CONTROLLING PROCESSES WITH REFERENCE TO COSTS, ITEM PRICE AND PROCESS EVOLUTION

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A thesis submitted in fulfillment of the requirements for the Master of Science Degree

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February, 1998

"Necessity to study the needs of the customer, and to provide service to product, was one of the main doctrines of quality taught to Japanese management in 1950, and onward"

W. Edward Deming

"Quality, Productivity and Competitive Position", 1982

"A dissatisfied customer does not complain: he just switches"

Oliver Beckwith, 1947

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DECLARATION

I hereby declare that:

(i) the following thesis contains only original work which has not been submitted previously, in whole or in part, in respect of any other academic award, and

(ii) due acknowledgment has been made in the text of the thesis to all other material used.

Violetta Iwona Misiorek
February 19, 1998

ACKNOWLEDGEMENTS

I would like to express my gratitude for the guidance, support and help offered by my supervisor, Associate Professor Neil S. Barnett, during my period as a research student at Victoria University. He has provided many ideas as well as invaluable suggestions and comments on many aspects of this thesis.

Special thanks are due to all the staff and research students of the former Department of Computer and Mathematical Sciences, Victoria University, for their assistance and for making my time at the University so enjoyable. In particular I wish to acknowledge the assistance of Dr. Neil Diamond with the S-Plus code for approximation of equation (5.1). His friendly advice, helpfulness and encouragement are much appreciated. I am grateful also to my co-supervisor, Associate Professor Peter Cerone, for his many useful remarks and suggestions.

I am thankful to CSIRO, Mathematical and Information Sciences, for their support in the final stages of this research. Special thanks are due to Dr. John Field for his permission to use his method for calculating the probability of failing the weights and measures requirements in EEC and USA. This assistance is gratefully acknowledged.

Finally, I wish to extend my most sincere thanks to my husband Piotr and my children, Gabriella and Piotr S., for their understanding, love and patience.

A special credit is owed to my mother for her help during this work.

The research for this thesis was supported by a Department of Computer and Mathematical Sciences studentship. This assistance is gratefully acknowledged.

ABSTRACT

This thesis presents some recent work of the author in developing the analysis of a number of process control models that take into account statistical, economic and other practical issues. Special attention is paid to the problem of optimum selection of the initial process mean setting, with particular reference to filling/canning processes. As there are many different situations that involve different cost parameters, this leads to the consideration of various models each with their own particular solution. The effects of change of the process variance on the optimal solution as well as on the expected profit are discussed. Implications to 'Weights and Measures' requirements of following this optimality path are provided, with particular reference to loss in expected profit per item.

Chapter 1 provides a brief introduction and is followed by a literature review in Chapter 2.

Chapter 3 deals with the issue of selecting the optimum process mean by presenting a simple model and emphasising the dependencies between the process parameters.

Chapter 4 further investigates the problem presented in Chapter 3 and presents several models for which the selection of the most profitable process setting is considered, concentrating on a canning problem. Various industrial filling processes are described and some of the issues considered include: waste, overfill, top-up, and the penalty costs for items that initially fail to meet specifications.

Chapter 5 discusses Weights and Measures requirements in connection with a canning process. Both Australian requirements and OIML International

recommendations are discussed. The Australian requirements are also compared with the requirements of the European Economic Community as well as the United States.

Chapter 6 illustrates the potential use of the models developed in chapter 4 by giving an industrial example and again discussing the implications to Weights and Measures requirements.

In Chapter 7 the problem of an optimal selection of the initial process mean is examined for a process with a linear shift. Special focus is on the economic benefits obtained from reducing process standard deviation and the rate of change of the mean.

Conclusions and some suggestions for future work are provided in chapter 8.

Parts of chapter 4, 5 and 6 form the contents of a paper, 'Mean Selection for Various Types of Filling Process with Implications to 'Weights and Measures' Requirements', undergoing revision for publication in the Journal of Quality Technology.

CHAPTER 1

Introduction

1.1 Origins of quality control

The origin of modern industrial quality control dates from May, 1924 when a young, Walter Shewhart, developed the first sketch of a process control chart.

Shewhart was born on March 18, 1891 and received his Ph.D. in physics from the University of California in 1917. He worked with the Western Electric Company before joining Bell Laboratories in 1925. It was there that his ideas regarding application of statistical methods to the analysis of physical and engineering data started to stimulate the interest of other scientists. His methods became more and more accepted, boosted by the presentation of his paper on the economic control of quality of manufactured product to the Royal Statistical Society, in 1931.

Shewhart was the first to prescribe methods by which a process could be gauged to have reached a state of statistical control. Statistical methods for quality control have been widely used since then, being elevated in importance by the post war industrial success of Japan. Many western countries were too complacent, and totally unaware of the industrial competition they were shortly to face with the dramatic effects this would have, to have had an intimate interest in the industrial application of statistics. Deming and Juran contemporaries of Shewhart, became strong advocates of the use of statistical techniques in the industrial environment in order to assist efforts of process control and improvement of product quality. Deming, in particular, espoused such techniques to the Japanese who claim a strong connection between this and their climb to industrial prominence from the ashes of destruction of 1945. Statistical methods in quality control are now widely accepted throughout the world, as they are adapted to suit advancing technology.

Statistical Process Control (SPC) is a statistical approach to production that is used to ensure that processes are maintained in a state of statistical control. Such control helps to ensure that manufactured products are consistent in their vital parameters, although, of course, all processes are subject to a certain amount of variability. Walter Shewhart made the distinction between common and special causes of variation. The latter are often referred to as assignable causes and the former, which are considered solely due to chance, are called common (random) causes. Any random cause results in process and product variation but generally such causes cannot be economically eliminated. Assignable variation, however, is assumed to be due to specific "findable" causes and action to eliminate these is usually economically justifiable. Ideally, only random causes should be present in a process. Control charts, which were invented by Shewhart, are often used to distinguish between assignable and random causes and for prompting process adjustment.

1.2 Design of control charts

There are two basic approaches to designing control charts, economic design and statistical design. Economic design is a method of selecting the design parameters in such a way as to either minimise the total cost of production or to maximise the producer's profit. The method requires the assessment of several cost and non-cost parameters associated with the process. Statistical design, on the other hand, is a method which selects a procedure more on its statistical merits than by directly considering economic factors although, of course, any practical application must have regard for matters of cost, at least implicitly. Mongomery (1980) summarised the work that had been published in the area of economic design of control charts up until then. This was followed by Ho and Case (1994) who reviewed the literature for the years 1980-1991.

As has been indicated by several authors, the economic method of designing control charts has certain weaknesses. One of the main drawbacks in the use of economic models is their poor statistical performance (e.g. insensitivity to small shifts in process parameters). In addition, total cost savings, that economic designs are supposed to yield, frequently do not accurately reflect reality, Woodall (1986). As Woodall (1986b) has shown, economic models generally do not agree with Deming's (1982) philosophy, which states that the main focus of process studies should be on improving the process.

A common assumption made when examining economic aspects of quality control is that the most desirable values of the process parameters are known and that the task is to optimize procedures for controlling the quality. The methods of quality control should not be applied to the process until the most economic values of the process parameters are selected. The role of process control methods should then be to ensure that the selected values of the parameters are achieved.

1.3 Changes in mean

The objective of any process control system is to make sound decisions on actions to be taken that affect the process. A process is said to be operating in statistical control when the only sources of variation are common causes. Most statistical process control (SPC) techniques assume that process data can be described in terms of statistically independent observations that fluctuate about a constant mean. In reality, however, many manufacturing processes exhibit systematic trend in one or more of the process parameters during the course of operation. This can often be linked to machine reliability. Such changes often occur in production processes that involve drilling, grinding or cutting (Taha (1966) as well as many others). This type of trend is often a consequence of tool-wear. The general approach of SPC is to establish stability (fixed mean and constant variability) and leave the process unadjusted whilst it produces items between certain established statistically desirable boundaries. The design of a process that manifests systematic trend, for example in its mean, requires first the selection of the initial setting of the mean level as well as some kind of control regime that would regularly require the process to be adjusted. Such an approach ignores the issue of assignable causes. With an economic approach what is required is the striking of a balance between different costs involved in the manufacturing process. Such costs include the setup costs, production costs and the financial penalty of actually producing defective items. In SPC, a quantum "jump" in the mean is usually interpreted as an assignable cause that needs to be rectified. If certain supplementary run rules are used in SPC charts these provide facility for detecting a trending mean but, generally, ensuing process adjustments are made on statistical rather than economic grounds.

1.4 Thesis objectives

The main thrust of this thesis is to present and bring together some of the work that has been published on SPC and economical design of control mechanisms.

Some new models are proposed for the optimum mean selection of the process quality characteristic with special attention given to provide the potential user with a simple method able to provide a useable numerical solution. The focus is on maximising the expected profit. The need to consider whether or not the obtained results meet Australian 'Weights and Measures' requirements is also discussed. The method of calculating the probabilities of meeting Legislation requirements is shown. Furthermore, OIML (The Organisation Internationale de Metrologie Legale) International Recommendations are described and the Australian requirements compared with other selected countries' Legislation.

In addition, a model, which is applicable to a process that exhibits a linear shift in mean, is presented.

The dependencies between process parameters are analysed in detail and displayed graphically. An industrial example is also given as an illustration of the methods proposed.

Once the selection of the optimal process parameters is complete a brief summary of statistical methods used to maintained the current process state is given.

Chapter 2 presents a review of the published work on the economics of process settings and adjustment when there are systematic changes in the process mean and / or variance.

CHAPTER 2

Literature Review

2.1. Introduction

The economic design of control charts has been the focus of a considerable amount of work during the last forty years. Comparatively little has appeared, however, on the economic selection of the parameters especially in relation to filling processes. Models that apply to processes that exhibit shift in mean or variance, although being discussed by many authors, consider mainly processes involving cutting, griding rather than filling processes.

This chapter gives a review of models and techniques proposed for selecting the most desirable quality characteristic with special attention given to canning problems. This is followed by a review of work published on controlling processes with an inherent shift in mean or variance.

2.2 Selection of the process mean

2.2.1 Selection of a process mean for a standard process

Pioneering work in the area of optimum setting of the initial process mean was published by Burr (1949) , Springer (1951) and Bettes (1962). They all considered related problems; the latter two took economic aspects into account and determined the optimal location of the mean, which minimises the total cost. The work of Springer was later extended by Nelson (1979).

Most work published on economic design concentrates on minimising the costs of production. This works well only for a simple pricing policy, i.e. if the product meets the specified requirements then it's sold at a regular price, if it does not then it is considered scrap. In reality, however, even if the product does not meet the specifications it may still be sold at a reduced price. In this situation the minimisation of the production costs could cause diminution in quality. What is more, the overall goal for most modern production processes is to manufacture with a "quality" that maximises total profit. Hunter and Kartha (1977) developed a method for determining the optimal target value of an industrial process so as to maximise the profit, taking process variability and production costs into account. The problem revolves around the situation where product above a certain dimensional threshold attracts a fixed selling price and product below the threshold attracts a reduced yet fixed selling price. In addition, product above the threshold implies 'give-away' which diminishes the net profit per item. The essential issue is to find the most suitable process setting (the process mean) so as to effectively trade off diminished profit due to 'give-away' with diminished profit induced by producing below the stipulated threshold. Besides successfully formulating the problem, Hunter and Kartha provide a graphical method of solution. The authors consider the problem under the assumption that once the initial setting is made, no other control actions are subsequently required. The assumed conditions in their model do not permit for an explicit optimal solution, however Nelson (1978) has found an approximating function which allows for about three-decimal accuracy in the solution. He has also included a plot of errors of this approximation.

A generalisation of this model was presented by Bisgaard, Hunter and Pallesen (1984) who developed a procedure for selecting optimal values for the process mean as well as the variance. They eliminated the assumption, made by Hunter and Kartha, that all under-filled items can be sold for a fixed price. They considered a situation where the under-filled items are sold for a price that is proportional to the amount of ingredient in the container. This assumption, however, is also not realistic since, due to government regulations, containers with a content below a certain proportion of the nominal weight or volume cannot be sold. A solution for a process having an approximate normal distribution was given together with a table that provides a simple way of getting the optimal process setting. Situations in which the distribution of the quality characteristic has lognormal and Poisson distributions were also discussed. The objective was to maximise the profit. Vidal (1988) has later examined this problem in more detail and given a mathematical discussion of the optimisation model. He explored the properties of the optimisation model, its stationary solution, found by a graphical method, and also gave examples of some special cases where the stationary points can easily be found. The proposed methods, however, were dependent on the assumption that the prices and costs are linear functions of the quality characteristic.

A model also similar to that of Hunter and Kartha was studied by Carlsson (1984). He analysed the choice of the process setting as well as the net expected income taking production costs, selling price, process variability, specification levels, and control plan all into account. An example from the steel construction industry, where rejects are either sold at a reduced price or reprocessed was given. The quality characteristic was assumed to be one-dimensional and normally distributed with known variance. Furthermore, one specification level , defined as the lower level, was assumed and the net income function was represented as a piecewise linear function of the quality characteristic. The situations where a customer is willing to pay extra for good quality as well as when a producer may have to compensate the customer for bad quality, are both discussed.

2.2.2 Selection of the process mean under a sampling plan.

All of the studies discussed so far, have addressed the problem of quality selection assuming 100% inspection of product. Carlsson (1989) and Boucher and Jafari (1991) have extended this line of research by evaluating the problem under a sampling plan.

Carlsson (1989) considered a case of acceptance sampling where the reject criterion was based on the sample mean. Special attention is given to the MIL-STD-414 acceptance sampling plan. He assumed just one specification limit, given as the lower limit, and that the quality characteristic follows a normal distribution with known variance. The rejected lot is sold at a secondary market or reprocessed. The expected income function proposed is similar to that of Carlsson (1984). As the solution is not explicitly obtainable an approximation is given. He noted that the approximation accuracy improves as the sample size increases.

Boucher and Jafari (1991) considered the case in which the rejection criterion is based on the number of nonconforming units in the sample. Special attention is given to a filling process where it is not possible or economically justifiable to inspect every unit of product. They consider a sampling plan in which a sample of size n is drawn from daily lots of size N. An accepted production lot is sold at a fixed selling price A. Whenever a lot is accepted then there is a cost of excess quality per item. A solution for determining the process setting so as to maximise profit per lot filled, is developed.

2.2.3 Simultaneous selection of a variables and an attribute target mean

Some industrial processes involving paper, plastic, glass and fabric have to satisfy both variables as well as attribute quality characteristics. The former of the two might correspond to weight or volume, hardness, or size, the latter usually relates to cracks, abrasions, and marks. Arcelus and Rahim (1994) discussed such cases. The objective of the model developed was the maximisation of the expected profit. The variable quality characteristic was assumed to be normally distributed and to have a lower specification only, with the attribute quality characteristic being Poisson and having a single upper specification limit. The two quality characteristics were assumed to be independent. The authors have adopted a Taguchi like loss function by penalising deviations from the target as well as excess of non-conformities. The optimal solution was found by using an iterative approach based upon the ZEROIN routine of Forsythe, Malcolm and Moller (1977). For ease of computation the attribute characteristic was assumed to be modelled by a real-valued variable rather than a discrete variable.

2.2.4 Selection of the process mean with special reference to a canning problem.

Operations that involve placing any ingredient into containers, be it fluid or solid, are typical of the 'canning problem'. Models to be used for optimally setting the mean have to take into account not only the nature of the process but also different cost parameters.

The models developed by Hunter and Kartha (1977) as well as Bisgaard, Hunter and Pollesen (1984) and discussed in the previous section, can also be applied to canning processes, however the assumptions they made for under-filled containers are frequently unrealistic. It is in breach of the law to sell under-filled items for either reduced price or at a price proportional to the amount of ingredient in the container.

Some of the earliest work in the area was presented by Golhar (1987) who addressed the issue of finding the most economic setting of a process mean, concentrating specifically on a canning problem. He modelled a situation where only correctly or overfilled product can be sold at a regular price and under-filled cans are emptied and refilled with added cost. The capacity of the containers was implicitly assumed infinite.

The canning problem analyzed by Gohlar was latter discussed by Schmidt and Pfeifer (1989), who explored the cost reductions achievable through a reduction in the process standard deviation. A linear relationship between the percentage reduction in cost and process standard deviation was found. They also noted that the final equations were independent of item price, as all containers are sold for the same amount. Revenue per can was constant thus minimization of expected cost is equivalent to maximisation of expected profit.

Golhar later extended his model (Golhar & Pollock (1988)), for a process where ingredients are expensive and where both the process mean and the upper limit can be controlled. The concavity of the solution function was shown only by a numerical example that covered a range of the process parameters. Their model was appropriate only if the filling process capacity can be varied and if the relationship between the cost of 'reworking' the overflowed container and the number of such containers is strictly linear.

A similar problem was discussed by Schmidt & Pfeifer (1991) and they have extended the above model to situations where a fixed capacity container was assumed.

In both of the models discussed above the profit per fill attempt or item, P(X), (as defined by Schmidt & Pfeifer and Golhar & Pollock) was as follows:

If the item falls within specifications then

P(X)= Revenue per acceptable container - material cost

Otherwise it incurs a loss. No other variations of the profit function were discussed.

2.3. Processes with a trend or shift in mean

Many manufacturing processes exhibit some sort of trend in one or more of the process parameters during the course of operation. Often this trend is of a systematic nature and its relationship with time is also frequently linear. Trends can be negative or positive. The former of these occurs ,for example, when the nozzle of a filling machine is clogged, the latter is a characteristic of tool-wear. Such changes usually occur in production processes that involve drilling, grinding or cutting (Taha (1966) and others) as well as filling. The process is kept in control by regular adjustments, replacements of some parts of the tool or, in the case of nozzles, by cleaning.

2.3.1 Optimum selection of the process mean

2.3.1.1 Linear shift in mean and tool-wear.

The first consideration of the effects of a linear trend on the process mean due to tool wear was made by Manuele (1945). His approach was subsequently popularised by Duncan (1974), Vaughan (1974), and Grant and Leavenworth (1980).

As the tool wears the average of the process increases. There is always a limit to how big the process average can get before the amount of defective items is intolerable. On the other hand, resetting is usually quite expensive. A balance must be obtained between these two costs. The following summarises the literature describing methods for the optimal selection of the length of the process run.

Gibra (1967) considered the case of a process that was known to exhibit a linear trend in the mean while having a constant variance. He obtained the optimal production run between process adjustments by controlling the initial setting of the mean for both stable and unstable processes. The case of statistical stability involved minimisation of the sum of the resetting cost and the financial loss due to manufacturing defective items. The cost was defined as a step function. For the unstable case, continuous process monitoring was prescribed to be necessary. The x-bar chart was used as a monitoring device. The final step in the solution was to find the parameters a of the x-bar chart for a given average proportion of time that the process was in state of statistical stability. For selected values of this average proportion it was demonstrated that the optimal production run between adjustments, the corresponding resetting cost and loss due to producing defective items, the optimal parameters of the x-bar chart, and the monitoring cost, could all be calculated. Subsequently, the other optimal parameters, such as production run length, sample size, control limit width and the initial setting, could all be found. The solutions for both the single and two-sided specification cases were provided.

Taha (1966) also discussed the problem of tool-wear with special reference to cutting tools. The assumption was made that any increase in the number of defective items was due to tool wear only. The overall objective of his work was to determine the optimal length of time before maintenance should occur. This requires minimising the sum of the reworking and/or scrapping costs of defective items and the cost of adjustments.

A slightly different and more general approach was demonstrated by Smith and Vemuganti (1967). They assumed that the wear of the tool was a linear function of time and that the distribution of the initial mean, as well as the rate of wear of the tool, is known. Solution to the problem is tantamount to finding the break-even point, i.e. the time at which the cost of machine adjustment is the same as the expected cost of producing items below the specification limit in one unit of time. The break-even point is then the optimal adjustment time. To update the distribution of the initial mean, as well as the rate of wear, a new sample is taken. The distribution of the above parameters can then be used to calculate a new optimal time for adjustment.

A negative trend in mean was investigated by Rahim and Lashkari (1982), for which they have described the determination of the optimal production length. Two cases were considered, one when there are not assignable causes during the production process and one when they exist. In their cost function the authors included the cost of resetting, the cost of rejected items as well as the cost associated with lost production due to resetting. Derivatives were used to obtain the optimum length of the production run so that the total expected cost per unit item can be minimised. They concluded that the optimal production run is dependent not only on the magnitude of the drift but also on its direction. They did not, however, investigate how other process parameters affect the optimal solution.

2.3.1.2 Quadratic loss function.

All of the methods above followed, what is sometimes called, the 'meeting specifications' approach, which fundamentally means that there is equal utility of product provided important characteristics are within specifications and total loss of utility if they are outside the specifications. The alternative philosophy promoted by Taguchi (for example 1986) states that the loss in utility (reflected in increased costs) occurs gradually and increasingly as the distance from target increases. He also postulated that this loss can be represented by a quadratic function.

The loss or cost, defined as a quadratic function of the deviation of the process mean from a given target value, was considered by Drezner and Wesolowski (1989a). They considered a problem similar to that of Gibra in that a least cost solution is sought. The objective was to minimise the total loss. The authors developed a procedure for determining the initial setting of the process mean and the optimal time between adjustments, when the rate of wear of the tool is linear. They solved the problem of finding the optimal solution for the case of a quadratic function that is symmetrical about the target. An asymmetric loss function about the target was also considered. The quadratic loss function was also considered by the same authors later (1989b) but is discussed in the next section.

2.3.1.3 Minimisation of costs versus maximisation of profit.

In all of the studies discussed so far the optimal decision rule was to minimise the total manufacturing cost. However, in situations where the undersized items can be sold at a discount rate (i.e., as "seconds") and oversized items can be reworked (or, if more profitable, sold as scrap) the above decision rule is not appropriate. What is more, the overall goal for most modern production processes is to manufacture with a "quality" that maximises total profit. This philosophy was adopted by Arcelus and Banerjee (1985) and (1987) .

Arcelus and Banerjee (1985) considered the problem of selecting the most profitable initial target value for the mean of a production process that undergoes a linear shift. The profit function is assumed to be a step function and is equal to the expected profit from a given run minus the setup cost divided by the run size. An item that falls below a given specification level, is sold at a discount, otherwise it is sold at the regular price. The objective was to maximise the total profit whilst the variance of the quality characteristic is assumed to be known. The solution algorithm involves finding the initial setting for a run size of n and then determining the run size that maximises the expected profit per unit. The solution algorithm was coded in Fortran.

The same authors used a similar approach in 1987 to solve a problem with the added assumption of a non-negative shift in both the mean and the variance. The objective was to maximise the unit profit, which was equal to the sum of the revenue from acceptable parts, the scrap value of all parts rejected as undersized and the reward of oversized parts minus the total cost of producing the run of size n, all divided by n. The decision rule starts by controlling the initial setting of the mean for any run of size n. The optimal initial setting is the one that maximises the profit per part (not just profit per acceptable part) for a given run size, and the optimal run size is the size that yields the highest unit profit.

2.3.1.4 Non-linear systematic and random shifts in mean.

A case where the process mean is subject to any systematic behaviour was considered by Gibra (1974) (the result obtained can also be applied to linear trends). He discussed the determination of the optimal lapsed time before adjustment. The objective was to minimise the costs involved with each production run. These costs include: set-up costs, down time costs and loss due to production of defective items. All other costs were assumed to be constant. Both single and double specification limit cases were considered. A graphical solution was used in the final step.

Drezner and Wesolowski (1989b) extended the above problem by considering a process which shifts randomly during production. The random shifts are approximated by discrete "jumps" in the setting of the mean . This assumption is used to develop models for unidirectional drift (i.e. when the process mean can only change in the positive direction), as well as the two directional drift case. The cost of deviation from the optimal setting is still assumed to be quadratic. The process is monitored continuously and may be reset at a fixed cost. For the case of two directional drift, the authors employed a gradient search method where the starting points can be obtained from Drezner and Wesolowski (1989a).

A situation when the deterioration of the process mean in a given time interval is assumed to be a random variable and where the trend is negative was discussed by Schneider, Tang and O'Cinneide (1990). They developed a procedure for the optimal selection of the starting level of the process mean as well as the lower level at which the process should be adjusted to the initial setting. The two process parameters were called optimum when they minimised the total expected production costs. The costs considered were: production and adjustment cost and loss due to defective items. One of the main differences between their model and that developed by authors previously mentioned is that they assumed that the process is regularly inspected. Only on the basis of this inspection the process is adjusted. They concluded that for a process with a large variance their model gives a more accurate solution than when the trend in mean is assumed to be linear.

As an extension, Jensen and Vardeman (1993) considered a situation involving random adjustment error. The optimal adjustment policy is developed by dynamic programming. They determined a prescription for machine adjustment that is optimal with respect to an appropriate cost criterion. This criterion involves the number of items produced while this control strategy is in effect, as well as using the fixed cost of making an adjustment. The authors discussed the effects of adjustment cost, adjustment variance, and drift rate on the optimal policy.

A similar case to that of Jensen and Vardeman (1993) was earlier considered by Adams and Woodall (1989) with the difference that they did not allow for the adjustment error. Crowder (1992) as well as Vander (1991) also analysed the problem of optimal discrete adjustments for a process with tool-wear with emphasis on results that apply to short-runs.

2.3.2 Controlling processes with mean trend

As the tool wears the average of the process increases. The classical control chart often cannot be applied to such a process as the distance between specification limits would most likely be much greater than 6σ.

