New Methods for Reliability Evaluation and Enhancement of Power Systems

By

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Declaration

“I, Rahamathulla Mohammad, declare that the PhD thesis entitled ‘New Methods for Reliability Evaluation and Enhancement of Power Systems’ is no more than 100,000 words in length including quotes and exclusive of tables, figures, appendices, bibliography, references and footnotes. This thesis contains no material that has been submitted previously, in whole or in part, for the award of any other academic degree or diploma. Except where otherwise indicated, this thesis is my own work”.

Signature ______________________________ Dated:

Table of Contents

List of Figures

List of Tables

List of Acronyms

AFTM	Accelerated Failure Time Model
CB	Circuit Breaker
CE	Cumulative Exposure Model
CPU	Central Processing Unit
EHV	Extra High Voltage
EWS	Early Warning System
FTS	Fault-Tolerant System
GMSSkn	Generalized Multi-State k-out-of-n System
HVDC	High Voltage Direct Current
i.i.d	Independent and Identical Distributed
LSS	Load Sharing System
M2M	Machine to Machine
ME	Memory Element
PE	Processor Element

PHM	Proportional Hazards Model
PMS	Phased-Mission System
RAPS	Remote Area Power Supplies
rms	Root Mean Squared
SCADA	Supervisory Control And Data Acquisition
SS	Sub-system
S&T	Signalling and Telecommunication
TFR	Tampered Failure Rate
UPS	Uninterrupted Power Supplies
WSS	Warm Standby Sub-system

Nomenclature

t	Mission time
$\boldsymbol{n}$	Number of components in system
k	Minimum number of working components required for system success
p_j	Probability of having exactly *i* component failures in a [logical] location
P_j	Probability of having exactly *i* component failures in system
Q_j	Probability of failure sequence *i*
λ	Parameter of Rayleigh distribution
$f(t)$	Probability density function (pdf)
$F(t)$	Cumulative distribution function (cdf)
$R(t)$	Component reliability
p_{sw}	Switch success probability on demand
$R_{Sys}(t)$	System reliability
$x!!$	Double factorial function; If x is even: $x!! = x(x-2)\ldots3.1$
$F_N(t)$	Cdf of Nakagami distribution
$I(a,x)$	Regulated gamma function
M	Number of phases
τ_j	Duration of phase j; $j = 1,\ldots,M$;
t	Mission time: $t = \tau_1 + \tau_2 + \cdots + \tau_M$

S	Total number of sub-systems
n	Number of components in a given sub-system; It can vary for each sub-system.
k_j	Minimum number of working components required for a given sub-system in phase j; It can vary for each sub-system.
m_j	Defined as: $m_j = n - k_j + 1$ Minimum number of failed components required for a given sub-system to fail in phase j;
λ_{oj}	Failure rate of an operating component in phase j
λ_{dj}	Failure rate of a dormant component in phase j
d_j	Defined as: $d_j = \lambda_{dj}/\lambda_{oj}$, dormancy factor of a component in phase j.
k'_j	Defined as: $k' = k_j/d_j$
n'_j	Defined as: $n' = n_j - k_j + k'_j$
R_l	Reliability of sub-system l
R_{PMS}	Mission reliability of PMS
n	Number of components in the system
k	Minimum number of components required for successful operation
z_i	Load on each component when i components failed; $z_i = z_{0.}\frac{n}{n-1}$
$h_0(t)$	Baseline failure rate of PHM
$\lambda_i(t)$	Failure rate of each component
δ_i	Failure rate multiplication factor
$\Lambda(t)$	Transition rate matrix

$p_{0(t)}$ Initial probability state vector

$R(t)$ System reliability

M Number of phases

t_j Duration of phase j; $j = 1,\ldots,M$;

T Mission time: $T = t_1 + t_2 + \cdots + t_M$

N Total number of sub-systems

n Number of components in a given sub-system;

It can vary for each sub-system. Hence, $n = n_l$ for sub-system l.

k_j Minimum number of working components required for a given sub-system in phase j;

It can vary for each sub-system. Hence, $k_j = k_{li}$ for sub-system l in phase j.

m_j Defined as: $m_j = n - k_j + 1$

L_T Total load on a sub-system in a given phase;

It can vary for each sub-system. Hence, $L_T = L_T(l,j)$ for sub-system l in phase j.

$\lambda(L)$ Failure rate function in load L;

$\lambda(L) = \lambda_l(L)$ for the components in sub-system l

λ_i Failure rate of a component when there are i failures in the sub-system;

It can vary for each phase and sub-system. Hence, $\lambda_i = \lambda_i(l,j)$ for sub-system l in phase j.

S_i State-i of a sub-system; it is equivalent to the sub-system state with i failures.

γ_i Transition rate of a sub-system from state-(i) state-($i+1$);

It can vary for each phase and sub-system. Hence, $\gamma_i = \gamma_i(l,j)$ for sub-system l in phase j.

$P_{a,i}$ Conditional probability that a given sub-system is in state-i at the end of a phase given that was in state-a at the beginning of the phase;

It can vary for each phase and sub-system. Hence, $P_{a,i} = P_{a,i}(j,l)$ for sub-system l in phase j.

Φ System structure function

R_l Mission reliability of sub-system l

R_{PMS} Mission reliability of the entire system

ABSTRACT

Modern Power systems are smart, interconnected, interdependent, load sharing and phased mission systems. Reliability of such complex power systems is very important in design, planning, installation and maintenance to provide electrical energy as economical as possible with an acceptable degree of reliability. In this thesis four new methods for reliability evaluation and enhancement of power systems are presented and further an innovative cost effect cloud service based smart early warning system using machine to machine **(M2M)** technology to improve the reliability of power systems is presented.

