MEAN REVERSION IN ASSET PRICES AND ASSET ALLOCATION IN INVESTMENT MANAGEMENT

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Abstract

This thesis examines the predictability of asset prices for an Australian investor. Evidence supporting the mean reversion alternative to the random walk hypothesis is presented, with a discussion of potential models, both linear and non-linear. The normality and homoscedasticity assumptions are investigated and their use in asset models is validated. A study of fund performance is carried out and value is found to be added by timing asset allocation but not by stock selection, though there is no correlation between past and present rankings of managers. The difficulty of proving mean reversion or reversion to trend , other than for large deviations or extremes, and the actual performance by managers, implies a strategy of allocation at these extremes. That is, managers should adhere to their policy portfolios and let markets run short term; making appropriate large strategic moves when markets have moved to extremes.

Glossary of Terms

Attribution analysis: The process that attempts to attribute performance to different components in the overall returns. The information is culled from the respective funds management databases and collected and collated into useable form.

Asset Liability Study: The process by which the assets and liabilities of a particular fund are examined. Legal liabilities in terms, say, of pension benefits are evaluated against the assets available to pay for those benefits at some time in the future.

Efficient Markets Hypothesis (EMH): A security market in which market prices fully reflect all known information is called efficient. Thus an investor cannot make a gain from a mispriced security.

Financial Reserving: The technical process whereby, usually actuaries, set the level of reserves needed to meet future claims. These claims, which may not have actually arisen yet, are forecast on a statistical basis.

Fundamental Analysis: An analysis of security or asset prices based upon a detailed study of the asset under question to discern a difference between the value of the asset and its price. This assumes that markets do not price assets correctly; hence they are 'inefficient'.

Market Timing: The approach by fund managers to attempt to add value to a portfolio by moving in or out of asset classes, when the relative performance of one class is expected to be better than that of the one moved out of.

Mean Reversion: The approach that postulates that security prices will revert to long run mean levels. Thus security prices will have a transitory component. In this document this hypothesis provides the alternative to the random walk.

Policy portfolio: This is the benchmark allocation among asset classes determined by the manager, representing the managers stance in the market or particular client objectives.

Random Walk: The concept that movement in a variable is unaffected by previous values of the variable. Hence the best forecast of the next value of the variable is the preceding value.

Random Walk Hypothesis: The theory that security prices contain no memory, in that the best forecast of the next price is the preceding price. It provides the null hypothesis for alternative hypotheses.

Return from market timing: This is the difference between the actual and benchmark asset allocation for each sector multiplied by the sector benchmark return. This is done each month and summed.

Return from security selection: This is the difference between the return of the fund and the sector benchmark return multiplied by the actual asset allocation. This is done each month and summed.

Security Selection or Selectivity: The approach by fund managers to attempt to add value to a portfolio by superior selection skills of securities within an asset class.

Sector benchmark return: This is the return of an appropriate index representing movements in the asset class as a whole. For example, the All Ordinaries accumulation index would represent a suitable benchmark for Australian equities.

Semi-Strong Form Market Efficiency: All publicly available information is in the current market price of a security, hence an analysis of such information cannot be used as a basis for earning abnormal returns.

Strong Form Market Efficiency: All public or private available information is in the current market price of a security, hence an analysis of such information cannot be used as a basis for earning abnormal returns.

Technical Analysis: An analysis of security or asset prices based upon historical charts of security prices, hence it is also called 'charting'. Chartists attempt to predict future prices based upon particular chart patterns. Essentially future prices are held to be predictable from past history, without any reference to the underlying security.

Variance Ratio Test: The test employed to investigate mean reversion. Essentially it tests for small autocorrelations at long lags which become significant by accumulation. This uses the feature that under the null hypothesis the variance is linear in the lags.

Weak-form Market Efficiency: The information contained in the past sequence of prices of a security is fully reflected in the current market price, hence an analysis of past prices cannot be used as a basis for earning abnormal returns. The random walk hypothesis is not the same as weak form efficiency.

Chapter 1: Introduction and Overview

1.1 Background

In recent years there has been a sharp increase in the study of financial markets. The growth in financial services, itself an outpouring both of the growth in computing power and growth in world trade, and hence integration of international capital markets, is the fundamental driver of this activity. Hence the range of available assets has grown in line with these developments, presenting those who manage these assets with both the opportunities offered by this choice and the problems inherent in dealing with them.

From an historical perspective the major topic of interest has usually been share prices, though it is fair to say that the general public or professional fund managers, have only become active in this asset class in more recent times. Physical property and securitised property (via property trusts) are also relatively recent additions. Finally, the whole arena of international investing has opened up in the last ten years (as indeed is the case with derivative markets). Thus logically investors are now presented, not only with the question of which particular security to buy, but which type of security, that is which *asset class*[1] to be in, and when.

Thus we are brought to a consideration of timing. Which asset class should we be in and when? If security prices are random then *prima facie* it is futile to attempt to time movements between asset classes. If however security returns exhibit a long term mean, then there will be periods of time or financial eras when returns are well above long term averages. So, to compensate there will be periods when returns are well below long term averages. If this is so, then prices can be said to be *mean reverting*. Then it also follows that if an investor is wise enough to be able to anticipate these

[1] The asset class means a broad categorisation by type of security enjoying certain characteristics, such as common shares or interest paying debt obligations. In reality there are many hybrids or entities with mixed features, created for tax or other capital market reasons.

periods of over or under performance, then that investor will be able to take advantage of this propensity.

The topic of mean reversion or long memory is much debated in the field of econometrics. The persistence or otherwise of "shocks" to the economic system are central to debates about the nature of unemployment for example. Is there a 'natural' level of unemployment or does the overall level vary? The same is true for many other economic variables as well as financial asset prices. The answers to these have profound implications for economic management to say nothing of the particular case of institutional funds management. Thus in some senses part of what we are considering is a subset of a much larger topic. We must be aware of this but cannot take the discussion too far, since it will detract from our central objective.

1.2 Aims and Objectives

In a very real sense investment management is a practical pursuit. Money must be managed. Trustees of superannuation funds, as the law stands, have an obligation to provide an investment strategy. Hence they need a basis upon which to make a policy decision. This is no less true for a small private client than for a manager of the largest funds in the country. What advice can we therefore offer? What conclusions does the evidence suggest? What do investors actually do and are they successful at it?

We may specify this in more concrete form. Then we will be in a position to outline our 'plan of campaign' to achieve our objectives. The final chapter will measure how well we have done.

Our objective is to be able to offer some practical advice, based upon an examination of theory and practice. We will need to distinguish between what we know with a great deal of confidence, what may well be indicated by the evidence yet not be conclusive and that which we are not just uncertain of, but where we are fully aware of that which we cannot know. In effect we need to be able to ascribe a degree of confidence in our conclusions as supported by the evidence, including any potential cost if we are wrong in our assessments. The very nature of our pursuit implies that we must build any case we may present from a range of considerations or ways of viewing the topic. By this we mean that perhaps an activity is feasible, yet if the evidence suggests that no one has actually been able to achieve that activity then logic may well indicate that it is simply too hard to do. A good example here would be whether or not professional managers can outperform a given index for an asset class, say the Australian All Ordinaries Index, for shares. The evidence is, and herein we find no differently, they can't. So why waste time trying? Clearly it is feasible, and hope, no doubt, springs eternal, but cannot the time spent on this be more productively used elsewhere?

This approach is essentially a decision theoretic based one. Conceptually within our decision trees, we are able to ascribe high probabilities to some routes and low ones to others. We cannot however give numeric values to these quantities; the subjective element is there. It would be spurious to attempt too much formality in the conclusions; nevertheless order of magnitude tendencies will indicate directions we must go.

We do not necessarily aim to develop definitive strategies. They will always be dependent on what objectives are sought by individual investors, what level of risk they are prepared to bear and so on. What we do wish to do is to bring together more closely theory and practice. If we cannot prove something then at the very least it needs to be recognized. Action may well be taken on the basis of uncertainty, but that is so often the case in the real world where one is dealing with applied economics.

Our strategy within this document must then include quite a wide ranging review of the facets which go together to build the total picture in the process of asset allocation in funds management. Hence if we are to consider the kind of modelling process used as an aid in developing strategies then we need to understand some of the sensitivities within the underlying assumptions. If the normal distribution of price changes is an assumption, then we must check to see whether they are or not and then how important it is if prices are not normally distributed.

1.3 Outline of the Document

Herein we outline the plan of campaign to achieve our objectives. This will provide 'signposts' on the way to provide a clear indication of why we have dealt with topics in the way we have, and why some things are in and others are left out. It is to be noted at the outset that many of the individual topics are more than rich enough in terms of research to provide enough for a thesis in themselves. Sometimes therefore we may take an issue a certain distance, hopefully enough for our purposes, but no farther. At those appropriate times, any further research is indicated.

We commence with a review of the literature. This is fairly self-contained, and does serve to put the endeavour into some kind of perspective. The following chapter then proceeds with a description of the data, both as to sources and any problems there may be. The main body of the work then commences with, after some introductory analysis, a determination of the autocorrelation structure of the major financial asset class time series. Short run positive autocorrelation and long run mean reversion are investigated, along with a discussion of their significance. If mean reversion does not exist then attempting to time financial markets is not a worthwhile exercise. We may just as well set our overall parameters and leave the asset allocation largely alone.

The next chapter then starts by considering the use of various classes of linear models as an aid in explaining the underlying generating mechanism. Some detailed consideration is given to the nature of the process and a model is developed alongside a simulation study to understand better some of the issues involved in significance testing. We then look briefly at the class of non-linear models, as well as other approaches, notably how an actuary might deal with the task of determining assets and liabilities as part of the *financial reserving* [2] process. Finally, as part of the task of checking assumptions the relevant financial series are reviewed for changes in mean and variance as well as the distribution of asset prices.

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[2] For an explanation of the term see the glossary.

The penultimate chapter moves on to an in-depth study of actual performance. It is vital to understand what managers can actually do. If they have shown an ability to time markets then we need to recognise that, even if theory would suggest it is not provable statistically. As such the chapter attempts to review a wide range of performance statistics from overall predictability of performance, to abilities at the asset class level and the relationship between aggressiveness of moves and timing ability. The chapter concludes with a look at the asset allocation modelling process, by pulling together the various strands of the preceding work and seeing which elements are critical and which are not.

In our conclusion, we hope to bring together all the work in preceding chapters, so that we may achieve our objective of providing some advice to various types of investor. It is worthwhile reiterating that we are, more by the nature of the uncertainties involved, building a case from a range of evidence rather than providing a particular clinching piece of evidence. Thus we may hope to add some more to this vital topic, which is one of great current interest.

Chapter 2: Survey of the Literature

2.1 Introduction and Early Studies

At the outset it is important to note that this survey, whilst attempting to lay out research in chronological order, must of necessity cover contiguous areas of research. Thus we may follow some of the directions that the research has taken, always keeping in mind the ambit of our thesis. Of course, the researchers have not operated in isolation; fruitful areas have shown flurries of activity, as new methodologies or lines of research have flowered. Certain aspects, as in say price distribution have quietened, while non-linear modeling and mean reversion currently show great activity. Nevertheless, spin-offs from one area often can rejuvenate another. Work in this field certainly exhibits those characteristics. Hopefully, we may be able to outline the main thrusts of activity in the field and see the linkages, and also their relation to our study.

The random walk hypothesis and it's corollary the efficient markets hypothesis (EMH), has a long and detailed history. The topic goes back to Bachelier (1900), who first analysed speculative prices in some detail, proposing the distribution of price changes as independent identically distributed normal variables (as against a log-normal distribution, see Alexander (1961). Apart from the Cowles Commission in the United States in the 1930's (reporting in 1939) relatively little was done on the topic until after WWII. One of the first to show interest was Kendall (1953) who considered the autocorrelation structure of various financial time series and concluded they were random walks, with the famous exception of the cotton series where he had used averages, thus bringing forward the brief note from Working (1960), where he demonstrated that first differences of averages in a random chain exhibit significant autocorrelation.

Further interest was now generated with authors like Alexander (1961), Roberts (1959), Cootner (1964) and Larsen (1960) all conducting various tests on series (usually US stock prices, thereby establishing a continuing tradition), in an attempt to see, for example, whether various trading rules could be applied to successfully "beat the market". Key articles were collected by Cootner (1964) and published. Fama in his review of financial markets, pulled together some of these concepts, with his definitions of forms of market efficiency (see the glossary for a more detailed exposition), for example, weak form efficiency (or can a profit be made solely based on the previous price history of a particular security). This is also very closely connected to so called technical analysis in asset prices, where predictions are made based solely on charts, and whether this is a futile activity or not.

We also see a move towards the application of other time series techniques to speculative prices by Granger, applying spectral analysis to stock price time series, for example, Granger and Morgenstern (1963). Granger has written comprehensively on a wide range of issues connected with speculative prices, over a long period of time. The eight references given, are a small portion of his research, logically extending into the recent developments in non-linear modeling and fractional differencing.

2.2 The Distribution of Stock Prices

Research was also directed towards an investigation into the distribution of stock prices with various authors beginning to question the assumption of normality. Alexander carried out investigations into the distribution of prices, at the same time as his study on the viability of profitable trading rules, suggesting non-normality (he also pointed out the distinction between the use of percentage changes and logs). Mandelbrot (1963) proposed the class of *stable Paretian* [1] distributions as a better alternative. Fama (1963) replied to Mandelbrot's thesis by pointing out, for example, the difficulties posed by a class of distribution with an infinite variance (this is one of Fama's earliest contributions in what has been a lifelong involvement in the area). Since then many other authors, and the list is by no means complete, have carried out similar studies. Praetz (1972), Blattberg and Gonides(1974), Ali and Giacotto (1982), Officer(1972), Hsu, Miller and Wichern (1974), have observed that prices have "fat

[1] The distribution of asset price changes is covered in detail in section 4.6, where this class of distributions is defined.

tails" and are more "peaked" than that predicted by the normal distribution. Most recently Sterge (1989) has also found evidence of non-normality in futures prices, where one must be aware of premia or discounts to fair value (that is the 'spot' adjusted for the costs of carry) which will colour the issue. Suggestions for various alternative plausible distributions such as the Student t-distribution or the generalised error distribution, have been put forward by some authors. Praetz suggests the Student-t, as it converges to the normal, and would thus fit the behaviour of individual stocks which then sum to a index, the index's behaviour then being approximated by normality.

A comprehensive review of possible distributional forms is found in Ali and Giacotto, along with an analysis of the impact of changing mean and variance of share prices on the overall distribution. This links closely to the topics of heteroscedasticity in share prices and changes in the overall level of the mean. Clearly, these changes of location and scale can be the cause of non-normality. Ali and Giacotto's study follows that of Boness, Chen and Jatusipitak (1974), who, in a concluding statement, commented that "The results of our analyses strongly support the hypothesis that parameters of price change processes vary with capital structure changes" (p.534). Ali and Giacotto, in their study find no significant evidence of changes of location through time but do find changes of scale (albeit for individual stocks, but over time periods including months). Perhaps the rationale given by Boness *et al* explains their findings, as indeed it may for the results of non-normality observed by other authors.

In an Australian context the topic is covered well. Apart from the above mentioned works by Praetz and Officer, there are papers by Stokie (1982), who, for Australian stocks, finds that there are no conclusive grounds for rejecting the normal distribution and one by Beedles (1986), who found asymmetry in stock prices. These can both be found in the excellent compendium put together by Ball, Brown, Finn and Officer (1989). Within the compendium there are collected papers based on Australian data covering a number of areas of research pertinent to this study.

2.3 Market Efficiency

Fama's next most significant contribution was the Efficient Markets Hypothesis or EMH (Fama (1970)). This was proposed in his review of financial markets, mentioned above, where he attempted to synthesize the known work on asset markets into one hypothesis.[2] These concepts give a framework for analysis of speculative prices, and serve to illustrate the nature of the research. Hence the various autocorrelation studies of short run persistence and long run mean reversion can be considered as tests of the weak form efficiency of markets. Thus, in an Australian context, Officer (1975)[3] reviewed the seasonality and general market efficiency of Australian capital markets. This is a fairly detailed study of various aspects of efficiency, where Officer concluded in favour of seasonality for shares but not for bonds. In a more recent paper Groenewald and Kang (1993) review the weak and semi-strong form efficiency of the Australian share market. Their conclusion is in favour of weak form efficiency but undecided on the question of semi-strong form.

An even more comprehensive sequel was published by Fama in 1991. This is a review of work in the broad area of financial economics, as it pertains to speculative prices, bringing together the major results, some of which are directly relevant to this study, many of which are not. It does show both the scope of the field but even more so the enormous growth in interest and research conducted since he first wrote in 1970. It is fair to say that in this later work of Fama, he is much less sure about the efficiency of markets. He cites many examples of the predictability of share prices (see for example Fama and French (1988a)). In his latest commentary, in an applied journal, Fama (1995) looks at specific practice in stock markets, notably technical and fundamental analysis. This short paper is aimed at the practitioner, where Fama finds

[2] Weak form efficient means prices cannot be predicted solely on the basis of previous prices. Semi-strong form means that all *publicly* available information is in the price and Strong form means that all information both *public* and *private* is in the price.

[3] From an article in the *Journal of Financial Economics* 2 (1975), 29-51 , reprinted in Ball, Brown, Finn and Officer (1989).

too little connection between academic and practitioner. He clearly feels that much market practice has little merit; stock market analysts need to expose their predictions to the ultimate test of demonstrating a track record. In his studies most fund managers certainly have not demonstrated this ability.

This activity should not be seen in isolation from developments in portfolio theory (Markowitz, Sharpe *et al*) nor from the seminal work on option theory from Black and Scholes. These issues whilst not core topics for this research *per se* are nevertheless critical. Modern funds management bases its asset allocation to a large degree on the ideas of portfolio theory, using the mean-variance optimiser to set asset allocation guidelines in the context of a specific asset-liability framework. Clearly mean reversion in asset prices and non-normality of the underlying price distribution or any potential heteroscedasticity are likely to have a profound effect upon any conclusions. Furthermore, the option pricing models of Black and Scholes (see Cox and Rubenstein (1976) p.205) rely on the EMH, and the i.i.d. normal distribution for the logarithm of asset prices.

2.4 Mean Reversion and the Use of Linear Models

The explosive growth in financial services has been fundamental in driving further research, particularly in the above mentioned areas. The advent of the Box-Jenkins approach, using the ARIMA class of linear models has encouraged much research into mean reversion in general economic issues as well as in the specific area of asset prices.

Granger has picked up this topic where he has recognised the limitations on the standard ARIMA linear model. Hosking (1981) in a paper introduced the idea of fractional differencing (or alternatively fractional integration) where he extended the ARIMA class to fractional models that is, using a differencing parameter which is a fraction, say $\frac{1}{4}$. These models exhibit slowly declining , but nevertheless small, levels of autocorrelation which accumulate to a significant size, typical of long memory processes. Granger then extended these concepts in two papers in the early 1980's.

These ideas are outlined in some detail in Granger (1980) and Granger and Joyeux (1980), where the characteristics of such models are determined. The use of these models as an alternative or a complement to the use of unit root tests (that is tests where a decision is needed as to whether or not a differencing of one or unity is statistically sufficient for stationarity, and thus whether or not shocks persist) has been picked up by econometricians, for use in developing their models.

In more recent times work has been conducted into *shock persistence* in a wide range of macroeconomic aggregates. Cochrane (1988) undertook a study into GNP attempting to determine whether an economy operating at a given level which is below it's potential due to a shock will over-correct to return to it's long-run potential path, hence shock persistence. That is, do these aggregates mean revert. Cochrane found little shock persistence in GNP suggesting a random walk for that variable. Other authors have found different results. Mayadunne, Evans and Inder (1995), in a very recent study, looked at a wide range of economic time series to investigate such persistence. Generally speaking their results were inconclusive. When considering the order of integration of the series, they found many with orders of integration above and below one. Put another way they found it very hard to decide whether or not shock persistence existed.

Poterba and Summers (1988) addressed this topic using U.S. stock prices and found mean reverting behaviour, using the variance ratio test. That is they showed that there is initially positive autocorrelation in stock prices followed by negative correlation at long lags. The degree of this negative autocorrelation is small at any given lag but can add to a significant total. Fama and French (1988a,1988b) in two separate papers addressed this same issue and found similar results, though the extent of mean reversion in the long run was very dependent on the inclusion of the 1930's period. On the other hand Lo and McKinlay (1988) concluded that although stock prices were not a random walk, they were also not described as a random walk plus a transient, mean reverting component. If there exists both a short term component of prices and a long term component which is temporary in nature a next problem is to discover and measure it. Eckbo and Liu (1993), attempt to find a lower bound on the proportion of total stock return variance caused by the predictable component. They find the proportion low, ".. ranges from 7-17% for the equal weighted NYSE index , with generally lower values for the value weighted index" (p.175).

More recently Chou and Ng (1995), extended the mean reversion approach to an analysis of international stock markets, and in particular the short term and long term correlation structure. On the *presupposition* that stock prices have a temporary (mean reverting) , as well as permanent (random walk) component, they decomposed international stock price indices for six major markets (U.S., U.K., Japan, France, Germany and Canada). They were concerned as to how the correlation structure changed over time; based upon the mean reversion hypothesis they concluded that the correlations increased over time. Furthermore, they found by dividing the period up into two sub-sample time periods that the correlations had increased. That is, there was evidence of convergence in the international series because the correlations had increased.

One of the features of long-memory is the wide range of results achieved by experimenters in the field. The reason for this is the difficulty of rejecting the null hypothesis of a random walk at a statistically significant level. Thus research has moved towards attempting to obtain more powerful testing techniques, in other words a better balance between Type I and Type II errors. One example of this approach is used by Lo (1991), where he uses the range to standard deviation test first proposed by Hurst (1951) on his work in hydrology (finding optimal dam size for use on the Nile river - and thus showing the very general nature of the mean reversion issue and it's applicability to many areas of study). Using this approach Lo finds that mean reversion is not at all clear. He concludes that there is no evidence of long range dependence once short range dependence is taken into account. Further details on this alternative statistic for testing for mean reversion or long memory characteristics can be found in Davies and Harte's (1987) paper which provided some of the ideas for Lo's alternative look at the topic. Most recently there has been a move to further re-examine some of the claims made. Chow and Denning (1993), extend the Lo and MacKinlay approach to generate a statistic to deal with *joint* hypothesis testing (see also Eckbo and Liu who similarly developed a joint test statistic). To not consider joint significance, in their view, would be selective bias or 'data snooping' as they term it. The net effect of the joint test statistic would be to significantly widen the confidence interval, thus making significant variance ratios no longer so. Poon (1995), also employed the joint significance approach applying it to UK data, finding similar difficulties with the mean reversion alternative hypothesis. Thus we may see the continuity and development of research in this field.

This study exemplifies the problems. There are many possible tests that can be used. All that is needed is a sampling distribution, obtained either by analytical means or, more likely, using Monte Carlo simulation techniques, then the test statistic can be calculated and the hypothesis tested. Given the difficulty of finding "patterns" in stock prices as against all the noise going on in the market place it is not surprising the results are often inconclusive. This is the more so when it is considered that authors nearly always use the same data, U.S. stock prices, from the CRSP database. Summers (1986), gives an excellent discussion on this topic, where he considers in some depth the nature of the difficulties of the issues under study concluding that "..we must face the fact that most of our tests have relatively little power against certain types of market inefficiency" (p.598).

2.5 Non-Linear Modelling

Having reviewed the literature on linear models, the next logical step is the relatively new field of non-linear models. These models are needed because of the difficulty in finding suitable linear models which are capable of representing the facts. That is we need models where the observations cannot be expressed as a linear combinations of current and past disturbances. If mean reversion is presumed to exist we would like to have models to account for this time varying nature of the overall price level. That is, we are postulating different economic periods where overall returns are on average higher than other periods. To replicate this we may consider non-linear switching models such as in Tyssedal and Tjostheim (1988), where they consider an autoregressive model with suddenly changing parameters. Perhaps it is more realistic to use slowly changing parameters to reflect the slower changes in economic circumstances which are likely to exist in reality. There are of course many other potential candidates (covered in depth by Granger and Terasvita (1993) in their most recent book).

The field of applied time series analysis or econometrics has developed an array of potential models in the ARCH class (Autoregressive Conditional Heteroscedasticity). The ARCH model exhibits a form of serial correlation in its variance, thus instead of the variance being constant, it is a function of previous disturbances, say of the form $\mathrm{var}(y_t) = \alpha + \beta\varepsilon_{t-1}^2$, where $\varepsilon_t = y_t - \mu_t$, and the conditional distribution of y_t is assumed normal with mean μ_t. Harvey (1990) covers the field of econometric analysis in his recent text with an excellent exposition of these recent developments.

In Abhyankar, Copeland and Wong (1995), we find an attempt to deal with non-linearities in financial time series. Other authors of interest in the field of stock prices include Hiemstra and Jones (1994), Nelson (1991) and Scheinkman and LeBaron (1989).

In the Australian context non-linear modeling is pursued by Kearns and Pagan (1993). Their paper is an in-depth study into the volatility of the Australian All Ordinaries index over the entire period of its existence, that is, from 1875. This paper is itself a logical sequel to extensive studies carried out by Schwert (1989) on the US market, though he did not apply non-linear techniques. Schwert restricted himself - if this is the correct word - to a wide ranging consideration of market volatility and its possible causes, though his conclusions do suggest the potential of non-linear models. Kearns and Pagan also recognising the heteroscedasticity of their series apply various models to the series for comparative purposes. GARCH (generalised ARCH models) and EGARCH (exponential GARCH) are used alongside an iterative two-step procedure. They can then use these models to check for the persistence of shocks. They find that the shocks persist, both for small and large shocks and furthermore that

"..there is no evidence that the persistence is due to structural change; over long periods it has remained remarkably constant" (p.1993).

Before completing our review of the modeling work available it is worth noting a practical contribution from Wilkie (1992) in the actuarial field, using a 'cascade' style model. Wilkie allows for mean reversion in his real interest rate model, which then helps drive returns from other asset classes and links together the asset-liability side of the picture. The merit of the approach is its internal consistency. It also, reflects the fact that insurance companies must have a basis upon which to plan and create their reserves.

2.6 Asset Allocation: Theory and Practice

With our background in the modelling process, we must now shift our focus to asset allocation, the ultimate objective of this research, where we review the EMH and look at empirical work on asset allocation and review actual performance.

Fama (1991) in the follow up survey to his initial formulation of the EMH in 1970, mentioned previously, recognises that in many ways markets are not efficient. There are plenty of indicators which can lead to gain (back to the early studies!). As regards long-memory he feels results are inconclusive. He also makes reference to recent research extending factor models to attempt to take into account economic conditions. This is an extension of the Sharpe-Lintner models, where we are concerned with links between asset classes, rather than just one asset class, and closely linked to the timing issue.

Once again we should not see the areas of academic interest being divorced from developments in the market place (a lot of study increasingly comes from Business Schools rather than the statistics/operations research field). Both academics and practitioners have become heavily involved in assessing the performance of fund managers, for example, carrying out comprehensive surveys to see if value can be added. Fama (1991), in his most recent survey of efficient markets has pointed out the known inability of sector managers to outperform their respective benchmarks. The use of, so called *passive management*[4] approaches has now become widespread in the U.S. and this is likely to spread elsewhere where identical results apply. It should be noted that this of itself is not a proof of the EMH (perhaps it has more to say about crowd psychology).

In the academic sphere, apart from the contribution from Fama (1972) discussing the components of investment performance, later papers from Kon and Jen (1979) and Kon (1983) evaluating the investment skills of mutual fund managers, are worthy of mention. It should be noted though, only in the context of equity funds where timing is to be measured against the benchmark of being either in cash or equities. Kon and Jen used a statistical procedure to deduce beta estimates in the equity portfolio (and thus giving changes in tack in the management of the portfolio). This was necessary, for at least two reasons. Firstly, at that time funds were not invested in a wide range of asset classes, bonds and equities being the extent of the allocation[5]. Secondly, they lacked the segmental data to be able to determine which area the performance had come from, so they were forced via a model process to induce when and to what extent the asset allocation switches had occurred. Using this approach with different beta estimates[6] they could then deduce how successful the managers were by comparing their beta switches with the market results. They could then evaluate the contributions of timing and selectivity. They found that of 37 funds tested, 25 had positive selectivity, 5 of which were significant at the 5% level. With respect to timing they found only 14 of the 37 had positive timing estimates and none were statistically significant at the 5% level. Some detailed theoretical discussions on

[4] With passive management managers do not pick securities in an asset class, but merely use an index. Thus they are guaranteed the performance of the index, usually at low cost as well.

[5] In modern parlance such funds with assets invested in a wide range of asset classes are called 'balanced funds'. Hence, most superannation funds are balanced funds, thus the manager deals with the asset allocation. Large industrial company or public sector funds may well take the asset allocation task 'in-house', and contract out sector management to individual fund managers.

[6] For a definition of beta refer to Appendix 2, at the end of the Chapter 5. The concept arises from the Capital Asset Pricing Model and thus, for example, a beta of 0.9 would mean that if the market rose by 10% then the portfolio would rise by 9%.

the topic, are put forward in the paper by Admati, Bhattacharya, Pfleïderer and Ross (1986), which widens the issue of how to evaluate timing and selectivity issues via different models in the absence of a detailed breakdown of the relevant performance data. In the absence of data which can , in some way, attribute performance to individual sectors, the only alternative is to develop an underlying model structure and apply it to the data. Of necessity this will lead to statistical problems, at the very least, with the conclusions.

Boudoukh (1995) looked at timing ability by examining a sample of asset allocation funds (thereby sidestepping the issue of timing versus selectivity). He was interested in assessing the timing ability of the managers by seeing whether or not they correctly predicted actual moves in the market. Interestingly, he finds evidence of market timing ability, but "..the evidence is often consistent with such timing ability, but rarely significant from a statistical viewpoint" (p.14). A very similar situation to the examination of the random walk hypothesis.

Whilst there have been many papers from academics there is also a very significant and indeed probably the major contribution, from practitioners, particularly from the field of asset consulting, as well as the funds management business. A wide range of these are available, particularly in the applied or industry journals, such as the *Financial Analysts Journal* or *Journal of Portfolio Management*, many of which are given in the references. There will be many more not listed, many of which are unpublished other than to clients, for example, of stockbroking firms (like Morgan Stanley or Solomon Brothers in the US or J.B.Were and Ord Minett in Australia).

With respect to the returns to market timing, they are large, as indicated in Sy (1990), (writing in reply to Sharpe's original contention that the advantage of a buy-and- hold strategy is so large that you need vastly superior, and unattainable, forecast accuracy to beat it), claiming that you don't have to be right anything like 75% of the time. Droms (1989), who also conducted a study of the level of predictive accuracy required, claims that forecasting accuracy is more important in bull markets than bear ones. Clarke, Fitzgerald, Berent and Statman (1989) present a model which they claim leads to significant outperformance over a buy-and-hold approach; and that generally only " ..modest amounts of information can bring substantial advantage" (p.36). Klemkosky and Bharati (1995) also attempt a model using various predictive variables, as an aid to timing. Using their model they claim gains, even when transaction costs are taken into account, thereby supporting active portfolio management.

Many practitioners feel that not only can market timing work in theory but that it actually does in practice. Lee and Rahman (1991), examined a sample of mutual fund managers. Using their model of returns to identify timing and selectivity they find that "...there is some evidence of superior forecasting ability on the part of fund managers (p. 82). Vandell and Stevens (1989) conducted an empirical examination of the Wells Fargo timing system. They are a very large fund manager in the U.S., with a particular strength in the field of passive funds management. Thus, and most importantly, they were studying an actual fund operation with working systems based upon extensive experience. They concluded that portfolio performance can be improved by market timing. It is noted that the Wells Fargo approach really times extreme markets, that is, any lost opportunities in good markets were more than compensated for when the dramatic downturns occurred. They comment, "A soundly conceived and disciplined approach to timing can reduce downside risk and improve average performance over a cycle" (p.42).

Other papers reviewed include a series initiated by work from Wagner, Shellans and Paul (1992), a team of industry based practitioners -albeit the asset advice industry-who claimed market timing skills based upon a study of managers that were used by institutions using data from the research firm run by one of the authors. Brocato and Chandy (1994), replied, pointing out the effect of survivorship bias, in that of the sample many firms that had dropped out were thus not included. This means that the firms that are unsuccessful *and this could happen by chance* tend to disappear, and so we are left with the ones who may well have just been lucky. The debate was continued with a paper from Larsen and Wozniak (1995), giving a discrete regression model, using various predictive variables, which when used, according to their claims, gives superior timing results. On the issue of measurement of market timing strategies, it is worth mentioning a paper by Beebower and Varikooty (1991), who point out how difficult it actually is to measure market timing ability. Put in an alternative way they essentially try to show how long it would take to be able to differentiate statistically between genuine skill and just good fortune. Note that one is often trying to measure perhaps 1-2% superior or excess return, and very substantial though that quantity is, it is hard to measure against a background of often quite high nominal returns and large variability of returns. It can indeed take many years to be able to recognise genuine outperformance.

In the Australian context, there are several papers reprinted in the excellent compendium edited by Ball, Brown, Finn and Officer (1989), mentioned previously. A paper by Robson (1986), Chapter 30, examines the performance of unit trusts over the period January 1969 to December 1978, including balanced funds. He found that the overall performance was below his calculated benchmark and there was no consistency in performance. However he did find stability in risk levels and correspondence between risk and objectives. Bird, Chin and McCrae (1982), in the following chapter examined superannuation funds and found applying the Jensen measure of performance, whereby the portfolio performance is adjusted for the level of risk taken, that managers were unable to outperform a passive indexation strategy.

