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Venkatesan Gopalachary

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VICTORIA UNIVERSITY OF TECHNOLOGY
P O BOX 14428
MELBOURNE CITY MC VIC 8001
AUSTRALIA

TELEPHONE (03) 9688 4492
FACSIMILE (03) 9688 4050

Footscray Campus

MODEL OF PROCESS CONTROL SYSTEM APPLIED TO PRODUCT QUALITY

VENKATESAN GOPALACHARY

Department of Computer and Mathematical Sciences, Victoria University of Technology, PO BOX 14428MCMC, Melbourne 8001, Australia

ABSTRACT

This paper is aimed at finding a solution to the commonly occurring product quality control problem by suitably modelling a process control system. The techniques from engineering and statistical process control overlap at the interface of the two process control methodologies. Problems connected with feedback (closed-loop) stability, controller limitations and dead-time compensation to obtain minimum variance (mean square) control at the output are encountered while applying statistical process monitoring and feedback control adjustment. The focus in this paper is to model a control system by application of both the process control techniques.

1 INTRODUCTION

In recent years, statisticians and control engineers have focused their attention on bringing the statistical process control (SPC) and the automatic process control (APC) methodologies closer together. Automatic process control techniques have been used to control process variables such as feed rate, temperature, pressure, viscosity and to product quality variables as well. Conventional practices of engineering control use the potential for step changes to justify an integral term in the controller algorithm to give (long-run) compensation for a shift in the mean of a product quality variable. Application of techniques from the fields of time series analysis and stochastic control to tackle product quality problems is also common. The literature on stochastic-dynamic process control is replete with contributions from (statistical) process control specialists, for example Box [1957], Box and Jenkins [1962, 1963, 1965, 1968, 1970, 1976], Astrom [1970], Box and MacGregor [1974], MacGregor [1987, 1988], Harris [1989], Harris and MacGregor [1987], Harris and MacGregor and Wright [1982]. This work covers topics that relate to the analysis of closed-loop dynamic-stochastic systems, assessment of control loop performance, on-line process control, discrete stochastic and linear quadratic controllers etc. These contributions, along with the work done independently by control engineers in automatic process control, focus only on particular aspects of process control. This paper is in the direction of modelling a process control system applied to product quality.

2 STOCHASTIC DISTURBANCE MODEL AND FEEDBACK CONTROL DIFFERENCE EQUATION

APC aims to maintain certain key process variables as near their set points (targets) for as much of the time as possible in order to satisfy certain production objectives. One of the production objectives is to produce material of desired quality by having an acceptable level of variation (product variability) in the measured output characteristics. *Disturbance (noise) causes variability in the output or outputs of an otherwise stable process by producing undesirable changes in the (output) mean.* Disturbances cause a process to wander and drift off target resulting in shifts in the mean of the output product quality away from target. The error, the difference between the output and target values, is used to determine a process adjustment. If no compensatory adjustments are made and no (feedback) control actions are taken, the output follows the course of the disturbance. The effect of an input adjustment (control action) is delayed in its effect due to dead time, (time taken to deliver material from the point of adjustment to the sample point), in the process. The presence of dead time in the process requires that forecasts of the output deviation (error) are made over the delay period.

An ARIMA (Autoregressive integrated moving average) model is used to forecast the behaviour of the time series describing the disturbance. The integrated moving average model has the property that the forecasts for all future time is an exponentially weighted moving average (EWMA) of current and past values of the disturbance. The EWMA provides the forecasts over the dead time period (time delay). Moreover, the time series controller gives (b+1) periods ahead forecast error variances over the time delay (dead time) period in a process.

The ARIMA (0,1,1) stochastic time series model characterising the drifting behaviour of the process disturbance is given by

$$z_t = z_{t-1} + a_t - \Theta a_{t-1}, \quad (1)$$

where 'z' is the *stochastic variable* and 'a_t', the *random variable*. *Z'* represents the stochastic disturbance and $\{a_t\}$ represents the sequence of *random variables*.

Z_t, is the output of a (linear) process control system, when subjected to a sequence of uncorrelated random shocks $\{a_t\}$, where a_t follows a Normal distribution with mean 0 and standard deviation σ_a, represented by $a_t \sim N(0,\sigma_a^2)$. Θ is the integrated moving average (IMA) parameter.

Figure 1 describes the relationship between the output (Y_t) and the input (X_{t+}) in the feedback control model.