A common approach to control processes with a linear shift in mean is to alter the traditional control chart so that the control limits are parallel to the tool-wear trend line (where the rate of change is assumed to be known). As long as the sample averages fall within those two trend limits, it can be said that the tool-wear is in control, otherwise the process should be adjusted. Alternatively, some maintenance must be applied to the process. The above method was described in detail by, for example Manuele (1945) and Duncan (1974)

Another approach to control tool-wear can be the use of the regression control chart. The pioneering work on the regression chart was done by DiPaola (1945), although his method was only useful to control a process where there was, by necessity, simultaneous control of two variables. His work was later extended by Jackson (1956) and Weis (1957). Mandel (1967) and (1969) discussed the regression control chart in more detail, however, he illustrated its use in the context of managerial applications only, like controlling man-hours, scheduling manpower resources and budgeting. It has been pointed out in the literature that the above methods assume that the costs associated with any adjustments to the process are large. This essentially focuses attention on minimisation of the total number of adjustments. Only sufficient adjustments are made so that items are kept within the specification limits. Far less attention is given to the reduction of variability.

Kamat (1976) developed a smoothed Bayes' procedure for the control of a variable quality characteristic in the presence of a non-random linear shift in mean. He defined the concept of control in terms of the acceptable fraction defective and divided the random variation into two components, one related to the variation within a lot sample and the other describing variation between lots. He compared his method with the standard X-bar chart as well as the cumulative sum chart. He used sampling to estimate the linear variation via linear regression. The same data was also used to estimate the two additional variations mentioned above. Double exponential smoothing was used to estimate the slope and the basic level at a given time.

In addition to the above methods, Quesenberry (1988) proposed a fixed interval compensator for a process where tool wear can be modelled over an interval of tool life by a regression model. The compensator is calculated by using the mean adjustment of a particular batch plus the estimated wear of the tool since the last adjustment. The main objective was to find a method of adjusting the process so as to minimise the expected mean square of deviations of part measurement from the nominal target value.

2.4. Shifts in process variance.

All of the studies discussed so far have considered processes, which exhibit some kind of change in mean with the assumption that the variance of the process remains constant. In practice, however, processes are often influenced by factors that induce changes in the process variance. Problems of this type were investigated by Arcelus, Banerjee and Chandra (1982). They investigated a process with a non-negative shift, be it linear or non-linear, in both the mean and the variance of the critical product characteristic. The aim was to find the optimum production schedule for the production of shafts of diameter within specified tolerance limit(s) where there is a specified minimum number of acceptable shafts required (H). Only the situation where there is a double specification limit was considered. The optimal production plan was determined by controlling the initial mean setting of the process. The objective was to minimise the total cost of producing an expected number at least as large as the minimum number of acceptable shafts. Two cases were studied: when H is infinite and when H is finite. In the first situation the problem reduces to finding the run size that minimises the per unit cost of acceptable items. In the second situation the problem may be formulated as a knapsack problem. The solution involves finding the optimal initial mean setting for a given run size and the determination of the optimal run size that minimises the total cost. The cost model considered by the authors is not general enough to handle discount for undersized items or cases where the oversized items can be reworked.

2.4 Conclusion

In this chapter a review of models and techniques proposed for selecting the most desirable quality characteristic have been presented. The work published on controlling processes with an inherent shift in mean or variance has also been included. Although a lot has appeared in the above two areas there are still issues that need to be addressed. It is the belief of the author that the maximisation of profit is a far better approach to that of the minimisation of production costs. The available profit models, especially in regard to filling processes, are suitable for only some of the types of industrial processes. The probability of whether or not working with the obtained optimum will meet the weights and measures requirements has yet to be discussed.

CHAPTER 3

Factors Affecting the Choice of Target

3.1 Introduction

Selection of the most economic values of the process parameters prior to applying methods of quality control is very important but, in industry, one that frequently receives insufficient attention. The use of non-optimum values, especially for the process mean, will result, not only in profit reduction but also in unnecessary process adjustments.

The main thrust of this chapter is to present a simple model for the optimum selection of a process setting with a view to maximising the expected profit per manufactured item. The emphasis is not on the development of the model itself but rather on the resultant dependencies between the process parameters. Process operations that involve placing fluid into containers typically illustrate the area where problems of this nature most commonly occur. It is, therefore, in this setting that the model is framed. In this context, a production item represents the amount of product placed into a particular container (eg. volume or weight).

The model presented is an extension of the work of Hunter and Kartha (1977) in which they determine the initial (and assumed static) setting of an industrial process with a view to maximising the expected profit per manufactured item. The problem revolves around the situation where a product above a certain dimensional threshold attracts a fixed selling price and a product below the threshold attracts a reduced yet fixed selling price. In addition, a product above the threshold implies 'give-away' which diminishes the net profit per item. The essential issue is to find the most suitable process setting (the process mean) so as to effectively trade off diminished profit due to ‘give-away’ with diminished profit induced by producing below the stipulated threshold. Besides successfully formulating the problem, Hunter and Kartha provide a graphical method of solution. They consider the problem under the assumption that once the initial setting is made, no other control actions are subsequently required.

The following considers a similar problem, the main focus being on a model where production between two dimensional values can be reprocessed at a cost but where items produced below the lower of these is unsaleable. As before, items initially produced above the upper threshold attract a fixed selling price but involve ‘give-away' product. The problem, once again, is to obtain the optimal process setting so as to maximise the expected profit per item. The problem is formulated, the solution discussed and the nature of dependencies of the solution on the problem parameters illustrated. The existence of more sophisticated computational tools, than those available in 1977, when Hunter and Kartha published their work, removes the necessity or desirability of relying on graphical methods of solution. None-the-less, graphical displays are shown to be powerful indicators of parameter dependencies.

3.2. Factors affecting the choice of target for a process with 'top-up' and 'give-away'.

3.2.1 Methodology

Consider a process where containers are filled, with quantity q, as close to L as possible. If $q \geq L$ then they are sold at a fixed selling price, with

Profit = A - Production Cost.

where A = Selling Price - Material Cost.

If, however, $L_0 \leq q < L$ the item can be 'topped-up' and sold at the same price providing

Profit = B - Production Cost,

where B =A - Additional Processing Cost.

When a container needs to be 'topped-up' it is assumed that this can be done exactly. A container such that $q < L_0$ is not 'topped-up', above all, for economic reasons, although the material does not have to be considered lost under such circumstances. The production cost p, which is the cost of filling, is assumed to be constant regardless of the amount placed in the container.

Whenever $q > L$ there is 'give-away' product and the cost of this excess per unit measure is denoted by e. The aim is to find the mean setting of the process (assumed to be stable) so that the profit per container is maximised.

Figure 3.1 illustrates the inter-relationships between L_0, L and T, the target dimension.

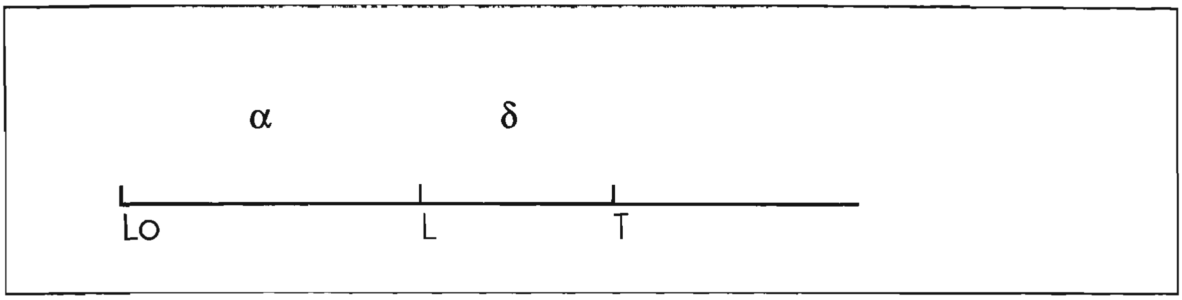


Figure 3.1
Shows the inter-relationships between lower limit of the process (L), the "secondary " lower limit (L_0) and the target dimension (T).

In practical applications, the target value is ordinarily above L but it is possible for it to be placed below L.

Unless $$3\sigma > \alpha + \delta,$$

where σ is the process standard deviation, then the problem is not significantly different from that considered by Hunter and Kartha. The inequality is thus assumed to hold.

In the analysis it is assumed that q is normally distributed with mean T and with known variance σ^2 . In any practical situation it would be expected that the optimal value of δ, which is the focus of attention, is greater than 0, however this depends on α, the ratio between A and B, and/or the standard deviation of the process. Again, from practical considerations, $B/A < 1$, where $A, B > 0$. The target value, $T=L+\delta$ is called optimum if it maximises the expected profit per container.

3.2.2. Theoretical analysis

The profit from a single item may be written as follows:

$$P(q)=\begin{cases}(A-p)-e(q-L), & q\geq L\\ B-p, & L_0\leq q\leq L\\ -p, & \text{otherwise.}\end{cases}$$

Thus the expected profit per item, denoted by E[P(q)], is

$$E[P(q)] = (A\text{-}p)\int_L^{\infty} f(q)dq - \int_L^{\infty} e(q-L)f(q)dq + (B\text{-}p)\int_{Lo}^{L} f(q)dq - p\int_{-\infty}^{Lo} f(q)dq \qquad (1)$$

where f(q) is the p.d.f. of q, i.e.

$$f(q)=(2\pi\sigma^2)^{-1/2}\exp\{-(q-T)^2/2\sigma^2\}$$

The objective is to obtain the value of δ that maximises E[P(q)] ; where $\delta = T - L$.

Let

$$\phi(x) = (2\pi^2)^{-1/2}\exp(-x^2/2)$$

and

$$\Phi(q) = \int_{-\infty}^{q} \phi(x)dx$$

Equation (1) can be simplified to

$$E[P(q)] = A-e\delta-p+(B-A-e\delta)\Phi\left(\frac{-\delta}{\sigma}\right)-B\Phi\left(\frac{-\delta-\alpha}{\sigma}\right)-e\sigma\phi\left(\frac{-\delta}{\sigma}\right).$$

Differentiating with respect to δ,

$$E'[P(q)]=\frac{(A-B)}{\sigma}\phi\left(\frac{-\delta}{\sigma}\right)+\left(\frac{B}{\sigma}\right)\phi\left(\frac{-\delta-\alpha}{\sigma}\right)-e\Phi\left(\frac{\delta}{\sigma}\right) \qquad (2)$$

Setting equation (2) to zero, gives:

$$\Phi\left(\frac{\delta}{\sigma}\right)^{-1}\left(\phi\left(\frac{-\delta}{\sigma}\right)+\frac{B}{A-B}\phi\left(\frac{-\delta-\alpha}{\sigma}\right)\right)=\frac{e\sigma}{A-B} \qquad (3)$$

The second derivative with respect to δ from (2) gives,

$$E''[P(q)] = -\left(\frac{(A-B)\delta}{\sigma^3}\right)\phi\left(\frac{\delta}{\sigma}\right)-\left(\frac{B(\delta+\alpha)}{\sigma^3}\right)\phi\left(\frac{\delta+\alpha}{\sigma}\right)-\frac{e}{\sigma}\phi\left(\frac{\delta}{\sigma}\right).$$

If $E''[P(q)] < 0$ (with $\delta = \delta_0$) that is if

$$-\frac{\delta_0}{\sigma}-\frac{B(\delta_0+\alpha)\phi\left(\frac{\delta_0+\alpha}{\sigma}\right)}{(A-B)\phi\left(\frac{\delta_0}{\sigma}\right)}<\frac{e\sigma}{A-B} \qquad (4)$$

then δ_0 is optimal. The solution to (3) will then give a setting for the target that will maximise the expected profit.

From practical considerations $\alpha > 0$ and if $\delta < 0$ then $-\delta < \alpha$ and so $\alpha + \delta > 0$, thus inequality (4) is true. If $\delta > 0$ (4) holds since $\sigma > 0$.

3.2.3 Dependencies between the process parameters

To investigate the relationships between the variables, as well as to study the effects of various model parameters on the target mean and the expected profit, several data sets were generated using *Mathematica*. The following graphs were obtained using *Mathematica* and *SPSS*.

Unless otherwise stated, each analysis is based on the example given by Hunter & Kartha. Some additional values, believed to be suitable, are also chosen by the authors. For $q \geq L$, A, the selling price –material cost, is 67 and the "give-away" cost is 55. The difference between L_0 and L is 0.1 and L=1.

Discussion commences with a study of the relationship between the process standard deviation and δ_0 the optimal value of δ. The data used is shown in Table 3.1 (the present model) and 3.1a (Hunter&Kartha model, as described in chapter 2). A graphical comparison is made for the current model with that prescribed by Hunter and Kartha.

SIGMA	Optimal DELTA
0.10	0.15
0.15	0.20
0.20	0.24
0.25	0.27
0.30	0.30
0.35	0.31
0.40	0.32
0.45	0.32
0.50	0.32
0.55	0.31
0.60	0.29
0.65	0.27
0.70	0.24
0.75	0.21

Table 3.1
Shows the data generated using the present model.

SIGMA	Optimum DELTA		SIGMA	Optimum DELTA
0.01	0.03		0.25	0.18
0.03	0.06		0.27	0.17
0.05	0.09		0.29	0.16
0.07	0.11		0.31	0.16
0.09	0.13		0.33	0.15
0.11	0.15		0.35	0.13
0.13	0.16		0.37	0.12
0.15	0.17		0.39	0.10
0.17	0.17		0.41	0.08
0.19	0.18		0.43	0.06
0.21	0.18		0.45	0.04
0.23	0.18		0.47	0.02

Table 3.1a
Shows the data generated using the Hunter & Kartha model.

The optimal values of δ (δ_0) plotted against σ (keeping A, B, α and e constant at A=67, B=0.5A, α=0.1, e=55)) are shown in Figure 3.2a. Several observations are worth noting. As is clearly shown, a single optimal δ value arises from two distinct σ's. Figure 3.2b illustrates the same phenomena for the Hunter & Kartha model.

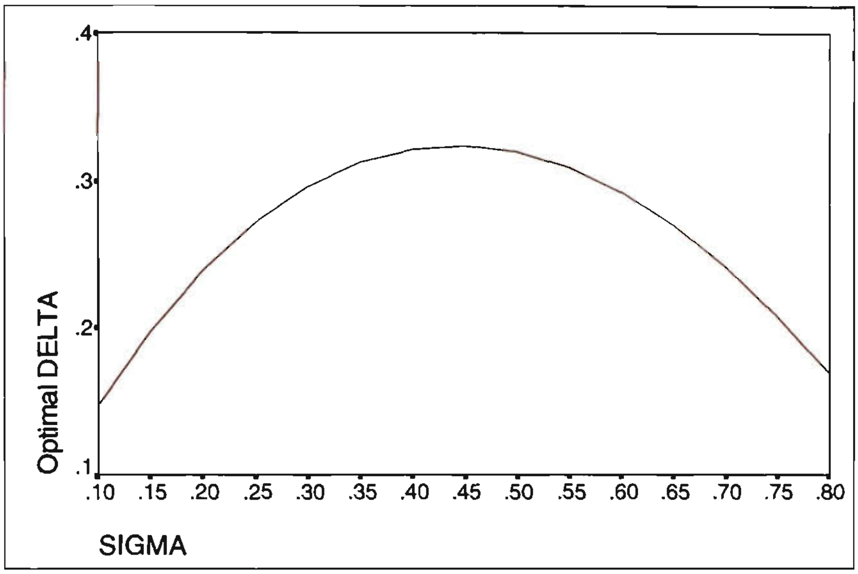


Figure 3.2a
Shows the optimal delta values for sigma ranging from 0.1 to 0.8.
The results were obtained using present model.

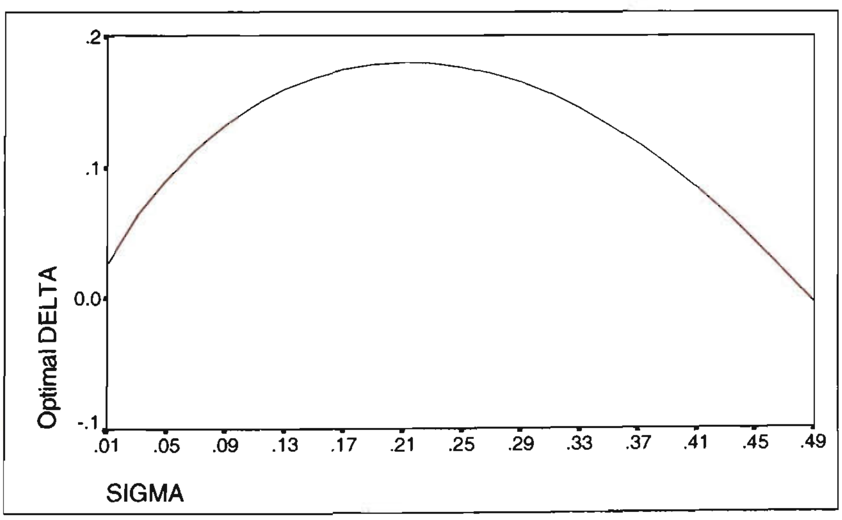


Figure 3.2b
Shows the optimal delta values for sigma ranging from 0.01 to 0.5.
The results were obtained using Hunter&Kartha model.

The second observation concerns the effect of various combinations of α, the ratio between A and B and the percentage increase in σ on the optimal value of δ. This is illustrated in Table 3.2, which gives the percentage change in optimal δ due to a shift in σ. For the particular chosen values of α, 0.1 and 0.3, we choose B/A to be 0.4, 0.5 and 0.6. Smaller shifts in σ (33% increase) cause nearly the same change in optimal δ as a shift by 66%. If the process standard deviation shifts by 100% the optimal setting of process target is not significantly affected. The bigger the ratio B/A, however, the bigger the affect of shift in standard deviation on optimal δ.

		shift in σ		
α	**B/A**	**0.3 to 0.4** (33% increase)	**0.3 to 0.5** (66% increase)	**0.3 to 0.6** (100% increase)
0.1	**0.4**	8.0%	7.0%	2.5%
	0.5	8.6%	7.9%	1.1%
	0.6	12.6%	8.8%	0.2%
0.3	**0.4**	7.7%	6.9%	2.6%
	0.5	9.6%	10.0%	1.0%
	0.6	12.3%	14.2%	5.5%

Table 3.2

Shows the percentage change in optimal delta due to 33%, 66%, and 100% change in σ, where the distance between L and L_0 increases from 0.1 to 0.3 and the ratio between A and B is equal to 0.4, 0.5 and 0.6.

It should be pointed out that the behaviour of σ in relation to δ_0, observed in Figure 3.2a, does not imply that the generated profit will be the same for the two different σ values that provide the same value of δ_0. The relationship between these three variables, using the above values, is shown in

Figure 3.3a.

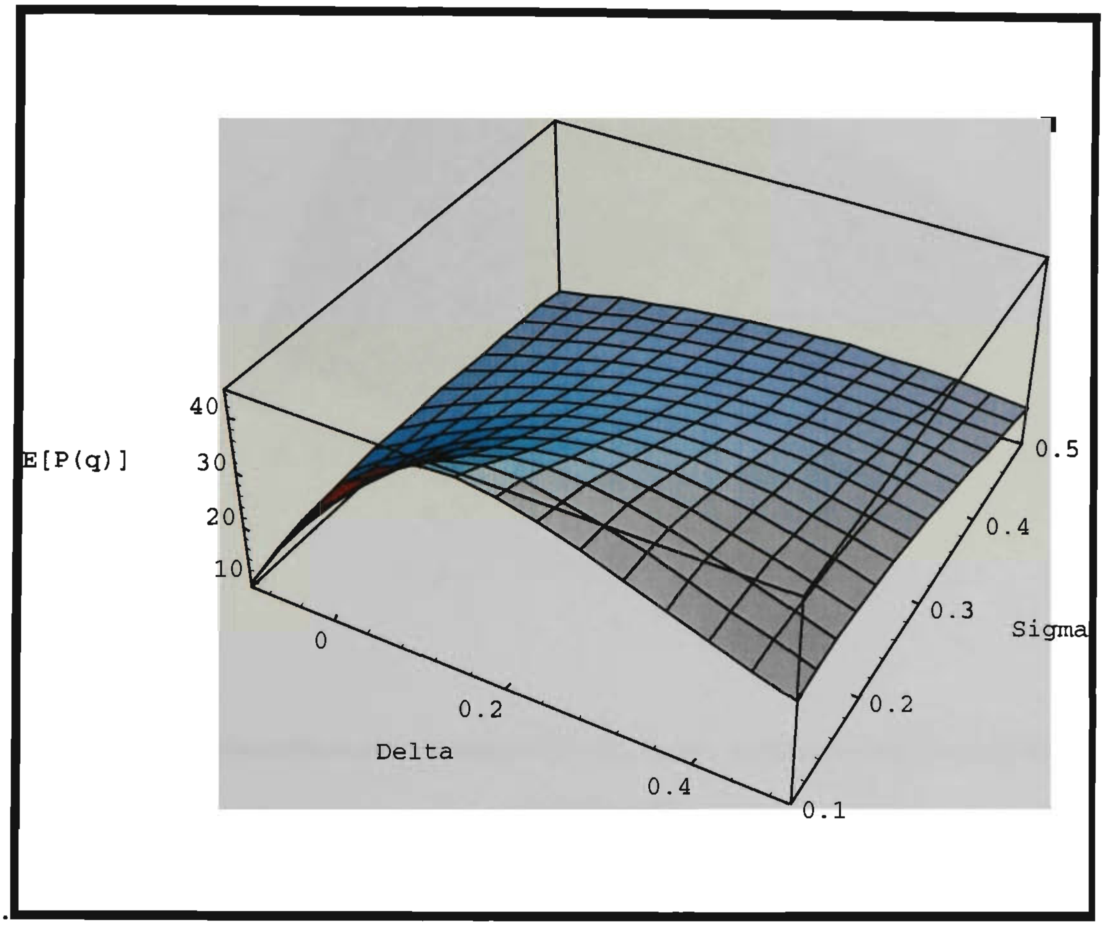


Figure 3.3a
Shows the relationship between the optimal delta, the process standard deviation (ranging from 0.1 to 0.5) and the maximum profit.

It is to be expected that an increase in sigma leads to a decrease in profit. This is shown clearly in Figure 3.3b. The flatness of the optimal profit curve as the standard deviation of the process gets bigger, should be noted.

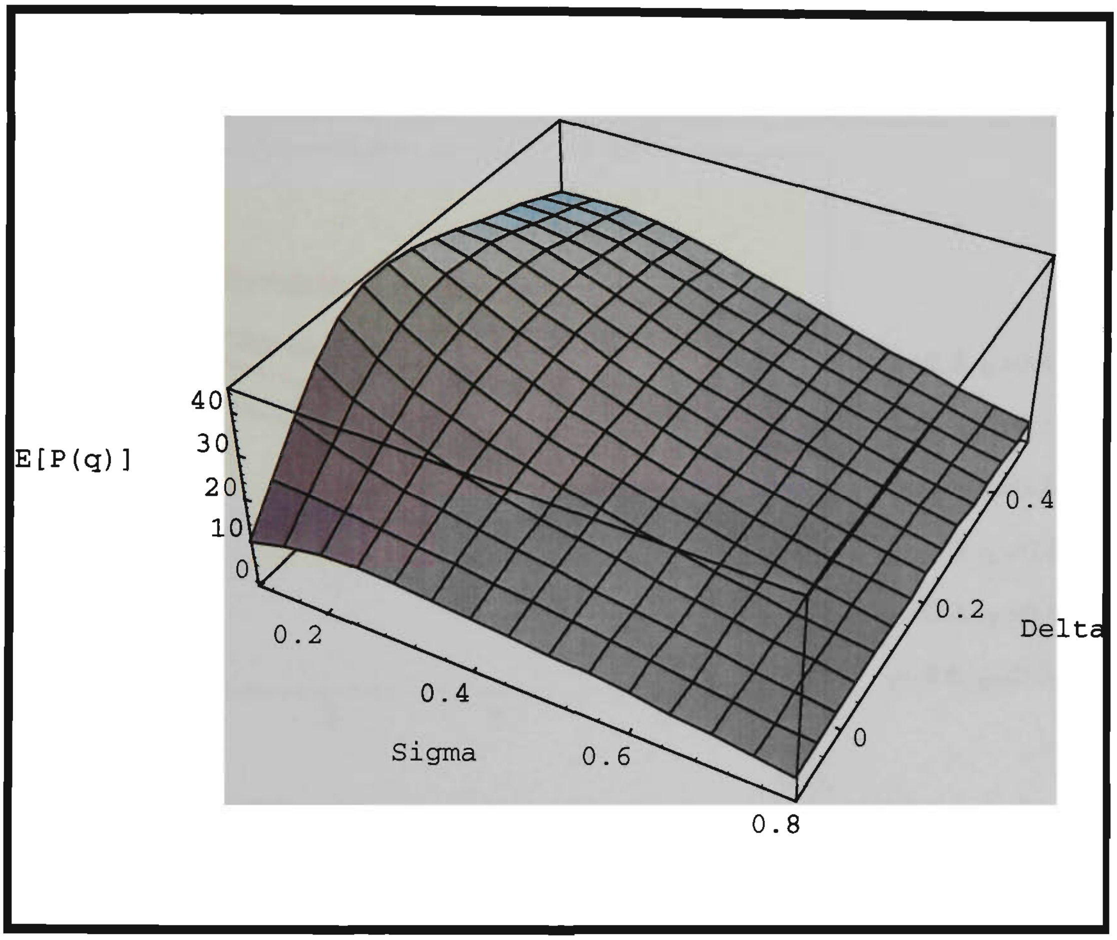


Figure 3.3b
Shows the relationship between the optimal delta, the process standard deviation (ranging from 0.1 to 0.8) and the maximum profit.

Figure 3.4 illustrates the effect of change in the ratio B/A on the optimal process setting. It should be observed that for small α (in this case 0.1) the optimal target setting seems to be approximately constant regardless of changes in B/A or the standard deviation of the process. Note that the result obtained from figure 3.2 is also clearly visible in Figure 3.4.

