*Dedicated to my Parents*

# DECLARATION

I, Dang Quang NGUYEN, hereby declare that the materials embodied in this thesis have not been submitted in part or full to any other university or institute for award of any degree.

[redacted]

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# SUMMARY

This study extends the Evolutionary Structural Optimization (ESO) method for application to multi-storey buildings. The objective is to find the optimal topologies of multi-storey buildings subject to overall stiffness or displacement constraints. It emphasizes the derivation of a methodology to help the structural designers to choose the optimal topology among many topologies that are generated during the evolutionary optimization process. Other problems of the ESO method such as the termination condition, sharp change in structural mean compliance or constrained displacements are also investigated.

The new added features provide the ESO method with the capability of dealing with structures containing different types of finite elements. For the structure being considered, only continuum elements are allowed to be removed during the optimization process while beam elements are assumed to be fixed and are referred to as a non-design domain. By having all the topologies with the same weight as the initial structure, the performance of these topologies can be evaluated by comparing the mean compliance or constrained displacements.

The results of this study show the extended ESO method can effectively find efficient bracing systems for multi-storey buildings.

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# CONTENTS

# CHAPTER 1: INTRODUCTION

## 1.1 STRUCTURAL OPTIMIZATION

Structural optimization aims to find the best design for a loaded structure which has the minimum weight or cost, while satisfying the requirements of strength, stiffness, stability and functionality. It is motivated by the growing realization of the scarcity of natural resources. In general, it is desirable that every design should be optimized. For example, a car must be designed such that minimum fuel consumption is achieved while maintaining the highest performance, an aeroplane is designed in such a way that it costs the least for material or capital while it maintains good in-flight performance during its lifetime. Structural optimization can be divided into three main categories, namely, size, shape and topological optimization. Among them, topological structural optimization is considered the most challenging because the topology and shape of the structure are both changed during the optimization process. Topological structural optimization will seek the pattern or the configuration of structural components which form an optimal structure.

In the building industry, the task of integrating structural optimization into a building design is an issue of significant concern. A typical building structure contains columns, beams and shear-walls. The sizes and locations of these elements need to be determined during the design process. Because of the complexity of the overall behaviour of multi-storey building structures, finite element method (FEM) is usually used for the structural analysis.

The evolutionary structural optimization (ESO) method introduced by Xie and Steven (1997) effectively bridges the gap between Finite Element Analysis (FEA) and structural optimization. In the ESO method, the structural optimization is carried out in an iterative manner based on the simple idea of systematically removing inefficient elements, with the result that the residual structure evolves toward an optimum. Currently, the ESO method has been developed to solve those structural optimization problems which are subject to overall stiffness, displacement, frequency and buckling constraints.

In the ESO method, by removing inefficient elements, there are a series of topologies generated during the evolution. The problems of choosing the optimal topology and appropriate termination condition become important. Additionally, sharp changes in the constrained function values are often encountered during the iterations. These shortcomings of the ESO method have been considered in the related research of the ESO method. Chu et. al. (1996) used a prescribed volume limit as the termination condition in the optimization process. In this technique, the structural optimization process is terminated when the ratio of the volume of the current structure to that of the initial structure reaches a prescribed limit. This termination condition is quite arbitrary because it cannot guarantee the optimal structure is reached in the process. Liang (2001) introduced the Performance Index to monitor the performance of topologies generated and terminated the process when the performance of the current structure is worse than that of the initial structure. This method is very efficient for generating the optimal topology and load carrying mechanism of two-dimensional (2D) structures. The applicability of this method, however, is limited to 2D structures because of the basic assumption that the global stiffness matrix is a linear function of the design variables.

The purpose of this thesis is to extend the ESO method for applications to multi-storey building structures. Weaknesses in the ESO method such as the termination conditions, the sharp changes in the constrained function values and maintenance of structural symmetry are examined. A review of current literature identifies a lack of application of structural optimization to practical building structures, particularly to three-dimensional (3D) structures.

A multi-storey steel frame building is considered in this thesis. The steel frame is a 2D plane frame structure. Two topological structural optimization problems will be carried out to determine the optimal bracing system for the building: (1) for the overall stiffness constraint subject to multiple lateral load cases; (2) for the top deflection constraint subject to multiple lateral load cases. An example of 3D frame is also considered.

## 1.2 AIMS OF RESEARCH

The aim of the thesis is to extend the ESO method to multi-storey buildings with stiffness and displacement constraints. The model of the building contains a combination of beams and continuum elements under multiple cases of lateral loading.

In order to achieve this general aim, the following specific aims are considered:

- Review the existing methods of structural optimization, their advantages, disadvantages and difficulties when applied to solve practical problems. Study topological optimization in detail. Particular attention will be paid to the Evolutionary Structural Optimization (ESO) method.

- Derive an algorithm to determine the optimal topology among the whole series of topologies generated during the evolutionary optimization process. Based on the algorithm the termination condition of the procedure will be derived.

- Derive an algorithm to handle sharp changes in constrained function values.

- Develop a program to carry out the structural optimization automatically. This program will use the output data of the finite element analysis package STRAND6$^{TM}$.

- Test the program on several simple topological structural optimization problems and compare optimal topologies with previous results in the literature.

- Apply the topological structural optimization technique to multi-storey buildings and determine the optimal topologies subject to overall stiffness or displacement constraints under multiple load cases.

## 1.3 SIGNIFICANCE OF THE RESEARCH

In this research, the ESO method is extended and is applied to multi-storey buildings subject to overall stiffness or displacement constraints. This extension has not been satisfactorily investigated in the discipline of structural optimization. The outcome of this research will make a contribution to building structural design in particular and the application of structural optimization in general.

## 1.4 LAYOUT OF THESIS

This thesis consists of seven chapters:

- *Chapter 2*: Literature review. A review of the development and application of structural optimization methods will be presented. Topological optimization will be examined in more detail. Particular attention will be focused on the Evolutionary Structural Optimization (ESO) method.

- *Chapter 3*: The extended ESO method for overall stiffness constraint. An extended ESO method for continuum topology optimization subject to overall stiffness constraint is developed. Concepts and definitions of the proposed method will be introduced. The issues related to the extended ESO method such as the termination condition, choosing optimal topology and sharp change in constrained function values will be addressed and handled. Several numerical examples including a 3D structure will also be presented to demonstrate the effectiveness of the method.

- *Chapter 4*: The extended ESO method for multi-storey frame buildings subject to overall stiffness constraint. The proposed method will be applied to a multi-storey steel frame subject to overall stiffness constraint. The structural optimization problem is to determine an efficient bracing system for the frame under multiple lateral load cases. The issue of maintaining structural symmetry will be also considered.

- *Chapter 5*: The extended ESO method for displacement constraints. The theoretical basis of the extended ESO method for displacement constraints will be developed. Several numerical examples will be given at the end of the chapter to verify the method.

- *Chapter 6*: The extended ESO method for multi-storey buildings subject to deflection constraint. The structural optimization method developed in Chapter 5 will be applied to a multi-storey steel frame building. The structural optimization problem is to find an optimal topology for the bracing system for the steel frame subject to constraint on the deflection at the top under multiple lateral load cases.

- *Chapter 7*: Conclusions and recommendations. Conclusions and recommendations of the thesis will be presented. Limitations of the research will be addressed. Further research will also be suggested.

# CHAPTER 2: LITERATURE REVIEW

In this chapter, a review on the development and applications of structural optimization methods will be presented. Although this thesis focuses on topological structural optimization, sizing and shape structural optimization methods are also reviewed in this chapter.

## 2.1 DEFINITIONS

- **Design variables**

A structural system can be described by a set of quantities. Based on practical experience, some quantities are chosen before the optimization process. Those quantities are considered as pre-defined variables and they are fixed during the optimization process. Other quantities, which are allowed to vary for optimization purposes, are considered as *design variables*. From a physical point of view, design variables can be divided into the following categories:

- **Material properties**: Material properties of structural elements such as modulus of elasticity $E$ and material density $\rho$ are design variables.

- **Structural topology**: The patterns of connections as well as the number of elements in the structure are design variables.

- **Structural geometry**: The geometrical dimensions of the structures are design variables. Geometrical dimensions include the height of the roof or the width of the bay in a frame building or the cross sectional dimensions of structural elements.

Depending on specific problems, design variables may be treated as discrete or continuous.

- **Constraints**

Every structural system has to satisfy the design requirements. These requirements may be given by the design practice code, the availability of material, the feasibility of construction, the behaviour or other considerations. Sets of design variables that meet all the requirements are called a feasible design. Restrictions on the design variables or the structural response are called constraints. There are two types of constraints:

➢ **Side constraints:** These constraints are imposed upon the design variables in explicit or implicit forms. Constraints such as minimum height of the beam for electrical conduit placement or minimum thickness of the plate are typical examples of side constraints.

➢ **Behavioural constraints:** These constraints are derived from the behavioural requirements of the structure. Limitations on the maximum stresses, displacements or buckling are typical examples of behavioural constraints.

$$\sigma^L \leq \sigma \leq \sigma^U \text{ (Stress constraints)} \qquad (2.1)$$

$$D^L \leq D \leq D^U \text{ (Displacement constraints)} \quad (2.2)$$

where:

$\sigma$: the stress of structural members.

$\sigma^L$, $\sigma^U$ : the lower bound and upper bound of the stress respectively.

$D$: the displacement of the structure.

$D^L$, $D^U$ : the lower bound and upper bound of displacement respectively.

Along with the number of design variables, the number of constraints in optimization problems has a significant impact on the time and effort of solution process. Therefore, the simplification of constraints needs to be carefully considered in order to reduce the solution effort.

Optimization problems, which have no constraints, are called unconstrained optimizations or considered as constrained optimizations otherwise.

- **Objective functions**

In order to evaluate the efficiency of designs, objective functions are defined and minimized during the optimization process. Objective functions may be the functions that represent the weight or the cost of the structures. The weight of the structure is the most commonly used due to the fact that it is readily quantified. Although a cost is of more practical importance, it is often difficult to obtain sufficient data for the construction of the real cost function. The objective function representing the weight of the structure can be expressed as:

$$f(X) = \sum_{i=1}^{n} W_i \tag{2.3}$$

where:

$f(X)$: objective function.

$W_i$: the weight of the $i^{th}$ element of the structures.

$n$: the number of total elements.

## 2.2 CLASSICAL METHODS IN STRUCTURAL OPTIMIZATION

The development of mathematical optimization started with the introduction of calculus by Newton and Leibniz during the latter part of the 17th century. Given a continuously differentiable objective function $f(X)$, the necessary condition for the minimization of $f(X)$ at $X^*$ is:

$$\nabla f^* = 0 \tag{2.4}$$

where:

$\nabla f$: the vector of the first derivatives, or the gradient vector of the objective function calculated at $X^*$.

$$\nabla f^{*T} = \left\{ \frac{\partial f^*}{\partial X_1}, \frac{\partial f^*}{\partial X_2}, \ldots, \frac{\partial f^*}{\partial X_n} \right\} \tag{2.5}$$

$X$: the vector of the design variables.

$n$: the number of design variables.

The sufficient condition for a local minimum of $f(X)$ at $X^*$ involves the calculation of the matrix of the second derivatives $H$.

$$\Delta X^T \times H^* \times \Delta X > 0 \tag{2.6}$$

where:

$\Delta X$: the vector of changes in the design variables:

$$\Delta X = X - X^* \tag{2.7}$$

$H$: the matrix of the second derivatives or the Hessian matrix defined as:

$$H \equiv \begin{Bmatrix} \frac{\partial^2 f}{\partial X_1 \partial X_1}, \frac{\partial^2 f}{\partial X_1 \partial X_2}, \ldots, \frac{\partial^2 f}{\partial X_1 \partial X_n} \\ . \qquad\qquad\qquad\qquad . \\ . \qquad\qquad\qquad\qquad . \\ . \qquad\qquad\qquad\qquad . \\ \frac{\partial^2 f}{\partial X_n \partial X_1}, \frac{\partial^2 f}{\partial X_n \partial X_2}, \ldots, \frac{\partial^2 f}{\partial X_n \partial X_n} \end{Bmatrix} \tag{2.8}$$

Equation (2.6) requires that the Hessian matrix $H^*$ is a positive definite matrix.

An extension of the simple differential calculus is the introduction of the Lagrangian function, which consists of both an objective function, and a constrained function, with additional variables called Lagrange multipliers. The Lagrangian function is defined as:

$$L(X,\lambda) = f(X) + \sum_{j=1}^{n_h} \lambda_j h_j(X) \tag{2.9}$$

where: $h_j(X) = 0$ $(j=1...n_h)$: the equality constraints.

$\lambda_j$: Lagrange multipliers.

At the optimum, the differential change in the Lagrangian function $L(X)$, in terms of differential change in design variables $X$ and Lagrange multipliers $\lambda$, must be equal to zero.

$$\frac{\partial L}{\partial X_i} = 0 \qquad i = 1,2,\ldots,n$$
$$\frac{\partial L}{\partial \lambda_j} = 0 \qquad j = 1,2,\ldots,n_h \tag{2.10}$$

For general optimization problems, where there are equality and inequality equations in the constraints, the Kuhn-Tucker condition can be used to test for relative minimum at a given point. The Kuhn-Tucker condition is simply expressed as:

$$\nabla f + \sum_{j=1}^{J} \lambda_j \nabla g_j = 0$$
$$\lambda_j \geq 0 \tag{2.11}$$

where:

$g_j(X) \leq 0$: the inequality constraints.

$J$: the number of active constraint $g_j$ that are evaluated at the point being tested.

## 2.3 MODERN METHODS IN STRUCTURAL OPTIMIZATION

### 2.3.1 MATHEMATICAL PROGRAMMING METHODS (MP METHODS)

The basic concept of MP methods in optimization is quite simple. It employs numerical search techniques, which involve a point-to-point search for the optimum in an *n-dimensional* design space (Venkayya *et. al.* 1968, 1973). First of all, an initial design is selected in the design space. Based on the initial design, a procedure for evaluation of the objective function is carried out.

Having obtained the value of the objective function, the current design is compared with all of the preceding designs. Finally, a rational way to select a new design is presented and the whole process is repeated.

The development of numerical search techniques has attracted particular attention to the linear programming methods (LP methods) proposed by Dantzig (1963). In LP methods, the objective functions and all of the constraints are linear functions of design variables. LP methods have significant advantages and were reviewed by Kirsch (1993):

- Within a finite number of steps, the exact global optimum is reached.
- Good reliability and efficiency in computational programming.
- Some non-linear problems can be approximated by a linear formulation and solved by LP algorithms.

Although LP algorithms are reliable and efficient, their applications to practical design problems reveal many difficulties. As the number of design variables increases, the computational effort involved in the solution process becomes prohibitively high and is the main drawback of these algorithms. In addition, the dependence of the optimal result on the initial design is one of the numerical uncertainties in the procedure. In practical design problems, differentiability and continuity of the objective functions and constraints are not satisfied easily. To reduce the number of functional evaluations and thus easing computational effort, many transformation and approximation techniques have been proposed. One of the commonly used transformation techniques is the dual and primal problem. The solutions of the two problems are identical. If one of the two problems is solved, we can find the solution of the other. Since the computational effort in solving LP problems is a function of the number of constraints, it is desirable to reduce this number. The number of constraints in the dual problem equals the number of design variables in the primal problem, thus we can solve the problem with a smaller number of constraints.

The development of non-linear programming methods is motivated by the fact that most practical design problems are formulated as non-linear functions in terms of design variables. In general, no single non-linear programming method can solve efficiently all optimization problems. Generality, simplicity, and easy adaptability to computers are the compelling features of linear and non-linear programming methods. Schmit (1960) integrated non-linear programming to finite element analysis. Since then, many numerical techniques for solving as well as improving convergence speed have been developed, for example, the penalty-function method (Zangwill, 1967), the feasible direction method (Zoutendijk, 1960) and the gradient-projection method (Rosen, 1961).

### 2.3.2 OPTIMALITY CRITERIA METHODS (OC METHODS)

The Optimality Criteria methods were first introduced by Prager and his co-workers in the 1960s (Prager *et. al.* 1967, 1974, 1977 and 1978). Optimality Criteria methods and Mathematical Programming methods are basically similar in concept to objective functions and constraints, but they differ in the redesign step. In MP methods, the objective function is optimized directly until a convergent condition is satisfied by several numerical search techniques, whereas in OC methods a priori criterion is derived before the optimization process and the optimal result is reached when that criterion is satisfied. According to the derivation of the priori criterion, OC methods can be divided into two main categories:

- **Intuitive OC methods**, where a priori criterion is defined based on the intuition and experience of the designers. The recurrence relations of the design variables are formulated explicitly based on approximations of the constraints. Initially, the methods are applied to problems with stressed constraints, for example Schmit (1960) and Reinschmidt (1975) and were later extended for displacement constrained problems by Berke (1970) and Venkayya and Berke (1973).

- **Mathematical OC methods**, in which the Kuhn-Tucker conditions for a minimum point are employed to define the condition of optimality as discussed by Falk (1967) and Fleury and Braibant (1986). Kuhn-Tucker conditions naturally lead to the definition of the Lagrangian function and Lagrange multipliers, which need to be computed.

For a general optimization problem subject to several constraints, the mathematical formulation can be expressed as follows:

$$\text{Minimize } f(x) \tag{2.14}$$

$$\text{Subject to: } g_j(x) \geq 0 \quad j = 1,2,\ldots,n \tag{2.15}$$

The Kuhn-Tucker conditions are:

$$\lambda_j^* g_j(x^*) = 0 \qquad j = 1,2,\ldots,n_g \tag{2.16}$$

$$\lambda_j^* \geq 0 \qquad j = 1,2,\ldots,n_g \tag{2.17}$$

$$\frac{\partial f}{\partial x_i} - \sum_{j=1}^{n_g} \lambda_j^* \frac{\partial g_j}{\partial x_i} = 0 \qquad i = 1,2,\ldots,n \tag{2.18}$$

Equation (2.18) can be re-written as:

$$\sum_{j=1}^{n_g} \lambda_j e_{ij} = 1 \qquad i = 1,2,\ldots,n \tag{2.19}$$

$$\text{where : } e_{ij} = \frac{\partial g_j}{\partial x_i} / \frac{\partial f_j}{\partial x_i} \qquad i = 1,2,\ldots,n \qquad j = 1,2,\ldots,n_g \tag{2.20}$$

is the effectiveness of the $i^{th}$ design variable with respect to the $j^{th}$ constraint. Since Lagrange multipliers are the measure of the importance of the constraints in terms of their effect on the optimum value of the objective function, equation (2.19) indicates that: at the optimum, the effectiveness of all design variables, weighted by Lagrange multipliers, are the same. Venkayya (1973) referred to the term $e_{ij}$ as virtual strain density of the member, which equals unity in the case a of single constraint. Based on these findings, several derivations of recurrence formulation for design valuables have been proposed. The most commonly used recurrence formulation was presented by Venkayya (1973) which employs the over-relaxation ν greater than unity to improve the convergence.

$$x_i^{k+1} = x_i^k \left( \sum_{j=1}^{n_g} \lambda_j e_{ij} \right)^{\upsilon} \tag{2.21}$$

where k denotes iteration cycles.

A special form of intuitive optimality criteria method is the fully stressed design method. It has been traditionally used as the optimality criteria for the optimal design for skeletal structures; see Schmit (1960), Reinschmidt, Cornell and Brotchie (1966) and Razani (1965). Since the optimal criterion is imposed on stress, the method is applicable to structures that are subject only to stress constraints. The basic concept of the fully stressed design (FSD) method can be simply stated as:

*At the minimum weight design, each member of the structure sustains its allowable stress $\sigma_i^U$ under at least one loading condition.*

In the recurrence formulation, the design variable in the next iteration is calculated by the redesign rule.

$$x_i^{k+1} = x_i^k \frac{\sigma_i^k}{\sigma_i^U} \tag{2.22}$$

The underlying assumption in the concept of the FSD method is that the primary effect of adding or removing material from a structural member is to change the stresses only in that member. This assumption is true for statically determinate structures. However, for statically indeterminate structures, where changes in a member will affect the stresses in other members, fully stress design procedure may not lead to minimum-weight designs as pointed out by Razani (1965). In this case, fully stress design procedure has to be applied repeatedly until convergence to any desired tolerance is achieved. Due to it's simplicity and fast convergence, the fully stress design method has been extensively used as a starting point for other structural optimization methods. Stroud (1982) considered another intuitively optimality criteria method based on the simultaneous failure mode approach, which assumed that the lightest design is obtained when two or more modes of failure occur simultaneously. It is also assumed that the failure modes that are active at the optimum design are known in advance. Later on, Xie and Steven (1993) presented an evolutionary structural optimization method which utilizes the fully stress design and element elimination concept based on the Von Mises stress. Baumgartner et al. (1992) proposed a topology optimization method by changing Young's modulus based on local stress levels.

Compared to MP methods, OC methods are more efficient and most suitable for large-scale structures. OC methods usually have a good convergence rate in structures with low order of indeterminacy. The convergence of the MP methods may be stable but usually slower near the optimum. Furthermore, the iteration number and computational effort required in MP methods may be prohibitively high when solving practical problems with multiple types of constraints and a large number of design variables. However, MP methods are more general and rigorous than OC methods. To overcome the shortcomings of each method, many approximation concepts have been proposed based on their positive features to establish better solution methods. While the efficiency of MP methods has been increased significantly by using approximation concepts (Kirsch, 1981, 1982), OC methods have been extended to more general problems and more rigorous optimality criteria. Different approximation formulation problems have been reviewed by Fleury (1978).

### 2.3.3 GENETIC ALGORITHM METHOD (GA METHOD)

GA method is a stochastic direct search strategy that mimics the process of genetic evolution. It has its philosophical basis in Darwin's postulates of the "survival of the fittest". The developments in the field of genetic algorithms were originally started by Holland (1975), but the concept of analysis and design based on principles of biological evolution may be attributed to Rechenberg (1965). It is interesting that the method is suitable for a combination of sizing optimization and structural layout optimization problems.

In the GA search procedure, at the beginning, an initial population of designs is randomly created by the design variables represented by strings of digital bits. Each bit in the design variables has a physical meaning that can be interpreted when the optimum is reached. After generating the initial population of designs, the GA search then establishes the fitness of each design by evaluating the fitness function. The definition of a fitness function requires that the objective and constraint functions be represented as a single composite function. Hajela *et. al.* (1993) defines the fitness function by using the usual exterior penalty function form $P_i = \sum_{j=1}^{ncon} \langle g_j \rangle^2 \quad i = 1,2,...,M$

where $\langle g_j \rangle = \begin{cases} g_j & if\ g_j \geq 0 \\ 0 & otherwise\quad j = 1,2,...,ncon \end{cases}$ are the '*ncon*' constraints on kinetic stability.

By conducting structural sensitivity analysis and adopting a first-order Taylor series expansion to approximate the magnitude of the displacements, Grierson and Pak (1993) proposed the fitness function as.

$$F = F_{\max} - \sum_i A_i L_i - \begin{Bmatrix} 0 & ,if\ u_r^l \leq \overline{u_r} \\ c\left(u_r^l - \overline{u_r}\right)^2 & ,if\ u_r^l < \overline{u_r} \end{Bmatrix} \qquad (2.23)$$

where:

$F_{max}$: An arbitrarily large positive value that ensures fitness F never becomes negative.

$A_i$, $L_i$: Member cross-section area and length, respectively.

c: a multiplier which is selected so as to heavily penalize a serious constraint violation while penalizing a minor violation more lightly.

$u_r$, $u_r^l$: Displacement and first–order Taylor series expansion approximation of displacement of the structure.

A typical GA search procedure contains three basic operators, namely reproduction, crossover and mutation. The GA search procedure starts with a reproduction stage, in which the fittest members of the current population are simply allowed to contribute to a larger extent to the progeny population. In the crossover operation, design characteristics of mating members are exchanged to produce the next generation based on a probability number. Eventually, the mutation operator is carried out with a low probability and at a randomly selected site on the chromosomal string of the chosen design to prevent the premature loss of some genetic information from the population.

Recently, the applications of the GA method have been developed for combined sizing and layout structural optimization of truss structures. Grierson and Pak (1993) used the method for combined size and geometry optimization of a simple frame structure. Hajela *et. al.* (1993) modified the penalty terms in fitness function to overcome the difficulties in numerical computation of distinguishing between good and poor designs from the population. Using the ground structure approach, sizing and topological optimization of the truss structure is solved successively subject to stress and displacement constraints. Koumousis and Arsenis (1994) applied the GA method to the optimal detailed design of reinforced concrete members of multi-storey buildings. The method decides the detailed design on the basis of a multi-criterion objective that represents a compromise between a minimum weight design, a maximum uniformity and the minimum number of bars for a group of members. Due to the large design space, a method is adopted to search for near optimum solution. It is worth knowing that most of the applications of the GA method relate to truss or detailed structural members. This is because of the difficulties involved in the solution process of the method, especially the re-analysis task to evaluate the objective functions and the constraints. In other words, GA methods are generally not as efficient as classical MP and OC methods.

### 2.3.4 HOMOGENIZATION METHOD

The homogenization method was first introduced by Bendsoe and Kikuchi (1988). The main idea for solving a class of optimization problems involving topology is to introduce an 'infinite' number of micro-scale voids to form a porous medium. This is because it is difficult to define a structural topology optimization problem by using a finite number of parameters according to Cheng and Olholf (1982). The optimization problem is formulated in terms of the design variables, which are represented by the geometry parameters of these voids. Three common ways to construct microstructure and voids are rectangular micro-scale voids, ranked layer material cells and artificial materials.

- **<u>Rectangular micro-scale voids</u>**: Figure 2.1 illustrates a square cell with a rectangular cavity.

where a, b are the design variables.

$$a = b = 0, \text{ solid}$$

$$\left.\begin{matrix} 0 < a < 1 \\ 0 < b < 1 \end{matrix}\right\} \text{ porous}$$

$$a = b = 1 \quad \text{void}$$

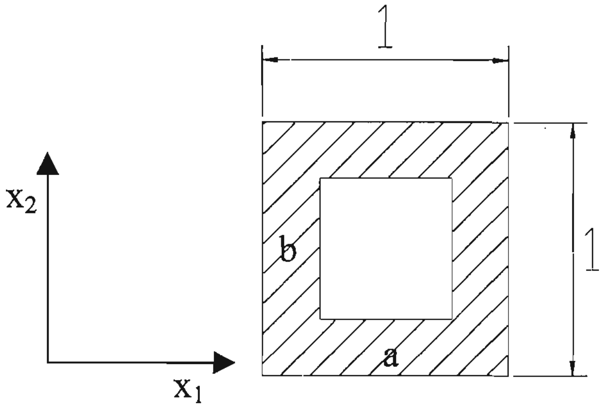



Figure 2.1

- **<u>Ranked layered material cells</u>**: Each cell of this type of microstructure is constructed from layers of different material. Figure 2.2 illustrates the construction of rank-2 bi-material composite.

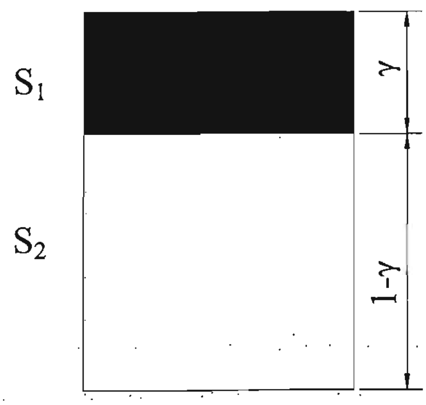



Figure 2.2

- **<u>Artificial material</u>**: The structure is described by a discrete function as

$$\chi(x) = \begin{cases} 1 \text{ if } x \in \Omega & \text{material} \\ 0 \text{ if } x \notin \Omega & \text{no material} \end{cases}$$

where $\Omega$ is the design domain.

The solution of the optimization problem involves the determination of effective (homogenized) material properties of microstructures. There are two methods for finding the effective material properties of microstructures and they can be expressed as follows:

- **Numerical approach**: In this approach, finite element models for the microstructure are constructed and appropriate boundary conditions for the periodicity are applied. A series of finite element analyses are carried out for voids of different sizes. In order to obtain a continuous variation of the homogenized material properties with respect to the void sizes, an interpolation process can be applied to the "discrete" results from the finite element analyses.

- **Analytical approach**: In this approach, explicit expressions for the effective elastic tensor can be obtained by establishing the optimal upper and lower bounds for the complementary elastic energy density of the perforated material. These microstructures are known as "external" microstructures in the sense that achieve in the Hashin-Shtrikman bounds on the effective properties of composite materials. This method can be applied to both 2D and 3D layered material cells of finite rank.

Once the effective material properties of the microstructure are determined, they will be used as input data for the normal finite element analysis of the structure, which is to be optimized. The unknowns in this problem are an array of densities (and orientations of the holes for some types of microstructures). In the optimization module, considering the geometrical parameters of the presumed material model in finite elements as design variables, the total potential energy is adopted as the objective function. The volume of material is considered as the global constraint that has to be active. Using the optimality criteria method, an updating scheme is constructed.

### 2.3.5 EVOLUTIONARY STRUCTURAL OPTIMIZATION METHOD (ESO METHOD)

By slowly removing inefficient material from a structure, the residual shape of the structure evolves towards an optimum. That is the simple concept of the ESO method. The ESO method was first introduced by Xie and Steven (1993, 1994a). At the beginning, the ESO method was developed for topology and shape design of continuum structures based on fully stressed design. By gradually removing elements, which have the lowest Von Mises stress, the remaining structure evolves towards an optimum. Chu *et. al.* (1996) extended the ESO method to layout design of continuum structures with stiffness and displacement constraint. During the optimization procedure, the sensitivity number of each element was computed. It represents the change in structural behaviour due to element removal. Xie and Steven (1994b, 1996, and 1997) also used the ESO method to optimize structural frequencies and buckling loads. The frequency of a structure can be shifted in a desired direction by removing material from the design domain based on a sensitivity analysis. A typical ESO method procedure is given as follows:

- **Step 1:** Model the structure using finite elements. Assign material properties, applied loads and boundary conditions. The design and non-design domains are defined.
- **Step 2:** Carry out finite element analysis (FEA) for the structure to obtain the structural behaviour i.e. element stresses, mean compliance or constrained displacements.
- **Step 3:** Compute the sensitivity number for each element. The sensitivity number is a number representing the change in structural behaviour due to element removal.
- **Step 4:** Remove elements that have the lowest sensitivity numbers.
- **Step 5:** Repeat Steps 2 to 4 until one of the constraints is violated.

The ESO method can be used to solve a wide range of practical engineering problems. The main assumption in the ESO procedure is that the pattern of the global stiffness matrix of the structure is not changed dramatically within an iteration. In other words, only a small number of elements are removed in a single iteration in order to maintain the smooth change between two successive iterations. Although the ESO method proves to be a reliable and efficient tool, there are drawbacks in the procedure that need to be improved. The deficiencies of the ESO method are listed below:

- Because of the assumption that the global stiffness matrix of the structure does not change significantly in a single iteration, only a small number of elements can be removed at each iteration.
- There is no rationale to determine the removal ratio for a specific problem. The removal ratio is the number of elements removed in each iteration over the number of all elements in the initial design or the number of all elements in the current design. So far, this number has been assigned based on the experience of the ESO users.
- There is no method for deciding which topology generated in the evolutionary process is the optimum. Previous studies decide the optimum topology when the prescribed number of iterations is reached or the specified amount of material that is allowed to be removed from the design is reached. These assumptions are quite arbitrary.
- So far, the application of the ESO method to a 3D model with combination of discrete and continuum elements has been very limited. This is due to

the fact that different types of elements in the model cause it to become highly non-linear.

