VICTORIA UNIVERSITY OF TECHNOLOGY

DEPARTMENT OF COMPUTER AND MATHEMATICAL SCIENCES

Mean Selection for a Filling Process

with Implications to 'Weights and Measures' Requirements

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Abstract

This paper considers the problem of optimally choosing the mean of a filling process for three model variations. Optimality is defined as that setting which maximises expected profit. Issues considered include waste, overfill, top-up and the additional filling costs of items not initially meeting requirements. Model solutions are displayed graphically. The effect of change of the process variance on the optimal solution as well as on the expected profit are both discussed. Implications to 'Weights and Measures' requirements of following this optimality path are provided with particular reference to loss in expected profit. The algorithms to obtain both the optimal mean and the probability of meeting Weights and Measures Legislation requirements, are also shown.

Key words: Filling Process , Overflow, 'Top-up', Optimal Mean, Weights and Measures requirements.

Mean Selection for a Filling Process with Implications to 'Weights and Measures' Requirements.

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Introduction

This paper considers a problem concerned with the selection of the mean of a filling process, where the focus is on maximising the expected profit. Other authors have addressed a similar problem assuming that containers have infinite capacity. In practice overflow can occur during filling, with or without a loss of material. Such filling processes are standard in both the chemical and food industries.

Early work in the area was presented in [1], [2] and [3]. These all considered related problems; the latter two took a number of economic aspects into account and determined the optimal location of the mean, for minimisation of total cost. The work in [2] was later extended in [4] with the development of a graphical method to select the optimal setting.

Hunter and Kartha, [5] developed a method for determining the optimal target value of an industrial process to maximise the expected profit taking process variability and production costs into account. The problem they considered, which related to a filling process, revolved around the situation where product above a certain dimensional threshold attracts a fixed selling price and product below the threshold attracts a reduced, yet fixed, selling price. In addition, product above the threshold implies 'give-away' which diminishes the net profit per item. Besides successfully formulating the problem, they provided a graphical method of solution. The authors considered the problem under the assumption that once the initial process mean setting is made, no other control actions are subsequently necessary. The assumed conditions in their model do not facilitate provision of an explicit optimal solution, however Nelson, [6] found an appropriate approximating function, which allowed for a close approximation to a solution to be obtained. He also included a plot of errors of this approximation.

A generalisation of this model was presented by Bisgaard et.al, [7] where the authors, again making the connection to a filling problem, developed a procedure for selecting optimal values for the process mean as well as the variance. They eliminated the assumption made in [5], that all undefilled items are sold at a fixed price. They considered a situation where underfilled items are sold for a price that is proportional to the amount of ingredient in the container. A solution for a filling process that is approximately normal was given together with a table that provides the optimal process setting. Situations in which the distribution has a lognormal or Poisson distribution were also discussed. The objective was to maximise the expected profit.

A model also similar to that of [5], was studied by Carlsson, [8]. In this paper the author analysed the choice of the process mean as well as the net expected income taking production costs and selling price into account. The discussion was based on an example from the steel construction industry, where rejected items are either sold at a reduced price or reprocessed. The net income function was represented as a piecewise linear function of the main quality characteristic. The situations where the customer is willing to pay extra for good quality as well as when the producer may have to compensate the customer for bad quality were both discussed.

Gohlar, [9] also addressed the issue of finding the most economic setting of a process mean for a canning problem. He modelled a situation where the overfilled product could only be sold in a regular market and underfilled cans emptied and refilled, with the penalty of an extra cost. The capacity of the container was implicitly assumed infinite.

The canning problem analysed in [9] was later discussed by Schmidt and Pfeifer, [10] who explored the cost reductions achievable through a reduction in the process standard deviation. A linear relationship was found between the percentage reduction in both cost and process standard deviation. Revenue per can was constant, impling that minimisation of expected cost is equivalent to maximisation of expected profit.

Bai and Lee, [11] concentrated on a process where a lower specification limit of the quality characteristic is given and each container is inspected on a surrogate variable correlated with the primary variable. The method of selection of the optimal target values for the mean of the quantity of material in the container as well as the cut-off value on the surrogate variable were presented. The authors assumed that overfilled containers, no matter how much the overfill, are sold for a fixed price. The same situation was discussed by Tang and Lo, [12] who besides formulating the problem, performed sensitivity analyses.

Lee and Kim, [13] extended the model discussed in [11] by including a controllable upper limit. The underfilled as well as the overfilled containers were both assumed to be emptied and refilled. The developed profit model took into consideration selling price as well as the costs of filling, inspection, rework and penalty costs.

Misiorek and Barnett, [14] focused on a model where production between two dimensional values can be reprocessed at a cost, but where items produced below the lower of these is unsaleable. Items initially produced above the upper threshold attract a fixed selling price but involve 'give-away' product. The objective was also to maximise the expected profit.

The purpose of this current paper is to present various filling process models for which it is desired to select the optimal process mean. Filling, where overflowing material is either recaptured or lost are both considered. The objective, having allowed for a fixed filling cost, is to maximise the expected profit. Following analysis, implications to Australian Weights and Measures Legislation requirements are considered.

General Problem

Consider an automatic filling process where containers are filled with some ingredient, let this amount be denoted by a random variable X, and without loss of generality assume this to be a measure of volume. It is assumed that X is normally distributed with mean T and known variance σ^2. The nominal amount of material in each container (on the label) is L. According to Weights and Measures Legislation in Australia containers with a minimum proportion of 0.95 of the stated label content can be legitimately sold at the regular price. For generality reasons let this quantity be hL, where $0 < h < 1$. An automatic device rejects containers with content below hL. The cost of product in the container is denoted by gx, where g is the cost of material per unit of volume.

The aim is to fix the filling mean of the process so as the expected profit per container is maximised. The Target value, $T = hL + \delta$ ($\delta > 0$), is called optimal if it maximises the expected profit per container.

In this paper three variations of the filling process model are considered. Two concern a filling process where overflowed material is captured at no additional cost and in the event of underfilling there is no loss of container or its contents. The final model, describes a situation where overflowed, as well as material in underfilled containers, are both lost.

