# Stochastic Modelling and Optimal Control of Compartment Fires

by

**Mehmet Ali TAT, BSc.(Hon)**

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Approved by
Dissertation Committee:

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To my children Deniz, Pınar, and my nephews and nieces Meral, Vural, Emel, Sibel, Neşet and Enes.

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Mehmet Ali TAT

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# Stochastic Modelling and Optimal Control of Compartment Fires

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Mehmet Ali TAT, Ph.D.
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Supervisor: Abraham M. Hasofer

**Abstract**

Compartment fires are defined as fires in enclosed spaces. They are labeled as oxygen driven fires and are non-stationary growth phenomenon. A gap exists in the knowledge of deterministic fire growth models and stochastic fire growth models. In this thesis we develop non-stationary stochastic models in an endeavor to bridge the gap.

The class of Epidemic models for infectious diseases are non-stationary growth models. In the first part of the thesis the Deterministic Simple Epidemic, Deterministic General Epidemic and the Stochastic General Epidemic models are investigated to develop equations for the growth of compartment fires by drawing analogies between the epidemic variables and the compartment fire variables. The Percolation and Contact processes are investigated for the spread of compartment fires. A mechanism for converting deterministic differential equations to stochastic differential equations based on the theory of Martingales is presented.

In part two of the thesis, two deterministic models based on the risk assessment model of the National Research Council Canada (NRCC) are developed and

calibrated. One of the deterministic models is a fuel driven model and the other is an oxygen driven model. The oxygen driven deterministic model is converted to a stochastic model, based on the theory of Martingales, and used as an input to calculate a fire severity measure, called Heat Load. Statistical tests are applied to the Heat Load data set to determine its distribution. A non-parametric statistical test, W-Test, is used to calculate the upper quartiles of the heat load.

A third model based on the NRCC model is built. This model is closer to the Epidemic models and its parameters do not require tedious optimisation algorithms to calculate. They are evaluated from the initial conditions of the physical process. In this model we make the assumption that the gas temperature inside the compartment is a function of the burning rate and develop a two variable model based on the burning rate and oxygen fraction. A change of variable is applied to simplify the differential equations, the equations are solved implicitly and their parameters evaluated using the initial conditions. The temperature equation is modelled using a first order differential equation with the burning rate and is solved separately.

Finally part three of this thesis investigates automatic sprinkler systems and the mathematical theory of optimal control. Optimal control theory is applied to automatic sprinkler systems to model sprinklered compartment fires. To reduce water damage inside a compartment due to sprinkler activation from small fires, we model the water spray rate. Two cases are considered, the first when the water damage is proportional to the total amount of water and the second when the water damage is proportional to the integral of the square of the water flow rate. Pontryagin's principle is used to solve the integrals and obtain the water spray rate equations.

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# Preface

Chapters (4) and (6) was published in Tat and Hasofer (1995). Parts of chapters (7) and (8) appeared in Hasofer and Beck (1995). Chapters (11) and (12) appeared in Hasofer and Tat (1997).

# Chapter 1

# Introduction

Compartment fires continue to pose problems on risk to public life and material costs to building owners. The methods of statistical analysis described here are aimed at improving our understanding of compartment fires and their growth and spread through buildings, with the hope that such additional knowledge will help in the control of these costs.

A major motivation for developing stochastic models for compartment fire data is that one can gain knowledge which is useful for determining strategies for the control of the fire. One useful information is to determine the mechanism of growth and then use this to estimate the mean duration's of the fire inside a compartment. Another useful information is to be able to determine the extent of variations in these durations and then use this to determine the minimum required precautions to prevent major fires.

An overview of fires and more specifically of compartment fires will initially be briefly discussed. It is envisaged that this will provide the reader with some familiarity with the topic and enable a better understanding of the information contained within the following chapters.

## 1.1 Combustion In The Diffusion Flame Phenomenon

Fire is defined primarily as rapid oxidation accompanied by heat and light. In general, oxidation is the chemical union of any substance with oxygen. The rusting of iron is oxidation but it is not fire because it is not accompanied by light. Heat is generated, but so little that it can hardly be measured. Burning can occur as a form of chemical union with chlorine and some other gases, but for our purpose we need only consider fire that involves oxygen.

### 1.1.1 The Classic Triangle Concept of Fire

Fire can usually take place only when three things are present: oxygen in some form, fuel (material) to combine with the oxygen, and heat sufficient to maintain combustion. Removal of any one of these three factors will result in the extinguishment of fire. The classic "fire triangle", see figure (1.1), is a graphical symbolisation of the recognised elements involved in the combustion process. Opening the triangle by removing one factor will extinguish a growing fire, and keeping any one factor from joining the other two will prevent a fire from starting.

### 1.1.2 The Tetrahedron Concept of Fire

Recent research suggests that the chemical reaction involved in fire is not as simple as the triangle indicates and that a fourth factor is present. This fourth factor is a reaction-chain where burning continues and even accelerates, once it has begun.

Haessler(1974), in his study of fire, formulated the theory of the diffusion flame combustion phenomenon as a tetrahedron. Haessler preferred to symbolise his concept of fire as a tetrahedron instead of a square because in the tetrahedron the four entities are adjoining and each is connected with the other three entities.

This reaction-chain is caused by the breakdown and recombination of the molecules that make up a combustible material with the oxygen of the atmosphere. A piece of paper, made up of cellulose molecules, is a good example of a combustible

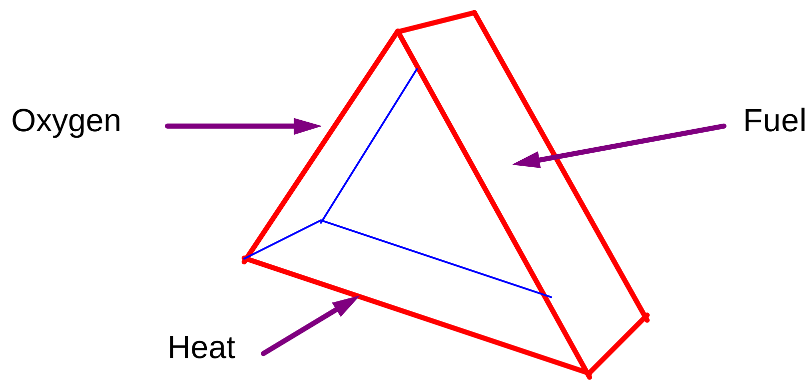


Figure 1.1: Fire Triangle

material. Those molecules that are close to the heat source begin to vibrate at an enormously increased rate and, almost instantaneously, begin to break apart. In a series of chemical reactions, these fragments continue to break up, producing free carbon and hydrogen that combine with the oxygen in the air. This combination releases additional energy. Some of the released energy breaks up still more cellulose molecules, releasing more free carbon and hydrogen, which, in turn, combine with more oxygen, releasing more energy and so on. The flames will continue until fuel is exhausted oxygen is excluded in some way, heat is dissipated, or the flame reaction-chain is disrupted.

Supporting this concept has led to the discovery of many extinguishing agents that are more effective than those that simply manage to open the triangle. Because of this discovery, we must modify our fire triangle into a three-dimensional pyramid, known as the "tetrahedron of fire", see figure (1.2). This modification does not eliminate old procedures in dealing with fire but it does provide additional means by which fire may be prevented or extinguished.

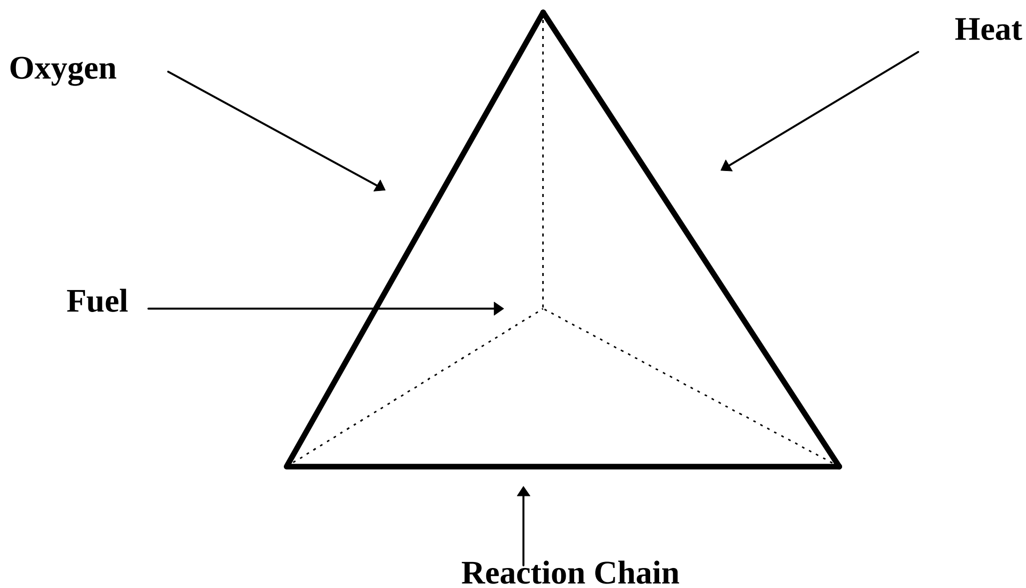


Figure 1.2: Fire Tetrahedron

## 1.2 Fire Spread

The rate at which fire will develop will depend on how rapidly flame can spread from the point of ignition to involve an increasingly large area of combustible material, Flame spread is considered as an advancing ignition front in which the leading edge of the flame acts both as the source of heat, to raise the fuel ahead of the flame front to the fire point, and as the source of pilot ignition. There are various factors which are known to be significant in determining the rate of flame spread over combustible solids: Material factors and Environmental factors.

Environmental factors consist of composition of atmosphere, temperature, imposed heat flux and air velocity. Composition of the atmosphere refers to the oxygen concentration. Combustible materials will ignite more readily, spread flame more rapidly and burn more vigorously if the oxygen concentration is increased. Higher rates of flame spread are observed with effective oxygen enrichment which enhances flame stability at the surface. Temperature refers to the temperature of the fuel. Increasing the temperature of the fuel increases the rate of flame spread, the higher the initial fuel temperature the less heat required to raise the unaffected fuel to the fire point ahead of the flame. An imposed radiant heat flux causes an increase in the rate of flame spread, by preheating the fuel ahead of the flame front. Confluent air movement enhances the rate of flame spread over a combustible surface. Friedman (1968), reports that the rate will increase quasi-exponentially up to a critical level at which extinction will occur.

Material factors are further divided into chemical and physical factors. The chemical factors consist of composition of fuel and presence of retardants. The physical factors consist of initial temperature, surface orientation, direction of propagation, thickness, thermal capacity, thermal conductivity, density, geometry and continuity. As an example, of surface orientation effect, Alpert and Ward (1984), point out that the spread of a flame along a vertical surface accelerates exponentially.

For theoretical models of flame spread, the rate of heat transfer across a surface determines the rate of fire spread. The 'fundamental equation of fire spread' is a simple energy conservation equation, William (1977).

$$\rho V \Delta h = q \tag{1.1}$$

where $q$ is the rate of heat transfer across the surface, $\rho$ is the fuel density, $V$ is the rate of spread, and $\Delta h$ is the change in enthalpy a unit mass of fuel is raised from its initial temperature ($T_o$) to the temperature ($T_i$) corresponding to the fire point. If it is possible to identify $q$ in a given fire spread situation, some insight can be gained into the factors that affect the rate of flame spread.

## 1.3 Compartment Fires

Compartment fires are defined as fires in enclosed spaces, typically thought of as rooms in buildings. Compartment fires are discussed usually in growth stages. These can be categorised as:

1. Ignition

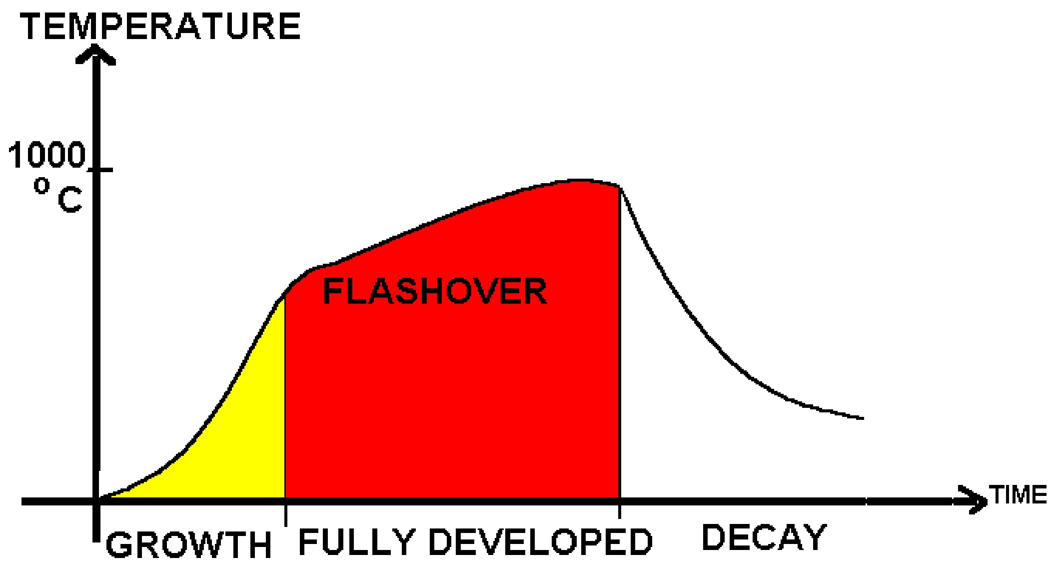


Figure 1.3: Graph of the change in temperature over time in compartment fires.

2. Growth

3. Flashover

4. Fully developed fire and

5. Decay.

Figure (1.3) shows an idealised form of temperature variation with time described by the growth stages.

The growth period is described as the point from where fire is initiated to the stage where flashover occurs. Walton (1990), defines a flashover as the "transition from a growing fire to a fully developed fire within the compartment in which all combustible items are involved in the fire".

All fires manifest an ignition stage but, beyond that, may fail to grow through all or some of the growth stages listed.

## 1.4 Deterministic and Non-Deterministic Modelling

Modeling the growth and spread of fire can be categorised under two general headings: deterministic and non-deterministic. Deterministic models attempt to derive equations which model the experimental data gathered on some of the chemical and physical interactions involved in a fire. They are generally too complicated to accommodate stochastic variation. Non-deterministic models attempt to model the inherent variability of fire, using simplified approximations to the fire phenomenon. Ramachandran (1991) provides a description of deterministic and non-deterministic models and an explanation of the type of questions/problems that can be modelled in compartment fire research using these models.

A Stochastic Process (non-deterministic model) is the mathematical abstraction of an empirical process whose development is governed by probabilistic laws. From a non-mathematician's point of view a stochastic process is any probability process, that is, any process running along in time and controlled by probabilistic laws. Numerical observations made as the process continues indicate its evolution. Ramachandran (1995) classifies stochastic models as dynamic as they are capable of predicting the course of fire development in a particular building. In these models, the various states, realms, or phases occuring sequentially in space and time during fire growth are specified together with the associated probability distributions. A set of deterministic equations can be turned into a set of stochastic differential equations by adding on the right-hand side forcing functions which are white noise multiplied by some function of the variables and/or endowing the parameters with probability distributions.

Ramachandran (1991) provides an overview on the application of several probabilistic and stochastic models in non-deterministic modelling of fire spread. The probability models reviewed are probability distributions, logic trees and the probabilistic version of a deterministic model. Several papers in each of the probability models have been reviewed and Ramachandran provides an explanation for the appropriate application of the probability models in compartment fire research. He points out that probabilistic models, treat critical events during fire spread as random and independent. They model final outcomes such as extent of spread, area damaged and financial loss. These models are ideal for fire protection and insurance problems concerned with collective risk in a group of buildings.

The stochastic models reviewed are the state transition stochastic model, Markov models, Network theory, Epidemic theory, Branching processes, Random walk and Percolation processes. The state transition stochastic model for a compartment fire with four states is developed using a probability tree to describe the development of a fire through the states. For the Markov model, Network theory, Epidemic theory, Branching process and Percolation process several papers in the application of compartment fires is reviewed. Ramachandran draws an analogy between random walk and fire spread. Using the exponential form of the random walk and making the assumption that damage is proportional to heat output he develops an equation for the damage exceeding a given value, as a pareto distribution. In this paper Ramachandran points out that stochastic models describe the critical events occurring sequentially in space and time, hence, a particular building with specific design features and fire protection measures can be modelled. Ramachandran also points out that most of the stochastic models, except for the simple version, involve computations more complex than probabilistic models.

Ramachandran (1995) discusses stochastic modelling of fire spread and two types of stochastic models are discussed in detail: (1)Markov Chains and (2) Networks. Then some attention is given to other stochastic models such as random walk, diffusion process, percolation theory, epidemic models and branching processes. Ramachandran points out that probability models used to model the growth of a fire involves complex calculations when the spread to other compartments are tried to be incorporated. He suggests using network models for fire spread through a building to simplfy the calculations.

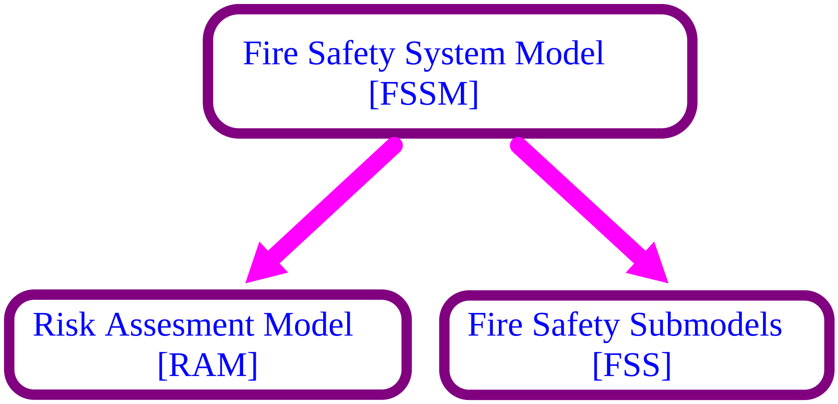


Figure 1.4: Structure of the Fire Safety System Model.

## 1.5 Thesis Objectives

The Fire Safety and Engineering Project report, published by the Warren Centre for Advanced Engineering, Beck (1992), suggests a significant gap exists in the knowledge of modelling of fire risk in buildings.

The Centre for Environmental Safety and Risk Engineering (CESARE) at Victoria University of Technology is conducting a program of research to improve fire safety and develop cost-effective design solutions for fire safety systems in buildings.

One of the projects is to develop an integrated risk assessment model for fire risk in buildings. CESARE is currently developing a Fire Safety System Model (FSSM). This is a mathematical model that can be used to provide a systematic methodology to identify those combinations of building subsystems that provide the requisite level of safety to the occupants in a cost-effective manner.

The FSSM consists of two parts; the Risk Assessment Model (RAM), and the Fire Safety Submodels (FSS), see figure (1.4). The FSS are used to represent the physical processes associated with fire growth and spread, human behavior, and building design. The results obtained from the FSS are transferred to the RAM. The RAM is used to integrate the results of the submodels and to calculate two parameters: expected risk-to-life safety and fire-cost expectation.

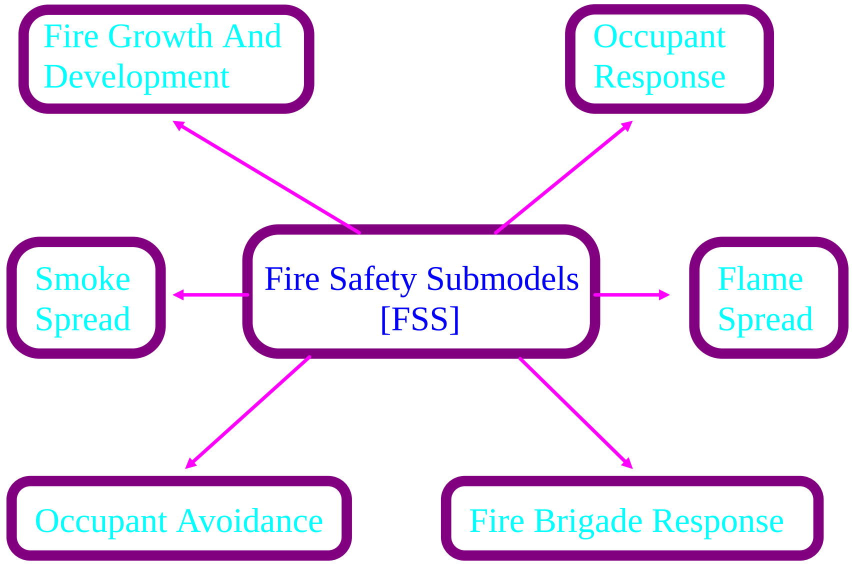


Figure 1.5: Structure of the Fire Safety Submodels.

Beck (1992), has defined the submodels within the FSS as follows, also see figure (1.5):

- Fire Growth and Development,
- Smoke Spread,
- Flame Spread,
- Occupant Response,
- Occupant Avoidance and
- Fire Brigade Response.

One essential component of the FSS is a stochastic model for compartment fires. This is because multiple interactions of physical and chemical processes and the composition of a variety of burning material and their geometric arrangements in a substance produce inherent randomness in fire development.

This research will look at the development of models to predict the time dependent, non-stationary and stochastic behavior of fire growth and development in multistory buildings for the submodel Fire Growth and Development. Including a stochastic model which accommodates the inherent randomness in fire development will produce a more realistic and accurate model for fire development. The stochastic models incorporate uncertainty which enables them to estimate the variability as well as averages of output parameters. Once the submodels are refined and tested the FSSM will be routinely used by professionals to identify cost-effective fire safety system designs for buildings.

Current time-dependent models used to describe the spread of a fire in a compartment are deterministic mechanical models without appropriate risk components. Platt (1989), see section (2.1), has developed a time dependent probability model to attempt to estimate the cumulative probabilities of fire spread in buildings over infinite time.

Ling and Williamson (1986) model postflashover fire spread from room to room using a stochastic analysis beginning with the development of a probabilistic network and followed by a method for solving the network for discrete probability distributions. These authors Ling and Williamson have made the assumption that the fire in a room has to flashover before it can spread.

The authors Ling and Williamson have developed the probabilistic network to represent fire spread using the following three steps:

(i) The floor plan of a building is transformed into a graph grid, where rooms are nodes and walls and other fire barriers are links between the nodes. Each link in the graph represents a possible route of fire spread.

(ii) The graph is then transformed into a probabilistic network by introducing one node for representing the preflashover state and another node for representing the postflashover state of each room with the link between them representing the probability of flashover and the time characteristic to flashover. Three different types of links are identified: 1. fire growth in a compartment, 2. the fire breaching the barrier element, and 3. the fire spreading along the corridor.

(iii) The fire spread probabilistic network is transformed into an equivalent network which has multiple links between the nodes to represent the uncertainty intrinsic to fire spread. Mirchandanis equivalent network and procedure is used for the shortest path calculation.

Finally a numerical example is solved in which the source node is the room in which the fire originated and the sink node is a section of the corridor. The analysis examines the possible flows through equivalent networks and the probability of a source node connecting with the sink node as well as the expected shortest travel times are calculated. Ling and Williamsons modelling offers a number of advantages over deterministic fire spread models. It allows a quantitative comparison of the fire scenarios. It directly addresses uncertainty and allows quantitative assessment of risk when generating equivalent safety in building designs which do not meet the traditional code categories. It can also be used as a framework for analysing building and fire codes.

This thesis will look at using stochastic growth models to model the time dependent growth and spread of a fire. We have divided the thesis into three parts:

In part one we investigate the class of Epidemic models of infectious diseases for fire growth. By an infectious disease we mean a disease which is infectious in the sense that an infected host passes through a stage, called his infectious period, during which he is able to transmit the disease to a susceptible host, either by a direct 'sufficiently close' host-to-host contact or by infecting the environment and the susceptible host then making 'sufficiently close' contact with the environment. In chapter (5) we demonstrate how to convert deterministic differential equations to stochastic differential equations by making use of the fact that forcing functions are Martingale Differences.

Part two of the thesis focuses on building deterministic models based on the risk assessment model from the National Research Council Canada (referred to in this thesis as NRCC). The deterministic models are converted to stochastic models using the theory of Martingales in chapter (5). Using the stochastic differential equations we simulate the heat load, see chapter (8), of a compartment fire. Then in chapter (9) we derive a two variable model closer to the General Epidemic model.

Finally in part three we investigate sprinklered compartment fires since increased application of sprinkler protection through a building can reduce fire losses significantly. However, in many small compartment fires the water damage is far more extensive than the flame damage. We examine the possibility of controlling the flow of water from sprinklers in an optimal way so as to minimise the water damage and the overall property damage. Two cases are considered. The first when the water damage is proportional to the total amount of water and the second when it is proportional to the integral of the square of the water flow rate.

# Chapter 2

# Literature Review

There appears to be no textbook which describes the modelling of fire growth and spread using non-deterministic mathematical models. However, some papers have been published in some of the fire journals which attempt to incorporate probability in their models.

The probability and statistical contents of the papers by Platt (1987), Takeda and Yung (1992) and Berlin (1990) have been summarised to show the level of statistics and probability theory used in the growth and spread of fire modelling. Although these papers have very little relevance with this research project they do show a gap between elementary probability models and stochastic models in the fire research area.

## 2.1 Time Dependent Probability Model

Platt (1989), has developed a time dependent probability model. He has made an attempt to estimate the cumulative probabilities of fire spread in buildings over infinite time. Three steps were used to calculate the probability of fire spread, they are:

1. The initial calculations are the probabilities for the three ways in which fire can spread from a compartment to any of the adjacent compartments, $P_{FS}$.

   Platt divides the way in which fire spreads into three sets to calculate the probability of fire spreading from a compartment to any of the adjacent compartments. The three sets are:

   - Spread through open doorways [$D$]. The probability of fire spread via an open door is assumed to be the probability that the door is open, $P_0$.
   - Spread vertically up the facade of a building via external windows [$W$]. The probability of fire spreading via external windows is the probability that the height of the external flame is greater than or equal to the height of the spandrel.
   - Spread by the failure of an internal barrier [$B$]. The probability of the fire spreading via an internal barrier is the probability that the fire resistance of the barrier is less than the fire severity.

     $$P_B = P[R < S]$$

     where:

     $R$ = barrier resistance [time] and

     $S$ = compartment fire severity [time].

     He has assumed that $R$ and $S$ are independent lognormal variables.

   The three ways fire can spread are ranked to form three mutually exclusive sets, see figure (2.1). Then the probability fire spreads to an adjacent compartment is the sum of the individual probabilities.

   $$P_{FS} = P[D \cup (W \cap \bar{D}) \cup (B \cap \bar{W} \cap \bar{D})]$$

   $$= P[D] + P[(W \cap \bar{D})] + P[(B \cap \bar{W} \cap \bar{D})]$$

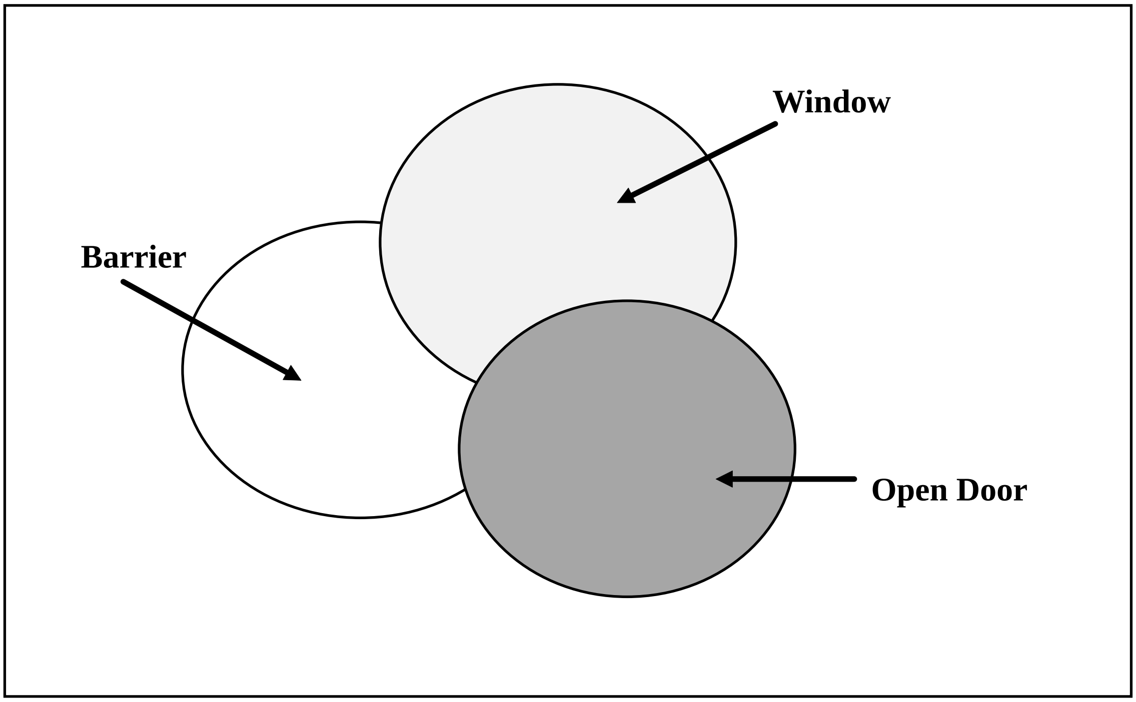


Figure 2.1: Fire Spread Events - Venn Diagram

Assuming the three events are independent the above sum can be simplified to

$$P_{FS} = P[D \cup W \cup B] = 1 - (1 - P[D])(1 - P[W])(1 - P[B]).$$

2. In the second step Platt uses conditional probability for calculating the expected time it will take for the fire to spread from a compartment to any of the adjacent compartments, given that it does spread, $E(T/FS)$.

3. In the third step Platt calculates the probability of a fire spreading from a compartment to any other compartment, via any path in a given length of time, given that a fire starts in compartment $i$; Platt employs a stochastic network representation of fire spread with spaces represented as nodes and the barriers between nodes being represented as probabilistic links of the network.

## 2.2 Deterministic Fire Growth Models

### 2.2.1 Computer Models For Fire Protection

A paper was compiled by Friedman (1992) that categorises the 62, and an additional 12 more recently available, computer programs for fire protection identified in a report based on an international survey presented at the 1989 Forum for International Cooperation on Fire Research. The categories are zone models for compartment fires, field models for compartment fires, submodels for fire endurance, submodels for building evacuation, submodels for actuation of thermal detectors, fire-sprinkler interaction models and other fire models and submodels. Friedman also provides a general discussion of models dealing with a growing and interacting fire in an enclosure and describes the features and advantages of field models and zone models.

Friedman (1992) in his discussion of computer models for a fire in a compartment overviews compartment inputs and fire inputs that are required for the computer models. For the fire inputs he talks about a fire being specified with various complexities: (a) the heat release rate continuing at a constant rate for a specified interval then stopping, (b) heat release rate varying in a known manner, (c) burning rate reducing according to the percentage of oxygen decrease in the compartment, and (d) the radiative feedback of energy from the compartment to the burning surface. Friedman highlights the difficulties that prevent the accurate modelling of the burning rate or spread rate.

After the outputs from the computational models are listed, he outlines additional uncertainties incorporated with compartment fires and makes some remarks on the method of validating computer models. Finally, Friedman points out that due to the complexity of compartment fires, computer models need to incorporate variations in the output parameters due to the inherent uncertainties.

Friedman classifies the NRCC1 model as a zone model for compartment fires. As the NRCC model is used to develop our non-deterministic models, a detail account of this model follows.

### 2.2.2 NRCC Model

Takeda and Yung (1992), have developed a simplified one-zone compartment fire growth model which is used in the NRCC computer model. The model incorporates vitiated oxygen conditions and combustion efficiency based on compartment size.

This model includes the two main external factors affecting the flame spread rate on a burning object in a compartment. The two external factors are the radiative heat flux from the hot ceiling layer and the oxygen concentration in the compartment. The lateral flame spread rate measured by Quintiere and Harkleroad (1984) is used to include the effect due to the radiative heat flux. The varying oxygen concentration is built into the model using the finding by Tewarson and Pion (1976), that the burning rate decreases with decreasing oxygen concentration and flaming combustion ceases when the oxygen concentration is lower than 11% regardless of what the external heat flux may be.

An equation described by Tewarson and Pion (1976) for the mass burning rate is used but, with the burning surface area being calculated assuming that the flame spreads radially on the burning object with a speed $V_f$, where $V_f$ is the speed based on Tewarson and Pion (1976).

The oxygen concentration in the gas mixture at any time is obtained using two different equations, one when the concentration is above 16% and the other when the oxygen concentration falls below 16%. This ensures that the oxygen concentration does not drop too fast and thus cause the fire to be extinguished prematurely.

The heat loss rate through the compartment walls is obtained by solving the one-dimensional heat conduction equation with some boundary conditions.

The air ventilation rate through the compartment opening is calculated using the empirical equations derived by Steckler and Quintiere (1982) and Prahl and Emmons (1975) with a correction factor to take into account the effect of compartment size.

The $CO$ and $CO_2$ concentrations in an air ventilated fire are obtained using a stoichiometric equation, experimental data and the mass ratio of $CO$ to $CO_2$.

In a smoldering fire the burning rate, $R$, at any time, $t$, is determined by Quintiere and Birky (1982) as

$$R = 0.1t + 0.0185t^2$$

The $CO$ and $CO_2$ concentrations are calculated using the following equations for the smoldering fire

$$Y_{CO} = 0.05Y_{PRO}$$

$$Y_{CO_2} = 0.56Y_{PRO}$$

where $Y_{CO_2}$ is the concentration of $CO_2$ (weight %), $Y_{CO}$ is the concentration of $CO$ (weight %) and $Y_{PRO}$ is the product gas concentration (weight %).

This model of Takeda and Yung (1991) is implemented using the algorithm in figure (2.2) in the NRCC's, risk cost assessment model for evaluating the fire risk and protection costs in apartment buildings

## 2.3 Stationary Stochastic Models in Fire Research

Berlin (1990) describes a number of simple probability models which can be applied to fire protection problems. He describes how they can be used in estimating the annual losses due to fires from a randomly selected transformer in terms of the three outcome measures: property damage, injuries, and deaths.

Markov chains are described relating to how they could be used to model a fire in a residence given by the Building Fire Simulation Model. This is done by dividing the time over which a fire develops into a finite number of stages and assigning probabilities to the likely transition from one stage to another. From this model it is possible to answer questions like

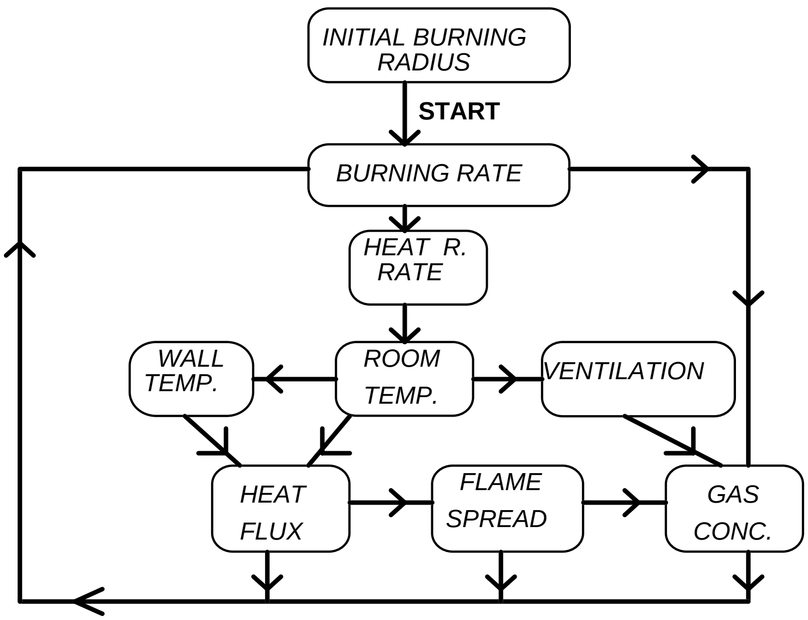


Figure 2.2: Computation algorithm of the NRCC fire growth model

1. What is the maximum extent of the fire growth?

2. What is the expected number of transitions between any stage and another?

The major weakness of this model is the assumption of constant transition probabilities.

Queuing models are described modelling a Fire Brigades availability, where the arrival process corresponds to fire alarms, and the service rate is the time to respond to the emergency and then return to the station.

Stress-Strength models are used for the situation where a component accomplishes its intended function, provided it is strong enough to sustain the opposing forces of the operating environment.

The section on Markov Chains by Berlin (1990) has some relevance to this research. The probability and statistical models described by Berlin, have not been applied to experimental data to estimate model parameters.

Stochastic fire and smoke spread models available at present are time dependent, Elms and Buchanan (1981), Beck (1987), Beck (1988), Ramachandran (1990).

Existing models are extremely limited in their ability to accurately predict the levels of risk to life safety because the time-dependent nature of fire spread is not included.

Fire growth and spread are phenomena that exhibit all the hallmarks of growth, intrinsic randomness and high variability between one occurrence and the next. It is necessary to develop new models specially adapted to the description of fire growth and spread. As a stepping stone into the modelling of fire growth and spread in buildings this work will examine modelling compartment fires using several non-stationary stochastic growth models and introduce the use of Percolation Processes and Contact Processes for the spread phenomena.

## 2.4 Stochastic Growth Models

There are several models in the class of Epidemic processes, see chapter (3) for a definition, which could be used to model the growth of compartment fires. The essential characteristics of any epidemic process is the transfer of infection. The equivalent characteristic in compartment fires is the transfer of flame/fire. We will initiate our research with an examination of the Epidemic models and then try to equate compartment fires to these equations. Detailed accounts of the epidemic models are given by Becker (1989) and Bailey (1957).

Hammersley (1957) defines a percolation process as typically the spread of a fluid through a medium under the influence of a random mechanism associated with that medium, see chapter (4) for a review and a graphical representation of the percolation process. This model has analogies with fire spreading along a level of a building, this is investigated further in chapter (4).

Bezuidenhout and Grimmett (1991), define a Contact process as a stochastic model for the spread of disease amongst the members of a population distributed about a d-dimensional space, $Z^d$ see chapter (4) for a review and a graphical representation of the Contact Process. A contact process is a type of oriented percolation process. This model has analogies with fire spreading through the levels of a building, this is investigated further in chapter (4).

The three non-stationary stochastic models described above have two common characteristics which are also revealed in the physical properties of compartment fires:

1. They are non-stationary growth models,

2. They have a threshold theorem.

With fire the threshold appears to be the point at which flashover is reached, but further investigation and modelling is required to be more conclusive.

# Part I

# NON-STATIONARY MODELS FOR COMPARTMENT FIRES

There are several models in the class of non-stationary processes which could be used to model the growth and spread of fires inside compartments. Three epidemic models are considered for the **growth** of a fire inside a compartment: The Deterministic Simple Epidemic, the Deterministic General Epidemic and the Stochastic General epidemic. Two additional non-stationary stochastic models are considered for the **spread** of a fire through a building: The Percolation process and the Contact process. In the final chapter of this part we develop a methodology for converting deterministic equations into non-deterministic (stochastic) equations.

# Chapter 3

# Epidemic Models

## 3.1 Introduction

The essential characteristic of any epidemic process is the transfer of infection. Before looking at some specific epidemic models, we will make a number of assumptions which will be common to all of these models:

- The disease is transmitted by contact between an infected individual and a susceptible individual
- There is no latent period for the disease, hence the disease is transmitted instantaneously upon contact
- All susceptible individuals are equally susceptible and all infected individuals are equally infectious.
- Susceptibles and infectives mix together homogeneously.

## 3.2 Deterministic Simple Epidemic Model

The simple epidemic model describes the spread of a relatively mild infection through a finite population in which none of the infected individuals is removed from the population by isolation, recovery or death. This means that no births, deaths or migration occurs.

To calculate the equations of the model we let time be represented by $t$ and a small change in time by $\delta t$. The number of infected individuals at time $t$ is represented by $I(t)$ and the number of susceptible individuals (healthy individuals) at time $t$ is represented by $S(t)$. The average number of contacts between susceptible and infected individuals which lead to a new infective per unit of time per infective per susceptible in the population is represented by $\beta$.

Since everyone in the population is either susceptible to the disease or else infected with the disease. We have $S(t) + I(t) = N$, where $N$ is the total number in the population.

It is a simple matter to deduce the number of susceptible individuals at time $t + \delta t$ in terms of the number of susceptibles at time $t$. Clearly, $S(t + \delta t)$ is just the number of susceptibles at time $t$, $S(t)$, minus the number of susceptibles who contract the disease in the time interval from $t$ to $t + \delta t$. In mathematical notation,

$$S(t + \delta t) = S(t) - \beta S(t)I(t)\delta t.$$

Epidemics are discrete phenomena. However, we approximate them with continuous variables due to large populations.

Next, rearrange the equation into the form:

$$\frac{S(t + \delta t) - S(t)}{\delta t} = -\beta S(t)I(t)$$

and then let $\delta \mathrm{t} \to 0$. This yields

$$\frac{dS(t)}{dt} = -\beta S(t)I(t).$$

Using $S(t) + I(t) = N$

$$\frac{dS(t)}{dt} = -\beta S(t)[N - S(t)].$$

This is a nonlinear differential equation, but the equation can be solved using the separation of variable method of solution as for a linear differential equation. Rewriting as

$$\frac{1}{S(t)[N - S(t)]}\frac{dS(t)}{dt} = -\beta.$$

Expanding the left-hand side using partial fractions we obtain

$$(\frac{1}{NS(t)} + \frac{1}{N[N - S(t)]})\frac{dS(t)}{dt} = -\beta$$

Integrating both sides with respect to $t$,

$$\int \frac{1}{NS(t)}dS(t) + \int \frac{1}{N[N - S(t)]}dS(t) = \int -\beta dt$$

gives

$$\frac{1}{N}[\ln S(t) - \ln(N - S(t))] = -\beta t + c$$

or

$$\frac{S(t)}{N - S(t)} = ke^{-\beta N t}.$$

Simplifying further gives

$$S(t) = \frac{N}{1 + \frac{1}{k}e^{\beta N t}}.$$

Solving for the integration constant, $k$, from initial conditions, $S(0) = N - \mathrm{I}_0$, gives

$$k = \frac{N - I_0}{I_0}$$

$$S(t) = \frac{N(N - I_0)}{(N - I_0) + I_0 e^{\beta N t}}.$$

In addition, since the total population size is always $N$, and since all individuals are either susceptible or infected, using $I(t) = N - S(t)$ we can solve for $I(t)$,

$$I(t) = \frac{NI_0}{I_0 + (N - I_0)e^{-\beta N t}}.$$

A typical solution as a function of time is shown in the figure (3.1):

The graph suggests that in a large population with a small initial number of infectives, at first the epidemic (as measured by the total number of infectives) grows exponentially, and then, as fewer susceptibles are available, the rate of growth

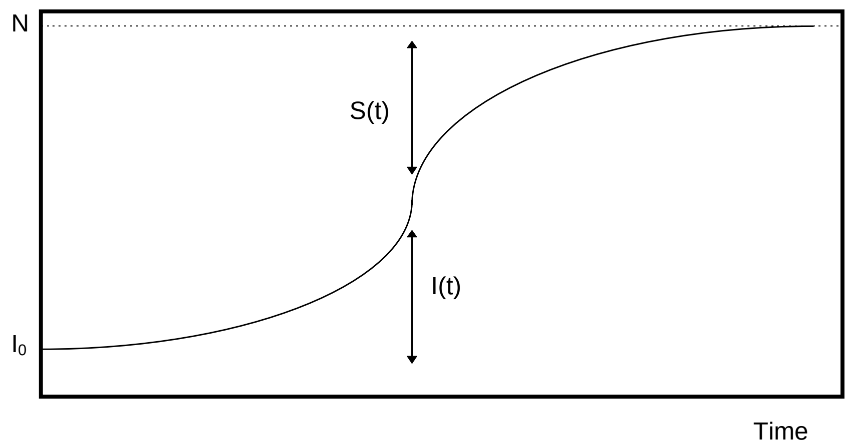

Figure 3.1: Change in Susceptibles into Infectives over time.

decreases, but the epidemic does not end until everyone in the population has contracted the disease.

The more usual quantity to report is the 'epidemic curve' which records the rate at which the disease spreads in the population. For the present model the epidemic curve, $W(t)$, is the rate of change in the number of infectives, thus

$$
\begin{aligned}
W(t) &= \frac{dI(t)}{dt} = \beta S(t) I(t) \\
&= \frac{\beta N^2 (N - I_0) I_0 e^{\beta N t}}{[(N - I_0) + I_0 e^{\beta N t}]^2}
\end{aligned}
$$

A graph of this function is in figure (3.2).

This model is an extremely simple first model, and has a rather unrealistic aspect. Notice that whenever an epidemic gets started, everyone in the population ultimately contracts the disease. The reason for this can be traced to the fact that infectives remain infected forever. A more realistic model must take into account that for most diseases infectives either recover or else they die.

### 3.2.1 Deterministic Simple Epidemic Fire Growth Model

To adapt the Simple Epidemic model to describe the growth of a fire inside a compartment table (3.1) draws an analogy between the variables in the fire growth

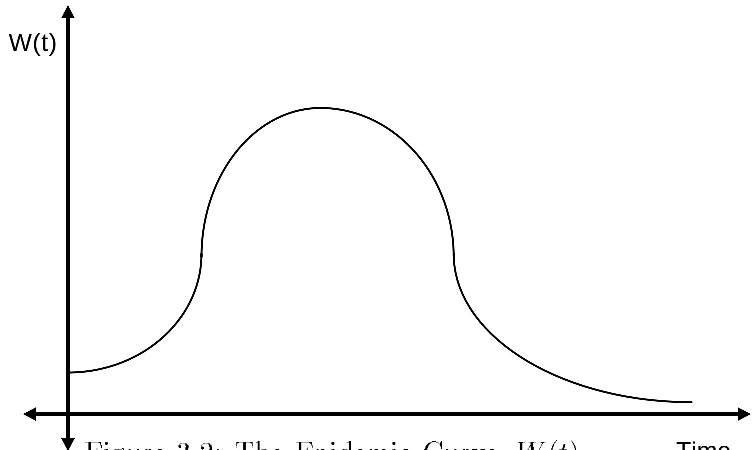


Figure 3.2: The Epidemic Curve, $W(t)$.

model and the Simple Epidemic model.

To calculate the equations of the model we let time be represented by $t$ and a small change in time by $\delta t$. The amount of burning material at time $t$ is represented by $F(t)$ and the amount of combustible material at time $t$ is represented by $M(t)$. As fire spreads by the contact between flames and the combustible material, the average number of contacts between combustible and burning material which lead to a new burning material per unit of time per combustible per burning material in the compartment is represented by $\beta$.

| **FIRE** | **EPIDEMIC** |
|---|---|
| Combustibles | Susceptibles |
| Burning Material | Infectives |

Table 3.1: Analogy between fire growth and Simple Epidemic variables

Assumptions of the deterministic fire growth model:

1. When a material catches fire it is immediately capable of spreading the fire (no latent period).

2. Combustible material is subject to homogeneous mixing. We can relax this

assumption by using the modification introduced by Becker (1977) for non-homogeneous mixing.

For the deterministic simple epidemic model for fire growth, if we let

$N$ = Total amount of combustible material in a compartment

$t$ = epoch time

$M$ = combustible material remaining

$F$ = amount of material on fire (burning material)

The average number of contacts between combustibles and burning materials which lead to a new burning material per unit of time per burning material per combustible in the compartment be represented by $\beta$.

We have $M(t) + F(t) = N$.

