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A WEIGHTED VERSION OF OSTROWSKI INEQUALITY FOR MAPPINGS OF HÖLDER TYPE AND APPLICATIONS IN NUMERICAL ANALYSIS

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Abstract. In this paper we establish a weighted version of Ostrowski inequality for mappings of Hölder type and apply it in Numerical Integration. Some Examples for the most popular weights: Legendre, Logarithm, Jacobi, Chebyshev, Laguerre and Hermite are also given.

1 Introduction

In 1938, A. Ostrowski proved the following integral inequality [2, p. 468]

Theorem 1.1. *Let $f:[a,b]\to\mathbf{R}$ be continuous on $[a,b]$ and differentiable on (a,b) and whose derivative $f':(a,b)\to\mathbf{R}$ is bounded on (a,b), i.e., $\|f'\|_\infty:=\sup_{t\in(a,b)}|f'(t)|<\infty$. Then*

$$\left|f(x)-\frac{1}{b-a}\int_a^b f(t)\,dt\right|\le\left[\frac{1}{4}+\frac{\left(x-\frac{a+b}{2}\right)^2}{(b-a)^2}\right](b-a)\left\|f'\right\|_\infty \tag{1.1}$$

for all $x\in[a,b]$.

The constant $\frac{1}{4}$ is the best possible.

For some applications of Ostrowski's inequality to certain numerical quadrature rules, we refer to the recent paper [1] by S.S. Dragomir and S. Wang.

In this paper we establish a weighted version of Ostrowski inequality for mappings of $r-H$-Hölder type and apply it in Numerical Integration.

Some examples for the most popular weights: Legendre, Logarithm, Jacobi, Chebyshev, Laguerre and Hermite are also given.

For other results in connection to Ostrowski inequality, the reader is advised to consult [1-11].

2 The Results

The following theorem holds:

Theorem 2.1. *Let $f,w:(a,b)\subseteq\mathbf{R}\to\mathbf{R}$ be so that $w(s)\ge 0$ on (a,b), w is integrable on (a,b) and $\int_a^b w(s)\,ds>0$, f is of $r-H$-Hölder type, i.e.,*

$$|f(x)-f(y)|\le H|x-y|^r \qquad \text{for all } x,y\in(a,b) \tag{2.1}$$

where $H>0$ and $r\in(0,1]$ are given. If $wf\in L_1(a,b)$, then we have the inequality:

$$\left|f(x)-\frac{1}{\int_a^b w(s)\,ds}\int_a^b w(s)f(s)\,ds\right|\le H\cdot\frac{1}{\int_a^b w(s)\,ds}\int_a^b|x-s|^r w(s)\,ds \tag{2.2}$$

for all $x\in(a,b)$.

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The constant factor $C = 1$ in the right hand side of the inequality can not be replaced by a smaller one.

Proof. As f is of $r - H-$Hölder type, we can state that

$$|f(x) - f(s)| \leq H |x - s|^r \qquad \text{for all } x, s \in (a, b). \tag{2.3}$$

Multiplying by $w(s) \geq 0$ and integrating over s on $[a, b]$, we get

$$\int_a^b |f(x) - f(s)| w(s) ds \leq H \int_a^b |x - s|^s w(s) ds \tag{2.4}$$

for all $x \in (a, b)$.

On the other hand, by the integral's properties, we have

$$\int_a^b |f(x) - f(s)| w(s) ds \geq \left| \int_a^b (f(x) - f(s)) w(s) ds \right| \tag{2.5}$$

$$= \left| f(x) \int_a^b w(s) ds - \int_a^b f(s) w(s) ds \right|.$$

Now, using (2.4) and (2.5), we get the desired inequality (2.2).

To prove that the constant factor $C = 1$ is sharp, let us assume that (2.2) holds with a constant $C > 0$, i.e.,

$$\left| f(x) - \frac{1}{\int_a^b w(s) ds} \int_a^b w(s) f(s) ds \right| \leq H \cdot \frac{C}{\int_a^b w(s) ds} \int_a^b |x - s|^r w(s) ds \tag{2.6}$$

for all $x \in (a, b)$.