Perhaps on a less formal level, Samuelson (1989,1990), when surveying the results of many years of investment practice, for example managing endowments over a working lifetime, questions the merits of attempting to time markets. In his view there is a lot to be said for what amounts to developing the appropriate asset mix and largely sticking to it. He also interestingly enough, makes the comment that, "I side with Schiller and Modigliani and am prepared to doubt *macro* market efficiency". Once again it should be noted that even if this statement is true and that mean reversion might exist, we would still need to find the measure of it and understand it otherwise we could not efficiently exploit it (then presumably lose the edge as others copied our strategems).

As has been indicated above, to be able to achieve our goal of assessing the value of asset allocation or market timing we must be able to measure it. The techniques applied are based upon the concepts of attribution analysis outlined in the paper by Brinson, Hood and Beebower (1986), and the sequel Brinson, Singer and Beebower (1991). The basic idea is to be able to separate out sector from asset class performance by attributing to each particular asset, it's return. This enables us to unbundle, as it were, the returns to see the effect of timing, that is the contribution to the total return obtained from being in the right asset at the right time. Hence one must have the programs, and other tools necessary to extract the data from fund management information systems. This is a substantial task, particularly given the incompatibility of most systems. The results of the above two papers, using this methodology, indicate that active management *cost* the average sponsor money. They note that whilst some plans added value, by far the most important issue was the overall investment policy, that is, the strategic asset allocation. The returns from the policy portfolio dwarfed other components. Adding value from timing even where it was achieved is not the *key* issue. Booth and Fama (1992) in their paper discussing certain technical aspects of returns also suggest that incremental returns may actually be lost by active management.

Ankrim (1992) extends the work of Brinson et al, by pointing out that managers often, in practice, do not stick to their policy portfolios. He then suggests a risk adjustment procedure that compensates for the extra risk taken (or lower risk if appropriate) by deviating from the relevant benchmark. In essence he compares the portfolio averages to their benchmarks. Whilst his sample is small it is certainly suggestive of a significant level of deviation.

Hensel, Ezra and Ilkiw (1991) in their paper reviewing the importance of the asset allocation decision, point out that asset allocation policy to be accurately assessed depends upon the alternative. If this is a diversified portfolio rather than say Treasury bills then the asset allocation decision may not be so important. However given that returns from different assets do vary widely over time and that there is little doubt that being in the correct asset class does deliver outperformance, the next obvious questions are why, and can we take advantage.

The paper by Benari (1990) (who is a practitioner) is very interesting as he suggests there are various eras which are more or less appropriate to different asset classes and these eras persist for a long time (although eventually the system, as it were, corrects itself). So, he outlines three periods 1966-72;1973-82 and 1983-88. The key to these eras are the incidence of price inflation. The first period can be characterised as one stable but rising inflation. The second as accelerating inflation, and the third as declining inflation. Different types of asset did well under each scenario. Hence one needed to be able to, correctly, recognise the changed circumstances and act accordingly.

On a more technical level, the paper by Jorion (1992), considers the mean variance process by which assets are put together in the most efficient manner possible. He points out that the process of portfolio optimisation, in practice, involves dealing with measurement error, and this means that the optimal allocation obtained should really consist of a scatter diagram of points rather than one clean line.

2.7 Impact on Investment Strategy

Two papers by Thorley (1995) and Reichenstein and Rich (1994), pull together many of the ideas covered in this review. They are interested in what the results of the studies into stock returns, efficiency and mean reversion actually imply for market practice. Thorley points out that the use of mean-variance optimisers needs to be treated with caution. He posits that time diversification (risk can be diversified through time), is not only widely practised, but should be in principle. Using arguments centred on return probabilities he argues for an increasing allocation to risky assets, when a longer time horizon is available. Reichenstein and Rich, justify such a position, not only based upon studies showing the predictability of stock prices, but also on mean reversion. They feel that an equity range of 35-65%, is optimal, reckoning that diversification benefits outweigh timing ones. However they go on to claim that mean reversion implies that the actual shortfall risk (of T-notes beating stocks), is much lower than previously thought. This is a function of the lack of independence of yearly returns; and hence they conclude that investors with long time horizons should take a much higher level of risky assets than previously would have been considered appropriate.

This survey then sets the context for the overall objective of the thesis. That is, having reviewed the random walk hypothesis, mean reversion and the modelling of asset prices, we can then look at the actual tactical asset allocation performance and see if it is reasonable to expect managers to be able to time markets and whether they, in fact, do so. Then we may be in a position to offer some advice.

Chapter 3: Data Analysis and Autocorrelation: Persistence and Reversion

3.1 Introduction

This chapter commences the main body of the thesis by outlining the sources of information and how they are developed to provide our base series for investigation. After considering some key features of the series we move on to look at the major issues of autocorrelation. We are attempting to find out as much as possible about the structure of our financial series and how they relate to one another. We need to consider the extent of any patterns in the series, in particular if they have any long memory characteristics. Within this context we must also provide some estimation of the significance of the results, that is how much confidence can we place on our conclusions.

3.2 Sources of Information

The Australian Stock Exchange (ASX) provides information on the All Ordinaries Index (AOI) as far back as January 1875 on a monthly basis. At that time the index was known as the Commercial and Industrial Index and it continued until June 1936. From July 1936 until December 1979, the index was the Sydney All Ordinaries Index; then when the ASX became fully national, the current AOI was created. These earlier indices are thus comparable to the current All Ordinaries Index. Potter Warburg[1] provided an AOI series back to 1900, and original data was added on to take the series back to 1875. The Potter Warburg series being re-based was validated against the Commercial and Industrial Index. An accumulation index is not available over that time. However, one commencing in October 1960 was provided by

[1] Where available the indices that were used were validated against each other. We have available data from Potter Warburg, J.B.Were and the 'SuperCMS' database. Thus we are dependent on the integrity of the data, and whilst checks were performed it is notable that, for example, we have a difference between all three sources and that of Kearns and Pagan (1993), for October 1987. This may well be due to the likely use of average prices by Kearns and Pagan, as against month end prices.

Potter Warburg. Therefore the task was to find an appropriate dividend yield series to fill in the missing portion. It is noted that the accumulation index used was with month end prices, rather than the average prices given, for example, in the *Stock Exchange Journal*. Whilst we can be confident about the integrity of the Potter Warburg series from 1960, further back we cannot be so certain, particularly with the old Commercial and Industrial Index, where the information is in typed foolscap sheets from a ledger. However, the analysis following in 3.3.2, leads us to conclude that it is very likely so. A month end AOI was made available, by courtesy of J.B.Were, for the period of the new index, that is December 1979 up to the current time. Prior periods are more difficult; a weekly series from 1968 to 1979 was available. Unfortunately, though it was adjusted to be comparable to the monthly series[2], the weeks ending do not coincide with the month end. Thus we have variously 4 and 5 week months, so they are not comparable.

A dividend yield series was available from the ASX for the period 31 October 1882 to 30 September 1983 which was, however, an *unweighted quarterly* series. This series was first converted to a monthly basis by taking the quarterly rate on an annualised basis and determining the corresponding monthly rate, that is, raising to the power (1/12). Thus the monthly rates for the months in each quarter are identical. An accumulation index was then formed by applying the dividend yield series to the AOI determined above. Finally the accumulation index is reconciled to the current All Ordinaries Accumulation Index which is based upon a *weighted* dividend yield. We will denote the All Ordinaries Accumulation Index as AOIA, as distinct from the AOI capital series.

The steps of reconciliation were as follows. Firstly the ratio of the two series was taken for the period of overlap being 31/10/1960 to 30/9/83. This series shows a decreasing trend, due to the bias in unweighted series. By applying the unweighted yield to the original AOIA series significant shift is apparent by 1983. The reason for this is that some larger companies have low dividend yields (examples would be News

[2] The 1968/79 Sydney All Ordinaries multiplied by 0.6757 (adjustment factor), to link with the ASX Index at 31 Dec 1979.

Corporation and BHP) and thus the unweighted yield overstates the true yield across the market.

This trend in the series was thus projected back over the previous years, from 31/10/1960 to 31/10/1882. And so the original unweighted yield series was adjusted by applying this trend to it. There may be some bias in this, because of the increasing importance of generally low dividend paying mining stocks in the index from the mid-1960's. However most of the variance in the series comes from changes in prices not dividends which were extremely stable throughout the period. In any case, a statistical analysis into both series was conducted to see whether any results would be affected by this. The evidence suggests there is little difference.

The consumer price index (CPI) was obtained from the Australian Bureau of Statistics (ABS). Annual data was available from 1850 to 1993 using a long term linked series generated by the ABS (originally the series was called the Retail Price Index and contained only basic items like food, clothing and rent). Quarterly data was available from 1948. This was converted to monthly data by the same method as was used on the dividend yield series.

To obtain *excess* [3]returns it was necessary to find a risk free rate. An obvious choice, widely used in funds management, is the 90-day Treasury note. This series was made available by the Reserve Bank of Australia (RBA). The earliest date that this series started was November 1959. Of course, in the early days the note rate was virtually constant, remaining in the 3-4% band until April 1965. Whilst the series showed a slowly rising trend during the 1970's, it did not exhibit significant volatility until the 1980's, particularly after the $A was floated (and thus exposing short term rates to market forces).

Data for the Commonwealth Bank Bond Index (CBBI) All Maturities and the Morgan Stanley Capital International Index (MSCII) was made available via the "SUPERCMS " database. This is an actuarial style database provided by consulting actuaries from a division of Rainmaker Australia based in Sydney . This database

[3] Excess returns are those after deflation by the risk free rate (in our case the 90 day T-note).

provides a large variety of financial time series of varying periods . However despite the relative shortness of many of the series (the CBBI starts in December 1976 and the MSCII in December 1969) they are nevertheless essential if one is to consider alternative asset classes. It is also of note that only the last 20 years of data, for the bond market, is relevant due to the controlled nature of the market prior to that period. It could also be argued that the floating of the A$ meant that prior periods were so different as to make comparisons very difficult.

3.3 Data Analysis

3.3.1 Preliminaries

As a preliminary, basic statistics on the nature and distribution of the AOIA accumulation series was determined. It is noted that we will focus throughout on the *total returns*, given by the accumulation index for each asset class, because these are benchmarks for the asset classes, discussed later. Hence, bond returns given by the CBBI are inclusive of interest payments. Thus we will define throughout :-

$$r_t = \frac{(z_t - z_{t-1})}{z_{t-1}} \times 100\% \text{ and } \bar{r} = \frac{1}{n}\sum_{t=1}^{n} r_t \text{ is the mean return.}$$

where z_t is the accumulation return at time t, corresponding to the capital value denoted p_t with dividends re-invested.

The series were examined for non-normality, heteroscedasticity (note that these will be dealt with in greater depth in Chapter 4) and the existence of significant autocorrelations, all of which could cause problems with the assumptions in financial models. The following initial results are for the AOIA:

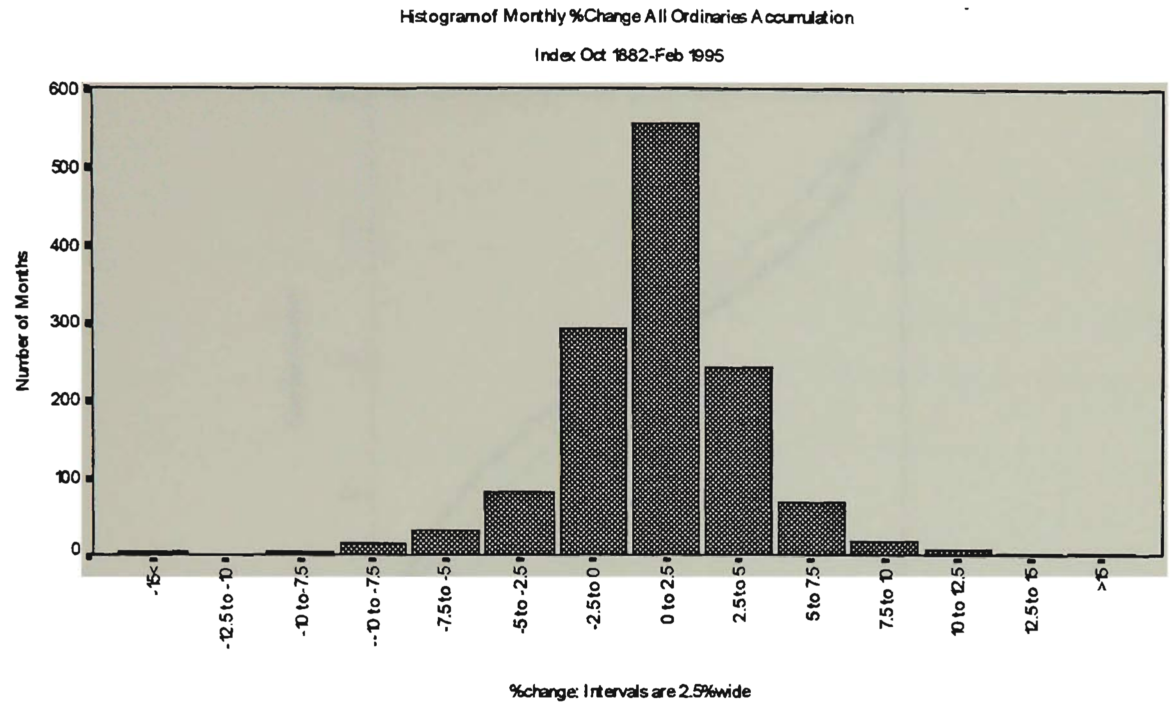

Figure 3.1 Histogram AOIA monthly : Intervals are 2.5 % wide

As can be seen from the above histogram the distribution of monthly percentage changes in the accumulation index has a lack of symmetry suggestive of non- normality. Note that the category range is for intervals of 2.5% in width. The choice of the number of intervals represents a compromise between having sufficient entries in a class and introducing too many classes, and follows the suggestion by Scott (1992) that a lower bound for the number of intervals should be at least $\sqrt[3]{2n} \cong 14$, in this case. Nevertheless it well illustrates the point. A closer look at a normal probability plot revealed the following.

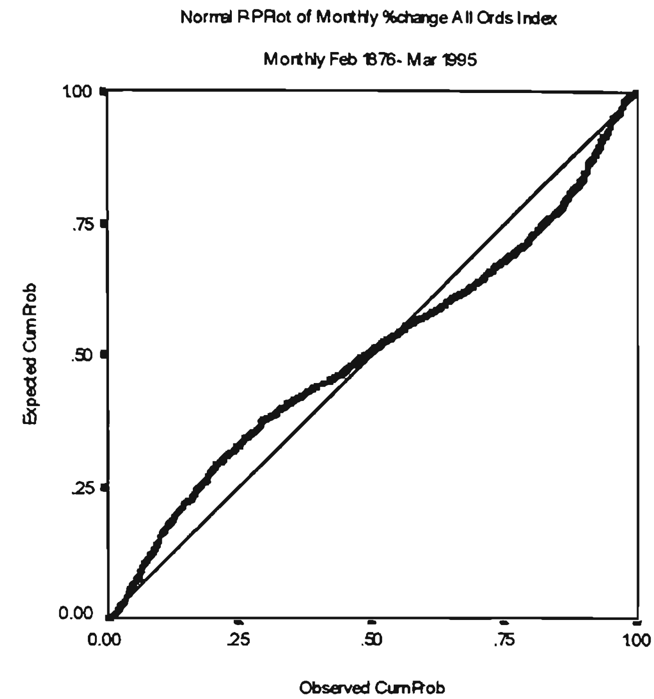

Figure 3.2 Normal Probability Plot AOIA

The cumulative plot shown above gives an alternative way of viewing the preceding histogram. There are too many observations centred close to the mean and too many extreme values. In part this is likely due to the change in volatility over time. However to assess this, a check on the assumption of constant variance was made.

We then need to consider how we are to best measure the variance. A recursive style estimate was made, corresponding to an increasing sample size as more data points are added and we allow time for the series to run-in. The estimate is then:-

$$\hat{\sigma}_t^{\,2} = \frac{1}{t}\sum_{i=1}^{t}\left(r_t - \bar{r}\right)^2$$

Given our estimator there will inevitably be time needed for it to settle down as, clearly, data points have a much greater ability to move the series in the earlier period. After some consideration, it was felt that the best way to capture the longer term movements was to determine the 5 year simple moving average of the variance, centred with 30 months before and after. The estimator[4] then becomes:

[4] Using the Excel V5 definition of sample variance. Given the sample size, any sample bias would be small, in any case.

$$\hat{\sigma}_t^{\,2} = \frac{1}{60}\sum_{i=t-30}^{t+30}\left(r_i - \bar{r}\right)^2$$

There are many other alternative estimators that we could use, say, a 12-month rolling estimator, or one based upon the variance within a calendar year. These shorter term estimates would be very volatile with smoothing required. The recursive estimator used does have the virtue of incorporating all the information in the latest estimates. The resulting plots are shown in Figures 3.3 and 3.4 below.

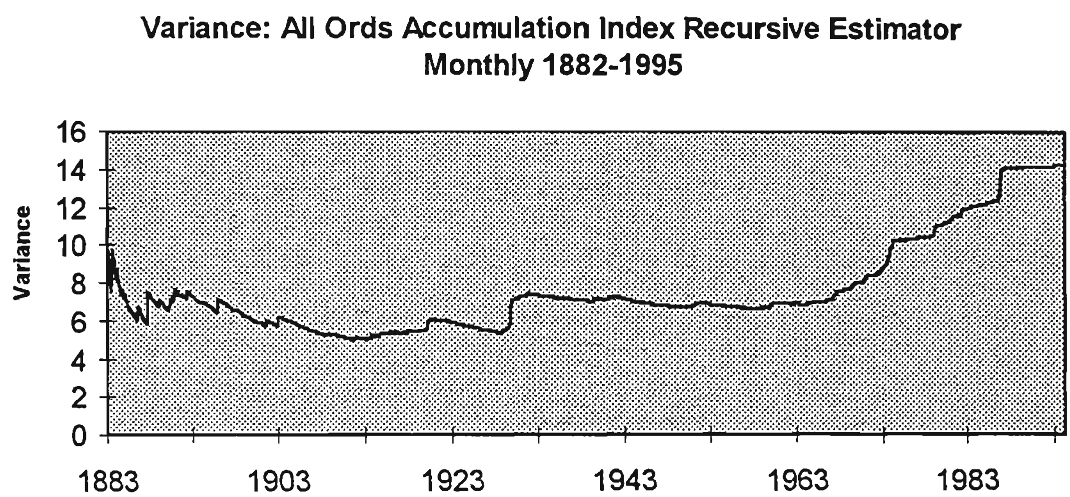

Figure 3.3 Plot of Variance AOIA; Recursive Estimator

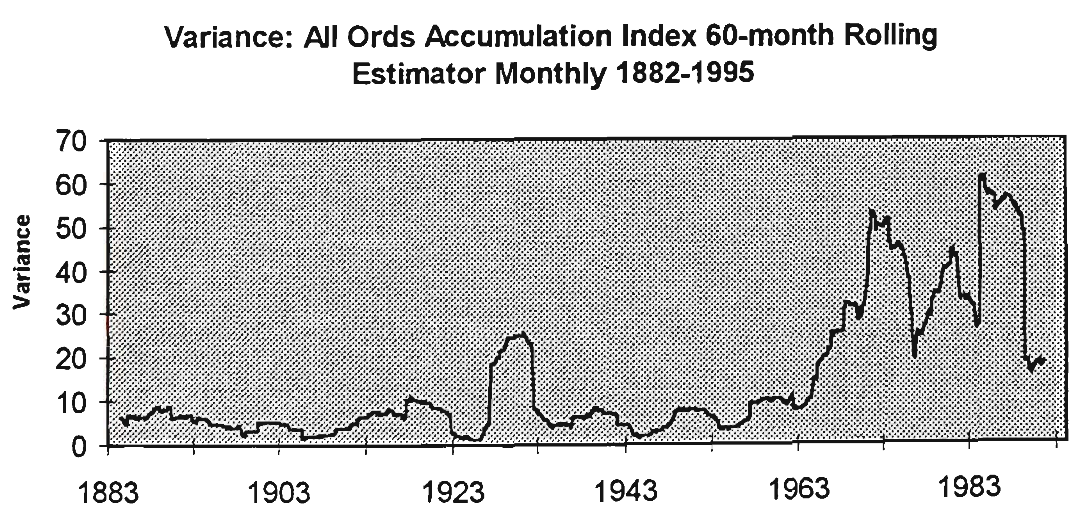

Figure 3.4 Plot of Variance AOIA; 60-month Rolling Estimator

Apart from the period during the 1930's when the volatility[5] was substantially higher, the variance has fluctuated between 5% and 10% until the 1960's. Indeed it is noteworthy the way it returned to the long term trend after the "shock" of the crash in 1929 died away. It is clear that the volatility had increased significantly during the 1960's (likely related to the increase in the mining component of the AOI). The period of rapid ascent took place in the late 1960's (Poseidon and the nickel boom), punctuated by the increases due to the fallout of 1974 ; the "commodity boom " during the late 1970's and most recently the stock market crash of 1987.

It would therefore appear that there has been a permanent increase in volatility in the Australian market. And thus we may divide up the history into two distinct periods. If we, somewhat arbitrarily, take our dividing line as when the variance first exceeded 20%, September 1967, then up until then the variance was 7.0% and since then it has been 36.6%, though clearly there is much more variation than this. We will review this in more detail in 4.5.3, where at a micro level one can discern greater variation. The paper by Kearns and Pagan (1993) also covers this topic in considerable depth.

As we shall see later, when considering the variance ratio results, this should not affect the results from 1959 onwards. That is, the period of most interest, the most recent past, *has* shown a reasonably constant variance, even if at a higher level. Indeed the results are similar for both the more recent period and the whole period.

3.3.2 Autocorrelation and Persistence

The next issue to be considered for the series was to look for any significant autocorrelations. The *autocorrelation* at lag k is defined by:-

[5] Risk, volatility and standard deviation are terms often used somewhat synonymously. In simple terms, if we measure risk as the dispersion of returns about the expected level, then we can use many alternatives, say range or mean absolute deviation. Then risk and volatility are the same thing, and in our case we use variance estimators as our measures of volatility. Given that that the variance is the square of the standard deviation, we are applying the usual measure. See Radcliffe (1994), Chapter 6, for a detailed discussion of this and related concepts.

$$\rho_k = \frac{\mathrm{cov}(z_t, z_{t-k})}{\{\mathrm{var}(z_t).\mathrm{var}(z_{t-k})\}^{1/2}}, \quad k = 0, 1, \ldots\ldots, \quad \text{hence } \rho_0 = 1$$

A correlogram was plotted with the following results.

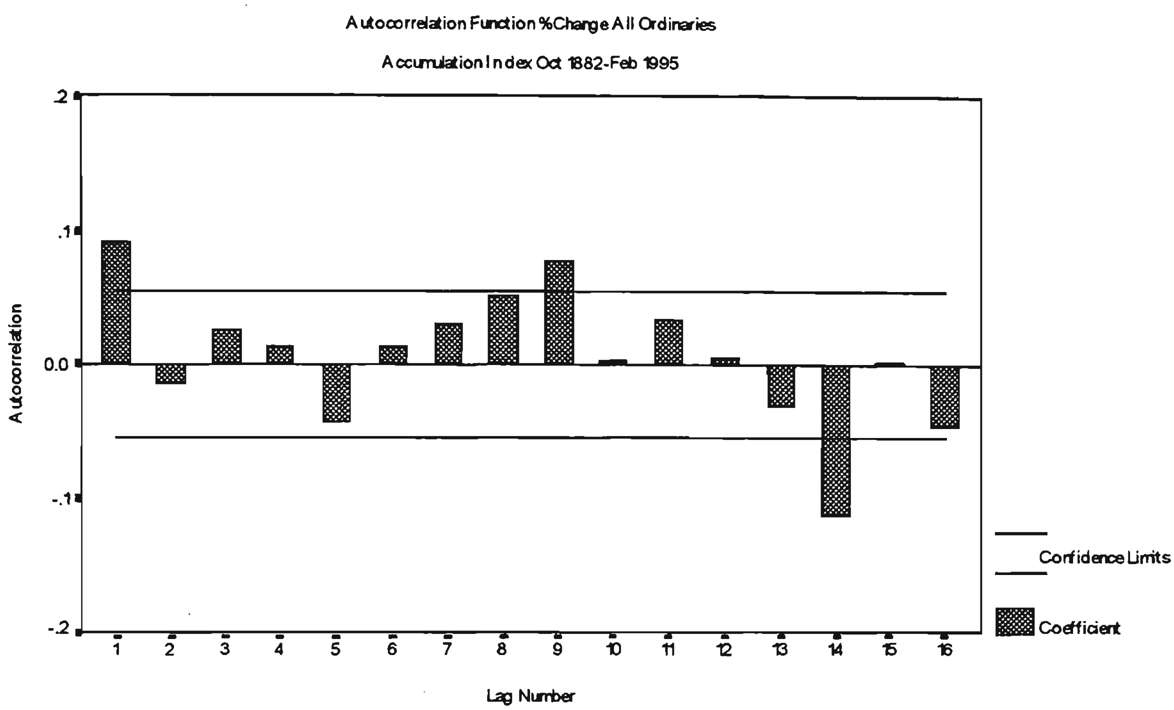

Figure 3.5 Correlogram AOIA 1882-1995

There are significant autocorrelations at lag 1, with a value of $\rho_1 = 0.091$, and a standard error of 0.027, and again at lags 9 and 14. The value of the Box-Ljung statistic for joint significance of the first 12 autocorrelations $Q(12)$ was 29.853, as compared to the critical value of 21.03 for $\chi^2_{0.05}$ with 12 degrees of freedom. This further test is a modification of the 'portmanteau' diagnostic test of Box and Pierce (1970), which tests the joint null hypothesis

$$H_0 : \rho_1 = \rho_2 = \ldots\ldots = \rho_k$$

whereby Box and Ljung (1978) show that the statistic:-

$$Q(K) = n(n+2)\sum_{i=1}^{K} \frac{\hat{\rho}_i^{\,2}}{(n-i)}$$

approximately follows the distribution $\chi_{(K-m)}^{2}$, where m is the number of parameters in the model. In our case $m = 0$.

However before we attempt to impose some kind of structure on the series we must consider in order :-

1. The partial autocorrelation function (PACF), which measures the correlation between members of a series where dependence on the intermediate terms has been removed. Hence for an AR(1) all members of the series will be correlated, hence the correlogram will be an exponentially declining series. The PACF removes that dependence which affects correlations at lag 2, 3,....

2. Is there any difference when we divide up the series into the two periods of differing variances. As an aside the boundary line was recast back to 30/11/59 to coincide with the period over which the variance ratio tests were conducted.

The *partial autocorrelation* at lag k is defined by:-

$$\phi_{kk} = \frac{|P_k^*|}{|P_k|}, \quad \text{where } P_k \text{ is the } (k \times k) \text{ autocorrelation matrix}$$

and P_k^* is P_k with the last column replaced by $(\rho_1, \rho_2, \ldots\ldots\rho_k)'$

Firstly we can view the PACF.

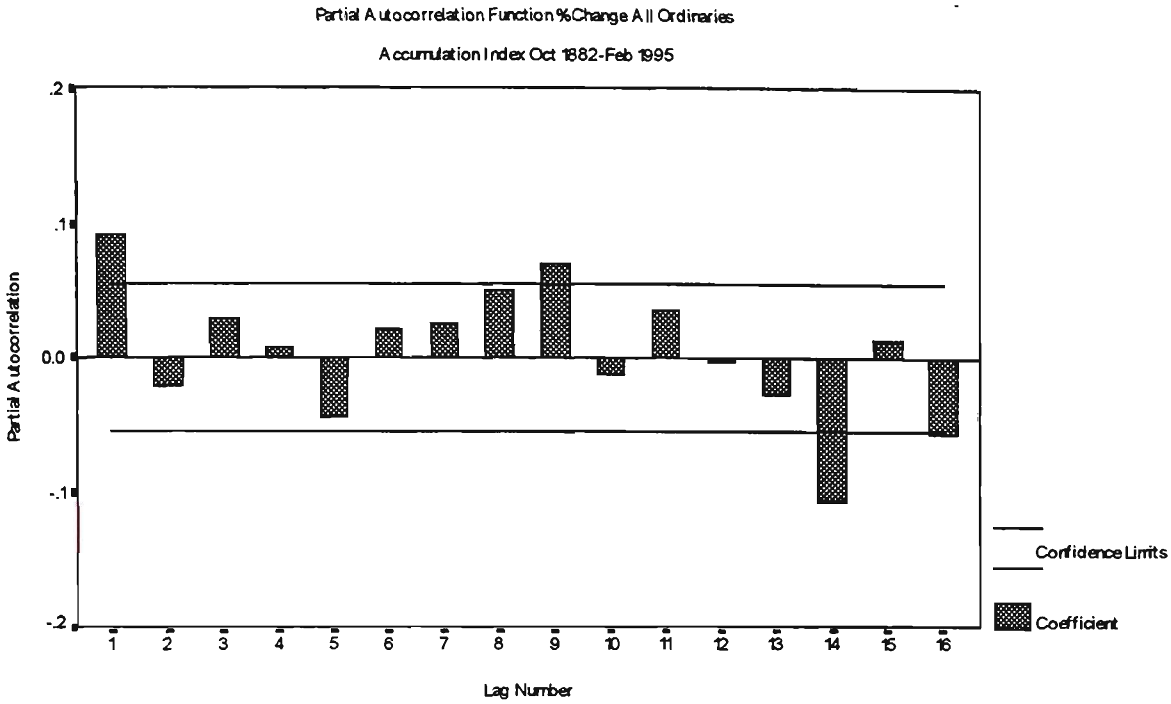

Figure 3.6 Partial Correlogram AOIA 1882-1995

This, in conjunction with the significant autocorrelation at lag 1 in Figure 3.5, suggests that an AR(1) model may be appropriate, however let us consider item 2 and see if there is any stability in the autocorrelations. One would expect *a priori* some changes over time.

We have not considered the capital series, the All Ordinaries Index[6], over the longer time horizon (see the comments on this aspect above), however we should not expect a big difference[7], given the stability of the dividend yield series, ranging between a minimum of 4.45% and a maximum of 10.29%. Indeed the variance of the dividend yield series was a mere 1.48% versus a value of 12.37% for the AOI series, that is the capital component. This difference is even more pronounced for the more recent periods, where, both the volatility of the capital series has risen and the dividend yield series has, if anything, become more stable, due to the tendency for dividend

[6] Note that the ASX stated series uses average prices. Based upon the results of Working (1960), where he showed that that the expected first-order serial correlation of first differences between averages of terms in a random chain approximates 0.25 (and using 20 working days per month), we would expect positive autocorrelation. In fact, an analysis gave a value of 0.306 for the period Nov 1959- Feb 1995. We must therefore use month end data, for any analysis.

[7] In fact extending the series to Dec 1995 gave a value for ρ_1 of 0.033, for the capital series available, versus 0.038 for the, albeit shorter, accumulation series. See summary Table below.

policy to "smooth" dividends. To the extent that we have dividend imputation, and thus an incentive for companies to pay out franked dividends and an increase in smaller investors, this tendency will likely increase. Not surprisingly, a quick review of the unweighted quarterly dividend yield series, referred to in 3.1, gave a first order autocorrelation of 0.95, a classic AR(1) representation.

