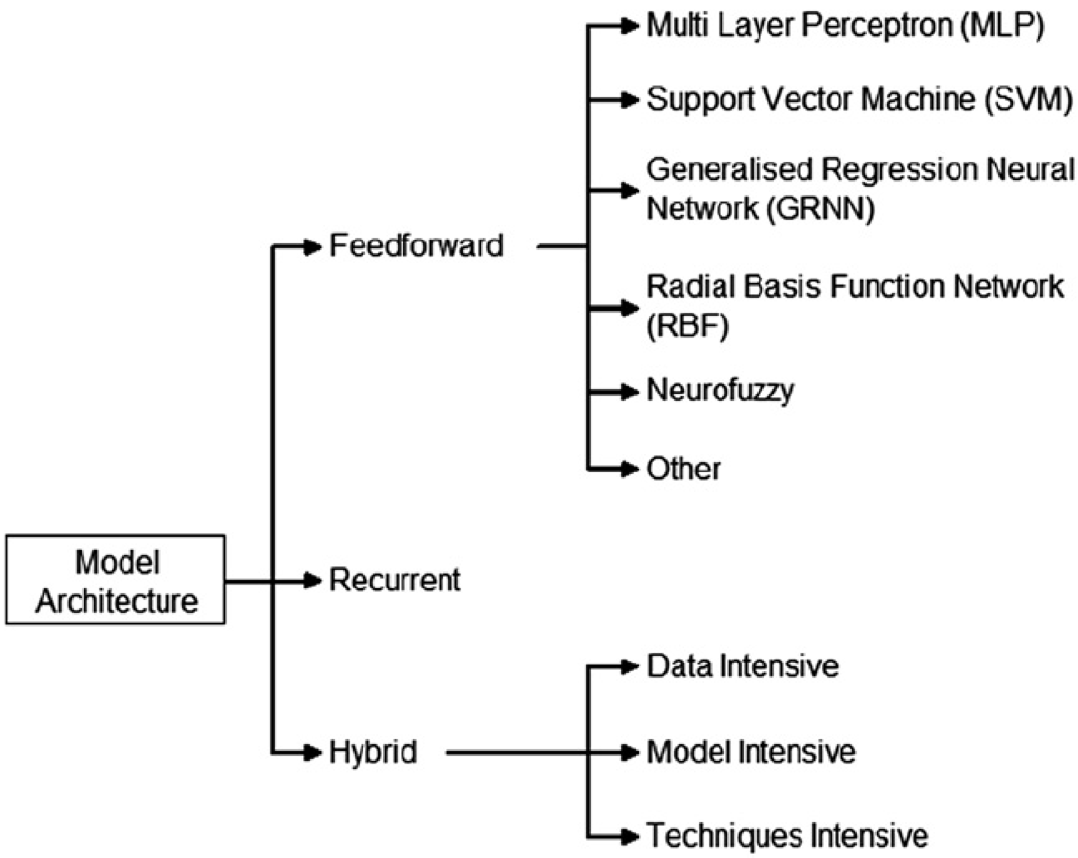


Figure 2.4 Taxonomy of ANN model architectures (Maier *et al.*, 2010)

2.3.6. Adaptive Neuro-Fuzzy Inference System Model

The Fuzzy Logic (FL) approach proposed by Zadeh (1965) is based on the linguistic uncertainty expression rather than on numerical uncertainty. The Fuzzy Inference System (FIS) is a rule based system consisting of three conceptual components: (1) a rule-base, containing fuzzy if-then rules, (2) a data-base, defining the Membership Function (MF) that transforms the input value into a membership degree (i.e., a value between 0 and 1), and (3) an inference system, combining the fuzzy rules to produce the system results (Firat and Gungor, 2007, 2008). The Adaptive Neuro-Fuzzy Inference System (ANFIS), consisting of the combination of ANN and FIS, has been used by many researchers to organize the network structure itself and to adapt the parameters of the fuzzy system for many engineering problems, such as modeling of a hydrological time series (Nayak *et al.*, 2004; Firat and Gungor, 2007, 2008). The ANFIS has the potential to capture the benefits of both ANN and FIS approaches in a single framework (Nayak *et al.*, 2004; Sen and Altunkaynak, 2006; Firat and Gungor, 2007, 2008), and eliminates the basic problem in the fuzzy system design (i.e., defining the MF parameters and the design of fuzzy if–then rules) by effectively using the learning capability of ANN for automatic fuzzy rule generation and parameter optimization.

The ANFIS is developed by Jang (Jang, 1993; Jang *et al.*, 1997) which has a feed-forward ANN structure, where each layer is called a neuro-fuzzy system component (Buyukbingol *et al.*, 2007). Bacanli *et al.* (2008) introduced the ANFIS modeling approach for developing early drought warning system using SPI values. It simulates Takagi-Sugeno-Kang (TSK) fuzzy rule of type-3 (Sugeno and Kang, 1988) where the consequent part of the rule (i.e., the layer 4 in Figure 2.5 which will be described shortly) is a linear combination of input variables and a constant. The basic structure of an ANFIS is shown in Figure 2.5. The ANFIS model is briefly described below for a simple case in which only two input variables (x and y) and one output $f(x, y)$ are considered. Suppose that the rule base consists of two fuzzy if-then rules of TSK type:

Rule 1: IF x is A_1 and y is B_1, THEN $f_1 = f(x, y) = p_1 x + q_1 y + r_1$,

Rule 2: IF x is A_2 and y is B_2 THEN $f_2 = f(x, y) = p_2 x + q_2 y + r_2$.

where A_i and B_i are the linguistic labels such as low, medium, high, etc., and p_i, q_i and r_i are the consequence parameters. Bacanli *et al.* (2008) used *SPI(t* - 1*)* and *P(t* - 1*)* which are the SPI and rainfall values respectively at time *(t - 1)* as the two inputs (x and y) and SPI(t) which is the SPI value at time t as the output $f(x, y)$.

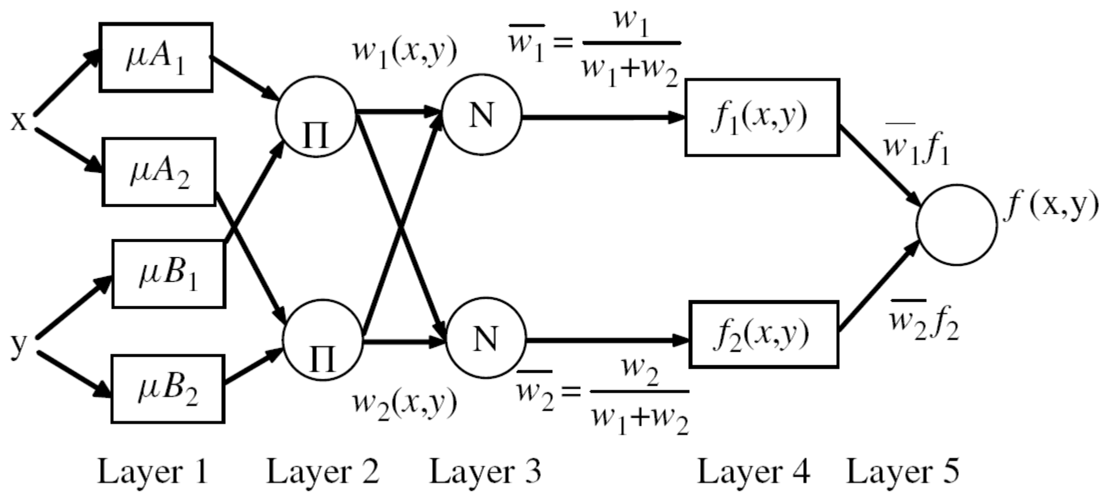


Figure 2.5 Basic structure of Adaptive Neuro-Fuzzy Inference System (Firat and Gungor, 2008)

Layer 1 - called fuzzification layer and the output from this layer is given by Equation (2.11).

$$O_i^1 = \mu_{A_i}(x) \tag{2.11}$$

Where x is the input to node i, and A_i is the linguistic label (low, medium, high, etc.) associated with this node function. The generalized bell MF (gbell MF) is usually used as the MF with maximum equal to 1 and minimum equal to 0:

$$\mu_{A_i}(x) = \frac{1}{1+\left[\left(\frac{x-c_i}{a_i}\right)^2\right]^{b_i}} \tag{2.12}$$

where a_i, b_i and c_i are the parameters of the function. These are adaptive parameters and their values are adapted during the model calibration.

Layer 2 – called rule layer, every node in this layer is labeled with "Π" which multiplies the incoming signals and sends the product out. Each node output represents the firing strength of a rule. For instance,

$$O_i^2 = w_i(x, y) = \mu_{A_i}(x) * \mu_{B_i}(y), \qquad i = 1,\ 2 \tag{2.13}$$

Layer 3 – called normalization layer and every node in this layer is labeled with "**N**". The i^{th} node calculates the ratio of the i^{th} rule's firing strength to the sum of all rules' firing strength:

$$O_i^3 = \overline{w_i} = \frac{w_i(x, y)}{w_1 + w_2}, \quad i = 1,\ 2 \tag{2.14}$$

Layer 4 – called consequent layer and output from this layer is given by Equation (2.15).

$$O_i^4 = \overline{w_i} \cdot f_i = \overline{w_i}\left(p_i x + q_i y + r_i\right), \qquad i = 1,\ 2 \tag{2.15}$$

Layer 5 – called output layer that computes the overall output as the summation of all incoming signals. It is the last step of the ANFIS, i.e.,

$$Q_i^5 = overall\ output = f(x, y) = \sum\nolimits_i \overline{w_i} \cdot f_i \tag{2.16}$$

ANFIS uses the hybrid learning algorithm to calibrate the network. The combination of back-propagation algorithm with least-squares approximation or back-propagation algorithm is used in the hybrid learning algorithm for optimizing parameters in layers 1 and 4 respectively. The mathematical details of these algorithms are given in Jang *et al.* (1997), Nayak *et al.* (2004), and Bacanli *et al.* (2008).

The study conducted by Bacanli *et al.* (2008) showed that the ANFIS model can be successfully used to develop accurate and reliable drought forecasting models. They compared the performances of several ANFIS models which were developed for forecasting 1-month ahead drought condition with different time scale (i.e., 1-, 3-, 6-, 9- and 12- month) SPI time series at 10 stations. They found that the model with the higher time scale (i.e., 12- month SPI) at all stations was better than the other models. The reason for this is related to the SPI values calculated for longer time scales contain long periods of dry and wet periods. Passages between positive and negative values occur more frequently in shorter time scale of SPI and this also results in instability in calculations.

2.4. Summary

There are several drought assessment tools that have been used in the past, and among those tools Drought Indices (DIs) have been used most commonly by researchers and professionals around the world to define drought conditions. Some of these DIs and their usefulness were discussed in this chapter. Majority of these DIs (e.g., Percent of Normal (PN), Deciles and Standardized Precipitation Index (SPI)) were defined with the rainfall as the single variable. However, many researchers and professionals have suggested that DIs should include all significant hydro-meteorological variables that affect droughts, to define wider drought circumstances. Based on this idea, the Surface Water Supply Index (SWSI) had been developed considering a number of hydro-meteorological variables (i.e., rainfall, streamflow/snow water content and storage reservoir volume). However, the SWSI did not consider other important hydro-meteorological variables such as soil moisture content and potential evapotranspiration that affect droughts. The Aggregated Drought index (ADI) was the most recent DI that was developed using several significant hydro-meteorological variables that affect droughts (i.e., rainfall, streamflow, storage reservoir volume, soil moisture content, snow water content and potential evapotranspiration) which describe the fluctuations in the hydrologic cycle. The strength of the ADI is that it describes wider drought circumstances (i.e., total shortage of water resources within the catchment) compared to other DIs describing individual meteorological, hydrological and agricultural droughts.

In the review of the existing DIs, it was found that majority of the DIs had been developed for specific region and some DIs are better suited than others for specific uses. Consequently, any of existing DI may not be directly applicable to other regions due to different hydro-climatic conditions and many other factors. Therefore, a quantitative assessment of existing DIs is considered necessary, to see whether they are applicable to a region for which these DIs are not specifically developed for. This is conducted in Chapter 4, evaluating PN, Deciles, SPI, SWSI and ADI for use in the Yarra River catchment in Australia.

Various drought forecasting modeling techniques have been used in the past by researchers and professionals. The literature review conducted in this chapter highlighted that the ARIMA model showed better drought forecasting only for 1-month ahead forecasts, and due to the seasonality effects in the data these models are not able to forecast for longer lead times. The SARIMA model, on the other hand, was able to give reasonably good results up to 2 months ahead drought forecasts. It was also found that the drought forecasting with the SARIMA model had given better results when higher time scale SPI time series (i.e. 9-, 12-, and 24- month) were used and were able to give good results for up to 3 month ahead forecasts. This might be due to increase in rainfall accumulation in higher time scale SPI time series which reduced the noise more effectively. The log linear modeling approach had also shown good capability for forecasting drought conditions up to 2 months ahead. However, the approach had not been recommended for forecasts of more than 2 months due to the increasing number of model parameters (beyond 2 months forecasts), which makes the respective contingency tables complex and difficult in the interpretation of the model results. The Markov chains modeling approach, on the other hand, showed that it can be used to forecast drought conditions satisfactorily up to 3 months ahead. However, it was found that the predictive performance in the Markov Chains model decreased greatly as the severity of drought increased and the predictive performance was not always satisfactory for drought state transitions. The ANFIS have shown suitability for 1-month ahead drought forecasting with longer time scale (i.e., 12- month) SPI values and the forecasts results were not satisfactorily with short time scale (i.e., 1-, 3-, 6-month) SPI values. As was seen with the SARIMA model, the reason for this is related to the SPI values calculated for long periods containing longer periods of dry and wet periods. Passages between positive and negative values occur more frequently in shorter time scale of SPI and this also results in instability in calculations. It should be noted that the ANFIS modeling technique has not yet been tested for forecasting drought conditions beyond 1-month forecasts.

The ANN modeling approach was found to be the most widely used and successful drought forecasting approach around the world. It has shown great ability in modeling and forecasting nonlinear and non-stationary DI time series, due to its innate nonlinear properties and flexibility for modeling. The results of the past studies have shown that the ANNs were capable to forecast drought conditions for longer lead time steps (i.e. 6 months ahead) than any other model. Therefore, the ANN modeling approach was used in this study for developing the drought forecasting model for the Yarra River catchment in Australia which will be presented in Chapter 6.

3. YARRA RIVER CATCHMENT, DROUGHT HISTORY, AND DATA SOURCES AND PROCESSING

Overview; Yarra River Catchment; Drought History in Yarra River Catchment; Hydro-meteorological Data Sources; Hydro-meteorological Data Processing; Summary

3.1. Overview

As stated in Chapter 1, the Yarra River catchment located in Victoria (one of the states in Australia) was used as the case study in this thesis. The Yarra River is one of the rivers in Victoria and is a major source of water supply for Melbourne residents (EPA Victoria, 1999). The water resources management of this catchment is important in terms of a wider range of water uses as well as downstream user requirements and environmental flows. However, due to frequent droughts and increasing water demand in recent years, pressure on the water resources management activities have increased within the Yarra River catchment (Tan and Rhodes, 2008). Many initiatives have taken place recently for protecting the environmental health of the waterways of this catchment, especially during drought periods (EPA Victoria, 1999; Melbourne Water, 2007).

Hydro-meteorological data of the Yarra River catchment were used in this research for developing drought indices (DIs) and for the development of the drought forecasting model. Historical drought records in Victoria were also used throughout the research work for evaluating DIs.

This chapter first describes the Yarra River catchment with respect to land use conditions, its importance and water resources. Then the drought history in Victoria is described, followed by sources of hydro-meteorological data which were used in this thesis. The hydro-meteorological data processing is then presented. Finally, a summary is presented at the end of the chapter.

3.2. Yarra River Catchment

3.2.1. Description of Yarra River Catchment

The Yarra River catchment is located in the eastern part of Victoria and is shown in Figure 3.1. The Yarra River flows from east to west, and has a total catchment area of 4,044 square kilometers, and a stream course of 245 kilometers from its source in the Great Dividing Range to the end of its estuary at Port Phillip Bay, as shown in Figure 3.2 (Melbourne Water, 2010). This figure also shows the location of several storage reservoirs within the catchment and the Melbourne Central Business District (CBD) area. The Yarra River catchment consists of a series of sub-catchments based on major tributaries, whose total length is about 1,800 kilometers (EPA Victoria, 1999). Due to natural divisions and different landuse activities in the catchment, the Yarra River catchment has been divided into three reaches – *Upper*, *Middle* and *Lower*, as shown in Figure 3.2 (EPA Victoria, 1999). Each reach has its own distinct characteristics.

The *Upper* reach of the catchment, from the Great Dividing Range to the Warburton Gorge, consists of mainly dense and extensive forested area with less human population. Major tributaries of the upper reach flows through these forested and mountainous areas, which have been reserved for water supply purposes for more than 100 years (Melbourne Water, 2010).

The *Middle* reach of the catchment, from the Warburton Gorge to Warrandyte, flows mainly through rural floodplains and valleys. There is limited urban development within this reach. It is notable as the only part of the catchment with an extensive flood plain area. The river gradient decreases and valley widens as the river approaches downstream. There are several gorges in this segment, which restrict the flow of the river, in particular, the Yering Gorge as indicated in Figure 3.2. Majority of the land in this reach is used for agricultural purposes (Gardner, 1994).

The *Lower* reach of the catchment, downstream of Warrandyte, flows through the mainly urbanized floodplain area of Melbourne. Large areas of this reach consists of hard surfaces such as paved roads, roofs, car parks, concrete channels, etc. (EPA Victoria, 1999). Most of the land along rivers and creeks of the lower reach have been cleared for urban development.

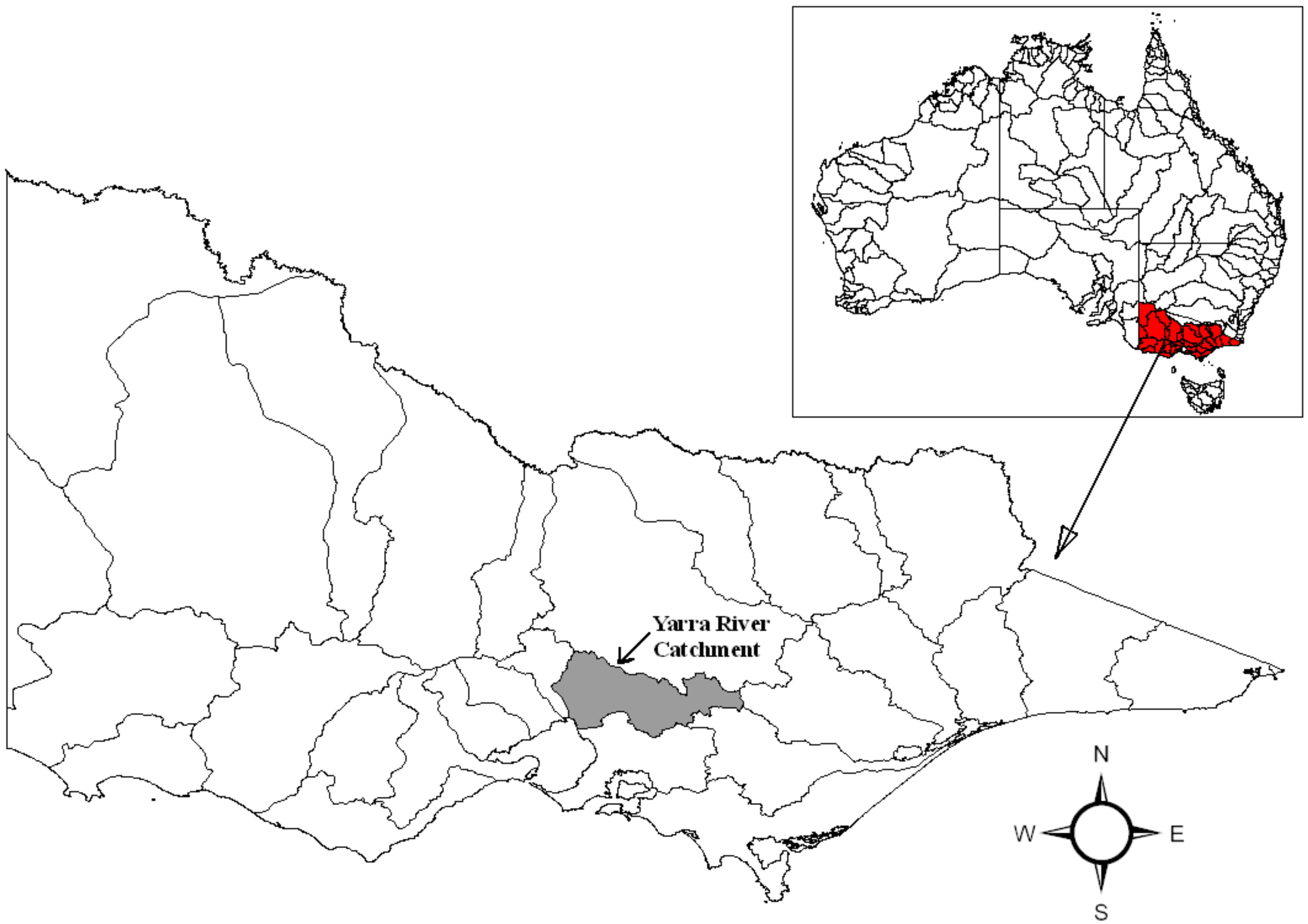


Figure 3.1 Yarra River catchment

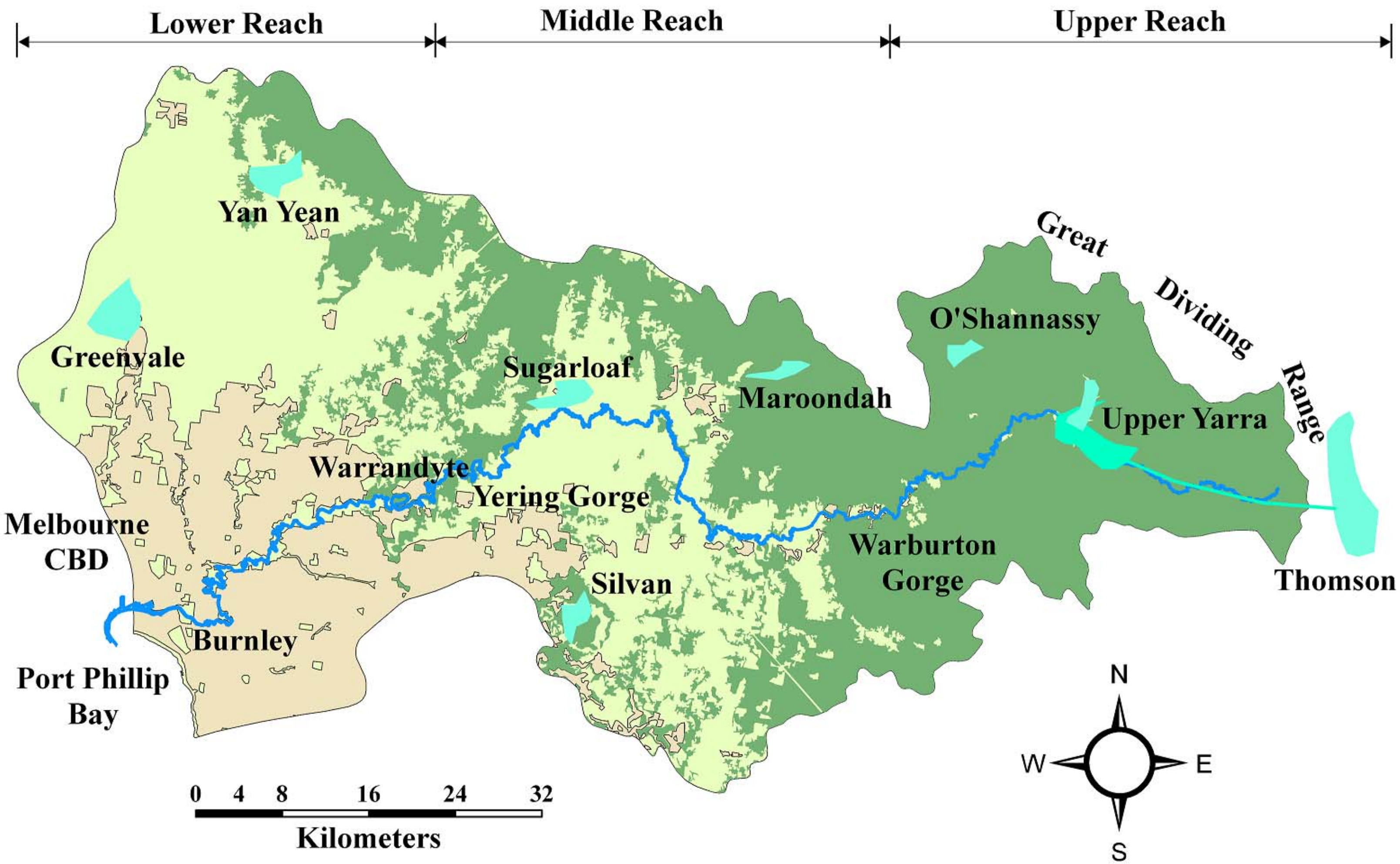


Figure 3.2 Details of Yarra River catchment (adapted from Melbourne Water, 2010)

3.2.2. Importance of Yarra River Catchment

The Yarra River catchment is an important water resources catchment for Victoria, where over one-third of Victoria's population (approximately 1.5 million) lives in this catchment. Although the Yarra River is not large by Australian standards, it is a very productive catchment as it generates the fourth highest water yield per hectare of catchment in Victoria (Melbourne Water, 2009). The catchment water resources support a range of uses valued by the Melbourne's community, including urban water supply, agricultural, horticultural industries and downstream user requirements, as well as flow requirements for maintaining environmental flows. There are eight major storage reservoirs within the catchment as shown in Figure 3.2. They are Upper Yarra, Sugarloaf, Silvan, Yan Yean, Greenvale, Maroondah and O'Shannassy within the catchment, and Thomson reservoir, which is outside the catchment. However, water from Thomson is transferred to Upper Yarra reservoir mainly for urban water supply purposes. Extensive harvesting of potable water occurs from the tributaries of the *Upper* reach of the Yarra River through Upper Yarra and O'Shannassy reservoirs, which have altered the natural flow regime of the Yarra River and its tributaries (EPA Victoria, 1999). There are many licensed water extraction points in the Yarra River and its tributaries including 200 licensed users within the Yarra River main stream (Melbourne Water, 2007). There are also numerous farm dams within the catchment, and water extraction from the rivers and creeks for agriculture is prevalent. A range of recreational activities, metropolitan parks and biodiversity conservation is also located around the catchment waterways.

3.2.3. Sources of Water Resources in Yarra River Catchment

Like many other catchments, rainfall is the main source of water resources in the Yarra River catchment. However, the catchment receives some amount of water from the Thomson reservoir (which is in the Thomson River catchment) to the Upper Yarra reservoir. The average annual rainfall varies from 1080 mm at Upper Yarra reservoir near Warburton, to 615 mm at Burnley near Melbourne (Melbourne Water, 2009). However, the annual average rainfall has declined during the last decade within the Yarra River catchment compared to the long-term historical average (Muttil *et al.*, 2009). Figure 3.3 shows the annual catchment average rainfall for the Yarra River catchment based on the 22 rainfall measuring stations (which will be discussed in Section 3.4) for the period from 1960 to 2008. This figure shows that the annual catchment average rainfall was around 982.7 mm when consider all years from 1960 to 2008. This figure also shows that the annual catchment average rainfall from 1960 to 1996 was around 1031.9 mm, and it was only around 831.1 mm for the period from 1997 to 2008. It shows that the annual catchment average rainfall has dropped within the Yarra River carchment by more than 200 mm 1997 onwards. This reduction in rainfall puts pressure on the availability of water resources from its sources.

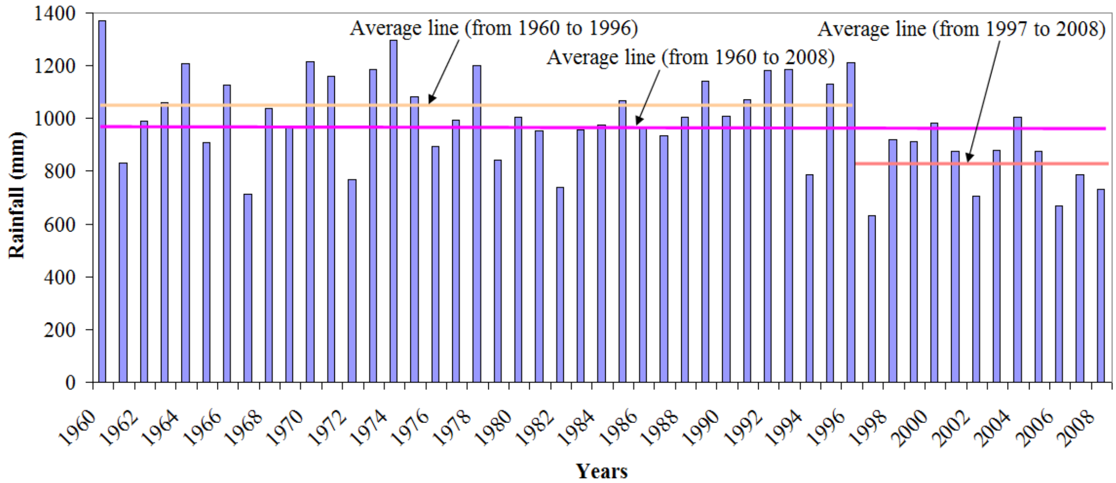


Figure 3.3 Annual catchment average rainfall for the Yarra River catchment

3.3. Drought History in Yarra River Catchment

Several droughts have occurred in the past in Victoria including 1967-1968, 1972-1973, 1982-1983, 1997-1998, 2003-2004 and 2006 onwards. These historical droughts were recorded in Keating (1992) and Tan and Rhodes (2008) after considering rainfall and storage records at that time and comparing them against their average values. They had also recorded that during these droughts there were severe deficiency in water resources in terms of rainfall and storage reservoir volume, and also had negative socio-economic impacts due to the shortage of water. Based on the hydro-meteorological data of the Yarra River catchment (which will be discussed in Section 3.4), the following observations are made:

1) During the 1967-1968 drought, the monthly catchment-average rainfall had fallen below 5% of its normal (i.e. monthly catchment average rainfall for the period from 1960 to 2008) (in February 1968) and the storage volume fell below 25% capacity (in April 1968).