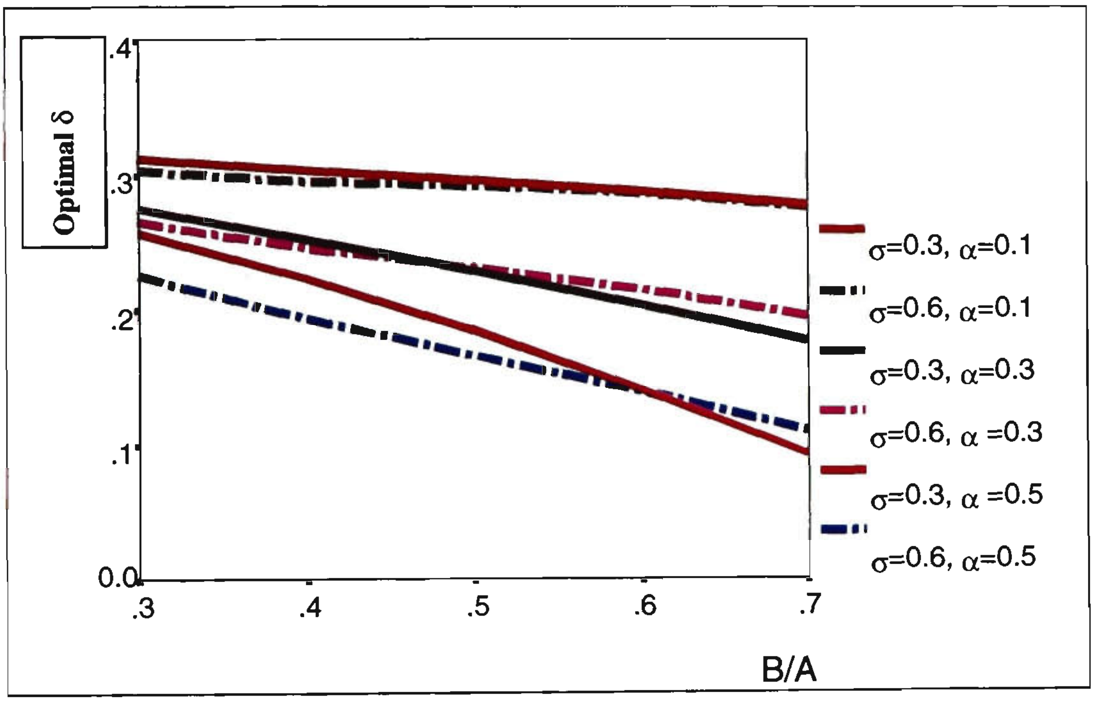


Figure 3.4

Shows 3 pairs of curves. Each pair has assigned the same two values for standard deviation (0.3 and 0.6) but different α values (0.1, 0.3 and 0.5); the same for both curves within each pair. The graph shows the effect of change in the ratio between A and B on the optimal target setting (optimal δ) .

A more precise analysis of the relation between the optimal target value of the process and B/A is shown in Table 3.3, which illustrates the percentage change in δ_0 due to change in pricing policy. It can be observed that for relatively small α, if σ increases by 100% then even a large increase in the ratio of B over A has a minor effect on optimal δ.

α	σ	**Change in B/A from 0.4 to 0.6**	**Change in B/A from 0.3 to 0.7**
0.1	**0.3**	5.1%	10.8%
0.1	**0.6**	2.8%	8.7%
0.3	**0.3**	19.2%	35.5%
0.3	**0.6**	12.5%	26.1%
0.5	**0.3**	37.0%	63.8%
0.5	**0.6**	28.2%	51.2%

Table 3.3
Shows the percentage change in optimal process setting due to change in the ratio B to A. The distance between L and L_0 varies from 0.1 to 0.5, the process standard deviation shifts from 0.3 to 0.6 and B/A changes from 0.4 to 0.6 and from 0.3 to 0.7.

The further L_0 is from L (i.e. the larger the α) the bigger the effect of B/A on optimal δ. These changes will be larger for smaller σ.

Figure 3.5 illustrates the result of relaxing or tightening the distance between L and L_0 on the optimal solution. An increase in α reduces the value of the optimal δ i.e. brings it closer to L_0. This effect is more significant for small values of α as well as bigger ratios of B/A. As α increases, this effect diminishes.

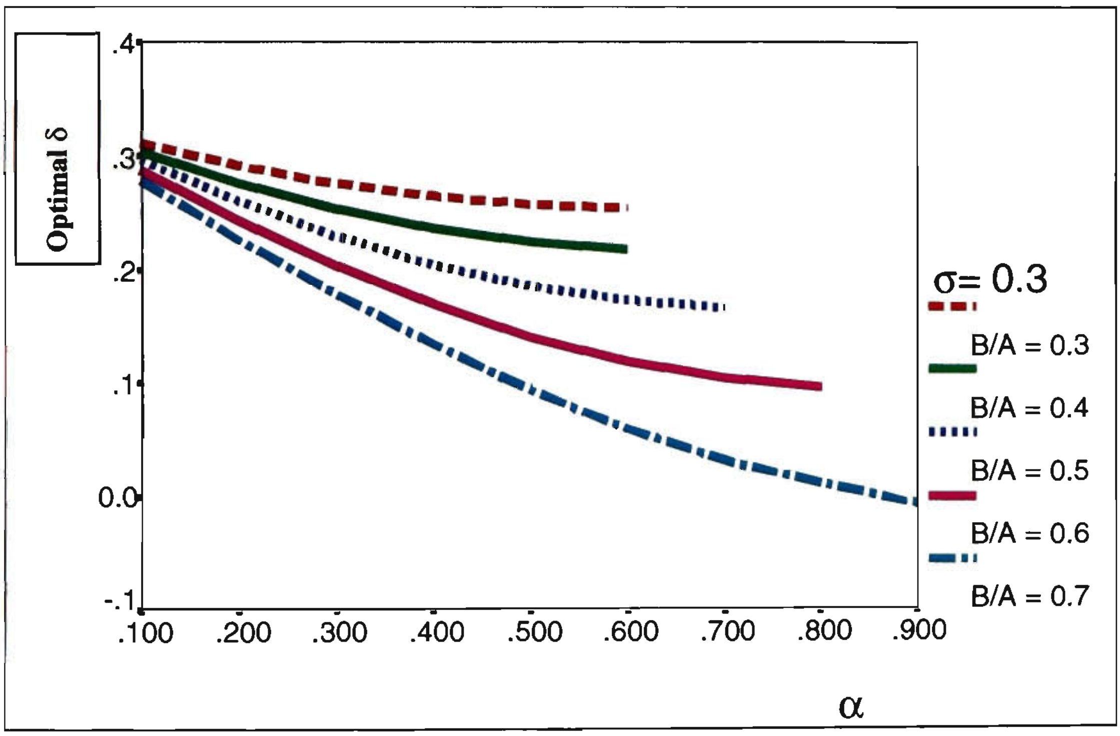


Figure 3.5
Shows curves of optimal delta values against values of alpha for sigma = 0.3 and B/A from 0.3 to 0.7.

3.3 Concluding remarks

The results would seem to indicate that even if the process variance increases there is little gain in adjusting the mean per se but a more appropriate strategy would be to concentrate on reducing this variability as the increase will diminish profit per item. Furthermore, if there is an increase in α, which would likely be accompanied by a decrease in B (ie. the ratio B/A will decrease) then there is again little advantage in adjusting the process setting.

CHAPTER 4

Mean Selection for a Filling Process

4.1 Introduction

Many types of filling processes exist in practice, that considered here, is with the intent of obtaining the most appropriate mean setting to maximise profit per item . Overflowed material can either be recaptured or lost; under-filled containers can either be 'topped-up' and sold for a regular price, or emptied out and material put back into the process or they can simply be discarded. The latter would be typical of the food or pharmaceutical industries where material cannot be re-used, mainly for hygienic reasons. Other authors have addressed a similar problem, however they have assumed containers to have infinite capacity and their cost to be negligible. The issue of whether or not the obtained optimum meets 'Weights and Measures' requirements has not previously been discussed. In practice, overflow can occur during filling, with or without a loss of material and meeting weight (or volume) specifications is enforced by law. Overfilling is a real problem in industry, not only due to excess material that is 'given-away' when the containers are overfilled but also in some instances, eg. wine and spirits, where there are penalties for overfilling associated with excise tax. In addition, any container that is hermetically sealed cannot be filled to capacity. In the food industry under-filled containers are invariably discarded. In chemical processes under-filled containers are frequently topped-up or emptied out and the material re-used. Such situations involve different costs and make a significant difference to the model and its solution.

This chapter explores different types of filling processes and presents appropriate models and theoretical analysis for the selection of the most profitable process settings. Special attention is drawn to the finite capacity of containers. Issues considered include waste, overfill, 'top-up' and the additional filling costs of items not initially meeting requirements, as well as extra costs per unit of recapturing the overflow, topping-up the containers or putting the material back into the process. The cost of the containers is also considered as an important part of the model. Model solutions are displayed graphically. The effect of change of the process variance on the optimal solution as well as on the expected profit are both discussed.

In all cases problem solutions are illustrated graphically and results obtained using numerical methods. The latter are obtained on Mathematica. The codes for solution, of each of the models proposed, are shown in the appendix and have proven to be both very easy to use and quick to evaluate.

4.2 General Problem

Consider an automatic filling process where containers are filled with some ingredient, let this amount be denoted by a random variable X, and without loss of generality assume this to be a measure of volume. It is assumed that X is normally distributed with mean T and known variance σ^2. A common method of checking that the process follows a normal distribution is to take a random sample from the process and draw a histogram. Once achieved, control charts can be used to monitor that the process remains in statistical control. The nominal amount of material in each container (on the label) is L. According to Weights and Measures Legislation in Australia, containers with a minimum proportion of 0.95 of the stated label content can be legitimately sold at the regular price. For generality reasons let this quantity be hL, where $0 < h < 1$. An automatic device rejects containers with content below hL. The cost of product in the container is denoted by gx, where g is the cost of material per unit of volume.

The aim is to fix the filling mean of the process to maximise the expected profit per container. The Target value, T = hL + δ ($\delta > 0$), is called optimal if it maximises the expected profit per container. The inter-relationships between hL, L, L+k and T are shown in Figure 4.1.

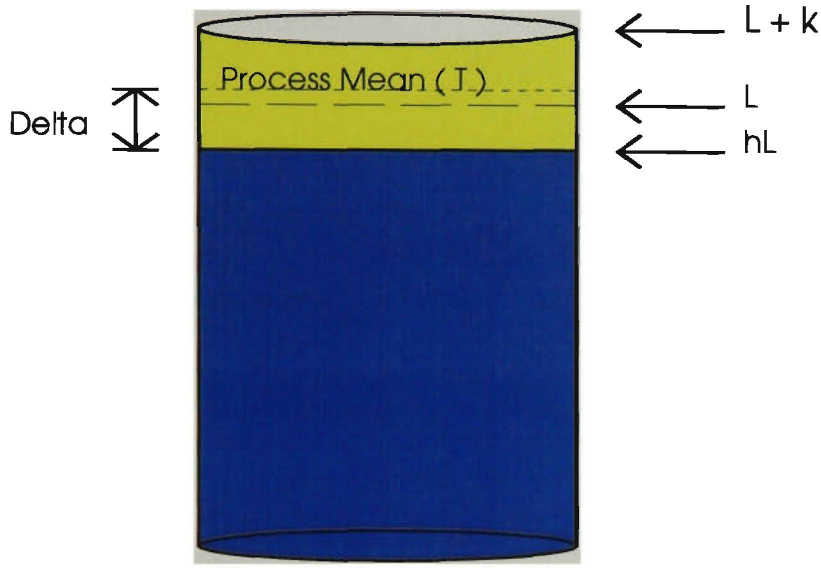


Figure 4.1
Inter-relationships between hL, L, L+ k and T.

In this chapter a number of variations of the filling process model are considered. Two concern a filling process where overflowed material is captured at no additional cost. In the event of under-filling one considers a case where the containers are emptied-out and the material put back into the process and the other considers a situation where 'top-up' occurs. The final model describes a situation where there are additional costs involved in the filling process.

4.3 Theoretical Analysis

The following provides the development of a number of different models for filling processes together with their analyses. Each solution for the initial setting of the process is displayed graphically, showing a single optimum solution for all meaningful parameter values. Each solution, as well as the expected profit function, were evaluated using Mathematica.

4.3.1 Finite capacity containers

In these cases overflow is recaptured and reused.

4.3.1.1. Model 1

Consider a filling process where, in the event of under-filling, the product is reused. This is often the case in the chemical industry for products like dish-washing liquid, washing powder, hair-care products, and paint. In the case of a powder, in order to get the material from inside the box, the boxes have to be cut up and thrown away. In instances of liquids two scenarios are common. If the containers are difficult or very expensive to wash, for example as in the paint industry, the containers are discarded. For containers that are easily washable, like dish-washing liquid or shampoo, the bottles are washed and reused. Two model variations are proposed,

for both cases it is assumed that the containers have a capacity of L+k and any overflow is captured at no additional cost.

Model 1-Case 1

The following considers the case where the containers from the under-filled items are discarded.

Hence if $hL \leq x$,

Profit = Selling Price - Filling Cost(including the cost of the container) - Material Cost.

Thus profit from a single fill attempt may be written as:

$$P(x) = \begin{cases} S - C_f - C_c - g(L+k) & x \geq L+k \\ S - C_f - C_c - gx & hL \leq x < L+k \\ -C_f - C_c & x < hL \end{cases}$$

where S is the selling price and C_f and C_c are the filling cost and the cost of the container, respectively.

The expected profit per fill attempt, denoted by E[P(x)], is

$$E[P(x)] = -(C_f + C_c)\int_{-\infty}^{hL} f(x)dx + \int_{hL}^{L+k}(S - C_f - C_c - gx)f(x)dx + (S - C_f - C_c - g(L+k))\int_{L+k}^{\infty} f(x)dx \qquad (1)$$

standardising,

$$E[P(x)] = -(C_f + C_c)\int_{-\infty}^{\frac{hL-\mu}{\sigma}} \phi(z)dz + (S - C_f - C_c)\int_{\frac{hL-\mu}{\sigma}}^{\frac{L+k-\mu}{\sigma}} \phi(z)dz - g\int_{hL}^{L+k} xf(x)dx + (S - C_f - C_c - g(L+k))\int_{\frac{L+k-\mu}{\sigma}}^{\infty} \phi(z)dz$$

and putting $\mu = hL + \delta$ gives,

$$
\begin{aligned}
E[P(x)] = &-(C_f + C_c)\Phi\left(\frac{-\delta}{\sigma}\right) + \\
&(S - C_f - C_c)\left(\Phi\left(\frac{L+k-hL-\delta}{\sigma}\right) - \Phi\left(\frac{-\delta}{\sigma}\right)\right) - \\
&g\int_{hL}^{L+k} xf(x)dx + (S - C_f - C_c - g(L+k))\left[1 - \Phi\left(\frac{L+k-hL-\delta}{\sigma}\right)\right]
\end{aligned}
\tag{2}
$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ are the p.d.f. and distribution functions respectively of the standard normal distribution.

Further simplifications lead to:

$$
\begin{aligned}
E[P(x)] = &(S - C_f - C_c - g(L+k)) + g(L+k)\Phi\left(\frac{L+k-hL-\delta}{\sigma}\right) - S\Phi\left(\frac{-\delta}{\sigma}\right) - \\
&-g\int_{hL}^{L+k} xf(x)dx
\end{aligned}
$$

where,

$$
g\int_{hL}^{L+k} xf(x)dx = g\int_{\frac{hL-\mu}{\sigma}}^{\frac{L+k-\mu}{\sigma}} (\sigma z + \mu)\sigma\phi(z)dz =
$$

$$
g\sigma(hL+\delta)\left(\Phi\left(\frac{L+k-hL-\delta}{\sigma}\right) - \Phi\left(\frac{-\delta}{\sigma}\right)\right) + g\sigma^2\left(\phi\left(\frac{-\delta}{\sigma}\right) - \phi\left(\frac{L+k-hL-\delta}{\sigma}\right)\right)
$$

The value of this function for any parameter values can be found by using, for example, Mathematica (an example is shown in the Appendix). The goal is to find the target value, T=hL+δ, that maximises the expected profit. Hence, one is interested in the value of δ, say δ_0, that maximises (2). Numerical methods can be used to find a solution to (2) and when doing so it is convenient if the first derivative gives only one optimal solution and the second derivative is always negative thus ensuring that the obtained optimum is a maxima (i.e. the optimum value of the process setting obtained from setting the first derivative equal to zero and solving it for δ by the use of numerical methods on Mathematica would always return a maxima). An algorithm to obtain the optimal mean setting, which can be used for practical purposes, can be found in the Appendix. To find δ_0 differentiate equation (2) with respect to δ. Setting $E'[P(x)] = 0$, gives:

$$E'[P(x)] = \frac{S}{\sigma}\phi\left(\frac{\delta}{\sigma}\right) + g\left(\Phi\left(\frac{-\delta}{\sigma}\right) - \Phi\left(\frac{L+k-hL-\delta}{\sigma}\right)\right) - \frac{ghL}{\sigma}\phi\left(\frac{-\delta}{\sigma}\right)$$

$$\frac{\frac{1}{\sigma}\phi\left(\frac{\delta}{\sigma}\right)}{\Phi\left(\frac{L+k-hL-\delta}{\sigma}\right) - \Phi\left(\frac{-\delta}{\sigma}\right) + \frac{hL}{\sigma}\phi\left(\frac{\delta}{\sigma}\right)} = \frac{g}{S} \qquad (3)$$

As shown in the next section, (3) can be used for the graphical illustration of the solution. Further,

$$E''[P(x)] = -\frac{S\delta}{\sigma^3}\phi\left(\frac{\delta}{\sigma}\right) + \frac{g}{\sigma}\phi\left(\frac{L+k-hL-\delta}{\sigma}\right) - \frac{g}{\sigma}\phi\left(\frac{\delta}{\sigma}\right) + \frac{ghL\delta}{\sigma^3}\phi\left(\frac{\delta}{\sigma}\right)$$

If $E''[P(x)] < 0$ (with $\delta = \delta_0$) i.e. if

$$\frac{-\delta}{\sigma} + \frac{\delta ghL}{S\sigma} + \frac{g\sigma\phi\left(\frac{L+k-hL-\delta}{\sigma}\right)}{S\phi\left(\frac{\delta}{\sigma}\right)} < \frac{g\sigma}{S} \qquad (4)$$

then δ_0 is optimal i.e. the solution to (4) will give a setting for the mean that will maximise the expected profit. It is shown in Figure 2 that $E''[P(x)]$ is always negative.

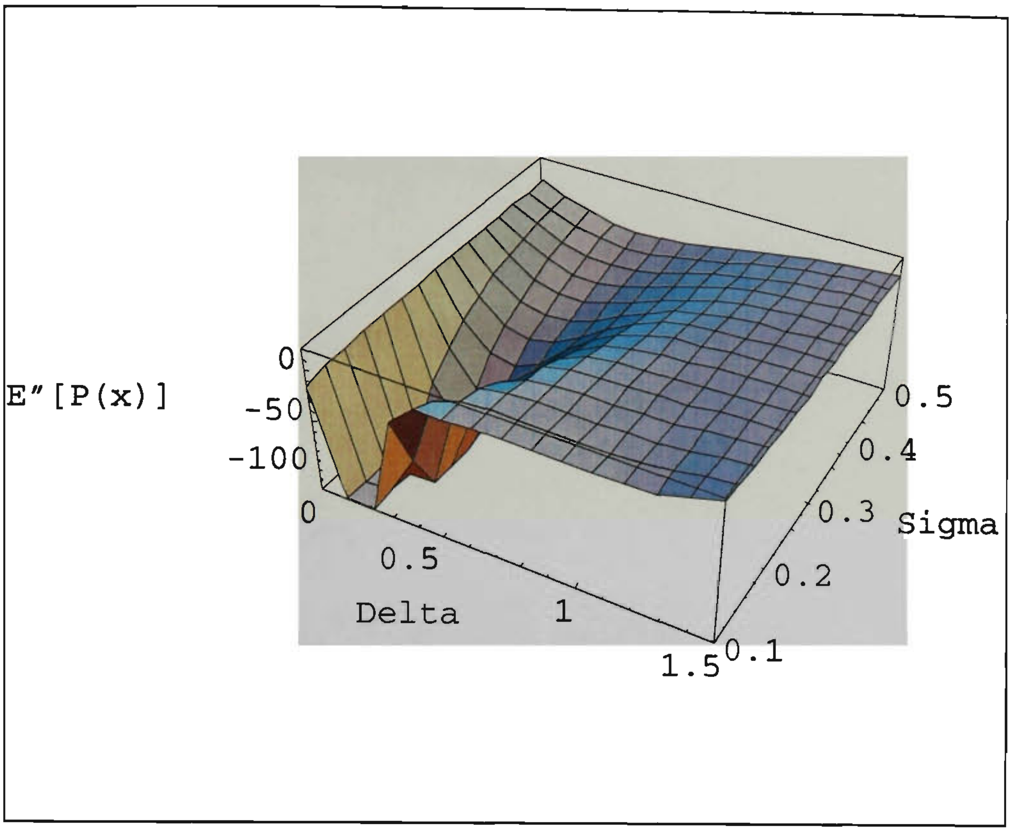


Figure 4.2
E"[P(x)] for $0<\delta<1.5$ and $0.1<\sigma<0.5$ with L=10, h=0.95, k=1, S=67, g=3.5.

Model 1-Case 2

The following considers the case where the containers from the under-filled items are reused. Hence if $hL \leq x$,

Profit = Selling Price - Filling Cost - Material Cost.

Thus profit from a single fill attempt may be written as:

$$P(x) = \begin{cases} S - C_f - C_c - g(L+k) & x > L+k \\ S - C_f - C_c - gx & hL \leq x \leq L+k \\ -C_f - C_w & \text{otherwise} \end{cases}$$

where S is the selling price and C_f , C_c and C_w are the filling cost, the cost of the container and the cost of washing the containers from under-filled items, respectively. The expected profit per fill attempt, denoted by E[P(x)], is

$$E[P(x)] = -(C_f + C_w)\int_{-\infty}^{hL} f(x)dx + \int_{hL}^{L+k}(S - C_f - C_c - gx)f(x)dx + (S - C_f - C_c - g(L+k))\int_{L+k}^{\infty} f(x)dx \quad (1)$$

Standardising and putting $\mu = hL + \delta$ gives,

$$E[P(x)] = -(C_f + C_w)\Phi\left(\frac{-\delta}{\sigma}\right) + (S - C_f - C_c)\left(\Phi\left(\frac{L+k-hL-\delta}{\sigma}\right) - \Phi\left(\frac{-\delta}{\sigma}\right)\right) - g\int_{hL}^{L+k} xf(x)dx + (S - C_f - C_c - g(L+k))\left[1 - \Phi\left(\frac{L+k-hL-\delta}{\sigma}\right)\right] \quad (2)$$

Further simplifications lead to:

$$E[P(x)] = (S - C_f - g(L+k)) + g(L+k)\Phi\left(\frac{L+k-hL-\delta}{\sigma}\right) - (S + C_c - C_w)\Phi\left(\frac{-\delta}{\sigma}\right) - g\int_{hL}^{L+k} xf(x)dx$$

where,

$$g\int_{hL}^{L+k} xf(x)dx = g\int_{\frac{hL-\mu}{\sigma}}^{\frac{L+k-\mu}{\sigma}} (\sigma z + \mu)\sigma\phi(z)dz =$$

$$g\sigma(hL+\delta)\left(\Phi\left(\frac{L+k-hL-\delta}{\sigma}\right) - \Phi\left(\frac{-\delta}{\sigma}\right)\right) + g\sigma^2\left(\phi\left(\frac{-\delta}{\sigma}\right) - \phi\left(\frac{L+k-hL-\delta}{\sigma}\right)\right)$$

Then to find δ_0 differentiate equation (2) with respect to δ:

$$E'[P(x)] = \frac{S - C_c - C_w}{\sigma}\phi\left(\frac{-\delta}{\sigma}\right) - g\left(\Phi\left(\frac{-\delta}{\sigma}\right) - \Phi\left(\frac{L + k - hL - \delta}{\sigma}\right)\right) - \frac{ghL}{\sigma}\phi\left(\frac{-\delta}{\sigma}\right)$$

Setting $E'[P(x)] = 0$, gives the optimal solution.

$$E''[P(x)] = -\frac{S - C_c - C_w}{\sigma^3}\phi\left(\frac{\delta}{\sigma}\right) + \frac{g}{\sigma}\phi\left(\frac{L + k - hL - \delta}{\sigma}\right) - \frac{g}{\sigma}\phi\left(\frac{\delta}{\sigma}\right) + \frac{ghL\delta}{\sigma}\phi\left(\frac{\delta}{\sigma}\right)$$

If $E''[P(x)] < 0$ (with $\delta = \delta_0$) then δ_0 is optimal.

4.3.1.2 Model 2

Consider now a filling process where all under-filled containers are 'topped-up'. This is, for example, the case in the production of carbon black. The grade of product used in printer inks, for example, is dispatched in 40kg bags. The filling process involves the product being dispensed from a dual headed filler into the bags which, when filled, pass, by conveyor, to an automatic weighing device where they are automatically rejected if underweight and subsequently topped up by hand. The cost of the bag is far less than the contents and can be considered negligible. The capacity of the container is L+k. Hence, if $hL \leq x \leq L + k$,

Profit = Selling Price - Filling Cost - Material Cost.

If, however, $x<hL$ the container can be 'topped-up' so that in this instance,

Profit = Selling Price - Filling Cost-Additional Processing Cost - Material Cost.

Excessive material is assumed captured at no additional cost.

The profit from a single fill attempt may be written as:

$$P(x) = \begin{cases} S - C_f - g(L+k), & x > L+k \\ S - C_f - gx, & hL \le x \le L+k \\ S - C_t - C_f - gL, & x < hL \end{cases}$$

where C_t represents the additional filling cost associated with containers that have to be 'topped-up'. It is assumed that each under-filled container can be 'topped-up' exactly to the label specification, L.

