EFFECTIVENESS OF USING

STRUCTURED AIDS

IN LEARNING NUMBER CONCEPTS

FOR MIDDLE PRIMARY SCHOOL CHILDREN

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This thesis is submitted

to fulfill the requirements of the degree

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DEDICATION

This work is dedicated to children and teachers in their search for the truth and beauty and order of mathematics.

Special thanks are due to my parents who have aided me on my journey and also to my dear wife Mary-Rose and our family who have been supportive at all times.

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Statement of Sources

I declare that the work presented in this thesis is to the best of my knowledge and belief original, except as acknowledged in the text and that the material has not been submitted in whole or in part for a degree at this or any other university.

Abstract

This study set out to evaluate the effectiveness of using *Blockaid* material for the teaching and learning of number concepts and of operations in middle primary school.

Blockaid is base 10 material designed to model number in multiple representations. The different representations facilitate the modelling of procedures which follow the same steps of the formal algorithms of addition, subtraction, multiplication and division.

The hypothesis is that the external connections formed through the manipulation of structured materials aids the construction of internal cognitive networks by the learner in the building of conceptual knowledge. Knowledge formed in this way is rich in relationships and leads to better understanding of number and operational concepts.

While the results of the study indicate that the procedures with *Blockaid* improved performance only with subtraction, more than the traditional direct instruction approach, the implications are that with improved modelling better procedures can be devised using structured materials to improve teaching and learning of all operations.

Ongoing research is needed the evaluate the benefits of using of multi-representational materials, rather than single unit structures, to represent the base 10 number system. Further research should focus on establishing better procedures for modelling the operations of multiplication and division. These operations seem to involve higher levels of processing load than addition or subtraction.

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CHAPTER ONE

INTRODUCTION

This study was driven by a passion to seek better ways to facilitate the effective teaching and learning of mathematical concepts in primary schools. The thesis is on mathematics education, in the field of children's cognition. It aimed to establish through an experimental study, whether knowledge of facts and skills, and of even more importance, understanding in learning, can be improved significantly when pupils in middle primary school are exposed to a range of experiences using structured learning apparatus specifically designed by the researcher for modelling number concepts.

This apparatus, called *Blockaid*, utilises modelling procedures based on historic representational forms and contemporary models of number such as Dienes base 10 blocks. It was trialled and refined in the classroom over a period of about thirty years by the researcher. *Blockaid* is a complete teaching package consisting of multi-representational material for concept of number development together with teachers' manuals, pupil work cards and blackline masters (Carney & Barrow, 1993).

The approach planned utilised *Blockaid* and took full cognisance of the work by Montessori, Dienes, and Piaget. The research of Fuson, Hiebert and Carpenter relating to the effective use of structured materials for modelling base ten structures was of particular relevance to the study.

The need for this investigation was evidenced by the following:

1. Concern expressed in the media about lack of perceived number skills in primary school pupils. In the editorial in The Age newspaper on April 8th., 1994:

> "... the evidence, sketchy though it is, is that many students are slipping through the system without a functional grasp of the three basic skills: reading, writing and arithmetic."

2. Anxiety amongst parents, and teachers, worried about numeracy standards. From personal experience I have been asked on numerous occasions to speak to both

parents and teachers on this subject. The anxiety is often associated with fear about employment opportunities.

3. Demands for educational accountability and the recent trends in Australia to 'launch' head-long into state-wide testing programs in basic skill areas.

> "... beginning next year, South Australian students will be assessed for basic literacy and numeracy skills" (Painter, 1994, p. 3).
>
> In line with the Curriculum Frameworks to be introduced in Victoria in 1995, ... Last month the government announced details of its Learning Assessment Project in which young children will be assessed twice during their seven years of primary schooling" (MacLean, 1994, p. 10).

4. Debate in Australia, regarding national curricula for schools and, in particular, the need to design these in accord with sound educational principles based on proven research (Gaudry, 1994).

5. My own investigations regarding understanding of basic number concepts by children in Victorian schools, both private and state. These investigations over nearly thirty years, as a teacher and in the role of mathematics consultant, have revealed significant weaknesses in key areas of number concept development.

Aims of the Project.

In mathematics education, there is a major gulf between the implications drawn from educational and psychological research, and actual classroom practice found in most schools (Copeland, 1979). This study aimed to utilise and build on previous research to design instruction to narrow this gap. Such research gives insight into the manner in which mathematical knowledge is constructed and strategies for facilitating the acquisition of conceptual knowledge that Hiebert and Carpenter (1992) associate with understanding.

The thesis aimed to establish whether learning of number concepts, and of operations on number, can be improved significantly when pupils in middle primary school are exposed to a range of experiences using structured learning apparatus specifically designed for those particular purposes.

The basic need for concrete representations of number has a long history and is evident in the use of tallying devices from earliest times; in the evolving of a base ten number system that is structured on the number of fingers on the human species; in the abacus as the earliest computing machine; in the educational approaches advocated by Pestalozzi in the late eighteenth century, Froebel in the early nineteenth century, and more recently in the base ten structures devised by Montessori, Dienes and Carney (Carney & Barrow, 1993).

The specific objective focused on the problem whereby basic number concepts are fundamental prerequisites to higher order mathematical skills such as operations on number and fractional concepts. In this regard Dienes (1971a) was adamant that the concept of number is perhaps one of the most difficult and elusive concepts that we require young children to learn. Dienes (1971a), in elaborating the concept, identified three elements - the cardinal ordinal link, the idea of grouping and thirdly the concept of place value.

The above elements, constituting the concept of number, would seem to imply structure and it is this structure that was modelled in this study in an attempt to lead the pupils to internalise mathematical concepts through a guided discovery approach using concrete equipment. The study used models of the number system to demonstrate to children a system that orders relationships. While Dienes pointed out the complexities of this concept, he then established a hierarchy whereby the arithmetic operations on those number concepts are at higher levels of conceptualisation again. Attempts were made in the modelling to bring out this hierarchy and the interrelatedness of the operations of addition, subtraction, multiplication and division.

In attempting to facilitate the acquisition of these concepts by children, the learning process and resultant outcomes are categorised as either skills or understanding, or perhaps more appropriately, as procedural knowledge and conceptual knowledge. Hiebert and Carpenter's (1992) notion of procedural and conceptual knowledge and how they relate to understanding will be dealt with in Chapter 2.

In Dienes' stages of learning at the level of search for commonality of rules, he claims that the child makes the connections between the various models advocated in his 'multiple embodiment approach' and their representations (Dienes & Golding, 1971). The hypothesis of this thesis is that if the prerequisite models for concept building are varied and interconnected then the abstractions from the concrete models, and their alternative representations, are more likely to be clearly constructed by the child with connected networks that facilitate understanding.

The nature of the discourse that maximises the communication of mathematical knowledge between teacher and child is central to this study. The factors of teacher, child and mathematical knowledge itself however need to be examined in the light of current research and in the light of the movement towards national curriculum and national standards in education.

The study attempted to assess the effectiveness of the *Blockaid* apparatus and the methodology advocated in the teacher's manual in using miltivariant base ten structures and in utilising such structures in modelling the formal algorithms of addition, subtraction, multiplication and division. Assessment of effectiveness was for both achievement and understanding. The study achieved this aim. The strengths and weaknesses of the approach advocated will be discussed in Chapter 8.

Hopefully such studies will have ramifications not only for children in general but also for teachers, parents, and society at large.

CHAPTER TWO

LITERATURE REVIEW

Introduction.

This review attempts to glean from the literature clues that should give direction to this quest for creating the best possible conditions for mathematics learning for all students. The focus is on the use of structured aids for representation of number concepts, and the action on such representations corresponding to the basic operations in arithmetic.

The National Statement on Mathematics for Australian Schools suggested that:

> There is no definitive approach or style for the teaching of mathematics. The teaching of any particular mathematical concept will be influenced by the nature of the concept itself and by the abilities, attitudes and experiences of students. In general however, teaching should be informed by a thorough understanding of how learning occurs and of the nature of the mathematical activity (Australian Education Council, 1991, p. 18).

In his research investigating the effects of manipulative materials in mathematics instruction, Sowell (1989) obtained results that showed:

> ...mathematics achievement is increased through the long term use of concrete instructional materials and that students' attitudes towards mathematics are improved when they have instruction with concrete materials provided by teachers knowledgeable about their use (p. 498).

These statements underpin the importance of an understanding of the nature of teaching and learning.

Analysis of the literature will be addressed under four headings:

- Historical perspective for the use of structured aids for number ideas;
- Fundamentals of education;
- Concept of number and algorithms; and
- Current situation in Victorian mathematics education.

Emphasis in the first of the above sections will be on the work of Montessori, Hill and Dienes; the second section will focus largely on Piaget and Vygotsky; the third section will describe the work of Hiebert, Carpenter and Fuson; and the last section will touch on current curriculum statements.

1. Historical perspective for the use of structured aids for number ideas.

The manner by which early man constructed number systems may give insight into devising structured aids that allow young children to construct number concepts in permanent and meaningful ways. It was recognised by Sowell (1989) that the teaching of mathematics using manipulative materials has a long history. In the nineteenth century, Pestalozzi (Cubberley, 1948) advocated their use. Emphasis on using concrete materials was evident in primary schools in Victoria the 1960's.

As discussed below some of the main proponents advocating the use of such material for furthering mathematics learning in schools were Montessori, Hill and Dienes.

(a) Manipulative and structured aids.

Tallying devices. A range of methods for tallying, or recording numbers, have been found in different ancient civilizations: knots in string, collections of pebbles, notches in bone or wood (Groza, 1968). According to Barrow (1992) one of the oldest examples of a tallying device, dating back to Paleolithic times, was a tally stick found in Moravia. It was a piece of bone engraved with 57 deep notches, arranged in groups of five.

> This extraordinary relic dating from about 30,000 BC, displays systematic tallying and perhaps some appreciation of grouping in fives, inspired no doubt by the fingers on the hunters hand. Presumably the tally was a record of the hunter's kills (p. 31).

Abacus. The oldest computing machine known is the abacus. Around 450 BC, the Greek historian Herodotus wrote that the Egyptians calculated with pebbles. That pebbles were used for counters in Roman times is suggested by our word "calculate", which is derived from the Latin "calculus" which means "pebble" (Groza, 1968).

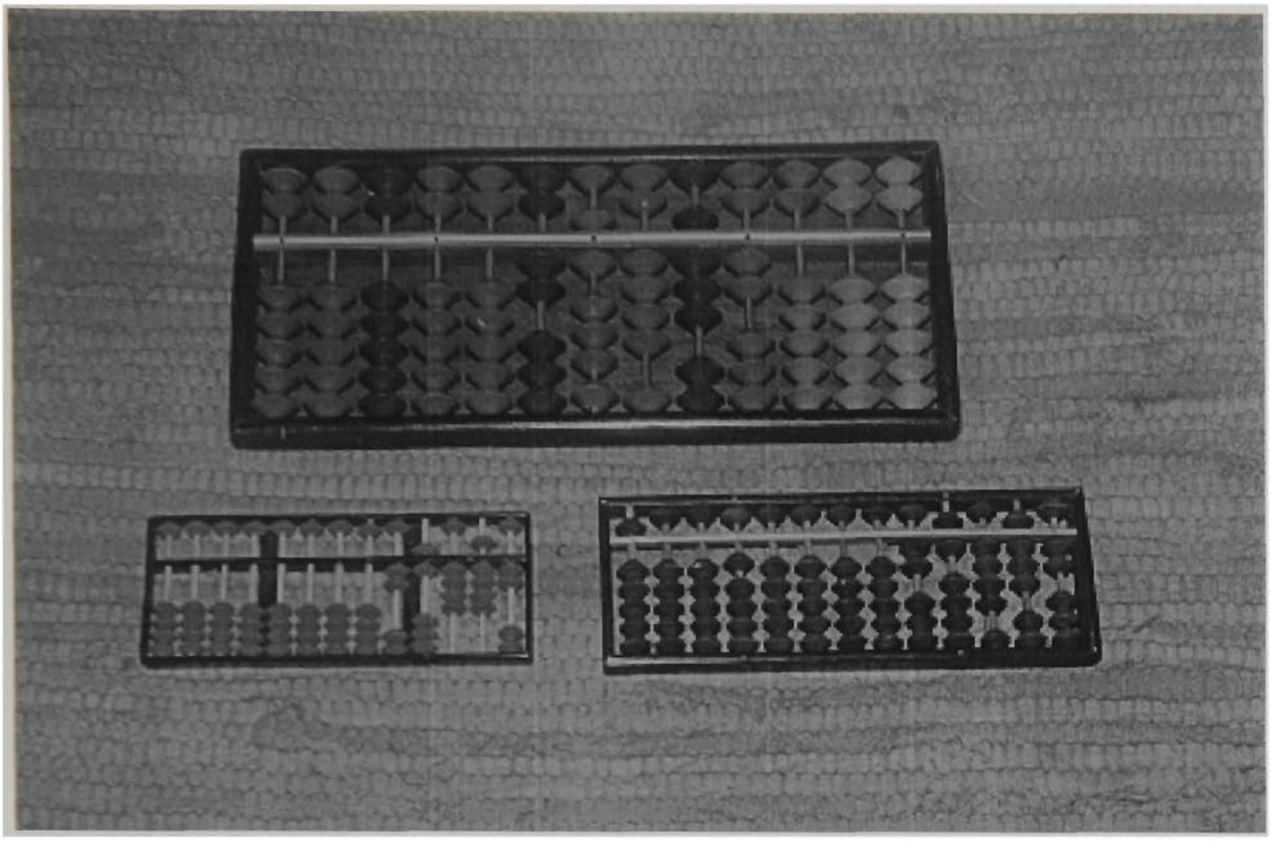

Photo 1. Variations of Chinese and Japanese abaci showing 26,482.

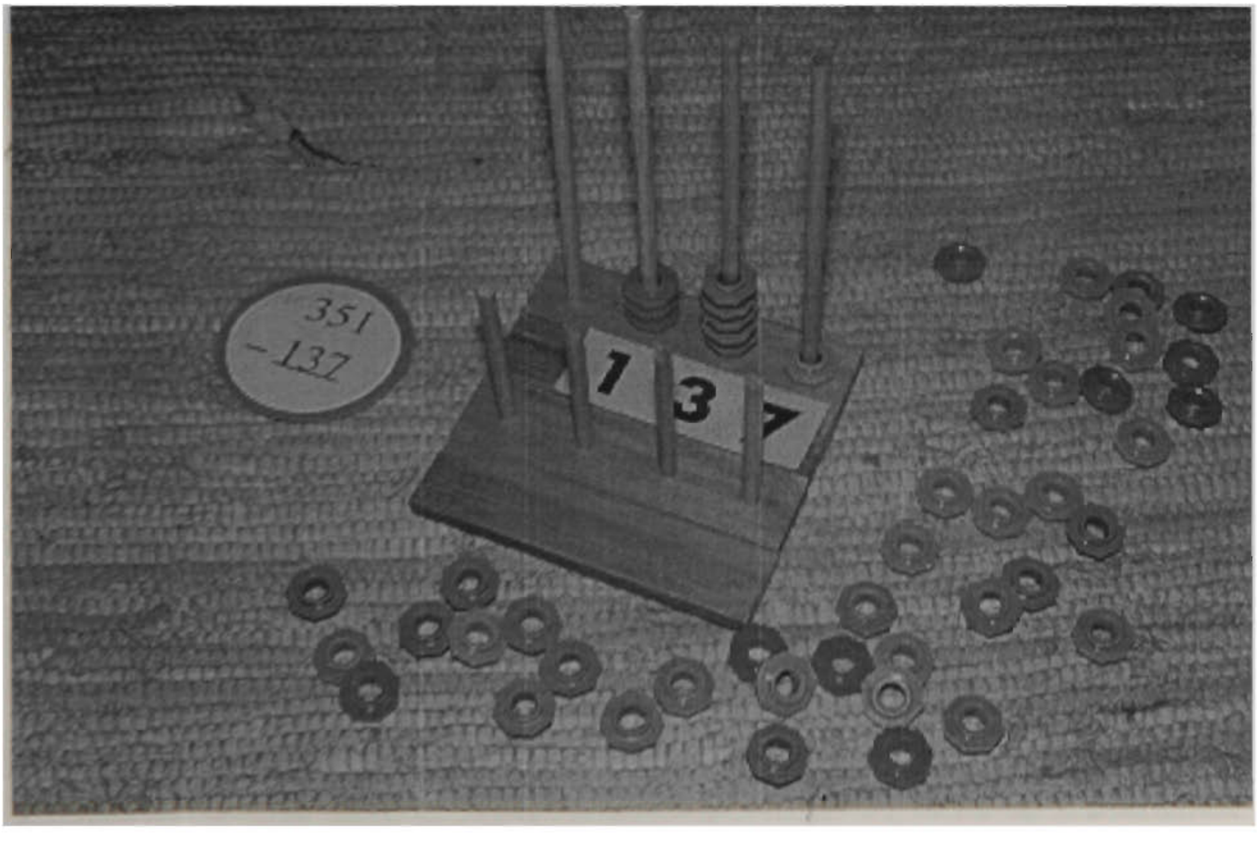

Photo 2. Spike abacus set out for subtraction problem 351 -137.

It is thought that the abacus was invented by the Greek mathematician, Pythagoras. While the Greek device required up to nine counters in each column the simplified Roman version utilised a sub-grouping of five, represented by a single button, which worked in an upper short groove at the top of each column on the counting board (Stephens, 1972). This sub-grouping structure, using a single button to represent a handful (five), is in direct relationship to the Roman system of notation that incorporates sub groupings of 5, 50, 500, ...symbolised by the letters V, L, and D, and so on. Although by convention the Roman symbols were written in order from those representing the largest groupings of the number to be represented first to the unit representation last, the early system of recording was categorised as a simple grouping numeral system (Groza, 1968).

The abacus is still used today in different forms: the Russians' choty, the Chinese suan pan (See Photo 1) and the Japanese soroban (Groza 1968). The Japanese abacus is a bead frame generally with twelve rods and a crosspiece. Each rod has five beads arranged with one above the crosspiece and four below. Beads above are worth five times the beads below, while from right to left the beads increase in powers of ten. For a number to be displayed the appropriate beads are moved to the crosspiece. This semi-abstract representational system links with the structure of the Chinese counting sticks and written numerals described below.

The Chinese abacus has an extra bead above and below the crosspiece in each column which acts as a memory bank, thus eliminating some of the recall which is necessary in the Japanese version when performing any of the arithmetic operations (Stephens, 1972).

Montessori's (1973) description of her abacus utilised ten beads on each wire and the wires are arranged horizontally instead of vertically. In order to display a particular number the appropriate beads are moved to the left of the frame. Resulting numbers such as 4729 are read from bottom to top. While the horizontal arrangement of the beads is similar to that of the counting boards used in Europe in the middle ages the numbers on the latter were, according to Stephens (1972), read from top to bottom.

Most of the abaci currently used in Victorian schools are either looped abacus or spike abacus varieties. The loop abacus allows the beads required to represent a number such as 275 to be looped over a board that hides the remaining counters from view.

Chinese stick numerals. According to Grozia (1968),

> . . . the Chinese mathematician, Sun-Tsu Suan-Ching in his treatise of 100 AD., wrote 'In making calculations we must first know positions of numbers. Unity is vertical and ten is horizontal, the hundreds stand while the thousand lies; and the thousand and the ten look equally, and so also the ten thousand and the hundred' (p. 50).

The system of stick placement and the accompanying symbolisation uses a sub-grouping of five in a similar way to such sub-grouping in the Chinese abacus called the suan pan. The digits one to nine are displayed with sticks or written as follows:

$\mid$	$\mid\mid$	$\mid\mid\mid$	$\mid\mid\mid\mid$	$\mid\mid\mid\mid\mid$	$\overline{\mid}$	$\overline{\mid\mid}$	$\overline{\mid\mid\mid}$	$\overline{\mid\mid\mid\mid}$
1	2	3	4	5	6	7	8	9

The written symbols that copy these stick positions, known as the Scientific Chinese Numerals, are a true place value or positional form of representation of number. The symbol "O" was used for zero.

Number rods. There have been various attempts to design sets of material to help children represent the numbers one to ten by means of number rods. These materials attempt to represent the magnitude of the numbers 1 to 10 by rods of different length. Although of varying designs, these consist of a set of rods that form a series that can represent multiples of the smallest rod. Montessori was perhaps the first to design such apparatus with her material being based on a unit piece of two centimetre square cross-section and one decimetre in length. The ten rod was thus represented by a stick one metre in length. Her material was generally called the long stair (Montessori, 1917).

With the Cuisenaire rods (Gattegno, 1960) the smallest piece was a cubic centimetre and the longest piece was ten centimetres in length. The Montessori coloured beads on wires pre-dated the multi coloured Stern (Sealey, 1961a) and Cuisenaire materials.

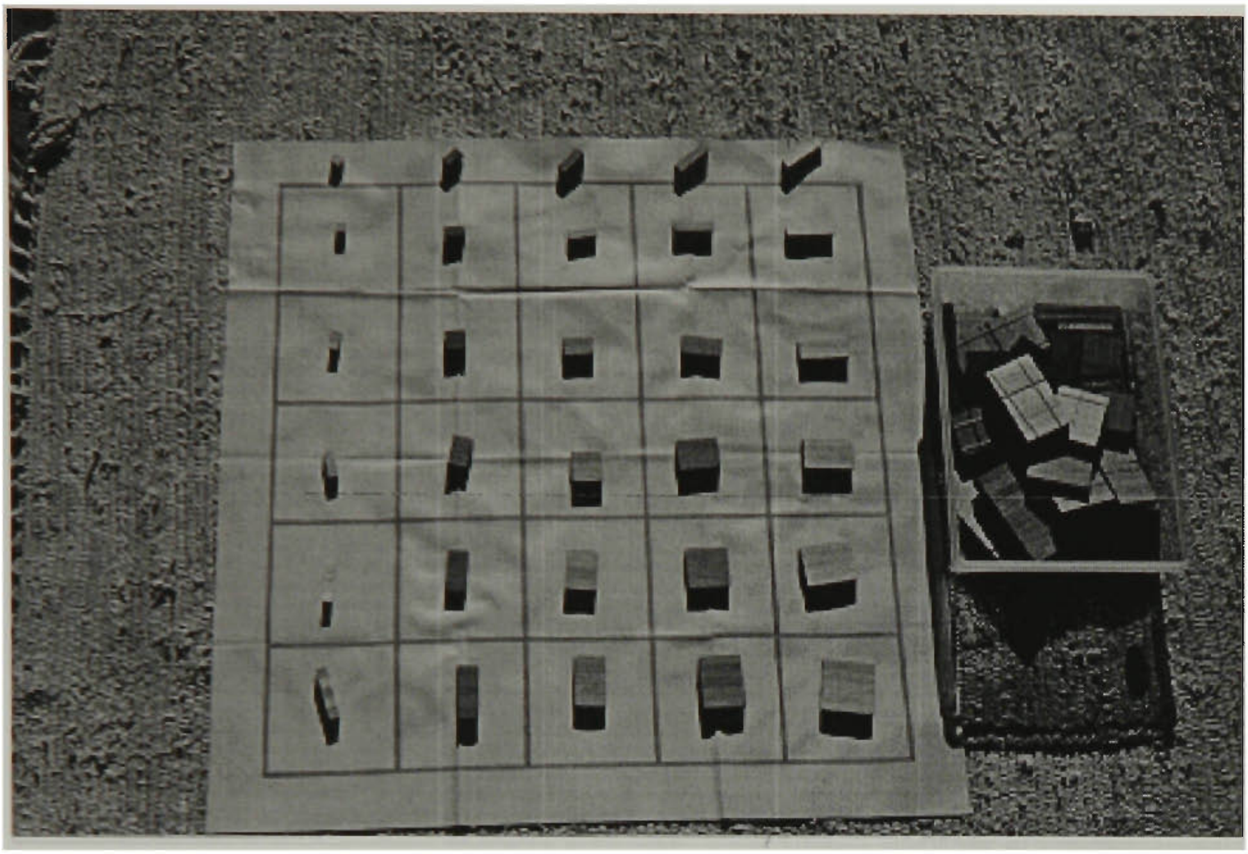

Photo 3. Cuisenaire rods structured to display multiplication table to 5 x 5.

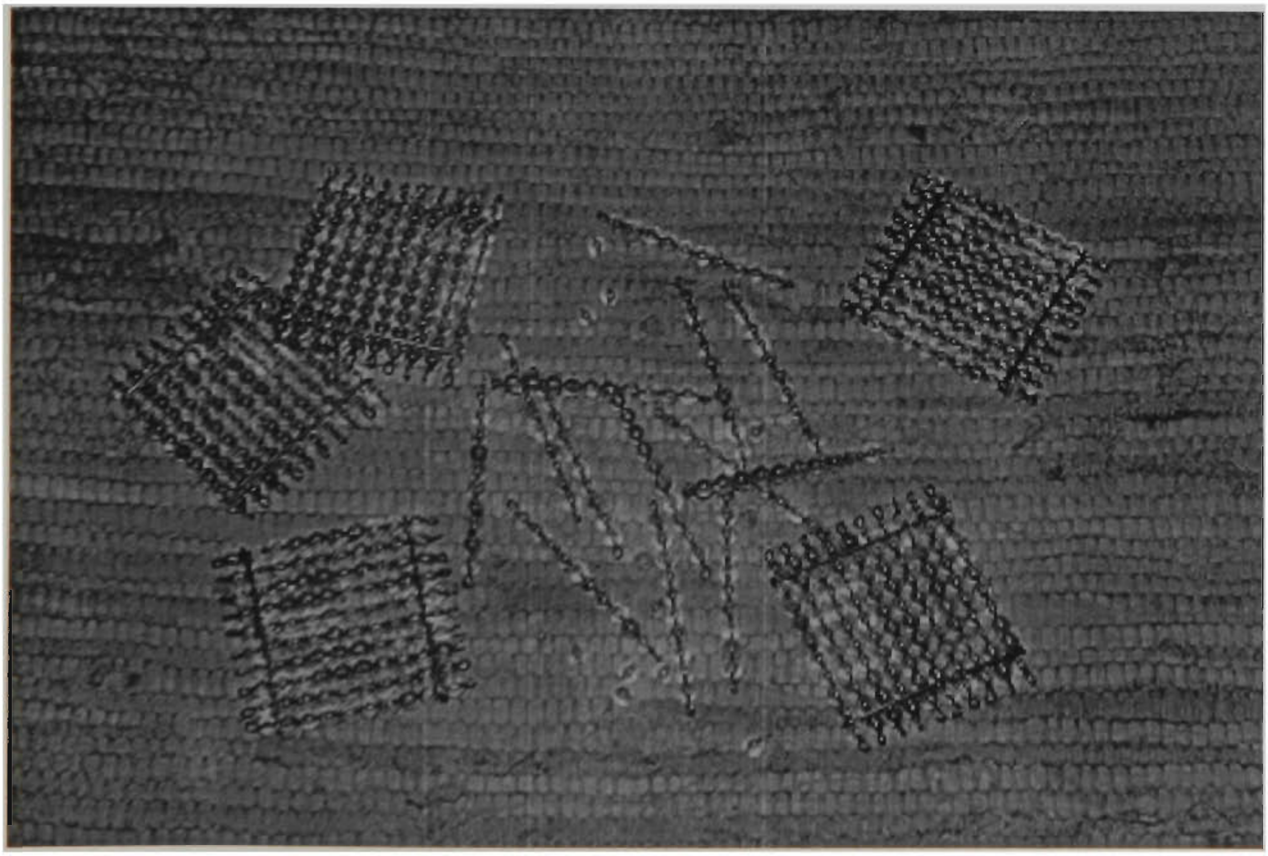

Photo 4. Montessori Golden Material.

Cuisenaire's material 'Numbers in Colour' consisted of coloured rods, charts and cards (Gattegno, 1960). It became widely used throughout Australia and many other countries and was generally known as Cuisenaire rods and associated material (Sealey, 1961a). The materials are still found in schools but they are generally no longer used by teachers for teaching number concepts. Sudam and Weaver (1975) however, in reporting research focusing on the effectiveness of using Cuisenaire materials, claim that Crouder and Brownell found aspects of improved learning by those pupils participating in the Cuisenaire program when compared to those taught by conventional means.

Gattegno argued that some teaching practice can do more to hinder learning than encourage and assist it. His published works include a series of texts on the coloured rods devised by Cuisenaire (Gattegno, 1960).

Bead apparatus. The literature suggests that Montessori's first design of equipment to model the base ten number system consisted of chains of beads. (See Photo 4). All of the beads are the same colour, a beautiful translucent amber, leading to the name 'golden materials'. Each link of the chains of beads was a wire on which were strung ten beads. Thus the components for the structuring of multi digit numbers up to 1999, consisted of one chain of beads containing 1000 beads strung in series of 100; nine 100 beads strung in tens; nine 10 bead links and nine single beads (Montessori, 1917).

These chains of beads were linked in such a manner to allow zig zag folding of the hundreds chain to form a square. The thousand chain could be folded in a similar manner to make a rectangle, ten chains wide with 100 beads in each. This rectangle could then be folded in ten vertical layers to construct the cube. Thus the links can be established between exponents and powers, the square of ten being 100, and the cube of ten being 1000.

Permanently "folded structures" are available as an alternative to the chain structures and are simpler to use by children. The "folded structures" material (researcher's term) consists of single beads, ten-bead bars, hundred-bead squares, and thousand-bead cubes wired in those configurations. The Montessori golden bead material is, according

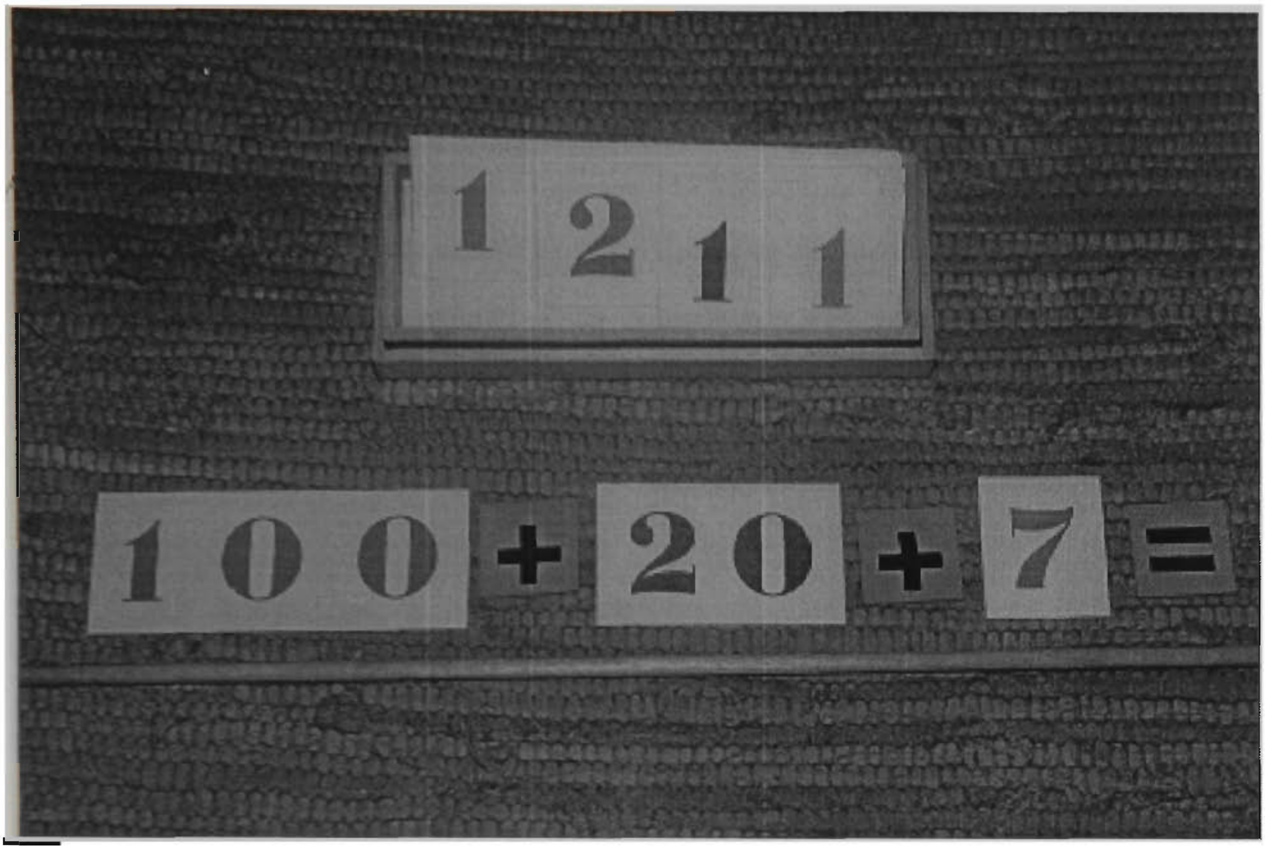

Photo 5. Montessori place/face value cards in box and example, 127.

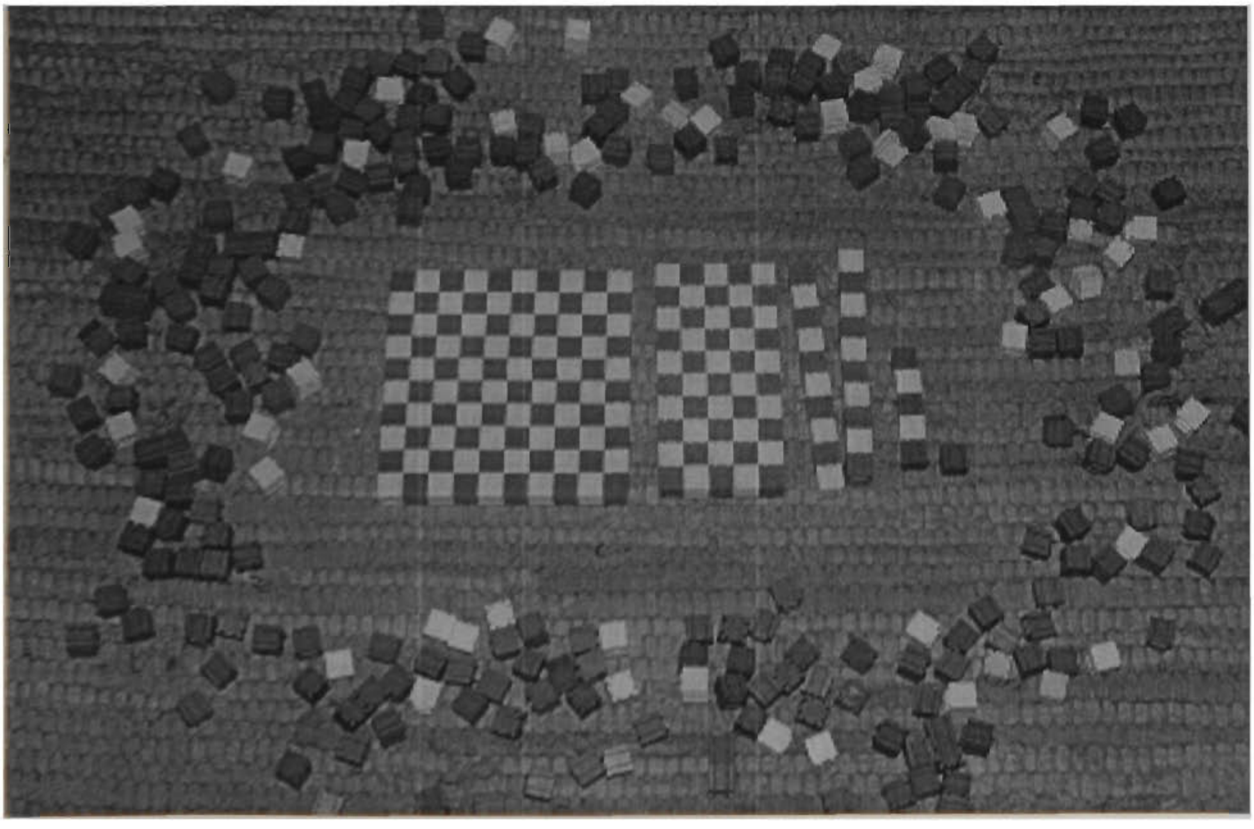

Photo 6. Interlocking centimetre cubes to build Base 10 structures.

to Hainstock (1971), an integral part of Montessori mathematics. The Montessori bead materials were also strung in multibase groupings.

Number symbol cards. These cards are used in Montessori schools in conjunction with the golden material (Hainstock, 1971). They facilitate the labelling of representations of numbers that the children construct using the golden material.

The cards have the numerals printed in three different colours; green, blue and red and increase in size according to the power of ten (See photo 5). The unit cards coloured green are numbered 1 to 9. The tens cards in blue are twice as long and are numbered 10 to 90. The hundreds cards coloured red are three times as long as the units cards and are numbered 100 to 900. The thousand cards numbered 1000 to 9000 are four times as long and are coloured the same as the units. These cards allow the children to construct multi-digit numbers such as 127, by using the following cards - 100, 20 and 7 and then placing them on top of one another starting with the 100, to make the numeral that is written as 127 in the conventional place value notation. It is of interest to note that that this dual strategy of symbolisation for number has historic roots. According to Stuart (1970), "...two forms in which a number such as 782 may have been written, even to the beginning of the fifteenth century, were .7.8.2. and .700.80.2. The latter form is often sighted in children's early attempts to record numbers" (p.78).

Interlocking cubes. Labinowicz (1980), in applying Piagetian principles to the structuring of a hierarchy of levels of representation, advocated the use of a wide range of concrete materials leading to the abstract symbolic level required at the formal level of written symbols in the Hindu-Arabic notation. Interlocking plastic cubes such as Unifix can assist in bridging the gap between discrete collections such as bean seeds grouped in cake-cups and continuous structures such as Dienes' base ten blocks or *Blockaid*, as described below. The blocks can be grouped in tens that have a unit structure relating to the collection but within that structure there remain ten separate cubes that are in one to one correspondence with any collection of ten objects.

Base ten blocks. Base 10 blocks, generally made of wood with minor variations in design have been used to model the magnitude of the powers of ten in the number system by means of a volume representation. (See Photos 7 and 8).

Although base 10 block materials were popularised in the early 1960's, by Dienes in his Adelaide Mathematics Project at Cowandilla, their immediate predecessors were almost certainly found prior to that time in schools using the Montessori approach. The structures are equivalent to the bead structures designed by Montessori and her later block material. From the researcher's reading of the literature however, it is not evident when her base ten block models were first devised. There is no mention of them in Montessori's writing up to 1917.

The closeness of this date to 1920, allows one to speculate whether the first such models of the base ten structure were devised in Australia by W.R. Hill prior to the marketing of his Hill's Constructive Counters and the handbook "Mathematics Simplified". Although no date appears on his book published by George Philip and Son of 451 Pitt Street Sydney, Hill's Constructive Counters have printed on the wooden box a patent number and date. the latter being 1920.

Such speculation about Hill's material being the first of its type is of special interest to the researcher as a set of Hill's Constructive Counters was given to him by the Infant Mistress at Albert Park Primary School in 1958, immediately prior to his appointment to his first rural school.

Since that time this set of apparatus has been used to demonstrate the structure of the number system to hundreds of children and teachers. In 1995 when talking about this material to a group of mathematics educators, one amongst the group, Dr Marj Horne, declared that she had the book that was a teacher guide to the Hill's Constructive Counters. The question "Would you like to see it"? was quickly answered in the affirmative. One of the reasons for indulging in discussion on this particular material is that there are features of this equipment and methodologies proposed by its author that have relevance to this study and indeed to all levels of primary and secondary education. Such relevance is indicated in the full title of the book; "Mathematics simplified: Arithmetic, Algebra, Geometry, Trigonometry and Calculus for Kindergarten, Infant, Primary and High Schools".

The Dienes' base materials, promoted in the 1960's, were part of a wide range of apparatus devised for the modelling of number ideas and known as the Multibase

Arithmetic Blocks (MAB). Supplementary to this material were the Multibase Extension Materials and Variations Material on Multibase which were used to represent number in the various bases. Dienes also developed an extensive array of other manipulatives across the whole mathematical field; Logic tracks and attribute blocks for logical reasoning, and algebraic experience material for algebraic notions, among these. Of the wide range of materials designed to develop number ideas, the only one remaining popular today in schools is the base ten set from the MAB range of apparatus. This base ten set has generally become known in Victorian schools as MAB.

Blockaid. (See Photo 9 and 10). Carney (1995a) claims that *Blockaid* is a multi-purpose aid designed to give insight into the base ten number system and the four operations on number. It also has applications in practical measurement of length, area and volume and in developing fractional concepts. The apparatus contains three forms of representation of number; value related foam pieces that model the base ten system by volume, pictorial representations of the foam pieces and face value numeral cards in three sizes to represent the groupings of hundreds, tens and units. The foam pieces and the pictorial cards use a sub-grouping of fives and fifties which was used to advantage in some early number systems. The use of these intermediate representations allows the material to be used without the necessity of counting, simplifies mental calculations and speeds up the use of the materials in the same way as intermediate coins and notes do such as the 50 cents and the $5 and $50 in our decimal currency system. (See section on equipment - Chapter 3).

Based on Dienes' claim that man performs no better than the chimpanzee in "recognising" or subitizing the numerosity of small collections of objects, Carney and Barrow (1993) state " ... that the majority off us simply cannot recognise groups larger than five. Beyond five we either combine subgroups such as 5 and 3 to make 8, or we count" (p. 6).

> Subitizing (the instant recognition of the number associated with a configuration) is the first skill to be acquired and is part of the basis on which the child's understanding of number develops (Bergeron & Herscovics, 1990, p. 33).

Money. When introducing arithmetic, Montessori (1964) stresses the importance of counting every-day objects such as buttons, plates, money, and so on.

> No form of instruction is more practical than that tending to make familiar with the coins in common use, and no exercise is more useful than that of making change. It is closely related to daily life that it interests all children intensely (Montessori, 1964, p. 326).

While Hiebert and Carpenter (1992) state that the values of particular pieces of money are assigned arbitrarily, and that children bring with them knowledge of the value of each piece, the researcher does not agree with their claim that children's familiarity with money suggests that it may be an especially useful representation of place value.

The coins have a face value arbitrarily established by convention but do not embody a concept of place value. This face value of course can be linked to and can reinforce the more abstract concept of place value. The same is also true for block representations of number as detailed under the above heading - 'Base 10 Blocks' which is value related material by volume.

Informal materials. (See Photo 11). Baratta-Lorton devised a wide range of activities using simple materials to focus attention on a single concept in order to draw out the generalisation of that concept. She advocated that the children be given the opportunity to express something about their experiences with the manipulative materials (Baratta-Lorton, 1972, 1976). Carney and Barrow (in press) advocate the use of the number "junk box" using everyday materials such as sticks, stones, pegs and buttons from the child's world. The use of children's toys such as farm yard sets and toy trains was advocated by Carney and Carney (1993, 1994) as a means of structuring meaningful learning situations to promote learning of number concepts with understanding.

(b) Advocates for structured aids in promoting mathematics learning.

Montessori was the originator of a system of teaching young children by the use of equipment which provides a multi sensory approach to learning. Her ideas seem to predate the work of Hill, Stern, Cuisenaire and Dienes and some of her ideas are not inconsistent with the constructivist view point later elaborated by Piaget. While Piaget claims that knowledge is constructed by the child from within, Montessori defines intelligence "... as the sum of those reflex and associative or reproductive activities which enable the mind to construct itself, putting it in relation to the environment" (Montessori, 1964, p.198).