2) During the 1972-1973 drought, the monthly catchment-average rainfall had fallen below 6% of its normal (in December 1972) and the storage volume fell below 55% capacity (in January 1973). This drought was relatively short in duration.

3) Of all droughts recorded until 1990, the worst drought occurred in Victoria was in 1982-1983. This drought has affected most areas in eastern Australia, and sparked the Ash Wednesday bushfires, which burnt 13,000 hectares of Melbourne's water supply catchments, and caused massive dust storms (Keating, 1992). During this period, the monthly catchment-average rainfall had fallen below 8% of its normal (in February 1983) and the storage volume fell below 20% capacity (in May 1983).

4) During the period of 1997-2008, inflows into Melbourne Water's four major harvesting reservoirs (i.e. Thomson, Upper Yarra, O'Shannassy and Maroondah shown in Figure 3.2) have been below the long-term average. Three major droughts (1997-1998, 2002-2003 and 2006 onwards) have occurred during this twelve-year extended dry period (Tan and Rhodes, 2008). It should be noted that this long-term dry circumstances is still continuing and therefore drought from 2006 is noted here as the 2006 onwards drought. The monthly catchment-average rainfall during 1997-2008 dry period dropped to the approximately 69 mm, whereas the average monthly rainfall prior to this period (1960-1996) was around 86 mm. The storage volume also fell below 22% capacity (in May 2007).

Along with the reduction in annual average rainfall, and the diversity of water uses and activities (Section 3.2), pressure upon water resource management within the Yarra River catchment has become more intense in recent years especially during the droughts. Therefore, the management of water resources in terms of droughts is important within the Yarra River catchment.

3.4. Hydro-meteorological Data Sources

Data required for this project to compute the DIs and the drought forecasting model development work were rainfall, potential evapotranspiration, streamflow, storage reservoir volume and soil moisture content. Justifications for selecting these variables for this study will be given in the various sections of Chapters 4, 5 and 6. They were collected from a number of organizations such as Bureau of Meteorology (BOM) (i.e., rainfall data), SILO database (Jeffrey *et al.*, 2001) (i.e. potential evapotranspiration data) and Melbourne Water Corporation (i.e., streamflow and storage reservoir volume data). Soil moisture content data were not available for the catchment, and therefore a two-layer water budget model of Palmer (1965) was adapted to determine the soil moisture content in the catchment, which is further elaborated in Section 3.5.4. Data measurement locations for rainfall, evaporation, streamflow and storage reservoir volume are shown in Figure 3.4. Twenty two rainfall, six evaporation and one streamflow stations were considered in this study. Of the six evaporation stations, two stations are outside of the catchment area; however they were considered in this study as they are very close to the study area and no other evaporation gauges are present in the southern part of the catchment. These evaporation data have also been used to calculate potential evapotranspiration in the SILO database (Jeffrey *et al.*, 2001).

Data ranged from 1960 to 2008 (49 years) were used in this study which were either available or estimated for all variables (i.e., rainfall, potential evapotranspiration, streamflow, storage reservoir volume and soil moisture content). These data were available in different time scales (i.e., daily and monthly) for different variables. However, DIs were developed at monthly time scale, since monthly DIs are suitable for operational purposes and have lower sensitivity to observational errors (McKee *et al.*, 1993; Mishra and Singh, 2010). Therefore, data processing was carried out to obtain the catchment representative monthly values. The procedures that were used in data processing are described in Section 3.5.

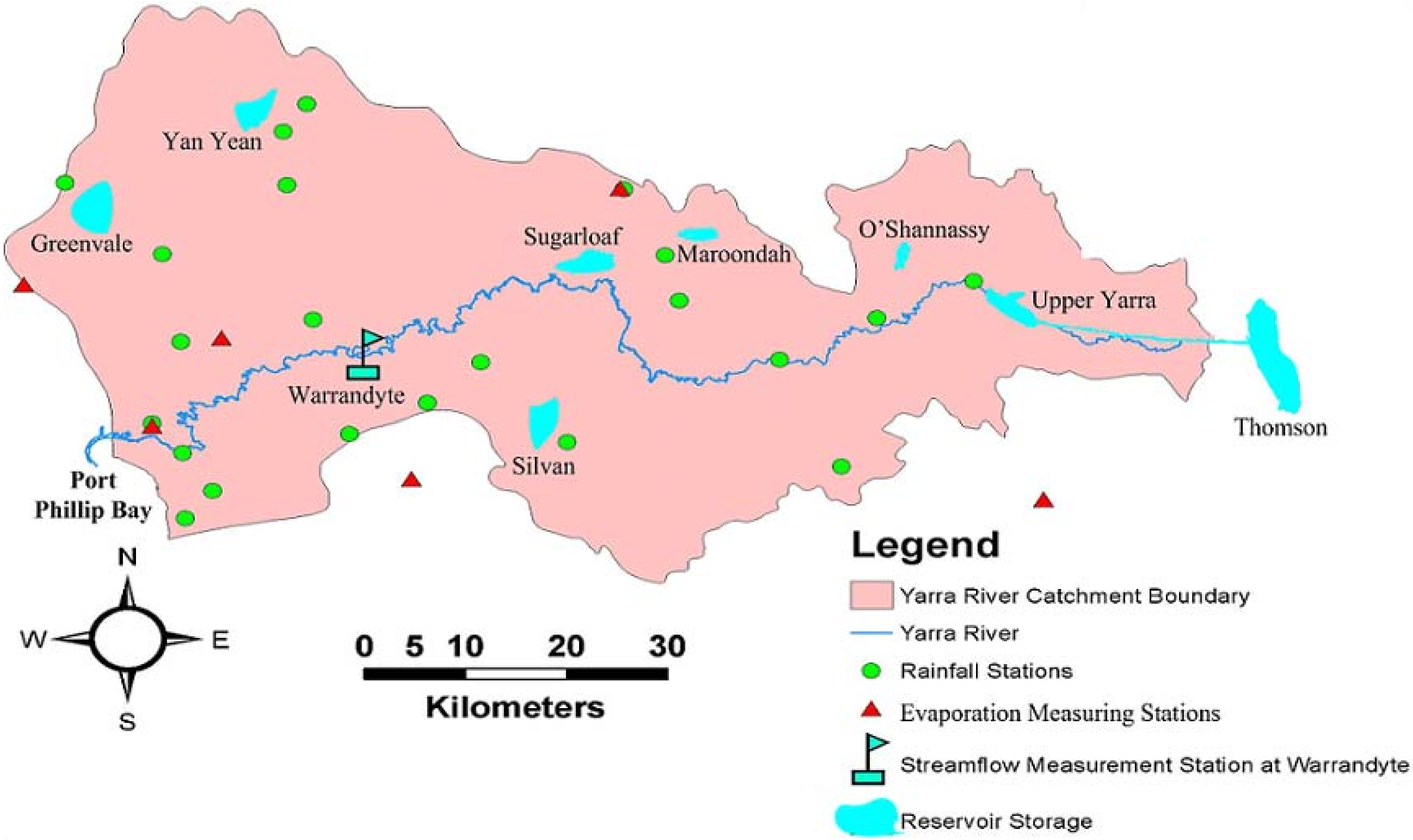


Figure 3.4 Locations of the hydro-meteorological data measurements

3.5. Hydro-meteorological Data Processing

3.5.1. Rainfall and Potential Evapotranspiration Data

As mentioned in Section 3.4, twenty two rainfall and six evaporation stations (Figure 3.4) were used to compute the monthly catchment values. The station numbers, names and geographical coordinates of each of the measuring stations are presented in Tables 3.1 and 3.2 for rainfall and evaporation measuring stations respectively, and their spatial locations are shown in Figure 3.4. Data from these stations were used, as all these stations had long historical records from 1960 to 2008.

Table 3.1 Description of rainfall measuring stations

Station No.	Station Name	Latitude (^{o}S)	Longitude (^{o}E)
86018	Caulfield (Racecourse)	-37.88	145.04
86027	Croydon (Samuel Street)	-37.79	145.28
86033	Brighton Bowling Club	-37.91	145.00
86035	Eltham	-37.70	145.15
86036	Epping	-37.63	144.99
86066	Lilydale	-37.75	145.34
86070	Maroondah Weir	-37.64	145.55
86071	Melbourne Regional Office	-37.81	144.97
86073	Mickleham	-37.55	144.88
86074	Mitcham	-37.82	145.19
86090	O'Shannassy Reservoir	-37.71	145.79
86094	Powelltown Dnre	-37.86	145.74
86095	Prahran (Como House)	-37.84	145.00
86096	Preston Reservoir	-37.72	145.00
86106	Silvan	-37.83	145.44
86117	Toorourrong	-37.48	145.15
86121	Warburton	-37.75	145.68
86125	Whittlesea	-37.51	145.12
86131	Yan Yean	-37.56	145.13
86142	Mt St Leonard DPI	-37.57	145.50
86219	Coranderrk Badger Weir	-37.69	145.56
86271	Upper Yarra Dam	-37.67	145.89

Table 3.2 Description of evaporation measuring stations

Station No.	Station Name	Latitude (^oS)	Longitude (^oE)
85277	Noojee (Slivar)	-37.90	145.97
86071	Melbourne Regional Office	-37.81	144.97
86104	Scoresby Research Institute	-37.87	145.26
86142	Mt St Leonard DPI	-37.57	145.50
86282	Melbourne Airport	-37.66	144.83
86351	Bundoora (Latrobe University)	-37.72	145.05

Two commonly used averaging methods namely arithmetic average and Thiessen polygon (Thiessen, 1911) were used to compute the monthly rainfall and potential evapotranspiration values for the catchment using data for twenty two rainfall and six evaporation stations. Figure 3.5 and Figure 3.6 show the comparisons between the two methods used to compute the monthly catchment total rainfall and evapotranspiration respectively. Figure 3.5 shows that the Thiessen polygon estimates for rainfall are relatively higher than the arithmetic average especially for higher rainfalls. However, both methods produce similar results for potential evapotranspiration (Figure 3.6). Since the Thiessen polygon method is an area-weighted method, the monthly catchment total values obtained from the Thiessen polygon method were considered in this study for both rainfall and potential evapotranspiration.

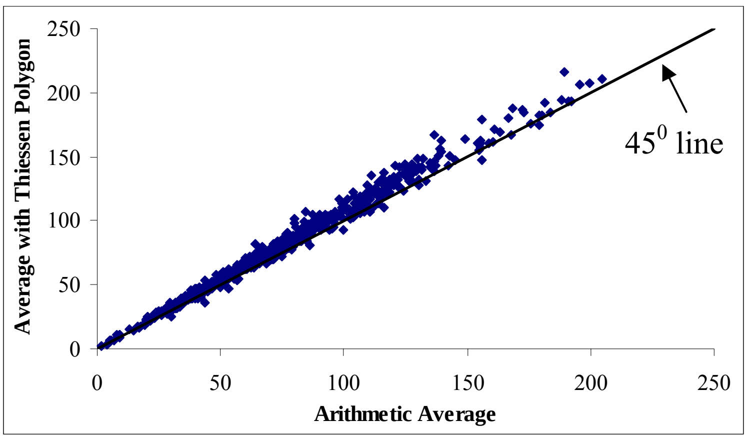


Figure 3.5 Monthly catchment total rainfall (mm)

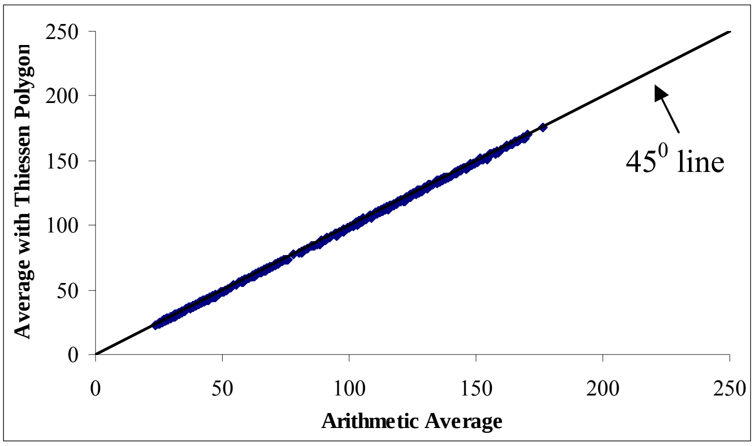


Figure 3.6 Monthly catchment total potential evapotranspiration (mm)

3.5.2. Streamflow Data

There are approximately fifteen streamflow gauging stations in the Yarra River mainstream. However, only the gauging station at Warrandyte (Figure 3.4) was used in this study, since it is the only station that had long records (i.e. 1960 to 2008) of streamflow measurements. It is also a good representative station in the Yarra River mainstream, because of its location close to the centroid of the catchment. The station number of the Warrandyte streamflow gauge station is 229200B, and the geographical coordinates are -37.74^{o} S and 145.21^{o} E. Daily streamflow data at Warrandyte are used to compute average daily streamflow for each month; they were considered as the catchment representative data, to account for the fluctuations in streamflow discharge.

3.5.3. Storage Reservoir Volume Data

Seven storage reservoirs exist within the study area as shown in Figure 3.4. In addition, Thomson reservoir, which is located outside of the catchment area, transfers water to the Upper Yarra reservoir (Figure 3.4). Of these eight storage reservoirs, only Upper Yarra and Thomson reservoirs contribute to supply water to downstream users in the Yarra River catchment especially for maintaining environmental flows. These releases could influence the drought conditions of the catchment. Therefore, these two storage reservoirs were considered in this study. Percent full of daily storage reservoir volume data were used to obtain the average percent full of storage reservoir volume for each month. Percent full is defined here as the percentage storage volume with respect to the total storage capacity within the system. Percent full was considered instead of the absolute storage reservoir volume, as Thomson was operational only since 1983 and percent full will give consistent values throughout the analysis period.

3.5.4. Soil Moisture Data

Soil moisture content was used to account for the fluctuations in water stored in the plant root zone. As outlined in Section 3.4, the two-layer water budget model of Palmer (1965) was adapted in this study to calculate the monthly basin average soil moisture content. The model considers the soil to have two arbitrary layers (i.e., the surface layer and underlying layer) as shown in Figure 3.7.

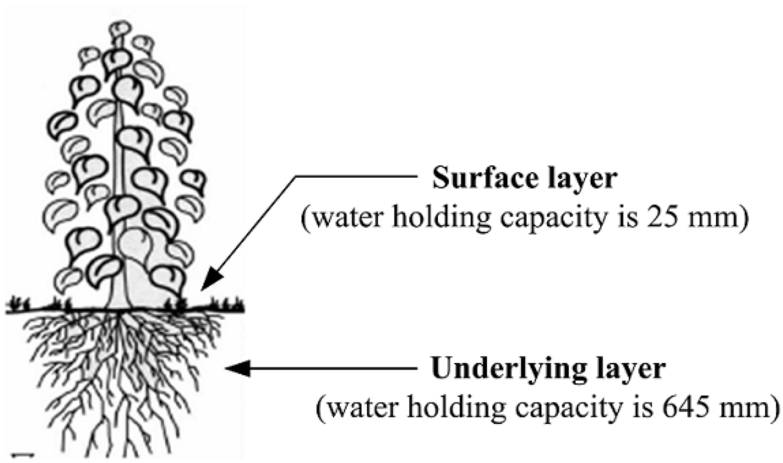


Figure 3.7 Two layer water budget model for the Yarra River carchment (adapted from Palmer, 1965)

The upper layer (called surface layer) is roughly equivalent to the plough layer. This is the layer which receives rainfall and from which the evaporation takes place. In moisture accounting, it is assumed that evapotranspiration takes place at the potential rate from this surface layer until all available moisture in the layer is removed. Only then, moisture can be removed from the underlying layer of soil. Likewise, it is assumed that there is no recharge to the underlying layer until the surface layer soil moisture is brought to field capacity. The available water holding capacity of the soil in the underlying layer depends on the depth of the effective root zone and the soil characteristics in the area under study.

Watson (1999) in a study of forest water yield in the Yarra River catchment used the maximum water holding capacity (both in the surface layer and underlying layer) as 670 mm (water) / 1000 mm (soil) under saturated conditions. This assumption was made following the analyses presented by Campbell (1999) using soil moisture tension data. Therefore, 670 mm (water) / 1000 mm (soil) under saturated conditions was used as the total available water holding capacity (*AWC*) in the two-layer water budget model in this study. It should be noted that there are some non-forest area within the catchment; however, the selection of the above *AWC* value depends on soil characteristics rather then whether forest or non-forest area. Similar to Palmer (1965), 25 mm was used as the water holding capacity for the surface layer and the remaining 645 mm was used for the underlying layer. Monthly water balance was then conducted in the two layers as was done by Palmer (1965), considering catchment average rainfall and potential evapotranspiration calculated in Section 3.5.1. In the water balance model, it was assumed that the surface runoff was generated only when available water was greater than the water holding capacity in both layers. Moreover, the loss from the underlying layer depends on initial moisture content (i.e., available moisture content at beginning of the month) as well as on the potential evapotranspiration (*PE*) and the *AWC* of the soil system. Therefore, the moisture losses from surface and underlying layers were calculated using Equations (3.1) and (3.2) respectively.

$$\mathrm{L_s} = S_s^{'} \text{ or } (PE - P) \quad \text{whichever is smaller, and} \tag{3.1}$$

$$\mathrm{L_u} = (PE - P - L_s)\frac{S_u^{'}}{AWC}, \qquad \mathrm{L_u} \le S_u^{'} \tag{3.2}$$

where $\mathrm{L_s}$ = moisture loss from surface layer (mm),

$S_s^{'}$ = available moisture stored in surface layer at start of month (mm) (i.e., $\le$ 25 mm),

PE = potential evapotranspiration for the month (mm),

P = rainfall for the month (mm),

$\mathrm{L_u}$ = loss from underlying layer (mm),

$S_u^{'}$ = available moisture stored in underlying layer at start of month (mm) (i.e. ≤ 645 mm), and

AWC = total available water holding capacity of the both surface and underlying layer (i.e. 670 mm (water) / 1000 mm (soil) under saturated conditions).

The model was run with a start month (i) which considered the catchment soil moisture content equal to the AWC. Therefore, the catchment soil moisture content at the end of month i, W_i:

$$W_i = AWC = 750 \text{ mm} \tag{3.3}$$

For the following month i+1, the available moisture stored in the surface layer at the start of month, $S_{s,\,i}^{'} = 25$ mm and underlying layer, $S_{u,\,i}^{'} = 645$ mm. The catchment soil moisture content at the end of month i+1, W_{i+1}, was then calculated as:

$$W_{i+1} = \begin{cases} \left(S_{s,\,i}^{'} + S_{u,\,i}^{'}\right) - \left(L_s + L_u\right), & \text{when } PE > \mathrm{P} \\ \left(S_{s,\,i}^{'} + S_{u,\,i}^{'}\right), & \text{when } PE = \mathrm{P} \\ \left(S_{s,\,i}^{'} + S_{u,\,i}^{'}\right) + \left(P - PE\right), & \text{when } PE < \mathrm{P} \end{cases}, \quad W_{i+1} \le AWC \tag{3.4}$$

Equation (3.4) implies that when PE is larger than P then there will be moisture loss from the system, while when P is larger than PE then there will be moisture added in the system until it reaches the AWC. If PE is equal to the P in any month then there will not be any change in soil moisture content in the catchment for that month. The calculations were continued using Equations (3.1), (3.2) and (3.4) for the rest of the months.

The catchment average total yearly rainfall and potential evapotranspiration for the Yarra River catchment is shown in Figure 3.8. It shows that the year, 1960, was relatively wet and the rainfall was much higher than the potential evapotranspiration. Moreover, it was found that the rainfall was continuously much higher than the potential evapotranspiration for six months from April to September in 1960. Therefore, for running the water budget model, the start month was considered as September 1960 and the *AWC* for this month was considered as 670 mm. The catchment soil moisture content for the months following September 1960 were calculated using Equations (3.1), (3.2) and (3.4). As mentioned above, the months from April to September in 1960 were relatively wet, therefore the soil moisture content for the months from April to August in 1960 were also considered at *AWC* (i.e., 670 mm) and back calculations were carried out to calculate the soil moisture content for the months from January to March in 1960.

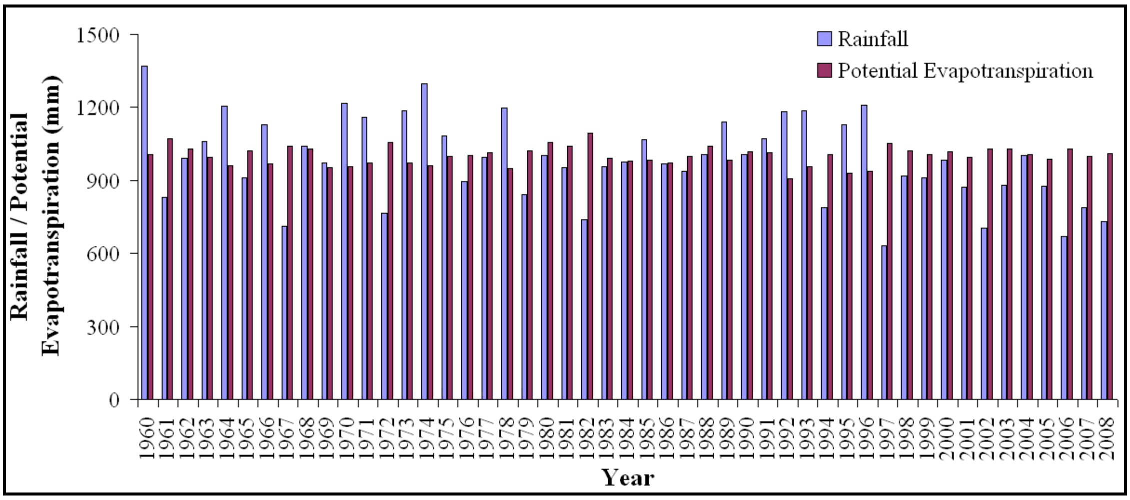


Figure 3.8 Catchment average total yearly rainfall and potential evapotranspiration for the Yarra River catchment

3.6. Summary

The Yarra River catchment located in Victoria (Australia) is a valuable asset to all Melbourne residents. The water resources from this catchment are important in terms of a wide range of water uses as well as downstream user requirements and environmental flows. Many initiatives have been taken by several water authorities including EPA Victoria and Melbourne Water Corporation for protecting catchment waterways and mitigating water demand in drought circumstances. However, frequent droughts and increasing water demand in recent years have increased pressure upon water resources management within the catchment, and therefore, the management of water resources in terms of droughts is important within the Yarra River catchment.

Data on several hydro-meteorological variables (i.e., rainfall, potential evapotranspiration, streamflow and storage reservoir volume) were collected for the Yarra River catchment from Bureau of Meteorology, SILO database and Melbourne Water Corporation to compute the Drought Indices (DIs) for use in the catchment and for drought forecasting model development work. However, the soil moisture content data were not available for the catchment, and therefore the two-layer water budget model of Palmer (1965) was adapted to determine the soil moisture content in the catchment.

Data were collected or estimated for the period from 1960 to 2008 (49 years). These data were available in two time scales (i.e., daily and monthly) for these hydro-meteorological variables. However, DIs and drought forecasting model were developed using a monthly time step as the monthly drought forecasting was suitable for operational purposes, and also monthly data have lower sensitivity to observational errors. Therefore, the data that were not on monthly basis were converted to represent the monthly time scale. These data (i.e., rainfall, potential evapotranspiration, streamflow, storage reservoir volume and soil moisture content) were collected and/or estimated, and analyzed for three specific purposes in this thesis for use in the Yarra River catchment:

1. To evaluate the existing drought indices (Chapter 4),
2. To develop the Nonlinear Aggregated Drought Index (Chapter 5), and
3. To develop the drought forecasting model (Chapter 6).

4. EVALUATION OF SELECTED DROUGHT INDICES

Overview; Study Area and Data Used; Methodology Used for Development of Drought Indices; Analysis of Drought Indices; Evaluation of Drought Indices; Summary

4.1. Overview

Drought Indices (DIs) have commonly been used around the world to quantify drought conditions. As discussed in Section 2.1, in most cases, DIs were developed for a specific region, and therefore they may not be directly applicable to other regions due to inherent complexity of the drought phenomena, different hydro-climatic conditions and different catchment characteristics (Redmond, 2002; Smakhtin and Hughes, 2007; Mishra and Singh, 2010). Suitability of some of the existing DIs had been analyzed for different climatic regions around the world, of which details can be found in Keyantash and Dracup (2002), Heim Jr (2002), Smakhtin and Hughes (2004) and Morid *et al.* (2006). However, no such study had been conducted in Australia which is often referred to as the driest inhabited continent on the Earth (Davidson, 1969). Pigram (2006) noted that Australia not only has the lowest rainfall and runoff in proportion to its area, but also the lowest percentage of runoff to rainfall. He also noted that evapotranspiration is high in Australia and, on average, consumes 87% of all moisture that reaches the ground, compared to about 60% for North America and Europe. Frequent droughts are common in Australia than elsewhere in the world, and these droughts have become more recurrent in the last 50 years, for instance in the south-eastern part of Australia (Keating, 1992; Tan and Rhodes, 2008). Because of these reasons, it is worthwhile to study the suitability of the existing DIs for use in Australia for drought management; these DIs are predominantly developed for use in other parts of the world.

The work presented in this chapter is the first part of this research project, which is aimed to develop a *Drought Forecasting Model* for management of water resources within the Yarra River catchment. The management of water resources in this catchment has great importance to the Melbourne's community as described in the Section 3.2.2. Selection of an appropriate DI that can be used for defining drought conditions within the Yarra River catchment is therefore an important issue for this project. Five existing DIs from different drought perspectives (i.e., meteorological, hydrological and agricultural) were evaluated in this study. They were: widely used rainfall based DIs, namely, the Percent of Normal (PN), Deciles and Standardized Precipitation Index (SPI) as the meteorological DIs; Surface Water Supply Index (SWSI) as the hydrological DI; and the Aggregated Drought Index (ADI) which considers all three categories of droughts (i.e. meteorological, hydrological and agricultural droughts). PN, Deciles and SPI had been used in Australia in the past as mentioned in Section 2.2 (Gibbs and Maher, 1967; Osti *et al.*, 2008), but to date SWSI and ADI have not been used in Australia. This is the first study conducted in Australia to evaluate the suitability of existing DIs for drought management in an Australian catchment. This is also the first study where the ADI was tested outside the area where it was originally developed in the U.S. Qualitative assessment of the abovementioned selected DIs were discussed in Section 2.2. The current chapter aims to conduct a quantitative assessment of these DIs for the Yarra River catchment.

The chapter begins with some reference to the study area and the data used in this study (which was detailed in Chapter 3), followed by the methodology used for the development of the selected five DIs. Analysis of the DIs are then presented, followed by the evaluation of the DIs. The summary drawn from the study are presented at the end of the chapter.

4.2. Study Area and Data Used

The Yarra River catchment was used in this study. The details of the catchment and importance of its water resources for Victorian people were elaborated in Chapter 3. Five hydro-meteorological variables (i.e., rainfall, potential evapotranspiration, streamflow, storage reservoir volume and soil moisture content) which have effects on droughts were used in the evaluation of DIs. Measurement locations of these data were shown in Figure 3.4, and the data processing to obtain the monthly values was also described in Chapter 3. Data from 1960 to 2008 (49 years) were used in this study which were either available or computed for all five variables.

4.3. Methodology Used for Development of Drought Indices

4.3.1. Percent of Normal

Percent of Normal (PN) is expressed as the actual rainfall in percentage compared to the normal rainfall (Hayes, 2003; Morid *et al.*, 2006). Usually, the long-term mean or median rainfall value has been considered as the normal (Smakhtin and Hughes, 2004). However, Hayes (2003) commented that the use of PN implies a normal distribution where the mean and median are the same, and the use of the mean can easily be misunderstood and can give different indications of drought conditions as the mean rainfall is often not the same as the median rainfall in a long-term record. Nevertheless, the mean was used in this study, since the difference between mean and median for the catchment average monthly rainfall data was not statistically significant at 5% level. As was mentioned in Section 2.2.2, PN can be calculated in a variety of time scales ranging from a single month to a group of months representing a particular season, calendar year or water year. However, PN in this study was calculated using the monthly time step. As was mentioned in Section 2.2.2, no threshold ranges had been published for this DI; usually, lower PN (PN<100%) values indicate dry circumstances. However, in this study, drought classifications for PN and their threshold values were proposed to define drought conditions which are presented in Table 4.1. Descriptions of drought classifications are proposed in PN similar to the other DIs. Each of the drought classifications in PN are considered with equal intervals as in Deciles except for further splitting of the lowest and highest classifications into two additional classifications to be consistent with the other DIs (i.e. SPI, SWSI and ADI).

Table 4.1 Drought classification based on PN

> 180% of normal rainfall	Extremely wet
> 160% to ≤ 180% of normal rainfall	Very wet
> 120% to ≤ 160% of normal rainfall	Moderately wet
> 80% to ≤ 120% of normal rainfall	Near normal
> 40% to ≤ 80% of normal rainfall	Moderate drought
> 20% to ≤ 40% of normal rainfall	Severe drought
20% ≤ of normal rainfall	Extreme drought

4.3.2. Deciles

As was discussed in Section 2.2.3, Deciles is a meteorological drought measurement tool which uses rainfall. In calculating Deciles, the long-term monthly rainfall records were first ranked from highest to lowest to construct a cumulative frequency distribution. This distribution was then split into ten parts (or deciles) based on equal probabilities (Gibbs and Maher, 1967). The threshold ranges of Deciles used to define drought conditions are presented in Table 4.2 (Gibbs and Maher, 1967).