Proceeding as previously, the expected profit per fill attempt is:

$$E[P(x)] = S - C_f - g(L+k) + g[L(1-h)+k]\Phi\left(\frac{L-k-hL-\delta}{\sigma}\right) + g\delta\Phi\left(\frac{-\delta}{\sigma}\right)$$
$$+ [gL(h-1) - C_t]\Phi\left(\frac{-\delta}{\sigma}\right) + g\sigma\left(\phi\left(\frac{L+k-hL-\delta}{\sigma}\right) - \phi\left(\frac{-\delta}{\sigma}\right)\right)$$
$$- g\delta\Phi\left(\frac{L+k-hL-\delta}{\sigma}\right)$$

The expected profit, plotted against σ and δ, is shown in Figure 4.3.

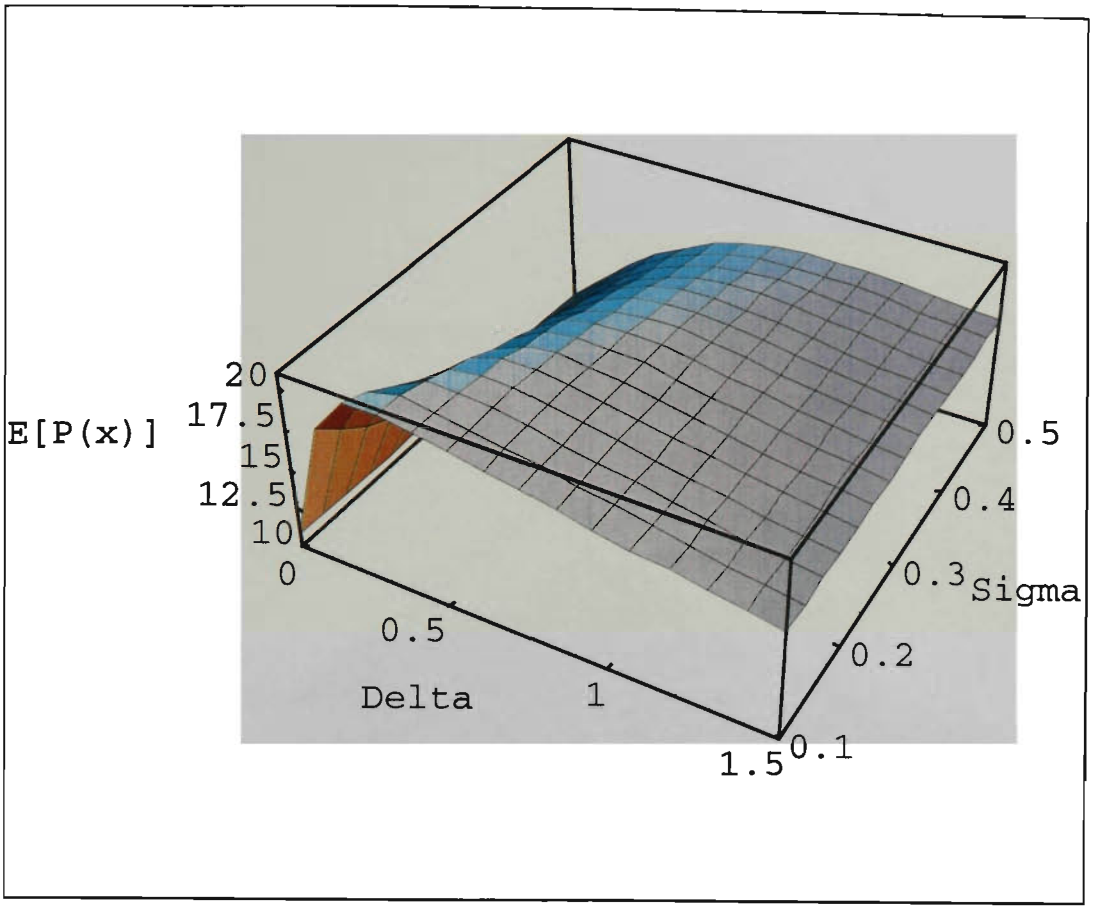


Figure 4.3
E[P(x)] against δ and σ with S=67, C_f=12, g=3.5, C_t=20, L=10, k=1, h=0.95.

Differentiation with respect to δ gives:

$$E'[P(x)] = -g\Phi\left(\frac{L+k-hL-\delta}{\sigma}\right) + g\Phi\left(\frac{-\delta}{\sigma}\right) - \frac{gL(h-1)}{\sigma}\phi\left(\frac{-\delta}{\sigma}\right) + \frac{C_t}{\sigma}\phi\left(\frac{-\delta}{\sigma}\right)$$

Setting $E'[P(x)] = 0$, gives:

$$\frac{\frac{1}{\sigma}\phi\left(\frac{-\delta}{\sigma}\right)}{\Phi\left(\frac{L+k-hl-\delta}{\sigma}\right) - \Phi\left(\frac{-\delta}{\sigma}\right)} = \frac{g}{C_t - gL(h-1)}$$

As Figure 4.4 indicates, there is more than one optimum solution (one maximum and one minimum). One of these solutions (the minimum) however, as shown in greater detail in Figure 4.5, is a point of inflection and also it would provide a value for process setting beyond the capacity of the container. There is thus only one feasible optimum solution for the process setting.

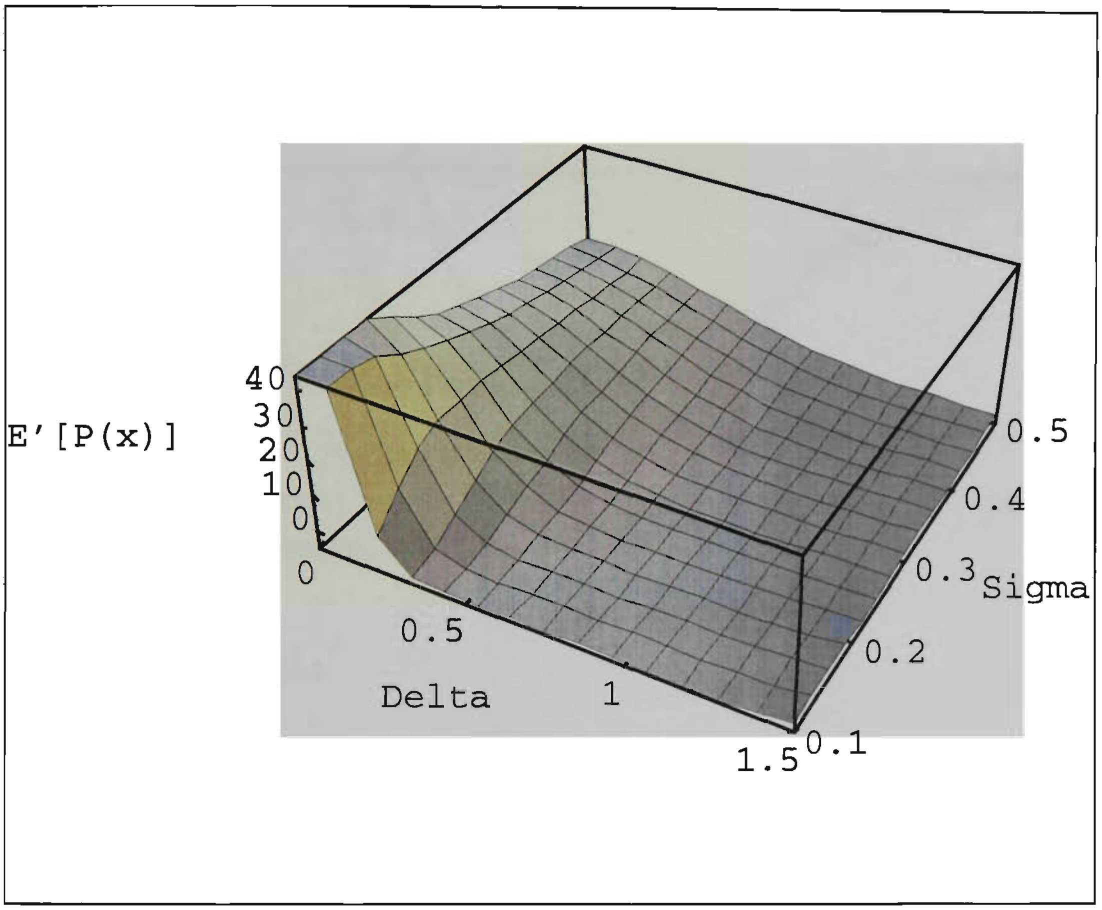


Figure 4.4
E'[P(x)] against σ and δ with L=10, k=1, h=0.95,g=3.5 and C_t=20.

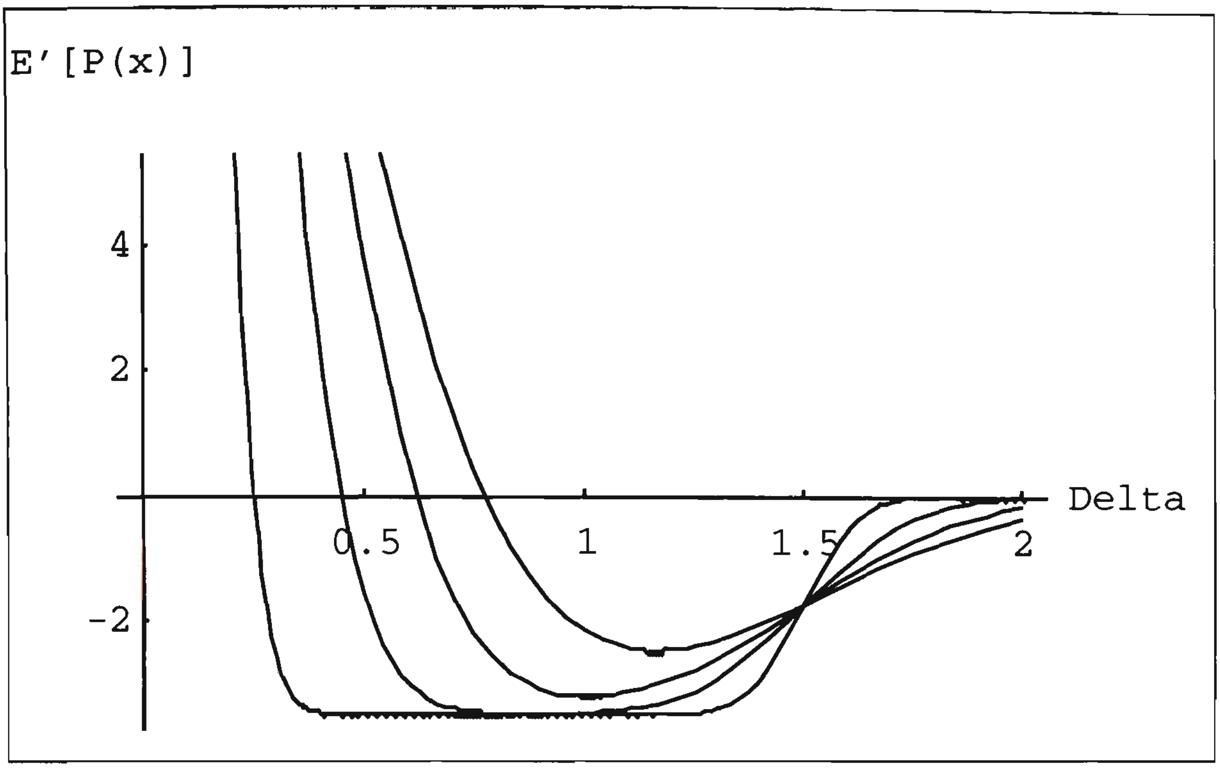


Figure 4.5

E'[P(x)] against δ with L=10, k=1, h=0.95,g=3.5 and C_t=20. The curves shown correspond, from left, to σ = 0.1, 0.2, 0.3 and 0.4.

and if $E''[P(x)] < 0$ i.e.

$$\frac{-\delta\phi\left(\frac{-\delta}{\sigma}\right)}{\phi\left(\frac{L+k-hL-\delta}{\sigma}\right)+\left(1-\frac{L(h-1)}{\sigma^2}\right)\phi\left(\frac{-\delta}{\sigma}\right)} < \frac{-g\sigma^2}{C_t}$$

then δ_0 is optimal. As would be expected, the optimal solution does not depend on S.

As shown in Figure 4.9 the second derivative is negative for all meaningful values of δ and σ.

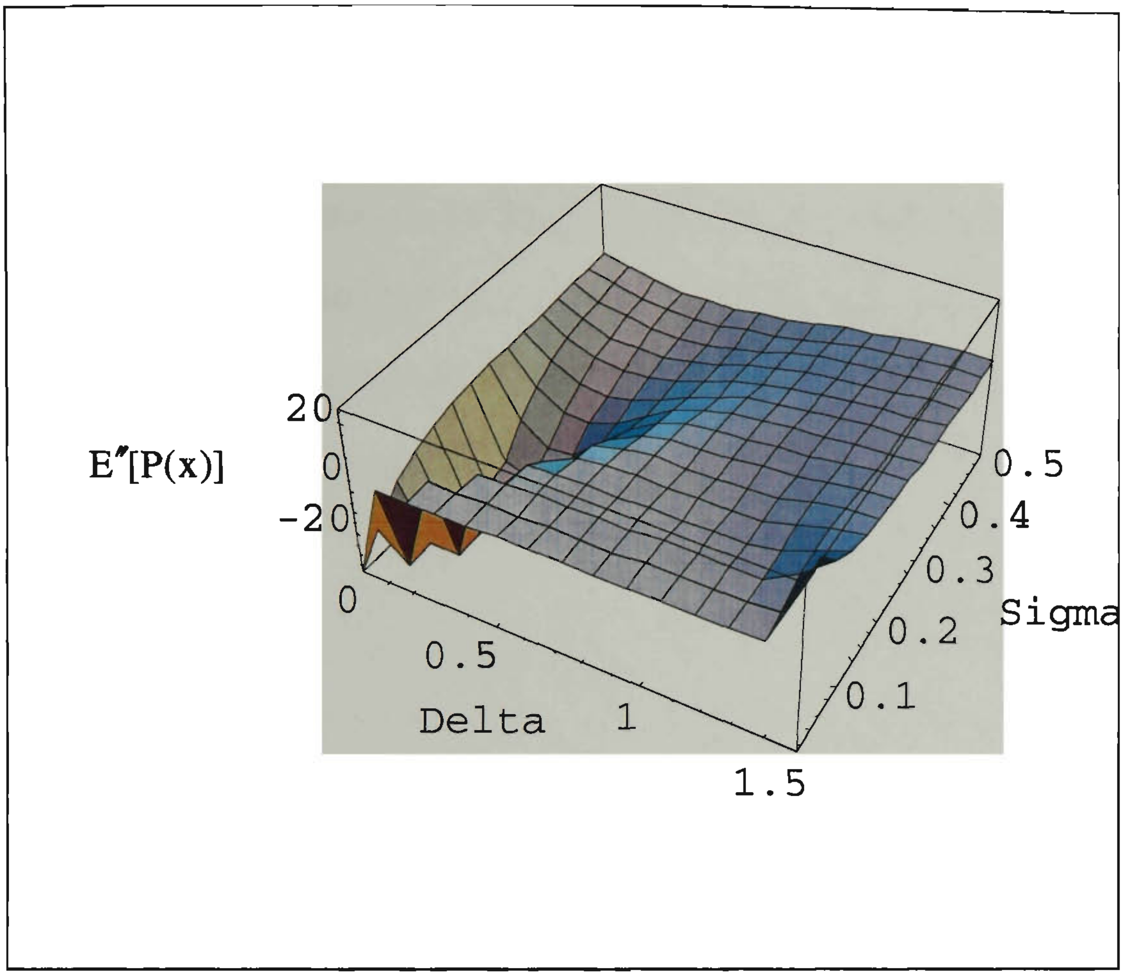


Figure 4.6
E″[P(x)] against δ and σ with C_t=20, g=3.5, L=10, k=1 and h=0.95.

4.3.2 When Overflow is not an Issue

In order to focus on the effects on the optimum setting of treating under-filled containers differently, two variations are considered and the problem of determining the most profitable process setting explored when container overflow during filling is not an issue.

4.3.2.1 Model 3

Consider a model similar to model 1 with the assumption that there is no overflow. This would be a common situation in, for example, paint manufacturing, where a volumetric filling operation is often in use, hence overflow is not an issue but under-filling is still a distinct possibility. Due to high costs of the ingredient the under-filled cans are emptied-out and the material is reused with the containers being discarded mainly due to difficulties involved in washing them. Care is taken to maintain the quality of the product. The profit from a single fill attempt may be written as:

$$P(x) = \begin{cases} S - C_f - C_c - gx, & x \geq hL \\ - C_f - C_c, & x < hL \end{cases}$$

Thus the expected profit per fill attempt is:

$$E[P(x)] = -(C_f - C_c)\int_{-\infty}^{hL} f(x)dx + (S - C_f - C_c)\int_{hL}^{\infty} f(x)dx - g\int_{hL}^{\infty} xf(x)dx$$

$$= S - C_f - C_c - g(hL + \delta) - (S - g(hl + \delta))\Phi\left(\frac{-\delta}{\sigma}\right) - g\sigma\phi\left(\frac{-\delta}{\sigma}\right)$$

The relationship between the expected profit, delta and sigma can be seen in Figure 4.7. The slow decrease in expected profit, for delta bigger than the optimal setting, should be noted.

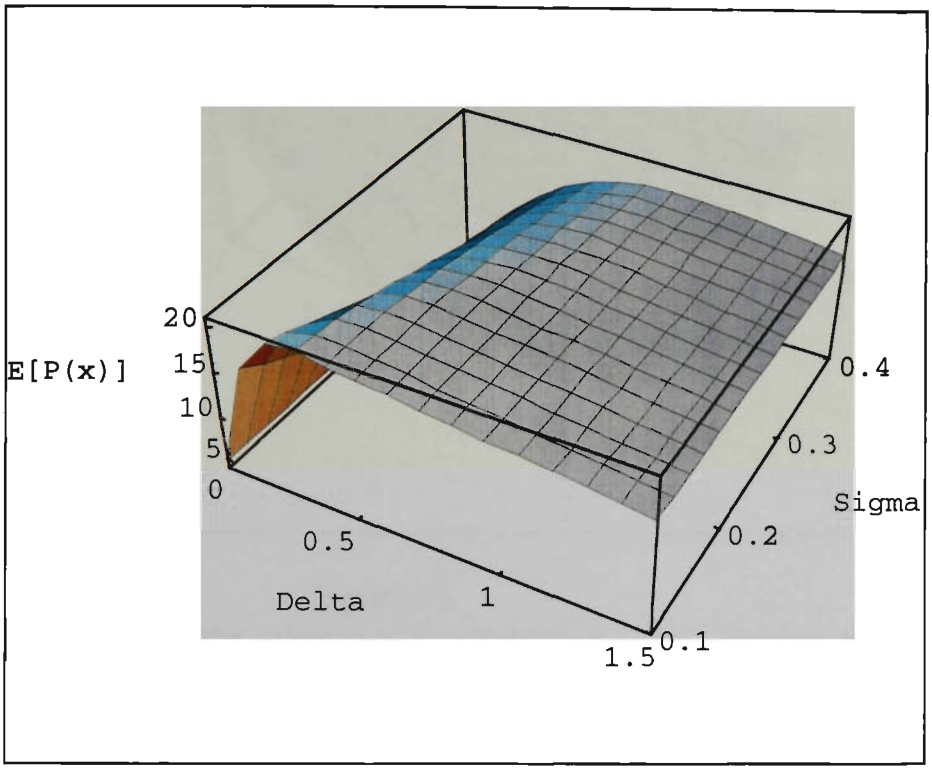


Figure 4.7
Shows the Expected Profit plotted against Delta and Sigma for $0<\delta<1.5$ and $0.01<\sigma<0.5$. The graph was constructed by using: S=67, C_f=12, C_c=0.2g=3.5, L=10, k=1, h=0.95.

Differentiating with respect to δ:

$$E'[P(X)] = -g + \frac{S}{\sigma}\phi\left(\frac{\delta}{\sigma}\right) - \frac{ghL}{\sigma}\phi\left(\frac{\delta}{\sigma}\right) + g\Phi\left(\frac{-\delta}{\sigma}\right)$$

Setting $E'[P(X)] = 0$; $\dfrac{\frac{1}{\sigma}\phi\left(\frac{\delta}{\sigma}\right)}{1 + \frac{hL}{\sigma}\phi\left(\frac{\delta}{\sigma}\right) - \Phi\left(\frac{-\delta}{\sigma}\right)} = \dfrac{g}{S}$.

Figure 4.8 shows the solution to the problem for particular values of process parameters.

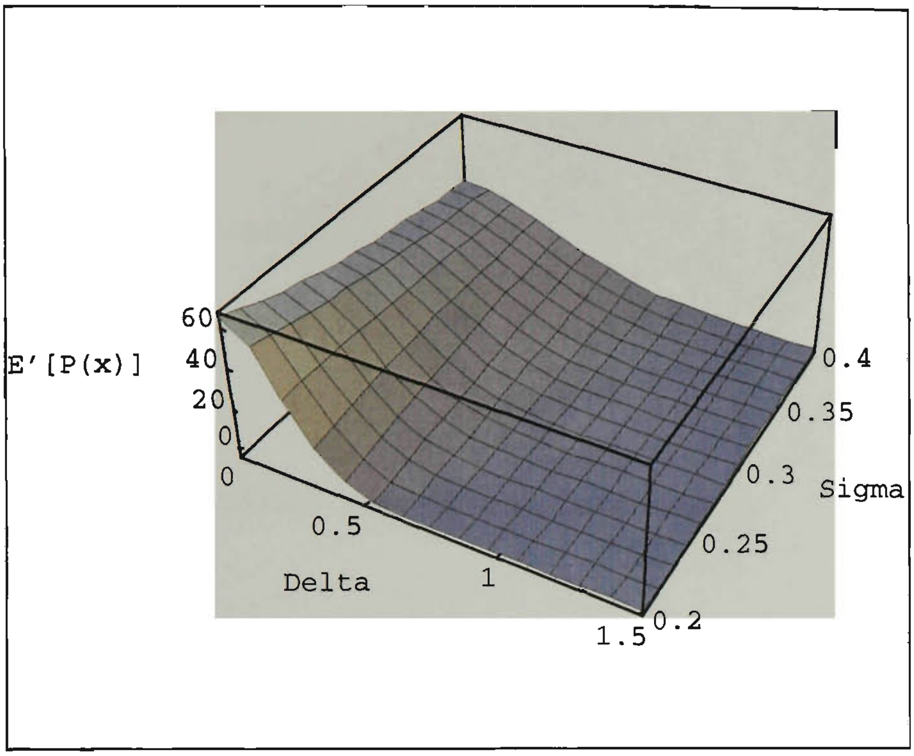


Figure 4.8
Shows the solution to the problem for the following parameter values: L=10, k=1, h=0.95 and the ranges for optimal δ and σ 0 to 1.5 and 0.01 to 0.5 respectively.

Further, $E''[P(X)] = \left[\frac{\delta}{\sigma^2}(ghL - S) - g\right]\frac{\phi\left(\frac{\delta}{\sigma}\right)}{\sigma}$

Thus the expected profit function has maximum, $\delta = \delta_0$, if $-\frac{\sigma^2}{\delta} > \frac{S}{g} - hL$.

As shown in Figure 4.9 the second derivative is negative for a wide range of the process parameters.

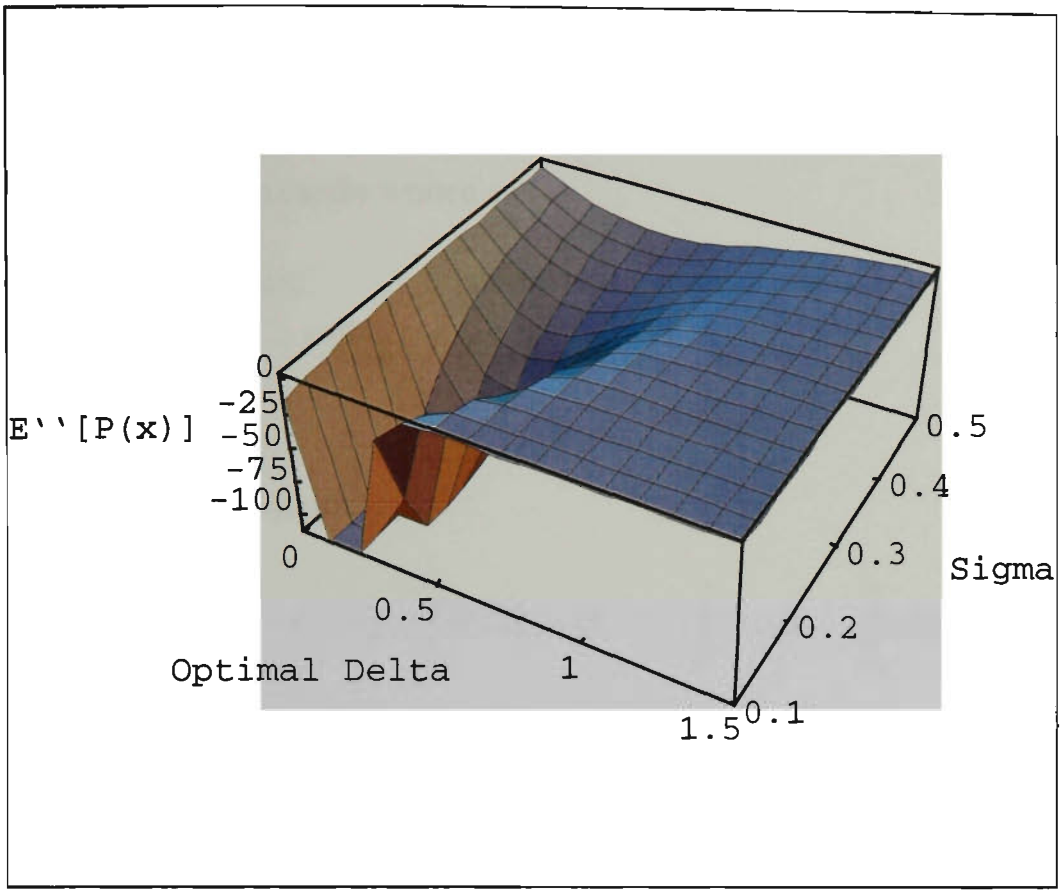


Figure 4.9
Shows the graphical representation of the second derivative for for $0<\delta<1.5$ and $0.1<\sigma<0.5$. The graph was constructed by using: S=67, g=3.5, L=10, and h=0.95.