As indicated the series was divided up and the ACF and PACF, for the most recent period were determined with the following results:-

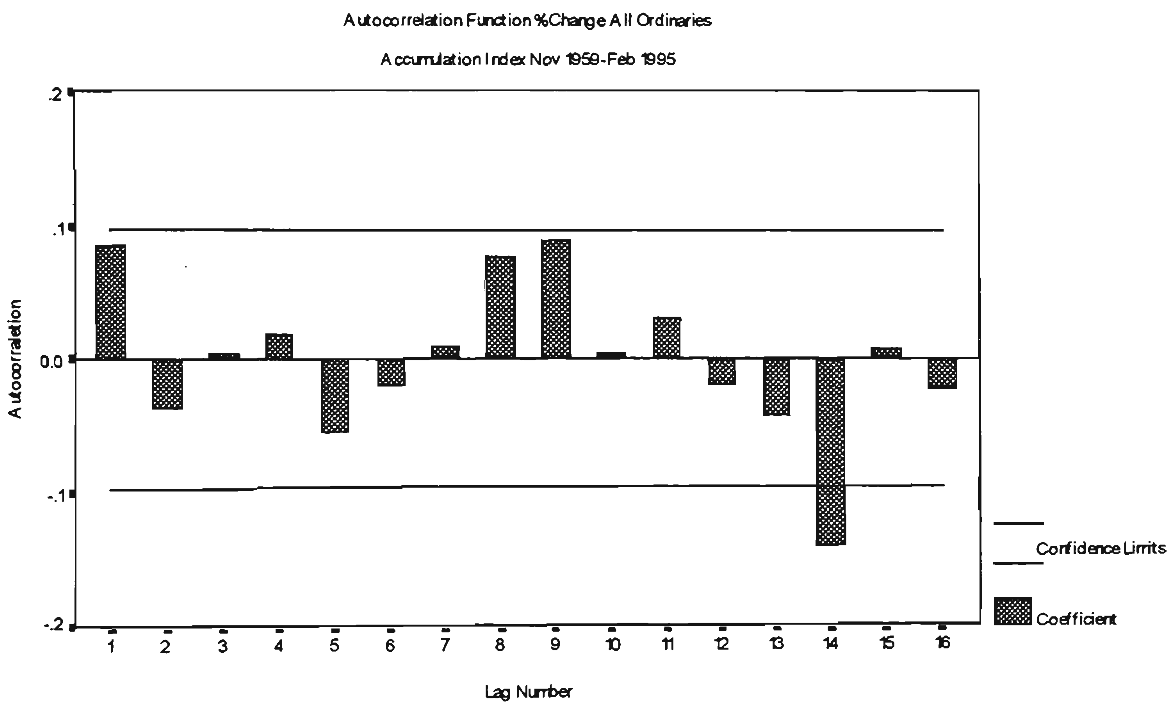

Figure 3.7 Correlogram AOIA 1959-95

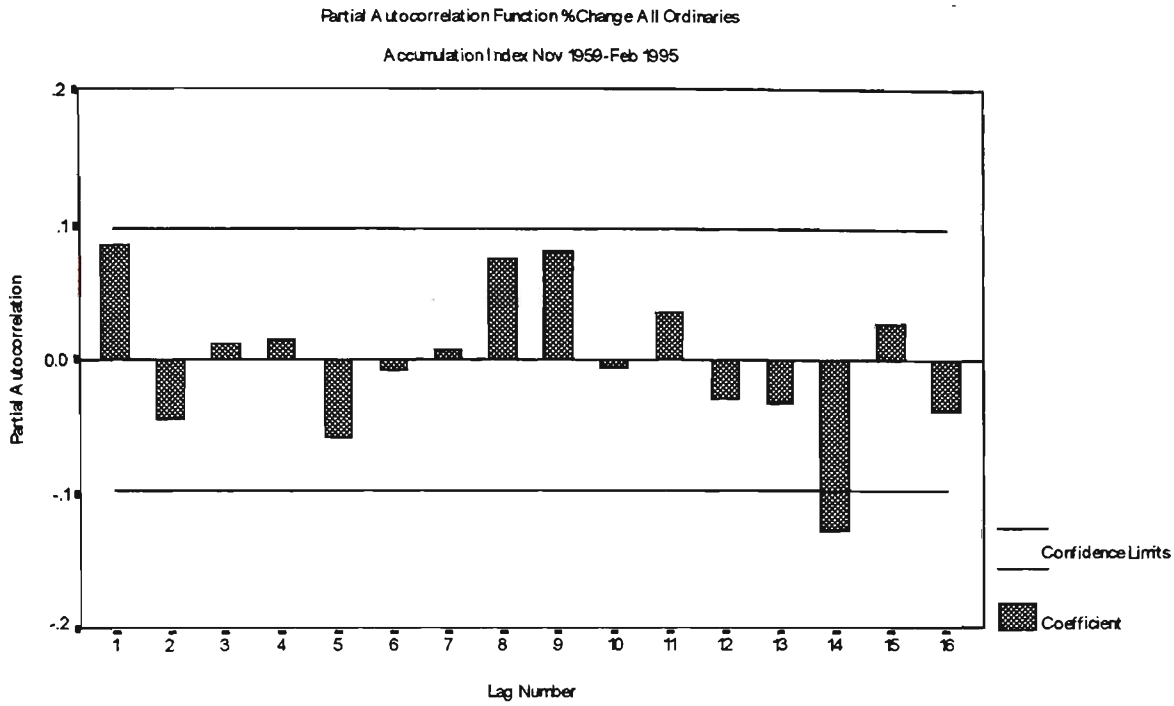

Figure 3.8 Partial Correlogram AOIA 1959-95

The ACF in Figure 3.7 exhibits a non-significant level of autocorrelation at lag 1. The actual value for the series is $\rho_1 = 0.085$ with a standard error of 0.048. The much longer series of 1018 observations from Oct 1882 to Oct 1959 gave a value of $\rho_1 = 0.102$[8], which is significant at the 5% level using the standard error calculated of 0.036. This is not greatly different, particularly when one takes account of the October 1987 value, outlined below. Therefore, in this sense it would appear that the structure and operation of the Australian equity market has not undergone a significant change. The results would suggest positive autocorrelation or persistence, similar to the results of Poterba and Summers (1988) for the US market. The result is also consistent with the variance ratio results covered in the next section, indicating that perhaps the degree of positive short run autocorrelation is not as strong as that for other smaller markets. This result needs some further qualification, particularly when one considers the results of Groenewald and Kang (1993). Taking the very much shorter time period of January 1980 (the commencement of the current AOI) to June

[8] The fact that this value is similar to the that for the most recent series, implies that the series is likely not a series of *average* prices. When that was investigated we obtained 0.306 for the first order serial autocorrelation, see note 3 above.

1988 they find no significant autocorrelations, or put alternatively the market is *weak form efficient.*[9]

A closer examination of the data reveals some interesting features (these are of course closely related to the heteroscedasticity observed in stock prices that is covered in 4.5.3).

1. There are likely differences in data, so the results herein may well show corresponding differences.

2. As has been shown in the US by Schwert (1989), and in Australia by Kearns and Pagan, large moves in the market tend to be followed by reversals. The series from 1980 to 1988, includes swings of +17.43 to -14.30 to +12.54 separated by a month each time.

3. The period includes the October 1987 large value.

By way of demonstration of the importance of October 1987 in a series of 102 data points, the October value was replaced by the mean of the two either side (as in 4.5.2[10]). The value of ρ_1, the first order autocorrelation moved up from 0.038 to 0.111, and ρ_2 went from -0.159 to -0.224 which is significant with a standard error of 0.097. One would expect the reversals outlined in item 2 above has a lot to do with the significance of ρ_2. At the same time the Box-Ljung statistic $Q(12)$, as defined above, went from 7.736 to 16.132. In effect then we may need to differentiate between 'ordinary' and 'extraordinary' events. Tentatively one might suggest that over the longer period, when individual events are smoothed out or when extraordinary events do not occur, that persistence can be observed. Large reversals or extraordinary events break this pattern to such a degree that they are no longer statistically observable. Finally, one may note the somewhat better results in Groenewald and Kang of the

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[9] As outlined in Fama (1970), various forms of efficiency are defined as part of the Efficient Markets Hypothesis (EMH).

[10] Note: The value of the change in the accumulation index was -42.13. It was the same in the Potter Warburg , J.B.Were and SuperCMS data, *but* different from the data exhibited in Kearns and Pagan (1993), Table 1, p.165.

accumulation series as against the capital ones. This is consistent with the strong positive autocorrelation observed in the dividend yield series, and should be born in mind when considering the results presented here.

We may summarise these results along with those for the other asset classes, the CBBI and MSCII, as in the following table. The results for the other asset classes are interesting. The CBBI exhibits a first-order autocorrelation of 0.116, which is 1.73 times the standard error, not significant, though the Box-Ljung statistic is. The MCSII is far less 'comforting', neither statistic being significant. Of course both series have been subject to considerable change, for the MSCII devaluations in the 1970's, then the collapse from 1983-85, let alone any reversals. This would merit detailed investigation.

Table 3.1 Significance of First - Order Correlation and Joint Significance First 12 Lags for Key Asset Classes

Series	ρ_1	Std.Err	$Q(12)$
AOI Accum. Oct 1882- Feb 1995	0.091*	0.027	29.853*
Ditto - ex Oct 1987	0.116*	0.027	51.549*
AOI Accum. Oct 1882- Oct 1959	0.102*	0.033	32.246*
AOI Accum. Nov 1959- Feb 1995	0.085	0.048	11.833
Ditto - ex Oct 1987	0.122*	0.048	20.732
CBBI Jan 1977- Feb 1995	0.116	0.067	21.386*
MSCII Jan 1970- Feb1995	0.066	0.057	13.323
AOI Dec1979-Dec 1995	0.033	0.072	8.303
Ditto - ex Oct 1987	0.092	0.072	14.205
AOI Weekly Jan 1968-Dec 1979	-0.098**	0.040	6.099

* Significant at the 5% level.

** Significant *negative* correlation, at the 5% level, though $Q(12)$ not so. We will not address this result herein, but it is certainly interesting[11].

Given the results in the Table above, one would be inclined to suggest an AR(1) for the stock series, using the long run series coefficient as parameter; that is a model of the form:-

$z_t = 0.091 z_{t-1} + \varepsilon_t \;,\; \varepsilon_t \sim N\left(0, {\sigma_{\varepsilon_t}}^2\right)$ and $E(z_t) = 0$ (or else this is transformed by subtracting $\bar{z}$).

[11] Perhaps this is an ex-dividend effect. For a fuller discussion of this and related data issues see Officer (1975), Chapter 15 in *Share Markets and Portfolio Theory*. The data is relatively old but Officer gives a detailed analysis as it pertains to the Australian situation. This has general applicability to the data as well as some of the results given herein. The time periods do overlap.

Note: An alternative representation would be an MA(1), given the cut-off in the ACF and damped oscillating PACF. The parameter would then be given by the solution of:-

$$\rho_1 = \frac{\theta}{\left(1+\theta^2\right)} \quad \text{or} \quad \theta = \frac{1 \pm \sqrt{1-4\rho_1{}^2}}{2\rho_1} \quad \text{where } \rho_1 = 0.091$$

Yielding the model $z_t = \varepsilon_t + 0.0918\varepsilon_{t-1}$, which when inverted using the backshift operator gives the infinite AR, $z_t = 0.0918z_{t-1} - 0.0083z_{t-2} + \ldots + \varepsilon_t$. This is not greatly different, indicating the alternative models available in practice.

We will not consider any further potential for short term model fitting, as it would lead us away from our primary objectives, though it is a very relevant consideration in market timing. Obviously, this also is a subject which would merit further investigation, particularly given the degree of instability in the autocorrelation function and its sensitivity to individual 'outliers' . In our discussion in 4.5.4, this is further expanded upon. We must now look at further aspects of the structure of the series by viewing the variance ratios, for a range of time periods and transformations using both real and excess prices, as well as nominal ones.

3.4 The Variance Ratio Tests

Following the work, particularly of Poterba and Summers (1988) and also related work by Fama and French (1988b), it was felt that the variance ratio test would be most useful in evaluating autocorrelations in the various accumulation series over the long term. Note that only for stock prices do we have a sufficiently long time series to be able to use annual data. The essential idea behind the variance ratio is that, if a series is a random walk, then successive elements in the series are uncorrelated and thus the variance of the sum of elements in the series should grow in proportion to the number of terms. If we then standardise this sum by dividing by the number of terms in the sum and compare this with the 12-month value we will obtain a measure of the degree of correlation over a long time period. Thus, as a measure, it is better able to capture, particularly for long horizons, small autocorrelations which may be insignificant in themselves but which may sum to a significant level.

For monthly returns the statistic is (Poterba and Summers (1988) p.30) :

$$VR(k) = \frac{\mathrm{var}\left(R_t^k\right)/k}{\mathrm{var}\left(R_t^{12}\right)/12} \qquad \text{where } R_t^k = \sum_{i=0}^{k-1} r_{t-i} \text{ and } r_t \text{ is the return at time } t.$$

If the series is a random walk then the expected value of the statistic will be unity. *Positive* short run autocorrelation will yield a value below one, for values below 12 and *negative* long run autocorrelation will also yield a value below one, because of the nature of the ratio. Appendix 1 of chapter 4, shows the relationship between the autocorrelation coefficients and the variance ratio $VR(k)$.

Hence we may determine whether or not there is negative autocorrelation over the long term. That is we can consider the existence or extent of *mean reversion*[12] in the series. We then have the random walk hypothesis (H_0) as the null hypothesis, versus the alternative being mean reversion (H_1). Clearly this is a very critical input to the approach that needs to be taken in asset allocation. As it stands domestic institutions use tactical asset allocation or moving between asset classes, as a means of adding value to their *policy portfolios* (that is long run portfolios based on their asset/liability mix; for a detailed discussion and definitions see section 5.3).

Furthermore, positive autocorrelation in the short run combined with negative long run autocorrelation would then eliminate the possibility of a simple ARIMA model as representing the underlying generator of the series. This aspect will be considered in more detail in the following chapter, see 4.1

Given the long run nature of our endeavour and its importance in the overall asset allocation of a fund it was decided that it would be useful to look at both annual and monthly data for the AOIA, particularly since we have available such a long time series. Further this meant that given the very long CPI series we could investigate

[12] In layman's terms mean reversion is the assumed propensity of markets to correct over long time periods. That is, good economic times are followed by bad ones; similarly for assets. Hence if a market has performed particularly well, one would assume that it will do poorly to bring average performance back in line with long term trends. This of course leaves aside the key issue of just what long term mean returns actually are.

mean reversion for real prices as well as nominal. Monthly results for key asset classes are covered below.

Thus the variance ratio test was carried out for nominal, real and excess[13] returns on monthly and annual data. The data was also further split up to investigate both the pre and post 1928 situations. The rationale for this is the importance of the Great Depression and the period up to the end of the second World War as has been mentioned in Jones (1993), Fama and French (1988b), Poterba and Summers *et al.* Evidence from their work suggests significantly higher levels of mean reversion (lower variance ratios at long time periods) for earlier time periods than the more recent post-war period.

It was also felt necessary to gain some measure of the significance of the results by gaining an appreciation of the sampling distribution of the variance ratio test for the random walk hypothesis. Monte Carlo simulations were therefore conducted based upon the null hypothesis of a random normal distribution. Samples of length 110 were generated (being the period of the annual time series from 1883-1995 allowing for the year ending being June). 1000 such series were generated and the test was performed. Thus an estimate of the standard error was found.

Estimates of the standard error were similar to those found in Poterba and Summers. Thus it was not felt necessary to conduct a large number of simulations . Further unlike Lo and MacKinlay (1988) where it was possible using their test statistics to find an analytic form for the sampling distribution, we do not provide here a formal statistical inference procedure. Lo and MacKinlay do not generally support the mean reversion alternative to the random walk hypothesis, feeling that the alternative fits the evidence no better than the null. They do conclude that "... the sum of a random walk and a mean reverting process cannot be a complete description of stock price behaviour" (p.61)

More recently some authors have further questioned the validity of the conclusions drawn by Poterba and Summers. There has been a move to re-examine

[13] *Real* returns are returns after inflation (using the CPI as the deflator), and *excess* returns are those after deflation by the risk free rate (in our case the 90 day T-note).

some of the claims made. Chow and Denning (1993), extend the Lo and MacKinlay approach to generate a statistic to deal with *joint* hypothesis testing, by defining a joint confidnce interval for the $VR(k)$ (p.389-390). Put simply they feel that instead of simply rejecting the random walk hypothesis (H_0) when *any* of the $VR(k)$ is significantly different from 1, we must only reject H_0 when the $VR(k)$ are jointly different from 1, "However when H_0 is rejected further information concerning whether the individual variance ratios or all ratios are different from one is desirable" (note 1, p.386). To not consider joint significance would be selective bias or 'data snooping' as they term it. The net effect would be to significantly widen the confidence interval, thus making significant variance ratios no longer so. Poon (1995), referring to the work published by Chow and Denning (1993), writes that "... many previous studies that employed the variance ratio (VR) tests of the random walk hypothesis have typically calculated multiple sets of VR estimates which should have been tested jointly." (p.2) Interestingly, Poon in the Appendices (Fig. 1(b)) using aggregate data (the Financial Times All Share Index for the UK) does show ratios initially rising then declining to values well below one over a time horizon of up to ten years.

In this context it is perhaps worth reviewing Poterba and Summers original arguments. They took the view that there exists a trade-off between Type I and Type II errors. In effect they maintain there is no reason to have such a strong attachment to the random walk hypothesis, and using classical inference is both conservative and biased against alternatives, given the problems that arise in financial time series (insufficient data, rational and irrational behaviour and so on). One would like to see, if mean reversion exists, a stronger rejection of the null at longer lags. We do not necessarily get this. But then the confidence interval is wider. However to the extent we have rejection of the null at longer lags but not shorter ones is quite consistent with 'stronger rejection'. This then is not necessarily viewed as 'data snooping' but a reflection of the sheer size of the variability in the series.

In summary, the difficulties arise because of the size of the effect we are trying to measure relative to the size of the variability in the series as a whole. Herein, the view is not taken that there is one definitive 'clincher'. Strong criteria, such as applied to physical sciences will always present difficulties of proof, one way or the other. So we must present supporting information from other areas to allow us to build a case. We must also be aware that there is a penalty in that, if we incorrectly accept the alternative we will incur unnecessary costs and underperform. Obviously incorrectly accepting the random walk null would also incur very significant underperformance costs. We do not know which alternative is the more expensive but we do need to bear these facts in mind when we come to our conclusion, since the most appropriate strategy for an individual client will depend very much on total payoffs.

Even if we allow for the weaker testing criteria, other points from Poon are valid. Firstly, given positive short run autocorrelation the denominator in the expression for $VR(k)$ (given at the start of this section), being a 12 month number will be higher than it would be if the returns were independent. Thus the longer term variance ratios will be lower, even if there is in reality no negative autocorrelation. Secondly, we must allow for heteroscedasticity in returns (which the heteroscedasticity-robust statistic, given in Lo and MacKinlay (1988) does). However, we do have a good sample size, even though one can argue that changes over time mean that the generating process has changed. We cannot do much about that.

What is comforting is that at the aggregate level (rather than at the sub-index or individual firm level where micro-economic factors can strongly come into play), we see the same kinds of variance ratio patterns, typically hump-shaped, *even if there are serious problems with the degree of significance*.

The results for the variance ratio tests using annual data were as follows:-

Table 3.2 Variance Ratio Tests: Annual Data[14] Various Periods for AOIA

variance ratio test results - varying time periods and data series annuals							
time lag years	2	3	4	5	6	7	8
nominal-1928-94	1.0933	1.0077	0.8673	0.8617	0.8092	0.7144	0.6490
nominal-1884-94	1.1092	1.0231	0.9401	0.9568	0.9367	0.8816	0.8429
excess-1960-94	1.0231	0.8701	0.6945	0.6772	0.5960	0.4461	0.3155
real-1928-94	1.0966	0.9988	0.8736	0.8805	0.8255	0.7158	0.6120
real-1884-94	1.1260	1.0468	0.9442	0.9416	0.8919	0.7964	0.7166
standard error	0.0976	0.1459	0.1824	0.2113	0.2352	0.2565	0.2762

As we can see above the annual data exhibits positive autocorrelation in the short term, though generally not significant given the size of the standard error. The nominal data is showing fairly weak negative autocorrelation over the longer term for the entire period 1884-1994, but much stronger levels over the shorter period, that is from 1928-1994. Both periods include the 1930's, but it is much more important in the shorter series. This thus confirms the importance of this period in the overall result.

The situation for the series in real terms is of a stronger level of negative autocorrelation for the longer period but a very similar result for the shorter term. Again we need to be careful about our conclusions, given that the size of the standard error means a value of 0.45 for the variance ratio at year 8 is required to be statistically significant. Probably the more interesting result, albeit on a very short time period, is the result for the excess returns. This shows an extremely low value by year 8. However given the shortness of the series and any potential small sample bias we must look to the monthly data to be able to draw firm conclusions. As an aside, one of the difficulties in coming to any definitive conclusions about such a long term phenomenon (if it exists) is that the situation continually changes. For example, the

[14] The variance ratio formula given above is modified for annual data by making the denominator simply $\mathrm{var}(r_t)$.

changes in the Australian domestic markets over the last 20 years have been enormous. In many cases therefore, particularly the other major asset classes, we only really have meaningful data from the early 1970's or indeed from the floating of the A$ in 1983.

Thus when we consider mean reversion, given that in an individual country we only have one share price index (and those only where a market economy and developed capital market has been in existence for a substantial time), we need to be able to draw samples elsewhere from markets that are not highly correlated with each other. The AOI and the US S&P 500 are of course highly correlated. Thus we must look at other asset classes. In particular, if the cause of the suspected mean reversion is indeed the discount rate effect, as suggested by Fama and French (1988a), then clearly it will affect all the other asset classes (via valuation affects as well as representing alternative asset classes for capital flows).

3.5 Monthly Results

The results of the variance ratio test for the monthly series are as per Table 3.3. We can see similar results to the annual series. Once again there is consistent positive autocorrelation in the short term with negative correlation in the longer term. Again we see the strong result for excess returns. Of additional interest are the results for the other asset classes. The CBBI All Maturities series exhibits strong negative correlation over the 72 month period (at a similar level to the excess returns) and positive autocorrelation in the short run. It does however show a much greater degree of persistence in that it takes 36 months until the variance ratio falls below unity.

The MSCII series (which starts on 31/12/69) exhibits strong positive autocorrelation out as far as 72 months. This is however an aggregate series representing a global index adjusted back into $A. Furthermore the $A was not a floating currency prior to 1983. Unlike the more domestically oriented series which are correcting in the internal economy, the currency has shown a steady downtrend, certainly in trade weighted terms. It does however explain why domestic (that is $A institutions) have such a heavy offshore weighting in their efficient frontiers.

Table 3.3 Variance Ratio Tests: Monthly Data Various Periods for Key Asset Classes

time lag months	1	2	3	4	5	6	7	8	9	10	11	12	24	36	48	60	72
nominal-1959-94	0.8558	0.9307	0.9353	0.9579	0.9491	0.9402	0.9281	0.9221	0.9332	0.9570	0.9777	1.0000	0.9316	0.8127	0.7518	0.7607	0.7269
nominal-1928-94	0.7471	0.8270	0.8510	0.8711	0.8903	0.8933	0.8959	0.9044	0.9231	0.9511	0.9754	1.0000	0.9669	0.8064	0.7125	0.7047	0.6717
nominal-1884-94	0.7891	0.8613	0.8784	0.8974	0.9122	0.9111	0.9124	0.9199	0.9351	0.9585	0.9787	1.0000	0.9824	0.8694	0.8041	0.7917	0.7526
excess-1959-94	0.8635	0.9394	0.9432	0.9484	0.9568	0.9464	0.9326	0.9253	0.9354	0.9585	0.9784	1.0000	0.9067	0.7366	0.6247	0.5811	0.5132
real-1948-94	0.7150	0.8012	0.8288	0.8515	0.8749	0.8848	0.8932	0.9049	0.9265	0.9548	0.9779	1.0000	0.9715	0.8638	0.7868	0.7474	0.7070
real-1959-94	0.8213	0.9015	0.9120	0.9194	0.9307	0.9263	0.9197	0.9184	0.9320	0.9569	0.9778	1.0000	0.9464	0.8272	0.7531	0.7163	0.6778
cbbi all mats	0.6474	0.7255	0.7559	0.8043	0.8427	0.8431	0.8468	0.8823	0.9062	0.9446	0.9745	1.0000	1.0238	1.0005	0.8303	0.6259	0.5147
mscii x Aust in $A	0.7423	0.7909	0.8064	0.8208	0.8245	0.8508	0.8615	0.8870	0.9157	0.9488	0.9825	1.0000	1.1078	1.2066	1.2671	1.2620	1.1226
mscii x Aust in $US	0.8878	0.9527	0.9504	0.9669	0.9696	0.9911	0.9794	0.9715	0.9650	0.9709	0.9883	1.0000	1.0268	1.0516	1.1237	1.0887	0.9884
A$/US$ rate	0.9877	0.9752	1.0327	1.0400	0.9990	0.9506	0.9286	0.9175	0.9274	0.9516	0.9764	1.0000	1.0642	1.0929	0.9713	0.7904	0.6277

CBBI all maturities is representative of the fixed interest asset class and is the appropriate “benchmark” for that class in, for example, performance monitoring. Likewise the Morgan Stanley Capital International Index (MSCII) in $A is the benchmark return for international equities for an Australian investor. The variance ratio tests are as in Poterba and Summers (1988) .That is the ratio of the variances being successive variance of sums of returns divided by the variance of the 12 month returns :-

$$VR(k)=\frac{\mathbf{var}\left(R_t^k\right)/k}{\mathbf{var}\left(R_t^{12}\right)/12} \qquad \text{where } R_t^k=\sum_{i=0}^{k-1} r_{t-i} \text{ and } r_t \text{ is the return at time } t\,.$$

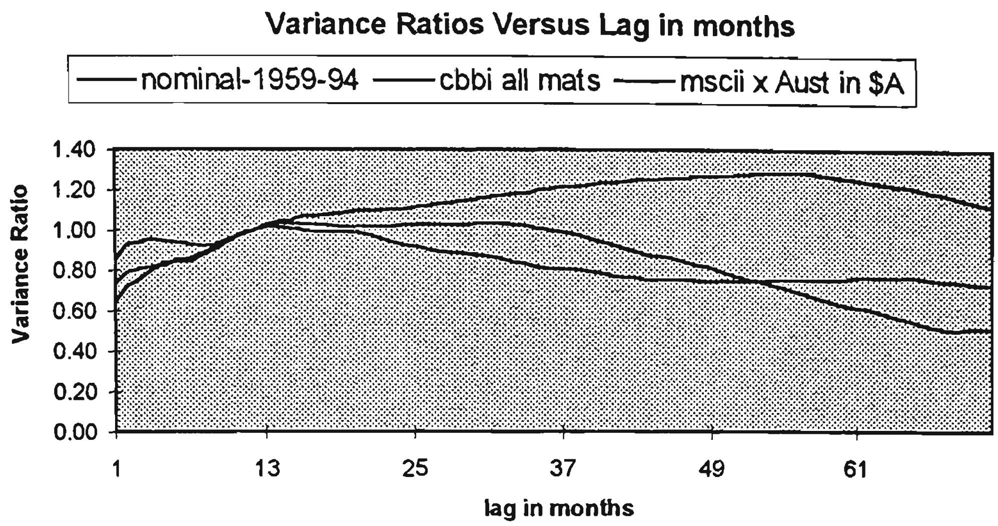

Figure 3.8 Plot of Variance Ratios Monthly for Key Asset Classes

Perhaps the most general conclusion one can draw is that there does appear to be at first positive autocorrelation (asset class prices are subject to trends) followed by negative correlation over the long run. We must however, incorporate the results from our correlation studies above, that is, that only the first-order autocorrelations for the stock series are significant. In aggregate the CBBI exhibits positive correlation, only the MSCII being a long way from significance. This is quite consistent with the level of confidence we see in the variance ratio series. If we apply the level of standard deviations determined in Poterba and Summers, Table 4, p.40-41, then the values determined herein for the CBBI and MSCII are not significant. In all cases, though, the prices appear to rise to a peak followed by a slow decline towards a value well below unity, that is showing a typical hump shape as seen by other authors. We can view this graphically by plotting time against the variance ratio, as in Figure 3.8.

It is to be noted that the data series chosen are "benchmarks" for the major asset classes available for investment. This will be of value later when we will consider the actual performance of institutional fund managers in Australia based upon their policy portfolios and use of tactical asset allocation.

The next issue to deal with is the cross correlations between each of the three benchmark series, that is the correlations between the series at various lags. This will both help us with mean reversion, by assessing the degree of independence between the asset classes, as well as providing an insight into the correlation matrix used in finding the efficient frontier. The *cross autocorrelation* at lag k is defined by:-

$$\rho_{u,z}(k) = \frac{\operatorname{cov}(u_t, z_{t-k})}{\{\operatorname{var}(u_t).\operatorname{var}(z_{t-k})\}^{1/2}}, \quad k = 0, \pm 1, \pm 2, \ldots\ldots,$$

This is a logical extension of the autocorrelation function. In general it is not symmetric, that is $\rho_{u,z}(k) \neq \rho_{u,z}(-k)$.

The first cross-correlogram Figure 3.9 below correlates bonds with domestic equities.

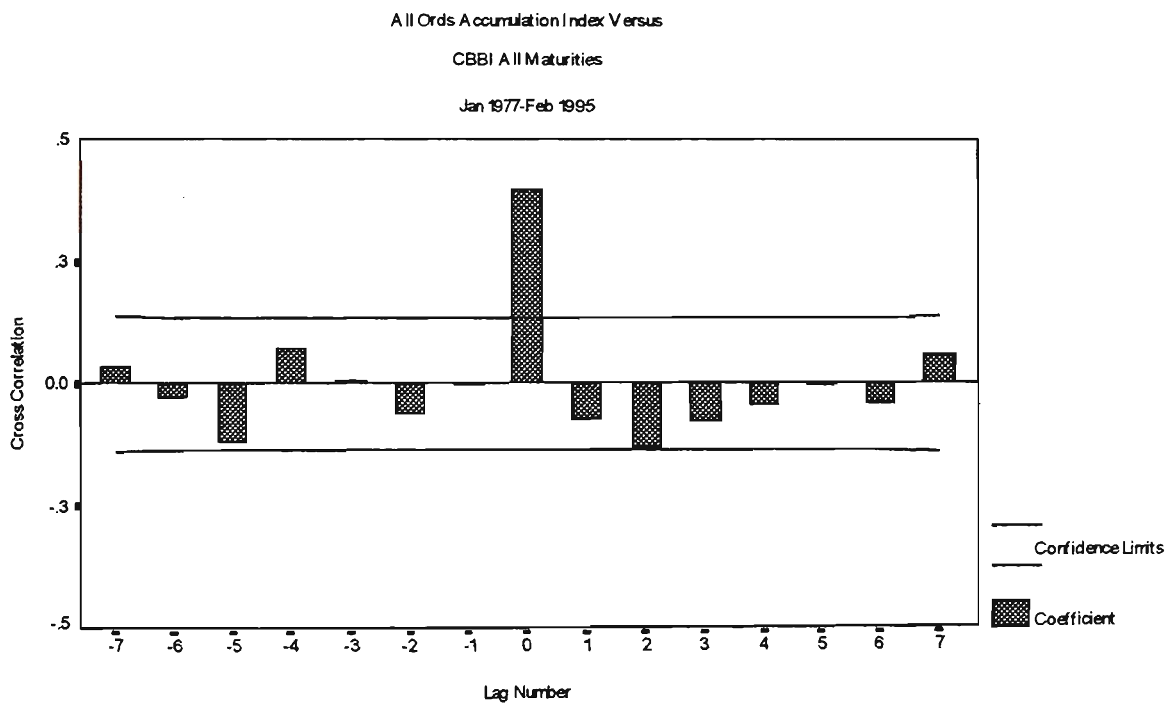

Figure 3.9 Cross correlogram AOIA versus CBBI 1977-95

As can be seen from Figure 3.9, the only significant correlation is at lag 0 with a value of 0.399 and corresponding standard error of 0.034. This is not surprising since an increase in the discount rate should lower the value of equities (via the discount rate in the dividend discount model or through a contraction in the price earnings ratio using a more market oriented approach). Whilst this is a short run effect it is perhaps not unexpected then that the series should show similar longer term features, though this doesn't prove the discount rate effect is the cause.

The next cross-correlogram matches the CBBI with the MSCII as in Figure 3.10. As can be seen there is no significant correlation at all. The value at lag 0 is 0.065 which is the value to be input to the correlation matrix.

The final cross-correlogram, Figure 3.11, relates the AOI Accumulation index to the MSCII. Here we can see significant correlations at both lag 0 and lag 1. Again this is not unreasonable due to the importance of the US market in the overall MSCII and the correlation between the domestic market and the S&P 500. Indeed most Western markets are highly correlated, markets like Japan far less so. Perhaps more of a surprise is the value at lag 1 where we find a value of 0.224 as compared with 0.304 at lag 0. This result indicates that it takes a certain time for the movement on Wall Street to translate itself to Australia (that certainly reflects local wisdom). Alternatively perhaps it takes overseas players a while to appreciate the fact, they are the most important investors in moving the local market one way or another.

Of even more interest is whether or not this observation is incorporated in the correlation matrix used in determining the efficient frontier.