Weights and Measures Legislation within Australia requires that for a sample of 12 containers all must contain at least 0.95 of the label content (hL where h=0.95) and the average of the 12 must equal or exceed the amount appearing on the label. Since the automatic rejection device ensures the former of these two conditions, this leaves only the condition $\bar{x}_{12}$>L to be satisfied. Unless every container has x> L this condition cannot be guaranteed. It remains desireable, therefore, when seeking to maximise the expected profit per container, to have $\bar{x}_{12}$>L with a large probability. The actual probability is a special case of

$$\Pr(\overline{X}_n > L \,/\, x_1, x_2, \ldots, x_n > hL) = \frac{\Pr\left(\sum_{i=1}^{n} x_i > nL,\ x_1 > hL, \ldots, x_n > hL\right)}{\left[1-\Phi\left(\frac{hL-T}{\sigma}\right)\right]^n},$$

giving,

$$\Pr(\overline{X}_n > L \,/\, x_1, x_2, \ldots, x_n > hL) =$$

$$\frac{\left[1-\Phi\left(\frac{hL-T}{\sigma}\right)\right]^n - \int\limits_{x_1=hL}^{nL-(n-1)hL} \cdots \int\limits_{x_i=hL}^{nL-(n-i)hL-(x_1+\ldots+x_{i-1})} \cdots \int\limits_{x_n=hL}^{nL-(x_1+\ldots+x_{n-1})} \phi(x_1)\ldots\phi(x_n)\,dx_1\ldots dx_n}{\left[1-\Phi\left(\frac{hL-T}{\sigma}\right)\right]^n}$$

If $z = \frac{x-\mu}{\sigma}$ and $\mu = hL + \delta$, then the above equation can be simplified to:

$$= \frac{\left[1-\Phi\left(\frac{-\delta}{\sigma}\right)\right]^n - \int\limits_{-\delta/\sigma}^{(nL(1-h)-\delta)/\sigma}\cdots\int\limits_{-\delta/\sigma}^{(nL(1-h)-i\delta-\sigma(z_1+\ldots+z_{i-1}))/\sigma}\cdots\int\limits_{-\delta/\sigma}^{(nL(1-h)-n\delta-\sigma(z_1+\ldots+z_{i-1}))/\sigma}\cdots \phi(z_1)\ldots\phi(z_n)dz_1\ldots dz_n}{\left[1-\Phi\left(\frac{-\delta}{\sigma}\right)\right]^n}$$

When n=12, h=0.95 for any particular L, σ and T this provides the probability of not breaching Weights and Measures Legislation on a single sampling of size 12.

Theoretical Analysis

***(Model 1)* Overfilled material is captured, underfilled containers are emptied out and the product is put back into the process.**

Consider a filling process where, in the event of underfilling, the containers and the product are reused. Containers have a maximum content of L+k and any overflow is captured at no additional cost.

Hence, if $hL \leq x$,

Profit = Selling Price - Filling Cost - Material Cost.

Thus profit from a single item may be written as follows:

$$P(x) = \begin{cases} A - M - g(L+k) & x \geq L+k \\ A - M - gx & hL \leq x < L+k \\ -M & x < hL \end{cases}$$

where A is the selling price and M is the filling cost. If the cost of a container is significant it can simply be included in A since, in the case where the contents are emptied because x<hL, the containers are re-used.

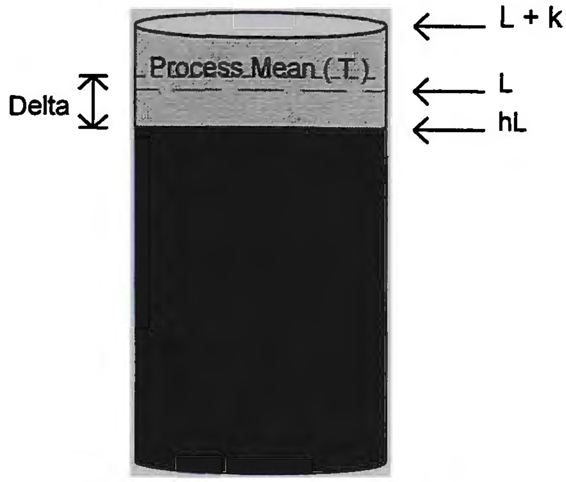

Figure 1
Inter-relationships between hL, L, L+ k and T.

The expected profit per item, denoted by E[P(x)], is

$$E[P(x)] = -M\int_{-\infty}^{hL} f(x)dx + \int_{hL}^{L+k}(A-M-gx)f(x)dx + (A-M-g(L+k))\int_{L+k}^{\infty} f(x)dx \qquad (1)$$

Standardising and putting $\mu = hL + \delta$ gives,

$$E[P(x)] = -M\Phi(\frac{-\delta}{\sigma}) + (A-M-g(L+k)[1-\Phi(\frac{L+k-hL-\delta}{\sigma})] + (A-M)\int_{\frac{hL-\mu}{\sigma}}^{\frac{L+k-\mu}{\sigma}}\phi(z)dz - \int_{hL}^{L+k} gxf(x)dx$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ are the p.d.f. and distribution functions respectively of the standard normal distribution.

Further simplifications lead to:

$$E[P(x)] = (A-M-g(L+k)) + (L+k)\Phi(\frac{L+k-hL-\delta}{\sigma}) - A\Phi(\frac{-\delta}{\sigma}) - g\int_{hL}^{L+k} xf(x)dx$$

Differentiating with respect to δ, using the general formula,

$$\frac{\partial}{\partial\delta}\left[g \int_{\frac{hL-\mu}{\sigma}}^{\frac{L+k-\mu}{\sigma}} z\phi(z)dz \right] = \Phi\left(\frac{L+k-hL-\delta}{\sigma}\right) - \Phi\left(\frac{-\delta}{\sigma}\right) - \frac{(L+k)}{\sigma}\phi\left(\frac{L+k-hL-\delta}{\sigma}\right) + \frac{hL}{\sigma}\phi\left(\frac{-\delta}{\sigma}\right)$$

and setting $E'[P(x)] = 0$, gives:

$$\frac{\frac{1}{\sigma}\phi\left(\frac{\delta}{\sigma}\right)}{\Phi\left(\frac{L+k-hL-\delta}{\sigma}\right) - \Phi\left(\frac{-\delta}{\sigma}\right) + \frac{hL}{\sigma}\phi\left(\frac{\delta}{\sigma}\right)} = \frac{g}{A} \tag{2}$$