4.3.2.2 Model 4

Consider now a situation, similar to model 2, where the under-filled containers are 'topped-up' and sold at a regular price, but overflow does not occur. Hence, if the amount in the container is below hL,

Profit = Regular Selling Price - Additional Processing Time Cost - Filling Cost - Material Cost.

It is again assumed that when a container needs to be 'topped-up' it can be done so exactly. If $x \geq hL$, profit generated is as in Model 2.

The profit per item can be written as:

$$P(x) = \begin{cases} S - C_f - gx, & x \geq hL \\ S - C_t - C_f - gL, & x < hL \end{cases}$$

Thus the expected profit is:

$$E[P(x)] = (S - C_t - C_f - gL)\int_{-\infty}^{hL} f(x)dx + (S - C_f)\int_{hL}^{\infty} f(x)dx - g\int_{hL}^{\infty} xf(x)dx$$

$$= S - C_f - g(hL + \delta) - [g(L(h+1) + \delta) - C_t]\Phi\left(\frac{-\delta}{\sigma}\right) - g\sigma\phi\left(\frac{\delta}{\sigma}\right)$$

To find the optimal process setting, equating the first derivative of E[P(x)] with respect to δ to zero, gives:

$$\frac{\frac{1}{\sigma}\phi\left(\frac{\delta}{\sigma}\right)}{1 + \frac{L(h-1)}{\sigma}\phi\left(\frac{\delta}{\sigma}\right) - \Phi\left(\frac{-\delta}{\sigma}\right)} = \frac{g}{C_t}.$$

Further, if $E''[P(X)] < 0$ (with $\delta = \delta_0$) i.e. if

$$\frac{-\delta}{\sigma^2} < \frac{g}{(C_t + gL(1-h))}$$

then the solution is optimal.

4.3.2.3 Model 5

Consider now a situation where there is a loss of material for both overfilled and under-filled containers, and containers are assumed to be of negligible value compared with other costs. Hence, all under-filled containers incur a loss equal to the sum of the material and filling costs. This is a common situation in the food and pharmaceutical industries. The sum of the filling and container costs will again be denoted by C.

The profit from a single fill attempt can then be written as:

$$P(x) = \begin{cases} -C - gx , & x < hL \\ S - C - gx , & x \geq hL \end{cases}$$

and the expected profit per item,

$$E[P(x)] = \int_{-\infty}^{hL} (-C - gx) f(x) dx + \int_{hL}^{\infty} (S - C - gx) f(x) dx$$

$$= S - C - g(hL + \delta) - S\Phi\left(\frac{-\delta}{\sigma}\right).$$

The expected profit function is shown in Figure 4.10

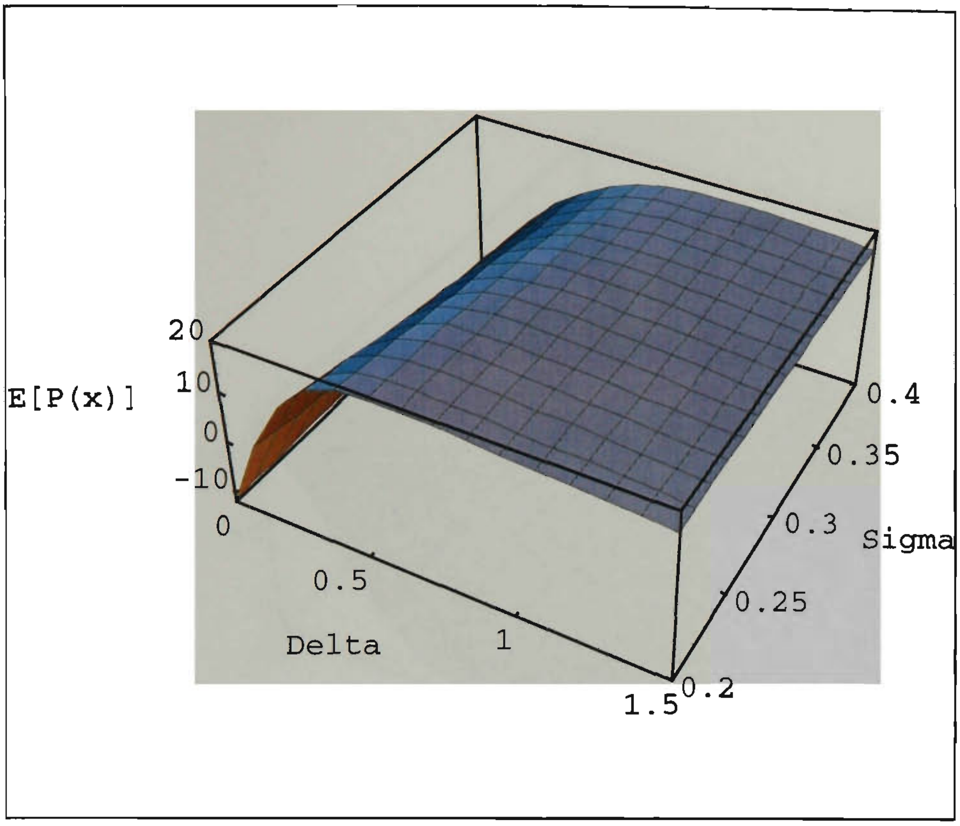


Figure 4.10
E[P(x)] against δ and σ with S=67, C=12.2, g=3.5, L=10, h=0.95.

There is a very rapid decrease in profit as the process mean is set closer to hL. Differentiating with respect to δ and setting $E'[P(x)] = 0$ gives:

$\frac{1}{\sigma}\phi\left(\frac{\delta}{\sigma}\right) = \frac{g}{S}$ which has a single optimum solution as shown in Figure 4.11.

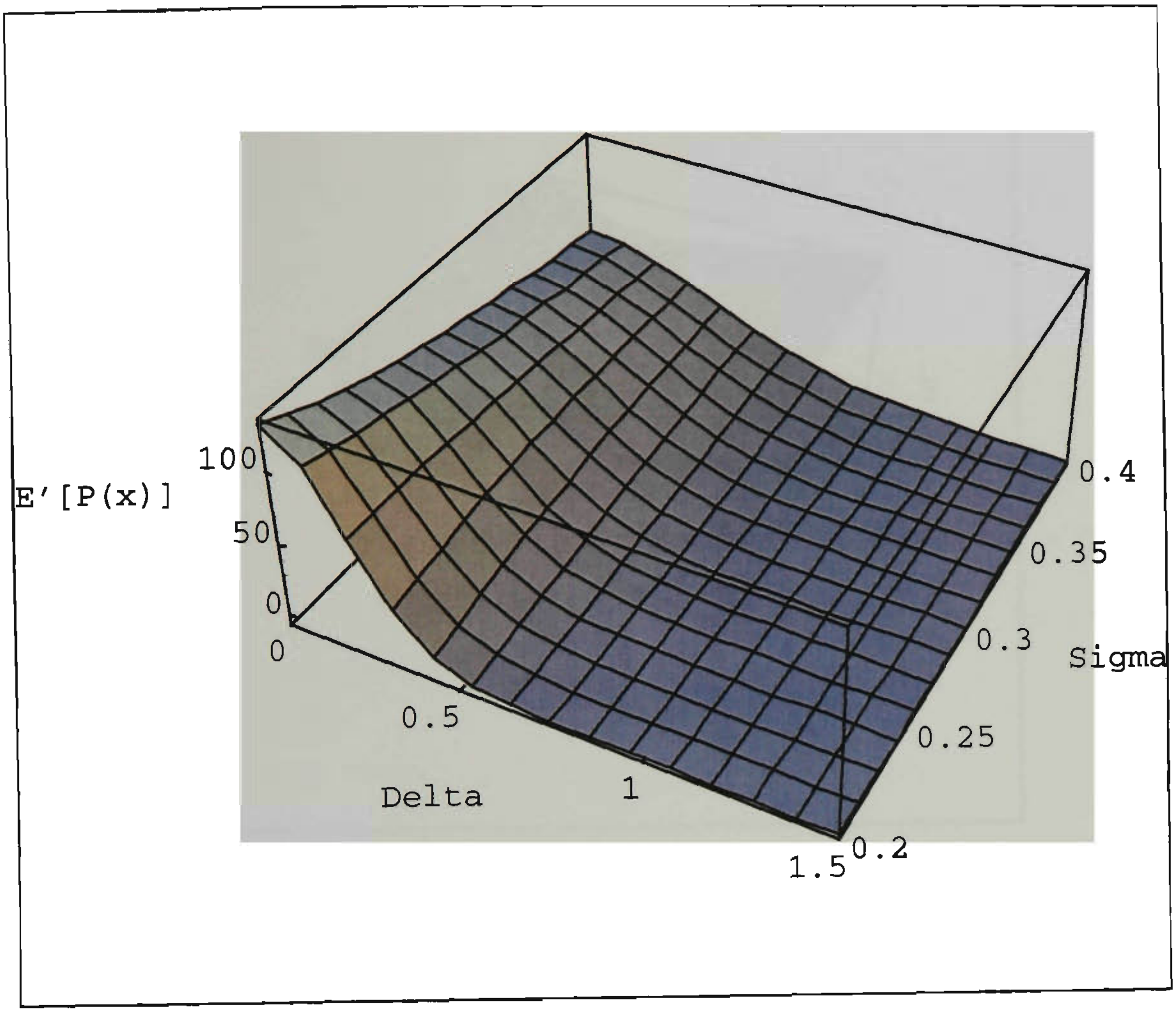


Figure 4.11
E'[P(x)] against δ and σ with L=10, h=0.95, k=1, g=3.5, S=67.

Further, if $E''[P(x)] < 0$ (with $\delta = \delta_0$) i.e. if $-S\delta\sigma^{-3}\phi\left(\frac{\delta}{\sigma}\right) < 0$ then the solution is optimal. Graphical illustration of the second derivative is shown in Figure 4.12

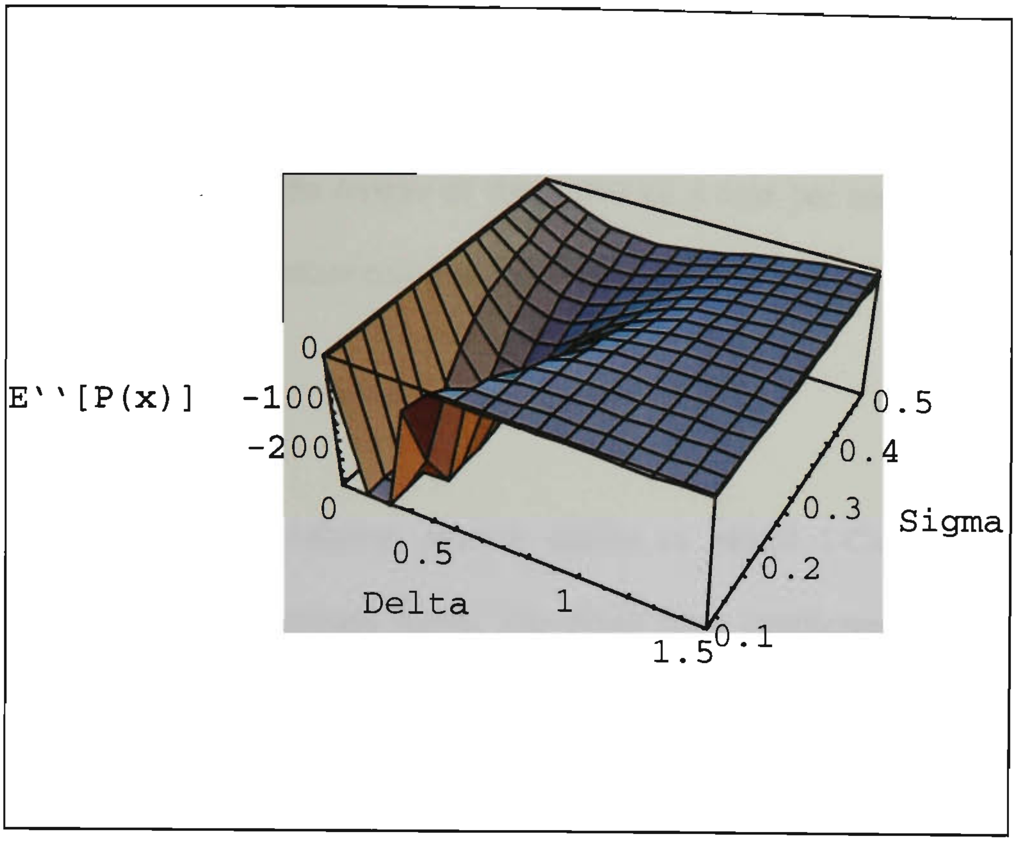


Figure 4.12
E″[P(x)] against δ and σ with S=67, g=3.5.

4.4 Modeling filling processes with additional costs

As filling processes are common in various manufacturing operations, each following unique guidelines and regulations, there are likely to be costs involved in the production that are not included in the simple models, previously considered. The addition of extra cost items to these models is quite straightforward.

This section will illustrate the effect of additional costs on the complexity of the model and on the optimum solution

The extra costs considered will be expended in recapturing overflowed material and emptying out under-filled containers and putting the material back into the process. Although the former of these will be a cost per unit the latter will be assumed to be a constant cost per under-filled container.

4.4.1 Model 6

Consider a canning process similar to Model 1-Case 1 but with the additional costs discussed above. The profit from overflowed containers would then be

Profit = Selling Price - Filling Cost -Cost of Container - Material Cost - Cost of Reusing the Overflow

If, however, x<hL the container can be emptied-out at a constant cost so that in this instantce,

Profit = Filling Cost - Cost of the Container - Cost of Emptying Out the Containers and Reusing Material.

Since the filling cost and the cost of the containers are common to all items regardless of the amount of the ingredient in them, for simplicity reasons C will represent the sum of these two costs, i.e. $C = C_f + C_c$

Thus profit from a single fill attempt may be written as follows;

$$P(x) = \begin{cases} S - C - g(L+k) - c_1(x - L - k), & x \geq L + k \\ S - C - gx, & hL \leq x < L + k \\ -C - c_2, & x < hL \end{cases}$$

The expected profit per fill attempt is:

$$E[P(x)] = (-C - c_2)\int_{-\infty}^{hL} f(x)dx + \int_{hL}^{L+k} (S - C - gx)f(x)dx +$$

$$(S - C - g(L+k))\int_{L+k}^{\infty} f(x)dx - c_1 \int_{L+k}^{\infty} (x - L - k)f(x)dx$$

Proceeding as previously, gives:

$$E[P(x)] = (S - C - g(L+k) + c_1(L + k - hL - \delta) +$$

$$(g(hL + \delta) - S - c_2)\Phi\left(\frac{-\delta}{\sigma}\right) - g\sigma\phi\left(\frac{-\delta}{\sigma}\right) -$$

$$(c_1 - g)(L + k - hL - \delta)\Phi\left(\frac{L + k - hL - \delta}{\sigma}\right) +$$

$$\sigma(g - c_1)\phi\left(\frac{L + k - hL - \delta}{\sigma}\right)$$

Figure 4.13 shows the relationships between expected profit, σ and δ. Due to the added costs, the expected profit is more sensitive to changes in process setting in comparison to the other models involving overflow. This is shown in greater detail in Figure 4.14.

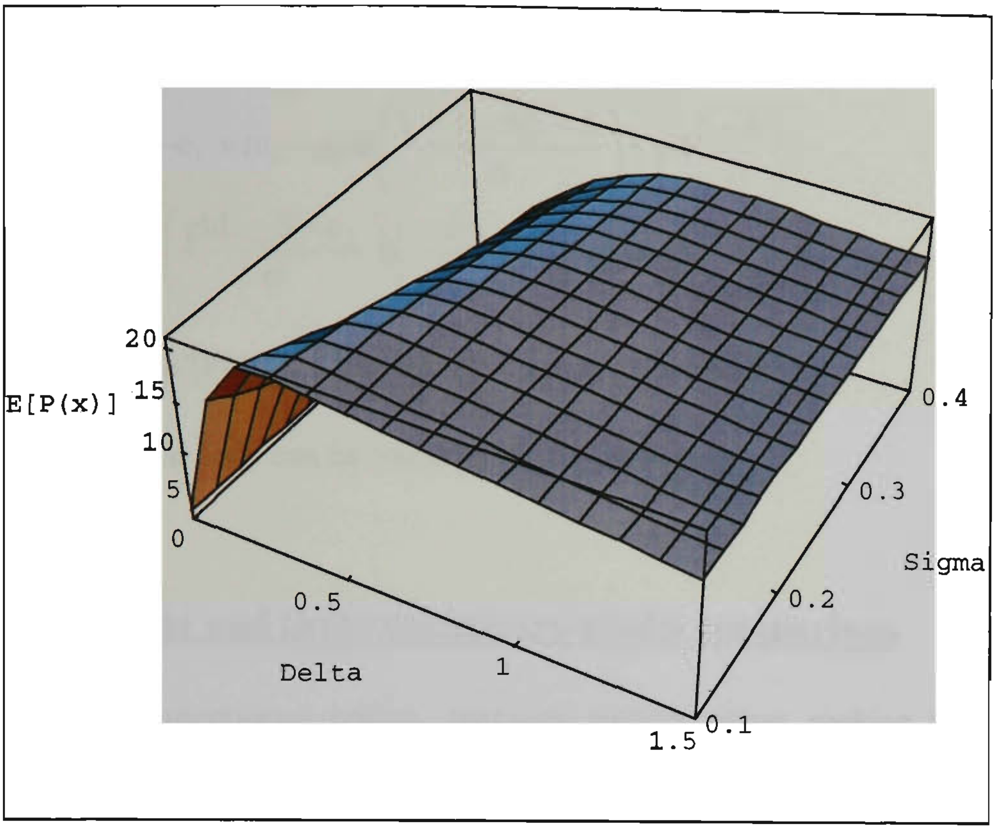


Figure 4.13
Shows Expected profit against δ and σ. The parameter values are: S=67, C=12, g=3.5, L=10, k=1, h=0.95, c_1=0.3, c_2=1.5.

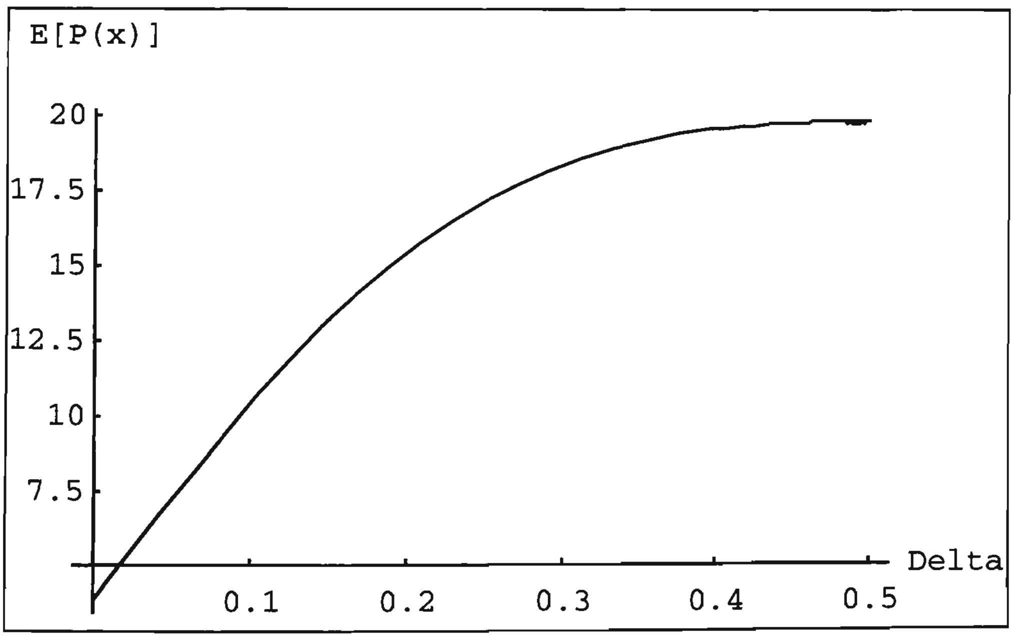


Figure 4.14
Shows Expected profit against δ and σ. The parameter values are: S=67, C=12, g=3.5, L=10, k=1, h=0.95, c_1=0.3, c_2=1.5.

Differentiating with respect to δ:

$$E'[P(x)] = -c_1 + (c_1 - g)\Phi\left(\frac{L + k - hL - \delta}{\sigma}\right) + g\Phi\left(\frac{-\delta}{\sigma}\right) - \left(\frac{ghL - S - c_2}{\sigma}\right)\phi\left(\frac{-\delta}{\sigma}\right)$$

By setting $E'[P(x)] = 0$ the optimum solution is obtained.

Mathematica code can be found in the Appendix.

4.5 Infinite and finite containers-model comparison

As mentioned before, previous authors, when seeking to optimise the expected profit, have made the assumption of no overflow. This approach overestimates the optimum process setting and the consequential reduction in expected profit is quite significant.

Table 4.1 compares two models, both having the same features except for the matter of capacity.

It should be noted, that the Expected Profit is per fill attempt.

	Finite capacity Model 1		Infinite capacity Model 3	
σ	Optimum δ	E[P(x)]/n	Optimum δ	E[P(x)]/n
0.15	0.38	20.2	0.42	20.1
0.20	0.49	19.8	0.54	19.7
0.25	0.59	19.4	0.66	19.3
0.30	0.68	19.0	0.77	18.8
0.35	0.77	18.6	0.88	18.4
0.40	0.86	17.9	0.98	17.7

Table 4.1
Finite and infinite capacity models-difference in optimum δ and E[P(x)].

4.6 CONCLUDING REMARKS

In this chapter the problem of selecting the appropriate mean for several different filling models has been articulated and analysed. The economic gains and losses caused by changs in the process parameters have been illustrated as well as consideration given to finding the mean setting to maximise expected profit.

The assumption of infinite capacity containers (i.e. no possible overflow) causes over estimation of the optimum process setting, which in turn reduces the expected profit.

The algorithms for solution, developed by the author and shown in the Appendix, are both easy to use and quick to execute.

CHAPTER 5

Weights and Measures Requirements

5.1 Introduction

There is a lack of uniformity in Weights and Measures requirements across national boundaries and sometimes across state boundaries. In Australia, the agreement between states on common weights and measures requirements was not achieved until the early 1960s. There are, however, still major differences between countries and uniform legislation is long overdue.

The complexities of modern industry have necessitated the development of a variety of weights and measures requirements.

"Weights and Measures" legislation exists, in part, to ensure that if a producer stamps on his product that it weights 300 grams (for example), then the purchaser has redress at law if the product weight varies appreciably from this. Informed producers will consider the variation that they have to contend with, consider the legislation and stamp their product accordingly. They will, additionally, look closely at the variation that exists, assess what it is costing and seek to reduce it. In so doing, they will continue to meet legislative requirements, but at a minimal cost. There is always a risk that a producer will, inadvertently, breach 'Weights and Measures' legislation, one would hope that the likelihood of this is small.

5.2 Australian requirements and the probability of not breaching the legislation

In Australia penalties for breaches of the Trade Measurement Act 1995 or the Trade Measurement Regulations 1995 are between $A10,000 and $A20,000 for individuals, and between $A50,000 and $A100,000 for a corporate body upon conviction. The direct monetary cost of breaching 'Weights and Measures' legislation, however, is only part of the issue and the total monetary consequences of deficient product reaching customers are not ascertainable.

'Weights and Measures' legislation within Australia requires that for a sample of 12 containers all must contain at least 0.95 of the label content (hL where h=0.95) and the average of the 12 must equal or exceed the amount appearing on the label. If there is an automatic rejection device this will ensure the former of these two conditions, leaving only the condition $\bar{x}_{12} \geq L$ to be satisfied. Unless every container has $x > L$ this condition cannot be guaranteed. It remains desirable, therefore, when seeking to maximise the expected profit per container, to have $\bar{x}_{12} \geq L$ with a large probability. It is common in industry to desire the probability of meeting government regulations to exceed 0.95 with some manufacturers seeking a probability of 0.995. Provided there is a degree of stability in the operation of the filling machine, for any given setting, the probability of a sample of twelve breaching the legislation can be calculated. By 'stability' of the filling operation is meant that the process variability can be estimated, usually by calculating the standard deviation of the volumes or weights of a substantial sample of filled containers. If this stability is not present, then the risk of breaching legislation is uncalculable. It is assumed that the volumes or weights are normally distributed, with mean T, when the filler is set at T, and with standard deviation σ. It is also assumed that variation of final product is a consequence mainly of variation of the filler and not the containers, this can easily be checked if there is any doubt. Having made a decision on T, the actual probability of meeting the requirements, assuming sample observations are independent, is a special case of :-

$$\Pr(\overline{X}_n > L \,/\, x_1, x_2, \ldots, x_n > hL) = \frac{\Pr\left(\sum_{i=1}^{n} x_i > nL,\ x_1 > hL, \ldots, x_n > hL\right)}{\left[1 - \Phi\left(\frac{hL - T}{\sigma}\right)\right]^n},$$

giving,

$$\Pr(\overline{X}_n > L \,/\, x_1, x_2, \ldots, x_n > hL) =$$

$$\frac{\left[1 - \Phi\left(\frac{hL - T}{\sigma}\right)\right]^n - \int_{x_1 = hL}^{nL-(n-1)hL} \cdots \int_{x_i = hL}^{nL-(n-i)hL-(x_1+..+x_{i-1})} \cdots \int_{x_n = hL}^{nL-(x_1+\ldots+x_{n-1})} \phi(x_1)\ldots\phi(x_n)\,dx_1\ldots dx_n}{\left[1 - \Phi\left(\frac{hL - T}{\sigma}\right)\right]^n}$$

If $z = \frac{x - \mu}{\sigma}$ and $\mu = hL + \delta$, then the above equation can be simplified to:

$$= \frac{\left[1 - \Phi\left(\frac{-\delta}{\sigma}\right)\right]^n - \int_{-\delta/\sigma}^{(nL(1-h)-\delta)/\sigma} \cdots \int_{-\delta/\sigma}^{(nL(1-h)-i\delta-\sigma(z_1+\ldots+z_{i-1}))/\sigma} \cdots \int_{-\delta/\sigma}^{(nL(1-h)-n\delta-\sigma(z_1+\ldots+z_{i-1}))/\sigma} \phi(z_1)\ldots\phi(z_n)\,dz_1\ldots dz_n}{\left[1 - \Phi\left(\frac{-\delta}{\sigma}\right)\right]^n} \quad (5.1)$$

When n=12, h=0.95 for any particular L, σ and T this provides the probability of not breaching Weights and Measures legislation on a single sample of size 12.