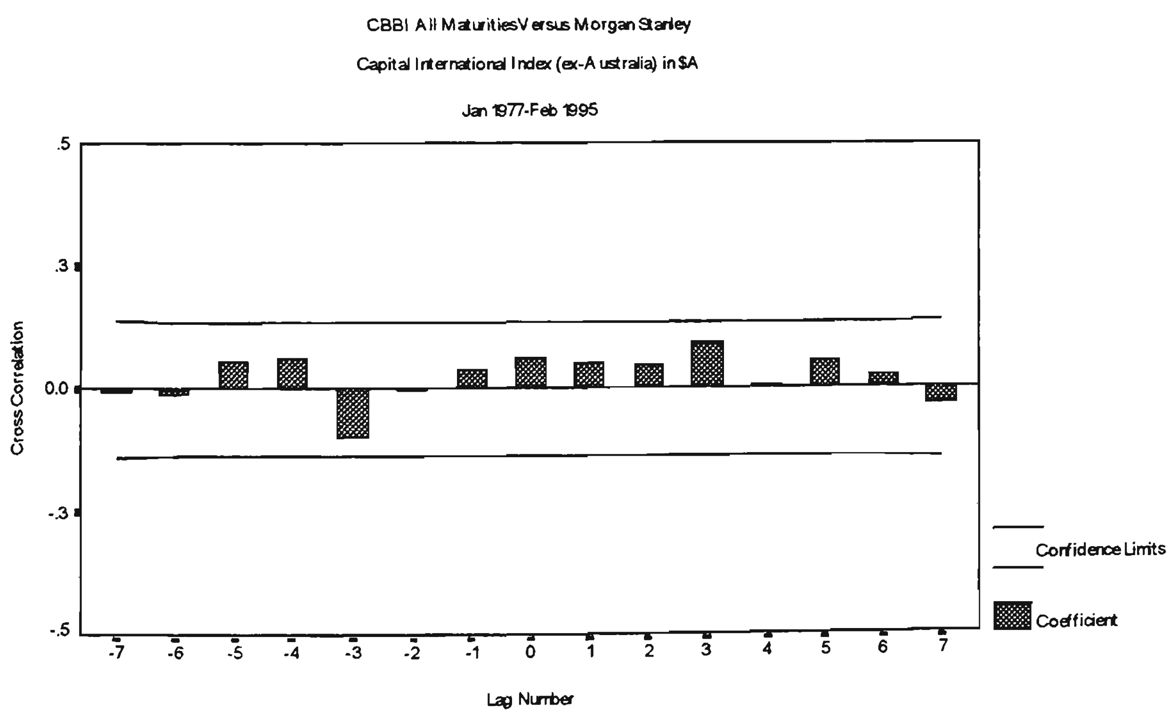

Figure 3.10 Cross correlogram MSCII versus CBBI 1977-95

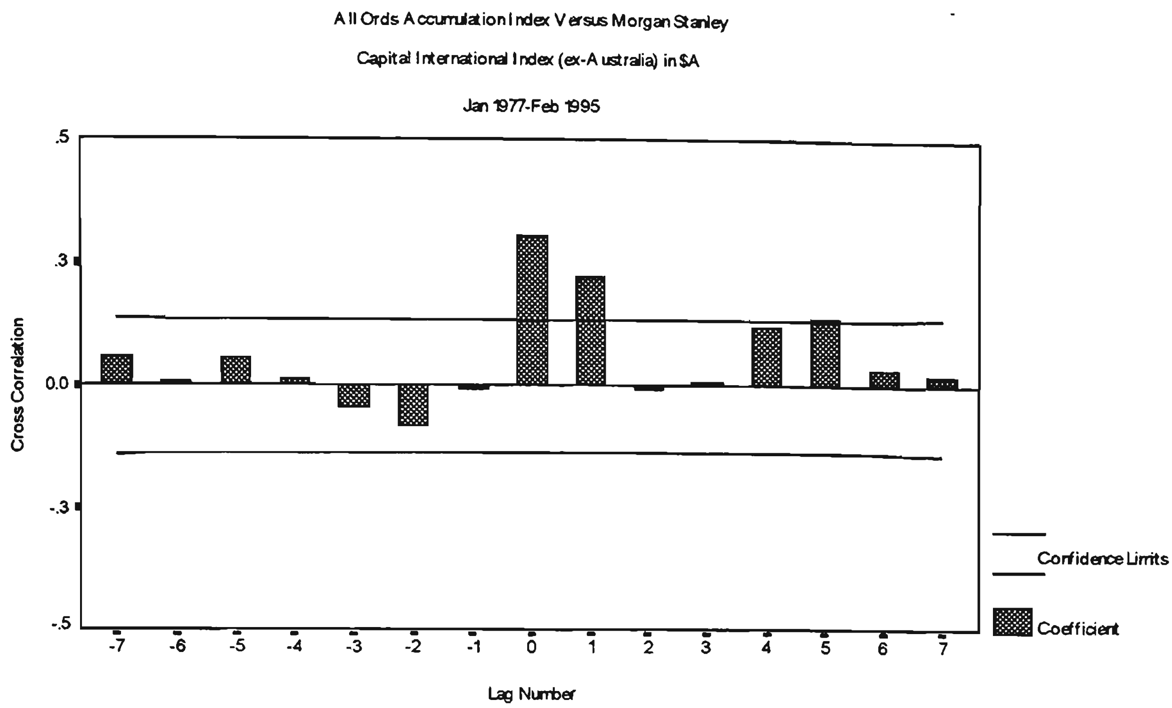

Figure 3.11 Cross correlogram AOIA versus MSCII 1977-95

3.6 Validity of the Results :A Sign Test

One of the constant problems in mean reversion is the ability statistically to differentiate between a null hypothesis of a random walk and long range dependence. Much of the literature revolves around finding a suitable test, determining its power against the null and then setting about an empirical examination. Lo (1991) is another attempt to do this, this time by using the "range over standard deviation" or R/S statistic first developed by Hurst (1951) where he developed an approach to examine dam capacities on the river Nile. Interestingly here Lo finds no evidence for the mean reversion alternative hypothesis once short run dependence is taken into account.

The approach here is twofold. Firstly, the cross correlations indicate that the asset classes considered have low correlations, though these vary between the various classes, and we have seen very similar results. Hence we have extra supporting samples. Secondly, another way to consider the issue is to examine more closely the variance ratio and see whether or not it is monotonically increasing or decreasing over its range; Figure 3.8 displays this graphically. On the basis of the null hypothesis of a random walk we would expect the *change* in the variance ratio to be as likely to be up as down. That is the probability of a positive difference would be the same as a negative difference at a 1/2. (This avoids any problems associated with finding the distribution of the changes in the variance ratio, assuming a random walk in prices.) Hence we need to consider "runs" of values. This was done with the following interesting results (note a '+' = '1' and a '-' = '0'):-

Table 3.4 Changes in Variance Ratio Showing 'Runs' Key Asset Classes

All Ords Accum	CBBI	MSCII
3 x 1	13 x 1*	20 x 1
4 x 0	2 x 0	1 x 0
6 x 1*	1 x 1	33 x 1*
38 x 0	2 x 0	17 x 0
5 x 1	1 x 1	
2 x 0	2 x 0	
5 x 1	4 x 1	
8 x 0	1 x 0	
	4 x 1	
	38 x 0	
	3 x 1	

* Maximum reached at the end of this particular 'run'.

We may, of course use the normal binomial tests to see whether or not we would find such a result by chance, but a quick perusal shows that we have extremely long runs in all cases, runs of lengths 38, 38 and 33 are highly unlikely. We may indeed see that the ratios achieve a clear maximum (remembering that numerically before 12 months a value below 1 means positive autocorrelation) before slowly declining away. In the case of the MSCII even given the long decline the ratio is still above 1.

We may calculate the respective probabilities of a rise prior to the maximum and a fall thereafter, from Table 3.4. For example, with the 'All Ords Accum' column there are 3+4+6 = 13 changes in the variance ratio, of which 3+6 = 9, represent increases, prior to the maximum. Thus the probability of a rise before the maximum is simply 9/13 = 0.69. We may tabulate the calculations for the three key asset classes in the following table :-

Table 3.5 Probabilities of a Rise Before Maximum and Fall Thereafter

	All Ords Accum	CBBI	MSCII
BEFORE MAX	9/13 = 0.69	13/13 = 1*	53/54 = 0.98*
AFTER MAX	48/58 = 0.83*	45/58 = 0.78*	17/17 = 1*

Significant[15] at the 1% level.

These probabilities are much greater than the expected value of 1/2. We therefore have further support for the mean reversion hypothesis. Thus on this basis we will proceed to the issue of model building, with an attempt to discover what kind of process could lead to the results actually found, that is, as if mean reversion was a fact.

3.7 Conclusion

We have examined in some detail in this chapter, our base series where we looked at the autocorrelation structure of the individual series. We reviewed both short term dependence and long term dependence, via the variance ratio tests, including the significance or otherwise of the results. We have also examined the links between the series via the cross correlograms. These results and the main conclusions to be drawn will be reviewed later on, when we bring together the next two chapters into the final concluding chapter.

[15] Based upon a simple proportion test. The null is $p = q = \frac{1}{2}$ versus the alternative $p > \frac{1}{2}$ and the standard error is $\sigma_p = \sqrt{\frac{pq}{n}}$ and using the t distribution.

Chapter 4: Mean Reversion Models and the Distribution of Prices

4.1 Introduction

In the previous chapter we considered the structure of our series. We must next look at possible models to help explain the behaviour of the series, and in particular understand the kind of process which has long memory characteristics. The empirical modelling, in 4.2 - 4.4, is an attempt at understanding some of the issues of significance testing raised in the discussion in 3.3. Finally we look at some of the assumptions that are part of the mean variance optimization process covered later in 5.8, such as constant mean and variance, and the normality of the distribution of asset prices.

4.2 ARIMA and Integrated Linear Models

There are, of course a very large range of possible stochastic models which are candidates to display the kind of variance ratios actually observed. In so doing we need to recognise:-

1. Whatever tests we actually use, be it variance ratios, range to standard deviations or some form of cumulative sum, they are all variants on a theme. There are limits to what the various "resolutions" of the series can add (Summers (1986), gives an excellent discussion on this topic). Nevertheless different tests will have different powers against the particular null hypothesis under consideration. Therefore this document has concentrated on the variance ratio, on the basis of the degrees of equivalence, between the approaches.

2. Given the variance ratio we can clearly work backwards to determine the underlying correlation structure and hence attempt to consider what model form could generate such a picture. That is, can the ARIMA class of linear models actually

generate the observed variance ratios. If so then we can put it into a Box-Jenkins style format, using lag operators.

We may determine the autocorrelations from a given variance ratio (see the Appendix for a derivation of the formula) and therefore we can examine whether a given model form is possible.

Under the principle of "parsimonious parametrisation" we seek an ARIMA(p,d,q) where generally $d \leq 2$ and $(p + q) \leq 2$ where p, d and q are integers. We are looking for a representation which can give small negative autocorrelations at long lags, implied by mean reversion. Let us consider each of the possibilities in turn (where d is given as 1). Graphical representation of these models and their features are shown in Box and Jenkins (1970) on the pages indicated.

ARMA(0,1): ARMA(0,2). The autocorrelation function cuts out for an MA(q) at lag q. Hence this is not a possible representation, since mean reversion implies small negative autocorrelations at long lags.

ARMA(1,0): ARMA(2,0). For the AR(1) the autocorrelogram is a declining exponential series (either all positive or else alternate signs). The AR(2) is more complex (refer p.59 Box and Jenkins (1970)) with damped exponentials and/or damped sine waves. Within the *region of stationarity*[1], however, it is evident that where there is an element of periodicity (and hence the possibility of positive then negative autocorrelation) that the autocorrelogram oscillates too fast and damps down far too soon. In either case it will not give us our observed variance ratios.

ARMA(1,1). In this last case (refer p.78 Box and Jenkins (1970)) as a mixture of the above, the function decays exponentially from the first autocorrelation. It is

[1] A stochastic process is called strictly stationary if its properties are unaffected by a change of time origin. Hence, in particular, the process has a constant mean and variance and so does not 'explode'. This condition places restrictions on possible parameters for a model, thus defining a feasible region.

evident that no combination, within the stationarity region, will give us what we require. The best possibility is $0 \leq \theta \leq 1, \theta > \phi$, θ being the moving average parameter and ϕ being the corresponding autoregressive parameter. Even there the cut-off is too sharp and the decay too rapid.

We may indeed extend our arguments to the more general model, ARMA(p,q), after the logic put forward by Granger and Joyeux (1980). Firstly we recognise that in any general ARMA model the moving average contribution to the autocorrelations will cut out at lag q. Thus eventually, given that q is not excessively large, the AR portion will provide the higher order autocorrelations and these will be declining exponentially, then, using the recurrence relation for the autocorrelation function:-

$$\rho_k = \phi_1 \rho_{k-1} + \phi_2 \rho_{k-2} \ldots\ldots + \phi_p \rho_{k-p} \qquad k \geq q+1$$

and $\quad \rho_k \cong C\phi^k \quad |\phi| < 1$

and since all the ϕ cannot be close to 1, this term must decline rapidly.

Because the autocorrelations are exponentially declining they are far too small to be able to represent a long-memory process where the autocorrelations, though small are still significant (or else they could not generate mean reversion). This argument is given more merit in the following section where we will consider a fractional differenced representation to model the long-memory process. Thus no model of these forms will give us the variance ratios observed[2].

[2] In a recent draft paper, Zhou (1996) tackles this very issue. He puts forward a 'state space' model introducing another underlying state variable to modify the autocorrelated (AR(1)) transitory component. This is similar to the 'attack' taken in the empirical approach following. The empirical study is meant to exhibit more 'naturalness', in that it is based upon an interpretation of market behaviour.

Let us now consider the situation of a decomposition of the series into random walk and transitory components. As is outlined by Box and Jenkins (Chapter 4, *Time Series Analysis* and Appendices) the sum of two independent moving average processes is another moving average process. Furthermore, in the general case the sum of two independent ARIMA processes is itself an ARIMA process (with certain bounds on the parameters)- see Anderson *Time Series Analysis and Forecasting* (1976) Chapter 14.

Now a random walk can be considered as an IMA(0,1,1), that is of the form:-

$\nabla z_t = (1\text{-B})z_t = (1\text{-}\theta\,\text{B})a_t$ where a_t are "white noise" and $\theta = 0$.

Thus describing a long memory process as a random walk with a transitory component, would imply that the transitory component cannot be a simple additive ARIMA linear model.[3] If the combined series is not of ARIMA form, then we cannot decompose it into two components both of which are.

The above specifications assume that the processes are independent, which would seem unlikely in practice. That is we may expect that there is an interaction between the random walk component and any mean reverting portion. For example if the effect of random moves were to take prices to an extreme level then the mean reverting component would come in[4]. Thus there would be a degree of dependence in the components, such that even if the component series were ARIMA there is no reason why the aggregate should be. In a more general sense again, Granger suggests "

[3] It is interesting to note that in a comparison of the ARIMA approach with Brown's quadratic forecasting method for IBM stock prices Box and Jenkins, p.166-170, find an IMA(0,1,1) gives the best forecast and for a value of $\theta = 0.1$, of course, close to a random walk. These are short - term forecasts.

[4] Most recently I have become aware of a paper by Bleaney and Mizen (March 1996), wherein they discuss a modelling process applied to exchange rate dynamics. They suggest a cubic rather than linear model for mean reversion, reflecting "..... the intuition that the rate of mean reversion increases with distance from equilibrium"(p.36).

...that integrated processes, and long- memory relationships are likely to occur from aggregation of dynamic models, it should be pointed out that they by no means necessarily arise " (Granger (1980) p.238).

Finally, we can consider two other possibilities, that of a fractionally differenced stochastic model or some form of deterministic drift or trend. It is recognised that there may be an issue as to the economic basis for fractionally differenced models. Granger spends considerable time considering where they may arise. The fractionally differenced models do have the kinds of long-memory properties that are desirable in any mean reversion study. Furthermore, as a linear model they do provide completeness to our considerations of the general ARIMA class and therefore have merit on that basis. It is also true that L, the lag operator, is an operator like D the differential operator or the integral operator, and so on. As such it obeys the laws of algebra (commutative law, associative law etc), thus the extension of the operator L to fractions also follows in a similar manner to that applying for other linear operators.

In the case of 'drift' we do not expect the transient term to be deterministic but we should consider it for completeness. Indeed, as is commented upon by Granger and Terasvirta (1993) "Observed series are thus *not* simply signal plus noise, where noise is just measurement error which can perhaps be filtered out. Actual series may contain deterministic components, such as cycles and trends, and this possibility will have to be considered in any analysis" (p.2).

1. **Fractionally Differenced**. These are models of the form $(1-L)^d x_t = \varepsilon_t$, where d is some fraction, say 1/4 (see Granger & Joyeux (1980) or Lo (1991)). Thus using the lag operator approach we may expand the expression $(1-L)^d x_t = \varepsilon_t$, and using the binomial expansion pick out the autoregressive / moving average coefficients (Lo, p.1285), that is the $AR(\infty)$ and $MA(\infty)$ representations. We may further determine the autocorrelations and hence calculate the variance ratios. This is done by way of example for $d = +0.1$ and -0.1.

We may use the formula from Granger and Joyeux (1980) :-

$$\rho_k = \frac{\Gamma(1-d)}{\Gamma(d)} \frac{\Gamma(k+d)}{\Gamma(k+1+d)} \quad \text{for } d < 1/2 \text{ and } \rho_k = 0 \;\; k > 0 \text{ if } d = 0$$

and noting that $\Gamma(-d) = -\frac{1}{d}\Gamma(1-d)$.

Now we may use Stirling's approximation for large k ie

$$\Gamma(n+1) = n! = \sqrt{(2\pi)}e^{-n}n^{n+\frac{1}{2}}$$

to deduce the autocorrelations:-

$$\rho_k = \frac{\Gamma(1-d)}{\Gamma(d)} \frac{\Gamma(k+d)}{\Gamma(k+1+d)} = C(d)k^{-(1+d)}$$

and hence we see the autocorrelations are declining much more slowly than exponentially.

Thus further we arrive at the following table (at lags 1- 6,12,24,36,48,60 and 72) :-

Table 4.1 Autocorrelation Function and Variance Ratios for Fractionally Differenced Models: d = ± 0.1

Lag k	$\mathrm{MA}(\infty)$	ρ_k ($d=$ -0.1)	$VR(k)$	$\mathrm{AR}(\infty)$	ρ_k ($d=$ +0.1)	$VR(k)$
1	0.100	-0.091	1.500	-0.100	0.111	0.647
2	0.055	-0.039	1.364	-0.045	0.064	0.719
3	0.038	-0.024	1.279	-0.028	0.047	0.771
4	0.030	-0.017	1.219	-0.021	0.037	0.812
5	0.024	-0.013	1.173	-0.016	0.031	0.846
6	0.021	-0.010	1.136	-0.013	0.027	0.875
12	0.011	-0.005	1.000	-0.006	0.015	1.000
24	0.006	-0.002	0.876	-0.003	0.009	1.146
36	0.004	-0.001	0.810	-0.002	0.006	1.242
48	0.003	-0.001	0.766	-0.001	0.005	1.315
60	0.003	-0.001	0.733	-0.001	0.004	1.375
72	0.002	-0.001	0.707	-0.001	0.004	1.425

As we can see the model shows long range dependence due to the slowly declining autocorrelations. We may compare this with the results observed from the All ordinaries Index nominal 1959-94 series. The ratio at lag 1 was 0.8558 and 0.7269 at lag 72. The differenced series $d=$ -0.1 , represents a good candidate at long lags giving a ratio of 0.707. To get a ratio at lag 1 of 0.8558 requires a value of d of around +0.04. The CBBI and MSCII series would require different values of d again to get a fit.

Of course, the variance ratios show clear maxima followed by declining ratios. To obtain this behaviour we would have to consider some form of mixed or integrated model. We must then either consider d as itself a stochastic process, or else use some combination of the linear filters.

If we use successive linear filters then a primary application of the autoregressive model will be largely cancelled out by the following moving average. That is, if we apply a filter to z_t to form $x_t = (1-B)^{-d'_1} z_t$,and the x_t to form y_t , where $y_t = (1-B)^{-d'_2} x_t = (1-B)^{-(d_1+d_2)} z_t$. In our case values of d_1 of +0.1 and d_2 of -0.1 will cancel each other out leaving a white noise series.

2. **Deterministic or Stochastic Drift** When we consider the plot of the variance ratio series, *VR* against time, and its' derivative ∇VR then it would appear that a cubic would fit well *VR* and correspondingly a quadratic would fit ∇VR. On this basis, we are able to determine the theoretical ρ_k and hence find the appropriate model in lag operator form.

Not surprisingly, the model form for a cubic would need fourth-differencing to achieve stationarity or $\nabla^4 z_t$, with a moving average component being a cubic. Most economic models only need differencing at most twice to achieve stationarity. Indeed over-differencing leads to a rapid increase in variance in the series. This is, of course, the standard technique for helping to decide the degree of differencing needed for a series known as the 'variate difference' method. The essential idea is that, if a series consists of a polynomial plus random component then successive differencing will give us estimates of the variance of the reduced order polynomial plus random component. Once the polynomial is eliminated, we are left with the variance of the random term, which should remain the same even after further differencing, that is, it will settle down. (see Kendall *Time Series* p.47-52). For interest this was done for the All Ords 1959-95 series as follows.

Table 4.2 Application of the Variate Difference Method to the All Ordinaries Series 1959-95

differencing		∇z_t	$\nabla^2 z_t$	$\nabla^3 z_t$	$\nabla^4 z_t$
variance	4713467	33520	68826	207826	697029
variate difference		16760	11471	10391	9958

where the variate difference is $V_d = \dfrac{\operatorname{var}\left(\nabla^d z_t\right)}{\binom{2d}{d}}$, see Kendall p.47.

The minimum variance is achieved on the first differencing and the variate difference has 'settled down', thus indicating that only one differencing is necessary. Hence we can eliminate the possibility of a polynomial trend. Further from an economic viewpoint it would be hard to believe that the trend factor is deterministic.

We may however logically extend the above to a determination of the optimal degree of differencing[5] if we allow for non-integer or fractional differencing. Using the binomial expansion we can pick out the moving average coefficients to apply to the All Ords series. If $(1-L)^d = \sum_{k=0}^{\infty} a_k L^k$ then (again using the gamma function expression for the binomial coefficients):-

$$a_k = \frac{\Gamma(k-d)}{\Gamma(-d)\Gamma(k+1)} = -\frac{d(1-d)\Gamma(k-d+1)}{(k-d)\Gamma(2-d)\Gamma(k+1)}$$

using the relation $\Gamma(n) = (n-1)\Gamma(n-1)$, to ensure a positive gamma function in the optimal range. In effect, the difference operator was applied to successive lags in the series where it was found that the coefficients, a_k, died out very quickly. It was found that values of a_k for values of $k > 5$ were insignificant. A grid search[6] of the above was carried out, revealing a minimum at $d = 1.005$. The function was very smooth with the minimum variance being very close to the value at the optimum.

[5] I have recently been made aware of further work in this area from a seminar given by N. Olekalns entitled "Fractional Integration and Non-Parametric Measures of Persistence" at the University of Melbourne, Feb 1996.

[6] This approach has not been observed elsewhere by the author. However given that this aspect of econometrics is not central to this thesis, it was felt that exploring techniques for finding optimal differencing would detract from the focus of market timing and asset allocation. Clearly, this is not to suggest that such an activity would not be extremely rewarding, particularly in the context of the wider economic issues at stake.

This result is by no means surprising, as it underscores the fact that in the above section we saw that the first half of the series with positive autocorrelation approximated with a small negative value of d. Then the second half with negative autocorrelation approximated with a small positive value of d. Hence given values of d of roughly the same size they obviously cancelled each other out. Since we were dealing with percentage changes, that is first differences, then an aggregate d of almost 1 is the net result.

This approach is related to the 'unit root' methodology for determining whether or not an economic time series is a random walk and hence whether 'shocks' persist. $d = 1$ is a random walk, so shocks persist; $d = 0$ is white noise so shocks die out straight away; in between the process is mean reverting - though this can take a long time. In the above we would not reject the null hypothesis of a unit root, but closer analysis shows why this is so; short run positive autocorrelation is hiding the long run negative autocorrelation. If we ignore the positive short run autocorrelation and fit the series as in Table 4.1 then we have an aggregate value of d of order 0.9 (that is an application of $d = 1$ followed by $d \cong -0.1$).

4.3 Model Building - Some Empirical Considerations

If we assume that in the short run asset price changes are random and approximately normally distributed then we need to consider the nature of a process which is very difficult to discern . That is the autocorrelations generated by any mean reverting process may well be small and insignificant on their own, but accumulate to a significant sum. Further, of course, these autocorrelations are at first positive then negative and presumably they must eventually disappear as the process completes the reversion.

Criteria which we would like a model to satisfy, based upon features observed in the data[7], would include :-

1. The more recent the trend then the more likely it is to continue. This is necessary to capture the positive autocorrelations in the mean reverting process. However the process itself is swamped by the random component. Put another way the variance of a component of the process in the short term is very small relative to the variance of the random component.

2. On reversal of the existing trend then the new underlying trend is likely to continue. This is necessary to explain the negative autocorrelations at longer lags. We can also postulate this kind of behaviour particularly in currency markets. It makes sense if one takes the point of view of market traders who are more likely to bet on the continuation of an existing trend.

3. The further that the cumulative returns deviate from the long run average return the more likely is a return to the mean on the assumption that the process is stationary. We need to consider cumulative returns because a large return on its own many not take the overall process too far from the mean.(whilst there is some evidence that large daily price changes are followed by large daily changes, see Neiderhoffer and Osborne (1966), as mentioned in Fama (1970), we are concerned with monthly changes at a minimum).

We may also posit that provided the deviations are not too far from the long run mean then the "pull" effect is likely to be very small to negligible put another way within broad limits we may say that the particular asset is neither particularly over-valued nor under-valued. This approach is similar to the ideas in Cootner (1964) where he outlines a model of stock behaviour with prices being a random walk within reflecting barriers. These reflecting barriers are formed when prices deviate too far

[7] Chapter 3 refers. We are designing here a process that will have mean reversion, and be plausible, then attempt to test for it. Hence we may better observe how well our tests perform.

from the mean and thereby bring in professional investors (for example the followers of Benjamin Graham[8] like Templeton Funds management[9]). Some kind of exponential function may be appropriate or indeed where say within one standard deviation of the mean the effect is zero. That is, if we standardise the returns by subtracting the mean and dividing by the standard deviation we can then form the cumulative sums and compare these to the one and two standard deviation limits.

Let us consider the progress of an asset price through time. At time t the return is r_t . Let us assume that $\left(r_t - \bar{r}\right)\rangle 0$ then Prob{ $\left(r_{t+1} - \bar{r}\right)\rangle 0$ }>0.5 but the value of the probability will decrease the longer the trend has persisted. In effect a reversal will occur with the probability falling below 0.5 and generating a value below the long run mean to which the process seeks to revert.

To complicate matters further we must take into account the overall level of the market as indicated by its cumulative deviation from the long run mean[10] (which we won't know until it happens) though we may use the current mean as a suitable proxy. If we assume { $\left(r_t - \bar{r}\right)/ s$ }<1.5, where s is the global standard deviation (see over 4.3), then this will have no impact at all. Only when the market deviates significantly will this effect cut in. We can postulate a function of the form $\exp\left\{\left[\left(r_t - \bar{r}\right)/ s\right] \pm 1\right\}$ such that, at say two and a half standard deviations from the mean, the probability of a large reversal converges on unity.

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[8] The former Professor of Finance at the University of California Graduate School of Business Administration who wrote widely on the principles of 'value investing' via fundamental analysis. In practice, value investors tend to be 'counter-cyclical'. See *Security Analysis Principles and Techniques* by Graham, Dodd and Cottle, Mc Graw Hill, 4th Ed. (1962).

[9] The management company formerly associated with Sir John Templeton, which still carries his name and adheres to the approach espoused by Graham.

[10] We assume that the process is stationary, and so has a long run mean level. This would then suppose that reversion would take place over a finite span, which is short relative to the observed series.

On the basis of the above considerations we see that any mean reverting process must be stochastic in nature as any moves made will be dependent on the level of the system at any time which is itself the product of a largely random process. It is only the very persistence of the underlying mean reverting process which is modifying this out-turn. A deterministic process would not allow for the effects of the likelihood of runs of large returns building into an over or under-valued situation. Put another way, there does not seem to be any particular periodicity to the returns so that the mean reverting behaviour may *eventually* occur but we have no idea how long it may actually take.

One of the difficulties all researchers have found is that any attempt to determine the mean of the process via a moving average will lead to a series whose autocorrelations will be a significantly declining series. Thus we find that the use of any ARIMA model will generate a correlogram which looks nothing like the ones observed. The closest class to this property of slowly declining, but nevertheless at no point significant autocorrelations, comes from the fractionally differenced models mentioned above.

4.4 Model Building

Given the above considerations, rather than attempt some *a priori* model it was thought best to borrow from quality control the ideas of cumulative sum techniques[11] (as outlined, say, in Woodward and Goldsmith (1964), one of the excellent series of ICI Monographs published during Kendalls period at ICI PLC). This constituted, on the assumption of a stationary process, standardising the deviations from the mean and plotting the cumulative sums. The story is as follows:-

[11] I have not seen this used elsewhere in finance. It is a good example of how techniques can be borrowed from one area and applied to new ones. Lo (1991), does much the same thing using Hurst's methodology originally applied to dam capacities.

Let $s_k = \left(\sum_{i=1}^{k} \left(r_i - \bar{r} \right) / s \right)$ where $s = \sqrt{\left(\frac{1}{(n-1)} \right) \sum_{i=1}^{n} \left(r_i - \bar{r} \right)^2}$ and $m = \bar{r} = \frac{1}{n} \sum_{i=1}^{n} r_i$

that is the s_k, the partial or cumulative sums and a global mean and standard deviation are used. We may then plot this 'CUSUM' and its two-standard deviation control limits with the result as in Figure 4.1. The use of the control lines gives us some feel for the extent of the deviation. Of course, by the very definition of the cusum, we assume a global mean, and hence a line to which the cusum must revert.

We may see from this periods of large departure from the long run mean, being 1974 and the period of high inflation through to the 1980 "resource boom". The swing the other way reached the upper limits in October 1987. Based upon our previous considerations it is of some interest to consider how important these long swings are in the overall mean reverting process. That is, can we consider mean reversion as just occurring because of the "bubbles"[12] in stock prices (also we must remember the importance of the 1930's, a period of deep price swing, on the overall existence or otherwise of mean reversion).

[12] The term "bubble" essentially means a very over-valued situation. Prices are driven to such a level that some, often relatively insignificant event, can trigger a collapse in prices. That is, bursting the "bubble". The origin likely goes back to the famous 'South Sea Bubble', a swindle driven by intense speculation during Walpole's Prime Ministership in England in 1720. We are using the term somewhat losely here, suggesting that deep market lows are 'bubbles' of depression. What these extremes do have in common though, are often 'irrational' behaviour, where prices are driven by intense gloom or euphoria. A central tenet of economics is 'rational' behaviour, perhaps this suggests that investors are as prone to crowd psychology as anyone else.

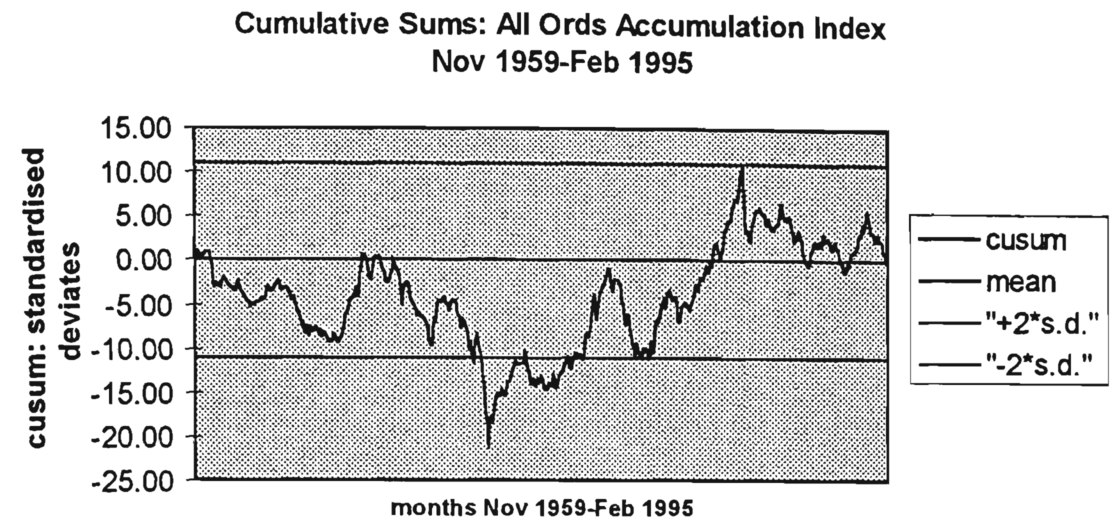

Figure 4.1: Plot of Cumulative Sums and 2 Standard Deviation Control Lines

If stock prices are mean reverting then the degree of over or under-valuation will be relative to a higher or lower mean. Put another way 1974 can be viewed as an extreme in pessimism against an overall weak period for stock prices. Likewise 1987 would be over exuberance in a generally buoyant period. Thus a moving average was used initially (48 months prior) to get a feel for this trend. Then the returns were standardised against this trend and using the global standard deviation determined against this trend to generate the cusums. In effect all we have done is replace a global mean with one which follows the trend, as follows:-

$$z_t = \frac{1}{s}\left(r_t - \frac{1}{48}\sum_{i=t-47}^{t} r_t\right) \quad \text{where } s \ = \text{global standard deviation}$$

This leads to the CUSUM's $s_k = \sum_{i=1}^{k} z_t$ and we test for $|s_k| \rangle 1.5s$.

This resulted in a set of outlier standardised variates, corresponding to the time periods 1974, 1980 and 1987. This is reasonable, based on our knowledge of events, and so they were accepted as being outliers. Interestingly, a look at the distribution of these events showed a exponential tendency (the mean and standard deviation were approximately equal). The CUSUM portion lying outside the $1.5s$ limits was eliminated, then by working backwards a set of adjusted returns were generated, as below:-

$$z'_t = (s'_k - s'_{k-1})s + m$$

$$\text{where } s'_k = s'_{k-1} + (s_k - s_{k-1}) - (|s_k| - 1.5s) \text{ for } |s_k|\rangle 1.5s$$

$$= s'_{k-1} + (s_k - s_{k-1}) \quad \text{otherwise}$$

The adjusted data was then tested using the variance ratio test. The net result showed that in the modified series over 72 months the variance ratio was 0.7382 versus a value of 0.7269 in the original series. And likewise throughout. Thus we can see that there is negligible impact from individual extreme events. One can therefore conclude that if mean reversion exists then it is the result of relatively small but much more powerful movements persisting over a long period[13]. This is itself not surprising when one considers the statistical process, since individual extreme events *do* affect first order autocorrelations as we saw in 3.2.2. The variance ratio, at lag 72, will incorporate the other 71 observations, and in effect smooth the 'peak' out.

4.5 An Empirical Study In Mean Reversion

In our review of economic considerations we considered the process of a bias in an existing trend. What we now seek to do is to mimic this. In our thinking we need to be mindful of the fact that positive autocorrelation will be generated between members of a random series where a moving average has been applied. This, the Slutsky-Yule effect (see Kendall *Time Series* Chapter 3), means that we can determine the autocorrelations of the derived series[14]. Thus if we seek low but positive

[13] If we take the Great Depression period it was not the fall in October 1929 which caused the real impact but the continuation of the bear market to its nadir in April 1932. In the US some issues had, by then, fallen to 10% of their peak price. Interestingly average prices in Australia fell by 38.3% over this period, less than the Oct 1987 fall, which surely gives that fall a clearer context.

[14] This result is similar to that pointed out by H.Working in a "Note on the Correlation of First Differences of Averages in a Random Chain" , Econometrica ,Vol.28, No.4, Oct.1960; pp.916-918, continued overleaf

autocorrelation then a model should incorporate such an average and such a component should be small.