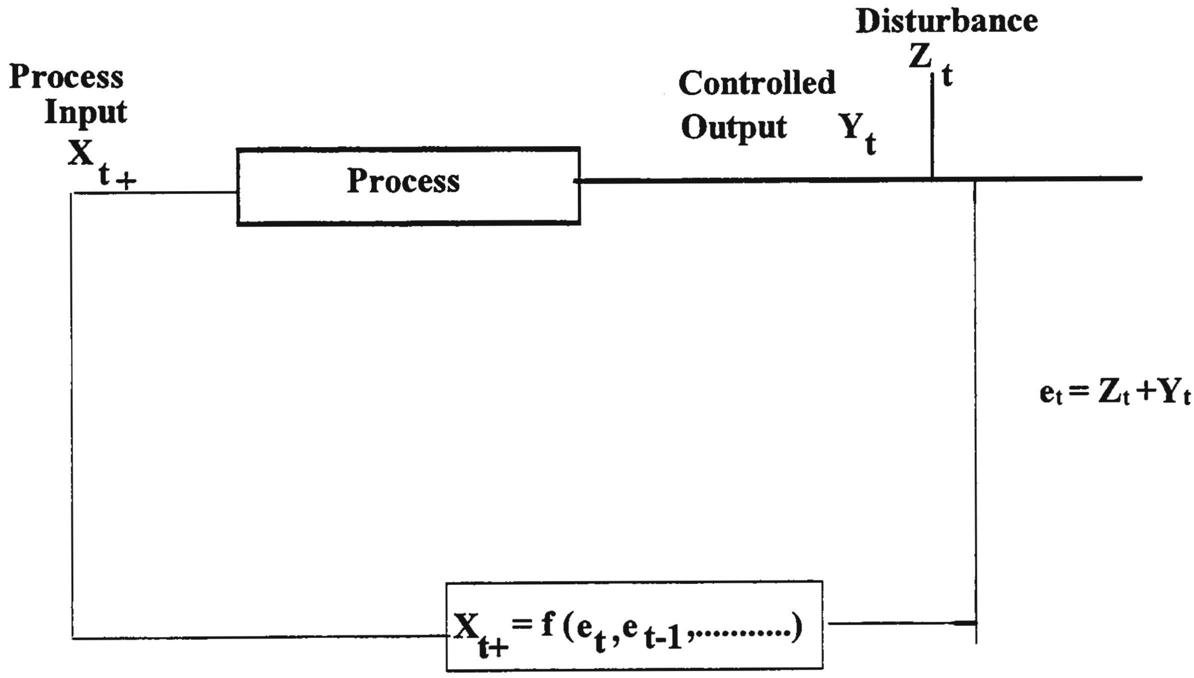

Figure 1 Block Diagram for the Feedback Control Model

The 'stochastic difference equation' for the feedback control model can be (approximately) represented by the following second-order dynamic model (transfer function) of the form:-

$$Y_t (1 - \delta_1 B - \delta_2 B^2) = \omega B^{b+1} X_t, \quad (2)$$

where

X_{t+}, the input put manipulative variable, is a linear function of e_t, the forecast error and of integral over time of past errors,

Y_t is the output or controlled variable,

δ_1 and δ_2 are the parameters that represent the dynamics (inertial) characteristics of the system, B is the backward shift operator; $BX_t = X_{t-1}$, $B^bX_t = X_{t-b}$.

ω is the magnitude of the process response to a unit step change in the first period following the dead time carrying over into additional sample periods.

b is the process dead time, b > -1. PG represents the process gain realised by total effect in output caused by a unit change in the input variable after the completion of the dynamic response.

e_t is the forecast error in Figure 1 and $e_t = Z_t + Y_t$.

It can be shown that (i) $PG = 1/(1-\delta_1-\delta_2) = g$, the steady-state gain for a critically damped second-order dynamic system when the conditions given below are satisfied and (ii) $\omega = PG(1-\delta_1-\delta_2) = 1$ for a critically damped system.

Equation (2) describes the critically damped behaviour of the second-order dynamic system for which the time constants are real and equal. The following inequality conditions are imposed for closed-loop stability.

$$\delta_1+\delta_2<1,$$

$$\delta_2-\delta_1<1,$$

$$-1<\delta_2<1,$$

$$\delta_1^2+4\delta_2 = 0.$$

3 EXPRESSION FOR FEEDBACK CONTROL ADJUSTMENT

From Equations (1) and (2) for the stochastic and dynamic models, an expression for feedback control adjustment is developed which minimizes the variance of the output controlled variable by making an input control adjustment at every sample point that exactly compensates for the forecasted disturbance. Figure 2 shows a feedback control scheme to compensate disturbance in a second-order dynamic model with delay (dead time). Refer to Venkatesan [1995] for derivation of the feedback control algorithm.