Yang (1999) developed the so-called bi-directional evolutionary structural optimization method (BESO method). In the BESO method, elements can be removed from as well as added to the model to obtain the optimum. Although BESO method starts with the simplest initial design connecting the load and supports, the maximum design domain in which the structure is allowed to occupy must also be defined. In other words, these elements are still stored in the data file but they do not physically exist as part of the structure in the initial design. By adding and removing elements simultaneously in the optimization procedure, the BESO method has many advantages according to Yang (1999).

- As the BESO method starts from the simplest initial design, the degree of dependency of the optimal result on the initial design may be less than the ESO method which starts from an over-sized full design.

- By starting from the simplest initial design, the computational time and cost needed to carry out the finite element analysis for the model in the BESO method is dramatically less than that in the ESO method, which starts from a full design. This fact is especially true when dealing with practical large-scale structures. The difference between the size of the simplest structure and the size of the full model is usually significant. However, as indicated by Yang (1999), if that difference is not large enough, it may lead to computational times in the BESO procedure greater than times for the ESO method.

- In the BESO method, elements that are removed prematurely can be recovered. This makes the method more reliable than ESO method.

The BESO method also has some disadvantages:

- In some cases, the designer may only need a better design rather than a theoretically optimal solution. On one hand, this may be satisfied with the ESO method that starts from a feasible design. On the other hand, the BESO method starts from an infeasible design and all the intermediate solutions lie in an infeasible region.

- Compared with the ESO method, the BESO procedure requires more parameters that have to be specified by the users.

Compared with other methods, the ESO method is likely to be the most efficient method with acceptable reliability. Liang (1999a, 1999b, and 2000) proposes a method called the performance-based optimization method for structural topology and shape design. It employs the ESO method in the element elimination procedure and uses a scaling technique at the end of each iteration to monitor and determine the optimal topology (Liang, 2000). This method has proved reliable for 2D continuum structures in which the global stiffness matrix is a linear function in terms of the design variables. However, for structures with a combination of discrete and continuum elements, Liang's performance index is no longer valid.

## 2.4 COMMENTS ON CURRENT RESEARCHES AND APPLICATIONS

Despite significant effort directed towards the study of structural optimization in the past, most studies have been restricted to the area of sizing or geometrical optimization problems. Much less effort has been spent on topological optimization that could result in most significant material savings. Furthermore, as stated by Liang (2001), structural optimization techniques could become more attractive to civil engineers if they are developed not only for saving materials but also for simplifying the designer's task by automating the major design process.

For building structures, the appropriate method for structural optimization needs to have the following features:

- Capable of dealing with large-scale structures. For example, buildings with more than 50 storeys and multiple bays. The data entry and output handling tasks need to be automated.
- Have an acceptable level of reliability i.e. the guarantee of convergence, the stability of numerical solution.

- Capable of solving structures with combination of discrete and continuum elements. For example, shear wall-frame building where beams and columns are modelled by beam elements and shear walls are considered as plate elements.
- Capable of dealing with both 2D and 3D models.
- Capable of dealing with multiple load cases.
- Capable of dealing with multiple support environments.
- Capable of dealing with multiple material environments.

Most of the commercial FEA packages available satisfy most of the items shown above. In this thesis, the ESO method will be extended for dealing with structures containing both beam and continuum elements. The method proposed will use the output data of the FEA package STRAND6™ to carry out the optimization process.

# CHAPTER 3: THE EXTENDED ESO METHOD FOR OVERALL STIFFNESS CONSTRAINTS

## 3.1 INTRODUCTION

A review on the development and applications of structural optimization methods has been presented in Chapter 2. In this chapter, the extended ESO method for *continuum topology optimization subject to overall stiffness constraints* will be developed. Firstly, the topological structural optimization problem is stated for seeking the optimal topology of a structure subject to overall stiffness constraints. The optimal topology will be the one which has the same weight as the initial structure, but has the maximum stiffness compared with all other topologies that are generated during the optimization process. Secondly, sensitivity analysis will be carried out to derive the element removal criteria. Thirdly, termination criteria and techniques to overcome sharp change in the mean compliance value of the structure will also be discussed in order to complement the method. Finally, three examples representing different types of finite element models will be presented to demonstrate the validity and effectiveness of the extended ESO method.

## 3.2 OPTIMIZATION PROBLEM STATEMENT

In the ESO method for overall stiffness constraints, the task of the designer is to find the stiffest structure. It is known that maximizing the stiffness is equivalent to minimizing the mean compliance value of a structure. Therefore, the constraint of the optimization problem for overall stiffness constraint can be mathematically expressed as: $C - C^{*} \leq 0$ where $C$ and $C^{*}$ are the mean compliance value and its prescribed limit for the structure, respectively. However, in practice, the mean compliance limit of the structure is usually unknown in advance.

By removing inefficient elements at each iteration, there will be a number of topologies generated during the optimization process. In order to determine the topology which has the maximum stiffness, it is natural to scale the topologies so that they all have the same weight and then their stiffnesses can be compared with each other. The topological structural optimization problem for overall stiffness constraint can be stated as:

*Starting from an initial structure, the topological structural optimization problem for overall stiffness constraint is to find the structural topology which has the same weight as that of the initial structure and has the minimum mean compliance value.*

In order to determine which elements are most inefficient, element removal criteria will be derived by undertaking a sensitivity analysis.

## 3.3 ELEMENT REMOVAL CRITERIA BASED ON STRAIN ENERGY DENSITY

Before developing the strain energy density formulation, it is useful to present some definitions and theoretical concepts used in the ESO method.

**The variable $x_i$:** As defined by Yang (1999), in the ESO method for topological design of structure, the design variable is a non-dimensional quantity. For beam elements, it is defined as $x_i = \frac{A_i}{A_{0i}}$ where $A_{0i}$ and $A_i$ are the sectional area of the bar before and after being removed respectively. This means that $A_i$ only receives value 0 (after elimination) or $A_{0i}$ (before elimination). Therefore, $x_i$ only receives value 0 or 1. Similarly, for continuum elements, $x_i$ is defined as $x_i = \frac{t_i}{t_{0i}}$ where $t_{0i}$ and $t_i$ are the thicknesses of the continuum elements before and after being removed, respectively.

In finite element analysis (FEA), the static behaviour of a structure is represented by the following equilibrium equation:

$$Ku = P \tag{3.1}$$

where $K$ is the global stiffness matrix, $u$ is the displacement vector and $P$ is the external load vector.

The strain energy of the structure, which is defined as

$$C = \frac{1}{2} P^T u \tag{3.2}$$

is commonly used as the inverse measure of the overall stiffness of the structure. $C$ is also known as the mean compliance of the structure.

Differentiating equation (3.1) with respect to the $i^{th}$ design variable, the result is:

$$\frac{\partial K}{\partial x_i} u + K \frac{\partial u}{\partial x_i} = 0 \tag{3.3}$$

$$\therefore \frac{\partial u}{\partial x_i} = -K^{-1} \frac{\partial K}{\partial x_i} u \tag{3.4}$$

From equations (3.2) and (3.4) we have:

$$\frac{\partial C}{\partial x_i} = \frac{1}{2} P^T \left( -K^{-1} \frac{\partial K}{\partial x_i} u \right) \tag{3.5}$$

Because the global stiffness matrix $K$ is a symmetrical matrix, thus

$$\frac{\partial C}{\partial x_i} = -\frac{1}{2} u^T \frac{\partial K}{\partial x_i} u \tag{3.6}$$

Using the first term of a Taylor series expansion, we obtain

$$\Delta C = C' - C = \sum_{i=1}^{m} \frac{\partial C}{\partial x_i} \Delta x_i = -\frac{1}{2} u^T \left( \sum_{i=1}^{m} \frac{\partial K}{\partial x_i} \Delta x_i \right) u \tag{3.7}$$

where $C'$ is the mean compliance of the structure after element removal and $m$ is the total number of elements removed in the iteration.

Because the ESO process is a 0-1 decision scheme, elements are gradually removed from the structure. Thus

$$\Delta C = -\frac{1}{2} u^T \left( \sum_{i=1}^{m} \frac{\partial K}{\partial x_i} (0-1) \right) u = \frac{1}{2} u^T \left( \sum_{i=1}^{m} K_i \right) u = \frac{1}{2} \sum_{i=1}^{m} u_i^T K_i u_i = \sum_{i=1}^{m} C_i \tag{3.8}$$

where

$u_i$ is the displacement vector of the $i^{th}$ element.

$C_i = \frac{1}{2} u_i^T K_i u_i$ is the strain energy of the $i^{th}$ element.

In the ESO method the continuum design domain is usually divided into a finite element mesh of identical elements, and all the elements have the same volume. Therefore the element strain energy above can be successfully used as a driving force for the optimization process. However, for a model with finite elements of different shapes or sizes, the strain energy density of the $i^{th}$ element is defined as

$$\alpha_i = \frac{\frac{1}{2} u_i^T K_i u_i}{w_i} \tag{3.9}$$

where $w_i$ is the weigh of the $i^{th}$ element.

In the ESO process for continuum topology design with stiffness constraint, the elements with the lowest strain energy densities will be automatically removed at each iteration. To ensure a smooth change between two consecutive iterations, only a small number of elements are removed from the model.

## 3.4 STRUCTURE UNDER MULTIPLE LOAD CASES

In the case of multiple load cases, the procedure of deriving the strain energy density for individual elements is much the same as the last section. The strain energy density of the $i^{th}$ element due to the $j^{th}$ loading condition can be re-written as

$$\alpha_{ij} = \frac{\frac{1}{2} u_{ij}^T K_i u_{ij}}{w_i} \tag{3.10}$$

where $u_{ij}$ is the displacement vector of the $i^{th}$ element due to the $j^{th}$ load case.

The element strain energy density under multiple load cases is defined as the sum of the strain energy density due to each load case.

$$\alpha_i = \sum_{i=1}^{J} \alpha_{ij} \tag{3.11}$$

where $J$ is the number of load cases.

## 3.5 UNIFORMLY CHANGING THE THICKNESS OF CONTINUUM ELEMENTS

Uniformly changing the thickness of continuum elements is often referred to as scaling the structure. Scaling technique has been proposed by many researchers (Kirsch, 1993; Morris, 1982 and Liang, 2000). As stated by Kirsch, the great advantage of the scaling technique is that it can convert an infeasible design into a feasible one. For example, in topological continuum optimization, by uniformly changing the thickness of the continuum elements, the topology of the structure unchanged, the designer can change the stiffness or displacements of a model from an unaccepted value to an acceptable one.

The scaling multiplier is simply defined as

$$\varphi = \frac{x'}{x} \tag{3.12}$$

where $x$ and $x'$ are the vector of design variable before and after scaling, respectively.

If the global stiffness matrix of the model is a linear function of the $z^{th}$ order of the design variable, i.e.

$$K = Cx^z \tag{3.13}$$

where $C$ is a constant, then the global stiffness matrix of the model after scaling

$$\therefore K' = Cx'^z = C(\varphi x)^z = \varphi^z Cx^z = \varphi^z K \tag{3.14}$$

From equations (3.1) and (3.14) and assuming the scaling does not affect the applied load, we have

$$u' = K'^{-1} P = \frac{K^{-1}}{\varphi^z} P = \frac{1}{\varphi^z} u \tag{3.15}$$

The mean compliance of the structure after scaling becomes

$$C' = \frac{1}{2} u'^T K' u = \frac{1}{2}\left(\frac{u}{\varphi^z}\right)^T (\varphi^z K)\left(\frac{u}{\varphi^z}\right) = \frac{1}{\varphi^z}\frac{1}{2} u^T K u = \frac{1}{\varphi^z} C \tag{3.17}$$

- **For truss elements:** The element stiffness matrix is a linear function of the width of cross-sectional area. Thus z=1

$$C' = \frac{1}{\varphi} C \tag{3.18}$$

- **For plane stress finite elements:** The element stiffness matrix is a linear function of the thickness of continuum elements. Thus this is similar to the case for truss elements, however the thickness of the element is used as the design variable instead of the cross-sectional area of the truss member.

- **For plate bending elements:** The element stiffness matrix is a linear function of the 3rd order of the thickness of the plate. Thus z=3

$$C' = \frac{1}{\varphi^3} C \tag{3.19}$$

- **For general plate and shell elements:** They have both in-plane and flexural action. For small deflections, these two actions are independent. Therefore, it is assumed that flexural deflections and rotations of the element are only related to the forces normal to the plane and the in-plane displacements are only related to the in-plane forces. The element stiffness matrix consists of two parts, namely in-plane behaviour term and bending behaviour term.

$$K = K_p + K_b \tag{3.19}$$

where $p$ denotes in-plane behaviour and $b$ denotes bending normal to the plane.

From equations (3.14) and (3.19)

$$K' = \varphi K_p + \varphi^3 K_b \tag{3.20}$$

For this type of elements, the problem becomes non-linear and the scaling technique cannot be used. This type of problem is also encountered when dealing with structures containing both beam and plate finite element combinations, in which only the continuum elements are removed from the structure while beam elements are assumed to be fixed.

Liang (2000) proposed a method to monitor and select the optimal topology by calculating the performance index of each design that is generated in the ESO process. He defined a performance index

$$PI = \frac{C_0 W_0}{C_i W_i} \tag{3.21}$$

where $PI$: performance index of the current structure.

$C_0$, $C_i$: mean compliance the initial structure (ground structure) and the current structure, respectively.

$W_0$, $W_i$: the weigh of the initial structure and the current structure, respectively.

According to Liang (2000), the optimal topology is the one which has the highest $PI$ value. However, this method is based on the assumption that the global stiffness matrix of the structure is a linear function of the $z^{th}$ order of the design variables. As shown in equation (3.20), this method is invalid when applied to structures containing general plate and shell elements, as of 3D structures or structures containing different types of finite elements.

In order to solve the above problem, this thesis proposes a new procedure for comparing the structural performance of general 2D and 3D topologies. In each iteration, after removing inefficient elements, the thickness of the plate elements will be uniformly changed ("scaled") to make the weight of the current structure (after scaling) equal to the weight of the initial structure. Then the performance of the structure is evaluated by its mean compliance value. Consider the structural model with a mesh of identical elements, the relationship between the element thickness of the current structure and of the initial structure can be expressed as

$$\rho A t_0 n_0 = \rho A t_k n_k \tag{3.22}$$

$$\therefore t_0 n_0 = t_k n_k \tag{3.23}$$

where $\rho$: the weight density of the material used.

$A$: the area of one continuum element, which is the same for all elements.

$t_0, t_k$: the thickness of the continuum elements of the initial structure and the current structure at $k^{th}$ iteration, respectively.

$n_0, n_k$: The total number of continuum elements of the initial structure and the current structure at $k^{th}$ iteration, respectively.

From equation (3.23), it can be shown that $t_k$ can be calculated from the element thickness of the previous iteration, i.e.

$$t_k = t_{k-1} \frac{n_{k-1}}{n_k} \tag{3.24}$$

After changing the thickness of the elements, a new finite element analysis will be carried out to compute the mean compliance of the current design. This value will be saved into a database. At the end of the optimization process, based on the mean compliance history, the optimal topology will be picked from among many topologies generated in the evolutionary process. The optimal topology is the one, which has the same weight as all the others, but has the lowest mean compliance.

## 3.6 TERMINATION CONDITIONS

In the ESO method, several criteria for stopping the optimization process have been proposed before. Xie and Steven (1994), in the original ESO method, proposes to stop the process when the volume of the current structure reaches a prescribed value, says 50% of the initial structure. This criterion is arbitrary because it does not guarantee that the optimal topology is included in the optimization process. The performance-based topological optimization method proposed by Liang (2000) states that the optimization process is terminated when the performance index of the current structure is greater than that of the initial structure. This termination condition is rigorous as it means the performance of the current structure, at that iteration, is even worse than the performance of the initial structure and hence the optimization process should be terminated. However, Liang's termination condition may not be applied to general 3D structures or structures with a combination of beam and plate elements.

**In this thesis, the optimization process is terminated when there is no more decrease in the mean compliance of the equally weighted topologies.** In the computer code, the optimization process will be terminated if the structural topology remains unchanged for over 20 consecutive iterations.

## 3.7 HANDLING SHARP CHANGE IN THE MEAN COMPLIANCE VALUES

The basic assumption of the ESO method for continuum topology optimization is that the global stiffness matrix of the structure is not changed significantly within an iteration. It means that a smooth change in the topology of the structure must be kept between consecutive iterations. If this requirement is violated, the values of the mean compliance will change sharply and the structure may become a mechanism (thus unstable). To keep the changes of the mean compliance between consecutive iterations to be small, it is necessary to restrict the number of elements removed in each iteration by specifying a removal ratio. *The removal ratio is defined as the ratio of the number of elements to be removed to the total number of elements of the initial or current structure.* If the removal ratio is based on the initial structure, the number of removed elements in each iteration is kept constant during the whole optimization process. If the removal ratio is based on the current structure, the number of removed elements in each iteration is gradually decreased during the optimization process and hence the computational cost is increased. However, it is worth using a removal ratio based on the current structure as the accuracy of solution is improved.

It has been observed that after a certain number of iterations sharp changes in the mean compliance value still occurs although the number of elements removed in each iteration is small. Consider two consecutive iterations $i$ and $i+1$ where there is a sharp change in the mean compliance values between them, the elements removed in the $i^{th}$ iteration must have played an important role in forming the global stiffness matrix of the structure. Therefore, to correct this problem, we can go back to the $i^{th}$ iteration and force the optimization process to remove other elements instead. In the computer code, the elements removed in the $i^{th}$ iteration, which have caused the sharp change, will be recovered temporarily and fixed and the program will look for other elements to remove. It is noted that, after temporarily recovering and fixing the elements, because the weight of the current structure has been increased, the thickness of the continuum elements needs to be uniformly reduced to keep the weight the same as the initial structure.

The technique of handling sharp changes in the mean compliance values can be illustrated in Figure 3.1.

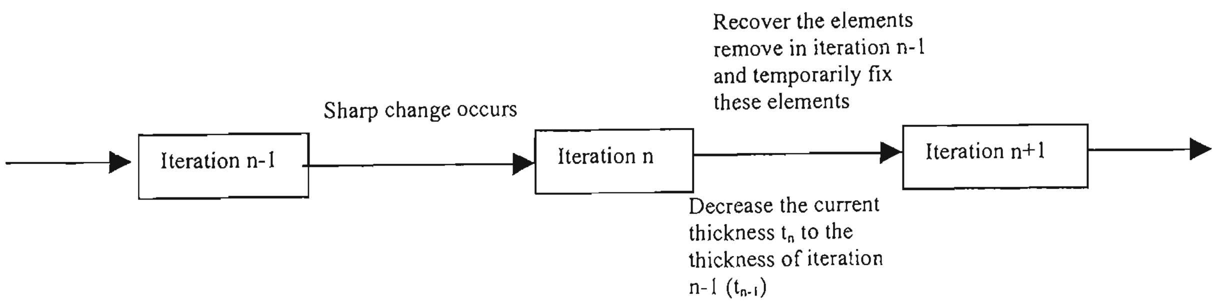



**Fig.3.1** Technique of handling sharp change in the mean compliance values

Those temporarily fixed elements are temporarily stored in a specific data file and they will be released later in the optimization process. In order to avoid structural mechanism collapse due to removing these elements, it is necessary that the temporarily fixed elements be released only if the optimization process finds other elements to remove without causing the sharp change in the mean compliance value. The temporarily fixed elements can only be released when one of the conditions listed below is met.

- **The number of temporarily fixed elements reaches a fixed ratio** Is defined by the ratio of the number of the temporarily fixed elements to the number of total elements of the current structure.

- **After a successful iteration involving element removal**. A successful iteration involving element removal is defined as an iteration in which elements are removed without causing sharp change in the mean compliance value. This means that the optimization process is successful in finding a new way of removing elements for evolution. The temporarily fixed elements now can be released and allowed to participate in the future evolution.

The technique of releasing the temporarily fixed elements is illustrated in Figure 3.2 and Figure 3.3.

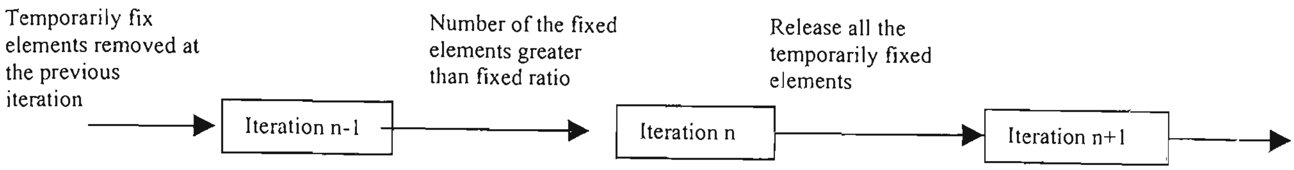



**Fig.3.2** Releasing temporarily fixed elements when the number of temporarily fixed elements is greater than fixed ratio

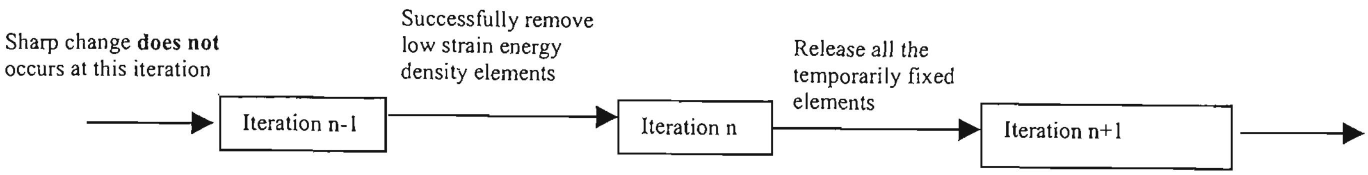



**Fig.3.3** Releasing temporarily fixed elements after an iteration involving element removal

## 3.8 DESIGN PROCEDURE

The design procedure for topological structural optimization for overall stiffness constraint is outlined as follows

**Step 1:** The structure is modelled using finite elements. The beam elements are considered as a non-design domain. Their sizes and shapes are not changed during the optimization process. Only continuum elements are considered as the design domain and allowed to be removed during the optimization process. This model is called an initial structure.

**Step 2:** Carry out the finite element analysis to compute the mean compliance of the current structure. The mean compliance of the structure is then saved in a database.

**Step 3:** Calculate the strain energy density of each continuum element by using equation (3.10).

**Step 4:** If there is a sharp change in the mean compliance value, temporarily fix those elements removed at the previous iteration. Return the thickness of continuum elements to the thickness value of the previous iteration. Repeat from Step 2.

**Step 5:** If the number of temporarily fixed elements is greater than or equal to the fixed ratio, release all the temporarily fixed elements. Repeat from Step 2.

**Step 6:** Remove elements which have the lowest strain energy density from the structure. The number of removed elements is equal to the removal ratio (*RR*) multiplied by the number of elements of the current structure.

**Step 7:** If there is no sharp change in the mean compliance value at the previous iteration, release all the temporarily fixed elements.

**Step 8:** Uniformly increase the thickness of continuum elements in the design domain by using equation (3.24).

**Step 9:** Save the current structure.

**Step 10:** Repeat from Step 2 to Step 9 until the termination condition in Section 3.6 is met.

**Step 11:** Plot the mean compliance history and select the optimal topology.

The design procedure can be illustrated in Figure 3.4

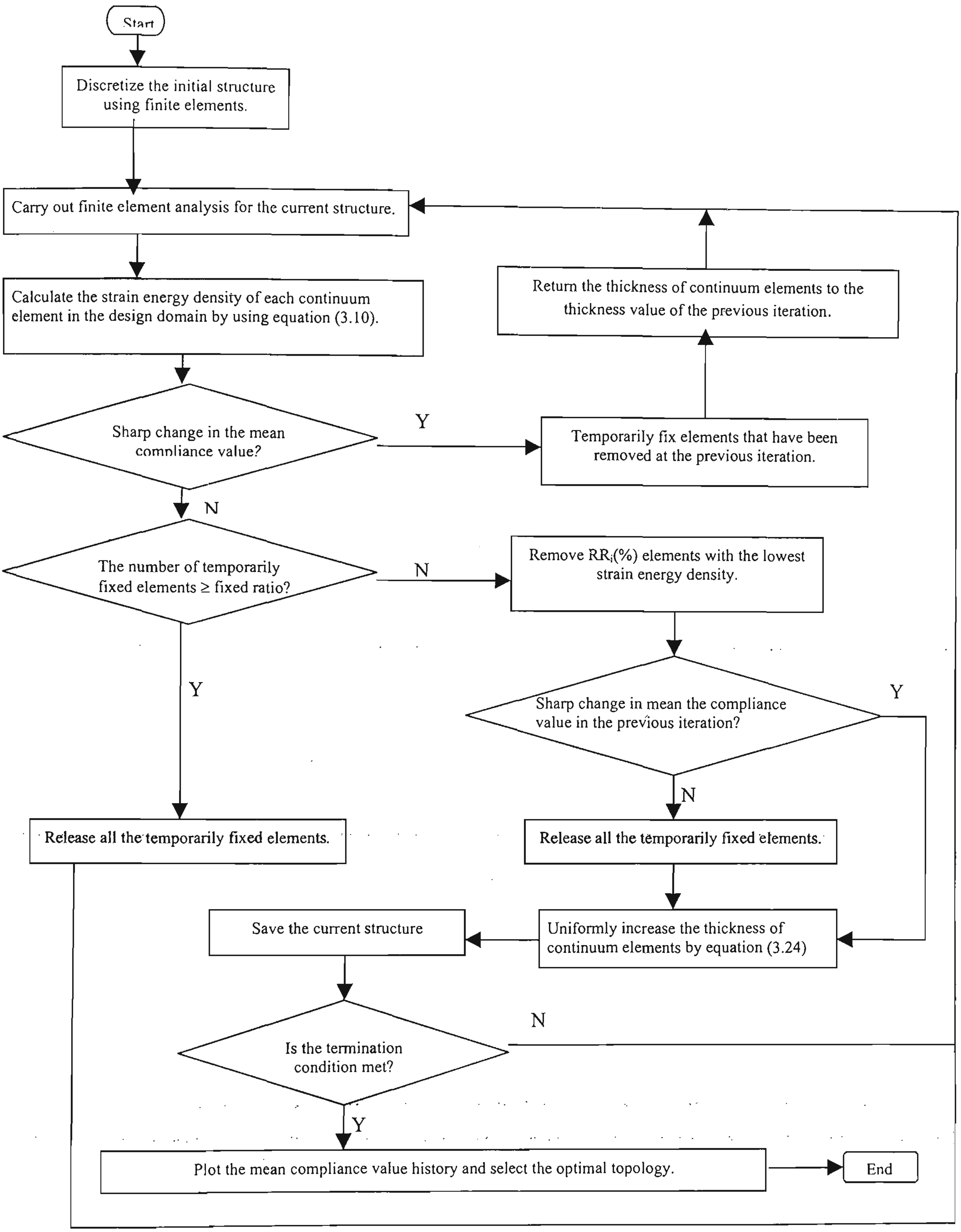



**Fig.3.4** Procedure for topological optimization for overall stiffness constraint

## 3.9 EXAMPLES

In this section, three examples are provided to demonstrate the effectiveness of the proposed method. Firstly, topological optimization for overall stiffness for a plane stress problem is solved. Secondly, a structure with plate elements (bending only) is considered. Finally, the topological optimization problem of a 3D structure containing both continuum and beam elements is solved.

### 3.9.1 A PLANE STRESS STRUCTURE (TWO-BAR FRAME)

The efficiency and reliability of the proposed method is first examined by solving the well-known two-bar frame problem shown in Figure 3.5. The structural optimization problem is to determine the optimal geometry of the frame under a point load P subject to overall stiffness constraint. This problem has been analytically solved by Rozvany (1976). If the frame structure is assumed to be a truss for the minimum-weight design, its optimal height $H$ is obtained as $H = 2L$. Later, Suzuki and Kikuchi (1991) obtained the same result by using the homogenization method.

Figure 3.6 shows the initial design for the optimization process. In order to achieve the optimal solution, it is necessary to have the initial design domain larger than the resultant two-bar frame structure. The continuum design domain is divided into a 30x80 mesh of four-node plane stress elements. A point load of 200 N is applied to the middle of the free end. The value of elastic modulus is 200 GPa and Poisson's ratio is 0.3. The initial value of the thickness of continuum elements is 1mm. The finite element analysis input and optimization parameters are listed in Table 3.1.

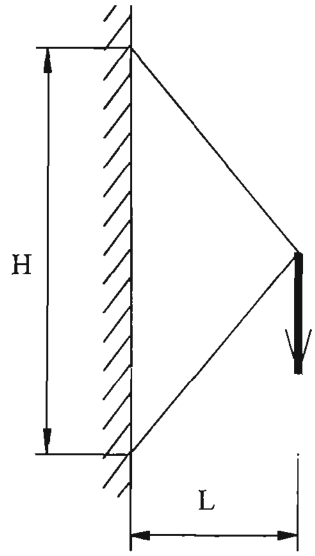



**Fig.3.5** Optimal geometry of the two-bar frame structure.

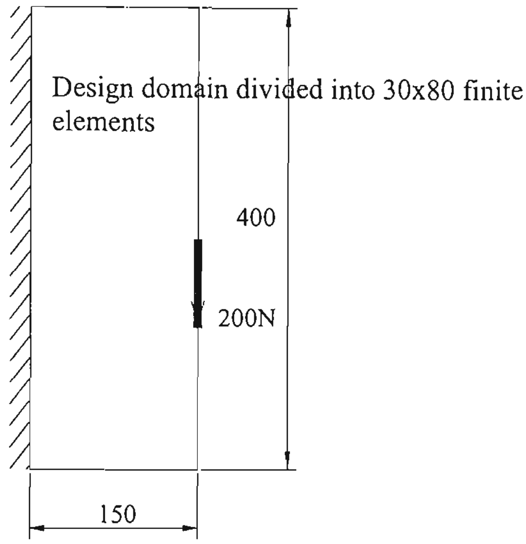



**Fig.3.6** Initial design of the optimization process.

**Table 3.1** Input values for the two-bar frame optimization.

| Finite element analysis input | Optimization parameters |
|---|---|
| • Height 400 mm<br>• Length 150 mm<br>• Continuum design domain: 30x80 mesh of four-node plane stress continuum elements.<br>• Load: $P = 200$ N<br>• Modulus of elasticity: $E$=200 GPa<br>• Poisson's ratio: $\nu$=0.3<br>• Initial plate thickness: $t$=1mm<br>• Static plane stress elastic analysis. | • Removal ratio: $RR$=1% of total number of continuum elements of the current structure.<br>• Topological structural optimization (cavities allowed).<br>• Maximum number of temporarily fixed elements, fixed ratio=20%.<br>• Difference in the change of mean compliance value that will be considered as sharp change, sharp change ratio=5 % between two adjacent iterations. |

The mean compliance history of the two-bar frame structural optimization subject to overall stiffness constraint is given in Figure 3.7. It is seen from Figure 3.7 that the mean compliance of the structure gradually decreases during the process. In other words, the overall stiffness of the structure is increasing during the optimization process. The straight line AB indicates that the optimization program cannot remove any more elements without causing the structure to collapse. Therefore, there is no improvement obtained during those iterations. The minimum mean compliance value was 0.167, which was obtained at iteration 93, and the corresponding topology is the optimal topology, as shown in Figure 3.8 (c). It is observed that the optimal topology also results in *H=2L*, which agrees well with the results of other researchers.