Table 4.2 Drought classification based on Deciles (Gibbs and Maher, 1967)

Deciles 1-2 (lowest 20%)	Much below normal
Deciles 3-4 (next lowest 20%)	Below normal
Deciles 5-6 (middle 20%)	Near normal
Deciles 7-8 (next highest 20%)	Above normal
Deciles 9-10 (highest 20%)	Much above normal

4.3.3. Standardized Precipitation Index

The Standardized Precipitation Index (SPI) is a drought monitoring tool and has been used to quantify rainfall deficit to monitor drought conditions. To calculate the SPI values, first the long-term rainfall record is fitted with a probability distribution. Tsakiris *et al.* (2002) and Sonmez *et al.* (2005) used the gamma distribution as it fits the rainfall time series well. The current study also used the gamma distribution to fit the long-term rainfall record; gamma distribution is defined by its probability density function of Equation (4.1).

$$f(x;\alpha,\beta)=\frac{1}{\beta^{\alpha}\Gamma(\alpha)}x^{\alpha-1}e^{-x/\beta} \quad \text{for } x,\ \alpha,\ \beta>0 \tag{4.1}$$

where, α and β are the shape and scale parameters respectively; x is the rainfall amount; and $\Gamma(\alpha)$ is the gamma function. The maximum likelihood method was used to estimate the optimal values of α and β parameters using Equations (4.2) and (4.3) respectively.

$$\alpha=\frac{1}{4A}\left(1+\sqrt{1+\frac{4A}{3}}\right) \tag{4.2}$$

$$\beta=\frac{\bar{x}}{\alpha} \tag{4.3}$$

where, sample statistic, $A=\ln\left(\bar{x}\right)-\frac{\sum\ln\left(x\right)}{n}$; $\bar{x}$ is the rainfall average; and n is the number of observations.

The resulting parameters were then used to derive the cumulative probability for non-zero rainfalls using Equation (4.4).

$$F(x;\alpha,\beta) = \int_0^x f(x;\alpha,\beta)\mathrm{dx} = \frac{1}{\beta^{\alpha}\Gamma(\alpha)}\int_0^x x^{\alpha-1}e^{-x/\beta}\mathrm{dx} \tag{4.4}$$

which can be expressed by Equation (4.5).

$$F(x;\alpha,\beta) = \frac{1}{\Gamma(\alpha)}\int_0^x t^{\alpha-1}e^{-t}\mathrm{dt} \tag{4.5}$$

where, $\mathrm{t} = x/\beta$

Since the gamma function is undefined for $x = 0$ and the rainfall time series data may contain zero rainfalls, the cumulative probability of zero and non-zero rainfalls, *H(x)* was calculated using Equation (4.6).

$$\mathrm{H}(x) = \mathrm{q} + (1-\mathrm{q})\mathrm{F}(x;\alpha,\beta) \tag{4.6}$$

where, *q* is the probability of zero rainfall. If *m* is the number of zeros present in a rainfall time series, then *q* is estimated by *m/n*.

The cumulative probability was then transformed into a standardized normal distribution so that the SPI mean and variance were zero and one respectively. Following McKee and Edwards (1997), Hughes and Saunders (2002) and Mishra and Desai (2006), the current study employed the approximate transformations provided by Abramowitz and Stegun (1965) to transform the cumulative probability distribution into a standardized normal distribution, which are given in Equations (4.7) and (4.8):

$$\mathrm{SPI} = -\left(\mathrm{k} - \frac{\mathrm{c_o} + \mathrm{c_1 k} + \mathrm{c_2 k^2}}{1 + \mathrm{d_1 k} + \mathrm{d_2 k^2} + \mathrm{d_3 k^3}}\right), \text{ when } \mathrm{k} = \sqrt{\ln\left(\frac{1}{\left(\mathrm{H}(x)\right)^2}\right)}$$

$$\text{for } 0 < H(x) \le 0.5 \tag{4.7}$$

$$\mathrm{SPI} = +\left(k - \frac{c_o + c_1 k + c_2 k^2}{1 + d_1 k + d_2 k^2 + d_3 k^3} \right), \text{ when } k = \sqrt{\ln\left(\frac{1}{\left(1 - H(x)\right)^2} \right)}$$

for $0.5 < H(x) \leq 1$ (4.8)

where, c_o = 2.515517, c_1 = 0.802853, c_2 = 0.010328, d_1 = 1.432788, d_2 = 0.189269, d_3 = 0.001308.

Similar to PN, the SPI values can be calculated for multiple monthly time scales (e.g. 3-, 6-, 12-, 24- and 48- month time scales) of interest as was discussed in Section 2.2.5. However, in this study SPI was calculated using a monthly time step. The SPI threshold ranges that are used to define drought conditions are presented in Table 4.3 (McKee *et al.*, 1993).

Table 4.3 Drought classification based on SPI (McKee *et al.*, 1993)

2.00 or more	Extremely wet
1.50 to 1.99	Very wet
1.00 to 1.49	Moderately wet
0.99 to -0.99	Near normal
-1.00 to -1.49	Moderate drought
-1.50 to -1.99	Severe drought
-2.00 or less	Extreme drought

4.3.4. Surface Water Supply Index

The Surface Water Supply Index (SWSI) is a technique to measure the surface water condition as the measure of the drought condition (Section 2.2.4). The SWSI considers rainfall, streamflow / snow water content (streamflow in summer and snow water content in winter) and storage reservoir volume. The snow water content was not considered in the current study, as it is not relevant within the study region. The mathematical formulation of SWSI is given in Equation (4.9).

$$\mathrm{SWSI} = \frac{\left[\left(\mathrm{a}\times PoN_{rn}\right)+\left(\mathrm{b}\times PoN_{sf/sn}\right)+\left(\mathrm{c}\times PoN_{rs}\right)-50\right]}{12} \tag{4.9}$$

where, *PoN* is the probability of non-exceedance (%); *rn, sf / sn* and *rs* refer to rainfall, streamflow / snow water content and storage reservoir volume components respectively; and *a, b and c* are weights for each component and must meet the condition $a+b+c = 1$. Subtracting 50 and dividing by 12 is a cantering and compressing procedure designed to make the SWSI values range between -4.2 and +4.2. The SWSI thresholds values are shown in Table 4.4 (Shafer and Dezman, 1982).

Table 4.4 Drought classification based on SWSI (Shafer and Dezman, 1982)

4.00 or more	Abundant supply
3.99 to 1.99	Wet
2.00 to -0.99	Near normal
-1.00 to -1.99	Incipient drought
-2.00 to -2.99	Moderate drought
-3.00 to -3.99	Severe drought
-4.00 or less	Extreme drought

4.3.5. Aggregated Drought Index

The Aggregated Drought Index (ADI) is a multivariate DI that comprehensively considers all physical forms of drought (i.e., meteorological, hydrological and agricultural) through selection of input variables that are related to each drought type as discussed in Section 2.2.6. The ADI considers several variables that define the hydrologic cycle, including the most important eight variables: rainfall, potential evapotranspiration, streamflow, storage reservoir volume, soil moisture content, snow water content, groundwater flow and temperature. Of these eight variables, six influential variables namely rainfall, potential evapotranspiration, streamflow, storage reservoir volume, soil moisture content and snow water content were used for ADI formulation by Keyantash and Dracup (2004). These variables except the snow water content were considered in this study, as snow water content is not relevant for the study area. Groundwater flow was excluded in this study for three reasons as explained by Keyantash and Dracup (2004): (1) data on historic groundwater levels are not easily accessible for this catchment; (2) groundwater flow into heterogeneous aquifers across the catchment is difficult to assess; and (3) groundwater recharge/depletion is a slow process and it occurs at longer time scales, which extends beyond the ADI time scale of one month used in this study. Keyantash and Dracup (2004) argued that drought generally represents shortage of surface water and that the drought effects should be observed more directly in water-related variables. Because of this reason, groundwater was not included in the ADI formulation in their study as it does not represent the water quantity. For similar reasons, temperature was also not included in this study.

Principal Component Analysis (PCA) was used to aggregate the aforementioned variables (i.e., rainfall, potential evapotranspiration, streamflow, storage reservoir volume and soil moisture content). Computation of the principal components (PCs) requires constructing a square (p x p, where p is the number of variables) symmetric correlation matrix to describe the correlations between the original data. Twelve correlation matrices were used, one for each month. These correlation matrices then underwent PCA. An advantage of the correlation-based PCA approach used in the development of ADI is that the ADI is not impacted by the measurement units of the input data, as all input variables are standardized before they are used in ADI computations.

PCs are a re-expression of the original p-variable data set in terms of uncorrelated components Z_j ($1 < j \leq p$). Eigenvectors derived through PCA are unit vectors (i.e., magnitude of 1) that establish the relationship between the PCs and the original data as shown in Equation (4.10).

$$Z = X\ E \tag{4.10}$$

where, Z is the (n x p) matrix of PCs (i.e., uncorrelated components), in which n is the number of observations, X is the (n x p) matrix of standardized observational data and E is the (p x p) matrix of eigenvectors.

As described by Keyantash and Dracup (2004), the ADI was considered as the first PC (PC1), normalized by its standard deviation as in Equation (4.11):

$$\mathrm{ADI}_{i,k} = \mathrm{Z}_{i,1,k} / \sigma_k \tag{4.11}$$

where, $\mathrm{ADI}_{i,k}$ is the ADI value for month k in year i, $\mathrm{Z}_{i,1,k}$ is the first PC for month k in year i, and σ_k is the sample standard deviation of $\mathrm{Z}_{i,1,k}$ over all years for month k.

The ADI utilizes only the PC1 because it explains the largest fraction of the variance described by the full p-member, standardized data set. Since PCs are orthogonal vectors, it is not mathematically proper to combine them into a single expression (Keyantash and Dracup, 2004). Considering all 12 months, PC1 described an average of 56.4% of the data set variance in this study. This figure was ranged from 58.0 to 63.0% in Keyantash and Dracup (2004) where they had developed ADI for three catchments in California, U.S.A. Once the ADI values were computed for each year and each month, they were reordered into a single time series in chronological order.

To determine ADI thresholds, the empirical cumulative distribution of above ADI values was constructed and is shown in Figure 4.1. The ADI thresholds values were then calculated for the study catchment using this empirical cumulative distribution function. These thresholds were used to classify the drought conditions. Keyantash and Dracup (2004) used the SPI thresholds to generate ADI thresholds. The SPI dryness thresholds are the Gaussian variates of -2, -1.5, -1 and 1 standard deviations (Table 4.3), which corresponds to *2.3*th, *6.7*th, *16.0*th and *84.0*th percentiles in SPI distributions. The ADI thresholds corresponding to these percentiles were -1.70, -1.41, -0.96 and 0.92 respectively for the study catchment (Figure 4.1). These ADI thresholds and relevant drought classifications for the study catchment are presented in Table 4.5.

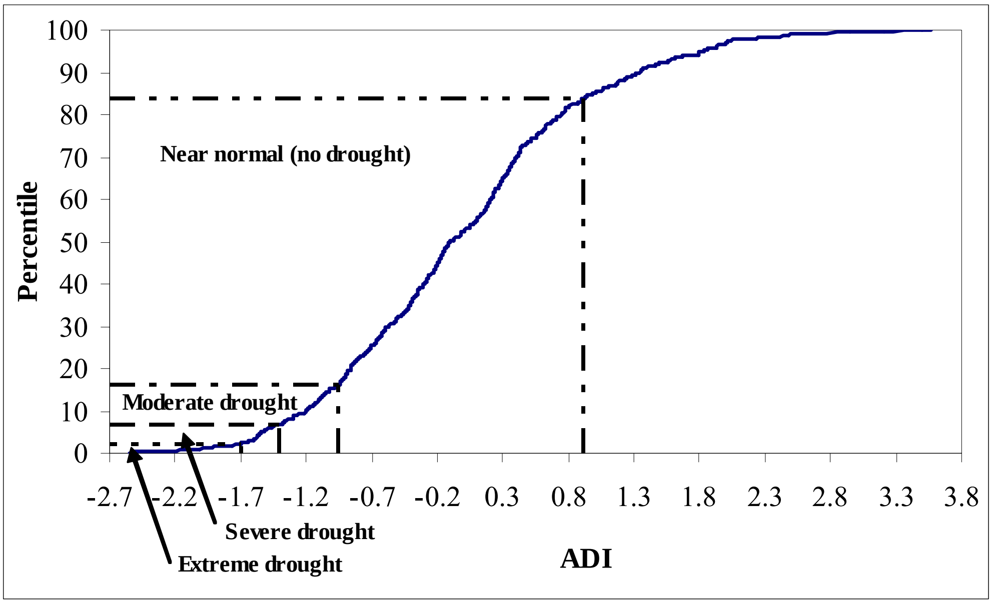


Figure 4.1 Computation of ADI thresholds for the study catchment

Table 4.5 Drought classification based on ADI for the study catchment

-0.95 to 0.92	Near normal
-1.40 to -0.96	Moderate drought
-1.69 to -1.41	Severe drought
-1.70 or less	Extreme drought

4.4. Analysis of Drought Indices

Drought Indices (i.e., PN, Deciles, SPI, SWSI and ADI) were computed for the Yarra River catchment using monthly data of input variables using the methodology described in Section 4.3. The input variables used were rainfall, potential evapotranspiration, streamflow, storage reservoir volume and soil moisture content, and the data from 1960 to 2008 (Section 4.2) were used to compute the above DIs.

As described in Section 3.3, there were several historical droughts recorded in Victoria, especially in the last 50 years (i.e. 1967-1968, 1972-1973, 1982-1983, 1997-1998, 2003-2004 and 2006 onwards). All these historical droughts were considered in this study to investigate how well the DIs analyzed were capable in defining drought conditions. Traditionally, the drought assessment has been done with drought severity defined by a DI (Yevjevich, 1967; Keyantash and Dracup, 2002). The drought severity (*S*) is the product of the drought duration (*D*) (during which the DI values are consistently below a truncation level) and the drought magnitude (*M*) (which is the mean departure of DI values from that truncation level during the drought period). Generally, the drought is a slow process and lasts for a longer period. In this case, the probability distribution of the departure of the DI values of these droughts from the truncation level follows a normal distribution and hence either the mean or the median value can be used to define *M*. However, in this study, the median value was used to define *M*, since there were some droughts with short durations and the number of data points describing the probability distribution of the departure of the DI values of these droughts from the truncation level is small, and also the median is not affected by the outliers (Helsel and Hirsch, 1993). The relationship between *S*, *M* and *D* is shown in Figure 4.2. Using this nomenclature, the five DIs considered in this study are analyzed in this section using the above historical droughts.

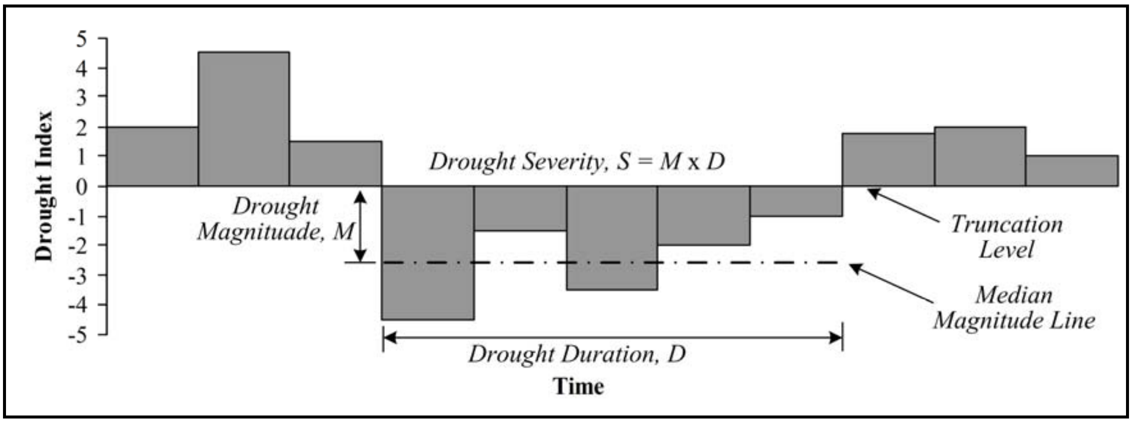


Figure 4.2 Relationship between Drought Severity, Drought Magnitude and Drought Duration (adapted from Yevjevich, 1967; Keyantash and Dracup, 2002)

Time series plots of the DIs are shown in Figure 4.3. In Figure 4.3, the shaded vertical areas represent the historical droughts detected by each DI (i.e., DIs values below the truncation line). This figure shows that the rainfall based DIs (i.e., PN, Deciles and SPI) have shown similarities in detecting historical droughts. These three indices showed rapid fluctuations over the whole period, and also even during the detected droughts. They have given some indication of the historical droughts. However, in Figure 4.3 it is difficult to see the historical droughts within these periods. This is because during the drought, when large rainfall events occurred for short periods, these indices have produced large index values indicating that the drought has ceased, which was clearly not the case. It can be also seen from Figure 4.3 that drought durations detected by these three indices were almost similar for all historical droughts except for the 1982-1983 drought. In Figure 4.3, both SWSI and ADI indices showed smoother transitional characteristics during droughts, and from dry to wet spells and vice versa. Because of this characteristic, the historical droughts detected by SWSI and ADI are clearer than those detected by PN, Deciles and SPI. Furthermore, the ADI showed the DI values were consistently below the drought defining threshold during the duration of the droughts compared to SWSI, which had few values above the threshold during the duration of the drought.

Characteristics of historical droughts as detected by PN, Deciles, SPI, SWSI and ADI are presented in Table 4.6. The table shows median and maximum intensity of droughts together with their drought classifications, in addition to their duration and severity. By comparing drought classifications in Tables 4.1 to 4.5, it can be seen that they are similar for four classifications (i.e. near normal, moderate drought, severe drought and extreme drought) in PN, SPI and ADI. The classification in Deciles on ‘Much below normal’ can be split into ‘severe’ and ‘extreme’ drought classifications considering the deciles 2 and 1 respectively, and the classification ‘below normal’ in Deciles can be considered as ‘moderate drought’. Similarly, the classifications on ‘near normal’ and ‘incipient’ in SWSI can be combined into the ‘near normal’ classification. The above discussion brings in a homogeneous classification of drought across all five DIs considered in this study. This homogeneous classification is used in Table 4.6 to classify drought conditions. In general, the classification based on median intensity is similar (within one level of classification) in all 5 DIs for all droughts, but SWSI shows markedly different classifications for 1972-1973 and 1997-1998 droughts for maximum intensity compared to the other DIs. Generally the drought durations detected by rainfall based DIs (i.e. PN, Deciles and SPI) for each drought are similar. These durations are also similar for SWSI and ADI, but quite different from those of rainfall based DIs.

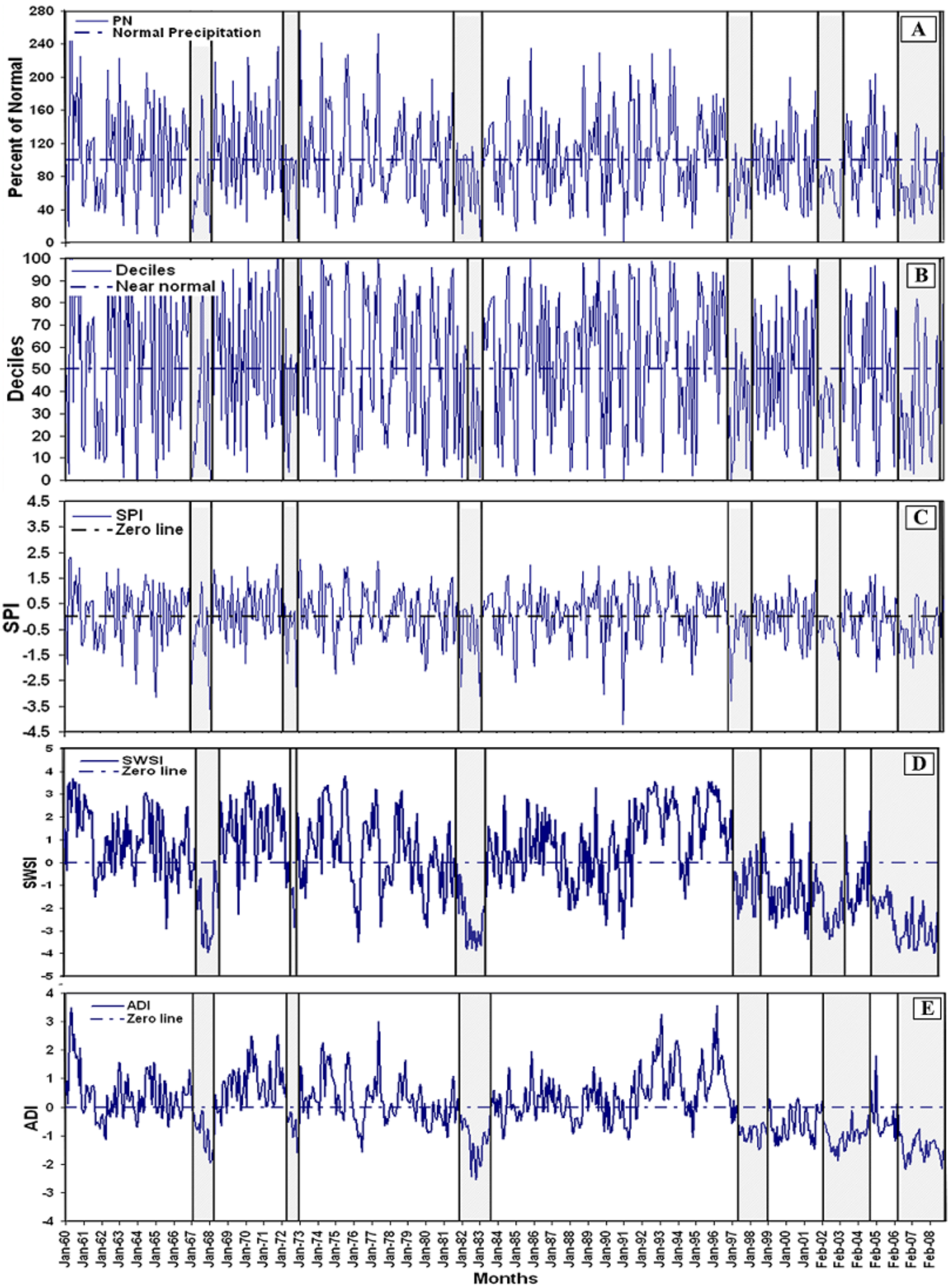


Figure 4.3 Time series plots of drought indices; A: Percent of Normal, B: Deciles, C: Standardized Precipitation Index, D: Surface Water Supply Index and E: Aggregated Drought Index

Table 4.6 Characteristics of historical droughts as detected by PN, Deciles, SPI, SWSI and ADI

	PN	**Deciles**	**SPI**	**SWSI**	**ADI**
			Historical drought known as 1967-1968 drought		
Median intensity, *M*	(53) Moderate	(19) Severe (close to moderate)	(-0.87) Near normal	(-2.33) Moderate	(-0.81) Near normal
Duration, *D* (months)	15	15	15	16	15
Severity, *S*	-705 [a]	-1215 [a]	-13.05	-37.28	-12.15
Maximum intensity, M_{max}	(4) Extreme (Feb, '68)	(0) Extreme (Feb, '68)	(-3.62) Extreme (Feb, '68)	(-3.97) Severe (Feb, '68)	(-1.96) Extreme (Feb, '68)
			Historical drought known as 1972-1973 drought		
Median intensity, *M*	(78) Moderate	(36) Moderate	(-0.26) Near normal	(-1.41) Near normal	(-0.52) Near normal
Duration, *D* (months)	11	11	11	5	9
Severity, *S*	-242 [a]	-704 [a]	-2.86	-7.05	-4.68
Maximum intensity, M_{max}	(6) Extreme (Dec, '72)	(1) Extreme (Dec, '72)	(-3.36) Extreme (Dec, '72)	(-2.84) Moderate (Dec, '72)	(-1.60) Severe (Dec, '72)
			Historical drought known as 1982-1983 drought		
Median intensity, *M*	(76) Moderate	(29) Moderate	(-0.33) Near normal	(-2.54) Moderate	(-1.10) Moderate
Duration, *D* (months)	20	11	17	21	22
Severity, *S*	-480 [a]	-781 [a]	-5.61	-53.34	-24.2
Maximum intensity, M_{max}	(8) Extreme (Feb, '83)	(1) Extreme (Feb, '83)	(-3.11) Extreme (Feb, '83)	(-3.86) Severe (Feb, '83)	(-2.55) Extreme (Nov, '82)

continue

	PN	Deciles	SPI	SWSI	ADI
		Historical drought known as 1997-1998 drought			
Median intensity, M	(71) Moderate	(32) Moderate	(-0.40) Near normal	(-1.03) Near normal	(-0.86) Near normal
Duration, D (months)	17	17	17	20	21
Severity, S	-493 [a]	-1156 [a]	-6.8	-20.6	-18.06
Maximum intensity, M_{max}	(7) Extreme (Feb, '97)	(1) Extreme (Feb, '97)	(-3.27) Extreme (Feb, '97)	(-2.40) Moderate (Aug, '98)	(-1.51) Severe (Aug, '98)
		Historical drought known as 2003-2004 drought			
Median intensity, M	(68) Moderate	(30) Moderate	(-0.48) Near normal	(-2.07) Moderate	(-1.04) Moderate
Duration, D (months)	18	16	16	23	32
Severity, S	-576 [a]	-1120 [a]	-7.68	-47.61	-33.28
Maximum intensity, M_{max}	(30) Severe (Feb, '03)	(5) Extreme (Feb, '03)	(-1.64) Severe (Feb, '03)	(-3.35) Severe (Nov, '02)	(-1.89) Extreme (Jan, '03)
		2006 onwards drought			
Median intensity, M^{b}	(67) Moderate	(29) Moderate	(-0.49) Near normal	(-2.66) Moderate	(-1.47) Severe (close to moderate)
Duration, D (months)	-- Still continuing --				
Severity, S	-- Still continuing --				
Maximum intensity, M^{b}_{max}	(23) Severe (Apr, '07)	(3) Extreme (Apr, '07)	(-2.00) Extreme (Apr, '07)	(-4.00) Extreme (Sept, '08)	(-2.19) Extreme (Oct, '06)

[a] M values were rescaled from 0 – 100 to -100 – 0 for calculating S values

[b] Median and maximum intensity are until 2008

Ranking of all historical droughts detected by PN, Deciles, SPI, SWSI and ADI based on median intensity, duration, severity and maximum intensity are presented in Table 4.7. All droughts were arranged in descending order in Table 4.7 for the above parameters, most severe drought being rank 1. These rankings were also compared with those of historical records (Keating, 1992; Tan and Rhodes, 2008). Note that the median drought intensities for these historical droughts have not been published in Keating (1992) and Tan and Rhodes (2008).

Table 4.7 Ranking of historical droughts*

	Historical**	**PN**	**Deciles**	**SPI**	**SWSI**	**ADI**
Median intensity, *M*	-	1972/73	1972/73	1967/68	1982/83	1982/83
		1982/83	1997/98	2003/04	1967/68	2003/04
		1997/98	2003/04	1997/98	2003/04	1997/98
		2003/04	1982/83	1982/83	1972/73	1967/68
		1967/68	1967/68	1972/73	1997/98	1972/73
Duration, *D*	Longest, *D* was in 2003/04	1982/83	1997/98	1972/73	2003/04	2003/04
		2003/04	2003/04	1982/83	1982/83	1982/83
		1997/98	1967/68	2003/04	1997/98	1997/98
		1967/68	1982/83	1967/68	1967/68	1967/68
		1972/73	1972/73	1972/73	1972/73	1972/73
Severity, *S*	Highest, *S* was in 2003/04	1967/68	1967/68	1967/68	1982/83	2003/04
		2003/04	1997/98	2003/04	2003/04	1982/83
		1997/98	2003/04	1997/98	1967/68	1997/98
		1982/83	1982/83	1982/83	1997/98	1967/68
		1972/73	1972/73	1972/73	1972/73	1972/73
Maximum intensity, M_{max}	Maximum intensity, M_{max} was in 1982/83	1967/68	1967/68	1967/68	1967/68	1982/83
		1972/73	1972/73	1972/73	1982/83	1967/68
		1997/98	1997/98	1997/98	2003/04	2003/04
		1982/83	1982/83	1982/83	1972/73	1972/73
		2003/04	2003/04	2003/04	1997/98	1997/98

*2006 onwards drought was not included in this ranking since it is still continuing

**Historical drought records were obtained from Keating (1992) and Tan and Rhodes (2008)

Table 4.7 showed that in ranking historical droughts, SWSI and ADI had produced exactly the same ranks for drought duration, whereas the rainfall based DIs, PN, Deciles and SPI had given the same ranks for maximum intensity. In addition, similarities were also found both in SWSI and ADI for top rank in median intensity, whereas PN, Deciles and SPI had produced the top rank for severity. The above findings show some similarities in terms of ranking in rainfall based DIs (i.e., PN, Deciles and SPI) and in SWSI/ADI. The reason for this is that the rainfall based DIs use only rainfall in defining DI, whereas multiple inputs describing the hydrologic cycle are used in SWSI/ADI. Table 4.7 also shows that all five DIs had produced the lowest drought duration and the lowest severity ranking for 1972-1973 drought. However, it is important to highlight that in comparison to the published details of historical droughts only the ADI ranking had produced the longest duration, the highest severity and the maximum intensity as published for historical droughts. The 2006 onwards drought is not included in the rankings in Table 4.7 as the duration and the severity of this drought cannot be calculated since the drought is still occurring.