We may use the result that for a moving average of order q, applied to a random series, with weights $\theta_1 ,......\theta_q$, then the k th autocorrelation of the series is given by:-

$$\rho_k = \frac{\sum_{j=1}^{q-k} \theta_j \theta_{j+k}}{\sum_{j=1}^{q} \theta_j^{\,2}} \quad , \quad 0 \le k \le q \ \text{ and } \ \rho_k = 0 \ , \ k > q$$

Thus , for example, a simple moving average of length 5, with equal weights $\theta_j = 1/5$, for all j , would have a series of linearly declining positive autocorrelations $4/5$, $3/5$,...... and so on with $\rho_5 = 0$.(Clearly, as the members of the series 5 apart are independent). Similarly we can consider longer moving averages. In the empirical study a prior simple moving average of 48[15] was used. Thus the autocorrelations expected were :-

$$\rho_k = \left(1 - \frac{k}{48}\right)$$

Our simple model was therefore of the form whereby the 'trend' represented by the moving average was added to a random variate, so as to modify the actual experience, as in conditions 1 and 2, discussed previously in 4.2. Thus we have a form:-

$$z_t = \varepsilon_t + \frac{1}{48}\sum_{j=0}^{48} \varepsilon_{t-j} \quad \text{where } \ \varepsilon_t \sim \text{N}\,(0,1)$$

---

where he showed that that the expected first-order serial correlation of first differences between averages of terms in a random chain approximates 0.25.

[15] The choice of 48, from an economic viewpoint, can be considered the period of the short-term interest rate cycle. We could have chosen the the period *a posteriori* to fit the first-order variance ratio observed or to minimise the sum of squares of the fit to the, say, first 12 lags in the variance ratio. As always, the exact choice is a matter of judgement.

and $\text{var}(z_t) = \frac{49}{48}\text{var}(\varepsilon_t)$. We can now determine the theoretical autocorrelations for this model and hence look at the variance ratio, as in the table below:-

Table 4.3 Autocorrelation Function and Variance Ratio Simple Moving Average Model

Lag k	ρ_k	$VR(k)$
1	0.039	0.708
2	0.038	0.736
3	0.038	0.763
4	0.038	0.790
5	0.037	0.817
6	0.037	0.844
12	0.034	1.000
24	0.029	1.292
36	0.025	1.556
48	0.000	1.792
60	0.000	1.967
72	0.000	2.083

We must also have mean reverting behaviour. In our case we have overlapping runs of length 48 which are cumulative normal sums (damped by the divisor of 48). In the long run they are as likely to be above the 'origin' as below. However random walk theory tells us that this could be a very long time. Thus we need to bring in condition 3, discussed previously in 4.2. We therefore use a cumulative sum test (CUSUM) to modify the series, z_t , obtained from our simple model. This entailed finding the CUSUM's and if the result lay outside 1.5 standard deviations from the mean, then adjusting the CUSUM by the square of the excess amount. Note that we need the sign function to ensure we add or subtract the mean reverting element appropriately. We may see this better, by using the symbolism outlined in 4.2 and following a similar process, where we let:-

s_k = the kth partial or cumulative sum

s = global standard deviation

m = global mean

We form the elements of our new series:-

$$z'_t = \left(s'_k - s'_{k-1}\right)s + m$$

where $s'_k = s'_{k-1} + \left(s_k - s_{k-1}\right) - \mathrm{sgn}\left(s_k\right).\left\{\frac{|s_k| - 1.5s}{s}\right\}^2$ for $|s_k| \rangle 1.5s$

$$= s'_{k-1} + \left(s_k - s_{k-1}\right) \quad \text{otherwise}$$

Note that we have our adjustment in terms of standard deviation units. We treat the function in this way so as to capture the increasing pull effect as we move further from the mean but adjusting the cumulative sum in not too excessive a way whilst doing so. In practice this meant obtaining a CUSUM larger than 2.5 s above the mean is very difficult, since say at 3.5 s the pull would be 4 units times s (=5.6%) or 21.4%. One can argue about this, though the observed series show these characteristics and allowing further deviation would have no noticeable impact on the observed variance ratios (see the previous comments on "bubble" events). For an alternative model see the cubic mean reversion approach of Bleaney and Mizen (1996).

A test of the above approach, was carried out with a small Monte Carlo sample. Note that we have designed here a process that will have mean reversion as well as positive short term autocorrelation, and is plausible. Now we are testing it (and hence we may better observe how well our tests perform). The results showed consistent positive short-run autocorrelation but a very unclear situation when considering longer lags, with mean reversion (a clear maximum for the variance ratio with a declining trend as the lag increased) being as likely as continuing positive autocorrelation, that is a variance ratio which continued to increase. This is a result of the carry over of the positive autocorrelations generated by the moving average process, in that they don't damp down rapidly enough. Therefore it was decided to consider a process whereby the autcorrelations damped down more quickly. An obvious candidate was an exponentially weighted moving average (EWMA). The details for the calculation of the theoretical autocorrelations and variance ratios are in the Appendices and follow the procedure outlined above, for the prior MA. The results are tabulated below for the case $\theta = 0.9$

Table 4.4 Autocorrelation Function and Variance Ratio Exponentially Weighted Moving Average Model

Lag k	ρ_k	$VR(k)$
1	0.110	0.531
2	0.099	0.590
3	0.089	0.644
4	0.080	0.695
5	0.072	0.742
6	0.065	0.787
12	0.034	1.000
24	0.012	1.257
36	0.003	1.399
48	0.000	1.482
60	0.000	1.533
72	0.000	1.567

It is noteworthy that we have now generated a set of autocorrelations and variance ratios close to those produced by the fractionally differenced model with a value of d = -0.1. Note also that, for example, we have reduced the variance ratio at lag 72 to 1.567 from the value 2.083 for the prior MA.

Having made the above changes to the model the next step was a simulation run. 1000 cases were used, using an identical approach to that outlined previously but with an EWMA providing the short term autocorrelation instead of the prior MA and mean reverting behaviour provided, as previously, by the CUSUM approach. The EWMA used was of the following form:-

$y_t = \varepsilon_t + (1-\theta)\sum_{j=0}^{47} \theta^j \varepsilon_{t-j}$,where we have truncated the MA at lag 47

(in practice $\theta^{47} \cong 0$), and the results are shown in Table 4.5.

Table 4.5 Simulation: Mean and Standard Deviation of Variance Ratios at Given Lags

lag	1	6	12	24	36	48	60	72
mean	0.692	0.872	1.000	1.092	1.084	1.045	0.986	0.922
std.dev.	0.140	0.084	0.000	0.140	0.233	0.291	0.329	0.354

As can be observed *for the averages* we have significant positive autocorrelation at short lags and negative autocorrelation at long lags, but obviously not at all significant. The results were viewed more closely and it was observed that the maximum had occurred in 474 cases by lag 24 and that the value at lag 72 declined from lag 60 in 769 cases. That is, each individual series showed negative autocorrelation, but the values at lag 72 were tremendously variable. Further analysis showed that in 658 cases a clear maximum was achieved, by which we mean that the variance ratio rose monotonically to a maximum, then declined monotonically to lag 72[16]. If we (as we did at the end of Chapter 3), consider the signs of the changes and a rise is given a"1" and a fall "0", then favourable permutations are 0,0,0,0,0 ; 1,0,0,0,0 up to 1,1,1,1,0. That is 5 out of 32 combinations or 0.156 versus the observed of 0.658. Hence we can conclude negative autocorrelation, as we would expect[17].

What conclusions can we thus draw from our empirical study? Clearly we know the generating mechanism and it has economic logic underlying it. If we consider the results from the data analysis in Chapter 3, and look at mean reversion in our three major series we observe a pattern not dissimilar to this. Whilst all three major asset class series show mean reversion the value of the variance ratio at lag 72 varies widely, being 0.727 for the AOIA; 0.515 for the CBBI and 1.123 for the MSCII. From what

[16] Acheived by taking each 12 month period as a change. Hence 12-24 months is one change and there are 5 in all up to lag 72.

[17] We could regard this as a binomial situation with p=0.156. A value of 0.658 is well outside any confidence interval, say at the 99% level, with a sample of 1000.

we have seen previously in our modelling the "trend" factor we built in had a profound effect on the mean reversion, that is the positive autocorrelations carried over to the longer term. If we take the MSCII as a case in point, from an economic standpoint the sharp devaluation in the early 1980's means that it is difficult for the series to be able to fully mean revert (put another way the devaluation has the form of a *permanent* development, as against economic events within an economy which can return to long run equilibrium).

Another clear result of our modelling is the difficulty, if not impossibility, of being able to distinguish between our series, for which we know the generating process and a random walk. It is difficult to think of any statistical test with the power to resolve out the true situation. If we were to pick, at random, any of the series generated above, then we could well find no mean reversion at all or a value close to unity and not significant by statistical test. The bottom line is that we have so few samples. Given that we need extremely long time series, by definition, we really can only have one sample per asset class. Other series within an asset class are obviously closely correlated. The only alternative is to consider asset classes in other major countries on the assumption that they are independent (which if one considers, for example, the October 1987 crash, they are not, though markets like Japan certainly have a low correlation with Western markets).

In this context a recent draft paper by Chou and Ng (1995) sheds some light. On the presupposition that stock prices have a temporary (mean reverting) , as well as permanent (random walk) component, they decomposed international stock price series for six major markets (U.S., U.K., Japan, France, Germany and Canada). They were concerned as to how the correlation structure changed over time; based upon the above hypothesis they concluded that the correlations increased over time. Further they determined that by considering two sub-sample time periods (1976-83, 1983-89) that there was evidence of convergence in the international series because the correlations had increased. Hence, despite Chou and Ng's assumption of a transitory component, it is going to be very difficult to increase our sample size to actually prove it; particularly when one considers the even more integrated nature of bond markets. Perhaps if we had readily available property prices we would be on safer ground. These markets would seem *a priori* the least integrated.

4.6 Non-Linear and Other Linear Models

4.6.1 Non-Linear Models

It is worthwhile here reflecting on a more general issue of model building relevant to our discussion. It is put very neatly by Harvey (1990) p.231, where he reflects upon the different approaches taken by Box -Jenkins modellers and econometricians. When discussing the examples used by Box and Jenkins he notes, "The observations are generated by a controlled experiment, and so the question of whether other explanatory variables should be included in the model does not arise. In econometrics the problem is to estimate a behavioural relationship from non-experimental data. Thus specification takes on two dimensions. As well as determining the dynamics of the model, there is the more fundamental question of which variables should be included in the first place." We, of course, face the situation of non-experimental data. The ARIMA models are very particular models, both in terms of the dependent variable and the assumption of linearity.

Thus far we have only considered linear models and have used as basic assumptions a constant mean and variance. There are thus several issues we need to consider in greater depth with respect to the modelling process. A first point that we must remember is that in this document we are really concerned with longer term issues; those that are strategic in nature. Much of the literature is more concerned with shorter term details, for example, such as affect option prices. Options in the market place are typically of 90-days duration. Longer term series exist, out to 9-months, but these are not often traded. Warrants do exist in the Australian market but where they do the originator often has a covered position. Thus we are not concerned with, for example, the contribution of trading days to volatility. Also we are primarily interested in indices or benchmarks which represent asset classes, as such they are an aggregation of individual stocks. Thus any microeconomic tendencies seen at the individual stock level will tend to be obscured.

Let us first review the mean level[18] of the process. If we consider the CUSUM chart, Figure 4.1, outlined earlier for nominal monthly returns we can seen turning points which can be considered to divide up the aggregate time period. The market bottom in 1974, and the top in 1987 stand out as focal points. The former in some senses was a final sell-off from the previous deterioration whilst the later was the final run-up from a period of increasing euphoria. Let us consider some values as per the following Table 4.6.

Table 4.6 Mean of Monthly Returns for AOIA

	nominal	real
mean Oct 1960- Sept 1974	0.40	0.03
mean Oct 1974- Sept 1987	2.22	1.43
mean Sept 1987- Feb 1995	0.39	0.06
mean for whole period*	1.09	0.56

* The variance of the nominal returns was 31.33 (standard deviation 5.60)

Clearly the mean level changed. This is due to at least two causes. Firstly the market had fallen to an extreme low in 1974 and thus in part the increased returns are a function of this. Secondly, the overall inflation level was much higher translating into higher nominal earnings growth. The period of disinflation in the early 1980's acted via the discount rate to value the increased nominal earnings stream more highly. However, it is to be noted that in real terms the returns are still much better for the comparable periods, though not as great. Thus one may tentatively postulate that it is

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[18] The mean return here is defined as per Chapter 3, to be $\bar{r} = \frac{1}{n}\sum_{i=1}^{n} r_t$.

the change in inflation (or expectations thereof), that has added the extra returns. This would then suggest a linear econometric style of model with various causal variables.

Such models are covered in Granger (1992), where he reviews work by several authors and concludes that dividend yields and interest rates (although one quarterly model surveyed used the inflation rate lagged by three quarters) provide good long-run forecasting abilities. We will not consider such relationships here but would note that the R^2 was not particularly high. They are also consistent with mean reverting behaviour, in each of the following cases:-

1. The explanatory or independent variable is itself mean reverting. In this context we may refer to Mayadunne, Evans and Inder (1995), where they examined *shock persistence*[19] in a range of economic time series, and in general were unable to come to any clear conclusions "...it is hard to conclude either on the existence or magnitude of persistence in economic time series" (p.146).

2. Or alternatively any mean reverting component is found in the residuals. This means that the model would not be fully specified and some form of mixed structure may be appropriate (it is always possible that some of the mean reverting component has been taken up in the explanatory variable so what's left may not be discernible).

An econometric model of Australian share prices (it may be necessary to separate out industrials from resources) would be an interesting and challenging task. In particular, it would be useful to try out the predictive power of dividends (or the dividend yield) and see the impact of inflation (or changes to the discount rate which is

[19] Persistence is defined by the long run effect of a shock on the level of a time series variable continuing indefinitely into the future. If shocks persist then the time series are therefore not mean reverting. For a further discussion see Mayadunne, Evans and Inder (1995) mentioned above, pp. 146-8. Note this is different from the short run persistence referred to in Chapter 3.

equivalent if it is assumed that there is a real interest rate premium with a constant mean).

A further alternative is that the different mean levels may be caused by a time dependent process. That is, the interaction effects of economic variables in different financial eras produce different results in the time series. Economic agents learn; thus responses to the same event may be quite different in different times; thus interest rate rises can be much more effective in a subdued or pessimistic time than in a more buoyant period. Therefore the parameters of our models will be different during the successive eras. Further the transition between these eras is likely to be over a reasonable time period. That is, some form of relatively smooth transition is in order.

Hence a further refinement is to allow a smooth transition from one state to another via the use of the exponential or logistic functions. These models are highly complex and researchers have simplified their assumptions by restricting their attention to, for example, two-state models. This gives us the smooth transition autoregressive model (STAR) of the form:-

$$z_t = \alpha f(z_{t-2}) z_{t-1} + \varepsilon_t \ ,$$

where $f(y)$ is a smooth non-decreasing function, such as the logistic function.

Another alternative is the switching model with a sharp or sudden change. This then embeds the process in a Markov chain of transition probabilities (rather than some other function). An example is Tyssedal and Tjostheim (1988) where they introduce a chain of transition probabilities between various states described individually by AR(1) processes with different parameters. That is:-

$$z_t = \theta_t z_{t-1} + \varepsilon_t \ , \qquad t = 0, \pm 1, \pm 2, \ldots..$$

and the $\{\theta_t\}$ are a Markov chain. They use this approach on the IBM stock price data from Box and Jenkins and use it to identify change points in the stock data which correspond to economic events.

For completeness we may mention other possibilities, for example NLMA(1), where:-

$$z_t = \varepsilon_t + \beta\varepsilon_{t-1}^{\ \ 2}$$

or moving average models and mixed processes (bilinear), or indeed situations with time varying parameters. For a comprehensive coverage of non-linear models refer to Granger and Terasvirta (1993), p.3-11.

We may also mention in this context the class of ARCH (Autoregressive Conditional Heteroscedastic) models and related variations, which are introduced to deal with potential heteroscedasticity in the variance (see 4.5.3). The p-th order ARCH form with serial correlation in the variance is :-

$$\operatorname{var}(y_t) = \sigma_t^{\ 2} = \alpha_0 + \alpha_1\varepsilon_{t-1}^{\ \ 2} + \ldots\ldots + \alpha_p\varepsilon_{t-p}^{\ \ 2}\text{, where } \varepsilon_t = y_t - \mu_t$$

which itself may be generalised to the GARCH model where a moving average of the variance is introduced so the conditional variance is:-

$$\sigma_t^{\ 2} = \alpha_0 + \alpha_1\varepsilon_{t-1}^{\ \ 2} + \ldots\ldots + \alpha_p\varepsilon_{t-p}^{\ \ 2} + \beta_1\sigma_{t-1}^{\ \ 2} + \ldots\ldots\ldots + \beta_p\sigma_{t-p}^{\ \ 2}$$

and so on, to other variants. For further details refer to Granger and Terasvirta (1993).

Given the possibility, indeed likelihood of changes in the level of the mean we should expect similar problems with higher order moments. Hence we will next consider the variance. In the data analysis section we postulated that there was only one change in the variance level over the whole time period, 1967 or thereabouts, due to the increasing prominence of the mining sector. We must inspect the data more closely to check for evidence of heteroscedasticity.

4.6.2 Outliers

A quick plot of the 24-month rolling variance (to smooth the statistic and pick up trend features) defined as :-

$$\hat{\sigma}_t = \left(\frac{1}{23}\right) \sum_{i=t-24}^{t} \left(r_i - \bar{r}\right)^2$$

revealed extreme peaks corresponding to the All Ordinaries Accumlation Index low in 1974 and the corresponding peak in October 1987. It was decided to remove these outliers. In the 1974 case we had monthly returns of -17.1% and +18.05% in September and October respectively. This was replaced by half the average of the two values for each month (but for the timing of the month end we may not have got such a large swing). For October 1987, where no reversal had taken place, we had a monthly return of -42.13%. This was replaced by the mean of the values either side.

It is worthwhile here commenting that Granger (1992), himself quoting from work of Friedman and Laibson (1989), points out their result that these extreme events can obscure simple and soundly based underlying relationships. Thus we separate out the more predictable 'ordinary' situation from the 'extraordinary' (as in our discussion in 3.2.2 above).

4.6.3 Heteroscedasticity

Having removed the outliers it is now interesting to view the plot of the series as follows:-

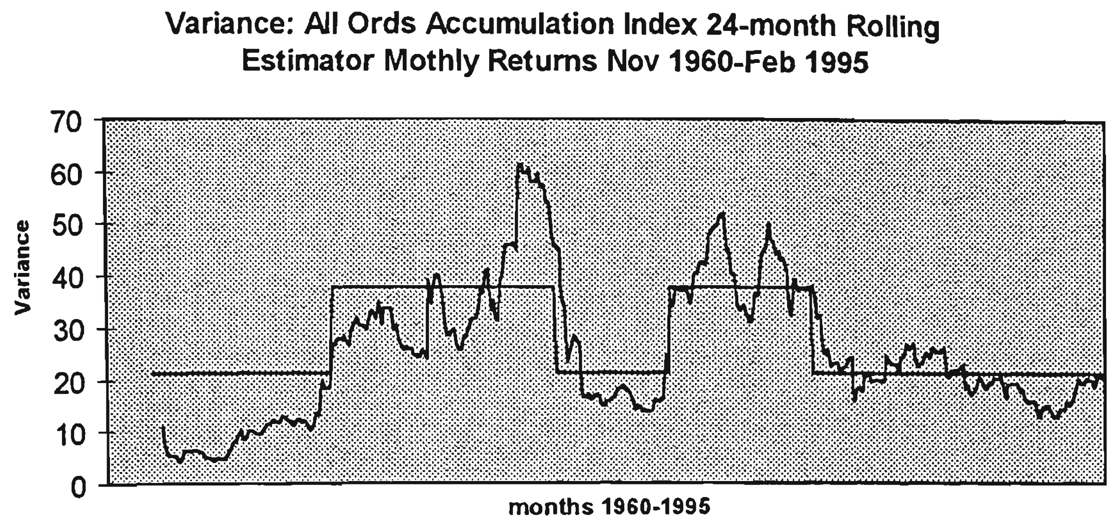

Figure 4.2 : Plot of 24 Month Rolling Variance of Stock Prices

As can be seen we can divide up the series into two distinct levels. These higher levels of the variance correspond to the periods in the market during which resource stocks were performing. That is the mining boom of the late sixties and the inflationary era corresponding to the move of the $US from the fixed parity of 1 oz gold being 32$US (and the oil shock of 1974), for the first period. The second period corresponds to the second oil shock and the run-up in oil prices (and of course Australian steaming and coking coal). This all makes good economic sense. Once the higher beta and thus more volatile resource stocks stopped performing, and thus had a much smaller weight in the All Ordinaries, then the volatility of the All Ordinaries fell to a level commensurate with the more stable industrial sector. The following table outlines this in more detail:-

Table 4.7 Volatility of AOIA Monthly Returns; Different Sub-Periods

Time Period	No. of observations	Variance of Monthly Returns
Oct 1960-Aug 1968	94	12.93
Sept 1968-Apr 1976	91	36.56
May 1976-Feb 1980	47	21.87
Mar 1980-Jan 1985	58	39.36
Feb 1985-Feb 1995	122	20.78
whole period	412	25.29
average low periods	169	21.08
average high periods	149	37.65

It would be valuable if we could compare the volatility of the All Industrials Index with that of the All Resources Index . Unfortunately we run into the kind of data difficulties outlined in section 3.1. We really only have our month ending, monthly accumulation indices for the most recent national index series, starting Dec 1979. Reviewing this very neatly in Table 4.8 summarises the situation and validates our contention.

Table 4.8 Volatility of Industrials Versus Resources
Different Sub-Periods

Time Period	All Ords	All Inds	All Res
Mar 1980-Jan 1985	39.36	20.82	77.75
Feb 1985-Feb 1995	20.78	20.36	37.30

We can thus consider this as a Markov style process with the two states being 'high' and 'low' variance. Of course the transition probability will depend upon external variables (for example, global GDP growth, which will make resource shares perform). The real point is not to develop a model, but to indicate both the possibilities and the necessity of generalising, particularly a short term model, to one of heteroscedasticity. Of course, the above approach would form a good starting point for investigating further the heteroscedasticity of Australian stock market prices.

As mentioned previously, Kearns and Pagan (1993) conducted a detailed study into the volatility of the Australian stock market over the same time period. They extended their considerations to an attempt at model building using ARCH style models, as well as an autoregressive iterative two-step procedure (refer to p.170-4 in that paper for further details). Using the models they develop they find "...that there is persistence of shocks in volatility and that this persistence is as true of small shocks as it is of large ones. Moreover, there is no evidence that the persistence is due to structural change; over long periods it has remained remarkably constant" (p.177). There are some points arising that need to be considered.

Firstly they appear to have used average prices for the month, that is the average of the daily closing prices. For the All Ords (and we focus on the accumulation index herein), for the period Sept-Oct-Nov 1987, the *average* prices went from 2238.7 to 1885.1 to 1280.0, whereas the *month end* closing prices went from 2249.2 to 1294.5 to 1329.5. Hence they attribute the large changes which occur in successive months in the average series to non-trading effects[20]. Given that the 50 leaders represents 75% of the index and these trade in large volume all the time, one would expect the index to react very rapidly indeed to any price changes, as indeed was the case. Further, using averages means that the variance is approximately 2/3 of the month end series, using a result due to Working (1960).

Secondly, while this is not a detailed study on trading patterns, the AOI does not contain all stocks. Hence those that do not trade generally are not in the index, as

[20] This non-synchronous trading is due to not all stocks being traded at the month end. Thus we may find some serial correlation, and hence predictability of returns.

also the weighting of stocks, like CRA, with its overseas parentage, are altered to reflect the fact that the stock is not available to trade (and also the lack of control premium and so on, which may affect fundamental value). Thus some care is necessary in assuming any non-trading effects; or indeed calendar ones. As Kearns and Pagan, quite rightly, point out, there are substantial differences between the US and Australian experiences[21].

In this document, model building has not been pursued, not because it is not worthwhile, it obviously is, and there is a lot to do (" a fruitful topic for future research"), but because it derviates from our objective.

4.6.4 An Actuarial Approach

For completeness in our 'review' of models it is useful to consider how an actuary might go about the modelling process. After all, a life company or indeed any superannuation fund must model its assets and liabilities. This is obviously necessary as part of the financial reserving process. Further these models are specifically long-term and by definition highly dependent on the asset class allocation process which will generate the long-run returns. An example of the approach is outlined in Wilkie (1992) using a 'cascade' style model. This is a logical and standard approach which has as its basis the recognition that the assets and liabilities are driven by the overall price level or rate of inflation. The liabilities are then determined by average weekly earnings (AWE) or some suitable proxy, which has as components real wages plus inflation. The assets side has stock prices which are driven by yields (dividends divided by dividend yield gives price) and interest rates driven by real yields plus inflation.

[21] Officer (1975), Chapter 15 in *Share Markets and Portfolio Theory* p.220, interprets a large positive first order serial correlation coefficient observed in his index data as due to non-trading effects, not weak form inefficiency. This is not the last word on the subject, but volumes of trade are likely much higher now than they used to be. One would thus expect the effect to be less now, though it is likely to be very relevant for the much earlier data.

The final leg of the model is the international or currency portion. This is driven by the purchasing power parity model, that is, determined by the relative levels of inflation between countries, with an adjustment process which recognises that currencies can depart from their 'fundamental' values. This is integrated as in the schema outlined below:-

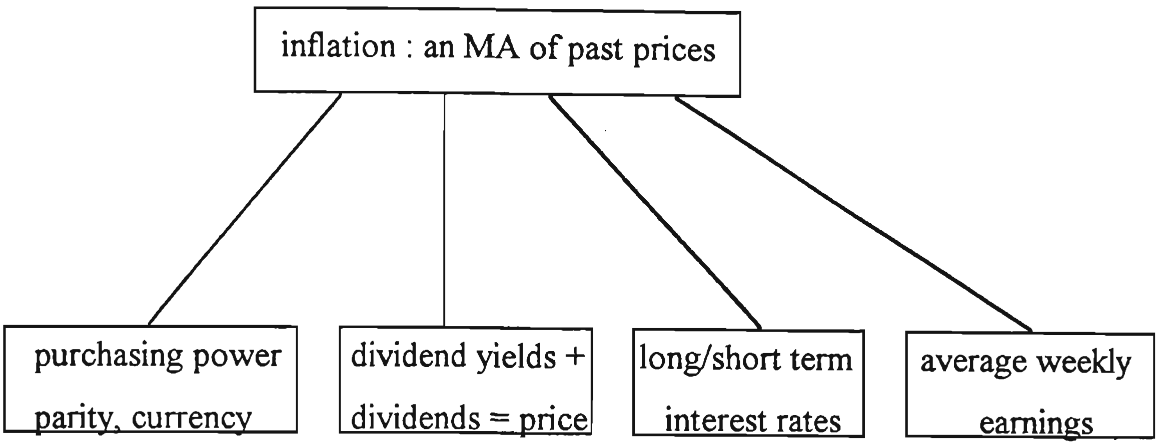

Figure 4.3 : Layout of Actuarial Model Building

Wilkie, in his model, generates the dividend yield series as an AR(1) and the dividends as a distributed lag of inflation. Given the inflation model is itself an AR(1), mean reversion is not allowed for. However, interestingly, mean reversion is allowed for in the real interest rate model (by allowing investors to estimate future inflation from the last 20 years experience).

Apart from the obvious consistency in this approach it also lines up very closely with the mean-variance optimisation issue as the optimisation would need to take place over different periods rather than condense them as if there were no different periods. It would be very useful to extend this model to Australian data and develop it with mean reversion in stock prices and consider the implications that this would have for the practice of mean-variance optimisation.

4.6.5 Summary

Each of the model approaches outlined above helps to shed light on the underlying process. As such all models are just approximations to reality, or views of the object under consideration from different angles, rather than being the actual process. From a *forecasting* viewpoint there is a lot to be said for developing a series of models. The random walk model is a short term approximation which is a very good one in practice. Improvements upon this in the non-linear class are best if treated as short term in nature. The econometric or causal model with independent variables could be regarded as medium term. It is concerned more with turning points and explaining the changes in, say, mean or variance rather than assuming their continuity in some model form. Finally we come to the longer term models, those that an actuary might use. These are very broad in ambit and are concerned as much with consistency as anything else. We are unable to anticipate social or economic trends which are 10 or 20 years away. Here we are more concerned with relativities, for example the *difference* between AWE and investment returns, rather than the absolute values *per se*.

Hence if one were to prepare a series of forecasts and scenarios for a stock broking house, it would be useful to develop all three approaches and integrate them together in a consistent framework.

4.7 The Distribution of Stock prices

Before reviewing this in more detail the first point to make is to explain the use of percentage changes rather than logs. Alexander (1961) brings out the fundamental difference between the two. When discussing the change in a \$100 stock he notes "..under the percentage form it is equally probable that a \$100 stock goes to \$101 or to \$99 in a given time, whereas in the logarithmic form it is equally probable that a \$100 stock goes to \$101 or \$99.01 in a given time. This difference of one cent in the dollar change from \$100 spells the difference between zero expectation of change and positive expectation". We use percentages for four good reasons:-

1. The standard in the marketplace is percentage change. Participants are not familiar with the use of logs. Thus all model inputs, for example in derivative pricing, are expressed as percentages.
2. The analysis herein deals with different asset classes and the risk free asset (the Treasury note). Benchmarks for these asset classes are all expressed as percentages.
3. Discussion of efficient frontiers , and optimisation takes as input percentage changes and means, covariances and variances thereof.
4. In practice there is virtually no difference between the two. Our changes are not, on average large, and by comparison with the inexactitude in what we are trying to determine (mean reversion, for example), any difference between the two measures can be safely ignored.

So far we have considered modelling the speculative price process. In the first instance we assumed a constant mean and variance, but have recognised that in practice these may not be valid. Common pricing and asset allocation models make the further assumptions that:-

1) Each individual price change is independent.
2) Individual price changes are identically distributed.
3) Individual price changes (whether percentage changes or logs of price) are normally distributed.

We have discussed at some length the serial independence issue and shown some evidence of positive short run autocorrelation and negative autocorrelation or mean reversion over the longer term. We must now review the distributional issue.

Our observation, as discussed in Chapter 3, is the same as that of all other authors, Fama (1963), Praetz (1972), Blattberg and Gonides(1974), Ali and Giacotto

(1982), Officer(1972), Hsu, Miller and Wichern (1974) and many others, who observed prices have "fat tails" and are more "peaked" than that predicted by the normal distribution. Regrettably we cannot just fit a distribution to the observed price changes since the nature of the underlying distribution is clearly not independent of any changes either in mean level or of scale. And we know from our discussion above that there are likely changes in mean, and certainly the variance is also not constant. If we allow that the distribution is normal but we have a change in mean or variance or both we can see the following permutations[22]:-

1) Different mean, same variance, then the overall distribution will have thinner tails.
2) Different variance, same means, then the overall distribution will have thicker tails, the exact amount depending on the mixture.
3) Different means, different variance then the overall distribution could go either way.

In the literature various approaches have been used to tackle some of these problems. The use of the Pareto class of stable distributions was introduced by Mandelbrot (1963) and Fama. These typically have "fatter tails" than the normal, and are a class of distributions defined by their characteristic function, as follows:-

$$\Phi(t) = \exp\left\{ i\delta t - |ct|^{\alpha}\left[1 + i\beta\left(\frac{t}{|t|}\right)\omega(\alpha,t)\right]\right\}$$

where

$$\omega(\alpha,t) = \begin{cases} \tan(\alpha\pi/2) & \alpha \neq 1 \\ (2\log|t|)/\pi & \alpha = 1 \end{cases}$$

and $0 < \alpha \leq 2; -1 \leq \beta \leq 1$. The parameters δ, c, α, β, indicate the location, dispersion, tailedness and skewness of the distribution. Choosing $\beta = 0$, makes the distribution

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[22] For a detailed discussion of these propositions see Boness, Chen and Jutusipitak (1974) p.521-2, on the class of 'contaminated distributions'. Praetz (1972) p49-52, considers the case of non-constant variance. His arguments lead to a scaled t-distribution for price changes; that is, "thick tails".

symmetric and hence gives the class of stable distributions. There are several points to be made:-

1. The value of $\alpha = 2$, gives the normal distribution, $\alpha = 1$, the Cauchy distribution. Thus we are really only interested in values between these limiting cases.
2. Of those in the above range, only the normal has finite second and higher moments.
3. We cannot find analytic forms for these distributions other than the normal and Cauchy.
4. By standardising the variates using the location and scale parameters, we may use a procedure due to Fama and Roll to estimate the exponent α, obtained from fitting this distribution to the relevant empirical distribution.

Whilst a lot of work has been carried out with these distributions (see Hsu, Miller and Wichern or Officer, for example), the non-existence of the variance is a very serious drawback. For our sample, referring to Figures 3.3 and 3.4, we can see the variance whilst increasing over time, shows fluctuations which are explicable. More recently the 60-month rolling variance has declined. Moreover, if the variance was infinite then we would not be able to draw conclusions from our sample variance, which is our fundamental risk measure. The implications would be profound indeed for our models. As put by Fama (1963), when discussing Mandelbrot's original hypothesis, "Moreover, if the variance is infinite, other statistical tools (eg. least-squares regression) which are based on the assumption of finite variance will, at best, be considerably weakened and may, in fact, give very misleading answers" p.298, Cootner (1964).

Perhaps, then, we need to consider leptokurtic distributions of which there is a large range. Another acceptable class is the Student distribution. This has the property of convergence to the normal as the number of degrees of freedom increases. Thus an approach is to see whether daily changes follow a Student distribution, then monthly observations, say 22-25 trading days, would be approximately normal. This element of stability in the distribution is clearly sought (as the changes are supposed to be identical). These arguments are covered well in Praetz, and Blattberg and Gonides.