The above probability can be calculated using S-Plus code shown in Appendix.

5.3 OIML international recommendations

"The Organisation Internationale de Metrologie Legale is a world-wide, international organisation whose main task is that of coordinating the metrological regulations and controls applied to the metrological services, or related organisations, of Member States". (OIML R 87, edition 1989 E).

Apart from international documents of an informative nature, OIML also publishes international recommendations, which are model regulations. It is recommended by this organisation that all OIML member states implement these recommendations as far as possible. As Australia is one of these it was recently suggested that it should also implement their standards. This section discusses these international recommendations and attempts to compare them with those currently used within Australia.

5.3.1 Statistical tests-general rules

The international recommendation "Net content in packages" (OIML R 87, Edition 1989 E) specifies legal metrology requirements for labelled packaged commodities with constant nominal content. It also presents recommended sampling plans applicable to both general and large lots. They are relevant, however, only to goods where net content is declared in units of mass or volume. Adoption to packages where the contents are declared in other quantities should be possible. The metrological requirements for packages consist of regulations for both average and individual item's content. The lots are subject to rejection based on the average net content as well as on the percentage of non-conforming units in the lot. For a lot to be accepted it would have to satisfy both of these conditions.

Average Content

It is recommended that a lot's average should be equal to or exceed the net content (let it be L) appearing on the label. If the average has to be estimated by sampling then the sample average has to fall within the sampling error of the nominal content. The test for the average follows a one-sided t-test and should have a significance level, α_μ, with $\alpha_\mu \leq 0.5\%$ for $\mu = L$.

Individual packages content

The amount of ingredient in the container should accurately reveal the amount meant to be in it. Any deficiencies are permitted only if they are due to standard process variability. An item is declared non-conforming when its contents fall below L-TD, where TD is a tolerable deficiency in the container. TD depends on the nominal net content L and its values are shown in Table 5.1.

In a case when sampling has to be used to determine the presence of non-conforming items, the tests should have a significance level α_p where $\alpha_p \leq 1\%$ for p=1%. Also lots that contain 16% of non-conforming items should be detected in at least 90% of cases.

NOMINAL NET CONTENT L g or ml	**TOLERABLE DEFICIENCY TD Percent of L**	**TOLERABLE DEFICIENCY TD g or ml**
5 to 50	9	_
50 to 100	_	4.5
100 to 200	4.5	_
200 to 300	_	9
300 to 500	3	_
500 to 1000	_	15
1000 to 10000	1.5	_
10000 to 15000	_	150
15000 to 25000	1.0	_

Table 5.1
Acceptable individual deficiencies.

5.3.2 Sampling plans-recommended examples

There are two sampling plans recommended by OIML that depend on the output per hour from a production line. One is suitable for lots of at least 150 items the other for lots of more that 4000 items.

It is assumed that the lots are homogeneous and that simple random sampling is used to ensure that each possible sample combination has an equal probability of being selected (ISO 3534 point 3.6). In a case when random sampling cannot be performed an alternative method may be used, chosen on the basis of agreement between the consumer and the producer. It is recommended that all lots that contain more than 10,000 units should be divided. It should be ensured that the process is not adjusted or corrected in any way apart from standard corrective procedures. The recommended tests are briefly summarised in Table 5.2.

Lot Size	**Average Test**	**Test for non-conforming packages**
At least 150	ISO 2854 (comparison of a mean with a given value) with $t_{0.095}(31)/\sqrt{32} \approx 0.485$	ISO 2859, code letter G. Normal inspection, Single sampling
More than 4000	ISO 2854 (comparison of a mean with a given value) with $t_{0.095}(79)/\sqrt{80} \approx 0.295$	ISO 2859, code letter J. Normal inspection, Single sampling

Table 5.2
Recommended ISO standards

Sampling plan for general use (lot size of least 150 items)

In a case of the lot size being between 150 and 4000 the recommended sample size is 32, with the condition for the average being that $\overline{x} \geq L - 0.485\sigma$. There shall not be more then two non-conforming items in the sample.

Sampling plan for large lots (lot size of more than 4000 items)

In a case of the lot size being larger than 4000 the recommended sample size is 80 packages, with the condition for the average being that $\overline{x} \geq L - 0.295\sigma$. There shall not be more then five non-conforming items in the sample.

5.4 Regulations in selected countries - comparison with australian requirements

As mentioned previously, Weights and Measures requirements differ between countries. At present, regulations in Australia and New Zealand are very similar. Other countries, however, like the United States and members of The European Economic Community have adopted, in different ways, the regulations proposed by OIML. The main differences between them and the Australian requirements are related to the condition for the average of the sample, the sample size, and the tolerable deficiency.

As discussed before, in Australia (and New Zealand) the average content *must be* equal to or exceed the nominal volume/weight indicated on the container. In the United States as well as the EEC, however, the average sample amount shall not be less than the stated content *by an amount exceeding the sampling error*. The sample sizes are also different. In Australia the required minimum sample size is 12 yet in the EEC it is 20 with the condition for the minimum batch size being at least 12 and 100 for Australia and EEC, respectively. In the United States this issue becomes more complex with the sample size having three possible values depending on the batch size. If the batch size is between 12 and 250 the sample size is 12, for batch sizes between 251 and 3200 the sample size increases to 24 and if the batch size is bigger than 3200 than the sample size is 48. The maximum tolerable deficiency in any container is also defined differently. In Australia the deficiency in any one container cannot exceed 5% regardless of the weight/volume of the container. In the EEC as well as the United States the maximum tolerable deficiency depends on the nominal weight/volume on the container and are similar to OILM recommendations (see Table 5.2). Further, there are also conditions to be met for the number of non-conforming packages in the sample, which depends on the lot and sample sizes and differs between the EEC and the United States.

The following tables illustrate the difference between the probabilities of *failing* Weights and Measures requirements in Australia and New Zeleand, the EEC and the United States. The sample volumes are chosen in such a way that there is one volume for each interval of nominal net content as given in Table 5.1. The probabilities for the last two intervals from Table 5.2 cannot be calculated due to EEC directives (75/106/EEC and 76/211/EEC) applying only to volumes between 5ml and 10 litres. Also, in general, the Australian Trade Measurement regulations and the New Zealand Weights and Measures Act apply to volumes between 15ml and 150 litres, and weights between 5g and 10kg, although various specified goods have different limits. Two or three values of the target mean are selected: one equal to and one or two bigger than the nominal amount in the container.

The choice of the values of the mean that are bigger than the nominal amount in the container, is arbitrary. The mean is used only for the purpose of comparison of the probabilities of failing the requirements.

Standard Deviation / Country		1.25	1.50	1.75	2.00	2.25	2.50	2.75	Target
Australia & New Zealand		0.825	0.903	0.943	0.965	0.977	0.984	0.988	30
		0.030	0.112	0.240	0.388	0.524	0.636	0.724	32
EEC		0.039	0.163	0.357	0.551	0.704	0.809	0.878	30
		0	0	0.003	0.017	0.059	0.137	0.248	32
USA	batch size								
	12-250	0.050	0.127	0.252	0.395	0.529	0.639	0.726	30
	251-3200	0.079	0.224	0.432	0.629	0.775	0.868	0.924	
	>3200	0.026	0.095	0.306	0.581	0.792	0.909	0.963	
	12-250	0	0.002	0.010	0.035	0.084	0.155	0.241	32
	251-3200	0	0.003	0.020	0.069	0.160	0.285	0.424	
	>3200	0	0	0	0.009	0.048	0.144	0.298	

Table 5.3

Shows probabilities of failing the requirements for volume=30.

Country \ Standard Deviation		1.5	2.0	2.5	3.0	3.5	4.0	4.5	Target
Australia & New Zealand		0.537	0.619	0.784	0.844	0.922	0.937	0.968	70
		0.002	0.035	0.156	0.339	0.519	0.663	0.765	72
		0	0	0.003	0.021	0.072	0.159	0.270	80
EEC		0.003	0.027	0.1633	0.398	0.618	0.773	0.867	70
		0	0	0.004	0.038	0.139	0.302	0.477	72
		0	0	0	0	0.003	0.017	0.060	80
USA	batch size								
	12-250	0.024	0.056	0.170	0.345	0.519	0.658	0.759	70
	251-3200	0.026	0.089	0.298	0.564	0.765	0.881	0.941	
	>3200	0.023	0.029	0.158	0.486	0.778	0.923	0.975	
	12-250	0	0.001	0.016	0.072	0.177	0.309	0.444	72
	251-3200	0	0.002	0.032	0.139	0.322	0.523	0.691	
	>3200	0	0	0.002	0.036	0.180	0.431	0.675	
	12-250	0	0	0.002	0.003	0.016	0.051	0.112	80
	251-3200	0	0	0	0.006	0.032	0.099	0.211	
	>3200	0	0	0	0.001	0.002	0.019	0.081	

Table 5.4

Shows probabilities of failing the requirements for volume=70.

Country \ Standard Deviation		2.5	3.0	3.5	4.0	4.5	5.0	5.5	Target
Australia & New Zealand		0.5056	0.528	0.575	0.638	0.704	0.765	0.816	150
		0	0.001	0.006	0.024	0.063	0.124	0.203	154
		0	0	0	0	0	0.003	0.009	160
EEC		0.004	0.027	0.102	0.237	0.398	0.551	0.678	150
		0	0	0	0.003	0.013	0.043	0.099	154
		0	0	0	0	0	0	0	160
USA	batch size								
	12-250	0.038	0.091	0.193	0.322	0.454	0.571	0.667	150
	251-3200	0.053	0.157	0.335	0.531	0.696	0.813	0.887	
	>3200	0.024	0.055	0.195	0.440	0.680	0.843	0.928	
	12-250	0	0	0.006	0.024	0.062	0.121	0.199	154
	251-3200	0	0.002	0.012	0.047	0.120	0.228	0.358	
	>3200	0	0	0	0.004	0.027	0.094	0.219	
	12-250	0	0	0	0	0	0.003	0.009	160
	251-3200	0	0	0	0	0.001	0.006	0.017	
	>3200	0	0	0	0	0	0	0	

Table 5.5

Shows probabilities of failing the requirements for volume=150.

Country \ Standard Deviation		5	6	7	8	9	10	11	Target
Australia & New Zealand		0.532	0.5923	0.6716	0.749	0.813	0.862	0.897	250
		0	0.007	0.032	0.086	0.170	0.272	0.272	257
		0	0.001	0.008	0.029	0.072	0.137	0.220	260
EEC		0.164	0.398	0.619	0.773	0.867	0.921	0.952	250
		0	0.003	0.024	0.087	0.199	0.343	0.343	257
		0	0	0.003	0.017	0.060	0.141	0.253	260
USA	batch size								
	12-250	0.1721	0.347	0.521	0.650	0.750	0.829	0.876	250
	251-3200	0.2998	0.565	0.766	0.882	0.941	0.970	0.984	
	>3200	0.160	0.488	0.779	0.923	0.976	0.992	0.998	
	12-250	0.002	0.016	0.059	0.137	0.241	0.355	0.355	257
	251-3200	0.004	0.032	0.115	0.256	0.424	0.585	0.585	
	>3200	0	0.002	0.025	0.117	0.298	0.518	0.518	
	12-250	0	0.003	0.016	0.051	0.112	0.195	0.290	260
	251-3200	0	0.006	0.032	0.099	0.211	0.351	0.496	
	>3200	0	0	0.002	0.019	0.081	0.212	0.393	

Table 5.6
Shows probabilities of failing the requirements for volume=250.

Country \ Standard Deviation		6	8	10	12	14	Target
Australia & New Zealand		0.502	0.533	0.615	0.715	0.802	400
		0	0.001	0.016	0.074	0.182	410
		0	0	0.003	0.021	0.072	415
EEC		0.398	0.398	0.704	0.867	0.939	400
		0	0.002	0.036	0.167	0.374	410
		0	0	0.003	0.034	0.132	415
USA	batch size						
	12-250	0.324	0.324	0.572	0.743	0.844	400
	251-3200	0.533	0.533	0.813	0.933	0.975	
	>3200	0.445	0.445	0.842	0.969	0.994	
	12-250	0	0.011	0.072	0.202	0.365	410
	251-3200	0	0.021	0.139	0.363	0.596	
	>3200	0	0.001	0.036	0.225	0.536	
	12-250	0	0.001	0.016	0.072	0.177	415
	251-3200	0	0.002	0.032	0.140	0.322	
	>3200	0	0	0.002	0.036	0.180	

Table 5.7

Shows probabilities of failing the requirements for volume=400.

Country \ Standard Deviation		5	7.5	10.0	12.5	15.0	Target
Australia & New Zealand		0.500	0.500	0.500	0.514	0.554	700
		0.083	0.178	0.245	0.302	0.374	702
		0	0.011	0.042	0.090	0.163	705
		0	0	0	0	0.005	715
EEC		0.003	0.077	0.398	0.704	0.867	700
		0	0.023	0.229	0.548	0.772	702
		0	0.003	0.078	0.318	0.588	705
		0	0	0	0.014	0.093	715
USA	batch size						
	12-250	0.023	0.042	0.173	0.394	0.596	700
	251-3200	0.023	0.061	0.301	0.625	0.834	
	>3200	0.023	0.025	0.162	0.575	0.867	
	12-250	0	0.010	0.097	0.287	0.495	702
	251-3200	0	0.017	0.180	0.488	0.742	
	>3200	0	0.001	0.060	0.381	0.747	
	12-250	0	0.002	0.041	0.170	0.356	705
	251-3200	0	0.004	0.080	0.311	0.585	
	>3200	0	0	0.012	0.169	0.519	
	12-250	0	0	0.001	0.018	0.079	715
	250-3200	0	0	0.003	0.036	0.151	
	>3200	0	0	0	0.003	0.043	

Table 5.8

Shows probabilities of failing the requirements for volume=700.

Country \ Standard Deviation		10	15	20	25	30	35	Target
Australia & New Zealand		0.500	0.500	0.500	0.508	0.535	0.587	1200
		0	0	0.006	0.035	0.110	0.233	1215
		0	0	0	0.004	0.020	0.062	1250
EEC		0.026	0.398	0.770	0.921	0.970	0.987	1200
		0.00	0.036	0.301	0.634	0.832	0.923	1215
		0.00	0.001	0.0522	0.263	0.536	0.740	1250
USA	batch size							
	12-250	0.024	0.099	0.340	0.587	0.750	0.851	1200
	251-3200	0.025	0.170	0.550	0.826	0.940	0.977	
	>3200	0.023	0.064	0.470	0.858	0.970	0.990	
	12-250	0	0.003	0.055	0.203	0.399	0.574	1215
	251-3200	0	0.006	0.106	0.365	0.639	0.819	
	>3200	0	0	0.021	0.227	0.598	0.851	
	12-250	0	0	0.026	0.127	0.295	0.471	1250
	251-3200	0	0.002	0.051	0.239	0.503	0.720	
	>3200	0	0	0.005	0.103	0.403	0.717	

Table 5.9

Shows probabilities of failing the requirements for volume=1200.

It can be observed form Tables 5.3-5.9 that the probability of failing the requirements is not always higher for one particular country and depends rather on the volume of the container, how close the target is to the nominal label content and the standard deviation of the process. In general, if the target value is very close to the value of the label content and the standard deviation is very small then Australian/New Zealand requirements are harder to pass. Australian/New Zealand requirements seem to be harder to pass if the volume is below 300mL, regardless of the value of the target. Once the volume is above 300mL then the EEC and USA requirements are harder to pass unless the process setting is very close to, or nearly equal to, the value of the nominal label content and the standard deviation is extremely small. As the volume increases and the process is set further from the label amount then the EEC and the USA requirements are harder to pass than the Australian requirements. Also, if the volume falls below 100 or is above 500 the differences between the Australian/New Zealand and the EEC and the USA requirements are large and get larger as the process standard deviation increases. It is also evident that the EEC and the USA requirements are more sensitive to changes in process standard deviation.

5.5 Conclusion

In this chapter the Australian Weights and Measures Requirements have been discussed together with the International Recommendations and regulations of some other countries. The probability of failing individual requirements were compared. The probability of meeting the Australian Legislation was calculated and the code to perform the calculations is given in the Appendix.

The disparity between Weights and Measures requirements of different countries can clearly be a barrier to trade. The manufacture has to make sure that not only domestic requirements are met but also the requirements of those countries into which goods are being exported. As there is no one single country that has the tightest test to pass, manufacturers have a complicated task of trying to make sure that all requirements are met. As mentioned before, if the process is set very close to the nominal weight/volume and the standard deviation is small then it is much harder to pass Australian/New Zealand requirements, which, in that case, provides better consumer protection. If, for the same small target, the process standard deviation increases, particularly for larger volumes, then the USA and the EEC tests protect the consumer better. There is the need for a uniform standard that is fair to both consumers and producers.

CHAPTER 6

Industrial Example

6.1 Introduction

The following example illustrates use of the methods developed in the previous two chapters. Actual industrial data is used to illustrate a potential application of the models developed in chapter 4 as well as to make implications to Weights and Measures requirements. The emphasis is on Australian requirements, however, satisfying others is also discussed.

6.2. Example - laundry powder manufacturing

The data comes from an industrial process manufacturing laundry powder. During the filling of powder into boxes all over-flowed material is recaptured. Under-filled boxes are cut open and thrown away and the material put back into the process. Model 1-Case 1 of chapter 4 is relevant in this situation.

The parameter values are:

Selling Price (S) = \$4.53.

Material Cost (g) = \$0.78 per kg.

Fill/Prod Cost (C_f) = \$0.21.

Packaging (C_c) = \$0.23.

Label weight (L) = 1kg.

Capacity of the box (L+k) = 1.38kg.

Standard Deviation (σ) = 0.02kg.

Minimum proportion of the stated label content (h) =0.95.

Last monthly target = 1.025kg

6.2.1 Some dependencies between process parameters

Before the optimum value of the process setting is calculated it is useful to explore some of the dependencies between the process parameters. Figure 6.1 shows the relation between the expected profit per fill attempt and δ for three different values of the standard deviation. As expected, the smaller the process standard deviation the smaller the optimum process setting and the larger the associated expected profit per fill attempt.

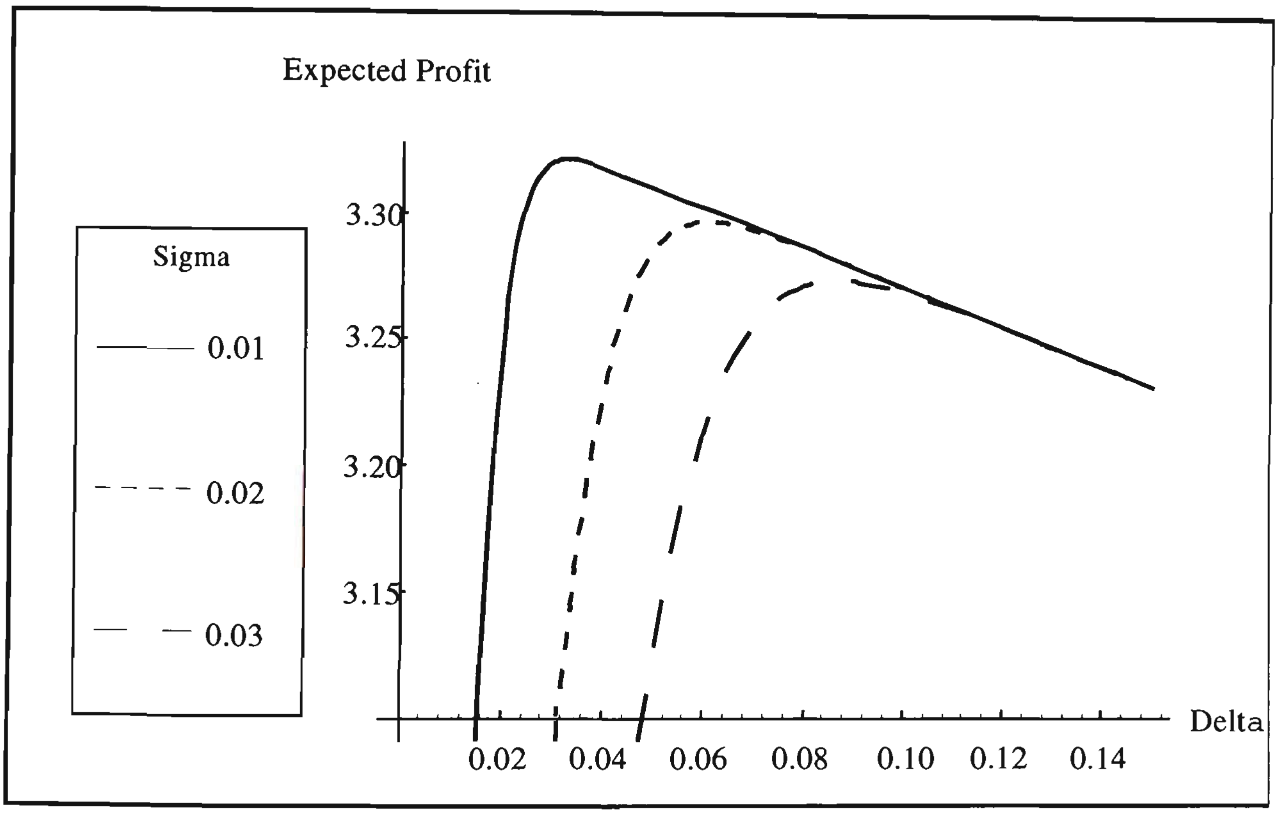


Figure 6.1

Expected profit per fill attempt against Delta when Standard Deviation=0.01, 0.02 and 0.03.

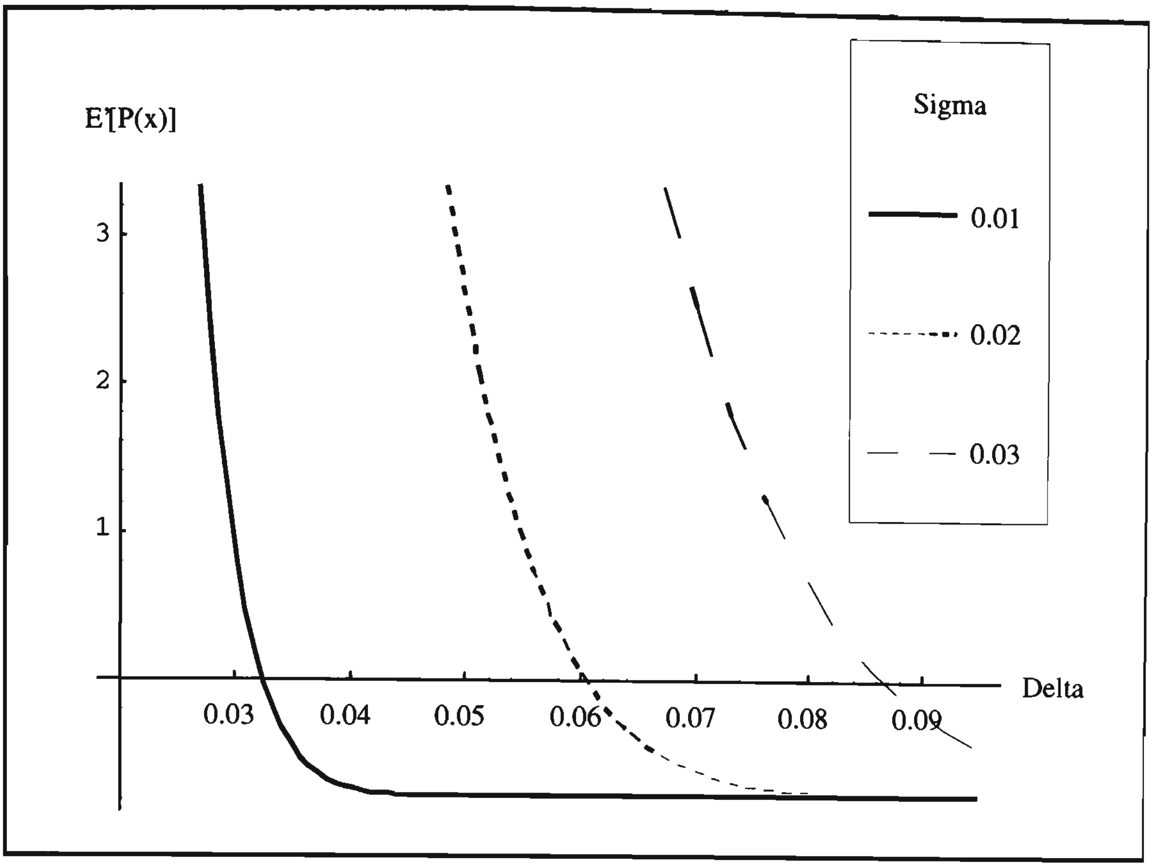


Figure 6.3
Graphical illustration of the solution for Standard Deviation equal to 0.01, 0.02 and 0.03.

