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A NEW TERTIARY MATHEMATICS SUBJECT

To Sofo

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(5 TEACH 2)

October 1990

TECHNICAL REPORT

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This paper is the text of a presentation made at the joint conference of the Mathematics Education Research Group of Australia and the Australasian Association of Mathematics Teachers.

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ABSTRACT

Many issues that one needs to come to grips with are explored in relation to the delightful and absorbing task of designing a mathematics subject from scratch. The subject was developed for incoming tertiary students with a variety of mathematical backgrounds. Some solutions are suggested and a plan is outlined for the implementation of the subject.

Although the mathematics subject was designed for a specific course, it is suggested that it may have implications for any introductory tertiary course containing a mathematics subject.

The aim of the subject is first and foremost to foster interest and confidence in mathematics for incoming tertiary students, then either to lay a solid foundation for future mathematical study or for it to remain a worthwhile terminal subject.

INTRODUCTION

During the 1960s, 1970s and 1980s education emerged as a major public issue at both the state and national levels. It seemed to have become a major news item to an extent not previously known: students, parents, teachers, employers and politicians joined together in highlighting the many defects of the education system. Today, education must aim to encompass not only the rectification of these defects but also the rapidly accumulating bodies of knowledge in widely diverse and largely incommensurable fields. Education must seek to identify and highlight the broader and more subtle complexities of the world in which we live - the warp and weft of the social fabric which define the relationship of the various parts to the whole and thereby give the fabric coherence and meaning. Probably the most essential thing which education should provide, with regard to a rapidly changing society, is the basis for understanding change, a willingness to anticipate change in the future and a confidence in being able to direct it, to participate actively in the process and not merely be subject to it.

Grubb (1987) argues that one real inadequacy of education lies in its failure to teach students to be flexible and adaptable to change.

Mathematics classes and syllabi must be sensitive to changes otherwise we may find that a programme loses its importance and viability, hence its very existence.

> *"As she said this, she looked down at her hands, and was surprised to see that she had put on one of the rabbit's little white kid gloves while she was talking. 'How can I have done that?' she thought. 'I must be growing small again'. She got up and went to the table to measure herself by it, and found that, as nearly as she could guess, she was now about two feet high, and was going on shrinking rapidly: she soon found out that the cause of this was the fan she was holding, and she dropped it hastily, just in time to save herself from shrinking away altogether. 'That was a narrow escape!" said Alice, a good deal frightened at the sudden change, but very glad to find herself still in existence."*
>
> (Carroll, 1975)

In this paper I would like to describe the mathematics programme being developed at Footscray Institute.

BACKGROUND

During 1988 the Victorian Post Secondary Education Commission instigated the idea of possibly offering some sort of Generalist Diploma at a number of Tertiary Institutions. It soon became apparent that the renamed Diploma of Tertiary Studies (D.O.T.S.) should offer studies in the areas of business, science and technology and arts/social sciences. So that what we were offering was some general academic training with a specialized industrial experience component. The Victorian Post Secondary Education Commission was clear in its intention for the academic component of the D.O.T.S. course, however the Commission was initially ambiguous in its direction for the industrial experience component.

> *"'Would you tell me please, which way I ought to go from here?'*
> *'That depends a good deal on where you want to get to,' said the Cat.*
> *'I don't much care where,' said Alice.*
> *'Then it doesn't matter which way you go,' said the Cat."*

One particularly pleasing aspect of the course is its stated intention to encourage participation by those groups of students currently under-represented in tertiary education and to increase access to specific existing tertiary education courses. Hence the government's objectives related to participation and equity could, at least, be partly met. Just as important was the fact that this course was placed at tertiary institutions willing and capable of using and taking advantage of this participation and equity programme.

The aims of the D.O.T.S. course at Footscray Institute are to provide:

- a broad general preparation, and to help students acquire a wide range of knowledge and skills,
- employable diplomats with a broadly based general preparation enabling students to study a number of subject areas with an emphasis on breadth rather than depth, and
- opportunities for articulation with credit and/or advanced standing in other more vocationally specific programmes.

The course is of three years duration comprising two years of coursework and one year of structured and supervised work experience. The first year of the course is common for all students, and subjects have been specifically designed. The subjects are broadly based and cover written and oral communication, Australian studies, mathematics, physical science, business studies and computing. It has been a specific aim to provide a broad general education, and in this respect we agree with Jones (1982)... "Australia's future will depend as much on its politicians, writers, artists and humanities teachers as on its engineers and chemists. Our primary emphasis in education ought still to be on the general rather than the specific and vocational."