Consider the mapping $f_0 : [0, 1] \to \mathbf{R}, f_0 = x^r, r \in (0, 1]$. Then

$$|f_0(x) - f_0(y)| = |x^r - y^r| \leq |x - y|^r,$$

for all $x, y \in [0, 1]$, which shows that f_0 is of $r-$Hölder type with the constant $H = 1$.

Writing the inequality (2.6) for f_0, we get

$$\left| \int_0^1 (x^r - s^r) w(s) ds \right| \leq C \int_0^1 |x - s|^r w(s) ds \tag{2.7}$$

for all $x \in [0, 1]$ and w as above. Letting $x = 0$ in (2.7), we deduce $C \geq 1$, which proves the sharpness of the constant. ∎

Remark 2.1. *If $r = 1$, i.e., the mapping f is Lipschitzian with, let us say, the constant $L > 0$, then we get*

$$\left| f(x) - \frac{1}{\int_a^b w(s) ds} \int_a^b w(s) f(s) ds \right| \leq \frac{L}{\int_a^b w(s) ds} \int_a^b |x - s| w(s) ds. \tag{2.8}$$

Now, if in (2.8) we assume that the weight function $w(t) = 1$, then we get Ostrowski's inequality for Lipschitzian mappings (see also [12]):

$$\left| f(x) - \frac{1}{b - a} \int_a^b f(s) ds \right| \leq \left[\frac{1}{4} + \frac{\left(x - \frac{a+b}{2}\right)^2}{(b - a)^2} \right] L(b - a), \quad x \in [a, b]. \tag{2.9}$$

The proof is obvious by (2.8) taking into account the fact that

$$\begin{aligned}\frac{1}{b-a}\int_a^b |x-s|\,ds &= \frac{1}{b-a}\left[\int_a^x (x-s)\,ds + \int_x^b (s-x)\,ds\right] \\ &= \frac{1}{b-a}\frac{(x-a)^2+(b-a)^2}{2} \\ &= \left[\frac{1}{4}+\frac{\left(x-\frac{a+b}{2}\right)^2}{(b-a)^2}\right](b-a)\,.\end{aligned}$$

Remark 2.2. *If the mapping f is differentiable on (a,b) and whose derivative f' is bounded on (a,b), i.e., $\|f'\|_\infty := \sup_{t\in(a,b)} |f'(t)| < \infty$, then instead of L in (2.8) we can put $\|f'\|_\infty$.*

The following corollary, which provides an Ostrowski type inequality for mappings of Hölder type holds.

Corollary 2.2. *Let $f : [a,b] \to \mathbf{R}$ be a mapping of $r-H-$Hölder type. Then we have the inequality*

$$\left|f(x) - \frac{1}{b-a}\int_a^b f(t)\,dt\right| \tag{2.9'}$$

$$\begin{aligned}&\le H\frac{1}{r+1}\left[\left(\frac{x-a}{b-a}\right)^{r+1}+\left(\frac{b-x}{b-a}\right)^{r+1}\right](b-a)^r \\ &\le \frac{H}{r+1}(b-a)^r\end{aligned}$$

The constant factor $C=1$ in the right hand side of the inequality can not be replaced by a smaller one.

Proof. Put $w(s)=1$ in (2.2) to get, in the right hand side, that

$$\begin{aligned}\frac{1}{b-a}\int_a^b |x-s|^s\,ds &= \frac{1}{b-a}\left[\int_a^x (x-s)^r\,ds + \int_x^b (s-x)^r\,ds\right] \\ &= \frac{1}{(b-a)}\left[\frac{(x-a)^{r+1}+(b-x)^{r+1}}{r+1}\right]\end{aligned}$$

and the inequality $(2.9')$ is proved. ∎

We give now some corollaries for the most popular weight functions.

2.1 Logarithm

Corollary 2.3. *Let $f : (0,1) \to \mathbf{R}$ be a differentiable mapping whose derivative is bounded and for which the integral $\int_0^1 \ln\left(\frac{1}{t}\right) f(t)\,dt$ is finite. Then*

$$\left|f(x) - \int_0^1 \ln\left(\frac{1}{t}\right) f(t)\,dt\right| \le \left[\frac{1}{4} - x + x^2\left(\frac{3}{2}-\ln x\right)\right]\|f'\|_\infty \tag{2.10}$$

for all $x \in (0,1)$.