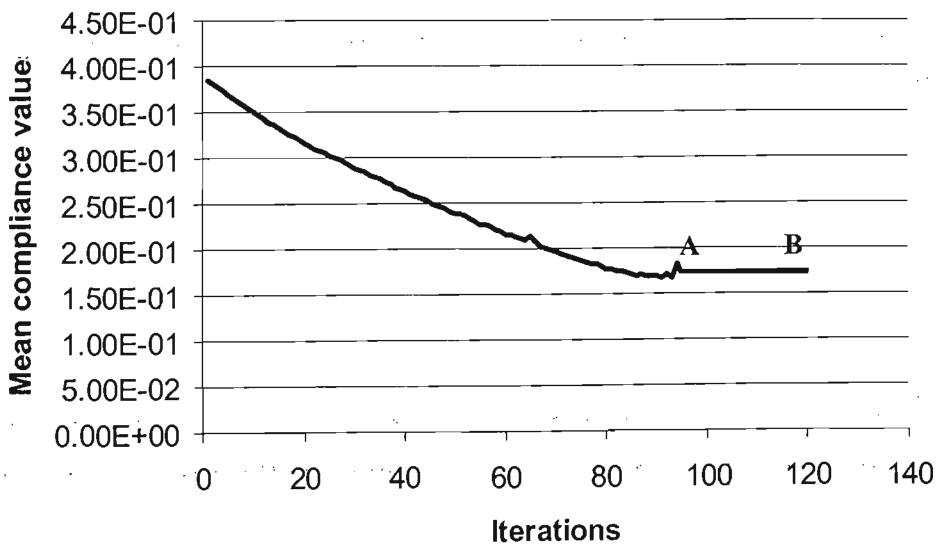
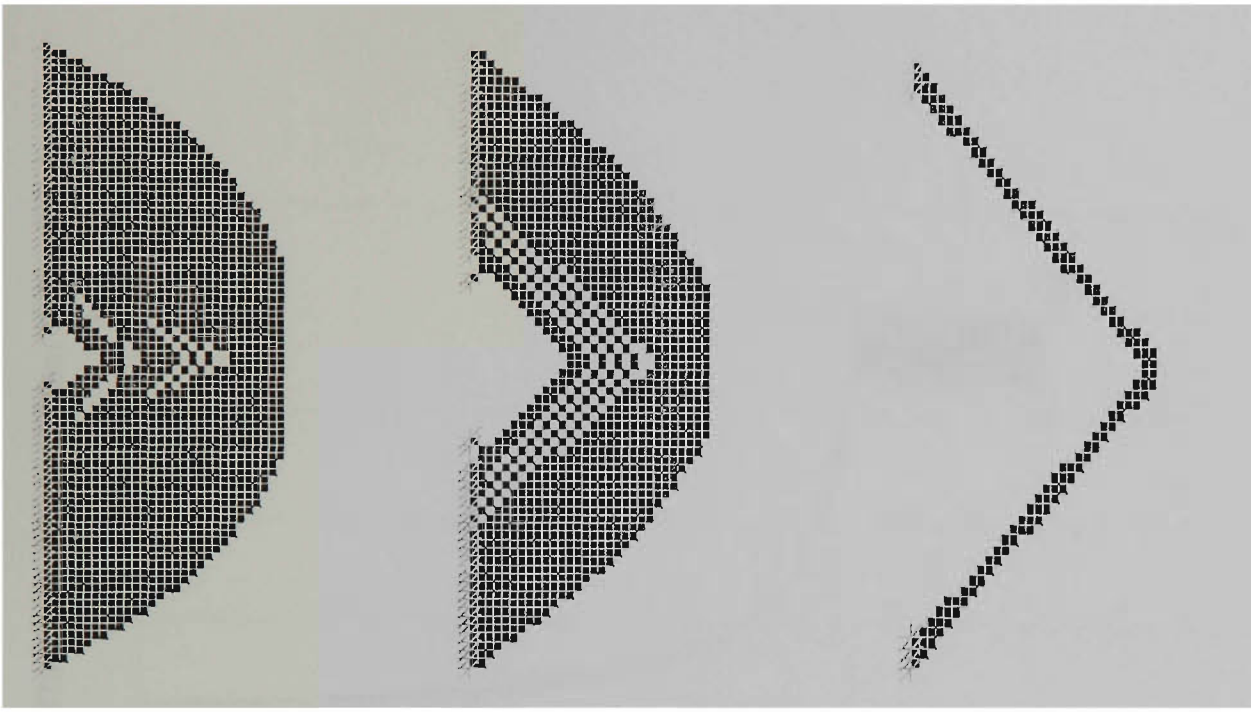



**Fig.3.7** Mean Compliance History



(a) Topology at iteration 30. Mc=0.316

(b) Topology at iteration 50. Mc=0.239

(c) Optimal topology at iteration 93. Mc=0.167

**Fig.3.8** Optimal topologies and mean compliance.

The thickness of the continuum design domain increases during the iterative optimization process to ensure the topologies generated have the same weight as that of the initial structure (see Figure 3.9). The zig-zag section AB indicates that the optimization program is only fixing and releasing elements during those iterations, hence, there is no improvement in structural topology.

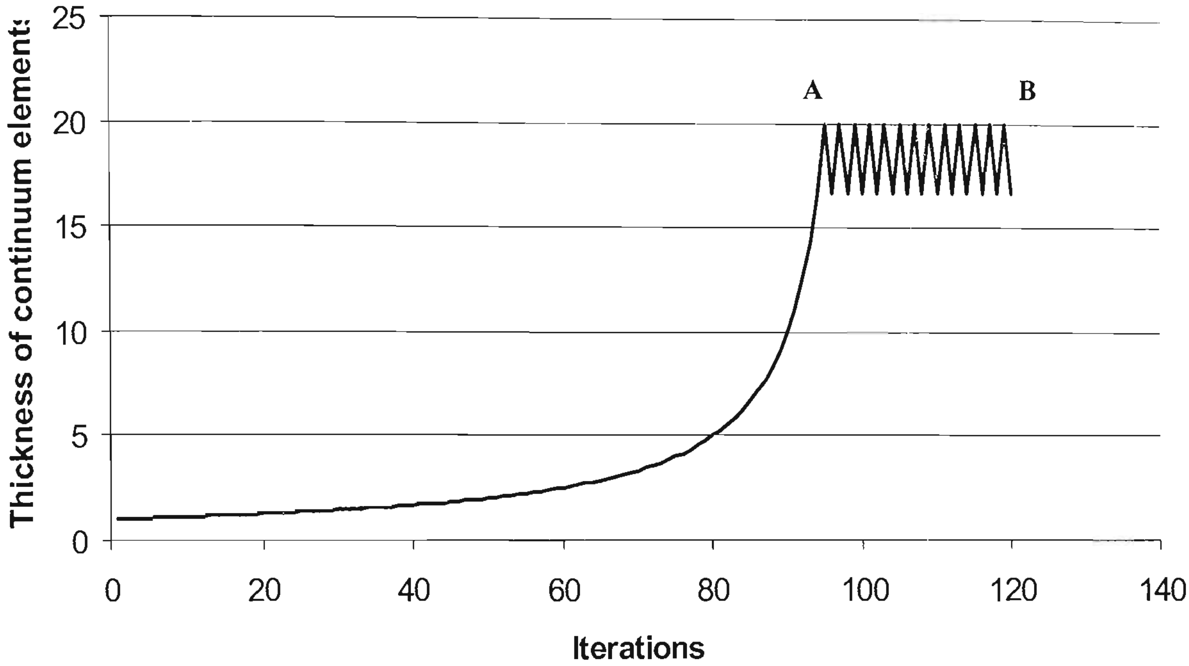



**Fig.3.9** History of element thickness

### 3.9.2 A PLATE IN BENDING

The extended ESO method is further examined by solving a plate in bending problem. A plate is fully fixed along its four edges and is loaded at the centre by a point load P=100N. Figure 3.10 shows the geometry of the plate under the concentrated load. The finite element analysis input and optimization parameters are listed in Table 3.2.

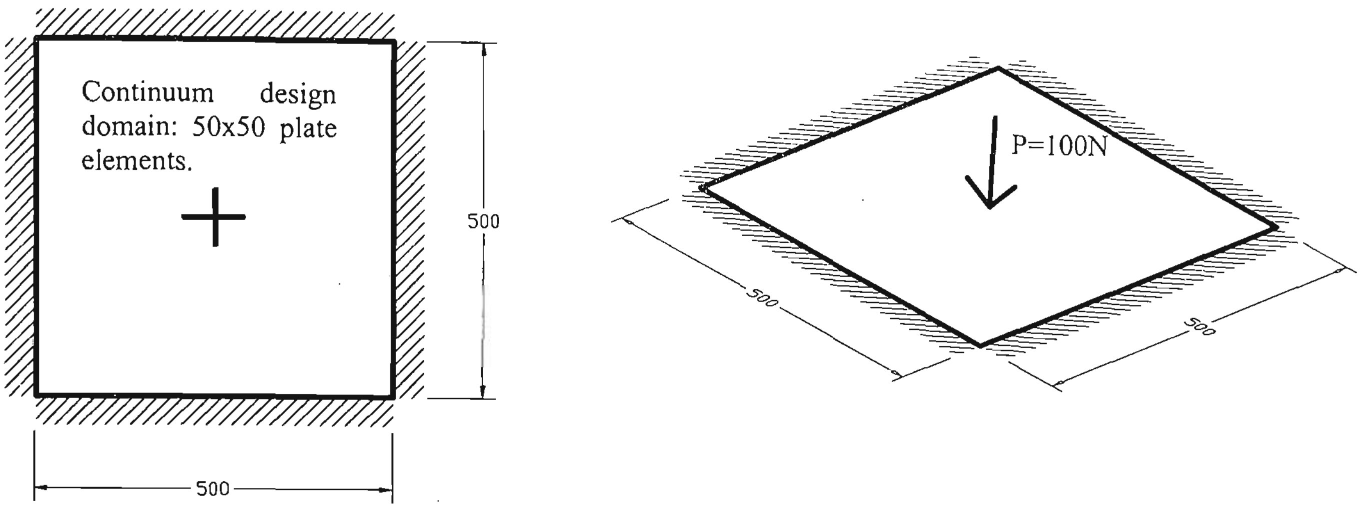



**Fig.3.10** Boundary and loading conditions of the plate.

**Table 3.2** Input values for plate in bending optimization subject to overall stiffness constraint.

| Finite element analysis input | Optimization parameters |
|---|---|
| • Plate side length 500 mm<br>• Continuum design domain: 50x50 mesh of plate elements.<br>• Load: $P = 100$ N<br>• Modulus of elasticity: $E$=200 GPa<br>• Poisson's ratio: $\nu$=0.3<br>• Plate thickness: $t$=1mm<br>• Static elastic analysis. | • Removal ratio: $RR$=1% of total number of continuum elements of the current structure.<br>• Topological structural optimization (cavities allowed).<br>• Maximum number of temporarily fixed elements, fixed ratio=20%.<br>• Difference in the change of the mean compliance value that will be considered as sharp change, sharp change ratio=5% between two adjacent iterations. |

Figure 3.11 shows the mean compliance history during the evolutionary process. A rapid decrease in the mean compliance value has been observed during the first 100 iterations. After iteration 151, the optimization process can no longer gain any further improvement. The optimal topology is reached at iteration 151, the mean compliance reduced from 380 to 6.75.

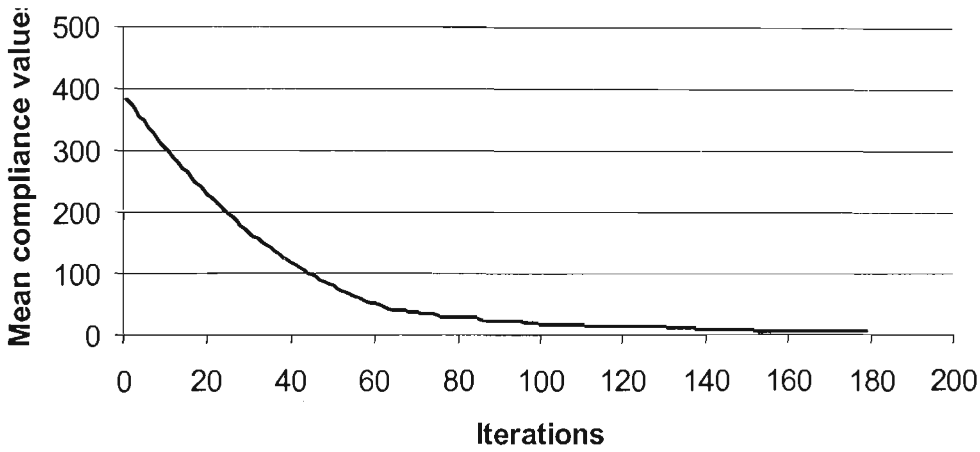
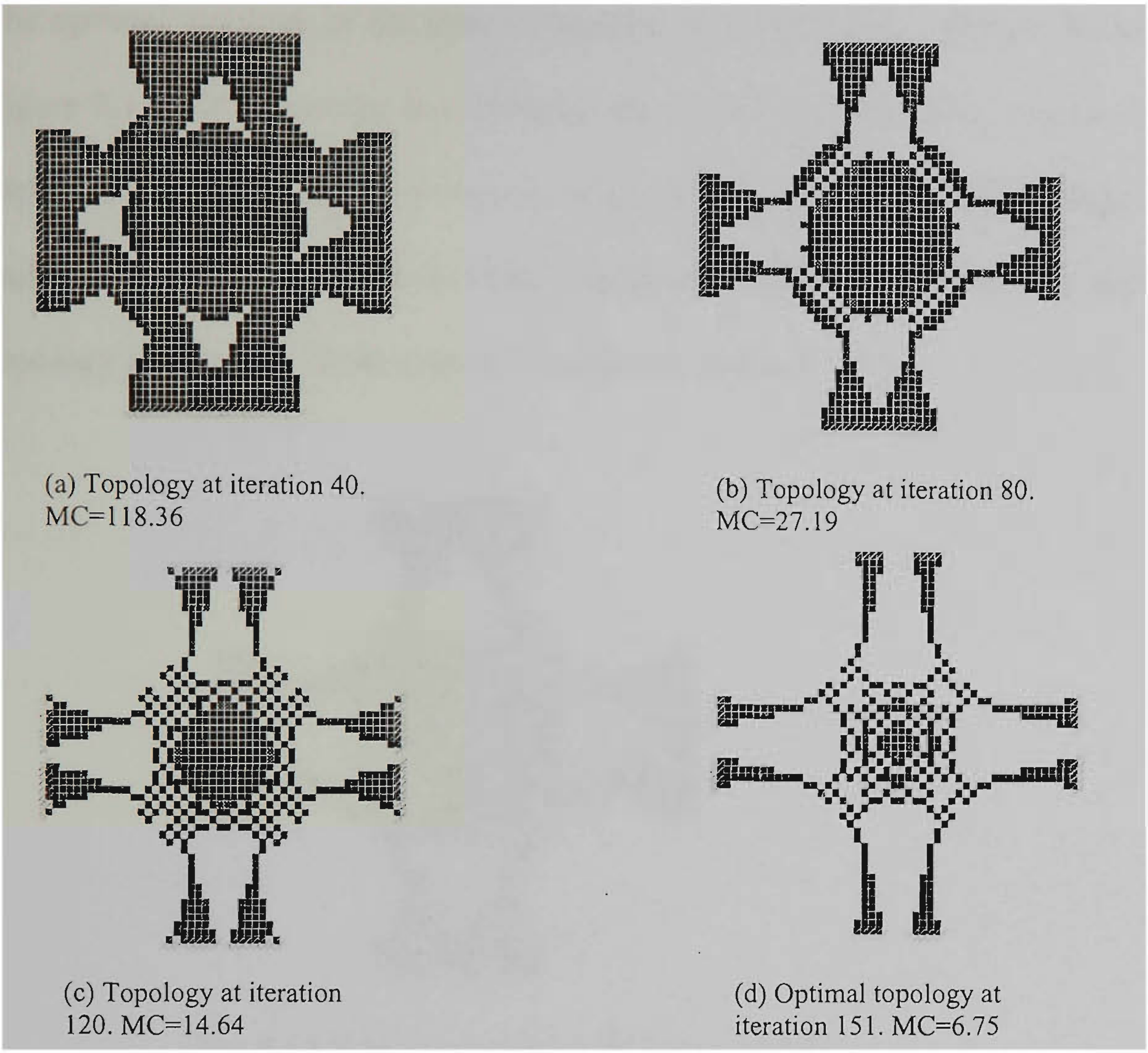



**Fig. 3.11** Mean Compliance History

Figure 3.12 shows various topologies and their corresponding mean compliance values during the optimization. The overall stiffness of the plate has increased dramatically during the first 60 iterations. The checkerboard pattern in the topology has occurred after the iteration 80. The history of the element thickness is illustrated in Figure 3.13. The thickness of elements, initially assigned to 1mm, has increased to the maximum value of 6.95mm.



(a) Topology at iteration 40. MC=118.36

(b) Topology at iteration 80. MC=27.19

(c) Topology at iteration 120. MC=14.64

(d) Optimal topology at iteration 151. MC=6.75

**Fig.3.12** Topologies of the plate in bending optimization

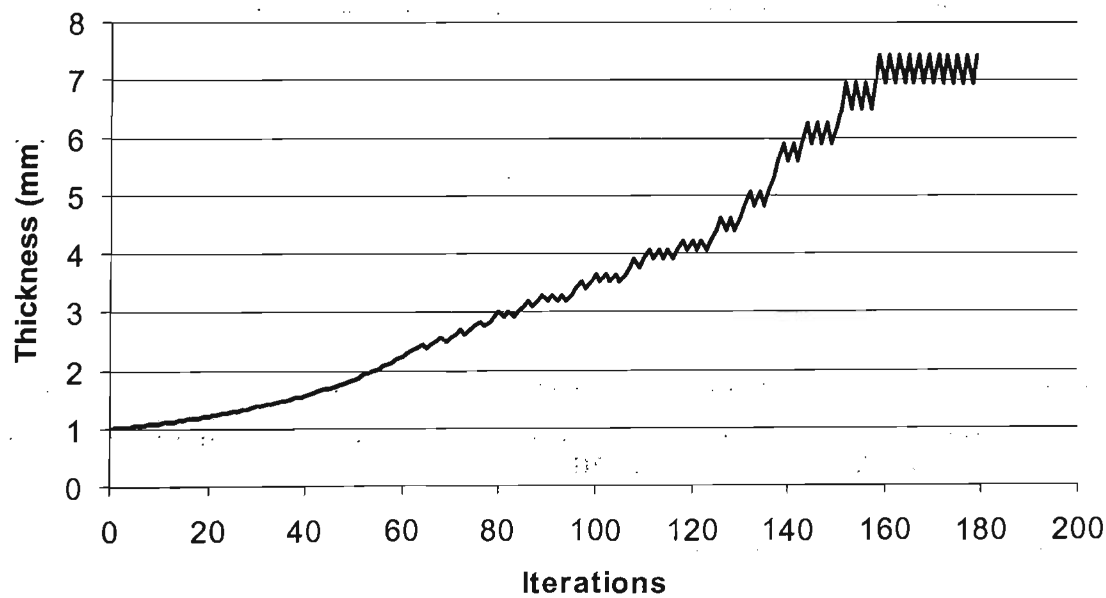



**Fig. 3.13** History of the element thickness

The optimal topology of the plate in bending given by Liang (2001) is shown in Figure 3.15. This topology was obtained by minimizing the mean compliance of the plate and using the performance index to choose the optimal topology. By using the technique to avoid the sharp change in the mean compliance, the optimal topology in Figure 3.12 (d) is better than that in Figure 3.14.

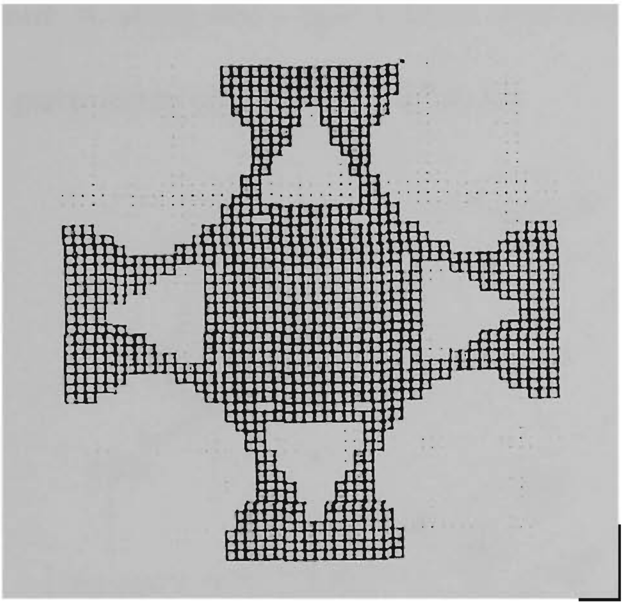


Fig.3.14 Optimal topology by Liang (2001)

### 3.9.3 A 3D STRUCTURE CONTAINING BEAM AND CONTINUUM ELEMENTS

In engineering applications, a 2D model is often an approximation of a real 3D structure. The task of finding the optimal 3D structure for a particular environment is far more challenging. In this section, the extended ESO method is applied to a 3D structure that contains both beam and plate elements under two lateral loads and subject to overall stiffness constraint. The geometry and boundary conditions of the 3D structure are given in Figure 3.16. Due to architectural requirements, only the face along the y axis of the frame is allowed bracings to be located. The structural optimization problem is to find an efficient bracing system at the face along the y axis of the frame. Because the lateral loads are often reversible, four lateral load cases are defined in the model depicting two load cases in the real structure, which is along the x and y axes. The finite element analysis input and optimization parameters are listed in Table 3.3.

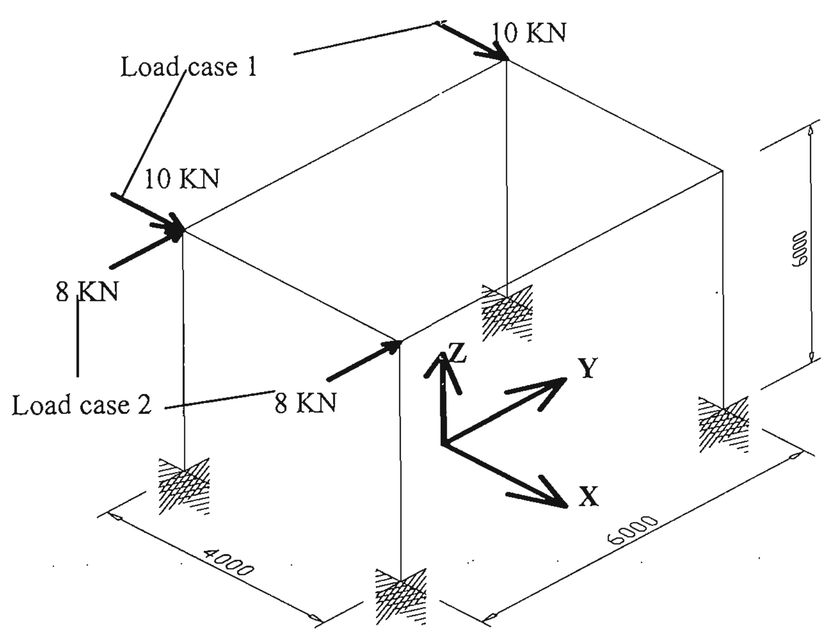



**Fig. 3.15** Geometry and boundary conditions of the 3D structure

**Table 3.3** Finite element analysis input and optimization parameters for the 3D structure

| Finite element analysis input | Optimization parameters |
|---|---|
| • Cross sectional geometry:<br>Column: 400x300 mm<br>Beam: 300x300 mm<br>• Continuum design domain:<br>Along X axe: 30x20 mesh of finite elements<br>Along Y axe: 30x30 mesh of finite elements.<br>• Load:<br>Load case 1: $P_1$ = 10 KN<br>Load case 2: $P_2$ = 8 KN<br>• Modulus of elasticity:<br>$E$=27.5 GPa<br>• Poisson's ratio: $\nu$=0.15<br>• Initial plate thickness: $t$= 30 mm<br>• Static elastic analysis. | • Removal ratio: $RR$=1% of total number of continuum elements of the current structure.<br>• Topological structural optimization (cavities allowed).<br>• Maximum number of temporarily fixed elements, fixed ratio=20%.<br>• Difference in the change of the mean compliance value that will be considered as sharp change, sharp change ratio =5 % between two adjacent iterations. |

The history of the mean compliance of the continuum elements due to two lateral load cases is shown in Figure 3.16. It can be seen that the minimum mean compliance values corresponding to two lateral load cases do not occur at the same iteration. It is clear from Figure 3.16 that the optimization process tends to narrow the difference between the mean compliance of the two load cases along the x and y axes. The optimal topology for the bracing system along the y axis is determined by monitoring the mean compliance caused by load case 2. Theoretically, the optimal topology is the one, which has the minimum mean compliance of the entire structure (beam and continuum elements). However, because the author does not have access to the source code of the beam elements, various topologies of the structure are given in Figure 3.17. The interpreted structure of topology at iteration 32 is also shown in Figure 3.18.

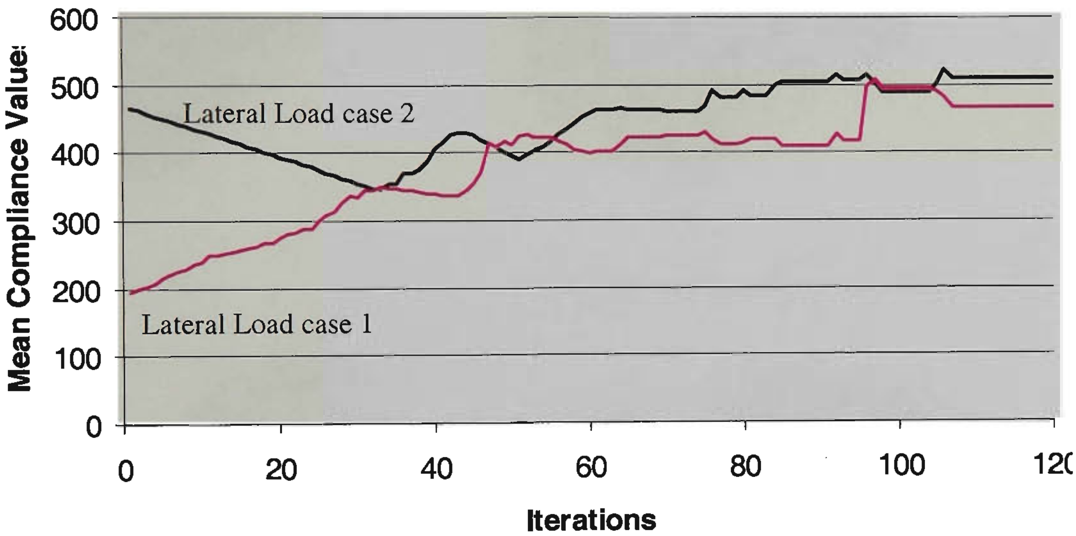
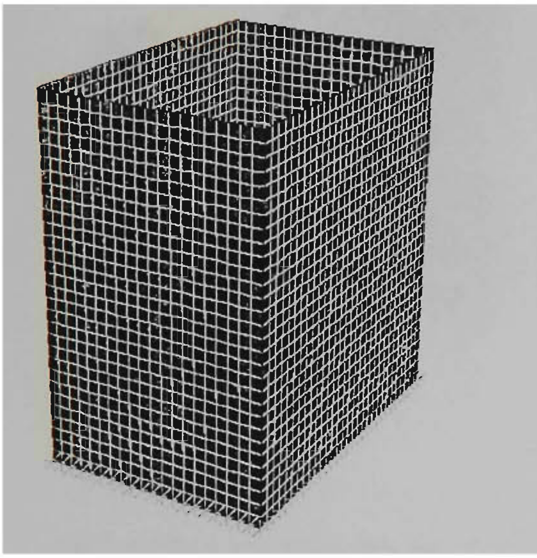
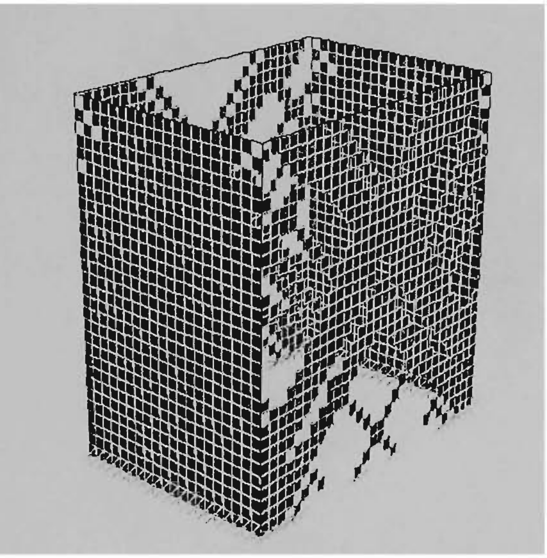


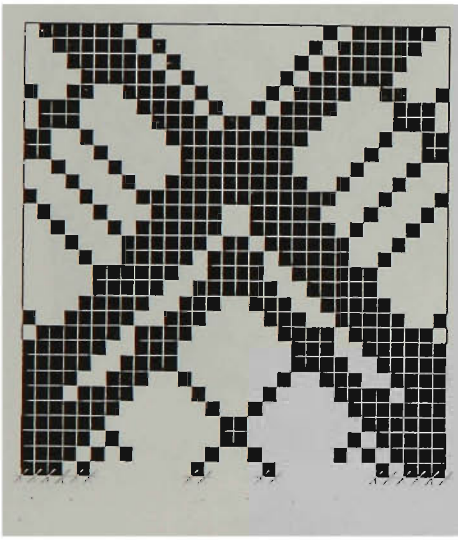

**Fig. 3.16** History of the mean compliance of continuum elements



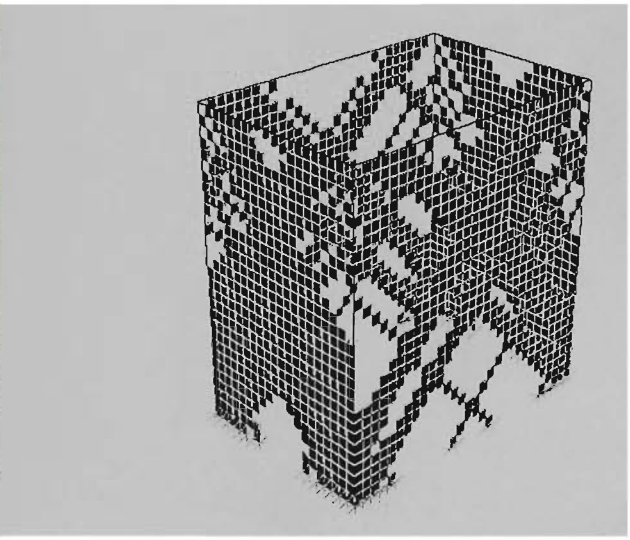
(a) Initial design



(b) Topology at iteration 32, $MC_2$= 347.06

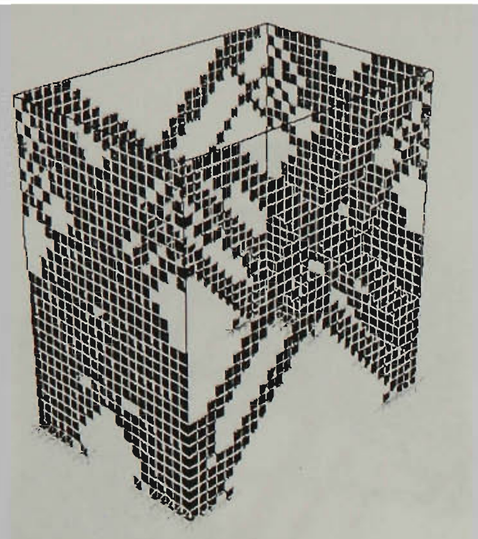


(c) Topology of the bracing system along y axe at iteration 32.