4.5. Evaluation of Drought Indices

In this section, DIs that were analyzed for the Yarra River catchment were evaluated to investigate how they satisfied desirable properties of a good DI and how they could be useful for this catchment. In judging the overall usefulness of the DIs, Redmond (1991) proposed a number of desirable properties. Keyantash and Dracup (2002) used six decision criteria from these desirable properties, namely *robustness, tractability, sophistication, transparency, extendability* and *dimensionality*, for evaluating some of the DIs used in the U.S. They believed that these criteria would give a reasonable framework for evaluation of the DIs without excessive complications. Except the *dimensionality* criterion, the decision criteria used by Keyantash and Dracup (2002) was used in this study. The *dimensionality* criterion was not considered, since it is mostly covered by the *transparency* criterion, as stated also by Keyantash and Dracup (2002). To account for the relative importance of each decision criterion, a set of weights called relative importance factors were used by Keyantash and Dracup (2002). However, these relative importance factors were determined subjectively without giving any justification. They stated that the determination of relative importance factors depends on professional experience and personal judgment, and researchers and practitioners are free to modify these weights to suit their own perspectives (Keyantash and Dracup, 2002). The current study considered that each decision criterion has equal importance for evaluation of DIs for the Yarra River catchment. However, it should be noted that the *Revised Simos Procedure* of Figueira and Roy (2002), which is a modified version of the original *Simos Procedure* (Simos, 1990), can be used to objectively determine the weights for decision criteria, eliciting preferences from drought researchers and professionals. This method has been successfully used by Kodikara *et al.* (2010) in determining weights for multi-objective operation of an urban water supply system.

A range of raw scores from 1 to 5 (5 being the most desirable) were assigned to each of the five selected decision criteria to evaluate the DIs for the Yarra River catchment. The raw scores were given based on the qualitative and quantitative assessment of DIs. The qualitative assessment are based on findings of various researchers in the past on evaluating DIs (described in Section 2.2) and the theoretical/computational aspects of DIs (described in Section 4.3). The quantitative assessment is based on how well these DIs modeled the historical droughts in the Yarra River catchment which was presented in Section 4.4. The sum of the weighted scores of each criterion (i.e., raw scores multiplied by relative importance factor) was the total weighted score for each index. These total weighted scores were used for comparative evaluation of DIs for the Yarra River catchment in this study. Since the relative importance factors were not considered in this study, they were considered as 1. Therefore, the maximum possible total weighted score any DI could have is 25. The total weighted scores were calculated for each index and are presented in Table 4.8. The rationale for the allocated raw scores of each criterion for the Yarra River catchment is given below.

4.5.1. Robustness

Robustness represents the usefulness of the DI over a wide range of physical conditions. Ideally a DI should be responsive, but not temperamental (Keyantash and Dracup, 2002). The ADI was quite responsive as well as it was not temperamental in detecting historical drought conditions (Figure 4.3). Therefore, a raw score of 5 was given for the robustness criterion to the ADI (Table 4.8). The SWSI developed as a hydrological DI showed good agreement with the ADI for detecting historical droughts (Figure 4.3). However, Figure 4.3 showed that the ADI was superior to the SWSI in detecting historical drought conditions in some cases (e.g., in detecting drought in 1997-1998), showing some temperamental characteristics in SWSI. Moreover, as seen before (Table 4.6), the drought classification detected in SWSI was markedly different to other DIs for 1972-1973 and 1997-1998 droughts in terms of maximum intensity. Therefore, the robustness score for the SWSI was given as 3 (Table 4.8). The rainfall based DIs (i.e., PN, Deciles and SPI) as mentioned earlier have shown similarities in detecting historical droughts. They were quite responsive to the rainfall variations as they are solely dependent on rainfall. However, it is seen that these DIs are very temperamental. Furthermore, they do not account for overall water resources variability within the catchment, rather they show the rainfall variability. The SPI nevertheless showed slightly better robustness characteristics than PN and Deciles (Figure 4.3). Therefore, the robustness score of 2 was assigned to SPI, and score of 1 each to both PN and Deciles.

Table 4.8 Comparative scores of DIs based on weighted evaluation criteria

Drought Index	**Raw scores (1-5)**					**Total weighted scores**
	Robustness	**Tractability**	**Transparency**	**Sophistication**	**Extendability**	
PN	1	5	5	1	5	17
Deciles	1	5	4	1	5	16
SPI	2	4	3	1	4	14
SWSI	3	3	3	3	4	16
ADI	5	2	3	5	3	18

4.5.2. Tractability

Tractability implies the practical aspect of the drought index. A tractable index requires low level of numerical computations, less number of input variables and less extensive database with historical data (Keyantash and Dracup, 2002). PN and Deciles require simple numerical computations and use only rainfall as the input variable. SPI also requires only rainfall, but the computations are complex than in PN and Deciles. SWSI and ADI require more input variables, most being in ADI. The level of computational difficulty is almost the same in SWSI and ADI. In terms of the database, all DIs require rainfall data. However, SWSI requires more hydro-climatic data and most for ADI. Therefore, raw scores for PN, Deciles, SPI, SWSI and ADI are assigned as 5, 5, 4, 3 and 2 respectively (Table 4.8).

4.5.3. Transparency

Transparency represents the clarity of the objective and rationale behind the drought index (Keyantash and Dracup, 2002). A DI is considered to be *transparent* if it is understandable by both the scientific community and the general public, and therefore transparency may represent general utility. PN and Deciles in general are understandable to both the scientific community and the general public, but PN more than Deciles, and therefore, the scores of 5 and 4 were assigned for PN and Deciles respectively (Table 4.8). Compared to PN and Deciles, the indices of SPI, SWSI and ADI are not relatively easy to understand by the general public, but they are understandable by drought researchers and professionals. The level of understandability seems to be the same in these three DIs. Therefore, a score of 3 was assigned to these three DIs for the transparency criterion (Table 4.8).

4.5.4. Sophistication

Sophistication considers the conceptual merits of the drought characterization approach (Keyantash and Dracup, 2002). Sometimes, the computational technique of the DI is complex and the DI itself might not be quite understandable (i.e., neither tractable nor transparent), but it may be sophisticated and appreciable from the proper perspective. The ADI is least tractable and less transparent, but it is more useful than PN, Deciles, SPI and SWSI in defining drought intensity, duration and severity (Figure 4.3 and Table 4.7). As outlined in Section 4.4, only the ADI was able to pick the most severe historical drought in terms of the longest duration, highest severity and maximum intensity, which was same as in the published records (Table 4.7). Therefore, ADI is more sophisticated than PN, Deciles, SPI and SWSI, and a score of 5 was assigned to ADI (Table 4.8). SWSI had detected the worst drought only in terms of the longest duration of historical droughts as in the published records. The rainfall based DIs (i.e., PN, Deciles and SPI) have not detected any of these characteristics of historical droughts as in published records. Therefore, raw scores of 1 were assigned to PN, Deciles and SPI, and 3 for SWSI.

4.5.5. Extendability

Extendability corresponds to the degree to which the DI may be extended across time to alternate drought scenarios (Keyantash and Dracup, 2002). For example, all DIs evaluated in this study use basic measured data (e.g., rainfall, streamflow and storage volume), and therefore were constructed for the period where historical data were available. They are extendable for future, provided these data will be available. When long records of future data will be available, they can be used to update the classification thresholds. These calculations are easy for PN and Deciles, and more complex for SPI, SWSI and ADI (almost with the same level of complexity). However, ADI requires further calculations to compute the soil moisture content, which is an input to ADI. Therefore, PN and Deciles were assigned a score of 5, SPI and SWSI were assigned a raw score of 4 and ADI was assigned a raw score of 3 (Table 4.8).

4.5.6. Sensitivity of Raw Scores on Ranking

It should be noted that a significant effort has been spent in this study to assign raw scores (which were discussed in Sections 4.5.1 to 4.5.5) in some objective form to each of the decision criterion, based on the results of past studies and modeling of historical droughts for the DIs that were investigated (in this study). However, the researchers and professionals may select nearby raw scores compared to the current scores in Table 4.8, which might give a different ranking. Therefore, a sensitivity analysis was conducted using the Monte Carlo simulation (Mooney, 1997) technique. In the sensitivity analysis, the current raw scores in Table 4.8, if they are not either 1 or 5, were considered have 50% probability and the remaining 50% probability was assigned equally to the adjacent raw scores. For example, the raw score 3 in Table 4.8 was considered have 50% probability, and 25% probability was assigned to adjacent raw scores of 2 and 4. The raw scores 1 and 5 in Table 4.8 were considered to have 75% probability, while the adjacent score was assigned 25% probability. For example, the raw score 1 was given 75% probability and the adjacent raw score 2 was assigned the remaining 25%. Then, 10, 000 possible raw score combinations were randomly generated for each of the 5 decision criteria of each DI with probabilities assigned to them as discussed above, and the total weighted score was computed for each combination of 5 criteria of each DI. The modes of these total weighted scores of each DI were then calculated as the measure of central tendency, as the mode is most suitable for measuring the central location when the most frequently occurring value is important and data consist of discrete values as in this study (Helsel and Hirsch, 1993; Gravetter and Wallnau, 2008). The modes of the total weighted scores from the sensitivity analysis were found to be exactly same as those of Table 4.8. This means that the subjective nature of assigning raw scores did not have any impact in the overall ranking of the DIs investigated in this study.

4.5.7. Overall Evaluation

According to Table 4.8, the overall rankings of the DIs in terms of the total weighted score are ADI, PN, SWSI and Deciles and SPI; however, the indices SWSI and Deciles show same ranking. Therefore, this study shows that the ADI is a better DI than PN, Deciles, SPI and SWSI in quantifying the drought conditions of the Yarra River catchment. As can be seen from Figure 4.3, the ADI was the most stable DI having smooth transitional characteristics during the droughts as well as during other periods. The ADI has modeled the characteristics of historical droughts better than any other DI. The SPI has produced the lowest total score within the group of rainfall based DIs. This is because of the complexity of SPI calculations and relative difficulty in understanding the SPI by the general public.

Based on the results in Table 4.8 and the sensitivity analysis, the ADI was found to be superior to the other DIs for the Yarra River catchment, followed by PN. However, the ADI requires more data than PN, but this has been already considered in the evaluation of the DIs for the Yarra River catchment. Nevertheless, if the results are to be generalized for other catchments especially in Victoria without detailed evaluation of the DIs, then there is an argument that PN can be considered as a better DI than the ADI for catchments, where data are not available or difficult to obtain to compute the ADI.

4.6. Summary

Drought assessment has been a challenging task among drought researchers and professionals. There are many Drought Indices (DIs) that have been developed around the world and are commonly used to quantify drought conditions as was discussed in Chapter 2. It was found that in most cases, DIs are developed for a specific region, and therefore may not be directly applicable to other regions due to inherent complexity of drought phenomena, different hydro-climatic conditions and catchment characteristics. Suitability of some of the existing DIs had been analyzed for different climatic regions around the world. However, no such study had been conducted in Australia which is the driest inhabited continent on the Earth. In this chapter, an examination of the performance of five DIs namely *Percent of Normal (PN), Deciles, Standardized Precipitation Index (SPI), Surface Water Supply Index (SWSI)* and *Aggregated Drought Index (ADI)* for modeling historical droughts within the Yarra River catchment in Victoria (Australia) is presented.

Historical droughts recorded in Victoria during 1967-1968, 1972-1973, 1982-1983, 1997-1998, 2003-2004, and 2006 onwards were used in this study to investigate how well the above DIs were capable in defining drought conditions. However, the 2006 onwards drought was not included in the ranking of historical droughts, since it is still continuing. The study showed that PN, Deciles and SPI have shown similarities in detecting historical droughts as was expected, because they were developed using rainfall as the single variable. These three indices also showed rapid fluctuations over the whole period, and also even during droughts indicating that the drought has ceased when large rainfall events occurred for short periods. It was difficult to see the historical droughts within these periods with those rainfall based DIs. Both SWSI and ADI showed smoother transitional characteristics during droughts, and from dry to wet spells and vice versa. However, SWSI has limitations in the wide range of applications as important parameters such as soil moisture content and evapotranspiration that directly affect the agricultural droughts have not been considered in the development of SWSI as was discussed in Section 2.2.4. In comparison to published details of historical droughts, ADI showed better agreement in detecting historical droughts than the other DIs.

Five decision criteria - *robustness, tractability, sophistication, transparency, and extendability* - were used in this study to evaluate the performance of the above DIs in modeling historical droughts and to use these indices in drought management of the Yarra River catchment. Analysis showed that the ADI was more robust and sophisticated than SWSI. The other three DIs were of the same level in terms of robustness and sophistication, except that SPI was little more robust than PN and Deciles. However, the rainfall based DIs (i.e. PN, Deciles and SPI) were less robust and sophisticated than ADI and SWSI. In terms of the tractability criterion, PN and Deciles had the same raw score and more tractable than SPI, SWSI and ADI in that order, the ADI being the least tractable. The analysis also showed that ADI, SWSI and SPI had the same raw score for the transparency criterion and less transparent than the PN and Deciles, PN having the highest transparency score. Finally, PN and Deciles were found to be more extendable than SPI, SWSI and ADI in that order with ADI having the least score and SPI and SWSI having the same score.

Overall, the study has found that the ADI was a better DI for modeling droughts and the management of droughts in the Yarra River catchment than the other DIs analyzed in this study. It has modeled the characteristics of historical droughts better than any other DI. It was also the most stable having smoother transitional characteristics during droughts as well as during other periods. Although a significant effort has been spent on reducing subjectivity in assigning raw scores for each of the decision criterion in relation to each DI, still there can be some subjectivity involved in these raw scores. A sensitivity analysis was conducted to investigate how the overall ranking changes with respect to likely values of raw scores; interestingly, it was found that there was no difference in the overall ranking. Furthermore, the evaluation in this study considered equal importance to all decision criteria. However, the evaluation can be improved by incorporating relative importance factors for each of the decision criterion, and this information can be obtained through *Revised Simo's Procedure*, eliciting information from drought researchers and professionals.

5. DEVELOPMENT OF NONLINEAR AGGREGATED DROUGHT INDEX

Overview; Study Area and Data Used; Methodology Used for Development of Nonlinear Aggregated Drought Index; Analysis of NADI for Drought Assessment; Comparative Evaluation of NADI and ADI; Summary

5.1. Overview

Defining drought conditions is an important task in drought management. There are many drought defining tools, and of these tools, Drought Indices (DIs) have been most commonly used to define the drought conditions in the world as discussed in Section 2.2. Some of the well known DIs were evaluated for their potential use in this research project (for the Yarra River catchment) and presented in Chapter 4. It was found that the Aggregated Drought Index (ADI) was the most suitable DI for this research project. As discussed in Section 4.3.5, the Principal Component Analysis (PCA), also known as the linear PCA (Lorenz, 1956; Barnston and Livezey, 1987; Lins, 1997; Hidalgo *et al.*, 2000) was used to combine the hydro-meteorological variables in ADI. However, the linear PCA assumes that the variables used in the ADI have linear relationships between them in formulating Principal Components (PCs). Ellis *et al.* (2006) and Linting *et al.* (2007) noted that if nonlinear relationships between variables exist, as is often the case with environmental data (Gauch, 1982), then the linear PCA technique may not perform well or correctly. Therefore, the Nonlinear Principal Component Analysis (NLPCA) which was originally developed by Guttman (1941) is introduced and used in the current study to avoid the limitation of the use of linear PCA used in ADI (Section 4.3.5), which has led to the development of a novel Nonlinear Aggregated Drought Index (NADI) in this research project.

The objective of this chapter is to present the findings of the development and evaluation of the NADI method of drought assessment. The chapter begins with a brief description of the study area and data used in this study, followed by the methodology used for the development of NADI. Analysis of NADI for drought assessment which was conducted to investigate how well the NADI define the historical drought conditions is then presented, followed by the comparative evaluation of NADI and ADI. Finally, summary drawn from the study are presented at the end of the chapter.

5.2. Study Area and Data Used

Similar to Chapter 4, the Yarra River catchment was used as the case study to demonstrate the applicability of NADI. The details of the Yarra River catchment and its importance were described in Chapter 3. Similar to the development of ADI, five hydro-meteorological variables (i.e. rainfall, potential evapotranspiration, streamflow, storage reservoir volume and soil moisture content) were used in the development of NADI. Monthly time step was used in the development of NADI time series. The same data that were described and used in Chapter 4 were used in the development of NADI for the Yarra River catchment.

5.3. Methodology Used for Development of Nonlinear Aggregated Drought Index

A flowchart showing the overall steps followed during the NADI development is presented in Figure 5.1. Details are given in Section 5.3.1 to 5.3.4 including an example. First two steps in Figure 5.1 are self-explanatory. The third step segregates (or groups) data by categories of months to model the seasonal effects. The NLPCA was then performed individually for each month and produced the PCs. The NADI values were then computed separately for each of 12 months based on the first PC (or PC1), normalized by its standard deviation as the NADI value, as was done with the ADI (Section 4.3.5). These monthly NADI values were then merged to a single NADI time series in chronological order. Finally, the NADI thresholds are computed using the computed NADI time series.

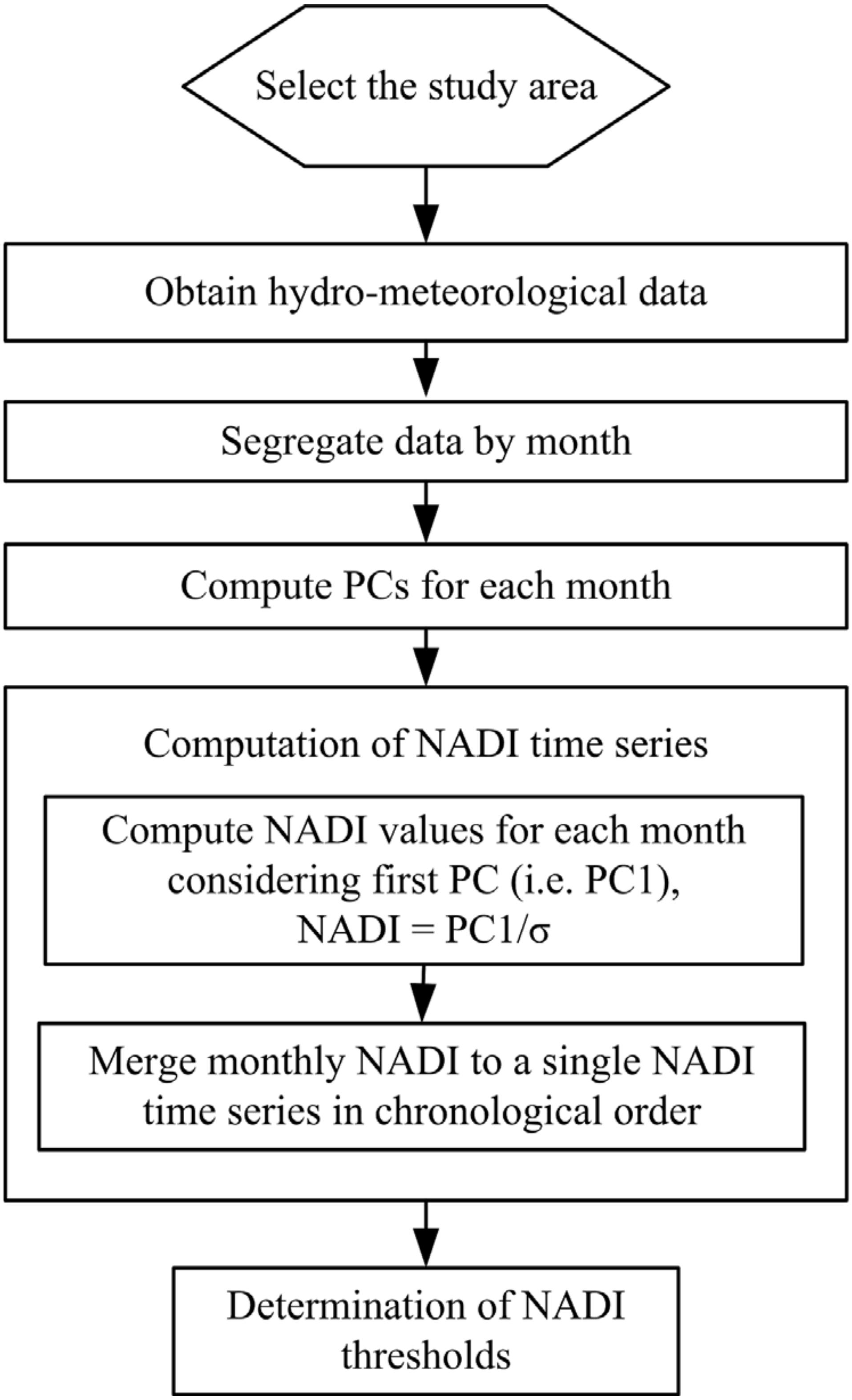


Figure 5.1 Flow chart of the NADI computational process

5.3.1. Computation of Principal Components using Nonlinear Principal Component Analysis

Generally, NLPCA has been used as a data redundant technique (Kramer, 1991; Monahan, 2000, 2001; Linting *et al.*, 2007). However, in this study, the NLPCA is introduced and adopted as the numerical approach to aggregate five hydro-meteorological variables (i.e., rainfall, potential evapotranspiration, streamflow, storage reservoir volume and soil moisture content) in the development of NADI. The use of NLPCA in the development of NADI is similar to the use of linear PCA in the development of ADI in Chapter 4 and used by Keyantash and Dracup (2004). The NLPCA is similar to the linear PCA in most aspects, with only difference being that in linear PCA the PCs are obtained through a linear combination of variables, whereas, in NLPCA the PCs obtained through a linear combination of transformed variables. The variable transformations are performed in NLPCA using an iterative process (which will be discussed shortly) where the observed data are replaced with new numeric values. By doing so, the PCs generated through NLPCA capture the nonlinear relationships between the variables and account for more variance in the data than the linear PCA (Linting *et al.*, 2007).

In linear PCA, PCs are a re-expression of the original m-variable data set in terms of uncorrelated components Z_j ($1 < j \leq m$). As was discussed in Section 4.3.5, eigenvectors derived through linear PCA are unit vectors (i.e., magnitude of 1) that establish the relationship between the PCs and the original data as shown in Equation (5.1).

$$Z = H\ E \tag{5.1}$$

where, Z is the (n x m) matrix of PCs (i.e. uncorrelated components), in which n is the number of observations, H is the (n x m) matrix of standardized observational data (also called component scores) and E is the (m x m) matrix of eigenvectors (also called component loadings).

Usually, in linear PCA, component scores and component loadings are obtained from a singular value decomposition of the standardized data matrix or an eigenvalue decomposition of the correlation matrix. However, the same results are obtained in the NLPCA through an iterative process as was mentioned above in which a least squares loss function is minimized. The loss to be minimized is the loss of information due to representing the variables by a small number of components: in other words, the difference between the variables and the component scores weighted by the component loadings. It should be noted that in NLPCA, the variable transformation task and the linear PCA model estimation (i.e., computation of component scores and component loadings) were performed simultaneously through the iterative process. Computer programs or modules that perform NLPCA are available in two major commercial statistical packages: the module PRINQUAL in SAS (SAS Institute, 1992) and the module CATPCA in SPSS (Meulman *et al.*, 2004; SPSS, 2006). The CATPCA (i.e., Categorical Principal Component Analysis) module of SPSS software was used in this study to perform the NLPCA and the way NLPCA was performed in CATPCA is described mathematically below.

Suppose (n x m) is an observational data matrix (**H**), where n is the number of observations and m is the number of variables. Each variable h_j in the j^{th} column of **H** has a vector of n (n x 1) observational data, with $j = 1, \ldots, m$. Twelve matrices of **H** were used separately for NLPCA, one for each month. Assume that after optimal iterative process in NLPCA, the matrix **H** is replaced by the (n x m) matrix of **Q**, containing the transformed variables q_j ($q_j = \phi_j(h_j)$). Various types (e.g., nominal, ordinal and spline) of nonlinear transformation can be chosen in the CATPCA module based on the data. Spline type transformation which is suitable for numeric and continuous data was used for nonlinear variable transformation in this study, since numeric and continuous data were used in the matrix **H**. If **X** is the (n x p) matrix of component scores, p is the number of components, and **A** is the (m x p) matrix of component loadings, with its j^{th} row indicated by a_{js} (where, $s = 1, \ldots, p$), then the loss function (L(Q, X, A)) that can be used in NLPCA for the minimization of the difference between the transformed data and the PCs can be expressed as:

$$L\left(\mathrm{Q}, \mathrm{X}, \mathrm{A}\right) = n^{-1}\sum_{j=1}^{m}\sum_{i=1}^{n}\left(q_{ij} - \sum_{s=1}^{p} x_{is}a_{js}\right)^2 \qquad (5.2)$$

In matrix notation, this function can be written as

$$L(\mathrm{Q}, \mathrm{X}, \mathrm{A}) = n^{-1} \sum_{j=1}^{m} \operatorname{tr}\left(q_j - \mathrm{X}a_j\right)' \left(q_j - \mathrm{X}a_j\right) \tag{5.3}$$

where *tr* denotes the trace function that sums the diagonal elements of a matrix. For example,

$$\operatorname{tr} B'B = \sum_{j=1}^{m} \sum_{i=1}^{n} b_{ij}^2 \tag{5.4}$$

It was proven that the loss function in Equation (5.3) is equivalent to Equation (5.5) (Gifi, 1990).

$$L(\mathrm{Q}, \mathrm{X}, \mathrm{A}) = n^{-1} \sum_{j=1}^{m} \operatorname{tr}\left(q_j a_j' - \mathrm{X}\right)' \left(q_j a_j' - \mathrm{X}\right) \tag{5.5}$$

The loss function given by Equation (5.5) is used in CATPCA, because the vector representations of variables as well as the representations of transformed data as a set of group points can be incorporated in the loss function in Equation (5.5) (Linting *et al.*, 2007).

Loss function in Equation (5.5) is minimized in an alternating least squares way by cyclically updating one of the three sets of parameters **Q**, **X** and **A**, while keeping the other two fixed. This iterative process is subjected to conditions below:

1) the transformed variables are standardized, so that

$$\mathrm{q_j}'\mathrm{q_j} = n \tag{5.6}$$

2) the component scores are restricted by requiring

$$X'X = nI \tag{5.7}$$

where I is the identity matrix.

3) the component scores are centered; thus,

$$1'X = 0 \tag{5.8}$$

where "**1**" indicates a vector of ones.

The condition presented by Equation (5.6) is required to solve the indeterminacy between q_j and a_j in the inner product $q_j a_j'$. The condition in Equation (5.7), on the other hand, is applied to avoid the solutions **A** = 0 and **X** = 0. Moreover, the conditions presented by Equations (5.7) and (5.8) imply that the columns of **X** containing the components scores whose mean is zero and the standard deviation is one, and the components are uncorrelated.

The abovementioned iterative process is continued until the improvement in the subsequent loss values is below some user-specified small value. In CATPCA, starting values of **X** are randomly selected.

As was mentioned above, the PCs that are generated through NLPCA are a re-expression of the m-variable transformed data set, in terms of uncorrelated components Y_j ($1 < j \leq m$). Therefore, the relationship between the PCs and the transformed data can be presented in NLPCA by Equation (5.9).

$$Y = Q\,E \tag{5.9}$$

where, Y is the (n x m) matrix of PCs (i.e., uncorrelated components), Q is the (n x m) matrix of transformed variables and E is the (m x m) matrix of eigenvectors.

5.3.2. Computation of NADI Time Series

As explained in Section 5.3.1, PCs were generated through NLPCA for each of 12 months separately. The NADI values were then calculated individually for each month of each year where the NADI values were considered as the first PC (PC1), normalized by its standard deviation:

$$NADI_{i,k} = \frac{Y_{i,1,k}}{\sigma_k} \tag{5.10}$$

where, $NADI_{i,k}$ is the NADI value for month k in year i, $Y_{i,1,k}$ is the PC1 during year i for month k and σ_k is the standard deviation of $Y_{i,1,k}$ over all years for month k.

The NADI utilizes only the PC1 because it explains the largest fraction of the variance described by the full m-member, standardized data set, as was the case with ADI (Section 4.3.5). Moreover, PCs are orthogonal vectors, and therefore it is not mathematically proper to combine them into a single expression (Keyantash and Dracup, 2004). Therefore, only the PC1, which is the dominant mode, was adopted to describe the bulk of water anomalies in the observational data. Considering all 12 months, PC1 which is generated through NLPCA, described an average of 60.0% of the data set variance in this study, whereas the PC1 that was generated through linear PCA described only 56.4% of the data set variance (Section 4.3.5). The total percent of variance accounted by PC1 in the individual month is shown in Figure 5.2. This figure shows that the PC1 accounted lowest variance in June (49.0%), while the highest variance was in October (67.1%).

The PC1 is standardized in Equation (5.10) to enable each month's NADI to represent a normalized expression of variability. Without standardization, months that routinely possess a higher degree of hydrologic variability cause the chronological plot of NADI values to highly jump (Keyantash and Dracup, 2004). It is shown in Appendix that the NADI time series for a single month has a mean of zero but possesses non-unit standard deviation.