Further,

$$E''[P(x)] = -\frac{A\delta}{\sigma^3}\phi\left(\frac{\delta}{\sigma}\right) + \frac{g}{\sigma}\phi\left(\frac{L+k-hL-\delta}{\sigma}\right) - \frac{g}{\sigma}\phi\left(\frac{\delta}{\sigma}\right) + \frac{ghL\delta}{\sigma^3}\phi\left(\frac{\delta}{\sigma}\right)$$

If $E''[P(x)] < 0$ (with $\delta = \delta_0$) i.e. if

$$\frac{-\delta}{\sigma} + \frac{\delta ghL}{A\sigma} + \frac{g\sigma\phi\left(\frac{L+k-hL-\delta}{\sigma}\right)}{A\phi\left(\frac{\delta}{\sigma}\right)} < \frac{g\sigma}{A} \tag{3}$$

then δ_0 is optimal i.e. the solution to (2) will give a setting for the mean that will maximise the expected profit. It is shown in Figure 2 that $E''[P(x)]$ is always negative.

An algorithm to obtain the optimal mean setting, which can be used for practical purposes, can be found in the Appendix.

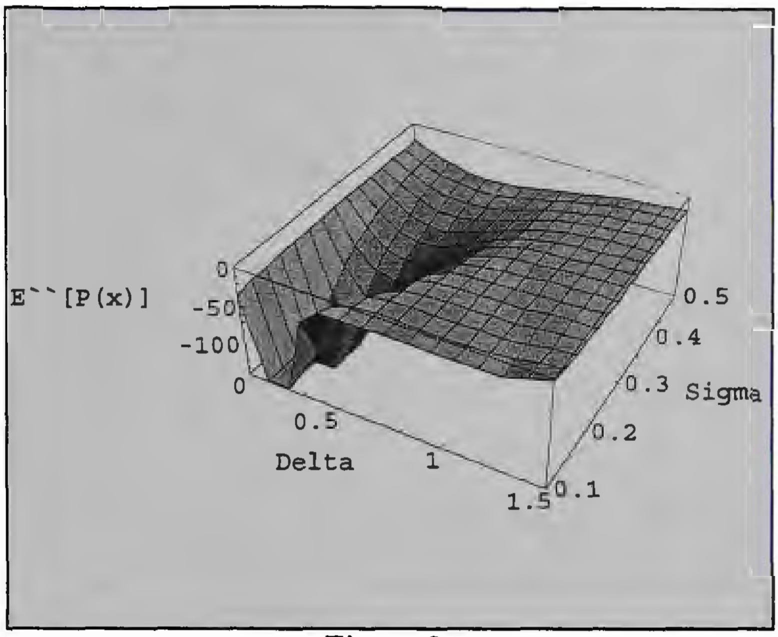

Figure 2
E″[P(x)] for $0<\delta<1.5$ and $0.1<\sigma<0.5$ with L=10, h=0.95, k=1, A=67, g=4.

Model 1 - Discussion

The following, explores the economic gains and losses caused by shifts in the filling mean and variance.

Figure 3 illustrates the effect of changes in δ on the expected profit for three different values of the process standard deviation (0.2, 0.3 and 0.4 respectively). As σ increases the optimal mean gets larger. The convergence of the three curves as δ increases should also be noted.

A more vivid appraisal of the relationship between the process mean, sigma and the expected profit is shown in Figure 4. It should be observed that for δ bigger than the optimal, the expected profit is not sensitive to changes in the process variance.

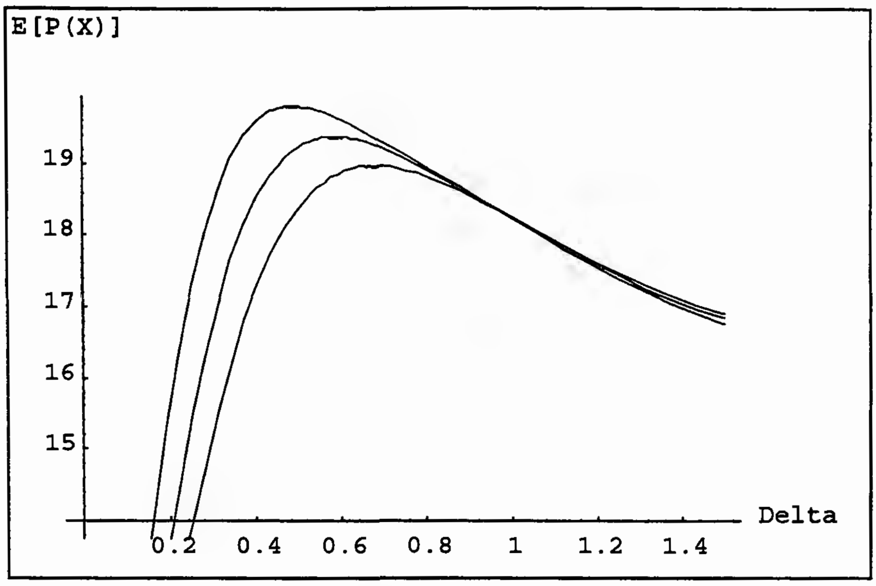

Figure 3
δ against E[P(x)] with σ= 0.2, 0.3 and 0.4 respectively and g=3.5, L=10, k=1, h=0.95, A=67 and M=12.