It is a simple matter to deduce the amount of combustible material at time $t + \delta t$ in terms of the amount of combustible material at time $t$ in exactly the same way as we did for the simple epidemic model.

$$M(t + \delta t) = M(t) - \beta M(t)F(t)\delta t$$

Solving the equation gives

$$M(t) = \frac{N}{1 + \frac{1}{k}e^{\beta N t}}$$

Solving for the integration constant, $k$, from initial conditions, M(0)=N-$F_0$, gives k=$\frac{N-F_0}{F_0}$

$$M(t) = \frac{N(N - F_0)}{(N - F_0) + F_0 e^{\beta N t}}$$

In addition, since the total combustible material is always $N$, and since all material are either combustible or burning, using $F(t) = N - M(t)$ we can solve for $F(t)$.

$$F(t) = \frac{NF_0}{F_0 + (N - F_0)e^{-\beta N t}}$$

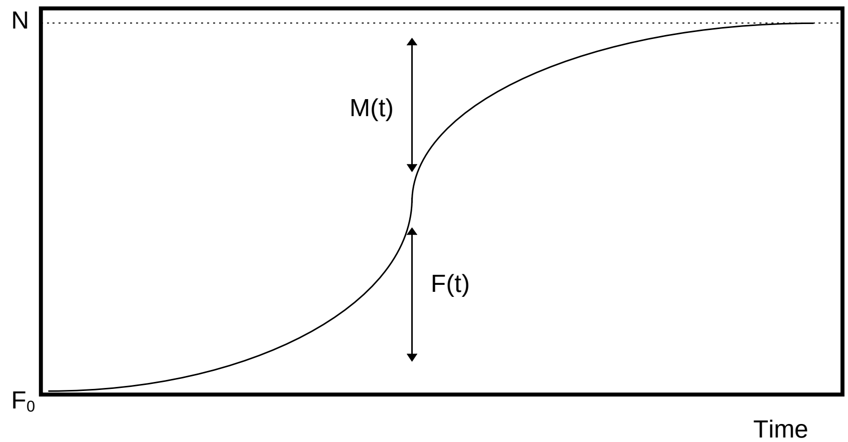

Figure 3.3: Change in Combustible Material into Burning Material over time.

A typical solution as a function of time is shown in the figure (3.3):

The graph suggests that in a large compartment with a small initial number of burning material, at first the fire (as measured by the total amount of burning material) grows exponentially, and then, as fewer combustible material is available, the rate of growth decreases, but the fire does not end until all of the combustible material in the compartment has contracted the fire.

A more useful quantity to report would be the 'fire curve' which records the rate at which the fire spreads in the compartment. For the present model the fire curve, $X(t)$, is the rate of change in the amount of burning material, thus

$$\begin{aligned} X(t) &= \frac{dF(t)}{dt} = \beta M(t)F(t) \\ &= \frac{\beta N^2(N-F_0)F_0 e^{\beta N t}}{[(N-F_0)+F_0 e^{-\beta N t}]^2} \end{aligned}$$

A graph of this function is in figure (3.4).

This model is a very simple first model and has a rather unrealistic aspect. Notice that whenever a fire gets started, everything in the compartment ultimately contracts the fire. The reason for this can be traced to the fact that burning material remain burning forever. A more realistic model must take into account that for most fires burning material either stop burning or else they burn out.

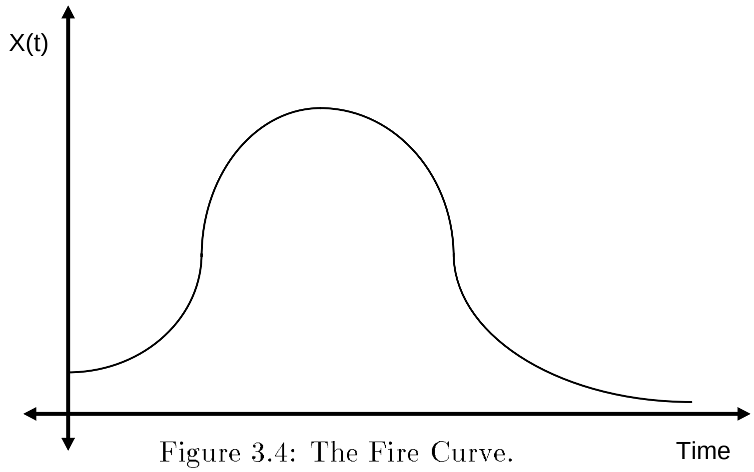

Figure 3.4: The Fire Curve.

## 3.3 Deterministic General Epidemic Model

Since for many diseases a natural immunity occurs, it is further assumed that former infectives enter a new class which is not susceptible to the disease, the Removals. The General Epidemic model considers the transfer of infection by contact between the members of the population, as well as the removal of infectives from the population by recovery, death or isolation.

By introducing a class of removed individuals, we have managed to avoid a precise statement of the severity of the disease being modelled. The removals may be recovered and immune, or they may be quarantined and thus out of circulation or they may be dead. All that is necessary is that the disease not be available to any individual more than once. Therefore, the basic parameters in the General Epidemic model are the infection rate, $\beta$, and the removal rate, $\gamma$. In addition to the variables defined in the Simple Epidemic model, let:

$R(t)$ = number of removed individuals at time t. Removed from the infected group and

$\gamma$ = average rate of removal of infectives from circulation per unit time per infective in the population.

Assumptions of the General Epidemic model are:

1. Any individual who has recovered from the disease has permanent immunity.
2. The disease has a negligible short incubation period, no latent period. When a susceptible is infected it is assumed that he immediately becomes infectious.
3. The assumption of homogeneous mixing.

Since the new class of individuals, the removals, in no way interacts with the susceptibles, the governing equation for the susceptibles is unchanged from the simple epidemic model. Thus the differential equation is

$$\frac{dS(t)}{dt} = -\beta S(t)I(t) \tag{3.1}$$

The differential equation developed previously for the number of infectives must be modified to take into account the removals. Using an argument similar to the one for the Simple Epidemic model, it is not hard to deduce the equation

$$\frac{dI(t)}{dt} = \beta S(t)I(t) - \gamma I(t) \tag{3.2}$$

The individuals who are removed from the ranks of the infectives then contribute to the number of removed individuals according to the relation

$$\frac{dR(t)}{dt} = \gamma I(t) \tag{3.3}$$

Since all individuals in the population are either susceptible, infected or removed and the population is constant in size,

$$S(t) + I(t) + R(t) = N \tag{3.4}$$

By differentiating this last expression with respect to time, it follows that the three governing equations must sum to zero (as they in fact do.) In addition, the last expression guarantees that once the size of any two of the classes is known, the size of the third follows by simple arithmetic.

To complete the specification of the model it is necessary to know the initial state of the population. Assume that at time $t = 0$ there are no removed individuals, a very small number, $I_0$, of infectives, and the remainder of the population, $S_0$, is susceptible. Thus, $S(0) = S_0 = N - I_0$; $I(0) = I_0 << N$ and $R(0) = 0$. Before attempting to find a solution to the set of governing equations, it is informative to look carefully at the equations. Specifically, look at the equation for the number of infectives in the form

$$\frac{dI(t)}{dt} = \beta[S(t) - \rho]I(t) : \rho = \frac{\gamma}{\beta}.$$

where $\rho$ is defined as the *relative removal rate.*

Clearly, since $I(t) > 0$, the sign of the term in square brackets is the same as the sign of $dI(t)/dt$, hence $dI(t)/dt > 0$ if and only if $S(t) > \rho$. Further, since $S(t)$ is a monotonically decreasing function of time (since susceptibles become infected and no new susceptibles are made) if $S(0) < \rho$ then $S(t) < \rho$ for all $t > 0$ and $dI(t)/dt < 0$ for all future time.

In other words, if the initial number of susceptibles is smaller than some critical number, $\rho$, there will not be an epidemic (where here the word epidemic is used in the technical sense of a large, one-time outbreak of the disease).

We proceed now to analyse the model in detail. To do so, begin by eliminating the explicit dependence on $I(t)$ between the first and the third of the governing differential equation to get

$$\frac{dS(t)}{dt} = -\frac{S(t)}{\rho}\frac{R(t)}{dt}$$

Separating variables, multiplying through by $dt$ and integrating leads to

$$S(t) = S_0 e^{-R(t)/\rho}.$$

Next, make use of the relation $S(t) + I(t) + R(t) = N$ in the equation for $R(t)$:

$$\frac{dR(t)}{dt} = \gamma I(t) = \gamma[N - R(t) - S(t)]$$

and then use the expression just derived to eliminate $S(t)$; thus

$$\frac{dR(t)}{dt} = \gamma I(t) = \gamma[N - R(t) - S_0 e^{-R(t)/\rho}]$$

Note that $R(t)$ is the only one of the dependent variables which appears in this equation. Although it is possible to solve this differential equation exactly, the methods are rather complicated. We therefore seek an approximate solution. Since this difficulty in solving the equation results from the presence of the exponential term, we proceed to replace the exponential by a polynomial. To do so we expand the exponential in a Taylor Series about the only point at which we know the value of $R(t)$. Specifically, we expand about $R(0) = 0$; this leads to:

$$e^{-R(t)/\rho} = 1 - (\frac{R(t)}{\rho}) + \frac{1}{2}(\frac{R(t)}{\rho})^2 - \frac{1}{6}(\frac{R(t)}{\rho})^3 + ...$$

Clearly, if one attempts to retain the entire infinite series, nothing has been gained. By truncating the series after the first few terms, a separable differential equation which is fairly easily solved will result. The question remains, how many terms should be retained? It is not difficult to show that if only terms up to the linear one are kept, only an absurd answer is possible. On the other hand, if terms up to the cubic one are kept, the resulting integration is very hard. We therefore choose to keep terms up to the quadratic one, thereby balancing realism against solvability. Following a bit of rearranging, the resulting equation is:

$$\frac{dR(t)}{dt} = \gamma[I_0 + (\frac{S_0}{\rho} - 1)R(t) - \frac{S_0}{2\rho^2}R(t)^2]$$

Separating variables and integrating leads to the expression:

$$R(t) = \frac{\rho^2}{S_0}[\frac{S_0}{\rho} - 1 + \alpha \tanh(\frac{\alpha\gamma t}{2} - \phi)]$$

where $\alpha = [(S_0/\rho - 1)^2 + 2S_0 I_0/\rho^2]^{1/2}$ and $\phi = \tanh^{-1}[(S_0/\rho - 1)/\alpha]$.

Developing this solution is straightforward, but does involve a considerable amount of rather messy algebra.

As with the Simple Epidemic model, we are really more interested in knowing the shape of the predicted epidemic curve, $W(t)$, than the cumulative number of removals, $R(t)$. Since cases of the disease are counted as victims seeking medical attention, and this is also the time at which individuals are removed from active circulation, it is customary to assume that

$$W(t) = \frac{dR(t)}{dt} = \frac{\gamma\alpha^2\rho^2}{2S_0}\text{sech}^2(\frac{\alpha\gamma t}{2} - \phi)$$

Note that this expression describes a function which rises to a single maximum at time $t = 2\phi/\alpha\gamma$ and then dies away symmetrically. This is very similar to the result for the epidemic curve in the Simple Epidemic model; however, in this model not all susceptibles need to be infected.

### 3.3.1 Deterministic General Epidemic Model For Fire Growth

To adapt the General Epidemic model to describe the growth of a fire inside a compartment we let the burning rate be denoted by $\beta$ as fire spreads by the contact between flames and the combustible material. The rate of removal or decrease of combustible material in the building can be represented by the burnout rate, $\gamma$. Table (3.2) draws an analogy between the variables in the fire growth model and the General Epidemic model.

| **FIRE** | **EPIDEMIC** |
|---|---|
| Combustibles | Susceptibles |
| Burning Material | Infectives |
| Burnt Material | Removals |

Table 3.2: Analogy between fire growth and General Epidemic variables.

Assumptions of the deterministic fire growth model:

1. Material burnt is removed permanently or immune to catch fire indefinitely

2. Independent isolated material of a given size is subject to homogeneous mixing

3. When a material catches fire it is immediately capable of spreading the fire (no latent period)

4. All combustible material are equally combustible and all burning material are equally capable of spreading fire.

For the deterministic general epidemic model for fire growth, if we let

$N$ = Total amount of combustible material in a compartment

$t$= epoch time

$M$ = combustible material remaining

$F$ = amount of material on fire (burning material)

$B$ = amount of material combust(burnt-out material).

Then $M(t) + F(t) + B(t) = N$ at any time t. At time $t = 0$ if one unit amount of substance is on fire, $F = 1$, $B = 0$ and the remaining $N - 1$ combustible material is available to catch on fire. Hence $N = M(0) + F(0) + 0$. If the rate of fire growth (burning rate) is proportional to both the amount of combustible material and the amount of material ignited, then the amount of new material burnt in the time interval δt can be written as $\beta MF\delta t$, where $\beta$ is the burning rate.

$$\delta M = -\beta MF\delta t$$

or

$$M(t) - M(t + \delta t) = \beta MF\delta t$$

writing this as differential equation

$$\frac{dM}{dt} = -\beta MF.$$

Likewise for the increase in the amount of burnt-out material

$$\frac{dB}{dt} = \gamma F$$

where $\gamma$ is the rate of removal or decrease of combustible material in the building, the burnout rate. Also for the increase in the amount of burning material

$$\frac{dF}{dt} = \beta MF - \gamma F.$$

Approximate solutions to the above equations can be found by assuming $\beta$ and $\gamma$ constant. If we let $\rho = \frac{\gamma}{\beta}$, the relative removal rate of combustible material then

$$F = N - B - M$$

$$M = M_0 e^{-B/\rho}$$

$$B = \rho^2 [\frac{M_0}{\rho} - 1 + \alpha \tanh(\frac{\alpha \gamma t}{2} - \phi)]/M_0$$

where $\alpha = [(M_0/\rho - 1)^2 + 2M_0F_0/\rho^2]^{1/2}$ and $\phi = \tanh^{-1}[(M_0/\rho - 1)/\alpha]$.

The equations for the variables of this compartment fire model can be determined similar to the deterministic General Epidemic model.

## 3.4 Stochastic General Epidemic Model

In the preceding models we have studied deterministic growth models. Such an analysis is usually satisfactory for the study of a reasonably large population. Deterministic systems are incompatible for modelling the changes in a small population. When one is interested in modelling the changes in small populations then the deterministic approach is not appropriate. When one is concerned with relatively few individuals, then the growth of the system may be strongly influenced by chance events. If the model is to be useful in connection with the explanation and prediction of observable phenomena, then these chance events cannot be ignored, and we are led naturally to consider stochastic models.

The predictions produced by the deterministic and stochastic models are intrinsically different. Whereas the deterministic model provides a function giving the size of the population for any specified time, the stochastic model gives a probability distribution of population size for each time. Thus our goal is to produce a family of probability distributions, one for each instant in time in which we are interested.

Studying deterministic models is in no way a waste of time. Using deterministic models we can gain some insight into the mechanism of large scale phenomena, and the results will suggest various features worth examining more carefully when we come to stochastic models.

A set of deterministic equations can be turned into a set of stochastic differential equations by either adding on the right hand side forcing functions which could be for example white noise multiplied by some function of the variables and/or endowing the parameters with probability distributions.

When dealing with deterministic equations we generally use continuous variables as it is okay to make this assumption when the populations are large. In the case of stochastic equations we use discrete variables as it gives new light for the deterministic case. Deterministic models can be viewed as an average of the stochastic model.

If at time $t$ there are $S(t)$ susceptibles, $I(t)$ infectives and $R(t)$ removals in the population, and if $N$ is the total population size we have $S(t)+I(t)+R(t) = N$. Then from Becker (1989), the General Epidemic model can be summarised in table (3.3) using the probabilities of the associated transitions for a time increment $(t, t+h)$, and the initial conditions $S(0) = k$, $I(0) = I_0$ and $R(0) = 0$.

Using these transition probabilities we are able to write the equations of the Stochastic General Epidemic model, see Bailey(1957).

Becker (1976), states that associated with the Stochastic General Epidemic model is the Stochastic Epidemic Threshold theorem, which states essentially that for large $k$

| TRANSITION | PROBABILITY |
|---|---|
| $(S, I, R) \rightarrow (S-1, I+1, R)$ | $\beta SI + o(h)$ |
| $\rightarrow (S, I-1, R+1)$ | $\gamma hI + o(h)$ |
| $\rightarrow (S, I, R)$ no change | $1 - \gamma hI - \beta SI + o(h)$ |

Table 3.3: Transition probabilities for the Stochastic General Epidemic model.

$$\text{Pr (minor epidemic)} = 1 - \text{Pr (major epidemic)}$$
$$= \min(1, (\frac{\gamma}{k\beta})^{I_0})$$

Here the initial infection rate, $\frac{k\beta}{\gamma}$, where $\beta$ is the infection rate and $\gamma$ is the removal rate, determines the probability of a major outbreak.

### 3.4.1 Stochastic General Epidemic Model For Fire Growth

The Deterministic General Epidemic Model for Fire Growth can be made into a Stochastic General Epidemic Model for Fire Growth from the transition probabilities of the model, as done in the Stochastic General Epidemic model. The probabilities of the associated transitions for a time increment $(t, t+h)$ for the General Epidemic Fire Growth model can be summarised in a table using the initial conditions $M(0) = M_0$, $F(0) = F_0$ and $B(0) = 0$, see table (3.4).

| TRANSITION | PROBABILITY |
|---|---|
| $(M, F, B) \rightarrow (M-1, F+1, B)$ | $\beta MF + o(h)$ |
| $\rightarrow (M, F-1, B+1)$ | $\gamma hF + o(h)$ |
| $\rightarrow (M, F, B)$ no change | 1 - $\gamma hF - \beta MF + o(h)$ |

Table 3.4: Transition probabilities for the Stochastic General Epidemic Fire Growth model

To write the equations of the Stochastic General Epidemic Model for Fire Growth suppose that at $t = 0$ there are $n$ units of combustibles and $a$ units of

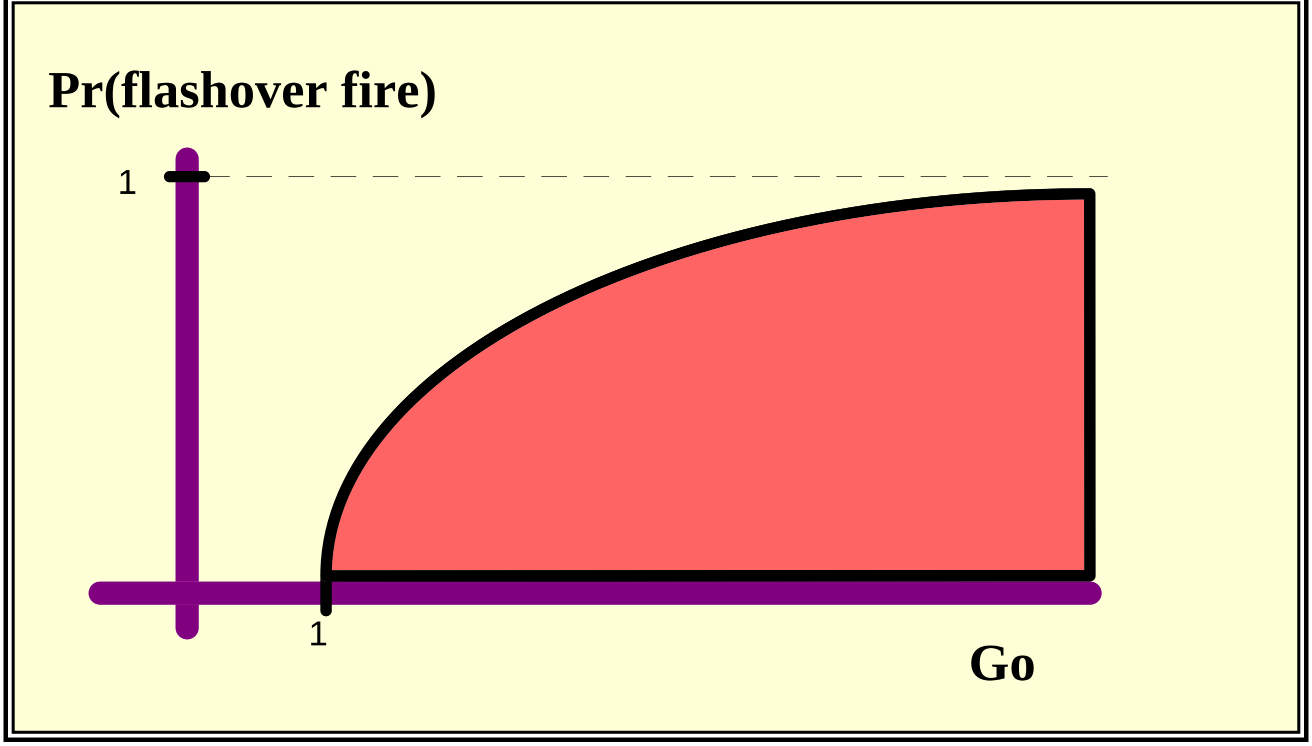


Figure 3.5: Probability of a flashover fire versus initial ignition rate.

burning material. If we write $P_{MF}(t)$ for the probability that at time, $t$, there are $M$ combustible units still to burn and $F$ units of burning material, and the relative burnt-out rate is given by, $\rho = \frac{\gamma}{\beta}$. Using the time-scale given by $\tau = \beta t$ instead of $t$ leads to the differential-difference equations given by Bailey(1957).

$$\frac{dP_{MF}}{d\tau} = (M+1)(F_1)P_{M+1,F-1} - F(M+\rho)P_{MF} + \rho(F+1)P_{M,F+1}$$

and

$$\frac{dP_{na}}{d\tau} = -a(n+\rho)P_{na},$$

where $0 \leq M+F \leq n+a$, $0 \leq M \leq n$, $0 \leq F \leq n+a$ and initial condition $P_{na}(0) = 1$ where $P_{na}$ is the probability that at time $t = 0$ there are $n$ units of combustible material and $a$ units of burning material.

These differential-difference equations cannot be solved exactly but, some asymptotic solutions based on the pure birth-death process exist for the General Stochastic Epidemic equations.

Using the Stochastic Epidemic Threshold theorem from Becker (1976) we can state the threshold for the Stochastic General Epidemic Model of Fire Growth.

Pr(non-flashover fire) = 1- Pr (flashover fire) = $min(1, (\frac{\gamma}{\beta M_0})^{F_0})$. Here the initial ignition rate, $G_0 = (\frac{\gamma}{\beta M_0})^{F_0}$ , determines the probability of a flashover fire. Pr(flashover fire) = 1-(1/$G_0$)

The graph of the above equation is in figure (3.5). It shows how the probability of a flashover fire increases with increasing initial ignition rate.

## 3.5 Conclusion

As a first approximation these epidemic models appear fine but for fire growth they have limited physical interpretation. From the fire literature the three main factors affecting the growth of a compartment fire are the gas temperature in the room, the burning rate and the oxygen concentration. In part II of this thesis we look at fire growth models which use these three factors.

# Chapter 4

# Fire Spread Using Percolation and Contact Processes

## 4.1 Introduction

The rate at which fire will develop will depend on how rapidly flame can spread from the point of ignition to involve an increasingly large area of combustible material, Flame spread is considered as an advancing ignition front in which the leading edge of the flame acts both as the source of heat, to raise the fuel ahead of the flame front to the fire point, and as the source of pilot ignition. There are various material and environmental factors which are known to be significant in determining the rate of flame spread over combustible solids, composition of atmosphere, temperature, composition of fuel and surface orientation are just some of them.

In this chapter we will attempt to model spread of a fire along a level of a building using a Percolation process and the spread of a fire through the levels of a building using a Contact process.

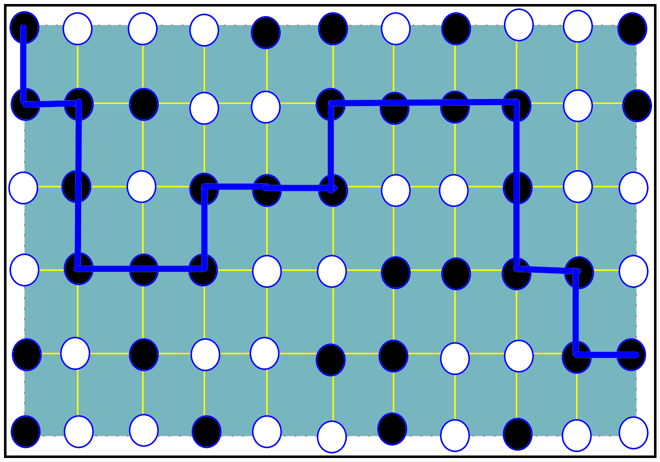

Figure 4.1: Two dimensional representation of a Percolation process.

## 4.2 Percolation Process

A percolation process is typically the spread of a fluid through a medium under the influence of a random mechanism associated with that medium.

Hammersley (1957), considers the medium to be made up of an infinite set of atoms bonded together with either an undirected bond or a directed bond. An undirected bond is defined as one which allows passage from either atom to the other and a directed bond is one which will allow passage from one atom to the other but not vice versa. Each bond has an independent probability $p$ of being undirected and $q = 1 - p$ of being directed.

The spread of a fluid in the medium can occur only via undirected bonds. Hammersley (1957), defines a fluid as wetting an atom in the medium when it spreads along the undirected bonds of the medium.

The percolation process is above its threshold if there is at least one path of undirected bonds where a fluid can travel from one end of the medium to the other. This model can be represented on a two dimensional lattice as in figure (4.1) where $\bigcirc$ represents atoms of the medium with directed bonds and the $\bullet$ represents atoms in the medium with undirected bonds. The path represented by the thick black line indicates the system is above the threshold of the medium.

In this sense a percolation process can be considered to be a restricted random walk.

### 4.2.1 Modelling the Spread of Fire Along a Level of a Building Using a Percolation Process

The spread of a fire through a level of a building under the influence of a random mechanism associated with the building can be thought of as a percolation process.

The level of a building (medium) can be considered to be made up of a set of compartments (atoms) separated by barriers (bonds). The barriers are considered either as walls with openings, doors and/or windows (undirected bonds); or solid fire rated walls without openings (directed bonds). The walls with openings can be defined as barriers which allow the passage of flame from one compartment to the other following or prior to flashover and the solid fire rated walls can be defined as barriers which do not allow the passage of flame from one compartment to the other. Each barrier has an independent probability $p$ of being a wall with an opening and $q = 1 - p$ of being a solid fire rated wall. The analogy is summarised in a table (4.1) as follows:

| **FIRE IN A BUILDING** | **SPREAD OF FLUID IN A MEDIUM** |
|---|---|
| Barrier | Bond |
| Fire | Fluid |
| Building | Medium |
| Walls with openings | Undirected Bonds |
| Solid fire rated walls | Directed Bonds |
| Compartment | Atom |

Table 4.1: Analogy between a fire in a compartment and a percolation process.

Table (4.1) provides an analogy between a percolation process defined by the spread of a fire on a level of a building and the spread of a fluid in a medium.

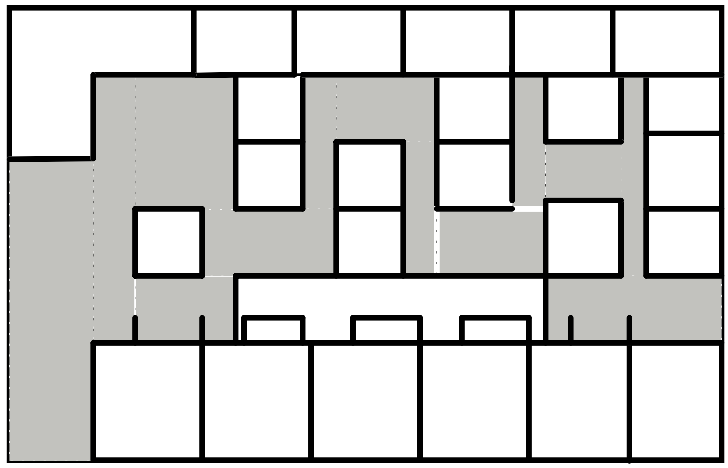

Figure 4.2: Two dimensional Percolation process for the Spread of Fire.

The spread of a fire in the level of a building can occur only via barriers which are walls with an opening. The building in this percolation process is above its threshold when there is at least one path of barriers which are walls with an opening where the fire can travel from one end of the level of a building to the other.

This model can be represented on a two dimensional diagram as in figure (4.2) where the solid lines, —, represents compartments of the building with solid fire rated barriers and the dotted lines, $\cdots$, represents compartments in the building with barriers which contain openings. The existence of a path represented by the shaded region indicates the system is above the threshold of the level of the building.

## 4.3 Contact Process

Bezuidenhout and Grimmett (1991), define a Contact process as a stochastic model for the spread of disease amongst the members of a population distributed about a d-dimensional space, $Z^d$.

If $\lambda$ is taken as the rate at which an individual infects their neighbour and $\delta$ is taken as the rate at which an infected individual is cured, then from Bezuidenhout

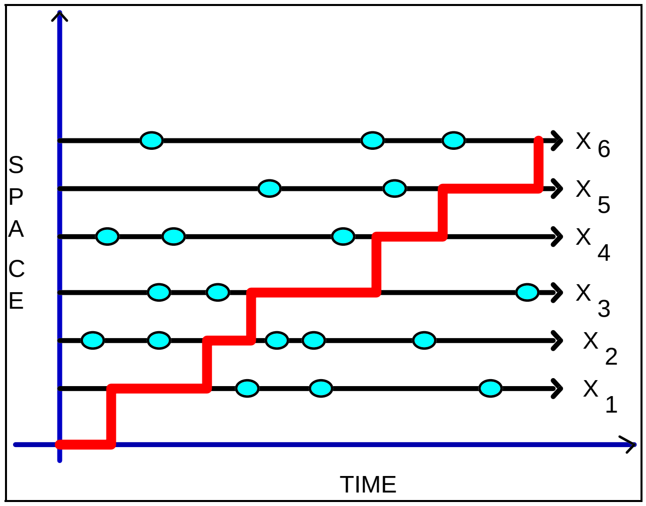


Figure 4.3: A graphical representation of a contact process.

and Grimmett (1991), there is a critical value $\rho_c$ of the ratio $\rho = \lambda/\delta$ such that the probability $\theta^0(\lambda, \delta)$ that the disease survives forever from a single initial infective satisfies

$$\theta^0(\lambda, \theta) \begin{cases} = 0 & \text{if } \lambda/\delta \leq \rho_c \\ > 0 & \text{if } \lambda/\delta > \rho_c \end{cases}$$

One technique for studying contact processes is via graphical representation, see figure (4.3). If we consider the graph $Z^d * [0, \infty)$, in which $Z^d$ represents the spatial component and $[0, \infty)$ represents time. Along each time line $X * [0, \infty)$ is positioned a Poisson process of points (with intensity $\delta$) called deaths and between each ordered pair $X_1 * [0, \infty)$ and $X_2 * [0, \infty)$ of adjacent time lines, there is a Poisson process (with intensity $\lambda$) of crossings oriented in the direction $X_1$ to $X_2$.

From this definition $\theta^0(\lambda, \delta)$ can be defined as the probability that there is an unbounded directed path from the origin of $Z^d * [0, \infty)$, using time lines in the direction of increasing time but crossing no deaths, together with crossings in the direction of their orientations.

The thick lines represent the crossings which have a Poisson process with parameter $\lambda$ and the shaded ovals represent the deaths, which have a Poisson distribution with parameter $\delta$, along each $X_i$.

A contact process is a type of oriented percolation process.

### 4.3.1 Modelling the Spread of Fire Through a Building Using a Contact Process

From Bezuidenhout and Grimmett (1991), we can define a Contact process as a stochastic model for the spread of a fire through the levels of a building.

If $\lambda$ is taken as the rate at which fire spreads from one level to the next and $\delta$ is taken as the rate at which burning material is burnt-out, then from Bezuidenhout and Grimmett (1991), there is a critical value $\rho_c$ of the ratio $\rho = \lambda/\delta$ such that the probability $\theta^0(\lambda, \delta)$ that the fire spreads through all levels from a single initial fire satisfies

$$\theta^0(\lambda, \theta) \begin{cases} = 0 & \text{if } \lambda/\delta \leq \rho_c \\ > 0 & \text{if } \lambda/\delta > \rho_c \end{cases}$$

If we consider the graph $Z^d * [0, \infty)$, in which $Z^d$ represents the building and $[0, \infty)$ represents time the fire travels along a specific level, then along each level $L * [0, \infty)$ is positioned a Poisson process of points (with intensity $\delta$) at which the fire burns out and between each level $L_1 * [0, \infty)$ and $L_2 * [0, \infty)$ of adjacent levels, there is a Poisson process (with intensity $\lambda$) of crossings, points where the fire spreads from one level to the next oriented in the direction $X_1$ to $X_2$. Using this definition we can represent the Contact process for the spread of fire in a building graphically, as in figure (4.4).

The thin horizontal lines represent the crossings, openings in the levels within a building where fire can spread, which have a Poisson process with parameter $\lambda$ and the thick lines represent the deaths, solid fire rated walls which can stand the load of the fire, which have a Poisson distribution with parameter $\delta$, along each $L_i$. The existence of a path represented by the shaded region indicates the system is

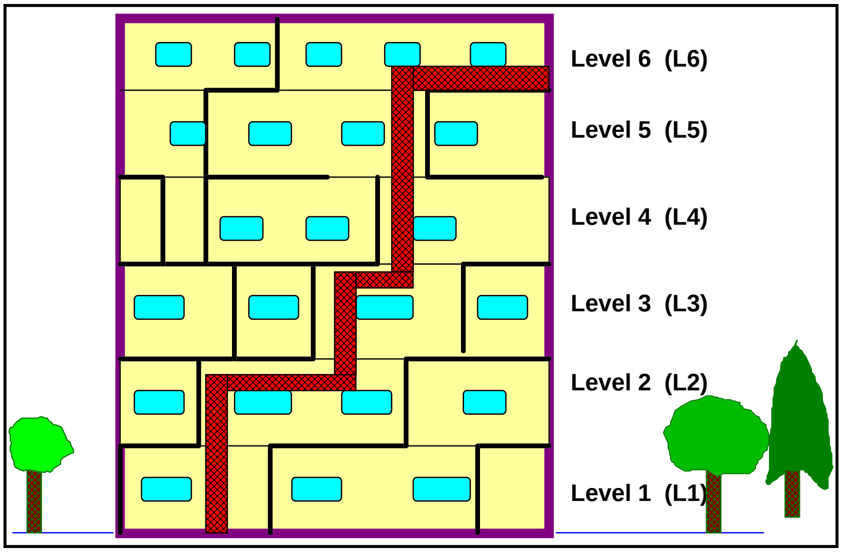


Figure 4.4: A graphical representation of a contact process for Fire Spread.

above the threshold of the building.

## 4.4 Conclusion

The two non-stationary stochastic models described above have two common characteristics which are also revealed in the physical properties of compartment fires:

1. They are non-stationary growth models, and

2. They have a threshold theorem.

These models can be studied using asymptotic theory, as $n$ approaches infinity. However, in modelling compartment fire spread there are only a finite number of compartments and levels. Hence the asymptotic theory of Percolation and Contact processes would not be applicable.

# Chapter 5

# Converting Deterministic Differential Equations to Stochastic Differential Equations

## 5.1 Introduction

The idea of using a mathematical model to describe the behavior of a physical phenomenon is well established. In particular, it is sometimes possible to derive a model based on physical laws, which enables us to calculate the value of some time-dependent quantity nearly exactly at any instant of time. If exact calculations were possible such a model would be entirely deterministic. Deterministic models attempt to derive deterministic differential equations which model the experimental data gathered on some of the chemical and physical interactions involved in a process.

Probably no phenomenon is totally deterministic, because unknown factors can occur. In many problems we have to consider a time-dependent phenomenon in which there are many unknown factors and for which it is not possible to write a deterministic model that allows exact calculations of the future behavior of the phenomenon. Nevertheless, it may be possible to derive a model that can be used to calculate the probability of a future value lying between two specific limits. Such a model is called a probability model or a stochastic model.

Deterministic differential equations may be thought of as a degenerate form of a stochastic differential equation in the absence of randomness. Hence deterministic models are a subset of stochastic models. In this paper we will present a method to convert ordinary differential equations into stochastic differential equations. But first it is useful to review some of the basic properties of a differential equation and a stochastic process.

## 5.2 Ordinary Differential Equations

Differential equations are separated into ordinary and partial as well as deterministic and stochastic. Ordinary differential equations are differential equations with only one independent variable,

$$\dot{x} = \frac{dx(t)}{dt} = r(t, x)x(t). \tag{5.1}$$

Partial differential equations are differential equations where there are two or more independent variables and partial derivatives are used. In our research we deal only with ordinary differential equations.

Equation (5.1) is the simple population growth model, where $x(t)$ is the size of the population at time $t$ and $r(t, x)$ is the relative rate of growth. We can write equation (5.1) in the symbolic differential form

$$dx(t) = r(t, x)x(t)dt \tag{5.2}$$

or, as an integral equation

$$x(t) = x_0 + \int_{t_0}^{t} r(s, x(s))x(s)ds \tag{5.3}$$

where $x(t) = x(t; x_0, t_0)$ is a solution satisfying the initial condition $x(t_0) = x_0$. Regularity assumptions, such as Lipschitz continuity, are usually made on $r$ to ensure the existence of a unique solution $x(t; x_0, t_0)$ for each initial condition. These solutions are then related by the evolutionary property

$$x(t; x_0, t_0) = x(t; x(s; x_0, t_0), s) \tag{5.4}$$

for all $t_0 < s < t$, which says that the future is determined completely by the present, with the past being involved only in that it determines the present. This is a deterministic version of the Markov property.

A stochastic process is a mathematical abstraction of an empirical process whose development is governed by probabilistic laws. Numerical observations made as the process continues indicate one realization of the stochastic process. With this background for guidance, Karlin (1975) defines a stochastic process as any family of random variables

$$X_t(\omega), \quad t \epsilon T,$$

where $X_t(\omega)$ is the observation at time $t$, $T$ is the time range and $\omega$ is the outcome space.

The distinguishing feature of a stochastic process $X_t(\omega)$ is the dependence structure the random variables, $X_t(\omega)$, for $t \epsilon T$. This dependence is, specified by giving the joint distribution function of every finite family $X_{t_1}(\omega), ...., X_{t_n}(\omega)$ of the variables of the process.

Stochastic processes are a function of two variables $t$ and $\omega$; that is, we can write $X_t(\omega)$ as $X(t, \omega)$. For a fixed value of $\omega$ it is just a function of $t$ and is called a sample function. If $T$ is the set of real numbers, the sample function is merely an ordinary function of a real variable. On the other hand, for a fixed value of $t$, the single observation is a random variable.

## 5.3 Heuristic Approach

To introduce stochastic variation into the simple population growth model, equation (5.1), we will assume that $r$ is not completely known, but subject to random effects. Hence we can replace $r(t,x)$ by $r(t,x) +$ "noise". Equation (5.1) can now be written as

$$\frac{dX(t)}{dt} = [r(x,t) + \text{"noise"}]X(t). \tag{5.5}$$

If we let $r(x,t)X(t) = a(t, X(t))$ and "noise"$X(t) = b(t, X(t))\xi_t$ then equation (5.5) can be written as

$$\frac{dX(t)}{dt} = a(t, X(t)) + b(t, X(t))\xi_t, \tag{5.6}$$

where $a(t, X(t))$ is the deterministic term, $b(t, X(t))$ is the space-time dependent intensity factor, $\xi_t$ is white noise and $b(t, X(t))\xi_t$ is the noisy diffusive term. Equation (5.6) can be written in a differential form as

$$dX(t) = a(t, X(t))dt + b(t, X(t))\xi_t dt. \tag{5.7}$$

At this point we can compare equations (5.1) and (5.2) with equations (5.6) and (5.7). The first term, $a(t, X(t))$ in the stochastic equation is an average drift term and is equivalent to the term, $r(t,x)x(t)$ in the deterministic differential equation. The stochastic equation has an extra term included, $b(t, X(t))\xi_t$. This extra term is the forcing function multiplied by white noise, it is responsible for introducing the stochastic variation. At this point we will define Gaussian white noise and its relationship to the Wiener process.

## 5.4 Gaussian White Noise and the Wiener Process

Gaussian white noise is an idealization of stochastic phenomena encounted generally in engineering systems analysis. Gard (1988) defines Gaussian white noise as being a model for a completely random process whose individual random variables are normally distributed. Gaussian white noise is mathematically defined as a scalar stationary Gaussian process $\xi(t)$ for $-\infty < t < \infty$ with $E(\xi(t)) = 0$ and a spectral density function $f(\lambda)$ which is constant on the entire real line, that is, if $C(t) = E(\xi(s)\xi(t+s))$

$$f(\lambda) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i\lambda t} C(t)dt = \frac{K}{2\pi}, \qquad \lambda \epsilon R,$$

for some constant $K$. Also since

$$C(0) = \int_{-\infty}^{\infty} f(\lambda)d\lambda = \infty,$$

the variance of $\xi_t$ is infinite, and such a process cannot be realised. Hence white noise is not a stochastic process in the usual sense. Furthermore, there is a relationship between Gaussian white noise and the standard Wiener process. The covariance of the derivative of the Wiener process is the covariance of white noise, see Gard(1985).

The Wiener process is the mathematical description of the physical process known as Brownian motion. The standard Wiener process, $W = W(t), \quad t \geq 0$, is defined by Kloeden (1994). It is a continuous Gausian process with independent increments such that

$$W(0) = 0, \qquad \text{w.p.1},$$

$$E(W(t)) = 0 \qquad \text{and}$$

$$\text{var}(W(t) - W(s)) = t - s$$

for all $0 \leq s \leq t$. Also from this definition $W(t) - W(s)$ is a Gaussian process, $N(0, t-s)$, for $0 \leq s \leq t$.

Using these definitions and descriptions we can now define the stochastic integral.

## 5.5 Rieman and Itô's Integral

The stochastic integral equation for equation (5.7) is

$$X_t(\omega) = X_{t_0}(\omega) + \int_{t_0}^{t} a(s, X_s(\omega))ds + \int_{t_0}^{t} b(s, X_s(\omega))\xi_s(\omega)ds. \tag{5.8}$$

$\xi_s(\omega)ds$ can be written as $dW_s(\omega)$ as white noise is the derivative of a Wiener process. Thus

$$X_t(\omega) = X_{t_0}(\omega) + \int_{t_0}^{t} a(s, X_s(\omega))ds + \int_{t_0}^{t} b(s, X_s(\omega))dW_s(\omega). \tag{5.9}$$

Even with this substitution, equation (5.9) still has the problem that a Wiener process $W_t$ is almost nowhere differentiable. Strictly speaking the white noise process $\xi_t$ does not exist as a conventional function of $t$. Thus the second integral in equation (5.9), cannot be an ordinary integral. A method known as Itô's Integral is used to address this issue.

Rieman integrals are the ordinary integrals we are taught in calculus in secondary school and first year university to solve deterministic problems. The Rieman integral is defined by Kaplan (1984) as follows:

Let $f(x)$ be defined for $a \leq x \leq b$. Then the definite integral

$$\int_a^b f(x)dx$$

is defined as a limit

$$\lim_{h \to 0} \sum_{i=1}^{n} f(x_i^*)\Delta x. \tag{5.10}$$

In this limit one is considering subdivisions of the interval $a \leq x \leq b$ by values $a = x_0 < x_1 < x_2 < ... < x_n = b$. The $\Delta_i x = x_i - x_{i-1}$, $x_i^*$ is the sampling number which is between $x_{i-1} \leq x_i^* \leq x_i$, $h$ is the largest of $\Delta_1 x, ..., \Delta_n x$; we call $h$ the mesh of the subdivision. The limit of equation (5.10) is said to exist and have value $c$ if for every $\epsilon > 0$, one can choose $\delta > 0$ so small that for every such subdivision of mesh $h$ less than $\delta$ and no matter how the $x_i^*$ are chosen in the interval $x_{i-1} \leq x \leq x_i$, one has

$$\left| \sum_{i=1}^{n} f(x_i^*)\Delta_i x - c \right| < \epsilon.$$

There is also a theorem, Kaplan (1984), which states that if $f(x)$ is a continuous real function of the real variable $x$ for $a \leq x \leq b$, then the Rieman integral

$$\int_a^b f(x)dx$$

exists.

Itô's integral is one of the bases for an analysis of solutions of stochastic equations along the lines of the approaches used in ordinary differential and integral equations.

The second integral of equation (5.9), $\int_{t_0}^{t} b(s, X_s(\omega))dW_s(\omega)$, can be approximated using a sum

$$\sum_{i=1}^{n} b(x_i^*, X(x_i^*))\Delta W_i, \tag{5.11}$$

where $\Delta W_i = W(t_i) - W(t_{i-1})$ and $x_i^*$ is the sampling number which is between $x_{i-1} \leq x_i^* \leq x_i$. This sum converges in the mean square sense to different values of the integral depending on the value of the $x_i^*$. If the $x_i^*$ are taken as $x_{i-1}$, then the Itô integral results. Comparing the Itô integral with the Rieman integral we see that for the Rieman integral the $x_i^*$ can be chosen anywhere in the interval $x_{i-1} \leq x \leq x_i$, but for the Itô's integral the $x_i^*$ must be taken at the beginning, $x_{i-1}$. Another base for the analysis of stochastic equations is the Stratonovich integral. The Stratonovich integral results when the $x_i^*$ in equation (5.11) is taken as the midpoint, $1/2(x_{i-1} + x_i)$. The stochastic integral of Itô's,

$$\int_a^b g(t)dW(t),$$

satisfies

$$E[\int_a^b g(t)dW(t)] = 0$$

and

$$E\left|\int_a^b g(t)dW(t)\right|^2 = \int_a^b E\left|g(t)\right|^2 dt.$$

The reason for using Itô's integral instead of the Stratonovich integral is that when the Itô integral is viewed as a function of the upper limit of integration, it forms a Martingale, see Gard (1988) for a more detailed discussion. Hence when necessary, the rich theory of martingales can be used for estimating parameters in the models.

## 5.6 An Example with the General Epidemic Model

Using the Deterministic General Epidemic and the Stochastic General Epidemic models we will pointing out the similarity with deterministic and stochastic equations when the stochastic equations are written in *Martingale* form.

A Martingale is a random process, which evolves over time, whose properties are specified in terms of conditional expected values, and indeed most Martingale theory is driven by expected values. The Martingale property is essentially determined by the fact that its expected value at any future point in time is equal to its current value.

In a more mathematically precise definition a Martingale is defined as follows: A process $M = \{M_t; t \in \tau\}$ is a Martingale if, for all $t \in \tau$,

$$\mathrm{E}(|M_t|) < \infty \tag{5.12}$$

this is a boundedness condition which generally applies in real world applications and

$$\mathrm{E}(M_{t+x}|H_t) = M_t \text{for all x} \in \tau. \tag{5.13}$$

This captures the character of a Martingale and is called the Martingale property.

Recall that for the General Epidemic model we have two independent equations as the third is derived from the relation $S(t) + I(t) + R(t) = 0$.

From equations (3.1) and (3.3) we can write the deterministic equations as

$$dS + \beta SIdt = 0$$

and

$$dR - \gamma Idt = 0.$$

The corresponding Martingale form of the stochastic general epidemic equations are

$$dS + \beta SIdt = dM_1$$

and

$$dR - \gamma I dt = dM_2.$$

Where $dM_1$ and $dM_2$ are Martingale differences. This provides a motivation to turning deterministic differential equations into stochastic differential equations by adding to the right hand side of the deterministic equations a forcing function $f_i(S)dW_i$ where $f_i(S)$ is an appropriately chosen increasing function of S and $dW_i$ is a standard Brownian motion with independent components (each with mean zero and variance $dt$). The forcing functions $f_i(S)dW_i$ are Martingale differences.