6.2.3 Numerical solution and implications to weights and measures requirements

The numerical solution is calculated using *Mathematica*. The optimum process setting associated with a particular standard deviation and followed by the value of expected profit per fill attempt, is shown in Table 6.1. The probabilities of meeting 'Weights and Measures' requirements when δ is optimal are given. In addition, in Table 6.2, the required mean setting is given should the priority be to meet 'Weights and Measures' requirements with a probability of 0.995 or 0.9995. The corresponding expected profit per item is then provided and can be contrasted with the optimum value and the difference weighted against the risk of not meeting the requirements.

Standard Deviation σ	δ_0 & Optimum Target (T)	Expected Profit per fill attempt In $	Probability of meeting Australian W&M Requirements
0.0050	0.0173 T=0.967	3.3345	0
0.0075	0.02499 T=0.9749	3.3279	0
0.0100	0.0324 T=0.9824	3.3215	0
0.0125	0.0397 T=0.9897	3.3152	0.0021
0.0150	0.0467 T=0.9967	3.3091	0.2207
0.0175	0.0537 T=1.0037	3.3030	0.7585
0.0200	0.0605 T=1.01	3.2971	0.9431
0.0225	0.06718 T=1.01718	3.2912	0.9792
0.0250	0.0738 T=1.0238	3.2850	0.9807
0.0275	0.0802 T=1.0302	3.2797	0.9789
0.0300	0.0866 T=1.0366	3.2741	0.9769
0.0325	0.09295 T=1.04295	3.2685	0.9749
0.0350	0.0992 T=1.0492	3.2629	0.9728
0.0375	0.105 T=1.055	3.2574	0.9698
0.0400	0.111 T=1.061	3.2519	0.9674

Table 6.1

Optimum Target and Probability of meeting the W.&M. Requirements. Summary of results.

Standard Deviation σ	Min. Target (t_1) for Pr. at least 0.995	Min. Target (t_2) for Pr. at least 0.9995	Expected Profit per fill attempt if Target= (t_1) In $	Expected Profit per fill attempt if Target= (t_2) In $
0.0050	1.004	1.005	3.3069	3.3061
0.0075	1.006	1.008	3.3053	3.3038
0.0100	1.008	1.010	3.3038	3.3022
0.0125	1.010	1.012	3.3022	3.3006
0.0150	1.012	1.015	3.3002	3.2983
0.0175	1.014	1.020	3.2986	3.2943
0.0200	1.018	1.029	3.2947	3.2872
0.0225	1.025	1.039	3.2889	3.2794
0.0250	1.034	1.048	3.2820	3.2724
0.0275	1.042	1.058	3.2757	3.2646
0.0300	1.050	1.068	3.2694	3.2568
0.0325	1.056	1.078	3.2642	3.2490
0.0350	1.067	1.087	3.2562	3.2420
0.0375	1.075	1.097	3.2499	3.2342
0.0400	1.1084	1.107	3.2429	3.2264

Table 6.2

Minimum Target values for a specific probability of meeting W.&M. requirements.

If the current standard deviation of the process is 0.02 then the difference between the expected profit when the process is set to the optimal target and the expected profit when the process is set to the target that would ensure meeting

Weights and Measures requirements with probability of 0.9995, is \$0.01. If the company's output per hour is 5000 then this difference adds up to \$50 per hour, which is approximately \$1138.5 per 23 production hours per day. The currently used process setting in the company was 1.025, which, according to our model, would give an expected profit per fill attempt of \$3.2902. This is lower than if the optimum process setting had been used. On the other hand the present process setting guarantees meeting the Australian Weights and Measures requirements with probability 0.998. The judgment of whether or not this probability is 'high enough' or 'too high' belongs to the company's management. The cost or financial gains, based on production costs however, can be easily calculated.

Reduction in process variance is one of the most important aspects of quality improvement. Such reduction not only will increase the overall quality of the process and/or product but will also be a significant factor in cost reduction. From the data it is estimated that σ=0.02. Consider also standard deviation of 0.01 and 0.03. The optimum initial settings are equal to 1.01, 0.9824, and 1.0366, respectively, with associated expected profits of 3.2971, 3.3215 and 3.2741, respectively. Consider now two possible changes in σ:

Case 1: that it will start to increase most likely due to an assignable cause.

Case 2: that the process is being improved and σ is being reduced. The problem now focuses on the significance of the financial benefit of reducing the present standard deviation. As shown in Figure 6.4, in case 1 there is a rapid decrease in expected profit per container. In case 2 the increase in E[P(x)] is very slow. The Target values, 0.9824, 1.01 and 1.0366, are the optimum target values if the standard deviation is 0.01, 0.02 and 0.03, respectively.

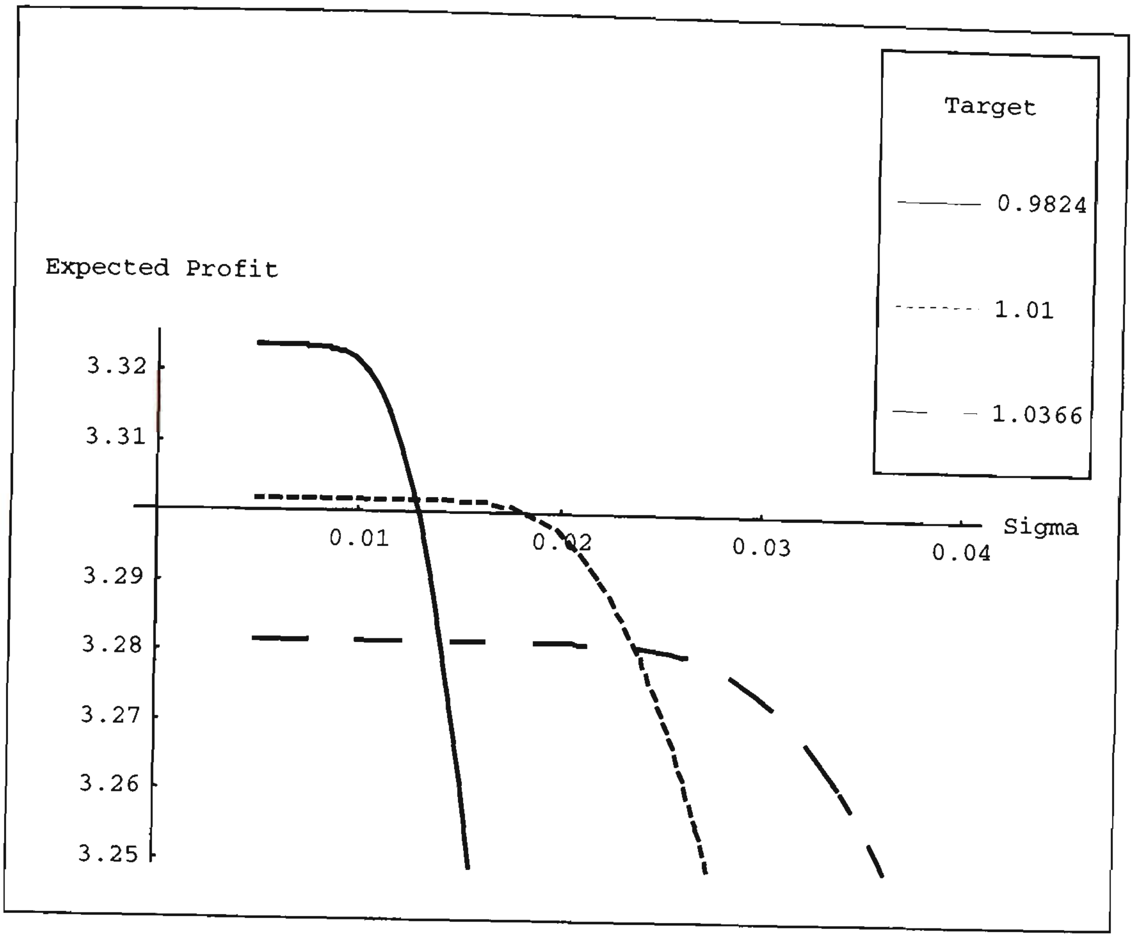


Figure 6.4
Expected profit per fill attempt against sigma for three different Target values.

6.2.3.1 Implications to other countries requirements

Meeting Weights and Measures requirements in Australia does not guarantee meeting overseas requirements. It is shown in Table 6.3 that for weight equal to 1kg (1000grams) and target equal to 1.025kg (1025grams) it is much “easier” to meet the Australian requirements than it is to meet the requirements of the United States or of countries that belong to the European Economic Community. Table 6.4 shows the probabilities of passing the requirements if the process setting is reduced to the optimal setting , which, for the present standard deviation, would be equal to 1.010.

It can be noticed that at the current process target it is easier to meet Australian/New Zealand requirements than the EEC and the USA. If the process is set very close to the nominal label weight then it will be harder to pass Australian requirements than it would be to pass those of the EEC and USA. This situation is, however, very unlikely, as in most cases the process is set above the nominal label content.

Standard Deviation / Country		0.0125	0.0150	0.0175	*0.0200*	0.0225	0.0250	0.0275	0.0300
Australia & New Zealand		100 *0.00*	100 *0.00*	99.9 *0.01*	99.89 *0.11*	99.48 *0.52*	98.37 *1.63*	96.16 *3.84*	92.6 *7.4*
EEC		99.98 *0.02*	99.51 *0.49*	96.59 *3.41*	88.65 *11.35*	75.7 *24.3*	60.43 *39.57*	45.81 *54.19*	33.56 *66.44*
USA	batch size								
	12-250	99.99 *0.01*	99.85 *0.15*	99.00 *1.00*	96.48 *3.52*	91.63 *8.37*	84.5 *15.5*	75.87 *24.13*	66.54 *33.46*
	251-3200	99.99 *0.01*	99.71 *0.29*	98.01 *1.99*	93.09 *6.91*	84.0 *16.0*	71.5 *28.5*	57.56 *42.44*	44.28 *55.72*
	>3200	100 *0.00*	100 *0.00*	99.92 *0.08*	99.09 *0.91*	95.24 *4.76*	85.6 *14.4*	70.16 *29.84*	52.1 *47.9*

Table 6.3

The probabilities, shown as percentages, of meeting (failing, shown in italic) the Weights and Measures Requirements in Australia/New Zealand, European Economic Community and United States, for volume equal to 1kg and the Target equal to 1.025kg.

Standard Deviation / Country		**0.0125**	**0.0150**	**0.0175**	***0.0200***	**0.0225**	**0.0250**	**0.0275**	**0.0300**
Australia &		99.2	98.92	97.26	94.31	89.63	83.15	75.25	66.59
New Zealand		*0.28*	*1.08*	*2.74*	*5.69*	*10.37*	*16.85*	*24.75*	*33.41*
EEC		91.61	72.23	49.14	30.59	18.35	10.96	6.64	4.12
		8.39	*27.77*	*50.86*	*69.41*	*81.65*	*89.04*	*93.36*	*95.88*
USA	batch size								
	12-250	99.18	95.5	87.42	75.86	63.03	50.83	40.3	31.71
		0.82	*4.50*	*12.58*	*24.14*	*36.97*	*49.17*	*59.70*	*68.29*
	251-3200	98.36	91.2	76.43	58.56	36.64	29.86	16.26	10.07
		1.64	*8.80*	*23.57*	*42.44*	*60.26*	*74.14*	*83.74*	*89.93*
	>3200	99.95	98.53	90.00	70.16	45.51	25.29	12.62	5.9
		0.05	*1.47*	*10.00*	*29.84*	*54.49*	*74.71*	*87.38*	*94.10*

Table 6.4

The probabilities, shown as percentages, of meeting (failing, shown in italic) the Weights and Measures Requirements in Australia/New Zealand, European Economic Community and United States, for volume equal to 1kg and the Target equal to 1.010kg.

6.3 Concluding remarks

In this chapter an industrial example has been used to illustrate the potential use of the models developed in chapter 4. The results from the optimal solution were displayed both graphically and numerically. Implications to Weights and Measures requirements were discussed with special reference to current Australian legislation. A comparison to other countries was also made.

CHAPTER 7

Linear Shift in Mean

7.1 Introduction

An important industrial problem in industry is that of controlling a process that is subject to a systematic drift in its mean. A most common change in process mean in relation to time is linear. The drift can be either positive or negative. Processes involved in grinding, cutting, drilling, and stamping are often subject to a positive trend in the mean, which is usually due to tool-wear. The tool wears and the dimension of parts will increase. Filling operations are frequently subject to either a positive or negative trend in the mean. The positive trend is usually due to the mechanical wear of parts, and the negative trend is most likely due to clogging of the nozzles. In the latter instance the amount of filling injected into a container will be reduced with time. The appropriate selection of the initial process setting is an important part of quality control and it can be a significant cost reduction factor.

In this chapter a new model for the most profitable selection of the process mean is proposed. Detailed attention is given to the relationships between different process parameters. Special attention is given to the economic benefits obtained from reducing the process standard deviation and the rate of change in the mean.

7.2 Optimum mean selection-Model 7

To illustrate the selection of the optimum process setting, for a process with a linear shift in mean, model 6 was selected.

Let the time to fill the container be t giving the total time to fill n containers equal to nt. Assume that the mean is constant during the filling of a single container and that the mean commences a linear trend immediately after the filling of the first container. Let the initial process setting be equal to μ_0 and after the filling of the jth container assume the process mean to be equal to $\mu_j = \mu_0(n)+(j-1)r$, where r is the rate of change in mean and $\mu_0(n)+(j-1)r<L+k$.

Thus profit per item is given by:

$$P(x_j) = \begin{cases} S - C - g(L+k) - c_1(x_j - L - k), & x_j \geq L+k \\ S - C - gx_j, & hL \leq x_j < L+k \\ -C - c_2, & x_j < hL \end{cases}$$

The expected profit per item per run of size n, is denoted by $\dfrac{E\left[\sum_{j=1}^{n} P(x_j)\right]}{n}$, where

$$E\left[\sum_{j=1}^{n} P(x_j)\right] = \Big(\sum_{j=1}^{n} \int_{L+k}^{\infty} (S - C - g(L+k) - c_1(x_j - L - k)) f(x_j) dx_j +$$
$$\sum_{j=1}^{n} \int_{hL}^{L+k} (S - C - gx_j) f(x_j) dx_j - \sum_{j=1}^{n} \int_{-\infty}^{hL} (C + c_2) f(x_j) dx_j\Big)$$

Standardising and setting $x_j = z_j\sigma + \mu_j$, gives:

$$E\left[\sum_{j=1}^{n} P(x_j)\right] = ((S - C + (c_1 - g)(L + k)\sum_{j=1}^{n} \int_{\frac{L+k-\mu_j}{\sigma}}^{\infty} \phi(z_j)dz_j -$$

$$c_1 \sum_{j=1}^{n} \int_{\frac{L+k-\mu_j}{\sigma}}^{\infty} (z_j\sigma + \mu_j)\phi(z_j)dz_j +$$

$$(S - C)\sum_{j=1}^{n} \int_{\frac{hL-\mu_j}{\sigma}}^{\frac{L+k-\mu_j}{\sigma}} \phi(z_j)dz_j - g\sum_{j=1}^{n} \int_{\frac{hL-\mu_j}{\sigma}}^{\frac{L+k-\mu_j}{\sigma}} (z_j\sigma + \mu_j)\phi(z_j)dz_j -$$

$$(C + c_2)\sum_{j=1}^{n} \int_{-\infty}^{\frac{hl-\mu_j}{\sigma}} \phi(z_j)dz_j)$$

Further;

$$E\left[\sum_{j=1}^{n} P(x_j)\right] = (S - C - g(L + k) + c_1(L + k)) + (g - c_1)(L + k)\sum_{j=1}^{n} \Phi\left(\frac{L + k - (\mu_0 + (j-1)r}{\sigma}\right) -$$

$$c_1 \sum_{j=1}^{n} (\mu_0 + (j-1)r) + \frac{(c_1 - g)}{n} \sum_{j=1}^{n} (\mu_0 + (j-1)r)\Phi\left(\frac{L + k - (\mu_0 + (j-1)r}{\sigma}\right) +$$

$$g\sigma\left[\sum_{j=1}^{n} \phi\left(\frac{L + k - (\mu_0 + (j-1)r}{\sigma}\right) - \sum_{j=1}^{n} \phi\left(\frac{hL - (\mu_0 + (j-1)r}{\sigma}\right)\right] -$$

$$c_1\sigma\sum_{j=1}^{n} \phi\left(\frac{L + k - (\mu_0 + (j-1)r}{\sigma}\right) + \frac{g}{n}\sum_{j=1}^{n} (\mu_0 + (j-1)r)\Phi\left(\frac{hL - (\mu_0 + (j-1)r}{\sigma}\right) -$$

$$(S + c_2)\sum_{j=1}^{n} \Phi\left(\frac{hL - (\mu_0 + (j-1)r}{\sigma}\right) + [(\mu_0 + (j-1)r)]$$

Differentiating with respect to the initial setting, μ_0, gives:

$$E'\left[\sum_{j=1}^{n}[P(x)]\right] = -c_1 - \frac{(g-c_1)(L+k)}{\sigma}\sum_{j=1}^{n}\phi\left(\frac{L+k-(\mu_0+(j-1)r)}{\sigma}\right)+$$

$$(c_1-g)\sum_{j=1}^{n}\Phi\left(\frac{L+k-(\mu_0+(j-1)r)}{\sigma}\right)-$$

$$\frac{c_1-g}{\sigma}\sum_{j=1}^{n}(\mu_0+(j-1)r)\phi\left(\frac{L+k-(\mu_0+(j-1)r)}{\sigma}\right)+$$

$$\sigma(g-c_1)\sum_{j=1}^{n}\frac{L+k-(\mu_0+(j-1)r)}{\sigma^2}\phi\left(\frac{L+k-(\mu_0+(j-1)r)}{\sigma}\right)-$$

$$g\sigma\sum_{j=1}^{n}\frac{hL-(\mu_0+(j-1)r)}{\sigma^2}\phi\left(\frac{hL-(\mu_0+(j-1)r)}{\sigma}\right)+$$

$$g\sum_{j=1}^{n}\Phi\left(\frac{hL-(\mu_0+(j-1)r)}{\sigma}\right)-\frac{g}{\sigma}\sum_{j=1}^{n}(\mu_0+(j-1)r)\phi\left(\frac{hL-(\mu_0+(j-1)r)}{\sigma}\right)+$$

$$\frac{(S+c_2)}{\sigma}\sum_{j=1}^{n}\phi\left(\frac{hL-(\mu_0+(j-1)r)}{\sigma}\right)$$

By setting the first derivative equal to zero the optimum solution is found, ie. the initial process setting that would maximise the expected profit per container per run. The graphical illustration of the first derivative shows that the function has a single optimum solution (Figure 7.1).

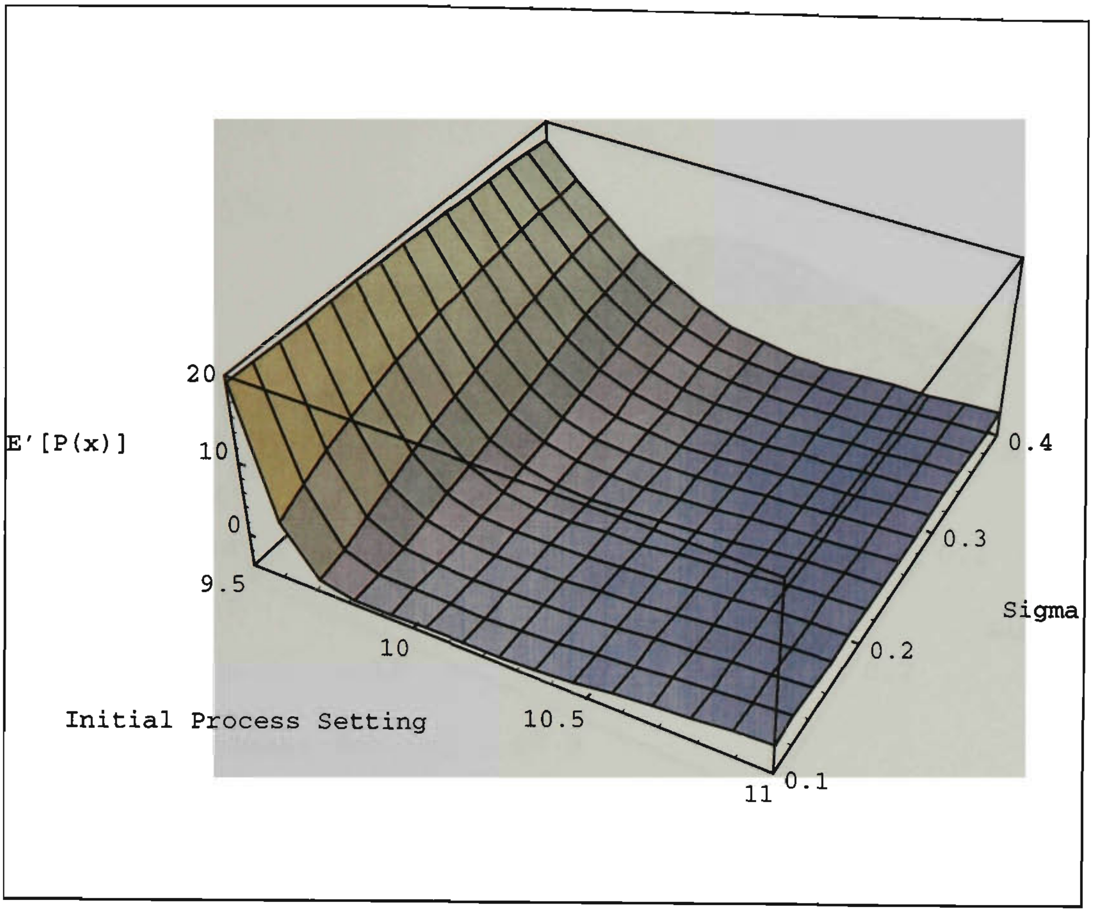


Figure 7.1
Graphical illustration of the optimum solution.

7.3 Dependencies between process parameters

7.3.1 E[P(x)] against initial process setting

Consider the effect of change in the initial process setting and process standard deviation on the expected profit per container per run.

It is shown in Figure 7.2 that μ_0 is quite sensitive to changes in process standard deviation.

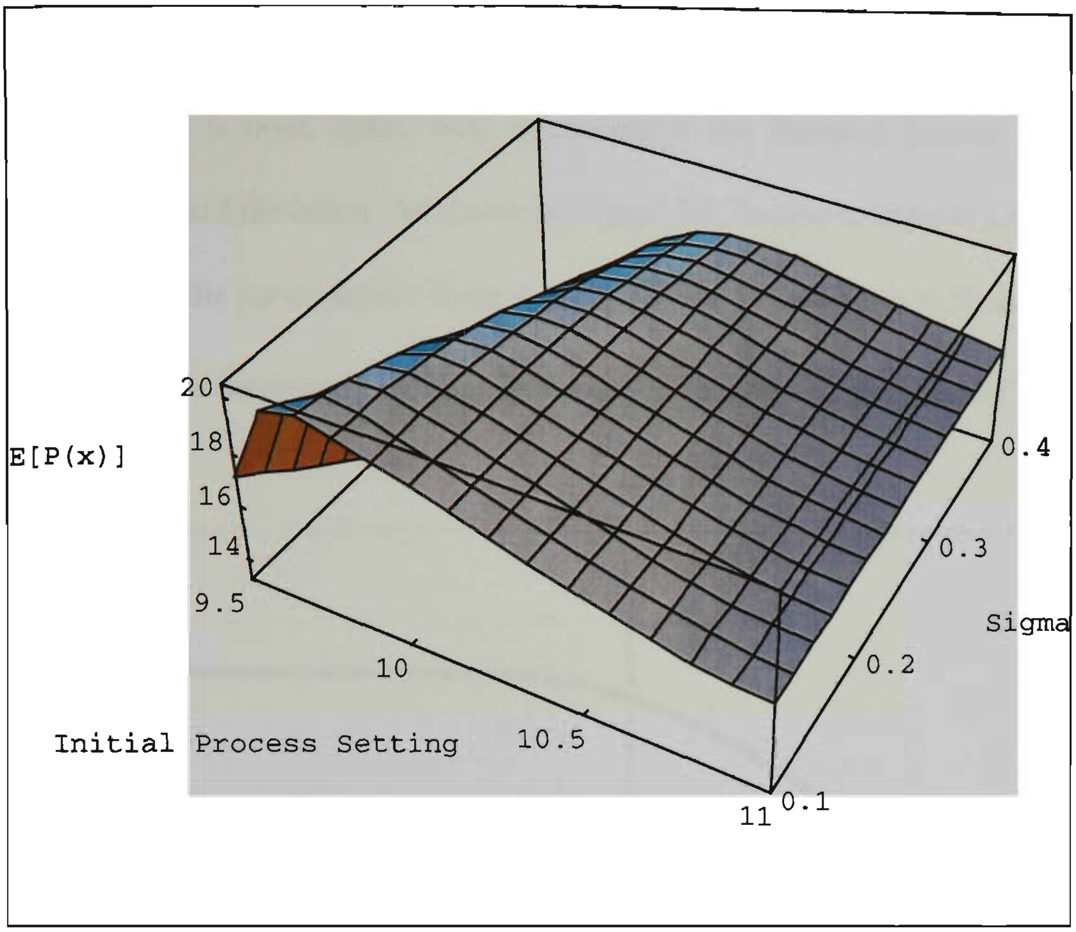


Figure 7.2
E[P(x)] against σ and δ, for :S=67, g=3.5, L=10, k=1, h=0.95, $c_1 = 0.3$, $c_2 = 2$, n=25, r=0.005.