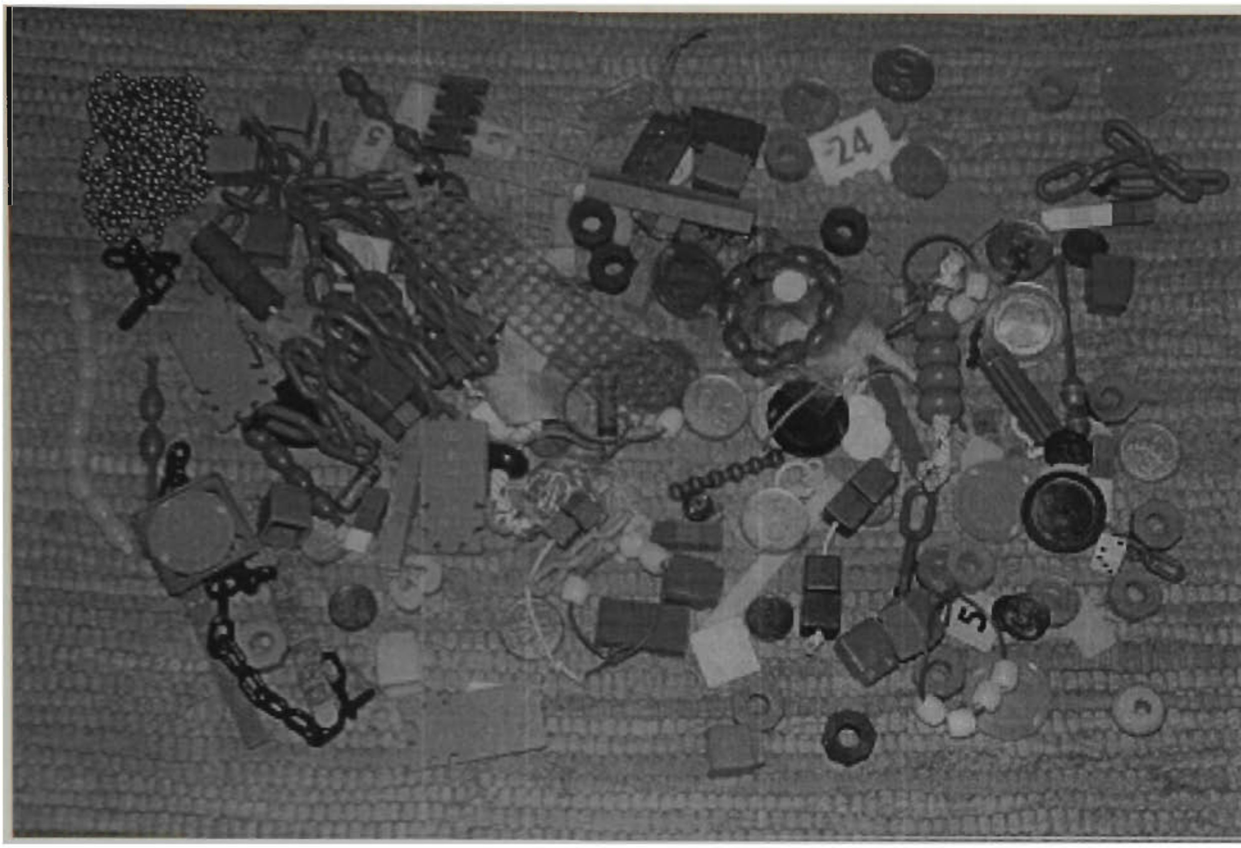

Photo 11. Junk-box for number ideas.

Photo 12. Structured number materials for modelling groups to five.

According to Montessori (1964), her method offers a system of instruction in arithmetic which fits the child. A summary of its main principles are:

- no formal teaching until the child's mind is seen to be ready for it;
- the very clear presentation of the decimal principle of our number system;
- the free use of concrete material until the child elects to abandon it;
- individual progress limited only by individual capacity;
- learning by experiment and thought rather than from teacher instruction.

Osborne and Nibbelink (1975) list the materials common in most Montessori programs as follows:

> Stairs (or number rods), sand-paper numerals, spindle box, counters, coloured bead material, bead frames, addition board for rods, golden bead material (base ten), number cards (place or face value), block material (made of wood in base ten), multibase bead materials, multiplication board, abacus equivalent, division board (p. 276).

Hill designed material that would seem to have significant advantages compared with present day designs of Base 10 apparatus. Its application extends beyond elementary arithmetic into algebra, geometry, trigonometry and calculus.

> The constructive counter system is so arranged as to suggest the underlying principles of the number system, and thus assist children to overcome many of the main difficulties met with in arithmetic. It helps the child until he can help himself (Hill, 1920, p. 8).

When elaborating the concept of construction, Hill (1920) quoted James:

> Constructiveness is as genuine and irresistible an instinct in man as in the bee or beaver. Whatever things are plastic to his hands those things he must remodel into shapes of his own, and the result of the remodelling, however useless it may be, gives him more pleasure than the original thing. The mania of young children for breaking and pulling apart whatever is given them is more often the expression of a rudimentary constructive impulse than a destructive one (p. 9).

Dienes in advocating a Piagetian approach to mathematics learning advocated the use of a wide range of structured materials to promote learning. Some of these materials were adaptations of equipment devised by Monteshsori and his attribute blocks were based on the earlier ideas of Vygotsky (1962). Dienes (1971), suggested "shifting the emphasis from teaching to learning mathematics, from our experiences to the children's, in fact from our world to their world" (p.11).

In his works he examined the continuum of mathematics learning and in doing so took account of the following:

- The learner. Dienes' principles of pedagogy as well as his six stages of learning were noteworthy. Integral to these stages of learning were the use of concrete representations that facilitate concept generalisation. He took full cognisance of the work of Piaget and Bruner in his writings. In the examination of the psychological aspect of mathematics learning, Dienes (1971a) argued that educators were just beginning to understand the process of abstract thinking.

- Curriculum content. Dienes emphasised the two distinct aspects of mathematical learning, namely the acquisition of techniques on the one hand and the understanding of ideas on the other:

  > There is, of course, a difference between understanding the workings of a technique and understanding its subject matter. A child may well be quite conversant with all the technicalities of linear equations without having much idea of what sort of thing a linear equation is (Dienes, 1971a, pp. 3-4).

- Methodology or educational aspect - Dienes proposed the creation of self-motivating learning situations with children working either individually or in small groups (Dienes, 1971a). Dienes has been called the twentieth century's mathematics teaching genius (Message, 1971, p. 2).

Theory of Mathematics learning framed by Dienes. Dienes (1971a) acknowledged and drew on the work of other educators such as Piaget, Bruner and Jeeves and worked in collaboration with Golding and Williams, in framing what he terms "a feasible skeleton theory of mathematics learning" (pp. 21-22) which he expounded in a number of principles.

- The Need for Basic Experiences. - Undoubtedly the best experiences to promote learning should be drawn from meaningful encounters with the real world but such encounters can not always be duplicated in the classroom, thus the need to structure situations and equipment (Dienes, 1965).

- The Dynamic Principle. - "Preliminary, structured and practice and/or reflective type of games must be provided as necessary experiences from which mathematical concepts can be built" (Dienes, 1971a, p. 30).

- The Multiple Embodiment Principle. - Here Dienes suggested that the child be "provided with experiences whose structure is as near as possible to that of the concept he is intending to learn" (Dienes, 1965, p. 3). In a later account - "Multiple embodiment means that every concept should be presented in as many different ways as possible" (Dienes and Golding, 1971, p. 55).

- The Mathematical Variability Principle. - "Concepts involving variables should be learnt by experiences involving the largest possible number of variables" (Dienes, 1971a, p. 31). For example, in the Multi-base Arithmetic Blocks the multiplicity of the different bases brings out the general structure of the concept of place value.

- The Deep End Theory. - This is based on the analogy of teaching a child to swim, where one method is to push the learner into the deep end of the pool. "This kind of presentation is at variance with traditionally accepted approaches, which required new structures to be presented simply and built through easy steps to the more complex forms" (Dienes and Golding, 1971, p. 57).

- The Constructivity Principle. - Children tend to think constructively, which means that they tend to build up their structures from separate components (Dienes and Golding, 1971, p. 58).

It was reported by Biggs in Collis and Biggs (1991)

> ...that children taught by the Dienes approach for two years were superior to a traditionally taught matched sample, in attitude to mathematics, conceptual understanding and computing skill. The results for the test of computing skills was unexpected as these were emphasised in the traditional classes but were not taught as such in the experimental classes (p. 202).

Carney. The approach to number advocated by Carney and Barrow (1993; in press) and Carney and Carney (1993, 1994) is very much rooted in Piaget, Bruner, Dienes and certainly Hill, but it also considers the acquisition of number concepts in the light of an historical perspective in man's coming to know such concepts. It also considers the advantages that may derive from other forms of representation evident in Asian cultures, particularly their oral counting word sequence and abbaci.

Carney and Barrow believe that:

> ...it is paramount for children to record in their own way the activities that they perform using the equipment and ideas set out in this series, and that they are allowed to suggest variations of the techniques involved (Carney & Barrow, in press, p. 8).

The approach is still in the process of development shaped very much by the combination of children's individual constructions in operating with materials and will continue to be shaped as a result of this study.

2. Fundamentals of education.

Investigating whether mathematics learning at the middle primary school level can be improved by using structured materials, requires consideration of all the elements that effect teaching and learning.

> The teacher on one side, pupil on the other side, knowledge between, discourse joining them. (The Upanishads, Vedic writings, 3000 BC).

Laborde and her colleagues (1990), in their concern for the relationship between language problems and the learning of mathematics, proposed a theoretical framework that takes into account the fundamental elements of the learning situation in the school: the content to be taught, the student viewed from a cognitive and social point of view, and the teacher.

Dienes (1970) in addressing ways of improving teacher education in mathematics raised questions of content to be taught and the methods applied. In relation to the child seen as central to educational discourse, Dienes (1971a), also examined the psychology of learning. These key factors were discussed by Carney (1995a), under the headings of the child, the method and the content.

The nature of the discourse that maximises the shared communication of mathematical knowledge between teacher and child is central to this study. The hypothesis is that this dialogue is richer when structured learning apparatus is used to model the concepts and procedures to be learned. In drawing on Dienes' view the researcher shows the links between the fundamental elements in learning in Figure 1.

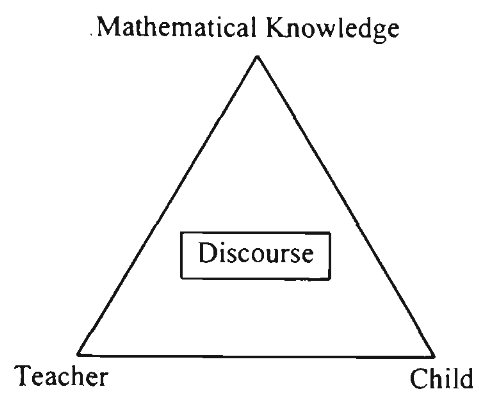

Figure 1. Fundamental elements of the learning situation.

The question is whether the above model has relevance to the nature of discourse in the light of modern educational theory that considers the constructivist viewpoint:

> Our epistemological conceptions of mathematical knowledge are closely linked to our teaching practices. In relation to this there exists two possible conceptions: we can think of mathematical knowledge as either a product or a process. In the first case, we attribute importance to the mathematical discourse. In the second, we give priority to the mathematical activity. The constructivist interpretation of mathematical teaching corresponds to the second conception (Barbin, 1993, p. 49).

In the light of current worldwide trends towards national curricula it is necessary to extend the above diagram to include the constraining factors influencing educational practice. Constraining factors such as parent, community and school administration influences, will not be elaborated on in this review.

The three fundamentals of education will be examined in more detail under the headings of psychology, epistemology and pedagogy (See Figure 2)..

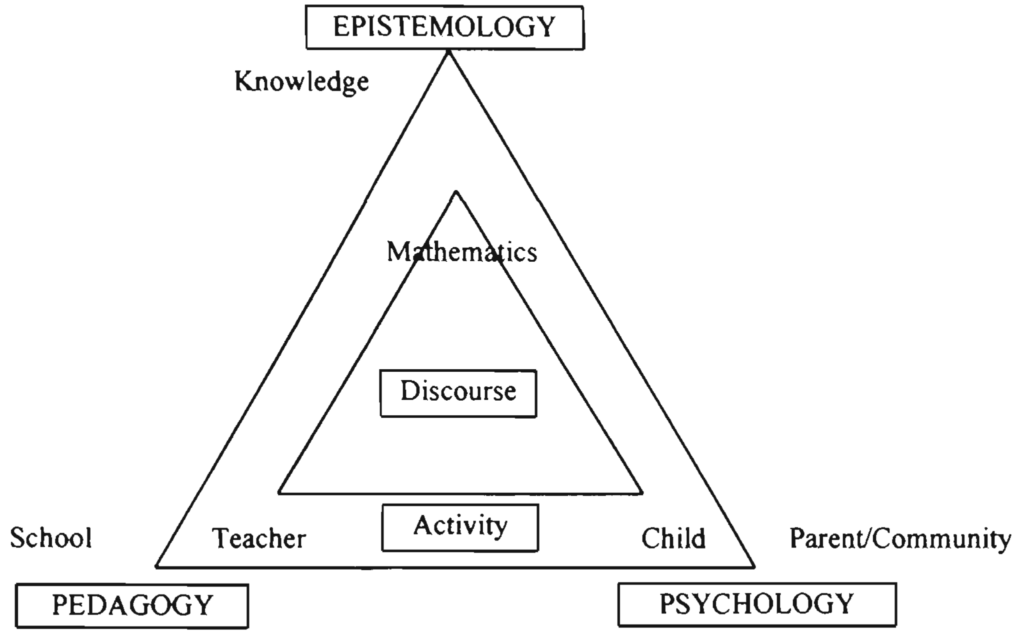

Figure 2 Factors modifying teacher practice - from Clemson and Clemson (1994).

(a) Psychology.

From the psychological perspective Vergnaud (1990) regarded the starting point to be Piaget, as he is probably the most important contributor to the epistemology of mathematics from the point of view of psychology. In giving an overview of educational psychology in Australia, Langford (1989) outlined five major theories in educational psychology, namely - behaviourism, humanistic psychology, Freudian depth psychology, information processing theory and cognitive structuralism.

While the influence of behavioural theory is still very evident in influencing classroom practice, the theory that would seem to have greatest relevance to mathematics education in primary school is the work of Piaget and his theory on cognitive structure. While all of the above psychological theories deal with cognition (Langford, 1989), Piaget's structuralism is the element that contrasts his theory with that of all the others. It is also the element that gives rise to his views on constructivism (Miller, 1983). These views are more fully addressed under the heading epistemology.

Structuralism is a system that attempts to uncover deep underlying structures in literature, philosophy and mathematics. "Structuralists look at how parts are organised into a whole, and they abstract patterns of change. In particular they are concerned with relationships - between parts and the whole and between prior and subsequent states" (Miller, 1983, p. 38). Piaget belonged to this school of thought, and proposed that a small set of types of reasoning underlies a wide variety of thinking episodes (Miller, 1983). Amongst the cognitive structures that Piaget identified are class inclusion, seriation (ordering), transitivity and topology (Copeland, 1974; Labinowicz, 1980).

> In Piaget's view, moment to moment specific encounters with objects or people lead to general ways of understanding the world. This understanding changes during development as thinking progresses through various stages from birth to maturity (Miller, 1983, p. 30).

Piaget is probably best known in educational circles for his work in identifying four basic stages of intellectual development through which he suggested that children pass. Copeland (1979) identified these states by name and approximate ages as:

Sensory-motor	0 to 2 years
Pre-operational	2 to 7 years
Concrete operational	7 to 11 or 12 years
Formal operational	11 or 12 years on.

These stages should be regarded as flexible as children at any particular chronological age often demonstrate thinking characteristics of more than one stage (Labinowicz, 1985).

It is the third stage, namely concrete operational, that has the most significant implications for primary school teachers. Traditionally a play or activity approach using a wide range of equipment has been the accepted teaching method found in most infant classes of schools in Australia. By age eight most pupils in Victoria enter the middle primary school years designated as grade 3 and grade 4. These grade levels are within the period identified as the concrete operational stage. This is the level at which the subjects of this thesis will be studied. According to Piaget this period, and the remaining upper primary school years, constitute a crucial period of intellectual development in which children's logical thinking must be based on actions performed on objects or things for them to formulate the reasoning process (Copeland, 1979).

Development.

The mechanisms for development are functional invariance of cognitive organisation, cognitive adaptation, and equilibration. Cognitive organisation refers to the tendency for thought to consist of systems or 'schema'. Cognitive adaptation refers to the interaction between the organism and the environment. (Labinowicz , 1985).

Equilibration is the mechanism whereby every organism strives towards equilibrium with the environment and equilibrium within itself. This mechanism is facilitated by the dual processes of assimilation and accommodation. Assimilation occurs when learners interpret inputs from the environment through previously built networks of ideas. The learner's organisation of ideas resists change causing discomfort and disequilibrium. The discomfort, confusion, or frustration of the disequilibrium state serves to motivate a search for better organisation of ideas to resolve the discrepancy resulting in accommodation (Palmer, 1970; Labinowicz, 1985).

For Piaget, equilibration is the process that combines the elements of maturation, experience with the physical environment, and experience with the social environment.

The cognitive adaptations resulting from such processes is what constitutes learning. Dienes (1971b) viewed learning as adaptation to the environment. His views were very much in accord with Piaget's ideas on both cognition and the nature of knowledge.

Behavioural theory.

What is the relevance of behavioural psychology in today's mathematics education programs in primary school? According to Labinowicz (1980), Piaget believed that, "...the great danger today is the use of slogans, collective opinions, ready made trends of thought" (p.266). In recent times the trends in education of damming past practice and advocating total change have not met with a great deal of success in facilitating change in schools.

Although many current curriculum standards frameworks purport to advocate modern learning theory, their emphasis on content rather than process is likely to ensure the continuation of traditional teaching styles.

Despite challenges to change practice in school mathematics, educators and politicians in most countries, for most of the twentieth century have continued to accept the premise that school mathematics should be based on externally prescribed curricula, formal textbooks and written examinations (Clements, 1995, p. 2).

Contrasting Piaget's theory is that of the traditional behaviourists.

At the primary school level, the behaviourist theories of Thorndike and Skinner have given direction to content based curriculum that is identified as traditionalist.

Traditional mathematics instruction is based on the transmission, or adsorption, view of teaching and learning. In this view students passively 'adsorb' mathematical structures invented by others. Teaching consists of transmitting sets of established facts, skills, and concepts to students (Clements & Batista, 1990, p. 34).

Traditional behaviourists, in their laboratory study of animals, found that a stimulus from the external environment produced a predictable immediate response in the animal. The advocates of this theory were Pavlov, Skinner and Thorndike (Lindgren, 1956). Behaviourists applied these findings to human subjects. According to Labinowicz, (1980) however:

Piaget observed that man's responses to stimuli are much less predictable than those of lower animals. ... man has both the ability to choose his responses and to initiate changes in the environment. Piaget therefore focused on the intermediary processes between the stimulus and the response, which could explain man's behaviour (p. 147).

Traditional behaviourist's focus

S ←-----------→ R

Stimulus (observable) — Response (observable)

Piaget's focus

S ←---- Intermediary processes ---→ R

Stimulus (observable) — (Unobservable) — Response (observable)

Figure 3. Contrasting Behaviourist theory and Piaget's theory of learning.

Labinowicz (1980), in contrasting these two theories suggests that no theory can be applied universally:

...behaviorist theory is powerful in explaining lower-level learning (memory learning). Piaget's theory is powerful in explaining higher level learning, such as understanding relations as in logical-mathematical knowledge (p. 155).

Piaget (1970) recognised the potential of behavioural theory, and its application in programmed instruction in transmitting certain kinds of information, but saw it as irrelevant in areas such as the understanding of relationships in mathematics and science. "It is possible to envisage a balance being struck, varying from subject to subject, between different parts to be played by memorizing and free activity" (Piaget, 1971, p. 78).

(b) Epistemology.

According to Piaget, there are three kinds of knowledge: physical knowledge, logical-mathematical knowledge and social knowledge. Each requires the child's actions (Wadsworth, 1989; Labinowicz, 1985).

Bloom's (1956) threefold division of educational objectives: cognitive, affective and psychomotor are compared to the Greek philosophers use of a tripartite organisation of human behaviour: cognition, conation, and feeling; thinking, willing and acting. Bloom (1956) in his Taxonomy of objectives in the cognitive domain lists the following: basic terms and facts, techniques and manipulative skills, understanding, application, analysis, synthesis, creativity, reasoning and evaluation.

Anderson's network model of memory discussed in Fox (1993) proposes a schema whereby knowledge is represented as a web or network. The network consists of nodes which are related by a series of links. A basic cognitive unit consists of a unit node. Cognitive units combine together in larger units known as 'tangled hierarchies'.

Hiebert and Carpenter (1992) define procedural knowledge, "...as a sequence of actions. The minimal connections needed to create internal representations of a procedure are connections between successive actions in the procedure. Examples of procedures are standard written algorithms in arithmetic". They define conceptual knowledge "...in a way that identifies it with knowledge that is understood. It is rich in relationships. ...Conceptual knowledge is equated with connected networks that can be pictured in the form of hierarchies or webs" (p. 79).

According to Eisenhart, Boroko, Underhill, Brown, Jones and Agard (1993) both procedural and conceptual knowledge are considered necessary aspects of mathematical understanding. They contend that teaching for understanding is stressed in current reform in mathematics teacher education. Hiebert (1986) suggested that it is an extremely complex process to teaching mathematics for understanding.

Romiszowski in Fox (1993) classified knowledge in four dimensions:

- Declarative - knowing about facts, events and people;
- Procedural - knowing what to do in a given situation;
- Concepts - knowing how to recognise and distinguish between groups of phenomena, or being able to define things;
- Principles - knowing how to link together concepts or facts in specific ways, thus enabling an explanation or prediction of events. (p.193)

Piaget was concerned with the classical issues in epistemology with what philosophers have considered the basic categories of thought, namely time, space, causality and quantity. Piaget (1972) claimed that knowledge is a process rather than a state:

> It is an event or a relationship between the knower and the known. ...In a sense a person 'constructs' knowledge. Cognitive man actively selects and interprets information in the environment. He does not passively soak up information to build a storehouse of knowledge (Miller, 1983, p. 37).

It is the construction of knowledge with understanding that is central to this study. The work of Hiebert and Carpenter (1992) suggests that the links that the child makes between the concrete models that embody a concept, assist in the abstraction and generalisation of the concept.

<u>Constructivism.</u>

The early work on constructivism in mathematics teaching grew out of Piagetian epistemology (Pateman, 1993). To Piaget all knowledge is a construction resulting from the child's actions.

Biggs and Moore (1993) defined constructivism as:

> ....a viewpoint of the nature of learning, which emphasises the relativity of knowledge; that knowledge is constructed by the individual, not transferred and that individual constructions vary according to previous knowledge. They saw constructivism as psychology, and also deep pedagogy (p. 524).

Constructivism is knowledge building and thus it is also essentially epistemology. Interpretations of the views on constructivism vary greatly, with the extreme position held by the radical constructivists such as von Glasersfeld (1987). Acceptance of his view of knowledge "requires that we shift the emphasis from the students 'correct' replication of what the teacher does, to the students successful organisation of his or her own experience" (von Glasersfeld, 1987, p.6). Jeong-Ho (1994) explained that radical constructivism is not generally supported by Piagetian followers in mathematics education, who trace their path to traditional constructivism.

The weaving together of actions, beliefs and thought in the constructivist paradigm allows one to understand mathematics in a very different way to the traditional approach (Malone, 1993).

Hill (1920) stated the case for an experiential, discovery approach:

> The child already has the disposition to learn by doing. He is by nature an observer and imitator. Nothing pleases him more than making things, and it is usually the constructions around him that he imitates. To utilise the constructive instinct to an educational advantage is aimed at in the use of these counters. Each pupil in the early stages of construction and number work is busy doing something - experimenting, observing and discovering (Hill, 1920, p. 9-10).

Carney and Barrow (in press) view the role of the teacher as that of a guide who supports students' invention of viable ideas rather than allowing "free for all" discovery based on hope.

Role of affect.

Affective factors play a central role in mathematics learning and instruction (McLeod, 1991). According to Wadsworth (1989), Piaget's theory of intellectual development is seen as having two components, one cognitive and the other affective. "Until recently educators have focused largely on Piaget's work on cognitive development and overlooked the role of affective development in intellectual growth" (p. 29). Affect includes feelings, interests, desires, tendencies, values and emotions in general.

One aspect of affect has to do with motivation or energising of intellectual activity. Wadsworth (1989) noted that:

> Piaget argued that *all* behaviour has both affective and cognitive aspects. There is no pure cognitive behaviour and no pure affective behaviour. The child who 'likes' mathematics typically makes rapid progress, The child who 'dislikes' mathematics typically does not make rapid progress (p. 31).

In considering the relationship between the affective and the cognitive domain Krathwohl, Bloom and Masia (1956) cite Scheerer: "No matter how we slice behaviour, the ingredients of motivation-emotion and cognition are present in one order or another" (p.123).

Children assimilate experience to affective schemata in the same way as they assimilate experience to cognitive structures. Piaget contended that the basis of social interchange is an interchange of attitudes and values between the child and others.

According to McLeod (1991) von Glasersfeld noted the powerful positive emotions that often go along with the construction of new ideas, and similarly he reports of Cobb having written of the importance of confidence and willingness to persist as students develop mathematical concepts. Hill (1920) asserted that the driving force throughout is the children's desire to learn. They like making discoveries. The teacher acts as the tactful guide, but the children lead the way and surmount each difficulty themselves.

Carney and Barrow (in press) in their approach to developing number ideas suggest that in adopting an activity approach using materials in small groups the teacher has the opportunity to learn along-side the child and to share in the sheer joy of discovery.

Role of language.

Language and thought are at the basis of mathematical thinking. Evidence suggests that mathematical thought and language are enhanced when pupils interact with the physical world. The researcher believes that such interaction can be structured through the use of apparatus such as *Blockaid* to promote dialogue that is rich in meaning.

In comparing the Vygotsky and Piaget notions of language, Fox (1993) stated that while Vygotsky believed that it is the gradual internalisation of speech that leads to the development of thinking, Piaget took a different view. He believed that children's development of thinking depends on their acting on the environment. Language may be involved to amplify or facilitate thinking, but is not essential for thinking. Vygotsky's theory, elaborated in Ernest (1991), hypothesised that for the individual, thought and language evolve together and that conceptual development depended on language experience.

Hiebert and Carpenter (1992) advanced the argument that concrete materials can provide a public representation to which attention can be drawn and be a focus for discussion:

> Because the materials are public, they can be shared by all the students in the class. To the extent that the same mathematical features are perceived by the students, the language in the classroom can be about the same things. Communication among the students and teachers is enhanced because all participants can focus their attention on the same entities and relationships between entities (p. 720).

Fuson and Briars (1990) rightly distinguished between the named-value system of English spoken number words and the system of written multidigit number marks in which values are indicated only by the relative position of those marks. They claim that the connections between these two systems of coding are more difficult to establish with English speaking children than their Chinese, Japanese or Korean counterparts due to the irregularity of the named values in the former.

Carney (1995b) advocated the introduction of a supplementary counting sequence to assist young children in making the connections between spoken English number words and the linked conceptual structures of the grouping by tens and powers of ten, that the child needs to construct. This structure of the base ten number system frequently remains hidden for many pupils. The grouping nature of our number system may be of more fundamental importance than the place value concept which is necessary only when the child begins to record numbers in numeral form applying the Hindu/Arabic notation.

The counting sequence advocated by Carney (1995b) applies the regularity of the counting sequence that commences only when we reach the standardised English named words four-tee, four-tee-one, four-tee-two, four-tee-three. This regularity ceases at fifty at which counting number children can be guided to see the irregularity. If such a discovery is made by the children, they may proceed to invent the regular substitute 'five-tee'. Discussion centering on the "tee" or "ty" or "t" soon establishes that this is an abbreviation of "ten", and a regular named sequence of oral counting words is soon established. The "t-count" then becomes 1t, 2t, 3t, 4t, 5t and so on that gives structure and meaning to our number system. The emphasis should be on a meaningful oral count.

Thomas (1995) stated that the relationship between language and mathematics is now recognised in schools but it is also clear teachers have little idea what this means in practice and research to support better teaching is far from complete.

Mathematical knowledge.

Mathematics is still generally thought of as a body of knowledge to be transmitted from one generation to the next, rather than as understandings which are constructed in the minds of learners (Mousley, 1993).

The arithmetic of earlier course of study documents was defined as the study of space, number and time (Victorian Education Department, 1946). Biggs (1971) defined mathematics as the study of truth and beauty and order. Mathematics is often defined as the science of space and number but according to the Australian Education Council (1991) a more apt definition in that same document is that mathematics is the science of patterns. The more open definition of mathematics as the structure of relationships (Dienes, 1971b) is one that connects the various strands of the study into a more coherent whole relevant at the primary school level.

A simplified version of Dienes mapping of the range of mathematical concepts is shown in Figure 4.

Mathematical knowledge is constructed through interaction with the environment (Labinowicz, 1985). Mathematics is derived not from the material world but from our particular way of systematising that world as we bring it into the social world (Watson, 1989). Dienes (1971b) regarded mathematics as the study of relationships; it is the study of relating elements in the environment.

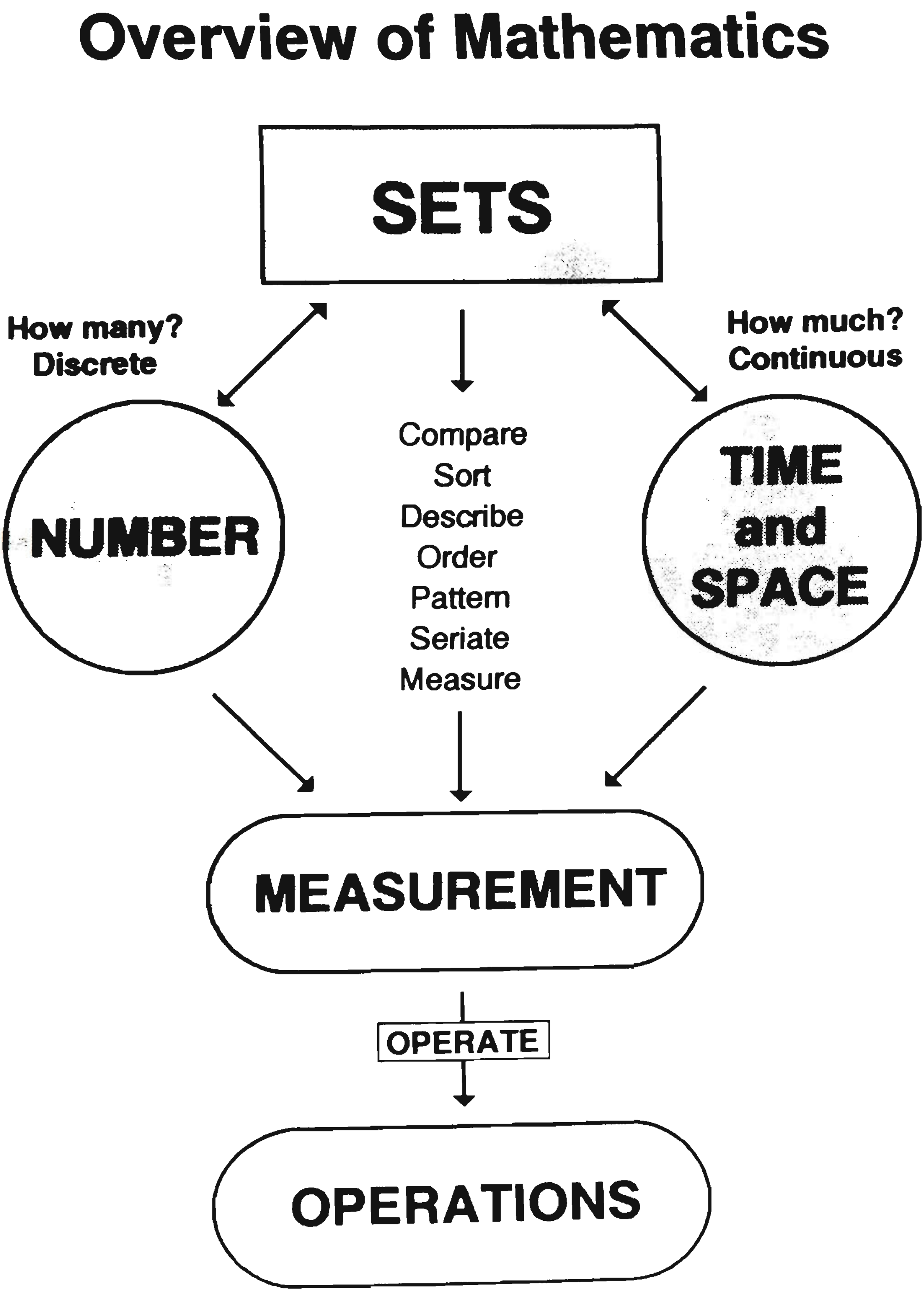

Figure 4. Dienes' mapping of mathematics concepts - simplified version.

Understanding.

Brown, Carpenter, Kouba, Lindquist, Silver and Swafford (1989) clearly showed that, although most students are reasonably proficient in computational skills, the majority do not understand many basic concepts and are unable to apply the skills they have learned in even simple problem-solving situations.

Fischbein (1990) suggested that the end product of the constructive process is a personal involvement leading into insight, a genuine intuitive understanding of the respective concepts. Vergnaud (1990) argued "that the meaning of mathematics comes essentially from problems to be solved, not from definitions and formulas" (p.21).

Although Bloom (1956) listed understanding at the third level in his hierarchy of educational objectives, it is an aspect of cognition that develops through the remaining higher levels of thinking. Hiebert and Carpenter (1992) stated that many of those who study mathematics agree that understanding involves recognising relationships between pieces of information.

Knowledge of Number Concepts.

In addressing the question, "What constitutes a number?" Bergeron and Herscovics (1990) suggested that such an epistemological consideration is of great importance if we wish to understand the theories underlying research on the number concept.

Bergeron and Herscovics (1990) compared two opposing theories: Piaget's theory, which perceived logical reasoning as the basis for the construction of the number concept; and the contra view postulated by Klahr and Wallace that the acquisition of number concepts is through quantification processes.

> 1. Logical reasoning - Piaget viewed number as a logico-mathematical concept that is *constructed* by the child rather than a physical concept that is discovered through sensory perceptions. An understanding of number requires a prior understanding of key logical concepts such as **conservation, class inclusion, and seriation** (researcher's emphasis. p. 35).
>
> 2. Klahr and Wallace's quantification processes: **subitizing, counting and estimating.** (researcher's emphasis) The function of these processes is to generate quantity or numerosity "symbols" of sets for mental manipulation. These processes or skills are hypothesized to develop in an invariant sequence (p.35).

Bergeron & Herscovics (1990) suggested that these two schools of thought may in fact be complementary and thus "hybrid" theories such as those of Ginsberg and of Gelman, have arisen. Cognitive psychology today recognises that higher mental processes are involved in the learning of early arithmetic.

Knowledge of Skills or Procedures of Four Basic Operations in Arithmetic.

In addressing the issue of learning and teaching with understanding, Hiebert and Carpenter (1992) investigated the understanding versus skill continuum which they equate to conceptual knowledge versus procedural knowledge. They cited the standard written algorithms in arithmetic as examples of procedural knowledge. Often in school mathematics, procedures prescribe the manipulation of written symbols in a step by step method. Hiebert and Carpenter proposed that one alternative method of giving meaning to algorithms is to connect the symbolic algorithm to other external representations that have meaning for students.

> One of the most commonly used materials for representing place value ...the blocks ...can be manipulated in a way that corresponds directly to the steps in standard addition and subtraction procedures by connecting each step in the algorithm to a corresponding action on the blocks. Because the connections are made explicitly ...this approach contrasts with encouraging invented procedures and allowing students' attention to be distributed across a range of potential connections (Hiebert & Carpenter, 1992, p. 84).

(See researcher's comment on place value versus value relations in section on money)

(c) Pedagogy.

Pedagogy is the function, or work or art of a teacher. Hiebert and Carpenter (1992) claimed that there is a close relationship between the ways teachers are taught and the instruction they implement in their classrooms. Teachers generally teach the way they were taught. Hiebert and Carpenter (1992) advocated that teachers develop strategies for integrating knowledge of mathematics and knowledge of children's thinking.

Teacher attitude and beliefs about mathematical knowledge and the nature of learning.

Teachers' beliefs about the nature of mathematical knowledge and of the way children learn determines teaching style. Barbin (1993) linked teaching style to the conceptions teachers hold to whether mathematical knowledge is a product (body of facts, skills, etc.) or a process of coming to know. Those holding the former view give priority to discourse, while the latter emphasise activity.

"Piaget did not provide us with a theory of education or a curriculum for teaching. What he did offer is a research-backed conception of how children acquire and develop knowledge" (Wadsworth, 1989, p. 183). Piaget (1972) suggested that the beginnings of enlightenment in epistemology have arisen from the field of child psychology. Starting from an epistemological base, the question was raised by Pateman (1993), "Can constructivism underpin a new paradigm in mathematics education" (p.69). In addressing this question he asserted that currently the dominant view is that mathematics is an objective body of knowledge to be delivered to students/consumers and that the best mode of delivery is essentially transitive. This is in direct contrast to the constructivist viewpoint. Pateman acknowledged that the school as a social institution exerts strong social pressure on teachers in the direction of conformity to existing practices.

In proposing a direction for change Taylor (1993) interpreted Pateman as advocating an approach that avoids the problem of individualism by adopting social constructivism as an epistemological referent, and supports methodologies such as Cobb (1994) whose research highlights the sociocultural contexts of classroom learning.

For those wanting to apply Piagetian and constructivist principles, DeVires with Kolberg in Wadsworth (1989) listed six principles as follows:

1. Psychological structures be developed before numerical questions are introduced.
2. Psychological structures must be developed before formal symbolism is introduced.
3. Automatized knowledge should not be stressed before implicit logic is understood.
4. Children must have the opportunity to invent (construct) mathematical relations rather than simply confront ready-made adult thought.
5. Teachers must understand the nature of children's mistakes.
6. An atmosphere of thinking must be established. (p.187)

Teaching Strategies.

Fox (1993) described the various teaching strategies in terms of a continuum with the extremes being an expository approach where at that end of the scale, the teacher is the source of knowledge and is responsible for instruction, and at the other end of the scale discovery strategies where the teacher presents to pupils opportunities to observe to discover and to act.

Between these two positions there is a continuum of methods for teaching:

Discovery

↑

Pure discovery

Free exploratory discovery (Bruner)

Guided discovery (Gagne)

Meaningful reception learning (Ausubel)

Direct instruction (Becker and Engelmann)

↓

Expositive

Figure 5. The expositive-discovery dimension (Fox, 1993, p.191).

In summary Fox's strategies are as follows:

- Pure discovery learning is not dependent in direct teaching. Pupils are encouraged to develop knowledge in the areas that are of interest to them.
- In Bruner's exploratory discovery, the broad learning goals are set and the pupils are asked to explore a particular concept in order to develop their learning processes.
- In Gagne's guided discovery approach, the objectives are fixed and the pupils are guided to generate the higher-order rules for themselves.
- In Ausubel's meaningful reception learning, the teacher gives, in final form, the information to be learned.

For direct instruction at the expositive end of the scale, the teacher applies drill and practice for facts and skills to be learned (pp. 191-192).

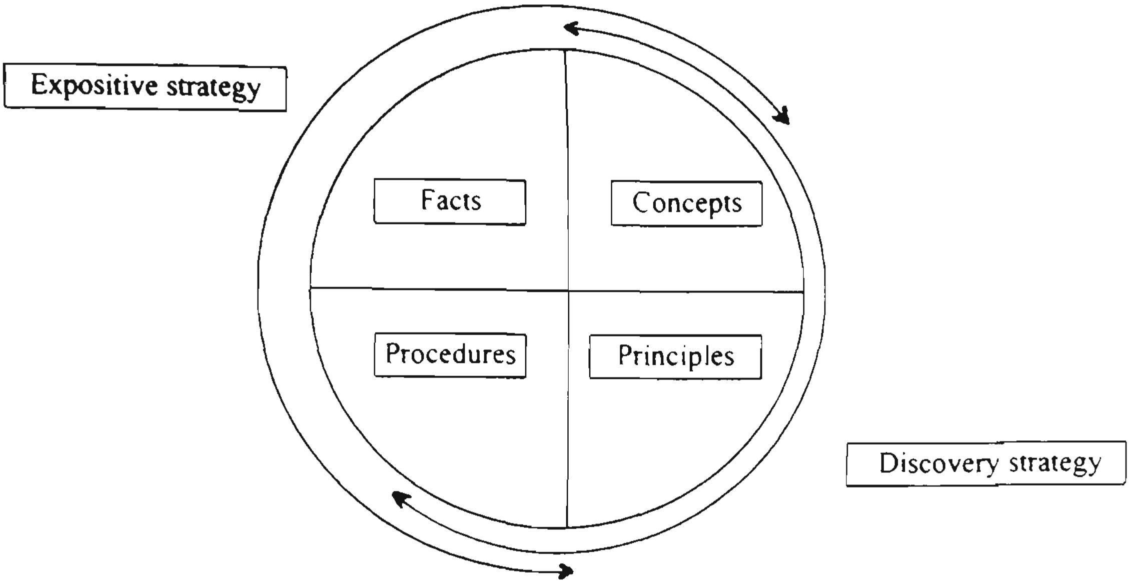

Figure 6. Strategies for teaching different kinds of knowledge (Romiszowski, 1984).

Romiszowski (1984) modified the above Figure 6 when attempting to match the knowledge areas for which individual and group learning are favoured. Individualised instruction and learning are applicable for the expository strategy continuum and he seems to suggest that it has particular relevance to fact learning.

> In the learning of discrete facts, or of simple factual information, there is comparatively little gained from group learning as opposed to self-instruction. The learning of facts is very much drill and practice. . The learning of principles on the other hand, has much to gain from well-organise group learning (Romiszowski, 1984, p. 63).

If we consider the implications of teaching from the behavioural and cognitive dimensions then according to Fox (1993) "the behavioural perspective emphasises the importance of the external environment. The cognitive perspective emphasises the internal mental processes that the pupils use to respond to the teaching situation - the discovery approach" (p.181).

<u>Discovery learning.</u>

The discovery approach developed by Bruner (1960) emphasised that the focus of teaching should be on developing the internal thought processes of the pupils - their cognitive representations of the world. Learning is seen as an active process whereby the pupil constructs models of the world. Such constructs are made in ways that are useful and make sense to the learner.

Pupil's conceptualisation develops through their manipulation of objects and materials. By manipulating objects, regularities are established between them and the cognitive structures that the pupil already has. In other words, a match takes place between what the pupil is doing in the outside world and some frameworks, templates or representations that he already has, cognitively, in his head. The manipulation of things in order to develop cognitive structures is termed 'discovery learning'.

3. Concept of number and algorithms.

In comparing the constructivist and traditional approaches to mathematics education Fischbein (1990) asked the questions "How do children count? How do they add, subtract, multiply or divide?"

> The traditionally accepted hypothesis was that the teacher teaches the child and that children understand and assimilate more or less correctly what they have been taught. The new point of view is that children invent their own methods of counting, adding and so forth; that these methods may be totally different from those usually presented by the teacher; and that children's methods may be more adequate to their own way of thinking than those proposed by the teacher (p. 7).

Fishbein's questions relating to counting and the operations of addition, subtraction, multiplication and division can be viewed in terms of the schema of knowledge proposed by Romiszowski. The researcher's belief is that counting and number ideas in general can be identified as concepts and that the steps in carrying out the formal algorithms of addition, subtraction, and division are identified as procedures. There is however, as discused in Hiebert and Carpenter (1992), an interplay between conceptual and procedural aspects of knowledge. The nature of the operation of subtraction for instance would be regarded as conceptual knowledge. The nature of the operations is very much a question of understanding.

The implications of Romiszowski's work are not so clear cut in devising instruction to cater for the other two quarters of his schema, namely concepts and procedures. It is within these domains that this study investigated number concepts and acquisition of skills in the four basic operations in arithmetic.

Romiszowski's diagram (1984) shown in Figure 6 suggested that for teaching concepts, a discovery approach was favoured but that an expositive strategy also had an influence.

For procedures on the other hand an expositive approach was the appropriate strategy but that discovery had an influence. This would seem to be on accord with the approach supported by Hiebert and Carpenter (1992) that suggested conceptual knowledge is built on procedural knowledge. It is this approach that was advocated by Carney and Barrow(1993) in the teaching basic written algorithms that are clearly defined procedures. The teacher knows the rules that have been refined over a long period of time. The paralleling of these same procedures with concrete materials is theorised to develop conceptual knowledge which is equated to understanding (Carney,1995a).

In regard to concepts Romiszowski (1984) suggested that there is little difficulty in selecting learning strategies for simple defined concepts. He defined such concepts as those that are only one step removed from the concrete, for example, the concepts of red. Dienes (1964) believed that the concepts of number are at a higher level of abstraction as such concepts can not, except in the case of unity, be attributed to single objects but rather are the generalised abstractions from groups of objects.