## 5.7 Vector form of Stochastic Integrals

Stochastic differential equations are usually written in differential form with the forcing function being a standard Brownian motion differential $d\mathbf{W}$ with independent components (each with mean zero and variance $dt$) multiplied by an appropriate function of the variables. The standard vector form is

$$d\mathbf{X} + \alpha(\mathbf{X}, t)dt = \beta(\mathbf{X}, t)d\mathbf{W} \tag{5.14}$$

where $\mathbf{X}$, $\alpha$ and $d\mathbf{W}$ are vectors of length $n$ and $\beta$ is an $n \times n$ matrix. Since the future behavior of the vector $\mathbf{X}$ is independent of its past values, given its present value, it is a Markov vector.

## 5.8 Conclusion

A set of ordinary (deterministic) differential equations can be turned into a set of stochastic differential equations by adding on the right-hand side forcing functions which are white noise multiplied by some function of the variables. Once this is done, the next step is to determine the amount of stochastic variation to add. This can be done by varying the forcing function so that the model is as close as possible to the experimental data.

# Part II

# MODELS BASED ON NRCC MODEL

The models developed in this chapter are derived from physical principles and the NRCC model. In chapter (7) the deterministic model is calibrated against the NRCC model and then it is made into a stochastic model using the motivation in chapter (5). The use of stochastic models is illustrated with the evaluation of the heat load for fire severity. Finally in this part we develop a two variable model by making an assumption of a relationship between the gas temperature and the burning rate inside the compartment.

# Chapter 6

# One-Zone Fuel Driven Fire Model

## 6.1 Introduction

A variety of models have been developed to represent the environment in an enclosure during a fire. For example, Takeda and Yung (1992) have developed a one-zone model which can be used to predict the fire environment for a range of fires including flaming non-flashover and post-flashover fires. Based on the model of Takeda and Yung a fuel driven model for the growth of a fire inside a compartment will be developed. To model the growth of a fuel driven fire we develop three differential equations for three variables: the gas temperature $T$ (degrees Celsius), the mass burning rate $R$ $(kg/min)$, and the amount of burnt-out material $B$. The three differential equations are solved in three stages to derive the equations for the three variables. Finally the parameters in the models are evaluated using the three stages.

## 6.2 Heat Balance Equation

If we assume the heat loss is directly proportional to the temperature, $T$, then we can write

$$\frac{dT}{dt} = \beta R - \gamma T$$

where $\beta R$ = rate of heat production (scaled), $\gamma T$ = rate of heat loss (scaled) and $T(0) = 0$.

## 6.3 Burning Rate Equation

Simplified from Quintiere and Harkleroad (1984) we have

$$\frac{dR}{dt} = kT \text{ as long as } R < R_{max},$$

where $k$ is a positive parameter and $R_{max}$ is the maximum burning rate. When $R$ reaches $R_{max}$, $dR/dt = 0$. Further changes in $R$ are controlled by $B$, hence we can write

$$\frac{dR}{dt} = f(T, R, B).$$

## 6.4 Burnt-Out Material Equation

$$\frac{dB}{dt} = R, \text{and } B(0) = 0$$

so $B(t) = \int_0^t R(u)du$. When $B(t) = B_{max}$, where $B_{max}$ is the total amount of combustible material,

$$\frac{dR}{dt} = -bR.$$

Thus the three differential equations are:

$$\frac{dT}{dt} = \beta R - \gamma T \quad (6.1)$$

$$\frac{dR}{dt} = f(T, R, B) \quad (6.2)$$

$$\frac{dB}{dt} = R \quad (6.3)$$

where

$$f(T,R,B) = \begin{cases} kT & \text{for } R < R_{max} \\ 0 & \text{for } R = R_{max} \\ -bR & \text{for } B = B_{max} \end{cases}$$

with initial conditions $T(0) = 0$, $R(0) = R_0$, and $B(0) = 0$.

## 6.5 Solving Equations (6.1), (6.2) and (6.3) in Three Stages

To solve the three differential equations we need to solve them in three stages, as the differential equation for $dR/dt$ has three parts.

### 6.5.1 Stage 1, When $R < R_{max}$ for $0 \leq t < t_1$

For small $t$, the three differential equations become

$$\frac{dT}{dt} = \beta R - \gamma T \tag{6.4}$$

$$\frac{dR}{dt} = kT \quad \text{and} \tag{6.5}$$

$$\frac{dB}{dt} = R. \tag{6.6}$$

Differentiating equation (6.4) we get

$$\frac{d^2T}{dt^2} = \beta\frac{dR}{dt} - \gamma\frac{dT}{dt} \tag{6.7}$$

Substituting differential equation (6.5) into differential equation (6.7) we get a linear homogeneous second order differential equation.

$$\frac{d^2T}{dt^2} + \gamma\frac{dT}{dt} - \beta kT = 0 \tag{6.8}$$

The characteristic equation of equation (6.8) is $x^2 + \gamma x - \beta k = 0$. There are two real roots to this equation, the first root is positive, $\alpha_1$, and the second root is negative, $-\alpha_2$.

By substituting the eigen values back into the characteristic equation, we will find relationships between the eigen values and the constants $\gamma, \beta$ and $k$. These relationships will be used to simplify equations further in our calculations of the solution. Since the eigen values are roots of the characteristic equation, we can write the characteristic equation as:

$$(x - \alpha_1)(x + \alpha_2) = 0.$$

Multiplying this out

$$x^2 + (\alpha_2 - \alpha_1)x - \alpha_1\alpha_2 = 0.$$

Now equating coefficients with the original characteristic equation.

$$\gamma = \alpha_2 - \alpha_1 \tag{6.9}$$

$$\beta k = \alpha_1\alpha_2 \tag{6.10}$$

Now continuing with the solution for temperature. The general solution for the temperature is

$$T(t) = Ae^{\alpha_1 t} + Be^{-\alpha_2 t}. \tag{6.11}$$

Differentiating equation (6.11), substituting it into equation (6.4) and using the initial conditions $T(0) = 0$ and $R(0) = R_0$, we can solve for $A$ and $B$:

$$A = \frac{\beta R_0}{\alpha_1 + \alpha_2} \text{ and } B = -\frac{\beta R_0}{\alpha_1 + \alpha_2}.$$

Hence

$$T(t) = \frac{\beta R_0}{\alpha_1 + \alpha_2}e^{\alpha_1 t} - \frac{\beta R_0}{\alpha_1 + \alpha_2}e^{-\alpha_2 t}$$

$$T(t) = \frac{\beta R_0}{\alpha_1 + \alpha_2}(e^{\alpha_1 t} - e^{-\alpha_2 t}). \tag{6.12}$$

To calculate the burning rate equation we have

$$\frac{dR}{dt} = kT.$$

Substituting equation (6.12) into equation (6.5) gives

$$R(t) = R_0 + k\int_0^t T(z)dz$$

$$= R_0 + \frac{k\beta R_0}{\alpha_1+\alpha_2}\int_0^t (e^{\alpha_1 z} - e^{-\alpha_2 z})dz$$

$$= R_0 + \frac{k\beta R_0}{\alpha_1+\alpha_2}[\frac{1}{\alpha_1}e^{\alpha_1 z} + \frac{1}{\alpha_2}e^{-\alpha_2 z}]_0^t$$

$$= R_0 + \frac{k\beta R_0}{\alpha_1+\alpha_2}[(\frac{1}{\alpha_1}e^{\alpha_1 t} + \frac{1}{\alpha_2}e^{-\alpha_2 t}) - (\frac{1}{\alpha_1} + \frac{1}{\alpha_2})]$$

$$= R_0 + \frac{k\beta R_0}{\alpha_1+\alpha_2}[\frac{\alpha_2 e^{\alpha_1 t} + \alpha_1 e^{-\alpha_2 t} - (\alpha_1+\alpha_2)}{\alpha_1\alpha_2}].$$

From equation (6.10), since $\beta k = \alpha_1\alpha_2$ we can write

$$R(t) = R_0 + R_0[\frac{\alpha_2 e^{\alpha_1 t} + \alpha_1 e^{-\alpha_2 t}}{\alpha_1+\alpha_2} - 1]$$

$$R(t) = \frac{R_0}{\alpha_1+\alpha_2}[\alpha_2 e^{\alpha_1 t} + \alpha_1 e^{-\alpha_2 t}]. \qquad (6.13)$$

Substituting equation (6.13) into equation (6.6) and integrating gives

$$B(t) = \frac{\alpha_1\alpha_2 R_0}{\alpha_1+\alpha_2}(\alpha_2^2 e^{\alpha_1 t} - \alpha_1^2 e^{-\alpha_2 t}) + K,$$

where $K$ is the constant of intergration. **Note:** $\frac{dT}{dt} = \frac{\beta R_0}{\alpha_1+\alpha_2}(\alpha_1 e^{\alpha_1 t} + \alpha_2 e^{-\alpha_2 t}) \geq 0$. Hence, $T$ is always greater than 0 in this stage and so is $R$ since $\frac{dR}{dt} = kT$.

### 6.5.2 Stage 2, When $R = R_{max}$ for $t_1 \leq t < t_2$

When $R$ reaches $R_{max}$ say at $t = t_1$, the differential equations become

$$\frac{dT}{dt} = \beta R - \gamma T \qquad (6.14)$$

$$\frac{dR}{dt} = 0 \qquad (6.15)$$

$$\frac{dB}{dt} = R. \qquad (6.16)$$

At $t = t_1$, $B(t = t_1) = B_1$ and $R(t = t_1) = R_{max}$, hence

$$B(t) = R_{max}(t - t_1) + B_1 \tag{6.17}$$

Since $R = R_{max}$ for $t_1 \leq t < t_2$, then differential equation (6.4) becomes a linear non-homogeneous equation

$$\frac{dT}{dt} + \gamma T = \beta R_{max} \tag{6.18}$$

with solution

$$T(t) = \frac{\beta}{\gamma} R_{max}(1 - e^{-\gamma(t-t_1)}) + T_1 e^{-\gamma(t-t_1)} \tag{6.19}$$

where $T_1 = T(t = t_1)$.

### 6.5.3 Stage 3, When $B = B_{max}$ for $t \geq t_2$

When $B(t)$ reaches $B_{max}$ say at $t = t_2$ we have $T(t_2) = T_2$ and the differential equations become

$$\frac{dT}{dt} = \beta R - \gamma T \tag{6.20}$$

$$\frac{dR}{dt} = -bR \tag{6.21}$$

$$\frac{dB}{dt} = 0 \qquad \text{since} B(t \geq t_2) = B_{max}. \tag{6.22}$$

Since $R = R_{max}$ for $(t_1 \leq t \leq t_2)$, this initial condition can be substituted into equation (6.21) once it is integrated to get

$$R(t) = R_{max} e^{-b(t-t_2)}.$$

To find the solution of $T(t)$ we substitute $R(t)$ into the differential equation (6.20)

$$\frac{dT}{dt} = \beta R_{max} e^{-b(t-t_2)} - \gamma T.$$

Using $T(t_2) = T_2$ we solve to get

$$T(t) = T_2 e^{-\gamma(t-t_2)} + \frac{\beta}{b - \gamma} R_{max} e^{-\gamma t_2}[1 - e^{-(b-\gamma)(t-t_2)}].$$

We see that both $R$ and $T$ decay exponentially to zero from their values at $t_2$.

## 6.6 Evaluation of Parameters

Assuming that the $T$ and $R$ curves are available, by inspection of the $R$ curve we obtain the five values $t_1$, $t_2$, $R_{max}$, $R_0$, and $t_3$.

From stage 1 ($0 \le t < t_1$)

$$k = \frac{R(t_1) - R_0}{\int_0^{t_1} T(z)dz}$$

From stage 2 ($t_1 \le t < t_2$)

$$\beta = \frac{T(t_1)\int_{t_1}^{t_2} T(z)dz - [T(t_2) - T(t_1)]\int_0^{t_1} T(z)dz}{\int_0^{t_1} R(z)dz \int_{t_1}^{t_2} T(z)dz - \int_{t_1}^{t_2} R(z)dz \int_0^{t_1} T(z)dz}$$

and

$$\gamma = \frac{T(t_1)\int_{t_1}^{t_2} R(z)dz - [T(t_2) - T(t_1)]\int_0^{t_1} R(z)dz}{\int_0^{t_1} R(z)dz \int_{t_1}^{t_2} T(z)dz - \int_{t_1}^{t_2} R(z)dz \int_0^{t_1} T(z)dz}.$$

From stage 3 ($t_2 \le t \le t_3$)

$$b = \frac{R(t_2) - R(t_3)}{\int_{t_2}^{t_3} R(z)dz}$$

## 6.7 Conclusion

A simplified one zone deterministic model for the growth of a compartment fire based on physical principles was constructed.

The equations derived in stages 1, 2 and 3 can be made into stochastic equations by endowing the parameters $k$, $b$, $\beta$ and $\gamma$ with probability distributions or introducing a stochastic forcing function on the right-hand sides of the equations. The method of using forcing functions was covered in chapter (5) and its use will be demonstrated in chapter (7).

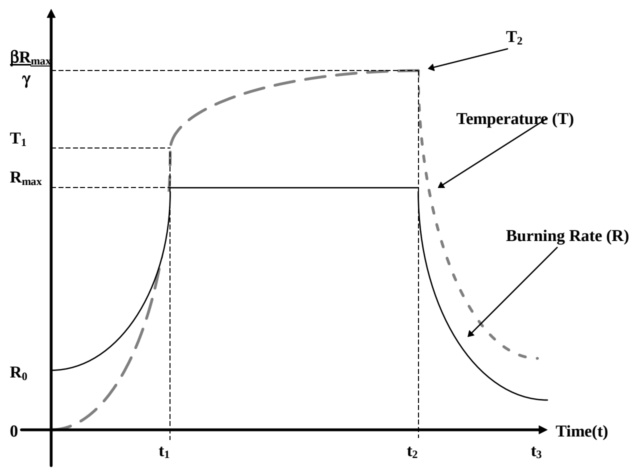

Figure 6.1: General features of the temperature and burning rate graphs over time.

The simplified one zone model developed in this chapter is a fuel driven compartment fire model. Generally compartment fires must be considered to be oxygen driven, as they burn in enclosures where the flow of air or the oxygen concentration are critical for the growth of the fire. For this reason in the next chapter we build an oxygen driven model.

# Chapter 7

# Oxygen Driven Compartment Fire Model

## 7.1 Introduction

In this chapter we develop an oxygen driven compartment fire model. Using the physical laws of conservation of mass, energy and momentum, deterministic differential equations are developed to model the three variables temperature, oxygen deficiency, and burning rate. The stability conditions of the differential equations are investigated and their parameters evaluated by comparing the equations to a run of the NRCC model. Finally the deterministic equations are made into stochastic equations using the method in chapter (5).

The compartment is considered to be a room rectangular in shape with equal and parallel floor and ceiling areas, and the room has a single vent that may or may not be open at any given time. A vent may be a door, window, leak, or other openings in the vertical boundaries of the enclosure. We assume that there are no horizontal vents, i.e., no openings in the floor or ceiling.

Fire, in a basic sense, is an object that releases heat energy into an enclosure. The rising plume gases collect below the ceiling and form a hot, smoky layer.

This layer may continue to grow while the fire exists. This hot, smoky gas layer's temperature is one of the variables to be described as a function of time.

The compartment fire model which we are going to develop is an Oxygen Driven Zone Model. Zone models are the most common type of physical models used in engineering. They are widely accepted and applied due to their relatively simplistic approach to the modelling problem. Other common names for zone models are *control volume* or *engineering models*. The compartment can be divided into several distinct zones, the greater the number of zones the more complex the deterministic equations become. For simplicity we will consider the compartment to be a single zone. The layer is considered to be isothermal and composed of homogeneous gases.

## 7.2 The Deterministic Model

The basic physical laws used to derive the equations here are also used by Takeda and Yung (1992) in the derivation of their model. Drysdale (1985), gives a detailed discussion of these laws in chapters 9 and 10. See in particular Section 10.3.2 "Mathematical models for compartment fire temperatures" where the compartment is regarded as a calorimeter and its temperature is obtained from a heat balance equation.

The model which we are about to develop describes the time-varying conditions produced by a fire within an enclosure. The model consisting of three variables: the gas temperature $T$ (degrees Celsius), the rate of fuel burning $R$ (g/min), and the oxygen fraction in the compartment $x$.We shall eventually convert $x$ into the percentage oxygen deficiency $D = 23 - 100x$. The initial temperature is $T_0 = 20^oC$ and the time $t$ is measured in minutes.

### 7.2.1 Notation

The notation used is given in this subsection. The values of the variable parameters are those used in the NRCC model run that were used for calibration. These particular values refer to a flashover fire. They were chosen because the emphasis of the model in this chapter is on post-flashover concepts.

**Variable Parameters**

| | | | |
|---|---|---|---|
| $V$ | = | volume of compartment $m^3$ | $= 21.6m^3$. |
| $S$ | = | inside surface area of compartment $m^2$ | $= 46.88m^2$. |
| $A$ | = | area of opening $m^2$ | $= 1.6m^2$. |
| $B_{max}$ | = | total fuel mass $kg$ | $= 172.8kg$. |
| $R_0$ | = | Initial burning rate $g/min$ | $= 8.38g/min$. |
| $h$ | = | height of opening $m$ | $= 2m$. |

**Fixed Parameters**

| | | | |
|---|---|---|---|
| $\rho$ | = | gas specific gravity $g/m^3$ | $= 490g/m^3$ at $500^0C$ |
| | | | $= 1,300g/m^3$ at $20^0C$ |
| $c_p$ | = | specific heat of gas $kJ/gK$ | $= 0.001kJ/gK$ |
| $\sigma$ | = | Stefan Boltzman constant | $= 3.4x10^{-9}kJ/minm^2K^4$ |
| $\epsilon$ | = | gas emissivity | $= 0.015$ |
| $\nu$ | = | mass of oxygen used up by $1g$ of fuel | $= 1.36g$ |

### 7.2.2 The Heat Balance Equation

Heat for ignition can come from many sources: open flame, the sun, electricity, friction, and so on. The intensity of heat required to start the chemical action of combustion varies with each type of fuel. This ignition temperature, is defined as the minimum temperature to which a substance (fuel) must be heated in order to initiate or cause self sustained combustion independent of another heat source. Most solid materials have an ignition temperature between $205^0C$ and $400^0C$. These temperatures, however, vary with conditions: time of exposure, size and shape of container, concentration of oxygen, humidity, and others. Wood, for instance, when subjected to $400^0C$ for a short time, will normally start to burn. But, if exposed to a much lower temperature, say $175^0C$ to $205^0C$ for about half an hour, it will begin to smoke and give off gases that are readily ignited.

For combustion to take place most substances must be heated rather rapidly. After ignition temperature has been reached, burning will continue as long as the fuel remains above this temperature. The heat to maintain the ignition temperature is usually produced by the chemical reaction between the oxygen in the air and the substance that is burning. The amount of heat produced is called the heat of combustion. Heat of combustion also varies with every type of fuel and is usually expressed in kilo Joules ($kJ$). While this unit is important in determining the amount of potential heat in a quantity of fuel, remember that it does not indicate the momentary intensity of the fire as it burns. Intensity depends upon the rate at which oxygen is supplied.

If we have a material that needs less heat to reach ignition temperature than the material will produce as heat of combustion, we have the possibility of a self-sustaining combustion. A few substances that are not combustible themselves may cause heating and if combustible material is present start a fire. The most common example of this is unslaked lime. When water is added to unslaked lime, the reaction generates considerable heat: 1150 $kJ$ per kilogram of lime, to be precise.

In simple terms the amount of heat $Q$ in a mass $m$ with a specific heat $c_p$ and temperature $T$ can be written as

$$dQ = mc_p dT. \tag{7.1}$$

Equation (7.1) can be written on a total or time rate of change basis. As a time rate of change basis the equation is

$$\frac{dQ}{dt} = mc_p \frac{dT}{dt}. \tag{7.2}$$

Considering the compartment as a calorimeter, its temperature can be obtained by solving the heat balance equation,

$$Q_{ROOM} = Q_c - (Q_l + Q_w + Q_r + Q_b), \tag{7.3}$$

where $Q_{ROOM}$ is the heat in the room, $Q_c$ is the heat release due to combustion, $Q_l$ is the heat loss due to replacement of hot gases by cold air, $Q_r$ is the heat loss by radiation through the openings, $Q_b$ is the heat stored in the gas volume and $Q_w$ is the heat loss through the walls, ceiling and floor.

Using equation (7.2), equation (7.3) can be simplified and written in a time rate of change basis as

$$mc_p \frac{dT}{dt} = \frac{dQ_c}{dt} - \frac{dQ_{LOSS}}{dt}.$$

where the left hand side is the rate of heat change in the compartment, $\frac{dQ_c}{dt}$ is the rate of heat change due to combustion and $\frac{dQ_{LOSS}}{dt}$ is the net rate of heat loss from the enclosure.

Let us denote the net rate of heat loss from the enclosure by $Q_L$ and replace $m$ by $\rho V$. Then the heat balance equation reads

$$c_p \rho V \frac{dT}{dt} = H_B R - Q_L$$

where $H_B$ is the net combustion heat per gram of fuel, $R$ is the burning rate in grams per minute and $\rho$ is taken to be some average gas specific gravity. We can rewrite the above equation as

$$\frac{dT}{dt} = \beta R - q(T) \tag{7.4}$$

where $\beta = H_B / c_p \rho V$ and $q(T) = Q_L / c_p \rho V$.

### The Heat Loss $Q_L$ Formula

In a general sense, heat transfer is the study of energy transfer that takes place between material bodies due to a temperature difference between the bodies. Heat transfer can occur due to conduction, convection, radiation, or any combination of the three. All three modes are present in compartment fires.

We will consider heat loss to be made up of two components, as the loss of heat via conduction is negligible relative to the other two forms.

The radiation loss rate $Q_R$. Thermal radiation involves transfer of heat by electromagnetic waves confined to a relatively narrow width, wavelengths between 0.4 and 100 $\mu$m, in the electromagnetic spectrum. From Drysdale (1985) using the Stefan-Boltzmann law and replacing $T$ by $(T + 273)$ to account for us using degrees Celsius instead of degrees Kelvin, the radiation loss rate is given by

$$Q_R = \epsilon\sigma[(T + 273)^4 - (T_0 + 273)^4](S + A),$$

where $S$ is the inside surface area of the compartment, $A$ is the area of the opening, $\epsilon$ is the effective emissivity of the gases and $\sigma$ is the Stefan-Boltzman constant.

The convection loss rate $Q_c$. Convection is the mode of heat transfer to or from a solid involving the movement of surrounding fluid or gas. From Drysdale (1985) the convective heat transfer coefficient is known to be a function of the fluid properties, the flow parameters and the geometry of the surface. From Drysdale (1985), using the empirical relationship first discussed by Newton, this is given by

$$Q_c = c_p m_a (T - T_0),$$

where $m_a$ is the ventilation rate in $g/min$.

The total heat loss rate is $Q_L = Q_R + Q_C$. Dividing by $c_p \rho V$ we can rewrite the formula for heat loss as:

$$q(T) = \Sigma[(T + 273)^4 - (T_0 + 273)^4] + \Phi(T - T_0) \qquad (7.5)$$

where $\Sigma$ and $\Phi$ are some calibration parameters.

### 7.2.3 The Oxygen Mass Balance Equation

Most fires draw their oxygen from the air, which is a mixture of approximately 23 percent oxygen 76 percent nitrogen and small amounts of other gases. If a fire burns in a closed room the oxygen will gradually be used up and the fire will diminish. If no additional supply is available, the fire will go out. However, if a limited but continuous supply is provided (which is often the case) the fire will smoulder.

Since oxygen is so readily available from the air, eliminating the oxygen side of the fire tetrahedron, refer to chapter (1) for an explanation, to prevent fires is not always possible. However, one common method of suppressing a growing fire is by removing the source of oxygen, referred to as "smothering" the fire.

Let the oxygen fraction in the incoming air be $y(=0.23)$. Then, assuming homogeneous mixing, from Takeda and Yung (1992), the oxygen concentration in the gas mixture inside the compartment is

$$x + dx = \frac{\rho x V + y m_a dt - \nu R dt}{\rho V + m_a dt + R dt}.$$

In the numerator the first term is the initial amount of oxygen, the second term is the amount of oxygen entering the compartment and the third term is the amount of $O_2$ used by the fire. In the denominator the first term is the initial amount of gas, the second term is the amount of gas entering the compartment and the third term is the amount of gas produced by the fire. This yields the differential equation

$$\frac{dx}{dt} = \frac{m_a}{\rho V}(y - x) - \frac{(x+\nu)R}{\rho V}.$$

Changing $x$ to $D = 23 - 100x$, we obtain the equation

$$\frac{dD}{dt} = \delta(k_1 - D)R - \mu D \qquad (7.6)$$

where $\delta = 1/\rho V$, $k_1 = 100(y+\nu)$ and, $\mu = m_a/\rho V$. It is assumed that equation(7.6) applies only as long as the oxygen concentration is above 7 percent. When this value is reached, it remains steady at that value until the burning rate starts diminishing, at which point the oxygen concentration recovers exponentially with the same parameter $\mu$ as in equation(7.6). In other words, the oxygen deficiency $D$ obeys the differential equation

$$\frac{dD}{dt} = -\mu D$$

when the burning rate starts diminishing.

It should be noted that the algorithm just described results in fire curves which are less smooth than those obtained from the NRCC model in the transition phase from increasing burning rate to steady burning rate.

### 7.2.4 The Burning Rate Equation

The differential equation for the burning rate $R$ is based on two assumptions:

1. R is an increasing function of the gas temperature $T$. but rises only slowly for low temperatures.

2. When the oxygen fraction falls below 0.126, the burning rate stops increasing.

Moreover, when most of the combustible material has been consumed, the burning rate quickly decreases exponentially.

The above features are incorporated in the equation as follows (as long as there is still some fuel not burning)

$$\frac{dR}{dt} = \alpha(k - D)Z \tag{7.7}$$

where $Z$ is some function of $T$ which is slowly increasing for small $T$. In this thesis we use the calibration formula

$$Z(T) = 22T(1 - \frac{1}{1 + (0.001T)^2}). \tag{7.8}$$

The amount of fuel burned ( in $kg$) is

$$B(t) = \frac{1}{1000}\int_0^t R(u)du.$$

After $B(t)$ reaches $B_{max}$, $R$ obeys the exponential decay equation

$$\frac{dR}{dt} = -bR.$$

The initial burning rate $R_0$ must be given.

It has been demonstrated that modelling air and smoke as ideal gases and by considering the pressure within the fire enclosure as a constant allows several attractive simplifications. The computer code to model this compartment fire is written in a statistical computer package, S-plus, a copy of the code can be found in appendix A.

## 7.3 Stability Conditions

The differential equations (7.4), (7.6) and (7.7) developed in section (7.2) are:

$$\frac{dT}{dt} = \beta R - \gamma q(T)$$
$$\frac{dD}{dt} = \delta(k_1 - D)R - \mu D$$
$$\frac{dR}{dt} = \alpha(k - D)Z$$

To investigate the stability condition of the above differential equations, the system of three non-linear differential equations are reduced to a system of three linear differential equations if we take $t$ to be small.

$$\frac{dT}{dt} \approx \beta R - \gamma^*(T - T_0) \tag{7.9}$$
$$\frac{dR}{dt} \approx \alpha k^*(T - T_0) \tag{7.10}$$
$$\frac{dD}{dt} \approx \delta^* R - \mu D \tag{7.11}$$

Equations (7.9), (7.10) and (7.11) can be written in matrix notation as:

$$\begin{bmatrix} dR/dt \\ dT/dt \\ dD/dt \end{bmatrix} = \begin{bmatrix} 0 & \alpha k^* & 0 \\ \beta & -\gamma^* & 0 \\ \delta^* & 0 & -\mu \end{bmatrix} \begin{bmatrix} R \\ T \\ D \end{bmatrix} + \begin{bmatrix} -\alpha k^* T_0 \\ \gamma^* T_0 \\ 0 \end{bmatrix} \tag{7.12}$$

The characteristic equation of equation (7.12) is

$$det(B - \lambda I) = 0,$$

where $B$ is the square matrix in equation (7.12), $\lambda$ is an eigen value and $I$ is the identity matrix. Simplifying the characteristic equation gives

$$(-\mu - \lambda) \begin{vmatrix} -\lambda & \alpha k^* \\ \beta & -\gamma^* - \lambda \end{vmatrix} = 0 \tag{7.13}$$

$$\Rightarrow (-\mu - \lambda)[-\lambda(-\gamma^* - \lambda) - \beta\alpha k^*] = 0$$

$$\Rightarrow (\mu + \lambda)[-\lambda\gamma^* - \lambda^2 + \beta\alpha k^*] = 0$$

$$\Rightarrow -(\mu + \lambda)[\lambda^2 + \lambda\gamma^* - \beta\alpha k^*] = 0 \tag{7.14}$$

From equation (7.14) we can find the eigenvalues of the characteristic equation:

$$\lambda_1 = -\mu$$

The second and third eigenvalues are found using the quadratic formula

$$\lambda_{2,3} = \frac{-\gamma^* \pm \sqrt{\gamma^{*2} + 4\alpha\beta k^*}}{2}.$$

Hence

$$\lambda_2 = -(\frac{\gamma^* + \sqrt{\gamma^{*2} + 4\alpha\beta k^*}}{2})$$

and

$$\lambda_3 = \frac{-\gamma^* + \sqrt{\gamma^{*2} + 4\alpha\beta k^*}}{2}.$$

From above we see that $\lambda_1$ is -ve, $\lambda_2$ is -ve and $\lambda_3$ is +ve. From the theory of homogeneous systems the solution approaches infinity if and only if at least one eigenvalue is $> 0$. Otherwise the solution approaches zero. See Zitecki (1986) for a detailed discussion on Nonhomogeneous Systems.

As $\lambda_3$ is +ve, the solution approches infinity and the system is said to be asymptotically unstable.

The general solution for an homogeneous system is of the form

$$\underline{x} = c_1\underline{u}_1 e^{\lambda_1 t} + c_2\underline{u}_2 e^{\lambda_2 t} + c_3\underline{u}_3 e^{\lambda_3 t}.$$

Finding the corresponding eigenvectors by substituting the eigenvalues into the characteristic equation (7.14), the general solution of our homogeneous system is

$$\begin{bmatrix} R \\ T \\ D \end{bmatrix} = c_1 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} e^{-\mu t} + c_2 \begin{bmatrix} 1 \\ \frac{-\gamma^* + \sqrt{\gamma^{*2}+4\alpha\beta k^*}}{2\alpha k^*} \\ \frac{-2\delta^*}{-2\mu+\gamma^*+\sqrt{\gamma^{*2}+4\alpha\beta k^*}} \end{bmatrix} e^{-(\frac{\gamma^*+\sqrt{\gamma^{*2}+4\alpha\beta k^*}}{2})t}$$

$$+c_3 \begin{bmatrix} 1 \\ \frac{\sqrt{\gamma^{*2}+4\alpha\beta k^*}-\gamma^*}{2\alpha k^*} \\ \frac{2\delta^*}{2\mu-\gamma^*+\sqrt{\gamma^{*2}+4\alpha\beta k^*}} \end{bmatrix} e^{(\frac{-\gamma^*+\sqrt{\gamma^{*2}+4\alpha\beta k^*}}{2})t}.$$

To find the $c$'s we solve the equation $\mathbf{E}(t) = c_1 \underline{u}_1 e^{\lambda_1 t} + c_2 \underline{u}_2 e^{\lambda_2 t} + c_3 \underline{u}_3 e^{\lambda_3 t}$. were $\mathbf{E}(t)$ is

$$\begin{bmatrix} -\alpha k^* T_0 \\ \gamma^* T_0 \\ 0 \end{bmatrix}.$$

The solution shows that the fire increases exponentially until the combustible material is burnt.

## 7.4 Comparison With The NRCC Model

The parameters of the three non-linear differential equations (7.4), (7.6) and (7.7) will be calibrated using a particular run of the NRCC model.

The data from the NRCC model was obtained by executing the program with the parameter values described in subsection (7.2.1). The output data consisted of time ($t$) incremented in 0.02 minutes, the gas temperature ($T$), the burning rate ($R$), and the oxygen concentration ($x$). See appendix E for the data from the NRCC program.

To calibrate the parameters of the non-linear differential equations (7.4), (7.6) and (7.7) they were made into difference equations, as they cannot be solved explicitly, and evaluated discretely.

- The difference equations for the time equation is:

$$t[r+1] = t[r] + dt,$$

were $dt = 0.02$ of a minute.

- The difference equations for the heat loss equation is:

$$QL[r] = \sigma((GT[r]+273)^4 - (GT[1]+273)^4) + \gamma GT[r], \tag{7.15}$$

were $QL[r]$ is the heat loss at interval increment $r$ and $GT[r]$ is the gas temperature at interval increment $r$.

- The difference equations for the heat balance equation (7.4) is:

$$T[r+1] = GT[r] + (\beta R[r] - QL[r])dt, \tag{7.16}$$

$$GT[r+1] = min(1000, T[r+1]), \tag{7.17}$$

were $T[r+1]$ is the temperature in the room at interval $r+1$. Equation (7.16) calculates the temperature for the next interval using the previous temperature, heat loss and burning rate values. Equation (7.17) is used to ensure that the gas temperature does not exceed 1000°C.

- The difference equations for the burning rate equation (7.7) is:

$$B[r+1] = B[r] + R[r]dt, \tag{7.18}$$

$$Z[r] = \alpha 22GT[r](1 - \frac{1}{1+(0.001GT[r])^2}), \tag{7.19}$$

$$if B[r] < B_{max}, \tag{7.20}$$

$$R[r+1] = R[r] + max(0, (k - D[r]))Z[r]dt, \tag{7.21}$$

$$\text{else} R[r+1] = R[r] - \text{RDECAY} R[r]dt, \tag{7.22}$$

Equation (7.18) calculates the amount of burnt material, $B[r+1]$, using the burning rate, $R[r]$. Equation (7.19) calculates the temperature function, $Z[r]$, for the interval $r$ which is used in the calculation of the burning rate value. Equation (7.20) tests if the combustible material is all burnt, $B_{max}$. Equation (7.21) calculates the burning rate value for the interval $r+1$ if equation (7.20) is satisfied. Equation (7.22) is used to calculate the decaying burning rate if equation (7.20) is not satisfied.

- The difference equations for the oxygen deficiency equation (7.6) is:

$$D[r+1] = min(16, D[r] + (\delta(k_1 - D[r]))R[r]) - \mu D[r]dt, \tag{7.23}$$

were $D[r]$ is the oxygen deficiency.

The above difference equations were run with varying values of the parameters through an optimisation algorithm. The values of the parameters were calculated by minimizing the distance function

$$\Delta = \Sigma c_1^2(T_N - T)^2 + c_2^2(R_N - R)^2 + c_3^2(D_N - D)^2, \tag{7.24}$$

where $c_1 = 10^{-3}, c_2 = 0.008$ and $c_3 = 23$. The values of the $c$'s were chosen so as to obtain comparable fits for the three variables at their maximum values. See appendix B for the computer code, written is S-plus, which optimises the distance function (7.24).

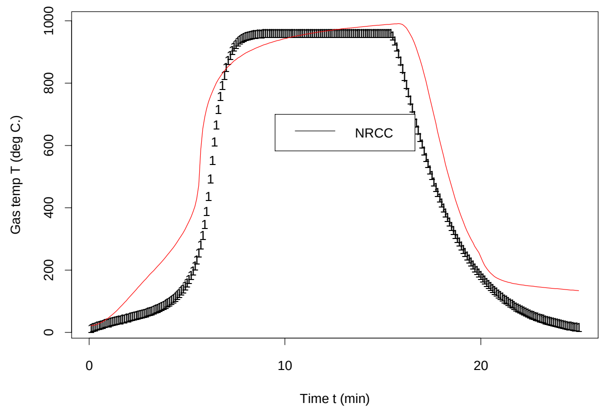


Figure 7.1: Comparison of the Gas Temperature.

The values of the calibration parameters obtained were as follows:

$$\begin{aligned}
\alpha &= 2.5\\
\beta &= 0.1125\\
\Phi &= 0.399\\
\Sigma &= 1.875x10^{-10}\\
\delta &= 1.5x10^{-5}\\
\mu &= 0.58\\
k &= 10.4\\
k_1 &= 158\\
B_{max} &= 82.5kg\\
R_0 &= 280g/min\\
b &= 0.956min^{-l}.
\end{aligned}$$

A comparison of the time-dependent variation of the burning rate $R$ the temperature $T$ and the oxygen percentage $23 - D$ with the corresponding output of the NRCC model is shown in figures (7.1), (7.2) and (7.3).

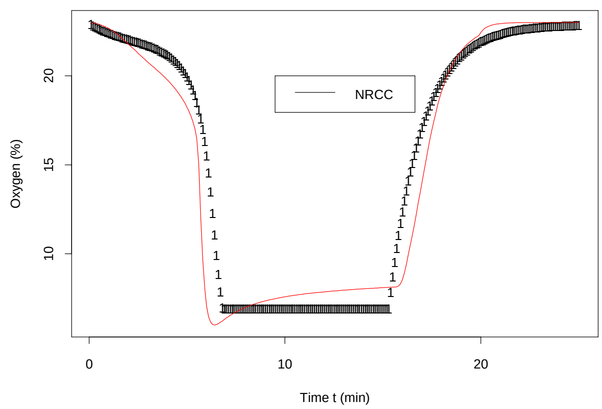


Figure 7.2: Comparison of the Oxygen Concentration.

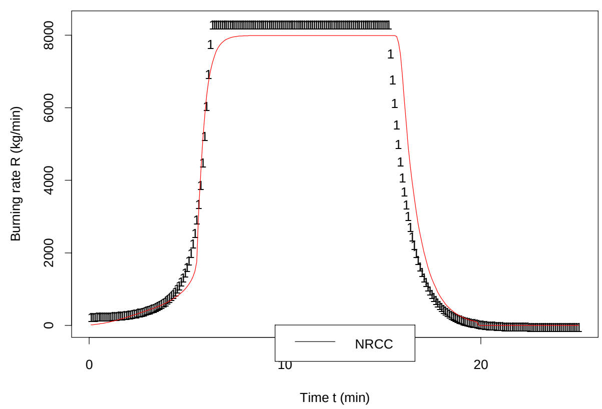


Figure 7.3: Comparison of the Burning Rate.

## 7.5 Discussion

Some of the more complex mathematical models evaluate the heat transfer activities within enclosures at each calculated time step. These models have detailed subroutines that calculate the heat transfer from flames to fuels, from the hot layer to fuel surfaces, from heated walls and ceilings to fuel surfaces, and so forth. The inclusion of subroutines increases the model's ability to successfully model an enclosure fire, but do so at the expense of long and complex computer codes and run times. Drysdale (1985) points out that a simple but acceptable engineering approximation of the conditions within an enclosure is desired. In this regard, this is a more simplistic approach to the variety of heat transfer activities that are continually occurring within enclosure fires.

In deriving the simplified equations of our model, we have replaced a number of parameters which are clearly highly variable by fixed values, and ignored the effect of certain variables on some other parameters, while retaining the effect of other variables. This approach has been justified by the fact that the general shape of the fire curves have been retained. For example, the gas specific gravity $\rho$ is well known to be inversely proportional to the gas absolute temperature, but is treated in our model as constant. Similarly the ventilation rate $m_a$ is highly dependent on the gas temperature, but that effect is ignored. What we propose is to consider only the dependence of $m_a$ on the area $A$ and the height $h$ of the opening in the form $m_a = m_0 A\sqrt{h}$. Such a formula is clearly purely nominal, but an appropriate choice of the calibrating constant $m_0$ as well as of the other calibrating constants enables us to mimic the fire curves of the NRCC model very closely.

The compartment dimensions which clearly affect the size of the fire are the volume V, the surface area of the walls $S$. the area of the opening $A$ and its height $h$. Thus, we propose the following nominal formulae for the parameters appearing in the three equations of our model:

$$\begin{aligned}
\beta &= \beta_0/V \\
\Sigma &= \Sigma_0(S+A)/V \\
\Phi &= \Phi_0 A\sqrt{h} \\
\delta &= \delta_0/V \\
\mu &= \mu_0 A\sqrt{h}/V.
\end{aligned}$$

The form of these formulae is obtained directly from the derivation of the model equations given above. The calibrating constants from the fitting described in Section (7.4) turn out to be:

$$\begin{aligned}
\beta_0 &= 2.43 \\
\Sigma_0 &= 0.84x10^{-10} \\
\Phi_0 &= 0.1763 \\
\delta_0 &= 32.4x10^{-5} \\
\mu_0 &= 5.54.
\end{aligned}$$

The decay constant for $R$ (taken here to be 0.956) appears to be simply based on experimental evidence.

As far as the total amount of fuel $B_{max}$ is concerned, the value that provides the best fit is, as mentioned above, $82.5kg$, which is slightly less than half the value used by the NRCC model. The discrepancy is due to the fact that in the NRCC model the flame is assumed to spread radially on the top surface of the burning object with a speed which is independent of geometry. This makes the burning rate a function of the radius of the burning area. In contrast, in our model ,the burning rate does not depend on the geometry of the burning surface, only on the temperature and the oxygen concentration.

A similar argument applies to the burning rate at the start of the fire. The value used in the NRCC model is $8.38g/min$ while the value we use is $280g/min$.

## 7.6 The Stochastic Model

The set of deterministic equations derived in Section (7.2) can be turned into a set of stochastic differential equations, as described in chapter (5), by adding on the right-hand side forcing functions which are white noise multiplied by some function of the variables. The purpose of these forcing functions is to model the intrinsic variability of the fire phenomenon due to the turbulent behaviour of the hot gases.

Stochastic differential equations are usually written in differential form with the forcing function being a standard Brownian motion differential $d\mathbf{W}$ with independent components (each with mean zero and variance $dt$) multiplied by an appropriate function of the variables. The standard form for describing the behaviour of a set of $n$ coupled variables is

$$d\mathbf{X} + \alpha(\mathbf{X}, t)dt = \beta(\mathbf{X}, t)d\mathbf{W}$$

where $\mathbf{X}, \alpha$ and $d\mathbf{W}$ are vectors of length $n$ and $\beta$ is an $nxn$. matrix. A typical example of a single-dimensional stochastic differential equation is the "Langevin" equation developed to model Brownian motion. The equation reads

$$mdv + \alpha vdt = \beta dW$$

where $m$ is the mass of a free particle, $v$ is the component of the particle velocity along the $x$-axis, $\alpha$ is the damping constant and $\beta dW$ represents the momentum due to the irregular force exerted on the particle by molecular collisions. See Soong (1973) for a more detailed exposition of the elements of Stochastic Differential Equations. Since the future behaviour of the vector $\mathbf{X}$ is independent of its past values, given its present value, it is a Markov vector.

For our purposes, we shall not make $\alpha$ and $\beta$ depend explicitly on the time, i.e., they will be functions of $\mathbf{X}$ only. Moreover, we shall assume at this preliminary stage that the forcing functions for each of $T, D, R$ are independent. This implies that the matrix, $\beta$ is diagonal. Furthermore, it is a plausible assumption that the randomness of the fire will increase with increasing temperature. Bearing in mind that the variance of a scalar forcing function differential $\beta dW$ is $\beta^2 dt$, we obtain the following set of stochastic equations to describe the fire:

$$dT - \beta R dt + q(T)dt = f_1(T)dW_1 \tag{7.25}$$

$$dD - \delta(k_1 - D)Rdt + \mu D dt = f_2(T)dW_2 \tag{7.26}$$

$$dR - \alpha(k - D)Zdt = f_3(T)dW_3 \tag{7.27}$$

where $q(T)$ is given by equation (7.5) and $Z(T)$ by equation (7.8). The functions $f_1(T)$, $f_2(T)$ and $f_3(T)$ are appropriately chosen increasing functions of $T$.

At this point of time, there is a great paucity of information about the intrinsic variability of enclosure fires, so that a more precise formulation for the functions $f_1$, $f_2$ and $f_3$ must await further experimental results specifically designed to identify these functions. Some experiments are being planned at present.

It remains to tackle the problem of the statistical variability whose source is our lack of knowledge of the parameters governing the fire. Assuming that we are dealing with a well-defined enclosure for which the geometry is defined and the amount and nature of the fuel is accurately known, the most important unknown parameter is the initial burning rate $R_0$. Since it is by definition a non-negative quantity, we propose to assign to it a lognormal distribution. However, at this point, it is not possible to give any guide-lines for choosing the parameters of that distribution (i.e. the mean and the variance).

Of course, if the other parameters affecting the fire, such as the geometry of the compartment, the size of the openings and whether they are open or shut and the nature, amount and position of the fuel load are also unknown, then further parameters of the model must be allocated a probability distribution to cater for the added uncertainty.

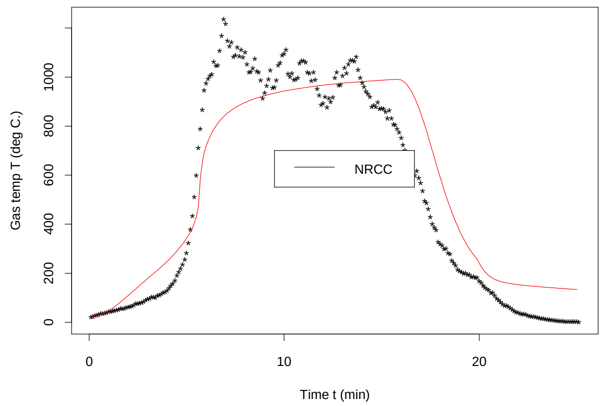


Figure 7.4: Stochastic output comparison of the Gas Temperature.

### 7.6.1 A simulation illustration

To illustrate the capacity of the model to simulate the intrinsic variability of a fire, a Monte-Carlo simulation of a fire with the parameters given in Section (7.2) was carried out, using equations (7.25), (7.26) and (7.27). The functions $f_i$ were chosen as follows:

$$f_1(T) = \phi T/1000$$

$$f_2(T) = 15\phi T/1000$$

$$f_3(T) = 12\phi T/10^6.$$

Figures (7.4), (7.5) and (7.6) are an examples of the output by running the stochastic equations and plotting the results with the NRCC model results. See appendix C for the S-plus code used to produce figures (7.4), (7.5) and (7.6).

By varying the parameter $\phi$, varying degrees of stochasticity may be achieved.

Figures (7.7) ($\phi = 50$), (7.8) ($\phi = 100$) and (7.9) ($\phi = 150$) illustrate the type of fire curve obtained.

As far as varying the value of $R_0$ is concerned, the general form of the result can be guessed by remembering that $(T, R, D)$ is a Markov vector and that initially

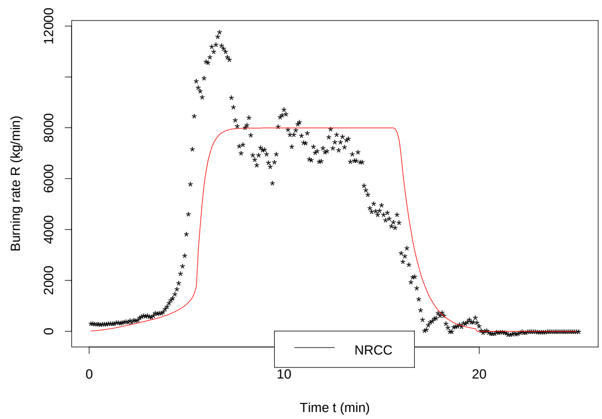


Figure 7.5: Stochastic output comparison of the Burning Rate.