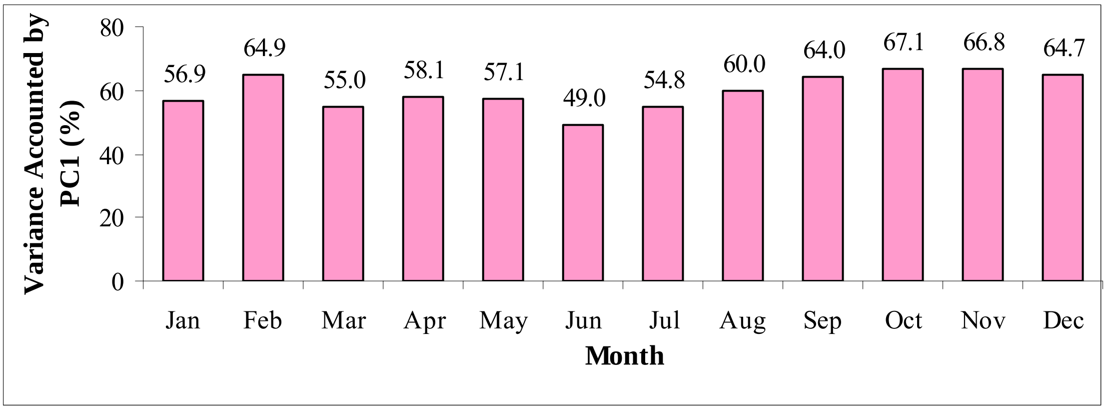


Figure 5.2 Percent of variance accounted by PC1 in different months

Once the computations of NADI values were completed for each year and each month (using Equation (5.10)), they were combined and reordered into a single NADI time series in chronological order.

5.3.3. Example Calculation of NADI Values for Month of January

Consider the computation of the NADI time series for the month of January. The 49 years of January data for rainfall (P), potential evapotranspiration (E), streamflow (Q), storage reservoir volume (V) and soil moisture content (W) are arranged columnarly into an (49 x 5) matrix of observations $\mathbf{H}_{\text{January}}$, which is shown in Figure 5.3. The monthly data in this figure for P, E, Q, V and W are reported in their original units of millimeters, millimeters, megaliters per day, percent full of total storage capacity and millimeters respectively. However, these different measurement units of the input data will not have any impact in the final NADI values as the input variables have gone through a standardization process in the NLPCA (condition (1) in Equation (5.6)).

The matrix $\mathbf{H}_{\text{January}}$ was then used in CATPCA module in SPSS to generate the (49 x 5) matrix of $\mathbf{Q}_{\text{January}}$ containing optimal transformed variables and the (5 x 5) matrix of eigenvectors related to the five PCs. However, only (5 x 1) matrix of $\mathbf{E}_{\text{January}}$ containing eigenvectors related to the PC1 is used in this study (Section 5.3.2). The matrix $\mathbf{Q}_{\text{January}}$ and $\mathbf{E}_{\text{January}}$ are shown in Figure 5.4.

$$
H_{\text{January}} = \begin{bmatrix}
51.5 & 165.4 & 417 & 93.09 & 658 \\
36.3 & 175.5 & 697 & 98.27 & 439 \\
63.0 & 154.9 & 293 & 85.40 & 363 \\
182.7 & 145.9 & 790 & 79.41 & 568 \\
10.7 & 157.2 & 276 & 86.54 & 403 \\
29.7 & 147.3 & 573 & 98.90 & 526 \\
61.6 & 158.8 & 276 & 71.50 & 458 \\
36.7 & 140.8 & 420 & 96.57 & 552 \\
43.3 & 167.3 & 143 & 35.76 & 350 \\
39.4 & 165.5 & 316 & 96.41 & 486 \\
107.0 & 129.1 & 571 & 97.00 & 529 \\
75.7 & 144.5 & 802 & 92.84 & 545 \\
51.0 & 133.9 & 708 & 98.96 & 545 \\
63.7 & 161.7 & 148 & 54.46 & 339 \\
62.8 & 160.1 & 391 & 96.96 & 497 \\
42.5 & 145.2 & 667 & 96.93 & 522 \\
40.1 & 148.9 & 523 & 83.05 & 486 \\
70.3 & 151.3 & 491 & 94.51 & 492 \\
67.7 & 140.4 & 308 & 73.64 & 409 \\
53.6 & 167.5 & 556 & 97.78 & 559 \\
71.8 & 136.9 & 281 & 78.97 & 439 \\
47.4 & 170.0 & 235 & 81.26 & 439 \\
81.7 & 161.0 & 271 & 78.44 & 439 \\
53.2 & 152.3 & 163 & 32.14 & 260 \\
71.2 & 132.9 & 287 & 89.09 & 423 \\
39.9 & 149.8 & 346 & 86.71 & 444 \\
65.1 & 137.3 & 902 & 91.23 & 585 \\
39.0 & 141.9 & 618 & 89.64 & 499 \\
84.5 & 161.5 & 537 & 82.42 & 506 \\
81.1 & 148.1 & 819 & 75.90 & 553 \\
6.9 & 154.0 & 343 & 67.58 & 410 \\
93.6 & 148.8 & 492 & 72.78 & 479 \\
69.1 & 123.6 & 1292 & 94.96 & 525 \\
80.5 & 145.1 & 1497 & 96.74 & 598 \\
79.7 & 142.8 & 2303 & 85.71 & 608 \\
92.7 & 140.5 & 519 & 81.02 & 431 \\
92.9 & 133.9 & 1058 & 87.43 & 579 \\
58.5 & 162.3 & 723 & 95.08 & 458 \\
74.0 & 155.7 & 319 & 66.50 & 275 \\
43.2 & 151.2 & 348 & 59.85 & 314 \\
70.2 & 136.3 & 463 & 49.22 & 350 \\
30.4 & 162.5 & 368 & 55.13 & 400 \\
58.9 & 136.6 & 452 & 53.94 & 420 \\
34.5 & 167.8 & 209 & 44.07 & 247 \\
68.3 & 142.3 & 310 & 47.74 & 332 \\
42.4 & 145.3 & 429 & 52.07 & 427 \\
72.9 & 152.3 & 342 & 49.24 & 422 \\
41.5 & 165.0 & 212 & 27.94 & 247 \\
56.0 & 165.0 & 226 & 32.37 & 270
\end{bmatrix}
$$

Figure 5.3 Observational data matrix **H** for the monthly of January

$$
Q_{January} = \begin{bmatrix}
-0.34 & 1.20 & 0.11 & 0.82 & 1.34 \\
-1.12 & 2.14 & 1.00 & 1.28 & 0.02 \\
0.19 & 0.29 & -0.63 & 0.26 & -1.04 \\
3.07 & -0.40 & 1.08 & 0.01 & 1.12 \\
-1.68 & 0.49 & -0.96 & 0.40 & -0.52 \\
-1.31 & -0.30 & 0.68 & 1.28 & 0.81 \\
0.06 & 0.69 & -0.96 & -0.41 & 0.25 \\
-1.12 & -0.79 & 0.11 & 1.13 & 1.01 \\
-0.76 & 1.40 & -2.22 & -1.91 & -1.16 \\
-1.01 & 1.20 & -0.63 & 1.13 & 0.46 \\
1.90 & -1.80 & 0.68 & 1.13 & 0.88 \\
0.69 & -0.50 & 1.08 & 0.82 & 0.95 \\
-0.48 & -1.39 & 1.00 & 1.28 & 0.95 \\
0.19 & 0.89 & -2.22 & -0.91 & -1.27 \\
0.19 & 0.79 & -0.09 & 1.13 & 0.65 \\
-0.89 & -0.50 & 0.94 & 1.13 & 0.81 \\
-1.01 & -0.10 & 0.56 & 0.13 & 0.46 \\
0.44 & 0.09 & 0.42 & 0.97 & 0.56 \\
0.32 & -0.89 & -0.63 & -0.32 & -0.38 \\
-0.34 & 1.40 & 0.68 & 1.13 & 1.07 \\
0.57 & -1.09 & -0.63 & -0.11 & 0.02 \\
-0.62 & 1.61 & -1.34 & 0.01 & 0.02 \\
1.03 & 0.89 & -0.96 & -0.11 & 0.02 \\
-0.34 & 0.09 & -2.22 & -2.10 & -1.96 \\
0.57 & -1.49 & -0.63 & 0.53 & -0.24 \\
-1.01 & -0.10 & -0.34 & 0.40 & 0.02 \\
0.19 & -1.09 & 1.17 & 0.68 & 1.17 \\
-1.01 & -0.69 & 0.78 & 0.68 & 0.65 \\
1.14 & 0.89 & 0.56 & 0.13 & 0.65 \\
0.92 & -0.20 & 1.11 & -0.22 & 1.01 \\
-1.70 & 0.29 & -0.34 & -0.58 & -0.38 \\
1.45 & -0.20 & 0.42 & -0.41 & 0.46 \\
0.44 & -2.22 & 1.42 & 0.97 & 0.81 \\
0.92 & -0.50 & 1.54 & 1.13 & 1.25 \\
0.92 & -0.69 & 1.71 & 0.40 & 1.28 \\
1.45 & -0.79 & 0.56 & 0.01 & -0.11 \\
1.45 & -1.39 & 1.28 & 0.40 & 1.17 \\
-0.07 & 0.99 & 1.00 & 0.97 & 0.25 \\
0.69 & 0.39 & -0.34 & -0.65 & -1.89 \\
-0.89 & -0.01 & -0.34 & -0.81 & -1.57 \\
0.44 & -1.19 & 0.28 & -1.02 & -1.16 \\
-1.31 & 0.99 & -0.09 & -0.87 & -0.52 \\
-0.07 & -1.19 & 0.28 & -0.91 & -0.24 \\
-1.22 & 1.40 & -1.34 & -1.31 & -2.02 \\
0.44 & -0.69 & -0.63 & -1.10 & -1.37 \\
-0.89 & -0.40 & 0.11 & -0.95 & -0.11 \\
0.57 & 0.09 & -0.34 & -1.02 & -0.24 \\
-0.89 & 1.20 & -1.34 & -2.52 & -2.02 \\
-0.21 & 1.20 & -1.34 & -2.10 & -1.89
\end{bmatrix}, \quad
E_{January} = \begin{bmatrix}
0.45 \\
-0.59 \\
0.90 \\
0.82 \\
0.90
\end{bmatrix}
$$

Figure 5.4 Matrices **Q** and **E** containing optimal transformed variables and eigenvectors related to the PC1

The (49 x 1) matrix of PC1 ($Y_{January}$) is then computed for the month of January using Equation (5.9) and is shown in Figure 5.5.

$$
Y_{January} = Q_{January}\ E_{January} = \begin{bmatrix}
-0.34 & 1.20 & 0.11 & 0.82 & 1.34 \\
-1.12 & 2.14 & 1.00 & 1.28 & 0.02 \\
0.19 & 0.29 & -0.63 & 0.26 & -1.04 \\
3.07 & -0.40 & 1.08 & 0.01 & 1.12 \\
-1.68 & 0.49 & -0.96 & 0.40 & -0.52 \\
-1.31 & -0.30 & 0.68 & 1.28 & 0.81 \\
0.06 & 0.69 & -0.96 & -0.41 & 0.25 \\
-1.12 & -0.79 & 0.11 & 1.13 & 1.01 \\
-0.76 & 1.40 & -2.22 & -1.91 & -1.16 \\
-1.01 & 1.20 & -0.63 & 1.13 & 0.46 \\
1.90 & -1.80 & 0.68 & 1.13 & 0.88 \\
0.69 & -0.50 & 1.08 & 0.82 & 0.95 \\
-0.48 & -1.39 & 1.00 & 1.28 & 0.95 \\
0.19 & 0.89 & -2.22 & -0.91 & -1.27 \\
0.19 & 0.79 & -0.09 & 1.13 & 0.65 \\
-0.89 & -0.50 & 0.94 & 1.13 & 0.81 \\
-1.01 & -0.10 & 0.56 & 0.13 & 0.46 \\
0.44 & 0.09 & 0.42 & 0.97 & 0.56 \\
0.32 & -0.89 & -0.63 & -0.32 & -0.38 \\
-0.34 & 1.40 & 0.68 & 1.13 & 1.07 \\
0.57 & -1.09 & -0.63 & -0.11 & 0.02 \\
-0.62 & 1.61 & -1.34 & 0.01 & 0.02 \\
1.03 & 0.89 & -0.96 & -0.11 & 0.02 \\
-0.34 & 0.09 & -2.22 & -2.10 & -1.96 \\
0.57 & -1.49 & -0.63 & 0.53 & -0.24 \\
-1.01 & -0.10 & -0.34 & 0.40 & 0.02 \\
0.19 & -1.09 & 1.17 & 0.68 & 1.17 \\
-1.01 & -0.69 & 0.78 & 0.68 & 0.65 \\
1.14 & 0.89 & 0.56 & 0.13 & 0.65 \\
0.92 & -0.20 & 1.11 & -0.22 & 1.01 \\
-1.70 & 0.29 & -0.34 & -0.58 & -0.38 \\
1.45 & -0.20 & 0.42 & -0.41 & 0.46 \\
0.44 & -2.22 & 1.42 & 0.97 & 0.81 \\
0.92 & -0.50 & 1.54 & 1.13 & 1.25 \\
0.92 & -0.69 & 1.71 & 0.40 & 1.28 \\
1.45 & -0.79 & 0.56 & 0.01 & -0.11 \\
1.45 & -1.39 & 1.28 & 0.40 & 1.17 \\
-0.07 & 0.99 & 1.00 & 0.97 & 0.25 \\
0.69 & 0.39 & -0.34 & -0.65 & -1.89 \\
-0.89 & -0.01 & -0.34 & -0.81 & -1.57 \\
0.44 & -1.19 & 0.28 & -1.02 & -1.16 \\
-1.31 & 0.99 & -0.09 & -0.87 & -0.52 \\
-0.07 & -1.19 & 0.28 & -0.91 & -0.24 \\
-1.22 & 1.40 & -1.34 & -1.31 & -2.02 \\
0.44 & -0.69 & -0.63 & -1.10 & -1.37 \\
-0.89 & -0.40 & 0.11 & -0.95 & -0.11 \\
0.57 & 0.09 & -0.34 & -1.02 & -0.24 \\
-0.89 & 1.20 & -1.34 & -2.52 & -2.02 \\
-0.21 & 1.20 & -1.34 & -2.10 & -1.89
\end{bmatrix} . \begin{bmatrix} 0.45 \\ -0.59 \\ 0.90 \\ 0.82 \\ 0.90 \end{bmatrix} = \begin{bmatrix}
1.12 \\ 0.19 \\ -1.38 \\ 3.62 \\ -2.06 \\ 1.97 \\ -1.35 \\ 1.89 \\ -5.78 \\ -0.40 \\ 4.25 \\ 3.11 \\ 3.40 \\ -4.33 \\ 1.05 \\ 2.39 \\ 0.63 \\ 1.82 \\ -0.50 \\ 1.52 \\ 0.26 \\ -2.41 \\ -0.99 \\ -5.69 \\ 0.78 \\ -0.36 \\ 3.39 \\ 1.79 \\ 1.19 \\ 2.27 \\ -2.07 \\ 1.24 \\ 4.31 \\ 4.15 \\ 3.85 \\ 1.54 \\ 4.01 \\ 1.30 \\ -2.46 \\ -2.78 \\ -0.73 \\ -2.44 \\ -0.04 \\ -5.48 \\ -2.10 \\ -0.94 \\ -1.15 \\ -6.20 \\ -5.43
\end{bmatrix}
$$

Figure 5.5 Matrix of first Principal Component (PC1) for the month of January

Finally, the NADI values for the month of January for each of 49 years are computed using Equation (5.10), and are shown in Figure 5.6 where the standard deviation of PC1 for the month of January for all 54 years is 2.88.

$$\mathrm{NADI}_{\mathrm{January}} = \frac{Y_{January}}{\sigma_{January\ for\ all\ 54\ years}} = \frac{1}{2.88} \cdot \begin{bmatrix} 1.12 \\ 0.19 \\ -1.38 \\ 3.62 \\ -2.06 \\ 1.97 \\ -1.35 \\ 1.89 \\ -5.78 \\ -0.40 \\ 4.25 \\ 3.11 \\ 3.40 \\ -4.33 \\ 1.05 \\ 2.39 \\ 0.63 \\ 1.82 \\ -0.50 \\ 1.52 \\ 0.26 \\ -2.41 \\ -0.99 \\ -5.69 \\ 0.78 \\ -0.36 \\ 3.39 \\ 1.79 \\ 1.19 \\ 2.27 \\ -2.07 \\ 1.24 \\ 4.31 \\ 4.15 \\ 3.85 \\ 1.54 \\ 4.01 \\ 1.30 \\ -2.46 \\ -2.78 \\ -0.73 \\ -2.44 \\ -0.04 \\ -5.48 \\ -2.10 \\ -0.94 \\ -1.15 \\ -6.20 \\ -5.43 \end{bmatrix} = \begin{bmatrix} 0.39 \\ 0.07 \\ -0.48 \\ 1.26 \\ -0.72 \\ 0.69 \\ -0.47 \\ 0.66 \\ -2.01 \\ -0.14 \\ 1.48 \\ 1.08 \\ 1.18 \\ -1.50 \\ 0.36 \\ 0.83 \\ 0.22 \\ 0.63 \\ -0.17 \\ 0.53 \\ 0.09 \\ -0.84 \\ -0.35 \\ -1.98 \\ 0.27 \\ -0.13 \\ 1.18 \\ 0.62 \\ 0.41 \\ 0.79 \\ -0.72 \\ 0.43 \\ 1.50 \\ 1.44 \\ 1.34 \\ 0.53 \\ 1.40 \\ 0.45 \\ -0.86 \\ -0.97 \\ -0.25 \\ -0.85 \\ -0.01 \\ -1.90 \\ -0.73 \\ -0.33 \\ -0.40 \\ -2.15 \\ -1.89 \end{bmatrix}$$

Figure 5.6 NADI values for the month of January

The NADI values for the other months were calculated separately in a similar manner.

5.3.4. Thresholds Determination of NADI

As mentioned in Section 5.3.2, the NADI values which were computed for each year and each month were combined and reordered into a single NADI time series in chronological order. Similar to the ADI thresholds calculation (Section 4.3.5), the NADI thresholds values were calculated probabilistically for the study catchment using the empirical cumulative distribution function of all NADI values. This is shown in Figure 5.7. These thresholds were used to classify the drought conditions. The SPI thresholds were used to generate NADI thresholds as was done for the ADI. The SPI dryness thresholds are the Gaussian variates of -2, -1.5, -1 and 1 standard deviations (Table 4.3), which corresponds to *2.3th*, *6.7th*, *16.0th* and *84.0th* percentiles in SPI cumulative distributions. The NADI thresholds corresponding to these percentiles were -2.27, -1.64, -0.84 and 0.88 respectively for the study catchment, as can be seen from Figure 5.7. These NADI thresholds and relevant drought classifications for the study catchment are presented in Table 5.1.

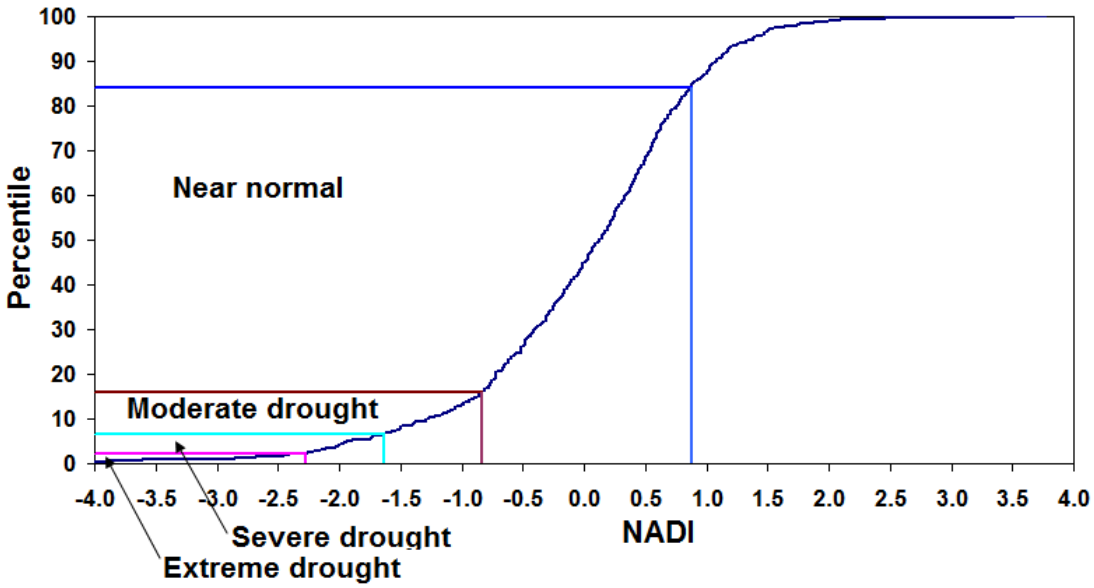


Figure 5.7 Computation of NADI thresholds for the Yarra River catchment

Table 5.1 Drought classification based on NADI thresholds

> -0.84 to ≤ 0.88	Near normal
> -1.64 to ≤ -0.84	Moderate drought
> -2.27 to ≤ -1.64	Severe drought
≤ -2.27	Extreme drought

5.4. Analysis of NADI for Drought Assessment

As was discussed in Section 4.4, drought assessment has been commonly done with drought severity defined by a DI (Yevjevich, 1967; Keyantash and Dracup, 2002), where the drought severity (*S*) is the product of the drought duration (*D*) (during which DI values are consistently below a truncation level) and the drought magnitude (*M*) (which is the mean departure of DI values from that truncation level during the drought). However, as was also discussed in Section 4.4, the median value was used in this study as the *M* value instead of the mean. The relationship between *S*, *M* and *D* was shown in Figure 4.2. Using this nomenclature, the historical droughts recorded in Victoria in the last 50 years (i.e. 1967-1968, 1972-1973, 1982-1983, 1997-1998, 2003-2004 and 2006 onwards; details of these droughts were presented in Section 3.3) were assessed with the NADI and are presented below. All these historical droughts were considered to investigate how well the NADI was capable in defining drought conditions for the Yarra River catchment.

The NADI time series plots for the Yarra River catchment is shown in Figure 5.8. In this figure, different levels of drought intensity (i.e., drought classes) are presented with the horizontal dotted lines based on the NADI thresholds of Table 5.1. The shaded vertical areas, on the other hand, represent various historical droughts that occurred in Victoria in 1967-1968, 1972-1973, 1982-1983, 1997-1998, 2003-2004 and 2006 onwards, which were well documented (Keating, 1992; Tan and Rhodes, 2008). However, the shaded areas in Figure 5.8 were determined based on the negative NADI values during historical drought periods.

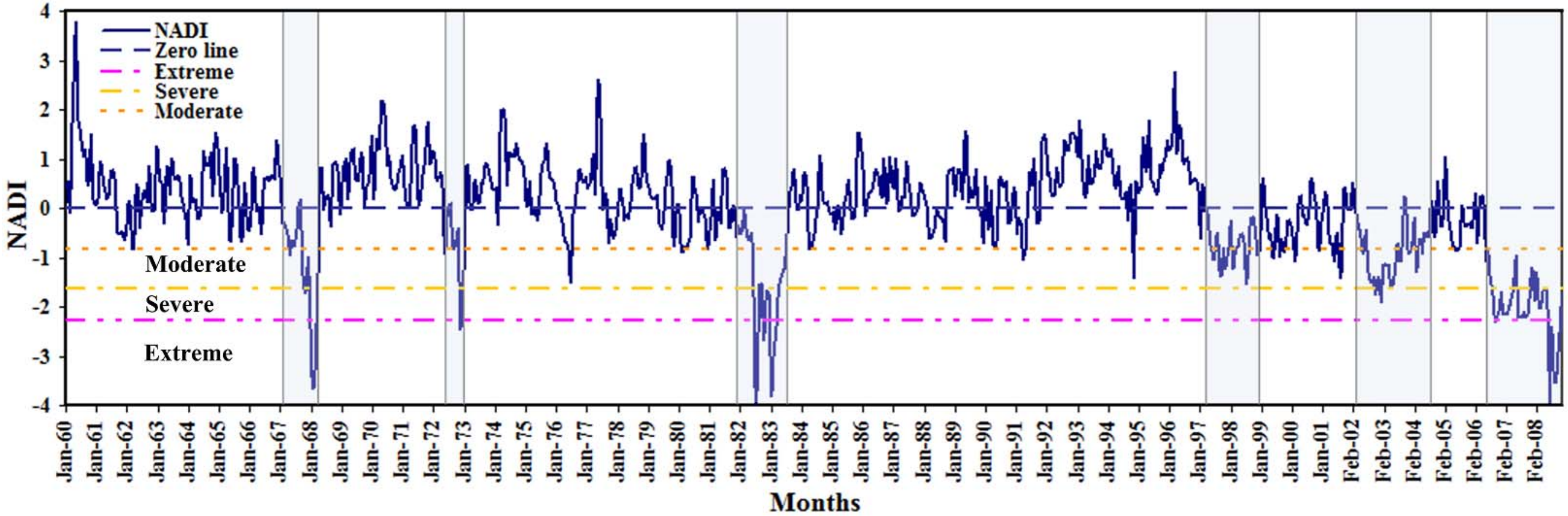


Figure 5.8 NADI time series for the Yarra River catchment showing severity levels and historic droughts

Figure 5.8 shows that the abovementioned historical droughts were well detected by NADI. From this figure, it can also be seen that the NADI showed smoother transitional characteristics during droughts, and from dry to wet spells and vice versa. Because of this characteristic, the historical droughts were defined clearly with the NADI. Moreover, the different levels of drought intensity in Figure 5.8 shows that the droughts in 1967-1968, 1972-1973, 1982-1983 and 2006 onwards have reached the level of extreme drought condition, whereas the droughts in 1997-1998 and 2003-2004 reached the level of moderate and severe drought condition respectively.

The period from 1997 onwards is also commonly known in Melbourne as the 12 years extended dry period or the longest dry circumstances in Victoria in the history where three major drought years (i.e. 1997-1998, 2002-2003 and 2006 onwards) have occurred during this period (Tan and Rhodes, 2008). Figure 5.8 shows that the NADI has detected this extended dry period well, although in few months the NADI values had gone above the truncation line (i.e. had positive NADI values). It should be noted that although the NADI calculations were done only until December 2008, the drought has continued until now.

Characteristics of historical droughts as detected by NADI are presented in Table 5.2. It shows the start and end months of these droughts as detected by NADI with negative NADI values during the historical droughts. The drought in 2006 onwards does not have an end month, since the drought is still continuing (however, data were used until 2008 in the analysis in this thesis). The table also shows that the 2003-2004 drought was started in March 2002 although the drought was known as the 2003-2004 drought (Tan and Rhodes, 2008). In addition, the 1982-1983 drought has started in December 1981 and the 1997-1998 drought has ended in January 1999. Table 5.2 also shows median (M) and maximum intensity (M_{max}) of historical droughts together with their drought classifications, in addition to their duration (D) and severity (S). It can be seen that the highest severity of drought as indicated by S was in 2003-2004 followed by 1982-1983, 1997-1998, 1967-1968 and 1972-1973. It is important to highlight that the NADI had detected the droughts with the longest duration, the highest severity and the maximum intensity as published for historical droughts (Keating, 1992; Tan and Rhodes, 2008). The published details of historical droughts were presented in Section 3.3.

Table 5.2 Characteristics of historical droughts as detected by NADI

	1967 - 1968	**1972 - 1973**	**1982 - 1983**	**1997 - 1998**	**2003 - 2004**	**2006 - Onwards**
Start month	February, 1967	June, 1972	December, 1981	June, 1997	March, 2002	June, 2006
End month	April, 1968	January, 1973	August, 1983	January, 1999	August, 2004	-
Median intensity, M	(-0.94) Moderate	(-0.78) Moderate	(-1.31) Moderate	(-0.77) Near normal	(-0.92) Moderate	(-1.99)* Severe
Maximum intensity, M_{max}	(-3.65) Extreme (February, 1968)	(-2.42) Extreme (December, 1973)	(-4.04)[c] Extreme (August, 1982)	(-1.55) Moderate (August, 1998)	(-1.90) Severe (January, 2003)	(-3.94)* Extreme (July, 2008)
Duration, D (months)	15	8	21	20	30[a]	----- **
Severity, S	-14.10	-6.24	-27.51	-15.40	-27.60[b]	----- **

*M and M_{max} are based on data until 2008

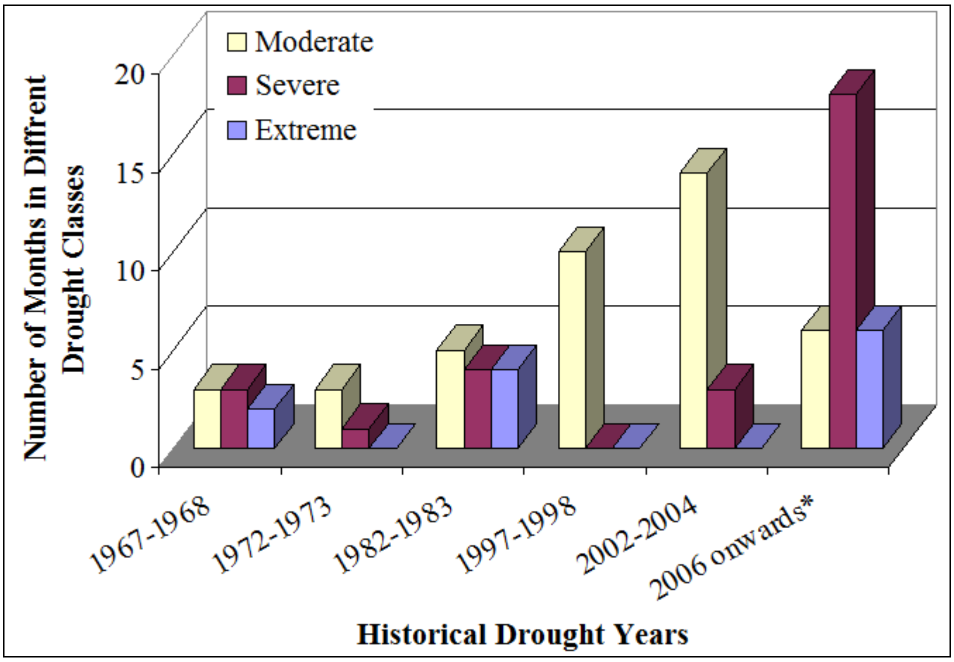
**D and S can not be calculated as 2006 onwards drought still continuing

[a]Longest duration

[b]Highest severity

[c]Maximum intensity

The total number of months detected with NADI having extreme, severe and moderate drought classifications for each of the historical drought are shown in Figure 5.9. This figure shows that the number of months was higher in moderate drought class in all records of historical droughts followed by severe and extreme drought classes, except in 2006 onwards drought where the number of months that was in severe drought condition was higher than those of the extreme and moderate drought conditions.