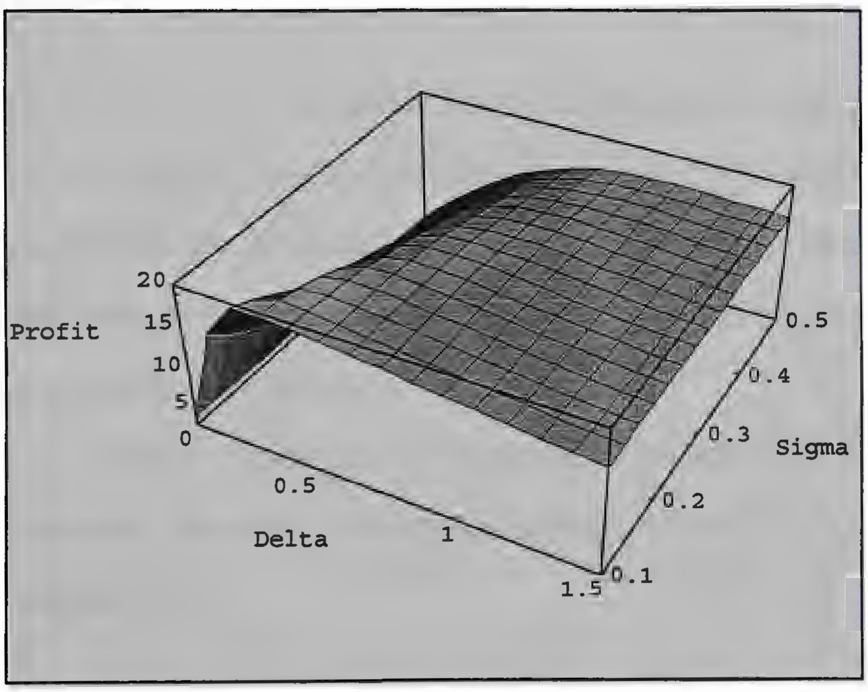

Figure 4
E[P(x)] against δ and σ with g=3.5, L=10, k=1, h=0.95, A=67 and M=12.

A graphical illustration of the optimal solution is shown in Figure 5.

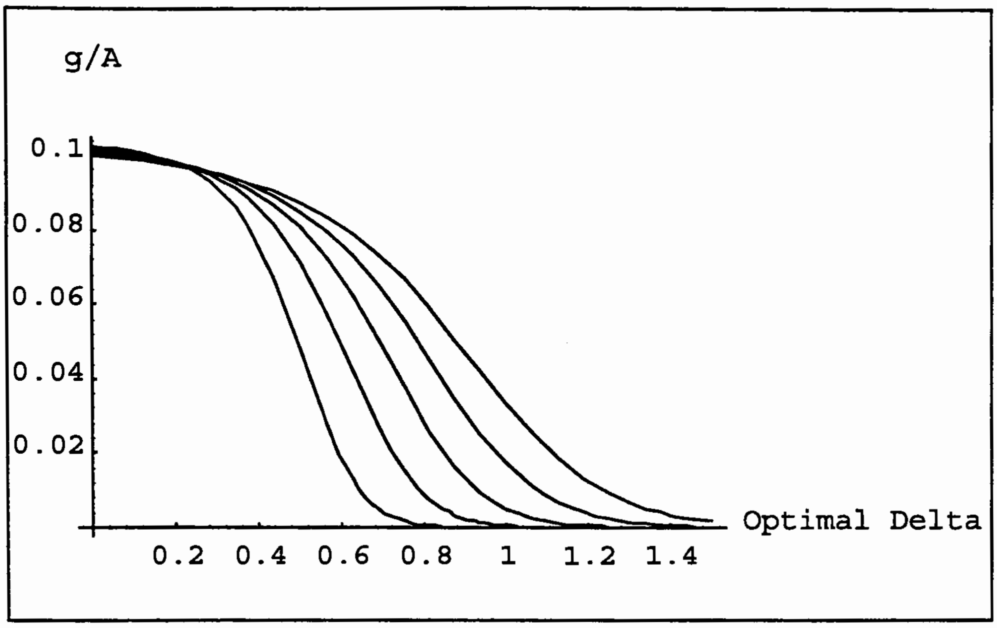

Figure 5
Solution of $E'[P(X)] = 0$ for $\sigma=0.2, 0.25, 0.3, 0.35$ and 0.4 and $L=10$, $h=0.95$, $k=1$

Each curve shows the relation between the optimal δ value and the ratio of g to A for five different values of σ. It is evident, that if A is assumed to be fixed, then if the cost of material is small the target can be set further from the lower limit hence decreasing the number of rejected containers. The larger the σ the further the target needs to be beyond hL.

Figure 6 illustrates the dependencies between optimal δ, σ and g/A. The larger the value of σ the less sensitive is the optimal solution to changes in the material cost.

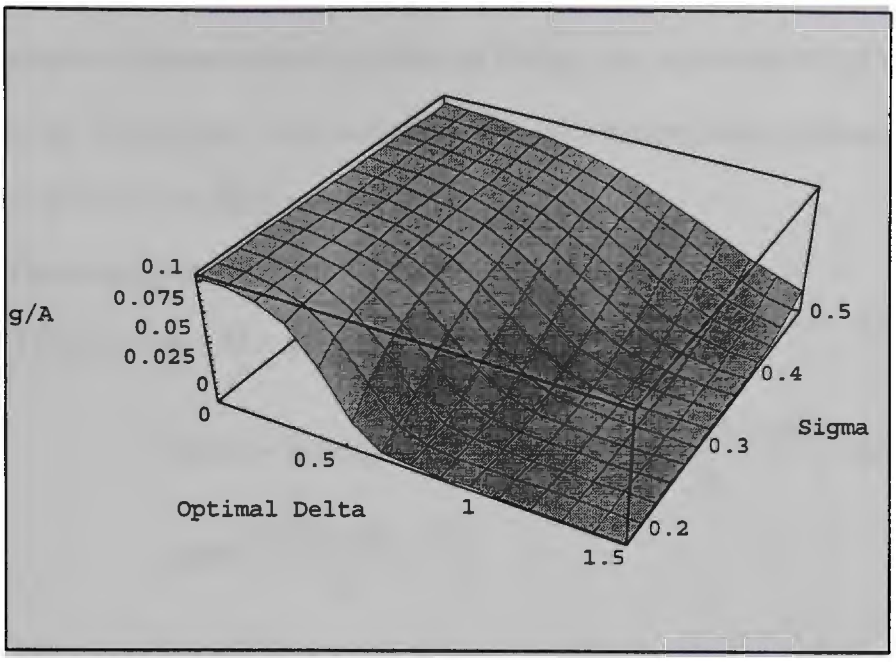

Figure 6
Optimal δ, σ and g/A with L=10, h=0.95, k=1.