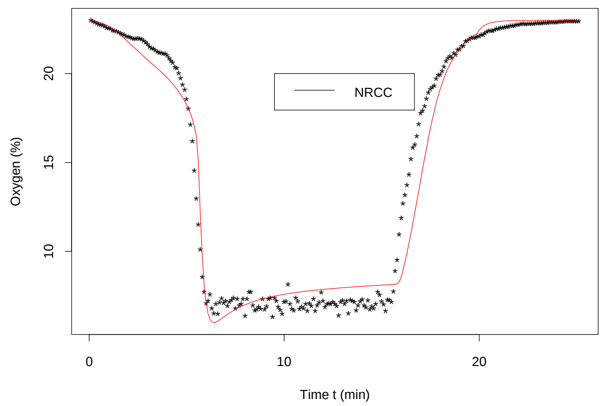


Figure 7.6: Stochastic output comparison of the Oxygen Concentration.

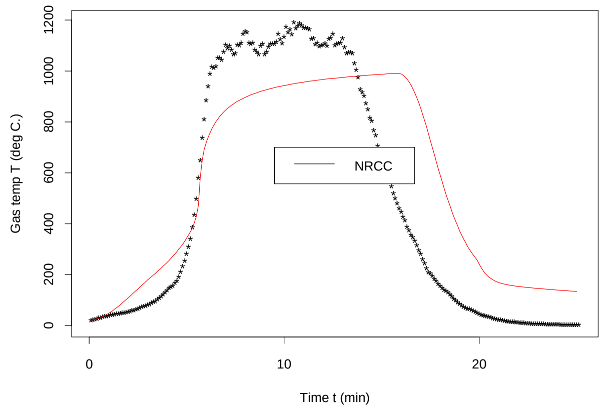


Figure 7.7: Stochastic output of the Gas Temperature - $\phi = 50$.

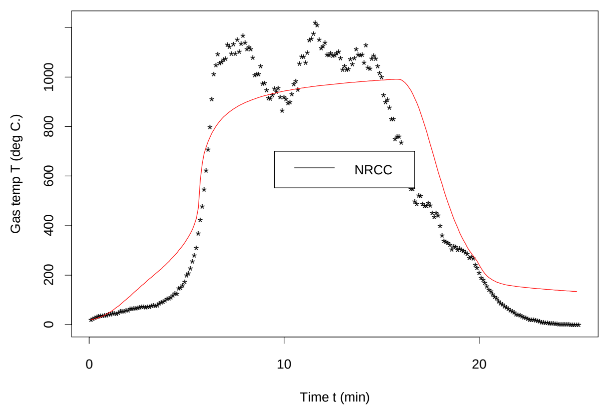


Figure 7.8: Stochastic output of the Gas Temperature - $\phi = 100$.

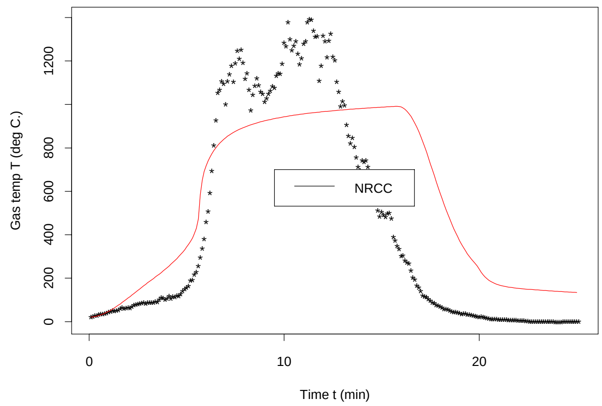


Figure 7.9: Stochastic output of the Gas Temperature - $\phi = 150$.

all three components increase monotonically, so that any decrease in $R_0$ will just shift all three curves to the right. Thus, choosing a random value for $R_0$ and a non-zero value for $\phi$ will produce curves for $T, R$ and $D$ similar to figures (7.1), (7.2) and (7.3), but shifted either to the right or to the left.

## 7.7 Conclusion

The main advantage of our proposed model over the present complex models based on fluid mechanics is that it is extremely easy to simulate and its output realistically models the observed behaviour of fires. By realistically we mean that the readings recorded in an experiment on the gas temperature, burning rate and oxygen fraction show non-smooth plots. With the addition of reasonable assumptions regarding the variability of the phenomenon (intrinsic or due to lack of knowledge), the probability of extreme values of the fire load can be estimated by a Monte-Carlo simulation and can be used as an input to the probabilistic fire risk analysis of the building under consideration.

# Chapter 8

# Calculating the Upper Quantiles of Heat Load using W-test

## 8.1 Introduction

The main projected use of the stochastic fire curves developed in the previous chapter is as an input to modules which will calculate the effect of the fire on proposed fire barriers, and subsequently trace the possible spread of the fire to adjacent compartments. As an illustration of the kind of result obtainable, one particular measure of fire severity, namely the "Normalized Heat Load" proposed by Harmathy and Mehaffy (1982) was studied. For a full discussion see Harmathy (1980). We ignored the normalising parameters and simply evaluated the variable $H$ defined by

$$H = \int_0^\tau f dt$$

where $f$ is heat flux penetrating the enclosure boundaries, $t$ is the time and $\tau$ is the time when the burning rate is reduced to a negligible value.

To calculate the upper quantile of heat load we will cover a statistical test in Non-parametric Estimation of Failure Probabilities, developed by Hasofer and Wang (1992), known as the 'W-Test'. The W-test enables the failure probability to be accurately estimated from a sample, provided the sample size is large enough, without making any assumptions about the underlying joint distribution.

In this chapter a brief outline of Extreme Value Theory is given, followed by a description of the W-test, and finally a simulation is run to calculate heat loads and apply the W-test.

## 8.2 Extreme Value Theory

Extreme value theory is concerned with probability calculations and statistical inferences connected with extreme values in random samples and stochastic processes. Environmental extremes and Structural reliability are the two main areas where Extreme Value theory is used. Examples of environmental extremes are river flow, wind speed, temperature and rainfall. Structural reliability is the study of the strength of materials where it is the maximum load, the weakest component or part of a system which is ultimately responsible for failure.

Other areas where extreme value concepts are being increasingly applied include financial calculations such as probabilities of large insurance claims, monitoring of air pollution (ozone, acid rain, etc.) and some rather more novel ones such as horse races and athletics records. Finally, there are applications of extreme value theory in other areas of statistics, such as testing for outliers and change point problems.

In recent years a lot of research has been conducted on the calculation of small probabilities of failure. Almost all of the work assumed that the distribution of the physical variables was multivariate normal. When the joint distribution is not multivariate normal, Hohenbichler and Rackwitz (1981) advocate the use of the Rosenblatt transformation to transform the variables to joint normality. It is then necessary to specify a family of conditional distributions to carry out the transformation. Provided one stays in the realm of theory, this can be done, at least in principle. If the vector of physical variables is known only through a sample, either by simulation or from field measurements, then it is not possible to calculate the required conditional probability distributions.

### 8.2.1 One Dimensional Case

Let the upper $\epsilon-$quantile $q(\epsilon)$ of a random variable $X$ be defined by $P[X > q(\epsilon)] = \epsilon$, where $\epsilon$ is some small number. Suppose that we are interested in evaluating the quantile $q(\epsilon)$, and the probability distribution of $X$ is not known explicitly. The only information we have about $X$ is either:

- a random sample of size $n$, or
- some algorithm for simulating such a random sample.

In classical statistics it is customary to postulate a parametric form for the distribution of $X$ depending on one or more unknown parameters. The validity of the form of the distribution would be tested by applying some goodness of fit test such as the chi-squared test. The parameters are then estimated from the sample and the estimators used to obtain an estimator $\hat{q}(\epsilon)$ of the required quantile.

A typical example of such a procedure would be to assume that $X$ is normally distributed with mean $\mu$ and standard deviation $\sigma$. The parameters $\mu$ and $\sigma$ are estimated by $\hat{\mu} = \sum X_i/n$ and $\hat{\sigma}^2 = \sum(X_i - \hat{X})^2/(n)$. The quantile $q(\epsilon)$ is then estimated by $\hat{q}(\epsilon) = \hat{\mu} + k(\epsilon)\hat{\sigma}$ where $k(\epsilon)$ is the corresponding quantile of the standard normal distribution.

### 8.2.2 Brief Mathematical Description of Extreme Value Theory

Suppose $X_1, X_2, \cdots, X_n$ is a random sample from the distribution function $F(x)$. Let $X_{1n}, X_{2n}, \cdots, X_{nn}$ be the corresponding descending order statistics, satisfying $X_{1n} > X_{2n} > \cdots > X_{nn}$. Then, of course, $X_{1n}$ is the maximum of $X_1, X_2, \cdots, X_n$, and

$$\lim_{n\to\infty} P(X_{1n} \leq x) = \lim_{n\to\infty} F^n(x) = \begin{cases} 0 & \text{if } F(x) < 1 \\ 1 & \text{if } F(x) = 1. \end{cases} \tag{8.1}$$

We are interested in finding a sequence of real constants $a_n$ and $b_n$, n= 1,2,$\cdots$ such that $F^n(a_n x + b_n)$ converges (weakly) to a non-degenerate distribution function $D(x)$. If such a sequence exists, the distribution function $F(x)$ is said to belong to the domain of attraction of the extreme value distribution $D(x)$. The sequences $a_n$ and $b_n$ are called the coefficients of attraction of $F(x)$.

The central result of extreme value theory is that the distribution $D(x)$ must be, up to a change of scale and origin, of the form

$$D_\alpha(x) = exp[-(1+\alpha x)^{-1/\alpha}].$$

- For $\alpha = 0$, $D_\alpha(x)$ is defined as

$$D_0(x) = \lim_{\alpha \to 0} D_\alpha(x)$$
$$= \exp[-\exp(-x)], \ -\infty < x < \infty$$

  This distribution is known as Type I (or Gumbel) extreme value distribution. The Negative Exponential, Normal, Exponential, Weibull, Lognormal, Rayleigh and Gamma (including chi-square), distributions belong to the domain of attraction of Type I

- For $\alpha > 0$,

$$D_\alpha(x) = \begin{cases} \exp[-(1+\alpha x)^{-1/\alpha}] & \text{for } x \geq -1/\alpha \\ 0 & \text{for } x < -1/\alpha. \end{cases} \tag{8.2}$$

  This distribution is known as Type II extreme value distribution. The Cauchy, Pareto and t-distributions belong to the domain of attraction of Type II.

- For $\alpha < 0$,

$$D_\alpha(x) = \begin{cases} 1 & \text{for } x > -1/\alpha \\ exp[-(1+\alpha x)^{1/\alpha}] & \text{for } x \leq -1/\alpha. \end{cases} \tag{8.3}$$

  This distribution is known as Type III extreme value distribution. The Uniform and Negative Gamma (i.e. the distribution of -Y where Y is Gamma distributed) belong to the domain of attraction of Type III.

It is easy to see that if $X$ has distribution $D_0(x)$ then

$$Y = [exp(\alpha X) - 1]/\alpha$$

has distribution $D_\alpha(x)$ and conversely, if $Y$ has distribution $D_\alpha(x)$, then

$$X = ln[(1 + \alpha Y)^{1/\alpha}]$$

has distribution $D_0(x)$.

The above theory has been developed in terms of the distribution of the maximum of a sample. If we are interested in the asymptotic properties of the minimum of the sample, we simply use the fact that $min(X_1, X_2, \cdots, X_n) = -max(-X_1, -X_2, \cdots, -X_n)$ and modify the equations appropriately.

It should be emphasised that although the Type I distribution belongs to the continuum of extreme value distributions, as shown above, it differs fundamentally from Type II and Type III and has to be treated separately. In essence, a Type I variable basically behaves like the logarithm of a Type II or Type III variable.

## 8.3 Wang's Procedure : W-Test

The W-Test is a non-parametric test in extreme value theory used to calculate the upper and lower quantiles of a measure. In this section the major concepts and the procedure of the W-Test from Hasofer and Wang (1992) is outlined.

### 8.3.1 Estimation of High Quantiles for Type I

Using Weissman's estimator which is an asymptotically minimum variance estimator $\hat{q}(\epsilon)$ for the quantile $q(\epsilon)$ based on the top $k$ order statistics $X_{1n}, X_{2n}, ..., X_{kn}$ when $F(x)$ belong to the domain of attraction of Type I. It is given by

$$\hat{q}(\epsilon) = \hat{a} \ln(k/n\epsilon) + X_{kn}$$

where

$$\hat{a} = (\sum_{i=1}^{k} X_{in})/k - X_{kn}.$$

Because it is linear in the order statistics, this estimator is also the Best Linear Unbiased Estimator ("BLUE") of $q(\epsilon)$.

### 8.3.2 Test for Extreme Value Domain of Attraction

Before Weissman's estimator can be applied, it must be determined that the hypothesis that $F(x)$ belongs to the domain of attraction of Type I is consistent with the relevant data, namely the high order statistics of the given sample. If that hypothesis is rejected, a suitable transformation of the variable studied must be carried out to fulfill the above requirement. Hasofer and Wang (1992) have developed a simple but effective statistic (based on the top $k$ order statistics) to test the hypothesis that $F(x)$ belongs to the domain of attraction of Type I. It is denoted by $W$ and is given by the formula

$$W = \frac{k(\bar{X} - X_{kn})^2}{(k-1)[\sum_{i=1}^{k}(\bar{X} - X_{in})^2]} \tag{8.4}$$

where

$$\bar{X} = (\sum_{j=1}^{k} X_{jn})/k.$$

The W-test can be considered as a generalisation of the Shapiro-Wilk test of normality. It is easy to see that $W$ is invariant under a linear transformation of $X$, (which will apply the same linear transformation to each order statistic).

The null hypothesis is that $F(x)$ belongs to the domain of attraction of Type I, while the alternatives are that $F(x)$ belongs to the domain of attraction of Type II or of Type III.

The critical regions of the test are as follows: Let $W_L$ and $W_U$ be the lower and upper chosen percentage points of $W$. Then if $W < W_L$ we accept $H_2 : X$ belongs to the domain of attraction of Type II, while if $W > W_U$ we accept $H_3 : X$

belongs to the domain of attraction of Type III. Otherwise, we accept that $X$ belongs to the domain of attraction of Type I and use Weissman's estimator as it is.

The rationale for the above choice of critical regions is as follows: We recollect that the extreme value domains of attraction can be indexed by a parameter $\alpha$ varying from $-\infty$ to $+\infty$. $\alpha > 0$ corresponds to Type II while $\alpha < 0$ corresponds to Type III. But it can be shown, see Hasofer and Wang (1992), that as $\alpha$ increases the distribution of $W$ shifts continuously towards the left. Thus large values of $W$ correspond to Type III, while low values correspond to Type II and intermediate values to Type I.

The upper and lower percentage points of $W$ (asymptotic for large $n$) are given as functions of the number $k$ of high-order statistics, see Table 1 of Hasofer and Wang (1992). For larger values of $k$, $W$ may be taken to be normally distributed with mean $1/k$ and standard deviation $2/k^{3/2}$.

### 8.3.3 Estimating quantiles for Type III

The most serious departure from Type I occurs when the $W$ test indicates a Type III domain of attraction. This is because in this case the variable $X$ must have an upper bound. Of course all physical variables are theoretically bounded, but often the bound is so high that it is of no practical significance (e.g. wind velocity) and the variable may be assumed to be unbounded. However, some load variables have a practically significant upper bound, e.g. rainfall and earthquake magnitude. On the other hand, practically all resistance variables have a significant lower bound, simply because they cannot be negative.

Let $X$ have a finite upper bound $\omega_0$, and let as before $X_{1n} \geq X_{2n} \geq \cdots \geq X_{kn}$ be the $k$ top order statistics from a sample of size $n$. Clearly we must have $\omega_0 \geq X_{1n}$.. It can be shown that the limiting distribution of the $Y_{in} = -ln(\omega_0 - X_{in})$ ($i$ =1,...,$k$), is after a transformation of scale and origin by a suitable pair of sequences, the limiting distribution of the top $k$ order statistics corresponding to Type I. We recollect that the statistic $W$ is invariant with respect to a linear transformation of the order statistics, and is thus only a function of the $X_{in}$ and of the additional parameter $\omega_0$.

To estimate $\omega_0$ we use a recently developed method called the "estimating equation method". This consists in finding a function $G$ of the sample and of the required parameters that has zero mean (when the parameters take the correct value) and then solving the equation $G = 0$ for the parameters. Special cases of the "estimating equation" method are the method of moments, the method of least squares and the method of maximum likelihood. One great advantage of the method is that it is often possible to find a $G$ that excludes some nuisance parameters, thus considerably simplifying the form of the estimators. Let $Y(\omega)$ be the vector $-ln(\omega - X_{1n}), \cdots, -ln(\omega - X_{kn})$. We shall take as an estimating function for $\omega_0$.

$$G = W[Y(\omega)] - E(W[Y(\omega)]/\omega = \omega_0).$$

Clearly $E(G) = 0$ when $\omega = \omega_0$. Also, for large $k$, we have approximately $E(W/\omega = \omega_0) = 1/k$. Simulation work indicates that this value can be used in the estimating equation for $k$ as low as 10.

The important fact that gives the proposed method the edge over other methods which have been proposed to estimate $\omega_0$ is that if the W-test rejects the null hypothesis that $X$ belongs to the domain of attraction of Type I in favour of the alternative that $X$ belongs to the domain of attraction of Type III with a significance level of less than 20 percent, then our estimating equation has always a unique solution. This follows from the properties of $W$ given in Hasofer and Wang (1993):

Indeed, it is then clear that $W[Y(\omega)]$ increases monotonically from $1/(k-1)^2$ which is less than $E(W[Y(\omega)]|\omega = \omega_0)$ to $W(X_{1n}, ..., X_{kn})$ which is greater than $W_U > E(W[Y(\omega)]|\omega = \omega_0)$ for $k \geq 3$, so that our equation has a unique solution. Moreover, it can be shown that the obtained estimator is consistent as $k$ tends to infinity, see Hasofer and Wang (1993). In the reference just quoted will be found numerous simulation results which indicate that the proposed estimation method for $\omega_0$ as well as for the high quantiles of the variable under study is efficient and easy to use.

High quantiles in the situation under consideration are obtained by carrying out the transformation

$$\hat{Y}_{in} = ln(\omega_0 - X_{in}),$$

where $\hat{\omega}_0$ is the obtained estimator of $\omega_0$, evaluating the required quantile of $\hat{Y}$ by using the Weissman estimator, and then carrying out the reverse transformation to obtain the estimator of the quantile of $X$.

### 8.3.4 Estimating quantiles for Type II

When the W-test indicates that $X$ is in the domain of attraction of Type II, this is not as critical as in the case of the domain of attraction of Type III. Indeed, the sequence $a_n$ may be taken to be null without affecting the asymptotic results. Thus, the limiting distribution of $Y_{in} = ln(X_{in})$ will be of Type I and the Weissman estimator may be applied to it.

However, in the practical application of the method, where the asymptotic theory is applied to finite samples, this approach is unsatisfactory on two counts:

1. A quantile estimator should be invariant under translation, in the sense that if the underlying variable is increased by some amount $X_0$ then the quantile estimator should be increased by the same amount. This is clearly not the case with the algorithm just described.

2. There is no good reason to believe that all sample values will be necessarily positive. For example, our sample may measure sea levels below some reference level. If some sample values are negative, taking logarithms will not be possible.

For the above reasons, we advocate the use of the transformation $Y_{in} = ln(X_{in} - \omega_0)$, with $\omega_0 \leq X_{kn}$, when the W-test rejects the null hypothesis that $X$ belongs to the domain of attraction of Type I in favour of the alternative that $X$ belongs to the domain of attraction of Type II. We use again the estimating function method to obtain an estimator of $\omega_0$. Let $Y(\omega)$ be the vector $ln(X_{1n} - \omega), ..., ln(X_{kn} - \omega)$. As previously, we shall take as an estimating function for $\omega$

$$G = W[Y(\omega)] - E(W[Y(\omega)]|\omega = \omega_0).$$

Clearly $E(G) = 0$ when $\omega = \omega_0$. Also, for $k > 10$, we still have approximately $E(W|\omega = \omega_0) = 1/k$.

Here $W(X_{1n}, \cdots, X_{kn})$ is less than $E(W[Y(\omega)]|\omega = \omega_0)$, since it is less than $W_L$ at a significance level of less than 20 percent. Also, $E(W[Y(\omega)]|\omega = \omega_0)$, which is approximately $1/k$ for large $k$, is less than unity and this ensures the existence of a unique solution for the estimating equation. As for Type III, it can be shown that the obtained estimator is consistent as $k$ tends to infinity, see Hasofer and Li (1999). In that reference will be found numerous simulation results which indicate that the proposed estimation method for $\omega_0$ as well as for the high quantiles of the variable under study is efficient and easy to use. Evidence is also brought out to show that the assumption that $\omega_0 = 0$ may lead to serious error in finite samples.

High quantiles in the situation under consideration are obtained by carrying out the transformation

$$Y_{in} = \ln(X_{in} - \hat{\omega}_0),$$

where $\hat{\omega}_0$ is the obtained estimator of $\omega_0$ evaluating the required quantile of $\hat{Y}$ by using the Weissman estimator, and then carrying out the reverse transformation to obtain the estimator of the quantile of $X$.

### 8.3.5 The choice of $k$

There have been two main approaches to the selection of the appropriate sample size in the one-dimensional case: the "threshold" method and the $k$ top order statistics method. In the threshold method some high level $u$ is chosen and only the sample values above $u$ are considered. In the alternative method it is the number $k$ of top order statistics which is chosen. In this chapter attention will be to focused on the $k$ top order statistics method.

The choice of an optimal value for the number $k$ of high order statistics to be used involves conflicting considerations. On the one hand, the standard deviation of the quantile estimator $\hat{q}$ is approximately equal to $1/k$. More precisely

$$\sigma(\hat{q}) = c_n[\zeta_n^2 \frac{(k-1)}{k^2} + \frac{\pi^2}{6} - R_k]^{1/2}$$

where $\zeta_n = \ln(k/n\epsilon)$, $c_n = q(1/ne) - q(1/n)$, $(e = 2.718...)$, and $R_k = \sum_{n=1}^{k-1}(1/n^2)$.

In the formula just given $q(\epsilon)$ denotes the true quantile corresponding to $\epsilon$, which is of course unknown. However, the standard deviation of $\hat{q}$ can be estimated from the above formula by replacing $q(1/ne)$ and $q(1/n)$ by their Weissman estimators.

Thus, to enhance the precision of the estimation, $k$ should be chosen as large as possible. However, when $k/n$ is taken too large, bias is introduced by departure from the asymptotic distribution. To balance the effects of variance decrease against bias increase the recommendation is, for $\epsilon \leq 0.05$, $k/n = 0.02$ for $50 \leq n \leq 500$ and $k/n = 0.1$ for $500 \leq n \leq 5,000$. In Hasofer and Wang (1992), investigations suggest that $k$ can be taken to equal approximately $1.5\sqrt{n}$.

It is worth noting that as long as $k > n\epsilon$, $\hat{q}$ is a monotonically increasing function of the order statistics, except $X_{kn}$. To ensure that the coefficient of $X_{kn}$ be positive as well, the additional condition $k > 2.718n\epsilon$ should be fulfilled. This will ensure that Lind's Principle of Reliability and Consistency, Lind (1987), will be satisfied.

A more sophisticated approach to the determination of the optimal $k$ has been proposed by Wang (1995). The proposed procedure treats as a whole the two problems of determination of the domain of attraction and selection of $k$. It consists in calculating for each value of $k = 3, 4, \cdots$ the value of $W$ based on $X_{1n} \geq$

$X_{2n} \geq \cdots \geq X_{kn}$. The values are then plotted against $k$ on a graph, together with the corresponding values of $W_L$ and $W_U$ for some chosen significance level. If the graph of $W$ shows an early downward or upward trend, this is taken as an indication that the domain of attraction is Type II or Type III respectively. An appropriate logarithmic transformation is then carried out and a new set of $W$ values is computed. These transformations, by their very nature, will bring back $W$ to the region between $W_L$ and $W_U$. We then take the optimal $k$ to be equal to the first value of $k$ for which $W$ leaves the region between $W_L$ and $W_U$ minus one. A further improvement would be to then recalculate the parameter of the logarithmic transformation and repeat the procedure.

## 8.4 Calculating the Upper Quantile of Heat Load

The destructive potential of compartment fires was traditionally measured assuming that the temperature history of the compartment fire is the primary descriptor of its severity. As a rule the time integral of the temperature-time curve above some arbitrarily selected level was taken. Harmathy (1980) rejected this traditional concept and introduced a measure known as 'heat load' ($H$) to measure the destructive potential of compartment fires. Heat load is defined as the total heat absorbed by a unit surface area of an enclosure during a fire, and is evaluated by

$$H = \int_0^{\tau} f dt,$$

where $f$ is heat flux penetrating the enclosure boundaries, $t$ is the time and $\tau$ is the duration of the fire.

Some key load-bearing elements of compartment boundaries for example steel and reinforced concrete depend highly or solely on the maximum temperature reached. Hence if we are able to quantify the maximum temperature we have a measure for the destructive potential of a compartment fire. Harmathy and Mehaffy (1982) state that Harmathy (1980) went on to prove that heat load is a measure of the maximum temperature attained by key load-bearing elements of the enclosure boundary as a result of fire exposure. Hence it is of great importance to be able to calculate the upper quantile of heat load as it is a measure of the destructive potential of a compartment fire.

If we were able to assume that the heat load was normally distributed we could use basic probability theory to calculate the $q(\epsilon)$ quantile of heat load. As this can be shown not to be the case, we demonstrate the use of the non-parametric estimator of extreme value theory known as, Wang's procedure to estimate the $q(\varepsilon)$ quantile of heat load.

### 8.4.1 Heat load generated by a fire in a compartment

In this subsection the "Normalized Heat Load" proposed by Harmathy and Mehaffy (1982) is studied. We ignored the normalizing parameters. The values of heat load are stored in a vector of length 1500 and the vector is refereed to as *hload*. The stochastic model for compartment fires developed by Hasofer and Beck (1995) has been used for the Monte-Carlo simulation of a compartment fire to calculate heat load, see appendix D for the S-plus code.

Figures (8.1) and (8.2) show histograms of the values of $H$ based on 1,500 simulations of the stochastic fire model when $R_0$ and $B_{max}$ are fixed and random respectively. It is interesting to note that for some simulations the fire was extinguished at the outset, and for some others the heat load remained low. However, the main body of the data is bell shaped and there are no particular anomalies at the upper end.

Using the data for figure (8.1) the mean of $H$ was 8063.5 and the standard deviation 849.8. A test of normality (Kolmogorov-Smirnov) was applied to the heat load vector and the normality hypothesis was rejected, even when the outliers at the lower end were removed. This was clearly due to the non-linearities in the compartment fire equations. As the normality assumption is rejected we will use

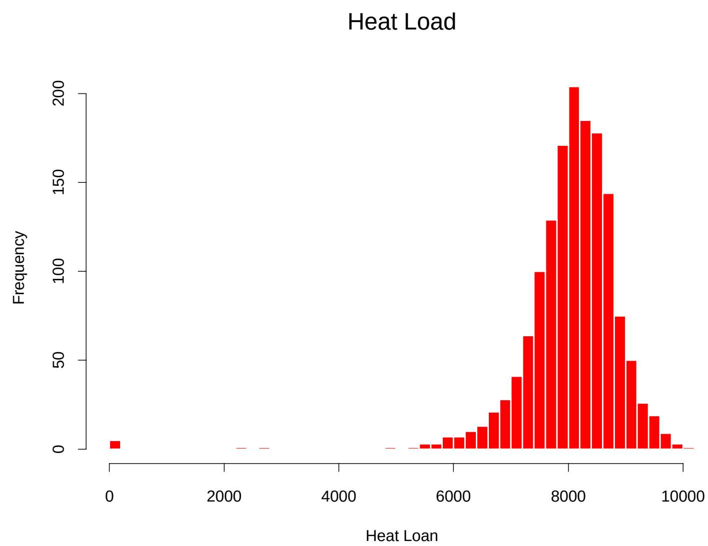


Figure 8.1: Histogram of Heat Load: $R_0$ and $B_{max}$ fixed.

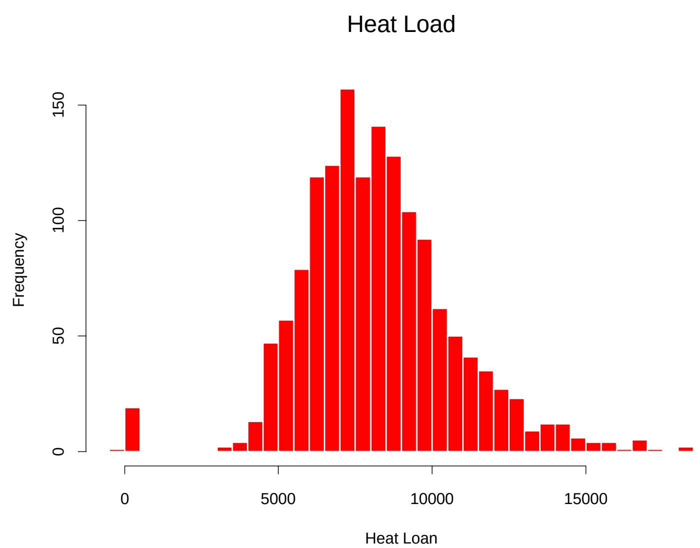


Figure 8.2: Histogram of Heat Load: $R_0$ and $B_{max}$ random.

the non-parametric method, W-Test, to calculate the upper quantile of heat load.

Using equation (8.4)

$$W = \frac{k(\bar{X} - X_{kn})^2}{(k-1)[\sum_{j=1}^{k}(\bar{X} - X_{in})^2]}$$

where

$$\bar{X} = \frac{(\sum_{j=1}^{k} X_{jn})}{k},$$

and $X_{kn}$ is the $k^{th}$ smallest order statistic.

To calculate the W-statistic in Wang's algorithm we used the statistical package 'S-plus'; the function is as follows:

```
> wtest
function(k)
{
lk <- length(k)
muk <- mean(k)
den <- sum((muk - k)^2)
W <- (lk * (muk - k[1])^2)/((lk - 1) * den)
W
}
```

To calculate the values of $W$ for $k = 2, 3, \cdots$ we wrote a function in S-plus called WANG

```
> WANG
function(hload, size)
{
Calculates values of W for WANG'S TESt
W <- rep(0, 30)
for(j in (1:30)) {
order <- hload[(1501 - size[j]):1500]
```

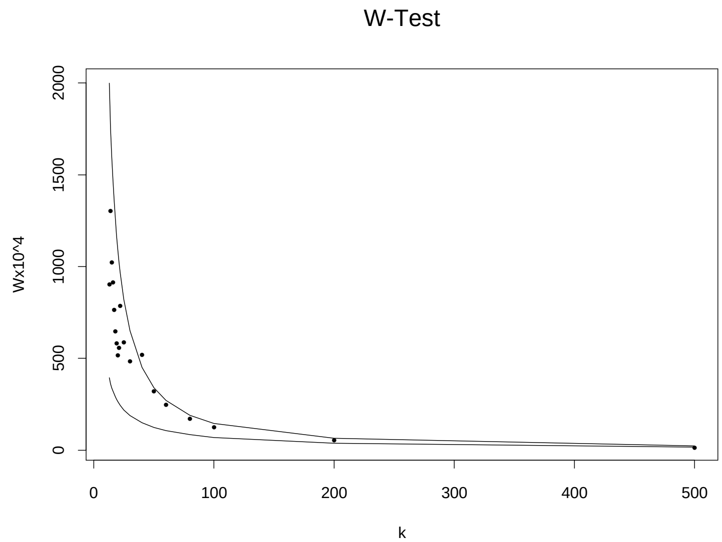


Figure 8.3: Graph of original values of $W$ versus $k$.

```
W[j] <- wtest(order)
}
W
}
```

To determine an appropriate value for $k$, the W-test was applied to the data with various values of $k$. The results, together with the 5 percent confidence limits for $W$, from Table (4.1) of Hasofer (1996), are plotted in Figure (8.3). From the plot, the $W$ values show an early upward trend, hence, we take it to be a domain of attraction of Type III. Domain of attraction of Type III is the most serious departure from Type I. The optimal $k$ appears to be $k = 40 - 1 = 39$ as the $40^{th}$ value crosses the upper bound. To estimate the finite upper bound, $\omega_0$, the 'estimating equation' method is used. If $Y(\omega) = \{\ln(\omega - X_{1n}), \ln(\omega - X_{2n}), \ldots, \ln(\omega - X_{kn})\}$ The estimating function of $\omega_0$ is

$$G = W[Y(\omega)] - E(W[Y(\omega)]/\omega = \omega_0).$$

Since for large $k$ $E(W/\omega = \omega_0) = 1/k$

$$G = W[\ln(X - Sample)] - 1/k.$$

This is written as a function in S-plus called W3 and is optimised using the function uniroot in S-plus.

```
> W3
function(x, sample)
{
W3 : W(log(x-sample))-1/k
k <- length(sample)
r <- length(x)
y <- rep(0, r)
for(i in 1:r) {
y[i] <- wtest(log(x[i] - sample)) - 1/k
}
y
}
```

The finite upper bound, $\omega_0$, was found to be 10,236 for heat load.

The next step is to carry out the transformation

$$H^* = -\log_e(10,236 - hload),$$

using the value of $\omega_0 = 10,236$, to transform hload to the domain of attraction of Type I.

Using the vector $H^*$ a new set of $W$ values, $W^*$ with various values of $k$ are plotted. The results are shown in figure (8.4) from which we now conclude that $H^*$ may be assumed to be in the domain of attraction of Type I.

From figure (8.4) the values of $W^*$ appear to belong to the domain of attraction of Type I. The optimum $k$ appears to be $k = 200 - 1 = 199$.

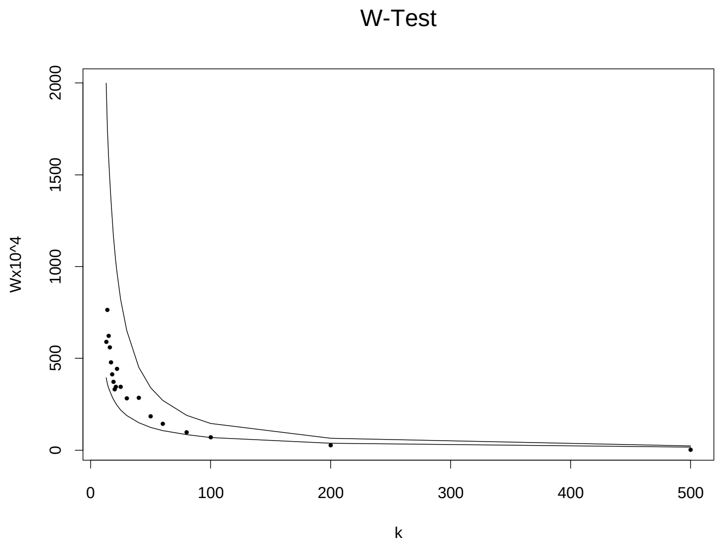


Figure 8.4: Graph of transformed values of $W^*$ versus $k$.

We will now calculate the upper quantile using Weissman's estimator as we assume that the cumulative distribution function, $F(x)$, belongs to the domain of attraction of Type I.

$$\hat{q}(\epsilon) = \hat{a}\ln(\frac{k}{n\epsilon}) + X_{kn}$$

where $\hat{a} = \frac{(\sum_{i=1}^{k} X_{in})}{k} - X_{kn}$

$$\hat{q}(0.01) = -6.521523.$$

Transforming $\hat{q}(\epsilon)$ back to the original heat load we find

$$\hat{q}(0.01) = 10,236 - \exp(-6.521523) = 9555.977.$$

Finally some quantiles of $H^*$ were calculated, using the Weissman estimator and the corresponding value of $H$ found. For comparison purposes the quantiles were also calculated on the assumption that $H$ belonged to the domain of attraction of Type I, and also on the assumption that $H$ was normally distributed with the mean and standard deviation given above. The results are given in Tables (8.1,8.2,8.3and 8.4).

| $\epsilon$ | Type I | Type III $\omega_0 = 10,236$ | Normal |
|---|---|---|---|
| 0.01 | 9551 | 9556 | 10040 |
| 0.02 | 9360 | 9397 | 9808 |
| 0.05 | 9108 | 9130 | 9461 |

Table 8.1: Quantiles when $k$=100 mean=8063.5 s.d.=849.8

| $\epsilon$ | Type I | Type III $\omega_0 = 10,236$ | Normal |
|---|---|---|---|
| 0.01 | 9599 | 9539 | 10040 |
| 0.02 | 9378 | 9387 | 9808 |
| 0.05 | 9087 | 9132 | 9461 |

Table 8.2: Quantiles when $k$=199, mean=8063.5, s.d.=849.8

| $k$ | $\epsilon = 0.01$ | $\epsilon = 0.02$ | $\epsilon = 0.05$ | $\epsilon = 0.10$ |
|---|---|---|---|---|
| 40 | 9538.499 | 9359.939 | 9123.895 | 8945.335 |
| 45 | 9537.836 | 9370.018 | 9148.173 | 8980.355 |
| 50 | 9538.812 | 9366.188 | 9137.991 | 8965.367 |
| 60 | 9540.862 | 9365.221 | 9133.036 | 8957.395 |
| 70 | 9550.402 | 9358.792 | 9105.498 | 8913.888 |
| 80 | 9546.818 | 9359.807 | 9112.592 | 8925.582 |
| 90 | 9551.911 | 9360.372 | 9107.171 | 8915.632 |
| 100 | 9550.511 | 9360.260 | 9108.761 | 8918.510 |

Table 8.3: Quantiles Assuming Type I

| $k$ | $\epsilon = 0.01$ | $\epsilon = 0.02$ | $\epsilon = 0.05$ | $\epsilon = 0.10$ |
|---|---|---|---|---|
| 40 | 9095.714 | 9049.518 | 8985.565 | 8934.906 |
| 60 | 9190.085 | 9112.799 | 9001.796 | 8910.591 |
| 70 | 9252.24 | 9157.739 | 9018.737 | 8901.796 |
| 80 | 9329.253 | 9215.526 | 9042.999 | 8893.353 |
| 90 | 9420.497 | 9287.178 | 9076.902 | 8887.383 |
| 100 | 9555.977 | 9397.594 | 9130.21 | 8872.603 |

Table 8.4: Quantiles Assuming Type III with $\omega_0 = 10,236$

From the above tables it can be seen that quantile estimation in this case is insensitive to the choice of domain of attraction as well as to the precise value of $k$. On the other hand, it is quite clear that quantile evaluation under the normality assumption is grossly in error. For $\epsilon = 0.01$ the quantile is overestimated by more than half a standard deviation! Of course, as is to be expected, the error diminishes as the tail probability increases, but since in reliability applications it is small tail probabilities which are of interest, the example illustrates dramatically the superiority of the method advocated in this paper over methods based on the entire sample.

## 8.5 Conclusion

In this chapter we provided a brief outline of Extreme Value theory and provided a detailed account of the W-Test. The measure Heat Load was defined and used to illustrate the superiority of the W-Test compared to assuming the normal distribution of the data.

# Chapter 9

# Simpler Oxygen Driven Compartment Fire Model

## 9.1 Introduction

Even though the deterministic General Epidemic model and the fire growth model proposed by Hasofer and Beck (1995) are not the same, they do show similarity. Since equation (9.4) relates the three differential equations together, the General Epidemic model is given by any two of the following three differential equations. The differential equation describing the susceptibles is

$$\frac{dS(t)}{dt} = -\beta S(t)I(t), \tag{9.1}$$

the differential equation describing the number of infectives is

$$\frac{dI(t)}{dt} = \beta S(t)I(t) - \gamma I(t), \tag{9.2}$$

and the differential equation describing the number of removals is

$$\frac{dR(t)}{dt} = \gamma I(t). \tag{9.3}$$

Since all individuals in the population are either susceptible, infected or removed and the population is constant in size,

$$S(t) + I(t) + R(t) = N. \tag{9.4}$$

The assumptions of the General Epidemic model are:

1. Any individual who has recovered from the disease has permanent immunity.

2. The disease has a negligible short incubation period, no latent period. When a susceptible is infected it is assumed that he immediately becomes infectious.

3. The assumption of an independent isolated group of given size, subject to homogeneous mixing.

The fire growth model proposed by Hasofer and Beck (1995) is comprised of:

- The heat balance equation

  $$\frac{dT}{dt} = \beta R - q(T), \tag{9.5}$$

  where

  $$q(T) = \Sigma[(T + 273)^4 - (T_0 + 273)^4] + \phi(T - T_0). \tag{9.6}$$

- The oxygen mass balance equation

  $$\frac{dD}{dt} = \delta(k_1 - D)R - \mu D, \qquad \text{where} \qquad D = 23 - 100x. \tag{9.7}$$

- And the burning rate equation

  $$\frac{dR}{dt} = \alpha(k - D)Z(T). \tag{9.8}$$

The deterministic General Epidemic model has a product term $S(t)I(t)$ appearing and the model by Hasofer and Beck (1995) has, $D(t)R(t)$ and $D(t)Z(T)$. The two sets of differential equations are a system of three non-linear differential equations and need numerical methods to be solved.

To make the fire growth model set of equations closer to the General Epidemic model set of equations, we simplify the model by Hasofer and Beck (1995) by making the assumption that the rate of increase in temperature is a function of the burning rate only. Thus we can ignore the gas temperature at first and write a set of differential equations which describe a compartment fire based only on the burning rate and the oxygen concentration. We then introduce the gas temperature by finding a relationship between gas temperature and burning rate. We chose to use this method to reduce the system of three non-linear differential equations to a system of two non-linear differential equations and to have only one product term which will make it more like the General Epidemic model. Aligning the Hasofer and Beck (1995) model closer to the deterministic General Epidemic model will reduce the computer processing time, and further more if the new set of equations is aligned close enough to the General Epidemic model we can use the available rich asymptotic theory of epidemic models.

## 9.2 Two Variable Oxygen Driven Model

Let $R$ (g/min) be the burning rate and $x$ be the oxygen fraction in the compartment. If we assume no air comes into the room, the rate of decrease of the oxygen fraction is directly proportional to the burning rate. Then we can write a differential equation for the oxygen fraction as

$$\frac{dx}{dt} = -k_1 R \tag{9.9}$$

where $k_1$ is a constant. Also since the rate of increase in burning rate depends on the current burning rate and the oxygen fraction, the differential equation for the burning rate can be written as

$$\frac{dR}{dt} = k_3 x R \tag{9.10}$$

where $k_3$ is a constant. Solving these two equations numerically and plotting them shows $x(t)$ has negative values, see figures (9.1) and (9.2). As they stand these

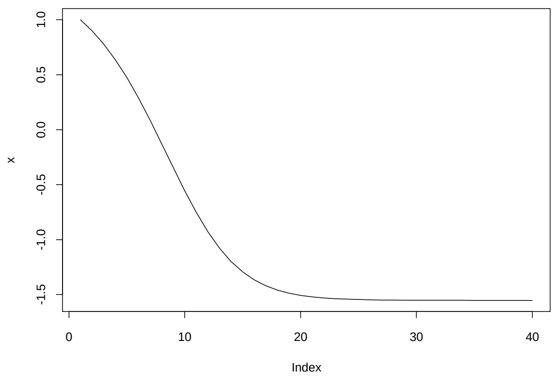


Figure 9.1: Plot of oxygen fraction

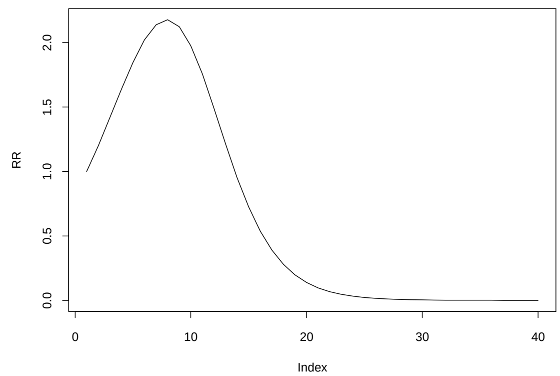


Figure 9.2: Plot of burning rate

equations are not appropriate for modelling the fire growth within a compartment.

To make physical sense, equations (9.9) and (9.10) are modified. Instead of writing the oxygen fraction equation as $dx/dt = -k_1R$, we modified it to

$$\frac{dx}{dt} = -k_1R + k_2(x_0 - x) \tag{9.11}$$

where $x_0$ is the oxygen fraction at time $t = 0$ and $k_2$ is a constant. This differential equation says the rate of the oxygen fraction is decreased due to increased burning rate, $-k_1R$, and is increased due to new oxygen coming into the compartment, $k_2(x_0 - x)$.

Also, instead of writing the burning rate equation, $dR/dt = k_3xR$, we modified it to

$$\frac{dR}{dt} = k_3(x - x_1)R$$

where $x_1$ is the oxygen fraction at which the fire is extinguished. This differential equation says that the rate of change in burning rate is increased with burning rate increase and oxygen fraction increase above the threshold $x_1$.

To stop $(x-x_1)$ from becoming negative we introduce max(0,$x$), the max(0,$x$) can be refined by introducing the function $pos(w, \beta)$. The $pos(w, \beta)$ is a smoothed

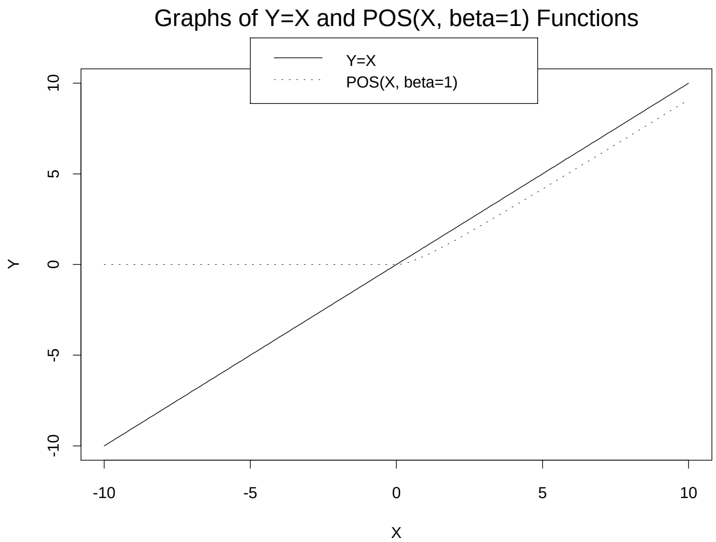


Figure 9.3: Plot demonstrating *pos* function against Y=X

version of max(0,$x$).

$$pos(w, \beta) = \max[0, w(1 - \frac{1}{(1+w)^{\beta}})],$$

see figure (9.3) for a graphical illustration of the $pos(w, \beta)$ function.

Hence the two differential equations for the compartment fire are:

$$\frac{dx}{dt} = -k_1 R + k_2(x_0 - x) \tag{9.12}$$

and

$$\frac{dR}{dt} = k_3 pos((x - x_1), \beta)R. \tag{9.13}$$

The initial conditions for equations (9.12) and (9.13) are $R(0) = R_0$ and $x(0) = x_0 = 0.23$. From Hasofer and Beck (1995) the oxygen concentration at which the fire is extinguished is $x_1 = 0.126$.