This course then may satisfy, in part, the Government's desire that more and more young people, (and perhaps mature students as well), take up the challenge of tertiary studies.
Its possible that some students could even succeed in their tertiary studies especially if one can make some progress in trying to overcome some of the problems of retention in higher education as noted by Ramsay and Clark (1990).

M.A.N.I.C.

The Diploma of Tertiary Studies requires that all students take 112 hours of M.A.N.I.C. - "Mathematics - A New Introductory Course", in their first year.

At the outset we probably need to ask ourselves 'why teach mathematics at all?' The answer is best given by Cockcroft (1981). In his report Cockcroft tells us that mathematics is useful for everyday life, for science, for commerce and for industry, because it provides a powerful, concise and unambiguous means of communication and because it provides means to explain and predict. It attains its power through its symbols, which have their own 'grammar' and 'syntax'. Moreover the report claims mathematics has aesthetic appeal and it develops logical thinking.

It is possible that the given reasons on why we teach mathematics may strike a harmonious chord with those of us involved in mathematical research or mathematical education, however I have an uneasy feeling that a great majority of these incoming tertiary students see mathematics as a necessary evil forced on them by the compulsory requirement of the course.

The D.O.T.S. course does not require year twelve mathematics as a prerequisite subject, in fact some students may only have mathematics to year 10 level, and some mature age students may have done some mathematics many many years ago and have certainly forgotten much of what they did. The great majority of school leavers have a bare pass in whatever mathematics they may have attempted, and mathematics does not figure prominently in their overall scheme of things. Indeed some students display an anxiety towards mathematics similar to that described by Munro (1985). We have this broad spectrum of student competence, ability and interest that one needs to cover. M.A.N.I.C. needs to be a suitable terminal course or indeed a suitable solid foundation in which future mathematical frameworks may be built.

> *"The Mock Turtle said with a sigh, 'I only took the regular course.'*
> *'What was that?' enquired Alice.*
> *'Reeling and Writhing, of course, to begin with,' the Mock Turtle replied; and then with different branches of Arithmetic - Ambition, Distraction, Uglification and Derision.'"*

Students come into tertiary colleges with varying points of view of what mathematics entails. Some of them think of it as nothing more than a set of rules that sometimes work and sometimes don't, and are never quite sure when they work and when they don't.

Others think of it as something with a strange language that doesn't always make sense: 'Transpose, differentiate, invert the divisor and multiply, integrate, crossmultiply, cancel, simplify, least upper bound, an independent variable, a feasible region, a null sequence, a uniform random variable, ' etcetera.

I guess that all teachers have experiences on the lack of students' understanding, I will mention just two such cases:

Once when asked to simplify $\frac{\tan x}{\sin x}$ a student obtained the result $\frac{ta}{si}$

and in another case when asked to differentiate the function $f(x) = x^2$, the student asked with determination "What does differentiate mean?"

They sometimes have little idea of how mathematics might be useful to them and certainly pure mathematics for its own sake would be remote from the minds of a great majority of students and probably some teachers. As Bertrand Russell once said when speaking about pure mathematics and mathematical theorems, "The theorems themselves are abstractions that belong in another realm remote from human passions - remote even from the pitiful facts of nature!"

ISSUES

How do we cover this broad spectrum of student competence, ability and interest? Is it possible that we may make mathematics appeal aesthetically in a way similar to music or art? Our response to music or art is influenced by personal differences, we cannot expect everyone to enjoy the same sort of music, and similarly we cannot expect everyone to enjoy the same sort of mathematics. It appears that a basis to our enjoyment of music or art and hopefully mathematics is our response to pattern. Consider the following: Take any two positive integers, say 8 and 5, permute them in pairs and add them, $58 + 85 = 143$, divide the result by the sum of the two integers, and the result is always 11.

Any three positive integers arranged in pairs, with the same operations as above will always result in 22. Any four positive integers will result in 33 etcetera. We would probably expect most students to rebel against tedious boredom in long winded calculations and hopefully search for an elegant and general pattern. Can we generalise the pattern further? Do we get 222 and 666 when we permute three or four positive integers in triplets?

As G.H. Hardy wrote, 'A mathematician, like a painter or a poet is a master of pattern'.

Is it possible that some students may enjoy mathematics because it is useful? This may be a justification for spending a great deal of time on mathematics, however one should not neglect its appeal based on intellectual or aesthetic responses similar to those of art or music.