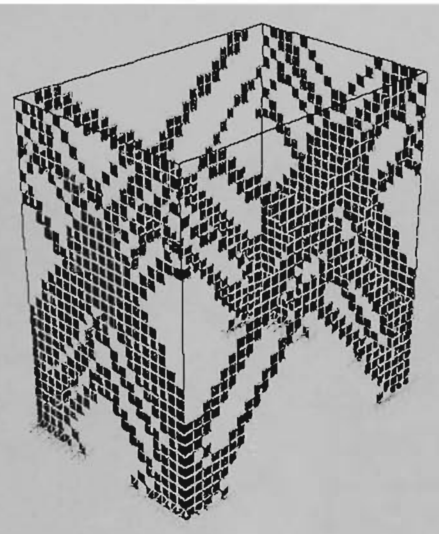


(d) Topology at iteration 42, $MC_1$= 335.81 $MC_2$= 429.25

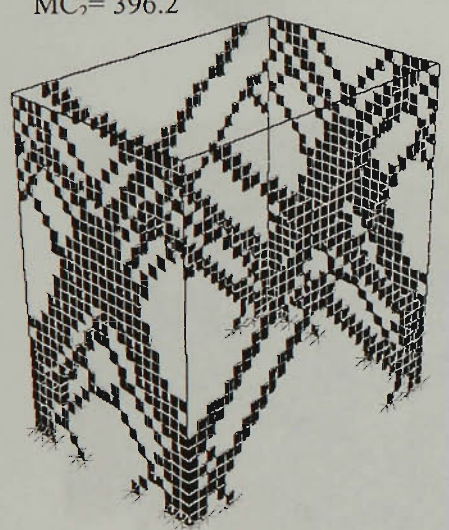
**Fig. 3.17** Topological history of the 3D structure (continued)



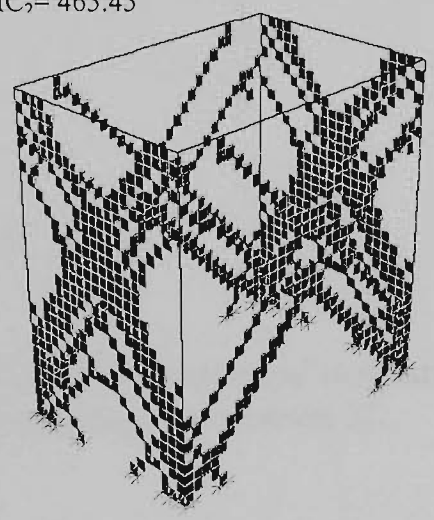
(e) Topology at iteration 50. $MC_1$= 411.61 $MC_2$= 396.2



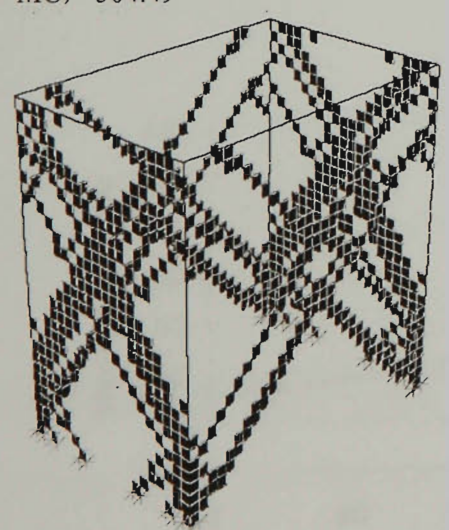
(f) Topology at iteration 65. $MC_1$= 422.7 $MC_2$= 465.45



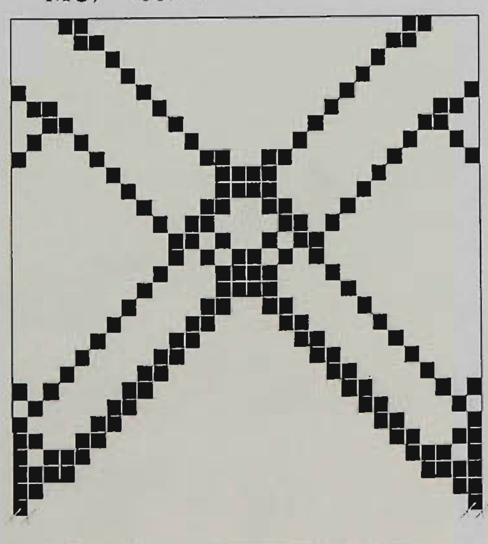
(g) Topology at iteration 90. $MC_1$= 409.51 $MC_2$= 504.49



(h) Topology at iteration 100. $MC_1$= 493.11 $MC_2$= 488.44

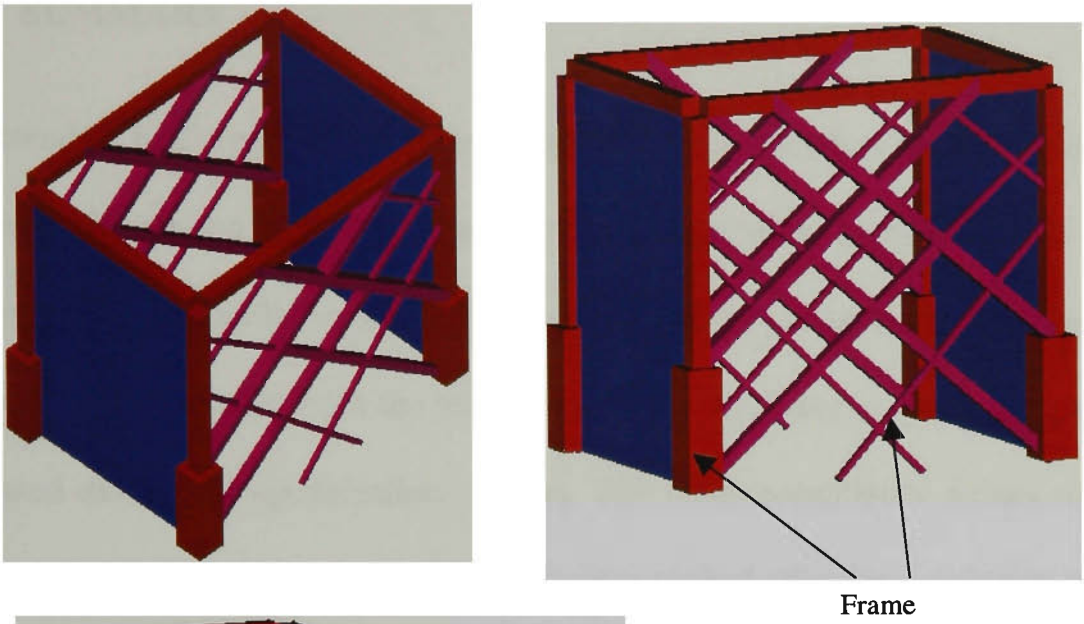


(i) Topology at iteration 120. $MC_1$= 466.59 $MC_2$= 409.43



(j) Topology of the bracing system along y axe at iteration 120. $MC_2$= 409.43

**Fig. 3.17** Topological history of the 3D structure



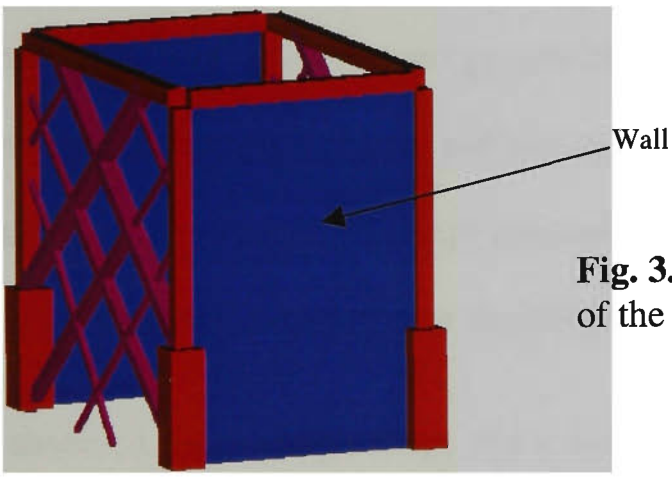




**Fig. 3.18** The interpreted structure of the topology at iteration 32.

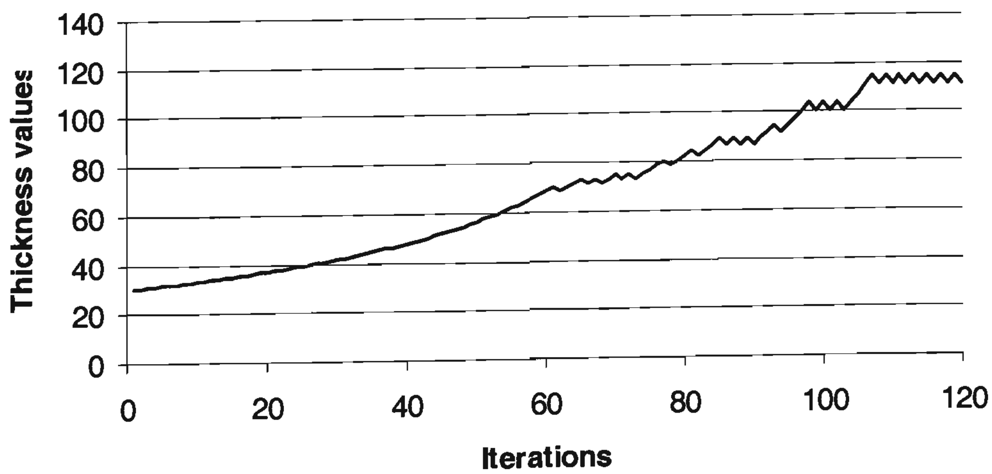



**Fig. 3.19** History of the continuum element thickness

## 3.10 SUMMARY

An extended ESO method for overall stiffness constraint has been developed in this chapter. Starting from a ground structure, continuum elements in the design domain were gradually removed according to their strain energy density. As elements were removed from the initial structure, there were a series of topologies generated during the optimization process. The mean compliance values of those topologies were stored in a database and were plotted after the evolution process was completed. The optimal topology was the one that had the highest stiffness i.e., the smallest mean compliance and had the same weight as the initial structure. The weight of the structure was kept constant during the optimization process by uniformly changing the thickness of the continuum elements.

It was observed that a sharp change in the mean compliance occurs after a certain number of iterations. This is because the connection patterns of continuum elements in the structure and the global stiffness matrix were changed significantly due to removal of elements that were required to maintain structural integrity. To correct this problem, when a sharp change in the mean compliance occurred, elements removed in the previous iteration were temporarily fixed and the next few iterations were carried out without removal of these elements. The temporarily fixed elements were released only if there was a successful iteration involving element removal without causing a sharp change or the number of temporarily fixed elements was greater than or equal to a fixed ratio. This technique could maintain a smooth change in the mean compliance in two adjacent iterations during the optimization process.

Three examples have been solved in this chapter to demonstrate the effectiveness of the extended ESO method. Firstly, the reliability of the method has been examined by solving the well-known two bar plane stress problem. The result obtained agreed well with results achieved from the homogenization method. Secondly, by solving a clamped plate in bending problem, the proposed method has been further examined when dealing with structures containing plate elements. It is noted that by using the "temporarily fix and release" technique, a sharp change in structural behaviour can be avoided and the result obtained is improved compared with those of previous methods in the ESO field. Thirdly, a 3D structure subjected to lateral loading has been solved to demonstrate the effectiveness of the method when dealing with 3D structures. The resultant topology gives the structural designer valuable information about the best bracing system and the locations for column size transitions.

Although the examples in this chapter were relatively simple, the theory of the proposed method and the computer program developed can be applied to practical engineering structures.

# CHAPTER 4: THE EXTENDED ESO METHOD FOR MULTI-STOREY BUILDINGS SUBJECT TO OVERALL STIFFNESS CONSTRAINTS

## 4.1 INTRODUCTION

A typical rigid frame building comprises parallel or orthogonally arranged frames consisting of columns and girders (Smith and Coull, 1991). The lateral load resistance of the frame is provided by the bending resistance of the columns, girders and joints. For buildings higher than about 30 stories or having a high slenderness ratio, rigid frame systems are not sufficient because the bending resistance of the column-girder intersection cannot provide enough stiffness for the building. A braced frame is used to reduce the column and girder bending factor by adding truss members such as diagonals between floor systems. A typical braced frame consists of columns and girders, whose primary purpose is to support the gravity loading, and diagonal bracing members that are connected so that the total set of members forms a vertical cantilever truss to resist the horizontal loading.

Diagonal bracing is inherently obstructive to the architectural plan and can pose problems in the organization of internal space and traffic as well as in locating window and floor openings. For this reason it is usually concentrated in vertical panels or bents that are located to cause a minimum of obstruction while satisfying the structural requirements of resisting the shear and the torque on the building

(Smith and Coull, 1991). Several types of bracing are shown in Figure 4.1. Generally, the types of the braced frames that respond to lateral loading by bending of the girders, or of the girders and columns, are laterally less stiff and, therefore, less efficient, weight for weight, than the fully triangulated trusses, which respond with axial member forces only.

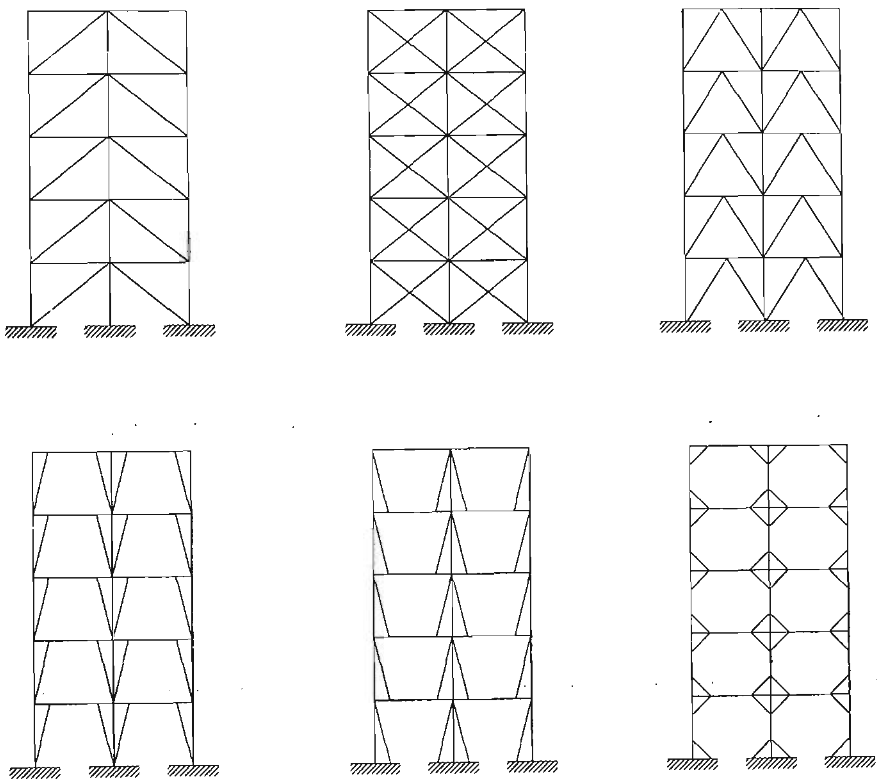


**Fig.4.1** Types of bracing in frame buildings.

During the past few decades, the traditionally storey-height, bay-width bracing systems have been extended to a larger modular scale, both within the building and externally across the faces. Sometimes, the massive diagonals at building faces have been emphasized as an architectural feature of the facades.

The basic theory of the extended ESO method for overall stiffness constraint has been developed in Chapter 3. It was noted that the method can be applied to structures containing both beam and continuum elements. In this chapter, the applicability and effectiveness of the proposed method to multi-storey buildings will be demonstrated. The aim of structural optimization is to find the optimal topology for the bracing system subject to overall stiffness constraints under multiple lateral load cases.

## 4.2 STRUCTURAL OPTIMIZATION PROBLEM

The topological structural optimization problem for the multi-storey buildings in this chapter is to determine the optimal configuration of the bracing system subject to overall stiffness constraints under lateral load cases. It is traditionally assumed that, in braced frames, the columns and girders are designed only for gravitational loads based on strength criteria, whilst the bracing is designed based on the overall stiffness performance of the building under lateral loads.

The structural optimization problem for multi-storey buildings studied in this chapter is illustrated in Figure 4.2.

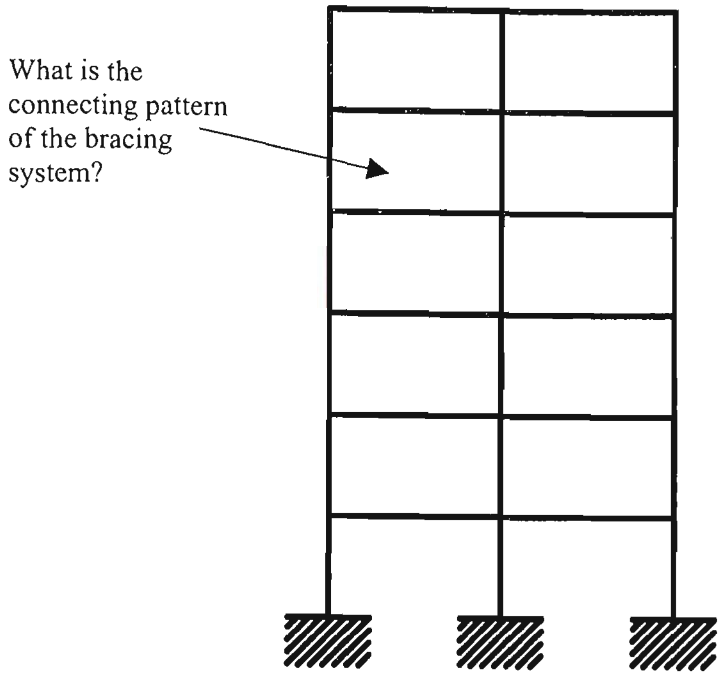



**Fig.4.2** Topology design for multi-storey buildings.

## 4.3 OPTIMIZATION PROCEDURE

The optimization procedure for multi-storey buildings can be divided into two main stages. At the first stage, the unbraced frame is designed to support the gravitational loads based on strength criteria. This task can be carried out on the individual structural component level. After the unbraced frame is designed for gravitational loads, the initial sizes of beams and columns are known. The second stage is to design the braces subject to overall stiffness constraints under lateral load cases. The braces of the frame will be modelled using a mesh of continuum finite elements. During the optimization process, the beams and columns of the frame will be fixed and referred to as the non-design domain. Only continuum elements are removed from the structure to obtain the optimal topology for the bracing system.

Maintaining symmetry of a building is often important for resisting the twisting moments produced by lateral loads. The symmetry of a bracing system can be maintained during the optimization process, if required. At a specific iteration, should the structure become unsymmetrical, the elements removed at the previous iteration will be temporarily fixed and the optimization process will be forced to remove other elements of lowest strain energy density values.

The optimization design procedure for multi-storey buildings is illustrated in Figure 4.3, and is set out below:

**<u>Step 1</u>**: Design the unbraced rigid frame for gravitational load cases based on strength criteria.

**<u>Step 2</u>**: Model the columns and girders of the unbraced frame by using beam elements. Lateral load cases, support conditions and material properties of the beam elements also assigned. These beam elements will not be removed during the optimization process and are referred to as the non-design domain.

**<u>Step 3</u>**: Use a fine mesh of continuum finite elements to model the bracing system. During the optimization process, these continuum elements will be gradually removed to get an optimal topology. The thickness of the continuum elements must be chosen such that their stiffnesses are comparable to that of the beam elements. Otherwise, the analytical solving module will become corrupted. Numerical problem will occur in the form of near-zero energy modes in the global stiffness matrix of the structure.

**Step 4**: Carry out finite element analysis to compute the displacements of the current structure.

**Step 5**: Calculate the strain energy density of each continuum element by using equation (3.10). Calculate the mean compliance of the current structure by adding the strain energy of each beam and continuum element. The mean compliance value is then stored in a database.

**Step 6**: If there is a sharp change in the mean compliance value, temporarily fix the elements removed at the previous iteration. Return the thickness of the continuum elements to the thickness value of the previous iteration. Repeat from Step 4.

**Step 7**: If the number of temporarily fixed elements is greater than or equal to a fixed ratio, release all the temporarily fixed elements. Repeat from Step 4.

**Step 8**: Remove continuum elements which have the lowest strain energy density from the continuum design domain. The number of removed elements is equal to the removal ratio (*RR*) multiplied by the number of elements of the current structure.

**Step 9**: If the current structure becomes unsymmetrical, temporarily fix the elements removed in the previous iteration. Return the thickness of continuum elements to the thickness value of the previous iteration. Repeat from Step 4.

**Step 10**: If there is no sharp change in the mean compliance value at the previous iteration, release all the temporarily fixed elements.

**Step 11**: Uniformly increase the thickness of continuum elements in the design domain by using equation (3.24).

**Step 12**: Save the current structure.

**Step 13**: Repeat from Step 4 to Step 12 until the termination condition is met.

**Step 14**: Plot out the mean compliance history of the optimization process from the saved database and select the optimal topology for the bracing system which corresponds to the lowest mean compliance.

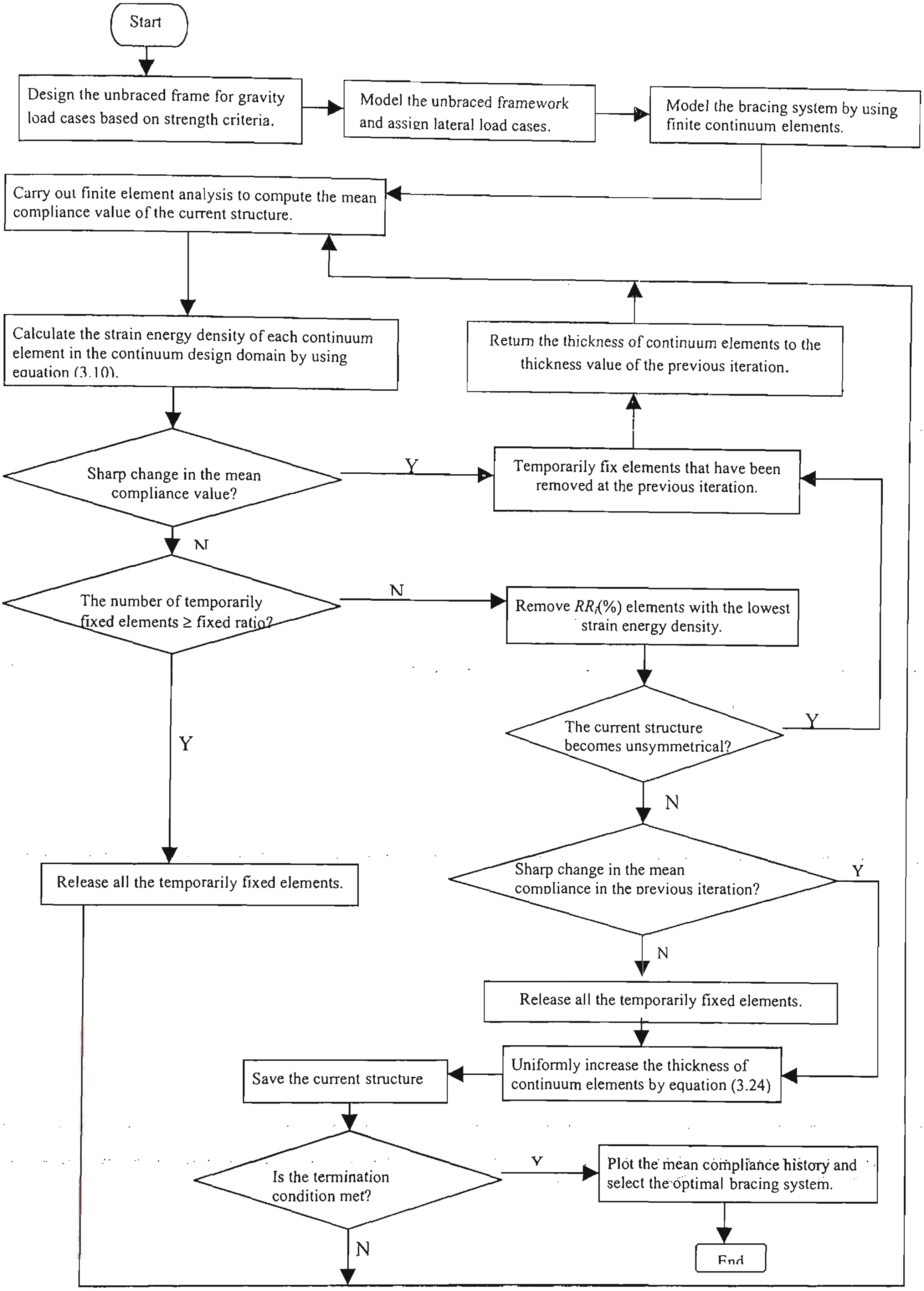



**Fig.4.3** Procedure for topological optimization subject to overall stiffness constraint of multi-storey buildings.

## 4.4 EXAMPLE

A three-bay twelve-storey steel frame shown in Figure 4.4 is to be designed to resist lateral loads. A similar structure has been investigated by Liang (2000) using standard steel sections subject to the overall stiffness constraint. Because lateral loads are usually reversible, two lateral load cases with the same magnitudes but opposite directions are considered - one from the left and the other from the right. Connections between columns, girders and the diagonal braces are assumed to be rigid. The support connections at points A, B and C are assumed to be fixed. All beams and columns are rigidly connected. The material used has Young's modulus $E$ = 200 GPa, shear modulus $G$ = 7,690 MPa and density $\rho$=7,850 kg/m$^3$. The unbraced multi-storey frame is designed for gravity loads based on structural component strength criteria. The BHP hot rolled standard steel sections are used for the unbraced frame. Section members are tabulated in Table 4.1.

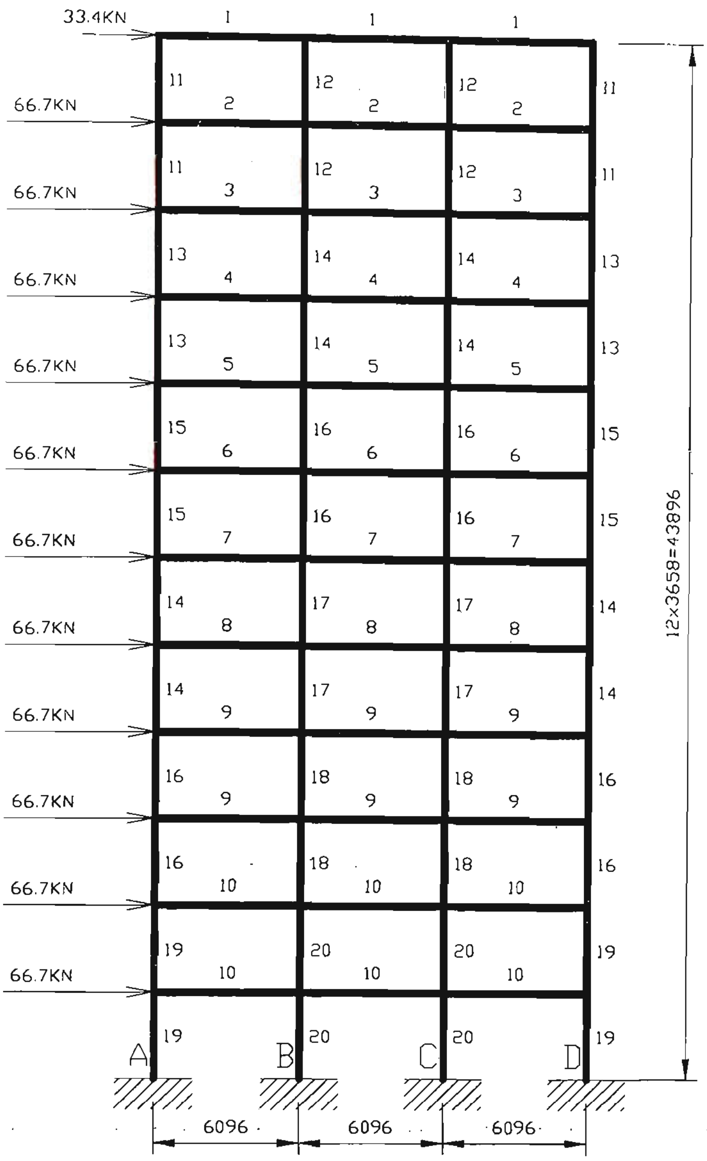


Fig. 4.4 Three-bay twelve-storey steel frame

**Table 4.1** Member section for three-bay twelve-storey steel frame

| Section number | Section |
|---|---|
| 1 | 150 UB 18.0 |
| 2 | 180 UB 18.1 |
| 3 | 200 UB 29.8 |
| 4 | 250 UB27.3 |
| 5 | 310 UB 40.4 |
| 6 | 360 UB 50.7 |
| 7 | 360 UB 56.7 |
| 8 | 410 UB 53.7 |
| 9 | 460 UB 67.1 |
| 10 | 460 UB 74.6 |
| 11 | 150 UC 23.7 |
| 12 | 150 UC 37.2 |
| 13 | 200 UC 46.2 |
| 14 | 200 UC 59.5 |
| 15 | 200 UC 52.2 |
| 16 | 250 UC 72.9 |
| 17 | 250 UC 89.5 |
| 18 | 310 UC 96.8 |
| 19 | 310 UC 118 |
| 20 | 310 UC 137 |

The bracing system of the frame is modelled using a mesh of 108x48 four-node plane stress finite elements. The removal ratio *RR*= 1% based on the current structure is used. The finite element analysis input and optimization parameters are listed in Table 4.2.