To simplify equations (9.12) and (9.13) we introduce a change of variables by dividing the time $t$ by $t_{cap}$ and $R$ by $R_0$.

$$\frac{dx}{d(t/t_{cap})} = -k_1(R/R_0)t_{cap} + k_2 t_{cap}(x_0 - x) \tag{9.14}$$

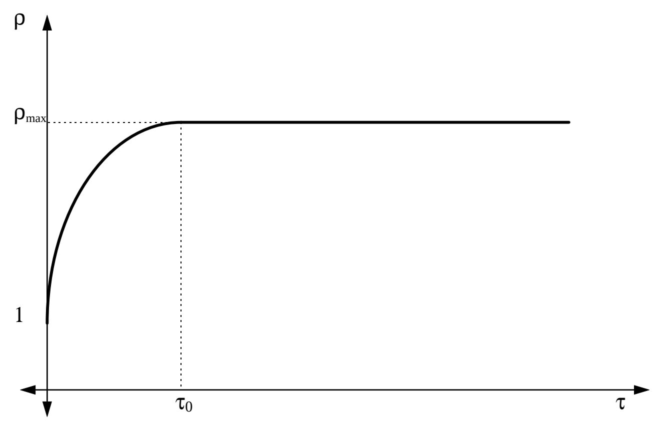


Figure 9.4: Illustation of the path of $\rho$

and

$$\frac{d(R/R_0)}{d(t/t_{cap})} = k_3 t_{cap} pos((x - x_1), \beta)(R/R_0). \tag{9.15}$$

We represent the new variables as $\tau = t/t_{cap}$ and $\rho = R/R_0$ and choose $t_{cap}$ such that $R_0 k_1 t_{cap} = p_1$. Also equating $k_2 t_{cap} = 1$ and $k_3 t_{cap} = p_2$, the equations reduce to:

$$\frac{dx}{d\tau} = -p_1 \rho + (x_0 - x) \tag{9.16}$$

and

$$\frac{d\rho}{d\tau} = p_2 pos((x - x_1), \beta)\rho. \tag{9.17}$$

Now the initial conditions imply $\rho(0) = 1$, since $\rho(t = 0) = \frac{R(t=0)}{R_0} = \frac{R_0}{R_0} = 1$; $x_0 = x(t = 0) = 0.23$ and $x_1 = 0.126$.

## 9.3 Behaviour

The variable $\rho$ increases from 1 to $\rho_{max}$ at time $\tau_0$, see figure(9.4). The parameters

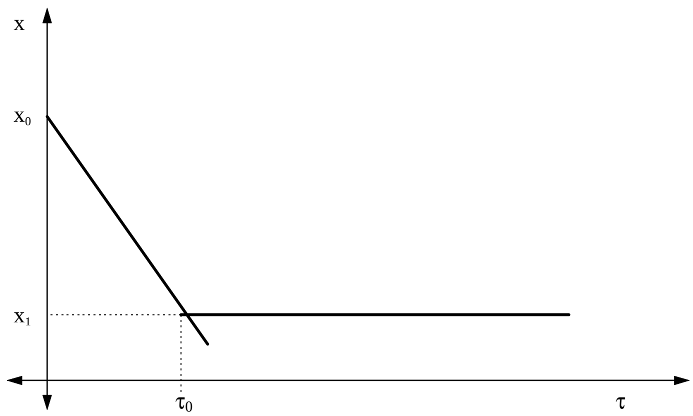


Figure 9.5: Simplified Illustation of $x$

$\tau_0$ and $\rho_{max}$ are both functions of $p_1$ and $p_2$. After $\tau_0$, $\rho$ remains constant at $\rho_{max}$ because $x < x_1$.

The variable $x$ decreases steadily from 0 to $\tau_0$. At $\tau_0$, $\frac{dx}{d\tau}$ is negative. We introduce $(\frac{dx}{d\tau})_{\tau=\tau_0} = -\Delta$. Then we have $-\Delta = -p_1\rho_{max} + (x_0 - x_1)$. Note $\Delta$ depends on $\rho_{max}$, $p_1$ and $p_2$. So $\rho_{max} = \frac{1}{p_1}[\Delta + (x_0 - x_1)]$.

After $\tau_0$ the behaviour of $x$ is governed by the equation

$$\frac{dx}{d\tau} = p_1\rho_{max} + (x_0 - x)$$

with initial condition $\tau = \tau_0$ and $x = x_1$.

The differential equation

$$\frac{dx}{d\tau} + x = -p_1\rho_{max} + x_0$$

has a transient solution $x = Ce^{-\tau}$. and a particular solution $x = x_0 - p_1\rho_{max}$. Hence the general solution is

$$x = Ce^{-\tau} + (x_0 - p_1\rho_{max}).$$

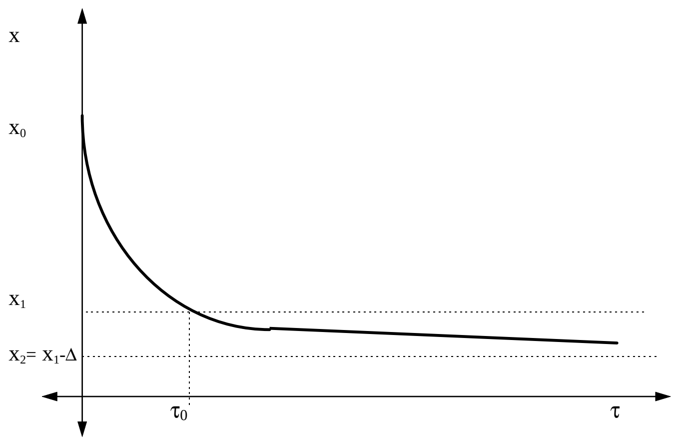


Figure 9.6: Illustation of the path of $x$

To find $C$ we substitute the initial condition into the above equation,

$$x_1 = Ce^{-\tau_0} + (x_0 - p_1\rho_{max})$$

$$C = e^{\tau_0}[p_1\rho_{max} - (x_0 - x_1)]$$

$$= e^{\tau_0}\Delta.$$

So

$$x = \Delta e^{-(\tau-\tau_0)} - \Delta + x_1$$

$$= \Delta e^{-(\tau-\tau_0)} + x_1 - \Delta$$

$$= x_1 - \Delta(1 - e^{-(\tau-\tau_0)}).$$

So the asymptotic value of $x$ is,

$$x_2 = x_1 - \Delta$$

$$= x_0 - p_1\rho_{max}.$$

See figure (9.6) for an illustration.

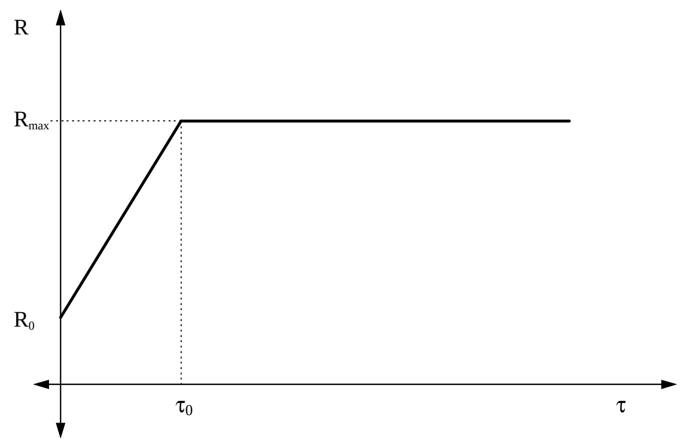


Figure 9.7: Simplified Illustation of $R$

## 9.4 Method For Evaluating The Parameters

From the data of the NRCC model run we observe at $t = 0$, $R = R_0$ and $x = x_0$; the oxygen concentration reaches its threshold at $t = t_1$, and at this point $R = R_1$ and $x = x_1$; the burning rate reaches it maximum value $R_{max}$ at $t = t_2$. Finally, the lowest asymptotic value the oxygen fraction approaches is $x = x_2$.

Now $\rho_{max} = R_{max}/R_0$, which we can calculate. Using $\rho_{max}$ and $x_2$ we calculate an approximate value of $p_1$.

$$x_2 = x_0 - p_1 \rho_{max},$$

which implies $p_1 = \frac{x_0 - x_2}{\rho_{max}}$.

Knowing $p_1$ we can find the value of $p_2$ using the S-Plus function “approx” on the simulation results of the S-Plus Two Parameter Model, see appendix (G) for the code.

Finally we find

$$t_{cap} = \frac{t_1}{\tau_0(p_1, p_2)}$$

where $t_1$ is observed. Using the relationship

$$\tau = \frac{t}{t_{cap}}$$

and the observed values when $t = t_1 = 5.66$ minutes, $x = x_1$. When $x = x_1$, $\tau = \tau_0$, see figures (9.6) and (9.7). Hence $\tau_0$ is found from the simulation run in calculating $p_2$ and $t_{cap}$ is calculated using

$$t_{cap} = \frac{t_1}{\tau_0}.$$

Using $t_{cap}$, $R_0$, $p_1$ and $p_2$ we can calculate the parameters $k_1$, $k_2$ and $k_3$ of the differential equations (9.12) and (9.13) using:

$$k_1 = \frac{p_1}{R_0 t_{cap}}$$

$$k_2 = \frac{1}{t_{cap}}$$

$$k_3 = \frac{p_2}{t_{cap}}.$$

Finally using these parameters we can solve the differential equations (9.12) and (9.13) numerically using difference equations.

## 9.5 Fitting The Equation To The NRCC Model

From Hasofer and Beck (1997), at $t = 0$, $x = x_0 = 0.2299$ and $R = R_0 = 8.38g/\text{minute}$; at $x = x_1 = 0.126$, $t = t_1 = 5.66$ minutes and $R = R_1 = 3,971.18g/\text{minute}$. Also at $t = t_2 = 8.74$ minutes, $R = R_{max} = 7991g/\text{minute}$ and the asymptotic value of $x = x_2$ is approximately $x_2 = 0.0727$.

$$\rho_{max} = \frac{R_{max}}{R_0} = \frac{7991}{8.38} = 953.580$$

$$p_1 = \frac{x_0 - x_2}{\rho_{max}} = \frac{0.2299 - 0.0727}{953.580} = 1.684 \times 10^{-4}$$

From the simulation run $p_2 = 15.7632$ and $\tau_0 = 5.32$.

To calculate $t_{cap}$, we have the relationship $\tau = t/t_{cap}$ or $t_{cap} = t/\tau$. At $x = x_1$, $t = t_1 = 5.66$ and $\tau = \tau_0 = 5.32$. Hence $t_{cap} = \frac{5.66}{5.32} = 1.0639$.

We can now evaluate the parameters $k_1$, $k_2$ and $k_3$ of the differential equation.

$$k_1 = \frac{p_1}{R_0 t_{cap}} = \frac{1.648\text{x}10^{-4}}{8.38\text{x}1.0639} = 1.84 \times 10^{-5}$$

$$k_2 = \frac{1}{t_{cap}} = \frac{1}{1.0639} = 0.9399$$

$$k_3 = \frac{p_2}{t_{cap}} = \frac{15.7632}{1.0639} = 14.8164.$$

Hence equation (9.12) and equation (9.13) can be written as

$$\frac{dx}{dt} = -1.84\text{x}10^{-5}R + 0.9399(x_0 - x) \tag{9.18}$$

$$\frac{dR}{dt} = 14.8164 pos((x - x_1), \beta)R \tag{9.19}$$

The deterministic differential equations (9.18) and (9.19) are non-linear autonomous systems. They cannot be solved explicitly, so we will use numerical solutions using S-plus.

Before we represent equations (9.18) and (9.19) as difference equations, we will find the relationship between the burning rate and the gas temperature in the compartment.

## 9.6 Gas Temperature Equation

Making the assumption that the rate of increase in temperature is a function of the burning rate, we can write

$$\frac{dT}{dt} = \beta R - \gamma T \tag{9.20}$$

This says that the rate of change in temperature increases with burning rate and decreases due to heat loss. We have assumed that the heat loss is a simple linear function of temperature. This assumption ensures that the differential equation is a first order differential equation which can be solved explicitly.

Rewrite the above equation

$$\frac{dT}{dt} + \gamma T = \beta R.$$

As $R(t)$ is the solution of a non-linear differential equation we will solve equation (9.20) numerically with equations (9.18) and (9.19).

### 9.6.1 Parameter Evaluation

When $dT/dt \approx 0$, $T = T_{max}$ and $R = R_{max}$. Substituting this condition into equation (9.20) we obtain

$$\gamma T_{max} = \beta R_{max}. \tag{9.21}$$

Hence

$$\beta = \frac{\gamma T_{max}}{R_{max}}.$$

$T_{max}$ and $R_{max}$ are constants, and can be obtained from the NRCC model. Due to this relationship only $\gamma$ needs to be evaluated.

A method for evaluating the parameter $\gamma$ of the model is to use ten points on the temperature curve of the NRCC model and minimise the sum of the squared differences between the temperatures at these points.

$$Sum = \sum_{i=1}^{10} (T_{NRCC(i)} - T_{PRED(i)})^2 \tag{9.22}$$

The S-plus algorithm for the above optimisation is given in appendix F.

### 9.6.2 Parameters of The Temperature Equation

From Hasofer and Beck (1995), at $t = 0$, $x = x_0 = 0.2299$ and $R = R_0 = 8.38g$/minute; at $x = x_1 = 0.126$, $t = t_1 = 5.66$ minutes and $R = R_1 = 3971g$/minute. At $t = t_2 = 8.74$ minutes, $R = R_{max} = 7991g$/minute and $x = x_2 = 0.0727$. The maximum temperature observed is approximately $990^oC$ at $t = 10$ minutes. Hence

$$\beta = \frac{990 \times \gamma}{7991}.$$

The ten time and temperature points respectively are (1.00, 48.5), (3.30, 200.2), (4.00, 250.0), (5.00, 345.3), (5.70, 601.5), (6.10, 743.4), (7.00, 848.1), (8.10, 900.7), (10.00, 943.3) and (14.00, 981.8). Using these points with the S-plus optimisation algorithm in appendix F gave the following results. Minsum in table (9.1) is the result of equation (9.22).

From the table (9.1) we see that the optimum estimate of $\gamma$ is

$$\gamma = 0.855$$

and as a result

$$\beta = 0.1059.$$

Hence the differential equation (9.20) is

$$\frac{dT}{dt} = 0.1059R - 0.855T. \tag{9.23}$$

To calculate the gas temperature in the compartment with the difference equations, we calculate $T$ at each time step using equation (9.23) with the value of $R$ at that time step.

## 9.7 Difference Equations

To compare the non-linear differential equations (9.18), (9.19) and (9.23) with the NRCC model they are made into difference equations, as they cannot be solved implicitly, and evaluated discretely.

- The difference equation for the time equation is:

  $$t[r+1] = t[r] + dt,$$

  were $dt = 0.02$ of a minute.

- The difference equations for the burning rate equation (9.19) is:

  $$B[r+1] = B[r] + R[r]dt, \tag{9.24}$$

| Values of $\gamma$ | Minsum |
|---|---|
| 0.1 | 1807086.85716672 |
| 0.2 | 898158.656669525 |
| 0.3 | 483682.646541452 |
| 0.4 | 271985.16012209 |
| 0.5 | 159185.734563122 |
| 0.6 | 99613.7102306882 |
| 0.7 | 70377.7139765986 |
| 0.8 | 59028.9260236807 |
| 0.82 | 58215.4160004428 |
| 0.84 | 57782.1155860546 |
| 0.84 | 57782.1155860546 |
| 0.845 | 57728.8359263009 |
| 0.85 | 57696.482140381 |
| 0.855 | 57684.5314347369 |
| 0.86 | 57692.4737846328 |
| 0.865 | 57719.8115890781 |
| 0.87 | 57766.0593363037 |
| 0.875 | 57830.7432794264 |
| 0.88 | 57913.4011219492 |
| 0.9 | 58414.9094940273 |
| 0.92 | 59169.7939643521 |
| 0.94 | 60153.3510461516 |
| 0.96 | 61343.1287965174 |
| 1.0 | 64261.4879879102 |
| 1.1 | 73940.3347811421 |
| 1.2 | 85801.9505316803 |
| 1.3 | 98797.2756379585 |
| 1.4 | 112254.766612557 |
| 1.5 | 125744.930343411 |
| 1.6 | 138995.857987799 |
| 1.7 | 151839.408233694 |

Table 9.1: Optimisation Results for the Parameter $\gamma$

$$\text{if} \quad B[r] < B_{max}, \tag{9.25}$$

$$R[r+1] = R[r] + (14.8164 * max(0, (x[r] - 0.126)) * R[r]dt), \tag{9.26}$$

$$\text{else} \quad R[r+1] = R[r] - \text{RDECAY} R[r]dt, \tag{9.27}$$

where RDECAY is a decay constant evaluated in Hasofer and Beck (1995). Equation (9.24) keeps a record of the amount of fuel burnt, $B$. If equation (9.25) is satisfied; the amount of fuel burnt is less than the total amount of fuel available, $B_{max}$; equation (9.26) is used to calculate the burning rate value for the interval $r+1$. If equation (9.25) is not satisfied; there is no unburnt fuel left; equation (9.27) is used to calculate the decaying burning rate value for the intervals following.

- The difference equation for the oxygen concentration equation (9.18) is: If equation (9.25) is satisfied; the amount of fuel burnt is less than the total amount of fuel available, $B_{max}$; equation

$$x[r+1] = -k_1 R[r] + u(x[r], \alpha)dt + k_2(0.2299 - x[r])dt, \tag{9.28}$$

is used to calculate the oxygen fraction value for the interval $r+1$. If equation (9.25) is not satisfied; there is no unburnt fuel left; equation

$$x[r+1] = x[r] - \mu(x[r] - 0.2299)dt, \tag{9.29}$$

is used to calculate the increasing oxygen fraction value for the interval $r+1$.

- The difference equations for the gas temperature equation 9.23) is:

$$T[r+1] = T[r] + (-0.855T[r]dt) + (0.1059R[r]dt), \tag{9.30}$$

## 9.8 Comparison With The NRCC Model

The three non-linear differential equations (9.18), (9.19) and (9.23) have been compared with the data of the NRCC model in section (7.4), see appendix E for the data

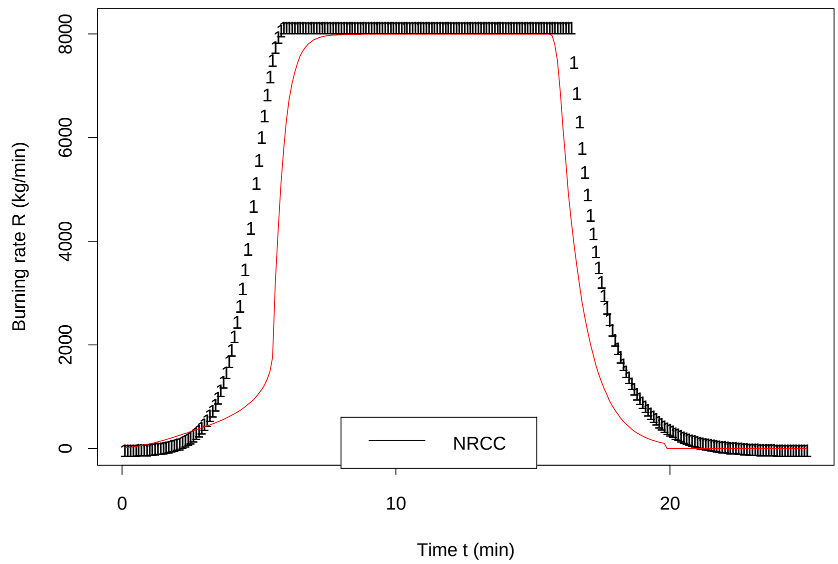


Figure 9.8: Comparison of the Burning Rate.

from the NRCC program and appendix G for the code of the difference equations using S-plus.

A comparison of the time-dependent variation of the burning rate $R$, the oxygen percentage $x*100$ and the gas temperature $T$ with the corresponding output of the NRCC model is shown in figures (9.8), (9.9) and (9.10).

Using the forcing functions for oxygen fraction, burning rate and gas temperature equations outlined in chapter (7) the above equations can be made into stochastic differential equations.

## 9.9 Conclusion

The advantage of the compartment fire model in this chapter over the one in chapter (7) is that this model is simpler i.e. it has fewer parameters to evaluate. The optimisation algorithm to estimate the parameters of the temperature equation is

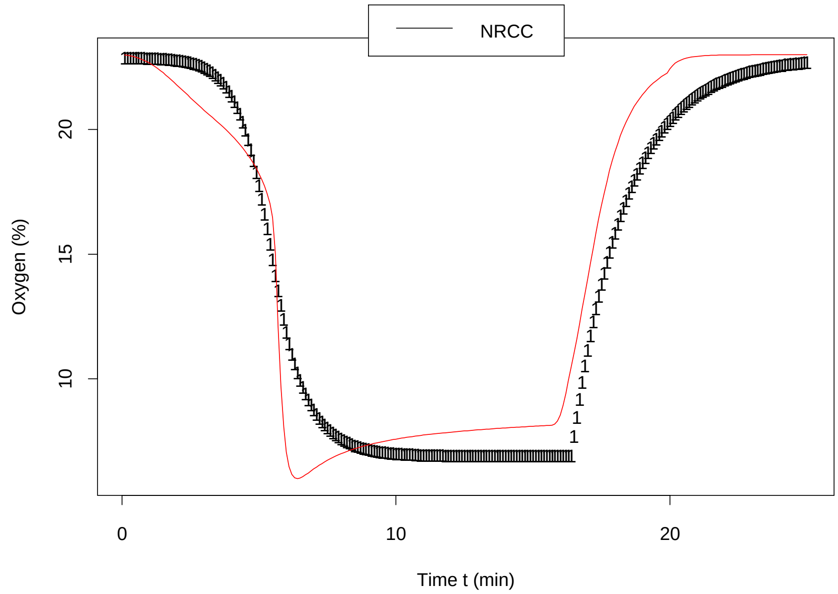


Figure 9.9: Comparison of the Oxygen Concentration.

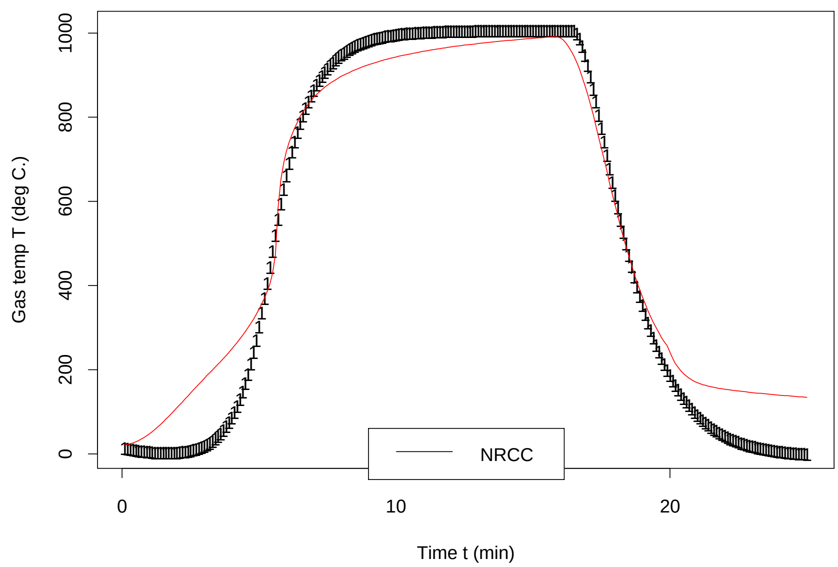


Figure 9.10: Comparison of the Gas Temperature.

extremely simple and quick in comparison to the optimation algorithm in Hasofer and Beck (1995). Hence it is easier and quicker to use with Monte-Carlo simulations in probabilistic fire risk analysis.

# Part III

# OPTIMAL CONTROL OF COMPARTMENT FIRES WITH SPRINKLERS

Automatic sprinkler systems are by far the most important type of fire protection. They provide a continuous protection against fires by both detecting and fighting a fire. This part of the thesis has three chapters. The first chapter covers a brief review of an automatic sprinkler system. In the next chapter we describe a topic in mathematical theory known as Optimal Control. Finally in the last chapter we combine the operation of sprinklers and the theory of optimal control to model the flow of water from sprinklers to minimise water damage.

# Chapter 10

# Automatic Sprinkler Systems

## 10.1 Introduction

An automatic sprinkler installation consists of a water supply connected by pipes to sprinkler heads. The sprinkler head is a heat sensitive valve, it is the sprinkler head which automatically detects the fire and as a consequence an alarm is given and water delivered to the seat of the fire. Thus, the fire is extinguished or kept under control until the fire brigade arrives. Figure (10.1) is a picture of one of the commonly used sprinkler heads.

When sprinkler systems are being designed for a building, the building is classified into one of three hazard classes:

- *Extra Light Hazard.* Buildings where the amount and combustibility of the material is low.
- *Ordinary Hazard.* Buildings involving the handling, processing and storage of material in which intensely burning fires are unlikely to develop in the initial stages.
- *Extra High Hazard.* Buildings with abnormal fire loads.

See FPA (1989) for a more comprehensive account of the hazard classes. Sprinkler

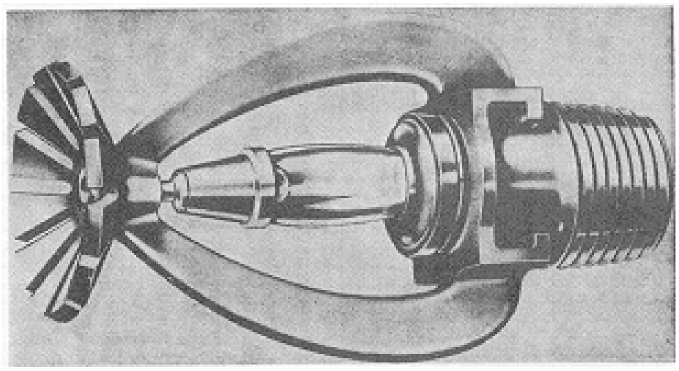

Figure 10.1: Medium velocity sealed and open spray sprinkler head.

heads are also rated into classes according to their operating temperature. See section (10.2) for a short outline or Bush and Mclaughlin (1979) for a detailed account.

The literature on sprinkler systems is very rich and as a result frequent reference is made to some of these publications for a detailed account of the topic. To help us with the chapters to follow some general features of automatic sprinkler systems are briefly described. This chapter is divided into three sections. The first section provides a brief description of sprinkler heads and their spacing. Then in the second section some time is spent on the pipework and the water supply. Finally, in section three some of the major factors which contribute to the fluctuation of the operating temperature of the sprinkler head are discussed.

## 10.2 Sprinkler Heads

A sprinkler head is a heat sensitive valve which opens when its heat sensitive element reaches a specific temperature. Changes in the design of sprinkler systems and their components have arisen in various ways over the years, both by experience from within the industry and by requirements from the users and the approving bodies.

The first "modern" sprinkler was the Grinell type of 1922. The standard sprinkler head is the result of a complete redesign of the deflectors during 1952 and 1953. This design creates a spray pattern shaped much like an umbrella with the open side down, the discharge covers a circular area. The first statement of the relationship between "response time and time constant" occurred in 1957. The heat sensitive detector was developed fifteen years later and was known as the "fast response" or "life safety" sprinkler. The second area of development started with the need to use the water discharged as economically and as effectively as possible, with the realisation that the spacing of the sprinklers in an array could be increased considerably without danger of the fire "getting away" once it was surrounded. The result was the so called "spray" sprinkler which had a much larger deflector plate than the conventional sprinkler, designed to throw the water further and more uniformly than the previous ones. The third area of development has arisen because of the requirement to use sprinklers in places where their appearance has been important, such as in hotels, and public places. This has led to more smaller and decorative sprinklers being made.

According to Bush and McLaughlin(1979), an ordinary sprinkler will operate at temperatures between 54.4°C and 73.9°C. This type is used where the ceiling temperatures do not exceed 37.8°C. Intermediate sprinklers are used when the ceiling temperatures do not exceed 65.6°C and operate between 79.4°C and 100°C. High, extra-high and very extra high rated heads may be obtained for unusual ceiling temperatures up to 246°C.

There are two basic types of operation, the Frangible bulb and the Soldered Strut, see figure (10.2). With the Frangible bulb, the sealed glass bulb contains liquid and a small gas bubble which can accommodate small changes in the volume of the liquid due to temperature changes. High temperatures cause the liquid to expand sufficiently to absorb the bubble, the resultant increase in pressure fractures the bulb, see figure (10.3), allowing water to flow through the pipe work.

With the Soldered Strut sprinkler head, heat melts the solder allowing the

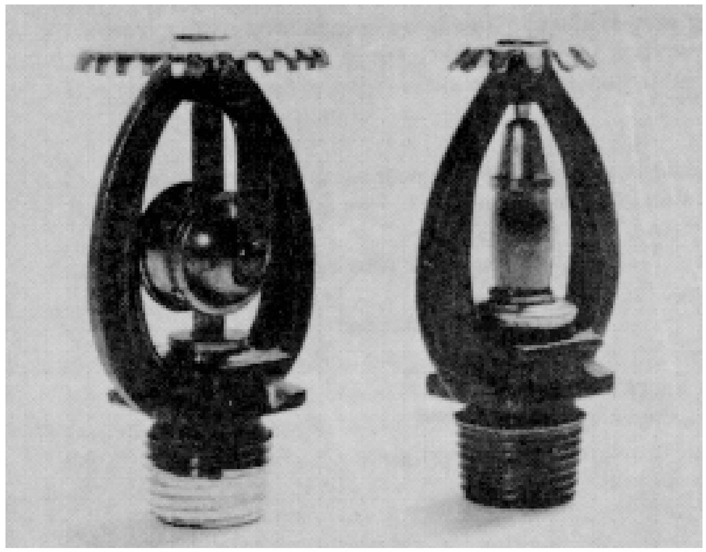

Figure 10.2: Soldered Strut and Glass Bulb sprinkler heads.

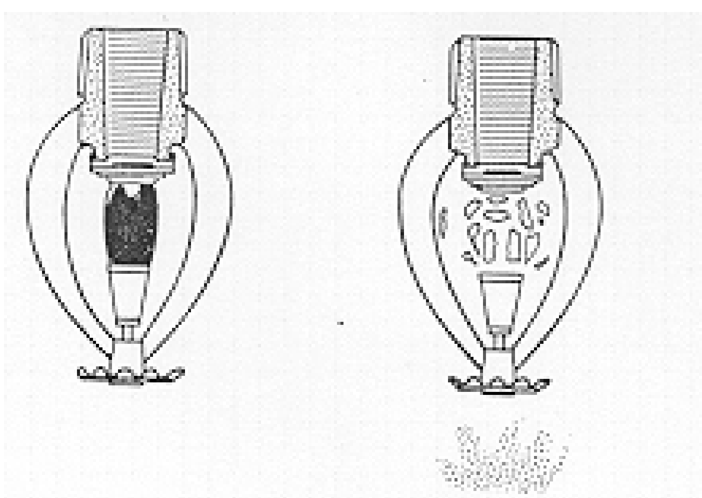

Figure 10.3: Breaking of the glass of the Frangible Bulb sprinkler head.

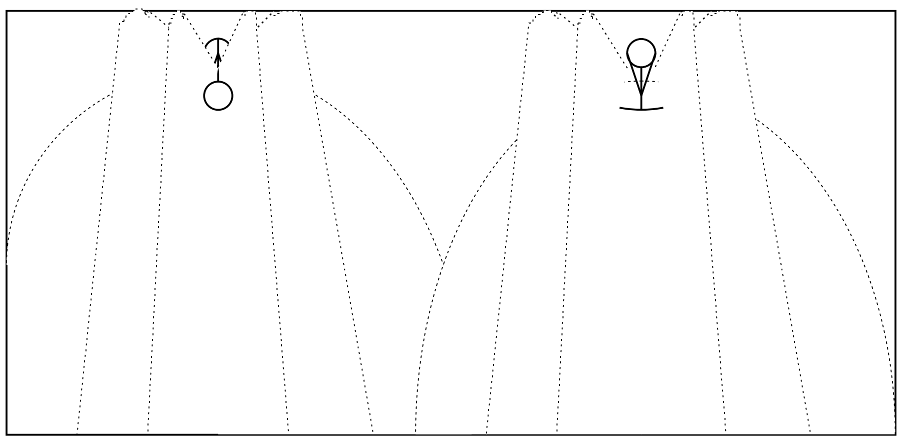

Figure 10.4: Spray patterns of Conventional type sprinkler heads.

strut of soldered strut types to part, letting water escape. Solders are alloys of tin, lead, cadmium, bismuth and antimony.

The water distribution pattern of the head depends on the type of deflector used. Conventional sprinklers produce a spherical discharge pattern with some of the water being thrown up towards the ceiling. See figure (10.4)for an illustration of the water spray pattern of a Conventional Sprinkler head. Spray sprinklers produce a hemispherical discharge below the sprinkler with little or no water reaching the ceiling. Side wall sprinklers are situated close to a wall. They deflect most of the water away from the wall. Figure (10.5) shows pictures of the three different deflector types.

Sprinkler heads should be spaced so that the area covered by each sprinkler overlaps that of its neighbour leaving no part of the floor unprotected. The standard method for arranging sprinklers is to locate them in square or rectangular formulation within the protected area, see figure (10.6). There are regulations according to hazard classes, specifying the maximum/minimum distance allowed between sprinkler heads and their height below the ceiling or roof, see FPA (1989) for details. The minimum distance is to prevent sprinklers from wetting adjacent heads which

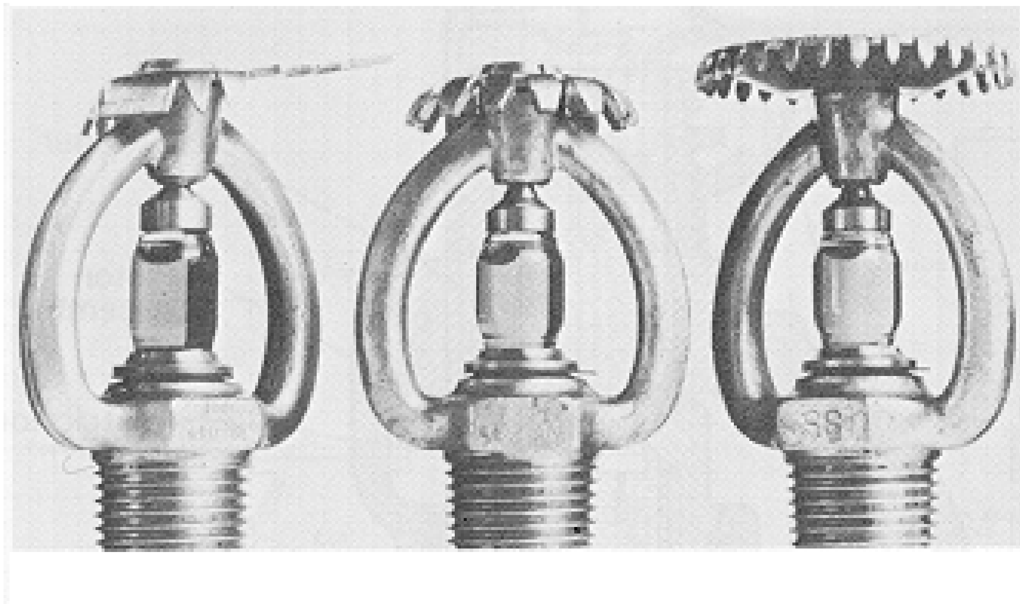

Figure 10.5: Types of sprinkler head deflectors: Side Wall, Conventional and Spray.

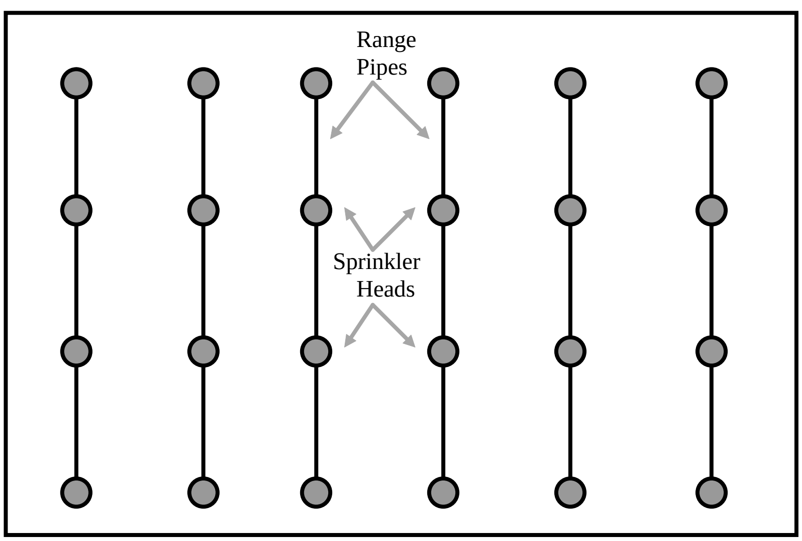


Figure 10.6: Square or Rectangular spacing of sprinkler heads.

might result in the cooling of these heads and so prevent their opening.

Sprinklers are suitable protection for all domestic and most industrial and commercial buildings. They have traditionally been used to protect property. Their ability to protect life is now also being increasingly recognized. With a few special exceptions sprinklers should be installed throughout a building because sprinkler systems are designed to control small fires and not designed to control large fires which have developed and spread from an unprotected area.

### 10.2.1 Pipework And Water Supply

Pipework above ground is normally medium grade steel tube. Pipes have different names according to their position in the system: Range pipes are pipes on which sprinklers are attached either directly or via short arms. Distribution pipes are horizontal pipes feeding range pipes. Risers are vertical pipes connecting installation valves with distribution pipes, or range pipes with distribution pipes. See figure (10.7) for an illustration of these pipes in the sprinkler system. The sizes of these pipes are determined by hydraulic calculations, either individually or by reference to precalculated tables to achieve the designed discharge density over the assumed maximum area of operation. They vary according to where they are positioned in the installation and the degree of hazard the installation is designed to meet. Typical sizes are 20-50 mm internal bore for range pipes and 32-150 mm internal bore for risers and distribution pipes, see FPA (1989) for a more comprehensive specification.

Every automatic sprinkler system must have at least one water supply with adequate pressure and volume to meet its demand. The standards provide a guide to the amount of water required. For a light hazard occupancy the amount of water required varies from $1140 l/min$ for a small system to $2840 l/min$ for a large installation. Ordinary hazards require from $2650 l/min$ to $5678 l/min$, or even more. Water supplies for a sprinkler system need to be reliable, at a suitable pressure,

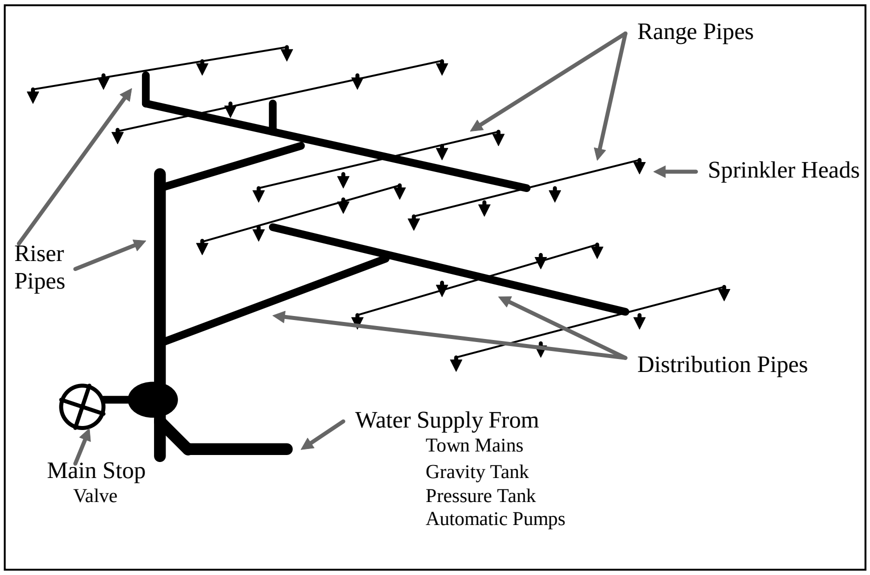


Figure 10.7: Sprinkler system - general layout.

and be able to supply a sufficient flow of water, long enough, to fight the largest expected fire.

Pressurized water for a sprinkler installation may be obtained from one or more of: Town mains, preferably fed from both ends. Gravity tank or elevated private reservoir where pressure is supplied by the height of the supply above the control valves.

### 10.2.2 Factors Affecting Sprinkler Operation

The temperature of operation is not constant even for similar sprinkler heads (sprinkler heads for the same hazard class). Several factors play an important role in the variation of the operating temperature of similar sprinklers. According to Nash (1978) these factors are:

1. Actual operating temperature of sprinkler

2. Thermal capacity of those parts of the sprinkler which affect operation

3. Ease of transfer of heat from the air to the affected parts of the sprinkler
4. Rate of growth of the fire in terms of its convective heat output
5. Height of the ceiling below which the sprinkler is mounted
6. 'Shape' of the ceiling, e.g. flat, paneled, concave, north-light
7. Thermal qualities of the ceiling assembly
8. Distance between sprinkler and ceiling
9. Horizontal distance of sprinkler from the fire
10. Any extraneous factors affecting the pattern of flow of the hot gases from the fire to the sprinkler, e.g. lift shafts and staircases, or venting arrangements
11. Rate of rise of air temperature surrounding the sprinkler.

Factors (1), (2) and (3) are controlled by the design of the sprinkler. Factor (4) depends upon the type of combustible material involved in the fire, method of stacking, aeration, etc. and can vary enormously with different classes of occupancy and types of goods. Factors (5), (6), (7), (8), (9) and (10) are controlled by the design of the building and the layout of the sprinkler array. Their influence, and that of factor (4) will result in the specific value of factor (11) which will determine, in conjunction with factors (1), (2) and (3), the actual time of operation of the sprinkler after the start of the fire.

The glass bulbs are made and filled in various ways by different manufacturers. The quantity of liquid in them, the thickness of glass in the walls, the shape of the bulbs and the type and condition of the glass used, can vary quite widely between manufacturers and between the bulbs made to any one 'nominal temperature rating' by one manufacturer. These disparities result in a variation of the actual operating temperatures of different samples of the same type of sprinkler.

Glass bulbs are also subject to some degree of 'aging' or crystallisation of the glass under the compression of the assembly and water pressure forces within the sprinkler; this may also, in time, further increase the range of operating temperatures of sprinklers of one nominal rating. Some glass bulb manufacturers include special devices in their bulbs to ensure uniform and regular operation. For example, carbon particles or a coil of wire in the liquid of the bulb will tend to ensure operation at a specific temperature, regardless of the rate of rise of temperature.

In the case of soldered-strut type sprinklers, the solder used for the struts is usually a eutectic alloy of low melting point components, generally bismuth, lead and tin, with small quantities of cadmium, silver and antimony according to the melting point required. The eutectic point is sharply defined and a small change in the composition of the alloy can result in a relatively large change in its melting point - and hence in the operating temperature of the sprinkler.

Aging of the solder can also produce variations. The first of these is due to the migration of one or more of the metallic components of the solder into the parent metal of the strut, with a consequent increase in the melting point of the remainder. In one extreme case of a soldered-strut type sprinkler manufactured in 1898, the Fire Research Station found in 1960 that this would not operate after prolonged heating because the solder had migrated completely into the parent metal of the strut so that the latter had become virtually a single piece. In other cases, the crystallisation of the solder - to which these fusible alloys are particularly prone - has weakened the solder to such an extent that it has collapsed under non-fire conditions.

## 10.3 Conclusion

There is a recognition that sprinklers provide a reliable method of controlling fire spread and raising the alarm, hence, their installation can mean that a lower standard of structural fire precautions is acceptable than would otherwise be necessary for building types which exceed normal compartment size limits. Still adequate means of escape must be provided, but a good case can be made for fewer and larger fire-resisting compartments (or compartments of a lower fire resistance standard) if sprinklers are provided. Greater flexibility in the construction of building and layout of plant can be achieved at the design stage of new projects if sprinklers are incorporated.

Two common factors in large fires are a delay in the discovery of the fire (because the premises were unoccupied) and/or a delay before fire fighting begins. Sprinklers overcome both factors and are thus extremely successful in keeping incipient fires small and flame damage to a minimum.

An automatic link between the sprinkler system and the fire brigade or a central alarm depot reduces the possibility of unnecessary water damage resulting from delay in calling the fire brigade to fires in unoccupied premises. For even if the fire took ten minutes to respond and if the fire was small enough for the sprinkler to extinguish the flame then 100's if not 1000's of litres of water would be dumped from the sprinkler into the compartment possibly causing an enormous amount of water damage.

In order to reduce the water damage after the fire is extinguished, we propose the use of solenoid valves to vary the water spray rate according to the burning rate inside the compartment.

# Chapter 11

# Optimal Control

## 11.1 Introduction

Optimal control is about controlling a system in some 'best' way. The optimal control strategy will depend on what is defined as the best way. This is usually achieved in terms of a performance index or criterion. For example consider sprinklers trying to extinguish a fire inside a compartment. A *control* problem would be that of choosing a set of parameters (rate of water spray, the number of sprinklers) so that some aim is achieved (fire is extinguished). An associated *optimal control* problem would be to choose the controls to achieve the aim with, for example, minimum water or minimum time.

## 11.2 Functionals

A large number of problems involve finding, subject to varying constraints, an extremum value of an integral of the form

$$J = \int_{t_0}^{t_2} F(x, \dot{x}, t)dt \tag{11.1}$$

where $F$ depends on the function $x(t)$, its derivative $\dot{x} = dx/dt$ and the independent variable $t$. The function $x(t)$ is the evolution of the system and is defined for

$t_o < t < t_2$. For a given function, say $x(t) = x_1(t)$, equation (11.1) gives the corresponding value of $J$, say $J = J_1$. For a second function, say $x(t) = x_2(t)$, equation (11.1) gives a value of $J$ say $J = J_2$. In general $J_1 \neq J_2$, and we call integrals of the form (11.1) functionals.

The above problem is solved using a branch of mathematics known as *Variational Calculus.*

If the function $x(t)$ is to have an extremum value, $F$ must satisfy the equation

$$F_x - \frac{d}{dt}(F_{\dot{x}}) = 0 \qquad (11.2)$$

where $F_x$ means $\partial F/\partial x$ and $F_{\dot{x}}$ means $\partial F/\partial \dot{x}$. Equation (11.2) is known as *Euler's equation.* For a more extensive discussion of Euler's equation see Burghes and Graham (1980).

## 11.3 General Problem

Using Variational calculus it is possible to define and solve the general optimal control problem when the controls are continuous and unbounded. Often, the problem is to find the optimal control $u(t)$, where $u$ is a function of $t$, which yields extremal values of

$$J = \int_0^{t_2} f_0(\mathbf{x}, \mathbf{u}, t)dt$$

subject to the differential constraint equations

$$\dot{x}_i = f_i(\mathbf{x}, \mathbf{u}, t) \qquad (i = 1, 2, ..., n), \qquad (11.3)$$

where $\mathbf{x} = [x_1 x_2 ... x_n]'$, and $\mathbf{u} = [u_1 u_2 ... u_m]'$.