The aesthetic appeal of mathematics is, in my opinion, in the eyes of the beholder. Aesthetic judgments may be transitory and may change from age to age. I doubt very much that the golden ratio, for example, will send a shiver down one's spine as it may have done in a past era. But it may be the case that an elementary excursion into chaos theory may help to ignite a passion not previously thought possible. Davis and Hersh (1987) put it beautifully:

> *'Blindness to the aesthetic element in mathematics is widespread and can account for a feeling that mathematics is dry as dust, as exciting as a telephone book, as remote as the laws of infangthief of fifteenth century Scotland. Contrariwise, appreciation of this element makes the subject live in a wonderful manner and burn as no other creation of the human mind seems to do.'*

A minor revolution occurred in mathematics teaching some twenty or thirty years ago, with the introduction of the 'new' mathematics. It has been stated that some use this new mathematics in teaching 'concepts' only, so that when we teach only for understanding, we get demands to teach calculating competence. When we teach only for calculating competence we get demands to teach understanding. My contention is that we need to do both, it's through both that we may be in a position to solve 'real' problems.

In the circumstance where students may come into tertiary life with little and/or shaky mathematical backgrounds, then it's probably not wise to offer them, in the first instance, a 'regular course' of bridging or remedial mathematics. One would need to build up their mathematical confidence and interest before any significant advancement is to be achieved. It would serve little purpose to suggest a mathematical text and work through it in some sort of structured order. We do not want a particular mathematics textbook to have a profound influence on the content of our course. However, consideration to the implementation of manageable class sizes we feel, is of crucial importance. Another crucially important issue is a need for the most senior and capable teachers available to instil a sense of worth and confidence in some of these mathematically maladapted students.

What, if any, learning theories should we place to the fore? According to Piaget learning is distinct from cognitive development, it takes place in relation to the relevant stages of cognitive development, but it is achieved through interaction with the environment. Skemp on the other hand, has proposed a theory of learning that takes into account the important question of goals and motivation. Learning, claims Skemp, is a goal directed change of state of a director system towards states which make for possible optimal functioning. Bruner, as a supreme optimist, suggests that any idea or body of knowledge can be presented in a form simple enough so that any particular learner can understand it in recognisable form. Learning, claims Bruner, consists essentially of concept formation, which is the multiple embodiment of an abstract idea in different physical forms. And finally, Dienes suggests that learning is a process of intricate play.

While one should be familiar with learning theories and perhaps be aware of their implications in the context of the classroom, one should not consider them as a panacea designed to cure all problems. One should be flexible and adaptable to change. One should also keep in mind the predicament of the Earl of Rochester, "Before I married I had six theories about bringing up children. Now I have six children and no theories."

The related issue of teaching mathematics is also very important and on this score Pateman (1989) argues in his beautifully written book, that it is not possible to engage in teaching mathematics effectively without some clear notion of the nature of mathematics and how that nature inevitably influences the art of teaching.

Taking these, and other issues into consideration we were, in short, keen to put into practice many notions about the 'ideal mathematics course' that we had formed over our years of experience in teaching the subject at post secondary level.

SOLUTION (STRUCTURE)

The structure of M.A.N.I.C.consists of two distinct parts, divided for ease, into semesters one and two.

Semester one is entirely motivational and because of the varying nature of students' mathematical ability and feelings, we try to make them feel confident and interested in the subject. We feel at this point that remediation or bridging would not serve that purpose. We try to convey the message that mathematics is useful and can sometimes be fun and may be applicable to the real world. One may then find that students can understand and do some mathematics.

The menu for the first semester mathematics course consists of three two week 'appetisers' followed by an intensive six week main 'option' and finished off with a student class presentation in the form of a talk. The content of the first semester may be structurally independent of whatever mathematics follows and therefore lecturers have the freedom to offer an appetiser which may be enlightening, captivating, interesting, non-threatening, historical or experimental. We hope that this may spark some mathematical enthusiasm in the students and allow the lecturer to become enthused in his own particular interest.

This year's appetisers have included Number Systems, Boolean logic, Greek and Egyptian contributions to the growth of mathematics and Pythagoras and the monochord. There is no shortage of material for the inclusion of stimulating appetisers and Marnell's (1987) book is an excellent reference text for ideas.

In the main option the lecturer again has the freedom to offer something 'different'. A topic is built up from scratch, but is not the usual excursion into quadratic equations or first order linear differential equations. Each lecturer would offer something different and the students would have a choice of which main option to attend, subject to manageable size groups.

One main option that I would consider offering is a 'real world' problem. A large paper manufacturing company requires a six metre diameter stainless steel sphere for its paper pulping plant. What mathematics do we need in the construction of this large sphere? Given that we require 30 mm thick stainless steel plates, manufactured in predetermined sizes by a large steel manufacturing company, this would allow one to discuss issues related to minimum trim loss of plates. Also we would have problems on projective geometry, two and three dimensional trigonometry, area of regular and irregular shapes, arc length and volume. The scope for investigating this type of problem is enormous - it would allow one to deal with certain aspects in detail while treating other aspects of this problem with less formal importance. Alternatively, perhaps, student generated interest would focus on a particular aspect of construction and that could be investigated fully. Hopefully, a class generated by students' interests rather than one generated by a dry syllabus, may be more stimulating for both lecturer and students alike.