***(Model 2)* Overflowed material is captured and underfilled cans are topped up.**

Consider now a filling process where all underfilled containers are 'topped-up'. The capacity of the container is L+k. Hence, if $hL \le x \le L+k$,
Profit = Selling Price - Filling Cost - Material Cost.
If, however, x<hL the container can be 'topped-up' so that in this instance,
Profit = Selling Price - Filling Cost-Additional Processing Cost - Material Cost.
Overfilled material is assumed captured at no additional cost.

The profit from a single item may be written as:

$$P(x) = \begin{cases} A - M - g(L+k), & x > L+k \\ A - M - gx, & hL \le x \le L+k \\ (A - m) - M - gL, & x < hL \end{cases}$$

where m represents the additional filling cost associated with containers that have to be 'topped-up'. It is assumed that each underfilled container can be 'topped-up' exactly to the label specification, L.

Proceeding as previously, the expected profit per item is:

$$\begin{aligned} E[P(x)] = {} & A - M - g(L+k) + g[L(1-h)+k]\Phi\left(\frac{L+k-hL-\delta}{\sigma}\right) + g\delta\Phi\left(\frac{-\delta}{\sigma}\right) \\ & + [gL(h-1)-m]\Phi\left(\frac{-\delta}{\sigma}\right) + g\sigma\left[\phi\left(\frac{L+k-hL-\delta}{\sigma}\right) - \phi\left(\frac{-\delta}{\sigma}\right)\right] \\ & - g\delta\Phi\left(\frac{L+k-hL-\delta}{\sigma}\right) \end{aligned}$$

The expected profit, plotted against σ and δ, is shown in Figure 7. It should be observed that the expected profit is relatively sensitive to changes in the process setting and that this sensitivity is reduced as the process standard deviation increases. Note that this is more pronouced than observed for *model 1*.

Differentiation with respect to δ gives:

$$E'[P(x)] = -g\Phi\left(\frac{L+k-hL-\delta}{\sigma}\right) + g\Phi\left(\frac{-\delta}{\sigma}\right) - \frac{gL(h-1)}{\sigma}\phi\left(\frac{-\delta}{\sigma}\right) + \frac{m}{\sigma}\phi\left(\frac{-\delta}{\sigma}\right)$$

Setting $E'[P(x)] = 0$, gives:

$$\frac{\frac{1}{\sigma}\phi\left(\frac{-\delta}{\sigma}\right)}{\Phi\left(\frac{L+k-hL-\delta}{\sigma}\right) - \Phi\left(\frac{-\delta}{\sigma}\right)} = \frac{g}{m-gL(h-1)}$$

As shown in Figure 8 the first derivative provides a single optimum solution.

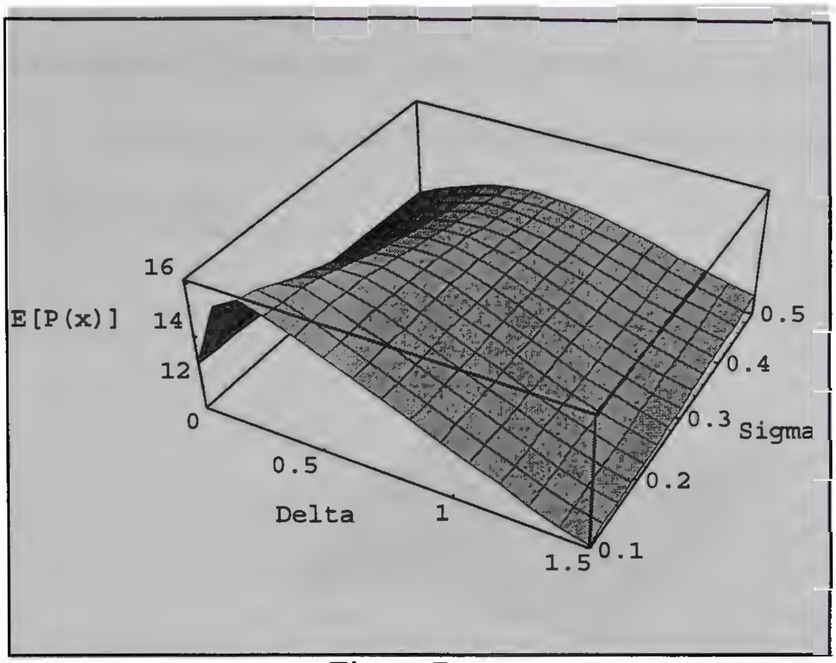

Figure 7
E[P(x)] against δ and σ with A=67, M=12, g=4, m=6, L=10, k=1, h=0.95.

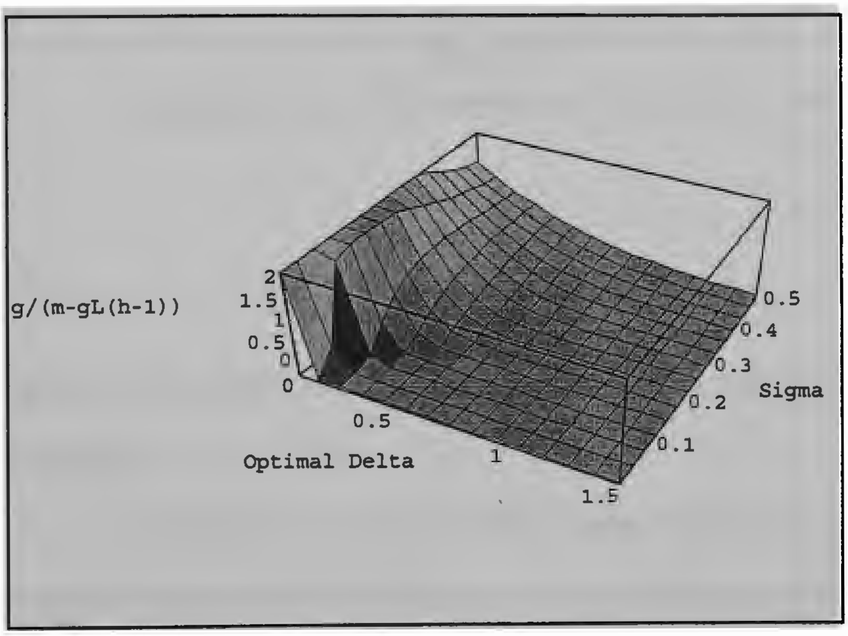

Figure 8
δ^* against σ and g/(m-gL(h-1)) with L=10, k=1, h=0.95.