Many fielded power systems use cold standby redundancy as an effective design strategy for improving system reliability. However, methods for analysing the reliability of ***k-out-of-n*** cold standby systems, particularly with components having age-dependent hazard (failure) rates, are limited. In this thesis the **first method** is proposed using the concepts of counting processes, an efficient approximate method to evaluate the reliability of '***k-out-of-n'*** cold standby systems is proposed. This proposed **new method** considers ***Rayleigh distributions*** for component life times and the effects of switch failures on system reliability. The main advantage of this counting process-based method is that it reduces a complex problem involving multiple integrals into an equivalent simple problem involving one-dimensional convolution integrals. Further eliminates the

need for one-dimensional convolution integrals using approximate closed-form expressions for computing the distribution of sum of Rayleigh distributed random variables. This ***new method*** shows that all steps involved in evaluating the reliability of ***k-out-of-n*** cold standby system with components having Rayleigh operational failure time distributions are simple and straightforward. The proposed method and its computational efficiency and accuracy are illustrated using a numerical example.

With the development of technology, sensor networks, and non-conventional power generators, became more and more complex, and their missions become more and more diversified. Further, many real world power systems operate in phased-missions where the system requirements and success criteria vary over consecutive time periods, known as phases. For mission success, all phases must be completed without failure. In order to ensure accomplishing missions successfully, many sub-systems adopt redundancy techniques to improve the mission reliability. Particularly, redundancy is an effective method to improve the reliability of mission critical power systems. Hence, there is a great need for accurate and efficient reliability evaluation of phased mission systems with phase dependent redundancy configurations and requirements.

The **second method** presented in this thesis is **a new method** for reliability analysis of phased-mission systems with warm standby sub-systems. In the analysis, multiple sub-systems were considered where each sub-system uses warm standby redundancy. The operational and standby failure rates of a component can vary with the phases. Similarly, the configuration of each sub-system can vary with the phases. The proposed algorithm is developed based on: (1) a modularization technique, (2) an easily computable closed-form expressions for conditional reliability of warm standby sub-system with phased dependent success criteria, and (3) a recursive formula for accounting the dependencies of sub-systems across the phases. As cold and hot standby configurations are special cases of warm standby configuration, the proposed method is also applicable for analysing the phased mission systems with cold and hot standby redundancies. The reliability evaluation algorithm is illustrated using an example of fault tolerant power system.

In the thesis **a third new model** for **load-sharing systems** using *k-out-of-n* structure is presented. It is assumed that the failure distribution of each component at a baseline load follows a general failure time distribution. Hence, the model can be used for analysing the systems where components' failure times follow Weibull, Gamma, Extreme Value and Lognormal distributions. In a load-sharing system, the system components experience different loads at different time intervals due to the load-sharing policy. Therefore, to analyse the reliability of load-sharing systems, the failure rate of each component must be expressed in terms of the current load and the current age of the component. In this thesis, the load-dependent time-varying failure rate of a component is expressed using Cox's proportional hazards model (PHM). According to the PHM the effects of the load is multiplicative in nature. In other words, the hazard (failure) rate of a component is the product of both a baseline hazard rate, which can be a function of time 't', and a multiplicative factor which is a function of the current load on the component. In addition, the load-sharing model also considers the switchover failures at the time of load redistribution. This research shows that the model can be described using a non-homogeneous Markov chain. Therefore, for the non-identical component case, the system reliability can be evaluated using well established methods for non-homogeneous Markov chains. In addition, when all components are identical, this thesis provides a closed-form expression for the system reliability even when the underlying baseline failure time distribution is non-exponential. The method is demonstrated using a numerical example with components following Weibull baseline failure time distribution. The numerical results from non-homogeneous Markov chains, closed-form expressions, and Monte Carlo simulation are compared.

The fourth method proposed in this thesis is a ***new*** efficient recursive algorithm for reliability evaluation of phased mission systems with load-sharing components. In the analysis, multiple sub-systems are considered, where each sub-system can have multiple load-sharing components. The proposed algorithm is developed based on: (1) a modularization technique, (2) an easily computable closed-form expression for conditional reliability of load-sharing sub-systems, and (3) a recursive formula for the reliabilities of sub-systems across the phases. The reliability evaluation algorithm in this thesis helps reliability engineers to accurately evaluate the reliability of phased mission systems with load-sharing components subjected to switch failure in an efficient way.

Finally, the development of a cost-effective, cloud service based smart early warning system for improving the reliability of power systems using M2M technology is presented.

The study presented in this thesis shows improvement in reliability of power systems using hardware and computationally efficient new mathematical algorithms. The usefulness of this research has been demonstrated by numerical examples and the analysis of data from power industry at different locations, among renewable and non-renewable power systems.

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CHAPTER 1[1]

THESIS OVER VIEW

1.1 Background

(The content of this Chapter has been presented at Reliability and Maintainability Symposium (RAMS) 2012 & 2013 conferences USA)

Reliability relates to the ability of a system to perform its intended function, in a qualitative sense. Planners and designers are always concerned with reliability. When qualitatively defined, reliability becomes a parameter that can be treated off with other parameters like cost. Necessity of qualitative reliability is an ever increasing complexity of systems, evaluations of alternate designs, cost competitiveness and cost benefit trade off. Due to fast extension of liberalization of power systems, innovation in technology, due to very short product development times, tightened budgets not giving enough time to do the reliability tests, there is no failure data occurs during those tests. In electrical engineering, Smart Grid connected power systems are facing above Reliability challenges. Therefore there is a big need to develop new methods for Reliability evaluations and enhancement of power systems based on not only an experimental data but also on technological and physical information available to the engineers. Overvoltage, Loads, Short-Circuits and weather conditions relates to wear and stress acting on the components/devices/equipment in power systems. A typical model of aging can be expressed mathematically by time behaviour of its hazard rate

[1] Some of the content in this Chapter has been presented in RAMS conferences 2011,2012 and 2013

function. Most electronic components are scarcely affected by aging and their failure is mainly accidental, so that a constant hazard rate function could be reasonably expressed. This leads to the adaptation of an exponential model in power systems, even in the absence of many data to support it on a statistical basis. Selection or correct identification of suitable probabilistic model for power system component reliability in the field of high reliability devices and large mission times should be better supported by probabilistic information that leads to reasonable modelling in medium voltage and high voltage components.