A further class that we need to review are the so called generalised error distributions (G.E.D.) referred to by Nelson (1991). Again like the stable Paretian class and the Student range they contain the normal distribution as a special case. This has the functional form (Harvey (1990), p.117) :-

$$p(z) = \frac{\exp\left\{-\left(\frac{1}{2}\right)\cdot\left|\frac{z}{\phi}\right|^{\theta}\right\}}{2^{\left(1+\frac{1}{\theta}\right)}\,\Gamma\left(1+\frac{1}{\theta}\right)\phi},$$

where θ, ϕ are positive scalar parameters and $E(z)=0$. Both the mean and variance are defined and Harvey p.118 gives an expression for the variance in terms of the parameters. The parameter θ is a measure of kurtosis, with a value of 2 being the normal. For values of θ >2, the distribution has thinner tails up to a value of $\theta=\infty$, where the distribution is uniform and thus has no tails. For values below 2, the distribution has fatter tails, with a value of $\theta=1$ representing the Laplace or double exponential distribution. This latter is a peaked distribution but values of θ closer to 2 could well give a good fit to the observed distributions.

4.8 Some Results from the Major Asset Classes

We are, of course, dealing with monthly returns for major indices. Thus we are smoothing out a lot of microeconomic detail. Previous authors, for example Officer, found that, for the U.S. stock market, monthly prices appeared to follow a stable Pareto distribution, but that the exponent differed between the pre-war and post-war periods. Our task is to consider this and related issues. That is, we need to consider the stability of the distribution and examine our stock price history to see if there are, as one would expect, significant changes in the generating process. Clearly over the entire period, (1882-1995), there have been great changes in technology, and society in general.

Firstly, for the most recent time period, the experience of the major asset classes was reviewed. We are constrained by the shortest series, when we wish to make comparisons over the same time period. Nevertheless we had available a series of 218 monthly observations from December 1976 to February 1995. As well as standard statistics, a chi-squared test was performed on each series. The variates were standardised and grouped into 8 classes. In practice, with only 218 observations and having classes 0.5 standard deviations (s.d.) wide it was found that going beyond 1.5 s.d. meant that there were classes with less than 5 observations. This would have meant that the test would not have been valid. As always, we must compromise between what we would like and what we actually have, which is, in any case, over 18 years data. The results are tabulated below (the abbreviations for the asset classes are as described previously). The full AOIA series (1348 observations back to 1882) is given for comparison.

Table 4.9 Summary Statistics Dec 1976-Feb 1995 Key Asset Classes

Asset	mean	std.dev.	kurtosis	S.E. kurt	skewness	S.E. skew	chi-square
AOIA	1.420	5.879	13.217	0.328	-1.784	0.165	10.414
AOIA full	0.963	3.177	16.066	0.133	-0.870	0.067	202.619
CBBI	0.967	1.600	1.249	0.328	-0.082	0.165	7.550
MSCII	1.362	4.670	1.006	0.328	0.246	0.165	5.617

Since we have 3 parameters to estimate and 8 classes in total we have a total of 5 degrees of freedom. The critical value at the 5% level is $\chi_{0.05}{}^{2} = 11.070$. Hence we can see that for all 3 asset classes we accept the null hypothesis of a normal distribution. It would seem that we have a kurtosis much above the expected 3 of the normal for the AOIA, but much lower numbers for the other two. The CBBI again appears very well behaved, as perhaps surprisingly is the MSCII. Perhaps all one can say there is that this series is in some sense a super-aggregation. Hencė with a series made up of many stock price indices translated via currencies one might expect a degree of convergence to the normal. It is somewhat comforting to find that this appears to be so.

The AOIA appears leptokurtic with much fatter tails and a significant degree of skewness, though this could well be due to a few outliers. Perhaps this cancels out to some degree maintaining the approximate normality of the distribution. However the full series has a very large χ^2 value. Thus given our available time series it was decided to divide up the series into six non-overlapping segments and look at the stability of the distribution. Four data points were dropped off at the start to give us 6x224 = 1344 data elements. Two series of χ^2 were calculated. The first was using a constant mean and variance for the whole series. In the second, the mean and variance were calculated separately for each segment, and standardised variates and classification based on a changing mean and variance. The results were as follows:-

Table 4.10 : Summary Statistics All Ordinaries Accumulation Index (AOIA) Six Equal Time Periods

period	mean	std.dev.	kurtosis	S.E. kurt	chi-square[23]	chi-square[24]
1	0.844	2.428	4.633	0.323	111.9053	21.9985
2	0.919	2.255	4.425	0.323	129.2185	22.8920
3	0.982	3.216	11.862	0.323	72.3646	37.9171
4	0.784	2.317	2.167	0.323	81.2438	13.5598
5	0.893	4.957	2.631	0.323	7.1683	17.1314
6	1.357	5.861	11.862	0.323	33.4018	9.6252

[23] mean and variance constant over the whole time period.

[24] mean and variance recalculated for each time period.

We may see a fairly stable mean apart from period 6. This is due to the effects of higher inflation, or more likely the lagged response of the market to that increase. The variance has risen. Interestingly the χ^2 has generally declined which tends to indicate that the later observations are more normal than earlier ones. Only the last period has a value of χ^2 below the critical value. One could speculate that this convergence towards the normal is a function of both technology and a more responsive market place. At the very least it is a very intriguing result. We could potentially fit a set of Paretian stable distributions to these different time periods. Undoubtedly one would find a series of exponents rising from say $\alpha = 1.5$ up to 2, which is the normal. These results closely mirror the results of Officer and also those of Hsu, Miller and Wichern, who used a studentised range test (Table 3, p.112).

The cause of the leptokurtic nature of the very long series is highly likely to be a changing variance (or scale parameter). This tentative conclusion is supported by the results of Ali and Giacotto, who find no significant evidence of changes of mean level through time but do find changes of variance (albeit for individual stocks, but over time periods including months). We can test for the equality of variance of our 6 equal periods using either Cochrane's test or Bartlett's test [25](see Walpole and Myers (1972) p.358-361). In the former the value of :-

$$G = \frac{\max\left(s_i^{\,2}\right)}{\sum_{i=1}^{k} s_i^{\,2}} = 0.4012,$$

where the $s_i^{\,2}$ is the sample variance which greatly exceeds the critical value of 0.2119, for 145 observations and 6 groups (and we have 224 observations so the test value is lower), given by tables.

[25] These tests assume that the samples are drawn from independent normal populations. Kearns and Pagan, note 4, p.169, dismiss the use of these tests as the assumptions are "plainly inappropriate". One would not argue with their comments, though they assume the conclusion; namely the distribution is non-normal. Perhaps the distribution is normal, for shorter periods, but heteroscedasticity and other factors affecting the mean make it appear not so.

In Bartlett's test, the value of b given by the formulas below follows approximately a χ^2 distribution with $(k-1)$ degrees of freedom.

If we let the 'pooled' variance $s_p^2 = \frac{\sum_{i=1}^{k}(n_i - 1)s_i^2}{N-k}$, where we have k sample variances $s_1^2, s_2^2, \ldots\ldots, s_k^2$ from samples of size $n_1, n_2, \ldots, n_k$ with $\sum_{i=1}^{k} n_i = N$. Then

$$b = 2.3026\frac{q}{h},$$

where

$$q = (N-k)\log s_p^2 - \sum_{i=1}^{k}(n_i - 1)s_i^2,$$

and

$$h = 1 + \frac{1}{3(k-1)}\left(\sum_{i=1}^{k}\frac{1}{n_i - 1} - \frac{1}{N-k}\right)$$

The value of b from Bartlett's test that is found is 5046.8[26] which is far above the critical value of 11.070. This is perhaps a better test since it tests more for the equality of all the variances rather than Cochrane's test which really is testing whether one of the variances is much larger than all the others.

As a final test of stability it was decided to look at non-overlapping sums of returns. Such sums, if they are independent should have a variance which is a linear combination of the individual stock price variance. That is :-

$$\sigma(sum) = \left(\sum_{i=1}^{n}\sigma_i^2\right)^{1/2} = n^{1/2}\sigma$$

[26] It is recognised that Bartlett's test is greatly affected by the kurtosis of the distribution, however we do have a very large value, which provides some comfort. In any case our plots of the variance clearly exhibit changes over time.

The following table outlines the results for non-overlapping sums from 2 months to 6 months:-

Table 4.11 : Sums of Returns for Non-Overlapping Months

No. months	2	3	4	5	6
variance	30.58	48.13	63.77	96.88	111.36
exp. variance	28.53	42.79	57.06	71.32	85.59
% difference	7.18	12.47	11.77	35.83	30.11
χ^2	70.40	47.41	35.89	37.84	18.29

The difference in variance from the expected is high for 5 and 6 month sums. This indicates a degree of instability. Of course, if there is positive autocorrelation then we might expect the variance to be higher than if there were none. As we can see above the distribution of 6-month sums gave a χ^2 value of 18.294, which is still above the critical level. Remembering that the χ^2 value for the whole series was 202.69, we can perhaps detect a better fit to the normal as we take increasingly longer sums. Again as our time period increases we seem to have a degree of convergence towards the normal. This would perhaps, more likely suggest a distributional form more like a Student for either shorter time periods or individual stocks/sub-indices. In all it is very difficult to be conclusive but one may suspect that the economic changes have been so substantial that there is some instability in the series taken as a whole. The more recent times, which we know to be more well behaved may show similar propensities in its distribution, that is, a quicker convergence towards a normal distribution. We simply lack sufficient monthly observations to combine them in non-overlapping sums of, say, order 6 months.

However, most importantly there is good evidence to suggest, based upon the more recent samples and our brief review of financial history, that in our asset optimisation models the use of the normal assumption is a reasonable one. Stokie (1982) (Chapter 2 in *Share Markets and Portfolio Theory*), using Australian share data finds also that there are no conclusive grounds for rejecting the normal distribution. However based upon the asymmetry found by Beedles (1986), Chapter 3 in the above compendium, and other research, for example, Blattberg and Gonides (1974), there must be some question as to the appropriateness of the use of the normal distribution in short range models. The study by Sterge (1989), albeit using futures prices rather than the underlying asset, indicated that the Treasury bond, 10-year Treasury note and Eurodollar futures price changes are non-normally distributed. He found also that even monthly price changes were non-normal though his calculated χ^2 improved at the longer time periods. Even if there is reason to believe futures prices may act slightly differently to the underlying asset price due to premia or discounts, short term models are still used, for example in derivative pricing. A specific example would be the Black and Scholes option pricing model (though perhaps given the way the market anticipates prices, a value to the nearest cent is probably not all that relevant). Finally it is noted that there is much more worthwhile research needed in this area. Not only do we need to investigate further other assets (there has been perhaps an over-concentration on share prices), but we need to consider the factors that differentiate the micro-financial details from the macro.

4.9 Conclusion

This chapter has considered some possible models available, for dealing with mean reversion or long memory. The difficulties with ARIMA models led us to fractional differencing and the topic of unit roots. An empirical investigation into model building was conducted exemplifying the difficulties observed in Chapter 3, with testing statistically the random walk hypothesis for a process whose characteristics we knew.

For completeness, a brief review of non-linear models was conducted where we considered some potential models applied to situations where the processes exhibited non-linearity. We then considered the assumptions of constant mean and variance that underlie most financial models, where the heteroscedasticity of share prices indicated a non-linear modelling approach.

Finally, we reviewed the distribution of returns, where we concluded that the asset class returns (at the index level) were well explained by a normal distribution. However, we did observe more problems at the individual share level or for shorter time periods than one month. Also it is reasonable to postulate changing levels of variance for the stock price series over an extended 113 year time period.

Appendices

Appendix 1

Let us first determine the correlations from a given variance ratio. Using the formula, given in W.Feller (1950) *An Introduction to Probability Theory and Its Applications* Chapter IX p.230 :-

If $X_1, \ldots\ldots X_n$ are random variables with finite variances $\sigma_1^2, \ldots\ldots\sigma_n^2$ and $S_n = X_1 + \ldots\ldots + X_n$ then :-

$$\operatorname{var}(S_n) = \sum_{k=1}^{n} \sigma_k^2 + 2\sum_{j,k} \operatorname{cov}(X_j, X_k)$$

the last sum extending over each of the $\binom{n}{2}$ pairs (X_j, X_k) with $j<k$.

Then letting ρ_k be the autocorrelation and $VR(k)$ be the variance ratio function at lag k, we find :-

$$\operatorname{var}(R_k^t) = \operatorname{var}\left(\sum_{i=0}^{k-1} r_{t-i}\right)$$

$$= \operatorname{var}(r_t + \ldots + r_{t-k+1}) +$$

$$2\{\operatorname{cov}(r_t, r_{t-1}) + \ldots + \operatorname{cov}(r_{t-k}, r_{t-k+1}) + \operatorname{cov}(r_t, r_{t-2}) + \ldots + \operatorname{cov}(r_{t-k-1}, r_{t-k+1}) + \ldots \operatorname{cov}(r_t, r_{t-k+1})\}$$

$$= k\sigma^2 + 2\sigma^2\{(k-1)\rho_1 + (k-2)\rho_2 + \ldots + \rho_k\}$$

$$\operatorname{var}(R_k^t)/k = \sigma^2 + 2\sigma^2\left((1-\tfrac{1}{k})\rho_1 + (1-\tfrac{2}{k})\rho_2 + \ldots\ldots (\tfrac{1}{k})\rho_k\right) \ldots\ldots (1)$$

$$= VR(k)\ (\operatorname{var}(R_{12}^t)/12) \text{ by definition. } \ldots\ldots\ldots (2)$$

So $\operatorname{var}(R_{12}^t)/12 = \sigma^2/VR(1)$, by letting $k=1$ and

$$\left(\left(\frac{VR(k)}{VR(1)}\right) - 1\right) = 2\left((1-\tfrac{1}{k})\rho_1 + (1-\tfrac{2}{k})\rho_2 + \ldots\ldots (\tfrac{1}{k})\rho_k\right)$$

Then $\rho_1 = \left(\left(\frac{VR(2)}{VR(1)}\right) - 1\right)$,

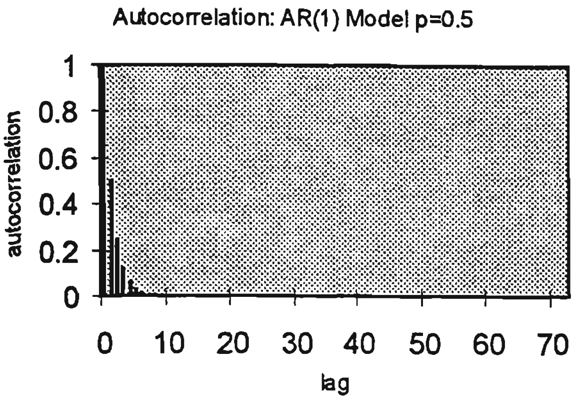
$$\rho_2 = \left(\frac{3}{2}\right)\left(\left(\frac{VR(3)}{VR(1)}\right) - 1\right) - 2\rho_1 \text{ and in general,}$$

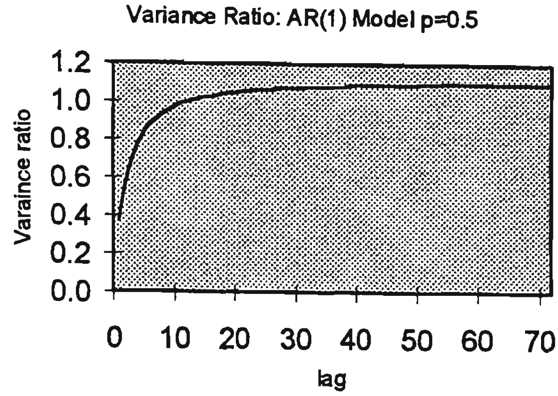
$$\left(\frac{k}{2}\right)\left(\left(\frac{VR(k)}{VR(1)}\right) - 1\right) - (k-1)\,\rho_1 - (k-2)\,\rho_2 - \ldots\ldots = 0$$

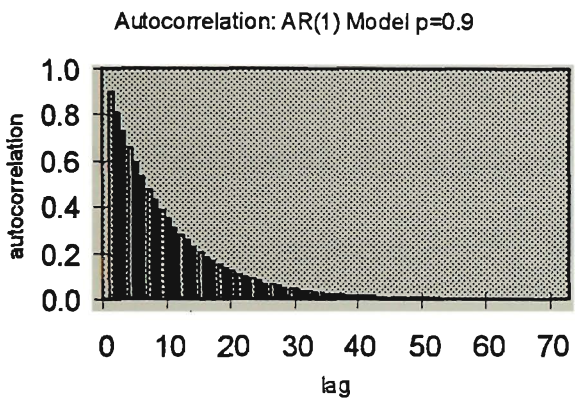
We may also find the $VR(k)$ from the ρ_k, from (2) above by substituting $k = 12$ in (1) and then re-arranging where σ cancels out leaving:-

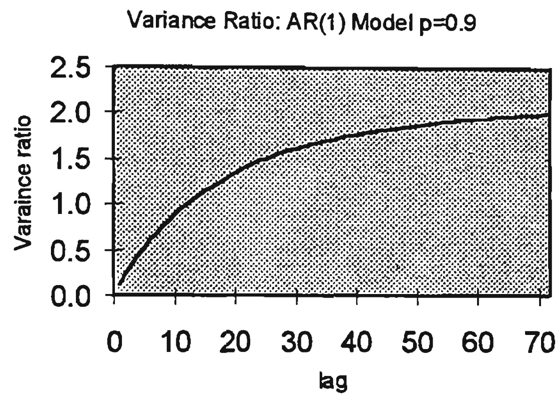
$$VR(k) = \frac{\left\{1 + 2\left(\left(1-\frac{1}{k}\right)\rho_1 + \left(1-\frac{2}{k}\right)\rho_2 + \ldots\ldots\left(\frac{1}{k}\right)\rho_k\right)\right\}}{\left\{1 + \frac{1}{6}\left(11\rho_1 + 10\rho_2 + \ldots\ldots\rho_k\right)\right\}}$$

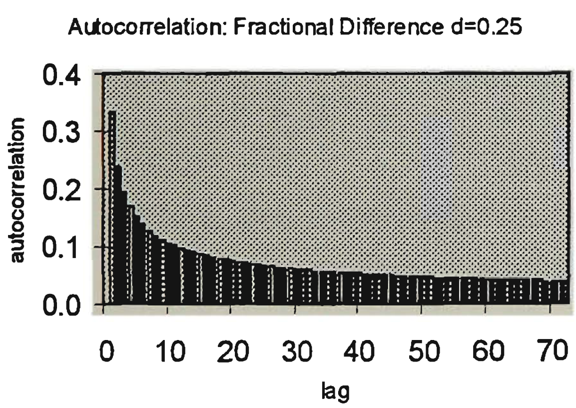
Examples of Variance Ratio Functions for Various Input Autocorrelations

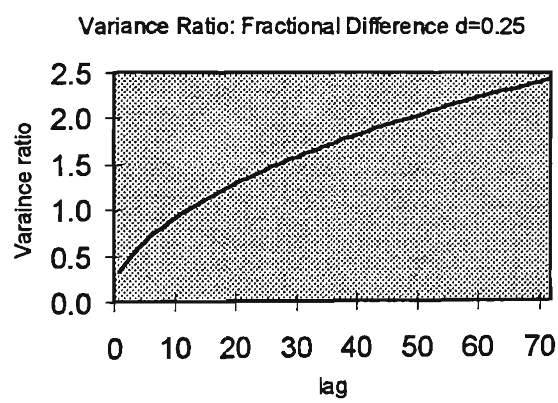


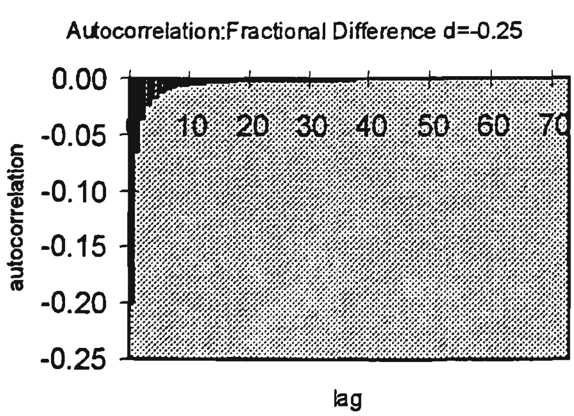


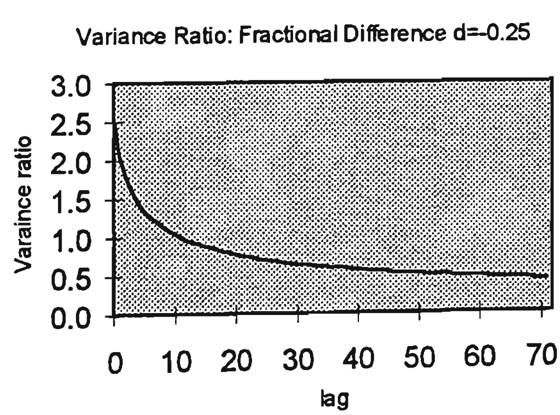












Appendix 2

As with the case of the prior simple moving average we need to determine the autocorrelations for the exponentially weighted moving average (EWMA). We may note the EWMA is in fact an ARIMA(0,1,1) and may be written $\nabla z_t = (1-B)z_t = (1-\theta B)a_t$ where the $a_t \sim N(0,1)$. Now we have for the EWMA the weights:-

$$(1-\theta)\ ,\ \theta(1-\theta)\ ,\ \theta^2(1-\theta)\ ,\ ....$$

where the value of z_t is updated by the formula:-

$$z_t = (1-\theta)\sum_{j=0}^{\infty}\theta^j z_{t-j}$$

We wish to modify the random variate by adding the EWMA to it ie

$y_t = \varepsilon_t + (1-\theta)\sum_{j=0}^{47}\theta^j \varepsilon_{t-j}$ where we have truncated the MA at lag 47 (in practice $\theta^{47} \cong 0$). Thus our weights are modified to $(2-\theta)$, $\theta(1-\theta)$......

Now $\rho_k = \dfrac{\sum_{j=1}^{q-k}\theta_j\theta_{j+k}}{\sum_{j=1}^{q}\theta_j^2}$, $\quad 0 \le k \le q$ and $\rho_k = 0\ ,\ k > q$

Therefore, after some algebra we obtain:-

$$\rho_1 = \frac{\theta(2-\theta)(1-\theta) + \theta^3(1-\theta)^2(1-\theta^2)^{-1}(1-\theta^{92})}{(2-\theta)^2 + \theta^2(1-\theta)^2(1-\theta^2)^{-1}(1-\theta^{96})}$$

and in general

$$\rho_k = \frac{\theta[\theta(2-\theta)(1-\theta) + \theta^3(1-\theta)^2(1-\theta^2)^{-1}(1-\theta)^{(94-2k)}]}{(2-\theta)^2 + \theta^2(1-\theta)^2(1-\theta^2)^{-1}(1-\theta^{96})}$$

and therefore we can iteratively obtain the ρ_k and thus the $VR(k)$ from the procedure outlined in Appendix 1.

It is of note that the "average age" of the data via the EWMA is given by :-

$$(1-\theta)\sum_{j=1}^{\infty} j\theta^{j-1} = (1-\theta)(1-\theta)^{-2} = \frac{1}{(1-\theta)}$$

which for $\theta = 0.9$ is 10 periods. This compares with 24 for the simple MA.

Chapter 5: Timing, Theory and Practice

5.1 Introduction

Thus far we have considered the general features of financial time series. In order to address our topic we must study the theory and practice of what is, in effect, applied portfolio management. The field of study contains concepts and terms which need to be understood, to gain a fuller appreciation of the results. Explanations are given as the terms arise. In addition, a glossary is provided which should act as a valuable reference.

The first section deals with some previous studies on the subject, by both academics and practitioners. The scene is then set in the following two sections with a discussion of the data and some definitions of terms, and a description of the process of how the information is analysed[1]. The next section, 5.4, reviews the overall performance of a collection of Australian funds using classic measurement tools. Section 5.5 introduces key elements of portfolio performance and the asset allocation issue, via the question, "Is portfolio performance predictable?" If it is, then we must find the source of that predictability, if not then this is an extremely important statement about strategy.

The next section breaks down the aggregate performance into its constituent parts, to attempt to come to terms with where, if at all, performance comes from. Various aspects of this are examined, with respect to the segmentation of performance. An attempt to understand the significance of the results is made, along with an assessment of the relationship between the aggressiveness of timing, (that is frequency and magnitude of moves) and the overall result.

The final section covers the actual modelling of the asset allocation process. As such it pulls together many features and results provided in earlier chapters. It

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[1] Called attribution analysis. This is defined further in 5.3 and is also explained in the glossary.

considers the sensitivity of the modelling process to variations in input assumptions. This then leads us to our conclusions in terms of the strategic direction we should take. From a philosophic view, we are not providing just one piece of evidence, but we are bringing together many strands in the evaluation. Each provides extra support, which all point towards the conclusions.

5.2 Asset Allocation

Whilst the literature review has covered this segment in part, nevertheless it is worth re-considering some of the previous studies, outlined there, and their conclusions and difficulties as they have a direct bearing on this chapter.

The field of asset allocation and the issue of market timing has become a major topic of research. The number of research papers has increased dramatically, which, given the importance of asset allocation, is not surprising. Of note are a contribution from Fama (1972), where he discusses the components of investment performance, and papers from Kon and Jen (1979) and Kon (1983). These later papers evaluate the investment skills of mutual fund managers, though it should be noted only in the context of equity funds where timing is to be measured against the benchmark of being either in cash or equities. Kon and Jen used a statistical procedure to evaluate the contributions of timing and selectivity[2]. They found that of 37 funds tested, 25 had positive or value adding selectivity, 5 of which were significant at the 5% level. With respect to timing they found only 14 of the 37 had positive timing estimates and none were statistically significant at the 5% level. A more theoretical discussion on the topic is presented in the paper by Admati, Bhattacharya, Pfleiderer and Ross (1986), which widens the issue of how to evaluate timing and selectivity issues via different models in the absence of attribution data, discussed in detail below.

[2] Formal definitions follow in 5.3. Timing refers to the selection of a particular asset class, whereas selectivity refers to the choice of particular securities *within* an asset class.

As well as the contribution from academics there is also a very significant input from practitioners, particularly in the field of asset consulting, as well as the funds management business, where they often have access to far more information. A raft of these are available, a small portion of which are given in the references. The returns to market timing are large as indicated in Sy (1990), see below; and many practitioners feel that not only should it work (see Droms (1989), Clarke, Fitzgerald, Berent and Statman (1989) or Klemkosky and Bharati (1995)), but that it actually does. Vandell and Stevens (1989) conducted an empirical examination of the Wells Fargo timing system (they are a very large passive manager in the U.S.), where they concluded that that portfolio performance can be improved by market timing. It is noted that the Wells Fargo approach really times extreme markets; they comment, "A soundly conceived and disciplined approach to timing can reduce downside risk and improve average performance over a cycle". Other papers include a series initiated by work from Wagner, Shellans and Paul (1992), who claim market timing skills. This was based upon a study of managers that were used by institutions, that is, a research firm kept all the relevant data. Brocato and Chandy (1994), replied, pointing out that *survivorship bias* is present. This means that the firms that are unsuccessful tend to disappear, and so we are left with the ones (who may well have just been lucky), but the results of which are more likely to be good over the reporting period, as they remain in business. Larsen and Wozniak (1995), continued the debate by putting forward a discrete regression model, giving them improved timing results. Finally, it is worth mentioning a paper by Beebower and Varikooty (1991), who point out how difficult it actually is to measure market timing ability.

There are several items in the compendium edited by Ball, Brown, Finn and Officer (1989), examining the topic of performance in Australia. A paper by Robson (1986), examines the performance of unit trusts over the period January 1969 to December 1978, including balanced funds. He found that the overall performance was below his calculated benchmark and there was no consistency in performance. However he did find stability in risk levels and correspondence between risk and objectives. Bird; Chin and McCrae (1982), examined superannuation funds and found, applying the Jensen measure of performance (see below section 5.4 for definitions and

Appendix 2 for an example), that managers were unable to outperform a passive indexation strategy.

To a large extent this is a practical debate ultimately resolvable in the market place. In the context of this document, we need to attempt to find answers to the two questions:-

1. Have fund managers shown an ability to time markets?
2. If not, then is it reasonable for them to be able to do so, for example on the basis of mean reversion?

A secondary issue is, 'What impact would the predictability or otherwise of markets have on the asset allocation process?' This has particular relevance to the Markowitz mean-variance optimisation procedure.

In our study we will focus on balanced funds, that is where investment is across all asset classes. It is noted that most previous studies have concentrated on equity mutual fund managers in the United States. Thus, it was possible to use the Capital Asset Pricing Model (CAPM) as the risk benchmark. Hence the studies asked the timing question of whether the manager was in cash or equities at the right time. Classic studies like those of Sharpe (1975) attempted to determine what benefits could be gained from timing, by considering bull and bear markets and the returns from 'perfect' timing, and thus how right an investor needed to be to add value. He concluded that the forecasts needed to be right 75% of the time. More recently Sy (1990) disputes this claiming that you don't have to be right anything like 75% of the time, and the gains are large. However, there was a choice of only two assets viz. the riskless asset, and the market (in this case defined as the S&P 500, or some suitable proxy). We have a much wider decision as to the appropriate benchmark to use to be able to judge timing. Our approach will therefore consist of taking different views of the results to build up a detailed picture of actual performance.

Before moving on to look at our data and discussing the issue we need some background considerations; that is, we need to be clear about our definitions and understand some of the limitations of the data before trying to read too much into them, or draw conclusions which may be unwarranted.

5.3 Data

The primary source of information for this study is an actuarial database called 'SUPERCMS'. The information provided (mentioned briefly in Chapter 3) comes from surveys conducted by the research house (Rainbow Corporation) working in close conjunction with an actuarial consulting house. The database is extremely comprehensive aiming to cater for a wide market of asset consultants, trustees, fund managers and so on. Thus, for example there is a very powerful software package provided which can present raw data in a more digestible form for further analysis. Hence the results have been taken under the assumption of integrity of the package. Further, whilst we can be fairly sure of the accuracy of the data on financial assets, some of the survey data, particularly as it relates to asset allocation policy, may well have some problems.

Another significant feature of the data is the changes that have occurred in the business. The number of funds under management has more than doubled over the last ten years. Thus, it is not surprising, that with such growth there has been significant changes in individual fund management teams. Indeed it is probably fair to say that there would be few, if any, management teams that have remained intact over the period. This means that there will be changes in style and approach which would make results hard to compare over time. Nevertheless, the issue for the investing public remains. We must test our hypothesis given the reality of a changing marketplace and players. It is difficult to see the industry dynamics changing in the near future.

Finally, fund names have been eliminated and, where appropriate, replaced with a number code. This maintains the confidentiality of the data, provided by the consulting group.

5.4 Some Definitions

At the outset we need to be clear about the difference between selectivity and timing, that is the difference between adding value by security selection within an asset class and adding value by being in the correct asset class at the right time. We must also separate out returns from active timing (so called tactical asset allocation) from the long run asset allocation or policy portfolio, based upon an assessment of the asset-liability structure of a particular client or the fund managers balanced fund approach. That is, at the wholesale level, the manager presents benchmark asset allocations for the policy portfolio, for example as an input to the determination of a clients' efficient frontier. Therefore a measure of value added in a timing sense can only be relative to a pre-set benchmark, which therefore determines the level of risk. Clearly, a more aggressive and thus risky asset allocation can lead to higher returns -if we assume increased risk leads to increased return-but value added by timing can only be assessed by what the intention actually was. In other words we need a background against which we can assess timing.

The following outlines the technical definitions, with some terms reproduced from the glossary, for ease of reference. An example is given to enhance the understanding of the terms.

Sector benchmark return: this is the return of an appropriate index representing movements in the asset class as a whole. For example, the All Ordinaries accumulation index would represent a suitable benchmark for Australian equities.

Policy portfolio: this is the benchmark asset allocation determined by the manager, representing the managers stance in the market or particular client objectives. The policy portfolio is thus determined in advance.

Return from market timing: this is the difference between the actual and benchmark asset allocation for each sector multiplied by the sector benchmark return. This is done each month and summed. See equation 1 below.

Return from security selection: this is the difference between the return of the fund and the sector benchmark return multiplied by the actual asset allocation. This is done each month and summed. See equation 2 below.

The difference between the total returns of the fund and that due to timing and selection, is set equal to the returns to the policy portfolio (or long run asset allocation). This means that since the 'total return' is known and that from 'timing' and 'security selection' can be calculated, formally as below, then that for 'policy allocation' is determined by subtraction and includes a small cross-product term. Using this approach the total return thus breaks down into three components:-

Total return = Timing + Security selection + Policy allocation

Schematically:-

		Selection	
		Actual	Policy
Timing	Actual	Actual Portfolio Return (A)	Policy and Timing Return (B)
	Policy	Policy and Security Selection Return (C)	Policy Return (D) (Passive Portfolio

Hence returns are

Timing = B - D

Selection = C - D

<u>Other = A - B - C + D</u>

Total = A - D

We may define this a little more clearly in mathematical terms, after Brinson *et al* - see below - by the following:-

Let us assume there are N asset classes, with n_i securities in each asset class. Further we will use the subscript a to denote an actual asset class or security, and p to denote that of the corresponding policy or benchmark portfolio. Then the actual return for the i th asset class is given by:-

$$R_{a,i} = \sum_{j=1}^{n_i} w_{aj,j}\, r_{aj,j}$$

where $w_{aj,j}$ = the weighting of the j th security in the i th asset class for the actual portfolio, and

$r_{aj,j}$ = the return of the j th security in the i th asset class for the actual portfolio.