and if $E''[P(x)] < 0$ i.e.

$$\frac{-\delta\phi\left(\frac{-\delta}{\sigma}\right)}{\phi\left(\frac{L+k-hL-\delta}{\sigma}\right)+\left(1-\frac{L(h-1)}{\sigma^2}\right)\phi\left(\frac{-\delta}{\sigma}\right)} < \frac{-g\sigma^2}{m}$$

then δ_0 is optimal.

As would be expected, the optimal solution does not depend on A.

As shown in Figure 9 the second derivative is negative for all meaningful values of δ and σ.

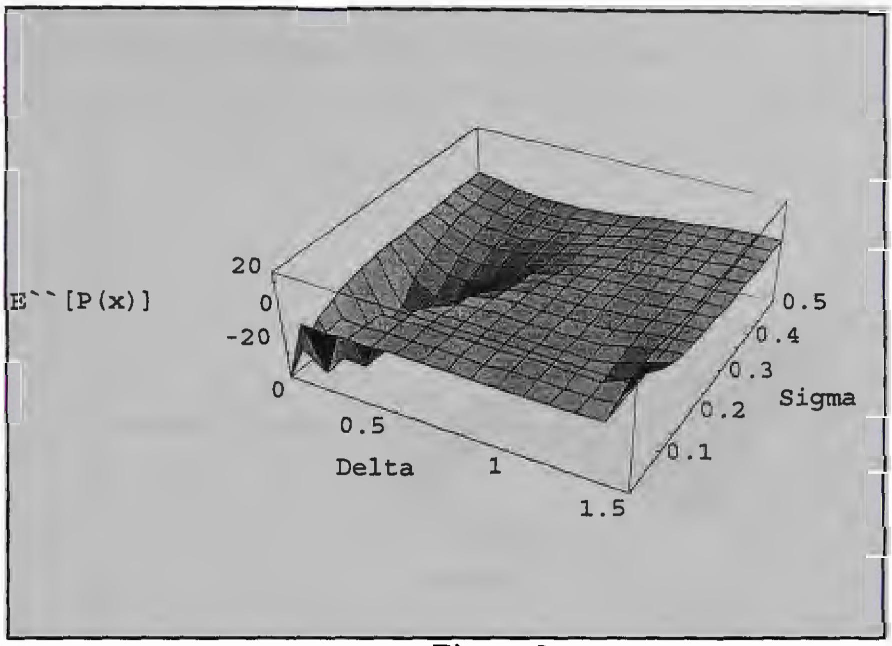

Figure 9
$E''[P(x)]$ against δ and σ with m=12, g=4, L=10, k=1 and h=0.95.

***(Model 3)* Overflowed material and material in underfilled containers are both lost.**

Consider now a situation where there is a loss of material for both overfilled and underfilled containers, and containers are assumed to be of negligible value. This may well be the case, for example, in the food or pharmaceutical industry where material cannot be put back into the process largely for hygienic reasons. Hence, all underfilled containers incur a loss equal to the sum of the material and filling costs.

The profit from a single item can then be written as:

$$P(x) = \begin{cases} -M - gx, & x < hL \\ A - M - gx, & x \geq hL \end{cases}$$

and the expected profit per item,

$$E[P(x)] = \int_{-\infty}^{hL}(-M - gx)f(x)dx + \int_{hL}^{\infty}(A - M - gx)f(x)dx$$
$$= A - M - g(hL + \delta) - A\Phi\left(\frac{-\delta}{\sigma}\right).$$

The expected profit function is shown in Figure 10.

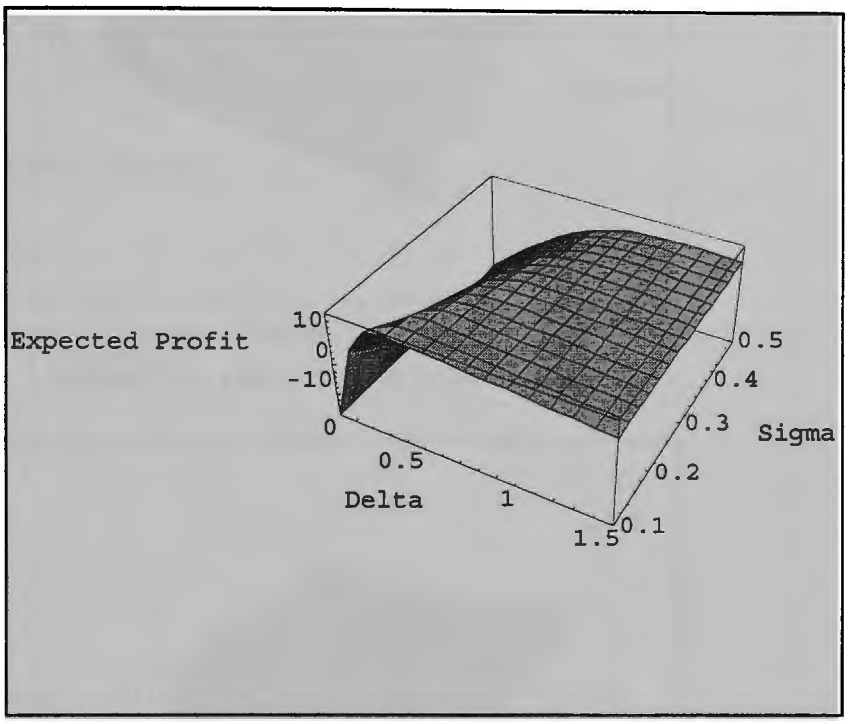

Figure 10
E[P(x)] against δ and σ with A=67, M=12, g=4, L=10, h=0.95.