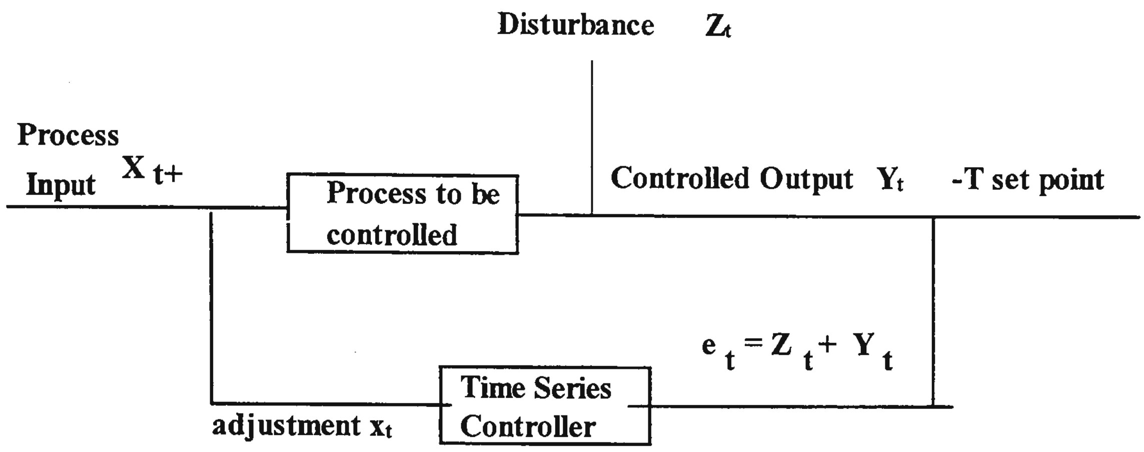

Figure 2 Feedback Control scheme to compensate disturbance Z_t

$$x_t = -\frac{(e_t - \delta_1 e_{t-1} - \delta_2 e_{t-2})(1-\Theta)}{PG(1-\delta_1-\delta_2)} - (1-\Theta)x_{t-b}. \qquad (3)$$

This feedback control algorithm gives information about when to make an adjustment and by how much. The control adjustment given by Equation (3) minimises the variance of the output controlled variable even in the face of dead time (time delay) and process dynamics (inertia).

The first term in the above stochastic feedback control algorithm represents the integral term and the second term, the dead-time compensator developed by Smith [1959] (Baxley [1991]). According to Palmor and Shinnar [1979], the Smith predictor is a direct result of minimal variance strategy and that minimal variance control for processes having dead times includes this type of dead-time compensation. At this stage, an intuitive conjecture is made that the inclusion of the dead-time compensation term of either the Smith predictor or the Dahlin's (on-line) tuning parameter, (whose values range from 0 to 1), in a feedback control algorithm will also result in a minimum variance strategy for processes with dead time.

This draws on the comparison made by Harris, MacGregor and Wright [1982] to the minimum variance controller they derived for the process with dead time (for which the number of whole periods of delay was equal to 2) and the Dahlin controller (Harris, MacGregor and Wright [1982]) given in their paper. The authors showed that the two controllers were identical upon setting the value of Dahlin's parameter, (the discrete time constant of the closed-loop process), equal to Θ, the IMA parameter in the stochastic disturbance model. They reconciled the different approaches by noting that the 'IMA parameter Θ provides information about the magnitude of the closed-loop time constant'.

Equation (3) is identical to Baxley's [1991] algorithm for a first-order model with dead time and includes both integral action and dead-time compensation terms, when $\delta_2 = 0$ and $\delta_1 = \delta$ and $b = 1$. The control obtained through the first term in Equation (3) is the discrete analogue of integral control. The IMA parameter Θ, (whose values range from 0 to 1), is set to match the disturbance as well as take care of Dahlin's parameter to compensate for the dead time. It is used as an on-line tuning parameter for dead-time compensation. So, it follows that *the variance of the output product variable achieved by using Equation (3) with integral action and dead-time compensation terms is a minimum. The dead-time compensation term (seemingly) removes the delay from stability considerations and definitely provides a stabilising effect on the feedback control system. These principles are used for designing (formulating) the discrete (sampled-data) time series controller.* Such a controller will maintain the mean of the process quality variable at or near target and will allow for a (rapid) response to process disturbances without much overcompensation or overcorrection.

4 SIMULATION AND ANALYSIS

The stochastic feedback control algorithm is simulated to find the time series controller performance measures, namely, CESTDDVN (control error standard deviation) or control error sigma (product variability) and the mean frequency of adjustment (MFREQ) of the time series controller. An advantage of dead-time simulation is that the inter-sample variances are compared at the sampling points. Table 1 gives the control error sigma (CESTDDVN) and the mean frequency of adjustment (MFREQ).