Proof. We apply (2.8) for $L = \|f'\|_\infty, a = 0, b = 1, w(t) = \ln\left(\frac{1}{t}\right)$.
We have

$$\int_0^1 \ln\left(\frac{1}{t}\right) dt = 1,$$

$$\begin{aligned}\int_0^1 |x-s| \ln\left(\frac{1}{s}\right) ds &= \int_0^x (s-x) \ln s ds + \int_x^1 (x-s) \ln s ds \\ &= x^2\left(\frac{3}{2} - \ln x\right) - x + \frac{1}{4}\end{aligned}$$

for all $x \in (0,1)$, and then (2.10) is obtained. ∎

Remark 2.3. *If $I(x) = x^2\left(\frac{3}{2} - \ln x\right) - x + \frac{1}{4}$, then $I'(x) = 2x(1-\ln x) - 1$, which shows that I has its minimum on $(0,1)$ at the point $x_{\min} \approx 0.1866823$. At this point $I(x_{\min}) \approx 0.1740840$.*

Consequently, the best inequality we can get from (2.8) is

$$\left| f(0.1866823) - \int_0^1 \ln\left(\frac{1}{t}\right) f(t)\, dt \right| \le 0.1740840 \left\|f'\right\|_\infty. \tag{2.11}$$

2.2 Jacobi

Corollary 2.4. *Let $f : (0,1) \to \mathbf{R}$ be a differentiable mapping whose derivative is bounded and for which the integral $\int_0^1 \frac{f(t)}{\sqrt{t}} dt$ is finite.*
Then

$$\left| f(x) - \frac{1}{2}\int_0^1 \frac{f(t)}{\sqrt{t}} dt \right| \le \frac{1}{6}\left(8x^{3/2} - 6x + 2\right)\left\|f'\right\|_\infty \tag{2.12}$$

for all $x \in (0,1)$.

Proof. We apply (2.8) for $L = \|f'\|_\infty, a = 0, b = 1, w(t) = \frac{1}{\sqrt{t}}$. We have

$$\int_0^1 \frac{dt}{\sqrt{t}} = 2,$$

$$\int_0^1 \frac{|x-s|}{\sqrt{s}} ds = \frac{1}{3}\left(8x^{3/2} - 6x + 2\right)$$

for all $x \in (0,1)$, and then (2.12) is obtained. ∎

Remark 2.4. *If $J(x) := \frac{1}{6}\left(8x^{3/2} - 6x + 2\right)$, then $J'(x) = 2\sqrt{x} - 1$, which shows that J has its minimum on $(0,1)$ at the point $x_{\min} = \frac{1}{4}$. $J(x_{\min}) = \frac{1}{4}$ and then, the best inequality we can get from (2.12) is*

$$\left| f\left(\frac{1}{4}\right) - \frac{1}{2}\int_0^1 \frac{f(t)}{\sqrt{t}} dt \right| \le \frac{1}{4}\left\|f'\right\|_\infty. \tag{2.13}$$

2.3 Chebyshev

Corollary 2.5. *Let $f:(-1,1)\to\mathbf{R}$ be a differentiable mapping whose derivative is bounded and for which the integral $\int_0^1 \frac{f(t)}{\sqrt{1-t^2}}dt$ is finite.*
Then

$$\left|f(x)-\frac{1}{\pi}\int_{-1}^{1}\frac{f(t)}{\sqrt{1-t^2}}dt\right|\le\frac{2}{\pi}\left(x\arcsin x+\sqrt{1-x^2}\right)\|f'\|_\infty \tag{2.14}$$

for all $x\in(-1,1)$.