There is a very rapid decrease in profit as the process mean is set closer to hL.

Differentiating with respect to δ and setting $E'[P(x)] = 0$ gives:

$\frac{1}{\sigma}\phi\left(\frac{\delta}{\sigma}\right) = \frac{g}{A}$ which has a single optimum solution as shown in Figure 11.

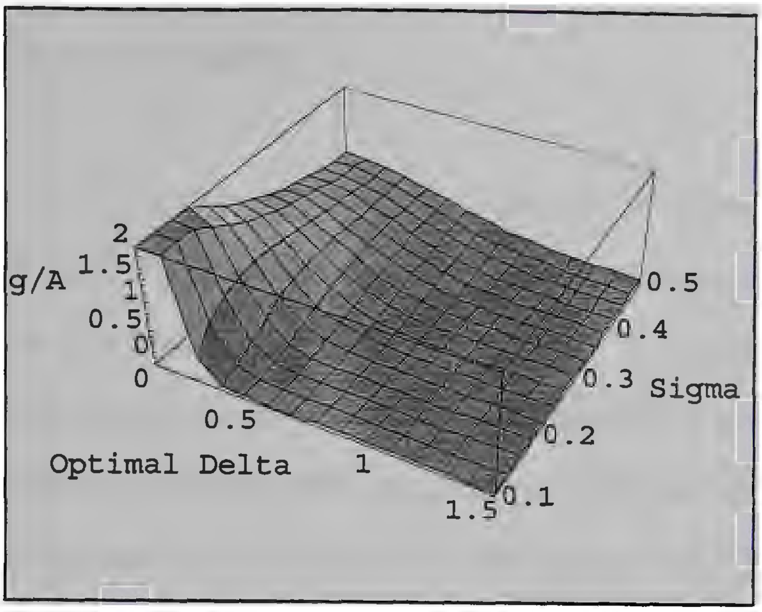

Figure 11
Optimal δ, σ and g/A with L=10, h=0.95, k=1.

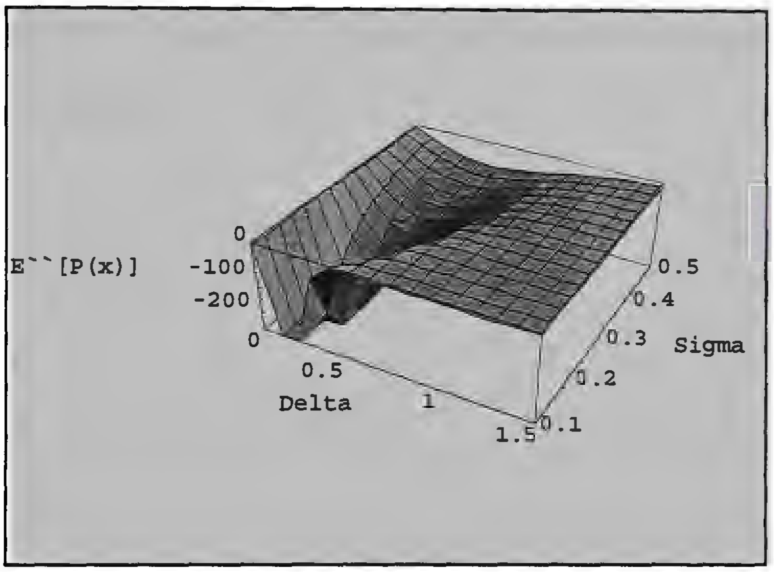

Figure 12
E"[P(x)] against δ and σ with A=67, g=4.

Further, if $E''[P(x)] < 0$ (with $\delta = \delta_0$) i.e. if $-A\delta\sigma^{-3}\phi\left(\frac{\delta}{\sigma}\right) < 0$ then the solution is optimal.

Weights and Measures Legislation and Maximisation of Expected

Profit - Discussion.

The following tables (I, II and III), corresponding to the three models, illustrate the implications to Weights and Measures requirements of optimising the mean setting based on expected profit. The probabilities of meeting Weights and Measures requirements when δ is optimal are given for the three models. In addition, the mean setting is given should the priority be to meet Weights and Measures requirements with a probability of 0.95. The corresponding expected profit per item is then given and contrasted with the optimal value. It can be observed that the loss in expected profit associated with $\Pr(\bar{x}_n > L \,/\, x_1, x_2, \ldots, x_n > hL) = 0.95$ for n=12 is largest for model 2, and smallest for model 3.

σ	δ^*_1	**E[P(x)]**	**Pr**$[\bar{x}_{12} > L / x_1 ... x_{12} > hL]$	Min. δ ($=\delta_2$) for Pr. at least 0.95	**E[P(x)] if** $T=hL+\delta_2$	**Difference in E[P(x)]**
0.15	0.38	20.23	0.93	0.40	20.2	0.03
0.20	0.49	19.8	0.91	0.53	19.7	0.1
0.25	0.59	19.4	0.89	0.66	19.3	0.11
0.30	0.68	19.0	0.87	0.80	18.8	0.2
0.35	0.77	18.6	0.85	0.93	18.4	0.2
0.40	0.86	18.2	0.83	1.05	18	0.2
0.45	0.96	17.9	0.82	1.18	17.7	0.2

Table I

Summary of E[P(x)] and Pr($\bar{x}_n$>L / $x_1, x_2, ..., x_n$ > hL) =0.95 for n=12 with L=10, k=1, h=0.95, A=67 and M=12. Model 1.