The aging failure in system components are a major concern and driving factor in system planning of many utilities. More and more system components are approaching their end of life stage. Hence, aging failure should be included in power system reliability evaluation in order to avoid severe underestimation of system rank as shown in [1], where ad hoc methods to incorporate aging failure in power system reliability evaluation are presented. Poisson model has many applications for describing the fault process in power systems [2-3]. Power devices in their actual service conditions are mostly subjected to time-varying stress levels. Therefore, there is a need to evaluate the reliability of power system components closer to real word situations. All components of power transmission and distribution grids exhibit almost the same rms current and voltage values at the same hours, during working days of a given period of the year under typical operation of the users [4]. Apart from the statistical fluctuation due to the random time-varying nature of the supplied loads and the deterministic fluctuations associated with the weekly and seasonal characteristics of the loads, such components in power systems are subjected to daily load cycles. Moreover, applied rms voltage is approximately constant with time apart from a generalized voltage increase when load decreases and vice versa; such variations, however under normal operating conditions are within + or – 10% of rated voltage of power systems. Thus time varying stresses are mostly associated with current variations in the form of daily load cycles. Therefore there is a great need to find new methods for reliability evaluation of load sharing system. The weakest part of a power device is its insulation, the predominant stress acting on insulation in service does commonly arise from the electric field associated with electric stress (Voltage) and the temperature associated with Joule losses in conducting elements plus dielectric losses in the insulation. Therefore, in general the maximum stress applied to a power system is maximum temperature and electric field in the insulation. In this framework, the life of power system subjected to load cycles is assumed here to end when its insulation fails because of the degradation caused by the maximum stress, that act all over its life as a consequence of a fixed stepwise constant daily load cycle.

1.2 Significance of the project

The significance of this project is that it presents the work towards the Reliability evaluation issues and solutions of power systems such as cold standby sub-system with components having linearly increasing hazard rates, warm standby sub-systems, load-sharing power systems, phased mission systems with load-sharing components subjected to switch failures in power systems, and development of a cost effective cloud service based, smart grid integration capable system to improve the reliability of power systems using M2M technology.

1.3 Scope and Objectives of the Project

From the literature it is clear that the existing methods for reliability evaluation are not adequate to enhance the reliability of modern smart-grid enabled power systems. The main objectives of this project are as follows:

1. By using the concepts of counting processes, an efficient approximate method to evaluate the reliability of k-out-of-n cold standby systems is proposed. This proposed method considers Rayleigh distributions for component life times and the effects of switch failures on system reliability.
2. A new method for reliability analysis of phased-mission systems with warm standby sub-systems is presented.
3. Reliability of load-sharing systems subject to proportional hazards model is presented. This method is demonstrated using a numerical example with components following Weibull baseline failure time distribution. The numerical results from non-homogeneous Markov chains, closed-form expressions, and Monte Carlo simulation are compared.
4. An efficient recursive algorithm for reliability evaluation of phased mission systems with load-sharing components subjected to switch failure is proposed.
5. A remote fault detection and identification system, for generation, transmission and distribution (GTD) system, for both renewable and non-renewable sources to minimize failures and their effects using innovative hardware and software system integration with M2M technology and cloud computing is presented.

1.4 Thesis Outline

This thesis comprises eight Chapters. Organization of the remaining seven Chapters is presented below:

Chapter 2 presents number of past efforts related to the current work. It presents literature review of past attempts in the area of Power System Reliability.

In Chapter 3, by using the concepts of counting processes, an efficient approximate method to evaluate the reliability of k-out-of-n cold standby systems is proposed. This proposed method considers Rayleigh distributions for component life times and the effects of switch failures on system reliability. The main advantage of this counting process-based method is that it reduces a complex problem involving multiple integrals into an equivalent simple problem involving one-dimensional convolution integrals. This research further eliminate the need for one-dimensional convolution integrals using approximate closed-form expressions for computing the distribution of sum of Rayleigh distributed random variables. Hence, this research shows that all steps involved in evaluating the reliability of k-out-of-n cold standby system with components having Rayleigh operational failure time distributions are simple and straightforward. This Chapter illustrates the proposed method and its computational efficiency and accuracy using a numerical example.

In Chapter 4, a new method for reliability analysis of phased-mission systems with warm standby sub-systems is presented. In the analysis, multiple sub-systems were considered where each sub-system uses warm standby redundancy. The operational and standby failure rates of a component can vary with the phases. The reliability evaluation algorithm is illustrated using an example of fault tolerant computing system.

In Chapter 5, the load-dependent time-varying failure rate of a component is expressed using Cox's proportional hazards model (PHM). According to the PHM the effects of the load is multiplicative in nature. In other words, the hazard (failure) rate of a component is the product of both a baseline hazard rate, which can be a function of time '*t*', and a multiplicative factor which is function of the current load on the component. In addition, the load-sharing model also considers the switchover failures at the time of load redistribution. Here first it is shown that the model can be described using a non-homogeneous Markov chain. Therefore, for the non-identical component case, the system reliability can be evaluated using the established methods for non-homogeneous Markov chains. In addition, when all components are identical, this Chapter provides a closed-form expression for the system reliability even when the underlying baseline failure time distribution is non-exponential. The method is demonstrated using a numerical example with components following Weibull baseline failure time distribution. The numerical results from non-homogeneous Markov chains, closed-form expressions, and Monte Carlo simulation are compared.

In Chapter 6, an efficient recursive algorithm for reliability evaluation of phased mission systems with load-sharing components subjected to switch failure is proposed. In the analysis, we considered multiple sub-systems where each sub-system can have multiple load-sharing components. The proposed algorithm is developed based on: (1) a modularization technique, (2) an easily computable closed-form expression for conditional reliability of load-sharing sub-systems, and (3) a recursive formula for the reliabilities of sub-systems across the phases. The reliability evaluation algorithm in this Chapter helps reliability engineers to accurately evaluate the reliability of phased mission systems with load-sharing components subjected to switch failure in an efficient way. We consider time-varying hazard rates are consider as future research work.

In Chapter 7, a remote fault detection and identification system, for generation, transmission and distribution (GTD) system, for both renewable and non-renewable sources to minimize failures and their effects using innovative hardware and software system integration with M2M technology and cloud computing is presented.

Chapter 8 summarizes the work as well as presents the future directions of research.