Then the total return from the actual portfolio may be written:-

$$A = \sum_{i=1}^{N} W_{a,i}\, R_{a,i}$$

where W_{ai} = the weighting of the i th asset class for the actual portfolio.

Hence we may write following these conventions the total return from the policy portfolio as :-

$$P = \sum_{i=1}^{N} W_{p,i} R_{p,i}$$

and the return from timing (T) and selection (S), dropping the range of summation, always 1 to N :-

equation (1) .... $T = \sum \left(W_{a,j} - W_{p,j}\right).R_{p,j} = \sum \left(W_{a,j}.R_{p,j}\right) - \left(W_{p,j}.R_{p,j}\right)$

equation (2) .... $S=\sum W_{p,j}.(R_{a,j}-R_{p,j})=\sum(W_{p,j}.R_{a,j})-(W_{p,j}.R_{p,j})$

Then $A-T-S=\sum W_{a,j}.R_{a,j}-\{\sum(W_{a,j}.R_{p,j})-(W_{p,j}.R_{p,j})\}-$
$\{\sum(W_{p,j}.R_{a,j})-(W_{p,j}.R_{p,j})\}$

$$=\sum W_{a,j}(R_{a,j}-R_{p,j})-\sum W_{p,j}(R_{a,j}-R_{p,j})+\sum W_{p,j}.R_{p,j}$$

$$=\sum(W_{a,j}-W_{p,j})(R_{a,j}-R_{p,j})+P$$

the 'cross-product' term is assumed small relative to the other terms, which is reasonable given weightings and returns are not that different, at least in order of magnitude terms. Referring to Brinson, Hood and Beebower (1986), they find a value of -0.07% for this term by comparison with -0.66% for timing and -0.36% for selection, based on their sample. This term is subsumed in the policy returns, and as such does not alter either the timing or selection returns. The analysis following therefore does not depend upon this variable.

Example

	Actual		Policy	
Bonds	Weight	Return	Weight	Return
Equities	0.5	5.0	0.4	4.0
Total	0.5	2.0	0.6	2.5
		3.5		3.1

Thus timing (T)

Bonds (0.5-0.4) x 4.0 = 0.40

Equities (0.5-0.6) x 2.5 = -0.25

Total = 0.15

Selection (S)

Bonds 0.4 x (5.0-4.0) = 0.40

Equities 0.6 x (2.0-2.5) = -0.30

Total = 0.10

Cross product term

(0.5-0.4)(5.0-4.0) + (0.5-0.6)(2.0-2.5) = 0.10 + 0.05 = 0.15

that is $(0.15+0.10+0.15)=(3.5-3.1)$ and this explains where the 0.4% outperformance has come from. Note that whilst the cross product term has smaller components, they add up, so that it is large in this example. Over a larger array of securities there would be much more cancelling out, in Brinson *et al* above this term only contributed 6% of the total difference between policy and actual returns.

There are other real world caveats. There will be some listed stocks held which are not in the appropriate index, though these should be small relative to the index. On the other hand there is highly likely to be unlisted stocks, for example in the technology or development capital areas or the likes of Optus Communications. Thus whilst we know that $\sum_{j=1}^{n_i} w_{pj} = 1$ we find that $\sum_{j=1}^{n_{ij}} w_{aj} \leq 1$. We are thus not strictly comparing like with like, but we must live with these data errors or inconsistencies.

This process is called *attribution analysis* as it attempts to attribute performance to different components (see Brinson, Hood and Beebower(1986) and the follow-up paper by Brinson, Singer and Beebower (1991) for a detailed discussion on this issue and a comprehensive survey of U.S. experience using this methodology). The principles of attribution analysis depend upon having available very detailed information, from many individual funds management operations at the individual security level. Thus, the information is culled from the respective funds management databases and collected and collated into useable form. For example, to attribute security selection skills we need for each manager the particular asset class portfolio and cash flows thereof to be able to determine the return from their asset class portfolio as compared to the index or benchmark for that asset class. This is very time consuming, and requires the skills to be able to interrogate many different databases with different structures, using different programming tools and so on. Fortunately we have available the detailed information necessary to perform this task.

5.5 Industry Background

The number of Australian funds under management, over the last ten years, has grown as given by the following table:-

Table 5.1 Growth in Funds under Management[3]

years	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994
no. funds	24	26	31	31	31	34	36	40	43	47

and this number had increased further to 49, as of April 1995. Clearly, there are growth problems involved with staff, for example mobility, which must be borne in mind.

Let us next consider some fundamental performance statistics. Measures of performance met in the literature are the Sharpe, Treynor and Jensen indices. Given the near equivalence of the Sharpe and Treynor indices we will only consider two. Further given we are assessing performance it is valuable to have a benchmark or null portfolio. It was possible to find a number of funds with a stated policy portfolio. Thus an average portfolio was formed, and a static asset allocation applied. Whilst it is clear that the actual funds may have different objectives it does give a totally passive benchmark (and so at the very least would be cheap to run - the task is programmable). This benchmark is thus not merely an averaging of performance since the point is to get some measure of overall value added. Note that one can safely assume that all

[3]The source of this information is the SuperCMS database.

balanced fund managers are market timers, that is timing a move from one asset class to another, since otherwise they would just be sector managers.

The table given in Appendix 1, the Sharpe Index Table, ranked by index score, was extracted from the system. Note that fund 19 was the average asset allocation (AAA) fund.[4] On the whole, a very ordinary set of results, particularly when one considers that the policy portfolio includes assets which have returned much better results than the 13-week T-note. This period, for example, produced one of the best bond markets for many years. Also of course these results are gross of fees, that is we would have to deduct fees from the returns. It is hard to see these being less than 1%, and of course for the retail public, far higher. Adding 1% to our neutral portfolio would give a return of 11.27% and rank it 7th. Clearly, most private clients would be well advised to manage their own funds by sticking to a simple strategy. Be that as it may, this is not our brief.

Let us now consider the Jensen performance index. This is based upon the CAPM :-

$$E(R_{p,t}) = R_{f,t} + \beta_t \left[E(R_{m,t}) - R_{f,t} \right]$$

where $R_{p,t}$ = the return on the portfolio at time t

$R_{m,t}$ = the return on the market at time t

$R_{f,t}$ = the return on the risk free asset at time t

β_t = the beta of the asset

which translates to the expected one period return at time t on a given investment being equal to the current risk free rate plus a risk premium. That premium is given by the beta of the asset times the excess of the expected return of the market over the

[4] The AAA fund had fixed weights being, 7% cash; 38% Australian equities; 21% international equities; 21% Australian fixed interest; 2% International fixed interest and 11% property. Other assets like indexed bonds were not included. Also property trusts and direct property were merged as often within the policy portfolio it was not clear which was to be used.

risk free rate. The market model then provides the benchmark for risk. Hence a regression of the fund return on this model delivers a measure of relative return for a given level of risk.

Jensen's alpha is thus given by the following regression equation:-

$$\left(R_{p,t} - R_{f,t}\right) = \alpha_p + \beta_p\left(R_{m,t} - R_{f,t}\right) + \varepsilon_t$$

estimated from the actual data and ε_t is an 'error ' term. The α_p term is a measure of the constant periodic return that a manager is able to earn above a passive portfolio of equal risk. Hence a positive alpha would represent value added by the fund, and vice versa. The beta is the standard measure, that is the leverage of the portfolio against the benchmark (for example the market; which in our case is the All Ordinaries Accumulation Index).

Jensen's alpha was originally used in the case of the equity market. We may then extend this concept to the case where the portfolio comprises a set of asset classes, rather than the equity asset class and cash only. Using the SUPERCMS package a number of regressions were run, using various benchmarks. The different benchmarks chosen were 13-week Treasury notes, wholesale cash and our AAA portfolio (see note 5). The cash based alternatives gave a very low R^2, which is not surprising as the rate is relatively constant and will not fluctuate with the portfolio, as background, so to speak. Of more interest is the null portfolio, AAA, the results of which are given in the attached Appendix 2 (since there are a lot of output tables in all). The same coding system was used as before. Hence we can conclude a similar result in terms of the rankings. The alpha's are generally positive with few funds showing negative values (only 5 out of 31 in the survey).

These results are of limited value however since they take no account of the risk inherent in the actual policy portfolio. Thus a comparison between a deliberately low risk and deliberately high risk fund is not a fair comparison. Further we have had reasonably good results from markets, so they will all look good by comparison with cash. Also, and of more note, is the fact that funds tend to move together. Managers are very aware of the competition, since there are plenty of surveys. Thus any manager who takes an allocation different from the pack is taking a great risk. If it turns out right, that particular manager will look like a 'star'; if not the fund will run out of money to manage. Indeed a detailed analysis of the returns reveals just that. The most highly regarded manager recently, ignored the stated policy guidelines and exited property entirely (indeed the paper by Ankrim (1992) suggests that managers "..systematically choose portfolios whose risk levels differ from their appropriate benchmarks.....Research so far suggests that for as many as one-third of certain style managers, this risk adjustment could be as large as 160 to 240 basis points[5] a year"). The larger funds won't take this risk. Therefore we should expect them *all* to do well or *all* to do badly. Hence we need to consider individual results. One way to do this is to look at their aggregate performance over many years.

5.6 Portfolio Performance and Ranking's

If we break down the total return into its components, as in 5.4, then the largest contribution is the policy portfolio allocation. For example, see the paper by Brinson, Singer and Beebower (1991), where their study of 82 large US pension plans showed that, on average 91.5% of the total returns came from the policy portfolio and 93.3% came from policy and allocation. Now the effect of the aggregate of policy allocation and any outperformance by superior timing or security selection should be predictable (we will look at contributions to the total return in 5.7). That is, we would expect the general public to be able to detect differences in total return over a suitably long time period. One way of testing this particular proposition is to look at the annual returns for each fund over a 7 and 10-year time period. We have sufficient historical data to do this for 31 funds over 7 years and 24 over 10 years. Although we have numerical returns, it is most useful also to consider rankings. The use of such non-parametric tests, whilst not using some information content, does not rely on the kind

[5] A term used in the bond market, where 1 basis point (b.p.) = 0.01%.

of assumptions needed for a t-test. The variances of the returns are not all equal, as we shall see. Further, it was found that, for example, there was very little difference in the correlations between the years, from using actual returns and the rankings. Note that in all results given below details are in the Appendices. Base data is available on request.

In the parametric case, an analysis of variance was carried out. This is to test the null hypothesis of the equality of the mean returns versus the alternative of at least two of the returns being unequal, on the assumption of equal variances. For the 7-year returns we find an F-value of 0.7987, based upon 31x6 = 186 degrees of freedom. This compares to a critical value of F(30,186) at the 0.05 level of significance of 1.5207. Correspondingly, for the 10-year returns we have an F-value of 0.0675 compared to a critical value of F(23,216) of 1.5794 at the 5% level of significance. On the basis of these results there is no significant difference in the average performance of the funds. We do need the assumptions that the samples are independent, normally distributed and have a common variance. We may accept the first two, though as we have previously noted managers do tend to act together and we have not tested the normality assumption, but we do need to check the common variance assumption.

A cursory view of the ANOVA table indicates that, especially in the 7-year case, there are a couple of outlier cases with very high variance relative to the remainder of the sample. We may use Bartlett's test, which was outlined earlier in our discussion of security price distributions (see section 4.7), to test for the equality of the variances. Here the value of b, given below follows a χ^2 distribution with $(k-1)$ degrees of freedom, k being the number of samples, in our case the number of funds.

Table 5.2 Bartletts Test Equality of Variances

No. Years	"b" value	deg. free.	χ^2 (0.05)
7	63.49	30	43.773
7*	10.45	28	41.337
10	34.01	23	35.172
10**	24.04	22	33.924

We can see the results of the test. In the 7-year case 7*, the two outlier funds were dropped (anecdotally they are very aggressive 'entrepreneurial' funds). The test value is now well within the 5% critical region. Similarly with the 10-year result, where only one of the two funds was represented-and a corresponding lower b value for the initial sample. Hence, in 10**, this fund was dropped and we have a result inside the critical region, though not as low as 7*.

A re-working of the one-way analysis of variance with the two funds dropped as in 7* and one fund as in 10* as above gave the following results. For the 7-year returns we find a new F-value of 0.6192, based upon 29x6 = 174 degrees of freedom, which compares to a critical value of F(28,174) of 1.5412, at the 5% level of significance. Correspondingly, for the 10-year returns we have an F-value of 0.0775 compared to a critical value of F(22,207) of 1.5939 at the 5% level of significance.

Thus, statistically we may draw a conclusion that there is no significant difference between the average results for the collection of managers in the survey. However, we must also be aware, as with tests of the random walk hypothesis, that this is a very demanding test. When discussing the measurement of market timing strategies, Beebower and Varikooty (1991) point out how hard it is to measure a manager's skills when the value is moderate, say at the 2% per annum level. In their paper they carried out a simulation study to generate various levels of value added by timers switching between the S&P 500 and US Treasury bills. They then measured the market timing abilities of these timers for various levels of outperformance. Thus a manager adding *no* value underperforms the S&P 500 about 42% of the time, whilst one adding 1.5-2% p.a. might be expected to *under*perform 35% of the time. Thus they conclude "Most common existing tests to detect statistically significant ability equal to about 2% excess return per year require time periods well beyond human life expectancy" (p.78). And it is also true that 2% per annum compound excess return over a 10-year period is a great deal of money, and well worth attempting to discover.

To expand this analysis we need to look at finer detail. One way of doing this is to calculate the correlation between yearly returns. That is we need to consider not just aggregate trends but whether there is any consistency over the years. Indeed there may well be reversion to the mean in funds management performance, due to different styles, for example, delivering better or worse results in periods more or less suited to that particular style. Rather than use actual values ranks were used instead. The use of a non-parametric approach eliminates any assumptions about underlying distributions. Results in any case, were very similar in those cases where a comparison was made.

Firstly the average rank correlation was calculated according to the formula from Mosteller and Rourke (1973) p.226 (see Appendix 3). Then Spearman's rank correlation coefficients were determined. The average rank correlation over 10 years was 0.0025, and over 7, 0.0945. These are extremely low numbers. The coefficients in Appendix 4, give an idea of the spread. On a two-tailed test only the correlations between YEAR4 and YEAR's 2 and 3 are significant. However given $m(m-1)/2=21$ combinations for $m=7$ years we would expect one or two significant at the 5% level on the basis of chance alone. For the 10-year data there are 45 different combinations. Of these there are 5 significant values, however two are negative and three are positive. In summary, this is good evidence in favour of the unpredictability of performance.

We may extend this argument further and apply the Friedman test. This is a test of the randomness of the correlations. When the null hypothesis of m random independent ranking's of I items, holds this statistic has a χ^2 distribution with

$(I-1)$ degrees of freedom, where the Friedman statistic is given by (Mosteller and Rourke, p.229):-

$$\chi_r^{\ 2} = \frac{12}{mI(I+1)}\sum_{i=1}^{I} R_i^{\ 2} - 3m(I+1),$$

and $\sum_{j=1}^{m} r_j = R_i$, where r_j is the j-th ranking of item I.

Using this test we obtain a value of 47.005 for the 7-year data ($m = 7, I = 31$) by comparison with the critical value of $\chi_{0.05}^{\ 2} = 43.773$ with 30 d.f. Hence we reject the null hypothesis of random ranking's at the 5% level though not at the 1% level, for which the critical value is 50.892. On the other hand using the 10-year data we find a value for the Friedman statistic of 23.524 ($m = 10, I = 24$), which compares to a critical value of 35.172 at the 5% level and thus accept the null hypothesis. The results are inconclusive, though one would be tempted to place more weight on the longer time period. To clarify the issue, it was decided to divide the 10-year period into two halves and see whether there was any relationship between them. The result of this was a rank correlation coefficient of -0.1504 (compared to a coefficient of correlation using actual returns of -0.1532) which is certainly not significant in a sample of 24.

In conclusion, at the aggregate or fund level, there would appear to be no predictability from past performance. We cannot conclude that necessarily good performances or a run of such, are followed by bad performances or a run thereof, merely there is no value in the current ranking as a guide to future rankings. Clearly, this does not tell us why this is so. We cannot tell whether the source is poor timing decisions, poor security selection or a combination (we must also remember the power of these tests to discriminate is also not high). Thus we must next consider attribution analysis, to attempt to disaggregate the sources of performance.

5.7 Attribution Analysis

At the outset, we need to note that there may well be a substantial difference between the policy portfolio and the average asset allocation over a short time period, say 2-3 years. However, they really should be the same over a large enough time period (and we have data for the period 31/12/88 to 30/4/95). As we shall see in the analysis this is not necessarily so. Thus whilst we can be reasonably happy with the attribution due to security selection, since it merely substitutes an appropriate index for the actual asset class choice with the given asset allocation, the same is not true for timing. Hence we really do need the stated policy portfolio to determine the value added by deviations from the allocation. Unfortunately, we only have this for a limited sample of the funds (13 out of 31), and it is fair to say that the data stated is by no means clear cut. It may well be a limitation of the survey, but one would have to question the clarity and discipline with which most fund managers are approaching this key issue. One would, perhaps somewhat cynically rather suspect that managers are driven more by what the competition are doing, or allowing valuation relativities to drive asset allocation, than by sticking to a strategic purpose (see Ankrim (1992) mentioned above). We will examine this aspect of the results.

Firstly, the average asset allocation was applied to each of the funds. We can then examine the returns, via attribution analysis. Thus, in particular, we can extract from this the security selection information, as we exhibited in Appendix 5. This information was also determined by dividing the time period in two, and thus providing two averages to deviate from. As we can see value was not added by security selection. In fact in aggregate -6.75% was added on average. We may use a standard t-test, to find if the value is significantly different from zero. The t-value obtained was -4.50 by comparison with a critical value of 2.056 at the 5% level of significance for 26 degrees of freedom, and so we can conclude, that managers' subtract value by security selection. At the 1% level the critical value is 2.779, still well below the t-value found. A non-parametric sign test was also used, to eliminate any assumptions about the distribution of the attribution. A value of $p = 0.0021$ was found using the sign or binomial test, with 22 scores being less than 0 and only 5 greater. We can thus be reasonably sure that this result is not by chance.

Next the data was divided into two time periods of equal length, from 31/12/88 to 31/1/92 and from 28/2/92 to 30/4/95. Once again, the returns were examined via attribution analysis for these two sections and similar tests applied. For the first time period 18 out of 27 of the returns were negative; however the t-value obtained was -1.42 which is not significant at the 5% level. For the second period 23 out of 27 funds had negative contributions from security selection, with a t-value of -6.84, which is highly significant. Further only two funds showed a positive contribution in both time periods (and only just). Taken together, these results must lead to the conclusion that fund managers do not add value by security selection but subtract it. And this is before costs.

Next we must consider the timing aspect of the attribution analysis. As foreshadowed, regrettably average asset allocations are a very poor benchmark. The results in Appendix 5, show too much variability for the average to be taken as representing a benchmark. The policy portfolio should show the majority of the returns, since tactical asset allocation should be just that; if the average over a period deviates sufficiently then the strategy is not being followed. Naively, we would find a positive t-value from the attribution, but we cannot assume that this is from timing alone. When we divide the period into two, value looks like it is being added, but not at a significant level. To help answer the timing question we must :-

1. Look at the policy portfolio as a benchmark.
2. Consider the extent to which those managers that give a policy portfolio, actually adhere to it.

Referring to the second table in Appendix 5, we have a batch of 14 managers only. One seems to be not adhering to the policy portfolio, it was decided to remove this result; there could be an error in the survey, for example. We are not in a position to investigate further, but must recognise the bias which would arise if we left it in. So we are down to 13 funds. The time period was split, as before, and t-tests performed on the attribution's, for all time periods. The table below summarises the position.

Table 5.3 Value Added from Market Timing based upon the Policy Portfolio (t and binomial tests)

Time Period	Mean	t-value	2-tail sig.	No. above 0	2-tail p
12/88-1/92	2.7100	3.22	.007	12	.0034
2/92-4/95	1.8538	2.59	.024	9	.2668
12/88-4/95	4.4700	3.17	.008	10	.0923

As can be seen, it would seem that value is indeed being added by timing for our sample. There are two caveats to this. Firstly we must be aware of the tendency of all funds to move together, so it could be argued they all went the right way at the right time. We must allow some benefit for this however. Secondly, we must acknowledge valuation benefits, as we shall see, letting the asset allocation drift can produce 'timing' returns, particularly if there is short term positive autocorrelation in asset classes. One can argue both ways; either this drift is a deliberate decision hence managers should get the benefit or inertia plus the costs of small short term changes makes this aspect of market timing involuntary. Nevertheless, in net, they appear to add value.

On the question of adherence to the policy portfolio, the actual asset allocations by month were extracted. This allows a calculation of mean and standard deviation of moves within an asset class. We can thus compare these features with the stated policy portfolio and check for significant deviations. The procedure was to determine the average asset allocations and subtract them from the stated policy allocations. These differences were then divided by twice the standard deviation of the actual asset allocations. This then gives a standardised measure of the significance of the deviations. Some approximations were necessary in matching volatility's to asset classes but these were minor. The situation is as seen in Appendix 6. Row totals do not add to 0, as some funds invested in minor asset classes, not always represented in other funds, for example indexed bonds. These are not large, though non-negligible, in any case, we are more interested in the larger issues; the major asset classes. Property is an interesting case. Some managers have used the listed market, rather than their stated intention of the direct market. Whilst there are clear differences between the two, one would not regard the substitution of one for the other as being as serious a matter as say, stocks for bonds. The correlations between direct and listed property should be close, particularly over a more extended time period. There are some very substantial *absolute* differences between the policy and average portfolios, particularly when it is remembered that the size of the allocation is not that large. In the case of international equities, for example, the average weighting was 12.6% below the stated policy portfolio, ranging from 31.3% below to 1.5% above. Australian equities on the other hand only averaged 2.0% above the stated policy portfolio, but this disguises a spread of 9.0% below to 20.4% above. This large deviation over such a substantial time period must beg the question as to quite what the policy portfolio actually represents. Whilst superannuation liabilities, like life and liability insurance are long term, just how long term is that supposed to be? It is by no means clear over what period the average asset allocation is supposed to match the stated policy allocation.

Next we can see how substantial are these deviations in the light of the volatility of the asset allocations. Property and property trusts were merged for the purposes of this calculation, for the reasons stated above. The second table in Appendix 6 details the findings. As can be seen, most funds are within 2 standard deviation limits of their policy benchmarks for each asset class. However, it is fair to say that the volatility is very high for most asset classes. To be as consistently far away from benchmark guidelines as some funds are, must be a cause for concern. If, as in the US studies, most returns come from the policy portfolio, then we will not see that result come through. That is, clients will not be getting what they expect. There were differences in the average weighting's given to different asset classes, but our results showed that whatever the differences might be, they ultimately were not detectable by consistency of performance. Further research into just what the results would be if funds, with different policy portfolios, rigidly adhered to them with benchmark returns for asset classes, would be most useful. We might, perhaps, risk adjust our performance attribution along the lines suggested by Ankrim (1992), and thus compensate for the deviations from the benchmark. It may help us decide what conclusions we can draw from performance numbers. That is, the degree of confidence one could have in, say, 5 year numbers ranking various managers.

In summary, it would seem managers do not add value by security selection, but do by timing even though they do not adhere too closely to their policy portfolios (that is when they are given). As noted above, however we must look a little more closely at valuation effects. On an *a priori* basis, there is reason to believe that *if* there is positive autocorrelation in markets, then we would expect to see value added by timing due to a favourable trend. Given the work we have done earlier on the three major asset classes; Australian equities, international equities and Australian bonds, with cash of course (the T-note), it was decided to form a portfolio simulation with these constituents, but no re-balancing.

The weighting's chosen were 8% cash, 25% Australian bonds, 24% international equities and 43% Australian equities. These were chosen on the basis of the average weighting's by fund managers scaled up to 100%, and are thus the same as we used in our AAA portfolio in 5.5 above. The returns were simply accumulated and the new weighting in the portfolio calculated. This, of course, ignores taxes and charges as well as the fact that income from investments is available to be re-invested in other asset classes, if needs be. Nevertheless there will be 'drift' in the portfolio; this will at least give us some feel for its possible importance. The graph below represents an area chart of the relative weighting's over time and gives a nice pictorial view of what is occurring.

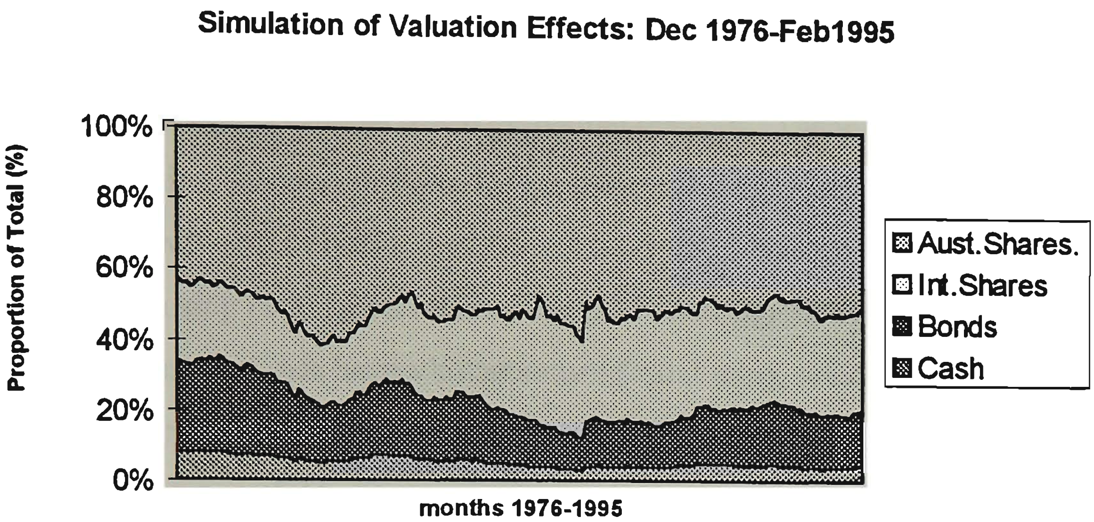

Figure 5.1 Valuation Effects on Weighting's, No Re-balancing

We can see some very clear trends in relative weights due to valuation over the period 12/76 to 2/95. Bonds went through a very poor period culminating in a proportion of 22.03% weighting in February 1984. Since then it has recovered to a weight of 29.46 as at February 1995. With the benefit of hindsight this was precisely when to re-weight to bonds. How many managers would have done that with the weight of so many poor years behind them? Further and most interestingly, at February 1984, Australian equities were at 54.08%, (up from 43% at the start), and reached a peak of 60.17 just before the 1987 crash in September 1987, and as at February 1995 stood at 50.21%. Nevertheless over a shorter period of time the graph demonstrates the benefits of following an existing trend. It may take years before the wisdom of a decision comes through and by then the manager may have no funds left under management. So called short-termism is thus a very sensible policy if the desire is to maintain one's salary.

We can thus see the valuation effect is very clear and substantial. We cannot, of course, differentiate between the intentional moves and those that happen on an involuntary basis. We assume that the move is intentional, even if the decision is to let the market or allocation run. However it is important to be aware of this.

Our final piece of analysis in this section is to assess whether there is any connection between the volatility of asset allocation and attribution from timing. To do this rather than take the volatility of the fund as a whole where individual effects would cancel out, the volatility's of the individual asset classes were added up. This gives us a measure of the aggressiveness of the individual manager. These totals were then correlated against the returns from market timing. the correlation was 0.5713, with a significance level of 0.041, that is at the 5% level, for 13 observations. A similar result was obtained using ranks; a correlation of 0.5934 and significant at the 5% level. That is, more aggressive funds have a better return from timing, which is an interesting result. Of course we do not have a large sample, but it would tend to suggest that not only is timing successful but some short term finessing may also be possible. We should perhaps not make too much of this, but it is certainly an area which would merit future review with more data available, particularly with respect to the relevant policy portfolios.

We must now move away from our review of the actual performance of funds to some consideration of efficient frontiers and models used in the asset allocation process.

5.8 The Modelling Process: Efficient Frontiers

In this section we are assessing the significance of various assumptions on the asset allocation optimisation process. We have come to some conclusions about the nature of our series; what we wish to assess is their importance from a strategic perspective.

The use of the mean-variance optimisation process for the efficient allocation of assets, is a widely used tool of modern finance theory. A superannuation fund manager will conduct an in-depth *asset-liability study*[6] to determine the long term financial environment. This sets the parameters within which the policy portfolio can be set. This will determine the appropriate risk/return trade-off which is required to meet a specific set of objectives determined from the asset-liability study. Thus, for

[6] See the glossary for a fuller explanation of this concept.

example, a fund may determine that the probability of a negative return should be below a certain level, (typically the case for an accumulation style superannuation fund or, say, a capital guaranteed style of product). In this case, we depend upon the validity of the model to be able to deduce suitable probabilities based upon historical data.

Essentially, the optimisation process takes a set of asset classes, with their mean returns, variances and correlations between the asset classes and aims to pick the asset class weights which will give the best possible return for a given level of risk. A plot of the set of points achieving this is called the *efficient frontier*. The optimisation process may or may not be constrained. Certainly negative weights are not a feasible solution (theoretically, one could be perpetually short an asset class but this is not allowable in practice). The optimisation process assumes a normal distribution for the returns.

The optimisation algorithm then calculates the returns and risk (standard deviation) for combinations of the assets selecting those with the highest return with a given level of risk, using the formulae:-

$$r_p = \sum_{i=1}^{n} w_i \, r_i$$

$$\sigma_p = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \rho_{i,j} \sigma_i \sigma_j}$$

where r_p = return of portfolio

r_i = return of asset i

w_i = weighting of asset i

w_j = weighting of asset j

σ_p = standard deviation of portfolio

σ_i = standard deviation of asset i

σ_j = standard deviation of asset j

$\rho_{i,j}$ = correlation coefficient between asset i and asset j

and there are n assets in total to choose from. We may put the above in matrix terms by letting $\sigma_{ij} = \rho_{ij}\sigma_i\sigma_j$, which forms the variance-covariance matrix, say, $\mathbf{M}$ and $\mathbf{w}$ is the vector of weights and $\mathbf{r}$ is the vector of returns then

$$r_p = \mathbf{w}'\mathbf{r} \quad \text{and} \quad \sigma_p = \mathbf{w}'\mathbf{M}\,\mathbf{w}$$

We may also modify returns, standard deviations or correlations of input variables. We can thus study the impact of changing the inputs, that is, if we have an incorrect estimate of the mean or standard deviation of an asset class.

We have seen that the major asset classes under study have the characteristics outlined below and we now wish to see what impact these characteristics have on the efficient frontier.

1. Persistent or positively autocorrelated in the short term. In this document the evidence of 3.2.2 and 3.3-3.4 suggests that there is persistence or that markets are weak form inefficient, at least for stock prices. We may refer to Tables 3.1 and 3.4, in particular. Note that we are concerned with trends *within* an asset class not relative trends *between* asset classes (see Figure 5.1).

2. Mean reverting or negatively autocorrelated in the long term. The significance level is a difficulty - again we may note Summers' (1986) comments referred to at the start of Chapter 4. Whilst evidence in favour is presented here in 3.3-3.5, we are not able to conclude at the 95% level of confidence for all asset classes. Our empirical revue in 4.2-4.4 revealed, in part, why this was so.

3. Heteroscedastic. There would appear to be general agreement about this for many financial time series, though the literature has focussed on stock prices (see Schwert (1989) for US stock returns). Our historical revue in 3.2.1 and

more detailed discussion of recent times in 4.5.3, showed very clear changes in the level of variance, for the AOIA series.

It is noted that, given the assumption of a normal distribution, we do not change the underlying distribution to make it more leptokurtic. Our review of this feature of asset returns indicated that we could be fairly relaxed about this particular assumption, for the major asset classes, as covered in 4.6 and 4.7. To the extent that heteroscedasticity is the cause of this, we can get some idea of the robustness of the assumption by varying input standard deviations.

Given that we are examining the three major asset classes and the risk free rate, our baseline efficient frontier was composed of those four asset classes. We have examined these in detail both above in this chapter and also in our discussion of mean reversion which is both the background and motivation for this document. Hence we have a degree of internal consistency within which to 'dimension' some of our variables as an input to the investment policy decision. Data from 31/12/79 to 30/4/95[7] was available for the asset classes mentioned, which is a sufficiently long period for use in policy portfolio formulation, particularly when it is remembered that the A$ exchange rate was controlled up to 1983, with all the attendant changes in the currency and bond markets. Note that this is *not* an in-depth study of the optimisation process[8]; our brief herein is much more limited. We seek to link the logic of mean reversion with portfolio management and practice with an eye to some concrete conclusions and potential advice, for example, for different classes of investor, based upon the evidence that this study has demonstrated.

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[7] The data for the asset classes is the same throughout, indeed they are obtained from the same database and as noted previously were checked for consistency. Note also that the data series as input to the optimiser commence in Dec 1979, that is the start of the ASX. The only difference from earlier analyses is thus the data period. Logically and for reasons outlined previouly, it is sensible to not use data from earlier times due to the changes in the markets, let alone data problems, for example, the availability of month-end accumulation indices for the All Ordinaries.

[8] There are studies considering this aspect in more detail, for example a paper by Jorion (1992), where he considers the impact on the optimisation process of measurement error in the input variables

The time period was divided in two, that is from 31/12/79 to 31/1/88 and 28/2/88 to 30/4/95. As background the performance statistics given in Table 5.4 were generated, where we define the measures as follows:

Annualised Return (% p.a.) : Ann Ret $= \left[\left(\frac{I_e}{I_s} \right)^{1/y} - 1 \right]$ -the geometric return.