**Table 4.2** Finite element analysis input and optimization parameters of three-bay twelve-storey steel frame building

| Finite element analysis input | Optimization parameters |
|---|---|
| • Continuum design domain: 108x48 four-node plane stress continuum elements<br>• Load:<br>Load case 1: lateral load case from the left.<br>Load case 2: lateral load case from the right.<br>• Modulus of elasticity: $E$=200 GPa<br>• Poisson's ratio: $\nu$=0.3<br>• Initial plate thickness: $t$= 25 mm<br>• Static 2D analysis. | • Removal ratio: $RR$=1% of the total number of continuum elements of the current structure.<br>• Topological structural optimization (cavities allowed).<br>• Maximum number of temporarily fixed elements, fixed ratio=20%.<br>• Difference in the change of the mean compliance that will be considered as sharp change, sharp change ratio =5% between two adjacent iterations. |

The history of the mean compliance of the continuum elements of the three-bay twelve-storey steel frame is shown in Figure 4.5. When a small number of continuum elements with the lowest strain energy density were removed from the continuum design domain, the mean compliance of the continuum elements decreased to the minimum value of 207391.5. After the minimum at iteration 93, the mean compliance gradually increased until iteration 240. After that, there was no further change in the topology of the structure. The minimum value of the mean compliance of the continuum elements was 207391.5, which occurred at iteration 93. Theoretically, the optimal topology of the bracing system could be found by monitoring the total mean compliance of the structure (beam and continuum elements). But at this stage the author does not have access to the source code for the beam elements. Therefore, the lowest point in Figure 4.5 may not correspond to the optimal topology, as the stiffness contributions from the beam elements were not included.

Various topologies of the bracing system are given in Figure 4.6. It is observed that the discrete-like result of the braces gradually emerges from upper levels to lower levels. The history of continuum element thickness is illustrated in Figure 4.8. An interpreted structure of the topology at iteration 93 is given in Figure 4.7.

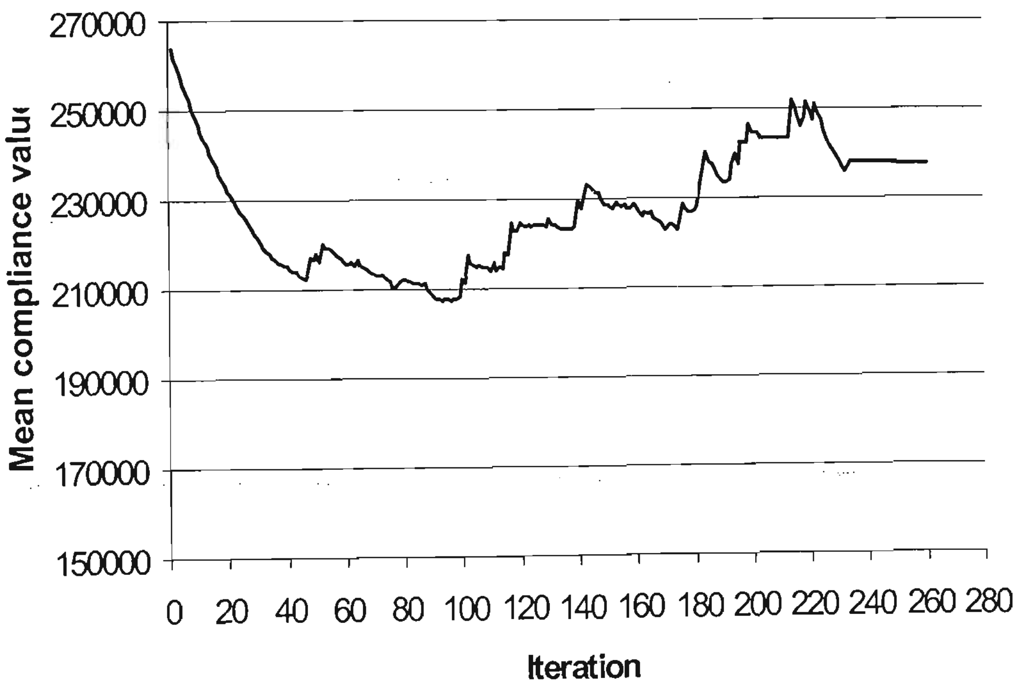
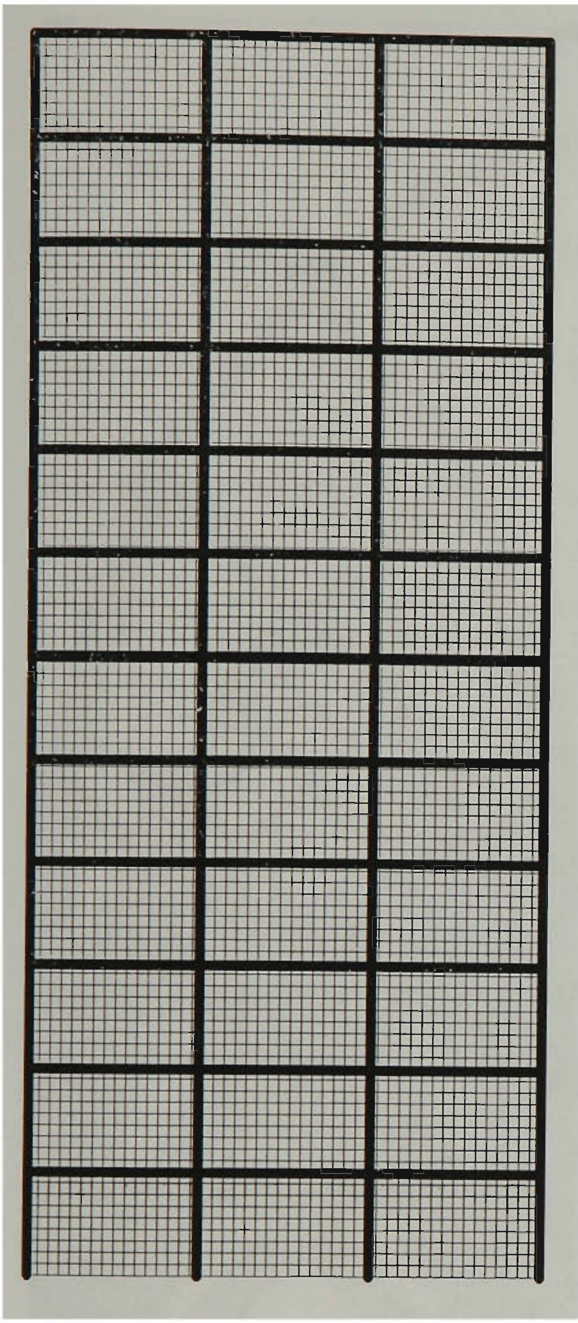
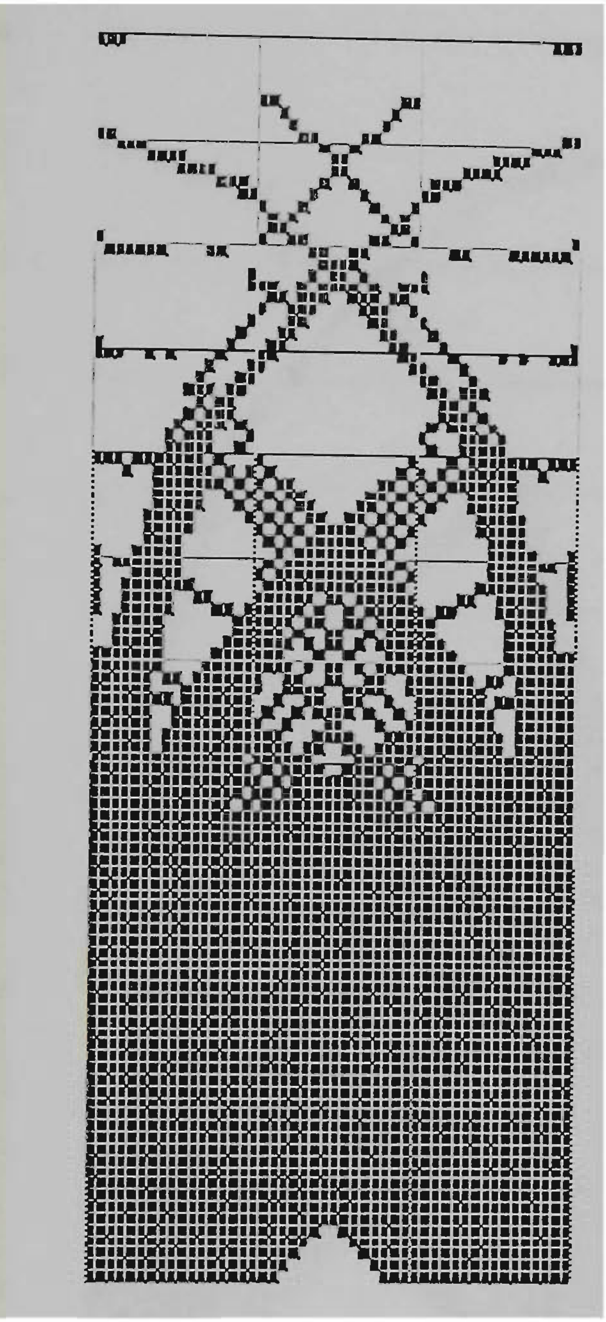



**Fig.4.5** Mean compliance value history for plane stress steel frame

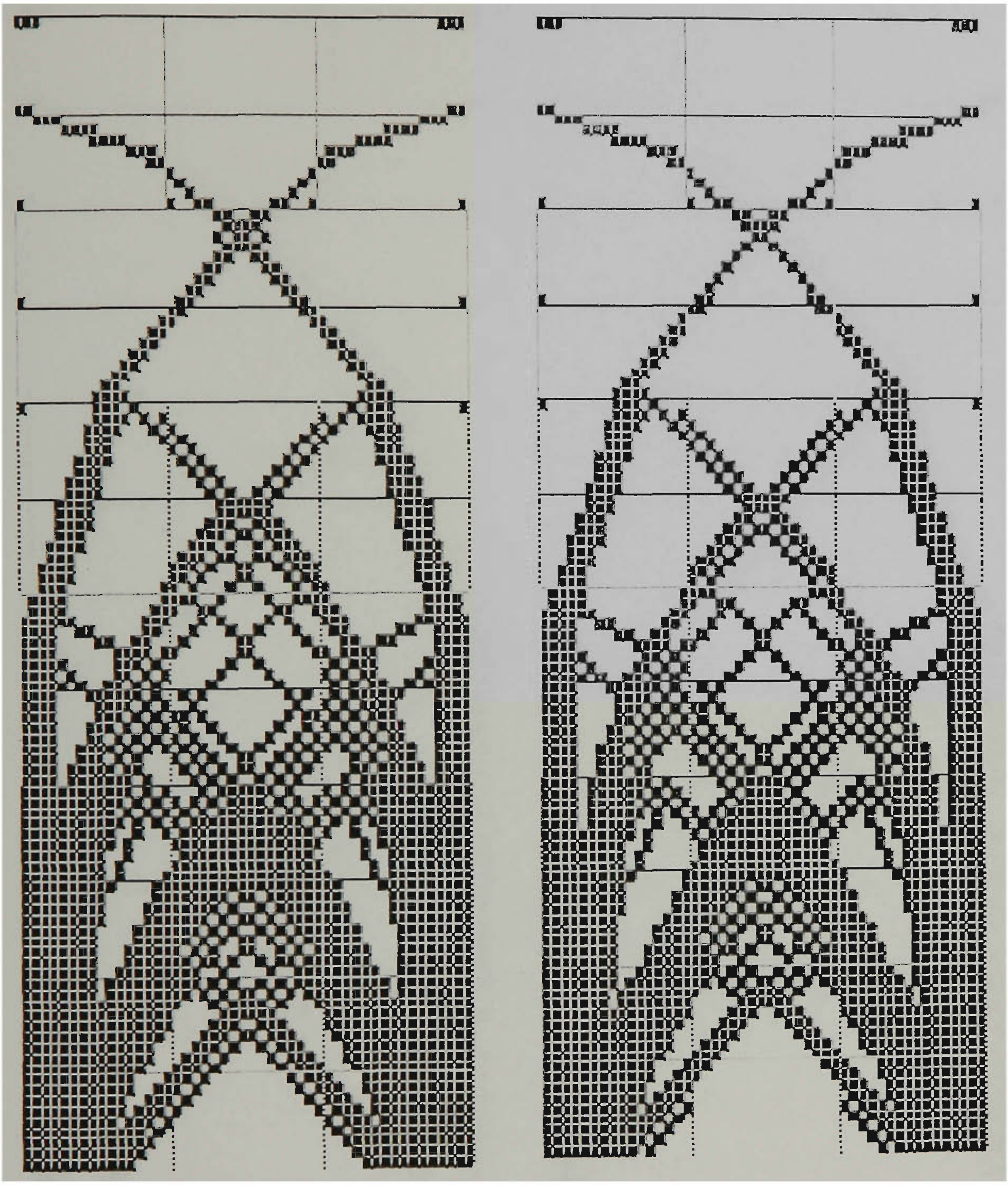


(a) Initial structure



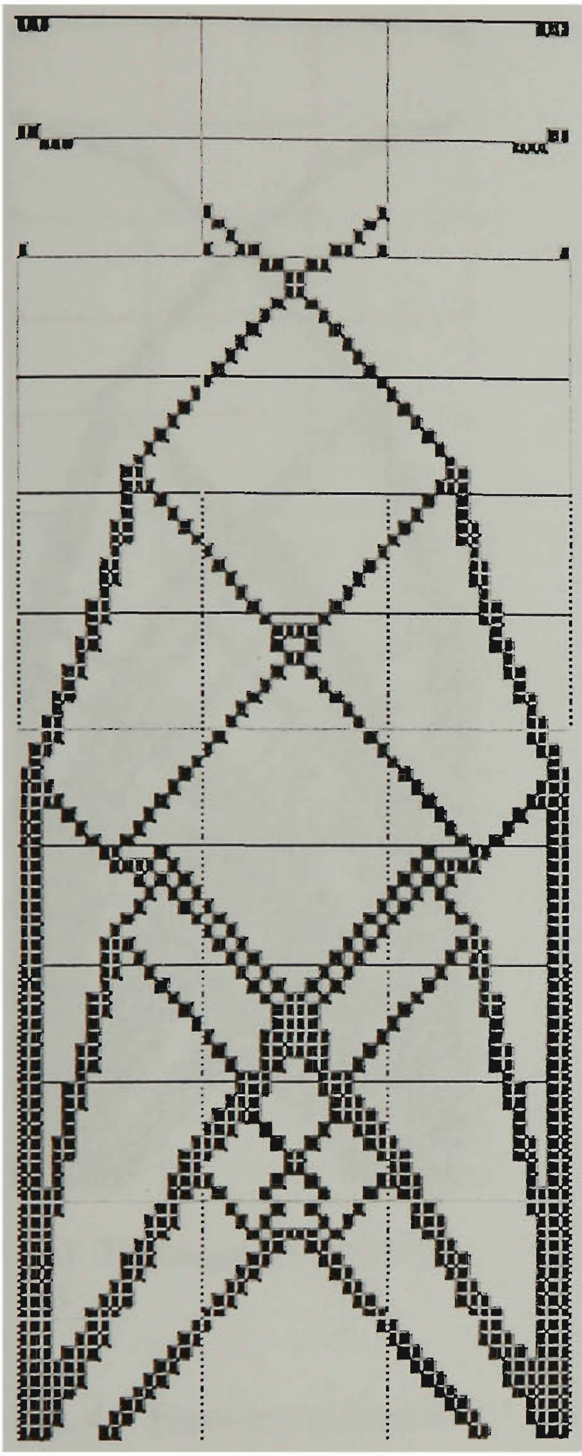
(b) Topology at iteration 50

**Fig. 4.6** Topology history of three-bay twelve-storey plane stress steel frame optimization subject to overall stiffness constraint (Continued)

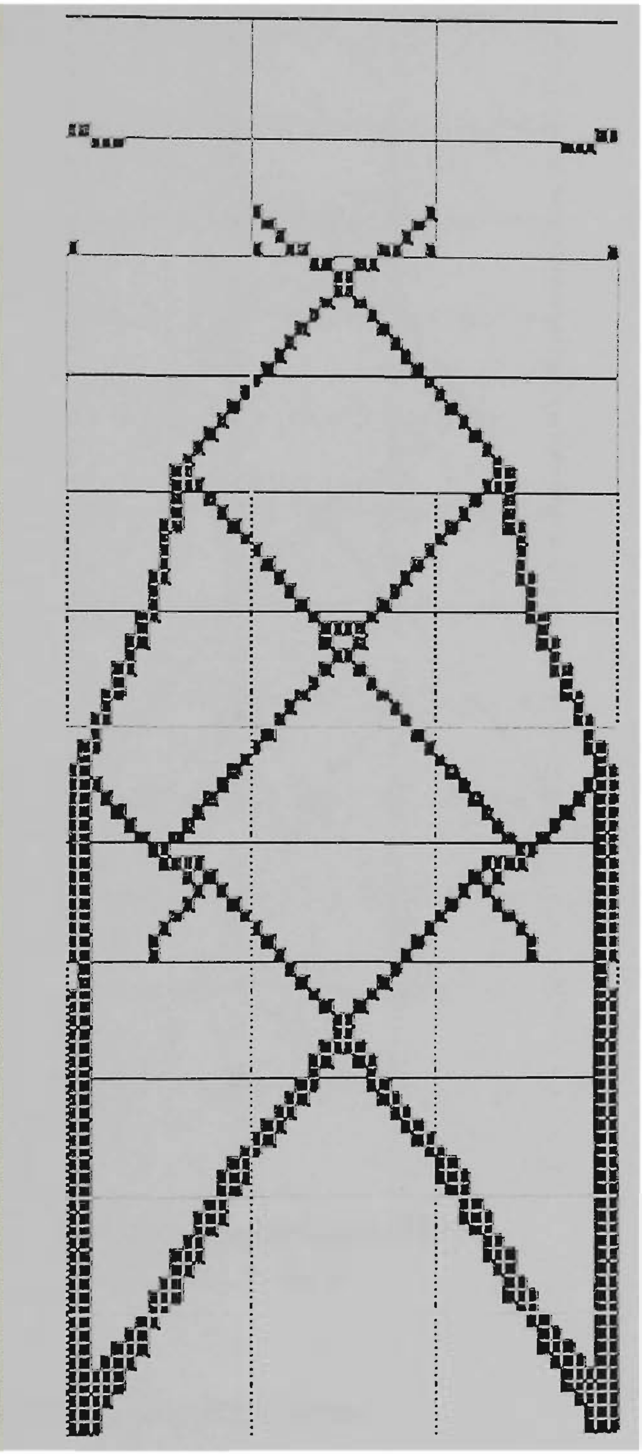


(c) Topology at iteration 93 (d) Topology at iteration 100

**Fig. 4.6** Topology history of three-bay twelve-storey plane stress steel frame optimization subject to overall stiffness constraint (Continued)

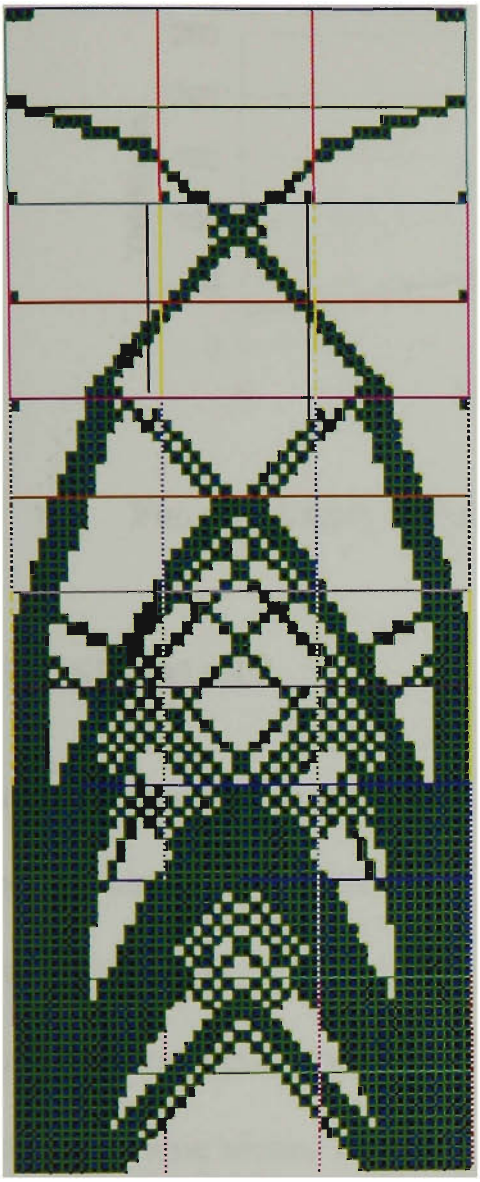


(e) Topology at iteration 180

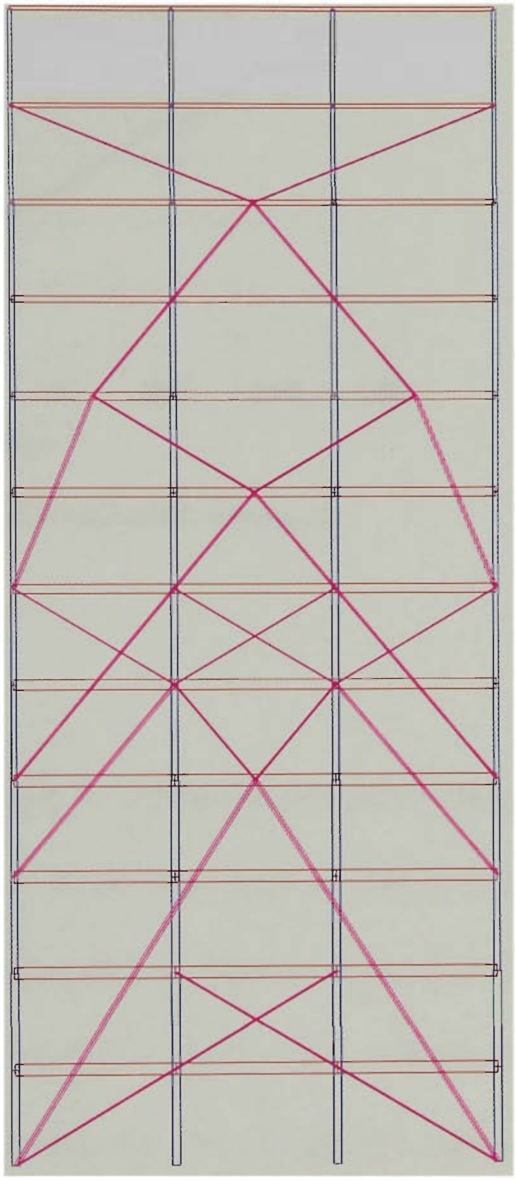


(f) Topology at iteration 240

**Fig. 4.6** Topology history of three-bay twelve-storey plane stress steel frame optimization subject to overall stiffness constraint



(a) Topology at iteration 93



(b) Interpreted structure of Figure 4.7 (a)

**Fig. 4.7** Topology at iteration 93 and its interpreted structure.

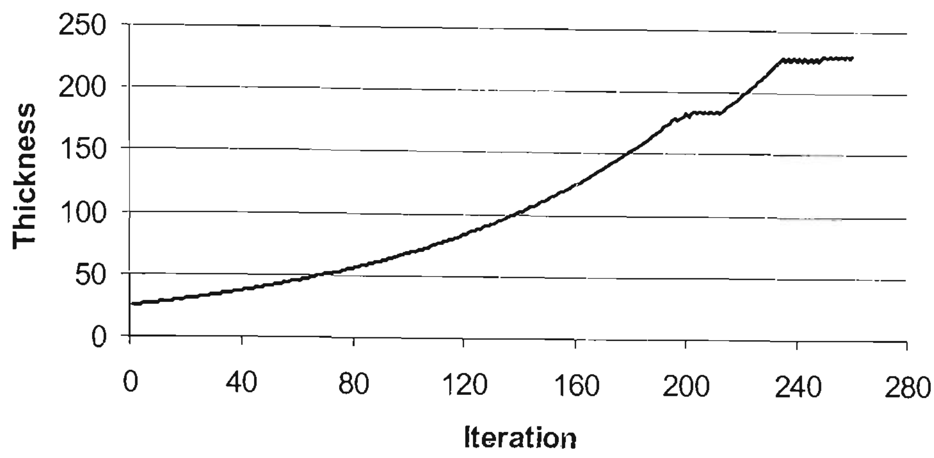



**Fig. 4.8** History of the thickness of continuum elements

## 4.5 SUMMARY

The application of the extended ESO method to a multi-storey steel frame has been presented in this chapter. The purpose of the optimization was to find the optimal topology of the bracing system subject to the overall stiffness constraint and under multiple lateral load cases. Gravitational loads were resisted by the unbraced frame alone. Therefore, the unbraced frame was designed for structural component strength under gravitational loads. The braces was modelled using a mesh of finite continuum elements. Only continuum elements are allowed to be removed during the search for the optimal topology for the bracing system.

By making all the topologies generated during the optimization process have the same weight, their stiffnesses can be compared and the optimal structure can be found. The optimal topology is the one with the lowest mean compliance. Maintaining the symmetry of the building form, and avoiding sharp changes in the mean compliance can be achieved by temporarily fix and release elements during the optimization process.

# CHAPTER 5: THE EXTENDED ESO METHOD FOR DISPLACEMENT CONSTRAINTS

## 5.1 INTRODUCTION

The optimal topology design method for *overall stiffness constraint* and its applicability to building structures have been demonstrated in Chapters 3 and 4. However, in practical engineering problems, it is often required that the maximum *displacement* of a structure or the displacement at a specific location be within a prescribed limit. For example, the maximum lateral deflection of a multi-storey building subjected to lateral loading is usually limited to 1/400 of the building height.

In this chapter, the theoretical basis of the extended ESO method for displacement constraints will be developed. Definitions and concepts presented in Chapter 3 will be adopted and modified for displacement constraints. Firstly, the topological optimization problem will be presented for seeking the optimal topology of the structure subject to multiple displacement constraints. The optimal topology will be the one which has the same weight as the initial structure, but has the lowest value of concerned displacement(s) compared with other topologies generated during the evolutionary process. Secondly, the element removal criteria will be derived from the sensitivity analysis. The virtual strain energy density of each element in the structure under a single load case as well as multiple load cases will also be formulated. The optimization process will be carried out in an iterative manner in which elements with the lowest virtual strain energy density will be gradually removed from the finite element model. The technique of changing the thickness of the continuum elements in the design domain is similar to that presented in Chapter 3. Thirdly, other effects of the proposed method such as sharp change in displacement values and termination criteria will be also investigated. Finally, three examples representing the different types of finite elements will be solved to illustrate the effectiveness of the proposed method at the end of this chapter. It is noted that the technique of handling sharp change in the displacement(s) is similar to that for the mean compliance developed in Chapter 3.

## 5.2 OPTIMIZATION PROBLEM STATEMENT

In the ESO method for displacement constraints, if a displacement constraint is imposed on the $j^{\text{th}}$ displacement component $u_j$, the mathematical expression of the displacement constraint may be given in the form $|u_j| - u_j^* \leq 0$ where $u_j^*$ is the prescribed limit for $|u_j|$. *Starting from an initial structure, the topological structural optimization problem for displacement constraints is to find the structural topology which has the same weight as that of the initial structure but has the lowest value of concerned displacement(s).*

The optimization process is carried out in an iterative manner by removing elements which have the least contributions to the constrained displacement(s) in each iteration. An element removal criterion will be derived in the next section.

## 5.3 ELEMENT REMOVAL CRITERION BASED ON VIRTUAL STRAIN ENERGY DENSITY

For structures under a single load case and subject to multiple displacement constraints, elements with the least effect on the change in the constrained displacements should be eliminated. These elements are underutilized in the design domain compared with other elements. To determine underutilized elements, a sensitivity analysis is undertaken on constrained displacements due to element removal.

From equations (3.1) and (3.4), the change of the nodal displacement vector due to the change of the design variable $x_i$ (the thickness of continuum elements) can be expressed as:

$$\Delta u_i = \frac{\partial u}{\partial x_i}\Delta x_i = \left(-K^{-1}\frac{\partial K}{\partial x_i}u\right)\Delta x_i \tag{5.1}$$

A usual approach to extracting the $j^{\text{th}}$ constrained displacement from the displacement vector is to use a virtual load vector $f_j$, which has all its components equal to zero except the one corresponding to the $j^{th}$ constrained displacement. The non-zero component is given a unit value and the unit virtual load is in the same direction as the $j^{th}$ constrained displacement. Multiply both sides of equation (5.1) by the unit virtual load value $f_j^T$, we have

$$f_j^T\Delta u_i = f_j^T\left(-K^{-1}\frac{\partial K}{\partial x_i}u\right)\Delta x_i = -\left(u_j^T\frac{\partial K}{\partial x_i}u\right)\Delta x_i \tag{5.2}$$

where $u_j$ is the displacement vector due to the unit virtual load $f_j$.

Recall that ESO is a 0-1 decision procedure. Elements will either remain in or be removed from the structure. When an element is removed,

$$\Delta u_{ij} = -\left( u_j^T \frac{\partial K}{\partial x_i} u \right)(0-1) = u_j^T K_i u = u_{ij}^T K_i u_i \qquad (5.3)$$

where $\Delta u_{ij}$ is change of the $j^{th}$ constrained displacement due to the $i^{th}$ element removal, $K_i$ is the stiffness matrix of the $i^{th}$ element, $u_i$ and $u_{ij}$ are the element displacement vectors containing the entries of $u$ and $u_j$, respectively, related to the $i^{th}$ element. Unlike the value $\alpha_i$ in equation (3.9), the value $\Delta u_{ij}$ in equation (5.3) can be either negative or positive which implies that the constrained displacement may change in two directions due to element removal. For a structure under a single load and subject to multiple displacement constraints, the virtual strain energy of the $i^{th}$ element can be defined as:

$$\beta_i = \sum_{j=1}^{m} \left| \Delta u_{ij} \right| \qquad (5.4)$$

where $m$ is the number of displacement constraints.

In the ESO method, the continuum design domain is often divided into a mesh of identical elements, and all the elements have the same volume and weight. Therefore, the virtual strain energy above can be used as the removal criterion for the optimization process. However, in situations where the finite element mesh has elements of different sizes, the virtual strain energy per unit volume of each element should be considered as the removal criterion. The virtual strain energy density of the $i^{th}$ element in the structure is defined as:

$$\gamma_i = \frac{\beta_i}{w_i} \tag{5.5}$$

where $w_i$ is the weight of the $i^{th}$ element.

The virtual strain energy density of the continuum element in equation (5.5) serves as the element removal criterion. To obtain the stiffest structure, it will be most effective to remove the elements which have the lowest virtual strain energy density number $\gamma_i$.

## 5.4 STRUCTURES UNDER MULTIPLE LOAD CASES

A structure is usually subjected to multiple load cases. From equation (5.2), considering the $l^{th}$ load case, the change in the $j^{th}$ constrained displacement due to the $i^{th}$ element removal can be expressed as:

$$\Delta u_{ijl} = u_{ij}^T K_i u_{il} \qquad (l = 1,...,L) \tag{5.6}$$

where $u_{il}$ is the displacement vector of the $i^{th}$ element under the $l^{th}$ load case, $L$ is the total number of load cases acting upon the structure. When a structure is under multiple load cases, the elements which have the least effect on the constrained displacements under all load cases should be considered as underutilized elements and could be removed. The virtual strain energy density of the $i^{th}$ element in structure under $L$ load cases can be simply defined as:

$$\gamma_i = \frac{\sum_{l=1}^{L}\sum_{j=1}^{m}\left|\Delta u_{ijl}\right|}{w_i} \tag{5.7}$$

The virtual strain energy density of each continuum element in the structure can be calculated at the element level after the finite element analysis. The optimization module extracts the displacement vector and the element stiffness matrix from the output data of the finite element package to calculate the virtual strain energy density of each continuum element.