σ	δ^*_1	E[P(x)]	$Pr[\bar{x}_{12} > L / x_1 ... x_{12} > hL]$	Min. δ (=δ_2) for Pr. at least 0.95	E[P(x)] if T=hL+ δ_2	Difference in E[P(x)]
0.15	0.28	20.52	0.69	0.4	20.32	0.2
0.20	0.35	20.2	0.61	0.53	19.86	0.34
0.25	0.41	19.9	0.52	0.66	19.4	0.5
0.30	0.45	19.63	0.44	0.79	18.95	0.68
0.35	0.50	19.37	0.38	0.92	18.52	0.85
0.40	0.54	19.12	0.32	1.05	18.13	0.99
0.45	0.57	18.9	0.27	1.18	17.8	1.1

Table II

Summary of E[P(x)] and Pr($\bar{x}_n$ >L / $x_1, x_2, ..., x_n$ > hL) =0.95 for n=12 with L=10, k=1, h=0.95, A=67 and M=12. Model 2.

σ	δ^{*}_1	E[P(x)]	$\Pr[\bar{x}_{12} > L / x_1 \ldots x_{12} > hL]$	Min. δ ($=\delta_2$) for Pr. at least 0.95	E[P(x)] if $T=hL+\delta_2$	Difference in E[P(x)]
0.15	0.42	20.1	0.99	n/a	n/a	n/a
0.20	0.54	19.6	0.96	n/a	n/a	n/a
0.25	0.65	19.21	0.94	0.66	19.21	0
0.30	0.76	18.71	0.93	0.79	18.70	0.01
0.35	0.87	18.27	0.92	0.93	18.23	0.04
0.40	0.97	17.84	0.91	1.06	17.71	0.07
0.45	1.07	17.42	0.90	1.19	17.31	0.11

Table III

Summary of E[P(x)] and $\Pr(\bar{x}_n > L / x_1, x_2, \ldots, x_n > hL) = 0.95$ for n=12 with L=10, k=1, h=0.95, A=67 and M=12. Model 3.

3.5 CONCLUDING REMARKS

In this paper the problem of selecting the appropriate mean for a filling process has been defined and analysed for three particular models. The economic gains and losses caused by change in the process parameters have been illustrated as well as consideration given to finding the mean setting to maximise expected profit. Further, the implications to Weights and Measures requirements of using this optimal mean, has been investigated.

The algorithm for solution, developed by the authors and shown in the Appendix, is both easy to use and quick to evaluate.

APPENDIX

The optimal delta (obtained by using numerical methods) as well as the solution to the equation for the expected profit was obtained using *Mathematica.*
The algorithm used is shown below.

Algorithm

In[1]:= <<Statistics`ContinuousDistributions`

In[2]:= ndist :=NormalDistribution[0,1]

In[3]:= f[x_] :=PDF[ndist, x]

In[4]:= F[x_] :=CDF[ndist, x]

(Model 1)

In[5]:= FindRoot[(f[d/s]/s)/(F[(L+k-h*L-d)/s]-F[-d/s]+(h*L/s)*f[d/s]) == g/A,
{d, d_0}]

In[6]:= P[d_,s_,g_,L_,k_,h_,A_,m_] := A-M-g*(L+k)-A*F[-d/s]+

g*(L+k)*F[(L+k-h*L-d)/s]-g*(h*L+d)*(F[(L+k-h*L-d)/s]-

F[-d/s])-g*s*(f[-d/s]-f[(L+k-h*L-d)/s])

In[7]:= P[d,s,g,L,k,h,A,m]

(Model 2)

In[8]:= FindRoot[((1/s)*f[-d/s])/(F[(l+k-h*L-d)/s]-F[-d/s]) == g/(m-g*L*(h-1)),

{d, d_0}]

In[9]:= P [d_,s_,g_,L_,k_,h_,A_,m,_,M_] := A-M-g*(L+k)+

g*(L*(1-h)+k)*F[(L+k-h*l-d)/s]-g*d*F[(L+k-h*L-d)/s]+

(g*L*(h-1)-m)*F[-d/s]+g*d*F[-d/s]+g*s*(f [(L+k-h*L-d)/s]-f[-d/s])

In[10]:= P[d,s,g,L,k,h,A,m,M]

(Model 3)

In[11]:= FindRoot(f[d/s]/s) == g/A, {d, d_0}]

In[12]:= P [d_,s_,g_,L_,h_,A_,m_] := A-m-A*F[-d/s]-g*(h*L+d)

In[13]:= P[d,s,g,L,h,A,m,M]

(where d=δ and s=σ).

****

To calculate the probability of not breaching Weights and Measures Legislation on a single sampling of size 12:

****

In[14]:= R[h_,L_,d_,T_,s_,n_]:=(((1-F[-d/s])^n)-

(Integrate[f[z1 s+T] f[z2 s+T] f[z3 s+T] f[z4 s+T]*

f[z5 s+T] f[z6 s+T] f[z7 s+T] f[z8 s+T] f[z9 s+T]* f[z10 s+T] f[z11 s+T] f[z12 s+T] s^n,

{z1, -d/s, (n*L-s*(z1+z2+z3+z4+z5+z6+z7+z8+z9+z10+z11)-12 (h L+d))/s},

{z2, -d/s, (n*L-(n-11) h L-s*(z1+z2+z3+z4+z5+z6+z7+z8+z9+z10)-11 T)/s},

{z3, -d/s, (n*L-(n-10) h L-s*(z1+z2+z3+z4+z5+z6+z7+z8+z9)-10 T)/s},

{z4, -d/s, (n*L-(n-9) h L-s*(z1+z2+z3+z4+z5+z6+z7+z8)-9 T)/s},

{z5, -d/s, (n*L-(n-8) h L-s*(z1+z2+z3+z4+z5+z6+z7)-8 T)/s},

{z6, -d/s, (n*L-(n-7) h L-s*(z1+z2+z3+z4+z5+z6)-7 T)/s},

{z7, -d/s, (n*L-(n-6) h L-s*(z1+z2+z3+z4+z5)-6 T)/s},

{z8, -d/s, (n*L-(n-5) h L-s*(z1+z2+z3+z4)-5 T)/s},

{z9, -d/s, (n*L-(n-4) h L-s*(z1+z2+z3)-4 T)/s},

{z10, -d/s, (n*L-(n-3) h L-s*(z1+z2)-3 T)/s},

{z11, -d/s, (n*L-(n-2) h L-s*z1-2 T)/s},

{z12, -d/s, (n*L-(n-1) h L -T)/s}]))/(1-F[-d/s]^n)

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