1.5 List of Publications

The following are publications related to this work:

1. **Mohammad, R**.; Kalam, A.; Amari, S.V., "Reliability evaluation of phased-mission systems with load-sharing components," *Reliability and Maintainability Symposium (RAMS), 2012 Proceedings- Annual* , vol., no., pp.1,6, 23-26 Jan. 2012. doi: 0.1109/RAMS.2012.6175468
2. **Mohammad R**, Kannan J, Kalam A and Zayegh A " Reliability Analysis of T-1000 1.2 kW PEMFC Power Generation System with Load-Sharing using MATLAB" International Conference in Renewable Energy Utilization, ICREU 2012, organised by ***Oklahoma State University, Stillwater USA*** and CIT India.
3. Kannan J , **Mohammad R**, Akhtar Kalam, Aladin Zayegh " Reliability Analysis of a Wind/Solar 4kW Micro-Generation System with Load Sharing using MATLAB" International Conference in Renewable Energy Utilization, ICREU 2012, organised by ***Oklahoma State University, Stillwater USA*** and CIT India.
4. Mohammad R, Kalam A, Amari V. Suprasad "*Reliability of k-out-of-n Cold Standby Systems with Rayleigh Distributions*" Proceeding of 18th ISSAT International conference on Reliability and Quality Design. July, 2012, **Boston USA**, pp 188-193
5. **Mohammad, R**.; Kalam, A.; Akella, R., "A cost-effective early warning system for improving the reliability of power systems," *Reliability and Maintainability Symposium*

(RAMS), 2013 Proceedings - Annual , vol., no., pp.1,6, 28-31 Jan. 2013 doi: 10.1109/RAMS.2013.6517731.

6. **Mohammad, R**.; Kalam, A.; Amari, S.V., "Reliability of load-sharing systems subject to proportional hazards model," *Reliability and Maintainability Symposium (RAMS), 2013 Proceedings - Annual* , vol., no., pp.1,5, 28-31 Jan. 2013 doi: 10.1109/RAMS.2013.6517708
7. **Mohammad, R**.; Kalam, A.; Amari, S.V., "Reliability of phased mission systems with warm standby sub-systems," *Reliability and Maintainability Symposium (RAMS), 2013 Proceedings - Annual* , vol., no., pp.1,5, 28-31 Jan. 2013 doi: 10.1109/RAMS.2013.6517753.
8. **Mohammad, R**.; Kalam, A., "Development of a Cost-Effective, Smart Early Warning System for Improving the Reliability of Electrical Substations", Submitted for a journal publication
9. **Mohammad, R**.; Kalam, A.; Amari, S.V., "Development of a Cost-Effective, Early Warning System for Improving the Reliability of standalone renewable power system by using M2M Technology", submitted for a journal publication.
10. **Mohammad, R**.; Kalam, A., "Reliability Evaluation of Phased Mission Systems with Load-Sharing Components with Switch failure", submitted for a journal publication.

CHAPTER 2

LITERATURE REVIEW

2.1 Literature Review[2]

(Some of the content in this Chapter has been presented in RAMS conference 2011)

The purpose of this Chapter is to provide the necessary background required for understanding the Reliability of Power Systems. It also highlights the concepts that related to recent developments in this field. This Chapter begins by presenting the need for this research and some of past research efforts in the field of Power systems Reliability.

Recent blackouts in Victoria, Queensland, New South Wales and in other parts of the Australia due to bush fires, floods and other natural disasters including Fukushima nuclear disaster have, however, focused attention on the need for an investigation and evaluations of Electrical Power Systems. Power system is a complex system, has numerous facilities and structures, systems and sub-systems, components and equipment, and has a complex communication among all those. The basic function of a power system is to supply energy as economically as possible and with a reasonable degree of continuity and quality its intended system. Power system reliability can be assessed based on system configuration, aging, component reliability and delivery of power to the load. Due to its complexity, power system has many issues in the field of power systems reliability.

Over the last 10 years, Victoria has suffered major blackouts each year. In February 2005, storms kept 410,000 Victorians off the power supply. In January 2006, high temperature and

[2] Some of the content in this Chapter has been presented in RAMS conference 2013, Orlando, USA

storms caused 618000, supply interruptions. In 2007, a bush fire forced rolling power shut down across Victoria. In April 2008, storms took 420000, Victorians off supply for days. In January 2009, break downs in distribution and transmission and problems with the bass link interconnect caused power supply loss more than 500,000 Victorians [5].

Reliability is one of the most important criteria, which must be taken into consideration during planning and operation phases of a power system. Electric power sector almost all over the world is undergoing considerable changes in regard to structure, operation and regulation which includes Smart Grids, Embedded Generation/Micro Generation (Roof top PVs, Wind forms, Geothermal and Gas-fired power stations), However, from the reliability point of view in this "new era", methods, algorithms and computer software capable of assessing at least the adequacy of systems much larger than in the past are needed [6,7].

To minimise the possibility of future blackouts, requires implementation of reliability policies that emphasize four factors essential to meeting the requirements of the new standards:

1. Continued development of sufficient electric generation resources, transmission delivery infrastructure, and demand response programs to reliably meet forecasted future electricity demands.
2. Effective and competent real-time operation and maintenance of that infrastructure to reliably produce and deliver electricity on a real-time basis, along with prompt restoration of adequate physical and cyber security to protect against malicious intrusion and attacks on critical facilities, diversity and redundancy of fuel supply.

None of these elements alone is sufficient to meet the world's electricity reliability requirements. The quality of electricity service is dependent upon maintaining a sufficient level of reserve capacity, in both generation supply and the transmission system, to be able to withstand unexpected outages of equipment, sudden increases in demand due to weather, and other emergencies. Reliability is enhanced through deploying these reserves in response to both planned and unforeseen changes to the system.

Many recommendations have already been addressed, as indicated in the final report of the task force on implementing recommendations in [8], but work remains in several areas that are more difficult to address. These include developing new or revised standards or guidelines in protective relay system design, application, maintenance, and testing; under voltage load shedding systems; and voltage and reactive planning and operation; as well as developing and implementing improved real-time system visualization tools for systems operators, including measurement systems; etc. [8]

The Australian Energy Market Operation (AEMO) is responsible for planning and directing the augmentation of the shared transmission networks. AEMO applies a probabilistic approach to planning the Victorian shared transmission network. Under that approach, investment only proceeds when the expected benefit exceeds the cost. The probabilistic approach involves the occurrence of plant outages occurring within the peak load season and weighting the cost of such an occurrences by its probability [9].