## 5.5 UNIFORMLY CHANGING THE THICKNESS OF CONTINUUM ELEMENTS

By removing elements with the lowest virtual strain energy density at each iteration, a series of topologies will be produced during the optimization process. To compare the performance of these topologies, it is convenient to make their weight equal so that their performance can be compared to each other by using the values of constrained displacement(s). Therefore, after removing elements with the lowest virtual strain energy density from the structure, the thickness of the remaining continuum elements will be uniformly increased by using equation (3.24) in Chapter 3 so that the weight of the current structure will be the same as that of the initial structure.

## 5.6 TERMINATION CONDITIONS

As the optimization module runs in an iterative manner, underutilized elements are gradually removed from the structure. The material of the structure is redistributed so that its performance is improved. The constrained displacements of the structure will decrease during the optimization process. The termination condition of the optimization process needs to ensure that the optimal topology is achieved. In other words, the optimal topology must be among the topologies generated in the evolutionary path. Therefore, for topological structural optimization subject to multiple displacements under multiple load cases, the optimization process will be terminated if there is no further decrease in the constrained displacements of the equally weighted topologies. When programmed into computer code, the optimization process will be terminated if there is no decrease in the constrained displacements for over 20 consecutive iterations.

## 5.7 HANDLING SHARP CHANGES IN THE CONSTRAINED DISPLACEMENT(S)

Like the topology optimization for overall stiffness constraint in Chapter 3, the temporarily fixed elements can only be released when one of the conditions below is met:

- **<u>The number of temporarily fixed elements reaches a fixed ratio</u>**, which is defined by the ratio of the number of temporarily fixed elements to the number of total elements of the current structure.

- **<u>After a successful iteration involving element removal</u>**. A successful iteration involving element removal is defined as an iteration in which elements are removed when the sharp change in the constrained displacements does not occur in the previous iteration.

The technique of releasing the temporarily fixed elements for topological optimization subject to displacement constraints is given in Figures 5.1 and 5.2 below.

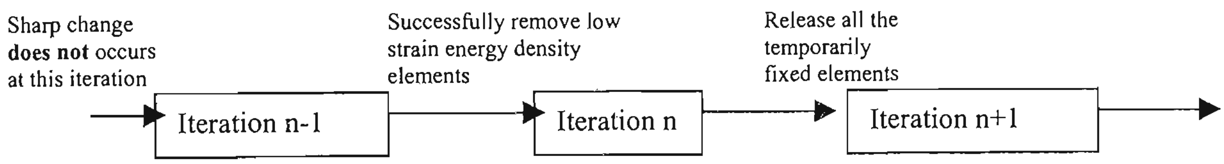



**Fig.5.2** Release temporarily fixed elements after a successful iteration involving element removal

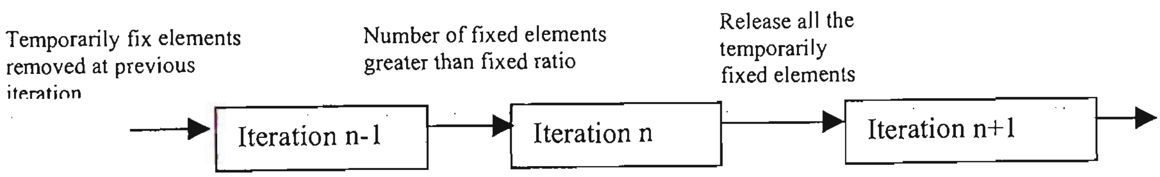



**Fig.5.1** Release all temporarily fixed elements when its number greater than fixed ratio.

## 5.8 DESIGN PROCEDURE

The design procedure for topological structural optimization subject to displacement constraints is illustrated in Figure 5.3 and explained as follows.

Step 1: Discretize the structure using finite elements. Discrete elements in the structure such as columns, beams, girders etc. are modelled using beam elements and considered as the non-design domain. Continuum elements are used for the design domain. The boundary conditions and applied loads are applied to the model. This model is the initial design.

Step 2: Apply the virtual unit loads on the constrained displacement locations in the structure.

Step 3: Carry out the finite element analysis to compute the constrained displacements and save the constrained displacements of the current structure in a database.

Step 4: Calculate the virtual strain energy density of each continuum element in the design domain using equation (5.7).

Step 5: If there is a sharp change in the constrained displacements, temporarily fix the elements removed at the previous iteration. Return the thickness of continuum elements to the thickness value of the previous iteration. Repeat from Step 3.

Step 6: If the number of temporarily fixed elements is greater than or equal to a prescribed fixed ratio, release all the temporarily fixed elements. Repeat from Step 3.

**Step 7:** Remove elements which have the lowest virtual strain energy density from the structure. The number of removed elements is equal to the removal ratio (*RR*) multiplied by the number of elements of the current structure.

**Step 8:** If there is no sharp change in the constrained displacements at the previous iteration, release all the temporarily fixed elements.

**Step 9:** Uniformly increase the thickness of continuum elements in the design domain by using equation (3.24).

**Step 10:** Save the current structure.

**Step 11:** Repeat from Step 3 to Step 10 until the termination condition in Section 5.6 is met.

**Step 12:** Plot the constrained displacement history of the optimization process from the saved database and select the optimal topology.

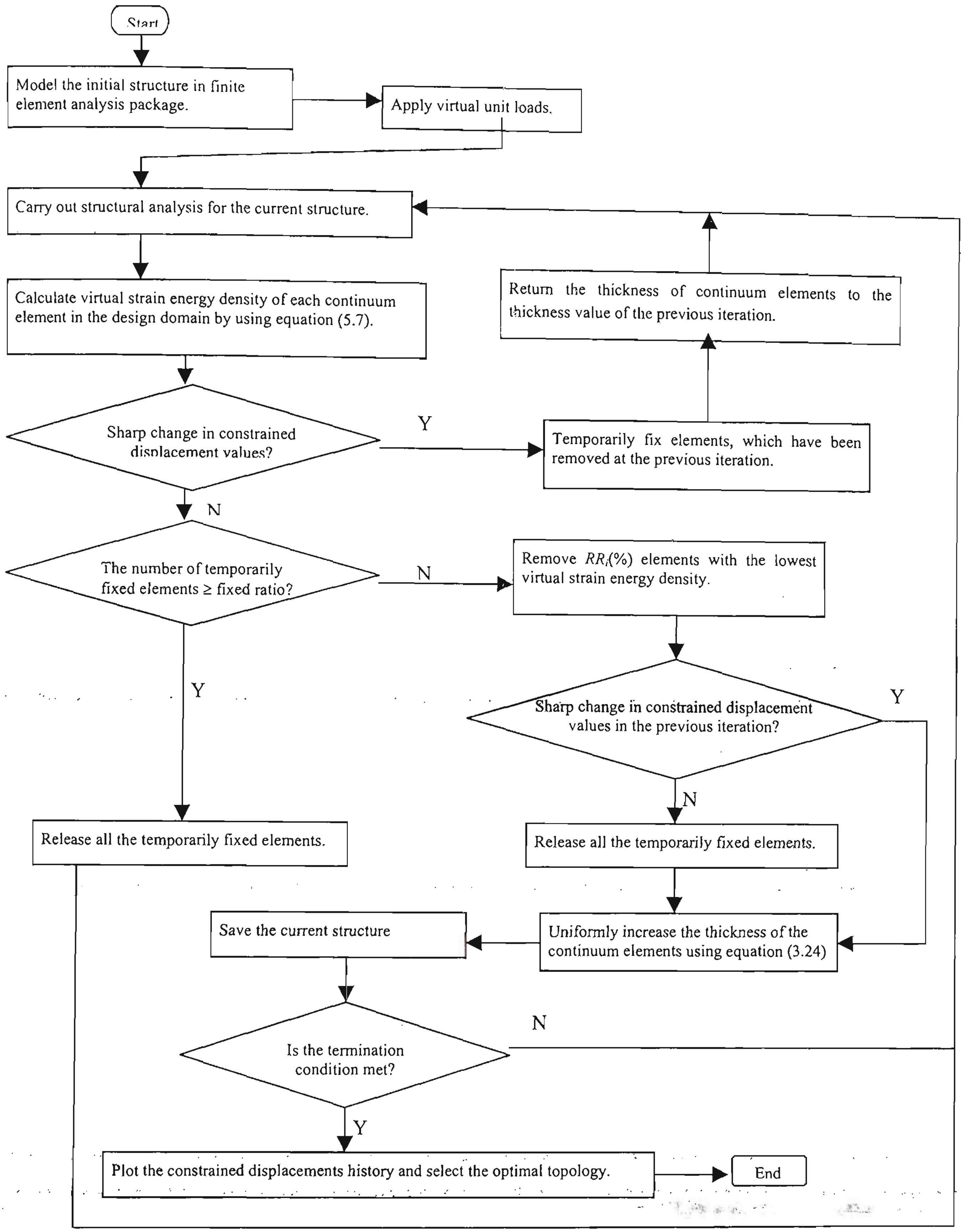



**Fig.5.3** Procedure of topological optimization for displacement constraints

## 5.9 EXAMPLES

In this section, three simple structures representing different types of finite elements are presented to demonstrate the proposed method. Firstly, a simply supported deep beam with three point loads is considered to find the optimal topology subject to multiple displacement constraints, which are imposed on the loaded points. The design domain is modelled by a fine mesh of continuum plane stress elements. Secondly, the validity of the proposed method is examined by solving a plate in bending problem, which has been solved for overall stiffness constraints in Chapter 3. The constrained displacement is imposed on the loaded point, at the centre of the plate. Finally, the effectiveness of the proposed method when applied to a 3D structure is demonstrated by solving a 3D structure. The 3D structure is subjected to multiple displacement constraints and multiple load cases.

For each example, the finite element package STRAND6™ is used to carry out the structural analysis. An optimization module, which serves as a post-processor of STRAND6™, is executed to remove elements with the lowest virtual strain energy density as described in the previous sections.

### 5.9.1 A PLANE STRESS STRUCTURE

This simply supported deep beam is designed to carry three concentrated point loads, each of 10kN, under the given boundary conditions shown in Figure 5.4. The displacement constraints are imposed on the loaded points in the vertical direction, and their limits are set to large values to ensure the optimum is included in the optimization process. The design domain is modelled using a mesh of 30x80 four-node plane stress elements. The material used has a Young's modulus of 200GPa, Poison's Ratio=0.3 and the initial thickness of the elements $t$=10mm. The removal ratio $RR$=1% based on the current structure is used in the optimization process.

The finite element analysis input and optimization parameters are listed in Table 5.1.

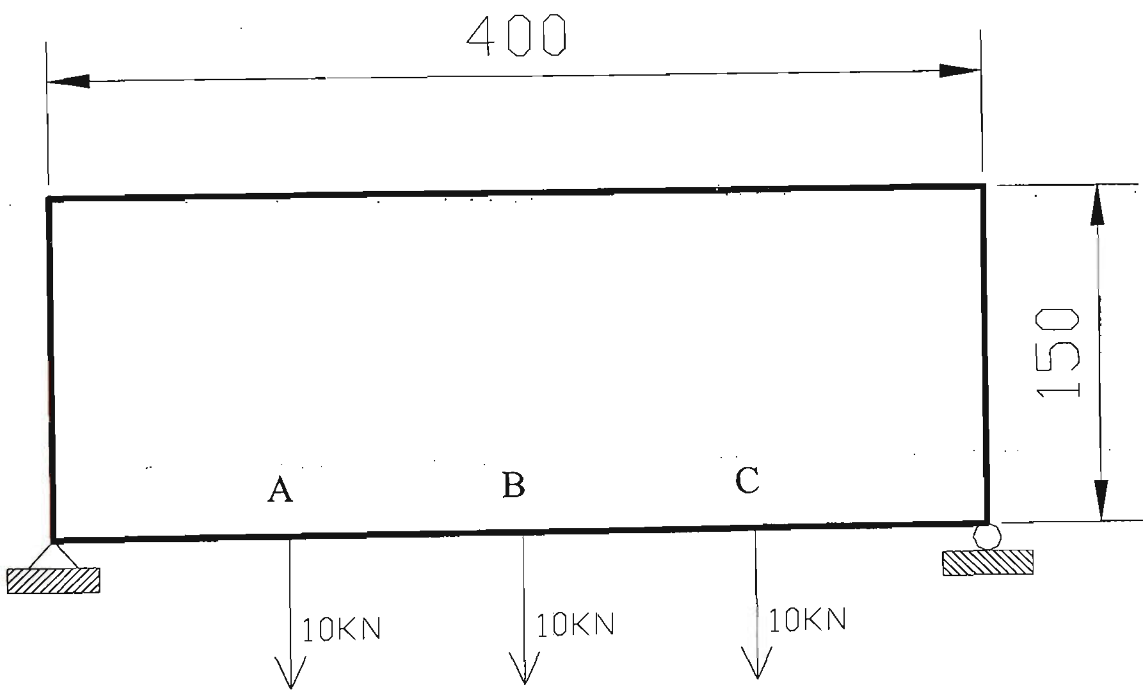



**Fig.5.4** Simply supported deep beam.

**Table 5.1** Input values for 2D plane stress deep beam optimization subject to displacement constraints

| Finite element analysis input | Optimization parameters |
|---|---|
| • Length of the beam 400 mm<br>• Depth of the beam 150 mm<br>• Continuum design domain: 30x80 four-node plane stress elements.<br>• Load: $P_1=P_2=P_3=$ 10KN<br>• Modulus of elasticity: $E$=200GPa<br>• Poisson's ratio: $\nu$=0.3<br>• Initial plate thickness: $t$=10mm<br>• Static elastic analysis. | • Removal ratio: $RR$=1% of the total number of continuum elements of the current structure.<br>• Topological structural optimization (cavities allowed).<br>• Maximum number of temporarily fixed elements, fixed ratio=20%.<br>• Limit of the constrained displacements: $u^*$ = 1000mm<br>• Difference in the change of constrained displacements that will be considered as sharp change, sharp change ratio=5% between two adjacent iterations. |

Figure 5.5 shows the history of constrained displacements. The negative numbers of constrained displacements mean that the direction of the displacements is downward. Due to symmetry, the displacements at points A and C are the same. By making all the topologies, which are generated during the optimization process, have the same weight, the deflections of the beam at three loaded points gradually decreases. The minimum deflections of point A and B are 0.0886mm and 0.1065mm respectively. The optimal topology corresponding to the minimum deflection occurs at iteration 73 for both of point A and B.

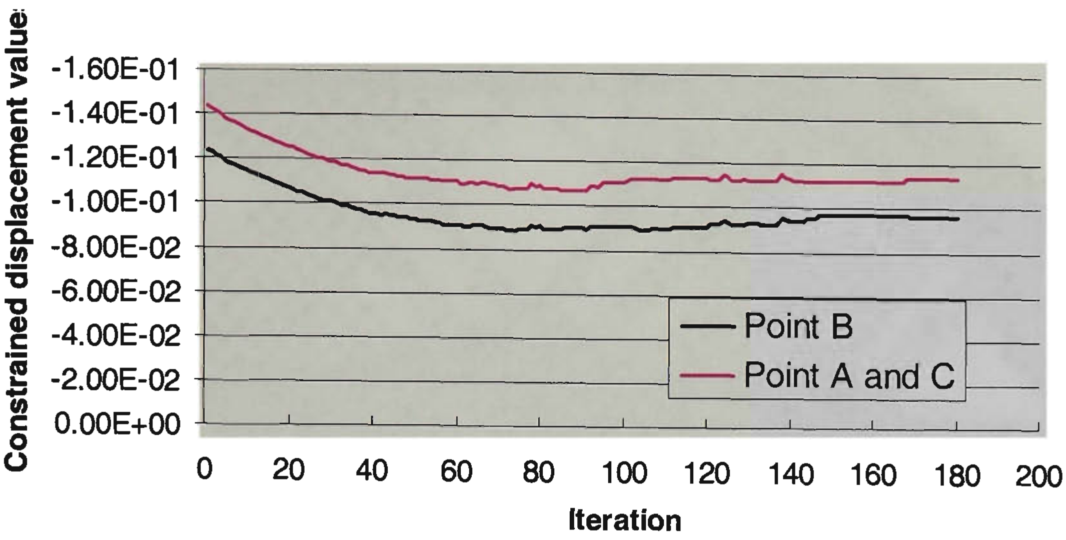
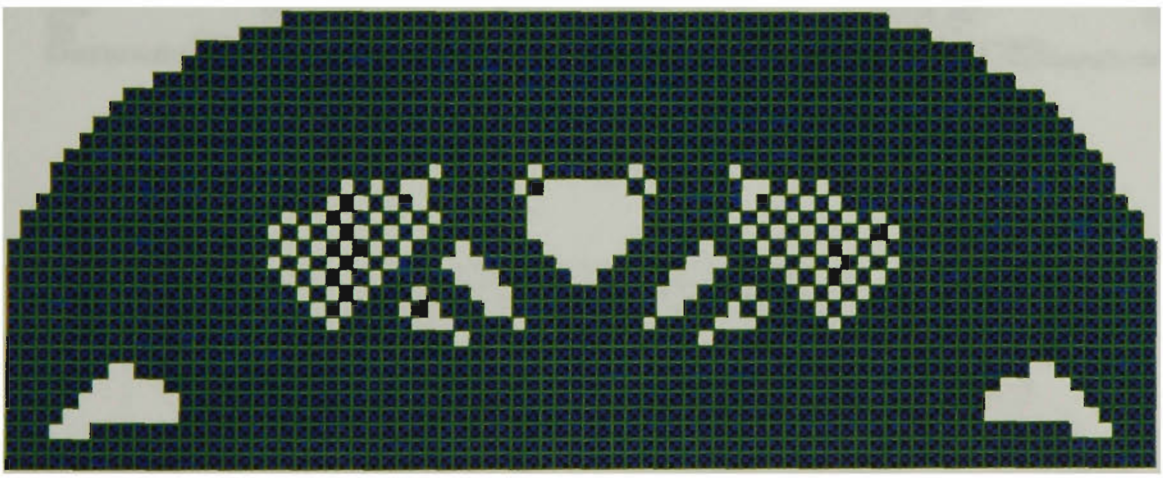


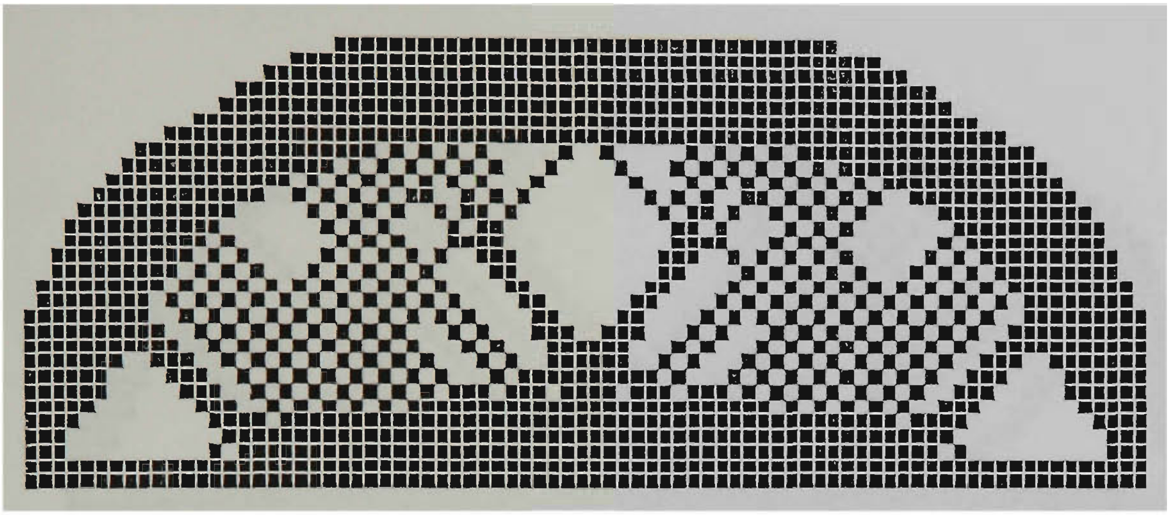

**Fig.5.5** History of constrained displacements

The topological optimization history of the simply supported deep beam subject to multiple displacement constraints is shown in Figure 5.6.

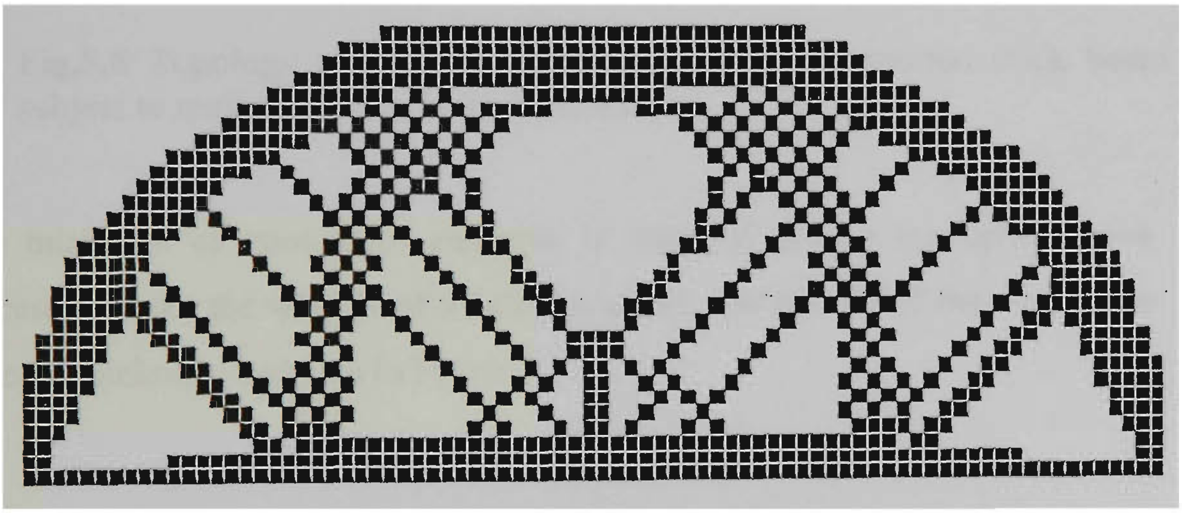


(a) Topology at iteration 20

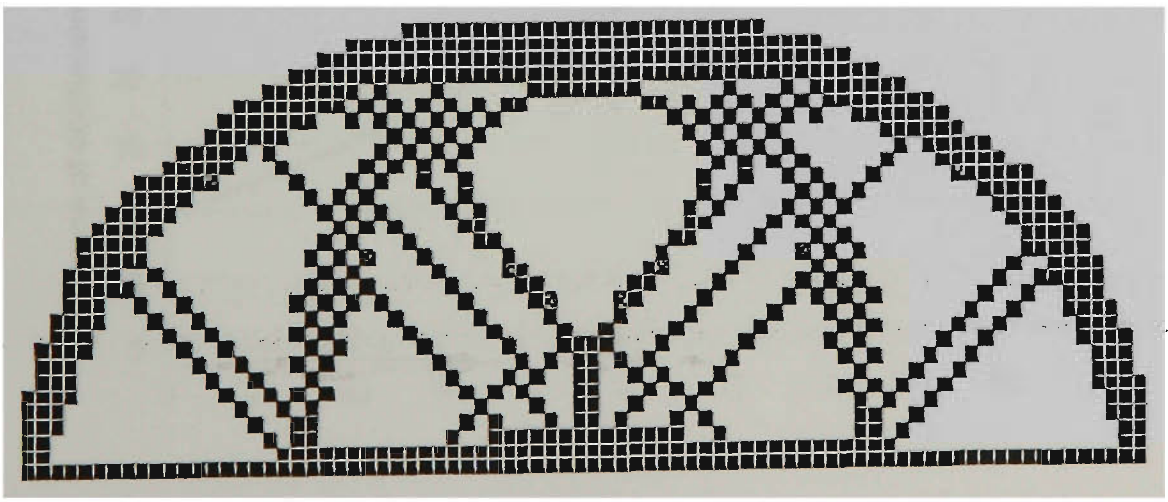
**Fig.5.6** Topology optimization history of simply supported thick beam subject to multiple displacement constraints (Continued)



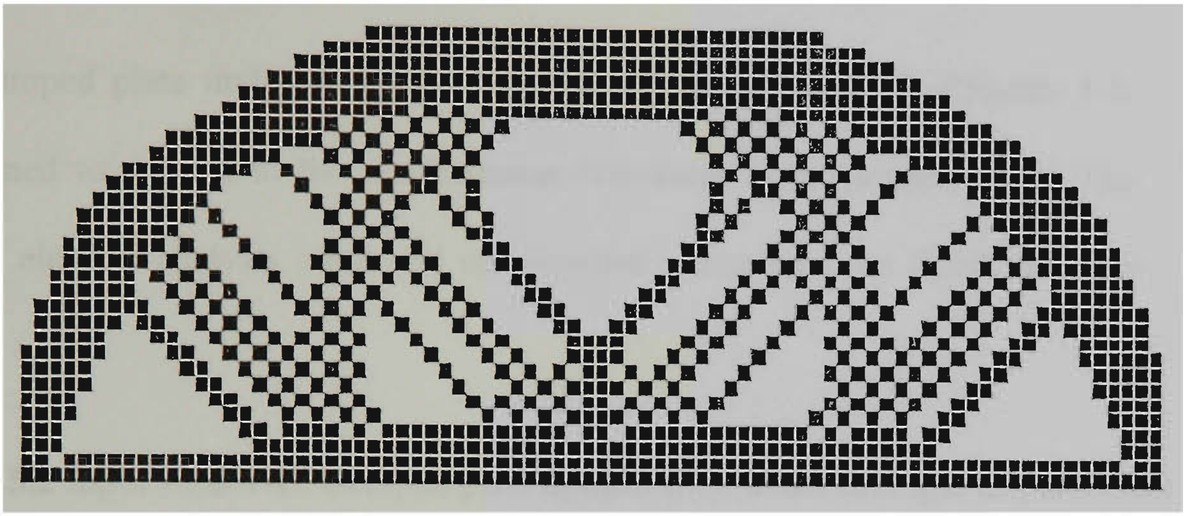
(b) Topology at iteration 50



(c) Topology at iteration 100



(d) Topology at iteration 150



(e) Optimal topology at iteration 73

**Fig.5.6** Topology optimization history of simply supported thick beam subject to multiple displacement constraints

The thickness of continuum elements is changed during the optimization process to make the weights of topologies equal. The history of the continuum element thickness is shown in Figure 5.7.

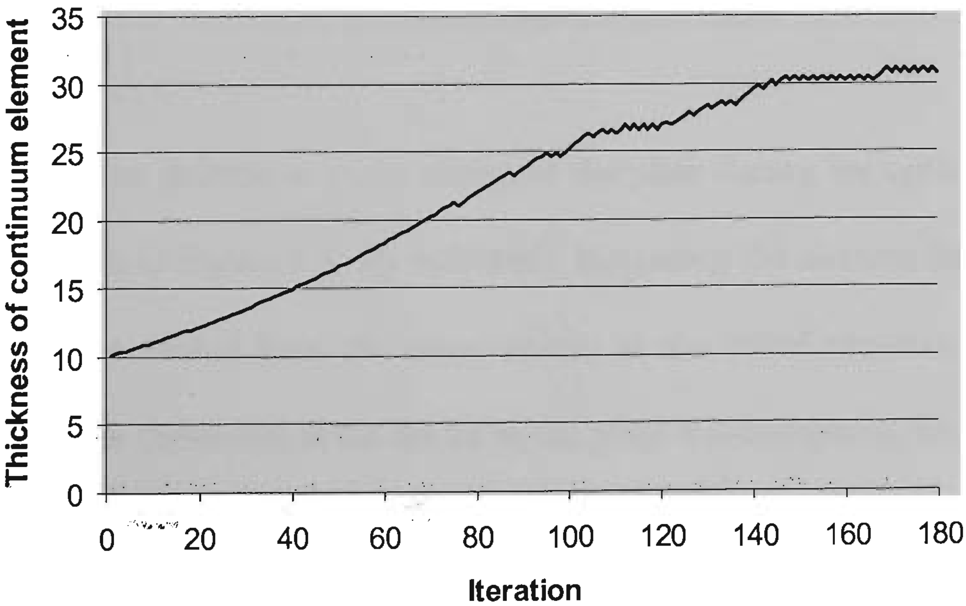



**Fig.5.7** The history of the continuum element thickness of the simply supported deep beam.

### 5.9.2 A PLATE IN BENDING

A clamped plate under a concentrated loading at the centre in Chapter 3 is designed to subject to the displacement constraint at the loaded point. The finite element analysis input and optimization parameters are listed in Table 5.2.

**Table 5.2** Input values for clamped plate optimization under multiple displacement constraints

| Finite element analysis input | Optimization parameters |
|---|---|
| • Plate side length 500 mm<br>• Continuum design domain: 50x50 mesh of plate elements.<br>• Load: $P = 100$ N<br>• Modulus of elasticity: $E$=200 GPa<br>• Poisson's ratio: $\nu$=0.3<br>• Plate thickness: $t$=1mm<br>• Static elastic analysis. | • Removal ratio: $RR$=1% of the total number of continuum elements of the current structure.<br>• Topological structural optimization (cavities allowed).<br>• Maximum number of temporarily fixed elements, fixed ratio=20%.<br>• Difference in the change of constrained displacement values that will be considered as sharp change, sharp change ratio=5% between two adjacent iterations. |

The history of the deflection at the centre of the plate during the optimization process is shown in Figure 5.8. By uniformly increasing the element thickness, all topologies generated have the same weight as the initial structure. It was observed that the deflection at the centre of the plate was decreasing throughout the optimization process. The straight line observed at the end of the optimization process indicates that there is no further improvement in the performance of the structure during those iterations.

Figure 5.9 shows the topology optimization history of the clamped plate. The optimal topology is obtained at iteration 180, which has the minimum value of deflection of 0.252mm (see Figure 5.10).