*until December 2008

Figure 5.9 Number of months in different drought classes detected by NADI

5.5. Comparative Evaluation of NADI and ADI

A comparative evaluation of NADI and ADI was conducted in this research and presented in this section. This comparative study has provided an important check on the appropriateness of the NADI findings. As was mentioned in Section 5.1, the ADI was the most suitable DI for defining drought conditions in the Yarra River catchment among the DIs that were evaluated in Chapter 4 (viz., PN, Deciles, SPI, SWSI and ADI). However, as was also discussed in Section 5.1, the use of NLPCA in NADI could improve variance representation by accounting nonlinear relationships between the variables and overcome the limitation of the use of linear PCA in the ADI development in which linear PCA is not able to capture the nonlinear relationships. It was shown in Section 5.3.2 that the NADI was able to represent 60.0% of the variance of the entire data set compared to 56.4% by ADI. A cross-validation approach was also applied to further justify this result by dropping each decade off one at a time and redo both assessments, with variance estimated over the dropped decade and averaged over all the decades that make up the historical record. In this case, the results show that the NADI represent 67.4% of the variance of the entire data compared to 59.4% by ADI. These results imply that the application of NLPCA technique was appropriate in this study rather than the use of linear PCA. Figure 5.10 shows the percentage variance of data explained by ADI and NADI for different months and the average percentage variance. This clearly shows that the fluctuations in the hydro-meteorological variables are more representative in the NADI than in the ADI.

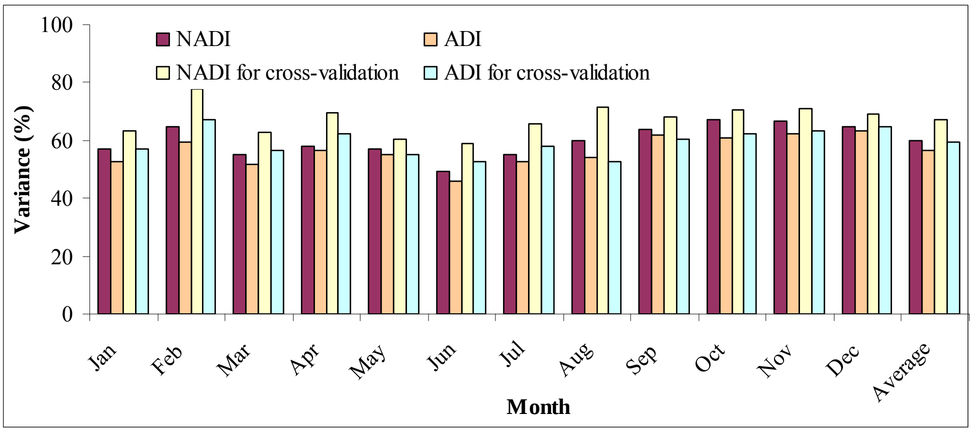


Figure 5.10 Percent variance accounted by NADI and ADI for different months

The time series of both ADI and NADI for the Yarra River catchment are shown in Figure 5.11. Different levels of drought intensity (i.e., drought classes) detected by their respective thresholds are presented with the horizontal dotted lines. The shaded vertical areas represented various historical droughts that occurred in Victoria in 1967-1968, 1972-1973, 1982-1983, 1997-1998, 2003-2004 and 2006 onwards, as detected by ADI and NADI with the negative values. This figure shows that both time series are quite similar. However, the figure shows that the NADI thresholds have a wider range than the ADI thresholds. Because of these larger threshold ranges between the drought classes, changes in drought intensities can be observed more clearly in the NADI time series than in the ADI time series, which makes easier to compare different historical drought conditions between each others.

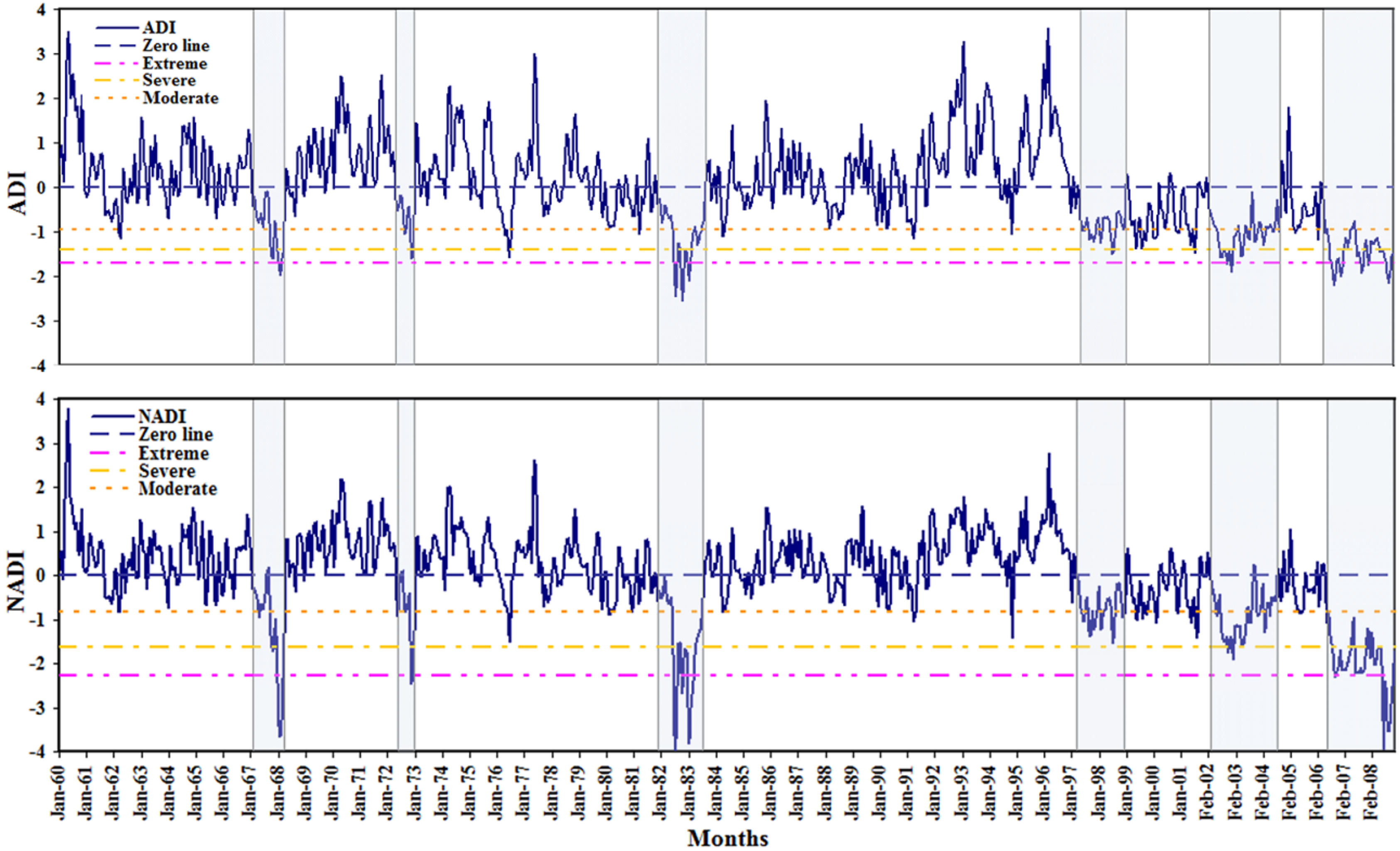


Figure 5.11 Time series of NADI and ADI for the Yarra River catchment

It should be noted that the NADI is similar to the ADI, since both use the same input variables and also a similar methodology, except the linear PCA was used in the ADI, whereas the NLPCA was used in NADI. This implies that the NADI will have similar characteristics to the ADI. Therefore the NADI and ADI were not evaluated and compared with respect to the five decision criteria (i.e. *robustness, tractability, sophistication, transparency, and extendability*) that were used in Section 4.5. However, it can be seen that there were some improvements in the NADI drought assessment approach compared to the ADI approach, especially in terms of the representation of fluctuations in the hydro-meteorological variables and larger threshold ranges.

5.6. Summary

Drought quantification is an important issue for drought management, and has always been a challenging task among the drought researchers and professionals. The suitability of the use of some existing Drought Indices (DIs) was investigated for the Yarra River catchment in Australia and presented in Chapter 4. It was found that the Aggregated Drought Index (ADI) was the most suitable DI for the Yarra River catchment. However, as was discussed in Chapter 2 (Section 2.2.6), the linear Principal Component Analysis (PCA) was used to aggregate the hydro-meteorological variables in the ADI development by accounting the variance in variables in which the linear PCA may not perform well when nonlinear characteristics exist in the hydro-meteorological variables which is often the case. To improve the limitation of the use of linear PCA in the ADI, a Nonlinear Principal Component Analysis (NLPCA) technique was introduced and successfully applied in this study which has led to the development of a novel Nonlinear Aggregated Drought Index (NADI) in this research.

The NADI was constructed using five hydro-meteorological variables that were used in the development of ADI: rainfall, potential evapotranspiration, streamflow, storage reservoir volume and soil moisture content. The NLPCA technique that has the capability to capture nonlinear characteristics was used to aggregate these variables through Principal Components (PCs). In NLPCA, the PCs are generated using the transformed variables compared to the PCs generated using the observed variables in the linear PCA. The CATPCA (i.e. Categorical Principal Component Analysis) module of SPSS software was used in this study to perform the NLPCA. The NADI was considered as the first PC, normalized by its standard deviation. Twelve NADI time series (one for each month) derived for the study catchment from 1960 to 2008 were chronologically re-ordered to produce a single time series. Finally, the NADI values were used to generate the thresholds, which were then used to investigate the appropriateness of the use of NADI in defining historical drought conditions within the Yarra River catchment.

The NADI time series successfully detected all the past major historical droughts that occurred in Victoria in 1967-1968, 1972-1973, 1982-1983, 1997-1998, 2003-2004 and 2006 onwards. The NADI was compared with the ADI. The results showed that the use of NLPCA technique in the NADI captured more variance of the input data set than the ADI, which implied that the fluctuations in the hydro-meteorological variables used were more representative in the NADI than in the ADI. Moreover, it was found that the changes in drought intensities can be observed more clearly in the NADI time series than the ADI time series due to the larger threshold ranges between the drought classes in the NADI. Therefore, the NADI was considered to be the best DI for the Yarra River catchment, and hence was used to develop the drought forecasting model in Chapter 6.

6. DEVELOPMENT OF DROUGHT FORECASTING MODEL

Overview; Data Used; Drought Forecasting Model Development; Results and Discussion; Summary

6.1. Overview

Drought forecasting plays an important role in the mitigation of impacts of drought on water resources systems. As discussed in Chapter 2 (Section 2.1), drought forecasting is commonly performed using a Drought Index (DI). This is because the use of a DI during decision making is believed to be far more functional than raw data (Hayes, 2003). Several DIs have been developed around the world in the past. Some of the well known DIs were discussed and evaluated for the Yarra River catchment in Victoria (Australia) in Chapters 2 and 4 respectively. Based on the conclusions of these chapters, a new Nonlinear Aggregated Drought Index (NADI) was developed for the Yarra River catchment in this research project and presented in Chapter 5. The NADI was evaluated using the historical droughts in Victoria and found to be more robust than the other DIs for defining drought conditions for the Yarra River catchment (Chapter 5). Therefore, the NADI was used to develop the drought forecasting model in this study, which will be presented in this chapter. The use of NADI in forecasting drought conditions forecast the overall dryness within the system as the NADI considers all significant hydro-meteorological variables (i.e., rainfall, potential evapotranspiration, streamflow, storage reservoir volume and soil moisture content) that affect the droughts. This study is different to the previous studies on drought forecasting which have used only the traditional rainfall based DIs.

There are several modeling techniques that have been used in the past to develop the drought forecasting models. All available techniques and their suitability for forecasting future drought conditions were discussed in Chapter 2 (Section 2.3). It was found that the Artificial Neural Network (ANN) modeling approach has shown great ability in modeling and forecasting nonlinear and non-stationary DI time series than the other techniques, due to its innate nonlinear properties and flexibility for modeling. ANN is an information processing system that resembles the structure and operation of the human brain; the basic concept of the ANN approach was described in Chapter 2 (Section 2.3.5). It was also found that the ANN approach can be used to model any relationship between a series of input and output variables by providing sufficient data and complexity (ASCE Task Committee on Application of Artificial Neural Networks in Hydrology, 2000; Maier and Dandy, 2000; Morid *et al.*, 2007; Maier *et al.*, 2010). Therefore, the ANN modeling technique was used in this study to develop the drought forecasting model to forecast NADI values for the Yarra River catchment.

The chapter begins with a brief description of the data used for the development of the drought forecasting model followed by a description of the drought forecasting model development. Then the results and the discussion are presented. The summary of the study is presented at the end of the chapter.

6.2. Data Used

As mentioned earlier, the Yarra River catchment in Victoria was used in this study. The details of the catchment and importance of its water resources for Victorians were elaborated in Chapter 3. Five hydro-meteorological variables (i.e. rainfall, potential evapotranspiration, streamflow, storage reservoir volume and soil moisture content) were used for the development of NADI time series for this catchment as was mentioned in Section 5.2. Monthly time step was used in the development of NADI time series. This monthly time series data of NADI was used for the development of drought forecasting model in this research. The NADI time series contains data for 49 years from 1960 to 2008.

6.3. Drought Forecasting Model Development

As mentioned in Section 6.1, the ANN technique was used to develop the drought forecasting model in this research project. A flowchart of the ANN based drought forecasting model development process is presented in Figure 6.1. This figure shows the overall steps followed during the model development; brief descriptions of each of these steps are given below. More details on the each of these steps are given later under several subsections.

The first step of the drought forecasting model development process involves the selection of ANN model structure and input variables. The choice of an appropriate ANN model structure and input variables is based on modeler's preference, and the optimal model structure and input variables generally needs to be determined using an iterative process (Maier *et al.*, 2010). These can be done based on a priori knowledge and the availability of data. In general, several potential input variables are selected from the probable input variables using a number of techniques including correlation tests. Thereafter, a number of models are developed based on the different combinations of the potential input variables. It should be noted that data pre-processing was involved before using them for models development.

The second step of the model development process involves the model calibration aiming to find a set of model parameters (i.e. hidden neurons, connection weights and biases) that enables a model with a given functional form to best represent the desired input/output relationship. The model calibration is not a simple task and in general it is done using a suitable optimization algorithm (Maier *et al.*, 2010). Back propagation (BP) training with minimizing Mean Squared Error (MSE) was used as the optimization algorithm in this study which will be discussed in Section 6.3.3.

Finally, the model development is completed by validating the calibrated model to investigate how the model performs using a separate data set that has not been used in the model calibration. This task generally conducted by investigating some model performance measures (e.g., correlation coefficient (R), Root Mean Squared Error (RMSE) and Mean Absolute Error (MAE)), and if the model outputs compare well with the desirable targets, then the model selected as a validated model. The model that performs best within all validated models was selected as the best drought forecasting model.

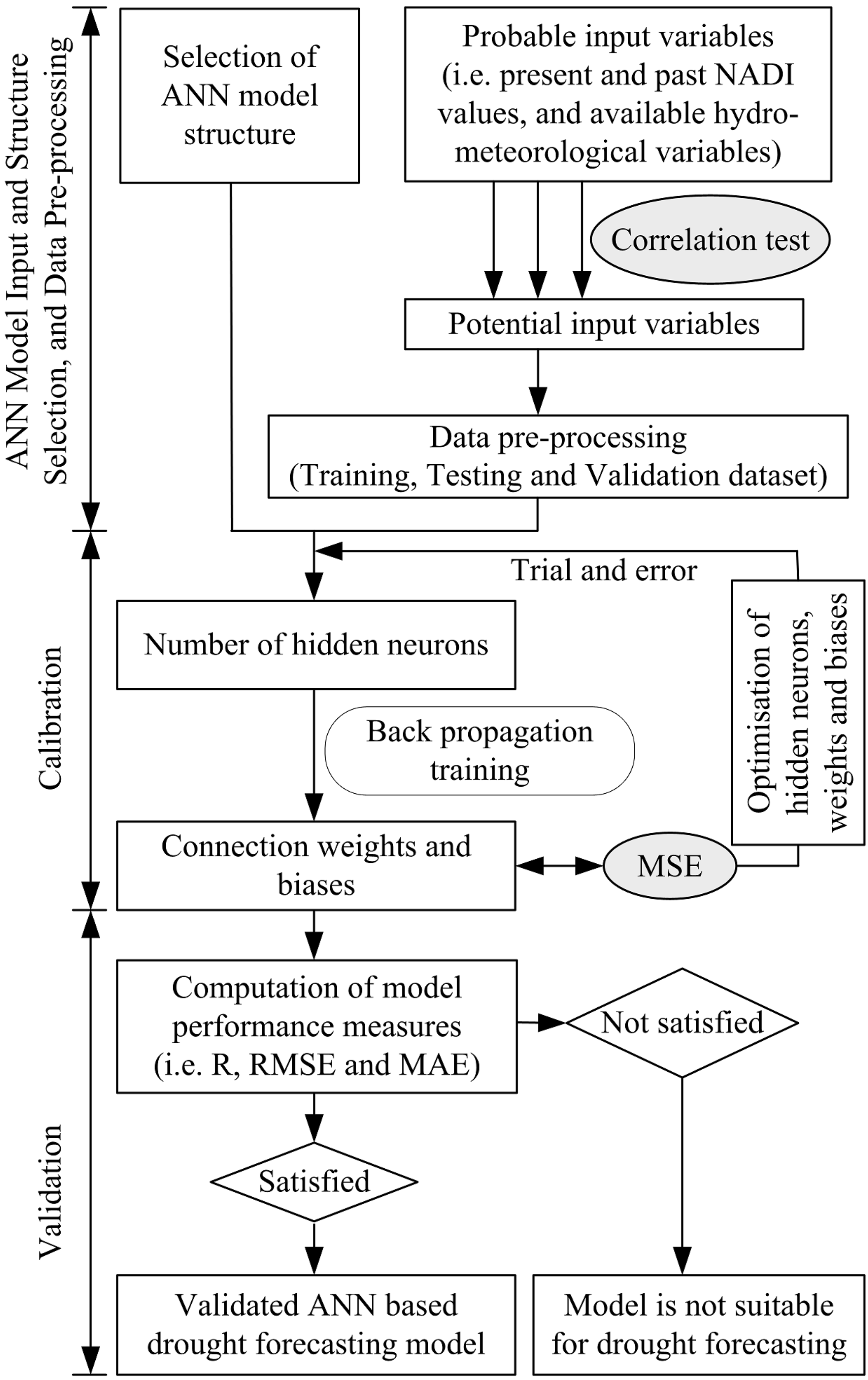


Figure 6.1 Flow chart of ANN based drought forecasting model development process

6.3.1. Selection of ANN Model Structure

There are a variety of ANN models developed for applications in applied science and engineering (Fausett, 1994; Samarasinghe, 2006). Maier and Dandy (2000) in their review of ANN models suggested that an ANN model with one hidden layer (i.e. three layer ANN model) can approximate any function, given that sufficient degree of freedom in terms of connection weights and biases are provided. Therefore, the widely used three-layer ANN model (Kim and Valdes, 2003; Mishra and Desai, 2006; Mishra *et al.*, 2007; Morid *et al.*, 2007) was used in this study.

To forecast NADI values with several lead times ahead, two different approaches namely, Recursive Multi-Step Neural Network (RMSNN) with only one output neuron and Direct Multi-Step Neural Network (DMSNN) with multiple output neurons were used in this study. The RMSNN approach was introduced and successfully used in drought forecasting by Mishra and Desai (2006), whereas DMSNN is the commonly used approach in most forecasting models (Sajikumar and Thandaveswara, 1999; Kim and Valdes, 2003; Mishra and Desai, 2006; Mishra *et al.*, 2007; Morid *et al.*, 2007; Ochoa-Rivera *et al.*, 2007). Details of these two types of ANN modeling approaches are presented below.

6.3.1.1 Recursive Multi-Step Neural Network

Recursive Multi-Step Neural Network (RMSNN) can have a single neuron or multiple neurons in both input and hidden layers. However, it consists of only a single neuron in the output layer, representing one month lead time forecast. A typical three-layer RMSNN is shown in Figure 6.2. In this figure, n and m are the number of neurons in the input and hidden layers respectively, and the forecasting is done for k months. The network is first designed and calibrated considering only 1 month ahead forecasts based on the present and several months of past NADI values as inputs. This network (with the same number of input variables) is then used for forecasting NADI values for multiple lead times recursively as shown in Figure 6.2. Forecasting is carried out in this figure at month t for k time steps from (t+1) to (t+k). First, forecast at (t+1) is computed based on n months of past NADI values including the NADI value at t. This forecast value ($NADI_{t+1}$) is then used with past NADI values of (n-1) months to forecast $NADI_{t+2}$. In this way, forecasting was carried out in this study recursively to obtain forecasts for twelve months. Beyond 12 months, forecasts were considered to have high errors. Note that as forecasting is carried out for multiple time steps away from the first time step, and more and more forecast values are introduced as inputs in RMSNN instead of known NADI values, thus introducing more errors in the forecasts beyond the first forecast.

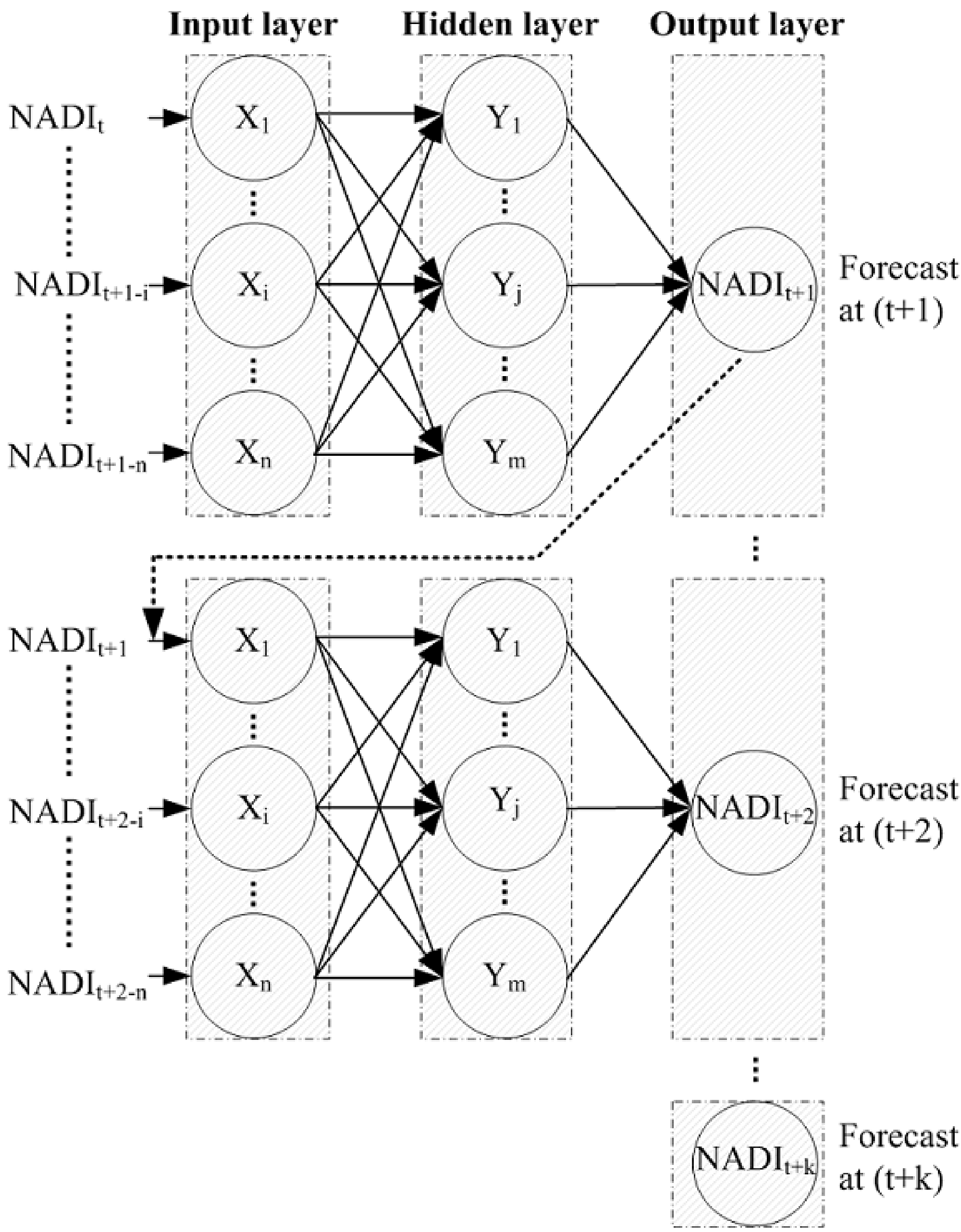


Figure 6.2 Typical three-layer RMSNN (adapted from Mishra and Desai, 2006)

6.3.1.2 Direct Multi-Step Neural Network

Similar to the RMSNN model, the three-layer Direct Multi-Step Neural Network (DMSNN) approach can have a single neuron or multiple neurons both in input and hidden layers. However, it can have several neurons in the output layer representing multiple month lead time forecasts as shown in Figure 6.3. In this figure, *n* and *m* are the number of neurons in the input and hidden layers respectively, and the forecasting is done for *k* months as was in the RMSNN model. The network in Figure 6.3 shows the *k* multiple lead time forecasts which are forecasted at month *t* (i.e. $NADI_{t+1}$ to $NADI_{t+k}$). Similar to the RMSNN model, the DMSNN model is designed and calibrated for forecasting drought conditions using the present and several months of past NADI values as inputs. However, in the DMSNN model of this study, in addition to present and past NADI values, other hydro-meteorological variables (that were used in calculating NADI) were also used as inputs to obtain the best forecasting model. These additional inputs could not be used in the RMSNN model, since forecast values of additional variables were required for the forecast model for forecasts beyond the first forecast, which will introduce more errors. The forecast values of these additional inputs are not required for the DMSNN model.

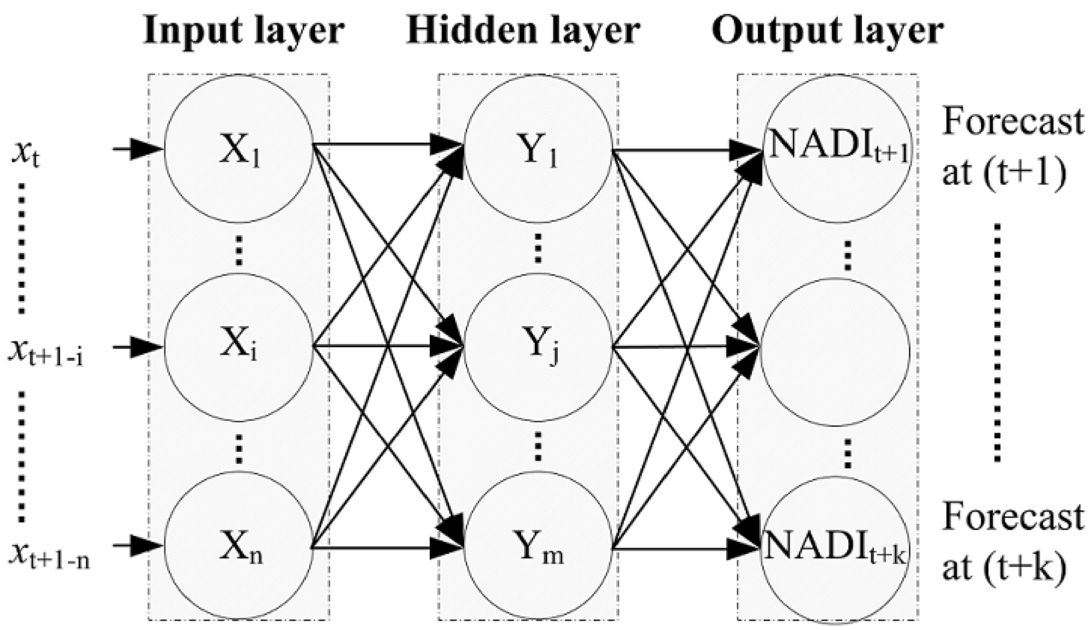


Figure 6.3 Typical three-layer DMSNN

6.3.2. Selection of Input Variables and Data Pre-Processing

Unlike the physically-based models, the set of input variables that needs to produce the ANN model output is not known prior (ASCE Task Committee on Application of Artificial Neural Networks in Hydrology, 2000). However, some previous studies have successfully used standard statistical correlation tests for selecting the model inputs for ANN models (Maity and Kumar, 2008; Tran *et al.*, 2009). Therefore, the potential input variables were selected for the ANN forecasting models in this study using the standard correlation test considering probable input variables (i.e. present and past 11 months NADI and hydro-meteorological variables that were used in calculating NADI). In the correlation test, the probable input variables that were highly correlated with one step ahead NADI at a predetermined level of statistical significance were used as potential input variables. Different combinations of these potential input variables were then used for developing several potential ANN drought forecasting ANN models. The details of these models are presented in Section 6.4.