> As we climb up the ladder of abstraction, however, the room for misunderstanding increases, as does the web of interrelationships between the concept being learned and other previously learned concepts. ... as the degree of abstraction of conceptual learning increases, so does the need for group learning techniques that capitalise on the communal store of insights and experiences, and share them in cooperative group discussions and activities (Romiszowski, 1984, p.64).

Understanding place value involves building connections between key ideas of place value - such as quantifying sets of objects by grouping by ten and treating the groups as units - and using the structure of the written notation to capture this information about groupings. Different forms of representation for quantities, such as physical materials and written symbols, highlight different aspects of the grouping structure.

Romiszowski (1984) seemed somewhat ambivalent in advocating a particular strategy for the learning of algorithms.

> We may argue, that when a procedure is learned through reception learning techniques, without investigation into the reasons, individual learning is quite adequate. When, however, the procedure is to be learned through discovery, deducing the steps of the procedure from the reasons of the procedure, group-based problem-solving strategies may be superior, as once again they provide greater opportunity for reflective discussion and the sharing of difficulties and insights (p. 64).

In their approach to structuring learning situations that will aid number concept development with young children Carney and Barrow (in press) follow a Dienesian approach and identify three important characteristics in such learning:

- The sequencing of number. Our number system is based on a "one more than sequence".
- Grouping in tens. We can also use sub groups of twos and fives for simplicity. An example of this is money.
- Place value.

Too often it is mainly the third aspect that is taught (p. 9).

Carney and Barrow (in press) advocate the use of a wide range of informal and formal (structured) materials in leading children to know number. In order to make the grouping nature of the base ten system become more apparent as a system:

> Children should be encouraged to invent their own counting scheme to help in understanding the logic behind our system. They could also be shown other counting systems ... Egyptian, Roman, Babylonian etc. One system which children can be led to explore is the "t" count (p. 18).

They suggest that children can be guided to "discover" such a count by regularising the counting sequence by tens backwards from 100 to get the numbers ninety, eighty, seventy, sixty, fivety, forty, threety, twoty and onety. Once established the grouping system becomes apparent as soon as we reach the number after nine which becomes one ten or onety. The sequence then continues onety one, onety two and so on. This structural count is very much in keeping with the spoken word count of many Asian countries. The count then becomes a mirror for the *Blockaid* materials in reflecting an image of the grouping structure.

In aiding the acquisition of number concepts through the use of base 10 material Hiebert and Carpenter (1992) claimed that if the written symbols are linked to the block representation, then those symbols are informed by the multiple associations which the learners have already constructed for the blocks.

> Only by thinking and talking about the differences and similarities between the symbol and block representation of whole numbers can students construct relationships between representations (p. 68).

Algorithms.

The word algorism or algorithm comes from the name of an Arabian mathematician - Al Khowarizmi - who wrote the first known book on arithmetical processes in about A.D. 825 (Stuart (1970). When his book was translated into Latin in the twelfth-century, his name was written as *Algorismi* and his methods of calculation started a battle between abacists and algorists that would endure for 400 years (Barrow, 1992).

The battle for computational methods continues in this technological age between the calculatorists (or computerists) and the algorists, with the former proponents arguing for the abandonment of formal paper and pencil computations.

Fuson and Briars (1990) cite several authors (including Cauley, Cobb & Wheatley, Davis & McNight, Labinowicz and Resnick & Omason) who claim that, in the United States, many children who carry out the algorithms correctly do so procedurally and do not understand reasons for crucial aspects of the procedure.

Addition and Subtraction.

If understanding is to be achieved children must solve addition and subtraction problems by directly modelling with physical objects the relationships embedded in the problem (Carpenter, Moser & Bebout, 1988).

A summary of Fuson and Briars (1990) approach is as follows:

- When adding and subtracting with the blocks, the blocks-to-written-marks links are made strongly and tightly.
- Links among English words, base-ten blocks, digit cards, and base-ten written marks are strengthened by the constant use of the three sets of words.
- Children work with the learning/teaching approach for many days; they are allowed to leave the embodiments and do problems in written form when they are ready.
- When children begin to do written problems without blocks, their performance is monitored to ensure that they are not practicing errors.
- Addition and subtraction both begin with four digit problems (or in some cases, these problems immediately follow initial work with two-digit problems).
- Children spend only 1 to 4 days on place-value concepts initially, much place value learning is combined with the work on multidigit addition and subtraction.

4. Current situation in Victoria.

A National Statement on Mathematics for Australian Schools (1991) suggests a dichotomy in that learning mathematics involves both its products and its processes. The document is divided into strands: some strands emphasising the body of facts concepts and skills to be learned while in other strands the processes or ways of knowing are emphasised.

In that same document the following statement is made:

> It is now widely accepted that learning is best thought of as an active process on the part of the learner. There are several important applications of accepting this simple proposition -
>
> - Learners construct their own meanings from, and for, the ideas, objects and events they experience.
> - Learning happens when existing conceptions are challenged.
> - Learning requires action and reflection on the part of the learner.
> - Learning involves taking risks. (Australian Education Council, 1991, pp. 16-17)

The emphasis in the National Statement on positive attitude, problem solving, communication and processes, illuminates the importance of connections amongst thoughts, feelings and actions.

The Curriculum and Standards Framework (CSF) for Mathematics in Victorian Schools (Board of Studies, 1995) does not include generic learning process strands. Processes are incorporated with content and concepts as they apply in each key learning area. This structure of the CSF, together with the assessment of skills through the operation of the Learning Assessment Project (LAP) on at least two occasions during the student's primary school years, clearly emphasises the product facet of learning rather than the process aspect. This has obvious implications that will undoubtedly influence the manner of instruction by teachers.

CHAPTER THREE

METHODOLOGY

A. DESIGN OF THE STUDY.

The study aimed to evaluate the importance of structured aids, and in particular *Blockaid*, as an educational aid to facilitate skill performance and understanding of numerical concepts in Mathematics.

Such outcomes would include not only the cognitive domain but also the affective. It was proposed that the evaluation data would be both quantitative and qualitative, that post-test and pre-test measures would be used for assessing performance on facts and skills, and questionnaires and interviews would be used to evaluate areas of understanding, attitudes and feelings.

In the planning the emphasis was on the cognitive domain, with an experimental design to test the hypothesis that the learning of concepts of number and of basic operations could be improved through the guided use of the *Blockaid* equipment.

Blockaid apparatus was to be used to represent numbers to 999 in three different forms - concrete, pictorial and abstract symbolic. These embodiments of the number concept are at different levels of representation, and are designed to assist children in their construction of number ideas. Labinowicz (1980) sees such representations forming a connection between objects in the real world and the abstraction and transformation of the properties of those objects.

The guided activities with the *Blockaid* materials were to be designed to map as closely as possible the same procedures that are carried out when performing the written algorithms of the four arithmetic operations, in the belief that such practice leads to understanding (Hiebert and Carpenter, 1992).

The design required the use of a control group and an experimental group that ideally would be randomly selected. As random selection of groups was not feasible the comparative methodology adopted was quasi-experimental (Burns, 1994).

1. Sample Selection.

The universe to which this study would have implications would be children in the early period of that stage of development termed by Piaget as the Concrete Operational stage (Copeland 1974) . The age range within this stage was approximately seven to nine years. The study was to be conducted in a school setting so the appropriate age range would be equated to that which is termed, in Victorian primary schools, the middle primary grades or grades three and four.

The population targeted was middle primary school children in the Western suburbs of Melbourne. From this population a representative sample was selected.

Sample size would comply with that recommended by Gay (1981) who cautions that 15 subjects per group would be regarded as a minimum for an experimental design with tightly controlled variables. Thirty subjects per group is generally recommended.

Ideally, in experimental research the subjects to be studied are randomly selected from the population and must be randomly assigned to groups, one of which will be the control group. This rigorous procedure is not involved in other methods of research. (Gay, 1981; Anderson, 1990). In the school setting, it is generally not possible for the experimenter to randomly assign subjects to groups (Burns, 1994).

The more practical solution in schools is to choose whole class groups. This constraint requires that the methodology of quasi-experimental design be adopted. This procedure is similar to the experimental design except that the subjects have not been randomly assigned to groups. With the involvement of control and experimental groups, the selection of such groups should be on the basis of matching them as closely as possible on the criteria of performance.

The schools from which such groups were to be selected should be representative of the population and chosen without bias. In order to improve the validity of the study, plans were initiated to replicate the study in a second school.

The choice of schools for this study was made by a process of deliberate selection. One school would be a Catholic primary school and the other a neighbouring State School.

Deliberate selection is a process whereby the researcher directly and deliberately selects specific elements of the population as the invited sample (Fox, 1969). The researcher was at this stage employed as a specialist mathematics teacher on a part-time basis at a Catholic primary school in the Western suburbs of Melbourne. The school was a smaller parish school with a total of about 270 pupils, in a suburb whose inhabitants were mostly of the lower middle class.

In the school there were two composite grade three and four classes with approximately 30 pupils in each. Students in these classes were assigned prior to the commencement of the 1994 school year from the various class groups of the previous year. They were chosen by the previous year's teachers on the criteria of mixed ability. Pupils were shared equally between the destination classes using a stratified sampling strategy to ensure a even distribution of pupils according to the characteristics of gender, age or grade level and ability. The grade four component of the two destination classes represented the whole population of children at that grade level in the school.

The number of grade three pupils to be placed in the two classes was small, so the decision was to select only boys. This was done in order to maintain some cohesiveness amongst the six or seven grade three members of the class. The selection was again stratified with pupils chosen systematically from the stratum according to perceived ability.

Only two of the pupils were born overseas, one in Hong Kong and the other in Brazil. Both were fluent in English and regarded as higher achievers. The majority of pupils were of Anglo Saxon origin with a mix from European and middle Eastern origins. There were no pupils falling into the category of special needs. One pupil however had 'Tourettes Syndrome'.

There was a bias factor in the sampling due to the placement of a small group of boys with a particular teacher because of behaviour problems. The researcher did not believe that this bias would have any significant affect on the overall balance of the two class groups in regard to ability. Although the number of grade three children in these classes was small, the overall balance of gender, grade numbers and ability appeared to suit the needs of the study to be undertaken.

The principal of the school was consulted and informed about the nature of the investigation. Tentative permission was given by the principal to use the two classes provided that the respective class teachers were also in agreement. Both teachers were consulted and approval to use both classes was granted. The teachers were fully informed of the comparative nature of the experiment in applying differing teaching styles to the teaching of particular mathematical concepts. They were aware that the classes would be pre-tested and post-tested to determine the effect of these different teaching strategies. They were shown copies of the tests.

Written permission was obtained from the principal for access to these classes for an hour each week for two full terms, or the equivalent, for the 1994 school year. Written authority was also obtained from the parents of all pupils involved in the study. The principal and the two class teachers concerned were supportive of the research project.

The designation of these two classes, one to the control group and the other to the experimental group was also made on the basis of deliberate selection rather than random choice determined by perhaps the toss of a coin. The selection was justified after having considered the characteristics of the two teachers as a possible extraneous variable that could affect the validity of the experiment. The deliberate choice of classes for each of the two groups was made in order that there would be no advantage to the experimental group.

Control Group. The class of thirty pupils chosen for the control group was composed of 10 girls and 20 boys. The grade three component of that class consisted of six boys. There were no transfers in or out of that class for the duration of the study. In this class there were a group of boys who were inclined to cause minor disruptions on occasions. The class teacher however seemed to have no great difficulty in obtaining a positive response from those same pupils on any of the occasions when the researcher visited the class. His position at the time of the investigation was Deputy Principal.

The experimental group consisted of a composite class of twenty nine grade three and four pupils being comprised of 10 girls and 19 boys. The grade three component of the class consisted of seven boys. One of the grade three pupils transferred from the school very early in term three. Two new grade four girls were enrolled in the class during the study period.

The experimental teacher was not as experienced as the teacher in charge of the control group. The principal of the school and the researcher had hoped that involvement in the study might assist the professional growth of this teacher.

The involvement of a further two groups to be used as control and experimental classes in a second school for the study was planned to ensure a balance in the sample representing the population and also to provide greater validity for the experiment. It would assist with some of the 'confounding' factors that could arise from extraneous variables that were to be controlled within limits. The term 'confounding' is used to refer to the fact that the effects of the independent variable may be confounded by extraneous variables such that it is difficult to determine the effects of each (Gay, 1981).

The deliberate choice rather than random choice of the second school was made for several reasons. A State School was chosen to ensure that the total sample was a better representation of the population nominated in the study. The choice seemed sensible because of the proximity of the chosen school to the Catholic school that had agreed to be involved.

The two composite classes at the grade three and four level had a good balance between grade levels and had been matched using stratified grouping strategies to balance the two classes on the criteria of gender, grade level and ability level to produce classes that could for the purpose of this study be regarded as more or less homogeneous. The researcher also knew the principal which would assist in gaining access to these classes. Finally, it was anticipated that some adjustment could be made to counterbalance the possible teacher effect in the other school.

Tentative permission was given at the beginning of 1994, by the principal of the State School, for two classes at the grade three four level to participate in the study. As discussed in Chapter Four this had to be abandoned.

Selection of sub-sample for interview. The initial plan, based on advice from colleagues, supervisors and Brian Doig at Australian Council of Educational Research, was to confine the interview to four students from the control group and a corresponding number from the experimental group. These were to be matched groups chosen by randomly selecting one child from each quartile according to mathematics ability determined by the battery of tests administered at the pre-study stage.

The selection would also consider the gender balance as well as being representative of the two grade levels that participated in the study. In order to achieve this the sample size needed to be increased to six. The subjects were to be randomly assigned from each hexile from the two class lists. Tabling of pupil names by gender and grade level were ranked in order according to those skills tested. (See Table 1)

Close examination of these tables facilitated the final choice of subjects for interview. The distribution of grade three pupils according to the tabled ranking was in fact similar at each hexile. This fact, combined with the preference to investigate children's understanding of number concepts at the early stage of middle primary school, resulted in the decision to choose the whole of the grade three population sample for interview. There were six children in each class at that grade level. All were boys.

2. Time Allocation.

As the experimental study involved interference in the regular schooling for all of the subjects involved, a degree of sensitivity in relation to the total time of exposure to the treatment administered on the subjects was required. The duration of treatment however needed to be of sufficient length in order to allow for the effect, that was hypothesised to occur, to be generated.

Total time for treatment was negotiated with the school principal and the teachers. An allocation of one hour per week for the two strategy classes extending over a period of two school terms or the equivalent being twenty weeks was regarded as a minimum time span for the experimental phase of the study. In the planning stage this equated to approximately four hours of supervised instruction for each of the sub skill areas to be investigated, namely - number ideas, addition, subtraction, multiplication and division.

CONTROL					EXPERIMENTAL				
		Boys		Girls			Boys		Girls
Pupil	Score	Gr.3	Grade 4		Pupil	Score	Gr.3	Grade 4	
1	72			X	1	65			X
2	72			X	2	61			X
3	66			X	3	61		X	
4	64		X		4	59		X	
5	62			X	5	58		X	
6	61		X		6	57			X
7	60		X		7			X	
8	57			X	8	51		X	
9	56	X			9	50	X		
10	55			X	10	48			X
11	54		X		11	47			X
12	53			X	12	46			X
13	51			X	13	44	X		
14	50		X		14	43			X
15	50		X		15	43		X	
16	49		X		16	42		X	
17	49			X	17	42		X	
18	47			X	18	42		X	
19	44		X		19	42			X
20	43	X			20	42	X		
21	42		X		21	41			X
22	42		X		22	40		X	
23	41		X		23	37		X	
24	40			X	24	36	X		
25	36	X			25	34	X		
26	35	X			26	29	X		
27	35		X		27	26			X
28	29		X		28	25		X	
29	16	X			29	14	X		
30	16	X			30				

Table 1. Rank order of pupils according to score on skills tested.

The time-table planned for the first ten week period is shown in Figure 7. It was to be used as the basis for planning for the second ten week period.

MATHEMATICS ACTIVITY PROGRAMME

MASTER LESSON TOPICS

Week 1. Number ideas 0 to 99 - beads on sticks and frames, *Blockaid*

Week 2. Addition

Week 3. Addition

Week 4. Subtraction

Week 5. Subtraction

Week 6. Number ideas - 0 to 999 - models for larger numbers

Week 7. Multiplication

Week 8. Multiplication

Week 9. Division

Week 10. Division

Figure 7. Time-table of topics for instruction for first ten weeks of program.

It was anticipated that the above topics would be part of regular instruction. On this basis it was agreed with the teachers that they would do some revision of the researcher controlled lessons each week.

3. Equipment.

The study involved the use of several structured learning aids designed by the researcher. Support material for these aids had been produced with the assistance of a co-author (Carney & Barrow, 1993). One of these, named Blockaid, utilises three different forms of representation in addition to the Hindu Arabic place value notation.

The first representation is value relations material by volume and is similar to Dienes' Multibase Arithmetic Blocks (Base 10) in its modelling of the Base 10 number system. The Dienes' Base 10 Blocks are currently used in many primary schools in Victoria.

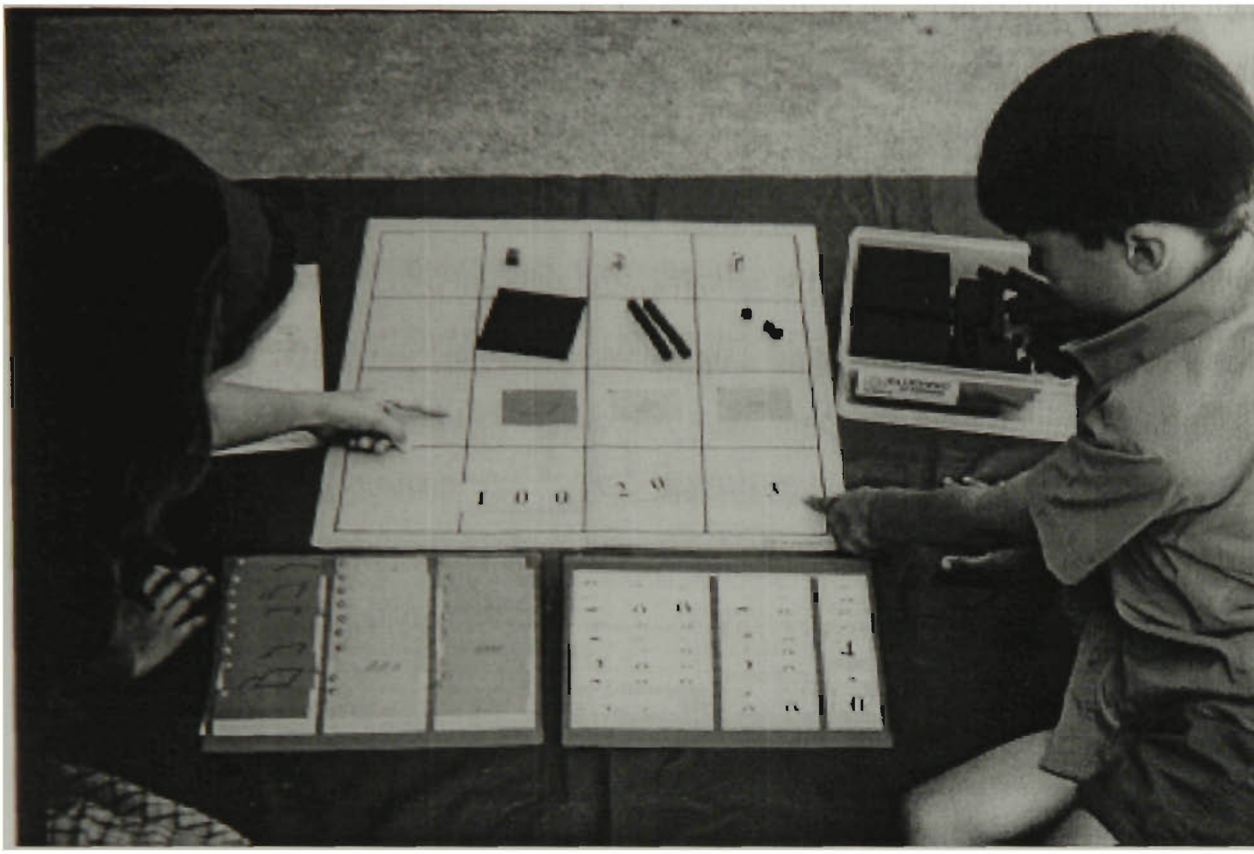

Photo 13. multiple embodiment or multi-unit structures for the number: 123.

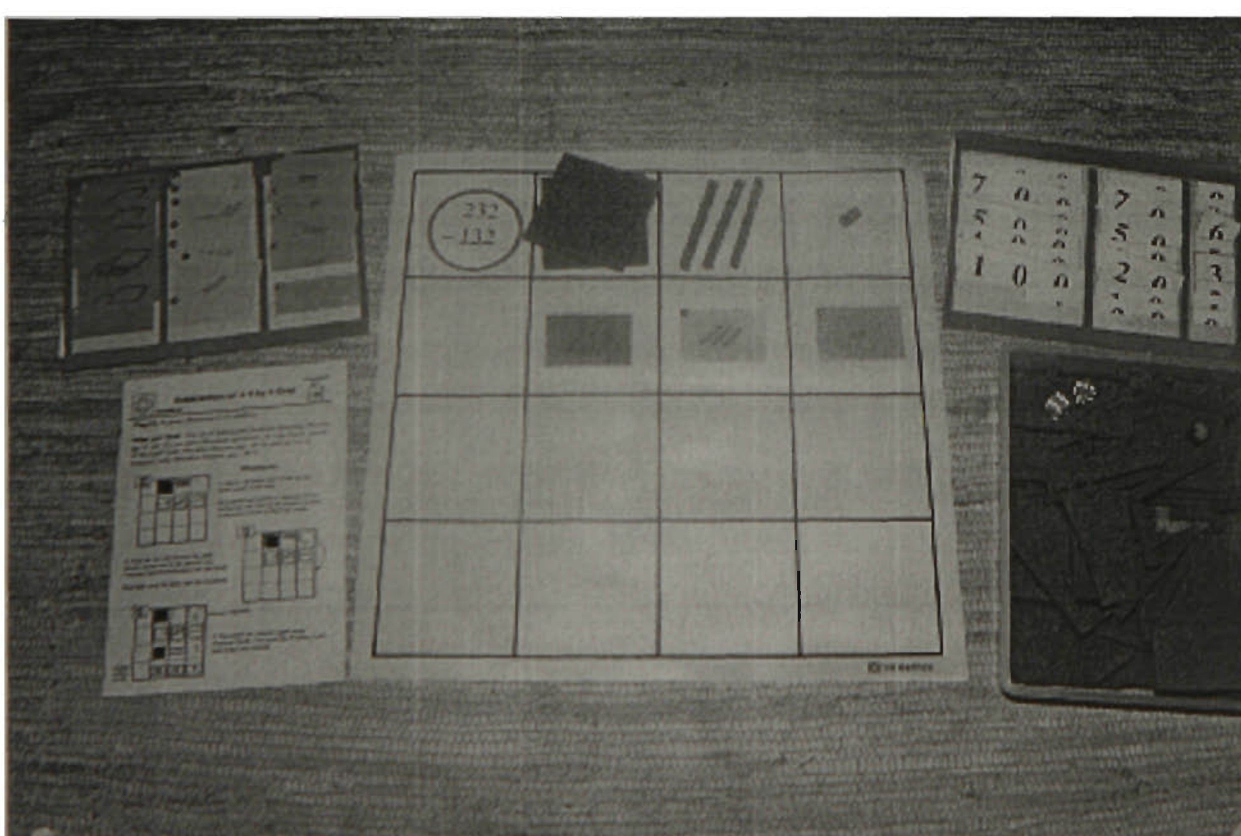

Photo 14. Blockaid materials set out for subtraction problem: 232 - 123.

However a special feature of *Blockaid* is the inclusion of intermediate pieces; one modelling the equivalent volume of five unit pieces and another piece with a value of fifty. This sub-grouping by fives, fifties, etc. was a characteristic of some ancient number systems and a feature evident today in Asian cultures where the soroban and suanpan are used as calculating devises.

The author's hypothesis is that this sub-grouping speeds up the task of constructing representations of number when using the pieces, and more importantly assists the visualisation and abstraction of the number concept. Extensions to the Blockaid models enabled numbers to be represented up to 99,999.

The second representation is in the form of pictorial cards illustrating the foam pieces. These cards are readily mapped onto the representations of number that are constructed using the foam pieces. The introduction of this alternate embodiment evolved from dialogue with children who were experiencing difficulty in performing the subtraction algorithm when using Base 10 materials. For subtraction problems using the Base 10 blocks, the steps in the procedure advocated by Dienes (1965) and by Sealey (1961b) and similar guides suggested representing both the subtrahend and minuend with the blocks.

The researcher had found that many children had great difficulty in manipulating the subtraction model in this manner and that this approach did not lead to understanding of the 'take' concept of subtraction that was intended.. A negotiated solution to the dilemma was discovered in a school where the design of the *Blockaid* material evolved. The solution was proposed by a child who suggested that the amount of wood representing the number to be 'taken' from the minuend could be drawn on cards to help in the mapping of the steps taken. In that school this became the approach that most children preferred in their modelling of the subtraction algorithm. The *Blockaid* material includes a complete set of isometric representations of the foam pieces.

The third representation consists of modified Montessori face value numeral cards from which numerals are selected according to the base ten groupings of hundreds, tens or units. (See Number Symbol Cards in Chapter 2) The set of cards that is part of the *Blockaid* Kit is designed to construct numbers to the limit of 999. If the child wishes to

Photo 15. Beads on sticks and frames material to show number 167.

Photo 16. Base 10 extension blocks to model the number: 79,486.

represent the number 386, the following cards are first selected - 300, 80 and 6. The cards are designed in such a manner that they can be superimposed on top of one another with the 300 card placed first then the 80 card on top of that then the 6 card. The number then appears as 386. The cards provide the opportunity for a synthesis and an analysis of numbers by making the connection between face value and place value ideas.

Blockaid is a complete teaching package consisting of the above mentioned apparatus, and also colour coded number die, a teacher manual, pupil work cards and blackline masters (Carney & Barrow, 1993).

Other apparatus designed along similar lines include :—

- Beads on Sticks and Frames;
- Abacaid - a modified spike abacus;
- Base 10 Extension to represent numbers to 99,999.

In addition to the above, decimal currency coins and notes were used as another support model for the base 10 number system and four operations.

4. Multi-variables in a Quasi-experimental design.

The purpose of the study was to determine the effectiveness of structured aids, and in particular *Blockaid*, in enabling children in middle primary school to acquire concepts of number and to construct the operational procedures of addition, subtraction, multiplication and division at the formal algorithm level. For each of these five skill areas the design would attempt to measure both mechanical performance and conceptual understanding.

A comparative study using a control group and an experimental group would be required to determine cause/effect relationships. The *Blockaid* activities hypothesised to make a difference in instruction will be referred to as the 'cause' or treatment, or as the independent variable. The difference, or 'effect', which will be determined to occur or not occur will be referred to as the dependent variable.

Ten dependent variables were monitored in the study. These are generated by the combination of first, skill performance and second, understanding for each of the five aspects of number and operations being investigated.

The skill performance would be equated to procedural knowledge, and understanding equated to conceptual knowledge as outlined by Hiebert (1986).

The major concern in the planning stage was the problem of isolating and controlling the independent variable to be manipulated and avoiding contamination that would negate internal and/or external validity.

The nature of treatment that would focus on the independent variable, was difficult to define because of its association with extraneous variables that intervene between the independent variable and the dependent variable. These extraneous variables would include social and linguistic factors.

The use of the *Blockaid* materials in the manner prescribed requires diversity of language negotiated between all the members of the class group of which the experimenter and the teacher would be members. Instruction associated with the *Blockaid* materials was to be in three different modes - class lesson, small groups involved in manipulating structured materials, and individual practice exercises. Instruction at the class level was to be used for guiding children through various procedures using a discovery approach that would lead to a consensus of a rules approach to be adopted. The major emphasis was to be the modelling the procedures with manipulative materials. However the corresponding procedures at the algorithmic level were to be demonstrated.

Part of each lesson was to involve the whole class divided into groups of three or four pupils. Each group would manipulate the *Blockaid* materials following the procedures prescribed. The procedures with materials have been designed to map as closely as possible the same procedures that are performed when performing the formal algorithms. Children would be given opportunities in each lesson to practice a few examples of the algorithms at the formal paper and pencil level.

Variable elements involved in treatment. An analysis of factors involved in using the *Blockaid* materials, based on the researchers own experience of involving children using structured aids, suggests four possible elements.

1. The richness of language that facilitates concept development.

2. The active participation of children working in small groups.

3. The teacher's attitude, knowledge and beliefs regarding mathematics and their attitude, knowledge and beliefs regarding the use of structured materials to improve children's construction of specific conceptual knowledge.

4. The modelling of procedures using manipulatives in a manner that maps readily onto those procedures that are prescribed for the formal algorithms. It is this connecting of embodiments of procedures that are meaningful to the child that leads to understanding (Hiebert, 1986; Hiebert & Carpenter, 1992).

It was be this fourth element, the use of mathematical models for instruction, that would be the one that would be manipulated as the independent variable. Every endeavour would be made to control the other three elements so that their effect could be removed from the equation in terms of 'cause'.

The linguistic/social factors in elements one and two above would need to be compensated for by adjusting the instructional program planned for the control group.

The third element, the teacher factor, was to be controlled partly by having the researcher adopt the role of teacher for both groups for at least one hour of instruction each week on the dependent variables being investigated. This dual role of researcher and teacher will at times be referred to in this thesis as the experimenter role. Further manipulation of the teacher factor was planned with an offer to provide professional training sessions related to mathematics and the alternate approaches to the teaching of computational skills. These sessions were to be offered to the teachers participating in the study.

A fifth element, namely motivation, could be added to the above list. While the role of motivation is considered to be of paramount importance by the researcher in all aspects of teaching and learning, it is his belief that its origins are derived from a mix of the above elements and also from elements within the child.

5. Factors of Reliability and Validity.

Reliability of data would be achieved by the use of procedures that lead to consistent results in the evaluation of the facts and skill aspects of the study. The reliability of the Clinical Interviews for assessing understanding of the concepts under investigation would be achieved by time-tabling the interview sessions for subjects in matched pairs from each hexile ranking of the control and experimental groups. This would compensate for any variation between strategies used in the earlier interviews and those used for the last interviews. All interviews would be conducted by the researcher to avoid the problem of intra-judge reliability that can lead to inconsistency of observations of the subjects.

The validity of the quasi-experimental procedures would be dependent on the level of control exercised by the researcher in manipulating the independent variable and controlling the intervening variables, to the extent that the independent variable could be attributed as the cause of observed differences that arise in the dependent variable, and not to some other variable.

Aspects that would need to be considered to insure internal validity would include —

- item validity of assessment tools;
- differential selection of participants;
- control of motivational factors;
- the Hawthorn Effect or novelty effect of receiving special treatment;
- bias of the researcher in either treatment or evaluation aspects.

The first three of these aspects have been already been accounted for in the study. The Hawthorne affect is a phenomena arising when the subjects' knowledge that they are involved in an experiment, or the feeling that they are in some way receiving special treatment, becomes the cause of change in performance. While the professional obligation requires that the participants be fully informed about the experimental nature of the investigation, the researcher would endeavour to avoid being viewed by the subjects as a dispenser of 'Special' treatment.

While not being able to completely control the influence of the Hawthorne Effect by the use of strategies such as the placebo treatment or the use of a blind group, attempts would be made to observe for evidence of its occurrence. Notes of such observations would be made if they arose.

In regard to bias by the researcher, it must be stated clearly that the *Blockaid* materials have been designed by the researcher with assistance from a colleague. The purpose for which the *Blockaid* was designed was to help children learn more efficiently. This study would attempt to investigate in an unbiased manner whether the purpose for which the materials were designed could be justified.

Aspects relating to external validity that could have ramifications for the study were -

- sampling problems and generalisability of results;
- pre-test treatment interaction;
- experimenter effects.

Problems relating to sampling have been dealt with in the section on sampling.

Pre-test treatment interaction occurs when the participants respond to treatment in a manner that has been affected by the pre-test. Because of the intensity of the battery of tests planned, the effect of such pre-testing would need to be monitored. Because both groups would complete the same battery of tests, any possible effect resulting from pre-tests should be the same for both groups under investigation.

Experimenter effects occur when the experimenter gives unintentional cues or messages which sensitise subjects to hypothesised outcomes. It can lead to artificial scores for the group that was expected to do well. Care would be taken to avoid giving cues that would sensitise either the control or experimental group.

B. PROCEDURE FOR DATA GATHERING.

Measurement instruments. The methods to be used to evaluate any differences arising from alternate approaches to mathematics instruction were to include observations, interviews, check lists, questionnaires and tests.

Evaluation was to focus largely on knowledge of facts and skills and of understandings in mathematics learning. However, in order to establish whether motivational factors between the control group and the experimental group were similar, an attitudinal questionnaire was to be administered to both groups.

Interviews aimed at assessing possible differences in levels of understanding between the control and experimental group were to be undertaken with the grade three pupils only. Particulars relating to this data are dealt with later in this chapter. The areas of assessment, together with the form of evaluation tool, are discussed as follows :

- Attitudes — Questionnaire;
- Knowledge of facts and skills — Validated test;
- Understanding — Clinical interview - normative.

1. Attitudes.

Instruction was to be provided in a manner that would attempt to control attitudes towards mathematics with both the control group and the experimental group, so that the motivational factors would not be a significant variable in influencing levels of performance at the end of the trial period. Since motivational factors play a major role in learning, any change or difference in this domain would need to be evaluated.

Two sources that had elements appropriate to this aspect of assessment were - Fraser's Class inventory (1989) and Clarke's impact questionnaire (1985). The latter instrument provides children with the opportunity to regularly share their successes and concerns with their teachers (Clarke, 1985). Both of these instruments were designed for groups of pupils that were older than those in this study. However some ideas were gained from these models and a questionnaire was designed. (See Figure 8)

The purpose of this instrument was to monitor what happened to the control and experimental groups over the time period specified. It was not be designed to measure attitudinal gains of one group over the other.

2. Knowledge of facts and skills.

The majority of data to be gathered and analysed would be evidence obtained from both the control group and the experimental group, as a result of pre-tests and post-tests. Advice was sought from Brian Doig and John King at the Australian Council of Educational Research (ACER) regarding suitable test instruments, and sampling size for the study. As a result of that advice, for the facts and skills area under investigation, the "Diagnostic Mathematics Profiles" would be used as a test for the four operations.

No suitable instrument was recommended to suit the particular needs for the investigation into number knowledge at the grade three and four level of primary school. A test instrument consisting of 20 items covering number ideas including, numeration and notation, grouping and place value was designed by the researcher. (See Appendix 1) This test would be administered at pre and post-study stages of the investigation.

Concern about basic number bonds knowledge prompted the search for a test for speed and accuracy of basic number facts. No suitable instrument was found that could be administered in a short time span. The need for this test was evaluated against the background of already planning for a battery of tests that would take approximately two and a half hours to administer. The use of this test would be kept open as an option. The tentative plans were for fifty items compiled by the researcher, to be completed in a period of about ten minutes.

WHAT DO YOU THINK ABOUT SCHOOL?

1. What do you like most about school?

....

2. What do you think is your best subject?

....

3. What subject do you think your teacher likes best?

....

4. How do you feel about your work in mathematics?

....

5. Rank the following mathematics skills in order from the ones you find the least difficult (put one next to it) to the one you find the most difficult —

Multiplication .....

Subtraction .....

Fractions .....

Division .....

Addition .....

6. Tick the one that you think is true for your grade

In mathematics
- the girls are better than the boys .....
- the boys are better than the girls .....
- there is no difference between girls and boys .....

7. What job or occupation would you like to have when you leave school?

....

Name .... Grade ....

Figure 8. Questionnaire on attitudes towards school and mathematics learning.

Thus there was a total of six different tests considered to be used to measure the performance of the mathematics skills under investigation. They fall into two categories and are listed as follows -

1. Whole number knowledge - two tests.
   - Number Knowledge
   - Speed and Accuracy
2. Arithmetic Operations - four tests
   - Addition
   - Subtraction
   - Multiplication
   - Division

The above battery of tests was to be administered prior to the study period to the control group and also to the experimental group. Copies of the tests for whole number knowledge appear in Appendix 1. The same battery of tests was to be administered at the end of the study period to both groups. Pre-tests and post-tests were to be identical.

3. Understanding.

The main instrument for assessing conceptual knowledge was to take the form of interviews. The nature of the questioning would follow the strategies of clinical interview employed by Piaget. In Piaget's clinical interview method, the adult viewer involves the child in conversation through verbally presented problems in the context of physical materials (Labinowicz, 1985).

Six pupils from each class would to be interviewed prior to the treatment period and at the end of the experiment. The school principal was fully consulted and had given written approval for the planned procedures. Parental permission had been obtained for those pupils for whom an interview was planned.

The purpose of these interviews was to probe the depth of understanding and attempt to gain insight into children's thinking strategies in relationship to basic concepts of number and of the concepts of the basic arithmetic operations of addition, subtraction, multiplication and division. Because of the wide scope of concepts being investigated, the planning of interview questions would need to be well structured but at the same time allowing sufficient flexibility to follow the paths of children's intuitive thinking where possible.

The major difficulty would be the capturing not only the words of explanation by which attempts would be made to assess conceptual knowledge, but also to have some record of non verbal communication, and in particular the actions performed by the children in their use of structured apparatus. Ideally the situation required a computer controlled video audio link that would maximise data retrieval. Such an approach would almost certainly require skill training and engagement of a research assistant. Video taping was regarded as a more intrusive approach to data gathering which may have created difficulties in gaining approval for the subjects chosen and thus was not pursued.

Check-sheets to facilitate the recording of both verbal and some non verbal responses by the children were prepared. These would be helpful in writing out the transcripts of interview and in the interpretation of children's thinking strategies and depth of understanding. Several drafts of proposed interview protocols were prepared and some pilot interviews conductedbefore the final format of interview, with accompanying check-sheets to assist with non verbal data, was chosen. Copies of these check sheets appear in Appendix 2.

The initial plan was to attempt to confine interview time to about one hour. However, based on pilot interviews, it was anticipated that most of the subjects participating would be interviewed for approximately one and a quarter hours. Audio tapes with playing time of 90 minutes were obtained and a compact, voice activated, tape recorder used.

It was not considered wise to conduct the pilot interviews in a space that was too private, and thus the only suitable space for the pre-study interviews was the staff room. Consultation with the principal confirmed the decision that the pre-trial interviews would probably have to be conducted in the staff room. Staff would be asked to keep noise levels to a minimum.

C. TREATMENT.

The major considerations in planning the treatment were as follows -

- The role of the researcher and of the class teachers;
- Knowledge content - number/operations;
- Cognition - procedural (skills)/conceptual (understanding);
- Influence of motivation;
- Teaching and learning strategies - Control/Experimental.

The role of the researcher and of the class teachers.

Permission was given for the researcher to take charge of the two class groups for one hour each week for a period of twenty weeks. The respective class teachers were expected to participate in all such sessions and to do some follow up with revision lessons. The researcher regarded himself as a participant in the learning process and in doing so attempted to provide some balance between the two strategy groups by being a common factor of influence in relation to mathematical knowledge.

It was anticipated that there would be professional growth achieved by the participating teachers and the experimenter. Some of the procedures to be followed during the experimenter controlled lessons would be adopted by the class teachers in review lessons.

The understanding with the participating teachers was that there would be follow up to these model lessons provided that such revisions did not interfere greatly with the weekly program of instruction planned by the teacher. The suggestion was that perhaps one follow up lesson or the equivalence of one hours instruction would be given each week. This was not to be regulated or evaluated for professional reasons.

Knowledge content - number/operations.

The content of instruction was to focus on the development of the concept of number and of the four basic operations of addition, subtraction, multiplication and division. The instruction in both groups was intended to develop not only performance in these factual and skill knowledge areas but also understanding.

The facts and skill aspects were to be equated with that aspect of cognitive development named by Hiebert and Wearne (1991) as procedural knowledge while understanding was named conceptual knowledge. Conceptual knowledge is built on procedural knowledge. Conceptual knowledge is rich in relationships and is associated with hierarchies or networks or webs.

Cognition - procedural (skills)/conceptual (understanding).

The approach to promote procedural knowledge was to be the same for both groups, but considerably less time was to be spent on drill and practice exercises by the experimental group in the experimenter controlled lessons. The reason for this was to test the hypothesis that the increased time spent on modelling the number and operational concepts with concrete materials that have meaning for the children at that age leads more effectively to understanding than traditional classroom practice. Such understanding leads to more permanent and meaningful learning.

The balance between drill and practice and the modelling with structured aids was considered in the planning and it was decided that the practice phase of instruction would be normally carried out by classroom teachers as part of normal weekly instruction and thus, for the experimental group, reliance was placed on the classroom teacher to do this as follow up lessons.

Demonstration of the approach to drill and practice of the facts and skills in number and operation were to be given by the researcher. The extent to which these same procedures were to be as closely followed by the experimental teacher as by the control teacher was a matter of conjecture. The amount of time spent with the control class would certainly give their teacher more opportunity to reflect on their teaching practice when using a drill approach.

An example of how difficult it is to break old habits in teaching was experienced by the experimenter during the study when instruction was being given in subtraction. The final refinement at the formal level was to follow the State-Operator-State (SOS) approach in order to maintain consistency in reading written forms of the subtraction algorithm whether they be in the horizontal or vertical format.

The equation or number sentence 6 - 2 = 4 was consistently read as "Six take two equals four". However when this same problem was written in the vertical form, the researcher in demonstrating on some occasions reverted to the language with which he had been taught, namely "Two from six equals four". The pupils however, particularly those in the control group with whom more time was spent on such matters, quickly noted the variation in approach and the preferred SOS method was adopted more consistently by the researcher.

Skill exercises for number and the basic operations were to be emphasised in the follow up lessons by the respective classroom teachers. For the refinement of the skills at the level of formal algorithms, a critical path analysis approach of searching for the most efficient procedures was advocated. This approach was based on the assumption that the goal of having children gain proficiency in basic arithmetic skills was an important aspect of mathematical learning. The search for efficient procedures should at all times be based on the children's ability to connect the component steps of the procedure in a manner that leads to understanding.

While a specific rule based approach to the arithmetical operations may seem too prescriptive, there is evidence from research that the construction of knowledge can be enhanced by giving guidance to the learner in the modelling of concepts and procedures through concrete experience. These procedures were to be presented in ways that map the same procedures by means of an algorithm at the formal abstract level (Fuson & Briars, 1990).

Influence of motivation.

While the degree of influence of the characteristics of the classroom teachers was a consideration in the design of the study, the researcher was also concerned about another variable that may have a significant effect on the dependent variable central to the study. The concern was the question regarding the influence of motivation arising from the different teaching strategies applied to the two groups.

The hypothesis was that small group instruction involving the use of various pieces of structured apparatus would promote more positive attitudes towards learning. This teaching strategy was an essential part of the treatment for the experimental group. If this hypothesis were true, then the change in attitude generated by the use of the materials by the experimental group would be a variable that would affect the performance levels of the subjects and thus render the experiment invalid.

Attempts to control the affective aspects of children's development, to the extent that such aspects would not be a significant factor in influencing the performance of one of the strategy groups more than the other, were made by modifying lesson plans.

As a result of these modifications, three types of instruction were to be given.

1. Class lesson to control group with emphasis on drill and practice.
2. Class lesson to experimental group with emphasis on understanding.
3. Group activities for both control and experimental groups involving the use of various pieces of structured materials.

The lessons were to be divided into four-weekly cycles with a total of five cycles. Each monthly cycle would involve both strategy groups using structured materials. Some of the structured materials used by the experimental group were to be conceptually linked directly to the concepts that were central to the study. Total time allocation for involvement with the use of materials by the control group would be longer for the experimental group. Time allocation for the specific topic content central to this study would be the same for both groups.

CHAPTER FOUR

TREATMENT

Introduction.

The different teaching strategies were applied to the two groups as planned. The experimental period began in the first week of term three of the 1995 school year.

The purpose of the treatment for the experimental group was to determine whether skills and understanding of concepts of number and operations were improved through the use of the *Blockaid* materials in a guided discovery approach to learning as advocated by Bruner.