Table 1 Time series Controller Performance Measures

IMA Parameter Θ	System Dynamics δ_1	δ_2	Mean Frequency of Adjustment(MFREQ)	Adjust. Interval AI=1/MFREQ	Control error sigma CESTDDVN
0.05	-1.82	-0.83	0.000	0.00	1.000
0.05	-1.00	-0.25	0.102	9.80	1.039
0.25	-1.00	-0.25	0.050	20.0	1.030
0.25	-0.27	-0.02	0.075	13.3	1.073
0.75	-0.27	-0.02	0.000	0.00	1.000
0.75	-1.00	-0.25	0.000	0.00	1.000
0.90	-1.00	-0.25	0.402	2.48	1.002
0.90	-1.82	-0.83	0.121	8.26	1.000
0.95	-0.27	-0.02	0.213	4.85	1.006
0.95	-0.01	0.00	0.187	5.34	1.004

The above table shows a range of minimal control error sigmas (CESTDDVN) for values of Θ and the dynamic (inertial) parameters δ_1, δ_2. The values of CESTDDVNs are 1.0, when Θ = 0.75, the EWMA forecasts are effective and the EWMA has good control of the process. *Since the CESTDDVN values, (close to around the value of 1.0), are obtained for the second-order dynamic process with dead time b = 1, it is possible to achieve good (feedback) control possessing features such as (i) Permissible gain of the feedback (closed) loop, (ii) Stability of the feedback control loop and (iii) Precise regulation of loops containing dead time.* The range of control error sigmas (CESTDDVN) for corresponding values of Θ can be used to formulate process regulation schemes. These values of Θ and AI are used to formulate process regulation schemes as shown in the next Section.

A dead-time compensation scheme which provides a process gain (PG) in the feedback path whose value depends on both the process output and model has been devised. This scheme is suited to use in situations where the process dead time results from a measurement device in a laboratory and is a known quantity. A process modelling (control) approach to product quality based on discrete laboratory data has the potential for improvements (in product quality). A practical control strategy would then be (i) based on the use of quality control laboratory analyses and (ii) the process model based on a time series analysis of plant data collected from a designed closed-

loop experiment and using the laboratory data to update the set point of the minimum variance time series controller to verify the quality of outgoing product.

5 PROCESS REGULATION SCHEMES

The choice of feedback control process regulation schemes depends on 'how capable' the 'controlled process' (model) was of providing quality products within manufacturing specifications. If the process capability index was high, then, a moderate increase in the control error deviation (product variability) might be tolerated if this action resulted in savings in sampling and adjustment costs. Table 2 shows the adjustment interval (AI) and the corresponding CESTDDVN for some alternative schemes using combinations of control limits L = 2.98, 3.0 and 3.04. These schemes are denoted by A, B, ...O. These schemes are based on how much the CESTDDVN would need to increase to achieve the advantage of taking samples and making adjustments less frequently. This approach avoids the direct assignment of values to costs C_a (cost of adjustment and sampling) and C_t (cost of being off-target). The table 2 shows for various values of the standardised action limit, L/σ_a = 3.010, 3.000, 3.009 and the adjustment interval (AI), the percent of increase in CESTDDVN (ISTD.) with respect to σ_a and AI. The AI and CESTDDVN values show that for different sets of values of AI and CESTDDVN, the process regulation scheme varies according to the AI values.

Table 2 Alternative Process Regulation Schemes

Control Limits L(= 3 σ_a)											
2.98				3.00				3.04			
Scheme	*AI*	*CESTDDVN*	*ISTD.*	*Scheme*	*AI*	*CESTDDVN.*	*ISTD*	*Scheme*	*AI*	*CESTDDVN*	*ISTD.*
A	10.99	1.022	-	F	10.0	1.033	-	K	20.00	1.025	-
B	10.52	1.032	0.980	G	9.00	1.058	2.42	L	12.20	1.032	0.68
C	8.200	1.043	1.066	H	5.26	1.157	9.36	M	12.35	1.037	0.48
D	5.100	1.127	8.050	I	5.02	1.158	0.09	N	9.52	1.089	5.01
E	5.680	1.186	5.24	J	4.46	1.217	5.09	O	6.94	1.119	2.75
L/σ_a	3.010			3.000					3.009		

The alternative schemes are: (i) Scheme B: To set L = 2.98 and adjust process at 10.5 sample periods, with an increase in CESTDDVN (ISTD.) of 0.98 or
(ii) Scheme E: Adjust process at 5.7 sample periods and ISTD of 5.24 or (iii) Scheme J: by setting L = 3.0 and AI = 4.46 sample periods, the same ISTD. could be achieved with an AI of 4.5.