Proof. We apply (2.8) for $L=\|f'\|_\infty, a=-1, b=1, w(t)=\frac{1}{\sqrt{1-t^2}}$. We have

$$\int_{-1}^{1}\frac{dt}{\sqrt{1-t^2}}=\pi,$$

$$\int_{-1}^{1}\frac{|x-s|}{\sqrt{1-s^2}}ds=2\left(x\arcsin x+\sqrt{1-x^2}\right)$$

for all $x\in(-1,1)$, and then (2.14) is obtained. ∎

Remark 2.5. *If $K(x):=\frac{2}{\pi}\left(x\arcsin x+\sqrt{1-x^2}\right)$, then $K'(x)=\frac{2}{\pi}\arcsin x$, which shows that K has its minimum on $(-1,1)$ at the point $x_{\min}=0$. $K(x_{\min})=\frac{2}{\pi}$ and then, the best inequality we can get from (2.14) is*

$$\left|f(0)-\frac{1}{\pi}\int_{-1}^{1}\frac{f(t)}{\sqrt{1-t^2}}dt\right|\le\frac{2}{\pi}\|f'\|_\infty. \tag{2.15}$$

2.4 Laguerre

Corollary 2.6. *Let $f:[0,\infty)\to\mathbf{R}$ be a differentiable mapping whose derivative is bounded and for which the integral $\int_0^\infty e^{-t}f(t)\,dt$ is finite. Then*

$$\left|f(x)-\int_0^\infty e^{-t}f(t)\,dt\right|\le\left(2e^{-x}+x-1\right)\|f'\|_\infty \tag{2.16}$$

for all $x\in[0,\infty)$.

Proof. We apply (2.8) for $L=\|f'\|_\infty, a=0, b=+\infty, w(t)=e^{-t}$.
We have

$$\int_0^\infty e^{-t}dt=1$$

$$\int_0^\infty |x-s|\,e^{-s}ds=2e^{-x}+x-1$$

for all $x\in[0,\infty)$, and then (2.16) is obtained. ∎

Remark 2.6. *If $L(x):=2e^{-x}+x-1$, then $L'(x)=-2e^{-x}+1$ which shows that the mapping L has its minimum on $(0,\infty)$ at the point $x_{\min}=\ln 2$. $L(x_{\min})=\ln 2$ and then the best inequality we can get from (2.16) is*

$$\left|f(\ln 2)-\int_0^\infty e^{-t}f(t)\,dt\right|\le\|f'\|_\infty\ln 2. \tag{2.17}$$

2.5 Hermite

Corollary 2.7. *Let $f : \mathbf{R} \to \mathbf{R}$ be a differentiable mapping whose derivative is bounded and for which the integral $\int_{-\infty}^{\infty} e^{-t^2} f(t)\,dt$ is finite. Then*

$$\left| f(x) - \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-t^2} f(t)\,dt \right| \le \frac{1}{\sqrt{\pi}} \left[e^{-x^2} + \sqrt{\pi} x \operatorname{erf}(x) \right] \|f'\|_\infty \tag{2.18}$$

for all $x \in \mathbf{R}$.

Proof. We apply (2.8) for $L = \|f'\|_\infty, a = -\infty, b = +\infty, w(t) = e^{-t^2}$. We know

$$\int_{-\infty}^{\infty} e^{-t^2} dt = \sqrt{\pi},$$

$$\int_{-\infty}^{\infty} |x - s| e^{-s^2} ds = e^{-x^2} + \sqrt{\pi} x \operatorname{erf}(x)$$

for all $x \in \mathbf{R}$, and then (2.18) is obtained. ∎

Remark 2.7. *If $M(x) = \frac{1}{\sqrt{\pi}} \left(e^{-x^2} + \sqrt{\pi} x \operatorname{erf}(x) \right)$, then $M'(x) = \operatorname{erf}(x)$ which shows that M has its minimum at $x_{\min} = 0$ and $M(0) = \frac{1}{\sqrt{\pi}}$. Consequently, the best inequality we can get from (2.18) is*

$$\left| f(0) - \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-t^2} f(t)\,dt \right| \le \frac{1}{\sqrt{\pi}} \|f'\|_\infty . \tag{2.19}$$