Romiszowski's (1984) model of the expository/discovery dimension would suggest that, for the teaching of facts and skills, a direct or traditional approach to teaching would be a time-efficient approach. The teaching of the formal algorithms with understanding would have elements of procedures and concepts and thus a guided discovery teaching strategy would according to Romiszowski's model be appropriate. Attempts would be made with both the control and experimental groups to teach for understanding.

Understanding involves recognising relationships between pieces of information (Hiebert and Carpenter, 1992). It means building networks and making connections. For this study, the connections hypothesised to assist in leading children to understand number concepts were the links between the various forms of representation of the concepts. The representations with the *Blockaid* material include the tactile third dimensional models of the number system together with the pictorial cards and the face value numeral cards. Accompanying the use of the *Blockaid* equipment was the oral language that was at another level of representation. These multiple connections between representations were fundamental to Dienes' multiple embodiment principle (Dienes, 1965; Dienes & Golding 1971). They are also cental to the approach to the construction of multiunit conceptual structures for developing understanding of multidigit numbers and of operations advocated by Fuson (1990, 1992).

For the procedures of operations, the manipulation of the models was designed to mirror the same procedures as those carried out when performing the abstract algorithm. These links between concrete experience and the abstract were hypothesised to make learning more meaningful and to lead to understanding.

The teacher in charge of the lesson for both demonstration and application was the researcher (referred to as the experimenter). The class teacher was invited to participate in the negotiation process and was at times informed incidentally of features of the teaching style such as encouraging all children to brain-storm ideas without fear of making mistakes. Communication on these occasions was usually one directional.

One meeting was arranged with the participating teachers in the planning stage of the study, and the offer was made for follow up meetings. The offer was not taken up. Incidental involvement of the teachers during the experimenter controlled lessons was unfortunately the only time available to exchange ideas. At the completion of the experimental period a meeting was held with the whole staff to report some of the general findings.

The experimenter's role was at times viewed as that of a participant in the learning situation, grappling with the variety of children's interpretations of a particular problem and their construction of solutions. The social interaction in sharing the joy in those moments of discovery by individual pupils of the underlying concepts that the equipment was designed to embody was evident at times.

At other times the experimenter's role was that of a guide giving direction to the path, that through experience. had proved to lead to the desired goal in learning. Such directions were sometimes given by other child participants in the journey of discovery. At other times the directions leading to the skills desired were given by the experimenter.

Teaching strategies.

Different teaching strategies were applied to the control and experimental groups in order to determine if significant changes occurred in the knowledge of facts and skill, and of understanding, by the end of the experimental period.

The control group was taught in an expository or 'traditional' manner with instruction given from the blackboard by the teacher followed by paper and pencil practice exercises performed by the pupils. While the emphasis of this instruction was on skill performance there was at all times the endeavour to assist children in the understanding of the concepts taught.

The approach was not purely that outlined by Fox (1993) as Direct Instruction, but featured aspects of the method which he terms Meaningful Reception Learning. (See Figure 5, Chapter 2)

The experimental group were more active participants in the learning process and were involved in manipulating the *Blockaid* apparatus in either demonstration lessons with the whole class or in small group instruction.

Only a small amount of time was given to paper and pencil exercises in the lessons given by the experimenter. The expectation was that the practice phase would be given by the experimental group teacher in the same manner as that given by the control class teacher. The approach to instruction used with the experimental group was that categorised by Fox (1993) as 'guided discovery'.

Time-table.

Equal instructional time of four weeks was planned for both groups for each of the topics: number, addition, subtraction, multiplication and division. The topics were taught in that order with two successive weeks devoted to each topic for the duration of the ten weeks in that term, according to the time-table in Figure 7. The same time-table was followed for term four. However with disruptions to classes in the final weeks of term, and with the priority of interviews and testing to be carried out, only nine weeks of lessons were given by the experimenter in that term. The effect of this was that only three weeks of lessons were taken for division.

Control Group.

The teaching strategy used with the control group was what is generally termed a traditional or formal approach to instruction. The strategy involves the teacher demonstrating and describing the procedures to be followed, or the particular concept to be learned, and this then being followed by practice examples performed by the pupils and then corrected.

The instruction was to the whole class with no differentiation of examples given to the two grade levels nor to differing levels of ability. However, examples presented for both demonstration and practice were graded from simple to complex to cater for the wide range of ability.

The emphasis in instruction was on the procedure of the algorithm and on specific areas of difficulty such as reverse subtraction where the smaller number is subtracted from the larger number in each place value column in all cases, even when the subtrahend is larger than the minuend.

In the light of recent cross cultural research, (Stigler & Fernandez, 1993; Fuson, 1992) the balance between teacher instruction and pupil practice was modified to provide greater discussion and negotiation by the pupils to make the learning more meaningful. Oral language and written symbols were the means of representation of the concepts to be learned.

A single problem was usually presented first, followed by discussion between teacher and pupils. The diversity of approaches for a particular algorithm was generally narrowed to a single approach that was drilled by the whole class. While there was this aim for unification and simplification of the rules for the form of the final algorithm there was a deep concern at times by the researcher over lack of diversity, and lack of understanding, of basic concepts.

Group instruction through activities with materials.

The purpose of providing group instruction with materials to the control group was to bring about a balance between the two groups on motivational factors. Motivation would be significant in determining the effectiveness of instruction during the experimental period. The group activities with materials for the control group appear in Table 2.

Group Activities (Weeks 2 and 4)
1. Addition facts snap
2. Soma Cubes
3. Heinvetters number tutors
4. 3d Geo Shapes (construction)
5. Addition facts snap
6. Soma Cubes
3. Heinvetters number tutors
8. 3d Geo Shapes (construction)

Table 2. Group activities for control group.

The power of meaningful mathematics activities to motivate children in their learning of mathematics concepts is documented in much of the literature. The researcher's own experience has demonstrated that changes in children's attitudes do occur when children are involved in an activity based approach to mathematics learning with structured aids. This approach has involved the use of small groups, often in a games approach.

Experimental Group.

The emphasis in the lessons with the experimental group, was more on the process of learning, and the means by which children construct knowledge, than on the facts and skills to be learned.

The process of negotiation between teacher and pupils was aimed to allow greater flexibility in establishing links that are meaningful to each child where possible and thus, through such links, establish the connections between the procedures for performing a particular skill such as subtraction with structured materials and the corresponding procedures followed with the formal written algorithm. According to Hiebert and Carpenter (1992) the connections are made between the outer world of experience and the inner world of abstraction in the child's mind.

The hour lesson for each week was divided equally between whole class instruction and application in small homogeneous groups. For the latter, the class was divided into eight groups consisting of three to four pupils. As far as possible, each group had a spread of ability levels and a mix of gender and grade representation.

The first part of the lesson was with the whole class generally gathered in a circle and focused on the manipulation of concrete materials that were in the centre of the group. These materials were introduced to represent the number concepts, or the actions on those representations, that correspond with one of the four operations being learned at that particular time.

During these lessons the pupils were guided towards a set of procedures that resembled as closely as possible the procedures that were followed when carrying out the formal algorithm. When this rule-based stage of using materials was reached the use of materials was combined with the written procedure performed on large sheets of paper. The steps in the procedure with materials were followed or matched with the corresponding steps with the written algorithm in the manner advocated by Fuson (1992) and Carney and Barrow (1993). The procedures followed are very much based on models constructed by children in many schools over many years. The closeness of fit of these models for the various algorithms is a matter for closer investigation as a result of this study. The implications from this will be discussed in Chapter 8

It is this linking or connecting between the two parallel sets of procedures that Hiebert and Carpenter claim to be fundamental to teaching that leads to understanding. Multiple links lead to deeper levels of understanding. This presenting of a concept in a variety of ways that are in relationship to one another is central to the multi-embodiment principle advocated by Dienes (1965). There is however the danger of a mis-match between the model being presented and the concept or procedure.

The time-table provided for two one-hour class lessons on each topic each term. Part of the second lesson on each skill area involved a demonstration of the formal written algorithm on the blackboard with some follow up practice by the pupils. The content of these abstract exercises was connected to the content of the practical demonstrations as closely as possible. The examples presented were graded to cater for the range of ability within each of the two classes and across the grade levels.

The second half of each lesson with the experimental group involved the pupils working with concrete materials in small groups. The pupils were divided into eight groups and eight sets of materials were provided. The children rotated through the activities completing two activities each week. The activities for the first four weeks are set out in Table 3.

GROUP ACTIVITIES FOR EXPERIMENTAL CLASS	
TOPIC, NUMBER (Weeks 1 and 2)	TOPIC, ADDITION (Weeks 3 and 4)
1 Four ways game (*Blockaid* Card 4)	1. State Operator State Addition (*Blockaid* Card 5)
2. Addition facts snap	2. Addition facts snap
3. Make/break a block, (*Blockaid* Card 6)	3. Matrix Addition (*Blockaid* Card 7)
4. Soma Cubes	4. Soma Cubes
5. Four ways game (*Blockaid* Card 4)	5. State Operator State Addition (*Blockaid* Card 5)
6. Heinvetters number tutors	6. Heinvetters number tutors
3. Make/break a block, (*Blockaid* Card 6)	7. Matrix Addition (*Blockaid* Card 7)
8. 3d Geo Shapes (construction)	8. 3d Geo Shapes (construction)

Table 3. Group activity program for first four weeks.

Two of the group activities specific to the topic for the two weekly period were duplicated to provide a total of half of the activities. The duplicated activities for the topic of number during weeks 1 and 2 were the 'For Ways Game' and 'Make or Break a Block'. These topic specific activities were arranged in a manner whereby the children were able to complete both of them in the two week period devoted to the topic.

The group activities specific to number concept development and arithmetical operations procedures were aided by the use of the *Blockaid* workcards and also by the teacher's participation alongside the children in such activities. (See Appendix 3 - *Blockaid* cards for Number and Addition 4,5,6 & 7)

Other activities in the program relate either to the building of number bonds in addition and subtraction and multiplication or to other aspects of mathematics such as spatial relations. The inclusion of such activities was necessary because of the involvement of the control group in an activity based program. The inclusion of such activities also facilitated the implementation of the total program because the spatial relations activities such as construction with 3D geo shapes are activities that are generally self directing and require little or no supervision. Provision of these activities allowed the researcher and teachers of each of the groups to spend more time with those groups that required assistance in the number skills and operational procedures areas..

Discussion.

While the treatment applied to the two strategy groups was controlled as far as the limits of the situation reasonably allowed, it was suspected that the John Henry effect was influencing the study. The John Henry effect is the term used when a competitive element influences the outcomes of an experimental study (Anderson, 1990). By the end of term three, the researcher was aware of gains in performance by both groups. At that stage the control group responses in class to operational skills seemed to be much better than that of the experimental group. The level of improvement with this group seemed to greater than expected with this group for the time allotted to each topic.

In the seventh week of term four, when the topic of multiplication was reintroduced for the second series of two one hour lessons on that skill, the researcher noticed in the children's work books that the multiplication algorithm by multi-digit numbers had been taught to the pupils in the control class.

Multiplication by multi-digit numbers was not part of the planned instruction by the researcher and is not included in current curriculum statements, either state or national, at the Grade 3/4 level of primary school. There were items however on the pre-test instrument that involved the use of multi-digit multipliers. In hindsight it was unfortunate that these items were not excluded from the test schedule.

The teaching of this advanced multiplication skill by the control group teacher would probably improve the post-test performance of the control group students. The decision was made to teach multiplication by multi-digit numbers to the experimental group also. Instruction in this skill was still given to the control group.

As the researcher was not prepared for this dilemma, the value of the instruction given to both groups would certainly have to be questioned.

Although the researcher was attempting to be even handed with this 'confounding' factor the best solution would have been to exclude examples of multi-digit multipliers from the statistical analysis of results but this was not done.

Evidence of this lack of understanding of the basic concept of multiplication and of its oral and written interpretation was evident with the two groups in this study.

When the topic of multiplication was first introduced by the experimenter, the majority of pupils in both groups were confused about the nature of the multiplicand and multiplier in the reading of number sentences and the interpretation of the problem in the vertical algorithm form. The number sentence 6 x 2 = 12, was read variously as six twos, six times two or six lots or groups of two. Not one child volunteered the reading of the equation in a manner that was consistent with the definitions of multiplicand and multiplier given by de Klerk-Cougar (1983) in such equations. Those definitions require that the equation 6 x 2 = 12 be read as six (multiplicand) multiplied by two (multiplier) equals twelve.

It can argued that because of the commutative property of multiplication that it does not matter. In this school however it was found that the fixation on the reading of the equation from left to right was transferred to the reading of multiplication problems in the vertical algorithm. It is the researcher's firm belief that this phenomena is wide-spread and is a factor contributing to children's lack of understanding of the multiplication algorithm.

When 6 x 2 is written in the vertical form the 2 has to be the multiplier, but the pupils from both classes interpreted the 6 which was on the top line as the multiplier by reading the problem as 'six times two even if the problem was written as a problem in multiplying money amounts. $6 x 2 was read as 'six dollars times two is twelve dollars'. This erroneous approach was so wide-spread amongst the pupils in both the experimental and control groups that data on this phenomena was gathered prior to further teaching of the multiplication algorithm.

Since there is evidence that suggests that, conceptually, multiplication is perhaps the most complex of the four operations concerned in this study, and that the pupils involve in the study had difficulties with the concept, it was totally inappropriate that multiplication by multi-digit multipliers was taught first by the control group teacher, and subsequently by the experimenter. The researcher believes that this occurrence renders this aspect of the study on multiplication invalid.

Dickson, Brown and Gibson (1984) noted that a number of authors including Nesher and Katriel have pointed out that understanding of the meaning of the operations of multiplication and division is considerably more difficult than addition and subtraction. This complexity is reflected in the responses to children's ranking of their perceived levels of difficulty of the four arithmetic operations and also on their performance measured at both pre- test and post-test stages of the study.

The conceptual difficulties associated with multiplication are enlightened further by evidence from Brown, cited in Dickson, Brown and Gibson (1984). In Brown's study it was found that a greater percentage of 11 to 12 year old children were successful in identifying the division operation which matched the corresponding word problem, than was the case for multiplication. This suggests that, at least psychologically speaking, that the concept of division may be more readily understood than multiplication.

Subtraction. A further example of basic lack of understanding was witnessed when children in both the experimental and control groups were questioned about their interpretation of the equation 6 - 2 = 4. The dominant reading of the operational sign for subtraction was given as "take away" with a few interpreting it as "minus". In a lesson introducing the concept of subtraction there was not one child in either group who was able in the time given to see this equation as "the difference between six and two is four" or to relate a story of that concept.

The language of the subtraction algorithm that is used in most schools in Victoria reinforces the concept of subtraction as a "take away" concept and the "difference aspect" tends to be ignored.

Confounding Factors.

As discussed above, there were some problems that arose during the treatment period of twenty weeks that were beyond the control of the researcher. The term 'confounding' is used to refer to the fact that the effects of the independent variable may be confounded by extraneous variables such that it is difficult to determine the effects of each (Gay, 1981).

The replication of the use of a control group and an experimental group in a second school had to be abandoned due to the transfer of one of the teachers to be involved and also the principal of that school. This replication was planned in order to ensure a balance in the sample representing the population and also to provide greater validity for the experiment. It would help in identifying some of the 'confounding' factors that could arise such as the "teacher effect'.

The "teacher effect" almost certainly was a confounding factor in the study. A chance remark by the control group teacher at the end of the study left no doubt in the researcher's mind that a competitive element arose during the course of the investigation.

This element, that was suspected in the early stages of the study, undoubtedly led to an error of judgment on the part of the researcher in making a decision in the final weeks of the study to teach multiplication by multi-digit numbers to both the control and experimental groups in order to ensure that the experimental group were not disadvantaged.

Implications.

During the treatment phase, it seemed that some aspects of design could have been improved. These aspects included:

- Modification to the pre-testing and post-testing of skills to enable these to be carried out in a shorter time frame;
- A narrower focus on the content of instruction under investigation;
- A consensus reached between teachers and the experimenter regarding the content of instruction.
- More equitable involvement of participating teachers in the researcher's class lesson;
- Monitoring of follow up lessons taken by the classroom teacher;

The breadth of the study will have assisted in the evaluation of the strengths and weakness of the teacher support material in the form of worksheets and teacher manual.

CHAPTER FIVE

CHILDREN'S ATTITUDE TOWARDS MATHEMATICS

INTRODUCTION.

The questionnaire designed to monitor any changes of attitude resulting from the different teaching strategies applied to the two groups was administered as planned at the beginning and at the end of the experimental period. The same questionnaire was used on both occasions. It was administered after the testing of number and operational skills. A copy of the questionnaire appears in Figure 8.

The reason for administering the questionnaire was a belief that the treatment involving an activity approach might cause the Experimental Group to develop a more favourable attitude towards mathematics that would contribute towards improved performance in the skills tested.. Although instruction was devised for the Control Group to counter balance this, it was decided that some evaluation of children's attitudes should be made in order to determine whether these modifications to the instruction program were successful.

It is important to note the gender mix of the two class groups with the significantly higher number of boys in each. For questions Numbers, 2, 3, 6 and 7, the responses will be analysed to show the differences in responses according to gender

COMPLETED QUESTIONNAIRES							
CONTROL				EXPERIMENTAL			
	Boys	Girls	Total		Boys	Girls	Total
Pre-study	19	10	29	Pre-study	18	10	28
Post-study	18	9	27	Post-study	16	10	26

Table 4. Number of pupils completing the questionnaire.

The responses from the questionnaires do suggest that for these two groups of students with these teachers and at this particular time, that there were differences in attitude before the study and marked differences in attitudes at the end of the study between the groups. These differences have important implications for this investigation.

QUESTION 1. What do you like most about school?

Aims.

This question aimed to encourage children to record their feelings and attitudes towards various aspects of school in the questions that follow. Its purpose was also to determine aspects of school life most liked by the children concerned, and to determine whether there were differences between the two groups at the pre and post-study stages.

Comparison of control and experimental responses.

This open ended question elicited very diverse responses. The number of answers worded differently on either the pre-study or post-study for each of the two groups was approximately the same as the number of respondents in the respective groups. There were 30 different responses from the Control Group and 28 for the Experimental Group.

The range of responses was classified under five broad headings: Play, People, Buildings and Grounds, Subjects and Other; and a separate heading for Mathematics.

The 'play' heading incorporated the following - playtime, play, lunch-time, home time, big lunch, playing, recess, home bell, lunch, playing down-ball, playing games. Sports responses were included under this heading rather than under the heading Subjects.

The 'people' responses included, friends, my teacher, being with friends, making friends, having a good teacher, my teacher reading books to us, teachers and Miss X. (teacher's surname)

The impact of change to buildings and grounds that were being upgraded for the duration of the investigation was evident in a few responses. For the two classes involved, the pupils had already moved into new class rooms. Responses relating to this included, playground, new sink in room, the colours (of building interior), new classroom, new school, my classroom.

The total responses for those nominating mathematics as the 'most liked thing' was five out of the full total of 107 responses.

Pre-study responses to Question 1.

The responses classified as play are slightly greater in number for the Control Group than the Experimental Group. (See Table 5a)

The Experimental Group's subject oriented responses as grouped are much higher than those of the Control Group. If the sport responses were regrouped, for example, four for the Control Group and one for the Experimental Group; from the play heading to the subject heading, the overall balance between the two groups would be similar.

On the pre-study questionnaire the only child choosing mathematics as the thing most liked about school was a grade 4 girl in the Experimental Group.

The Other responses for the Control Group were, free time (in school), work, learning, excursions and 'good', and for the Experimental Group - can learn in class, library, knowing I can get a job, weekly check ups and learning.

	CONTROL	EXPERIMENTAL
Play	15	12.5
People	4	2
Buildings, grounds	1	1
Subjects excluding Mathematics	3	6.5
Mathematics	0	1
Other	6	5
TOTALS	29	28

Table 5a. Pre-study responses to question. What do you like most about school?

Post-study responses to Question 1.

The number of responses for 'Play activities' is greater for the Experimental Group than the Control Group at the Post-Study stage. Although small in number the expressed liking of mathematics by three pupils in the Control Group is of special interest. The responses may of course be due largely to chance or perhaps the 'teacher pleaser' effect, but this evidence of an increase in the number of pupils liking mathematics, by the end of the study period, combined with other data such as marked improvement in scores is worth noting. Only one child from the Experimental Group nominated mathematics as the most liked aspect of school.

	CONTROL	EXPERIMENTAL
Play	13.5	17
People	4	3
Buildings, grounds	3	1
Subjects excluding mathematics	2	3
Mathematics	3	1
Other	4	1
TOTALS	27	26

Table 5b. Post-study responses to question. What do you like most about school?

Comparison of pre-study and post-study responses.

The dominant feature of the grouped responses for the four sets of answers to this question was the importance of activities which have been classified under the heading of play. The totals for all of those responses show that 58 of the 110 responses or 53%, were testimony to children's expression of play as the 'thing' most liked about school.

If the play responses are combined with people responses, this non-formal schooling aspect becomes even more pronounced. Montessori (1912) advocated the learning of concepts be through a games or play approach. Most educational theorists recognise that learning takes place in a social context (Miller, 1983). This is the approach to mathematics learning through activity in a social setting that is being assessed in this study. It was also the prediction of this effect of a play or activity approach that led to compensatory lessons being provided for the Control Group.

The total number of pupils nominating formal curriculum subjects as the aspect of school most liked was 19.5 of the 110 responses or 18%.

The changes in responses related to play and to the formal school subjects are evident in Table 5c which shows the aspects of schooling nominated by each group as a percentage of the respective groups. In this table there is evidence of changes in attitude between the time of the pre-study questionnaire and the time of the post-study questionnaire.

For the Control Group the figures show a slight decline from 52% to 50% nominating aspects of school grouped under 'Play' while at the same time there was a substantial increase from 10% to 19% in pupils nominating formal subjects.

For the Experimental Group the figures show an increase from 45% to 65% in those nominating 'Play' and a decline from 27% to 15% in those nominating formal subjects. These changes would seem to be significant.

CONTROL			EXPERIMENTAL		
	Play	Formal subjects		Play	Formal subjects
Pre-study	52%	10%	Pre-study	45%	27%
Post-study	50%	19%	Post-study	65%	15%

Table 5c. Comparison of children's liking for 'play' and 'formal subjects'.

Implications.

The changes in the figures for those nominating mathematics as the most liked aspect of school is perhaps the most important evidence, obtained from this question, of changes in attitude on the part of the Control Group. There were no children from the Control Group who nominated mathematics as the most liked aspect of school at the beginning of the study. There were three pupils from that group that nominated mathematics at the end of the study. For the Control Group the only other subject specific aspects of children's 'likes' were for science and language with one child nominating each area on the post-study questionnaire.

The corresponding figures for the Experimental Group suggest there was no change in attitude towards mathematics between the time of administration of the two questionnaires, with only one child nominating mathematics on each occasion.

While aware of the danger of reading too much into these small numbers of responses for mathematics, the other danger would be to discount them as insignificant. It seems likely that the attempts to motivate both groups at a similar level through providing mathematical activities to both groups, may have been more successful with the Control Group. This matter has been dealt with more fully in Chapter 3.

The responses for two of the three children nominating mathematics in the Control Group related to the mathematics activity component of the study. Their responses were either playing maths games or maths games.

Although most of the diverse responses given for this question would appear to have little direct relevance to this study on children's learning in mathematics, they do give insight into children's ways of getting to know the world around them. This has implications for teaching.

QUESTION 2. What do you think is your best subject?

Aim.

The purpose of this question was to asses children's beliefs about their performance in regard to the curriculum subject areas and in particular to determine any differences in beliefs of the two groups relating to mathematics performance.

Comparison of control and experimental responses.

The totals of responses for mathematics being chosen as best subject for the control and Experimental Groups both at the pre-study and post-study was higher than expected. At the pre-study stage the percentage of pupils nominating mathematics as their best subject was nearly 50% of the combined control and Experimental Groups. This percentage dropped to 47% at the time of the post-study questionnaire. The possibility of responses being influenced by children responding in a manner that pleases the author of the questionnaire, cannot be discounted.

Notwithstanding this there are variations between the responses of the two groups on each of the two occasions that they were asked to complete the questionnaire that are worth noting.

Pre-study responses to Question 2.

At the pre-study stage the choice of Mathematics as the best subject was made by 12 subjects of the 29 respondents in the Control Group and 16 of the 28 respondents in the Experimental Group.

By combining the totals of children's responses for either Mathematics or Language we see 16 of the 29 Control Group respondents choosing one of those two core subjects, and 21 of the 28 for the same combined choice for the Experimental Group.

PRE-STUDY							
CONTROL				EXPERIMENTAL			
	Boys	Girls	Total		Boys	Girls	Total
Maths.	8	4	12		10	6	16
Language	1	3	4		4	1	5
Science	5	2	7		1	0	1
Art	4	1	5		0	0	0
Sport	1	0	1		0	0	0
Other	0	0	0		3	3	6
	19	10	29		18	10	28

Table 6a. Pre-study responses to Question 2. What do you think is your best subject?

Post-study responses to Question 2.

At the end of the study, 13 of the 27 children in the Control Group chose Mathematics as their best subject while the corresponding figures for the Experimental Group were 12 of the 26 pupils responding.

The high proportion of respondents nominating traditional core subjects was very pronounced at this period with 22 of the 27 Control Group making the choice of either Mathematics or Language, and for the experimental 24 of the 26 making the choice for those same subjects.

POST-STUDY							
CONTROL				EXPERIMENTAL			
	Boys	Girls	Total		Boys	Girls	Total
Maths.	11	2	13		6	6	12
Language	5	4	9		8	4	12
Science	2	1	3		1	0	1
Art	0	1	1		0	0	0
Sport	0	1	1		0	0	0
Other	0	0	0		1	0	1
	18	9	27		16	10	26

Table 6b. Post-study responses to Question 2. What do you think is your best subject?

Comparison of pre-study and post-study responses.

The period of the study was a period when the Learning Assessment Project proposed by the Ministry of Education in Victoria was being debated amongst educators. The proposal was not popular with many teachers and was a matter of contention in this school. The final decision for the school was for teachers to participate with the Learning Assessment Project in 1995. The grade 4 component of the two composite classes would be tested the following year with the Learning Assessment Project.

The question arises whether this decision was accompanied by an increased emphasis on the key learning areas of Mathematics and Language by the teachers. The researcher believes that the emphasis did change with the teachers but in markedly different ways between the control and experimental teachers. One would expect any such change in emphasis would have an affect on pupil attitudes towards the subject areas that they see themselves best at.

Control Group.

For the Control Group, the main trend over the time period studied was a move of choices away from Science and Art towards the core subjects of Mathematics and Language with the latter gaining the most ground.

The number of children in the Control Group choosing Mathematics at the end of the study had increased marginally from 41% to 48%. The Control Group at that time had two fewer respondents. (See Table 6c)

It is of interest to note that at the start of the study the ratio of boys to girls in their choice of mathematics is in close proportion to the gender ratio in the class. The mix of girls and boys choosing Mathematics as 'best' changed considerably in the post-study responses in comparison with the ratio of the gender mix in the class. (See Table 6a and 6b)

	CONTROL	EXPERIMENTAL
PRE-STUDY	12 (41%)	16 (57%)
POST-STUDY	13 (48%)	12 (46%)

Table 6c. Frequency for choice of Mathematics as best subject of respondents.

<u>Experimental Group</u>.

For the Experimental Group there was a drop off in the choice of Mathematics as the best subject and also the miscellaneous responses which included the following - Italian, religion, library and 'going on messages'. As a result Language was the subject chosen as best. The number of pupils responding in this manner moved from 5 to 12.

The figures for the Experimental Group indicate a decrease in the numbers nominating Mathematics as their best subject from 57% to 46%. The number completing the questionnaire had declined from 28 pre-study to 26 post-study. (See Table 6c)

Implications.

If the figures in Table 6c represent a true picture of change of attitudes and beliefs towards mathematics of the respective groups, then this has relevance to the investigation due to the link between the cognitive and affective aspects of intellectual development. The changes in pupil attitude seemed to be linked with children's perception of teacher attitude towards mathematics as evidenced by responses to Question 3.

QUESTION 3. What subject do you think your teacher likes best?

Aim.

This question was worded in a manner that attempted to gauge the children's perception of the teacher's feeling towards the subjects they teach. The important word here is 'likes'. A similar question might have been worded; "What subject do you think your teacher enjoys most?"

Comparison of control and experimental responses.

The pupil responses to this question reflected major differences in regard to children's perception of their teacher's attitude towards various subject areas. The responses between the two groups and the change on emphasis between the two time intervals were highly significant to this investigation. There was the danger that, for some children, the interpretation of this question may have been influenced by the previous question that aimed to focus on pupil ability rather than feelings.

Pre-study responses to Question 3.

The Control Group's pre-study responses to this question showed a spread of subjects liked best by the teacher. These included Mathematics, Language, Science, Art, Sport and under Other (homework, religion and teaching the class).

The dominant "like' for the Control Group was for Science with a total of 14 students designating that subject as the one they see as their teacher liking best. This was followed by Art nominated by five students. This teacher had a reputation for his interest in science. He had a habit of bringing to school a mystery box with some object or specimen to observe or manipulate in a scientific manner.

The Mathematics and Language combined were seen by five pupils in the Control Group as being the teacher's 'best' subjects in terms of enjoyment. The balance between these was in favour of Mathematics.

Pre-study responses from the Experimental Group for this question were more narrowly focused with responses as follows, Mathematics 11, Language 12, and Other 5. The Other responses included Italian, handwriting, religion and learning.

CONTROL				EXPERIMENTAL			
	Boys	Girls	Total		Boys	Girls	Total
Maths.	1	1	2		8	3	11
Language	2	1	3		6	6	12
Science	8	6	14		0	0	0
Art	3	2	5		0	0	0
Sport	2	0	2		0	0	0
Other	3	0	3		4	1	5
			29				28

Table 7a. Pre-study responses - What subject do you think your teacher likes best?

Post-study responses to Question 3.

Answers given to the same question at the end of the year indicated that more than half the control class believed their teacher enjoyed either Mathematics or Language best with a combined tally of 14, being comprised of nine choosing Mathematics and five Language. There was a spread of other choices as follows Science, 6; Art, 5; and Others, 2. One of the 'Other' responses was actually a 'No Reply' while the second was for General Studies.

CONTROL				EXPERIMENTAL			
	Boys	Girls	Total		Boys	Girls	Total
Maths.	5	4	9		4	1	5
Language	5	0	5		8	9	17
Science	5	1	6		0	0	0
Art	2	3	5		0	0	0
Sport	0	0	0		0	0	0
Other	2	0	2		4	0	4
			27				26

Table 7b. Post-study responses - What subject do you think your teacher likes best?

End of year responses for the Experimental Group were more narrowly focused on Mathematics with a tally of 5 and Language a tally of 17. The only other subjects nominated were combined under 'Other' with a tally of four. These 'Other' responses were Italian, religion, handwriting, and teaching jokes.

Comparison of pre-study and post-study responses.

The major shifts in the responses given by the children regarding their perceptions of teacher attitude towards mathematics was a factor that has had a significant influence on most of the results of the investigation.

The influence of teacher attitudes on pupil attitudes and performance is well documented in the literature. (Dienes, 1964; Montessori, 1912; Booker, Briggs, Davey & Nisbert, 1992; McLeod, 1992). This phenomenon will be referred to as the 'teacher effect'.

Control Group.

The 'teacher effect' was obvious in the above figures for science for the Control Group. Science was seen by 48% of pupils at the pre-study stage as the subject liked best by their teacher . At that same time 24% of pupils nominated that subject as their own best. (See Table 7c)

The post-study results for science indicated a marked change in the responses given to those same questions with the teacher's 'best' liked subject falling to 22% and the pupil 'best' falling in direct proportion to the pre-study figures by dropping to 11%.

	PRE-STUDY	POST-STUDY
Teacher 'best'	14 (48%)	6 (22%)
Pupil 'best'	7 (24%)	3 (11%)

Table 7c. Control Group responses relating to Science as ' best '.

There is a similar effect with the figures for Mathematics. While the increase from 7% to 33% for responses for the control teacher's 'best' are not matched with a proportional increase in 'pupil best' nevertheless there is an increase between pre and post-study stages for the latter from 41% to 48%.

The researcher has no evidence in the field notes that indicate that there was any change in emphasis from science to mathematics instruction by the control teacher between the time of the pre-study questionnaire and that which was given at the end of the study, but there was other evidence that suggests that the emphasis did change. Mention is made in Chapter 4 about the inappropriate teaching of multiplication by multi-digit multipliers.

	CONTROL		EXPERIMENTAL	
	Pre-study	Post-study	Pre-study	Post-study
Teacher 'best'	2 (7%)	9 (33%)	11 (39%)	5 (19%)
Pupil 'best'	12 (41%)	13 (48%)	16 (57%)	12 (46%)

Table 7d. Frequency of choice for Mathematics as 'best' subject.

The researcher believes that the combined effect of changes in pupil attitude shown in Table 6c and perceived teacher attitude as shown in Table 7d. played a major part in improving performance that favoured the Control Group.

<u>Experimental Group.</u>

The teacher effect was even more pronounced with the Experimental Group where a decrease in responses for teacher and pupil 'best' is seen. While Mathematics, nominated as the subject 'best liked' by the teacher, fell from 39% to 19% the corresponding figures for the student's 'best' subject fell from 57% to 46%.

<u>Implications.</u>

The above table suggest major shifts in children's perception in regard to their teachers' attitude towards mathematics between the time of the pre-study questionnaire and the post-study questionnaire. It is the researcher's belief that these shifts are not a product of his differential treatment during the experimental period but rather are the result of changes in emphasis on the part of the participating teachers.

The increased percentage (from 7% to 33%) of pupils in the Control Group who regarded mathematics as their teacher's best subject was accompanied by an increase in that subject being nominated as the pupil's best. The decrease in the percentage (39% to 19%) of pupils in the Experimental Group that believed mathematics was their teacher's best subject was accompanied by a decrease in the percentage of pupils nominating that subject as their best. (See Table 7d)

The researcher believes that the 'teacher effect' has undoubtedly had a major influence on children's attitude and performance in mathematics during the course of this study.

QUESTION 4. How do you feel about your work in Mathematics?

Aim.

The purpose of this question was to attempt to establish whether there were positive or negative attitudes towards mathematics by the students participating in the study.

Comparison of control and experimental responses.

Analysis of the responses would have been much easier if the responses were limited to a scale of fixed alternatives as outlined by Burns (1994). This option however would not have revealed the subtleties of feelings that resulted from the more open-ended approach adopted.

This open-ended question resulted in a wide variety of responses. A total of 20 different responses were used by the Control Group for either the pre-study or for the post-study responses. For the Experimental Group the corresponding number of different responses on the two questionnaires was 14.

The most frequent responses for the Control Group were: good, O.K. and all right. For the Experimental Group the 'good" response was the most common followed by 'O.K.' and a few choosing 'easy'.

Amongst the more interesting responses for the Control Group were: impressed, fun, I like Maths, I should improve, and dumb. Responses from the Experimental Group included 'great' and 'comfortable'.

In analysing these responses, the researcher classified the full range of responses into five different categories, four of them being scaled from Very Good, Good, O.K. through to Not so good.

The fifth category was termed 'Other' to include those responses for which allocation to one of the scaled categories was not as clear cut.

Any interpretation of the data summarised in the tables following, was made with serious reservation because of the high degree of latitude in the meaning of the words most frequently used, particularly, good, O.K., and all right.

For the researcher O.K. and all right seem to have a negative connotation.

Pre-study responses to Question 4.

On the pre-study questionnaire the combined 'Good' and 'Very Good' categories totalled 13 for the Control Group and 18 for the Experimental Group. These figures correlated closely with those for the responses to the pre-study question, 'What is your best subject?' where the majority of the Experimental Group nominated Mathematics as their best.

	Very Good	Good	O.K.	Not so Good	Other	Total
Control	4	9	12	1	3	29
Experimental	1	17	3	1	6	28

Table 8a. Pre-study responses - How do you feel about your work in Mathematics?

Post-study responses to Question 4.

Post-study responses categorised under 'Good' and 'Very Good' totalled nine for the Control Group and sixteen for the experimental. The method of scaling of responses indicated a more positive attitude towards Mathematics held by the Experimental Group than the Control Group.

This result seemed to be in contrast with the results for performance on the four basic operations in mathematics where the Control Group gained higher marks than the Experimental Group in three out of the four tests for basic operations in Arithmetic.

	Very Good	Good	O.K.	Not so Good	Other	Total
Control	3	6	12	2	4	27
Experimental	2	14	6	2	2	26

Table 8b. Post-study responses - How do you feel about your work in Mathematics?

From the scale of responses tabled there appeared to be for both groups a shift in attitude away from the positive end of the scale, between the time of completion of the first and second questionnaires.

Comparison of pre-study and post-study responses.

Control Group. With the Control Group, the scaling of responses resulted in the following frequency for each category: Very Good, 4; Good, 9; O.K., 12; and Not so good, 1 at the pre-study stage and correspondingly, 3; 6; 12; and 2 at the post-study period. The respective responses categorised as 'other' were pre-study, 3 and post-study, 4. (See Tables 8a and 8b)

The decline in responses for Very Good and Good between the pre and post study stages contrasts with the improved performance between the pre and post-tests which when analysed using the t-test, revealed a statistically significant difference between the two sets of scores for the four skills of operations. (See Chapter 6)

	Very Good	Good	O.K.	Not so Good	Other	Total
Pre-study	4	9	12	1	3	29
Post-study	3	6	12	2	4	27

Table 8c. Control Group - How do you feel about your work in Mathematics?

Experimental Group.. The scaling of responses for the Experimental Group at the pre-study stage was as follows - Very good, 1; Good 17; O.K., 3; and not so good, 1 and similarly, 2; 14; 6; and 2 for the post-study questionnaire. The number of responses tabled under 'Other' declined from 6 on the pre-study questionnaire to 2 post-study.

	Very Good	Good	O.K.	Not so Good	Other	Total
Pre-study	1	17	3	1	6	28
Post-study	2	14	6	2	2	26

Table 8d. Experimental Group - How do you feel about your work in Mathematics?

Implications.

If the 'Good'' and 'Very good' categories were interpreted as positive; and the 'O.K.', neutral; and the 'Not so good' as negative, then the overall attitude towards mathematics for the two groups interpreted from the questionnaires pre and post-study was distinctly positive.

The result from the method of scaling of responses used in this study, suggests that those in the Control Group are generally less positive in their feelings towards mathematics than those in the Experimental Group at both stages of the study.

Perhaps this apparent difference may have been more a product of the overlap of meaning of the words good and O.K., as expressed by the pupils and the ordering of interpretation of those same words by the researcher.

This trend away from positive feelings about mathematics was contrary to the trends towards improvement in performance exhibited by both groups for all areas tested in that subject. It was also at variance to the general trend of responses regarding attitudes towards mathematics that are evident in Questions 2 and 3.

The researcher can reach no conclusion from this data. Perhaps increasing concern or anxiety as a result of emphasis on performance on the number and operations tests may have been a factor in causing a trend towards less positive attitudes towards mathematics for both strategy groups.

QUESTION 5. Rank the following Mathematics Skills in order from the one you find the least difficult (put 1 next to it) to the one you find the most difficult (5).

- Multiplication ....
- Subtraction ....

- Fractions ....
- Division ....
- Addition ....

Aim.

The prime aim of this item on the questionnaire was to gauge children's perception of the relative order of difficulty of the four basic operations. The inclusion of the concept of fractions was an attempt to determine whether there was any connection between that concept and the partition aspect of division to which it is linked.

Comments.

Fractions, although ranked generally at the higher order of difficulty, was not the most frequently ranked concept at the fifth level of difficulty. This finding applied to both groups at the pre-study and post-study stages of the investigation. This result was surprising to the researcher.

Equally surprising was the fact that fractions were rated the least difficult of the concepts listed by four pupils on the pre-study questionnaire and by two pupils on the post-study questionnaire. If we compare this ranking with addition at the other end of the scale for the topic rated least difficult, we find that no child in any of the questionnaires answered rated that topic most difficult.

The researcher's belief is that there may be at least two influences that have had some bearing on the responses given.

First is the nature of children's interpretation of what is meant by fractions. This of course could vary greatly from ideas about simple vulgar fractions such as halves and quarters through to more complex concepts of operations on fractions both vulgar and decimals, and even the fraction as an operator.

The second influence would be in the nature of instruction given on this topic by the teacher and perhaps even the influence of activities with structured materials by children in the Control Group during the investigation.

Comparison of control and experimental responses.

In refining the results of this item, the topic of fractions was excluded from the items ranked by the pupils. The results of this ranking appear in Tables 9a and 9b.

Pre-study responses to Question 5.

At the pre-study stage the overall frequency of tallies for the ranking of the four operations in order of difficulty was very similar for both the control and Experimental Groups.

The tally of responses indicating the increasing order of difficulty resulted in the majority of children in each of the two classes naming Addition at the first level, Subtraction at the second level, Multiplication at the third and Division at the fourth level.

The order of difficulty from least difficult to most difficult may be abbreviated at times to simply order of difficulty.

The highest tallies taken from the Control Group for each level of difficulty were as follows - First, 24 for addition; second 18 for subtraction; third 16 for multiplication and fourth 22 for division.

This ranking tally was similar to that of the Experimental Group at the beginning of the study with the four operations in the same order tallying: 25 for addition, 16 for subtraction, 15 for multiplication and 20 for division.

The variations in frequency of responses expressing the levels of difficulty of skills from addition least difficult, to division as most difficult, was also evident. The skills with the highest frequency of responses for a particular designated order were Addition and Division.

Distribution of responses for Subtraction indicated a perception of greater ease by the Control Group for that skill.

This difference in perception of difficulty level for subtraction corresponded with the difference in performance on that skill as measured by the test instrument. For the pre-test the differences in the scores analysed using the t-test were statistically significant, indicating that the Control Group was better than the Experimental Group (See Table 14b, Chapter 6).

Tallies relating to multiplication and division will be examined more closely when making longitudinal comparisons of the two groups.

	CONTROL				EXPERIMENTAL			
	addition	subtract'n	mult'n	division	addition	subtract'n	mult'n	division
First	24	2	2	0	25	0	0	2
Second	3	18	5	2	2	16	8	1
Third	1	7	16	4	0	8	15	4
Fourth	0	1	5	22	0	3	4	20

Table 9a. Pre-study responses for the ranking of operations in order of difficulty.

Post-study responses to Question 5.

Addition and Division were the dominant tallies at the two ends of the scale of. difficulty levels for both groups. (See Table 9b)

The highest tallies for each of the operational skills resulted in an order of difficulty for those skills as expressed by pupil responses, the order being from least difficult to most difficult - Addition, Subtraction, Multiplication and Division.

The highest tally for the Control Group was 20 for Addition, followed by 15 for Division, 14 for Subtraction and 9 for Multiplication.

	CONTROL				EXPERIMENTAL			
	addition	subt'n	mult'n	division	addition	subt'n	mult'n	division
First	20	0	7	0	18	1	6	1
Second	5	14	5	3	6	14	4	2
Third	2	7	9	8	2	6	14	4
Fourth	0	6	6	15	0	5	2	19

Table 9b. Responses for post-study ranking of operations in order of difficulty.

The highest tallies for each of the skills ordered represented more than half the class for each of the groups in all cases except for the Multiplication tallies of the Control Group.

There was a close correspondence between the tallies of the two groups for the operations of Addition and Subtraction.

Reading of the table for the two independent groups, suggested that the tallies for Subtraction at the post-test stage are the closest set of tallies to this question relating to the relative order of difficulty of the operations.

The closeness in the ordering of Subtraction across the four levels of difficulty by the two groups, was matched by the closeness in performance on the subtraction post-test by those groups. (See Tables 14 and 15a)

The distribution of ordering preferences for the ranking of Multiplication was more evenly spread for that task than for any of the other skills. The spread was more pronounced with the Control Group responses ranging from seven pupils designating that skill as the least difficult while at the other end of the scale six pupils placed it as the most difficult of the four operations.

Multiplication was perceived to be the least difficult skill by six pupils in the Experimental Group. The case for multiplication as a relatively simple procedural task was strengthened by these figures.

Over many years as a teacher, the researcher has pondered about the difficulties experienced by children in their performance on the four operations at the formal algorithm level.

The researcher's hypotheses was that provided the pre-requisite table facts are known and the pre-requisite connections made between repeated addition and multiplication, that this operation should be for most children the second easiest of the formal algorithms.