The available data of potential input variables is generally divided into three parts for calibration and validation of ANN models i.e. training, testing and validation (ASCE Task Committee on Application of Artificial Neural Networks in Hydrology, 2000). It should be noted that the testing and validation terminologies sometimes uses opposite to each others. However, in this study, the training and testing data sets were used for model calibration, while the validation data set is used for the evaluation of the forecasting model performance (i.e. validation). In order to ensure that all variables receive equal attention during the calibration process and commensurate with the limits of the activation function used in the hidden layer, all inputs values were standardized in this study within a range between 0.1 and 0.9 before their use in the ANN models. The range from 0.1 to 0.9 was selected to avoid the extreme limits (from 0 to 1) of the nonlinear activation function that was used in the hidden neurons. Note that the nonlinear sigmoid and linear functions were used as the activation functions in the hidden and output neurons respectively in this study, as the combination of sigmoid and linear functions gives an advantage to extrapolate beyond the range of the training data (Maier and Dandy, 2000). Moreover, the selection of the standardization range within the extreme limits of the activation function prevents the chances of weights update to extremely small values and the flatspots during the calibration process (Maier and Dandy, 2000). However, the output values were not standardized as the linear transfer function was used in the output neurons (Karunanithi *et al.*, 1994).

6.3.3. Calibration

The calibration of the ANN model (which is sometimes referred to as training (Maier and Dandy, 1998)) determines the optimum model parameters, while maintaining the best generalization abilities of the model (i.e. avoiding over-fitting). The suitable number of neurons in the hidden layer and the connection weights and biases in the network are the parameters to be determined during the calibration process. This is generally done by minimizing the mean square error (MSE) of the training data set (Maier and Dandy, 2000; Tran *et al.*, 2009). The MSE is one of many ways to quantify the difference between original and forecast values. However, there is the danger of over-fitting (or overtraining) a network by continuous minimizing of MSE. Over-fitting results in a network that memorizes the individual examples, rather than trends in the data set as a whole. When this happens, the network performs very well over the data set used for training, but shows poor forecasting capabilities when supplied with data that were not used in training (ASCE Task Committee on Application of Artificial Neural Networks in Hydrology, 2000). To prevent over-fitting, the early stopping technique described in Bishop (1995) was used in this study. The goal of this procedure is to stop calibration when the network begins to over-fit. The testing data set was used in parallel with the training data set for this purpose during the calibration.

The number of hidden neurons depends on the nature of the relationship between inputs and outputs, and there were no specific rules found in the literature to select the number of hidden neurons. Therefore, at the beginning of the calibration process, only one hidden neuron was selected, and network weights and biases were initialized to small random numeric values (ASCE Task Committee on Application of Artificial Neural Networks in Hydrology, 2000). Thereafter, feed-forward training with standard back propagation (BP) algorithm (which will be described shortly) was used to adjust the connection weights and biases of the ANN models by minimizing the

MSE using the training data set, considering several iterations (or epochs). At each epoch, the initial weights and biases were considered as the adjusted weights and biases of the previous epoch. The pattern of the changes in the MSE for both training and testing data sets during the calibration (which consists of several epochs) is shown in Figure 6.4. As can be seen from this figure, during the initial epochs, the MSE for both training and testing data sets go down. After a certain number of epochs, the MSE for the training set continues to decrease, but the MSE associated with the testing data set begins to rise (Figure 6.4). This is an indication that further calibration will likely result in the network over-fitting the training data. When this occurs, the process of calibration is stopped (i.e. early stopping technique of Bishop (1995) was applied), and the current set of weights and biases are assumed to be the optimal values for the network under consideration. Thereafter, the number of hidden neurons was increased sequentially by adding a new neuron to create a new ANN model. The BP algorithm was then used to determine the connection weights and biases of this new ANN model. Again the early stopping technique of Bishop (1995) was used to select the optimal network, but now with respect to the number of hidden neurons. In this case, Figure 6.4 will have in the horizontal axis the number of hidden neurons.

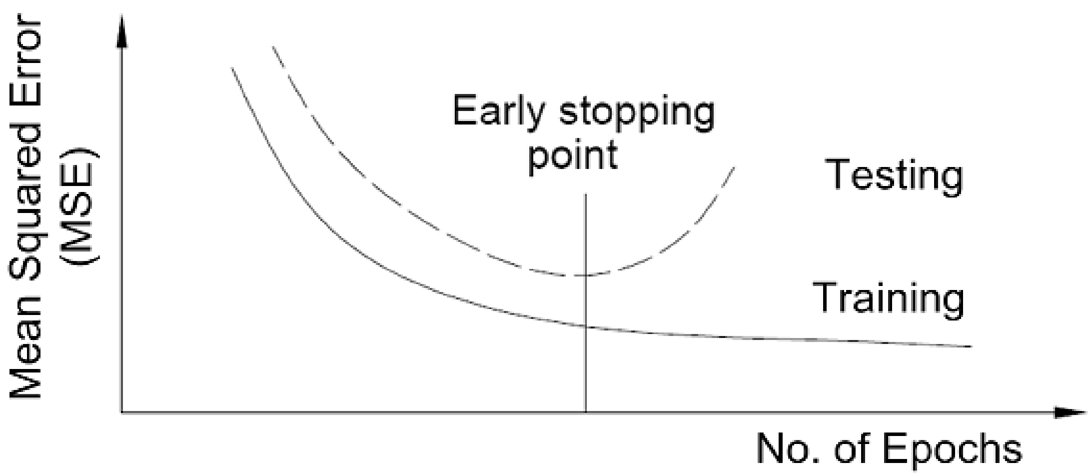


Figure 6.4. MSE versus epochs during calibration process

Back Propagation (BP) Training Algorithm

As was mentioned above, to calibrate (i.e. adjust the connection weights and biases) the ANN model, the most commonly used feed-forward training with standard BP algorithm (Lorrai and Sechi, 1995; Kim and Valdes, 2003; Mishra and Desai, 2006; Mishra *et al.*, 2007; Morid *et al.*, 2007) was used in this study. The BP algorithm was first introduced by Werbos (1974), but later became popular in its modified form developed by Rumelhart *et al.* (1986).

A typical three-layer feed-forward ANN model with BP algorithm is shown in Figure 6.5. In this model, the three layers are present: (1) input layer – where data are introduced to the network, (2) hidden layer – where data are processed, and (3) output layer – where results are produced for the given inputs. In Figure 6.5, the input neurons are shown with X_i (i = 1, . . ., n), where n is the number of neurons in the input layer representing input variables. The number of hidden neurons in the hidden layer, on the other hand, is one of the parameters to be determined during the calibration process. The hidden neurons are denoted by Y_j (j = 1, . . ., m), where m is the number of hidden neurons. A single neuron or a number of output neurons (denoted by Z_k) can be in the output layer depending on different drought forecasting lead times (e.g. if drought is to be forecasted is for up to 3 months lead time using a DMSNN, then there should be 3 output neurons). However, only one output neuron is shown in Figure 6.5 to provide a simple explanation of the BP algorithm. This figure shows an epoch (or an iteration) of the calibration process for an ANN with a fixed number of neurons in the hidden layer. This essentially means only the weights and biases are calibrated, and not the number of hidden neurons in the ANN model. In this figure, the initial (i.e. at the start of an epoch) connection weights between input and hidden neurons are shown as w_{ij} and the connection weights between hidden and output neurons are shown as w_{jk}, whereas updated (i.e. at the end of the epoch) connection weights between input and hidden, and hidden and output neurons are shown as w^*_{ij} and w^*_{jk} respectively. Similarly, the initial biases are presented with b_j and b_k, and the updated biases are presented with b^*_j and b^*_k for hidden and output neurons respectively (Figure 6.5). In this figure, given the sufficient numbers of hidden neurons in the hidden layer, the information received in the input layer processes through the hidden and output layers with the activation functions to get the output. As was mentioned in Section 6.3.2, the nonlinear sigmoid and linear functions were used in this study as the activation functions in the hidden and output neurons respectively. The mathematical relationship between inputs and outputs is given explicitly in Equation (6.1).

$$ADI_{forecast} = f_o \left[\sum_{j=1}^{m} w_{jk} . f_h (\sum_{i=1}^{n} w_{ij} x_i + b_j) + b_k \right] \qquad (6.1)$$

where, x_i is the input at i^{th} neuron in the input layer; w_{ij} is the weight connecting the i^{th} neuron in the input layer and the j^{th} neuron in the hidden layer; b_j is the bias for the j^{th} hidden neuron; f_h is the activation function of the hidden neuron; w_{jk} is the weight connecting the j^{th} hidden neuron in the hidden layer and the k^{th} neuron in the output layer; b_k is the bias for the k^{th} output neuron; and f_o is the activation function of the output neuron.

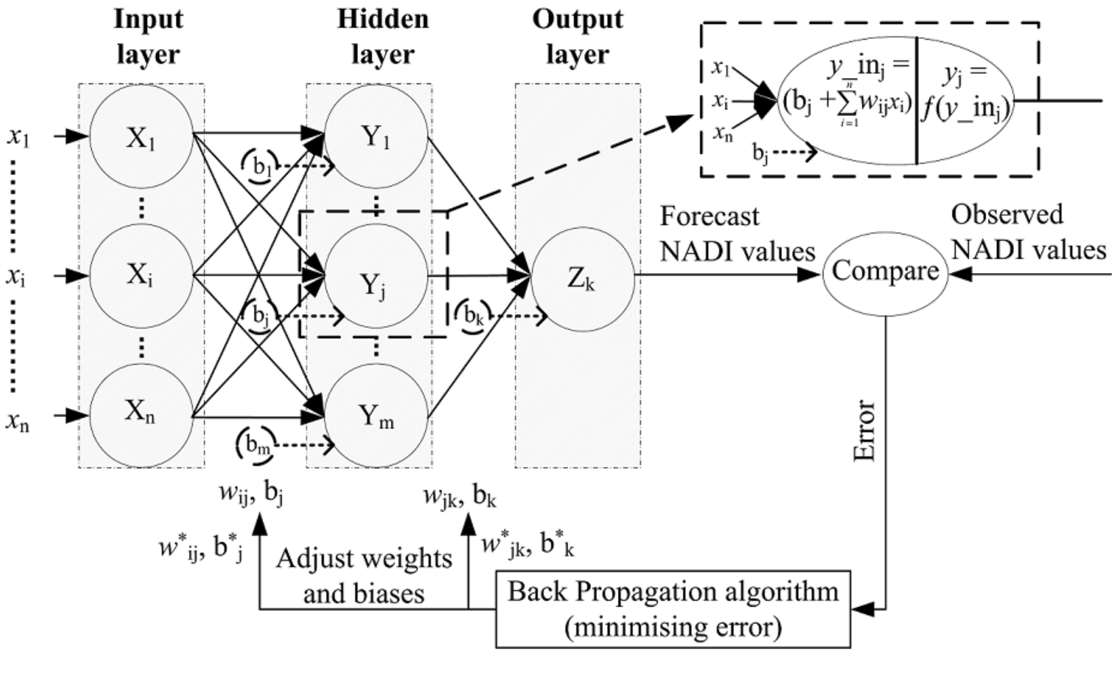


Figure 6.5 Typical three-layer feed-forward BP training algorithm (adapted from Kim and Valdes, 2003)

The outputs are compared with the observed or desired outputs and minimize the errors by adjusting weights and biases iteratively (Figure 6.5). All weights and biases are initialized to small random numeric values at the beginning of calibration (ASCE Task Committee on Application of Artificial Neural Networks in Hydrology, 2000) as was mentioned earlier. These weights and biases are then updated or modified iteratively using the steepest gradient descent method in the BP algorithm to minimize the errors between model outputs and observed values. Further mathematical details of this iterative process in the BP algorithm are described below.

The iterative process in the BP algorithm consists of two phases: 1) forward pass, during which the information is processed from the input layer to the output layer; and 2) backward pass, where the error from the output layer is propagated back to the hidden and the input layers by adjusting or modifying weights and biases (ASCE Task Committee on Application of Artificial Neural Networks in Hydrology, 2000). The calibration process is stopped when no appreciable change is observed in the values associated with the connection links (i.e., weights and biases) or some termination criterion (e.g., MSE) is satisfied as in Figure 6.4. The BP algorithm is adopted from Fausett (1994) and ASCE Task Committee on Application of Artificial Neural Networks in Hydrology (2000), and is described stepwise below. Note that the algorithm described here is for a network with a specified number of hidden neurons in the 3-layer network (Figure 6.5):

Step 1: Initialize weights and biases (i.e., weights and biases are set to small random values).

Step 2: While stopping condition is false, do Steps 3 – 10.

Step 3: For each training pair (i.e., combination of inputs and outputs), do Steps 4 – 9.

Feed-forward:

Step 4: Each input neuron (X_i, $i = 1, \ldots, n$) receives input information x_i and sends this information to all hidden neurons in the hidden layer.

Step 5: Each hidden neuron (Y_j, $j = 1, \ldots, m$) sums its weighted input information using Equation (6.2).

$$y\_in_j = b_j + \sum_{i=1}^{n} w_{ij} x_i \tag{6.2}$$

where w_{ij} is the connection weights between input and hidden neurons and b_j is the biases in the hidden neurons. The activation function is applied on "y_in_j" to compute the output from each hidden neuron using Equation (6.3). Sigmoid activation function was used for the hidden neurons in this study.

$$y_j = f(y\_in_j) \tag{6.3}$$

This output information from each hidden neuron is then sent to all neurons in the following layer (i.e., output layer in a 3-layer network).

Step 6: Each output neuron (Z_k, $k = 1$ in Figure 6.5) sums its weighted input information from the hidden neurons using Equation (6.4).

$$z\_in_k = b_k + \sum_{j=1}^{m} w_{jk} y_j \tag{6.4}$$

where w_{jk} is the connection weights between hidden and output neurons and b_k is the biases in the output neurons. The activation function is then applied on "z_in_k" to compute the output from each output neuron using Equation (6.5). Linear activation function was used for the output neurons in this study.

$$z_k = f(z\_in_k) \tag{6.5}$$

Back-propagation of error:

Step 7: Each output neuron receives a target value (t_K) corresponding to the input values and compares it with the computed output value (z_k) to compute its error information term using Equation (6.6).

$$\delta_k = (t_k - z_k)\, f'(z\_in_k) \tag{6.6}$$

This δ_k value is used to calculate weight and biases correction terms (used to update w_{jk} and b_k later) using Equations (6.7) and (6.8) respectively.

$$\Delta w_{jk} = \alpha \;\delta_k \; y_j \tag{6.7}$$

$$\Delta b_k = \alpha \;\delta_k \tag{6.8}$$

where, α is the momentum factor that is used to speed up the training process in very flat regions of the error surface and helps prevent oscillations in the weights.

Step 8: Each hidden neuron sums its delta inputs in Equation (6.6) (from neurons in the next layer i.e., output layer in a 3-layer network) using Equation (6.9).

$$\delta\_in_j = \sum_{k=1} \delta_k \; w_{jk} \tag{6.9}$$

This "δ_in_j" is then multiplied by the derivative of its activation function to calculate its error information term using Equation (6.10).

$$\delta_j = \delta\_in_j \, f'(y\_in_j) \tag{6.10}$$

This δ_j value is used to calculate weight and bias correction terms (used to update w_{ij} and b_j later) using Equations (6.11) and (6.12) respectively.

$$\Delta w_{ij} = \alpha \; \delta_j \; x_i \tag{6.11}$$

$$\Delta b_j = \alpha \; \delta_j \tag{6.12}$$

Update weights and biases:

Step 9: Weights between the hidden and the output neurons, and the biases in the output neuron are updated using Equations (6.13) and (6.14) respectively.

$$w^*_{jk} = w_{jk} + \Delta w_{jk} \tag{6.13}$$

$$b^*_k = b_k + \Delta b_k \tag{6.14}$$

Weights between the input and the hidden neurons, and the biases in the each hidden neuron are updated using Equations (6.15) and (6.16) respectively.

$$w^*_{ij} = w_{ij} + \Delta w_{ij} \tag{6.15}$$

$$b^*_j = b_j + \Delta b_j \tag{6.16}$$

Step 10: Test the stopping condition.

6.3.4. Validation

After optimizing model parameters through calibration, the forecasting models need to be validated using a validation data set (which is not used in the model calibration) as mentioned earlier. This was done in this study, and the performance validation was conducted using the correlation co-efficient (R), the mean absolute error (MAE) and the root mean squared error (RMSE) (i.e., square root of MSE) values, which are commonly used for such evaluation (Mishra and Desai, 2006; Morid *et al.*, 2007; Bacanli *et al.*, 2008). These measures are given in Equations (6.17) to (6.19).

$$R = \frac{\sum_{i=1}^{n_v}(NADI_{obs} - \overline{NADI_{obs}})(NADI_{\mathrm{mod}} - \overline{NADI_{\mathrm{mod}}})}{\sqrt{\sum_{i=1}^{n_v}(NADI_{obs} - \overline{NADI_{obs}})^2 \sum_{i=1}^{n_v}(NADI_{\mathrm{mod}} - \overline{NADI_{\mathrm{mod}}})^2}} \tag{6.17}$$

$$MAE = \frac{1}{n_v}\sum_{i=1}^{n_v}\left|NADI_{obs} - NADI_{\mathrm{mod}}\right| \tag{6.18}$$

$$RMSE = \sqrt{\frac{1}{n_v}\sum_{i=1}^{n_v}(NADI_{obs} - NADI_{\mathrm{mod}})^2} \tag{6.19}$$

where, $NADI_{obs}$ and $NADI_{\mathrm{mod}}$ are the observed and modelled NADI values respectively, $\overline{NADI_{obs}}$ and $\overline{NADI_{\mathrm{mod}}}$ are the average observed and modelled NADI values respectively, and n_v is the number of validation data points.

R is a measure of the strength of the linear relationship between observed and forecast NADI values; *R* varies from 0 to 1, where 0 and 1 values indicate poor and prefect forecasting capabilities of the model respectively. MAE and RMSE, on the other hand, measure the average magnitude of the errors in a set of forecasts. MAE is a linear score which means that all individual differences are weighted equally, whereas RMSE gives a relatively high weight to large errors and try to avoid these large errors by minimizing the RMSE value. Both the MAE and RMSE values increase from 0 for perfect forecasts to large positives values as the discrepancies between forecasts and observations become increasingly large (Kim and Valdes, 2003). In general, when a forecasting model has a large *R* value (e.g., close to 1) and low MAE and RMSE values (e.g., close to 0), then the model considered as a good (or validated) model.

6.4. Results and Discussion

Computational tasks of the ANN forecasting models were carried out using the Neural Network toolbox in MATLAB software (Demuth and Beale, 1994).

6.4.1. Potential and Best Drought Forecasting ANN Models

Selection of potential input variables is an important task to obtain the desired output from the ANN models. As mentioned in Section 6.3.2, the standard statistical correlation test was used to select the potential input variables for the ANN models. The correlation test was conducted between one step ahead NADI values, and the present and 11 months of past NADI values, and the data of present and 11 months of past hydro-meteorological variables that were used in NADI calculations (i.e. rainfall, potential evapotranspiration, storage reservoir volume, streamflow and soil moisture content). Generally, strong and weak relationships are considered to take values of the correlation coefficient between (0.85-1) and (0-0.5) respectively. The values between 0.5 and 0.85 are considered to have moderate relationships (Tran *et al.*, 2009). The correlation co-efficients (R) of the correlation test conducted in this study are presented in Table 6.1. In this table, the *R* values only up to past 5 months are shown, as poor correlations were obtained beyond past 4 months. The results show that none of the *R* values are strong and the *R* values decrease when lead time of the past input variables increases (which is to be expected). The present NADI and up to past 4 months NADI values show statistically significant moderate correlation at 0.01 level with one step ahead NADI (from its present value) and thereafter showing poor correlations. The present and the past 1 month storage reservoir volume, the present month streamflow, and the present and up to past 2 months soil moisture content also show 0.01 level statistically significant moderate correlations with one month ahead NADI (from its present value). Rest of the variables studied show poor correlations. Therefore, the potential input variables that were used in the DMSNN model were; the present and up to past 4 months NADI values, present and the past 1 month storage reservoir volume, the present month streamflow, and the present and up to past 2 months soil moisture content. However, only the present and up to past 4 months NADI values were used in the RMSNN model (Section 6.3.1).

Table 6.1 Results of correlation tests with one step ahead NADI

Input variables	**Present**	**Past**				
	t	**t-1**	**t-2**	**t-3**	**t-4**	**t-5**
NADI	**0.70***	**0.60***	**0.56***	**0.53***	**0.50***	0.43*
Rainfall	0.37*	0.31*	0.27*	0.26*	0.25*	0.23*
Potential evapotranspiration	-0.15	-0.17	-0.13	-0.12	-0.10	-0.08
Storage reservoir volume	**0.57***	**0.51***	0.45*	0.40*	0.36*	0.30*
Streamflow	**0.56***	0.48*	0.46*	0.41*	0.35*	0.28*
Soil moisture content	**0.60***	**0.55***	**0.50***	0.45*	0.38*	0.29*

*Correlation is statistically significant at the 0.01 level.

Bold values are statistically significant moderate correlations.

The full range of data from 1960 to 2008 for all potential input variables were divided into 3-parts for training, testing and validation of the ANN forecasting models. Data from 1970 to 2000 were used for training, as this series contained some extreme NADI values which are essential for better generalization ability of the ANN models. Data from 1960 to 1969 and 2001 to 2008 were used for testing and validation of the ANN forecasting models respectively.

In this study, several potential drought forecasting ANN models were developed as was outlined in Section 6.3.2 based on various possible combinations of the potential input variables. They are shown in Tables 6.2 and 6.3. It can be seen from Table 6.2 that the first potential RMSNN model (i.e., Model 1) was developed only based on the present NADI value, which had the strongest correlation (Table 6.1). The subsequent RMSNN models (i.e., models from 2 to 5) were developed by sequentially adding more past NADI values. Similar procedure was also followed for developing potential DMSNN models 6 to 10 (Table 6.3). It can be seen from Table 6.3 that only the present and past 2 NADI values were considered for rest of the models (from 11 to 28) along with the several combinations of the present and the past 1 month storage reservoir volume, the present month streamflow, and the present and up to past 2 months soil moisture content as input variables. Note that the NADI values beyond the past 2 months were not considered in the potential DMSNN models from 11 to 28 as no improvements were observed when sequentially added more than the past 2 months NADI values (i.e. potential DMSNN models 9 and 10), which will be discussed shortly.

All potential RMSNN and DMSNN models in Tables 6.2 and 6.3 were calibrated according to the procedure described in Section 6.3.3. The optimal number of hidden neurons found for each model during model calibration is also shown in Tables 6.2 and 6.3. It can be seen from the tables that the potential drought forecasting ANN models developed in this study required only 2 to 3 hidden neurons in the hidden layer for the RMSNN models, whereas only 2 to 4 hidden neurons were required for the DMSNN models. Optimum numbers of weights and biases were also obtained for each model in Tables 6.2 and 6.3 during the model calibration process. However, they are not presented here due to the presentation complexity.

Table 6.2 Drought forecasting RMSNN models considered

Model	Combination of input variables	Optimal number of hidden neurons
1	$NADI_t$	2
2	$NADI_t, NADI_{t-1}$	3
3	$NADI_t, NADI_{t-1}, NADI_{t-2}$	2
4	$NADI_t, NADI_{t-1}, NADI_{t-2}, NADI_{t-3}$	2
5	$NADI_t, NADI_{t-1}, NADI_{t-2}, NADI_{t-3}, NADI_{t-4}$	3

NADI is the nonlinear aggregated drought index;
t is the present month; and
t-1, *t*-2, *t*-3 and *t*-4 refer to past 1, 2, 3 and 4 months respectively.

Table 6.3 Drought forecasting DMSNN models considered

Model	Combination of input variables	Optimal number of hidden neurons
6	$NADI_t$	2
7	$NADI_t, NADI_{t-1}$	2
8	$NADI_t, NADI_{t-1}, NADI_{t-2}$	3
9	$NADI_t, NADI_{t-1}, NADI_{t-2}, NADI_{t-3}$	2
10	$NADI_t, NADI_{t-1}, NADI_{t-2}, NADI_{t-3}, NADI_{t-4}$	3
11	$NADI_t, NADI_{t-1}, NADI_{t-2}, V_t$	4
12	$NADI_t, NADI_{t-1}, NADI_{t-2}, V_t, V_{t-1}$	3
13	$NADI_t, NADI_{t-1}, NADI_{t-2}, Q_t$	2
14	$NADI_t, NADI_{t-1}, NADI_{t-2}, S_t$	2
15	$NADI_t, NADI_{t-1}, NADI_{t-2}, S_t, S_{t-1}$	2
16	$NADI_t, NADI_{t-1}, NADI_{t-2}, S_t, S_{t-1}, S_{t-2}$	2
17	$NADI_t, NADI_{t-1}, NADI_{t-2}, V_t, Q_t$	4
18	$NADI_t, NADI_{t-1}, NADI_{t-2}, V_t, S_t$	3
19	$NADI_t, NADI_{t-1}, NADI_{t-2}, V_t, S_t, S_{t-1}$	4
20	$NADI_t, NADI_{t-1}, NADI_{t-2}, V_t, S_t, S_{t-1}, S_{t-2}$	2
21	$NADI_t, NADI_{t-1}, NADI_{t-2}, V_t, Q_t, S_t$	3
22	$NADI_t, NADI_{t-1}, NADI_{t-2}, V_t, V_{t-1}, Q_t$	3
23	$NADI_t, NADI_{t-1}, NADI_{t-2}, V_t, V_{t-1}, S_t$	3
24	$NADI_t, NADI_{t-1}, NADI_{t-2}, V_t, V_{t-1}, S_t, S_{t-1}$	3
25	$NADI_t, NADI_{t-1}, NADI_{t-2}, V_t, V_{t-1}, S_t, S_{t-1}, S_{t-2}$	3
26	$NADI_t, NADI_{t-1}, NADI_{t-2}, V_t, V_{t-1}, Q_t, S_t$	2
27	$NADI_t, NADI_{t-1}, NADI_{t-2}, V_t, V_{t-1}, Q_t, S_t, S_{t-1}$	4
28	$NADI_t, NADI_{t-1}, NADI_{t-2}, V_t, V_{t-1}, Q_t, S_t, S_{t-1}, S_{t-2}$	4

NADI is the nonlinear aggregated drought index (NADI);
V is the storage reservoir volume;
Q is the streamflow;
S is the soil moisture content;
t is the present month; and
t-1, *t*-2, *t*-3 and *t*-4 refer to past 1, 2, 3 and 4 months respectively.

All calibrated models (presented in Tables 6.2 and 6.3) were validated as described in Section 6.3.4. It should be noted that forecasting was carried out over a 12-month period at each current time step of the validation period except for the last 12 months, and the R, RMSE and MAE values were computed for each potential drought forecasting model over this period. Forecasting during the last 12 months (in the year 2008) could not be compared against the computed NADI values, since there were no computed NADI values in 2009. Although the R, RMSE and MAE values were computed for each potential drought forecasting model over this period, in Table 6.4 the R, RMSE and MAE values are presented only for 1 month lead time forecasts. It should be noted that the *R* values for the rest of the forecasting lead times were sequentially decreased, and the RMSE and MAE values were sequentially increased.

Table 6.4 R, RMSE and MAE values obtained from validation of drought forecasting models

Models	R	RMSE	MAE
RMSNN models			
1	0.72	0.69	0.55
2	0.74	0.67	0.53
3	**0.75**	**0.61**	**0.49**
4	0.74	0.62	0.48
5	0.71	0.63	0.49
DMSNN models			
6	0.72	0.68	0.53
7	0.74	0.62	0.49
8	**0.75**	**0.61**	**0.48**
9	0.74	0.64	0.52
10	0.73	0.64	0.51
11	0.73	0.70	0.56
12	0.73	0.64	0.49
13	0.75	0.61	0.49
14	0.75	0.63	0.51
15	0.72	0.63	0.51
16	0.73	0.72	0.58
17	0.73	0.67	0.52
18	0.73	0.67	0.53
19	0.72	0.73	0.60
20	0.74	0.73	0.60
21	0.69	0.72	0.58
22	0.65	0.68	0.53
23	0.74	0.63	0.50
24	0.72	0.64	0.50
25	0.69	0.72	0.57
26	0.74	0.64	0.51
27	0.70	0.62	0.48
28	0.66	0.67	0.51

It is seen from the Table 6.4 that the models 3 and 8 give the best drought forecasts in terms of all three validation indices (i.e., R, RMSE, MAE) for RMSNN and DMSNN models respectively. It shows that the best forecasts were obtained from both RMSNN and DMSNN models when the combination of the present and up to past 2 months NADI values were used as model inputs (i.e., the models 3 and 8 of Tables 6.2 and 6.3 respectively). Moreover, the best forecasts provided by the RMSNN and DMSNN models required only 2 and 3 hidden neurons respectively (i.e. models 3 and 8 as shown in Tables 6.2 and 6.3 respectively).

6.4.2. Comparative Study between RMSNN and DMSNN Models

As mentioned in Section 6.4.1, forecasting was carried out over a 12-month period at each current time step of the validation period except for the last 12 months. It was found that performance of both RMSNN and DMSNN models (i.e., model 3 and 8 in Tables 6.2 and 6.3 respectively) decreased with increasing forecasting lead time in terms of forecasting ability. It was also found that the models performed poorly beyond the forecasts of first 6 months. Therefore, only the results up to 6 month forecasts were used for comparing the performance of the two models.

The comparison of the original NADI time series computed from observed data and the forecast NADI time series from both RMSNN and DMSNN models considering 6 month forecasts are presented in Figures 6.6 and 6.7 respectively. Each figure has six components, showing how 1, 2, 3, 4, 5 and 6 time step forecasts compared against computed NADI values. In these figures, comparative performance evaluations are presented for all three data sets (i.e. training, testing and validation) to show the optimum generalization ability of the forecasting models that was achieved in the model calibration process. The *R* values between the observed and the forecast NADI values are also shown in these figures. It can be seen from Figures 6.6 and 6.7 that the forecast time series matches well with the original NADI time series for all three data sets when forecast lead times are smaller and the differences become larger when the lead time increases. The R values show that they are close to each other for forecasts in all three data sets up to 3 months ahead, which implies the good generalization abilities of the forecasting models up to 3 months ahead forecasts.