We introduce the Lagrange multipliers, say $p_i(i = 1, 2, .., n)$ usually called the *adjoint variables* and form the *augmented functional*

$$J^* = \int_0^{t_2} \{f_0 + \sum_{i=1}^{n} p_i(f_i - \dot{x}_i)\}dt.$$

We also define the *Hamiltonian*, **H**, as

$$\mathbf{H} = f_0 + \sum_{i=1}^{n} p_i f_i$$

so that

$$J^* = \int_0^{t_2} (\mathbf{H} - \sum_{i=1}^{n} p_i \dot{x}_i) dt.$$

The integrand $F = \mathbf{H} - \sum_{i=1}^{n} p_i \dot{x}_i$ depends on $\mathbf{x}$, $\mathbf{u}$ and t, and we form $(n + m)$ Euler equations, namely

$$\frac{\partial F}{\partial x_i} - \frac{d}{dt}(\frac{\partial F}{\partial \dot{x}_i}) = 0 \qquad (i = 1, 2, ..., n)$$

that is,

$$\dot{p}_i = -\frac{\partial \mathbf{H}}{\partial x_i} \tag{11.4}$$

which are known as the *adjoint equations*; and

$$\frac{\partial F}{\partial u_j} - \frac{d}{dt}(\frac{\partial F}{\partial \dot{u}_j}) = 0 \qquad (j = 1, 2, ..., m)$$

that is,

$$\frac{\partial \mathbf{H}}{\partial u_j} = 0. \tag{11.5}$$

The optimal solution for $\mathbf{x}$, $\mathbf{u}$ and $\mathbf{p} = [p_1, p_2, ..., p_n]'$ is determined from the $(2n+m)$ equations given from (11.3), (11.4) and (11.5). If we assume the boundary conditions $x_i(0)$ $(i = 1, 2, ..., n)$ and $x_l(t_2)$ $(l = 1, 2, ..., q)$ are given, then the remaining values $x_{q+1}(t_2), x_{q+2}(t_2), ..., x_n(t_2)$ are free, and so we can apply the free end point condition

$$\frac{\partial F}{\partial \dot{x}_i} = 0 \qquad (k = q + 1, q + 2, ..., n) \qquad \text{at} \qquad t = t_2.$$

A more detailed explanation of the General Optimal Control problem can be found in Burghes and Graham (1980).

## 11.4 Pontryagin's Principle

In the above explanation we dealt with optimal control problems, where the controls were continuous and no restrictions were put on the range of possible values of the controls. For many practical applications, the control will be bounded, and it might well be possible to have discontinuities in the control values.

Pontryagin and his colleagues published a general principle (referred to as a minimum principle) which deals not only with continuous controls but also with unbounded and possibly discontinuous controls. Pontryagin's minimum principle states that on the optimal control the Hamiltonian, $\mathbf{H}$, is minimised with respect to the control variable, $\mathbf{u}$. Hence for discontinuous and/or bounded control variables the optimal control is obtained by minimising the Hamiltonian with respect to the control variable. This can occur at the boundary of the control region.

A typical example of a discontinuous control variable is the *Bang-Bang control* which is a control which has a switch (discontinuity) at time $t = t_d$ and the control takes only its maximum and minimum values.

# Chapter 12

# Optimal Control of Sprinklered Compartment Fires

## 12.1 Introduction

Compartment fires are defined as fires in enclosed spaces, typically thought of as rooms in buildings. As presented by Hasofer and Beck (1995), the three most important physical factors which describe a compartment fire appear to be the gas temperature, the burning rate, and the oxygen concentration inside the room.

The *gas temperature*, $T$, inside a compartment is discussed usually in growth stages. All fires manifest an ignition stage but, beyond that, may fail to grow through all or some of the growth stages listed in Tat and Hasofer (1995).

The *burning rate*, $R$, is an ambiguous, though useful expression. Quantitatively, it is expressed either as a mass loss rate, kg/min, or as a heat release rate, kW.

Most fires draw their *oxygen*, $O_2$, from the air, which is a mixture of approximately 23 percent oxygen, 76 percent nitrogen and 1 percent other gases. If a fire is burnt in a closed room the oxygen will gradually be used up and the fire will eventually diminish. If no additional supply is available, the fire will die out once the

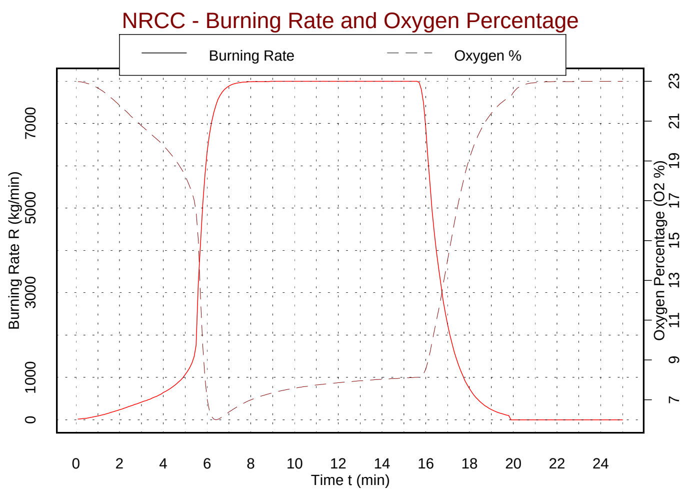


Figure 12.1: $R$ and $O_2$ results of the NRCC model

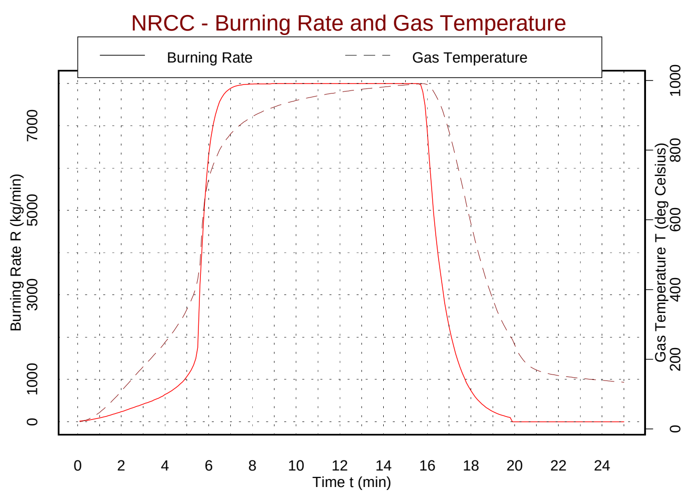


Figure 12.2: $R$ and $T$ results of the NRCC model

oxygen concentration reaches about 7 percent. However, if a limited but continuous supply is provided (which is often the case) the fire will smolder. Smoldering causes the fuel to vaporise into flammable gases that are only partly burned, since there is not enough oxygen for complete combustion.

Figures (12.1) and (12.2) show the three factors ($T$, $R$, and $O_2$) along in time as the fire inside a standard compartment develops. The data was obtained from a run of the NRCC model using the input parameters used by Hasofer and Beck (1995).

The oxygen concentration affects the growth of the fire at time $t \approx 5.9 \quad min$. At this time the gas temperature inside the room is over 600$^o$C and the room has already reached flashover, thus most of the damage by the fire has already been done. To prevent flashover inside a compartment, one measure of fire safety used is the installation of wet pipe sprinklers throughout a building.

## 12.2 Wet Pipe Sprinklers

The automatic sprinkler system is one of the most effective means of fire protection for both life and property. There is no doubt that increased application of sprinkler protection throughout a building can significantly reduce fire losses. The main function of a sprinkler is to detect and control a compartment fire, in the early stages, before it spreads beyond a limited area, and develops into a flashover fire.

By far the most common sprinkler system is the wet pipe type. A wet pipe sprinkler system consists of an automatic sprinkler attached to piping that holds water, and the pipes are connected to a water supply with adequate pressure and volume to meet its demand. When heat melts the sprinkler's fusible element, water is discharged immediately, extinguishing any fire in the area. One of the significant extinguishing properties of water is cooling, see Marryatt (1988) for the other extinguishing properties of water. Water cools the surface of the combustible material to below the point at which the material can produce vapor to support combustion. Water also absorbs the heat of combustion so that other material in the vicinity of the fire is not heated to participate in combustion. For water to have the maximum absorption effect it must reach the fire as droplets. This is because all things being equal the rate of heat transfer is proportional to the free surface of the liquid.

Present fire sprinklers are designed to activate when the temperature inside the sensor reaches a certain temperature. Once the sprinklers are activated they continue to spray water at a constant rate until the fire brigade arrives and turns the water tap off when they believe the fire is out.

With sprinklers operating in this way we can have two major problems. One is that if the sprinklers fail to activate, the fire will grow to flashover, damage the entire compartment and put at risk the other compartments in the building. On the other hand if the sprinklers are activated and spray out the fire, they are generally left on far too long, and the water causes a lot of property damage as well.

## 12.3 Property Damage

In the fire literature property damage, $PD$, in the event of a fire is categorised into three classes:

- Flame damage, $FD$, due to the flames of the fire,
- Water damage, $WD$, due to the water from sprinklers and
- Smoke damage, due to the smoke produced by the fire.

In this paper we will concentrate on the property damage due to flame damage and water damage. We will assume that property damage can be written as a linear sum of the flame and water damages.

$$PD = \theta FD + \phi WD \tag{12.1}$$

where $\theta$ and $\phi$ are some calibration parameters.

Ramachandran (1986), proposed an exponential model for flame damage. Ramachandran states that experimental evidence supports the scientific theory that heat output of a fire increases as an exponential function of time, t, and this implies that the area damaged by direct burning, flame damage area, has an exponential relationship with duration of burning;

$$A(t) = A(0)\exp[\beta_a t_a + \beta_b t_b]$$

where $A(t)$ is the floor area damaged in t minutes since ignition, $A(0)$ is the floor area initially ignited, $\beta_a$ is the fire growth parameter and $t_a$ is the time, from time of ignition to the time of fire brigade arrival at the scene of fire; $\beta_b$ is the fire growth parameter and $t_b$ is the time, fire brigade arrival at the scene of fire to the time when the fire is brought under control.

Using the flame damage model proposed by Ramachandran we can define the flame damage when sprinklers are involved as;

$$A(t) = A(0)\exp[\beta_i t_i + \beta_s t_s] \tag{12.2}$$

where $\beta_i$ is the fire growth parameter and $t_i$ is the time, from time of ignition to the time of sprinkler activation and $\beta_s$ in this case is the fire growth parameter and $t_s$ is the time, from time of sprinkler activation to the time when the fire is extinguished.

In the survey of the fire literature, there was no mention of a quantitative model for water damage. We will consider two functions of the water spray rate to quantify water damage. First we will assume water damage to be directly proportional to the integral of the water spray rate. This is because it makes sense to assume the water damage to be proportional to the total amount of water discharged. Secondly we assume water damage to be proportional to the integral of the square of the water spray rate. This is because when the sprinkler is activated the discharged water will absorb the heat produced by the fire, vaporise into steam and escape with the smoke; resulting in very little water damage at the start. Then as the fire is brought under control the surplus water will increase and cause the water damage to increase, hence the non linear function.

As cleaning up water damage is probably just as expensive as flame damage, one way to minimise the water damage would be to install fire sprinklers which can vary the amount of water being sprayed. As mentioned in chapter (10) one way of varying the amount of water would be by controlling the water flow rate using solenoid valves which are connected to a computer. Minimising the water damage is a problem of *optimal control*, which is a branch of mathematics very well researched. A brief review of the theory of optimal control was presented in the previous chapter.

## 12.4 Interaction of Burning Rate With Water Discharge Rate

Let $R(t)$ be the burning rate and $u(t)$ be the rate of discharge of water at time $t$. To model the use of a wet pipe sprinkler inside a compartment fire we would like the sprinkler to activate once the gas temperature reaches a given temperature, say $T_{act}$ at time $t_{act}$. This would correspond to a burning rate $R_1$. If the water flow and pressure is adequate to reduce the burning rate, we would assume that the burning rate will start decreasing from $R_1$ to become zero, fire extinguished, at time $t_{ext}$.

From the NRCC model graphed in figures (12.1) and (12.2) the burning rate has an approximate exponential growth until the oxygen concentration becomes important. This occurs when the gas temperature inside the compartment is over $600^oC$. Since the activation temperature of the ordinary sprinkler is to be taken as $73.9^oC$, as given by Bush and McLaughlin (1979), and this is well below the $600^oC$, it seems reasonable to assume that the uncontrolled burning rate increases exponentially with time. This assumption is also made in the Madrzykowski and Vittori equation, Madrzykowski and Vittori (1992) and the NIST equation, Fleming

(1993). Thus the uncontrolled burning rate can be modelled by the equation

$$\frac{dR}{dt} = \beta R \tag{12.3}$$

where $\beta$ is a positive constant.

We further assume that the sprinkler effect is to reduce the change in burning rate proportionally to the rate of water flow. We make this assumption as the availble results are not suited for modelling the relationship between the burning rate and the water spray rate. Equation (12.3) can be modified to

$$\frac{dR}{dt} = \beta R - \gamma u \tag{12.4}$$

where $\gamma$ is a positive constant and $0 \leq u \leq U_{max}$, where $U_{max}$ is the maximum flow rate determined by size of the pipe, as given in section (10.2.1).

Using the NRCC model, when we take the $T_{act}$ to be $73.9^0$C for an ordinary sprinkler, this corresponds to the sprinkler activating at approximately $t_{act} = 2.0 \quad min$ and at this stage the burning rate is approximately $R = 300 \quad kg/min$.

## 12.5 Optimal Control of Sprinklers

The time to control or reduce the burning rate, $R$, of a compartment fire is dependent on the activation time and the time required for the suppression agent to become effective. If the suppression agent is not capable of reducing the burning rate we get limited control. However, if the suppression agent is capable of stopping the burning rate from increasing due to the fire, then we get control and if the suppression agent is capable of decreasing the burning rate then we get extinguishment. This is illustrated in figure (12.3). The shape of the $R$ curve following $t_{act}$ will be determined by the agent and type of system. The paths sketched are there to indicate that $R$ can continue to increase, level out or decrease.

As mentioned, it would be useful to have the optimum amount of water to extinguish the fire with the minimum amount of water damage. The solution of the

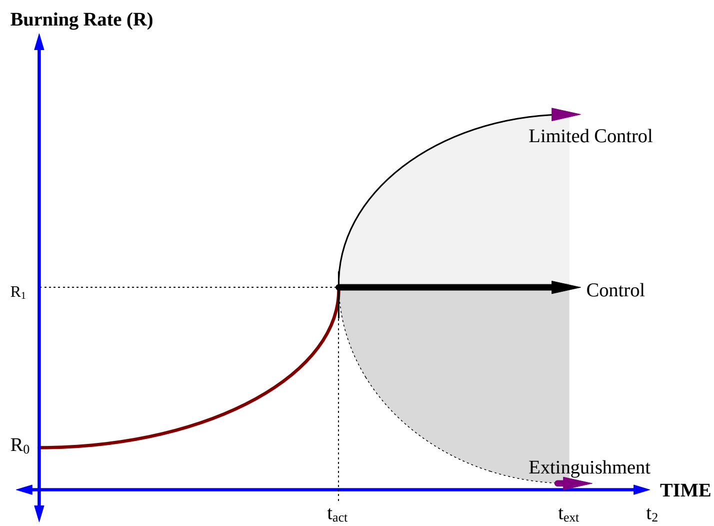


Figure 12.3: Burning rate versus time of a compartment fire with an automatic sprinkler

optimal control problem produces the function to be used to vary the amount of water being sprayed.

### 12.5.1 Water damage proportional to total amount of water

If we assume that water damage is directly proportional to the total amount of water discharged from the sprinklers, our functional, (see the previous chapter for the definition of a functional) would be the integral of the water discharge,

$$J = \int_{t_{act}}^{\infty} u(t)dt. \tag{12.5}$$

Given $R(t = t_{act}) = R_1$, where $t_{act}$ is the time when the sprinkler has activated and $t_{ext}$ is the time when the fire is extinguished, and $R$ is free at $t = \infty$, i.e. $t_{ext} \leq \infty$. We need to calculate $u(t)$, $\quad t_{act} \leq t \leq \infty$, such that the functional is minimised, subject to the equation (12.4)

$$\frac{dR}{dt} = \beta R - \gamma u.$$

Since there is no restriction on $R$ at the upper end point the optimal control problem is a free end point optimal control problem, see Burghes and Graham (1980) pg. 219. The free end point solution curve will also be the solution to the fixed end point problem which has the same values as the free end point solution, it will also satisfy the Euler Equation, see Burghes and Graham (1980) pg. 220.

We use the classical Hamiltonian approach to find the function $u(t)$ to extinguish the fire with the minimum amount of water. If we introduce a Lagrange multiplier, say $p$, we can define the Hamiltonian, $\mathbf{H}$, as

$$\mathbf{H} = u + p(\beta R - \gamma u).$$

Using Euler's equation (11.2) we get $\dot{p} = -\partial \mathbf{H}/\partial R = -p\beta$. Hence

$$p = p_0 e^{-\beta t}.$$

Since we are treating this problem as a free endpoint problem we apply the transversality condition $p(\infty) = 0$ which is automatically satisfied, see Burghes and Graham (1980) pg. 240. So the Hamiltonian can be written as

$$\mathbf{H} = u(1 - \gamma p_0 e^{-\beta t}) + p_0 \beta R e^{-\beta t}.$$

Using Pontryagin's principle, on the optimum control the Hamiltonian must be minimised. As $\mathbf{H}$ is a linear function of $u$, when we try to minimise $\mathbf{H}$ with respect to $u$ it will attain its minimum value at the boundary, either at $u = 0$ or $u = U_{max}$.

- Case 1

  $$(1 - \gamma p_0 e^{-\beta t}) > 0$$

  From figure (12.4), $\mathbf{H}$ is a minimum when $u = 0$, hence

  $$U_{opt} = 0.$$

  When $U_{opt} = 0$ equation (12.4) becomes

  $$\frac{dR}{dt} = \beta R.$$

  The general solution to this equation is $R = Ce^{\beta t}$. In this case $R$ will increase to infinity unless $C = 0$. This solution will apply for $t > t_{ext}$.

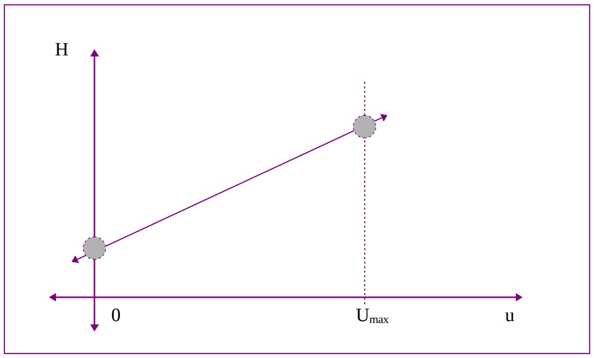


Figure 12.4: When $1 - \gamma p_0 e^{-\beta t} > 0$

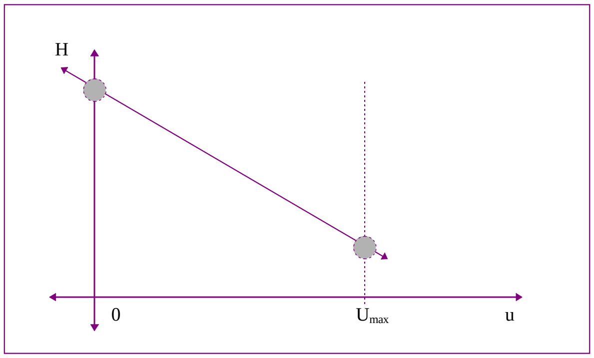


Figure 12.5: When $1 - \gamma p_0 e^{-\beta t} < 0$

- Case 2

$$(1 - \gamma p_0 e^{-\beta t}) < 0$$

From figure (12.5), **H** is a minimum when $u = U_{max}$, hence

$$U_{opt} = U_{max}.$$

If we take $t_{act}$ to be the reference point $t = 0$ and $U_{opt} = U_{max}$ equation (12.4) becomes

$$\frac{dR}{dt} = \beta R - \gamma U_{max}.$$

The general solution to the above equation is

$$R = Ke^{\beta t} + \frac{\gamma}{\beta} U_{max}.$$

Using the initial condition $R(t = 0) = R_1$, we can solve for $K$,

$$K = R_1 - \frac{\gamma}{\beta} U_{max}.$$

Hence

$$R = \begin{cases} (R_1 - \frac{\gamma}{\beta} U_{max}) e^{\beta t} + \frac{\gamma}{\beta} U_{max} & \text{when } 0 \leq t < t_{ext} \\ 0 & \text{when } t \geq t_{ext}. \end{cases} \tag{12.6}$$

The burning rate will go to zero provided $U_{max} > \beta R_1/\gamma$. This condition forces the coefficient of the exponential to be negative. The burning rate given by equation (12.6) is sketched in figure (12.6).

By rearranging equation (12.6) and substituting $R = 0$ we can calculate the time for the burning rate, $R$, to go to zero

$$t_{ext} = \frac{1}{\beta} \ln [\frac{\frac{\gamma}{\beta} U_{max}}{\frac{\gamma}{\beta} U_{max} - R_1}]. \tag{12.7}$$

The optimal solution is of the Bang-Bang type

$$u = \begin{cases} U_{max} & \text{when } 0 \leq t < t_{ext} \\ 0 & \text{when } t \geq t_{ext}. \end{cases} \tag{12.8}$$

The Bang-Bang solution of $u$ is illustrated in figure (12.7).

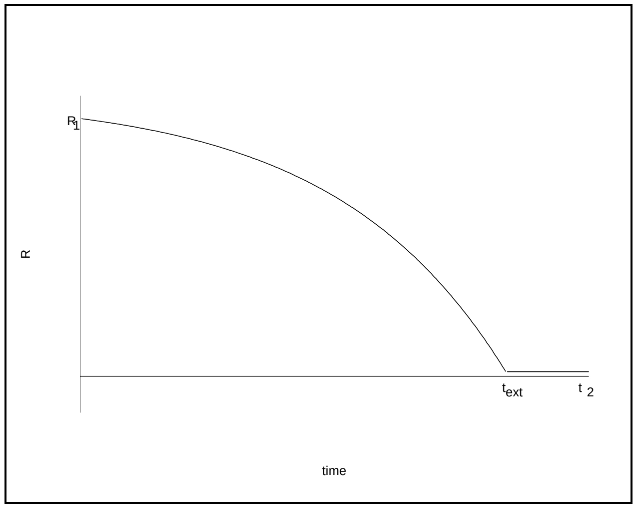


Figure 12.6: Path of the Burning Rate

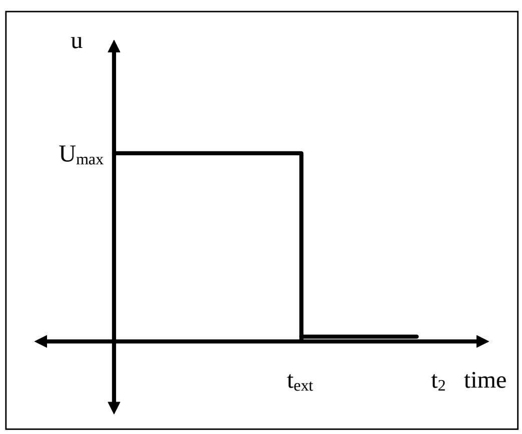


Figure 12.7: Bang-Bang Solution of u

Hence the optimum value of the water damage, $J$, can be obtained by substituting equation (12.8) into the integral equation (12.5),

$$J_{opt} = U_{max} t_{ext}.$$

### 12.5.2 Water damage proportional to the integral of the square of the water flow rate

Let $R(t)$ be the burning rate at time $t$ and $u(t)$ be the rate of discharge of water at time $t$. The functional can be written as the integral of the square of the water flow rate

$$J = \int_{t_{act}}^{\infty} [u(t)]^2 dt.$$

As before, $R(t = t_{act}) = R_1$, where $t_{act}$ is the time when the sprinkler has activated, $t_{ext}$ is the time when the fire is extinguished, and $R$ is free at $t = \infty$, i.e. $t_{ext} \leq \infty$. We need to calculate $u(t), \quad t_{act} \leq t \leq \infty$, such that the functional is minimised, subject to

$$\frac{dR}{dt} = \beta R - \gamma u \tag{12.9}$$

and

$$0 \leq u \leq U_{max}.$$

Since there is no restriction on R at the upper end point we have an optimal control problem with an upper free end point. Introducing the Lagrange multiplier, $p$, the Hamiltonian, $\mathbf{H}$, is defined as

$$\mathbf{H} = u^2 + p(\beta R - \gamma u).$$

From Euler's equation (11.2) $\dot{p} = -\frac{\partial H}{\partial R} = -p\beta$. Hence

$$p = p_0 e^{-\beta t}.$$

Here again the transversality condition $p(\infty) = 0$ is automatically satisfied and the Hamiltonian can be written as

$$\mathbf{H} = u^2 - (\gamma p_0 e^{-\beta t})u + p_0 \beta R e^{-\beta t}.$$

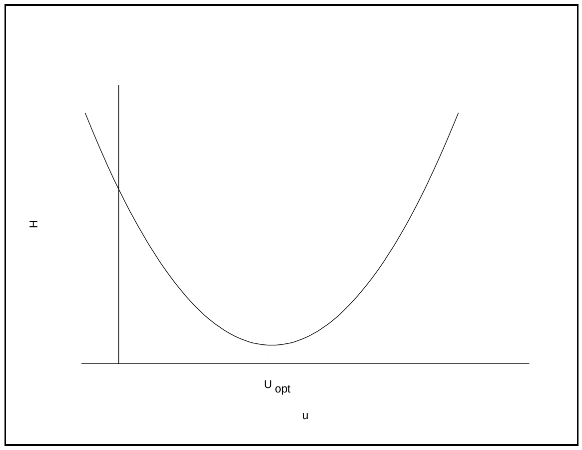


Figure 12.8: Sketch of the Hamiltonian with respect to $u$

The Hamiltonian is a quadratic in $u$. The location of the turning point has four different cases. The sketch of the appropriate curve is given in figure (12.8). For this quadratic the minimum is at

$$U_{opt} = \frac{\gamma}{2} p_0 e^{-\beta t}$$

where $p_0$ must be positive for a meaningful solution.

If we take $t_{act}$ to be the reference point $t = 0$ and $U_{opt} = p\gamma/2$ equation (12.9) becomes

$$\begin{aligned} \frac{dR}{dt} &= \beta R - \gamma \frac{p\gamma}{2}, \\ &= \beta R - \frac{\gamma^2}{2} p_0 e^{-\beta t}. \end{aligned} \tag{12.10}$$

The general solution to equation (12.10) is

$$R = K e^{\beta t} + \frac{\gamma^2}{4\beta} p_0 e^{-\beta t}.$$

Using the initial condition $R(t = 0) = R_1$, $K = R_1 - \gamma^2 p_0 / 4\beta$.

$$R = \begin{cases} R = (R_1 - \frac{\gamma^2}{4\beta} p_0) e^{\beta t} + \frac{\gamma^2}{4\beta} p_0 e^{-\beta t} & \text{when } 0 \le t < t_{ext} \\ 0 & \text{when } t \ge t_{ext}. \end{cases} \tag{12.11}$$

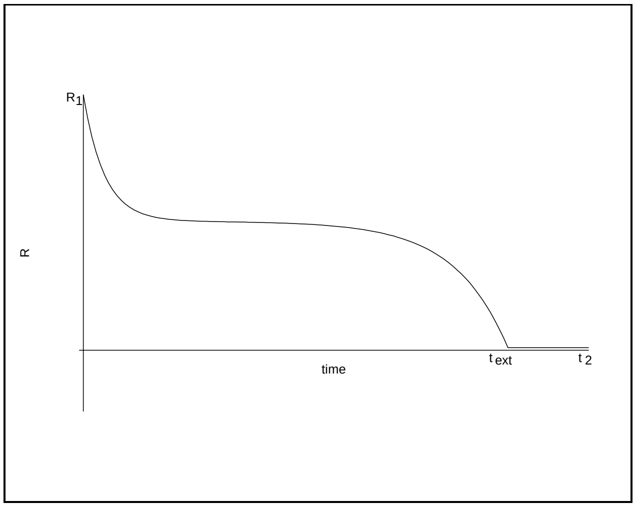


Figure 12.9: Path of the Burning Rate

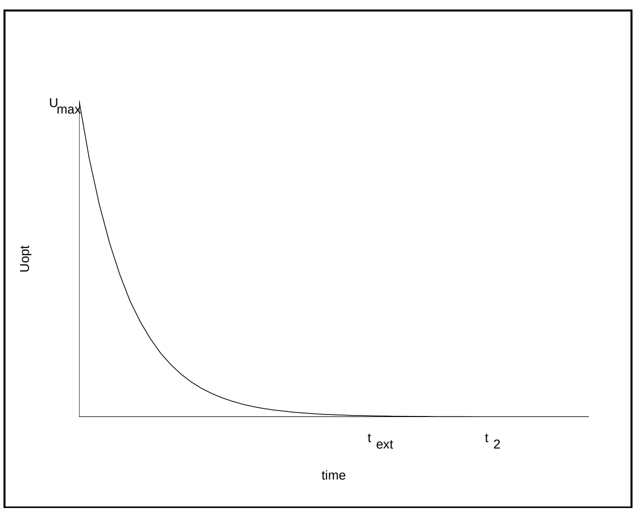


Figure 12.10: Exponential solution of u

For $R$ not to increase to infinity, we must have $p_0 > 4\beta R_1/\gamma^2$. By rearranging equation (12.11) and substituting $R(t = t_{ext}) = 0$ we can calculate the time for the burning rate, $R$, to go to zero

$$t_{ext} = \frac{1}{2\beta}\ln[\frac{p_0}{p_0 - 4\frac{\beta}{\gamma^2}R_1}].$$

Using the initial conditions we obtain

$$p_0 = \frac{4\beta R_1 e^{\beta t_{ext}}}{\gamma^2 \left(e^{\beta t_{ext}} - e^{-\beta t_{ext}}\right)}.$$

Substituting $p_0$ into $R(t)$

$$R = (R_1 - \frac{R_1 e^{\beta t_{ext}}}{e^{\beta t_{ext}} - e^{-\beta t_{ext}}})e^{\beta t} + (\frac{R_1 e^{\beta t_{ext}}}{e^{\beta t_{ext}} - e^{-\beta t_{ext}}})e^{-\beta t}. \qquad (12.12)$$

For equation (12.12) not to increase to infinity, we must have

$$\frac{e^{\beta t_{ext}}}{e^{\beta t_{ext}} - e^{-\beta t_{ext}}} > 1$$

which is always true. See figure (12.9) for the graphical illustration of equation (12.12). Now substituting $p_0$ into $U_{opt}$

$$U_{opt} = \frac{2R_1\beta e^{-\beta t}}{\gamma(1 - e^{-2\beta t_{ext}})} \qquad (12.13)$$

provided

$$U_{opt}(0) \geq \frac{2R_1\beta}{\gamma}. \qquad (12.14)$$

Hence the maximum value of $U_{opt}$, $U_{max}$ is given by

$$U_{max} = \frac{2\beta R_1}{\gamma(1 - e^{-2\beta t_{ext}})}. \qquad (12.15)$$

We can now write

$$U_{opt} = \begin{cases} U_{max}e^{-\beta t} & \text{when } 0 \leq t < t_{ext} \\ 0 & \text{when } t \geq t_{ext}. \end{cases} \qquad (12.16)$$

See figure (12.10) for the graphical illustration of equation (12.16).

Hence the optimum value of the water damage, $J$, is $J_{opt}$

$$J_{opt} = \frac{U_{max}}{2\beta}(1 - e^{-2\beta t_{ext}}) = \frac{R_1}{\gamma}.$$

## 12.6 Discussion

A large percentage of building fires require only a small number of sprinkler heads in operation. According to Marryatt (1988), of fires where automatic sprinkler systems have been involved in Australia and New Zealand 64.55% of fires required only one sprinkler head in operation and 80.41% of fires required two or fewer sprinkler heads for extinguishment or control. Marryatt (1988), supports a quote from the N.F.P.A. Handbook, 13 Edition, that "Fear of water damage comes in part from the thoughtless emphasis placed upon water damage in news reports of fires." However, a large percentage of fires, (according to him about 80%) are small fires, and the water damage in these cases is far more extensive than the flame damage. In this chapter we proposed the use of sprinkler heads with a solenoid valve connected to a computer which will continue spraying water only until the fire is extinguished. In this way we can optimise the amount of water used and minimise the water damage. Further research and experimentation is required to estimate the parameters of these models and to evaluate their appropriateness. In addition research is needed to clearly define water damage and develop appropriate equations to quantify water damage. For building owners and insurance companies this is an area of research which has the potential to save millions of dollars.

# Part IV

# THESIS CONCLUSION

The first two parts of this thesis has tried to satisfy the growing need to develop more realistic models which couples deterministic and stochastic methods to take account of uncertainties governing the growth of actual fires in compartments. In the final part of the thesis we have provided a solution to address the need to minimise water damage in small compartment fires with sprinklers.

In part 1 of the thesis I have covered the stochastic models suggested by Ramachandran (1991). The epidemic models would have been interesting to develop and apply as they are rich in theory. However, for fire growth in buildings they have limited physical interpretation. The percolation and contact processes as a first approximation to fire spread appear fine, but these models are studied using asymptotic theory, as $n$ approaches infinity. Since there are a small number of compartments and levels in a building these processes were not applicable for fire spread. In the final chapter of part 1 we present a method for converting deterministic equations to stochastic equations as the theory of deterministic models are extensive.

In part 2 of the thesis we have developed three deterministic models based on the three main factors affecting the growth of compartment fires. The first model is a one zone fuel driven model. This model does not incorporate the effect of decreasing oxygen in compartments, which is a critical factor in compartment fires. The second model is an oxygen driven model which is converted to a stochastic model using the method in part 1. This model is run through a Mounte Carlo simulation to calculate the upper quartile of the heat load. The upper quartile of heat load is calculated using the non-parametric statistic W-Test. This second model has a tedious optimisation algorithm to calculate the parameters of the model. The third model is also a one zone oxygen driven with simpler equation to solve. The equations are simplified by assuming that the oxygen fraction rate is dependent on the burning rate and the incoming oxygen, and the rate of change in burning rate increases with burning rate increase and oxygen fraction increase. These two equations are solved together. The rate of change of temperature is assumed to increase with burning rate and decrease with heat loss, where the heat loss is assumed to be a linear function of temperature. The advantage of this model is that this model has fewer parameters to evaluate, and even more, the estimation of the parameters is extremely simple and quicker than the optimisation algorithm in Hasofer and Beck (1995). As a result this model is easier and quicker to use in Monte Carlo simulations for probabilistic fire risk analysis.

In part 3 of the thesis we use optimal control theory to model the water spray rate from sprinklers to minimise the water damage in small compartment fires. The results show that the use of sprinkler heads with a solenoid valve connected to a computer which will continue spraying water only until the fire is extinguished will reduce the water damage in small fires. For building owners and insurance companies this is an area of research which has the potential to save them millions of dollars.

A problem with this thesis is the intended use of the developed models. It is relatively easy to include or remove effects from a model, making it more complex or simple as a result. However, methodological guidance is needed in making such choices. The choice of the degree of approximation is very much helped by knowing how the model is to be used, the output required of it and the quality and availability of its sources of data. In this thesis, a number of alternative models are proposed and described, but there is little basis given for comparison between them, as to the preferred context, for instance, of their use. In practice, it would be important that the parameters required for a model should be derivable from real data. Here, however, parameters have been generated artificially by fitting the model results to the results of a more complex computer model. The necessary connection with real data is absent. Without this being present, changing from deterministic to stochastic does not, on the face of it make sense, except as a demonstration that it can be done.

We have taken the approach we have in this thesis due to a lack of experimental and statistical data. We must remember that experimental data for compartment fires is very costly and hence slow coming. However, with all these models further research and analysis of experimental and statistical data is still necessary for applying and validating these models to practical problems in fire safety.

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# Appendix A

# Simulation of Oxygen Driven Compartment Fire Model

### FPAR7

```
list(alpha = 2.5, beta = 0.1125, gamma = 0.318948, rho = 1.25,
sigma = 1.875e-010, delta = 0.000015, mu = 0.58, k = 10.4, k1 = 158,
BMAX = 82500, RDECAY = 0.956, time = 25, R0 = 280, frac = 0, dt = 0.1)
```

### FCURVE7

```
function(FPAR7, tmax, dt)
{
FPAR <- FPAR7
alpha <- FPAR$alpha
beta <- FPAR$beta
gamma <- FPAR$gamma
sigma <- FPAR$sigma
delta <- FPAR$delta
rho <- FPAR$rho
mu <- FPAR$mu
```

```
k <- FPAR$k
k1 <- FPAR$k1
BMAX <- FPAR$BMAX
RDECAY <- FPAR$RDECAY
time <- tmax # FPAR$time
R0 <- FPAR$R0
n <- round(time/dt, 0)
tt <- rep(0, n)
GT <- rep(0, n)
R <- rep(0, n)
D <- rep(0, n)
B <- rep(0, n)
QL <- rep(0, n)
GT[1] <- 20
R[1] <- R0
D[1] <- 0
B[1] <- 0
tt[1] <- 0
for(r in (1:(n - 1))) {
tt[r + 1] <- tt[r] + dt
QL[r] <- sigma * ((GT[r] + 273)^4 - 273^4) + gamma *  GT[r]
gt[r + 1] <- GT[r] + (beta * R[r] - rho * QL[r]) * dt
GT[r + 1] <- min(1000, gt[r + 1])
B[r + 1] <- B[r] + R[r] * dt
Z[r] <- alpha * 22 * GT[r] * (1 - 1/(1 + (0.001 * GT[r])^2))
if(B[r] < BMAX) {
R[r + 1] <- R[r] + max(0, (k - D[r])) * Z[r] *  dt
D[r + 1] <- min(16, D[r] + (delta * (k1 - D[r]) * R[r] - mu * D[r]) * dt)
```

```
}
else {
R[r + 1] <- R[r] - RDECAY * R[r] * dt
D[r + 1] <- D[r] - mu * D[r] * dt
}
}
O2 <- 23 - D
x <- round(dt/0.02, 0) * (1:n)
graphics.off() #par(mfrow = c(2, 2))
y <- round(1/dt, 0)
GT.ts <- ts(GT, start = dt, frequency = y)
R.ts <- ts(R, start = dt, frequency = y)
O2.ts <- ts(O2, start = dt, frequency = y)
NGT.ts <- ts(FIRE2.list$GT[x], start = dt, frequency = y)
NR.ts <- ts(FIRE2.list$R[x], start = dt, frequency = y)
NO2.ts <- ts(FIRE2.list$O2[x], start = dt, frequency = y)
#postscript(file = "B:GT.ps")
win.graph()
tsplot(GT.ts, NGT.ts, type = "pl", lty = c(1, 1))
legend(9.5, 700, "NRCC", lty = 1)
title(xlab = "Time t (min)", ylab = "Gas temp T (deg C.)")
win.graph() #postscript(file = "B:R.ps")
tsplot(R.ts, NR.ts, type = "pl", lty = c(1, 1))
legend(9.5, 11, "NRCC", lty = 1)
title(xlab = "Time t (min)", ylab = "Burning rate R (kg/min)")
win.graph() #postscript(file = "B:O2.ps")
tsplot(O2.ts, NO2.ts, type = "pl", lty = c(1, 1))
legend(9.5, 20, "NRCC", lty = 1)
```

```
title(xlab = "Time t (min)", ylab = "Oxygen (%)")
}
```

# Appendix B

# Optimisation Algorithm for Oxygen Driven Compartment Fire Model

**fcurve8**

```
function(alpha, beta, gamma, sigma, rho, delta, mu, k, k1, BMAX, RDECAY, RO)
{
FPAR <- fpar8 #
#RDECAY <- FPAR$RDECAY
time <- FPAR$time
frac <- FPAR$frac #
#RO <- FPAR$RO
#gamma <- FPAR$gamma
#sigma <- FPAR$sigma
#delta <- FPAR$delta
#mu <- FPAR$mu
#k <- FPAR$k
```

```
#k1 <- FPAR$k1
dt <- FPAR$dt
n <- round(time/dt, 0) + 1
tt <- rep(0, n)
GT <- rep(0, n)
R <- rep(0, n)
D <- rep(0, n)
B <- rep(0, n)
QL <- rep(0, n)
GT[1] <- 20
R[1] <- R0
D[1] <- 0
B[1] <- 0
tt[1] <- 0
SD <- sqrt(dt)
for(r in (1:(n - 1))) {
dWGT <- (frac * GT[r] * rnorm(1, 0, SD))/1000
dWR <- (15 * frac * GT[r] * rnorm(1, 0, SD))/1000
dWD <- (12 * frac * GT[r] * rnorm(1, 0, SD))/1000000
tt[r + 1] <- tt[r] + dt
QL[r] <- sigma * ((GT[r] + 273)^4 - 273^4) + gamma * GT[r]
#gt[r + 1] <- GT[r] + (beta * R[r] - rho * QL[r]) * dt
GT[r + 1] <- GT[r] + rho * (beta * R[r] - QL[r]) * dt + dWGT #<- min(1000, gt[r + 1])
B[r + 1] <- B[r] + R[r] * dt
Z[r] <- 22 * GT[r] * (1 - 1/(1 + (0.001 * GT[r])^2))
if(B[r] < BMAX) {
R[r + 1] <- R[r] + alpha * max(0, (k - D[r])) * Z[r] * dt + dWR
D[r + 1] <- min(16, D[r] + (delta * (k1 - D[r]) * R[r] - mu * D[r]) * dt) + dWD
```

```
}
else {
R[r + 1] <- R[r] - RDECAY * R[r] * dt + dWR
D[r + 1] <- D[r] - mu * D[r] * dt + dWD
}
}
pred <- cbind(GT, R, D)
pred
}
```

**sum3**

```
function(alpha, beta, gamma, sigma, delta, mu, k, k1, BMAX, R0)
{
calc <- fcurve8(alpha, beta, gamma, sigma, delta, mu, k, k1, BMAX, R0)
minsum <-1e-006*(sum((tot[,1]-calc[, 1])^2))+0.000064*(sum((tot[, 2] -
calc[, 2])^2)) + 23^(-2)*(sum((tot[, 3] - calc[, 3])^2))
minsum
}
```

**sum4**

```
function(r0)
{
calc <- fcurve8(2.815511, 0.0900053, 0.318948, 1.5e-010, 1.25, 0.0000151634,  0.582113,
minsum<-1e-006 *(sum((tot[, 1]- calc[,1])^2)) + 0.000064*(sum((tot[, 2] -
calc[,2])^2))+23^(-2)*(sum((tot[, 3]-calc[, 3])^2))
minsum
}
```

**test**

```
function(x)
{
for(r in (1:(length(x)))) {
y <- c(x[r], sum4(x[r]))
cat(y, "\n")
}
}
```

# Appendix C

# Stochastic Oxygen Driven Compartment Fire Model

**fpar9**

```
list(alpha = 2.5, beta = 0.1125, gamma = 0.399, rho = 1.25, sigma
 = 1.875e-010, delta = 0.000015, mu = 0.58, k = 10.4, k1
 = 158, BMAX = 82500, RDECAY = 0.956, time = 25, R0 = 280,
frac = 100, dt = 0.1)
```

**fcurve7 (fpar9, 25, 0.1)**

```
function(fpar6, tmax, dt)
{
FPAR <- fpar6
RDECAY <- FPAR$RDECAY
time <- FPAR$time
frac <- FPAR$frac
BMAX <- FPAR$BMAX
R0 <- FPAR$R0
alpha <- FPAR$alpha
```

```
beta <- FPAR$beta
gamma <- FPAR$gamma
sigma <- FPAR$sigma
delta <- FPAR$delta
rho <- FPAR$rho
mu <- FPAR$mu
k <- FPAR$k
k1 <- FPAR$k1
dt <- FPAR$dt
n <- round(time/dt, 0) + 1
tt <- rep(0, n)
GT <- rep(0, n)
R <- rep(0, n)
D <- rep(0, n)
B <- rep(0, n)
QL <- rep(0, n)
h <- rep(0, n)
h[1] <- 0
GT[1] <- 20
R[1] <- R0
D[1] <- 0
B[1] <- 0
tt[1] <- 0
SD <- sqrt(dt)
for(r in (1:(n - 1))) {
dWGT <- (frac * GT[r] * rnorm(1, 0, SD))/1000
dWR <- (15 * frac * GT[r] * rnorm(1, 0, SD))/1000
dWD <- (12 * frac * GT[r] * rnorm(1, 0, SD))/1000000
```

```
tt[r + 1] <- tt[r] + dt
QL[r] <- sigma * ((GT[r] + 273)^4 - 273^4) + gamma * GT[r]
h[r + 1] <- h[r] + QL[r] * dt
GT[r + 1] <- GT[r] + rho * (beta * R[r] - QL[r]) * dt + dWGT
B[r + 1] <- B[r] + R[r] * dt
Z <- 22 * GT[r] * (1 - 1/(1 + (0.001 * GT[r])^2))
if(B[r] < BMAX) {
  R[r + 1] <- R[r] + alpha * max(0, (k - D[r])) * Z * dt + dWR
  D[r + 1] <- min(16, D[r] + (delta * (k1 - D[r]) * R[r] - mu * D[r]) *)+dWD
}
else {
R[r + 1] <- R[r] - RDECAY * R[r] * dt + dWR
D[r + 1] <- D[r] - mu * D[r] * dt + dWD
}
}
O2 <- 23 - D
x <- round(dt/0.02, 0) * (1:n)
graphics.off() #par(mfrow = c(2, 2))
y <- round(1/dt, 0)
GT.ts <- ts(GT, start = dt, frequency = y)
R.ts <- ts(R, start = dt, frequency = y)
O2.ts <- ts(O2, start = dt, frequency = y)
NGT.ts <- ts(FIRE2.list$GT[x], start = dt, frequency = y)
NR.ts <- ts(FIRE2.list$R[x], start = dt, frequency = y)
NO2.ts <- ts(FIRE2.list$O2[x], start = dt, frequency = y)
#postscript(file = "B:GT.ps")
win.graph()
tsplot(GT.ts, NGT.ts, type = "pl", lty = c(1, 1), pch = "*")
```

```
legend(9.5, 700, "NRCC", lty = 1)
title(xlab = "Time t (min)", ylab = "Gas temp T (deg C.)")
win.graph() #postscript(file = "B:R.ps")
tsplot(R.ts, NR.ts, type = "pl", lty = c(1, 1), pch = "*")
legend(9.5, 11, "NRCC", lty = 1)
title(xlab = "Time t (min)", ylab = "Burning rate R (kg/min)")
win.graph() #postscript(file = "B:O2.ps")
tsplot(O2.ts, NO2.ts, type = "pl", lty = c(1, 1), pch = "*")
legend(9.5, 20, "NRCC", lty = 1)
title(xlab = "Time t (min)", ylab = "Oxygen (%)")
}
```

# Appendix D

# Simulation of Stochastic Oxygen Driven Compartment Fire Model for Heat Load

**FCURVE8**

```
function(fpar6, tmax, dt)
{
FPAR <- fpar6
RDECAY <- FPAR$RDECAY
time <- FPAR$time
frac <- FPAR$frac
BMAX <- FPAR$BMAX
RO <- FPAR$RO
alpha <- FPAR$alpha
beta <- FPAR$beta
gamma <- FPAR$gamma
```

```
sigma <- FPAR$sigma
delta <- FPAR$delta
rho <- FPAR$rho
mu <- FPAR$mu
k <- FPAR$k
k1 <- FPAR$k1
dt <- FPAR$dt
n <- round(time/dt, 0) + 1
tt <- rep(0, n)
GT <- rep(0, n)
R <- rep(0, n)
D <- rep(0, n)
B <- rep(0, n)
QL <- rep(0, n)
h <- rep(0, n)
h[1] <- 0
GT[1] <- 20
R[1] <- R0
D[1] <- 0
B[1] <- 0
tt[1] <- 0
SD <- sqrt(dt)
for(r in (1:(n - 1))) {
dWGT <- (frac * GT[r] * rnorm(1, 0, SD))/1000
dWR <- (15 * frac * GT[r] * rnorm(1, 0, SD))/1000
dWD <- (12 * frac * GT[r] * rnorm(1, 0, SD))/1000000
tt[r + 1] <- tt[r] + dt
QL[r] <- sigma * ((GT[r] + 273)^4 - 273^4) + gamma *  GT[r]
```

```
h[r + 1] <- h[r] + QL[r] * dt
GT[r + 1] <- GT[r] + rho * (beta * R[r] - QL[r]) * dt + dWGT
B[r + 1] <- B[r] + R[r] * dt
Z <- 22 * GT[r] * (1 - 1/(1 + (0.001 * GT[r])^2))
if(B[r] < BMAX) {
R[r + 1] <- R[r] + alpha * max(0, (k - D[r])) * Z * dt + dWR
D[r + 1] <- min(16, D[r] + (delta * (k1 - D[r]) * R[r] - mu * D[r]) * dt) + dWD
}
else {
R[r + 1] <- R[r] - RDECAY * R[r] * dt + dWR
D[r + 1] <- D[r] - mu * D[r] * dt + dWD
}
}
y <- h[n]/1000
write(y, file = "hload", append = T)
}
```