There is however, evidence in the literature to suggest that the nature of the operation of multiplication is not understood by many primary school pupils (Dixon, Brown and Gibson, 1984). During the investigation anecdotal data was obtained that indicated that this was the case in regard to the pupils involved in the study. This data suggests that is possible to perform the mechanics of the multiplication algorithm without understanding what that operation means.

Hiebert and Carpenter (1992) regard the arithmetical operations as Procedural Knowledge that can be raised to the level of Conceptual Knowledge provided the connections are made that lead to giving the operation meaning.

The number of respondents from the Experimental Group placing Division at the most difficult level was 19. The corresponding number for that same skill from the Control Group was 15.

The difference in expressed levels of difficulty for the subtraction algorithm by the two groups corresponded to the significant difference in performance on the post-test by those same groups.

On the post-test for subtraction the Control Group performed better than the Experimental Group. Analysis of those post-test results using the t-test indicated that the differences in the raw scores of the two groups were statistically significant.

Comparison of pre-study and post-study responses.

Control Group. Close examination of the pre-study tallies and those from the post-study gave some evidence of shifts in the weighting of the orders of difficulty for the four operational skills.

The maximum tallies for each of the skills ordered have all fallen in the post-study questionnaire. The drop in maximum tallies was four for the cases of Addition and Subtraction and seven for Multiplication and Division.

The direction of shifts in responses for Addition and Subtraction suggested relatively greater levels of difficulty with those skills at the end of the study period. At the pre-study stage two children placed Subtraction at the lowest level of difficulty and one at the highest level. At the post-study stage no child nominated Subtraction as the simplest skill while the number at the highest level of difficulty had increased markedly to six.

For Multiplication and Division the response tallies suggested greater levels of ease in the performance of those skills by the Control Group than at the beginning of the study. This result is particularly surprising for the multiplication algorithm that was taught at a higher stage of performance than recommended by either State or National Curriculum documents.

Changes to the responses ranking Multiplication at a particular difficulty level were most apparent at the First and Third levels. At the first level, two respondents ranked Multiplication as the easiest skill compared with seven at the end of the study. At the third level, the number of pupils nominating Multiplication fell markedly from sixteen pre-study to nine post-study.

The number of respondents at the post-study stage was one less than that at the pre-study stage. This minor change did not affect the relative order of any of the shifts in the difficulty weighting allotted to the skills in question.

Comparison of responses from the pre-study and the post-study questionnaires revealed that shifts in responses for the First and Second levels were in the direction of expressing greater difficulty of addition and subtraction when compared with all the operations.

The fall from 22 responses placing Division as the most difficult at the pre-study period to a tally of 15 post-study was not surprising when comparisons were made with levels of performance by the Control Group as measured by the test instrument.

	PRE-STUDY				POST-STUDY			
	addition	subt'n	mult'n	division	addition	subt'n	mult'n	division
First	24	2	2	0	20	0	7	0
Second	3	18	5	2	5	14	5	3
Third	1	7	16	4	2	7	9	8
Fourth	0	1	5	22	0	6	6	15

Table 9c. Control Group responses ranking order of difficulty of operations.

Experimental Group. The trends suggested by the two sets of responses tabled were towards increasing levels of difficulty for addition and subtraction balanced by greater levels of ease for multiplication and division.

The maximum tallies for each of the skills ordered have all fallen in the post-study questionnaire. The drops in maximum tallies were seven for Addition two for Subtraction and one for Multiplication and Division.

The change in the difficulty order of Addition should be viewed in relationship to the increased ease with the other three skills where the propensity for improvement in learning is greater.

	PRE-STUDY				POST-STUDY			
	addition	subt'n	mult'n	division	addition	subt'n	mult'n	division
First	25	0	0	2	18	1	6	1
Second	2	16	8	1	6	14	4	2
Third	0	8	15	4	2	6	14	4
Fourth	0	3	4	20	0	5	2	19

Table 9d. Experimental Group responses ranking order of difficult of operations.

Implications.

Shifts in the perception of levels of difficulty for the operations of multiplication and division were more pronounced in the Control Group than the Experimental Group. The magnitude of these shifts was matched by increased performance on those same skills by the Control Group.

The change in the number of children ranking multiplication at the third level of difficulty from 16 pre-study to 9 post-study is particularly surprising. Multiplication by multi-digit numbers had been introduced by the Control Group teacher and the dilemma that this caused for the researcher and the John Henry effect were discussed in Chapter 4.

QUESTION 6.

Tick the one thing you think is true for your grade -

In mathematics	**- the girls are better than the boys,**	....
	- the boys are better than the girls,	....
	- there is no difference between girls and boys.	....

Aim.

The aim of this checklist was to determine whether the pupils in the study believed that there was a difference between boys and girls in mathematics ability and performance and if so to ascertain any variations of such perceptions between the control and Experimental Groups at the pre and post-study stages of the study.

Comments.

For this question the researcher wished to set aside such issues as sexual prejudices and culturally determined beliefs or culturally generated behaviours and to interpret the responses regarding gender differences as a measure of confidence or of self image of the respondents who believe differences exist.This issue is linked closely to motivation. A positive self image tends to generate success - a negative self image can lead to failure.

The girls mean average performance on the number and operational skills for both pre-test and post-test was better than the boys at the grade four level.

Comparison of control and experimental responses.

At both the pre-study and the post-study stages, the majority of pupils from both groups thought that there was no difference between girls and boys in mathematics. The overall responses indicated that 74% of the 105 responses were of the belief that there was no difference.

Of the 27 responses indicating a belief in gender difference in mathematics at either the pre or post-study stage, there were only five girls indicating such a difference. None of the girls from either group indicated a belief that girls were better than the boys.

Pre-study responses to Question 6.

Only three of the 27 respondents in the Control Group thought that there was any difference. Two of these were boys who thought that girls were better. The third respondent, a girl, thought that the boys were better. (See Table 10a)

The Experimental Group responses indicated that 18 of the 26 were of the opinion that there was no difference between girls and boys in mathematics. Of the six boys indicating a difference five stated that girls were better than the boys. The two girls who stated a belief in a difference, believed that boys were better than the girls.

PRE-STUDY							
CONTROL				EXPERIMENTAL			
	Boys better	Girls better	No difference		Boys better	Girls better	No difference
Boys	0	2	17	Boys	1	5	11
Girls	1	0	7	Girls	2	0	7
TOTAL	1	2	24	TOTAL	3	5	18

Table 10a. Pre-study responses to question regarding gender differences .

Post-study responses to Question 6.

The contrast between the control and experimental responses from the boys to the post-study questionnaire on this issue was most pronounced.

Of the six boys in the Control Group stating a difference, five believed that the boys were better than the girls. This represented 29% of the boys that are of this opinion. This suggests a positive attitude or a degree of confidence of those pupils responding in this manner.

The Experimental Group responses from the boys were evenly divided between those who thought that there was no difference and those who thought that there was a difference. Of the eight boys who believed there was a gender difference in mathematics performance, seven believed that the girls were better. These respondents comprised

44% of the boys in that group. Their responses seemed to indicate a lack of confidence or negative attitude regarding mathematics ability.

POST-STUDY							
CONTROL				EXPERIMENTAL			
	Boys better	Girls better	No difference		Boys better	Girls better	No difference
Boys	5	1	11	Boys	1	7	8
Girls	1	0	8	Girls	1	0	9
TOTAL	6	1	19	TOTAL	2	7	17

Table 10b. Post-study responses to question regarding gender differences.

Comparison of pre-study and post-study responses.

Control Group responses.

The combined pre and post-study responses indicate that 22% of the boys believed that there was a difference in mathematics ability. Only 12% of the girls indicated a belief that there was a difference according to gender.

On both the pre-study and post-study questionnaire, only one girl from the Control Group thought that there was a difference between girls and boys in mathematics. In each case the belief was that the girls were better.

The boy's responses indicated a change in attitude towards their ability in mathematics that developed between the two questionnaires. At the pre-study stage only two boys believed there was a difference. Both believed that the girls were better.

CONTROL							
PRE-STUDY				POST-STUDY			
	Boys better	Girls better	No difference		Boys better	Girls better	No difference
Boys	0	2	17	Boys	5	1	11
Girls	1	0	7	Girls	1	0	8
TOTAL	1	2	24	TOTAL	6	1	19

Table 10c. Control Group responses to question regarding gender differences.

The number of boys who thought that there was a difference in mathematics ability increased to six by the post-study stage. Five of these responses were indicating the opinion that the boys were better.

Experimental Group responses.

The combined pre and post-study responses from the Experimental Group, indicated that 42% of the boys believed that there was a difference in mathematics ability. Only 15% of the girls indicated a belief that there was a difference according to gender.

At each stage of the study only one boy indicated a belief that the boys were better than the girls. Five boys at the pre-study stage and seven at the post-study stage thought that the girls were better.

For the responses from the girls, no girl thought that they were better than the boys. At the pre-study stage two girls thought that the boys were better. Only one girl at the post-study stage thought that the boys were better.

EXPERIMENTAL							
PRE-STUDY				POST-STUDY			
	Boys better	Girls better	No difference		Boys better	Girls better	No difference
Boys	1	5	11	Boys	1	7	8
Girls	2	0	7	Girls	1	0	9
TOTAL	3	5	18	TOTAL	2	7	17

Table 10d. Experimental Group responses to question regarding gender differences.

Implications.

The most significant aspect of the responses to this checklist was the nature of the replies of the boys. The contrasts between the responses of the two groups at both the pre and post-study stages can be seen in Table 10e.

For both groups at the pre-study stage, of the boys who believed there was a gender difference in relation to mathematics ability, most were of the opinion that the girls were better than the boys.

Comparison of the boys' responses from the two groups at the pre-study stage suggest a difference in attitude on gender differences in mathematics ability by the two groups. For the Control Group, 11% of the boys believed that the girls were better. The corresponding figure for the Experimental Group was 29%. This may suggest a more negative attitude towards mathematics by the boys in the Experimental Group who were of that opinion. Such attitude would certainly have an influence on mathematics performance, and thus influence the results of this study.

This apparent difference in attitude at the beginning of the study would certainly not be advantageous to the Experimental Group in optimising performance in the mathematics skills to be taught. The evidence from the responses to this checklist was that the differences in attitude became more pronounced by the time the post-study questionnaire was administered.

For the Control Group 29% of the boys believed that the boys were better. This contrasts dramatically with the responses from the boys in the Experimental Group which indicate that 44% of the boys believe the girls are better in mathematics.

As both groups made significant gains in all skills tested, it seems unlikely that the reasons for such differences in attitude were the result of differential treatment by the researcher. It is probable that such changes were factors arising within the classroom setting.

The relative shifts in attitude of the two groups towards their perception of ability in mathematics, provided further evidence that extraneous factors beyond the control of the researcher were influencing the results of this study. Another extraneous factor apart from the perceived variation of change in attitude of the teachers was the gender of the teachers. The control teacher was male and the experimental teacher was female.

	PRE-STUDY			POST-STUDY		
	Boys better	Girls better	No difference	Boys better	Girls better	No difference
Control	0	2	17	5	1	11
Experimental	1	5	11	1	7	8

Table 10e. Responses from the boys to question regarding gender differences.

QUESTION 7. What job or occupation would you like to have when you leave school?

Aims.

The aim of this question was mainly to gain some insight into the socio-economic aspirations of the individuals that comprised the two groups.

Results.

While most pupils were able to give appropriate names to the occupations that were desired at the time of completing the questionnaire, others gave either the place of work or the nature of work to be performed.

One child at the pre-study stage wrote 'say bye', perhaps as a response to a different interpretation of the question. What do you want to do when you leave school? Two children at the post-study stage were unable to name a choice of occupation.

<u>Pre-study responses to question 7.</u>

The responses from the Control Group are listed as follows -

Inventor (2), scientist (2) pilot, teacher, nurse, policeman, police-lady, bank clerk, designer, cartoon artist, chef, work at the zoo, work at boarding kennels, army soldier, mechanic, cricketer, footballer, workman, 'say bye'?

The Experimental Group responses were as follows -

archaeologist (3), doctor, vet, teacher, movie star or teacher, electrician (2), author, policeman, fireman, mechanic, hairdresser (2), horse rider, footballer, army soldier, shop assistant, truck driver.

<u>Post-study responses to question 7.</u>

The responses from the Control Group are listed as follows -

Scientist, doctor, lawyer, architect, nurse/doctor (2), artist/pilot, teacher/artist, teacher, cook/pilot, chef/publisher, policeman, cartoonist, computer worker, pizza delivery, florist/office worker, basketballer, fireman, artist/painter, army soldier, bouncer, shop assistant.

The Experimental Group responses were as follows -

pilot/director, engineer, vet, teacher/dancing teacher, eye doctor, teacher, judge/vet, curator, scientist/engineer, policeman (2), policewoman, office worker hairdresser, "car" ??, elite special forces, army, special forces, NASCAR, cafe worker, actor, a beauty mum, don't know, not thought about it.

There was no further analysis or interpretation was made of this data.

CONCLUSION.

The overall assessment of differences in responses between the Control Group and the Experimental Group was that the attitudes towards formal schooling and towards mathematics changed considerably in the intervening time between the pre-study questionnaire and post-study questionnaire, and that such changes could be hypothesised to influence results in performance that would generally favour the Control Group. (See Table 11)

The assessment of the balance of attitude relating to Questions 1, 2 and 3 is evident in Tables 5c and 7d. The hypothesised effect of the difference in attitude suggested in Table 11 arose because of the link between attitude and performance. The bias of the difference in attitude on the first three questions was assessed as favourable in affecting performance by the Experimental Group at the time of the pre-study questionnaire. This situation changed dramatically by the time the post-study questionnaire was administered.

In regard to Question 3 it is the researcher's firm belief that the relative changes in teacher attitude perceived by the pupils in each of the two groups had a major influence in the overall performance of skills as measured by the test instruments. The magnitude of this influence in a way that favoured the Control Group was such that results from this experimental study could be regarded as invalid.

	Pre-study		Post-study	
	Control	Experimental	Control	Experimental
Question 1.	Unfavourable	Favourable	Favourable	Unfavourable
Question 2	Unfavourable	Favourable	Favourable	Unfavourable
Question 3	Unfavourable	Favourable	Favourable	Unfavourable
Question 4	Unfavourable	Favourable	Unfavourable	Favourable
Question 6.	Favourable	Unfavourable	Favourable	Unfavourable

Table 11. Hypothesised effect on performance of balance of attitude.

Analysis of Question 4 responses is summarised in the statement from that section which suggests that the Control Group responses to this question were "...generally less positive in their feelings towards mathematics than those in the Experimental Group at both stages of the study" (See Tables 8a and 8b). The balance towards more positive attitudes towards mathematics should have favoured that group at both stages of the study. Of all the questions designed to monitor whether the Experimental Group would be advantaged by the treatment involving an activity approach using structured materials, this is the only one for which responses indicated the possibility of this scenario.

Hypotheses for effects of differences in attitude suggested by responses to Question 6 relate to the imbalance of responses and in particular those of the boys from each group at the pre-study stage and the marked contrast of responses of boys at the post-study stage as discussed in that section relating to Table 10b. This change in attitude corresponds with the researcher's belief regarding the John Henry effect already discussed. The competitive nature of boys taught by a male teacher may have accentuated this effect. The contrast in attitude should have favoured the boys from the Control Group more than those from the Experimental Group. Boys comprised approximately two thirds of both classes.

Analysis of the mean scores of boys and girls on the subtraction test from "Diagnostic Profiles in Mathematics" indicated that' in comparison of gains by gender, greater improvement was made by the boys in the Control Group than the girls from that same group.

The comparison of mean average scores for both groups at the pre-study stage indicated about 3 points difference between boys and girls. This difference in performance between boys and girls at the post-study stage remained about the same for the Experimental Group. The difference in the mean average scores of boys and girls from the Control Group however narrowed to within 0.5 points at the end of the study period. No analysis of gains of boys performance over girls performance was made because of the small numbers involved.

CHAPTER SIX

TEST RESULTS FOR ARITHMETIC SKILLS

The same test batteries were administered at the pre-study and post-study stages as outlined in Chapter 3. Tests were administered for each of the following skill areas: number, addition, subtraction, multiplication and division. Analysis of results focuses on children's performance on "Diagnostic Mathematics Profiles." The test results from this tool generated the four sets of data appearing in the table below.

	CONTROL	EXPERIMENTAL
PRE-TESTS	Set Ac	Set Ae
POST-TESTS	Set Bc	Set Be

Table 12. Matrix elements of data for analysis.

Comparisons of the matrix elements are made horizontally and vertically across the matrix in order to discern differences in performances:

1. Of the Control Group and Experimental Group at each stage of the study.
    - Pre-test
    - Post-test
2. On pre-tests and post-tests.
    - Control Group
    - Experimental Group

The raw score test results were tabulated in a format that facilitates the first set of comparisons that would be made.

These sets of raw scores were used to generate descriptive data and for the analysis of difference using the t-test. The descriptive data included mean scores, standard deviation and range.

COMPARISON OF CONTROL AND EXPERIMENTAL PERFORMANCE.

A. PRE-TESTS. Diagnostic Mathematics profiles (Australian Council of Educational Research, 1990) consisting of four separate tests on the skills of arithmetic operations at the formal algorithmic level were administered prior to the study period. Raw scores obtained by subjects from the Control and Experimental Groups are presented in Table 13.

CONTROL					EXPERIMENTAL				
Number	Add'n.	Subtn	Multn.	Division.	Number	Add'n.	Subtn	Multn.	Division.
14	13	12	2	2	11	13	14	6	0
6	4	6	0	0	15	10	8	3	0
8	14	11	1	2	7	4	3	0	0
14	10	7	2	2	10	18	7	10	5
5	5	6	0	0	13	14	9	5	1
11	17	14	10	4	12	4	9	3	1
11	17	12	5	2	19	8	4	2	1
14	16	13	4	2	15	7	15	3	2
16	16	11	10	4	16	16	14	2	2
16	19	15	8	4	13	17	9	5	2
15	16	14	3	5	8	9	6	1	2
19	19	17	8	9	18	20	13	6	0
15	18	15	9	9	14	18	12	2	1
19	19	20	10	4	11	19	9	3	1
13	16	14	5	3	12	20	6	2	1
16	11	7	6	0	17	20	16	10	2
13	15	12	9	0	15	19	16	9	2
14	18	12	9	1	17	18	13	8	2
12	7	9	1	0	16	17	6	1	0
14	13	12	2	1	10	16	6	2	8
11	16	12	9	7	11	12	1	1	0
13	13	10	4	4	13	15	9	4	1
12	17	15	4	2	15	14	6	1	1
16	18	16	9	2	17	18	14	10	2
14	18	16	9	3	16	10	8	7	2
18	20	15	9	2	13	20	11	5	10
12	13	9	7	0	12	16	14	7	2
11	16	14	7	2	15	15	8	2	2
13	15	5	0	2		16	14	10	4
11	15	9	5	2					

Table 13. Pre-test scores - Control and Experimental Groups.

Mean scores for Addition, Subtraction, Multiplication and Division for the Control Group were 14.80, 12.00, 5.45 and 2.67 respectively; and for the Experimental Group 14.59, 9.66, 4.48 and 1.97. In each of the tests the mean score of the Control Group was higher than that of the Experimental Group. The greatest difference in mean scores was for the Subtraction test. (See Table 14a.)

	Addition	Subtraction	Multiplication	Division
Control	14.80	12.00	5.45	2.67
Experimental	14.57	9.66	4.48	1.97
Difference	0.23	2.34	0.97	0.70

Table 14a. Pre-test mean scores for four operations.

The rank order of levels of performance on the four tests on operations was the same for both groups. From highest to lowest the order was Addition, Subtraction, Multiplication and Division. The rank order of levels of performance on the four operations was identical to the overall pattern of children's perception of the relative levels of difficulty of those same tasks as charted from their responses on the attitude questionnaire. (See Table 9a, Chapter 5.)

This rank order consistency on both the mean scores and the children's perception of difficulty was, I believe, connected to the increased complexity of operations through that same order, being in a hierarchy. The nature of this hierarchy and the connections within were very much related to understanding. Understanding of the operations means seeing relationships and making connections. The interrelation of operations was central to this study.

The disparity between the mean performances of addition and subtraction on one hand, and multiplication and division on the other, means that the opportunity for gain on the higher mean scores for addition was less than for that on division where the mean scores were very low. The ceiling effect on the varying levels of performance on the pre-test would suggest the hypothesis that the propensity for gain on mean scores would be the reverse of the performance order.

The range of raw scores obtained by the control and the Experimental Groups was the same for addition (16 with a maximum of 20) and multiplication (10 with a maximum of 10). For subtraction the range of raw scores for the Control Group was (15 with a maximum of 20); and for the Experimental Group (15 with a maximum of 16). For division the range of scores was (9 with a maximum of 9) for the Control Group and (10 with a maximum of 10) for the Experimental Group.

The greater difference in range in subtraction suggests a skewing of distribution of the raw scores of the two groups in opposite directions that will be examined more closely.

In order to determine whether the two groups were essentially the same in respect to levels of performance on each of the four separate tests, the scores for each of the tests were compared directly using the 't-test' for unmatched groups, assuming unequal variances.

It was assumed that the two groups would be essentially at similar levels of performance at the pre-study period. Although this assumption was incorrect, there was no way of predicting which group, control or experimental, would perform better on any particular test. On this basis a two-tailed analysis would apply and the critical level of significance nominated would be at the 0.05 level.

The computed t-values for the pre-test data on addition and multiplication and division were in each case lower than the critical t-value on the table. The conclusion was that any differences in the mean scores of those skills tested were not statistically significant. Despite this conclusion, the researcher believes that the fact that all four mean scores were higher for the Control Group, although only marginally in some cases, suggests the likelihood that the Control Group were actually better performers on the skills tested at the pre-test stage.

	Addition	Subtraction	Multiplication	Division
t value calculated	0.87	2.33	1.23	1.15
t-value critical	1.67	1.67	1.67	1.67

Table 14b. t-test values for pre-test scores for Control and Experimental Groups.

For the data on subtraction the computed t-value for the two groups was higher than the critical t-value. The difference between the mean scores of the two groups on this test was statistically significant. The conclusion was that the Control Group was significantly better in performance on subtraction skills as measured by the test instrument, than the Experimental Group. This result has important implications for this study.

B. POST-TESTS. Raw scores obtained by subjects from the Control and Experimental Groups are presented in Table 15. The results from the post-tests revealed that, for both Experimental and Control Groups, a great deal of basic mathematics learning occurred during the study period.

CONTROL					EXPERIMENTAL				
Number	Add'n.	C/Subt'n	Mult'n.	Division.	Number	Add'n.	E/Subt'n	Mult'n.	Division.
15	19	13	6	6	13	16	17	6	3
14	20	15	10	11	15	15	16	8	4
18	19	17	17	9	9	8	5	2	2
18	18	14	8	7	14	19	20	11	5
11	18	4	14	6	17	19	20	17	8
18	16	20	14	18	17	19	20	4	2
18	18	17	16	12	10	16	17	15	9
13	20	19	13	12	14	20	18	15	6
19	19	20	16	15	10	18	13	8	6
17	20	18	17	13	15	19	20	12	2
14	20	17	9	8	15	18	18	15	5
20	20	20	14	15	11	20	16	7	9
13	19	16	10	13	15	14	20	17	4
18	15	15	17	13	18	19	20	16	9
16	18	18	16	5	18	19	17	8	5
17	18	18	9	11	19	17	20	4	5
19	17	16	9	5	14	18	15	6	3
15	17	14	14	6	11	18	4	11	0
18	15	13	7	13	18	17	10	6	14
13	17	14	12	9	19	14	14	14	2
14	19	20	11	6	17	20	19	9	5
19	18	16	11	8	12	17	11	13	11
18	19	13	14	9	16	18	15	10	5
18	16	18	16	8	18	19	14	9	2
17	17	16	18	11	18	19	19	8	4
13		19	5	3			20	16	17
15		20	14	12			19	17	4
15		13	8	10					
13		11	8						

Table 15. Post-test scores - Control and Experimental Groups.

Post-test mean scores for Addition, Subtraction, Multiplication and Division, for the Control Group were 18.08, 16.00, 12.17 and 9.79 respectively; and for the Experimental Group 17.44, 16.19, 10.52 and 5.59. The mean scores of the Control Group for the tests on addition, multiplication and division were higher than those of the Experimental Group. In subtraction the Experimental Group mean average was marginally better than the Control Group.

The greatest difference between the two sets of mean scores was for Division with a difference of 4.2.

	Addition	Subtraction	Multiplication	Division
Control	18.08	16.00	12.17	9.79
Experimental	17.44	16.19	10.52	5.59
Difference	0.64	- 0.19	1.65	4.20

Table 16a. Post-test mean scores for four operations.

It was seen when comparing the pre-test data on subtraction that a statistically significant level of difference existed between the two groups prior to the treatment period. The level of performance on Subtraction skills was significantly lower than the Control Group at that pre-test stage.

Subtraction was the only skill tested where the Experimental Group mean performance was better than the Control Group on the post-test. This result, which cut across all the other trends in the study, prompted further hypotheses and shifted the focus of the investigation towards performance on the subtraction algorithm.

The fact that the Experimental Group mean average at the post-test stage was higher than that of the Control Group, suggested that there was no need to do further statistical analysis in order to conclude that gains made by the former group, where the difference in mean average scores was significantly lower at the pre-test level, were a direct result of the treatment.

One could argue that the ceiling effect had a greater dampening effect on the Control Group than the Experimental Group. The pre-test mean scores for Subtraction were for the Control Group 12.00 and for the Experimental Group 9.66. It would seem that, because these mean scores were in the mid range, the ceiling effect would not be a major factor in limiting improvement of the Control Group.

The ceiling effect argument however can not be discounted. The scores for Subtraction, as examined earlier, were for the Control, in the range 5 to 20 and for the Experimental 1 to 16. Examination of the distribution of raw scores for this skill revealed that one child in the Control Group did receive the maximum score of 20 and thus would not be able to make any improvement on that score. The next highest score in that same group was 17, also obtained by one child only. (See charting of Subtraction scores, Table 15).

For subtraction an analysis of gains for all of the subjects using the t-test gave affirmative results showing that the gains were statistically significant.

For both groups, the rank order of levels of performance on the tests remained as at the pre-test stage; from highest to lowest being Addition, Subtraction, Multiplication and Division. This rank order for levels of performance corresponded with the rank order for the relative levels of difficulty of those same tasks as charted from pupil responses on the attitude questionnaire which were given by subjects from both groups at the end of the investigation. (See Table 16b)

The ceiling effect, with the disparity of pre-test mean scores ranging from a maximum of 14.80 in Addition for the Control Group, down to a minimum of 1.97 in Division for the Experimental Group, did in fact lead to a trend in post-test scores as hypothesised. The major improvements in scores were generally for the tests for multiplication and division.

The range of raw scores obtained by the Control Group was for addition (15 with a maximum of 20), subtraction (16 with a maximum of 20), multiplication (13 with a maximum of 18) and division (15 with a maximum of 18). For the Experimental Group the range was for addition (12 with a maximum of 20), subtraction (15 with a maximum of 20), multiplication (15 with a maximum of 17) and for division (15 with a maximum of 17).

The computed t-values for the post-test data on addition, subtraction and multiplication were in every case lower than the critical t-value on the table. The conclusion is that any differences in those skills tested were not statistically significant.

	Addition	Subtraction	Multiplication	Division
t value calculated	1.06	0.17	1.49	4.15
t-value critical	1.69	1.68	1.68	1.67

Table 16b. t-test values comparing post-test data for two independent groups.

For division the computed t-value comparing the two means was higher than the critical t-value. The difference between the mean scores of the two groups on this test was statistically significant. The conclusion was that the Control Group performed better than the Experimental Group on the post-test for division.

This result for division seriously questions the hypotheses that the use of structured aids assists the learning of all arithmetic operations more than traditional method of instruction by drill and practice. Possible reasons for the better performance of the Control Group may relate to a number of factors. These include, the John Henry effect already mentioned, the total amount of time devoted to this particular skill by the respective teachers, and also the information processing load. This according to Boulton-Lewis and Halford (1991) can at times hinder instruction rather than improve it. Such hindering is generally associated with the degree of 'goodness' or 'badness' of fit of the model to which the concept or procedure is connected.

COMPARISON OF PRE-TEST AND POST-TEST RESULTS.

The comparison of the pre-test and post-test results for the Control and the Experimental Groups indicates that considerable improvement was made in all four skills tested. The comparison of the two sets of results was assessed by two strategies.

1. Gain or difference scores. Gain or difference scores were calculated from the mean scores for the post-test and the pre-test. Normally one would expect that this would represent a gain in favour of the post-test mean score.

2. t-test analysis. Although an improvement in performance between the results of the pre-test and those of the post-test would be anticipated, the level of such improvement may not be statistically significant. The t-test was used to check for statistically significant differences for both groups between pre-test and post-test scores.

As the composition of the members of each of the groups involved in this study was essentially the same at the end of the study as at the beginning, the t-test for matched groups assuming equal variances applied. Thus the parameters were for a critical t-value for a one-tailed test and the level of significance set at 0.05.

It was predicted that some improvement during the time interval between pre-test and post-test would occur. The one-tailed analysis using the 'Microsoft Excel' computer software package allows for this situation by providing the option of nominating the improvement in mean scores.

'Diagnostic Mathematics Profiles' was not a standardised test instrument for which age or grade norms were available. Thus there was no way of determining the level of improvement that could be expected in each of the skill areas tested at the Grade 3 and Grade 4 level as a result of approximately six months schooling irrespective of any specific methodology in teaching.

Some thought was given to this matter and the conjecture, based on the pre-test results and the anticipated ceiling effect, was that improvements in mean scores expected might be of the following order.

Addition	10% improvement on the pre-test score.
Subtraction	20%
Multiplication	50%
Division	100%

These figures were based on the hypothesis that the propensity for improvement would be affected greatly by the ceiling effect which would allow greatest improvement to be made in Division, where scores were very low, through to least improvement where pre-test scores were high as in Addition.

The application of these strategies was applied to the Control Group and to the Experimental Group.

A. THE CONTROL GROUP.

(1). Gain or difference scores.

The mean average gains for the skills tested were as shown in Table 17a, Addition, 3.28; Subtraction, 4.00; Multiplication, 6.72 and Division, 7.12. The magnitude must be interpreted in terms of the pre-test scores which were respectively (14.80), (12.00), (5.45), and (2.67).

	Addition	Subtraction	Multiplication	Division
Post-test	18.08	16.00	12.17	9.79
Pre-test	14.80	12.00	5.45	2.67
Gain	3.28	4.00	6.72	7.12

Table 17a. Control Group mean scores for four operations.

The order of magnitude of these gains was consistent with the order of improvement hypothesised on the basis of the ceiling effect.

At the pre-test stage the highest mean score for addition was 14.80 which meant that the opportunity to make more than a few points score difference on the post-test was severely limited. The prediction made from the pre-test scores on the four skills tested was that least improvement would be made on addition. This prediction proved to be correct.

At the other end of the scale the pre-test mean score of 2.67 for division moved to a high of 9.79. This gain of 7.12 was the highest gain for the operations skills tested with the Control Group. The gain in mean scores for each of the tests was as hypothesized in an inverse relationship to the levels of achievement.

(2) t-test Analysis of Pretest and post-test mean scores.

The analysis of results, using the t-test, based on the hypothesis that there would be no difference between pre-test and post-test scores, produced for each of the four skills tested, a calculated t-value greater than the tabled critical t-value. The conclusion was that the probability that any improvement in performance between the pre-test and post-test was due to chance was less likely than five percent.

The results of the t-test analysis at the .05 level of significance computed t-values for each of the tests as follows - Addition, 3.85; Subtraction, 4.34; Multiplication, 7.23; and Division, 4.47. The corresponding critical t-values for a one-tailed test were, 2.01, 2.00, 2.00 and 2.00. Clearly all calculated t-values were higher than the critical t-values and thus the conclusion was that the performance of the Control Group on the post-test was significantly different from the performance on the pre-test.

	Addition	Subtraction	Multiplication	Division
t value calculated	3.85	4.34	7.23	4.47
t-value critical	2.01	2.00	2.00	2.00

Table 17b. t-test values comparing pre and post-test scores for Control Group.

The conclusion was that the improvement was a result of instruction, maturation or some other factor.

It is important to note that although the expectation was that instruction would improve performance to a level that was statistically significant, there were instances for both the experimental and Control Groups where this was not the case.

B. EXPERIMENTAL GROUP.

The comparison of the pre-test and post-test results for the Experimental Group indicated that considerable improvement. was made in all four skills tested. The comparison of the two sets of results was assessed by two techniques.

1. Gain or difference scores.

2. t-test analysis.

(1). Gain or difference scores. The mean average gains for the skills tested were as follows, Addition, 2.85; Subtraction, 6.53; Multiplication, 6.04 and Division, 3.62.

	Addition	Subtraction	Multiplication	Division
Post-test	17.44	16.19	10.52	5.59
Pre-test	14.59	9.66	4.48	1.97
Gain	2.85	6.53	6.04	3.62

Table 18a. Experimental Group mean scores for four operations.

The relevant magnitude of the gains when ranked in order was not consistent with the order of improvement hypothesized on the basis of the ceiling effect, except for the case of addition.

The lowest mean gain was 2.85 points for addition, which for the pre-test had the highest mean 14.59 for the Experimental Group. The prediction that this may be a skill that would show least improvement proved to be correct. The second highest gain of 6.06 points for multiplication meant that the prediction that that skill should show second highest improvement was correct.

The prediction that Division, with a very low pre-test mean of 1.97, should show the greatest difference of gain proved unfounded and, in fact, improvement in this skill was the second lowest after Addition. The actual gain of 3.62 points was however much better than that predicted for the half year program which suggested an improvement of 100% on pre-test scores should occur. The propensity for greater gains was almost certainly hindered by insufficient time to adequately teach the concept through a guided discovery approach.

For subtraction, the difference between pre-test and post-test scores achieved by the Experimental Group was a gain of 6.53 points which was the highest gain for skills tested. The magnitude of this gain was much better than the 20% gain predicted. The prediction that this skill would show second least improvement was incorrect.

This substantial change in predicted rank order for subtraction indicated that the treatment using Blockaid for that operation was successful.

(2) t-test analysis of pre and post-test scores.

The computations of the two sets of scores were performed in the identical manner to the procedure followed when comparing the pre-tests and post-tests for the Control Group. As shown in Table 18b the analysis gave t-values that were in all cases higher than the critical t-values that apply for a two sample study in which a prediction of difference or improvement was expected on the post-test.

	Addition	Subtraction	Multiplication	Division
t value calculated	2.28	5.74	5.81	4.30
t-value critical	2.01	2.00	2.00	2.00

Table 18b. t-test values comparing test scores for Experimental Group.

The conclusion was that there was a significant difference between the pre-test results for Addition, Subtraction, Multiplication, and Division and the post-test results for those same skills.

The improvement in mean average scores for division however was made on a very low pre-test mean so the actual gains in this area of competency need to be interpreted with caution. Other factors influencing the results such as teacher factors, total time allocation and information processing load will be addressed in Chapter 8.

The researcher believes that the method of instruction using structured materials had a significant bearing on this result.

CONCLUSIONS.

Table 19 shows that the improvement in performance in all skill areas tested was much greater than predicted. The percentage differences in improvement between the Control Group and the Experimental Group were greater for Subtraction and Division than for Addition and Multiplication. Although the improvement for Multiplication by the Experimental Group was greater than that for the Control Group there was not a statistically significant difference.

	Addition	Subtraction	Multiplication	Division
Predicted Improvement	10%	20%	50%	100%
Control Group Improvement	22%	33%	123%	266%
Experimental group improvement	19%	68%	135%	184%

Table 19. Comparison of percentage gains on pre-test mean scores over post-test.

The conclusion from the analysis of t-test values for pre and post test scores for both groups is shown in Table 20. While the Control Group performed better on Division, the Experimental Group performed better on Subtraction.

Group which performed better	Addition	Subtraction	Multiplication	Division
Control Group	No	No	No	Yes
Experimental Group	No	Yes	No	No

Table 20. Comparison of Pre and Post Test performance based on t-test analysis.

The very high percentage gains for division for both groups prompted further comparison of scores with the class of Grade 5 pupils. "Diagnostic Mathematics Profiles" were administered during the same pre-test period and post test periods. These tests were administered to provide test data for performance from an independent group and since there was no other group at the Grade 3 and 4 level, the Grade 5 class was tested. The results of that class are compared with those of the Control Group and the Experimental Group in Table 21.The overwhelming evidence is that the gains made by the Control Group consisting of Grade 3 and 4 pupils were better than the Grade 5 class and that the performance of the Control Group was marginally better than that of the higher grade at the post-study period. This is an extraordinary result which combined with other evidence indicated that the John Henry effect was a major influence on the Control Group results.

A t-test analysis was performed comparing the Experimental Group post-test results with the Grade 5 class pre-test results and there was found to be a statistically significant difference between the two sets of scores. This suggests that the gains made by the Experimental Group in Division were statistically significant.

	Control Group	Experimental Group	Grade 5 Class
Pre-test mean score	2.67	1.97	3.31
Post-test mean score	9.79	5.59	9.00

Table 21. Comparisons for Division - Control, Experimental and Grade 5 Class.

The statistically significant difference in performance in favour of the Experimental Group for Subtraction and for the Control Group for Division prompted closer analysis of results for the Grade 3 and Grade 4 population of each class.

The comparison of scores for subtraction and division shown in Table 22 shows that for both the Control Group and the Experimental group that the post-test scores for Grade 3 were higher than the pre-test scores for Grade 4.

	Control Group		*Experimental group*	
	Subtraction	*Division*	*Subtraction*	*Division*
Grade 4 pre-test mean score	*12.66*	*2.92*	*10.73*	*2.23*
Grade 3 post-test mean score	*13.80*	*9.50*	*15.60*	*4.00*

Table 22. Comparison of Grade 4 pre-test and Grade 3 Post-test mean scores.

In performance on Division for the Control Group the boys only Grade 3 post-test mean of 9.05 was close to their class average of 9.79. The Grade 4 boys outperformed the girls on the post-test suggesting a competitive element. These gender differences in performance were discussed in Chapter 5.

For the Experimental Group, where a less positive attitude was displayed on the part of the boys, the girls outperformed the boys on Subtraction with a mean average of 18.00. However the improvement by the boys in this class was greatest for the Grade 3. This raises the question of optimum time for teaching of particular skills and more importantly the value of providing physical embodiments on which actions can be performed that parallel the steps in the formal algorithm for subtraction. This is discussed in Chapter 8.

CHAPTER SEVEN

INTERVIEWS ON NUMBER AND OPERATIONS

The interviews were conducted as planned during the first two weeks of Term Three and during the last two weeks of Term Four. Twelve grade three pupils (six from each of the strategy groups) were interviewed at the commencement of the study and the same pupils re-interviewed at the end. The interviews followed the protocol of the checklists which appear in Appendix 2.

Each child was questioned on the five skill areas:

- Number ideas
- Addition
- Subtraction
- Multiplication
- Division

The interviews for all twelve subjects for each of these skill areas such as Number, were completed before the commencement of the next skill. Thirty interviews were arranged for the Control Group at the pre-study stage with the anticipated times that appear in Table 23. Arranging times for this number of interviews was difficult at the end of the year. A similar timetable applied for the Experimental Group.

	Number	Addition	Subtraction	Mult/Div.
Subject A	20 min.	20 min.	20 min.	20 min.
Subject B	20 min.	20 min.	20 min.	20 min.
Subject C	20 min.	20 min.	20 min.	20 min.
Subject D	20 min.	20 min.	20 min.	20 min.
Subject E	20 min.	20 min.	20 min.	20 min.
Subject F	20 min.	20 min.	20 min.	20 min.

Table 23. Pre-study interviews for Control Group.

ANALYSIS OF INTERVIEW DATA

A selective analysis of the data was executed. Partial transcripts were made for the cardinal/ordinal and grouping sub-skills in number. Full transcripts for the subtraction interviews were made and charting of these appear in Appendix 4.

The emphasis in the analysis was on the subtraction operation because of the improvement in performance in that skill by the Experimental Group. The difference in performance between the two groups for subtraction was statistically significant with the Experimental Group showing the greater gain at the end of the experimental period.

The subtraction algorithms that were set for the pre-study interviews were modified to provide appropriate tasks at two different levels of difficulty because of the wide range of ability of the subjects. With the improvement in skills, the same task was used for all subjects in the post-study interviews.

In the interviewing and analysis for the subtraction skill, the task was subdivided into the following phases:

- Computation and Explanation;
    - Non-standard procedures or invented strategies;
    - Error Paths;
- Demonstration;
- Connection;
- Verification.

These phases were used to facilitate analysis of the data in attempts to gain insight into children's understanding of the operation. The phases were outlined by Labinowicz (1985).

	CONTROL	EXPERIMENTAL
Subject A	Alex	Alan
Subject B	Bill	Ben
Subject C	Carl	Colin
Subject D	Don	David
Subject E	Ewen	Edward
Subject F	Fred	Frank

Table 24. Subjects interviewed. (real names not used)

DESCRIPTION OF THE PHASES

Computation and Explanation. The computation exercise was set out in the vertical algorithm form in large print. Subjects were asked to read the problem then perform the task. Depending on the diagnosed level of ability of the subjects as determined by the battery of tests already administered, subjects were given either Task A (243 - 61) or Task B (43 - 26) at the pre-study stage. Task A was given to all subjects at the post-study stage.

After completion of the task subjects were asked, "Tell me how you worked it out?" Further follow up questions were asked in order to assess children's level of understanding of concepts such as place value, for example, 'What does the seven stand for?' (seventy, seven tens, seven). During this phase the wide range of children's levels of understanding was evident. Although the teaching approach for subtraction in the school was by the decomposition algorithm, cases of invented strategies were evident. In some cases the error paths of children could be followed.

Demonstration. During this phase children were asked to demonstrate the same subtraction task as performed on the written computation phase with concrete materials and describe their actions. The models used for this demonstration were *Blockaid* materials for Task A and Unifix Cubes for Task B. For both tasks the steps in the procedure followed those set out in *Blockaid* Workcard, "Subtraction on a 4 by 4 Grid." (See Figure 9)

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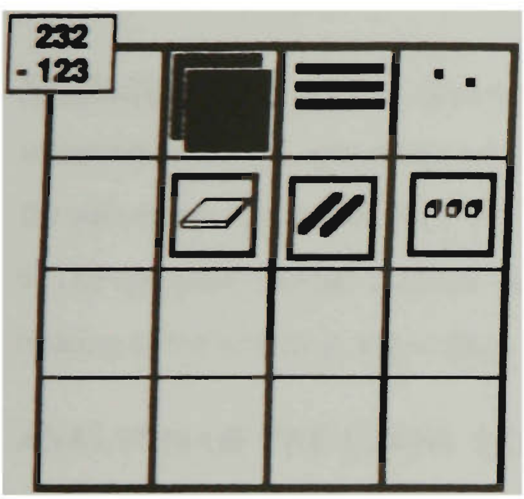
Subtraction on a 4 by 4 Grid

STANDARD SET **10**

Players. A group of between 3 to 5 children.

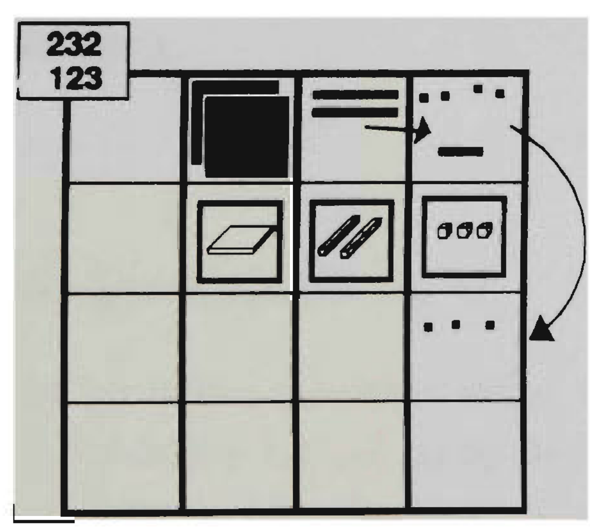
What you need. One set of Subtraction Problems (Blackline Masters, pgs 11 and 12), one set of Blockaid equipment, one 4 by 4 grid, one set of Blockaid Cards (Blackline Masters, pgs 4 to 6), and one set of Numeral Cards (Blackline Masters, pgs 1 to 3).



What to do.