The Australian Energy Market Commission (AEMC) Reliability panel sets the reliability standards for the National Electricity Market (NEM). The standard is the expected amount of energy at risk of not being delivered to customers due to lack of available capacity. To meet this standard, AEMO determines the necessary spare capacity that must be available for each region including via transmission interconnection to provide buffer against unexpected demand spikes and generation failures. Reliability panel is conducting a separate review of the Reliability and Emergency Reserve Trader (RERT) scheme, which expired in June 2012. Following the unserved energy events in south east Australia during the heatwave in 2009, the panel proposed to make the RERT arrangements more flexible to better address the risk of short term generation capacity short falls. [10]

Governments, AEMO and Private Investors are working closely in building wind forms, solar power generation stations, Geothermal power plants and other micro-generation plants, to connect to the electricity declared shared network, which creates greater need for the reliability analysis of these systems. **The current research effort is related to the need to know the reliability of this system accurately, efficiently and fast**.

Many research projects [11-76] have been conducted to evaluate the reliability of electrical power systems. In reference [11] a critical review on the reliability impacts of major smart grid resources such as renewable, demand response, and storage is carried out. In this research the author emphasised the need for more research into the reliability of Smart Grid. A study done in China on the Reliability Analysis of Electrical Distribution System Integrated with Wind Power concluded that wind power would generally improve reliability of the distributed system [12]. In reference [13] reliability network equivalent techniques are introduced to simplify the calculations. By these techniques, if just some new generators are added to the system, the impact of generation system including Wind Energy Conservation System (WECS) on the load point reliability can be separately determined, while the effect of transmission system remains unchanged. In reference [14] reliability modelling of hybrid energy systems involving photo voltaic (PV) and wind energy conservation systems were investigated. Disadvantages of the traditional methods for evaluating reliability evaluation like deterministic and probabilistic techniques are discussed, an improved method known as well-being analysis has been applied for evaluation of reliability of the system applying Monte Carlo Simulation technique. The reliability and cost implications of PV and wind energy utilization in hybrid energy systems designed for Indian conditions are discussed.

Liberalization of the power system and increasing level of technological innovation, which brings higher and higher reliability values for components due to shortage of failure data. Due to very short product development times and tightened budgets, reliability tests are conducted with severe time constraints and no failure data occurs during these tests. In particular smart grid connected power systems are facing above reliability challenges. Therefore there is a need to develop new methods for reliability evaluation and enhancement of power systems based on not only on experimental data but also on technological and physical information related to wear and stress such as over voltages, loads and short circuits acting on the components, devices and by time behaviour of their hazard rate function. Most electronic components in power systems are affected by aging, their failure is mainly accidental, so aging failures in power systems is a big concern and a driving factor in system planning in many power systems. More and more power system components are reaching their end of life period; therefore ageing failure must be included in power system reliability evaluation.

Electrical systems failure depends on factors such as loads, redundancies and configurations. There are different mathematical models to evaluate the reliability of existing power systems but non-of the existing models provide accurate reliability. The aim of this research is to develop advanced algorithms which are computationally efficient, provide practically and accurately evaluate the reliability of the existing and future Smart- Grid applications of Victorian power systems.

The most important part of the research is to develop new methods for reliability evaluation and enhancement of power systems. In this thesis power system is considered as a system with different sub-systems, sub-systems with different components such as electrical, electronic, mechanical and software.

2.2 Reliability and Hazard Functions

Reliability is the probability that a product will operate or service will be provided properly for a specified period of time under the design operating condition without failure. Reliability of a system is analysed based on the reliability analysis of components of that particular system on the basis of failure data from devices in-service. Performing a direct reliability analysis requires that the most adequate probability distribution for the reliability analysis to be chosen from a family of commonly employed distributions for such components is selected on the basis of a combination of all these aspects. The most adopted reliability models for electrical components of any power system are Gamma, Normal, Lognormal and Weibull. Recently Inverse Gaussian distribution, the Inverse Weibull distribution, the Birnbaum-Saunders distribution, the Log-logistic distribution and more are significantly used for electrical components reliability in power systems.

Hazard function *h(t)* is the conditional probability of failure in time interval 't' to (t+*dt*), given that there was no failure at time 't' divided by the length of the time interval *dt*.

$$h(t) = \frac{f(t)}{R(t)} \tag{2.1}$$

Where f(t) is probability density function and R(t) is reliability function.

The cumulative hazard function H(t) is the conditional probability of failure in the interval 0 to 't'.

If the total number of failures during the time interval 0 to t.

$$H(t) = \int_0^t h(\tau)d\tau \tag{2.2}$$

Hazard function is also referred as hazard rate or instantaneous failure rate in reliability theory. It is very important for power system design engineers, repair and maintenance people. Hazard rate is a function of time and it is a bathtub-shaped function shown figure 2.1:

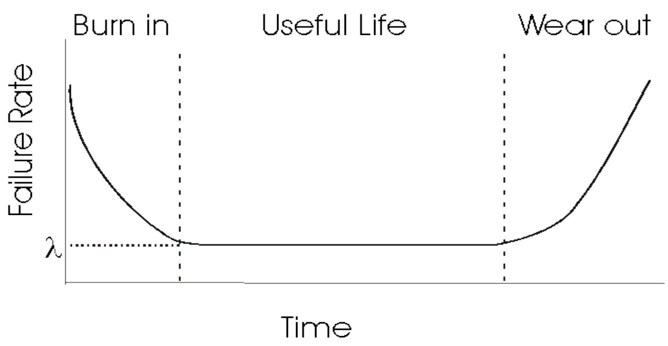

Figure 2.1: Bath tub shaped hazard rate function graph

The life of components follow three major periods:

1. Infant mortality period or decreasing failure rate period

$$h(t) = -\lambda t \qquad (2.3)$$

2. Useful life period or constant failure rate period

$$h(t) = \lambda \qquad (2.4)$$

3. Wear-out period or increasing failure rate period

$$h(t) = \lambda t$$

Many components in power systems exhibit constant failure rate during their lifetimes, this occurs at the end of the early failure region. Burn-in is performed by subjecting components to stress slightly higher than the expected operating stress for a short period in order to weed out the failure due to manufacturing defects. Most of the mechanical components in Power systems such as rotating shafts, valves and cams- exhibits linearly increasing hazards rate due to wear out, whereas components such as sprigs and elastomeric mounts exhibit linearly increasing hazard rate due to deterioration. Relays in power systems also exhibit linearly increasing hazard rate. Most power system components (both mechanical and electrical) exhibit decreasing hazard rates during their early lives.