3 Applications in Numerical Integration

Let $I_n : a = x_0 < x_1 < ... < x_{n-1} < x_n = b$ be a division of $[a, b]$ and $\xi_i \in [x_i, x_{i+1}]$ $(i = 0, ..., n-1)$ intermediate points. Let $f, w : [a, b] \to \mathbf{R}$ and define the sum

$$A(f, w, I_n, \xi) := \sum_{i=0}^{n-1} f(\xi_i) \int_{x_i}^{x_{i+1}} w(s)\,ds.$$

The following result holds:

Theorem 3.1. *Let f and w be as in Theorem 2.1. Then we have the following quadrature rule:*

$$\int_a^b f(s) w(s)\,ds = A(f, w, I_n, \xi) + R(f, w, I_n, \xi) \tag{3.1}$$

where $A(f, w, I_n, \xi)$ is given above and the remainder satisfies the estimate:

$$|R(f, w, I_n, \xi)| \le H \sum_{i=0}^{n-1} \int_{x_i}^{x_{i+1}} |\xi_i - s|^r w(s)\,ds \tag{3.2}$$

$$\le \frac{H}{2^r} \sum_{i=0}^{n-1} h_i^r \int_{x_i}^{x_{i+1}} w(s)\,ds \le \frac{H}{2^r} [\nu(h)^r] \int_a^b w(s)\,ds,$$

where $h_i := x_{i+1} - x_i$ and $\nu(h) = \max_{i=\overline{0,n-1}} h_i$.

Proof. We apply the inequality (2.2) on the interval $[x_i, x_{i+1}]$ $(i = 0, ..., n-1)$ to get

$$\left| f(\xi_i) \int_{x_i}^{x_{i+1}} w(s)\,ds - \int_{x_i}^{x_{i+1}} w(s) f(s)\,ds \right| \leq H \int_{x_i}^{x_{i+1}} |\xi_i - s|^r w(s)\,ds.$$

Summing over i from 0 to $n-1$ and using the generalized triangle inequality, we get the first part of (3.2).

The last part follows by the fact that

$$|\xi_i - s| \leq \frac{h_i}{2} \leq \frac{\nu(h)}{2}, \qquad i = 0, ..., n-1$$

and we omit the details. ∎

Suppose that the integral $\int_0^1 \frac{f(t)}{\sqrt{t}}dt$ is to be approximated. Let $\|f'\|_\infty := \sup_{t\in(0,1)} |f'(t)|$ and assume that $f' : (0,1) \to \mathbf{R}$ is bounded. If $I_n : 0 = x_0 < x_1 < ... < x_{n-1} < x_n = 1$ is a division of the interval $[0,1]$ and $\xi_i \in [x_i, x_{i+1}]$ are intermediate points, then

$$A\left(f, \frac{1}{\sqrt{\cdot}}, I_n, \xi\right) = 2\sum_{i=0}^{n-1} f(\xi_i)\left(\sqrt{x_{i+1}} - \sqrt{x_i}\right);$$

and

$$\begin{aligned}
\int_{x_i}^{x_{i+1}} |\xi_i - s|\, s^{-\frac{1}{2}} ds &= \int_{x_i}^{\xi_i} (\xi_i - s)\, s^{-\frac{1}{2}} ds + \int_{\xi_i}^{x_{i+1}} (s - \xi_i)\, s^{-\frac{1}{2}} ds \\
&= 2\left(\sqrt{\xi_i} - \sqrt{x_i}\right)\left[\xi_i - \frac{1}{3}\left(\xi_i + \sqrt{\xi_i \cdot x_i} + x_i\right)\right] \\
&\quad + 2\left(\sqrt{x_{i+1}} - \sqrt{\xi_i}\right)\left[\frac{1}{3}\left(x_{i+1} + \sqrt{\xi_i \cdot x_{i+1}} + \xi_i\right) - \xi_i\right] \\
&=: \delta(h_i, \xi_i);
\end{aligned}$$

and

$$\int_{x_i}^{x_{i+1}} w(s)\,ds = 2\left(\sqrt{x_{i+1}} - \sqrt{x_i}\right).$$

Consequently, we can approximate the integral $\int_0^1 \frac{f(t)}{\sqrt{t}}dt$ by