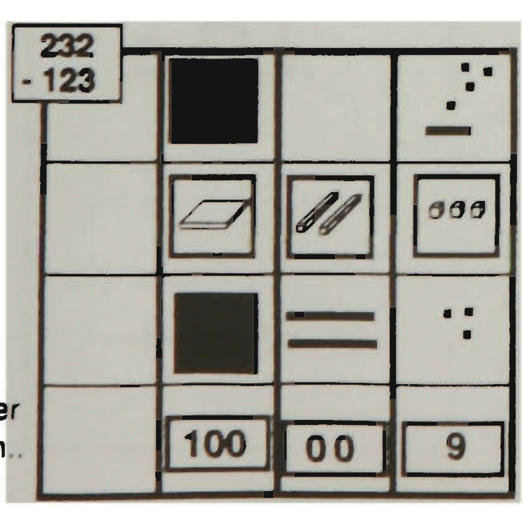
1. Place a problem card on the top left hand corner of the mat.

Set out the top number in Blockaid pieces on the top row. Set out the bottom number on the second row in Blockaid Cards.



2. From the top row remove the units shown on the card in the second row. You may have to exchange a ten (as here).

Now take away the tens, then the hundreds.



(answer in Blockaid)

answer
Num..
Cards)

3. The answer on the top row should already be show with the least number of pieces.

Represent the answer again on the fourth row using Numeral Cards. Turn over the Problem Card and check your answer.

Figure 9. Blockaid Workcard 10, Subtraction on a 4 by 4 Grid.

During this phase children were encouraged and prompted to describe their actions and the reasons for such actions. The prompting was standardised according to the interview protocol outlined in Appendix 2.

Connection. At the stage of regrouping or exchanging of materials as required when the subtrahend is larger than the minuend, the children were requested to reflect on their actions that they performed at the same stage in the written algorithm. They were asked to explain the connection between the regrouping or exchanging with materials and their renaming through decomposition in the written algorithm that was displayed (with answer hidden) at the top of the demonstration mat.

Verification. This phase attempted to assess the children's understanding of the interrelation of the operations of addition and subtraction as inverse operations, that is the subtraction operation 6 - 2 = 4 can be reversed by the addition operation of 4 + 2 = 6. The question whether addition can undo subtraction was followed by further inquiries relating to the practical and written performance of the tasks given.

ANALYSIS OF PRE-STUDY SUBTRACTION TASK A

$$\begin{array}{r} 243 \\ \underline{-\ 61} \\ \underline{} \end{array}$$

Figure 10.

Subtraction from a three digit number. 243 - 61

This was presented as a standard algorithm in vertical format as shown. Procedures followed during the interviews used the approach advocated by Labinowicz (1985) and Carney and Barrow (1994).

At the time of interview, six grade three pupils attempted this task, three from the Control Group and three from the Experimental Group..

EXPLANATION

In the reading of the problem presented in Figure 10, all of the children read the task as 243 take away 61. No other interpretation of the problem was given. Five were able to obtain the correct answer on their first attempt. Bill from the Control Group was unable to complete the paper and pencil task.

In their working of the problem the subjects applied the standard algorithm procedure by subtracting with the units column first, then going to the tens column, where most were able to carry out the usual borrowing steps and finally to the hundreds column.

The term borrowing used by Labinowicz (1985) refers to the process of renaming and regrouping of digits in the minuend when performing the subtraction algorithm using the decomposition method. The corresponding process when using structured materials to demonstrate the same operation may be referred to as the exchange process or "swap," depending on the natural language arising from a particular group of children. Both of the latter terms were used with the Experimental Group in this study. However the word exchange was the term that was generally applied.

The structure of the language used in explaining the procedures followed varied greatly. The syntax of the equation which follows the order 'a' take away 'b' equals 'c'. (a - b = c) will be identified as State-Operator-State format (SOS syntax). Non SOS syntax is identified where the language puts the operation first. Take 61 from 243 would be classed as non SOS syntax. Four of the pupils, two from each of the groups interviewed, used SOS syntax.

The mixing of place and face values, and at times giving incorrect face values, was evident but seemed to be a natural abbreviation of language that leads to a simplification of the formal algorithm. Of the six subjects attempting this task Alex was the only one who used a spontaneous place value tagging of numbers consistently and correctly throughout his explanation of his procedure.

Control Group

Alex. **(Correct answer with precise explanation of algorithm and spontaneous place values was given most of the time. State-operator-state syntax was used. His explanation was consistent with the form regarded by Labinowicz (1985) which gives insight into high level understanding that extends to the precise value of the number borrowed in the hundreds column and the value of the numbers in regrouping and renaming.** *4-6 you can't do, so I cross out the 2 in the hundreds column and make that a one, ... then I made the 4 ... 14 ... 14 tens and take six tens and that equals 8 tens.*

Bill encountered difficulties on "six from four, you can't do" stage of the process. This blockage in his thinking was reflected in his responses. **(Unable to complete the task when he got stuck on 4 - 6 = can't).** *I'm going to have to change it because it doesn't make sense ... I got up to 6 from 4 and you can't take 6 from 4, ...that's the problem.* **(Spontaneous place value was not given. Bill did not use state-operator-state syntax).**

Carl, one of the five successful computers, gave a non standard explanation of the procedure he followed in which he names the partial difference obtained in the tens column as negative 8. **(Correct answer with an unusual explanation, mostly in face value terms).** Tell me about this 4 - 6 is minus 8. *Because 6 is a higher number than 4, and you can't take 6 from 4, but there is another number that is 200, so I could take it away. ...3 take away 1 equals 2. You can't take 6 from 4.* **Place values were not given and S.O.S. syntax was generally used.**

Experimental Group

Alan and Colin both used the standard borrowing procedure taught in most schools throughout the state. It is of interest to note the conciseness of language used by Colin compared with Alan. Ben, at the borrowing stage used a bridging strategy when attempting to take six from four, whereby he took sixty directly from the hundreds column to get forty to which he added the forty in the tens column on the top line, giving 80 which he recorded as 8 in the tens column on the bottom line. While his use of place values was inconsistent his precision of language was commendable. His avoidance of the "you can't do" step in the more common procedure would appear to be worthy of further study.

The need to use precise language that reflects universal truth so far as that can be approximated should be respected by mathematics educators. The 'four take six = can't do' must at some stage give way during the child's schooling to "four take six equals negative two."

Alan. *243 minus 61. So 3-1=2, 4-6 you can't do, so you go to the front of 243 and cross that out and turn it into a 1 and you cross the 4 off and put it into the 14 ... and 6 take 14 equals 8 ... and the 2, you changed it into a 1 and you put the one down.* Right. What does that one there stand for? *..One hundred because it's in the 100 column.* **(S.O.S. syntax was generally used).**

Ben. *Well it was 243 take away 61, so I took 1 from the 3 and that equaled 2, ... and then I had to take 6 from 4 so I took the 6 away from one of the 100's, out of the 200's, and there was 40 ... and I added the four onto that and that equaled 8 ... I didn't have 200 but I only had 100 ... so I put 182.* **(S.O.S. syntax was not used and generally place values were not named).**

Colin. Tell me how you worked it out. *3 take away 1 equals 2 ... 4 - 6 you can't do that so cross off the 2, borrow one from the 100's and put in the 10's column ... 14 - 6 = 8, and 1 - 0 =1.* What does this one stand for? *100.* **(Spontaneous place values were not given. S.O.S. syntax was used).**

NON STANDARD PROCEDURE OR INVENTED STRATEGIES

Mention was made above on a variation of standard procedure utilised by Bill. Carl, while unable to verbalise the process of his thinking at the time of interview, seemed to use a similar strategy. These cases will not be elaborated further as non standard or invented procedures, as it is unknown whether the strategies employed were a direct result of earlier teaching or more unique strategies constructed by those children.

Control Group

There are many examples in the literature (for example Dixon, Brown & Gibson, 1984) of children inventing or constructing their own strategies in arithmetic tasks. Two such cases, Bill and Carl, both from the Control Group are of particular interest. Bill, the only unsuccessful computer in the paper and pencil task, after struggling with the problem suddenly gives the correct solution having adopted a backwards counting strategy.

Bill. - Can you read that out for me. *243 take away 61.* Away you go and do it. (After partial completion) *I'm going to have to change it because it doesn't make sense.* Do you think you will be able to change it for me? *I could have a go.* You can't take 6 from 4. Lets hear you read out the sum for me again. *243 - 61* Let's hear what you have got up to so far. *I got up to six from 4 and you can't take 6 from 4, ... that's the problem.* (It was apparent at this stage that Bill was unable to continue as he was unable to recall the borrowing procedure). Can we do this one or we can't? *No, we can't.* Is there any way we can get around it or is it impossible? *Just like the other one, you change the sum.* You've got to change the sum, because we can't do this one? *No.* So you don't think you can do this one? *No.* Do you want more time on it or you can't do it. *No, I can't.*

(After long pause). Do you think we should be able to take 61 from 242? *Yes, we can do it.* We can? *Another way is by counting backwards, like taking 60 by counting backwards, ... it would be less than 200 ... 182 or something.*

(Bill was regarded as one of the higher achieving pupils in the grade three group. It seems likely that Bill was able to apply this strategy successfully in solving subtraction problems but the formal algorithm was a complete mystery to him).

Carl. from the Control Group was able to complete the algorithm successfully on his first attempt but his explanation revealed a non-standard approach that was mathematically incorrect.

Let's hear how you worked it out. *3 take away 1 equals 2, 4 take away 6 equals* <u>*minus*</u> *8, and there was 200 so I put one hundred down and put the 8 down.* Tell me about this 4 take away 6 is minus 8. *Because 6 is a higher number than 4, and you can't take 6 from 4, but there is another number that is 200, so I could take it away.* Right. Do you normally show where you borrow or not? *Take aways, no. NEW PROBLEM.* You seem to do that problem in a different way to some children. I'd like you to do this problem, 243 - 71 for me. (Carl completed the problem using his own system and got the correct answer). Can you tell me what you did? *I took the one from the three, and that equalled 2 ... Four from 7 which equals minus 7, but the 2 is there so you could make it up ... 107 ... 70 ... you put the minus number down like that so I got the answer 172.* When you got that minus 7 where did you go from there? *I went to the next column to see if there was another number and there was, and I took that number down to 100 instead of 200.* Why did you take it down to 100 instead of 200? *Because you can't take 7 from 4, but there was another number to take it away. If it was only 43 take away 71 you couldn't do that. Now that there is another number there you can.* So what did you do - did you take 100 out of the 200? Do you ever show that as borrowing. *No.* Does your teacher show you borrowing or not. *We haven't done take aways this year.*

<u>Experimental Group</u>

<u>Ben.</u> *I took six away from one of the hundreds, out of the 200's and there was 40 ... and I added the four onto that and that equalled 8.*

ERROR PATHS

The major error was the occurrence of "reverse subtraction" (Labinowicz, 1985). This computational error occurred when the child reversed the order of minuend and subtrahend. For this particular problem the expected occurrence of this would be in the tens column when attempting the phase "4 tens take 6 tens - can't". The unsuccessful computers turned this about to make it six take four equals two.

One child in the Experimental Group, having carried out the borrowing procedure correctly, reversed the order in subtracting but still arrived at the correct answer.

Control Group

In the case of Bill, this phenomenon suggested to the researcher evidence of regression, whereby he reverted to earlier behavioral patterns. In his initial explanation Bill, blocked by the lack of recall of the borrowing procedure when attempting to take six from four in the tens column, seemed to have insight into the correct answer of 182. When pressed by further prompting he reverted to an algorithm he can apply using the reverse subtraction approach. He was then left with the dilemma whether 182, his answer using a backwards count strategy, or 122 the reverse subtraction answer was the correct one.

The researcher's suspicion is that there were other factors influencing Bill's ability to perform. This is only mentioned to illustrate that research into human behaviour is most complex, and the freedom to deal with the ethical considerations make the reporting of such behaviours even more difficult.

Bill. (Bill's initial explanation was that "you can't take six from four" When pressed further he tried to talk through an alternative solution). *Because 6-4 is 2 ... you would have to take 6 take away 4 ... is 2 ... 6-4 right is 2. So ... the and 3-1 is 2 and you have got 182.* So you are happy with the answer 182? *Yes. Now I've just been thinking as I've been talking through it. It could be 122 but I'm not too sure on that.*

Carl. **(Self-corrected near error)** - *Six take away ... 4 take away 6 you can't do.* **Carl's non-standard approach arrived at correct answer**. *4 - 6 equals minus 8, and there was 200 so I put 100 down and put the 8 down*

Experimental Group

Only one child used the "reverse subtraction" interpretation of the problem. Alan's imprecise or incorrect language did not seem to hinder his ability to arrive at a correct solution to the formal algorithm. However it seemed to suggest a lack of understanding of the meaning of 'take away' and this likelihood became more evident when Alan attempted the task with materials.

Alan ... *and 6 take 14 equals 8* ... (Alan made an error in the order of words used, thus reversing the order of subtrahend and minuend, but gave a response appropriate to the problem being solved).

DEMONSTRATION

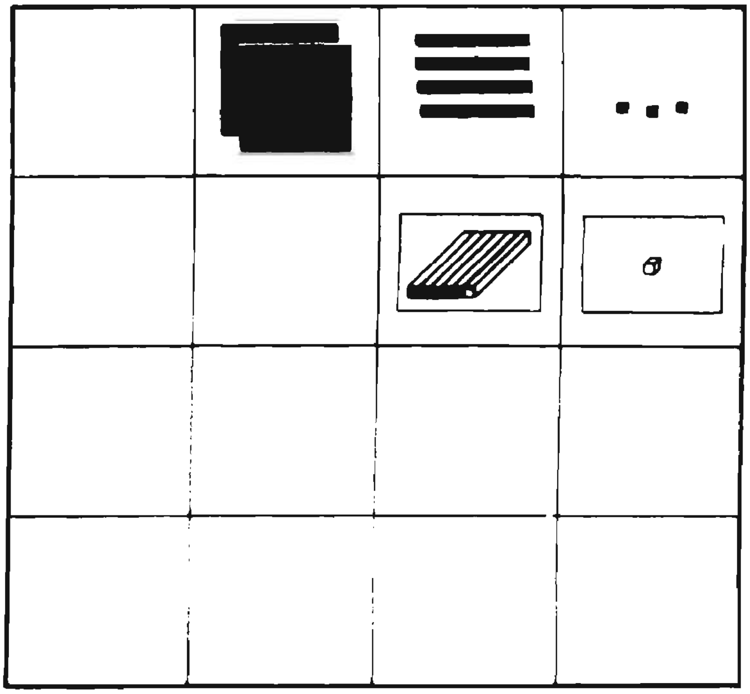


Figure 11.

Blockaid Mat with foam materials and Blockaid cards set out to show task.

Subtraction from a three digit number. 243 - 61

See Labinowicz (1985) pp. 350 - 357, also *Blockaid* Workcard Number 10.

The child's working of the written algorithm was next placed at the top of the plastic *Blockaid* mat on which the same subtraction procedure could be demonstrated through the use of structured apparatus. The written solution obtained by the child was hidden from view by slipping it under the mat. Subjects were aided through instruction to set out the task on the 4 x 4 mat as shown in Figure 11.

Three of the subjects were successful in demonstrating the use of materials to solve the task, with minimal prompting by the interviewer. Another child completed the task correctly after a great deal of prompting. Two of the subjects were unsuccessful in finding a correct solution to the problem, despite much prompting.

Minor problems due to lack of familiarity with some components of the structured materials, and the procedures associated with these, were generally overcome with minimal prompting by the interviewer.

Control Group

Alex's approach was clear and precise in solving the task. Bill tried to carry out an exchange of 100 for ten 10's but was confused with the procedure of using materials. He carried out an exchange successfully but got lost at the next phase. Carl seemed to focus more in finding the answer of 182 that he readily obtained using the formal algorithm. The process of taking away 61 got lost when he took 8 tens from one of the hundreds "because you need that for the answer". Further prompting however led to a correct solution. The insight gained was consistent with the Deep End Principle of learning advocated by Dienes and Golding (1971).

Alex. **(Good performance with minor error only in interpreting the number represented by the foam pieces after subtraction).** *You can't take 6 from the four so you put that there.* (Exchanges 100 for ten 10's). *Get 6 of these and take that away, then you take one and that's your answer.* What's your answer? *170 ... 2.* Are you sure? *182.*

Bill. **(After many prompts Bill performed an exchange but became confused with value relations or place value ideas when he combined two 50's borrowed from the 100's column and combined it with 4 in the tens column to obtain a partial minuend which he named 104 from which he took six. These errors were further compounded in his solution which he stated as 96 instead of 98).**

You think that sum is impossible? *Yes.* You don't think you can take 61 from 243? *As I said before by counting backwards but not with take away, sort of ...* Can we take 61 out of all that? *61 out of all that? (long pause)* Can we take the one out. *This part, yes.* Can you take 60 out? *60 out? Like this 60 out of here? Yes.* ... Is there anywhere we can get six tens? *Only if we change the sum.* ... You said before you thought you could take 61 from 243. *I think you can but not with figures.* Can we get more tens from anywhere so we can take six out? *Six tens from anywhere ... from the hundreds, yes.* ... Why can't we do that then? (With the interview running over the anticipated time, a degree of frustration was evident in the voice of the interviewer. What effect this had on the subject cannot be gauged. While the initial effect seemed to be positive Bill did not seem to grasp the tens grouping nature of the number system and the manner in which it is represented and interpreted by the use of place value). *You can do that? You can exchange the 100, you get two 50's.* Is that a fair exchange? *Yes ... These would go into here so it would be 104 take away 6 which leaves 104-6 ... leaves ... 96 and then ... how would you put like 96 (inaudible) 96 take away I mean, 96 we've got, put ... um ... 96 1 2 3 4 5 6 ... 96 that is ... that's 96 ...100... 100 90... 100 90 ... 8 and 8 ones, (198).* Is that right? *It does sound right.* OK Bill. (Interviewer runs out of time. The second phase of the task should have been 140 - 60 = 80 or simply in the tens column 14 - 6 = 8. The correct solution was 182).

Carl. *4 take away 6 you can't do, so take one of the 100's away and get 8 tens ... instead cause you need that for the answer. ...You can't do it. You need 4 more of those.* (After further prompts, he gave a clear explanation in a non standard form that suggested understanding of the problem to be solved. His newly discovered approach seemed to connect to his invented strategy which he applied to the formal algorithm. This explanation was free of the "minus eight" language error given in his explanation of the written algorithm) *We could take 60 from one of the 100's and then add 40 ... and that equals 180, ... so the answer equals 182.*

Experimental Group

Alan. What have you got to take away? *One from here and that leaves 2.* Can you take 6 away? *From the ... no.* How are you going to do it? (Inaudible response as Alan attempts to exchange one of the 100's for tens). ... *30 40 50 60 70 80* ...Hold it - I want you to put it there and talk about it. Can you show me here something that you did that shows that 1 and the 14 (on your paper)? *I took a 10 from the 100's and put it in the tens column, up the top ... then it's 14 take away 6 ...40 ... 4 take away.* Where is the 14? *Here at the top.* (After the exchange Alan had actually put ten 10's in the 100's column AND an additional ten in the tens column so that there were only five 10's in the top of the tens column instead of 14 tens). Have you got 14 here? *Yes.* Where? *I put it ... I think I need more tens ... I need one more ... I've got 200 here, I need one more ten.* Did you exchange properly in the first place. Let's start again.

(Alan again performed an exchange of ten tens for 100 - This time he put them in the hundreds column and took one of them and placed it in the tens column, Then removed the nine 10's from the 100's column and discarded them from the mat). *You need one 10 here ... you don't need to use these because that's only a hundred there.* And the answer is? *152* (Alan after he had exchanged unfairly 100 for one ten only had five tens from which to subtract six. He proceeded no further and read an answer from the value of the pieces of foam remaining on the top line). Are you happy with that answer? *Yes ... One of them is wrong.* Which one is wrong? *This one.*

Ben. *Take the 1 away.* (Interviewer guided Ben to place the one unit piece taken on the third line, rather than discard it from the mat). *and ...* (the child removed four tens from top line then proceeded to carry out exchange of one of the 100 pieces) *take away that ... and I've got to get that into tens.* What do you want from here? *Tens.* Read your answer. *182.*

Colin. Which ones will you take out first? *The one.* Now what do you take out? *Six, can't do it.* What will you do now? *Swap this.* What are you going to swap that for? *Two fives.* These are two 50's. What did you take out? *8 ... no 6.* What's your answer? *61* Is that your answer? *Oh ... up here ... 182.* (Minor confusion when working with materials, between subtrahend and answer or difference was corrected with prompt).

CONNECTION

This phase of the interview attempted to establish whether the interviewees were able to make the connection between the steps in the written solution with the algorithm and the corresponding steps in the procedure using the *Blockaid* material.

Control Group

Alex. *You can't take 6 from 4 so you need some more, ... so you take from the hundreds column.* Can you show where we did that on the paper? *Yes, we took the hundred out and put it over there.*

Bill. (A similar barrier arose in the written and practical approaches). You don't think you can take 61 from 243? *As I said before by counting backwards but not with take away, sort of* ... Is there anywhere we can get six tens? *Only if we change the sum.* ... You said before you thought you could take 61 from 243. *I think you can but not with figures.*

Carl. (As the subject showed no borrowing or decomposition figures in his calculation no link could be established between the paper and pencil working of the algorithm and the physical demonstration).

Experimental Group

Alan. Is there something here that you did that shows that 1 and the 14? *I took a 10 from the 100's and put it in the tens column, then it's 14 take away..* Where is the 14? *Here at the top.* (Alan put ten 10's in the 100's column and an additional 10 in the tens column so that there were five 10's instead of 14). Have you got 14 here? *Yes.* Where? *I think I need more tens*

Ben. *Well it was 243 take away 61, so I took 1 from the 3 and that equalled 2, ... and then I had to take 6 from 4 so I took the 6 away from one of the 100's, out of the 200's, and there was 40 ... and I added the four onto that and that equaled 8 ... I didn't have 200 but I only had 100 ... so I put 182.*

Colin. Can you show me something up there where you did that? *Crossed off that two ... there should only be 100 left ... I borrowed it and put it in the 10's column.* Where is the 10 you put across? ...When we did it on paper, we took 1 there and made it 14 - show me where you did that here. *There.*

VERIFICATION

The questioning during this phase of the interview was aimed at determining the level of the subjects understanding of the relationship between addition and subtraction.

Control Group

Alex. **(Seems to know the relationship between addition and subtraction)**. If I added these two numbers together, 61 and 182 what do you think I'd get? *243*. Why? *Because it's doing that sum backwards.*

Bill. Not questioned on this due to time factor.

Carl. (Carl when questioned about consistency of answers to the same problem using written and physical material approaches, indicated that answers should be the same, whatever the method).

Experimental Group

Alan. In this problem here, if I added these two numbers up, what do you think I would get? *243*. Why? *Because you are adding them up. 1+2 =3 ... 8+6=14* Will that always happen if we do a take away and then add these two together to get a total which is the first number? *Not always*

Ben. If we put all that foam back together, what would it come to? *243*. Let's look at your working of the problem on your paper, if I added these two numbers together, 61 and 182 what do you think I'd get? *243* Why did you say that? *There's 60 and 80, its got two 50's in it and that equaled 100 and with the other 100 ... etc.*

Colin. Look at all the *Blockaid* pieces on the mat, if we put them together what would they add up to? *243* Let's look at your working of the problem. If I added these two numbers up, what do you think I'd get? *243*

PRE-STUDY TASK B

43

- 26

______

Figure 12.

Subtraction from a two digit number: 43 - 26.

The example and the phases of the interview procedure followed the stages of the subtraction algorithm as outlined by Labinowicz (1985).

EXPLANATION

Control Group

Don. **(Correct answer, clear explanation with spontaneous place value given).** *Three take away 6 you can't do ... cross, um ... go to the tens column, cross out the 4 to the 3, put that ten in the 1's column so it's 13 - 6 and that equals ... 13 - 6 = 7 and 3 - 2 = 1 so it equals the whole answer equals 17.* **(S.O.S. syntax was used).**

Ewen. Answer = 23. Do you know where you went wrong there? *Yeah, ... I forgot to rename*. How could we have renamed? *Um ... I guess ... you can't take 6 away from 3, so you take the six away from the ten.* (further prompting). Let's hear you do it now. *13 - 6 um ... is 12.* 13 - 6 ? *Um ... I can't remember, ... seven.* Right. *3 - 2 is 1. (17)* **(S.O.S. syntax was used).**

Fred. Computation: 43 - 26 = 3. *Well 43 take away 26, now 3 - 6 you can't do so cross out 4 ... give it a three, give that a ten so that's 40 and get 30. I gave that a ten and that's 13 and that could take away that and that equals 7 and I got stuck on this one* *(the top number in the tens). ... you could use that number there and take away 6.* **(S.O.S. syntax was not used).**

Experimental Group

David. *You can't take six from three, so I crossed out the four, made that to three, carried the ten over to the three 1's, then made that 13 ... and 13 - 6 = 7 and so, then, so I wrote ˉ down and then 3 - 2 = 1 ...the sum equals 17.* What does this one stand for? *One ten.* **(Spontaneous place values were not given in explanation. S.O.S. syntax was generally applied).**

Edward. *I cross out that and put 13 in there and then that comes 3 and I took away 6 from the 13 and that equals 7 and I took one away and then I took away 3 from the 2, I mean 2 from the 3 and that equals one so my answer is that.* This one here, what does that stand for? *One ten.* **(Spontaneous place value was not given. S.O.S. syntax was not used).**

Frank. *I took away the 6 from the 3 equals 3 and 4 away from 2 which equals 2 and that equals 23.* Could we have borrowed there? *Borrowed?.* When you took 6 from 3 what did you get? *A three.* Are you sure? *Cause 3 + 3 is 6 and if you take 6 ... 3 from 6, it's 3.* Take 3 from 6 equals 3 what if you take 6 from 3? *Um ... zero, cause you can't do it.*

ERROR PATHS

Control Group

Ewen. *No, because you can't take 3 away from 6. Take 30 away from 20. 13 - 6 um ... is 12.* 13 - 6 ? *Um ... I can't remember, ... seven*

Fred. **(Fred's reversal error (6 - 3 = 3) in his partial solution was corrected in his explanation when he gave an account of borrowing).** *... now 3 - 6 you can't do so cross out 4 ... give it a three. ... I'm putting the 20 in 2 lots of 4.* Why did you do that? *Because I thought if I left 4 it would be a waste. I could do it easier 20, 20. ... That could be a ten, that could be a one*

Experimental Group

Edward. **(Edward self corrected minor error of reversal of minuend and subtrahend)...** *and then I took away 3 from the 2, I mean 2 from the 3 and that equals one so my answer is that.*

Frank. *I took away the 6 from the 3 equals 3 and 4 away from 2 which equals 2 and that equals 23.* Can you take 6 cubes from there? *Yeah but it will equal zero.* Why. *Because you can't, cause 6 is a higher number than 3. If there's 3 you can't take 6.*

DEMONSTRATION

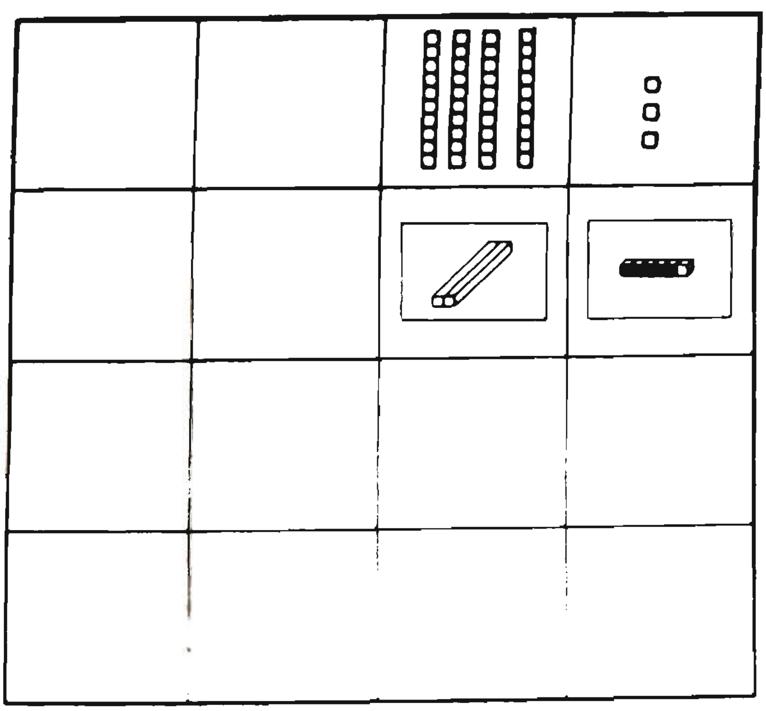


Figure 13.

Blockaid Mat with Unifix materials and Blockaid cards set out to show task 43 - 26.

Subtraction from a two digit number: 43 - 26 with structured materials.

Control Group

Don. **(Don regrouped 43 as 30 + 13 in order to complete the problem. He was then prompted to take the six by breaking the tens bar).**

Three take away 6 you can't do, so you take away one ten and put it in there and that equals three and that's 13 - 6, that's 13 - 6 ... that's 7.

Ewen. Can we do that? *No, because you can't take 3 away from 6.* So you can't take 3 away from 6, is that what you reckon? *Six away from 3.* What are you going to do?

Have to rename ... What are you going to rename. *I don't know how to.* Is there anywhere we could get six cubes from? (No reply - further prompt). *From the tens?*

Fred. *Yes ... six ... 3 - 6 ... oh no 6 -3 is 3 ... I just remembered it's that number take that.* Which number are we taking away? *Taking away that three ... no taking away six.* Point to the number we are taking away. *Six.* Can you do that? *Yes ... 6 - 3 is 3 and 30, no 40 - 6 is ... um 35. That's 35. Put down 5 and put 3 up there.*

Experimental Group

David. *This you can't do, cross this, take this one ... this ten 1's ... right ... now I can take this.* (David broke the tens bar into unit pieces). Have you finished? *No.* Where's your answer? *Up the top.* Can you write that answer or show the answer with those cards? *Seven.* Is that the answer you got there? *Yes.*

Edward. Where is your answer? *There.* (Edward exchanged one ten bar for tens separate cubes and placed them correctly). What answer did you get? *Seventeen.* Have you shown what you took out correctly here ... do you want to sort that out a bit? (Inaudible response). Where's the two 10's you took out? *There.* Where is the 6 you took out? *There.*

Frank. Can you take 6 cubes from there? *Yeah but it will equal zero.* Why. *Because you can't, cause 6 is a higher number than 3. If there's 3 you can't take 6.* Is there any way you can take 6 from what you have up there. *Yeah. Took those 2 away.* (Frank got a tens bar and removed two cubes and then another two cubes which he discarded).

CONNECTION

Control Group

Don. So show me again how you got 3 there and 13 there. *That's 4 ... when you couldn't take it away, you had to take ten from there so that equals three.*

Do you think you have the same answer here (as you worked out on paper)? *Yes.*

Ewen. (Ewen seemed to make some link between the subtraction operation using materials and the idea of renaming in the written algorithm. See reference to renaming below).

Fred. (Fred's partial solution suggested confusion between minuend and subtrahend and this difficulty was evident in the dialogue when using materials. There was also evidence in both the written and practical approaches of possible confusion with the multiplication algorithm - in both cases there was an attempt to take six units from both the units in the top line and also the four tens).

Experimental Group

David. With the 3 and the 13 here, did you do that up there? *Yep, there was four 10's and there was 43 and 26 and so you can't take six from three so I carried one ... a ten over to the ones then you could take six from that ... that equalled up to seven. Then I took away 20 from the ones I had left and that equaled 17.*

Edward. Did you do that with blocks here? *No cause I can't do it.* ... When you were taking away the six, did you take six out of there? *Yep.* How did you do that? *I carried it.* Where did you get six from, because you only had 3 (ones) up here. *I got 6 from 13.* Where did you get 13? *Cause I borrowed 10 from that.*

Frank. (The important connection for Frank was the meaning of the operation 'take away').

VERIFICATION

Control Group

Don. **(Did not spontaneously demonstrate recognition of addition/ subtraction link).** If you put all of the Unifix cubes on the mat together? *Um ... 69 or something.* If I added those two numbers together, 26 and 17. What would it come to? *Um ...* Not sure? *Two and one ... 43*

Fred. Say we got different answers, one on paper and the other with using materials, would that matter? *Yep.* Why. *That would be ... they can't be ... like, the same sum can't be two different answers, unless it's forty three take ... take away and a plus ...they could be different answers or a subtraction and a times.* OK.

Experimental Group

David. Look at all the Unifix pieces on the board. (David begins to count them). Without counting, how many do you think there are? Are there more than we started or the same or are there less? *Less.* (On your paper) what would all that add up to? *Um ... Zero.* If I added that and that? *Mm ... 33,* Thirty three? *I mean 43 ... yes 43* OK

Edward. How many cubes would there be altogether? (Edward begins to whisper through adding sequence). No adding.... *Oh, 4* If I added these two numbers (on your paper) up, that number 26 and that number 17 and added them together, what would I'd get? *43* Is there any way you could find that out without adding them up? *No, don't know.*

Frank. Without looking at the cubes, tell me how many cubes are there? *Um ... forty three.* Are you sure? *There's 23 up there and 20 here ... We started off with 43 and take away 26.* How did you know there was 43 there? *I just remembered.* Remembered what? *Remembered what the sum was.* So there were 43 cubes there? *Yep.*

POST-STUDY TASK A

All the pupils that were interviewed at the pre-study stage were presented with the same task during the post-study interviews. This task was identical to that presented to the higher achieving group at the pre-study stage.

EXPLANATION

The post-study tasks were identical to the pre-study tasks and they were presented in a similar manner consistent with Labinowicz (1985) and Carney and Barrow (1993).

Control Group

Alex. *Three take away one equals 2 ... 4-6, you can't do, take from the 100's column and I put 1 up here so its 14 tens - 6 tens = 8, and 1 and so that's 182.* What does this one stand for? *It stands for 100 or ten 10's.* And this one here? *It stands for 100 ... ten 10's.* **Spontaneous place value was given. Nearly always used S.O.S. syntax.**

Bill. *Three take away 1 equals 2 ... 4 - 6, you can't do, cross out the 2 make it 1, put 100 over next to the 4 ... 14 - 6 is ... 8, put down 8, 1 - 0 is 1.* Your answer is? *182* What does this one stand for? *10.* Is this one ten or is it something else? *It's 100 changed into ten 10's.* **Spontaneous place values were not given. S.O.S. syntax used.**

Carl. Tell me how you worked it out. *3 take away 1 = 2 ... 4 - 6, can't do cross out the 2, then make that a 1, put 1 on top of the 4 ... 14 - 6 = 8, put down the 8 ... 1 - 0 = 1. The answer is 182* OK What does this one stand for? *100 no ... yes 100.* **Spontaneous place value was not given. S.O.S. syntax was used.**

Don. Tell me how you worked it out. *Well 2 ... 3 take away 2 is 2 ... no 3 - 1 is 2 ... 4 - 6 you can't do, cross out the two and carry the one ... no, put 1 in the 10's column ... um, 14 - 6, that's 8 and 1 - 0 leaves 1.* What does this one here stand for? *One ten.* Is this one ten or is it something else? *That's one ...1 ... 100 ... um.* **Spontaneous place values were not given. S.O.S. syntax was applied.**

Ewen. Tell me how you worked it out. *3 take away 1 is 2 ... 4 - 6 you can't do, cross out 2, make it 1, carry the 1 ... 14 take 6 is 8, 1 - 0 is 1.* What does this one stand for? *100 ... 10* What does it stand for? *A 10.* **Spontaneous place value was not given. S.O.S. syntax was used.**

Fred. *3 take away 1 is 2, 4-6 you can't do, cross out 2, put 1 at the top of 2, put 1 next to 14 ... 14 -6 is 8 and ... oh, yep 1+ ...oh ...*Are you happy with that? *No.* Start again. *3 - 1 is 2, 4 - 6, you can't do ... oh, yeah I'm happy with that.* Happy with that. What does this one stand for? *Ten ... No ... 100.* **Spontaneous place values were not given. S.O.S. syntax was always used.**

Experimental Group

Alan. (Alan was able to talk through the various steps of the subtraction algorithm that he had performed correctly on paper. Although he got the correct answer, he reversed the subtrahend and minuend) ... *1 - 3 = 2* and then ... *6 - 4 you can't do.* What does this one stand for? *One Hundred.*

Ben. *You got to take 1 from 3 equals 2, 6 away from 4 you can't do, so you borrow ... exchange one of the 100's for 10's, and you take away 4 from 10, I mean 6 from 10 and left over from that is 4 ... you got 4 still and that equals 8 ... and you took one 100's away and you don't have to take any more 100's away and that equals 182.* **S.O.S. syntax was not used.**

Colin. *3 take away 1 equals 2, 4 - 6, you can't do, so you go to the 100's column and take away 100, cross off the 2 and put 1 into the tens column and that makes 14 tens take away 60 is 8 tens and then 100 take 0 is there.* This one here, what does that stand for? *Um ... 10.* What does it stand for? *Ten* One 10 is it? *No ten 10's.* **S.O.S. syntax was used.**

David. *Three take away one is 2 ... 4 - 6 you can't do so I took one from the 100's and put it on the forty, so that made it 100 and 4 ... 40 take away 6 and that equaled 80 and then there was nothing to take away from the 100's so I just put what was left.* (Spontaneous place value was given). What does this one stand for? *100.* **Spontaneous place value was given and S.O.S. syntax was used.**

Edward. *Um ... Well I took 1 away from 3 and that equaled 2 ... can't take 4 from 6 ... so, oh ... I can't take 6 from 4, so I borrow a ten from the 2 ... the 200 and put it in the tens column to make 14 ... 10's and I took away 6 from 14 that equals 8, then I took* *nothing from 1 and that equaled 1.* What is this one here? *One 10.* **Place value generally was given but S.O.S. syntax was not used.**

Frank. Let's hear you do that? *Ah ... I added, um ... I took away one from 3 equals 2. I took away 1 from 4 = 3 and I took away 1 take from 2 and that = 1.* **Spontaneous place values were not given and S.O.S. was not used.**

ERROR PATHS

Control Group

Bill. Can you take 61 out of that material? ... *Can I have 40 tens exchanged, please?* What do you want? *Forty tens exchanged.* Forty? ... *Ten 10's.* You are wanting to exchange ten 10's for 100, did you do it on paper? *I didn't exchange anything, except I crossed out the 2 and made it 100 instead of 200 and put the 100 over on the 14.*

Don. (Don initially removed the flat piece of foam worth 100 and placed it in the tens column. He then picked it up again and swapped it for two tens after he sought permission to exchange). *Can I swap this?* What are you swapping it for? *Two.* You're going to swap it for 2? *Yes, and ... oh yes.* (Time allowed to complete task)

Ewen. How many tens do you have to take away? *Six.* Can you do that? *Nope.* Is there anyway you can do that? *Yep* ... (Ewen took a flat with a value of 100 from the 100's column and placed it in the tens column. He then put one long worth 10 in the 100's column. He seemed unable to proceed beyond this point). *Yeah that's right ... one ... no.* Not sure of that? *No*

Fred **Seemed confused by the materials**. *3 take away 1 is 2 ... 6 - 4 you can't do. ... Oh 6 - 4 is* ... (It was not until the third error of '6 - 4, you can't' that the interviewer prompted). What are we taking away? *Four.* Which number are we taking away? Top or bottom number? *Top* ... We are taking the top number away. *No ... Yeah, bottom number.*

Experimental Group

Ben. *One take three equals two ... six take four you can't do.*

Frank. **(In written and practical forms Frank applied an inappropriate strategy borrowed from the multiplication algorithm).** *I took away one from 3 equals 2. I took away 1 from 4 = 3 and I took away 1 take from 2 = 1.* **Errors were more complex in the practical approach.** *I took 1 from 4, ... 1 from 40 = 10 and 1 from 100 = a 100 ... 120 ...* So what is the answer? *mm 111*

DEMONSTRATION

Control Group

Alex. Can you take away what's shown here? *3-1 leaves 2 ... Take away 6, you can't do it so you have to exchange that.* (Alex exchanged correctly and completed the task) What is the answer? *182.*

Bill. **Prompting after initial errors then** - Take away the number shown on the cards. (Bill began and took out the one unit piece. He then proceeded to exchange 100 for two 50's and completed the task successfully). *Took 60 off that.* Show me the number you took out. (He pointed to the pieces taken and placed on the third line). What answer did you get? *182*

Carl. Take away the number shown on the cards. *Took 60 off that.* Show me the number you took out. (Carl pointed to the pieces taken and placed on the third line). What answer did you get? *182 I carried one from there to there.* So you took 100 out of there and ... *put it there* What's that one there (on your paper)? *100.*

Don. ... So do you think your answer of 162 is right ... What have you got to do with that? *Um ... got to swap it.* For what? *For a four.* Is that fair? *Oh, no.* For four 10's is it? *Yes.* Is that fair though to swap that for four tens? *Oh no ... for just four um. No.* After further prompting. What is left behind. *181.* What did we take out there? *Took 2.* We should have taken out 1 shouldn't we.

Ewen. **Andrew seemed confused and unwilling to complete the task.** What have you got to take away in the units? *One.* What do you have to take away in the tens column? *Six.* Can we do that? *Yes.* Take away the numbers shown on the cards ... how many tens do you have to take away? *Six.* Can you do that? *Nope.*

Fred. What's the answer then. *60...? ... Shall I put it back in the box.* No. What's up the top? *100 ... 180 ...* 180? ... Anything else? *Oh ... and two 1's ...* What do we notice about that? *That's 182.* So where is our answer here? *It's up there.* *Right*

Experimental Group

Alan. Take away the number shown on the cards. Started with the units. (Alan exchanged ten tens for 100 and placed them correctly). What answer did you get here? *One hundred and eighty two.*

Ben. *Then you got to take 60 away from 40 so ...* (Interviewer interrupted). Did that happen here (on your paper) when we took 60 from 40? *We got 1 of the 100's and exchanged for 10.* Will you do that here? *Yes. Take one of the 100's, exchange for ten 10's. Got to take 6 from 140, took that away ... and that's all.* So what answer did you get there? *182*

Colin. What do we do now? *Take ... should be one more ... take 6 you can't ...* (whispered utterances as Colin exchanged two 50's for 100 and places them correctly, then went through the various steps of the operation). What answer did you get here? *Um ... 100 ... and 82*

David. Take away the number shown on the cards? (David took out the units correctly and proceeds to carry out an exchange of 100's for ten tens). Did you exchange correctly? *Yep.* (Pieces equivalent to 100 were taken for an exchange, but the 100 piece was not removed from the mat - error corrected without further prompting). You forgot about doing a swap didn't you?

Edward. Use those *Blockaid* cards to show the number we are taking away. What do these cards show? *Six tens and one unit.* Can you do that? *Take 1 from that ... then exchange one of these ...* (Interviewer interrupted to check links between approaches)

Frank. What did you take out then? *I took 1 from 2 is 1 ... and 1 from 6 ... I took 1 from 4, ... 1 from 40 = 10 and 1 from 100 = a 100 ... 120 ...* So what is the answer? *111 mm 111* Your answer is 111? *Do you do ...* Is your answer on the top or on the bottom? *On the top.* So what is your answer? *132.*

CONNECTION

Control Group

Alex. Can you point to something on the mat that this "one" stands for. ... *When we had to take it from here and put it over there when I exchanged it.*

Bill. Can you point to something on the mat that this "one" stands for. *It means 100.* Did we do that here (with materials) *How did we get that 100, we exchanged and got ten 10's to carry over to here.* OK.

Carl. Now you exchanged that for two 50's. When you did that was there something you did on paper that is like that? *I carried one from there to there.* So you took 100 out of there and ... *put it there* What's that one there (on your paper)? *100.*

Don. (Interviewer tried to see what link is understood from the written form to apply to the concrete materials approach in regard to the borrowing and exchanging aspects of the two approaches).

Ewen. (It seemed inappropriate for the interviewer to pursue the borrowing/exchange link).

Fred. (After much prompting an exchange of ten tens for the 100's flat piece was performed but due to the time factor no questioning was followed up on establishing the exchange/borrowing link used in the two methods).

Experimental Group

Alan. (Alan was able to make the connection between the borrowing steps in the written algorithm and the exchanging steps with the materials).

Ben. (Ben was able to make the connection between the borrowing steps in the written algorithm and the exchanging steps with the materials).