When Hazard rate function [h(t)] cannot be represented linearly with time then Weibull model is used where

$$h(t) = \frac{\gamma}{\theta}\left(\frac{t}{\theta}\right)^{\gamma-1} \tag{2.5}$$

where $\theta \ and \ \gamma$ are life and share parameters of the distribution.

When components or products experience two or more failure modes then hazard rate is described by Mixed Weibull model. If the hazard function is initially constant and then begins to increase rapidly with time then exponential model is used, where

$$h(t) = be^{\alpha t} \tag{2.6}$$

Where b is constant and e^{α} represents the increase in failure rate per unit time. Most of the mechanical components in power systems, subjected to repeated cyclic loads exhibit normal hazard rates. But there is no closed form expression for the reliability or hazard rate functions. The CDF of the life of a component is represented by:

$$F(t) = P[T \leq t] = \int_{-\infty}^{t} \frac{1}{\sigma\sqrt{\pi}}\left[-\frac{1}{2}\left(\frac{\tau-\mu}{\sigma}\right)^{2}\right] d\tau \tag{2.7}$$

And

$$R(t) = 1 - F(t) \tag{2.8}$$

Where $\mu \, and \, \sigma$ are mean and the standard deviation of the distribution. Unlike other distributions, the integral of the cumulative distribution cannot be evaluated in a closed form.

The pdf for the standard normal distribution is:

$$\emptyset(z) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right) \qquad -\infty < z < \infty \tag{2.9}$$

Where $z = \frac{\tau-\mu}{\sigma}$

The CDF is $\emptyset(\tau) = \int_{-\infty}^{\tau} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right) dz$ (2.10)

Therefore, when the failure time of a component is expressed as a normal distributed random variable T, with mean μ and standard deviation σ, then the probability the component will fail by time t is given by

$$P(T \le t) = P\left(\frac{T-\mu}{\sigma} \le \frac{t-\mu}{\sigma}\right) = \emptyset\left(\frac{t-\mu}{\sigma}\right). \tag{2.11}$$

The right side of this equation can be evaluated using the standard normal distribution. The hazard rate function h(t) of normal distribution is

$$h(t) = \frac{f(t)}{R(t)} = \frac{\emptyset\left(\frac{t-\mu}{\sigma}\right)/\sigma}{R(t)}. \tag{2.12}$$

One of the most widely used probability distribution is describing the life of data resulting from a Semiconductor failure mechanism in power systems is lognormal distribution. It is also used in predicting accelerated life test data. The pdf of lognormal distribution is:

$$f(t) = \frac{1}{\sigma t\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{ln\, t-\mu}{\sigma}\right)^2\right] \qquad -\infty < \mu < \infty, \sigma > 0, t > 0. \tag{2.13}$$

The reliability is

$$R(t) = P[T > t] = P[z > \frac{\ln t - \mu}{\sigma} \tag{2.14}$$

Thus, the hazard rate function is

$$h(t) = \frac{f(t)}{R(t)} = \frac{\emptyset\left(\frac{ln\, t - \mu}{\sigma}\right)}{t\sigma R(t)} \tag{2.15}$$

Gamma distribution is another widely used hazard rate function. It has decreasing, constant, or increasing hazard rates. The gamma distribution is suitable for describing the failure time of a component whose failure take place in *n* stages or the failure time of a system that fails when *n* independent sub failures have occurred. The gamma distribution is characterised by two parameters: shape parameter γ and scale parameter θ. When $0 < \gamma < 1$, the failure rate monotonically decreases from infinity to $\frac{1}{\theta}$ as time increases from 0 to infinity. When $\gamma > 1$, the failure rate monotonically increases from $\frac{1}{\theta}$ to infinity, when $\gamma = 1$, the failure rate is constant and equal to $\frac{1}{\theta}$.

The pdf of a gamma distribution is $f(t) = \frac{t^{\gamma-1}}{\theta^{\gamma}\Gamma(\gamma)} e^{\frac{i}{\theta}}$ (2.16)

The reliability function $R(t) = e^{\frac{-t}{\theta}} \sum_{k-0}^{n-1} \frac{\left(\frac{t}{\theta}\right)^k}{k!}$ (2.17)

The hazard rate of the gamma model, when γ is an integer n is:

$$h(t) = \frac{\frac{1}{\theta}\left(\frac{t}{\theta}\right)^{n-1}}{(n-1)! \sum_{k=0}^{n-1} \frac{\left(\frac{t}{\theta}\right)^k}{k!}} \tag{2.18}$$

2.3 Standby Systems

Safety critical power systems including the applications of phased-mission systems use either active or standby redundancy to improve the mission reliability. In general, there are three types of standby configurations, i.e., cold, hot and warm standby configurations. Cold standby implies that the inactive redundant components have a zero failure rate and cannot fail while in standby state. Hot standby implies that the redundant component has the same failure rate as active components while in standby state. Warm standby implies that an inactive component has a failure rate between cold standby and hot standby. Warm standby components are partially powered up when they are in standby mode. Therefore, they have a reduced failure rate in the standby mode. However, they are subject to the regular full failure rate when they are kept in operation to replace the faulty primary components. As compared to hot sub-systems, warm sub-systems do not consume much power when they are in standby mode. As compared to a cold standby system, the warm standby system does not need long initialization and recovery time.