Moreover, the *R* values within the three data sets decrease when forecast lead times increase showing the generalization abilities of the forecasting models decrease with the increase in forecasting lead time.

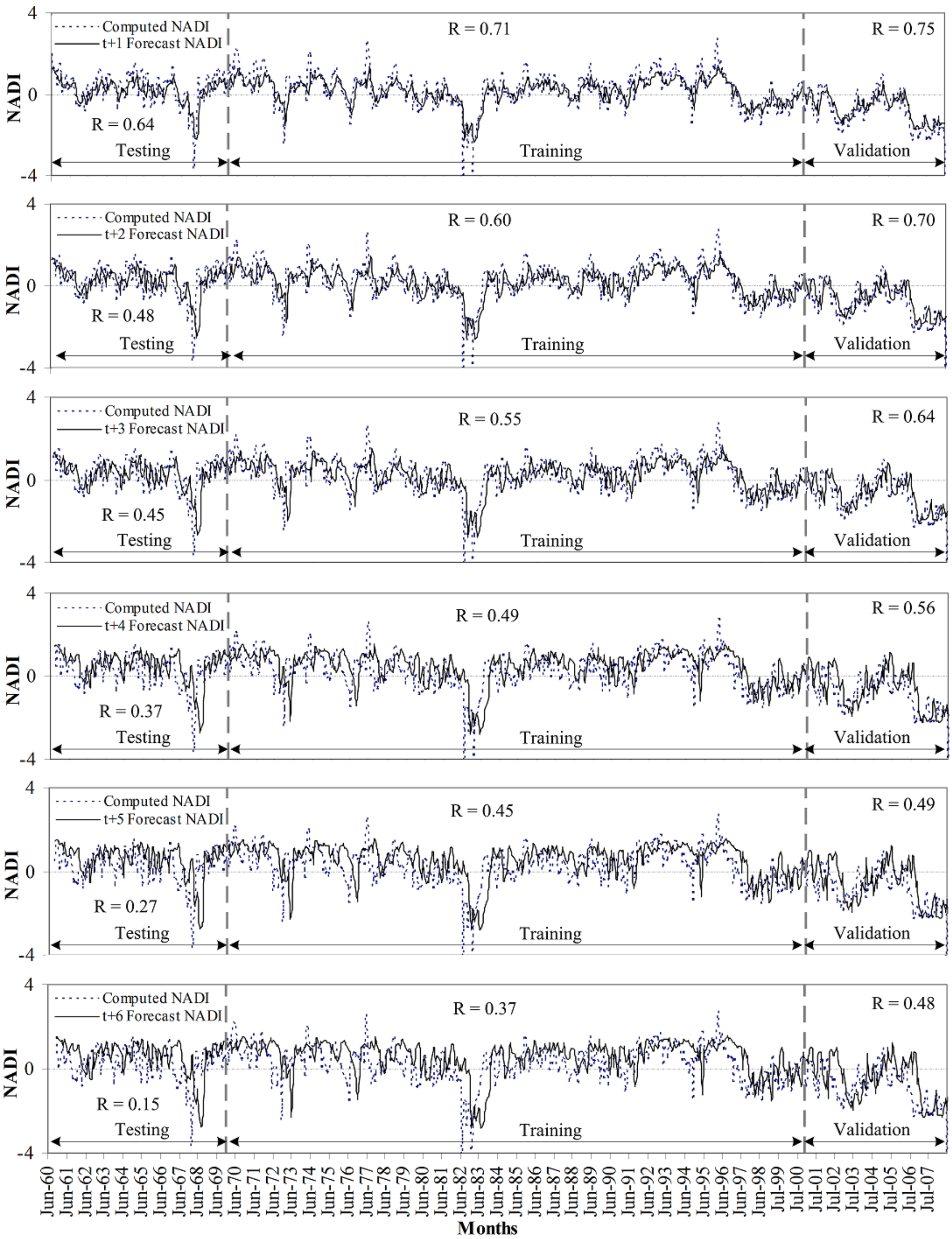


Figure 6.6 Comparison of computed NADI time series with forecast NADI time series from RMSNN model (i.e., Model 3)

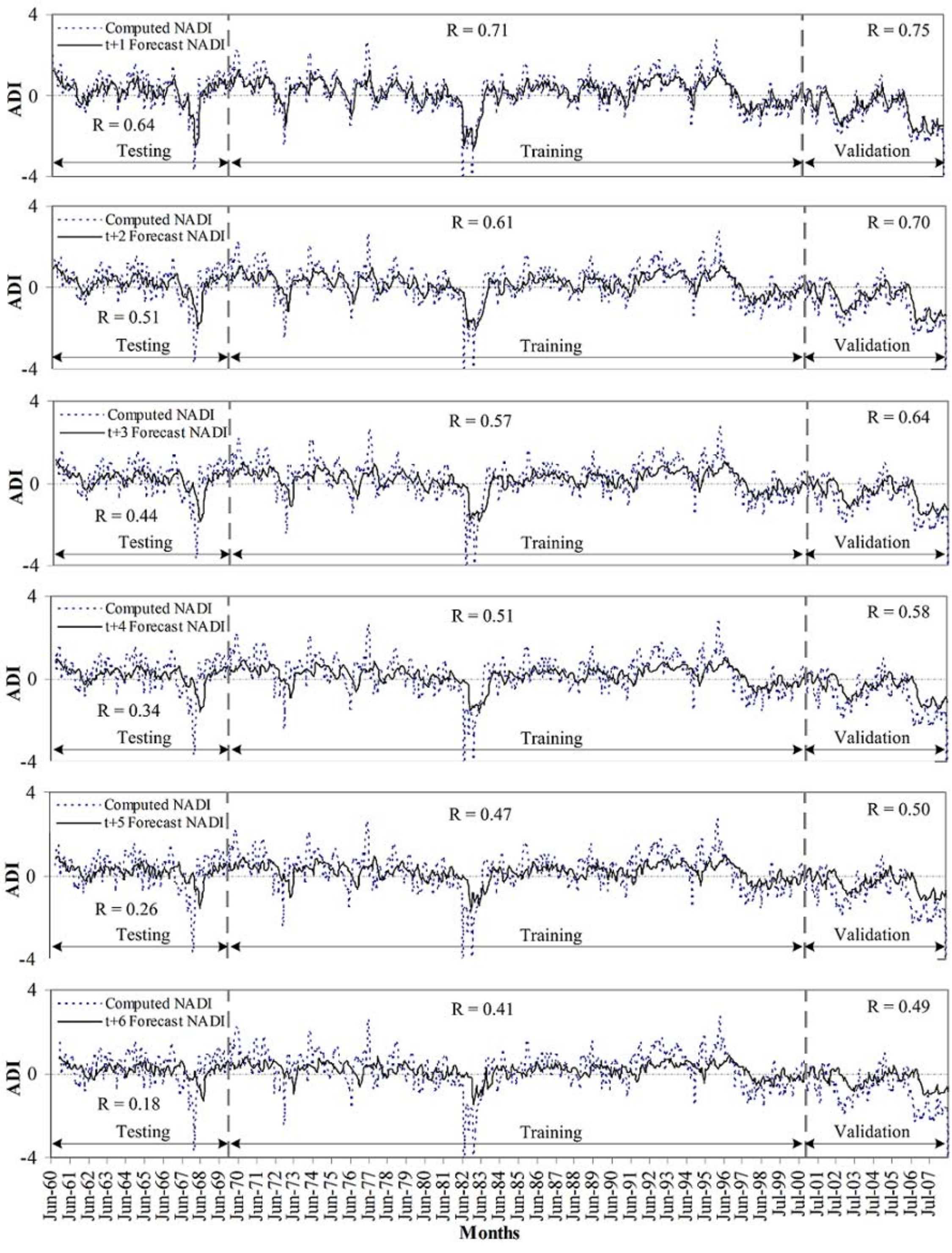


Figure 6.7 Comparison of computed NADI time series with forecast NADI time series from DMSNN model (i.e., Model 8)

The R, RMSE and MAE values of each lead time forecast obtained by both RMSNN and DMSNN models for the validation data set are presented in Table 6.5. It shows that in both RMSNN and DMSNN models, *R* values decrease, and RMSE and MAE values increase when forecast lead time increases, which are expected. Moreover, Table 6.5 shows that both models produce the same R, RMSE and MAE values for one 1-month lead time. However, the forecasting performance is slightly better in the RMSNN model than in the DMSNN model for 2 to 3 months ahead forecasts in terms of RMSE and MAE, although the performance of both models is the same in terms of their *R* values. However, the DMSNN model results are slightly better than those of the RMSNN model for forecasting lead times 4 to 6 months. In terms of the *R* values, the forecast NADI values were statistically significant at 0.01 level in both RMSNN and DMSNN models.

Table 6.5 R, RMSE and MAE values for validation data set

		t+1	**t+2**	**t+3**	**t+4**	**t+5**	**t+6**
R	RMSNN	0.75*	0.70*	0.64*	0.56*	0.49*	0.48*
	DMSNN	0.75*	0.70*	0.64*	0.58*	0.50*	0.49*
RMSE	RMSNN	0.61	0.67	0.76	0.93	1.05	1.08
	DMSNN	0.61	0.70	0.81	0.92	1.02	1.07
MAE	RMSNN	0.49	0.53	0.60	0.73	0.81	0.87
	DMSNN	0.49	0.56	0.64	0.72	0.80	0.86

*Correlation is statistically significant at the 0.01 level

The forecast values of NADI were also compared by classes of different dryness/wetness (or drought classifications in Table 5.1 in Chapter 5) to evaluate the accuracy of forecasts across events of different extremity, which is also ensure cross validation of the forecasting model. This approach was also used by Morid *et al.* (2007). For easy reference in this chapter, Table 5.1 is reproduced as Table 6.6, it should be noted that wetness thresholds are also included in this table as these thresholds were used in the evaluation of forecast accuracy study.

Percent of forecast accuracy in the validation data set is presented in Table 6.7 for forecasts up to 6 time steps. In this table, ‘1’ in the DC column (DC = Difference in Classes) means that there is only one difference in relation to the level of dryness/wetness between the original and the forecast NADI values (e.g., if ‘Extreme drought’ is observed in the original computed NADI series and ‘Severe drought’ is forecasted in the forecast NADI series, then DC = 1). Table 6.7 shows that when forecasting lead time increases the percent of forecast accuracy decreases, as expected. Moreover, for all 6 month forecasts, the DC values of the RMSNN model forecast with 0 (i.e. original drought classes are exactly same as forecast drought classes) are between 52% and 67%, whereas the corresponding figures in the DMSNN model are between 53% to 67%. However, if both 0 and 1 DC values are considered together, the percent of accuracy becomes between 88 to 98% and 85 to 98% for all 6 month forecasts for the RMSNN and DMSNN model respectively. As was seen before (Table 6.5), Table 6.7 also shows that both RMSNN and DMSNN models show same forecasting performance for 1-month lead time. Moreover, the table also shows that the RMSNN model gives slightly better forecasts than the DMSNN model for 2 to 3 months ahead forecasts, while the DMSNN model forecasts are slightly better than the RMSNN model forecasts for 4 to 6 months ahead forecasts.

Table 6.6 Drought classification based on NADI thresholds*

> 1.63	Extreme wet
> 1.20 to ≤ 1.63	Severe wet
> 0.88 to ≤ 1.20	Moderate wet
> -0.84 to ≤ 0.88	Near normal
> -1.64 to ≤ -0.84	Moderate drought
> -2.27 to ≤ -1.64	Severe drought
≤ -2.27	Extreme drought

*Reproduced from Table 5.1

Table 6.7 Percent of forecast accuracy in dry or wet classes – validation data set

DC[a]	**t+1**	**t+2**	**t+3**	**t+4**	**t+5**	**t+6**
			RMSNN			
0	67	66	64	53	53	52
1	31	30	31	43	38	36
2	2	4	4	3	8	11
3	0	0	1	1	1	1
(0+1)	98	96	95	96	91	88
			DMSNN			
0	67	64	58	57	54	53
1	31	32	35	31	32	32
2	2	4	5	11	12	13
3	0	0	1	1	2	2
(0+1)	98	96	93	88	86	85

DC[a]: Difference between original and forecast classes of dryness/wetness

6.5. Summary

Drought forecasting is important in terms of planning and operation of water systems, especially during continuing dry climatic periods. In the context of drought forecasting, researchers and professionals have used several techniques in the past for forecasting Drought Index (DI) values as the forecasts of future drought conditions. Based on the study on several existing DIs (Chapter 4), a new DI namely Nonlinear Aggregate Drought Index (NADI) was developed in this study which was presented in Chapter 5. The NADI defines the overall dryness as the drought condition within the system by considering all significant hydro-meteorological variables (i.e. rainfall, potential evapotranspiration, streamflow, storage reservoir volume and soil moisture content) that affect the droughts. The use of NADI in the drought forecasting model forecasts the overall dry condition within the catchment beyond the traditional forecast of rainfall based DIs as the forecast of drought condition. In this study, the most successfully used Artificial Neural Network (ANN) modeling technique was used to develop the drought forecasting model to forecast the NADI values.

Two types of ANNs namely: Recursive Multi-Step Neural Network (RMSNN) and Direct Multi-Step Neural Network (DMSNN) were used to forecast the NADI values for the Yarra River catchment in Victoria (Australia). The present and several months of past NADI values were tested as inputs in the RMSNN model, whereas the present and several months of past NADI values along with the present and several months of past hydro-meteorological variables that were used in NADI calculations were tested as inputs in the DMSNN model. It was found that the best forecasts were obtained from both RMSNN and DMSNN models, when the combination of the present and up to past 2 months of NADI values were used as model inputs. Moreover, the RMSNN model was required only two neurons in one hidden layer to get the best forecasts, whereas only three neurons were sufficient for the DMSNN model. The best developed RMSNN and DMSNN drought forecasting models were capable of forecasting drought conditions reasonably well up to 6 months ahead forecasts; these forecasts were statistically significant at 1% significant level. Moreover, it was found that both models show the same performance for forecasting 1-month lead time. However, the RMSNN model gives slightly better forecasts than the DMSNN model for lead times of 2 to 3 months and the DMSNN model gives slightly better forecasts than the RMSNN model for forecast lead times of 4 to 6 months. Poor forecasts were observed beyond the forecast lead time of 6 months.

7. Summary and Conclusions, and Recommendations

Summary and Conclusions; Limitations of the Study and Recommendations for Further Research

7.1. Summary and Conclusions

The main aim of this study was to develop a drought forecasting tool to forecast future drought conditions. Moreover, the study performed a detailed evaluation of existing Drought Indices (DIs) to investigate their usefulness under different climatic conditions for which they were not originally developed for. These were done using the Yarra River catchment in Victoria (Australia) as the case study. The aim of the study was achieved by undertaking the following tasks:

1. Selection of the study area, and data collection and processing
2. Review and evaluation of the existing Drought Indices
3. Review of the drought forecasting modeling techniques
4. Development of the Nonlinear Aggregated Drought Index
5. Development of the drought forecasting models

A brief summary and the conclusions drawn from each of these tasks are presented in the following sections.

7.1.1. Selection of Study Area, and Data Collection and Processing

The Yarra River catchment in Victoria, Australia was selected as the case study in this research. It was chosen because the management of water resources in this catchment has great importance, since it is a major source of water supply for Melbourne residents (EPA Victoria, 1999). Almost one third of Victorian population (approximately 1.5 million) depends on the water resources of this catchment. Moreover, the water resources of the catchment support a range of uses valued by the Melbourne's community, including urban water supply, agricultural and horticultural industries and downstream user requirements, as well as flow requirements for maintaining environmental flows. Therefore, the development of the drought forecasting model can be a useful tool for the management of water resources in the Yarra River catchment.

Data related to several hydro-meteorological variables (i.e. rainfall, potential evapotranspiration, streamflow, storage reservoir volume and soil moisture content) were collected for the Yarra River catchment. These data were required for the development of DI and the drought forecasting model. The required data were collected for this study from a number of organizations such as Bureau of Meteorology (BOM) (i.e. rainfall data), SILO database (Jeffrey *et al.*, 2001) (i.e. potential evapotranspiration data) and Melbourne Water Corporation (i.e. streamflow and storage reservoir volume data). Soil moisture content data were not available for the catchment, and therefore a two-layer water budget model of Palmer (1965) was adapted to determine the soil moisture content in the catchment.

Data used for the DIs and drought forecasting model development were from 1960-2008 (49 years). Data processing was carried out to obtain the catchment representative monthly values as DIs were developed in this study using a monthly time step. Monthly DIs are suitable for operational purposes and have lower sensitivity to observational errors (McKee *et al.*, 1993).

Several historical droughts recorded in Victoria including 1967-1968, 1972-1973, 1982-1983, 1997-1998, 2003-2004 and 2006 onwards were used in this study to evaluate the DIs. These historical droughts were recorded in Keating (1992) and Tan and Rhodes (2008) after comparing rainfall and storage records at that time against their average values.

7.1.2. Review and Evaluation of Existing Drought Indices

There are many DIs that have been developed in the past around the world to define drought conditions. No DIs have been used for the Yarra River catchment in the past, and therefore the usefulness of the existing DIs were first reviewed in this study. It was found that majority of the DIs had been developed for specific regions, and therefore may not be directly applicable to other regions due to different hydro-climatic conditions. Moreover, it was also found that the researchers and professionals are confronted with the ambiguity of the drought definition. Some researchers and professionals argue that drought is just deficiency in rainfall and should be defined with the rainfall as the single variable. However, many others believe that the definition of drought should consider significant components of the water cycle (such as rainfall, streamflow and storage reservoir volume), because the drought depends on numerous factors, such as water supplies and demands, hydrological and political boundaries, and antecedent conditions. Therefore, an evaluation study of the existing DIs was performed in this research to investigate whether they are applicable to a region, in this case the Yarra River catchment, for which these DIs were not specifically developed.

A quantitative assessment of five existing DIs selected from different drought perspectives (i.e. meteorological, hydrological and agricultural), was first conducted in this study to investigate how well these DIs can define the historical droughts in the Yarra River catchment. The selected DIs namely *Percent of Normal (PN), Deciles, Standardized Precipitation Index (SPI), Surface Water Supply Index (SWSI)* and *Aggregated Drought Index (ADI)*. Thereafter, an evaluation of these DIs was carried out based on both qualitative and quantitative assessments to select the most appropriate DI for defining drought conditions in the Yarra River catchment.

In assessing the overall usefulness of the DIs, five decision criteria - *robustness, tractability, sophistication, transparency, and extendability* - were used in this study. It was believed that these criteria consider the desirable properties of a DI and would give a reasonable framework for evaluation of the DIs without excessive complications. Overall, the study found that the ADI was a better DI for modeling droughts and the management of droughts in the Yarra River catchment. It modeled the characteristics of historical droughts better than the other four DIs. It was also the most stable DI having smooth transitional characteristics during droughts, and from dry to wet spells and vice versa. Although the ADI was the best DI in defining historical droughts amongst the studied DIs, it was found during the literature review that the linear Principal Component analysis (PCA) was used in the ADI development which assumes that the input variables have linear relationships between them (Monahan, 2000, 2001; Linting *et al.*, 2007). If nonlinear relationships exist between these variables, as is often the case with environmental data (Gauch, 1982), then the linear PCA technique may not perform well or correctly (Linting *et al.*, 2007). Therefore, the development of a new generic DI by overcoming the limitation of the ADI (i.e., use of linear PCA technique) was considered in this study.

7.1.3. Review of Drought Forecasting Modeling Techniques

Several drought forecasting modeling techniques have been used in the past for forecasting future drought conditions. A review on the existing drought forecasting techniques was conducted in this study to understand and select the appropriate drought forecasting modeling technique for use in the Yarra River catchment. It was found that there were two types of drought forecasting models that have been used around the world; (1) the deterministic forecast models such as Autoregressive Integrated Moving Average (ARIMA) models, Seasonal Autoregressive Integrated Moving Average (SARIMA) models, Artificial Neural Network (ANN) models and Adaptive Neuro-Fuzzy Inference System (ANFIS) models, where they were used to forecast DI values (at the current time step) for the future time steps, and (2) the probability based drought class transition forecast models such as Markov Chain and Loglinear models where they were used to estimate the probability of drought class transition from one state to another for future time steps at the current time step.

Amongst all the available drought forecasting modeling techniques, the ANN modeling approach was found to be the most widely used and successful drought forecasting modeling approach. It has shown great ability in modeling and forecasting nonlinear and non-stationary time series, due to its innate nonlinear properties and flexibility for modeling (ASCE Task Committee on Application of Artificial Neural Networks in Hydrology, 2000; Maier and Dandy, 2000; Maier *et al.*, 2010). The results of the past studies (e.g. Kim and Valdes, 2003; Morid *et al.*, 2007) have also shown that the ANNs were capable to forecast drought conditions for longer lead time steps (i.e. 6 months ahead) than other models. Therefore, the ANN modeling approach was selected for the development of the drought forecasting model in this study.

7.1.4. Development of Nonlinear Aggregated Drought Index

A generic DI, namely Nonlinear Aggregated Drought Index (NADI) was developed and evaluated for the Yarra River catchment. The aim of the development of NADI was to overcome a limitation of the ADI. The limitation of the ADI was the use of linear PCA for aggregating hydro-meteorological variables, which considers that hydro-meteorological variables have linear relationships among them, which is not often the case for environmental data. Nonlinear Principal Component Analysis (NLPCA) was introduced and used to aggregate the five hydro-meteorological variables (i.e. rainfall, potential evapotranspiration, streamflow, storage reservoir volume and soil moisture content) in this study to develop the NADI. The NADI time series was then developed for the Yarra River catchment and investigated to see how well NADI can define the historical drought records in Victoria. The results showed that the NADI time series successfully defined the past major historical droughts that occurred in Victoria in 1967-1968, 1972-1973, 1982-1983, 1997-1998, 2003-2004 and 2006 onwards. A comparative study between ADI and NADI was also conducted in defining historical droughts. It was found that the use of NLPCA technique in the NADI development captured more variance of the input data than the ADI. This implies that the fluctuations in hydro-meteorological variables were better represented by NADI than the ADI. It also implies that the variables used in the NADI have nonlinear relationship between them that was also considered in NLPCA. Although the NADI was developed and applied for the Yarra River catchment, it is a generic DI that can be applied to other catchments.

7.1.5. Development of Drought Forecasting Models

The NADI has proven to be the best DI in this study for defining drought conditions in the Yarra River catchment (Section 7.1.4). Therefore, the time series of NADI was used to develop the drought forecasting model in this research project to forecast the NADI values as the future drought conditions. To develop the drought forecasting model, the ANN modeling technique was used, as it was found to be the best suitable drought forecasting modeling technique (Section 7.1.3). Several drought forecasting models were developed with different combinations of the potential input variables using two types of ANN architectures namely Recursive Multi-Step Neural Network (RMSNN) and Direct Multi-Step Neural Network (DMSNN). The comparative performance evaluation of the developed models was then conducted to select the best drought forecasting models in both categories (i.e. RMSNN and DMSNN). The results showed that the best developed RMSNN and DMSNN drought forecasting models were capable of forecasting drought conditions reasonably well up to 6 months ahead forecasts, and the forecasts were statistically significant at 1% level. It was also found that both models show the same performance for 1-month lead time forecasts. However, the RMSNN model gave slightly better forecasts than the DMSNN model for lead times of 2 to 3 months, and the DMSNN model gave slightly better forecasts than the RMSNN model for forecast lead times of 4 to 6 months. Poor forecasts were observed beyond the forecast lead time of 6 months.

7.2. Limitations of the Study and Recommendations for Further Research

Based on the findings of this research project, several limitations of the present study were identified. In some cases, recommendations are suggested for future studies to alleviate these limitations. They are discussed below.

As was discussed in Section 7.1.1, the soil moisture data was not available as direct field measured data in this study, and therefore, a simple two-layer water budget model of Palmer (1965) was adapted to estimate the soil moisture content in the catchment. In this water budget model, several assumptions were made as discussed in

Section 3.5.4. They are: (1) evapotranspiration takes place at the potential rate from the surface layer until all available moisture in the layer is removed, and only then the moisture can be removed from the underlying layer of soil; (2) there is no recharge to the underlying layer until the surface layer is brought to field capacity, and the available water holding capacity of the soil in the underlying layer depends on the depth of the effective root zone and the soil characteristics in the area under study; and (3) the maximum water holding capacity in the surface layer and underlying layer throughout the Yarra River catchment was used as 670 mm (water) / 1000 mm (soil) under saturated conditions. The water budget model that was considered to model surface and ground water interaction is sufficient for this study. However, the surface and groundwater interaction system may not be a simple case, and therefore the application of a more complex water budget model which considers all necessary attributes of surface and groundwater interaction system may improve the quality of soil moisture data.

It should be noted that the NADI need to be developed for different catchments as hydro-meteorological variables varies from one catchment to other, and to analyze droughts for different catchments a comparative study using NADI with other drought indices (DIs) should be done before adopting the NADI approach of drought assessment.

In developing the drought forecasting models, the back propagation (BP) training algorithm was used to calibrate the models. The use of other training algorithms was not attempted in this study. However, the BP may sometime trap in local minima during model calibration (Siddique and Tokhi, 2001), and therefore the investigation of other available training algorithms such as Genetic Algorithm (GA) and Bayesian Markov Chain Monte Carlo (MCMC) simulation are recommended to test the robustness of parameters and the models.

Most of the forecasting models are complex by nature and involve many parameters and variables that cannot be determined precisely, and therefore forecasting always involves uncertainty. However, uncertainty analysis for any of the developed drought forecasting model was not attempted in this study. It is therefore recommended that an uncertainty analysis is conducted in the future for DI forecast.

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APPENDIX: Mean and Standard Deviation of Nonlinear Aggregated Drought Index

Overview; Zero Mean; Nonunit Standard Deviation;

A1. Overview

The NADI is computed from the standardized transformed hydro-meteorological data as was seen in Equation (5.10). The standardized data were considered for each of the 12 months separately, and therefore these standardized data for each month have a zero mean and a unit variance. The NADI similarly possesses a mean of zero, but generally has a nonunit standard deviation. These properties are described below.

A2. Zero Mean

Each column j of matrix **Q** (containing the transformed variables) is a column vector q_j of the standardized data of the j^{th} transformed hydro-meteorological variable. The sum of the column vector is zero over all n observations for a particular month as the NADI was computed for each of 12 months separately (Section 5.3):

$$\sum_{i=1}^{n} q_{ij} = 0 \tag{A.1}$$

The computation of a Principal Component (PC) for time i is the multiplication between a row of **Q** and the eigenvectors related to the first PC (PC1) (Equation (5.9)). Consider the calculation of the first two elements for PC1 (i.e., calculation of the PC values of a particular month of the first two years) (indicated as Y_1):

$$\begin{bmatrix} Y_{11} \\ Y_{21} \end{bmatrix} = \begin{bmatrix} q_{11} & q_{12} & q_{13} & q_{14} & q_{15} \\ q_{21} & q_{22} & q_{23} & q_{2} & q_{25} \end{bmatrix} \begin{bmatrix} e_{11} \\ e_{21} \\ e_{31} \\ e_{41} \\ e_{51} \end{bmatrix} \tag{A.2}$$

where, Y_{11} and Y_{21} are the PC1 values for the particular month for years 1 and 2 respectively, q_{ij} are the transformed hydro-meteorological variables; i = 1, 2 representing years 1 and 2 respectively, and j = 1 to 5 representing 5 individual variables, e_{j1} are the eigenvectors related to the PC1.

$$Y_{11} = q_{11}e_{11} + q_{12}e_{21} + q_{13}e_{31} + q_{14}e_{41} + q_{15}e_{51} \tag{A.3}$$

$$Y_{21} = q_{21}e_{11} + q_{22}e_{21} + q_{23}e_{31} + q_{24}e_{41} + q_{25}e_{51} \tag{A.4}$$

Sum the Equations (A.3) and (A.4):

$$\begin{aligned} Y_{11} + Y_{21} = e_{11}\left(q_{11} + q_{21}\right) + e_{21}\left(q_{12} + q_{22}\right) + e_{31}\left(q_{13} + q_{23}\right) \\ + e_{41}\left(q_{14} + q_{24}\right) + e_{51}\left(q_{15} + q_{25}\right) \end{aligned} \tag{A.5}$$

By induction, extend the pattern to include all n elements of Y_I:

$$\sum_{i=1}^{n} Y_{i1} = \sum_{j=1}^{5} e_{j1} \sum_{i=1}^{n} q_{ij} \tag{A.6}$$

The $\sum_{i=1}^{n} q_{ij}$ term is given in Equation (A.1) as zero. Therefore

$$\sum_{i=1}^{n} Y_{i1} = 0 \tag{A.7}$$

As was mentioned in Section 5.3.2, the NADI values calculated for a particular month using Equation (A.8).

$$NADI_i = \frac{Y_{i1}}{\sigma} \tag{A.8}$$

where, $NADI_i$ is the NADI value for a particular month in year i, Y_{i1} is the PC1 during year i for that month and σ is the standard deviation of Y_{i1} over all years for the same month.

Therefore, the mean of the NADI (i.e., $\overline{NADI_i}$) for a particular month (i.e., mean over all n years) is

$$\overline{NADI_i} = \frac{\frac{1}{\sigma}\sum_{i=1}^{n} y_{i1}}{n} \tag{A.9}$$

The numerator was given by Equation (A.7) as zero. Therefore the mean of the NADI for a particular month is zero:

$$\overline{NADI_i} = 0 \tag{A.10}$$

A3. Nonunit Standard Deviation

Consider the NADI sample standard deviation for a particular month, σ :

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}\left(NADI_i - \overline{NADI_i}\right)^2}{n-1}} \tag{A.11}$$

Equation (A.10) states $\overline{NADI_i} = 0$.

For σ to be 1,

$$\sum_{i=1}^{n} NADI_i^2 = n - 1$$

This is possible, but unlikely. Therefore, σ is generally not 1.