Colin. *I didn't have enough tens so I went to the 100's column, got one from there and exchanged it for two 50's and put it in the 10's column.* Explain how that worked on paper? *Explain it again?* Yes. *14 ... 4 - 6, you don't have enough tens, means you can't do it, so you went to the 100's column and gave one of the 100's and put it in the tens column.*

David. Now when you did that exchange, can you point to something on the paper where that happened? OK, can you explain that to me? *Um ... I crossed out 2 and made it one, and I put 1 on 4.* So what did we do with the foam? *Swapped it ... 100 for one 50 and five 1's.*

Edward. Did you do something (on paper) that you are doing now. *Yes*. Explain what you are doing. *When I did it there I had to cross out the 2 and put 1 up the top because there is no longer 200.* Are you doing the same here now? *Yep. Exchange that for one 50 and five 10's ... I got to take 6 away from that and that leaves ...* What's you answer? *182*

Frank. (Frank didn't identify the need to borrow in the written working of the problem nor the need to exchange when working with concrete materials and thus the borrowing/exchanging connection was not evident).

VERIFICATION

Control Group

Alex. Can we use addition to undo subtraction - *Yes* Look at all the *Blockaid* pieces on the mat. What is the total value of all the pieces? *243* Let's look at your working of the problem. If you added these two numbers together, what do you think you would get? *243*

Bill. Can we use addition to undo subtraction - *Yes, because you are adding instead of taking.* If we put all the *Blockaid* pieces on the mat together what would that come to? *The whole sum ... 243 take away 61 ... what would it all come to ... 243* On your paper if I added these two numbers up ...*You would have to do some changes in there ... 283*

Carl. What is the total value of the *Blockaid* pieces on the mat? *243 + 61* What would it come to? ... *if you added these two ...304.* (further prompt) ... just the *Blockaid* pieces. *They would add up to 143.* 143 or 243? *243* If you added this number and the bottom number what do you think you'd get? *243*

Don. If I put all those *Blockaid* pieces together on the mat what would they come to? *Not sure.* Can we use addition to undo subtraction? *Yes.* If I added these two numbers up, is there any number there on the sheet that would tell us what they come to altogether. *200 + 40 + 3 ... 243*

Ewen. Let's look at your working of the problem on your paper. If I added these two numbers up, what do you think I'd get? *243*

Fred. If you put all those *Blockaid* pieces together, what would it come to? *140* Can we use addition to undo subtraction? *I don't know.* If you put those back together, what would it would come to 140? I*t would come to 240.* (On your paper), if I added these numbers, what would we get? These numbers here. *61 and 182, um ... you would get 142 ... no 143.*

Experimental Group

Alan. Can we use addition to undo subtraction? *Yes, they are opposites.* Look at all the *Blockaid* pieces on the mat. What is the total value of all the pieces? *243.* Let's look at your working of the problem. If I added these two numbers up, what do you think I'd get? *243*

Ben. We have just done subtraction, can we undo that with addition? *Yes.* So if we put all the *Blockaid* pieces back together what do we get? *243* If you added those two numbers (on your paper) up, what would you'd get? *Um ...140, um ... yes 143* *underneath, you get 243 ... if you added it all back together.* 243? *You get the same answer.*

Colin. Can you use addition to undo subtraction. *Don't know.* Can you tell me what is the total of all the foam pieces on the board? ... *243* Can you use addition to undo subtraction? *Yeah.* Look at where we worked it out here (on paper), what would happen if we added these two numbers together? *Them two ... 243.* Are you sure? *Yeah.*

David. Can you use addition to undo taking away? *Yes.* Look at all the *Blockaid* pieces on the mat, if I put them back together what would they come to? *243.* If I added these two numbers together, the 61 and 182, what would they come to? *61 and 182 ... 243*

Edward. Can you use addition to undo subtraction? *Yes.* If I put all the foam pieces back together, what would it add up to? *243* If I added these two numbers together, the 61 and 182, what do you think it would come to? ... Do you know? *Nup.*

Frank. Can we use adding to undo subtracting. ... (No response) Is there any connection between adding and subtracting or are they totally different? *Totally different.* If I put those foam pieces together, what would it come to? *243.*

CONCLUSION

The purpose of the interviews was to attempt to assess the degree to which pupils had gained understanding of the mathematical concepts under investigation. The notion of understanding was pivotal to this study. The level of importance placed on attempting to assess such understanding can be gauged by the total time spent on conducting interviews which was nearly the same as that spent on instruction. Less than three and a half hours were devoted to each of the two groups on the subtraction algorithm. Approximately the same time was spent on interviews for the Control Group as the Experimental Group.

An assessment of mastery on the phases of interview is based on the full transcripts and analysed in the extracts of transcripts in this chapter. This assessment together with an appraisal of the concept and use of place value and, the "State-operator-State" procedure advocated for both groups is summarised in Table 25. The figures appearing in this Table were the number of pupils who were assessed as demonstrating mastery during interview of those aspects of learning listed. Error Path scores are negative scores in the table.

	Control Group		Experimental Group	
	Pre-study	Post-study	Pre-study	Post-study
Explanation	2	6	4	4
Demonstration	2	2	3	5
Connection	2	2	2	4
Verification	2	3	1	4
Place Value	2	1	0	2
State-operator-state	3	6	2	3
Error paths	-4	-4	-3	-2
TOTAL SCORE	9	16	9	20

Table 25. Scoring of phases of interview for procedure and understanding.

The scoring methodology applied equates the two groups on the totals for the procedures, the language and the errors at the pre-study stage. In the Explanation Phase of interview and in the State-operator-state language used at the post study stage, the Control Group scores for interview were better than the Experimental Group. The success of the Control Group in changing the language associated with the subtraction algorithm was outstanding. Such success may have been the result of standardising the subtraction algorithm and also more consistent drilling on the part of the Control Group teacher than that applied by the experimental teacher.

The total scores for interview were higher for the Experimental Group than the Control Group. The overall assessment, from the recollections of actual interviews, the transcribing of the tapes of interview, the reading of those transcripts by both the researcher and an independent observer and the scoring procedures applied to Table 25, was that there was a greater level of understanding shown by the Experimental Group.

If similar gains in understanding were made by the whole experimental class which the sample group represented then the conclusion must be that the treatment applied led to greater levels of understanding. The evidence suggests that this understanding facilitated gains in performance for subtraction by the Experimental Group that statistically were significantly better than those made by the Control Group.

CHAPTER EIGHT

CONCLUSIONS AND IMPLICATIONS

INTRODUCTION

The aim of this study was to determine whether a guided discovery approach to learning of number concepts and operational concepts using the *Blockaid* materials in the manner prescribed by Carney and Barrow (1993) leads to understanding and improved performance.

The *Blockaid* materials have evolved over a period of several decades through the interaction of the researcher (teacher) and his pupils in their attempts to construct meaning that is at the core of the number system. The approach for using the materials is designed specifically to promote understanding and proficiency of the formal whole number operations.

It was not envisaged that the study would attempt to link understanding with performance. However the evidence presented suggests that, in the case for subtraction, statistically significant improvement in performance was the result of the treatment using *Blockaid*. There was evidence in the data that suggested that greater understanding was achieved through the use of the *Blockaid* materials in other areas under investigation.

The study was ambitious and there were suggestions from the outset to narrow the focus to more specific outcomes. In hindsight it is probable that this very broad approach provided the best structure for a successful outcome for the study in which other factors including the "Teacher Effect" influenced the outcomes. A narrower focus would probably not have produced significant results in the time frame allowed.

The complexity of the study was compounded by attempts to monitor affective factors. These could arise from the difference between the two groups generated by the manner of instruction in which one group was involved in a guided discovery approach using materials and the other group in a modified directed instruction approach.

FACTORS AFFECTING THE PROJECT

There were a number of difficulties associated with the implementation of this study Some of these had a confounding effect on the extraneous variables which made it difficult to determine the effect of the treatment with the *Blockaid* equipment. Analysis of these difficulties raises a number of implications and avenues for further research.

The difficulties and implications are briefly summarised here:

Trial Period. There was probably insufficient time devoted to effectively teach the wide range of skills in the time allocated. This belief is confirmed by Sowell's (1989) study on concrete versus abstract instructional conditions for influencing mathematical performance:

> ... length of treatment was found to be related to achievement. When treatments lasted a school year or longer, the result was significant in favour of the concrete instructional condition. Treatments of shorter duration did not produce statistically significant results (p.503).

The breadth of the trial period allowed only 4 one-hour lessons to be conducted by the researcher for each of the five skill areas being investigated. Two such lessons were given for each topic in Term 3 and another two in Term 4. Time allocation to each topic could have been more effectively used without extending the treatment period.

Understanding Understanding

The central theme of this study was understanding. This theme has highlighted the researcher's ignorance regarding the nature of knowledge and the part played by understanding in knowledge construction.

The thesis has been a challenge to the researcher in his attempts to gain insight into the manner in which learners make links between procedural and conceptual knowledge, how understanding is built through establishing networks connecting different pieces of knowledge, and how concrete teaching aids can facilitate learning.

The quest for teaching practice which builds understanding is a search for understanding of the child, and the nature of learning, and also understanding of knowledge and how knowledge is acquired by the learner.

The work of Carney and Barrow (1993) in this field has been influenced very much by Piaget, Bruner and Dienes. More recent research of Fuson (1992) and Hiebert and Carpenter (1992) reinforce the importance of connecting mental imagery to physical models specifically designed to mirror targeted concepts or procedures.

For this study there was no written test for assessing understanding of the mathematics skills being investigated. The planning associated with the clinical interviews as the means for assessing understanding was the single biggest challenge for this study. It is a challenge for future study.

Information Processing Load. The mapping procedure for concrete modelling of operations can impose additional memory load on the learner which in some cases can hinder learning rather than assist it. Boulton-Lewis & Halford (1991) contend that:

> ... concrete teaching aids are useful only if children clearly recognise the correspondence between the structure of the material and the structure of the concept *(or in terms of analogue theory, if they can map one structure into another). Seeing this correspondence imposes a processing load. Success therefore requires that all other processing loads be minimised, to make as much capacity as possible available.* In practical terms this means the child must know the representations, concepts and procedures thoroughly, so that the recognition of the relevant relations is automatic (p.46).

Boulton-Lewis and Tait (1994) cautions teachers against some of the strategies recommended in curriculum guidelines because of the additional processing load that can interfere with the predicted transfer of learning from the concrete representation to the concept.

While the guided discovery approach prescribed in the *Blockaid* manual is designed to minimise this problem by leading the children through simple stages progressing from the concrete to the abstract, there is a need for further refinement and follow up research, particularly with the operations of multiplication and division.

Goodness of Fit.

Planning of lessons for the Experimental Group was difficult because the children often lacked the prerequisite skills that would ensure that the modelling with *Blockaid* was appropriate at a particular stage of learning.

> ... whether or not teacher-determined strategies and representations facilitate or interfere with learning depends on their 'goodness of fit' with the child's existing procedures, how much they increase the processing load for each individual child, how motivated the child is, and how well they are taught (Boulton-Lewis & Halford, 1991, p.47).

The goodness of fit criteria require that the steps of the procedure with the use of materials are mirrored by the same steps when performing the formal algorithm. When the steps in the procedures becomes precise and accurate the mental image for the formal algorithm can follow the same procedures readily.

Class Selection. It was important for this study to try and select two classes of comparable ability. As outlined in Chapter 3, the two classes at the beginning of the year were matched according to the characteristics of gender, grade level and ability using a stratified sampling strategy. However, when the pre-tests were given prior to the commencement of the study in Term 3, the pupils from the Control Group displayed superior performance on all skills tested. In the analysis of scores for subtraction there was a statistically significantly difference which indicated that the Control Group performed significantly better on that skill.

Teacher Effect. The Control Group and Experimental Group teachers were not of comparable competence. For this study the researcher had no knowledge of a school in which there were comparable class groups matched by teachers of like gender, experience and competency. Attempts to balance these factors by replication of the study in another school failed. To prevent bias that would favour the Experimental Group, deliberate selection was made in assigning the more experienced teacher to the Control Group.

The difference in competency of the two teachers, and the likely effect that their instruction would have on performance on their respective groups, was discussed with the principal. This discussion confirmed the researcher's strong belief that performance of the Control Group was greatly influenced by the "Teacher Effect". Evidence suggests that the this effect was accentuated further when an element of competition became evident.

John Henry Effect. The hypothesis that the John Henry Effect was unduly affecting performance of the Control Group was based on responses to the Attitudinal Questionnaire which showed evidence of marked changes of pupil and teacher attitudes between the pre-study and post-study periods. The significance of these changes in attitude are due to the link between affect and cognition as discussed in Chapter 2.

While there is some evidence from one question that could suggest there was a more favourable attitude towards mathematics by the Experimental Group, the balance of evidence from most questions suggested changes in attitude that favoured the Control Group in mathematics learning. The implications of this evidence were discussed in Chapter 5.

Revision Time. There was a difficulty in deciding the time allocation to be spent on revision lessons of the researcher conducted classes. The lesson conducted by the researcher was of one-hour duration. The recommendation was made that there would be a further one hour devoted to follow up revision with a drill and practice approach by both groups. There was however no provision made to monitor the total time spent on such skills that were part of the normal 6 hours a week (approximately) of mathematics instruction normally given in the school for those grades. Attempts by the researcher to meet with the teachers to negotiate such provisions prior to implementation of the study were unsuccessful.

Teaching Style. The researcher experienced some difficulty in maintaining the attention of the Control Group for the one hour lessons taken by him. The teaching strategy applied to this class was a modified traditional direct instruction approach which took account of cross-cultural studies of classroom practice such as Stigler and Fernandez (1993). Their study compared pupil performance of the Unites States and some Asian countries noting the differences in time given to class instruction and the emphasis given to dialogue between pupils and teacher to negotiate solutions to solving problems.

Similar emphasis was placed on children's thinking and problem solving with the Experimental Group. Maintaining attention of pupils in that group was not a problem.

End of Year Disruptions. Another major difficulty arose as a result of the study being conducted in Terms 3 and 4 instead of Terms 2 and 3 as originally planned. The end of the school year did not provide the best climate for taking classes, administering pupil questionnaires and tests, and for conducting more than 12 hours of interview.

Normal end of school year disruptions in the last week of school interfered with the planned program of lessons. Time tabling for interviews was very difficult at that time but all were conducted. Post-tests and the post-study questionnaire were give but there was no time to follow up pupils who missed tests and questionnaires.

RESULTS

Skill Performance. In spite of the limitations of the project, the Experimental Group performance for both subtraction and multiplication showed greater percentage gains on pre-test mean scores than the Control Group.

The conclusion reached in Chapter 6 was that the magnitude of the gains for subtraction made by the Experimental group were statistically significant and that the better performance by that group was the result of the treatment using the *Blockaid* materials.

On the post-test for multiplication, the Experimental Group made greater gains on mean average pre-test scores than the Control Group. Analysis of gains was not performed on these results as the level of gain was not sufficient to be statistically significant. However these gains are noteworthy in view of the various factors influencing performance in the study.

In the researcher's view the overall improvement in number concepts and algorithmic skills for both the Control and Experimental Groups was greater than would be expected in a twenty week period of middle primary schooling. In his view learning was accelerated. Analysis of results for skills tested using the t-test indicated that statistically significant improvement was made by both groups during the treatment period in all areas tested. The improved performance is shown in the graphing of pre-test and post-test mean scores for both classes. (See Figure 14)

Possible reasons for accelerated improvement of the facts and skills tested would include the "Teacher Effect" the "John Henry Effect" and the "Treatment". Attempts to adequately control such extraneous variables as the "Teacher Effect" and the "John Henry Effect" in order to assess the effect of the "Treatment" using *Blockaid*, failed.

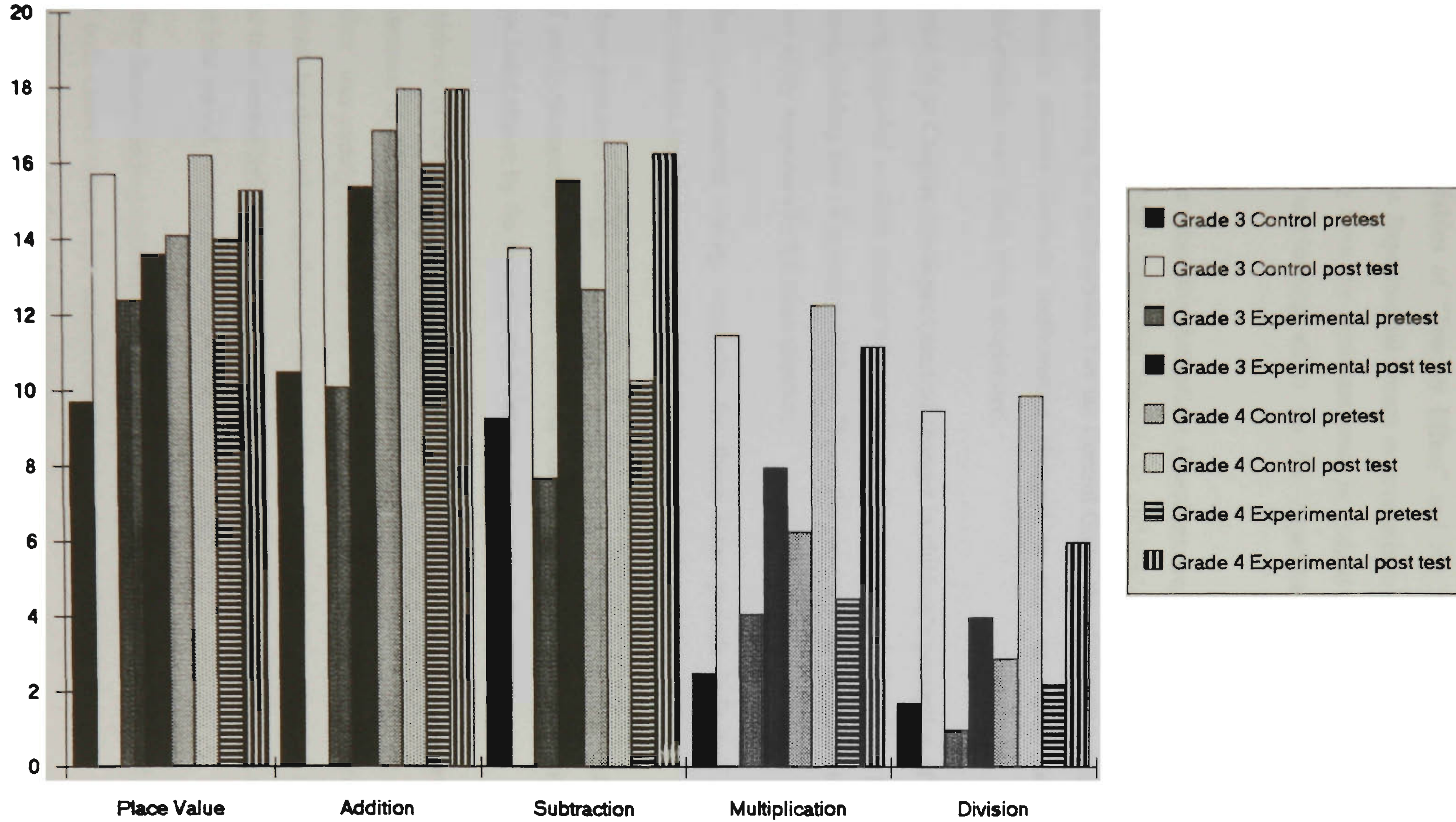


Figure 14. *Performance on number and operations, Control and Experimental Groups.*

Because those variables of "Teacher Effect" and the "John Henry Effect" were not advantageous to the Experimental Group in comparison to the Control Group then any difference resulting from the treatment must be judged in that light. The "Treatment" certainly had an accelerating effect on the Experimental Group's performance on subtraction.

Attitudes. Evidence from the Attitudinal Questionnaire, and other sources mentioned previously, leave little doubt that changes in children's attitude towards mathematics occurred during the study period. For the Control Group the children's perception of their teachers attitude towards mathematics changed to such an extent that results in performance were likely to be accelerated.

Table 7d in Chapter 5 indicated marked changes in children's perception of Mathematics being regarded as their teacher's best subject. For the Control Group the percentage of pupils holding this view increased from 7% on the pre-study questionnaire to 33% on the post-study responses to the same question.

The Experimental Group responses for those same periods indicated a decline in nominations for Mathematics as teacher's best subject from 39% to 19%.

These perceived changes in teacher attitude were accompanied by an increase in number of pupils nominating mathematics as 'best' subject by the control Group and a decline in that same choice by the Experimental Group.

Evidence of these changes in pupil attitude and perceived changes in teacher attitude was discussed in Chapter 5. It confirmed suspicion by the researcher that the "John Henry effect' was unduly influencing the level of performance of the Control group. A chance remark by the control teacher to the researcher made some weeks after the completion of the trial period left no doubt that an element of serious competition had developed during the trial period.

Other factors adding impetus to the John Henry effect may well have been the gender mix of both classes where boys outnumbered girls nearly two to one and also the quality of instruction given by the experienced male teacher in charge of the Control Group.

The assertion that achievement of performance on skills tested was probably affected by changes in teacher attitude is made on the basis of the intimate link between the affective and cognitive domains described in McLeod (1992).

Understanding. The results from this study support the hypothesis that understanding of a particular algorithm can be improved by establishing connections between the actions performed by written computation and the corresponding actions with the *Blockaid* materials for the same computation.

There can be little doubt that the improved performance of the Experimental Group for both subtraction and multiplication was the result of links made in the minds of the learners between the multiple representations; spoken, physical modelling both discrete and continuous, pictorial and written symbolic.

The evidence submitted in Chapter 7 showed that pupils from the Experimental Group were able to make the connections between the procedures for subtraction with materials and the formal algorithm more readily than those in the Control Group.

It is this linking process which transforms procedural knowledge into conceptual knowledge provided the model satisfies the criteria "goodness of fit". Results from the sample are an indication of the effect of treatment on the whole of the Experimental Group. The treatment for establishing connections resulted in statistically significant gains by the Experimental Group.

Attempts to facilitate understanding with the Control Group through dialogue without the support of concrete modelling were not as successful.

The conclusion is therefore that the treatment for subtraction using the *Blockaid* materials in the manner prescribed was superior to a direct instruction approach for those pupils investigated. The only explanation that the researcher has for this result is that the "goodness of fit" criterion for the subtraction model was realised with the Experimental Group using *Blockaid* materials and that the resultant learning with understanding did not require the same degree of drill and practice as did the Control Group.

It is the researcher's firm belief that *Blockaid* has the potential to facilitate improvement in performance and understanding of number and the operations of addition, subtraction, multiplication and division for middle primary school pupils. This study has however prompted critical analysis of the *Blockaid* materials which has implications for both future revision of the materials and for further research.

THE NEED FOR FURTHER RESEARCH

The history of civilisation is very much linked to the history of mathematics. The development of the concept of number and subsequently of calculations was closely linked to the creation of models, and of language and symbols. The earliest models for checking the numerosity of collections such as a flock of sheep involved tallying which involved making a one-to-one-correspondence between sheep and notches on a piece of wood or bone. Long before recorded history early man devised the concept of number and invented counting and calculating processes.

The history of numeration and calculating is complex, for example it is difficult to realise the importance of the invention of 'zero'. The modelling of number and the use of calculating devices such as the Chinese abacus is part of that history.

Number is a very abstract concept in the same way as colour. There is no such thing as a red nor is there such a thing as a three. The abstractions of colour and of number are derived from objects. These collections of objects are the starting point for model construction. Determining the numerosity of larger collections requires structuring through grouping and for formal recording, symbols and place value concepts.

The need in primary schools for number ideas to be represented with structured materials has a long history. Australia has been at the forefront in the work involving Base 10 material, including *Blockaid* which is the focus of this study. In 1922, the Hill's Constructive Counters described in Chapter 2, were probably one of the first sets of Base 10 material devised. This material was linked to the metric measurement system, not only in length, area and volume but also in capacity. Centimetre cubes could be stacked in trays and boxes which formed part of the apparatus. The design of this material and of the procedures advocated in the handbook by its author are still relevant today. The linking between number ideas and measurement ideas helps build conceptual structures in these two fields of mathematics.

Montessori schools here in Australia promote mathematics learning through the use of a wide range of structured materials including Base 10 blocks. Much of the early work in Base 10 modelling was inspired by Dienes and his work at the Cowandilla school in South Australia.

All of these earlier models have been devised to assist teachers in creating learning situations which help children grasp the multiplicity of abstractions which link together to form a concept. The Confucian proverb is relevant to child learning. "I hear and I forget, I see and I remember, I do and I understand."

According to von Glasersfeld (1987) "... the primary goal of mathematics instruction has to be the student's conscious understanding of what she or he is doing and why it is being done" (p.12).

While the notion that children should learn with understanding is widely accepted as an aim in mathematics education and is incorporated in current curriculum documents such as the National Statement on Mathematics (Australian Education Council, 1990) and the Curriculum Standards Frameworks (Board of Studies, 1995), there is very little evidence to suggest that this goal is being achieved.

Evaluation of the 'Outcomes Approach" to education in Victoria through State-wide testing has not established any significant improvement of mathematics performance and can not even attempt to gauge children's level of conceptual knowledge. The danger of such testing is that the emphasis in teaching will be placed on learning of facts and procedures and not on understanding.

There is an urgent need for ongoing research which can demonstrate to teachers a clear link between understanding and performance. The researcher believes that, for the case of subtraction, his study illustrates that link.

Concerns about educational standards in Australia and the United States are aggravated by cross cultural studies which demonstrate that pupils in some Asian countries outperform their United States counterparts in understanding and achievement of mathematical skills.

Differences in cross cultural performance have been researched by Stigler and Fernandez (1993):

> Recent studies have demonstrated beyond doubt that Japanese students far outperform their American counterparts on tests of academic achievement, especially in the areas of mathematics and science. In one study of fifth-grade elementary school students, for example, the highest performing school in the U.S. sample did less well on a test of mathematics achievement than the lowest performing school in either Japan or Taiwan. These differences were not limited to computational skills; they were equally large or larger when tests assessed novel problem-solving skills or conceptual understanding of mathematical principles (p. 13).

We have much to learn from studying good practice from all parts of the world. According to Gill & McPike (1995) the Japanese make extensive use of manipulatives and of small group.

Goodness of fit and processing load

Perry, Mulligan and Wright (1991) cite Boulton-Lewis's 1989 study;

> The results suggested that whether or not teacher-determined strategies and representations facilitate or interfere with learning depends on their goodness of fit with the child's existing procedures, how much they increase the processing load for each individual child, how motivated the child is, and how good the teaching is (p.187).

While there may be some evidence that suggests that some procedures associated with using of structured materials may hinder learning because of the processing load imposed on the learner, there is little evidence that contests Piaget's research which substantiates the belief that children learn through activity and experience with concrete materials.

CONCLUSION

The use of materials has a long history. This study demonstrated that properly designed materials do aid understanding of mathematics concepts. The need for further research is clear. In particular this should focus on evaluation of materials and procedures which satisfy goodness of fit criteria.

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Appendix 1. Whole Number Tests.

SPEED AND ACCURACY

1. 2 + 3 =
2. 8 - 5 =
3. 2 x 3 =
4. 8 / 2 =
5. 1/2 of 10 =
6. 6 + 6 =
7. 11 - 7 =
8. 3 x 4 =
9. 50 / 5 =
10. How many days in the week? . . . .
11. 9 + 7 =
12. 16 - 9 =
13. 5 x 4 =
14. 16 / 4 =
15. Centimetres in one metre = ...
16. 8 + 6 =
17. 17 - 8 =
18. 6 x 5 =
19. 40 / 10 =
20. 1/2 of 24 =
21. 17 + 7 =
22. 23 - 8 =
23. 3 x 6 =
24. 44 / 4 =
25. 1/4 of 20 =
26. 6 + 8 + 5 =
27. 48 - 6 - 5 =
28. 9 x 6 =
29. 49 / 7 =
30. How many days in July? . . . .
31. 25 + 36 =
32. 64 - 17 =
33. 7 x 9 =
34. 48 / 8 =
35. How many days in a year? . . . .
36. 98 + 6 =
37. 126 - 8 =
38. 6 x 7 =
39. 64 / 8 =
40. 1/2 + 1/2 =
41. 120 + 120 =
42. 154 - 99 =
43. 9 x 9 =
44. 42 / 7 =
45. 1/4 + 1/4 =
46. 1234 + 1234 =
47. 1896 - 500 =
48. 8 x 6 =
49. 63 / 7 =
50. 1/4 of 128 =

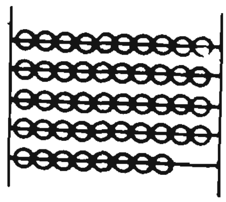
NAME: . . . .

SCORE: . . . .

TIME: Min . . . . Sec . . . .

DATE: . . . .

1. What number is shown on the bead frame ?

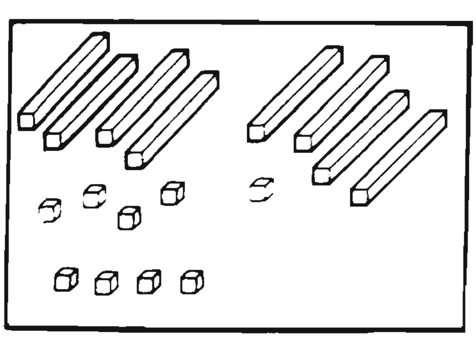


A 52
B 48
C 58
D 49

2. Another name for 43 is -
A 3 tens and 4 units
B 4 tens
C 4 tens and 3 units
D 4 tens and 4 units

3. Which of the following is a numeral for sixty-nine ?
A 609
B 69
C 96
D 6 009

4. The blocks illustrated show -

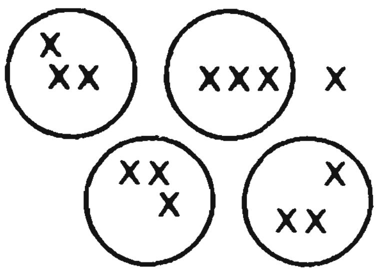


A 89
B 88
C 98
D 99

5. Which number is the largest?
A 608
B 580
C 680
D 588

6. We can write one hundred and thirty six in numerals as -
A 136
B 1 306
C 10 036
D 100 306

7. Which description below best describes the drawing ?



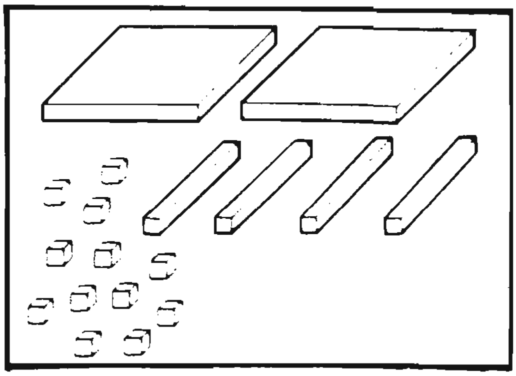
A 5 groups
B 4 groups of 3 and 1 more
C 4 groups of three
D 3 groups of 4 and 1 more

8. Which one shows the **smallest** number we can make with the figures - 3, 9, and 5?
A 359
B 395
C 539
D 953

9. Which number below is closest to the number 750 ?
A 700
B 860
C 740
D 850

10. Which of the following numbers is **larger** than ten thousand ?
A 10 000
B 9 999
C 1 999
D 10 001

11. The blocks illustrated can show -

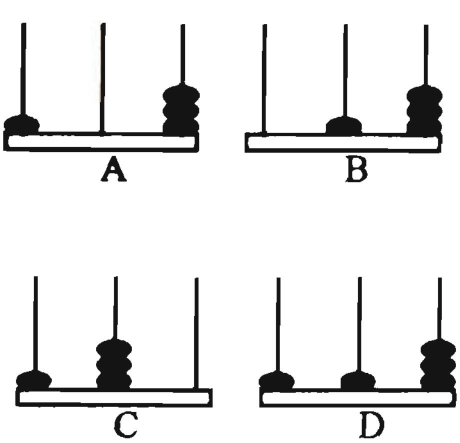


A 240
B 249
C 212
D 252

12. If  stands for 10 and  stands for 1, then -

stands for -
A 24
B 6
C 42
D none of these

13. Which abacus shows 103?



14. In the numeral 1847 which figure is in the tens place ?
A 1
B 8
C 4
D 7

15. Which of the following is the correct way of writing ten million in figures ?
A 10 000
B 10 000 000
C 1000 000 000
D 1 000 000

16. Which digit has the greatest value in the numeral 2 059 ?
A 9
B 5
C 0
D 2

17. Another name for forty-six tens is
A 46
B 4 60
C 4 610
D 4 600

18. The numeral 3 749 is less than -
A 3 479
B 3 749
C 3 794
D 3 497

19. How many zeros are there when the number ten thousand and seven is written in figures ?
A 4
B 2
C 1
D 3

20. 6 hundreds, 14 tens and 6 ones makes -
A 6 148
B 6 20
C 7 46

CONCEPT OF NUMBER 1

INTERVIEW ITEMS
FOR MIDDLE PRIMARY SCHOOL PUPILS

Counting - generating number-name sequences - oral counting.
(Labinowicz, 1985. pp 242-250)

1. Count forwards from eighty-seven.
   Count back by ones from fifty four.

2. Count forwards by tens, starting at thirty four.
   Count back from one hundred and thirty by tens.

3. Count forwards by fives starting at 73 - 123
   Count backwards by fives from 620 to 580

CONCEPT OF NUMBER 2

INTERVIEW ITEMS
FOR MIDDLE PRIMARY SCHOOL PUPILS

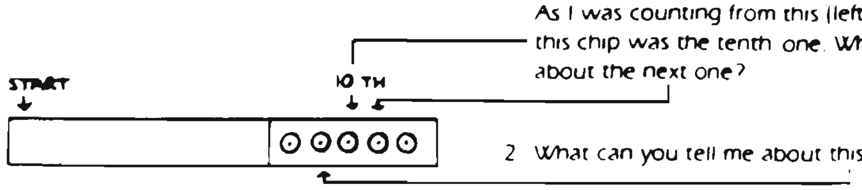
Cardinal / Ordinal Link

1. Put out 15 blocks - 7 red and 8 yellow.
   Let the child sort the blocks out.
   Tell me what you did. ...
   Are there more red blocks or are there more yellow or are the two groups the same? (if the child counts, ask how else we can decide which has more or less)
   Which group has less?
   How many more yellow blocks are there than red?
   Can we make the two groups the same number? How ?
   Can we do it another way ?

2. Put out collection that the child may arrange, to show numbers 1 to 6 in six colours.
   How can you sort these out ? Can you put them in order ?
   Tell me why you did that.
   Which is the first group ? How many in the third group ?
   Cover the cubes and ask "How many in the sixth group"?
   Tell me about this group - "four". Why did you put it there ?
   Could it go anywhere else ?

3. Constructing order and inclusion relations.
   Task A Labinowicz, 1985, p 61.

Task A: (Steffe, Herstein, and Spikes, 1976)



1 Yesterday I counted the chips on the board before covering some of them up.
As I was counting from this (left) end, this chip was the tenth one. What can you tell me about the next one?

2 What can you tell me about this one?

3 How many chips do we have on the whole board?
How did you decide?

4 How many chips are covered up?
How do you know?

5 Does the tenth one always come between the ninth and the eleventh one?
How come?

CONCEPT OF NUMBER 3

INTERVIEW ITEMS
FOR MIDDLE PRIMARY SCHOOL PUPILS

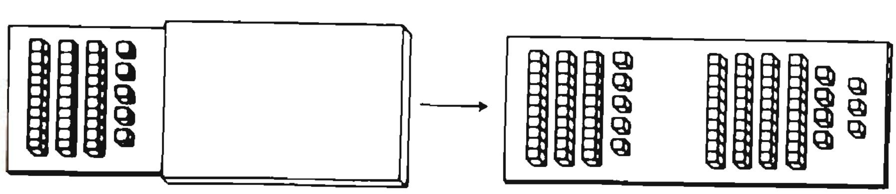
Grouping. See Labinowicz (1986), p 253

1. What is the number shown on the card? (show card "32")
   What does it mean?
   When I say "thirty two" what else do you think of?
   Count to see how many buttons we have in this container.
   Can you count them another way ? ... 2's **or groups of ten**

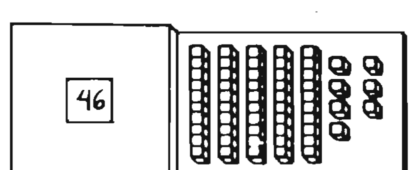
2. Using these coins put out 10c x 2 and 5c
   How much money do we have here?
   Put out 10c x 12 and 5c How much money now?
   (observe and note arrangement of coins by child)
   Can we show that amount of money another way?
   (have $1 & 50c coins available for possible exchange)

3. Grouping items from Labinowicz (1985, Page 242).

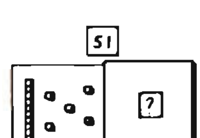
Count the value of the blockaid pieces showing. There are more pieces hidden. As I uncover them, count to find the total on the board.



Forty six is under here in blockaid pieces. Count on to find out how many on the board altogether.



There is fifty one in blockaid on this board. What number do you see ?
The rest are hidden here. Count to find the value of the hidden pieces.



CONCEPT OF NUMBER 4

INTERVIEW ITEMS
FOR MIDDLE PRIMARY SCHOOL PUPILS

Place Value. (See Labinowicz P 281)

1. Write the following numbers ; seventeen, seventy one, ten and two, two hundred and eighty nine, three thousand and ninety six.
   Read these numbers : 342 243 432
   What does the 3 stand for? How do you know ?
   Write the number that is one more than 342.
   Write the number that is ten more than 243.
   Look at these two numbers 231, 198
   Which of these two numbers is the larger ?
   How did you decide ?

2. Show the Montessori cards and ask -
   Have you seen these cards or cards like them before?
   Can you use the cards to make the number "57,
   Can you change the order of the cards around ?
   369, 1504" If I mixed those four cards up (1504), would the number change ?

3. Three spike abacus to be used with necessary beads.
   Have you seen equipment like this before?
   (If "no" give demonstration to show 12 .
   Ask what is the rule that makes this worth ten ?
   Show me "thirty six"
   Can "145" be shown?
   Now put out two thousand and eight.

CHECK LIST FOR NUMBER CONCEPTS PRESENTED AT INTERVIEW

CHILD"S NAME ....AGE ....
GRADE ....

Counting

1. 87 88 89 90 91 92 93 94 95 96 97 98 99 100 101 102 103 104
   54 53 52 51 50 49 48 47 46 45 44 43 42 41 40 39 38 37

2. 34 44 54 64 74 84 94 104 114 124
   130 120 110 100 90 80 70 60 50 40 30

3. 73 78 83 88 93 98 103 108 113 118
   620 615 610 605 600 595 590 585 580

Cardinal/ordinal

1. sorted by colour - put in two lines
   tell me what you did
   Reporting action - put them in two groups
   -put them in two lines
   - put 7r and 8y

   More red or more yellow responses - child counts both lots & replies ... Y / R
   - child matches one-to-one. Y / R
   Prompt if needed - match the two groups in line
   How many more yellow ? 1 7 8
   Which has less ? R / Y
   Can we make the two groups the same ? Y / N
   How ? (+ 1r) (-1y) How 2? (+1r) (-1y)

2.
   - sorted by colour - linked cubes of same colour
   - ordered by number L to R, (R to L) (other)
   Prompt for ordering if needed
   Tell me why you did it like that.
   First group Colour of third group (black)
   With groups covered ask "How many in sixth group ? (6)
   - tell me about this group (four) - one more than that one / one less than that
   - why did you put it there ? Could it go anywhere else?

3. Q2 - This one - 9th (other)
   Q3 - How many counters (12) (other)
   Q4 - Covered (7) (don't know)
   Q5 - Tenth position Y N
   Q6 - why

Appendix 2.

Grouping

1. 32 means - three tens and two units - (30 + 2) (10 + 10 + 10 + 2)
   - four groups of 8
   - thirty two cars

   count (45) - touch count
   - counting out - by twos, fives - into single pile
     - into groups of ten
     - into groups of five
   - another way
   - would putting them in tens help ? Why ?

2. Coins 25 cents
   $1.25 (other)
   Exchange of coins - 10c. x 10 for $1.00

3. Count on - 34, 44, 54, 64, 74, 75, 76, 77, 78, 79, 80, 81,
   From 46 - 56, 66, 76, 86, 96, 97, 98, 99, 100, 101, 102,
   Value of hidden pieces - to be counted out aloud

- Counting forwards by tens then units - use of fingers observed
- 15 (+10), 25 (+10) 35 (+10) 45 ... (+1) 46 (+1) 46 (+1) 48 (+1) 49 (+1) 50 (+1) 51
  30 + 6 =36
- or counting back from 51
- or computing in tens and units (tens first) (units first)
- no attempt

Place value

1. 17, 71, 12, 289, 3096

   342, 243, 432
   432 - three tens How do you know ?
   342 + 1 = 343
   243 +10 = 253

   Look at these numbers 231, 198 Which of these two numbers is the larger ?
   How did you decide ?

2. 57, If we separate the two cards can we put the seven first ?
   Is that the best way ?
   369,
   1504 If I mixed these cards up and put the thousand on top, what would the total of the cards be then ?

3. 36, 145 2008

ARITHMETIC OPERATIONS - ADDITION
INTERVIEW CHECK LIST FOR ....

Addition of two-digit numbers - 37 + 28 ... presented as standard algorithm, see Labinowitz P 308 -316 and Blockaid Workcard 7

Computation *Read the problem. Let's see you do it -*
correct .. incorrect (clerical, conceptual)
If child has difficulty with carrying, present example without carrying 66 + 23

Explanation *Tell me how you worked it out, or*
Tell me how you got the answer. (R to L) or (L to R) (carried 10)
Spontaneous place value explanation given (Y) (N)
What does the "one" stand for? (1) (10)
Does it stand for just one thing or does it stand for something else ( )
What does the three stand for ? (thirty) (three tens) (just three? Y N)
Why did you put the one up there and not the five?

Demonstration. COVER ANSWER TO WRITTEN COMPUTATION
Now lets see what you can do with these unifix cubes.
Show 37 red cubes on the mat.
(uses five Y N), puts materials in tens & units columns. Y N D
Now show 28 blue cubes.
What could you do to add them together
(Up: Down: Other) ..... (units first: tens first)
Spontaneous regrouping combining (Y) (N) (8+2 7+3 5+5 or exchange)
Pause *How many do you have altogether?* (65: ?)
Can we do something to the cubes so that we can see how many cubes there are altogether without counting? Regrouped Y N
Can you use these numberaid cards to show the answer (- 0 +)
Answer shown - (on the blocks) (elsewhere)
Can we show the answer somewhere ... bottom line?

Connection Display child's written computation with answer covered -
Point to something on the board to show what the one stands for-
Points to (groups of ten: the regrouped ten: points to one)
Pause *Does it stand for a particular one of these? Does it matter?*
If somebody told you that the one stands for this ten, what would you say about that? (- 0 +)

ARITHMETIC OPERATIONS - ADDITION
INTERVIEW CHECK LIST FOR ....

Addition of multi-digit numbers - 152 + 71 ... presented as standard algorithm, see Labinowitz P 316-322 and Blockaid Workcard 7

Computation *Read the problem. Let's see you do it -*
correct .. incorrect (clerical, conceptual)
If child has undue difficulty - proceed to simpler task 37 + 28

Explanation *Tell me how you worked it out, or*
Tell me how you got the answer. (R to L) or (L to R) (carried)
Spontaneous place value explanation given (Y) (N)
What does the "one" stand for? (1) (10) (ten tens or 100)
Does it stand for just one thing or does it stand for something else ( )
What does the seven stand for? (seventy) (seven tens) (just seven?Y N)
When you added the 7 and the five what did the 12 stand for?
Why did you put the one up there and not the two?

Demonstration. COVER ANSWER TO WRITTEN COMPUTATION
Now lets see what you can do with the blockaid material.
Use pieces to show 152 on the mat. (uses fifty Y N)
Puts materials in hundreds tens and units columns. (- 0 +)
Now show 71.
What could you do to add them together?
(Up: Down: Other) ..... (units first: tens first: hundreds first)
Spontaneous regrouping combining (Y) (N) (7+3 5+5 or exchange)
How many do you have altogether? (223: ?)
Can we do something to the pieces so that we can see how many there are altogether without counting? Regrouped: Y N
Use these numberaid cards to show the answer. (- 0 +)
Answer shown - (on the blocks) (bottom line)
Can we show the answer somewhere ... bottom line?