**fpar6**

```
list(alpha = 2.815511, beta = 0.0900053, gamma = 0.318948, rho = 1.25,
sigma = 1.5e-010, delta = 0.0000151634, mu = 0.582113,k = 9.7, k1 = 158,
BMAX = 82500, RDECAY = 0.956, time = 25, R0 = 279.85, frac = 100, dt = 0.1)
```

**fpar8**

```
list(alpha = 3, beta = 0.09, gamma = 0.3, rho = 0.9530336, sigma =1.5e-010,
delta = 0.000015, mu = 0.5, k = 11, k1 = 158, BMAX = 85000, RDECAY = 0.8,
time = 25, R0 = 280, frac = 0, dt = 0.2)
```

# Appendix E

# NRCC Data

| CNT | TIME(min) | TEMP | B.RATE | OXYGEN |
|---|---|---|---|---|
| 0001 | 0.000 | 20.020 | 8.380 | 22.999 |
| 0002 | 0.020 | 20.187 | 9.180 | 22.997 |
| 0003 | 0.040 | 20.367 | 10.017 | 22.995 |
| 0004 | 0.060 | 20.561 | 10.890 | 22.993 |
| 0005 | 0.080 | 20.769 | 11.799 | 22.991 |
| 0006 | 0.100 | 20.991 | 12.744 | 22.988 |
| 0007 | 0.120 | 21.228 | 13.725 | 22.986 |
| 0008 | 0.140 | 21.479 | 14.742 | 22.983 |
| 0009 | 0.160 | 21.746 | 15.795 | 22.980 |
| 0010 | 0.180 | 22.029 | 16.884 | 22.977 |
| 0011 | 0.200 | 22.328 | 18.008 | 22.974 |
| 0012 | 0.220 | 22.642 | 19.169 | 22.971 |
| 0013 | 0.240 | 22.974 | 20.364 | 22.967 |
| 0014 | 0.260 | 23.321 | 21.595 | 22.963 |
| 0015 | 0.280 | 23.686 | 22.862 | 22.959 |
| 0016 | 0.300 | 24.068 | 24.163 | 22.955 |
| 0017 | 0.320 | 24.467 | 25.500 | 22.950 |

| | | | | |
|---|---|---|---|---|
| 0018 | 0.340 | 24.883 | 26.871 | 22.946 |
| 0019 | 0.360 | 25.317 | 28.278 | 22.941 |
| 0020 | 0.380 | 25.769 | 29.719 | 22.936 |
| 0021 | 0.400 | 26.238 | 31.194 | 22.930 |
| 0022 | 0.420 | 26.725 | 32.704 | 22.925 |
| 0023 | 0.440 | 27.231 | 34.249 | 22.919 |
| 0024 | 0.460 | 27.754 | 35.827 | 22.913 |
| 0025 | 0.480 | 28.295 | 37.439 | 22.906 |
| 0026 | 0.500 | 28.854 | 39.085 | 22.900 |
| 0027 | 0.520 | 29.432 | 40.765 | 22.893 |
| 0028 | 0.540 | 30.027 | 42.478 | 22.886 |
| 0029 | 0.560 | 30.640 | 44.224 | 22.879 |
| 0030 | 0.580 | 31.271 | 46.003 | 22.871 |
| 0031 | 0.600 | 31.920 | 47.815 | 22.863 |
| 0032 | 0.620 | 32.587 | 49.660 | 22.855 |
| 0033 | 0.640 | 33.272 | 51.537 | 22.847 |
| 0034 | 0.660 | 33.974 | 53.446 | 22.838 |
| 0035 | 0.680 | 34.694 | 55.387 | 22.830 |
| 0036 | 0.700 | 35.431 | 57.360 | 22.820 |
| 0037 | 0.720 | 36.185 | 59.364 | 22.811 |
| 0038 | 0.740 | 36.957 | 61.400 | 22.802 |
| 0039 | 0.760 | 37.745 | 63.467 | 22.792 |
| 0040 | 0.780 | 38.551 | 65.564 | 22.782 |
| 0041 | 0.800 | 39.372 | 67.692 | 22.771 |
| 0042 | 0.820 | 40.211 | 69.851 | 22.761 |
| 0043 | 0.840 | 41.065 | 72.039 | 22.750 |
| 0044 | 0.860 | 41.936 | 74.257 | 22.739 |
| 0045 | 0.880 | 42.822 | 76.505 | 22.727 |

| | | | | |
|---|---|---|---|---|
| 0046 | 0.900 | 43.724 | 78.782 | 22.716 |
| 0047 | 0.920 | 44.642 | 81.088 | 22.704 |
| 0048 | 0.940 | 45.575 | 83.423 | 22.692 |
| 0049 | 0.960 | 46.522 | 85.786 | 22.679 |
| 0050 | 0.980 | 47.485 | 88.177 | 22.667 |
| 0051 | 1.000 | 48.461 | 90.596 | 22.654 |
| 0052 | 1.020 | 49.452 | 93.043 | 22.641 |
| 0053 | 1.040 | 50.457 | 95.517 | 22.627 |
| 0054 | 1.060 | 51.476 | 98.017 | 22.614 |
| 0055 | 1.080 | 52.509 | 100.545 | 22.600 |
| 0056 | 1.100 | 53.554 | 103.099 | 22.586 |
| 0057 | 1.120 | 54.613 | 105.679 | 22.572 |
| 0058 | 1.140 | 55.684 | 108.284 | 22.557 |
| 0059 | 1.160 | 56.768 | 110.915 | 22.542 |
| 0060 | 1.180 | 57.864 | 113.572 | 22.528 |
| 0061 | 1.200 | 58.972 | 116.253 | 22.512 |
| 0062 | 1.220 | 60.091 | 118.958 | 22.497 |
| 0063 | 1.240 | 61.222 | 121.688 | 22.481 |
| 0064 | 1.260 | 62.364 | 124.441 | 22.466 |
| 0065 | 1.280 | 63.517 | 127.219 | 22.450 |
| 0066 | 1.300 | 64.681 | 130.019 | 22.434 |
| 0067 | 1.320 | 65.855 | 132.842 | 22.417 |
| 0068 | 1.340 | 67.040 | 135.688 | 22.401 |
| 0069 | 1.360 | 68.233 | 138.556 | 22.384 |
| 0070 | 1.380 | 69.437 | 141.446 | 22.367 |
| 0071 | 1.400 | 70.649 | 144.358 | 22.350 |
| 0072 | 1.420 | 71.871 | 147.291 | 22.333 |
| 0073 | 1.440 | 73.102 | 150.245 | 22.315 |

| | | | | |
|---|---|---|---|---|
| 0074 | 1.460 | 74.341 | 153.220 | 22.298 |
| 0075 | 1.480 | 75.588 | 156.214 | 22.280 |
| 0076 | 1.500 | 76.844 | 159.229 | 22.262 |
| 0077 | 1.520 | 78.108 | 162.264 | 22.244 |
| 0078 | 1.540 | 79.379 | 165.318 | 22.226 |
| 0079 | 1.560 | 80.658 | 168.390 | 22.207 |
| 0080 | 1.580 | 81.944 | 171.482 | 22.189 |
| 0081 | 1.600 | 83.237 | 174.592 | 22.170 |
| 0082 | 1.620 | 84.536 | 177.720 | 22.152 |
| 0083 | 1.640 | 85.843 | 180.866 | 22.133 |
| 0084 | 1.660 | 87.155 | 184.029 | 22.114 |
| 0085 | 1.680 | 88.473 | 187.209 | 22.095 |
| 0086 | 1.700 | 89.798 | 190.405 | 22.075 |
| 0087 | 1.720 | 91.128 | 193.619 | 22.056 |
| 0088 | 1.740 | 92.463 | 196.848 | 22.036 |
| 0089 | 1.760 | 93.804 | 200.093 | 22.017 |
| 0090 | 1.780 | 95.149 | 203.354 | 21.997 |
| 0091 | 1.800 | 96.500 | 206.629 | 21.978 |
| 0092 | 1.820 | 97.855 | 209.920 | 21.958 |
| 0093 | 1.840 | 99.214 | 213.225 | 21.938 |
| 0094 | 1.860 | 100.578 | 216.544 | 21.918 |
| 0095 | 1.880 | 101.945 | 219.878 | 21.898 |
| 0096 | 1.900 | 103.316 | 223.224 | 21.878 |
| 0097 | 1.920 | 104.691 | 226.584 | 21.857 |
| 0098 | 1.940 | 106.070 | 229.957 | 21.837 |
| 0099 | 1.960 | 107.451 | 233.343 | 21.817 |
| 0100 | 1.980 | 108.836 | 236.741 | 21.796 |
| 0101 | 2.000 | 110.224 | 240.151 | 21.776 |

| | | | | |
|---|---|---|---|---|
| 0102 | 2.020 | 111.614 | 243.572 | 21.755 |
| 0103 | 2.040 | 113.007 | 247.005 | 21.735 |
| 0104 | 2.060 | 114.403 | 250.450 | 21.714 |
| 0105 | 2.080 | 115.800 | 253.905 | 21.694 |
| 0106 | 2.100 | 117.200 | 257.370 | 21.673 |
| 0107 | 2.120 | 118.602 | 260.846 | 21.652 |
| 0108 | 2.140 | 120.005 | 264.332 | 21.632 |
| 0109 | 2.160 | 121.410 | 267.827 | 21.611 |
| 0110 | 2.180 | 122.816 | 271.332 | 21.590 |
| 0111 | 2.200 | 124.224 | 274.846 | 21.569 |
| 0112 | 2.220 | 125.633 | 278.368 | 21.549 |
| 0113 | 2.240 | 127.043 | 281.899 | 21.528 |
| 0114 | 2.260 | 128.453 | 285.439 | 21.507 |
| 0115 | 2.280 | 129.865 | 288.986 | 21.486 |
| 0116 | 2.300 | 131.277 | 292.540 | 21.465 |
| 0117 | 2.320 | 132.690 | 296.103 | 21.445 |
| 0118 | 2.340 | 134.102 | 299.672 | 21.424 |
| 0119 | 2.360 | 135.515 | 303.248 | 21.403 |
| 0120 | 2.380 | 136.929 | 306.830 | 21.382 |
| 0121 | 2.400 | 138.342 | 310.419 | 21.361 |
| 0122 | 2.420 | 139.755 | 314.014 | 21.341 |
| 0123 | 2.440 | 141.167 | 317.614 | 21.320 |
| 0124 | 2.460 | 142.579 | 321.220 | 21.299 |
| 0125 | 2.480 | 143.991 | 324.831 | 21.278 |
| 0126 | 2.500 | 145.402 | 328.447 | 21.258 |
| 0127 | 2.520 | 146.812 | 332.067 | 21.237 |
| 0128 | 2.540 | 148.221 | 335.692 | 21.216 |
| 0129 | 2.560 | 149.630 | 339.321 | 21.196 |

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|---|---|---|---|---|
| 0130 | 2.580 | 151.037 | 342.953 | 21.175 |
| 0131 | 2.600 | 152.443 | 346.590 | 21.155 |
| 0132 | 2.620 | 153.847 | 350.229 | 21.134 |
| 0133 | 2.640 | 155.251 | 353.872 | 21.114 |
| 0134 | 2.660 | 156.652 | 357.517 | 21.093 |
| 0135 | 2.680 | 158.053 | 361.165 | 21.073 |
| 0136 | 2.700 | 159.451 | 364.815 | 21.052 |
| 0137 | 2.720 | 160.848 | 368.468 | 21.032 |
| 0138 | 2.740 | 162.242 | 372.122 | 21.012 |
| 0139 | 2.760 | 163.635 | 375.777 | 20.992 |
| 0140 | 2.780 | 165.026 | 379.434 | 20.971 |
| 0141 | 2.800 | 166.414 | 383.093 | 20.951 |
| 0142 | 2.820 | 167.800 | 386.751 | 20.931 |
| 0143 | 2.840 | 169.184 | 390.411 | 20.911 |
| 0144 | 2.860 | 170.565 | 394.071 | 20.891 |
| 0145 | 2.880 | 171.944 | 397.731 | 20.871 |
| 0146 | 2.900 | 173.321 | 401.391 | 20.851 |
| 0147 | 2.920 | 174.694 | 405.050 | 20.832 |
| 0148 | 2.940 | 176.065 | 408.709 | 20.812 |
| 0149 | 2.960 | 177.433 | 412.367 | 20.792 |
| 0150 | 2.980 | 178.798 | 416.024 | 20.773 |
| 0151 | 3.000 | 180.160 | 419.680 | 20.753 |
| 0152 | 3.020 | 181.519 | 423.335 | 20.733 |
| 0153 | 3.040 | 182.875 | 426.988 | 20.714 |
| 0154 | 3.060 | 184.228 | 430.639 | 20.695 |
| 0155 | 3.080 | 185.578 | 434.287 | 20.675 |
| 0156 | 3.100 | 186.924 | 437.934 | 20.656 |
| 0157 | 3.120 | 188.266 | 441.578 | 20.637 |

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|---|---|---|---|---|
| 0158 | 3.140 | 189.606 | 445.219 | 20.618 |
| 0159 | 3.160 | 190.942 | 448.858 | 20.599 |
| 0160 | 3.180 | 192.274 | 452.493 | 20.580 |
| 0161 | 3.200 | 193.603 | 456.125 | 20.561 |
| 0162 | 3.220 | 194.927 | 459.753 | 20.542 |
| 0163 | 3.240 | 196.249 | 463.378 | 20.523 |
| 0164 | 3.260 | 197.566 | 466.999 | 20.504 |
| 0165 | 3.280 | 198.880 | 470.616 | 20.486 |
| 0166 | 3.300 | 200.189 | 474.228 | 20.467 |
| 0167 | 3.320 | 201.495 | 477.836 | 20.449 |
| 0168 | 3.340 | 202.796 | 482.911 | 20.430 |
| 0169 | 3.360 | 204.120 | 487.137 | 20.411 |
| 0170 | 3.380 | 205.449 | 491.400 | 20.392 |
| 0171 | 3.400 | 206.784 | 495.700 | 20.373 |
| 0172 | 3.420 | 208.123 | 500.037 | 20.353 |
| 0173 | 3.440 | 209.469 | 504.412 | 20.334 |
| 0174 | 3.460 | 210.820 | 508.825 | 20.315 |
| 0175 | 3.480 | 212.178 | 513.278 | 20.295 |
| 0176 | 3.500 | 213.541 | 517.770 | 20.276 |
| 0177 | 3.520 | 214.910 | 522.302 | 20.256 |
| 0178 | 3.540 | 216.286 | 526.875 | 20.236 |
| 0179 | 3.560 | 217.668 | 531.490 | 20.217 |
| 0180 | 3.580 | 219.056 | 536.147 | 20.197 |
| 0181 | 3.600 | 220.451 | 540.847 | 20.177 |
| 0182 | 3.620 | 221.852 | 545.591 | 20.156 |
| 0183 | 3.640 | 223.260 | 20.136 | 550.379 |
| 0184 | 3.660 | 224.675 | 555.213 | 20.116 |
| 0185 | 3.680 | 226.097 | 560.092 | 20.095 |

| | | | | |
|---|---|---|---|---|
| 0186 | 3.700 | 227.526 | 565.018 | 20.074 |
| 0187 | 3.720 | 228.962 | 569.992 | 20.054 |
| 0188 | 3.740 | 230.406 | 575.015 | 20.033 |
| 0189 | 3.760 | 231.856 | 580.087 | 20.012 |
| 0190 | 3.780 | 233.315 | 585.209 | 19.990 |
| 0191 | 3.800 | 234.781 | 590.383 | 19.969 |
| 0192 | 3.820 | 236.255 | 595.609 | 19.947 |
| 0193 | 3.840 | 237.737 | 600.889 | 19.926 |
| 0194 | 3.860 | 239.227 | 606.223 | 19.904 |
| 0195 | 3.880 | 240.724 | 611.613 | 19.882 |
| 0196 | 3.900 | 242.230 | 617.059 | 19.860 |
| 0197 | 3.920 | 243.745 | 622.564 | 19.838 |
| 0198 | 3.940 | 245.268 | 628.128 | 19.815 |
| 0199 | 3.960 | 246.800 | 633.752 | 19.792 |
| 0200 | 3.980 | 248.341 | 639.438 | 19.770 |
| 0201 | 4.000 | 249.891 | 645.187 | 19.747 |
| 0202 | 4.020 | 251.450 | 651.002 | 19.723 |
| 0203 | 4.040 | 253.019 | 656.882 | 19.700 |
| 0204 | 4.060 | 254.597 | 662.830 | 19.676 |
| 0205 | 4.080 | 256.185 | 668.847 | 19.653 |
| 0206 | 4.100 | 257.783 | 674.936 | 19.629 |
| 0207 | 4.120 | 259.391 | 681.097 | 19.604 |
| 0208 | 4.140 | 261.010 | 687.333 | 19.580 |
| 0209 | 4.160 | 262.639 | 693.645 | 19.555 |
| 0210 | 4.180 | 264.279 | 700.036 | 19.530 |
| 0211 | 4.200 | 265.930 | 706.507 | 19.505 |
| 0212 | 4.220 | 267.592 | 713.062 | 19.480 |
| 0213 | 4.240 | 269.266 | 719.701 | 19.454 |

| | | | | |
|---|---|---|---|---|
| 0214 | 4.260 | 270.951 | 726.427 | 19.429 |
| 0215 | 4.280 | 272.648 | 733.243 | 19.403 |
| 0216 | 4.300 | 274.357 | 740.152 | 19.376 |
| 0217 | 4.320 | 276.079 | 747.155 | 19.350 |
| 0218 | 4.340 | 277.814 | 754.257 | 19.323 |
| 0219 | 4.360 | 279.562 | 761.459 | 19.295 |
| 0220 | 4.380 | 281.323 | 768.765 | 19.268 |
| 0221 | 4.400 | 283.098 | 776.179 | 19.240 |
| 0222 | 4.420 | 284.887 | 783.703 | 19.212 |
| 0223 | 4.440 | 286.690 | 791.342 | 19.184 |
| 0224 | 4.460 | 288.508 | 799.098 | 19.155 |
| 0225 | 4.480 | 290.342 | 806.977 | 19.126 |
| 0226 | 4.500 | 292.191 | 814.982 | 19.096 |
| 0227 | 4.520 | 294.056 | 823.117 | 19.067 |
| 0228 | 4.540 | 295.937 | 831.388 | 19.036 |
| 0229 | 4.560 | 297.836 | 839.800 | 19.006 |
| 0230 | 4.580 | 299.752 | 848.357 | 18.975 |
| 0231 | 4.600 | 301.686 | 857.066 | 18.943 |
| 0232 | 4.620 | 303.638 | 865.932 | 18.912 |
| 0233 | 4.640 | 305.610 | 874.961 | 18.879 |
| 0234 | 4.660 | 307.601 | 884.161 | 18.847 |
| 0235 | 4.680 | 309.613 | 893.538 | 18.813 |
| 0236 | 4.700 | 311.646 | 903.101 | 18.780 |
| 0237 | 4.720 | 313.701 | 912.857 | 18.745 |
| 0238 | 4.740 | 315.779 | 922.815 | 18.711 |
| 0239 | 4.760 | 317.880 | 932.985 | 18.675 |
| 0240 | 4.780 | 320.006 | 943.376 | 18.639 |
| 0241 | 4.800 | 322.156 | 954.000 | 18.603 |

| | | | | |
|---|---|---|---|---|
| 0242 | 4.820 | 324.334 | 964.868 | 18.566 |
| 0243 | 4.840 | 326.538 | 975.992 | 18.528 |
| 0244 | 4.860 | 328.771 | 987.387 | 18.489 |
| 0245 | 4.880 | 331.034 | 999.068 | 18.450 |
| 0246 | 4.900 | 333.328 | 1011.049 | 18.410 |
| 0247 | 4.920 | 335.654 | 1023.350 | 18.369 |
| 0248 | 4.940 | 338.014 | 1035.989 | 18.328 |
| 0249 | 4.960 | 340.410 | 1048.987 | 18.285 |
| 0250 | 4.980 | 342.844 | 1062.367 | 18.242 |
| 0251 | 5.000 | 345.317 | 1076.156 | 18.197 |
| 0252 | 5.020 | 347.832 | 1090.380 | 18.152 |
| 0253 | 5.040 | 350.391 | 1105.071 | 18.105 |
| 0254 | 5.060 | 352.996 | 1120.264 | 18.058 |
| 0255 | 5.080 | 355.650 | 1135.998 | 18.009 |
| 0256 | 5.100 | 358.357 | 1152.316 | 17.958 |
| 0257 | 5.120 | 361.119 | 1169.268 | 17.906 |
| 0258 | 5.140 | 363.940 | 1186.911 | 17.853 |
| 0259 | 5.160 | 366.826 | 1205.306 | 17.798 |
| 0260 | 5.180 | 369.779 | 1224.529 | 17.741 |
| 0261 | 5.200 | 372.805 | 1244.662 | 17.683 |
| 0262 | 5.220 | 375.911 | 1265.806 | 17.622 |
| 0263 | 5.240 | 379.104 | 1288.077 | 17.559 |
| 0264 | 5.260 | 382.391 | 1311.611 | 17.493 |
| 0265 | 5.280 | 385.782 | 1336.574 | 17.424 |
| 0266 | 5.300 | 389.286 | 1363.163 | 17.353 |
| 0267 | 5.320 | 392.917 | 1391.625 | 17.278 |
| 0268 | 5.340 | 396.689 | 1422.262 | 17.199 |
| 0269 | 5.360 | 400.620 | 1455.463 | 17.115 |

| | | | | |
|---|---|---|---|---|
| 0270 | 5.380 | 404.731 | 1491.724 | 17.027 |
| 0271 | 5.400 | 409.050 | 1531.708 | 16.932 |
| 0272 | 5.420 | 413.610 | 1576.316 | 16.831 |
| 0273 | 5.440 | 418.455 | 1626.828 | 16.721 |
| 0274 | 5.460 | 423.646 | 1685.134 | 16.600 |
| 0275 | 5.480 | 429.263 | 1754.202 | 16.465 |
| 0276 | 5.500 | 435.426 | 1839.071 | 16.313 |
| 0277 | 5.520 | 442.321 | 1949.336 | 16.135 |
| 0278 | 5.540 | 450.265 | 2106.780 | 15.916 |
| 0279 | 5.560 | 459.891 | 2379.284 | 15.615 |
| 0280 | 5.580 | 472.869 | 3235.572 | 15.017 |
| 0281 | 5.600 | 498.445 | 3505.640 | 14.376 |
| 0282 | 5.620 | 524.149 | 3587.918 | 13.779 |
| 0283 | 5.640 | 546.360 | 3744.784 | 13.202 |
| 0284 | 5.660 | 566.268 | 3971.181 | 12.631 |
| 0285 | 5.680 | 584.705 | 4185.509 | 12.074 |
| 0286 | 5.700 | 601.470 | 4388.412 | 11.539 |
| 0287 | 5.720 | 616.519 | 4580.499 | 11.030 |
| 0288 | 5.740 | 629.918 | 4762.347 | 10.551 |
| 0289 | 5.760 | 641.808 | 4934.500 | 10.103 |
| 0290 | 5.780 | 652.365 | 5097.478 | 9.688 |
| 0291 | 5.800 | 661.777 | 5251.768 | 9.303 |
| 0292 | 5.820 | 670.223 | 5397.833 | 8.950 |
| 0293 | 5.840 | 677.864 | 5536.111 | 8.625 |
| 0294 | 5.860 | 684.842 | 5667.019 | 8.329 |
| 0295 | 5.880 | 691.274 | 5790.948 | 8.059 |
| 0296 | 5.900 | 697.256 | 5908.272 | 7.814 |
| 0297 | 5.920 | 702.864 | 6019.341 | 7.592 |

| | | | | |
|---|---|---|---|---|
| 0298 | 5.940 | 708.160 | 6124.490 | 7.391 |
| 0299 | 5.960 | 713.193 | 6224.033 | 7.210 |
| 0300 | 5.980 | 717.999 | 6318.271 | 7.048 |
| 0301 | 6.000 | 722.608 | 6407.484 | 6.903 |
| 0302 | 6.020 | 727.043 | 6491.942 | 6.773 |
| 0303 | 6.040 | 731.322 | 6571.899 | 6.658 |
| 0304 | 6.060 | 735.459 | 6647.592 | 6.555 |
| 0305 | 6.080 | 739.467 | 6719.251 | 6.465 |
| 0306 | 6.100 | 743.354 | 6787.091 | 6.386 |
| 0307 | 6.120 | 747.128 | 6851.313 | 6.316 |
| 0308 | 6.140 | 750.820 | 6912.113 | 6.256 |
| 0309 | 6.160 | 754.412 | 6969.671 | 6.204 |
| 0310 | 6.180 | 757.908 | 7024.161 | 6.159 |
| 0311 | 6.200 | 761.310 | 7075.746 | 6.121 |
| 0312 | 6.220 | 764.623 | 7124.582 | 6.089 |
| 0313 | 6.240 | 767.850 | 7170.814 | 6.063 |
| 0314 | 6.260 | 770.992 | 7214.583 | 6.041 |
| 0315 | 6.280 | 774.055 | 7256.017 | 6.024 |
| 0316 | 6.300 | 777.040 | 7295.243 | 6.011 |
| 0317 | 6.320 | 779.950 | 7332.378 | 6.001 |
| 0318 | 6.340 | 782.790 | 7367.534 | 5.995 |
| 0319 | 6.360 | 785.560 | 7400.815 | 5.991 |
| 0320 | 6.380 | 788.262 | 7432.323 | 5.990 |
| 0321 | 6.400 | 790.899 | 7462.151 | 5.991 |
| 0322 | 6.420 | 793.473 | 7490.389 | 5.994 |
| 0323 | 6.440 | 795.986 | 7517.121 | 5.999 |
| 0324 | 6.460 | 798.439 | 7542.429 | 6.005 |
| 0325 | 6.480 | 800.836 | 7566.387 | 6.013 |

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|---|---|---|---|---|
| 0326 | 6.500 | 803.178 | 7589.069 | 6.022 |
| 0327 | 6.520 | 805.467 | 7610.542 | 6.032 |
| 0328 | 6.540 | 807.704 | 7630.869 | 6.043 |
| 0329 | 6.560 | 809.891 | 7650.113 | 6.055 |
| 0330 | 6.580 | 812.029 | 7668.332 | 6.068 |
| 0331 | 6.600 | 814.122 | 7685.579 | 6.081 |
| 0332 | 6.620 | 816.168 | 7701.906 | 6.094 |
| 0333 | 6.640 | 818.171 | 7717.364 | 6.108 |
| 0334 | 6.660 | 820.131 | 7731.998 | 6.123 |
| 0335 | 6.680 | 822.050 | 7745.851 | 6.138 |
| 0336 | 6.700 | 823.929 | 7758.966 | 6.153 |
| 0337 | 6.720 | 825.770 | 7771.382 | 6.168 |
| 0338 | 6.740 | 827.574 | 7783.136 | 6.184 |
| 0339 | 6.760 | 829.341 | 7794.263 | 6.200 |
| 0340 | 6.780 | 831.074 | 7804.797 | 6.215 |
| 0341 | 6.800 | 832.772 | 7814.770 | 6.231 |
| 0342 | 6.820 | 834.437 | 7824.211 | 6.247 |
| 0343 | 6.840 | 836.071 | 7833.149 | 6.263 |
| 0344 | 6.860 | 837.674 | 7841.610 | 6.279 |
| 0345 | 6.880 | 839.246 | 7849.621 | 6.295 |
| 0346 | 6.900 | 840.789 | 7857.204 | 6.311 |
| 0347 | 6.920 | 842.304 | 7864.383 | 6.327 |
| 0348 | 6.940 | 843.791 | 7871.180 | 6.343 |
| 0349 | 6.960 | 845.252 | 7877.614 | 6.358 |
| 0350 | 6.980 | 846.687 | 7883.705 | 6.374 |
| 0351 | 7.000 | 848.096 | 7889.472 | 6.390 |
| 0352 | 7.020 | 849.482 | 7894.931 | 6.405 |
| 0353 | 7.040 | 850.845 | 7900.099 | 6.420 |

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|---|---|---|---|---|
| 0354 | 7.060 | 852.185 | 7904.991 | 6.436 |
| 0355 | 7.080 | 853.504 | 7909.623 | 6.451 |
| 0356 | 7.100 | 854.800 | 7914.008 | 6.466 |
| 0357 | 7.120 | 856.075 | 7918.159 | 6.480 |
| 0358 | 7.140 | 857.330 | 7922.089 | 6.495 |
| 0359 | 7.160 | 858.564 | 7925.809 | 6.510 |
| 0360 | 7.180 | 859.778 | 7929.331 | 6.524 |
| 0361 | 7.200 | 860.972 | 7932.666 | 6.539 |
| 0362 | 7.220 | 862.148 | 7935.822 | 6.553 |
| 0363 | 7.240 | 863.306 | 7938.811 | 6.567 |
| 0364 | 7.260 | 864.446 | 7941.640 | 6.581 |
| 0365 | 7.280 | 865.568 | 7944.318 | 6.594 |
| 0366 | 7.300 | 866.674 | 7946.853 | 6.608 |
| 0367 | 7.320 | 867.763 | 7949.253 | 6.621 |
| 0368 | 7.340 | 868.835 | 7951.526 | 6.635 |
| 0369 | 7.360 | 869.893 | 7953.677 | 6.648 |
| 0370 | 7.380 | 870.935 | 7955.714 | 6.661 |
| 0371 | 7.400 | 871.962 | 7957.642 | 6.674 |
| 0372 | 7.420 | 872.975 | 7959.467 | 6.686 |
| 0373 | 7.440 | 873.973 | 7961.195 | 6.699 |
| 0374 | 7.460 | 874.958 | 7962.831 | 6.711 |
| 0375 | 7.480 | 875.928 | 7964.379 | 6.724 |
| 0376 | 7.500 | 876.886 | 7965.845 | 6.736 |
| 0377 | 7.520 | 877.831 | 7967.233 | 6.748 |
| 0378 | 7.540 | 878.763 | 7968.547 | 6.760 |
| 0379 | 7.560 | 879.683 | 7969.791 | 6.771 |
| 0380 | 7.580 | 880.590 | 7970.968 | 6.783 |
| 0381 | 7.600 | 881.486 | 7972.083 | 6.794 |

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|---|---|---|---|---|
| 0382 | 7.620 | 882.370 | 7973.139 | 6.806 |
| 0383 | 7.640 | 883.243 | 7974.138 | 6.817 |
| 0384 | 7.660 | 884.105 | 7975.083 | 6.828 |
| 0385 | 7.680 | 884.956 | 7975.979 | 6.839 |
| 0386 | 7.700 | 885.796 | 7976.827 | 6.850 |
| 0387 | 7.720 | 886.627 | 7977.629 | 6.860 |
| 0388 | 7.740 | 887.446 | 7978.389 | 6.871 |
| 0389 | 7.760 | 888.255 | 7979.108 | 6.881 |
| 0390 | 7.780 | 889.055 | 7979.789 | 6.892 |
| 0391 | 7.800 | 889.846 | 7980.434 | 6.902 |
| 0392 | 7.820 | 890.627 | 7981.044 | 6.912 |
| 0393 | 7.840 | 891.399 | 7981.622 | 6.922 |
| 0394 | 7.860 | 892.162 | 7982.168 | 6.932 |
| 0395 | 7.880 | 892.916 | 7982.686 | 6.941 |
| 0396 | 7.900 | 893.661 | 7983.177 | 6.951 |
| 0397 | 7.920 | 894.398 | 7983.641 | 6.961 |
| 0398 | 7.940 | 895.127 | 7984.080 | 6.970 |
| 0399 | 7.960 | 895.847 | 7984.496 | 6.979 |
| 0400 | 7.980 | 896.560 | 7984.889 | 6.989 |
| 0401 | 8.000 | 897.265 | 7985.262 | 6.998 |
| 0402 | 8.020 | 897.962 | 7985.615 | 7.007 |
| 0403 | 8.040 | 898.651 | 7985.949 | 7.016 |
| 0404 | 8.060 | 899.333 | 7986.265 | 7.024 |
| 0405 | 8.080 | 900.007 | 7986.564 | 7.033 |
| 0406 | 8.100 | 900.675 | 7986.848 | 7.042 |
| 0407 | 8.120 | 901.335 | 7987.116 | 7.050 |
| 0408 | 8.140 | 901.989 | 7987.371 | 7.059 |
| 0409 | 8.160 | 902.635 | 7987.611 | 7.067 |

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|---|---|---|---|---|
| 0410 | 8.180 | 903.275 | 7987.838 | 7.075 |
| 0411 | 8.200 | 903.909 | 7988.054 | 7.083 |
| 0412 | 8.220 | 904.536 | 7988.258 | 7.091 |
| 0413 | 8.240 | 905.157 | 7988.451 | 7.099 |
| 0414 | 8.260 | 905.771 | 7988.634 | 7.107 |
| 0415 | 8.280 | 906.380 | 7988.807 | 7.115 |
| 0416 | 8.300 | 906.982 | 7988.971 | 7.123 |
| 0417 | 8.320 | 907.578 | 7989.126 | 7.131 |
| 0418 | 8.340 | 908.169 | 7989.273 | 7.138 |
| 0419 | 8.360 | 908.754 | 7989.412 | 7.146 |
| 0420 | 8.380 | 909.333 | 7989.544 | 7.153 |
| 0421 | 8.400 | 909.907 | 7989.668 | 7.160 |
| 0422 | 8.420 | 910.475 | 7989.787 | 7.168 |
| 0423 | 8.440 | 911.038 | 7989.898 | 7.175 |
| 0424 | 8.460 | 911.596 | 7990.004 | 7.182 |
| 0425 | 8.480 | 912.148 | 7990.104 | 7.189 |
| 0426 | 8.500 | 912.695 | 7990.199 | 7.196 |
| 0427 | 8.520 | 913.238 | 7990.289 | 7.203 |
| 0428 | 8.540 | 913.775 | 7990.374 | 7.210 |
| 0429 | 8.560 | 914.307 | 7990.454 | 7.217 |
| 0430 | 8.580 | 914.835 | 7990.530 | 7.223 |
| 0431 | 8.600 | 915.358 | 7990.602 | 7.230 |
| 0432 | 8.620 | 915.876 | 7990.670 | 7.236 |
| 0433 | 8.640 | 916.390 | 7990.735 | 7.243 |
| 0434 | 8.660 | 916.899 | 7990.796 | 7.249 |
| 0435 | 8.680 | 917.403 | 7990.854 | 7.256 |
| 0436 | 8.700 | 917.903 | 7990.909 | 7.262 |
| 0437 | 8.720 | 918.399 | 7990.960 | 7.268 |

0438 8.740 918.891 7991.009 7.275

0439 8.760 919.378 7991.056 7.281

0440 8.780 919.861 7991.100 7.287

0441 8.800 920.340 7991.142 7.293

0442 8.820 920.815 7991.181 7.299

0443 8.840 921.286 7991.218 7.305

0444 8.860 921.754 7991.254 7.311

0445 8.880 922.217 7991.287 7.317

0446 8.900 922.677 7991.319 7.322

0447 8.920 923.133 7991.349 7.328

0448 8.940 923.585 7991.377 7.334

0449 8.960 924.033 7991.404 7.339

0450 8.980 924.478 7991.430 7.345

0451 9.000 924.919 7991.454 7.351

0452 9.020 925.357 7991.477 7.356

0453 9.040 925.791 7991.498 7.361

0454 9.060 926.222 7991.519 7.367

0455 9.080 926.649 7991.538 7.372

0456 9.100 927.073 7991.556 7.377

0457 9.120 927.493 7991.574 7.383

0458 9.140 927.911 7991.590 7.388

0459 9.160 928.326 7991.605 7.393

0460 9.180 928.737 7991.620 7.398

0461 9.200 929.145 7991.634 7.403

0462 9.220 929.550 7991.647 7.408

0463 9.240 929.952 7991.660 7.413

0464 9.260 930.351 7991.672 7.418

0465 9.280 930.747 7991.683 7.423

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|---|---|---|---|---|
| 0466 | 9.300 | 931.141 | 7991.693 | 7.428 |
| 0467 | 9.320 | 931.531 | 7991.704 | 7.433 |
| 0468 | 9.340 | 931.918 | 7991.713 | 7.438 |
| 0469 | 9.360 | 932.303 | 7991.722 | 7.442 |
| 0470 | 9.380 | 932.684 | 7991.730 | 7.447 |
| 0471 | 9.400 | 933.064 | 7991.738 | 7.452 |
| 0472 | 9.420 | 933.440 | 7991.746 | 7.456 |
| 0473 | 9.440 | 933.813 | 7991.753 | 7.461 |
| 0474 | 9.460 | 934.184 | 7991.760 | 7.465 |
| 0475 | 9.480 | 934.553 | 7991.767 | 7.470 |
| 0476 | 9.500 | 934.918 | 7991.773 | 7.474 |
| 0477 | 9.520 | 935.281 | 7991.779 | 7.479 |
| 0478 | 9.540 | 935.642 | 7991.784 | 7.483 |
| 0479 | 9.560 | 935.999 | 7991.789 | 7.488 |
| 0480 | 9.580 | 936.355 | 7991.794 | 7.492 |
| 0481 | 9.600 | 936.708 | 7991.799 | 7.496 |
| 0482 | 9.620 | 937.059 | 7991.803 | 7.501 |
| 0483 | 9.640 | 937.407 | 7991.808 | 7.505 |
| 0484 | 9.660 | 937.753 | 7991.812 | 7.509 |
| 0485 | 9.680 | 938.096 | 7991.815 | 7.513 |
| 0486 | 9.700 | 938.437 | 7991.819 | 7.518 |
| 0487 | 9.720 | 938.777 | 7991.822 | 7.522 |
| 0488 | 9.740 | 939.113 | 7991.825 | 7.526 |
| 0489 | 9.760 | 939.448 | 7991.828 | 7.530 |
| 0490 | 9.780 | 939.780 | 7991.831 | 7.534 |
| 0491 | 9.800 | 940.110 | 7991.833 | 7.538 |
| 0492 | 9.820 | 940.438 | 7991.836 | 7.542 |
| 0493 | 9.840 | 940.763 | 7991.839 | 7.546 |

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|---|---|---|---|---|
| 0494 | 9.860 | 941.087 | 7991.841 | 7.550 |
| 0495 | 9.880 | 941.408 | 7991.843 | 7.554 |
| 0496 | 9.900 | 941.728 | 7991.845 | 7.558 |
| 0497 | 9.920 | 942.045 | 7991.847 | 7.561 |
| 0498 | 9.940 | 942.360 | 7991.849 | 7.565 |
| 0499 | 9.960 | 942.673 | 7991.851 | 7.569 |
| 0500 | 9.980 | 942.984 | 7991.853 | 7.573 |
| 0501 | 10.000 | 943.294 | 7991.854 | 7.577 |
| 0502 | 10.020 | 943.601 | 7991.855 | 7.580 |
| 0503 | 10.040 | 943.906 | 7991.857 | 7.584 |
| 0504 | 10.060 | 944.210 | 7991.858 | 7.588 |
| 0505 | 10.080 | 944.512 | 7991.859 | 7.591 |
| 0506 | 10.100 | 944.811 | 7991.860 | 7.595 |
| 0507 | 10.120 | 945.109 | 7991.862 | 7.598 |
| 0508 | 10.140 | 945.405 | 7991.863 | 7.602 |
| 0509 | 10.160 | 945.700 | 7991.864 | 7.606 |
| 0510 | 10.180 | 945.992 | 7991.865 | 7.609 |
| 0511 | 10.200 | 946.283 | 7991.865 | 7.613 |
| 0512 | 10.220 | 946.572 | 7991.866 | 7.616 |
| 0513 | 10.240 | 946.859 | 7991.867 | 7.620 |
| 0514 | 10.260 | 947.144 | 7991.868 | 7.623 |
| 0515 | 10.280 | 947.428 | 7991.869 | 7.626 |
| 0516 | 10.300 | 947.710 | 7991.870 | 7.630 |
| 0517 | 10.320 | 947.990 | 7991.870 | 7.633 |
| 0518 | 10.340 | 948.269 | 7991.871 | 7.636 |
| 0519 | 10.360 | 948.546 | 7991.871 | 7.640 |
| 0520 | 10.380 | 948.822 | 7991.872 | 7.643 |
| 0521 | 10.400 | 949.096 | 7991.872 | 7.646 |

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|---|---|---|---|---|
| 0522 | 10.420 | 949.368 | 7991.873 | 7.650 |
| 0523 | 10.440 | 949.639 | 7991.874 | 7.653 |
| 0524 | 10.460 | 949.908 | 7991.874 | 7.656 |
| 0525 | 10.480 | 950.175 | 7991.874 | 7.659 |
| 0526 | 10.500 | 950.441 | 7991.875 | 7.662 |
| 0527 | 10.520 | 950.706 | 7991.875 | 7.666 |
| 0528 | 10.540 | 950.968 | 7991.875 | 7.669 |
| 0529 | 10.560 | 951.230 | 7991.875 | 7.672 |
| 0530 | 10.580 | 951.490 | 7991.876 | 7.675 |
| 0531 | 10.600 | 951.749 | 7991.876 | 7.678 |
| 0532 | 10.620 | 952.006 | 7991.876 | 7.681 |
| 0533 | 10.640 | 952.261 | 7991.876 | 7.684 |
| 0534 | 10.660 | 952.515 | 7991.877 | 7.687 |
| 0535 | 10.680 | 952.768 | 7991.877 | 7.690 |
| 0536 | 10.700 | 953.019 | 7991.877 | 7.693 |
| 0537 | 10.720 | 953.269 | 7991.878 | 7.696 |
| 0538 | 10.740 | 953.518 | 7991.878 | 7.699 |
| 0539 | 10.760 | 953.765 | 7991.878 | 7.702 |
| 0540 | 10.780 | 954.012 | 7991.878 | 7.705 |
| 0541 | 10.800 | 954.256 | 7991.878 | 7.708 |
| 0542 | 10.820 | 954.499 | 7991.878 | 7.711 |
| 0543 | 10.840 | 954.741 | 7991.879 | 7.713 |
| 0544 | 10.860 | 954.982 | 7991.879 | 7.716 |
| 0545 | 10.880 | 955.221 | 7991.879 | 7.719 |
| 0546 | 10.900 | 955.459 | 7991.879 | 7.722 |
| 0547 | 10.920 | 955.695 | 7991.879 | 7.725 |
| 0548 | 10.940 | 955.932 | 7991.879 | 7.727 |
| 0549 | 10.960 | 956.166 | 7991.880 | 7.730 |

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|---|---|---|---|---|
| 0550 | 10.980 | 956.398 | 7991.880 | 7.733 |
| 0551 | 11.000 | 956.630 | 7991.880 | 7.736 |
| 0552 | 11.020 | 956.861 | 7991.880 | 7.738 |
| 0553 | 11.040 | 957.091 | 7991.880 | 7.741 |
| 0554 | 11.060 | 957.319 | 7991.880 | 7.744 |
| 0555 | 11.080 | 957.546 | 7991.880 | 7.747 |
| 0556 | 11.100 | 957.771 | 7991.880 | 7.749 |
| 0557 | 11.120 | 957.996 | 7991.880 | 7.752 |
| 0558 | 11.140 | 958.220 | 7991.880 | 7.754 |
| 0559 | 11.160 | 958.442 | 7991.880 | 7.757 |
| 0560 | 11.180 | 958.664 | 7991.880 | 7.760 |
| 0561 | 11.200 | 958.883 | 7991.880 | 7.762 |
| 0562 | 11.220 | 959.102 | 7991.880 | 7.765 |
| 0563 | 11.240 | 959.321 | 7991.881 | 7.767 |
| 0564 | 11.260 | 959.538 | 7991.881 | 7.770 |
| 0565 | 11.280 | 959.753 | 7991.881 | 7.772 |
| 0566 | 11.300 | 959.967 | 7991.881 | 7.775 |
| 0567 | 11.320 | 960.181 | 7991.881 | 7.777 |
| 0568 | 11.340 | 960.394 | 7991.881 | 7.780 |
| 0569 | 11.360 | 960.606 | 7991.881 | 7.782 |
| 0570 | 11.380 | 960.816 | 7991.881 | 7.785 |
| 0571 | 11.400 | 961.025 | 7991.881 | 7.787 |
| 0572 | 11.420 | 961.233 | 7991.881 | 7.790 |
| 0573 | 11.440 | 961.441 | 7991.881 | 7.792 |
| 0574 | 11.460 | 961.647 | 7991.881 | 7.795 |
| 0575 | 11.480 | 961.853 | 7991.881 | 7.797 |
| 0576 | 11.500 | 962.057 | 7991.881 | 7.799 |
| 0577 | 11.520 | 962.260 | 7991.881 | 7.802 |

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|---|---|---|---|---|
| 0578 | 11.540 | 962.463 | 7991.881 | 7.804 |
| 0579 | 11.560 | 962.664 | 7991.881 | 7.806 |
| 0580 | 11.580 | 962.865 | 7991.881 | 7.809 |
| 0581 | 11.600 | 963.064 | 7991.881 | 7.811 |
| 0582 | 11.620 | 963.263 | 7991.881 | 7.813 |
| 0583 | 11.640 | 963.460 | 7991.881 | 7.816 |
| 0584 | 11.660 | 963.657 | 7991.881 | 7.818 |
| 0585 | 11.680 | 963.853 | 7991.881 | 7.820 |
| 0586 | 11.700 | 964.048 | 7991.881 | 7.823 |
| 0587 | 11.720 | 964.241 | 7991.881 | 7.825 |
| 0588 | 11.740 | 964.434 | 7991.881 | 7.827 |
| 0589 | 11.760 | 964.627 | 7991.881 | 7.829 |
| 0590 | 11.780 | 964.818 | 7991.881 | 7.831 |
| 0591 | 11.800 | 965.008 | 7991.881 | 7.834 |
| 0592 | 11.820 | 965.198 | 7991.881 | 7.836 |
| 0593 | 11.840 | 965.386 | 7991.881 | 7.838 |
| 0594 | 11.860 | 965.574 | 7991.881 | 7.840 |
| 0595 | 11.880 | 965.761 | 7991.881 | 7.842 |
| 0596 | 11.900 | 965.947 | 7991.881 | 7.845 |
| 0597 | 11.920 | 966.133 | 7991.881 | 7.847 |
| 0598 | 11.940 | 966.317 | 7991.881 | 7.849 |
| 0599 | 11.960 | 966.500 | 7991.881 | 7.851 |
| 0600 | 11.980 | 966.683 | 7991.881 | 7.853 |
| 0601 | 12.000 | 966.865 | 7991.881 | 7.855 |
| 0602 | 12.020 | 967.046 | 7991.881 | 7.857 |
| 0603 | 12.040 | 967.227 | 7991.881 | 7.859 |
| 0604 | 12.060 | 967.406 | 7991.881 | 7.861 |
| 0605 | 12.080 | 967.584 | 7991.881 | 7.863 |