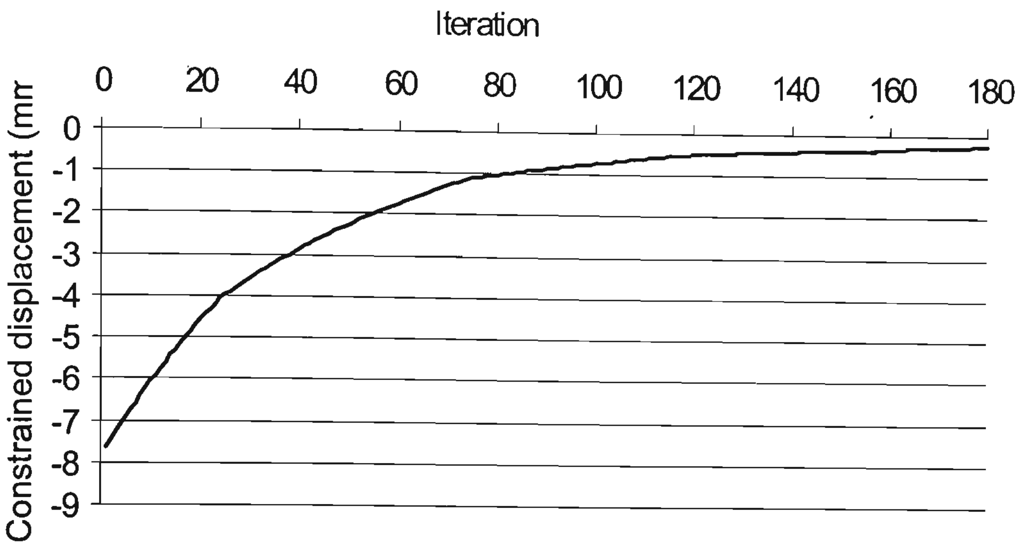
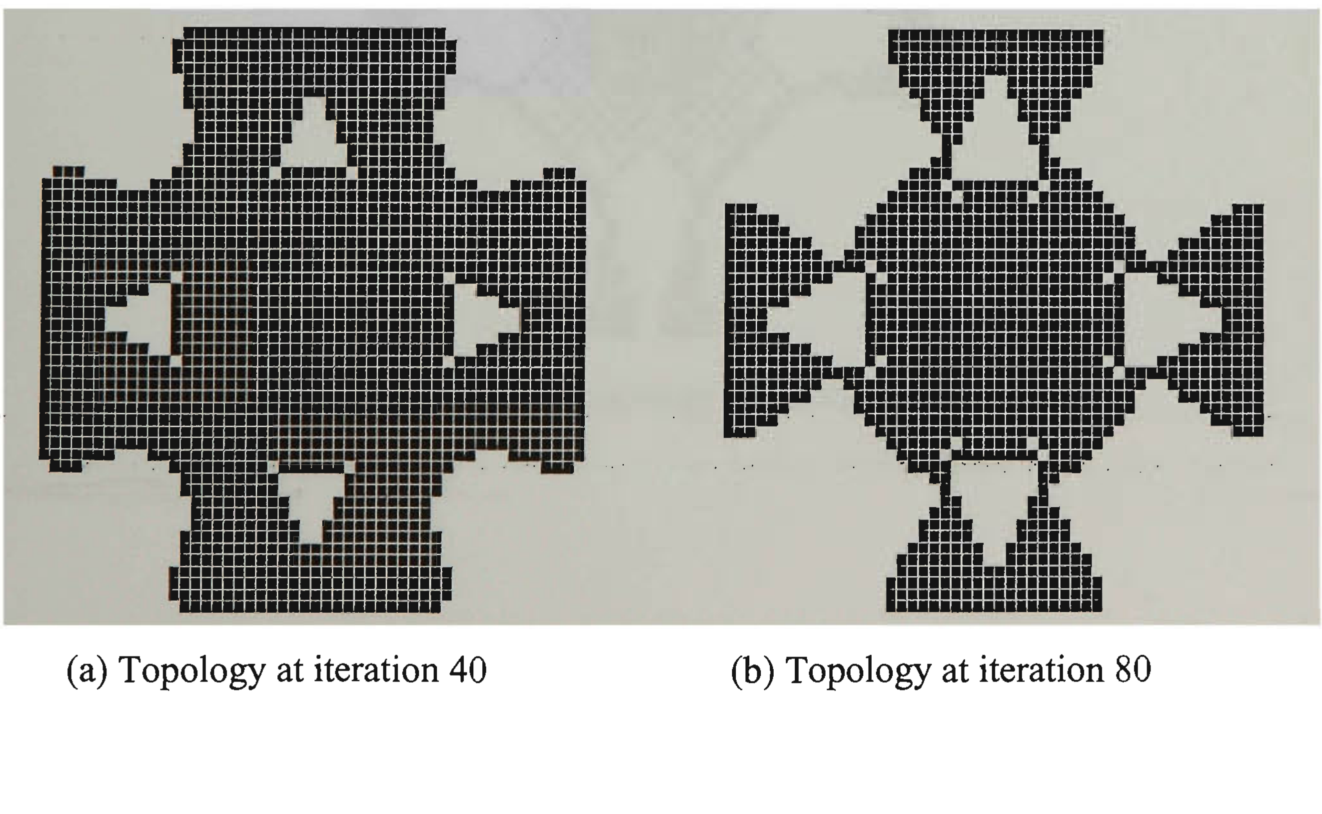
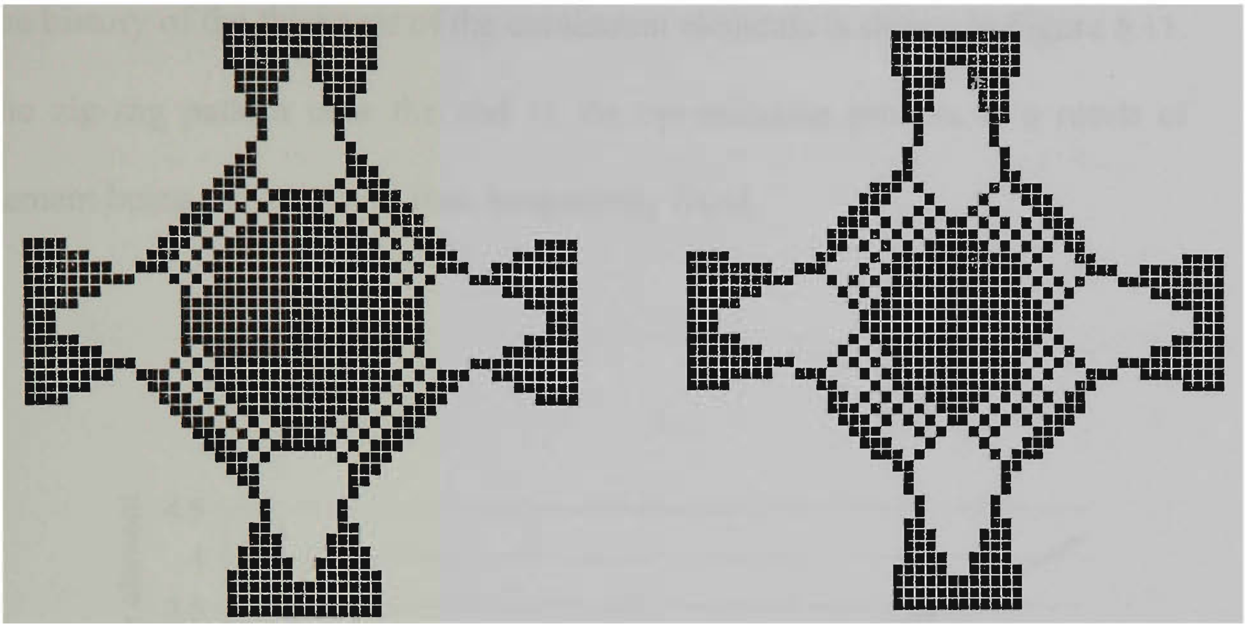



**Fig.5.8** Constrained displacement history of the clamped plate under concentrated loading subject to displacement constraint at loaded point



(a) Topology at iteration 40

(b) Topology at iteration 80



(c) Topology at iteration 120 (d) Topology at iteration 160

**Fig.5.9** Topology optimization history of the clamped plate under a concentrated load

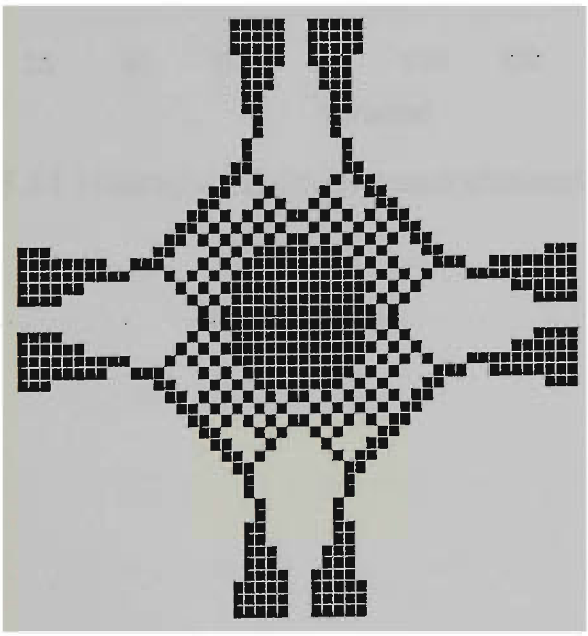


**Fig.5.10** Optimal topology (iteration 180)

The history of the thickness of the continuum elements is shown in Figure 5.11. The zig-zag pattern near the end of the optimization process is a result of element being removed and then temporarily fixed.

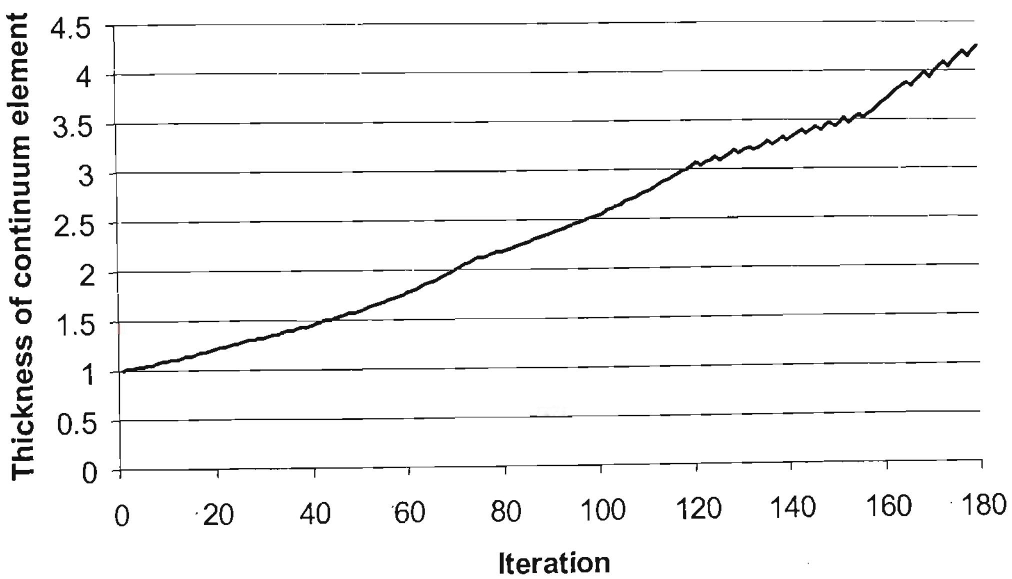



**Fig.5.11** History of the continuum element thickness

### 5.9.3 A 3D STRUCTURE

This example examines the capability of the proposed topological optimization method when dealing with a structure containing both discrete (beams) and continuum elements (plates). A 3D structure shown in Figure 3.15 in Chapter 3 is designed to resist two orthogonal lateral load cases in x and y directions. The optimization problem is to find the optimal topology of the bracing system in the facades along x and y directions subject to displacement constraints at the top of the frame. The four columns and four beams of the frame modelled by beam elements, and will not be removed during the optimization process. They form the non-design domain. The four faces of the frame are modelled using four-node plate elements. Only continuum elements are to be removed during the optimization process. They form the design domain of the optimization problem. The top deflection limit of the frame is set to a high value in order to ensure the optimal topology is included in the evolutionary procedure.

The history of the constrained displacements in x and y direction is shown in Figure 5.12. It is seen that while the displacement of the top of the frame along x-direction reduces gradually, the displacement along y-direction increases during the optimization process. It is clear that the optimization process tends to reduce the difference between the displacements along x and y direction. The optimum topology, which has the lowest difference between displacements in x and y directions, occurs at iteration 180. It is noted that there is no further improvement in the results since iteration 170, which is represented by a straight line in Figure 5.12.

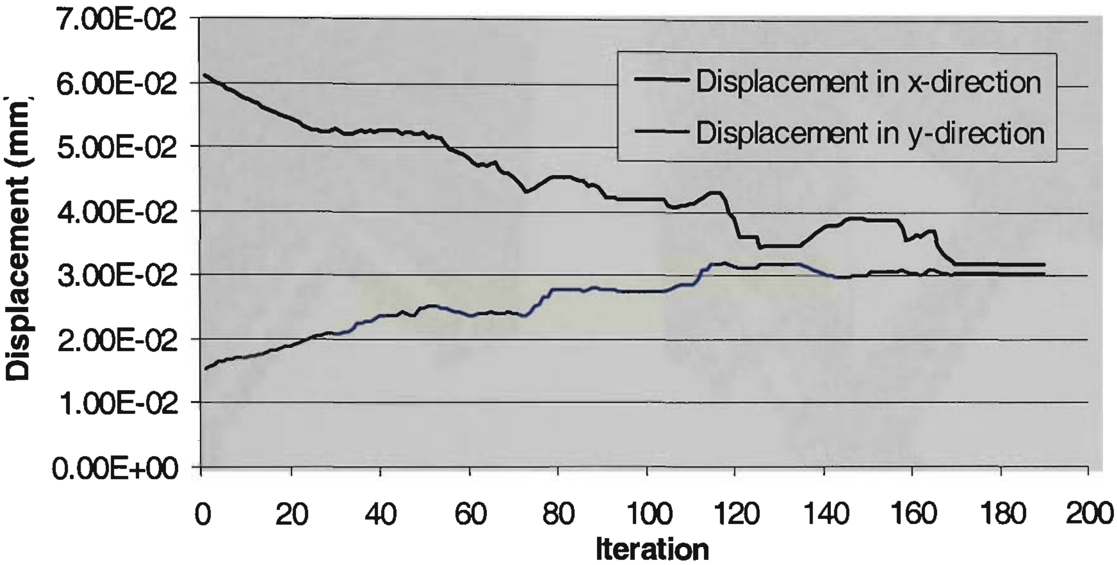
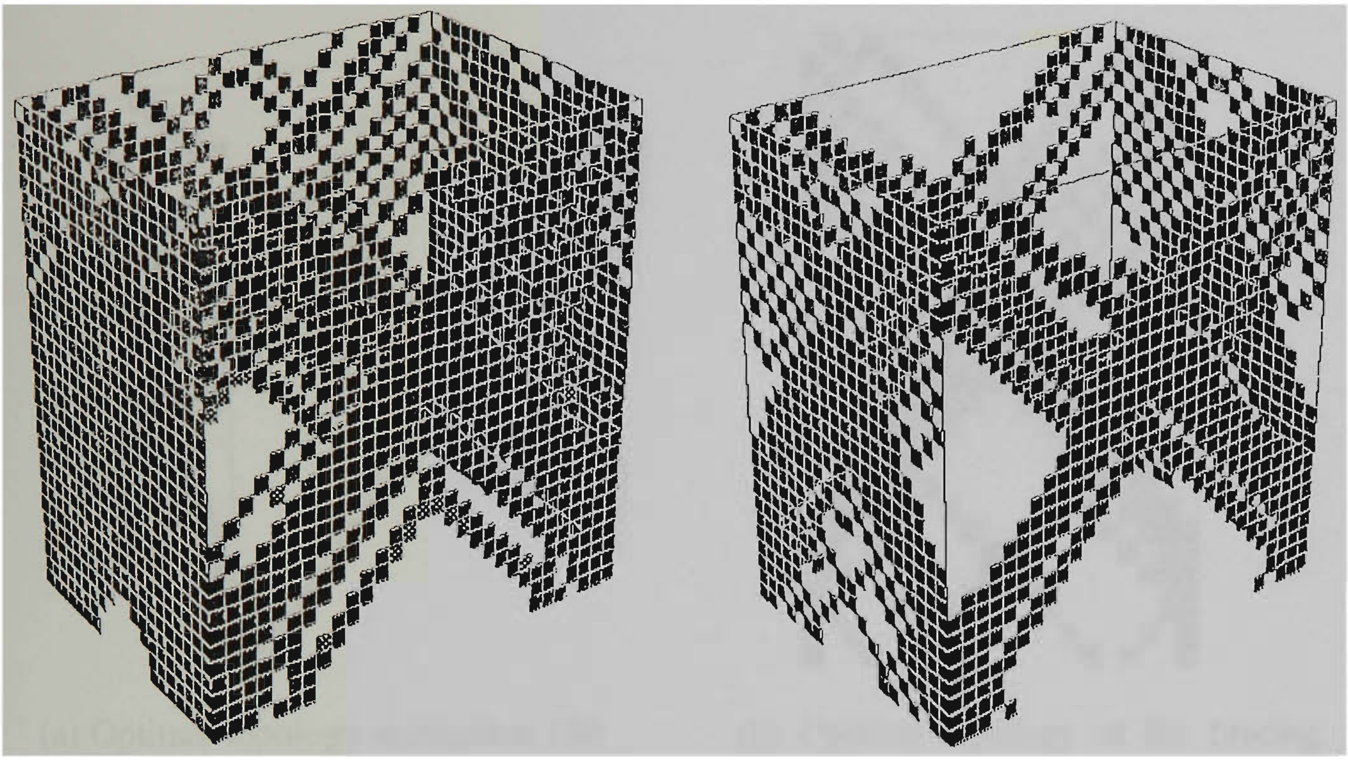
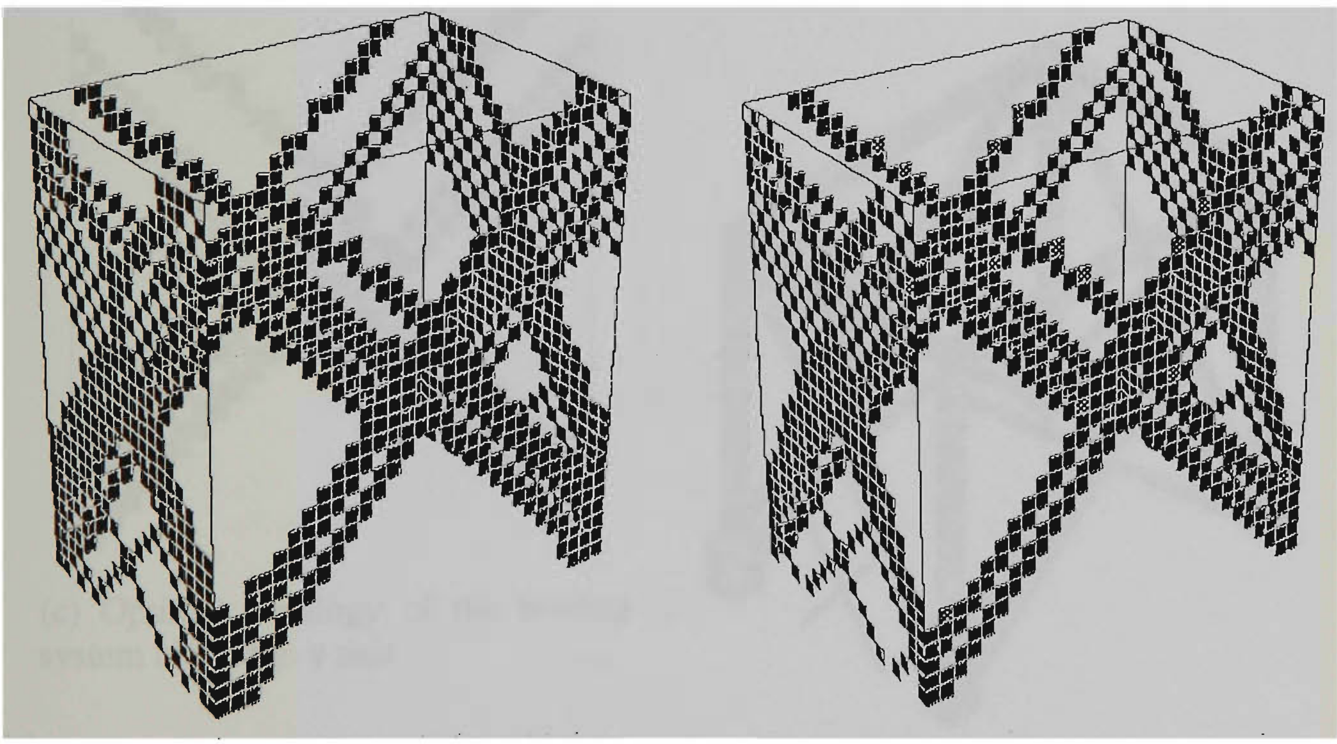



**Fig. 5.12** History of the constrained displacements of the 3D structure

The topological history of the 3D structure is shown in Figure 5.13. During the optimization process, the bracing system of the structure gradually evolves to a X-type bracing at the faces along both x and y axes. The interpreted model of the optimal topology is shown in Figure 5.14.

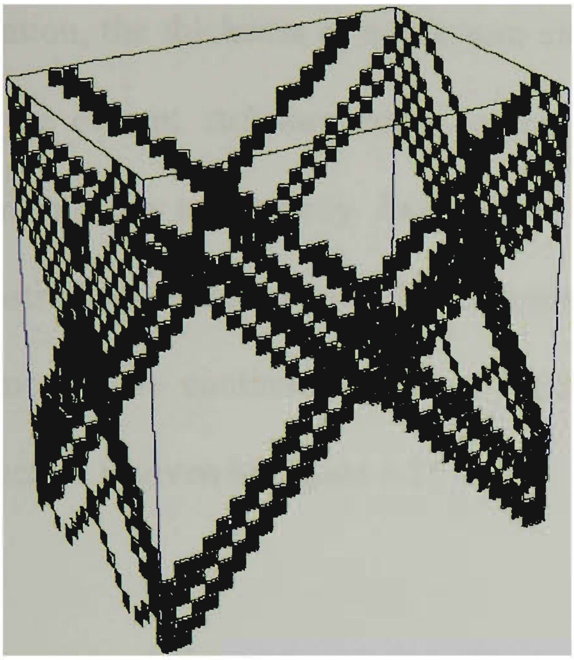


(a) Topology at iteration 40 (b) Topology at iteration 80

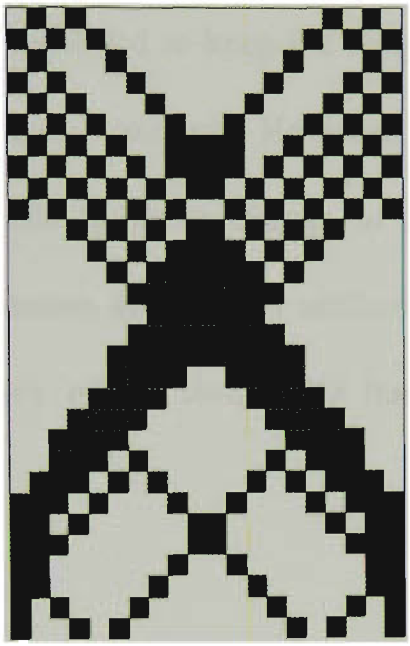


(c) Topology at iteration 120 (d) Topology at iteration 160

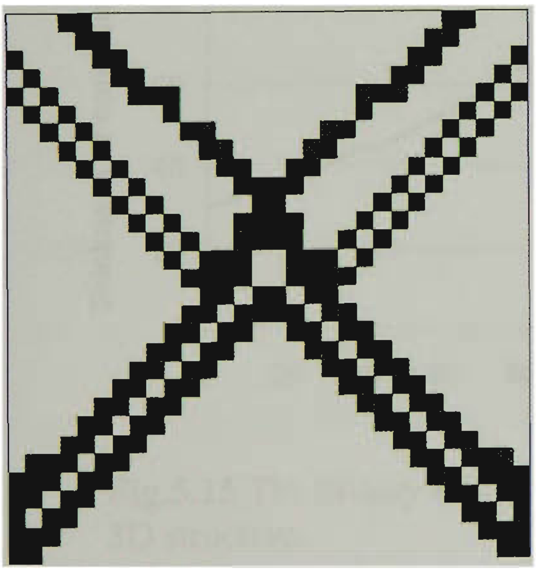
**Fig.5.13** Topology history of the 3D structure



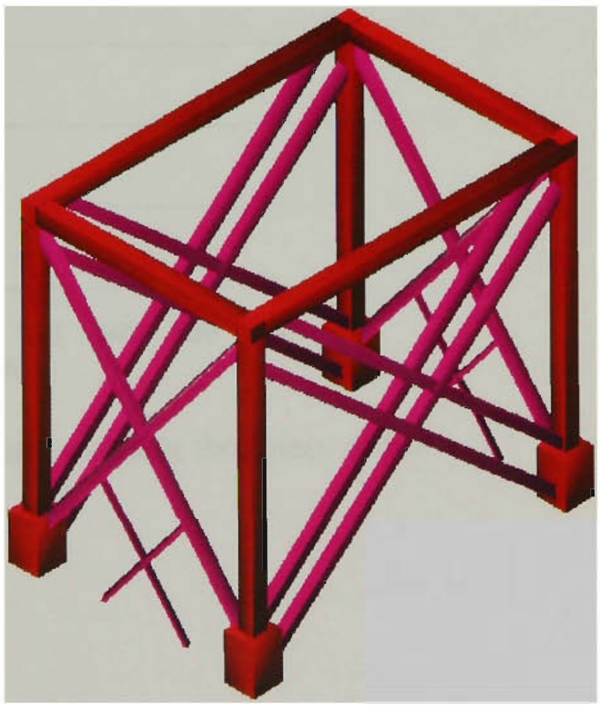
(a) Optimal topology at iteration 180



(b) Optimal topology of the bracing system along the x axis



(c) Optimal topology of the bracing system along the y axis



(d) Interpretation of optimal topology

**Fig. 5.14** Optimal topology of the 3D structure

After removing element with the lowest virtual strain energy density in each iteration, the thickness of continuum elements is increased to keep the weight of the current structure equal to that of the initial structure. However, if elements are temporarily fixed in an iteration due to sharp change in the constrained displacements, the thickness of continuum elements is uniformly reduced. The continuum element thickness history of the simple 3D frame structure is given in Figure 5.15 below.

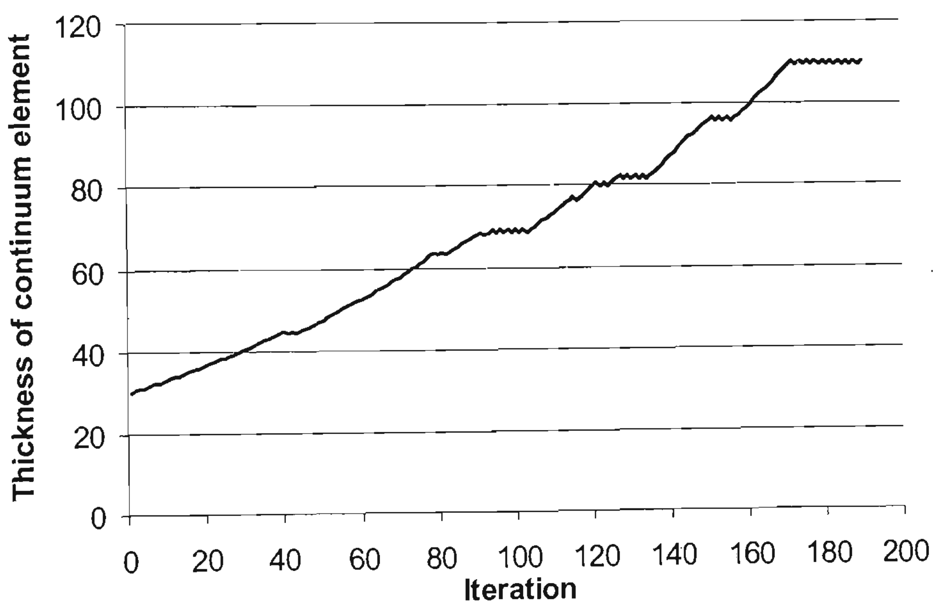



**Fig.5.15** The history of the continuum element thickness of the 3D structure.

## 5.10 SUMMARY

The extended ESO method for displacement constraints has been developed in this chapter. A sensitivity analysis has been undertaken in terms of constrained displacements to determine underutilized elements. After removing underutilized elements in each iteration, the thickness of continuum elements in the current structure was then uniformly increased to obtain the same weight as the initial structure. The performance of the current structure was evaluated by its constrained displacements.

Other effects of the method have also been discussed. They were the sharp change in the constrained displacements and the termination conditions of the process. Firstly, the sharp change in the constrained displacements occurs in a particular iteration because the elements removed in that iteration play an important role in forming the global stiffness matrix of the structure. Removal of these elements may lead to a collapse of the whole structure. Secondly, the termination conditions must ensure that the optimal topology was included in the evolution process.

Three examples representing different types of finite elements have been studied in this chapter. For plane stress and plate in bending examples, the optimal topology was the one having the lowest values of constrained displacements. For the 3D structure, which contains both beam and continuum elements, only continuum elements were allowed to be removed whereas the beam elements were fixed during the optimization process. It was observed that the optimization process tends to reduce the gap between two orthogonal constraint displacements at the top of the frame.

# CHAPTER 6: THE EXTENDED ESO METHOD FOR MULTI-STOREY BUILDINGS SUBJECT TO TOP DEFLECTION CONSTRAINTS

## 6.1 INTRODUCTION

In the structural design of buildings, the design criteria of serviceability usually requires that the deflection at the top of the building must be less than $H/200$ where $H$ is the overall height of the building from foundation level. In general, the lateral deflection of a building is caused by two contributing factors as in a cantilever beam. One factor is the bending deflection and the other is the shear deflection. They are referred to as the cantilever bending and shear racking deflections. In a rigid frame, the lateral load resistance is provided by the strength and stiffness from the non-deformability of the joints at the intersection of columns and girders. However, a rigid frame system may not be efficient for multi-storey buildings because the deflection produced by the bending of the columns and girders causes the building to drift too much. A braced frame attempts to improve upon the efficiency of a rigid frame action by virtually eliminating the column and girder bending factor. This is achieved by adding truss members such as diagonals between the floor systems.

The extended ESO method for displacement constraints has been presented in Chapter 5. Based on the procedures outlined in Chapter 5, a multi-storey steel frame will be designed to support lateral load cases in this chapter. It is modelled by using 2D plane stress finite elements. The structural optimization purpose is to determine the optimal pattern of the bracing system and the size transition locations of the columns subject to deflection constraints at the top of the building.

## 6.2 OPTIMIZATION PROCEDURE

The optimization procedure of the multi-storey buildings subject to displacement constraints is illustrated in Figure 6.1 and can be explained as follows.

**Step 1**: Design the pure rigid frame for gravity load cases based on strength criteria.

**Step 2**: Model the unbraced framework by using beam finite elements representing columns and girders. These beam elements will not be removed during the optimization process, and will be referred to as the non-design domain. Lateral load cases and support conditions are also assigned to the model.

**Step 3:** The bracing system of the multi-storey building is modelled by using a fine mesh of finite continuum elements. These continuum elements will be gradually removed during the optimization process to find the optimal topology for the bracing system.

**Step 4:** Assign the virtual unit load at the top of the multi-storey building.

**Step 5:** Carry out finite element analysis to compute the deflection at the top of the building.

**Step 6:** Calculate the virtual strain energy density of each continuum element in the design domain by using equation (5.5).

**Step 7:** If there is a sharp change in the top deflection values, temporarily fix the elements removed at the previous iteration. Return the thickness of continuum elements to the thickness value of the previous iteration. Repeat from Step 5.

**Step 8:** If the number of temporarily fixed elements is greater than or equal to a prescribed fixed ratio, release all the temporarily fixed elements. Repeat from Step 5.