Connection Display child's written computation with answer covered -
Point to something on the board to show what the one stands for-
Points to (that 100: the hundreds: groups of ten: points to one)
Pause *Does it stand for a particular one of these? Does it matter?*
If somebody told you that the one stands for this one hundred, what would you say about that? (- 0 +)

Alignment of Numerals - 17 + 362 + 5 verbal format, non aligned

Read these numbers.
I wonder if you could add these numbers up?
Read your answer.
Does it sound right ? Does it look right?

ARITHMETIC OPERATIONS - SUBTRACTION
INTERVIEW CHECK LIST FOR ....

Subtraction from a two digit number 43 - 26 ... presented as standard algorithm.
See Labinowitz pp 340 -350 and Blockaid Workcard 10

Computation. *Read the problem. Lets see you do it -*
Correct: Incorrect (clerical, conceptual,) (reverse subtraction) (other)
Prompt - *do you know how to "borrow"*
If task is too difficult for child, do following task 67 - 34

Explanation *Tell me how you worked it out.* (R to L: other)
Spontaneous place value given (yes no)
Other - ten from one column and put it in the next
One ten borrowed here and ten ones over there
What does this one stand for? (- 0 +)
Prompt - *Is this one ten or is it something else?*

Demonstration. *Now lets see if we can do that problem with unifix cubes?*
(COVER THE ANSWER)
Show forty three cubes on the top line. (- 0 +)
Use these blockaid cards to show the number we are taking away. (- 0 +)
See connection ***
Show me how you can take away the number shown on the cards. T U (- 0 +)
Child regroups or exchanges

Connection. *** After regrouping/renaming and prior to taking ask -
Tell me how you got the 3 and the 13 here?
You changed things around on the board and also on the paper.
Point to something on the board that this "one" stands for.
If time permits -- for incomplete regrouping in computation --
Can you show on paper what you did with the unifix cubes?

Comparison.
(correct) *you got the same answer both ways. What does that mean?*
(incorrect) *Is one answer better than the other or are they both just as good?*
Which one of these answers is right or are they both right? (A B Both)

Verification. Can use addition to undo subtraction - Y N
Look at all the unifix cubes on the board.
How many cubes are there on the board? (43: ?) (- 0 +)
Lets look at your working of the problem.
If I added these two numbers up, what do you think I'd get? (43: ?) (- 0 +)

ARITHMETIC OPERATIONS - SUBTRACTION
INTERVIEW CHECK LIST FOR ....

Subtraction from a two digit number 243 - 61 ... presented as standard algorithm.
See Labinowitz pp 350 - 357) also Blockaid Workcard Number 10

Computation. *Read the problem. Lets see you do it -*
Correct: Incorrect (clerical, conceptual,) (reverse subtraction) (other)
Prompt - *Do you know how to "borrow"?*

Explanation *Tell me how you worked it out.* (R to L: other)
Spontaneous place value given (Yes: No)
Other - ten from one column and put it in the next
One ten borrowed here and ten ones over there
What does this one stand for? (- 0 +)
Prompt - *Is this one ten or is it something else?*

Demonstration. *Now lets see if we can do that problem with blockaid?*
(COVER THE ANSWER)
Show two hundred and forty three on the top line. (- 0 +)
Use these blockaid cards to show the number we are taking away 61. (- 0 +)
See connection ***
Take away the number shown on the cards. Starts H: T: U (- 0 +)
Child regroups or exchanges

Connection. *** After regrouping/renaming and prior to taking ask -
Tell me how you got the 1 and the 14 here?
You changed things around on the mat and also on the paper.
Point to something on the mat that this "one" stands for.
If time permits -- for incomplete regrouping in computation --
Can you show on paper what you did with the blockaid?

Comparison.
(correct) *you got the same answer both ways. What does that mean?*
(incorrect) *Is one answer better than the other or are they both just as good?*
Which one of these answers is right or are they both right? (A B Both)

Verification. Can use addition to undo subtraction- Y N
Look at all the blockaid pieces on the mat.
What is the total value of all the pieces? (243: ?) (- 0 +)
Lets look at your working of the problem.
If I added these two numbers up, what do you think I'd get? (243: ?) (- 0 +)

Subtraction with borrowing across Zero 702 - 368 ...
presented as standard algorithm (334: ?) (- 0 +)
With materials (334: ?) (- 0 +)
See Labinowitz pp 362 - 366)

Appendix 2.

CHECKLIST FOR MULTIPLICATION AND DIVISION - A.

Identifying the operation which best matches word problem

John has 4 cars. Jane has three times as many. How many cars did Jane have ?
John has five cars. He looses three. How many does he have left ?
John has 8 cars. If he puts them in 2 equal rows, how many would be in each ?
John has 3 red cars and 2 blue cars. How many cars does he have altogether ?

Multiplication A

1. What does this sign mean?
Are there any other words you could use ? (times, lots of, multiplied by, other)
3 x 2 = Read that problem. Lets see you do it.
What is the answer? 6 (Y N)
What does 3 x 2 = mean ? (Y N ?) ....
How did you work it out ?
Can you tell me a story that goes with that ? (Y N)
If you put out three lots of two cars, how many cars would you have put out ?
(Y N) If necessary child puts out cars.

2. Read this "3 x 6 = " (and vertical format) Paper and pencil.
Can you work it out ? (Y N ?) Tell me what you did.
Can you show that with lollies? (Y N ?)
Can you show that with blockaid pieces? (Y N ?) Subject uses 5 piece (Y N)
Prompt if necessary. - three lots of six, six put out three times.
Does this sheet help? (Y N) (balloon Diagram, see Blockaid Workcard 11)

Division A.

1. What does this sign mean?
Are there any other words you could use? divide, share how manys, goes into.
Read this problem 6 / 2 = Can you work it out ?
What's the answer ?
What does 6 / 2 = mean ?
How did you work it out ? (... in search of partition and quotition aspects)
Can you tell me a story that goes with that ?
If 6 cars were shared equally between two friends, how many cars would each person get ?

2. Read the problem "18 /3 = " (algorithm format) and work it out.
Paper and pencil. Tell me what you did.
Can you show that with lollies on balloon diagram?
Can you show that with blockaid pieces ?
Put 18 in rectangle on balloon diagram. Exchange performed.
Prompt if necessary. Simplest form - eighteen shared three ways.
(see Blockaid Workcard 13)

3. Multiplication /Division link 42 / 3 = 14 x 3 =

CHECKLIST FOR MULTIPLICATION AND DIVISION - B.

Identifying signs + - x /

Identifying the operation which best matches word problem

John has 4 cars. Jane has three times as many. How many cars did Jane have ?
John has five cars. He looses three. How many does he have left ?
John has 8 cars. If he puts them in 2 equal rows, how many would be in each ?
John has 3 red cars and 2 blue cars. How many cars does he have altogether ?

Multiplication B.

1. What does this sign mean?
Are there any other words you could use ? (times, lots of, multiplied by, other)
3 x 4 = Read that problem. Lets see you do it.
What does 3 x 4 = mean ? (Y N ?) ....
What is the answer? 12 (Y N)
How did you work it out ?
Can you tell me a story that goes with that ? (Y N)
If you put out three lots of four cars, how many cars would you have put out ?
(Y N) If necessary child puts out cars.

2. Read this "14 x 3 = " (and vertical format) Paper and pencil.
Can you work it out ?
Can you show that with blockaid pieces? (Y N ?) Subject uses 10 piece (Y N)
Prompt if necessary. - three lots of four, four put out three times, etc.
Does this sheet help? (Y N) (balloon Diagram, see Blockaid Workcard 11)

Division B.

1. What does this sign mean?
Are there any other words you could use? divide, share how manys, goes into.
Read this problem 12 / 3 = Can you work it out ?
What does 12 / 3 = mean ?
What's the answer ? 4 (Y N)
How did you work it out ? (... in search of partition and quotition aspects)
Can you tell me a story that goes with that ?
If twelve cars were shared equally between three friends, how many cars would each person get ?

2. Read the problem "42 /3 = " (algorithm format also) and work it out.
Paper and pencil.
Can you show that with blockaid pieces ? Put 42 in least number of pieces.
Exchange performed. (Y N) two fives or ten ones, error in exchange.
Prompt if necessary. Simplest form - eighteen shared three ways.
Does this sheet help? (balloon Diagram, see Blockaid Workcard 13)

Appendix 3. Blockaid Workcards 4,5,6 and 7.

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BLOCKAID

The 4-ways Game

STANDARD SET **4**

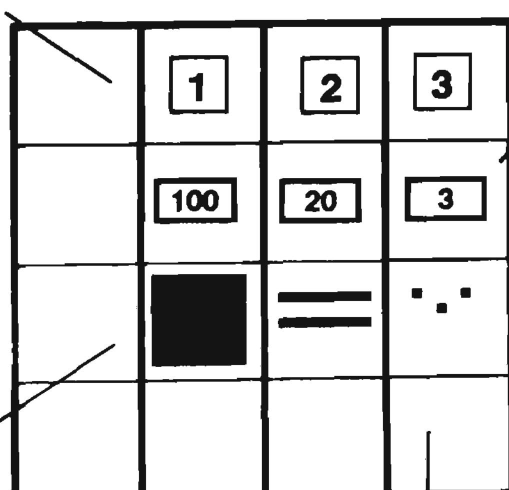
Players. Four players.

What you need. One four by four mat, one set of Montessori Dice, one set of Number cards, one set of Blockaid, and one set of other Base Ten material.

What to do.

Player 1.
I have thrown the Montessori colored dice and I have placed the red in the hundreds, the blue in the tens and the green in the units

Player 2.
Now I have shown the same number on the next line using Number Cards



Player 3.
I have made up the same number on line three using the Blockaid blocks.

Player 4.
Now I am going to make up the same number on the last line using the fourth set of material.

When everyone has completed the first round, they all move on to be the next number player, Player 4 becoming Player 1.

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BLOCKAID

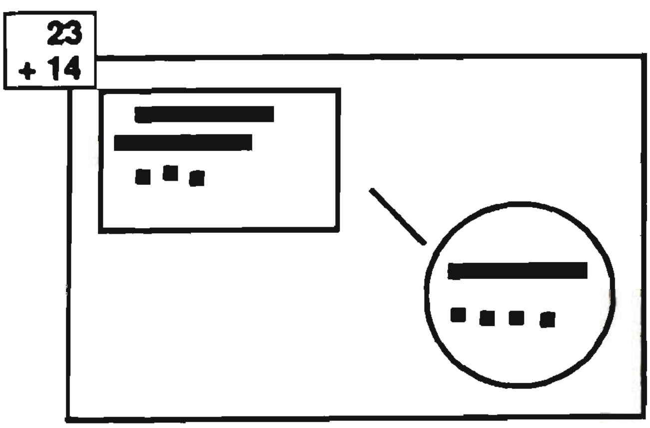
State-Operation Addition

STANDARD SET **5**

Players A group of between 3 to 5 children.

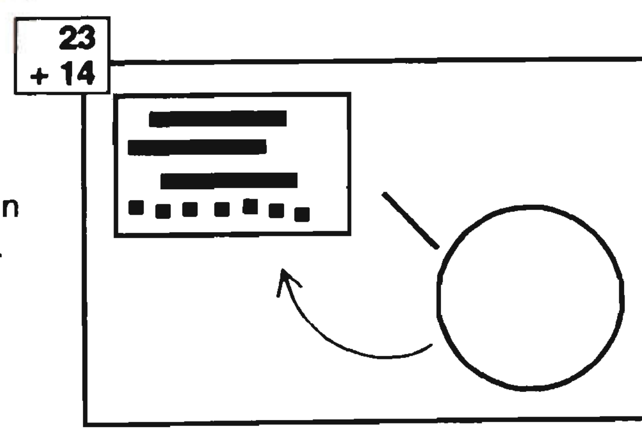
What you need. A state-operation-state mat (Blackline Masters, pge 27), Addition Problem cards (Blackline Masters, pgs 7 and 8), and one set of Blockaid pieces.

What to do.

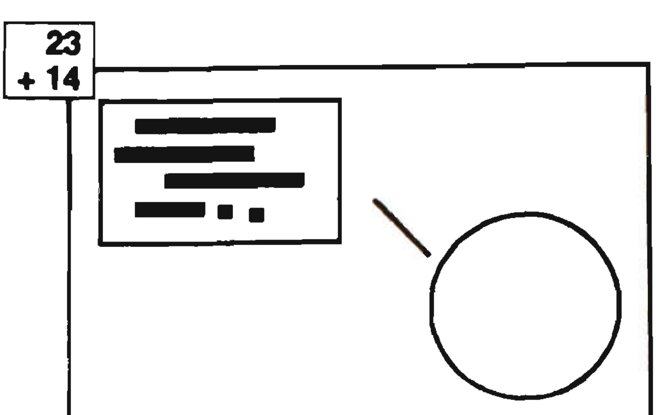


1. Place an Addition Card on the top left hand corner of the mat.

Set out the top number in the rectangle, and the bottom number in the circle.



2. Slide the Blockaid pieces in the circle up to the rectangle.



3. If necessary, exchange Blockaid pieces to show the answer in the least number of pieces.

Check your answer with the answer on the back of the Problem card.

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Make a Flat

STANDARD SET **6**

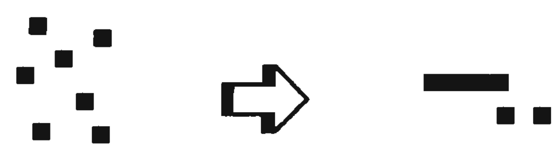
Players. A group of between 3 to 5 players.

What you will need. One box of Blockaid, and two dice per group.

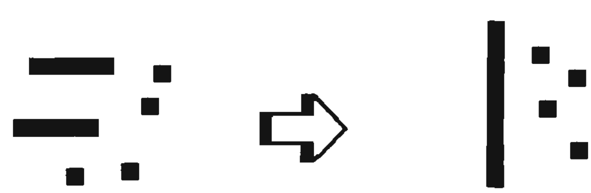
What to do. Throw the dice, and add their total to your score. Show your score in Blockaid pieces. ALWAYS show your score in the LEAST number of Blockaid pieces.

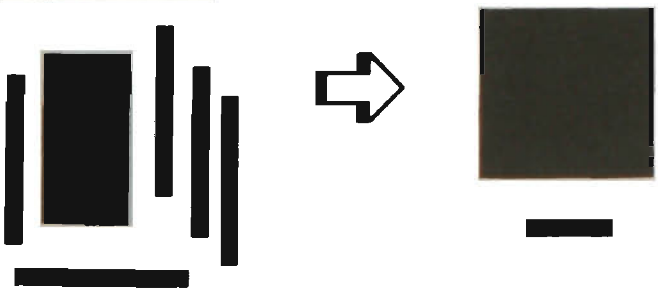
The first person to exchange their pieces for a flat (100) wins !

I had seven pieces, so I changed five of them for a five piece.



I had 14, so I changed two of my five pieces for a ten piece.





I had 105, so I changed 100 for a flat! I won, and I had 5 more than I needed!

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BLOCKAID

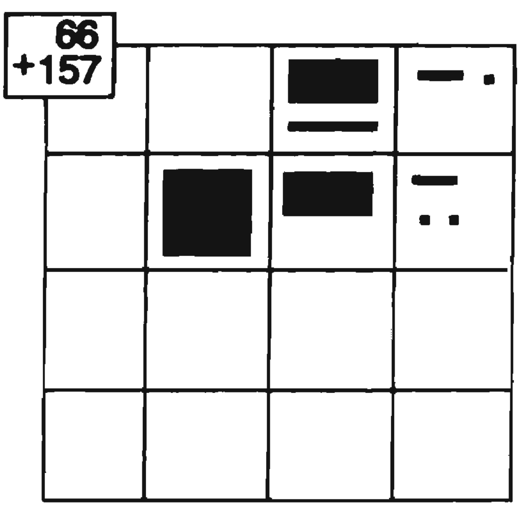
Addition on a 4 by 4 grid

STANDARD SET

7

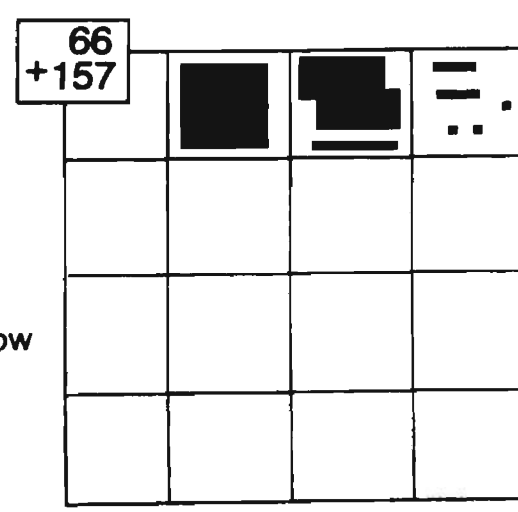
Players A group of between 3 to 5 players.

What you will need. One set of addition problems (Blackline Masters, pgs 7 to 9), one set of Blockaid pieces, one 4 by 4 grid on which to work, and one set of Numeral Cards (Blackline Masters, pgs 1 & 2).

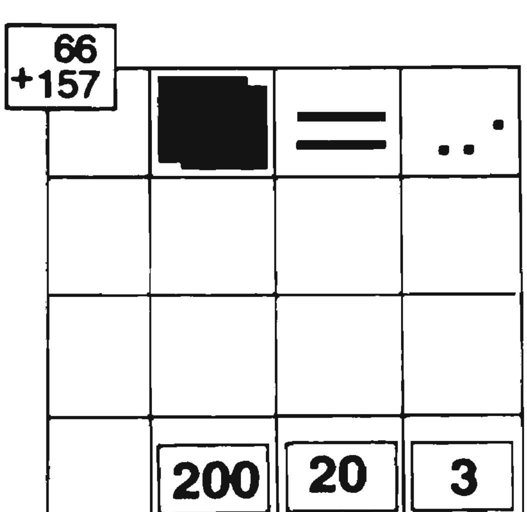


What to do.

1. Place a problem card on the top left hand corner of the grid. Set out the problem as shown.



2. Push the pieces on the second row up to join the pieces on the top row.



3. Regroup the pieces if necessary.

Now represent your answer on the fourth row using numeral cards.

Turn the problem card over and check the answer shown there with your answer.

Appendix 4. Extracts of transcripts of interview on subtraction.

PRE TRIAL INTERVIEW EXTRACTS FOCUSSING ON SUBTRACTION TASK 243 - 61 = ? EXPERIMENTAL GROUP - Alan, Ben, Colin			
WRITTEN ALGORITHM	DEMONSTRATION	CONNECTION	VERIFICATION
243 minus 61. So 3-1=2. 4-6 you can't do, so you go to the front of 243 and cross that out and turn it into a 1 and you cross the 4 off and put it into the 14 ... and 6 take 14 equals 8 ... and the 2. you changed it into a 1 and you put the one down. Right. What does that one there stand for? *..One hundred because it's in the 100 column.*	*One from here and that leaves 2.* Can you take 6 away? *From the ... no.* How are you going to do it? (Inaudible response as child attempts to exchange one of the 100's for tens) *... 30 40 50 60 70 80* ... Hold it - I want you to put it there and talk about it	Is there something here that you did that shows that 1 and the 14? *I took a 10 from the 100's and put it in the tens column, then it's 14 take away* .. Where is the 14. *Here at the top.* (The child puts ten 10's in the 100's column AND an additional 10 in the tens column so that there are five 10's instead of 14). Have you got 14 here? *Yes.* Where? *I think I need more tens*	In this problem here if I added these two numbers up what do you think I would get? *243.* Why? *Because you are adding them up. 1+2 =3 ... 8+6=14* Will that always happen if we do a take away and then add theses two together to get a total which is the first number? *Not always*
Well it was 243 take away 61, so I took 1 from the 3 and that equaled 2, ... and then I had to take 6 from 4 so I took the 6 away from one of the 100's, out of the 200's, and there was 40 ... and I added the four onto that and that equaled 8 ... I didn't have 200 but I only had 100 ... so I put 182.	*Take the 1 away.* (Interviewer guides the child to place the one unit piece taken on the third line, rather than discard it from the mat). *and* ... (the child removed four tens from top line then proceeded to carry out exchange of one of the 100 pieces) take away that ... *and I've got to get that into tens.* What do you want from here? *Tens.* Read your answer. *182.* .	*Well it was 243 take away 61, so I took 1 from the 3 and that equaled 2, ... and then I had to take 6 from 4 so I took the 6 away from one of the 100's, out of the 200's, and there was 40 ... and I added the four onto that and that equaled 8 ... I didn't have 200 but I only had 100 ... so I put 182.*	If we put all that foam back together, what would it come to? *243.* Let's look at your working of the problem on your paper, if I added these two numbers together, (61 &182) what do you think I'd get? *243* Why did you say that? *There's 60 and 80, its got two 50's in it and that equaled 100 and with the other 100 ... etc.*
Tell me how you worked it out. *3 take away 1 equals 2 ... 4 - 6 you can't do that so cross off the 2, borrow one from the 100's and put in the 10's column ... 14 - 6 = 8, and 1 - 0 =1.* What does this one stand for? *100*	Now what do take out? *Six, can't do it.* What will you do now? *Swap this.* What are you going to swap that for? *Two fives.* These are two 50's. What did you take out? *8 ... no 6* What's your answer? *61* Is that your answer? *Oh ... up here ... 182*	Can you show me something up there where you did that? *Crossed off that two ... there should only be 100 left ... I borrowed it and put it in the 10's column.* Where is the 10 you put across? ... When we did it on paper, we took 1 there and made it 14 - show me where you did that here. *There.*	Look at all the Blockaid pieces on the mat, if we put them together what would they add up to? *243* Let's look at your working of the problem. If I added these two numbers up, what do you think I'd get? *243*

PRE TRIAL INTERVIEW EXTRACTS FOCUSSING ON SUBTRACTION TASK 43 - 26 = ? EXPERIMENTAL GROUP - David, Edward, Frank			
WRITTEN ALGORITHM	DEMONSTRATION	CONNECTION	VERIFICATION
You can't take six from three, so I crossed out the four, made that to three, carried the ten over to the three 1's, then made that 13 ... and 13 - 6 = 7 and so, then, so I wrote 7 down and then 3 - 2 = 1 ...the sum equals 17. What does this one stand for? *One ten.*	*This you can't do, cross this, take this one ... this ten 1's ... right ... now I can take this.* (Michael broke the tens bar into unit pieces). Have you finished? *No.* Where's your answer? *Up the top.* Can you write that answer or show the answer with those cards? *Seven.* Is that the answer you got there? *Yes.*	With the 3 and the 13 here, did you do that up there up there? *Yep, there was four 10's and there was 43 and 26 and so you can't take six from three so I carried one ... a ten over to the ones then you could take six from that ... that equaled up to seven. Then I took away 20 from the ones I had left and that equaled 17.*	Look at all the Unifix pieces on the board. (child begins to count them). Without counting, how many do you think there are? Are there more than we started or the same or are there less? *Less.* (On your paper) what would all that add up to? *Um ... Zero.* If I added that and that? *Mm ... 33,* Thirty three? *I mean 43 ... yes 43* OK
I cross out that and put 13 in there and then that comes 3 and I took away 6 from the 13 and that equals 7 and I took one away and then I took away 3 from the 2. I mean 2 from the 3 and that equals one so my answer is that. (Spontaneous place value not given). This one here, what does that stand for? *One ten.*	Where is your answer? *There.* (Child exchanges one ten bar for tens separate cubes and places them correctly) What answer did you get? *Seventeen.* Have you shown what you took out correctly here ... do you want to sort that out a bit? (Inaudible response). Where's the two 10's you took out? *There.* Where is the 6 you took out? *There.*	Did you do that with blocks here? *No cause I can't do it.* ... When you were taking away the six, did you take six out of there? *Yep.* How did you do that? *I carried it.* Where did you get six from, because you only had 3 (ones) up here. *I got 6 from 13.* Where did you get 13? *Cause I borrowed 10 from that.*	How many cubes would there be altogether? (Child begins to whisper through adding sequence). No adding. .... *Oh, 43* If I added these two numbers (on your paper) up, that number (26) and that number (17) and added them together, what would I'd get? *43* Is there any way you could find that out without adding them up? *No, don't know.*
I took away the 6 from the 3 equals 3 and 4 away from 2 which equals 2 and that equals 23. Could we have borrowed there? *Borrowed?.* When you took 6 from 3 what did you get? *A three.* Are you sure? *Cause 3 + 3 is 6 and if you take 6 ... 3 from 6, it's 3.* Take 3 from 6 equals 3 what if you take 6 from 3? *Um ... zero, cause you can't do it.*	Can you take 6 cubes from there? *Yeah but it will equal zero.* Why. *Because you can't, cause 6 is a higher number than 3. If there's 3 you can't take 6.* Is there any way you can take 6 from what you have up there. *Yeah. Took those 2 away.* (Frank gets a tens bar and removes two cubes and then another two cubes which he discards) *and threw them away.*	(The important connection for Frank is the meaning of the operation 'take away').	Without looking at the cubes, tell me how many cubes are there? *Um ... forty three.* Are you sure? *There's 23 up there and 20 here ... We started off with 43 and take away 26.* How did you know there was 43 there? *I just remembered.* Remembered what? *Remembered what the sum was.* So there was 43 cubes there? *Yep.*

Experimental Group

POST TRIAL INTERVIEW EXTRACTS FOCUSSING ON SUBTRACTION TASK 243 - 61 = ? EXPERIMENTAL GROUP - Alan, Ben, Colin			
WRITTEN ALGORITHM	DEMONSTRATION	CONNECTION	VERIFICATION
(Alan was able to talk through the various steps of the subtraction algorithm that he had performed correctly on paper. Although he got the correct answer, he reversed the subtrahend and minuend with the following sequence of words spoken) - *One take three equals two ... six take four you can't do ...* What does this one stand for? *One hundred.*	Take away the number shown on the cards. Starts with units. (Child exchanges ten tens for 100 and places them correctly) What answer did you get here? *One hundred and eighty two.*	Tell me how you got the 1 and the 14 here? Point to something on the mat that this "one" stands for. (Alan was able to make the connection between the borrowing steps in the written algorithm and the exchanging steps with the materials).	Can use addition to undo subtraction? *Yes, they are opposites.* Look at all the Blockaid pieces on the mat. What is the total value of all the pieces? *243* Let's look at your working of the problem. If I added these two numbers up, what do you think I'd get? *243*
You got to take 1 from 3 equals 2, 6 away from 4 you can't do, so you borrow ... exchange one of the 100's for 10's, and you take away 4 from 10, I mean 6 from 10 and left over from that is 4 ... you got 4 still and that equals 8 ... and you took one 100's away and you don't have to take any more 100's away and that equals 182.	*Then you got to take 60 away from 40 so ...* (Interviewer interrupts) Did that happen here (on your paper) when we took 60 from 40? *We got 1 of the 100's & exchanged for 10.* Will you do that here? *Yes. Take one of the 100's, exchange for ten 10's. Got to take 6 from 140, took that away, and that's all.* So what answer did you get there? *182*	(Ben was able to make the connection between the borrowing steps in the written algorithm and the exchanging steps with the materials).	We have just done subtraction, can we undo that with addition? *Yes.* So if we put all the (Blockaid) pieces back together what do we get? *243* If you added those two numbers (on your paper) up, what would you'd get? *Um ... 140, um ... yes 143 underneath, you get 243 ... if you added it all back together.* 243? *You get the same answer.*
3 take away 2 equals 2. 4 - 6, you can't do, so you go to the 100's column and take away 100, cross off the 2 and put 1 into the tens column and that makes 14 tens take away 60 is 8 tens and then 100 take 0 is there. This one here, what does that stand for? *Um ... Ten.* What does it stand for? *Ten* One 10 is it? *No ten 10's*	What do we do now? *Take ... should be one more ... take 6 you can't ...* (whispered utterances as child exchanges two 50's for 100 and places them correctly, then goes through the various steps of the operation). What answer did you get here? *Um ... 100 ... and 82*	*I didn't have enough tens so I went to the 100's column, got one from there and exchanged it for two 50's and put it in the 10's column.* Explain how that worked on paper? *Explain it again?* Yes. *14... 4 - 6, you don't have enough tens, means you can't do it, so you went to the 100' column and gave one of the 100's and put it in the tens column.*	Can you use addition to undo subtraction. *Don't know.* Can you tell me what is the total of all the foam pieces on the board? ... *243* Can you use addition to undo subtraction? *Yeah.* Look at where we worked it out here (on paper), what would happen if we added these two numbers together? *Them two ... 243.* Are you sure? *Yeah.*

POST TRIAL INTERVIEW EXTRACTS FOCUSSING ON SUBTRACTION TASK 243 - 61 = ? EXPERIMENTAL GROUP - David, Edward, Frank			
WRITTEN ALGORITHM	DEMONSTRATION	CONNECTION	VERIFICATION
Three take away one is 2 ... 4 - 6 you can't do so I took one from the 100's and put it on the forty, so that made it 100 and 4 ... 40 take away 6 and that equaled 80 and then there was nothing to take away from the 100's so I just put what was left. (Spontaneous place value was given). What does this one stand for? *100*	Take away the number shown on the cards? (Child takes out the units correctly and proceeds to carry out an exchange of 100's for ten tens). Did you exchange correctly? *Yep.* (Pieces equivalent to 100 were taken for an exchange, but the 100 piece was not removed from the mat - error corrected without further prompting). You forgot about doing a swap didn't you?	Now when you did that exchange, can you point to something on the paper there where that happened? OK, can you explain that to me? *Um ... I crossed out 2 and made it one, and I put 1 on 4.* So what did we do with the foam? *Swapped it ... 100 for one 50 and five 1's.*	Can use addition to undo taking away? *Yes.* Look at all the Blockaid pieces on the mat, if I put them back together what would they come to? *243.* If I added these two numbers together, the 61 and 182, what would they come to? *61 and 182 ... 243*
Um ... Well I took 1 away from 3 and that equaled 2 ... can't take 4 from 6 ... so, oh ... I can't take 6 from 4, so I borrow a ten from the 2 ... the 200 and put it in the tens column to make 14 .. 10's and I took away 6 from 14 that equals 8, then I took nothing from 1 and that equaled 1. (Place value generally given). What is this one here? *One 10*	Use those Blockaid cards to show the number we are taking away. What do these cards show? *Six tens and one unit.* Can you do that? *Take 1 from that ... then exchange one of these ...* (Interviewer interrupts to check links between approaches)	Did you do something (on paper) that you are doing now. *Yes.* Explain what you are doing. *When I did it there I had to cross out the 2 & put 1 up the top because there is no longer 200.* Are you doing the same here now? *Yep. Exchange that for one 50 & five 10's. I got to take 6 away from that and that leaves.* What's you answer? *182*	Can use addition to undo subtraction? *Yes.* If I put all the foam pieces back together, what would it add up to? *243* If I added these two numbers together, the 61 and 182, what do you think it would come to? ... Do you know? *Nup.*
Let's hear you do that? *Ah ... I added, um ... I took away one from 3 equals 2. I took away 1 from 4 = 3 and I took away 1 take from 2 and that = 1.* (Spontaneous place value not given here).	What did you take out then? *I took 1 from 2 is 1 ... and 1 from 6 ... I took 1 from 4, ... 1 from 40 = 10 and 1 from 100 = a 100 ... 120 ...* So what is the answer? *111 mm 111* Your answer is 111? *Do you do ...* Is your answer on the top or on the bottom? *On the top.* So what is your answer? *132.*	(Frank didn't identify the need to borrow in the written working of the problem nor the need to exchange when working with concrete materials and thus the borrowing/exchanging connection was not evident).	Can we use adding to undo subtracting. ... (No response) Is there any connection between adding and subtracting or are they totally different? *Totally different.* If I put those foam pieces together, what would it come to? *243.*

Appendix 4. Extracts of transcripts of interview on subtraction.

PRE TRIAL INTERVIEW EXTRACTS FOCUSING ON SUBTRACTION TASK 243 - 61 = ? CONTROL GROUP - Alex, Bill, Carl.			
WRITTEN ALGORITHM	DEMONSTRATION	CONNECTION	VERIFICATION
3 take away 1 equals 2, ... and then 4-6 you can't do, so I cross out the 2 in the hundreds column and make that a one, ... then I made the 4 ... 14 ... 14 tens and take six tens and that equals 8 tens, then I put down the eight, ... and then I put down the one.	Take away the number shown on the cards. Take 1 away from the 3, you can't take 6 from the four so you put that there. (Child takes a 100's piece and exchanges it for tens). Get 6 of these and take that away, then you take one and that's your answer. What's your answer? 170 ... 2. Are you sure? 182.	You can't take 6 from 4 so you need some more, ... so you take from the hundreds column. Can you show where we did that on the paper? Yes, we took the hundred out and put it over there.	What's the total value of all the pieces? ... 243 Are you sure? Yep. If I added these two numbers together, (61 & 182) what do you think I'd get? 243 Why? Because it's doing that sum backwards.
I'm going to have to change it because it doesn't make sense ... I got up to 6 from 4 and you can't take 6 from 4, ... that's the problem. Do you think we should be able to take 61 from 242? Yes, ... Another way is by counting backwards, like taking 60 by counting backwards ... it would be less than 200 ...182 or something.	Can you take 61 from 243? As I said before by counting backwards but not with take away, sort of ... Is there anywhere we can get six tens? Only if we change the sum. ... You can exchange the 100, you get two 50's. ... These would go into here so it would be 104 take away 6 which leaves 96 and then ... 100 90 ... 8 and 8 ones	(A similar barrier arises in the written and practical approaches). You don't think you can take 61 from 243? As I said before by counting backwards but not with take away, sort of ... Is there anywhere we can get six tens? Only if we change the sum. ... You said before you thought you could take 61 from 243. I think you can but not with figures.	
3 take away 1 equals 2, 4 - 6 equals minus 8, and there was 200 so I put 100 down and put the 8 down. Tell me about this 4 - 6 is minus 8. Because 6 is a higher number than 4, and you can't take 6 from 4, but there is another number that is 200, so I could take it away.	Take that away equals 2 (units) ... 6 take away ... 4 take away 6 you can't do, so take one of the 100's away and get 8 tens ... instead cause you ... need that for the answer. So what's the answer? ... 182 You can't do it. You need 4 more of those.	(As the subject shows no borrowing or decomposition figures in his calculation no link could be established between the paper and pencil working of the algorithm and the physical demonstration).	(Carl when questioned about consistency of answers to the same problem using written and physical materials approaches, indicated that answers should be the same, whatever the method).

PRE TRIAL INTERVIEW EXTRACTS FOCUSING ON SUBTRACTION TASK 43 - 26 = ? CONTROL GROUP - Don, Ewen, Fred			
WRITTEN ALGORITHM	DEMONSTRATION	CONNECTION	VERIFICATION
Answer 17. Three take away 6 you can't do ... cross, um ... go to the tens column, cross out the 4 to the 3, put that ten in the 1's column so it's 13 - 6 That equals ... 13 - 6 = 7 and 3 - 2 = 1 so it equals the whole answer equals 17. (Spontaneous place value is given). What does this one here stand for? One ten.	What number are we taking? Twenty Six. Away you go. Three take away 6 you can't do, so you take away one ten and put it in there and that equals three and that's 13 - 6, that's 13 - 6 ... that's 7. That's six there (that you have taken out). ... 30, no 3 - 2 no/yes 3 - 2 = 1 Your answer is ? Um ... 17	So show me again how you got 3 there and 13 there. That's 4 ... when you couldn't take it away, you had to take ten from there so that equals three. Do you think you have the same answer here (as you worked out on paper)? Yes.	**Verification.** If you put all of the Unifix cubes on the mat together what would it come to altogether? Um ... 69 or something. (Looking at your working of the problem). If I added those two numbers together, 26 and 17, what would it come to? Um ... Not sure? Two and one ... 43
Answer = 23 Do you know where you went wrong there? Yeah, ... I forgot to rename. How could we have renamed. Um ... I guess ... you can't take 6 away from 3, so you take the six away from the ten. (further prompting). Let's hear you do it now. 13 - 6 um ... is 12. 13 - 6 ? Um ... I can't remember, ... seven. Right. 3 - 2 is 1....17	Can we do that? No, because you can't take 3 away from 6. So you can't take 3 away from 6? Six away from 3. What are you going to do? Have to rename. Rename? I don't know how to. ... Is there anywhere we could get six cubes from? (No reply - further prompt) From the tens? (Further confusion and prompting with -) Take 30 away from 20.	(Ewen seemed to make some link between the subtraction operation using materials and the idea of renaming in the written algorithm.	
Computation - 43 - 26 = 3. Well 43 take away 26, now 3 - 6 you can't do so cross out 4 ... give it a three, give that a ten so that's 40 & get 30. I gave that a ten and that's 13 and that could take away that and that equals 7 & I got stuck on this one (the top number in the tens). ... you could use that number there away 6, and take	Yes ... six ... 3 - 6 ... oh no 6 -3 is 3 ... I just remembered it's that number take that. Which number are we taking away? Taking away that three ... no taking away six. Point to the number we are taking away. Six. Can you do that? Yes ... 6 - 3 is 3 and 30, no 40 - 6 is ... um 35. That's 35. Put down 5 and put 3 up there.	Fred was unable to complete the problem 43 - 26 = ? on paper. His partial solution suggests confusion between subtrahend and minuend and this difficulty is evident in the dialogue when using materials. There is also evidence in both the written and practical approaches of possible confusion with the multiplication algorithm - in both cases there is an attempt to take six units from both the units in the top line and also the four tens.	Say we got different answers, (one on paper and the other with using materials) would that matter? Yep. Why. That would be ... they can't be ... like, the same sum can't be two different answers, unless it's forty three take ... take away and a plus they could be different answers or a subtraction and a times. OK.

Control Group.

POST TRIAL INTERVIEW EXTRACTS FOCUSSING ON SUBTRACTION TASK 243 - 61 = ? CONTROL GROUP - Alex, Bill, Carl			
WRITTEN ALGORITHM	DEMONSTRATION	CONNECTION	VERIFICATION
Three take away one equals 2 ... 4-6, you can't do, take from the 100's column and I put 1 up here so its 14 tens - 6 tens = 8, and 1 and so that's 182. (Spontaneous place value given) What does this one stand for? *It stands for 100 or ten 10's.* And this one here? *It stands for 100 ... ten 10's*	Can you take away what's shown here? *3-1 leaves 2 ... Take away 6, you can't do it so you have to exchange that.* (Alex exchanges correctly and completes the task) What is the answer? *182.*	Can you point to something on the mat that this "one" stands for. ... *When we had to take it from here and put it over there when I exchanged it.*	Can we use addition to undo subtraction - *Yes* Look at all the Blockaid pieces on the mat. What is the total value of all the pieces? *243* Let's look at your working of the problem. If you added these two numbers together, what do you think you would get? *243*
Three take away 1 equals 2 ... 4 - 6, you can't do, cross out the 2 make it 1, put 100 over next to the 4 ... 14 - 6 is ... 8, put down 8, 1 - 0 is 1. Your answer is? *182* What does this one stand for? *10.* Is this one ten or is it something else? *It's 100 changed into ten 10's.*	Can you take 61 out of that material? ... *Can I have 40 tens exchanged, please?* What do you want? *Forty tens exchanged.* Forty? ... *Ten 10's.* You are wanting to exchange ten 10's for 100, did you do it on paper? *I didn't exchange anything, except I crossed out the 2 and made it 100 instead of 200 and put the 100 over on the 14.*	Can you point to something on the mat that this "one" stands for. *It means 100.* Did we do that here (with materials*)* *How did we get that 100, we exchanged and got ten 10's to carry over to here.* OK.	Can we use addition to undo subtraction - *Yes, because you are adding instead of taking.* If we put all the Blockaid pieces on the mat together what would that come to? *The whole sum ... 243 take away 61 ... what would it all come to ... 243* On your paper if I added these two numbers up ...*You would have to do some changes in there ... 283*
Tell me how you worked it out. *3 take away 1 = 2 ... 4 - 6, can't do cross out the 2, then make that a 1, put 1 on top of the 4 ... 14 - 6 = 8, put down the 8 ... 1 - 0 = 1. The answer is 182* OK (Spontaneous place value not given). What does this one stand for? *100 no ... yes 100*	Take away the number shown on the cards. (Child begins and takes out the one unit piece. He then proceeds to exchange 100 for two 50's and completes the task successfully). *Took 60 off that.* Show me the number you took out. (Child points to the pieces taken and placed on the third line). What answer did you get? *182*	Now you exchanged that for two 50's, When you did that was there something you did on paper that is like that? *I carried one from there to there.* So you took 100 out of there and ... *put it there* What's that one there (on your paper)? *100.*	What is the total value of the Blockaid pieces on the mat? *243 +61* What would it come to? ... *if you added these two ...304.* (further prompt) ... just the Blockaid pieces. *They would add up to 143.* 143 or 243? *243* If you added this number and the bottom number what do you think you'd get? *243*

POST TRIAL INTERVIEW EXTRACTS FOCUSSING ON SUBTRACTION TASK 243 - 61 = ? CONTROL GROUP - Don, Ewen, Fred			
WRITTEN ALGORITHM	DEMONSTRATION	CONNECTION	VERIFICATION
Tell me how you worked it out. *Well 2 ...3 take away 2 is 2 ...no 3 - 1 is 2 ... 4 - 6 you can't do, cross out the two and carry the one ... no, put 1 in the 10's column, 14 - 6, that's 8 and 1 - 0 leaves 1.* What does this one here stand for? *One ten.* Is this one ten or is it something else? *That's one ...1 ... 100 ... um*	(Andrew initially removes the flat piece of foam worth 100 and places it in the tens column. He then picks it up again and <u>swaps it for two tens</u> after he seeks permission to exchange). *Can I swap this?* What are you swapping it for? *2* You're going to swap it for 2? *Yes, and ... oh yes.* (Time allowed to complete task) .. So do you think your answer of 162 is right. *No.*	(Interviewer tries to see what link is understood from the written form to apply to the concrete materials approach in regard to the borrowing and exchanging aspects of the two approaches. See below).	If I put all those Blockaid pieces together on the mat what would they come to? ... *Not sure.* Can we use addition to undo subtraction? *Yes.* If I added these two numbers up, is there any number there on the sheet that would tell us what they come to altogether. *200 + 40 + 3 ... 243*
Tell me how you worked it out. *3 take away 1 is 2 ... 4 - 6 you can't do, cross out 2, make it 1, carry the 1 ... 14 take 6 is 8, 1 - 0 is 1.* (Spontaneous place value not given). What does this one stand for? *100 ... 10* What does it stand for? *A 10*	How many tens do you have to take away? *Six.* Can you do that? *Nope.* Is there anyway you can do that? *Yep ...* (Ewen takes a flat with a value of 100 from the 100's column and places it in the tens column. <u>He then puts one long worth 10 in the 100's column.</u> He seems unable to proceed beyond this point). *Yeah that's right ... one ... no.* Not sure of that? *No*	(It seemed inappropriate for the interviewer to pursue the borrowing/exchange link).	Let's look at your working of the problem on your paper. If I added these two numbers up, what do you think I'd get? *243*
*3 take away 1 is 2, 4 - 6 you can't do, cross out 2, put 1 at the top of 2, put 1 next to 14 ... 14 -6 is 8 and ... oh, yep 1+ ...oh ...*Are you happy with that? *No.* Start again. *3 - 1 is 2, <u>4 - 6</u> you can't do ... oh, yeah I'm happy with that.* Happy with that. What does this one stand for? *Ten ... No ... 100*	*3 take away 1 is 2 ... <u>6 - 4</u> you can't do. ... Oh <u>6 - 4</u> is ...* (It was not until the third error of "6 - 4, you can't" that the interviewer questioned further). What are we taking away? *<u>Four.</u>* Which number are we taking away? Top or bottom number? *<u>Top</u> ...* We are taking the top number away? *No ...Yeah, bottom no.*	(After much prompting an exchange of ten tens for the 100's flat piece was performed but due to the time factor no questioning was followed up on establishing the exchange/borrowing link used in the two methods).	If you put all those Blockaid pieces together, what would it come to? *<u>140</u>* Can we use addition to undo subtraction? *I don't know.* If you put those back together, what would it would come to 140? *It would come to 240.* (On paper), if I added these, what would we get? These numbers here. *61 & 182, um ... you would get <u>142</u> ... no <u>143.</u>*