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|---|---|---|---|---|
| 0606 | 12.100 | 967.762 | 7991.881 | 7.866 |
| 0607 | 12.120 | 967.940 | 7991.881 | 7.868 |
| 0608 | 12.140 | 968.116 | 7991.881 | 7.870 |
| 0609 | 12.160 | 968.292 | 7991.881 | 7.872 |
| 0610 | 12.180 | 968.466 | 7991.881 | 7.874 |
| 0611 | 12.200 | 968.640 | 7991.881 | 7.876 |
| 0612 | 12.220 | 968.814 | 7991.881 | 7.878 |
| 0613 | 12.240 | 968.986 | 7991.881 | 7.880 |
| 0614 | 12.260 | 969.158 | 7991.881 | 7.882 |
| 0615 | 12.280 | 969.329 | 7991.881 | 7.884 |
| 0616 | 12.300 | 969.500 | 7991.881 | 7.886 |
| 0617 | 12.320 | 969.669 | 7991.881 | 7.888 |
| 0618 | 12.340 | 969.838 | 7991.881 | 7.889 |
| 0619 | 12.360 | 970.006 | 7991.881 | 7.891 |
| 0620 | 12.380 | 970.174 | 7991.881 | 7.893 |
| 0621 | 12.400 | 970.341 | 7991.881 | 7.895 |
| 0622 | 12.420 | 970.507 | 7991.881 | 7.897 |
| 0623 | 12.440 | 970.672 | 7991.881 | 7.899 |
| 0624 | 12.460 | 970.836 | 7991.881 | 7.901 |
| 0625 | 12.480 | 971.000 | 7991.881 | 7.903 |
| 0626 | 12.500 | 971.163 | 7991.881 | 7.905 |
| 0627 | 12.520 | 971.326 | 7991.881 | 7.907 |
| 0628 | 12.540 | 971.488 | 7991.881 | 7.908 |
| 0629 | 12.560 | 971.649 | 7991.881 | 7.910 |
| 0630 | 12.580 | 971.810 | 7991.881 | 7.912 |
| 0631 | 12.600 | 971.969 | 7991.881 | 7.914 |
| 0632 | 12.620 | 972.129 | 7991.881 | 7.916 |
| 0633 | 12.640 | 972.288 | 7991.881 | 7.918 |

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|---|---|---|---|---|
| 0634 | 12.660 | 972.446 | 7991.881 | 7.919 |
| 0635 | 12.680 | 972.603 | 7991.881 | 7.921 |
| 0636 | 12.700 | 972.759 | 7991.881 | 7.923 |
| 0637 | 12.720 | 972.915 | 7991.881 | 7.925 |
| 0638 | 12.740 | 973.071 | 7991.881 | 7.927 |
| 0639 | 12.760 | 973.226 | 7991.881 | 7.928 |
| 0640 | 12.780 | 973.380 | 7991.881 | 7.930 |
| 0641 | 12.800 | 973.534 | 7991.881 | 7.932 |
| 0642 | 12.820 | 973.687 | 7991.881 | 7.934 |
| 0643 | 12.840 | 973.839 | 7991.881 | 7.935 |
| 0644 | 12.860 | 973.990 | 7991.881 | 7.937 |
| 0645 | 12.880 | 974.142 | 7991.881 | 7.939 |
| 0646 | 12.900 | 974.292 | 7991.881 | 7.940 |
| 0647 | 12.920 | 974.442 | 7991.881 | 7.942 |
| 0648 | 12.940 | 974.592 | 7991.881 | 7.944 |
| 0649 | 12.960 | 974.741 | 7991.881 | 7.946 |
| 0650 | 12.980 | 974.889 | 7991.881 | 7.947 |
| 0651 | 13.000 | 975.036 | 7991.881 | 7.949 |
| 0652 | 13.020 | 975.183 | 7991.881 | 7.951 |
| 0653 | 13.040 | 975.330 | 7991.881 | 7.952 |
| 0654 | 13.060 | 975.476 | 7991.881 | 7.954 |
| 0655 | 13.080 | 975.621 | 7991.881 | 7.956 |
| 0656 | 13.100 | 975.765 | 7991.881 | 7.957 |
| 0657 | 13.120 | 975.910 | 7991.881 | 7.959 |
| 0658 | 13.140 | 976.053 | 7991.881 | 7.961 |
| 0659 | 13.160 | 976.196 | 7991.881 | 7.962 |
| 0660 | 13.180 | 976.339 | 7991.881 | 7.964 |
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| 0702 | 14.020 | 981.875 | 7991.881 | 8.026 |
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| 0704 | 14.060 | 982.118 | 7991.881 | 8.029 |
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| 0714 | 14.260 | 983.311 | 7991.881 | 8.042 |
| 0715 | 14.280 | 983.428 | 7991.881 | 8.044 |
| 0716 | 14.300 | 983.545 | 7991.881 | 8.045 |
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| 0720 | 14.380 | 984.007 | 7991.881 | 8.050 |
| 0721 | 14.400 | 984.122 | 7991.881 | 8.052 |
| 0722 | 14.420 | 984.237 | 7991.881 | 8.053 |
| 0723 | 14.440 | 984.351 | 7991.881 | 8.054 |
| 0724 | 14.460 | 984.465 | 7991.881 | 8.055 |
| 0725 | 14.480 | 984.578 | 7991.881 | 8.057 |
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| 0729 | 14.560 | 985.027 | 7991.881 | 8.062 |
| 0730 | 14.580 | 985.138 | 7991.881 | 8.063 |
| 0731 | 14.600 | 985.249 | 7991.881 | 8.064 |
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| 0734 | 14.660 | 985.580 | 7991.881 | 8.068 |
| 0735 | 14.680 | 985.690 | 7991.881 | 8.069 |
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| 0744 | 14.860 | 986.661 | 7991.881 | 8.080 |
| 0745 | 14.880 | 986.767 | 7991.881 | 8.081 |

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| 0747 | 14.920 | 986.979 | 7991.881 | 8.084 |
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| 0753 | 15.040 | 987.605 | 7991.881 | 8.091 |
| 0754 | 15.060 | 987.708 | 7991.881 | 8.092 |
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| 0765 | 15.280 | 988.824 | 7991.881 | 8.104 |
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| 0767 | 15.320 | 989.023 | 7991.881 | 8.106 |
| 0768 | 15.340 | 989.121 | 7991.881 | 8.107 |
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| 0775 | 15.480 | 989.806 | 7991.881 | 8.115 |
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| 0780 | 15.580 | 990.286 | 7991.881 | 8.120 |
| 0781 | 15.600 | 990.381 | 7991.881 | 8.121 |
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| 0783 | 15.640 | 990.571 | 7988.904 | 8.124 |
| 0784 | 15.660 | 990.662 | 7979.970 | 8.126 |
| 0785 | 15.680 | 990.743 | 7965.082 | 8.130 |
| 0786 | 15.700 | 990.809 | 7944.238 | 8.135 |
| 0787 | 15.720 | 990.857 | 7917.438 | 8.142 |
| 0788 | 15.740 | 990.883 | 7884.683 | 8.152 |
| 0789 | 15.760 | 990.883 | 7845.973 | 8.164 |
| 0790 | 15.780 | 990.859 | 7801.307 | 8.179 |
| 0791 | 15.800 | 990.804 | 7750.686 | 8.196 |
| 0792 | 15.820 | 990.718 | 7694.108 | 8.217 |
| 0793 | 15.840 | 990.597 | 7631.576 | 8.240 |
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| 0795 | 15.880 | 990.239 | 7488.646 | 8.297 |
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| 0803 | 16.040 | 985.862 | 6442.425 | 8.760 |
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| 0805 | 16.080 | 983.716 | 6160.855 | 8.934 |
| 0806 | 16.100 | 982.512 | 6024.719 | 9.026 |
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| 0813 | 16.240 | 971.806 | 5152.375 | 9.718 |
| 0814 | 16.260 | 969.969 | 5038.524 | 9.822 |
| 0815 | 16.280 | 968.057 | 4927.188 | 9.927 |
| 0816 | 16.300 | 966.069 | 4818.313 | 10.032 |
| 0817 | 16.320 | 964.005 | 4711.843 | 10.139 |
| 0818 | 16.340 | 961.866 | 4607.726 | 10.247 |
| 0819 | 16.360 | 959.650 | 4505.909 | 10.355 |
| 0820 | 16.380 | 957.360 | 4406.342 | 10.464 |
| 0821 | 16.400 | 954.992 | 4308.976 | 10.574 |
| 0822 | 16.420 | 952.546 | 4213.761 | 10.685 |
| 0823 | 16.440 | 950.024 | 4120.650 | 10.796 |
| 0824 | 16.460 | 947.424 | 4029.596 | 10.908 |
| 0825 | 16.480 | 944.748 | 3940.554 | 11.021 |
| 0826 | 16.500 | 941.996 | 3853.480 | 11.134 |
| 0827 | 16.520 | 939.167 | 3768.330 | 11.249 |
| 0828 | 16.540 | 936.263 | 3685.062 | 11.363 |
| 0829 | 16.560 | 933.282 | 3603.633 | 11.479 |

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| 0830 | 16.580 | 930.225 | 3524.004 | 11.595 |
| 0831 | 16.600 | 927.093 | 3446.135 | 11.712 |
| 0832 | 16.620 | 923.885 | 3369.986 | 11.829 |
| 0833 | 16.640 | 920.602 | 3295.519 | 11.947 |
| 0834 | 16.660 | 917.243 | 3222.698 | 12.066 |
| 0835 | 16.680 | 913.810 | 3151.487 | 12.185 |
| 0836 | 16.700 | 910.303 | 3081.849 | 12.305 |
| 0837 | 16.720 | 906.721 | 3013.749 | 12.425 |
| 0838 | 16.740 | 903.065 | 2947.155 | 12.545 |
| 0839 | 16.760 | 899.337 | 2882.032 | 12.667 |
| 0840 | 16.780 | 895.535 | 2818.348 | 12.788 |
| 0841 | 16.800 | 891.663 | 2756.071 | 12.910 |
| 0842 | 16.820 | 887.719 | 2695.170 | 13.032 |
| 0843 | 16.840 | 883.704 | 2635.615 | 13.155 |
| 0844 | 16.860 | 879.620 | 2577.376 | 13.277 |
| 0845 | 16.880 | 875.467 | 2520.424 | 13.400 |
| 0846 | 16.900 | 871.247 | 2464.731 | 13.523 |
| 0847 | 16.920 | 866.959 | 2410.268 | 13.647 |
| 0848 | 16.940 | 862.606 | 2357.008 | 13.770 |
| 0849 | 16.960 | 858.190 | 2304.926 | 13.894 |
| 0850 | 16.980 | 853.712 | 2253.994 | 14.017 |
| 0851 | 17.000 | 849.174 | 2204.188 | 14.140 |
| 0852 | 17.020 | 844.575 | 2155.482 | 14.263 |
| 0853 | 17.040 | 839.918 | 2107.852 | 14.387 |
| 0854 | 17.060 | 835.204 | 2061.275 | 14.509 |
| 0855 | 17.080 | 830.433 | 2015.728 | 14.632 |
| 0856 | 17.100 | 825.609 | 1971.186 | 14.754 |
| 0857 | 17.120 | 820.733 | 1927.629 | 14.876 |

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| 0858 | 17.140 | 815.806 | 1885.034 | 14.997 |
| 0859 | 17.160 | 810.831 | 1843.381 | 15.118 |
| 0860 | 17.180 | 805.809 | 1802.648 | 15.239 |
| 0861 | 17.200 | 800.744 | 1762.815 | 15.358 |
| 0862 | 17.220 | 795.635 | 1723.862 | 15.477 |
| 0863 | 17.240 | 790.487 | 1685.770 | 15.596 |
| 0864 | 17.260 | 785.300 | 1648.520 | 15.713 |
| 0865 | 17.280 | 780.077 | 1612.093 | 15.830 |
| 0866 | 17.300 | 774.820 | 1576.470 | 15.945 |
| 0867 | 17.320 | 769.533 | 1541.635 | 16.060 |
| 0868 | 17.340 | 764.215 | 1507.570 | 16.174 |
| 0869 | 17.360 | 758.871 | 1474.257 | 16.287 |
| 0870 | 17.380 | 753.502 | 1441.681 | 16.398 |
| 0871 | 17.400 | 748.111 | 1409.824 | 16.509 |
| 0872 | 17.420 | 742.700 | 1378.671 | 16.618 |
| 0873 | 17.440 | 737.270 | 1348.207 | 16.727 |
| 0874 | 17.460 | 731.825 | 1318.416 | 16.833 |
| 0875 | 17.480 | 726.367 | 1289.283 | 16.939 |
| 0876 | 17.500 | 720.898 | 1260.794 | 17.043 |
| 0877 | 17.520 | 715.420 | 1232.934 | 17.146 |
| 0878 | 17.540 | 709.935 | 1205.690 | 17.248 |
| 0879 | 17.560 | 704.444 | 1179.048 | 17.348 |
| 0880 | 17.580 | 698.950 | 1152.995 | 17.447 |
| 0881 | 17.600 | 693.456 | 1127.517 | 17.545 |
| 0882 | 17.620 | 687.962 | 1102.602 | 17.641 |
| 0883 | 17.640 | 682.471 | 1078.238 | 17.735 |
| 0884 | 17.660 | 676.985 | 1054.412 | 17.828 |
| 0885 | 17.680 | 671.504 | 1031.113 | 17.920 |

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| 0886 | 17.700 | 666.031 | 1008.329 | 18.010 |
| 0887 | 17.720 | 660.568 | 986.048 | 18.099 |
| 0888 | 17.740 | 655.116 | 964.259 | 18.186 |
| 0889 | 17.760 | 649.676 | 942.952 | 18.272 |
| 0890 | 17.780 | 644.250 | 922.116 | 18.356 |
| 0891 | 17.800 | 638.839 | 901.740 | 18.439 |
| 0892 | 17.820 | 633.444 | 881.814 | 18.521 |
| 0893 | 17.840 | 628.066 | 862.329 | 18.601 |
| 0894 | 17.860 | 622.707 | 843.274 | 18.680 |
| 0895 | 17.880 | 617.367 | 824.640 | 18.757 |
| 0896 | 17.900 | 612.048 | 806.418 | 18.833 |
| 0897 | 17.920 | 606.750 | 788.599 | 18.908 |
| 0898 | 17.940 | 601.476 | 771.173 | 18.981 |
| 0899 | 17.960 | 596.225 | 754.133 | 19.053 |
| 0900 | 17.980 | 590.998 | 737.469 | 19.123 |
| 0901 | 18.000 | 585.797 | 721.173 | 19.193 |
| 0902 | 18.020 | 580.622 | 705.237 | 19.261 |
| 0903 | 18.040 | 575.473 | 689.654 | 19.328 |
| 0904 | 18.060 | 570.352 | 674.414 | 19.393 |
| 0905 | 18.080 | 565.258 | 659.512 | 19.458 |
| 0906 | 18.100 | 560.194 | 644.939 | 19.521 |
| 0907 | 18.120 | 555.160 | 630.688 | 19.583 |
| 0908 | 18.140 | 550.155 | 616.751 | 19.644 |
| 0909 | 18.160 | 545.182 | 603.123 | 19.704 |
| 0910 | 18.180 | 540.239 | 589.796 | 19.762 |
| 0911 | 18.200 | 535.329 | 576.763 | 19.820 |
| 0912 | 18.220 | 530.451 | 564.019 | 19.876 |
| 0913 | 18.240 | 525.605 | 551.555 | 19.932 |

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| 0914 | 18.260 | 520.793 | 539.368 | 19.986 |
| 0915 | 18.280 | 516.015 | 527.449 | 20.040 |
| 0916 | 18.300 | 511.271 | 515.794 | 20.092 |
| 0917 | 18.320 | 506.561 | 504.397 | 20.143 |
| 0918 | 18.340 | 501.886 | 493.251 | 20.194 |
| 0919 | 18.360 | 497.246 | 482.352 | 20.244 |
| 0920 | 18.380 | 492.642 | 471.694 | 20.292 |
| 0921 | 18.400 | 488.074 | 461.271 | 20.340 |
| 0922 | 18.420 | 483.542 | 451.078 | 20.387 |
| 0923 | 18.440 | 479.047 | 441.110 | 20.433 |
| 0924 | 18.460 | 474.590 | 431.363 | 20.478 |
| 0925 | 18.480 | 470.170 | 421.832 | 20.522 |
| 0926 | 18.500 | 465.788 | 412.510 | 20.566 |
| 0927 | 18.520 | 461.443 | 403.395 | 20.609 |
| 0928 | 18.540 | 457.138 | 394.481 | 20.651 |
| 0929 | 18.560 | 452.870 | 385.765 | 20.692 |
| 0930 | 18.580 | 448.642 | 377.240 | 20.732 |
| 0931 | 18.600 | 444.452 | 368.904 | 20.772 |
| 0932 | 18.620 | 440.302 | 360.753 | 20.811 |
| 0933 | 18.640 | 436.191 | 352.781 | 20.849 |
| 0934 | 18.660 | 432.119 | 344.986 | 20.887 |
| 0935 | 18.680 | 428.087 | 337.363 | 20.923 |
| 0936 | 18.700 | 424.095 | 329.908 | 20.960 |
| 0937 | 18.720 | 420.143 | 322.618 | 20.995 |
| 0938 | 18.740 | 416.231 | 315.489 | 21.030 |
| 0939 | 18.760 | 412.359 | 308.518 | 21.064 |
| 0940 | 18.780 | 408.528 | 301.701 | 21.098 |
| 0941 | 18.800 | 404.736 | 295.034 | 21.131 |

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| 0942 | 18.820 | 400.985 | 288.515 | 21.163 |
| 0943 | 18.840 | 397.274 | 282.139 | 21.195 |
| 0944 | 18.860 | 393.602 | 275.905 | 21.226 |
| 0945 | 18.880 | 389.971 | 269.808 | 21.257 |
| 0946 | 18.900 | 386.381 | 263.846 | 21.287 |
| 0947 | 18.920 | 382.830 | 258.016 | 21.317 |
| 0948 | 18.940 | 379.321 | 252.315 | 21.346 |
| 0949 | 18.960 | 375.851 | 246.740 | 21.375 |
| 0950 | 18.980 | 372.422 | 241.287 | 21.403 |
| 0951 | 19.000 | 369.033 | 235.956 | 21.430 |
| 0952 | 19.020 | 365.684 | 230.742 | 21.457 |
| 0953 | 19.040 | 362.375 | 225.643 | 21.484 |
| 0954 | 19.060 | 359.106 | 220.657 | 21.510 |
| 0955 | 19.080 | 355.877 | 215.781 | 21.536 |
| 0956 | 19.100 | 352.687 | 211.013 | 21.561 |
| 0957 | 19.120 | 349.536 | 206.350 | 21.586 |
| 0958 | 19.140 | 346.425 | 201.791 | 21.610 |
| 0959 | 19.160 | 343.353 | 197.332 | 21.634 |
| 0960 | 19.180 | 340.320 | 192.971 | 21.657 |
| 0961 | 19.200 | 337.326 | 188.707 | 21.680 |
| 0962 | 19.220 | 334.370 | 184.537 | 21.703 |
| 0963 | 19.240 | 331.452 | 180.460 | 21.725 |
| 0964 | 19.260 | 328.573 | 176.472 | 21.747 |
| 0965 | 19.280 | 325.731 | 172.573 | 21.769 |
| 0966 | 19.300 | 322.927 | 168.759 | 21.790 |
| 0967 | 19.320 | 320.160 | 165.030 | 21.811 |
| 0968 | 19.340 | 317.431 | 161.384 | 21.831 |
| 0969 | 19.360 | 314.738 | 157.817 | 21.851 |

0970 19.380 312.081 154.330 21.871

0971 19.400 309.461 150.920 21.890

0972 19.420 306.878 147.585 21.909

0973 19.440 304.330 144.324 21.928

0974 19.460 301.817 141.135 21.946

0975 19.480 299.339 138.016 21.964

0976 19.500 296.896 134.966 21.982

0977 19.520 294.488 131.984 21.999

0978 19.540 292.113 129.068 22.017

0979 19.560 289.773 126.216 22.033

0980 19.580 287.466 123.427 22.050

0981 19.600 285.192 120.699 22.066

0982 19.620 282.952 118.032 22.082

0983 19.640 280.743 115.424 22.098

0984 19.660 278.567 112.874 22.113

0985 19.680 276.423 110.379 22.129

0986 19.700 274.310 107.940 22.144

0987 19.720 272.229 105.555 22.158

0988 19.740 270.178 103.223 22.173

0989 19.760 268.158 100.942 22.187

0990 19.780 266.168 98.711 22.201

0991 19.800 264.208 96.530 22.215

0992 19.820 262.277 94.397 22.228

0993 19.840 260.375 92.311 22.241

0994 19.860 258.502 90.271 22.254

0995 19.880 256.658 0.000 22.267

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| 0998 | 19.940 | 247.977 | 0.000 | 22.379 |
| 0999 | 19.960 | 244.684 | 0.000 | 22.411 |
| 1000 | 19.980 | 241.507 | 0.000 | 22.442 |
| 1001 | 20.000 | 238.443 | 0.000 | 22.471 |
| 1002 | 20.020 | 235.486 | 0.000 | 22.498 |
| 1003 | 20.040 | 232.635 | 0.000 | 22.523 |
| 1004 | 20.060 | 229.886 | 0.000 | 22.547 |
| 1005 | 20.080 | 227.235 | 0.000 | 22.570 |
| 1006 | 20.100 | 224.678 | 0.000 | 22.591 |
| 1007 | 20.120 | 222.213 | 0.000 | 22.611 |
| 1008 | 20.140 | 219.837 | 0.000 | 22.630 |
| 1009 | 20.160 | 217.545 | 0.000 | 22.647 |
| 1010 | 20.180 | 215.337 | 0.000 | 22.664 |
| 1011 | 20.200 | 213.207 | 0.000 | 22.680 |
| 1012 | 20.220 | 211.155 | 0.000 | 22.695 |
| 1013 | 20.240 | 209.176 | 0.000 | 22.709 |
| 1014 | 20.260 | 207.269 | 0.000 | 22.723 |
| 1015 | 20.280 | 205.430 | 0.000 | 22.736 |
| 1016 | 20.300 | 203.657 | 0.000 | 22.748 |
| 1017 | 20.320 | 201.947 | 0.000 | 22.759 |
| 1018 | 20.340 | 200.299 | 0.000 | 22.770 |
| 1019 | 20.360 | 198.710 | 0.000 | 22.781 |
| 1020 | 20.380 | 197.177 | 0.000 | 22.790 |
| 1021 | 20.400 | 195.699 | 0.000 | 22.800 |
| 1022 | 20.420 | 194.273 | 0.000 | 22.809 |
| 1023 | 20.440 | 192.898 | 0.000 | 22.817 |
| 1024 | 20.460 | 191.572 | 0.000 | 22.825 |
| 1025 | 20.480 | 190.292 | 0.000 | 22.833 |

| | | | | |
|---|---|---|---|---|
| 1026 | 20.500 | 189.057 | 0.000 | 22.840 |
| 1027 | 20.520 | 187.865 | 0.000 | 22.847 |
| 1028 | 20.540 | 186.715 | 0.000 | 22.853 |
| 1029 | 20.560 | 185.604 | 0.000 | 22.860 |
| 1030 | 20.580 | 184.532 | 0.000 | 22.866 |
| 1031 | 20.600 | 183.497 | 0.000 | 22.871 |
| 1032 | 20.620 | 182.497 | 0.000 | 22.877 |
| 1033 | 20.640 | 181.531 | 0.000 | 22.882 |
| 1034 | 20.660 | 180.598 | 0.000 | 22.887 |
| 1035 | 20.680 | 179.695 | 0.000 | 22.892 |
| 1036 | 20.700 | 178.823 | 0.000 | 22.896 |
| 1037 | 20.720 | 177.979 | 0.000 | 22.900 |
| 1038 | 20.740 | 177.164 | 0.000 | 22.905 |
| 1039 | 20.760 | 176.375 | 0.000 | 22.908 |
| 1040 | 20.780 | 175.612 | 0.000 | 22.912 |
| 1041 | 20.800 | 174.874 | 0.000 | 22.916 |
| 1042 | 20.820 | 174.159 | 0.000 | 22.919 |
| 1043 | 20.840 | 173.467 | 0.000 | 22.923 |
| 1044 | 20.860 | 172.797 | 0.000 | 22.926 |
| 1045 | 20.880 | 172.148 | 0.000 | 22.929 |
| 1046 | 20.900 | 171.519 | 0.000 | 22.932 |
| 1047 | 20.920 | 170.909 | 0.000 | 22.934 |
| 1048 | 20.940 | 170.318 | 0.000 | 22.937 |
| 1049 | 20.960 | 169.745 | 0.000 | 22.940 |
| 1050 | 20.980 | 169.188 | 0.000 | 22.942 |
| 1051 | 21.000 | 168.649 | 0.000 | 22.944 |
| 1052 | 21.020 | 168.125 | 0.000 | 22.946 |
| 1053 | 21.040 | 167.616 | 0.000 | 22.949 |

| | | | | |
|---|---|---|---|---|
| 1054 | 21.060 | 167.122 | 0.000 | 22.951 |
| 1055 | 21.080 | 166.642 | 0.000 | 22.953 |
| 1056 | 21.100 | 166.175 | 0.000 | 22.954 |
| 1057 | 21.120 | 165.721 | 0.000 | 22.956 |
| 1058 | 21.140 | 165.280 | 0.000 | 22.958 |
| 1059 | 21.160 | 164.850 | 0.000 | 22.960 |
| 1060 | 21.180 | 164.432 | 0.000 | 22.961 |
| 1061 | 21.200 | 164.025 | 0.000 | 22.963 |
| 1062 | 21.220 | 163.628 | 0.000 | 22.964 |
| 1063 | 21.240 | 163.241 | 0.000 | 22.966 |
| 1064 | 21.260 | 162.865 | 0.000 | 22.967 |
| 1065 | 21.280 | 162.497 | 0.000 | 22.968 |
| 1066 | 21.300 | 162.139 | 0.000 | 22.969 |
| 1067 | 21.320 | 161.789 | 0.000 | 22.971 |
| 1068 | 21.340 | 161.447 | 0.000 | 22.972 |
| 1069 | 21.360 | 161.113 | 0.000 | 22.973 |
| 1070 | 21.380 | 160.787 | 0.000 | 22.974 |
| 1071 | 21.400 | 160.468 | 0.000 | 22.975 |
| 1072 | 21.420 | 160.157 | 0.000 | 22.976 |
| 1073 | 21.440 | 159.852 | 0.000 | 22.977 |
| 1074 | 21.460 | 159.554 | 0.000 | 22.978 |
| 1075 | 21.480 | 159.261 | 0.000 | 22.979 |
| 1076 | 21.500 | 158.976 | 0.000 | 22.979 |
| 1077 | 21.520 | 158.695 | 0.000 | 22.980 |
| 1078 | 21.540 | 158.420 | 0.000 | 22.981 |
| 1079 | 21.560 | 158.151 | 0.000 | 22.982 |
| 1080 | 21.580 | 157.887 | 0.000 | 22.982 |
| 1081 | 21.600 | 157.627 | 0.000 | 22.983 |

| | | | | |
|---|---|---|---|---|
| 1082 | 21.620 | 157.373 | 0.000 | 22.984 |
| 1083 | 21.640 | 157.123 | 0.000 | 22.984 |
| 1084 | 21.660 | 156.878 | 0.000 | 22.985 |
| 1085 | 21.680 | 156.636 | 0.000 | 22.985 |
| 1086 | 21.700 | 156.399 | 0.000 | 22.986 |
| 1087 | 21.720 | 156.166 | 0.000 | 22.987 |
| 1088 | 21.740 | 155.936 | 0.000 | 22.987 |
| 1089 | 21.760 | 155.710 | 0.000 | 22.988 |
| 1090 | 21.780 | 155.488 | 0.000 | 22.988 |
| 1091 | 21.800 | 155.269 | 0.000 | 22.988 |
| 1092 | 21.820 | 155.053 | 0.000 | 22.989 |
| 1093 | 21.840 | 154.841 | 0.000 | 22.989 |
| 1094 | 21.860 | 154.631 | 0.000 | 22.990 |
| 1095 | 21.880 | 154.424 | 0.000 | 22.990 |
| 1096 | 21.900 | 154.220 | 0.000 | 22.990 |
| 1097 | 21.920 | 154.019 | 0.000 | 22.991 |
| 1098 | 21.940 | 153.820 | 0.000 | 22.991 |
| 1099 | 21.960 | 153.624 | 0.000 | 22.991 |
| 1100 | 21.980 | 153.431 | 0.000 | 22.992 |
| 1101 | 22.000 | 153.239 | 0.000 | 22.992 |
| 1102 | 22.020 | 153.050 | 0.000 | 22.992 |
| 1103 | 22.040 | 152.863 | 0.000 | 22.993 |
| 1104 | 22.060 | 152.678 | 0.000 | 22.993 |
| 1105 | 22.080 | 152.496 | 0.000 | 22.993 |
| 1106 | 22.100 | 152.315 | 0.000 | 22.993 |
| 1107 | 22.120 | 152.136 | 0.000 | 22.994 |
| 1108 | 22.140 | 151.958 | 0.000 | 22.994 |
| 1109 | 22.160 | 151.783 | 0.000 | 22.994 |

| | | | | |
|---|---|---|---|---|
| 1110 | 22.180 | 151.609 | 0.000 | 22.994 |
| 1111 | 22.200 | 151.437 | 0.000 | 22.995 |
| 1112 | 22.220 | 151.267 | 0.000 | 22.995 |
| 1113 | 22.240 | 151.098 | 0.000 | 22.995 |
| 1114 | 22.260 | 150.930 | 0.000 | 22.995 |
| 1115 | 22.280 | 150.764 | 0.000 | 22.995 |
| 1116 | 22.300 | 150.600 | 0.000 | 22.996 |
| 1117 | 22.320 | 150.437 | 0.000 | 22.996 |
| 1118 | 22.340 | 150.275 | 0.000 | 22.996 |
| 1119 | 22.360 | 150.114 | 0.000 | 22.996 |
| 1120 | 22.380 | 149.955 | 0.000 | 22.996 |
| 1121 | 22.400 | 149.796 | 0.000 | 22.996 |
| 1122 | 22.420 | 149.639 | 0.000 | 22.996 |
| 1123 | 22.440 | 149.483 | 0.000 | 22.997 |
| 1124 | 22.460 | 149.328 | 0.000 | 22.997 |
| 1125 | 22.480 | 149.175 | 0.000 | 22.997 |
| 1126 | 22.500 | 149.022 | 0.000 | 22.997 |
| 1127 | 22.520 | 148.870 | 0.000 | 22.997 |
| 1128 | 22.540 | 148.719 | 0.000 | 22.997 |
| 1129 | 22.560 | 148.569 | 0.000 | 22.997 |
| 1130 | 22.580 | 148.420 | 0.000 | 22.997 |
| 1131 | 22.600 | 148.272 | 0.000 | 22.997 |
| 1132 | 22.620 | 148.125 | 0.000 | 22.998 |
| 1133 | 22.640 | 147.978 | 0.000 | 22.998 |
| 1134 | 22.660 | 147.833 | 0.000 | 22.998 |
| 1135 | 22.680 | 147.688 | 0.000 | 22.998 |
| 1136 | 22.700 | 147.544 | 0.000 | 22.998 |
| 1137 | 22.720 | 147.401 | 0.000 | 22.998 |

| | | | | |
|---|---|---|---|---|
| 1138 | 22.740 | 147.258 | 0.000 | 22.998 |
| 1139 | 22.760 | 147.117 | 0.000 | 22.998 |
| 1140 | 22.780 | 146.976 | 0.000 | 22.998 |
| 1141 | 22.800 | 146.835 | 0.000 | 22.998 |
| 1142 | 22.820 | 146.696 | 0.000 | 22.998 |
| 1143 | 22.840 | 146.557 | 0.000 | 22.998 |
| 1144 | 22.860 | 146.418 | 0.000 | 22.998 |
| 1145 | 22.880 | 146.280 | 0.000 | 22.999 |
| 1146 | 22.900 | 146.143 | 0.000 | 22.999 |
| 1147 | 22.920 | 146.007 | 0.000 | 22.999 |
| 1148 | 22.940 | 145.871 | 0.000 | 22.999 |
| 1149 | 22.960 | 145.735 | 0.000 | 22.999 |
| 1150 | 22.980 | 145.601 | 0.000 | 22.999 |
| 1151 | 23.000 | 145.466 | 0.000 | 22.999 |
| 1152 | 23.020 | 145.333 | 0.000 | 22.999 |
| 1153 | 23.040 | 145.199 | 0.000 | 22.999 |
| 1154 | 23.060 | 145.067 | 0.000 | 22.999 |
| 1155 | 23.080 | 144.935 | 0.000 | 22.999 |
| 1156 | 23.100 | 144.803 | 0.000 | 22.999 |
| 1157 | 23.120 | 144.672 | 0.000 | 22.999 |
| 1158 | 23.140 | 144.541 | 0.000 | 22.999 |
| 1159 | 23.160 | 144.411 | 0.000 | 22.999 |
| 1160 | 23.180 | 144.281 | 0.000 | 22.999 |
| 1161 | 23.200 | 144.152 | 0.000 | 22.999 |
| 1162 | 23.220 | 144.023 | 0.000 | 22.999 |
| 1163 | 23.240 | 143.895 | 0.000 | 22.999 |
| 1164 | 23.260 | 143.767 | 0.000 | 22.999 |
| 1165 | 23.280 | 143.639 | 0.000 | 22.999 |

| | | | | |
|---|---|---|---|---|
| 1166 | 23.300 | 143.512 | 0.000 | 22.999 |
| 1167 | 23.320 | 143.386 | 0.000 | 22.999 |
| 1168 | 23.340 | 143.259 | 0.000 | 22.999 |
| 1169 | 23.360 | 143.134 | 0.000 | 22.999 |
| 1170 | 23.380 | 143.008 | 0.000 | 22.999 |
| 1171 | 23.400 | 142.883 | 0.000 | 22.999 |
| 1172 | 23.420 | 142.758 | 0.000 | 22.999 |
| 1173 | 23.440 | 142.634 | 0.000 | 22.999 |
| 1174 | 23.460 | 142.510 | 0.000 | 23.000 |
| 1175 | 23.480 | 142.387 | 0.000 | 23.000 |
| 1176 | 23.500 | 142.264 | 0.000 | 23.000 |
| 1177 | 23.520 | 142.141 | 0.000 | 23.000 |
| 1178 | 23.540 | 142.018 | 0.000 | 23.000 |
| 1179 | 23.560 | 141.896 | 0.000 | 23.000 |
| 1180 | 23.580 | 141.775 | 0.000 | 23.000 |
| 1181 | 23.600 | 141.653 | 0.000 | 23.000 |
| 1182 | 23.620 | 141.532 | 0.000 | 23.000 |
| 1183 | 23.640 | 141.412 | 0.000 | 23.000 |
| 1184 | 23.660 | 141.291 | 0.000 | 23.000 |
| 1185 | 23.680 | 141.171 | 0.000 | 23.000 |
| 1186 | 23.700 | 141.052 | 0.000 | 23.000 |
| 1187 | 23.720 | 140.932 | 0.000 | 23.000 |
| 1188 | 23.740 | 140.813 | 0.000 | 23.000 |
| 1189 | 23.760 | 140.695 | 0.000 | 23.000 |
| 1190 | 23.780 | 140.576 | 0.000 | 23.000 |
| 1191 | 23.800 | 140.458 | 0.000 | 23.000 |
| 1192 | 23.820 | 140.341 | 0.000 | 23.000 |
| 1193 | 23.840 | 140.223 | 0.000 | 23.000 |

| | | | | |
|---|---|---|---|---|
| 1194 | 23.860 | 140.106 | 0.000 | 23.000 |
| 1195 | 23.880 | 139.989 | 0.000 | 23.000 |
| 1196 | 23.900 | 139.873 | 0.000 | 23.000 |
| 1197 | 23.920 | 139.757 | 0.000 | 23.000 |
| 1198 | 23.940 | 139.641 | 0.000 | 23.000 |
| 1199 | 23.960 | 139.525 | 0.000 | 23.000 |
| 1200 | 23.980 | 139.410 | 0.000 | 23.000 |
| 1201 | 24.000 | 139.295 | 0.000 | 23.000 |
| 1202 | 24.020 | 139.180 | 0.000 | 23.000 |
| 1203 | 24.040 | 139.065 | 0.000 | 23.000 |
| 1204 | 24.060 | 138.951 | 0.000 | 23.000 |
| 1205 | 24.080 | 138.837 | 0.000 | 23.000 |
| 1206 | 24.100 | 138.724 | 0.000 | 23.000 |
| 1207 | 24.120 | 138.611 | 0.000 | 23.000 |
| 1208 | 24.140 | 138.497 | 0.000 | 23.000 |
| 1209 | 24.160 | 138.385 | 0.000 | 23.000 |
| 1210 | 24.180 | 138.272 | 0.000 | 23.000 |
| 1211 | 24.200 | 138.160 | 0.000 | 23.000 |
| 1212 | 24.220 | 138.048 | 0.000 | 23.000 |
| 1213 | 24.240 | 137.936 | 0.000 | 23.000 |
| 1214 | 24.260 | 137.825 | 0.000 | 23.000 |
| 1215 | 24.280 | 137.714 | 0.000 | 23.000 |
| 1216 | 24.300 | 137.603 | 0.000 | 23.000 |
| 1217 | 24.320 | 137.492 | 0.000 | 23.000 |
| 1218 | 24.340 | 137.382 | 0.000 | 23.000 |
| 1219 | 24.360 | 137.272 | 0.000 | 23.000 |
| 1220 | 24.380 | 137.162 | 0.000 | 23.000 |
| 1221 | 24.400 | 137.052 | 0.000 | 23.000 |

| | | | | |
|---|---|---|---|---|
| 1222 | 24.420 | 136.943 | 0.000 | 23.000 |
| 1223 | 24.440 | 136.834 | 0.000 | 23.000 |
| 1224 | 24.460 | 136.725 | 0.000 | 23.000 |
| 1225 | 24.480 | 136.617 | 0.000 | 23.000 |
| 1226 | 24.500 | 136.508 | 0.000 | 23.000 |
| 1227 | 24.520 | 136.400 | 0.000 | 23.000 |
| 1228 | 24.540 | 136.292 | 0.000 | 23.000 |
| 1229 | 24.560 | 136.185 | 0.000 | 23.000 |
| 1230 | 24.580 | 136.077 | 0.000 | 23.000 |
| 1231 | 24.600 | 135.970 | 0.000 | 23.000 |
| 1232 | 24.620 | 135.863 | 0.000 | 23.000 |
| 1233 | 24.640 | 135.757 | 0.000 | 23.000 |
| 1234 | 24.660 | 135.650 | 0.000 | 23.000 |
| 1235 | 24.680 | 135.544 | 0.000 | 23.000 |
| 1236 | 24.700 | 135.438 | 0.000 | 23.000 |
| 1237 | 24.720 | 135.333 | 0.000 | 23.000 |
| 1238 | 24.740 | 135.227 | 0.000 | 23.000 |
| 1239 | 24.760 | 135.122 | 0.000 | 23.000 |
| 1240 | 24.780 | 135.017 | 0.000 | 23.000 |
| 1241 | 24.800 | 134.912 | 0.000 | 23.000 |
| 1242 | 24.820 | 134.808 | 0.000 | 23.000 |
| 1243 | 24.840 | 134.704 | 0.000 | 23.000 |
| 1244 | 24.860 | 134.599 | 0.000 | 23.000 |
| 1245 | 24.880 | 134.496 | 0.000 | 23.000 |
| 1246 | 24.900 | 134.392 | 0.000 | 23.000 |
| 1247 | 24.920 | 134.289 | 0.000 | 23.000 |
| 1248 | 24.940 | 134.186 | 0.000 | 23.000 |
| 1249 | 24.960 | 134.083 | 0.000 | 23.000 |

1250 24.980 133.980 0.000 23.000

1251 25.000 133.878 0.000 23.000

# Appendix F

# Parameter Estimation For Simplified Oxygen Driven Compartment Fire Model

```
S-PLUS : Copyright 1988, 1994 MathSoft, Inc.
S : Copyright AT&T.
Version 3.2 Release 1 for MS Windows 3.1 : 1994
Working data will be in _Data

> EPGAMMA
[1] 0.840 0.845 0.850 0.855 0.860 0.865 0.870 0.875 0.880

> EPRESULT
function(x)
{
for(r in (1:(length(x)))) {
y <- c(x[r], EPSUM(x[r]))
```

```
cat(y, "\n")
}
}
> EPSUM
function(GAM)
{
calc <- EP4(GAM)
minsum <- ((48.5 - calc[11, 2])^2 + (200.2 - calc[34, 2])^2 + (250 - calc[41, 2])^2
+ (345.3 - calc[51, 2])^2 + (601.5 - calc[58, 2])^2 + (743.4 - calc[62, 2])^2
+ (848.1 - calc[71, 2])^2 + (900.7 - calc[82,  2])^2 + (943.3 - calc[101, 2])^2
+ (981.8 - calc[141, 2])^2)
minsum
}

> EP4
function(GAM)
{
RMAX <- 7991
BMAX <- 96250 #FPAR$BMAX
RDECAY <- 0.8 #FPAR$RDECAY
mu <- 0.5 #FPAR$mu
time <- 25 #tmax FPAR$time
dt <- 0.1
R0 <- 8.38
x0 <- 0.2299
x1 <- 0.126
GT0 <- 20.02
n <- round(time/dt, 0)
```

```
tt <- rep(0, n)
GT <- rep(0, n)
R <- rep(0, n)
B <- rep(0, n)
x <- rep(0, n)
GT[1] <- GT0
R[1] <- R0
B[1] <- 0
x[1] <- x0
tt[1] <- 0
for(r in (1:(n - 1))) {
tt[r + 1] <- tt[r] + dt
GT[r + 1] <- GT[r] + ((0.1239 * GAM) * R[r] * dt) - (GAM * GT[r] * dt)
B[r + 1] <- B[r] + R[r] * dt
if(B[r] < BMAX) {
    R[r + 1] <- R[r] + (14.8164 - 0) * max(0, (x[r] - x1)) * R[r] * dt
    x[r + 1] <- x[r] + ((-0.0000184 * 1) * R[r] * dt) + (0.9399 * (x0 - x[r])dt)
}
else {
R[r + 1] <- R[r] - RDECAY * R[r] * dt
x[r + 1] <- x[r] - {mu * (x[r] - 0.23) * dt}
}
}
pred <- cbind(tt, GT)
pred
}

> EPRESULT(0.4)
```

```
0.4 271985.16012209

> EPGAMMA<-seq(0.1,0.9,0.1)

> EPRESULT(EPGAMMA)
0.1 1807086.85716672
0.2 898158.656669525
0.3 483682.646541452
0.4 271985.16012209
0.5 159185.734563122
0.6 99613.7102306882
0.7 70377.7139765986
0.8 59028.9260236807
0.9 58414.9094940273

> EPGAMMA<-seq(1,1.9,0.1)

> EPRESULT(EPGAMMA)
1 64261.4879879102
1.1 73940.3347811421
1.2 85801.9505316803
1.3 98797.2756379585
1.4 112254.766612557
1.5 125744.930343411
1.6 138995.857987799
1.7 151839.408233694
Dumped
```

```
> EPGAMMA<-seq(0.8,1,0.02)

> EPRESULT(EPGAMMA)
0.8 59028.9260236807
0.82 58215.4160004428
0.84 57782.1155860546
0.86 57692.4737846328
0.88 57913.4011219492
0.9 58414.9094940273
0.92 59169.7939643521
0.94 60153.3510461516
0.96 61343.1287965174
Dumped

> EPGAMMA<-seq(0.84,0.88,0.005)

> EPRESULT(EPGAMMA)
0.84 57782.1155860546
0.845 57728.8359263009
0.85 57696.482140381
0.855 57684.5314347369
0.86 57692.4737846327
0.865 57719.8115890781
0.87 57766.0593363037
0.875 57830.7432794264
0.88 57913.4011219492
```

# Appendix G

# Two Variable Model for Comartment Fires

### EPCURVE

```
#EP3
> EP3
function(FPAR, tmax, dt)
{
RMAX <- 7991
BMAX <- FPAR$BMAX
RDECAY <- FPAR$RDECAY
mu <- FPAR$mu
time <- tmax # FPAR$time
R0 <- 8.38
x0 <- 0.2299
x1 <- 0.126
GT0 <- 20.02
n <- round(time/dt, 0)
```

```
tt <- rep(0, n)
GT <- rep(0, n)
R <- rep(0, n)
B <- rep(0, n)
x <- rep(0, n)
GT[1] <- GT0
R[1] <- R0
B[1] <- 0
x[1] <- x0
tt[1] <- 0
for(r in (1:(n - 1))) {
tt[r + 1] <- tt[r] + dt
GT[r + 1] <- GT[r] + ((0.1239 * 0.855) * R[r] * dt ) - (0.855 * GT[r] * dt)
B[r + 1] <- B[r] + R[r] * dt
if(B[r] < BMAX) {
    R[r + 1] <- R[r] + (14.8164) * max(0, (x[r] - x1)) * R[r] * dt
    x[r + 1] <- x[r] + ((-0.0000184 * 1) * R[r] * dt) + (0.9399 * (x0 - x[r])*dt)
}
else {
R[r + 1] <- R[r] - RDECAY * R[r] * dt
x[r + 1] <- x[r] - {mu * (x[r] - 0.23) * dt}
}
}
O2 <- 100 * x
w <- round(dt/0.02, 0) * (1:n)
graphics.off() #par(mfrow = c(2, 2))
y <- round(1/dt, 0)
GT.ts <- ts(GT, start = dt, frequency = y)
```

```
R.ts <- ts(R, start = dt, frequency = y)
O2.ts <- ts(O2, start = dt, frequency = y)
NGT.ts <- ts(FIRE2.list$GT[w], start = dt, frequency = y)
NR.ts <- ts(FIRE2.list$R[w], start = dt, frequency = y)
NO2.ts <- ts(FIRE2.list$O2[w], start = dt, frequency = y)
#postscript(file = "B:GT.ps")
win.graph()
tsplot(GT.ts, NGT.ts, type = "pl", lty = c(1, 1))
legend(9, 60, "NRCC", lty = 1)
title(xlab = "Time t (min)", ylab = "Gas temp T (deg C.)")
win.graph() #postscript(file = "B:R.ps")
tsplot(R.ts, NR.ts, type = "pl", lty = c(1, 1))
legend(8, 600, "NRCC", lty = 1)
title(xlab = "Time t (min)", ylab = "Burning rate R (kg/min)")
win.graph() #postscript(file = "B:O2.ps")
tsplot(O2.ts, NO2.ts, type = "pl", lty = c(1, 1))
legend(9, 25, "NRCC", lty = 1)
title(xlab = "Time t (min)", ylab = "Oxygen (%)")
}
```

**POS**

```
function(x, alpha)
{
#POS: smoothed version of max(0,x)
n <- length(x)
y <- rep(0, n)
for(r in 1:n) {
v <- x[r]
```

```
if(v > 0)
u <- v * (1 - 1/(1 + v)^alpha)
else u <- 0
y[r] <- u
}
y
}
```

# Vita

...

Permanent Address: Department of Communication and Infomatics
Victoria University of Technology
Footscray
P. O. Box 14428,
MCMC
Melbourne,
Victoria 8001
Australia

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[1] LaTeX $2_\varepsilon$ is an extension of LaTeX. LaTeX is a collection of macros for TeX. TeX is a trademark of the American Mathematical Society. The macros used in formatting this dissertation were written by Dinesh Das, Department of Computer Sciences, The University of Texas at Austin.