Warm sub-systems are commonly used in sensor networks, power generation, transmission and distribution systems using wind generators, tidal power generators, geothermal power system, solar power and other backup power generators. Warm standby components are subjected to different failure rates while they are in standby and operational modes. Such a state dependent failure behavior makes the reliability analysis of warm standby system a challenging task. Redundancy is an important concept in enhancing the reliability of systems but fault tolerant and safety critical system cannot achieve required reliability without using redundancy. Active redundancy and standby redundancy are two basic types of redundancies. As stated above standby redundancy is further classified as cold, warm and hot standby. An accurate analysis of reliability and related measures of a power system with redundant components/sub-system/systems is important to assess whether the power systems meet safety and reliability requirements and to determine the optimal redundancy configurations and other design alternatives [20].

2.4 Reliability of Phased Mission Systems

Power systems are everywhere in 21st century. The operation of mission encountered in power systems (nuclear power generation and geothermal power plants) involves several different tasks or phases that must be completed in sequence. The systems used in these missions are usually called phased mission systems (PMS). In PMS mission consists of multiple, consecutive, non-overlapping phases. For the mission to be a success, the system must operate successfully during each of the phases. In each phase, the system has to accomplish a specific task and may be subject to different stresses. Thus, the system configuration, success criteria, and component failure behaviour may change from phases to phase, the state of a component at the beginning of a new phase is identical to its sate at the end of the previous phase in a non-repairable PMS [39] . The consideration of these changes and dependencies poses unique challenges to existing reliability analysis methods. Over the past decade considerable research efforts have been expanded in the reliability analysis of PMS. In general there two classes of approaches to the evaluation of PMS: analytical modelling [39-63] and simulation. In analytical modelling there are three classes: state space oriented models, combinatorial methods, and a phase modular solution hybrid method. The state space oriented approaches are based on Markov chains and/or Petri nets. They are flexible and powerful in modelling complex dependencies among system components but they suffer from state explosion problem when modelling large scale power systems. Combinatorial model: Binary Decision Diagrams (BDD), Cut Sets and Disjoint reliability analysis methods suffer from combinatorial explosion due to dummy repeated variables. Reliability analysis of PMS by Simulation offers greater generality in system representation, but it is often more expensive in computational requirements [39]. This is particularly a concern with the crude Monte Carlo simulation for analysing safety-critical systems, especially those with ultra-high reliabilities often found in nuclear industry.

Therefore, all existing methods for PMS reliability analysis are limited to either small-scale problems (analytical methods) or non-critical systems with moderate reliability requirements (crude Monte Carlo simulation). New methods for reliability evaluation of phased mission systems to overcome the existing issues are investigated in this thesis.

2.5 K-out-of-n Systems

The k-out-of-n system structure has wide range of applications in reliability engineering. It is a common practice to use redundancy techniques to improve the system reliability and availability. A system will be working as long as k components are working in a system out of n components. If ‘k+1’ components fail out of ‘n’ components then the system will fail [8]. For example 1-out-of-4 remote area power system, at least one of the solar panels must be working out of 4 panels for the power system to function.

The reliability of a k-out-of –n system with identical components is evaluated by using binomial distribution.

$$R(t) = \sum_{i=k}^{n} \binom{n}{i} p^{i}(t).q^{n-i}(t) = \sum_{i=0}^{n-k} \binom{n}{i} p^{n-i}(t).q^{i}(t) \qquad (2.19)$$

Where p(t), q(t), and f(t) are the reliability, unreliability, failure (hazard) rate, and probability density function (pdf) of each component at time ‘t’.

In [16] several algorithms to compute the reliability of k-out-of-n system with non-identical components are proposed which have $O(n.(n-k+1))$ computational complexity and requires less memory than other algorithms proposed in other research papers. In [14] efficient reliability evaluation algorithms for binary k-out-of-n system with independent component is provided as:

$$R(n,k) = p_n.R(n-1,k-1) + q_n.R(n-1,k) \qquad (2.20)$$

Where R(n,k) is the recursive function to evaluate reliability of k-out-of-n system. p_n is the reliability of component n, and $q_n=1-p_n$.

The boundary conditions are:

$$R(n,0) = 1,$$

$$R(n,k) = 0, for\ 0 < n < k.$$

In [20] binary k-out-of-n system has been generalized with binary weighted k-out-of-system, with a recursive equation shown below:

$$R(i,j) = p_j.R(i-u,j-1) + q_i.R(i,j-1) \quad (2.21)$$

Where *R(i,j)* is the probability that the system with 'j' components can output a total weight of at least 'i'. The boundary conditions are:

$$R(i,j) = 1, \quad for\ i \leq 0, \qquad j \geq 0,$$

$$R(i,0) = 0, for\ i > 0,$$

There may be more than two different performance levels in some practical systems such as: a power generator in a power station can work at full capacity, which is its nominal capacity, say 100MW, when there are no failures at all. Certain type of failures can cause the generator to fail completely, while other failures will lead to the generator working at reduced capacity say 40MW. On the system level, it can be considered that the power generating system consisting of several power generators. The abilities of the system to meet high power load demand, normal load demand and lower power load demand can be regarded as different system states. The reliability evaluation of such system is done through multi-state k-out-of-n system modelling and evaluation. In [15] the first multi-state *k-out-of-n* system model is defined. Here the system state was defined as the state of the k^{th} best component. At any state *j*, for the system to be in state *j* or above, there should be at least *k* components in state *j* or above. That is, the *k* value is the same with respect to all states. In[16] a generalized multi-state *k-out-of-n:G* system model was proposed. In this model there can be different *k* values with respect to different states. In [17] an efficient recursive algorithm for reliability evaluation of generalized multi-state *k-out-of-n* system with identically and independently distributed (iid) components. This model has only a few practical application.