6 ENGINEERING CONTROL APPLICATION TO PRODUCT QUALITY

To control the quality of a product at the output, the set point of a product-quality controller is adjusted so that the product remains within its specification limits following expected load changes or disturbances. In product quality control, the product-quality set point is adjusted away from the specification limit in proportion to the peak deviation expected to be yielded by the controller. Again, the adjustment is in a direction that increases operating costs. Deviation in the 'safe' direction increases operating costs in proportion to the deviation. The quality-controller's set point is positioned relative to the specification limit so that the limit will not be violated for most upsets (load changes). Since the average output product quality will be equal to the set point, the product will be more expensive to make than if the set point were positioned exactly at the specification limit. Excess manufacturing cost is proportional to the difference between the set point and the specification limit and so, proportional to the peak deviation expected. By limiting the peak deviation, excess manufacturing cost and product quality are controlled in engineering control. Peak deviation of the controlled variable from set point is significant when excessive deviation will cause an incident such as rejecting product due to failure to meet specifications.

Process control provides the operating conditions under which a process will function safely, productively and profitably. Ineffective control can be costly in causing amongst other things such as plant shutdown, in allowing off-specification product to be made, etc. For a particular control loop, it is often possible to relate operating cost to deviation of the output controlled variable. In product-quality control loop, the cost function is usually found to be different on opposite sides of set point.

Process operators frequently place a large margin between the measured quality of a product and its specification. This is done in order to counteract the changes in economic performance when a product specification is violated. It will cost more to produce higher-quality product. Maximum profit can be realized when product quality meets exactly the specifications; but variations in product quality are not equally acceptable on both sides of the specification. So, as a consequence, the quality set point must be positioned far enough without excessive operating costs, on acceptable side of the specification in order to reduce the likelihood of specification violation. The operating cost can be reduced by better control and smaller variation in quality allowing the set point to be moved to closer the specification.

The deviation between the output controlled variable and its set point can be related (linearly) to operating cost. For a specific production rate, each increment of time will correspond to a quantity of product manufactured over that time. So, integration of deviation (error) over time could be equated to accumulated (excess) operating cost. Under such circumstances, the control objective would be to minimize integrated error*. This criterion could be applied to control the quality of a product flowing into a storage tank, for example. This can be achieved by keeping the integrated error as low as possible and the quality of the product closer to the set point. *Integrated error can be estimated from the feedback control equation (3), being equal to $(e_t-\delta_1 e_{t-1}-\delta_2 e_{t-2})$. It is a function of the change required in the input manipulated variable and the setting of the integral mode of the (time series) controller. Integrated error can be significant in product-quality loops, where it may represent excessive operating cost such as product giveaway. Lag-dominant dynamics characterize, (similar to the second-order dynamic model with two exponential terms), most of the important plant loops such as product quality. For these processes, the integral error varies linearly with time.

7 STATISTICAL QUALITY CONTROL APPLICATION TO PRODUCT QUALITY

For the sample values of a product variable whose measurements are normally distributed, its mean will equal the set point if the integral of the error approaches zero over a period of time. In minimizing the deviation of the output controlled variable, the standard deviation is a transformation of that deviation over a statistically significant number of samples or time of operation. The economic incentive behind the standard-deviation criterion is that this criterion estimates the percentage of time the controlled variable violates the specification based on a normal distribution. If samples of an output controlled variable that is 'cycling' uniformly are averaged over a complete cycle to form a subgroup, then the mean of subgroups will lie on the set point and their standard deviation will approach zero (if there are no disturbances in the feedback control system).

The assignment of subgroup size should reflect the capacity of a process to absorb variations in product quality. The method used to average samples also needs to be selected to match the characteristics of the process. If the product is segregated into lots, then, the samples should be segregated into the same lots and averaged equally.

Different criteria can be applied to set point and load disturbances that affect a control loop. Different controller settings will be required to satisfy these criteria. Overshoot of the output controlled variable can be minimized by limiting the rate of the set-point changes that are likely to be introduced by the operator during the course of plant operation and process control.

8 CONCLUSION

In this paper, the objective of applying the engineering and statistical techniques to find a solution to the product quality control problem was explained. Techniques from the two different disciplines at the interface of the two process control methodologies were used to derive a feedback control difference equation and an expression for feedback control adjustment. An analysis of simulated results were given and also some process regulation schemes along with engineering and statistical control applications to product quality.

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