Annualised Standard Deviation (% p.a.) : Std Dev $= \left[\frac{12}{n} \sum_{i=1}^{n} (r_i - \bar{r})^2 \right]^{1/2}$

where I_e = index value at end date (the series z_t for the AOIA)

I_s = index value at start date

y = number of years in period

r_t = return for time t (as in 3.2.1)

$\bar{r}$ = average monthly return (as in 3.2.1)

Correlation (between asset x and asset y) :

$$corr = \left(\frac{1}{(S_x S_y)} \right) \left[\frac{\left(n \sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i \right)}{n(n-1)} \right]$$

where x_i = return of asset x for month i

y_i = return of asset y for month i

S_x = standard deviation of asset x

S_y = standard deviation of asset y

using the standard definition of the correlation coefficient and the relation (see Walpole and Myers (1972) p.159) :-

$$S_x^2 = \frac{n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2}{n(n-1)}$$

Table 5.4 Returns and Risk for Given Asset Classes

	31/12/79-31/1/88		28/2/88-30/4/95		31/12/79-30/4/95	
Asset	Ann Ret	Std Dev	Ann Ret	Std Dev	Ann Ret	Std Dev
aoiacc	16.81	25.62	12.09	15.31	14.46	21.37
cbbiamat	13.01	6.26	12.04	5.53	12.53	5.92
tnotes13	12.95	0.65	9.82	1.15	11.46	1.00
mscixa$a	26.77	17.29	8.23	16.56	17.96	17.09

Interestingly enough the correlation matrix was surprisingly stable. This was dealt with at some length in 3.3, where the cross correlations were given. The only significant correlations were between the 'aoiacc' and 'cbbiamat', approximately 0.4 and between the 'aoiacc' and 'mscixa$a', approximately 0.3.

The above table is in nominal terms, and we should note the much lower inflation in the second half. This really underscores the mean reversion results given in 3.3, and the simulation of valuation effects chart given in 5.6 above. The very high 'mscixa$a' return for the first period, was corrected by a very low return in the second, particularly when risk is taken into account. Bonds on the other hand after suffering for years really performed. We may also note our results from our examination of the performance of fund managers. It is commented in section 5.4 that the most highly regarded manager, notwithstanding the stated policy portfolio, *exited property entirely*, quite correctly, due to the excess returns generated in the prior period, particularly in the commercial property market. We will return to this counter-cyclical allocation issue, but note in passing the Wells Fargo approach documented in Vandell and Stevens (1989), referred to in 5.1.

For each time period, the actual efficient frontiers[9] [10] were generated, as shown in the plot in Figure 5.2. Next a sensitivity analysis was conducted by varying the level of various inputs to the optimisation process. Firstly, given heteroscedasticity in stock prices, the standard deviation was increased by 50%, then 100% and results generated for each time period. Then the returns on the 'aoiacc' was varied by 2% up and down to estimate the impact. These results are given below in Table 5.5.

This table is an attempt to dimension the inputs by considering the range of returns and risk as well as generating another statistic the *average ratio of risk to return*, by which we mean the average ratio over the points representing the efficient frontier. As such it informs us about the curvature of the efficient frontier. Hence, in the first row 1979-95, the optimiser determined 59 efficient portfolios which could then be placed as points on the frontier; we have the average of these 59 risk/reward ratios. In general, there will be a different number of points for each data set, though as a rule the longer the time period the more efficient portfolios there are. Therefore these statistics tell us essential features about the shape of the frontier without plotting graphs most of which would be little different and thus impossible to distinguish.

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[9] Given the optimiser generates risk/return sets, the actual data points are approximations on the scatter diagram. They are reasonably close and give a clear impression of the dramatic change in frontier.

[10] Steps of 10% for each asset class were used. Finer grading, at the 5% increment are possible, but we are looking to dimension this issue, considering broader issues. In any case the variation in asset mix for the portfolios are so wide in the sub-periods that smaller increments are really quite spurious in terms of greater accuracy.

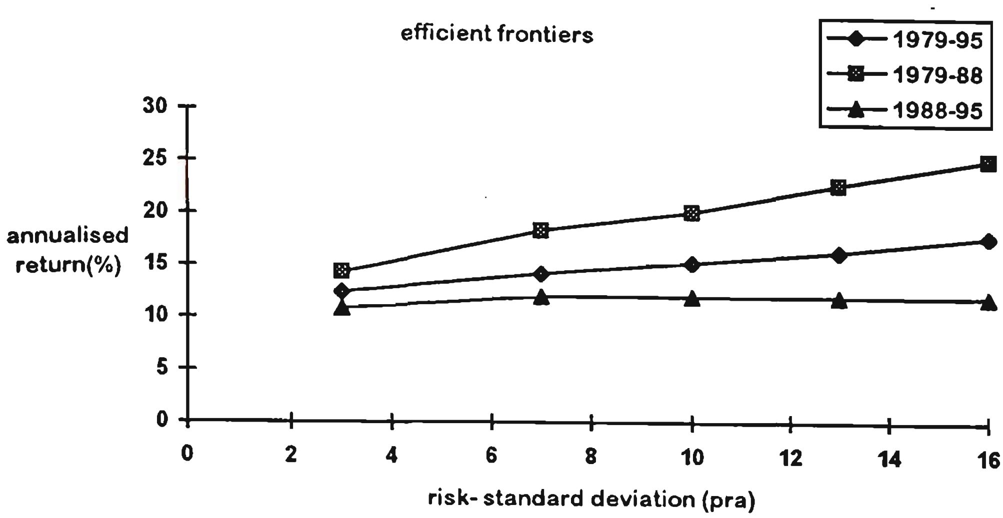

Figure 5.2 Efficient Frontiers for the Whole Period (1979-95) and the Two Sub Periods (1979-88) and (1988-95)

N.B. The dimensions for the standard deviation are strictly % per root annum (pra), rather than just %.

Table 5.5 Key Statistics of Efficient Frontiers

	dataset	range(return)	range(risk)	average ratio
	1 std.dev (s.d.)	6.499	16.083	2.642
1979-95	1.5 s.d.	6.499	16.083	2.676
	2 s.d.	6.499	16.083	2.849
	1 s.d.	13.823	16.640	3.252
1979-88	1.5 s.d.	13.823	16.640	3.601
	2 s.d.	13.823	16.640	3.757
	1 s.d.	2.268	14.158	3.004
1988-95	1.5 s.d.	2.268	21.813	2.882
	2 s.d.	2.268	29.473	2.793
1979-95	+2%	6.499	16.083	2.517
	-2%	6.499	16.083	2.849

Some points are quite clear. Firstly, there is a dramatic change in efficient frontier for the whole period as against the two sub-periods leading to radically different policy portfolios. Secondly, on a comparative basis even substantial deviations within a time period have a much smaller effect on the frontier. We may examine the first issue a little more closely by seeing the different asset mixes required for a given level of risk, for the various time periods. The following table gives the details:-

Table 5.6 The Asset Allocation Associated with Points Located on the Efficient Frontier with a Risk (Std Dev) of Approximately 10.0%

dataset	aoiacc	cbbiamat	tnotes13	mscixa$a	ann ret	std dev
1979-95	10	40	0	50	15.4362	10.0862
1979-88	10	30	10	50	20.2623	10.1513
1988-95	60	40	0	0	12.070	10.3524

The differences are so large as to suggest that great caution is needed to interpret results. Whilst one can argue that we need a long time period for the policy portfolio development, in practice the changes in the marketplace mean that we do not have that luxury. From a practical standpoint we are forced to adopt the actuarial approach of Wilkie (1992), and use long term estimates of the CPI (inflation), and real returns, which implies *acting as if mean reversion was an actual fact.* On the basis of this approach one would obviously discount the very high returns from the 'mscixa$a' series in the first period, on the basis that it would be corrected in the second -with a high degree of probability. Of course, if the returns from an asset class were good but not exceptional then one may not then make the assumption that a correction is in order.

In practice then we are much more concerned about extremes. As we have done in our empirical example in section 4.3 we only corrected when the market had deviated significantly from the mean. This is also very reflective of the finding that mean reversion is more pronounced in the pre-war period than the post. Given the crash of 1929, which must be regarded as an exceptional extreme, it is thus large enough to imply the statistical significance of mean reversion. This idea is not new; the observation has logic in the evidence.

Returning to our efficient frontier table, we can see that even increasing the standard deviation by 100% had little effect on the efficient frontier in all cases. Changing the returns by up to 2% either way also did not make much difference. Hence any inaccuracies here or errors in our assumptions are not key issues. Asset class relativities which generate the policy portfolio, where the returns are decided, obviously are.

Hence we may conclude our initial questions by letting markets and thus allocations run to take account of any positive short term autocorrelation or persistence, being relatively unconcerned about heteroscedasticity, but be prepared for bold strategic moves to take account of mean reversion, *particularly when markets have shown extreme behaviour*, that is deviated significantly from long term means.

Appendices

Appendix 1

Sharpe Index Table

RBA t-notes 13 weeks return (%pa): 9.47			
fund	ann ret	period ret	std dev
1	15.14	0.65	8.68
2	13.12	0.50	7.30
3	11.93	0.36	6.90
4	12.11	0.34	7.73
5	11.92	0.31	7.93
6	11.63	0.30	7.22
7	11.88	0.26	9.35
8	11.07	0.25	6.40
9	11.05	0.20	7.76
10	10.93	0.20	7.43
11	10.95	0.19	7.70
12	10.98	0.19	8.05
13	10.81	0.17	8.05
14	10.41	0.14	6.53
15	10.29	0.11	7.66
16	10.28	0.10	8.29
17	10.22	0.10	7.68
18	10.29	0.10	8.63
19	10.27	0.09	8.58
20	10.29	0.08	9.72
21	10.11	0.08	7.89
22	10.00	0.07	7.43
23	9.90	0.06	7.46
24	9.88	0.04	9.25
25	9.66	0.02	10.30
26	9.24	-0.03	7.65
27	9.18	-0.04	7.88
28	8.92	-0.07	8.21
29	8.51	-0.12	7.90
30	8.36	-0.18	6.18
31	7.48	-0.29	6.76
32	7.11	-0.34	6.93

The Sharpe index is defined as :

$$S = \left(r_p - r_f\right) / s_p$$

where r_p = annualised return from portfolio

r_f = annualised return from risk free benchmark

s_p = annualised standard deviation from portfolio

and is conceptually the return earned for bearing risk per unit of total risk

Appendix 2

Regression of Funds Against AAA
Jensen's Alpha

average asset allocation = policy neutral			
fund	alpha	beta	R squared
1	0.72	0.59	0.33
2	0.39	0.78	0.84
3	0.38	0.69	0.73
8	0.29	0.72	0.93
4	0.27	0.84	0.88
6	0.26	0.80	0.91
5	0.26	0.84	0.82
14	0.23	0.72	0.90
9	0.22	0.80	0.79
11	0.21	0.81	0.81
10	0.19	0.83	0.92
12	0.14	0.89	0.90
21	0.14	0.81	0.87
13	0.13	0.89	0.89
17	0.11	0.85	0.91
15	0.11	0.87	0.94
29	0.10	0.69	0.91
22	0.10	0.84	0.94
7	0.09	1.05	0.92
20	0.08	0.88	0.92
16	0.07	0.92	0.90
25	0.03	0.87	0.95
18	0.03	0.97	0.93
26	0.02	0.88	0.91
19	0.02	0.99	0.77
31	0.00	0.70	0.75
30	-0.01	0.75	0.90
27	-0.02	0.90	0.89
28	-0.03	0.88	0.91
23	-0.05	1.03	0.92
24	-0.17	1.16	0.94

Formulae:[1]

Beta(β). Measures the best fit relationship between two time series.

$$\beta=\sum_i\left[\left(r_{m,i}-r_m\right)\left(r_{p,i}-r_p\right)\right]\Big/\sum_i\left(r_{m,i}-r_m\right)^2$$

Alpha(α) Measures the average extent to which one time series differs from the other by a constant amount.

$$\alpha=r_p-\beta r_m$$

R Squared $\left(R^2\right)$ Measures the degree of fit between estimated and actual data.

$$R^2=\beta^2\sum_i\left(r_{m,i}-r_m\right)^2\Big/\sum_i\left(r_{p,i}-r_p\right)^2$$

where $r_{m,i}$ is the return from the market index for month i

$r_{p,i}$ is the return from the portfolio for month i

r_m is the average return from the market index

r_p is the average return from the portfolio

and the summations range over the selected time period, that is from i = 1 to n.

[1] Source: Super CMS Version 4.0 Manual. Address for correspondence Level 4, 111 Harrington St. Sydney, NSW 2000.

Appendix 3

Average Rank Correlation

If I items are ranked m times with $\sum_{j=1}^{m} r_j = R_i$, where r_j is the j-th ranking of item i, then the average of the $m(m-1)/2$ rankings is:

$$\frac{12\sum R_i^{\,2}}{m(m-1)I(I^2-1)} - \frac{1}{(I-1)}\left[\frac{2(2I+1)}{(m-1)} + 3(I+1)\right]$$

Appendix 4

Spearmans Rank Correlation Coefficients

7-Year ranks: 31 Funds

YEAR2	-0.0343					
YEAR3	-0.2589	-0.2319				
YEAR4	-0.0419	0.4540	0.4714			
YEAR5	0.2331	0.1403	0.1685	0.1859		
YEAR6	0.0778	0.2601	0.1246	0.0790	0.1492	
YEAR7	0.1911	-0.0722	-0.2266	-0.3460	0.0040	0.1927
	YEAR1	YEAR2	YEAR3	YEAR4	YEAR5	YEAR6

10-Year ranks: 24 Funds

YEAR2	-.0096								
YEAR3	-.1122	.2096							
YEAR4	-.0235	.4287	.3887						
YEAR5	.2026	.0870	.1913	.1148					
YEAR6	.2443	.0748	.0148	.0504	.1870				
YEAR7	.2191	-.3774	-.3678	-.6557	.0017	.0487			
YEAR8	.1174	-.4104	-.1496	-.1252	-.0565	-.1800	.1983		
YEAR9	-.2861	.4470	.1157	.1287	-.0539	.1191	-.2165	-.7374	
YEAR1 0	-.2348	.0435	.0487	-.0426	.1148	.1357	.1139	-.1374	.2443
	YR1	YR2	YR3	YR4	YR5	YR6	YR7	YR8	YR9

Appendix 5

Attribution Analysis 31/12/88-30/4/95

based upon average asset allocation 31/12/88 - 30/4/95				
fund	timing	stock	asset	total
1	66.61	-5.62	-8.74	52.25
2	-0.66	-6.82	69.42	61.94
3	22.93	-7.34	42.40	57.99
4	38.85	-21.41	36.46	53.90
5	2.64	-22.20	64.71	45.15
6	6.80	-7.25	68.76	68.31
7	6.46	-8.67	70.18	67.97
8	-3.13	4.26	62.07	63.20
9	110.12	-19.57	-28.76	61.79
10	6.07	-11.35	70.53	65.25
11	4.10	0.98	68.56	73.64
12	2.70	9.12	68.34	80.16
13	93.74	-0.70	-24.99	68.05
14	95.23	-0.37	-23.23	71.63
15	19.42	-11.47	54.37	62.32
16	8.05	-8.50	65.08	64.63
17	8.83	-14.98	53.43	47.28
18	9.76	-7.22	70.66	73.20
19	17.39	-4.02	54.38	67.75
20	1.20	-7.93	69.35	62.62
21	3.81	-6.39	69.91	67.33
22	-1.35	5.04	70.50	74.19
23	6.37	-16.07	67.49	57.79
24	4.96	-6.61	65.97	64.32
25	6.31	-0.20	62.47	68.58
26	4.31	-7.71	67.80	64.40
27	3.62	0.72	70.29	74.63
mean	20.19	-6.75	51.02	
std.dev.	32.04	7.80	32.10	
S.E. mean	6.05	1.47	6.07	

based upon policy asset allocation				
fund	timing	stock	asset	total
1	-1.15	-5.62	59.02	52.25
2	-0.08	-6.82	68.84	61.94
3	4.23	-7.34	61.11	58.00
4	8.79	-21.41	66.52	53.90
5	-1.90	4.26	60.84	63.20
6	148.42	-19.57	-67.07	61.78
7	1.08	9.12	69.97	80.17
8	6.19	-0.70	62.57	68.06
9	9.26	-0.37	62.74	71.63
10	3.40	-8.50	69.72	64.62
11	15.18	-7.22	65.24	73.20
12	0.59	-7.93	69.96	62.62
13	9.78	-6.61	61.15	64.32
14	2.74	0.72	71.16	74.62
mean	14.75	-5.57	55.84	65.02
std.dev.	38.78	8.19	35.61	7.88
S.E. mean	10.36	2.19	9.52	

Appendix 6

Table of Differences Between Stated Policy Portfolio and Actual Average Asset Allocation (N.B. Differences do not Add to 0)

fund	cash	eq(Aust)	eq(Int)	F.I.(Aust)	F.I.(Int)	prop(tr)	prop(dir)
1	1.49	1.05	-2.07	-7.24	3.29	0.01	-1.88
2	5.84	-3.00	-9.82	3.18	.77	0.00	0.00
3	5.08	-0.15	-3.58	-4.34	4.61	0.07	-5.57
4	-1.48	-1.36	-5.37	-0.18	1.45	3.69	-3.02
5	4.12	1.26	-2.43	-2.56	0.92	-14.60	12.71
6	8.82	-1.15	-0.61	-6.97	0.00	11.90	-12.00
7	-5.00	-1.65	0.29	-7.66	8.22	8.62	-8.19
8	-1.01	-1.64	-4.67	-5.95	5.23	4.74	-2.30
9	0.88	3.98	-1.78	0.79	-2.58	-2.61	0.06
10	2.37	7.15	-1.25	-0.91	-8.08	0.00	-7.56
11	4.02	-0.29	-0.61	-4.68	2.14	-1.74	0.00
12	3.93	5.84	-6.26	5.12	-2.49	0.97	-8.09
13	5.80	-3.16	-0.07	-0.08	-4.36	0.47	0.00

Table of the Ratio of differences to the Standard Deviation of Asset Allocation Changes

ratio of differences to std. dev. or volatility (aggression)						
fund	cash	eq(Aust)	eq(Int)	F.I.(Aust)	F.I.(Int)	prop(total)
1	0.42	0.49	1.05	2.43	1.15	0.31
2	1.11	0.83	1.64	0.62	0.74	0.00
3	1.08	0.05	1.19	1.28	1.28	1.53
4	0.29	0.42	1.44	0.03	0.45	0.21
5	0.74	0.47	0.85	0.63	0.43	0.23
6	2.31	0.54	0.46	2.72	0.00	0.13
7	0.00	1.08	0.29	0.74	1.41	0.24
8	0.14	0.51	0.86	1.06	1.41	0.78
9	0.14	1.31	0.50	0.18	0.92	1.32
10	0.32	1.34	0.26	0.17	2.19	3.29
11	0.50	0.11	0.22	0.68	0.58	1.24
12	0.62	1.53	1.17	0.63	1.71	1.49
13	1.42	0.60	0.02	0.02	3.93	0.26

Chapter 6: Conclusions and Future Directions

6.1 Summary of the Main Propositions

We now come to the final chapter where we aim to put together the results from the research carried out. In terms of the examination of the structure of the various financial time series, we find different results for the major asset classes. Table 3.1 summarises the persistence, or lower order autocorrelations (most importantly the first order) for the major classes. Here we find significant first-order autocorrelation in the All Ordinaries monthly percentage changes accumulation series for *all* time periods, including the most recent, *provided* we exclude the October 1987 value. Hence we would conclude an AR(1) representation may well be appropriate for a short term model. Referring to the EMH we may express this result by saying that the market is therefore weak form inefficient[1]. Two further significant comments can be made:-

1) We may well be advised to eliminate the effects of 'outlier' variables prior to any analysis. As has been indicated in the literature, including all data can distort sound underlying relationships.

2) The other two major classes, the CBBI and MSCII have really only been de-regulated since the floating of the A$ in 1983. Thus not only was there a controlled market prior to that date but the unwinding of that control created distortions.

When we consider the CBBI we find the level of first-order autocorrelation high but not significant. We do however, find evidence of joint significance, at the 5% level, using the Box-Ljung statistic. For the MSCII, we see no such patterns at all.

Our next major results concern the long term autocorrelation characteristics of the series which we review via the variance ratio test. Key results for the annual data are summarised in Table 3.2 and those for the monthly data in Table 3.3. The typical

[1] For a definition see the glossary.

'hump'-shaped curves, which we would expect under mean reversion, are exhibited in all major asset classes. Results are somewhat mixed as to the level of confidence we can have with the variance ratio measure of mean reversion, but we may state the main conclusions as follows:-

1) For the AOIA annual series, excess returns, those that is that are above the risk free rate, have a variance ratio at lag 72 which is significant at the 5% level of confidence. This is however a short time series. No other series shows a significant level of mean reversion, as measured by the variance ratio.

2) For the monthly series we find a variance ratio at lag 72 of 0.7526 for the 110 year stock series. Using Monte Carlo based estimates for the standard error, as per Poterba and Summers however, indicates that this value is not significant at the traditional 5% level. No series is significant under interpolation of the Poterba and Summers estimates.

3) The CBBI series demonstrates the most pronounced mean reversion. However the length of the series is only just over 17 years, a considerable period in itself, but fairly short when we are testing for the significance of lags of 72 months or 6 years.

4) The MSCII series shows reversion, but does not reach a peak until lag 54. Very strong positive short term autocorrelation has an impact on the longer lags. This may be a reflection of the very substantial devaluation through the early 1980's, and possibly represents a permanent change in mean level.

Given the difficulty of proving mean reversion at an appropriate level of significance, it was decided to review the matter by considering two related factors and these are:-

1) By considering the independence of the major series and concluding that we had three separate samples exhibiting jointly similar characteristics. This is a less likely event than any one doing so separately.

2) By considering a simple sign test and considering its pattern over the range of lags. Tables 3.4 and 3.5 showed clear 'runs' in changes in the variance ratio.

The argument of the requirement for testing of joint significance in the ratios, as per the Chow and Denning view cannot be rejected, but it is recognised that it represents a very rigorous set of criteria. To further elucidate this particular problem an empirical study into mean reversion was conducted. This represents an attempt to interpret the process based upon an understanding of the kind of generating process underlying asset prices. The idea of using reflecting barriers for cumulative prices which deviate significantly from the underlying mean is not, of itself, novel. This was suggested, for example, by Cootner in his 1964 publication.

In summary, we took a process with known tendencies (short term positive autocorrelation and long term mean reversion); generated a sample of such series and then applied tests to be able to re-interpret the underlying generating process. They showed very similar patterns to the major asset classes. However, in terms of the mean reverting process using the variance ratio test, we were unable to distinguish between our series, for which we *knew* the generating process and a random walk. Indeed only by our sign test interpretation could we conclude negative autocorrelation at long lags. This does not prove mean reversion. What it does do is add further understanding to the nature of the speculative price process, and is suggestive of the likelihood that within quite wide bounds speculative prices are indeed best seen as a random walk. Put alternatively, provided that valuations are not at an extreme, then asset prices or changes thereof follow a pattern which is indistinguishable from a random walk.

This document does not purport to give actual models to determine 'overvaluations', but the above does suggest that perhaps we may need to consider market extremes and look in more detail at these. In practice such valuation models that are used will have a mean reverting process embedded within them anyway, via long term inflation or real interest rate estimates.

Our next results reviewed the other assumptions used in the mean-variance optimisation process; constant mean and variance and the underlying distribution. Asset prices, most notably for the long AOIA series, were clearly heteroscedastic. This had significant implications for the distribution of share price changes. Our results suggested that the cause of the leptokurtic nature of distribution was most probably this hetroscedasticity. However, for all 3 major asset classes, over the period December 1976 - February 1995, the distribution of monthly price changes was normal at the 5% level of significance. Hence, and not withstanding the leptokurtic nature of the long series, we may conclude that the use of the normal assumption in our modelling process is a reasonable one. It is noted that evidence on the price distribution for individual stocks or for shorter time periods may well produce evidence of non-normality. This may imply an underlying distribution at the micro-financial level, which leads to normality at the monthly index level. This particular aspect has not been pursued in this document.

Next actual funds management practice was considered in detail, where in particular we looked for the ability to time markets. Key results obtained in the study in Chapter 5 were:-

1) Using standard measures, the Sharpe and Jensen indices, most funds performed poorly against a neutral policy portfolio, particularly once costs were taken into account. The neutral portfolio, when reasonable added costs are allocated, ranked 7th out of 32 funds.

2) Past performance of a fund has no predictive value for future performance of a fund. Whilst we cannot conclude that a run of good performances will be followed by a run bad ones, certainly current rankings are no guide to the future.

3) With a high degree of significance, we may conclude that managers subtract value by security selection; and that before costs.

4) Managers would appear to add value by timing decisions. However we need to be aware that there were considerable difficulties in determining the actual policy portfolio for most funds in the sample.

5) Based upon an analysis of actual asset allocations versus policy benchmarks there is good evidence to suggest that managers do not adhere to their policy guidelines. There would appear to be lack of clarity of strategic

purpose for most funds. Performance ranking may need to be reviewed in the light of this observation.

6) Positive 'drift' in markets due to valuation effects, as seen in Figure 5.1, may well be introducing some bias into the favourable timing results of managers. Hence what we observe as a timing effect may just be the consequence of valuation effects.

We next considered the importance of each of our assumptions in the determination of the efficient frontiers using actual data for the period 1979- 1995. The inputs were varied to test the importance of the fact of heteroscedasticity in stock prices and possible changes in mean returns. Overwhelmingly, the most important feature was not these factors but the strikingly different returns from each asset class in different time periods. Most notably, the returns for the MSCII had dropped from 26.8% in the period 31/12/79-31/1/88 to 8.2% in the period 28/2/88- 30/4/95. Effectively returns from this asset class had reverted to a level in line with other asset classes.

6.2 Impact of the Conclusions

We must review our conclusions in the light of work done by others in the field, as well as trying to draw some recommendations for action by differing investors, based on what we have found. As indicated in the introduction our approach is an *informal* decision theoretical one, whereby an element of subjective judgement is used in assessing the probability of an event. Thus, by way of an example, we ascribe a high probability to the proposition that managers cannot outperform sector indices. Thus it is not a useful area for managers to spend time upon. Resources are better employed elsewhere, where the evidence suggests a better result has been attained.

We have not found mean reversion, as measured by the variance ratio, at statistically significant levels of confidence, merely supporting evidence for the conjecture. Without repeating the arguments, it would seem that the effect is more marked at times of market extremes[2] than during more normal periods. In this context we should note:-

1) In reviewing our manager performance the best results came from the manager who had, quite correctly, exited property entirely due to the overvaluation in commercial property during the late 1980's.

2) The apparent success of the Wells Fargo approach documented by Vandell and Stevens, based upon timing extreme markets. This is an empirical evaluation of the system using ex-ante decisions.

3) There is evidence from our survey of market timing ability, but in general, no selectivity ability.

4) Mean reversion observed in the MSCII had such a powerful impact on our efficient frontier that in practical terms it cannot be ignored.

Our modelling work in Chapter 4 also supports this contention. We have herein not defined what an extreme or over-valued market is. We *do* make the point that adjusting asset allocations short term cannot be justified on the evidence we see. Nor would there be any benefit in fund management operations putting the kind of effort they currently do into stock selection.

It would then follow that short run tactical asset allocation is not likely to be a worthwhile exercise, particularly after costs. It would, on the evidence we see, be better to stick to the stated policy portfolio and only adjust the asset allocation when significant over-valuation occurs. We may well allow for significant drift in weighting's due to valuation effects. Within quite broad bounds these can be left to run.

This, of course, ignores the thorny issue of costs. To alter an allocation even for a large fund will cost of order 1% (brokerage; stamp duty and administration costs, though competition is reducing these) and even more for property. To say nothing of the time and effort involved by the management team. For a private client the situation

[2] We have not defined what an extreme market is. We can see extremes on an ex post basis, see 4.3 for example. Ex ante, like Wells Fargo or the manager who exited property entirely, they are using valuation indicators obtained from studies of historical patterns. The point is they appear successful.

is far worse. Two-way brokerage plus stamp duty would be up to 5%, let alone managed funds (up front fee 6%, plus annual charges of up to 1.5%).

We also have tax, particularly capital gains tax. Franked credits won't stretch far for all investors[3]. Hence we are looking at rates being effectively marginal rates for the respective taxpayer. At 15% for a super fund it would be extremely difficult to justify a tactical asset allocation move. Our conclusions above are *before* costs. For a private client at 47% it is out of the question.

Asset allocation as we argue the case is a strategic issue. Thus we do have time to make our moves. If we accept Benari's view then these eras persist for a long time. We really have a long while to get set. So we can let 'natural', events do it for us. Cash flow, either net new money or income flows and capital released from takeovers bond maturities and so on is very large indeed. To understand this more a brief study was carried out.

The average weightings of the major asset classes were taken along with the income return for that class and dividends successively re-invested in the chosen asset class to see how long it took to move the allocation 10% (ignoring any valuation effects). Table 6.1 outlines the situation.

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[3] A consideration of the mix of assets show that all super funds will pay tax. There is indeed a tax on contributions at 15%, as well as other asset classes with no imputation benefit. The top rate of tax for private clients is 47%, above the company rate. Thus they are bound to pay tax. We can safely ignore the odd pensioner with a share portfolio.

Table 6.1 Time in Years to Move More than 10% in Allocation

with Re-investment of Income and No Valuation Effects

Asset Class	Cash	Bonds	Aust. Equities	Int. Equities
original wt.(%)	8	25	43	24
income yld(%)	7	10	5	3
time for 10%(yrs)	1.99	2.44	3.34	2.55

Remembering that takeovers, redemption's and so on will also occur, means that we can move at least 5% in equities, say, in just over 3 years and more likely closer to 10% in practice. Given any likely feasible set of asset ranges generated in an asset liability study are likely to be no wider than 10% from the central value (for example a range of 35% to 55% for Australian equities) then this can be done in 3 years.

There is then little need to incur costs to achieve our objectives. Though given our observation of value added from timing we would need to look at any likely cost/benefit, before we make a decision. Let us now conclude this section with advice for our classes of investor.

For the managed fund, there needs to be a recognition that the evidence is that asset prices are micro efficient. The random walk model is a good approximation (this mirrors the Samuelson view). Any macro inefficiency is not inefficiency in terms of asset class relativities, but due to the long memory characteristics of asset price changes. The focus of attention must then shift to strategic asset allocation and broad timing issues away from, the often more exiting, stock picking arena where value is not added. Larger quantum moves, less frequently is the way forward, with a clear focus on:

1. Sticking to the strategy.

2. Reducing costs wherever possible.

For the private client, timing is no less easy than for the large fund. In any case, given the high costs, tactical asset allocation cannot be justified. Any moves can be readily accommodated by natural processes, thereby reducing costs to a minimum. If we also accept the time diversification principles outlined in Thorley (1995), and Reichenstein and Rich (1994)[4], then we may find mean reversion implies a higher level of risky assets is appropriate anyway. By this we mean that much year-to-year volatility is off-setting; because high returns are followed by low returns and vice versa. Hence the long term probability distribution is much more concentrated than would be expected on the assumption of independent yearly returns. Therefore the timing issue falls away, if the private client stays in say equites all the time, thus gaining the benefits of *time* diversification as against short term *asset* diversification. This is the view of the Graham school anyway (referred to previously).

6.3 Future Directions

There are so many potential areas for research that can be observed that it is not easy to enumerate them all. Where appropriate, indications for future research are commented upon in the body of this text. However we may be able to indicate potential research under, perhaps somewhat arbitrarily, four main areas:-

1. Mean Reversion

   a) Explore fractional differencing, most particularly the degree of differencing, that is, finding optimal values of d. A better understanding of the null against various alternatives, with an extension of the empirical modelling process, would also aid with fractional differencing.

   b) Look in more depth at the role of short term autocorrelation and outliers and their impact on mean reversion, that is, referring back to Lo's work and seeing what impact the short term correlation has on the power of the variance ratio test.

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[4] See Chapter 2, for an outline of the papers. The ideas are relevant here as we address the results of our studies into asset returns, just as they do.

2. Distribution of Prices

Review the distributional form, particularly the links between the macro and micro financial details, that is the relationship between distributions for different time periods and various levels of aggregation. Futures pricing would also be considered as part of understanding the nature of speculative pricing, with a view to the appropriateness of financial models.

3. Financial Modelling

Develop and integrate the various forecasting methodologies; short term autocorrelation based approaches with the econometric and actuarial styles. The aim being to develop consistency of method between the policy setting processes (mean-variance optimisation and actuarial assessments) and short term asset management. Amongst our considerations, such features as asset class relativities and the role of the bond rate in determining valuation parameters for other assets, would be necessary.

4. Performance Survey Studies

a) The role of the policy portfolio needs more clarity. An understanding of the degree to which funds are sticking to their policy portfolios and the impact of this on risk adjustment is needed. A comprehensive review is warranted of the ability of funds to be able to time markets. It is important to disentangle both policy issues, referred to above, and the role of involuntary timing.

b) Given the poor performance of managers in security selection a close look at the role for indexation in funds management is required. In conjunction with this a detailed study of costs is essential to deliver the requirements to the mass market of investors at a far cheaper price than is currently available.

Whilst the above set of future work is a solid sample of what one would like to see, probably as important is that the main players in the field act together. There is a need to link more closely the role of actuaries and asset consultants with those of market participants or fund managers and those of the academic community. In many ways it would seem that the directions that are taken by the groups are divergent and often contradictory. A closer linking between theory and practice with each group incorporating the results and insights obtained by each group must surely advance our knowledge.

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