The last part of the first semester entails the delivery of a 5 to 10 minute mathematical related talk by each student. We feel that students must be able to present mathematical results or discuss mathematical ideas with some confidence and be subjected to class questioning. The topics for presentation are not of vital importance, may be wide ranging and can be chosen by the student or suggested by the lecturer. The first semester may be further spiced with relevant films and guest spot lecturers with the unique talent to stimulate mathematical interest.

Assessment is by periodic class tests spread throughout the semester. The first semester is not about failing students, it is about encouraging students to participate in mathematics and stimulating interest so that they may develop a confident outlook towards mathematics. We make it difficult for students to fail, however, should they do so, they may redeem themselves in semester two.

Semester two is based on self paced units organised along the lines of the Keller plan. Students work through these units on their own and in their own time, however a number of lecturers are available for consultation at specific times. Students may work through these units during class times or indeed take units home to complete. Each unit comprises about four to eight hours work and when the plan is complete, there will be over 80 units to choose from. Some units fall naturally into sequences and students may choose which units to attempt subject to advice from lecturers, and to obvious logical orderings between some units. Progress through the units is based on 'mastery learning' at the 80% level and a final result is determined by the number of units completed.

This structure allows for very wide differences in entry level and units will be available that allow students to go right back to basics. Moreover, this structure allows for a wide variety of interests and goals, and certain selections of units are marked as being suitable for students wishing to proceed to further study in the areas of science, statistics, engineering, computing or business.

Currently our task is to design and produce suitable units. We hope to produce units which capture the spirit of the first semester, that is, units which may be stimulating and at the same time deal with mundane topics such as quadratic functions or basic calculus.

The use of computers in this structure is important and efforts are being made to acquire suitable hardware and software. There is no doubt that the impact on students in the use of computers in mathematics teaching has been profound. Research has indicated that beyond the understandable enthusiasm that students have shown towards computers, there are increased benefits for the students in the development of mathematical intuition and a significant rate of assimilation.

CONCLUSION

In its first year of operation, M.A.N.I.C. has proved to be successful with the majority of both lecturers and students. Of the seventy or so students that finished the course, only two were not successful, and their mortal wounds were, in the main, self inflicted.

The structure of M.A.N.I.C. is obviously extremely flexible - parts of it may be augmented or diminished. For example the first part, the motivational content, may be shortened to some suitable fraction of a semester. Units may be amended, updated, added to or deleted. Appraisal of units can be monitored very closely and accordingly advice by lecturers can be given accurately. Eventually the available units will cover the whole of any reasonable first-year mathematics course, and the fact that units will be logically related will greatly facilitate the design of new courses and the modification of old ones.

Because M.A.N.I.C. has no prerequisite mathematics requirement, it has afforded opportunities to students who otherwise would not normally have continued with scientific studies to pursue further their academic wishes. Moreover M.A.N.I.C . is intentionally designed to cater for the increasing nonuniformity of different school backgrounds, and an increase in school-based curriculum and assessment. It is our belief that M.A.N.I.C. will meet the challenge of change and is flexible enough to cater for it.

REFERENCES

Carroll, L. (1975) 'The Annotated Alice' Penguin books.

Cockcroft, Report (1981) Mathematics Counts, HMSO

Davis, P and Hersh, R (1987) 'The Mathematical Experience' Birkhauser Boston

Grubb, W.N. (1987) 'Responding to the constancy of change: New technologies and future demands on U.S. education' in Burke, G. and Rumberger, R.W. (eds). 'The Future Impact of Technology on Work and Education' Lewes: The Falmer Press, pp 118-140.

Jones, B. (1982) 'Sleepers, Wake! Technology and the Future of Work'.

Marnell, G. (1987) 'Mind Stretchers' Angus and Robertson Publishers.

Munroe, J. (1985) 'Mathematics Anxiety: Its nature and identification. Why mathematics Anxiety? Practical ways of reducing mathematics anxiety. Success No. 2.'

Pateman, N. (1989) 'Teaching Mathematics - a Tantalising Enterprise' Deakin University Press.

Ramsay, W. and Clark, E. (1990) 'Problems of Retention in higher education' Preprint.

ACKNOWLEDGEMENTS

We greatly appreciate a grant from the Victorian State Government.
To H. Mays and M. Baker. The seeds we have sown have blossomed into a healthy and lively project.