**Step 9:** Remove the continuum elements which have the lowest virtual strain energy density from the structure. The number of removed elements is equal to the removal ratio (*RR*) multiplied by the number of elements of the current structure.

**Step 10:** If the current structure becomes unsymmetrical, temporarily fix the elements removed at the previous iteration. Return the thickness of continuum elements to the thickness value of the previous iteration. Repeat from Step 5.

**Step 11:** If there is no sharp change in the top deflection values at previous iteration, release all the temporarily fixed elements.

**Step 12:** Uniformly increase the thickness of continuum elements in the design domain using equation (3.24).

**Step 13:** Save the current structure.

**Step 14:** Repeat from Step 5 to Step 13 until the termination condition described in section 5.6 is met.

**Step 15:** Plot the deflection history of the optimization process from the saved database and select the optimal topology for the bracing system.

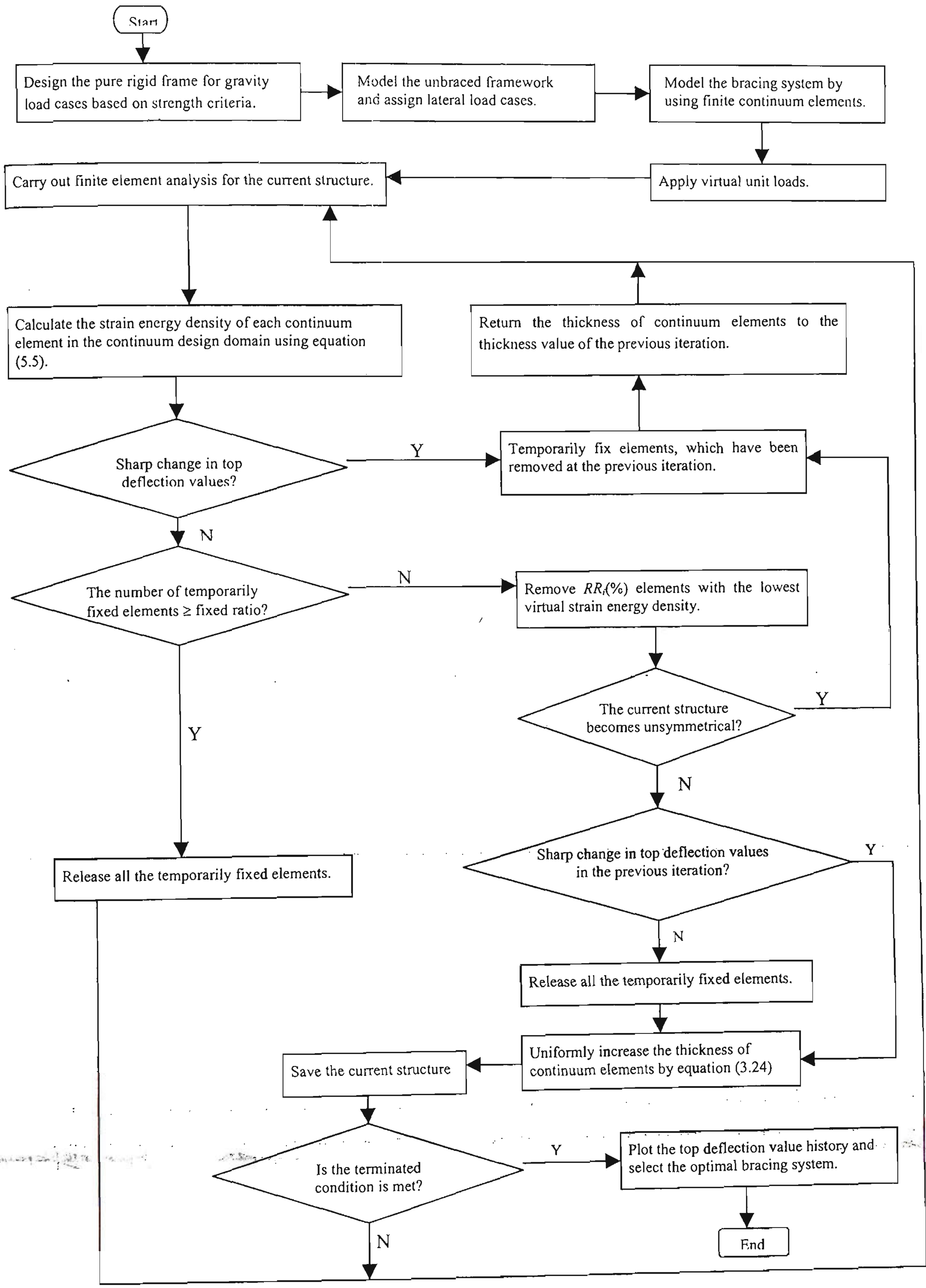



**Fig.6.1** Procedure for topological optimization subject to displacement constraints of multi-storey buildings

## 6.3 2D PLANE STRESS MULTI-STOREY STEEL FRAME

The three-bay twelve-storey plane stress multi-storey steel framework, which has been given in Figure 4.4, is considered in this section. The structural optimization purpose is to find an efficient bracing system to resist lateral load cases subject to top deflection constraint. To ensure the optimal topology of the bracing system is included in the optimization process, the limit of top deflection of the multi-storey framework is set to a high value. The finite element analysis input and optimization parameters are given in Table 6.1 below.

**Table 6.1** Finite element input and optimization parameters of plane stress multi-storey framework.

| Finite element analysis input | Optimization parameters |
|---|---|
| • Continuum design domain: 108x48 four-node plane stress elements<br>• Modulus of elasticity: $E$=200GPa<br>• Poisson's ratio: $\nu$=0.3<br>• Plate thickness: $t$= 25 mm<br>• Plane stress static analysis. | • Removal ratio: $RR$=1% of the total number of continuum elements of the current structure.<br>• Topological structural optimization (cavities allowed).<br>• Maximum number of temporarily fixed elements, fixed ratio=20%.<br>• Difference in the change of top deflection values that will be considered as sharp change, sharp change ratio=5% between two adjacent iterations. |

The history of the deflection at the top of the multi-storey framework is shown in Figure 6.2. It can be seen that the top deflection of the frame decreases as inefficient materials are removed from the design domain. The minimum top deflection value occurs at iteration 320 which corresponds to the optimal topology at that iteration.

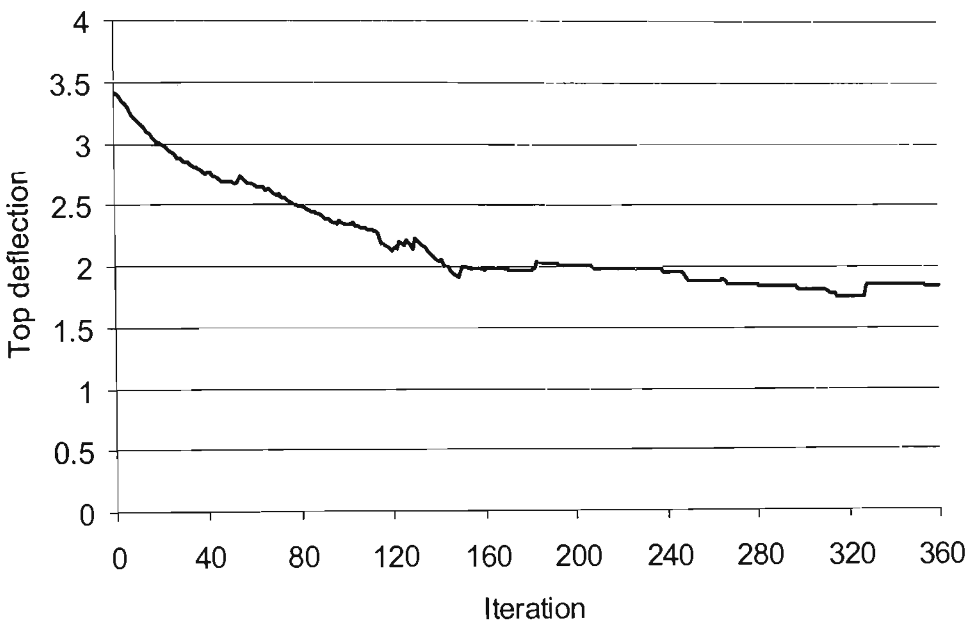
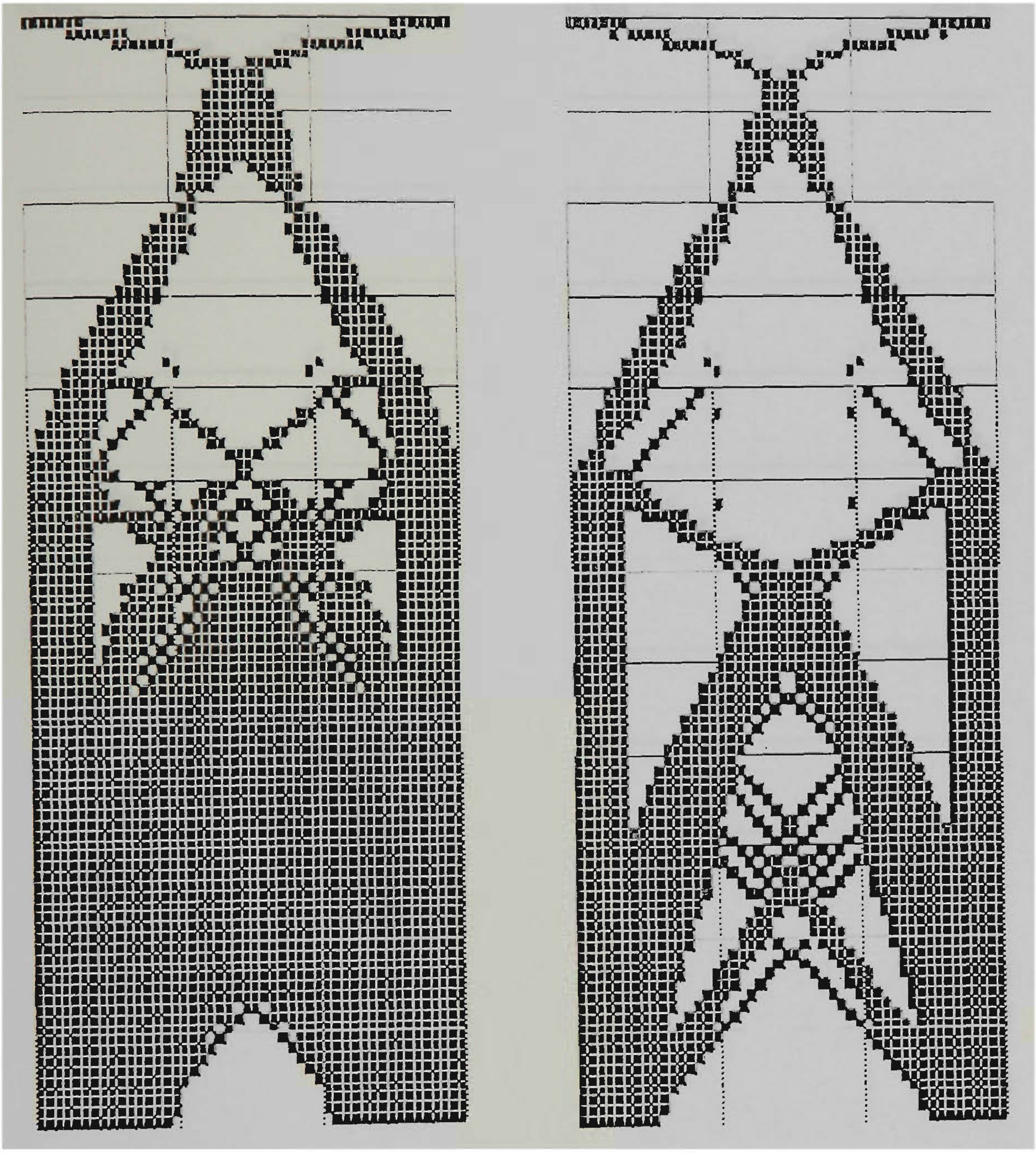
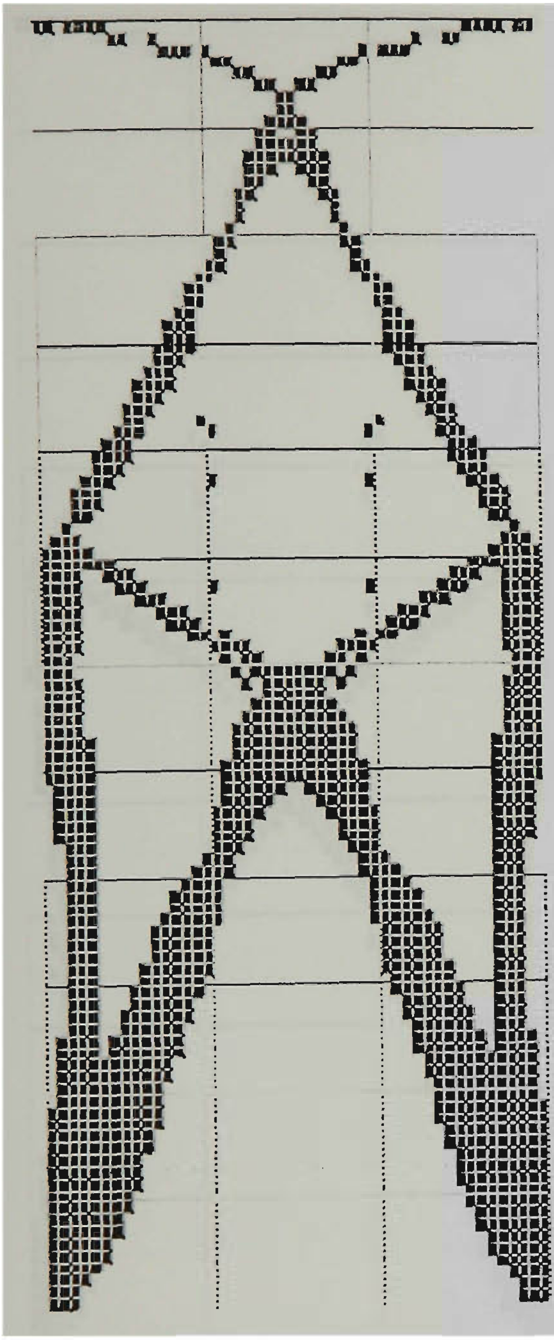



**Fig. 6.2** Top deflection history of the plane stress multi-storey framework

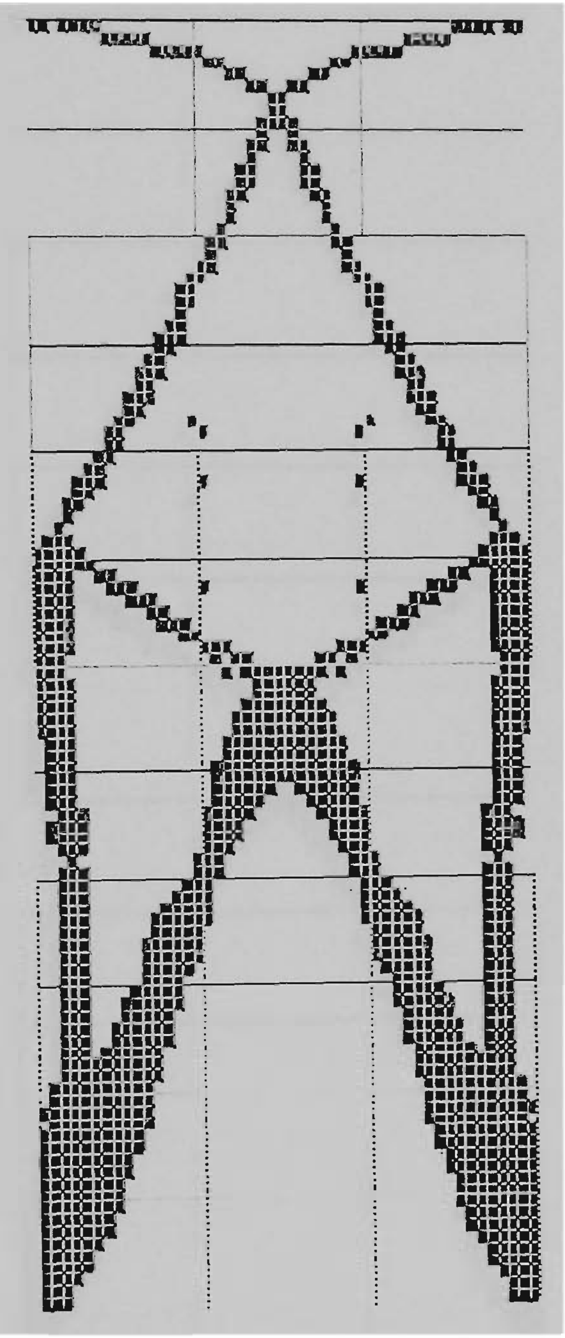
The topological history of the multi-storey framework is shown in Figure 6.3. It is seen that a discrete-like bracing system is obtained by removing inefficient elements during the optimization process. Figures 6.3 (c) and (f) are typical examples where a sharp change occurs in the top deflection due to discontinuity of the bracing at the top floor. The optimal topology and its interpreted structure are shown in Figure 6.4. Note that the column sizes at the ground levels need to be increased. The thickness of continuum elements in the design domain is increased during the optimization process to make the weight of the current structure equal to that of the initial structure (see Figure 6.5). The performance of equally weighted topologies is evaluated by their top deflection values.



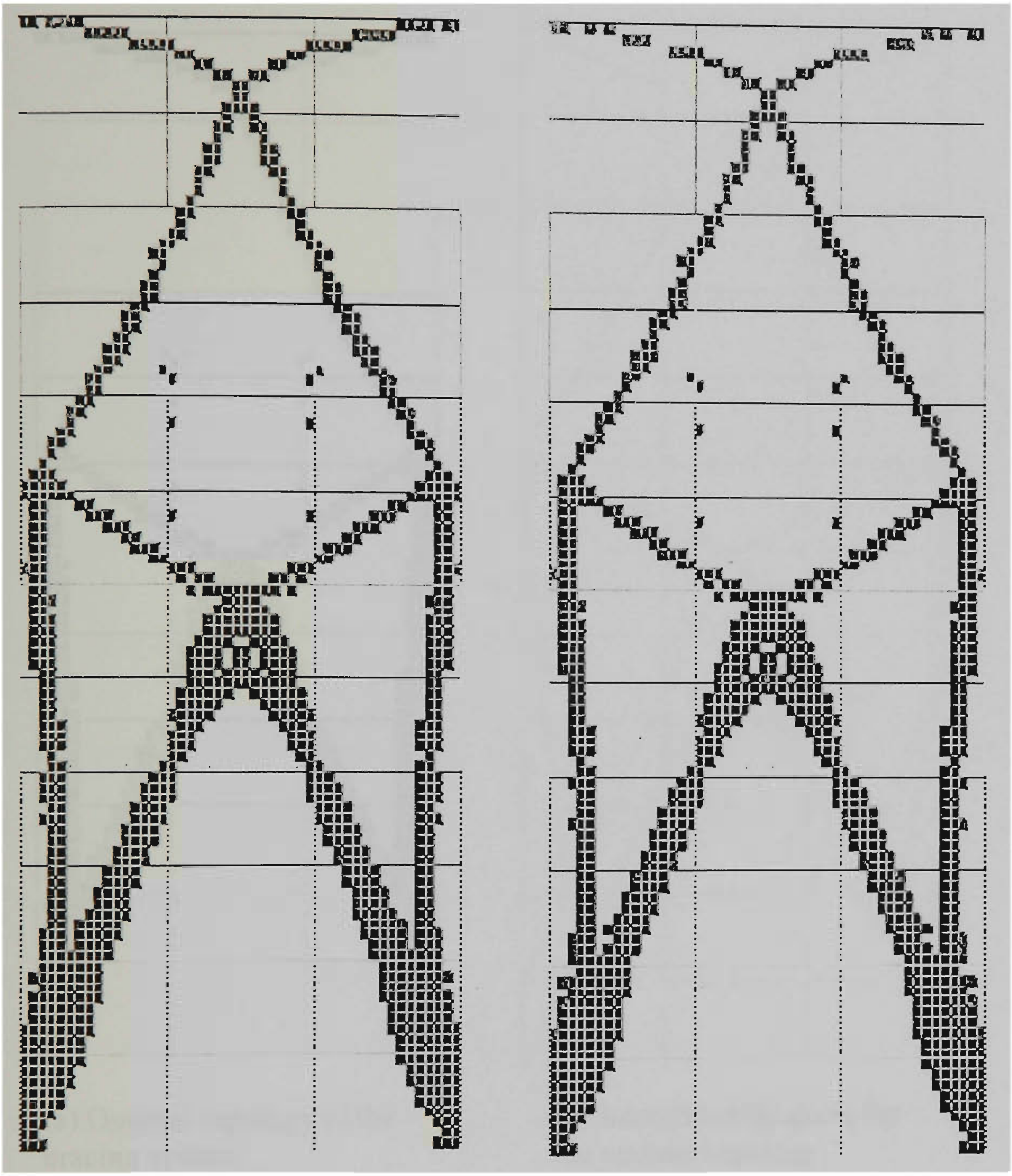
(a) Topology at iteration 50 (b) Topology at iteration 100



(c) Topology at iteration 150

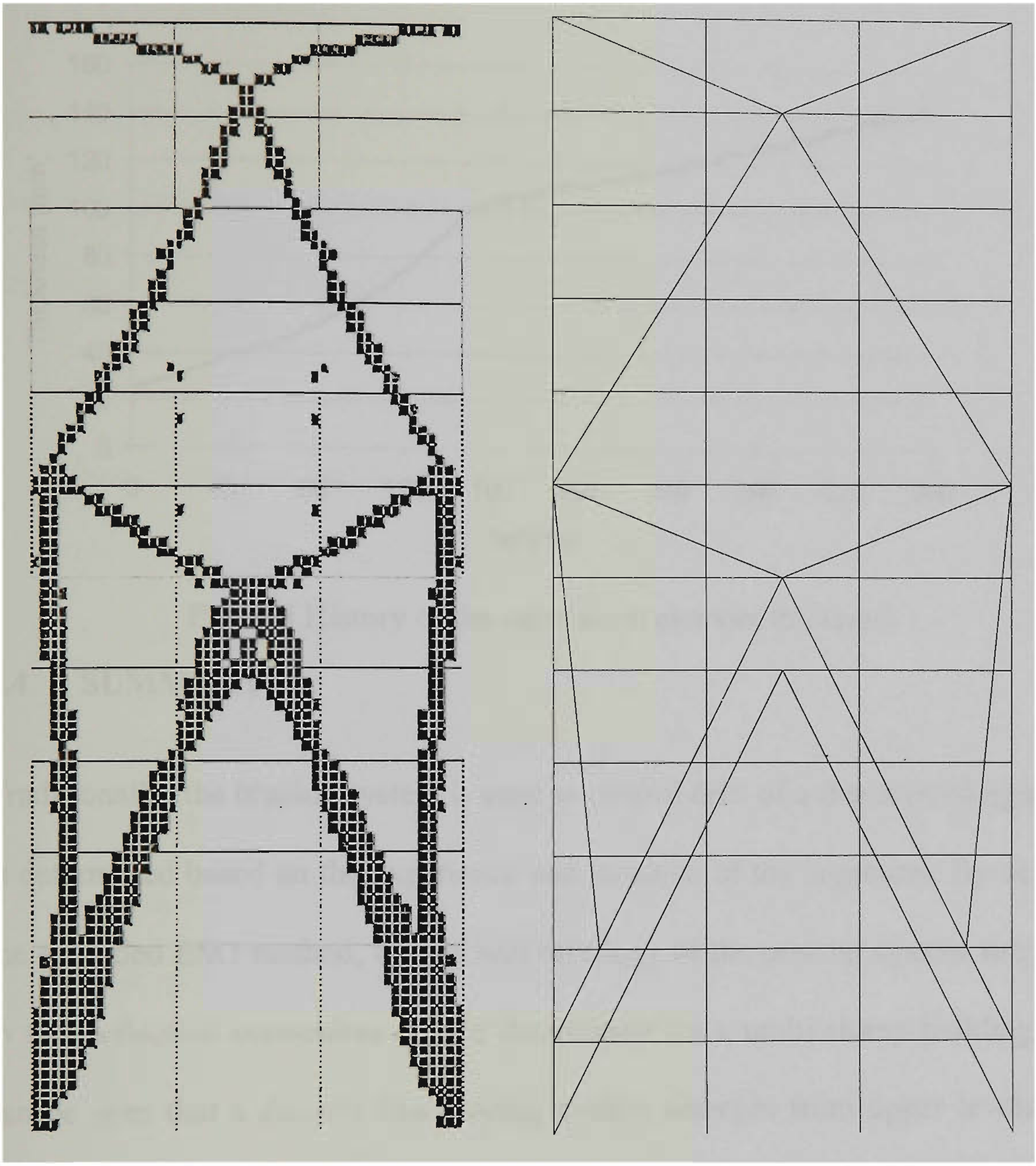


(d) Topology at iteration 200



(e) Optimal topology at iteration 320 (f) Topology at iteration 350

**Fig. 6.3** Topology history of plane stress framework optimization subject to top deflection constraint.



(a) Optimal topology of the bracing system

(b) Interpreted structure for the optimal topology

**Fig.6.4** Optimal topology for the plane stress multi-storey framework subject to top deflection constraint

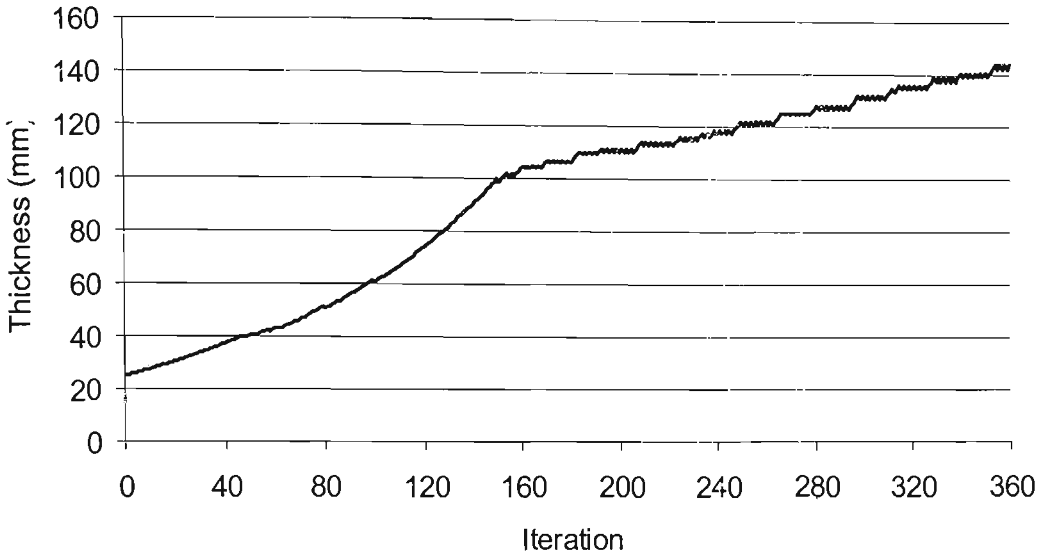



**Fig. 6.5** History of the continuum element thickness

## 6.4 SUMMARY

Traditionally, the bracing system is used to control drift of a frame building and is determined based on the experience and intuition of the engineers. By using the extended ESO method, the optimal topology of the bracing system subject to top deflection constraints can be determined for a multi-storey building. It can be seen that a discrete-like bracing system emerges from upper levels to lower levels. After obtaining the optimal topology of the bracing system, which contains continuum elements, the structural designers can translate it into a discrete structure basing on the construction and architectural requirements.

# CHAPTER 7: CONCLUSIONS AND RECOMMENDATIONS

## 7.1 CONCLUSIONS

This thesis has developed an extended ESO method for continuum topological structural optimization subject to overall stiffness or displacement constraints. The achievements of this thesis are summarized as follows:

1. For topological structural optimization subject to overall stiffness constraints, this thesis has introduced a technique to deal with the sharp change in the mean compliance of the whole structure so that the improved topologies can be obtained.

2. A technique has been developed to select the optimal topology from among a series of topologies, which were generated during the ESO procedure. This technique has been applied successfully to plane stress problems, plate bending elements and 3D structural optimization problems.

3. Using the extended ESO method, an efficient bracing system for a multi-storey frame subject to overall stiffness constraints has been found.

4. The extended ESO method has been developed for topological structural optimization for displacement constraints.

5. By using the technique of "fix and release elements", the ESO method for displacement constraints has been implemented to take into account

the sharp change in the constrained displacement values. This new feature can improve the topology result.

6. This thesis has incorporated the scaling technique into the ESO method to choose the optimal topology. The scaling technique is used on uniformly changing the continuum element thickness at the end of each iteration. This new feature enables the ESO users to monitor the evolutionary procedure and choose the optimal topology from among a series of generated topologies.

7. The extended ESO method for displacement constraints has been applied to a multi-storey steel frame. The proposed method can find an optimal topology for the bracing system of the building subject to top displacements.

## 7.2 LIMITATIONS OF THIS RESEARCH

Most of the difficulties encountered in this research come from the numerical accuracy related to large-scale structures and floating-point numbers. Additionally, due to the limited access to the source code of the finite element package used, especially the beam elements and the solution module, therefore the size and number of degrees of freedom of the structures in this thesis have been significantly limited. The limitations of this research are pointed out as follows:

1. The optimization process depends on many parameters which are determined by the experience of the users. These parameters are: removal ratio, sharp change ratio and fixed ratio. Further investigation is required to determine these parameters so that the optimization procedure will be carried out automatically.

2. The solution time depends on the FEA method. As ESO method requires hundreds of FEA run times, the time solution of the whole procedure will increase significantly with the increasing size of the models. Furthermore, for large-scale 3D structures, the number of the degree of freedom of the finite element model is very large which makes the solution time very high.

3. For structures containing beam and continuum elements, only the stiffnesses of the continuum elements have been counted for, although, theoretically, the stiffness of the whole structure (beam and continuum elements) must be included to select to optimal topology. This is because the author, at this stage, does not have access to the source code of the beam elements.

4. Only top deflection constraint has been considered for multi-storey buildings. In tall building design, the inter-storey drifts and acceleration (perception of motion) are more likely the governing serviceability criteria. Therefore, to achieve a more optimal topology, the optimization

method should include inter-storey drifts and acceleration criteria in the constraints.

5. The bracing system of the frame is allowed to freely occupy the building's facades. Because of the lack of information and for the sake of simplicity, architectural and constructional requirements have not been considered in this thesis.

## 7.3 RECOMMENDATIONS FOR FURTHER RESEARCH

Further research should aim at making the proposed method more general and of practical use for engineers. Recommendations for further research include the following.

1. Automatically generate optimization parameters based on FEA output. This would make the whole process of FEA and optimization automatic.

2. Automatically generate the mesh of continuum finite elements for continuum design domain and further eliminate the effects of finite element mesh and checker board pattern.

3. Complement the proposed method with sizing optimization. After obtaining the optimal topology, it is more efficient if the optimization program can provide the structural designers with the size of each structural component.

4. For tall building structures, limitation of inter-storey drift is an important criterion in structural design. Therefore, a technique to consider inter-storey drift constraint needs to be developed.

5. Extend the proposed method to dynamic problems.

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