7.3.2 Trends in standard deviation

Consider now the effects of change in process standard deviation. Assume that sigma of the process is 0.2. The other parameters necessary for a solution are assumed to have the following values:

S=67, g=3.5, L=10, k=1, h=0.95, $c_1 = 0.3$, $c_2 = 2$, n=25, r=0.005.

The solution to the first derivative gives the optimum initial setting equal to 9.945 and the associated expected profit of 19.75. As discussed in chapter 6, there are two possible changes in σ:

Case 1: that it will start to increase most likely due to assignable cause.

Case 2: that the process is being improved and a reduction in σ is being achieved.

The problem is now, again, how significant is the financial benefit of reducing the present standard deviation. As shown in Figure 7.3, in case 1, there is a rapid decrease in Expected Profit per container (note that the current value of σ is 0.2). In case 2, there is no appreciable increase in E[P(x)]. The exact values are shown in Table 7.1.

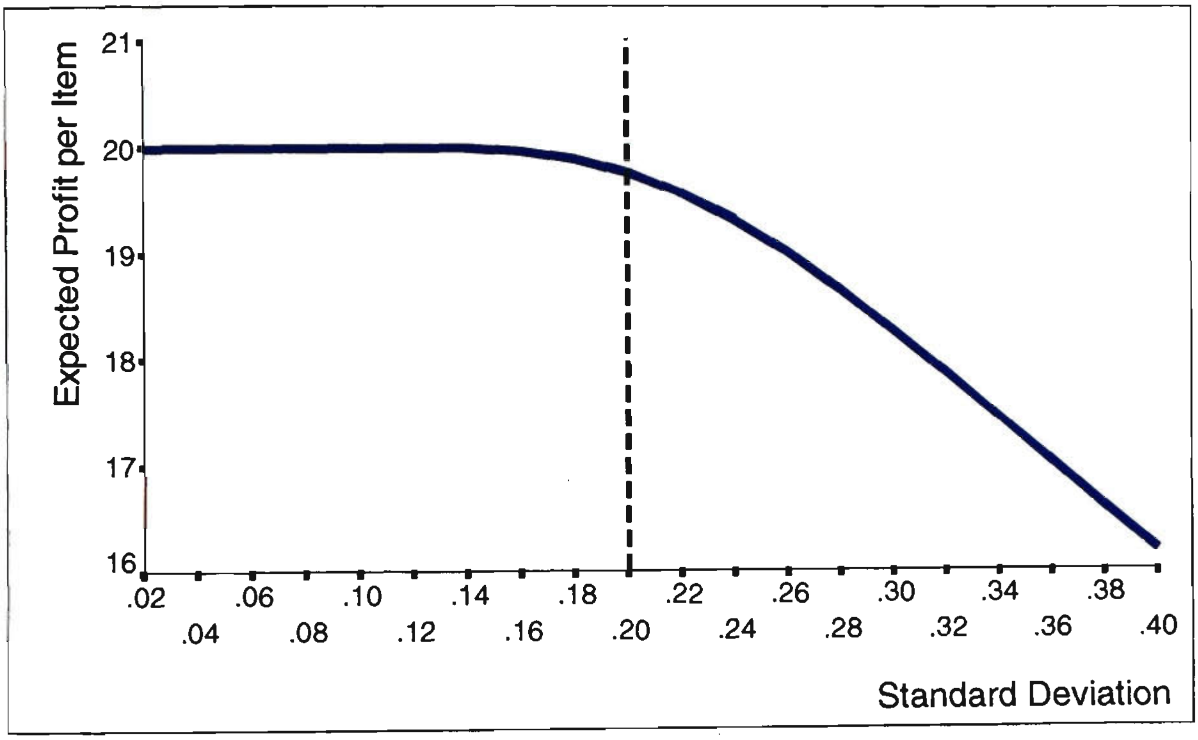


Figure 7.3
Changes in E[P(x)] due to shifts in σ for :S=67, g=3.5, L=10, k=1, h=0.95, $c_1 = 0.3$, $c_2 = 2$, n=25, r=0.005.

Sigma	E[P(x)]/n	Sigma	E[P(x)]/n
0.02	19.983	**0.22**	19.559
0.04	19.983	**0.24**	19.307
0.06	19.983	**0.26**	19.001
0.08	19.983	**0.28**	18.652
0.10	19.983	**0.30**	18.270
0.12	19.982	**0.32**	18.866
0.14	19.974	**0.34**	17.450
0.16	19.946	**0.36**	17.030
0.18	19.876	**0.38**	16.600
0.20	19.75	**0.40**	16.186

Table 7.1

Changes in E[P(x)] due to shifts in σ for :S=67, g=3.5, L=10, k=1, h=0.95, $c_1 = 0.3$, $c_2 = 2$, n=25, r=0.005.

7.3.3 Changes in shift size of the mean

Consider now a situation where it is possible to reduce the rate of change in the mean through for example better machine maintenance, change of equipment parts etc. How would this type of improvement affect earnings from the process? Consider a case where the process sigma and r are known to be 0.2 and 0.005 respectively. The other parameter values are the same as in the previous section. The associated optimum process setting would then be 9.945 and the generated expected profit 19.7497.

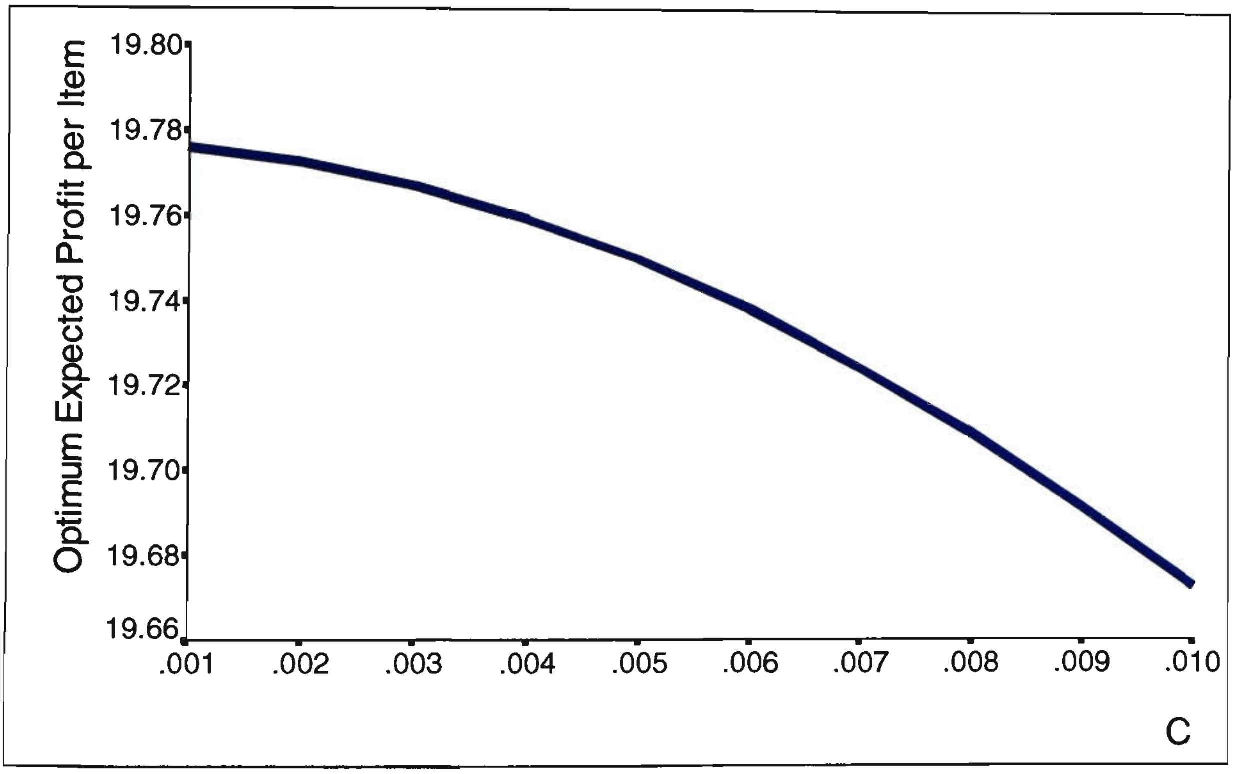


Figure 7.5
Changes in E[P(x)] due to change in r , for :S=67, g=3.5, L=10, k=1, h=0.95, $c_1 = 0.3$, $c_2 = 2$, n=25,σ=0.2.

If reduction in r is achieved then, as expected due to constant standard deviation, the optimum process setting would increase. It is shown in Table 7.2 and graphically displayed in Figure 7.5, that in this case the increase in E[P(x)]/n is small. An increase in r, however, would cause a significant decrease in E[P(x)]/n.

r	δ^*	**E[P(x)]/n**
0.001	9.987	19.776
0.002	9.976	19.773
0.003	9.965	19.767
0.004	9.955	19.759
0.005	**9.945**	**19.750**
0.006	9.936	19.738
0.007	9.928	19.724
0.008	9.919	19.709
0.009	9.912	19.691
0.010	9.905	19.673

Table 7.2
Changes in E[P(x)] due to change in r , for :S=67, g=3.5, L=10, k=1, h=0.95, $c_1 = 0.3$, $c_2 = 2$, n=25,σ=0.2.

7.4 Concluding remarks

In this chapter the problem of the optimum selection of the initial process setting in the presence of a linear shift in mean was discussed. A theoretical analysis was described in detail and an evaluation example provided. In addition, relationships between some of the process parameters were explored and illustrated by an example.

CHAPTER 8

Conclusion and Suggestions for Future Work

8.1 Summary and conclusion

In this thesis some models for the selection of the most profitable initial process mean setting have been presented and analysed. All of the models discussed focus on processes that involve filling containers with some ingredient. Different types have been discussed with special emphasis being placed on processes with 'give-away', 'top-up' and where there are additional costs related to overflow and under-filled containers. Some models have been followed by detailed analysis of the dependencies between process parameters. Special attention has been given to profit reduction or increase due to change in the process standard deviation and/or the process mean. In addition, situations where the process exhibits a linear mean trend have been modelled and analysed. Graphical displays of the solution and parameter dependencies, for some of the discussed models, have been presented. The important issue of meeting 'weights and measures' regulations has been discussed. The economic consequences associated with not complying with the requirements have been illustrated by an industrial example. Comparisons with other existing regulations have been made with the current Australian legislation.

Usable solutions have been shown to be easily available using *Mathematica.*

8.2 Suggestions for future work

(a) Optimum selection of the process initial setting when there is a shift in the mean and variance.

When considering processes with a shift in the mean it is common to assume that the variance remains constant. For most types of industrial processes, however, a significant shift in the mean is frequently accompanied by a change in the variance. There is a need for a model that will provide a simple solution for an optimal process setting considering both shift in the mean as well as in the variance.

(b) Economic Control of Filling Processes with Multiple Heads

Filling processes may have either one or several filling heads. In the case of only one head, or if the heads are perfectly correlated, the methods developed in this thesis can be applied. In the case of a multiple-head machine, where the heads are either weakly correlated or not at all, alternative methods have to be developed.

There is an absence of models that deal with optimal adjustment policies for multiple filling operations when there are linear drifts or even jumps in the filling mean.

(C) Statistical Control of Filling Processes with the Multiple Heads

SPC methods that have been applied to multiple stream filling processes have depended on the strength of correlation between the various heads. Depending on the speed of the machine, the filled container cannot always be identified with the filling head that was used to dispense its contents. When controlling any multiple-head operation the objective is, not only to detect a global shift in the mean of the machine, but also to pick up changes in individual heads. The latter is often due to clogging or mechanical deterioration, the former due, possibly, to a change in raw material. Any adjustments necessary can usually be applied globally. Some methods for identifying useful differences in the behaviour of the filling heads for multiple-head machines were discussed by Ott and Snee (1973).

Statistical methods of process control for multiple-head filling processes were discussed by Mortell and Runger (1995). In the case when the heads are perfectly correlated only one control chart is used for the entire process. In the case of weakly correlated heads, a group control chart is regarded as an effective tool. The shift in the mean of the entire machine is detected by taking a sample from all of the heads but plotting only the highest and the lowest means. The average range across all subgroups provides an estimate of the standard deviation. Each maximum and minimum sample mean plotted on a control chart is identified so that any shift in mean in an individual stream can be detected. If one of the sample means, plotted as a maximum or a minimum, comes from the same head more than a predetermined number of times in a row, then it is concluded, using a formula by Montgomery (1991), see below, that there has been a shift in the mean of that particular head.

$$\mathrm{ARL} = \frac{s^r - 1}{s - 1}$$

where: s-number of the heads

r-number of times the maximum or the minimum sample mean is consecutively identified with the same head.

If the amounts dispensed from each head are uncorrelated separate charts can be used for each head. This, however, is often not practicable, alternatives need to be sought.

(d) Sources of variation

When modelling filling processes it is often assumed that the sources of variation are associated with the amount of material dispensed. It is known, however, that some other sources of variation may significantly affect the process. In particular, the thickness of the container being filled, can contribute significantly to the total variation of the process. This is the case when the control of the contents in the containers is based on a volumetric checking device. A more practical model would likely take into consideration, not only the possibility of over-flow, but also possible under-filling due to the variation in the thickness of containers.

APPENDIX

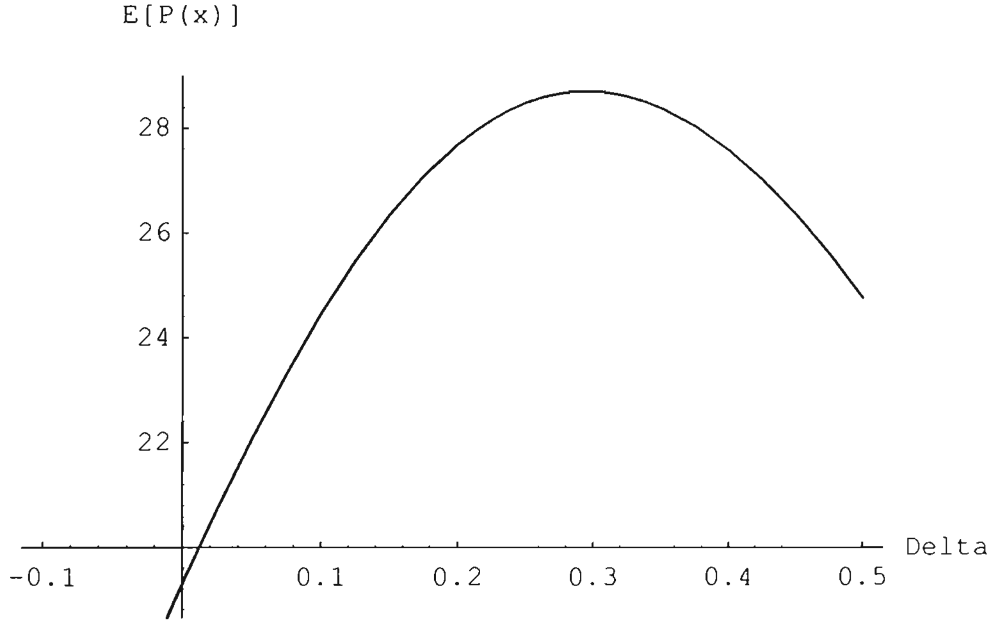
The optimal delta (obtained by using numerical methods) as well as the solution to the equation for the expected profit was obtained using *Mathematica*. The *complete* code used is shown below. The first derivative equation can be solved using *Mathematica* by either entering the function directly and solving as shown in example 1 or by defining the function first and then solving for particular parameter values (as shown in example 3). The probabilities of meeting 'Weights and Measures' requirements were calculated using S-Plus.

```
====================
CHAPTER 3 - MODEL A
====================

<<Statistics`ContinuousDistributions`
ndist:=NormalDistribution [0,1]
f[x_]:=PDF[ndist, x]
F[x_]:=CDF[ndist, x]


EXPECTED PROFIT/ITEM
--------------------

General Expression
In[] P[A_,d_,s_,B_,a_,e_,L_,p_]:=
A (1-F[-d/s])+B (F[-d/s]-F[(-d-a)/s])-
e (s f[-d/s]+d (1-F[-d/s]))-p
Evaluation of E[P(x)] per item - Example
In[] P[67,0.3,0.3,33.5,0.1,55,1,12]
Out[]
28.7548
```

PLOTS-EXAMPLE

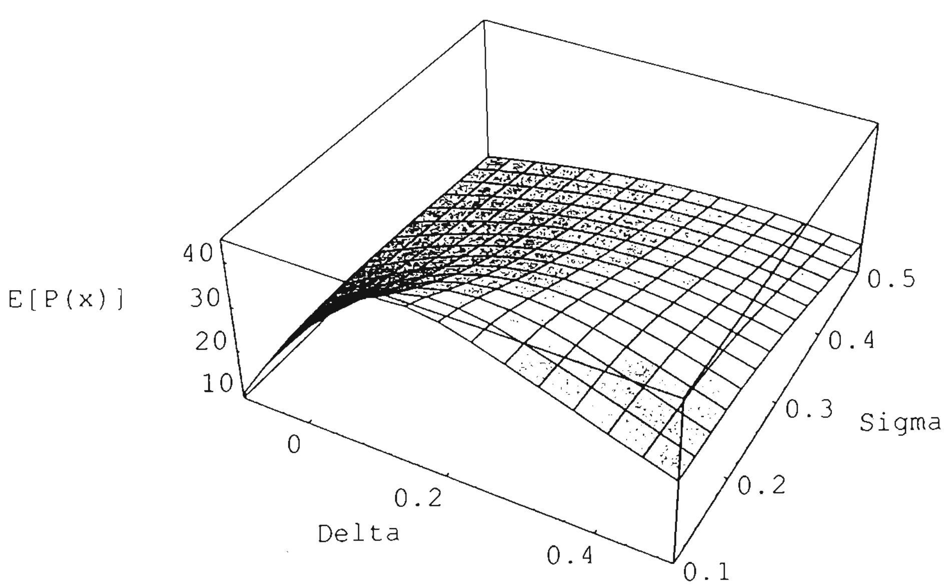
2-Dimensional plot-example

```
In[] Plot[P[67,d,0.3,33.5,0.1,55,1,12],
 {d,-0.1,0.5},
AxesLabel->{"Delta", "E[P(x)]"}]
Out[]
```



3-Dimensional Plot-example

```
In[] Plot3D[P[67,d,s,3.5,0.1,55,1,12],
 {d,-0.1,0.5}, {s,0.1,0.5},
AxesLabel->{"Delta ", "  Sigma", "E[P(x)]   "}]
Out[]
```



```
==========
CHAPTER 4
==========

In[] <<Statistics`ContinuousDistributions`
In[] ndist :=NormalDistribution[0,1]
In[] f[x_] :=PDF[ndist, x]
In[] F[x_] :=CDF[ndist, x]

-------
MODEL 1
-------

EXPECTED PROFIT
---------------

General Expression

In[] P[d_,s_,g_,L_,k_,h_,A_,M_]:=
A-M-g (L+k)+
g*(L+k)*F[(L+k-h L-d)/s]-A F[-d/s]-
g*(h L+d)*(F[(L+k-h L-d)/s]-
F[-d/s])-g*s*(f[-d/s]-f[(L+k-h L-d)/s])


Evaluation - Example

P[0.728,0.35,3.5,10,1,0.95,67,12]
 18.5663

First Derivative
----------------
Evaluation - Example

In[]
FindRoot[f[d/0.25]/(0.25*(F[(10+1-9.5-d)/0.25]-
F[-d/0.25]+(9.5/0.25)*f[d/0.25]))==3.5/67, {d,0.8}]
Out[]
 {d -> 0.58559}

-------
MODEL 2
-------

EXPECTED PROFIT
---------------
General Expression

In[] T[A_,m_,M_,d_,s_,g_,h_,L_,k_]:=
```

```
A-M-g*(L+k)+
g (L (1-h)+k)*F[(L+k-h*L-d)/s]-
g*d*F[(L+k-h*L-d)/s]+
(g L (h-1)-m)*F[-d/s]+g d F[-d/s]+
g s (f[(L+k-h*L-d)/s]-f[-d/s])
```

Evaluation- Example

```
In[] T[67,6,12,0.6667,0.35,3.5,0.95,10,1]
Out[] 19.1866
```

First Derivative

Evaluation - Example

```
In[]
 FindRoot[f[-d/0.35]/(0.35*(F[(10+1-9.5-d)/0.35]-
F[-d/0.35])) == 3.5/(6-3.5*10*(0.95-1)), {d,0.5}]
Out[]
{d -> 0.497194}

Plot[T[67,6,12,d,0.25,3.5,0.95,10,1],
 {d,0,1.5},
AxesLabel->{"Delta ", "E[P(x)] "}]
```

Model 3

EXPECTED PROFIT

General Expression

```
In[] T[A_,m_,d_,s_,g_,h_,L_]:=
A-m-g*(h*L+d)-(A-g*(h*L+d))*F[-d/s]-g*s*f[-d/s]
```

Evaluation- Example

```
In[] T[67,12,0.42,0.15,3.5,0.95,10]
Out[] 20.1934
```

First Derivative

Evaluation - Example

```
In[]
FindRoot[(1/0.4)*f[d/0.4]/(1+(10*(0.95-1)/0.4)*f[d/0.4]-
F[-d/0.4])==3.5/67, {d, 0.5}]
Out[]
```

```
{d -> 0.976916}
```

```
   -------
   Model 4
   -------
```

```
EXPECTED PROFIT
---------------


General Expression

In[]
B[A_,M_,G_,h_,L_,d_,s_,m_]:=
A-M-G*(h*L+d)-(m+G*L)*F[-d/s]+
G*(h*L+d)*F[-d/s]-G*s*f[-d/s]

Evaluation-Example

In[] B[67,12,3.5,0.95,10,0.965,0.4,6]
18.3074

First Derivative
----------------
Example

In[]
FindRoot[(f[-d/0.2])/0.4*(1+(10*(0.95-1)/0.4)*
f[-d/0.4]-F[-d/0.4])==3.5/67, {d, 0.5}]
Out[] {d -> 0.965601}
```

```
   -------
   Model 5
   -------
```

```
EXPECTED PROFIT
---------------
General Expression

In[] W[A_,M_,G_,L_,h_,d_,s_]:=
A-M-A*F[-d/s]-G*(h*L+d)

Evaluation - Example

In[] W[67,12,3.5,10,0.95,1.5,0.35]
Out[] 16.4994
```

First Derivative

Evaluation - Example

```
FindRoot[(67-12-67*F[(-d/0.2)]-3.5*(9.5+d))
 == 18, {d,1}]
{d -> 1.07143}
```

MODEL 6

OVERFLOWED MATERIAL IS RECAPTURED.
UNDERFILLED CONTAINERS ARE EMTIED-OUT
AND MATERIAL PUT BACK INTO THE PROCESS.

m1- COST/UNIT OF RECAPTURING THE OVERFLOWED
MATERIAL.
m2- (CONSTANT) COST OF EMPTING-OUT
THE UNDERFILLED CONTAINER.
m0- initial setting

EXPECTED PROFIT

General Expression

```
In[] T[A_,M_,g_,L_,k_,h_,d_,s_,m1_,m2_]:=
A-M-g (L+k)+
m1 (L+k-h L-d)+
(g-m1) (L+k) F[(L+k-h L-d)/s]+
(m1-g) (h L+d) F[(L+k-h L-d)/s]-
m1 s f[(L+k-h L-d)/s]-(A+m2) F[-d/s]+
g (h L+d) F[-d/s]-
g s f[-d/s]+g s f[(L+k-h L-d)/s]
```

Evaluation - Example

```
In[] T[67,12,3.5,10,1,0.95,0.5,0.2,0.3,1.5]
Out[] 19.7797
```

First Derivative

General Expression

```
In[] W[A_,g_,L_,k_,h_,d_,s_,m1_,m2_]:=
-m1+(m1-g) F[(L+k-h L-d)/s]+
```

```
(-1/s) (g h L-A-m2) f[-d/s]+g F[-d/s]
```

Solution to the first derivative - Example

```
In[]
FindRoot[W[67,3.5,10,1,0.95,d,0.2,0.3,1.5]==0,
 {d,0.2}]
Out[]
{d -> 0.490495}
```

```
=====================
CHAPTER 6 - MODEL 7
=====================

In[] <<Statistics`ContinuousDistributions`
In[] ndist:=NormalDistribution [0,1]
In[] f[x_]:=PDF[ndist, x]
In[] F[x_]:=CDF[ndist, x]


LINEAR SHIFT IN MEAN
---------------------

mj(n)=m0+(j-1)c


EXPECTED PROFIT
---------------

General Expression
In[] P[A_,g_,L_,k_,M_,m0_,c_,s_,n_,h_]:=
(n A-n g (L+k)-n M+
g (L+k) Sum[F[(L+k-m0-(j-1) c)/s], {j, 1, n}]-
A Sum[F[(h L-m0-(j-1) c)/s], {j, 1, n}]-
g s Sum[f[(h L-m0-(j-1) c)/s], {j, 1, n}]+
g s Sum[f[(L+k-m0-(j-1) c)/s], {j, 1, n}]-
g Sum[(m0+(j-1) c) F[(L+k-m0-(j-1) c)/s],
{j, 1, n}]+
g Sum[(m0+(j-1) c) F[(h L-m0-(j-1) c)/s],
{j, 1, n}])/n

Evaluation of E[P(x)] per item - Example

In[] P[67,3.5,10,1,12,9.99,0.015,0.2,50,0.95]
Out[]
18.7283


First Derivative
----------------

General Expression

In[] R1[A_,g_,L_,k_,m0_,c_,s_,n_,h_]:=
((-g (L+k)/s) Sum[f[(L+k-m0-(j-1) c)/s],
 {j, 1, n}]+
(A/s) Sum[f[(h L-m0-(j-1) c)/s], {j, 1, n}]-
(g/s) Sum[(h L-m0-(j-1) c) f[(h L-m0-(j-1) c)/s],
 {j, 1, n}]+
```

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