$$A\left(f, \frac{1}{\sqrt{\cdot}}, I_n, \xi\right) = 2\sum_{i=0}^{n-1} f(\xi_i)\left(\sqrt{x_{i+1}} - \sqrt{x_i}\right)$$

having an error $R\left(f, \frac{1}{\sqrt{\cdot}}, I_n, \xi\right)$ which satisfies the bound:

$$\left| R\left(f, \frac{1}{\sqrt{\cdot}}, I_n, \xi\right) \right| \leq \|f'\|_\infty \sum_{i=0}^{n-1} \delta(h_i, \xi_i) \tag{3.3}$$

$$\leq \|f'\|_\infty \sum_{i=0}^{n-1} h_i\left(\sqrt{x_{i+1}} - \sqrt{x_i}\right) \leq \|f'\|_\infty \nu(h).$$

Consider the integral $\int_{-1}^1 \frac{f(t)}{\sqrt{1-t^2}}dt$ is to be approximated and assume that $f' : (-1,1) \to \mathbf{R}$ is bounded and $\|f'\|_\infty := \sup_{t\in(-1,1)} |f'(t)|$.

If $I_n : -1 = x_0 < x_1 < ... < x_{n-1} < x_n = 1$ is a division of the interval $[-1, 1]$ and $\xi_i \in [x_i, x_{i+1}]$ are intermediate points, then

$$A\left(f, \frac{1}{\sqrt{1-(\cdot)^2}}, I_n, \xi\right) = \sum_{i=0}^{n-1} f(\xi_i)(\arcsin x_{i+1} - \arcsin x_i),$$

and

$$\int_{x_i}^{x_{i+1}} |\xi_i - s| \frac{1}{\sqrt{1-s^2}} ds$$

$$\begin{aligned}
&= \int_{x_i}^{\xi_i} \frac{\xi_i - s}{\sqrt{1-s^2}} ds + \int_{\xi_i}^{x_{i+1}} \frac{s - \xi_i}{\sqrt{1-s^2}} ds \\
&= \xi_i \left[\arcsin s|_{x_i}^{\xi_i}\right] + \frac{1}{2} \frac{\left(1-s^2\right)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\Bigg|_{x_i}^{\xi_i} \\
&- \frac{1}{2} \frac{\left(1-s^2\right)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\Bigg|_{\xi_i}^{x_{i+1}} - \xi_i \left[\arcsin s|_{\xi_i}^{x_i}\right]
\end{aligned}$$

$$= 2\xi_i \left(\arcsin \xi_i - \frac{\arcsin x_i + \arcsin x_{i+1}}{2}\right)$$

$$+2\left(\sqrt{1-\xi_i^2} - \frac{\sqrt{1-x_i^2} + \sqrt{1-x_{i+1}^2}}{2}\right) =: \beta(h_i, \xi_i)$$

and

$$\int_{x_i}^{x_{i+1}} w(s)\, ds = \arcsin x_{i+1} - \arcsin x_i.$$

Consequently, we can approximate the integral $\int_{-1}^{1} \frac{f(t)dt}{\sqrt{1-t^2}} dt$ by

$$A\left(f, \frac{1}{\sqrt{1-(\cdot)^2}}, I_n, \xi\right) = \sum_{i=0}^{n-1} f(\xi_i)(\arcsin x_{i+1} - \arcsin x_i)$$

having an error $R\left(f, \frac{1}{\sqrt{1-(\cdot)^2}}, I_n, \xi\right)$ which satisfies the bound

$$\begin{aligned}
\left|R\left(f, \frac{1}{\sqrt{1-(\cdot)^2}}, I_n, \xi\right)\right| &\le \|f'\|_\infty \sum_{i=0}^{n-1} \beta(h_i, \xi_i) \\
&\le \frac{\|f'\|_\infty}{2} \sum_{i=0}^{n-1} h_i (\arcsin x_{i+1} - \arcsin x_i) \\
&\le \frac{\|f'\|_\infty \pi}{2} \nu(h).
\end{aligned}$$

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