In [18] another multi-state weighted k-out-of-n model is proposed with more practical applications. This model has more flexibility in modelling systems involving *weighted-k-out-of-n* structure. In [19] Universal Generating Function (UGF) approach is developed to evaluate multi-state systems. In the binary weighted k-out-of-n system, UGF for the components is:

$$U_i(z) = (1 - p_i)z^{0\ X\ u_i} + p_i z^{u_i} \tag{2.22}$$

To obtain the UGF of the system based on the individual UGF of the components, the following composition operator Ω is used:

$$U_s(z) = \Omega(U_1(z), U_2(z), \ldots., U_n(z)) \tag{2.23}$$

Where

$$\Omega\big(U_1(z) \ldots U_k(z), U_{k+1}(z) \ldots., U_n(z)\big) = \Omega\big(U_1(z) \ldots U_{k+1}(z), U_k(z), \ldots., U_n(z)\big) \tag{2.24}$$

$$\Omega\big(U_1(z) \ldots U_k(z), U_{k+1}(z) \ldots., U_n(z)\big) = \Omega(U_1(z) \ldots U_k(z)), \Omega(U_{k+1}(z), \ldots., U_n(z))) \tag{2.25}$$

$$\Omega\big(U_1(z) \ldots U_k(z)\big) = \Omega\left[\sum_{j=1}^{J} p_{1j} z^{g1j} . \sum_{l=1}^{L} p_{2l} z^{g2l}\right] = \sum_{j=1}^{J}\sum_{l=1}^{L} p_{ij} p_{2lz}{}^{(g_{1j}+g_{2l})} \tag{2.26}$$

From the above equation system reliability is as shown below for an arbitrary k using an operator δ_A :

$$R_s = \delta_A(U_s(z), k) = \delta_A(\textstyle\sum_{k=1}^{K} p_k z^{G_k}, k) = \sum_{k=1}^{K} p_k \alpha\,(G_k - k) \tag{2.27}$$

Where $\alpha(x)$ in the above equation is:

$$\alpha(x) = \begin{cases} 1\,, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

The recursive algorithm for reliability evaluation of the multistate weighted k-out-of-n systems given for two models. Recursive function for the probability of the system to be in stat j or above as $R_j^I\big(k_j, i\big)$ and the recursive function for the probability of the n component system to have sum of useful weights of at least k when one is evaluating the probability for the system to be in state j or above as $R_j^{II}(k,n)$

$$R_j^I(k_j,i) = \sum_{r=0}^{r=M} p_{i,r}.R_j^I(k_j - w_{i,r}, i-1) \tag{2.28}$$

Where

$$R_j^I(k,0) = 0, when\ 0 < k \le k_j$$

$$R_j^I(k,i) = 1, when\ i \ge 0\ and\ k \le 0$$

The UGF of each multi-state component is given by:

$$U_i(z) = p_{i,0}z^{\,w_{i,0}} + p_{i,1}z^{\,w_{i,1}} \ldots \ldots .\, p_{i,M}z^{\,w_{i,M}} \tag{2.29}$$

$$R_j^{II}(k_j,n) =$$

$$q_{n,j}.R_j^{II}(k_{j,n}, n-1) + \sum_{r=j}^{r=M} p_{n,r}.R_j^{II}(k_j - w_n, n-1) \tag{2.30}$$

Where

$$R_j^{II}(j,0) = 0, \quad for \quad j = 1,2,3, \ldots \ldots .\, k_j$$

$$R_j^{II}(k,i) = 1, for \;\; k \le 0 \quad and \quad i = 0,1,2, \ldots .\, n.$$

The UGF for the individual component is as shown below:

$$U_i(z) = q_{i,j}z^0 + p_{i,j}z^{\,w_{i,j}} + \cdots \ldots .\, p_{i,k}z^{\,w_{i,k}} \ldots \ldots .\, p_{i,M}z^{\,w_{i,M}} \tag{2.31}$$

Where

n: the number of components in the system

M: the highest possible state of each component

$w_{i,j}$: the weight of component I when it is in state j

$p_{i,j}$: P_r{Component i is in state j}

$q_{i,j}$: P_r{Component i is in state below j}, $q_{i,j} = \sum_{l=0}^{j-1} p_{i,l}$.

k_j the minimum total weigh required to ensure that the system is in state j or above.

In [17] an example to illustrate the modelling of power system as a decreasing multi-state k-out-of-n:G model is shown. In this example three power generators are considered. Each generator is treated as a component and there are 3 components in this system. Each generator may be three possible states, 0, 1, and 2. When a generator is state 2, it is capable of generating 10 MW; in state 1, 2MW; and in state 0, 0MW. The total power output of the system is equal to the sum of the power output from all three generators. The system may also be in three different states: 0,1, and 2. When the total output is greater than or equal to 10MW, in state 1; otherwise, in state 0. The reliability of the cluster of power generators in Smart-Grid can be calculated with the help of formulae shown above. The methods for analysing and evaluating the reliability of ‘k-out-of-n’ systems shown above are complex. In this thesis new methods have been proposed. These methods are simple, computationally efficient and accurate [90].

2.6 PHM and Its Significance To This Research

Modern Power Systems' management requires the accurate assessment of current and the prediction of future health condition is crucial in the era of Smart-Grid. Suitable mathematical models that are capable of predicting Time-to-Failure (TTF) and the probability of failure in future are very important. The life of power system is influenced by different risk factors called covariates. The basic idea in reliability theory is the failure time of a system and its covariates. These covariates change stochastically, may influence and indicate the failure time of power systems. Until now, a number of statistical models have been developed to estimate the hazard of a system with covariates in reliability field. Most of these models are developed based on the Proportional Hazard Model (PHM) theory which was proposed by cox [51]. This model provides an estimate of the maintenance effect on survival after adjusting for other explanatory variable. It allows the engineers to estimate the failure (hazard) of a component or sub-system or system, given their predictive variables. Cox's PHM for statistical explanatory variable is expressed as $h(t,\Omega) = h_0(t)\varepsilon(\tau\Omega)$. Where, $h_0(t)$ is the unspecified baseline hazard function which is dependent on time only and without influence of covariates. The positive function term, $\varepsilon(\tau\Omega)$, is dependent on the effects of different factors, which have multiplicative effect on the baseline hazard function.

The proportionality assumption in PHM is that:

$$\frac{h(t;\Omega_X)}{h(t;\Omega_Y)} = \frac{h_0(t)\, exp(\Omega_X\tau)}{h_0(t)\, exp(\Omega_Y\tau)} = exp[\tau(exp(\Omega_X - \Omega_Y))] \qquad (2.32)$$

The hazard at different 'Ω' values are in constant proportion for all '$t > 0'$, hence the name PHM. There are several research papers dealing with PHM and Load-Sharing systems separately, but no research has been undertaken to evaluate the reliability of load-sharing systems subjected to PHM. In this thesis the research work undertaken to solve the problem of aging load-sharing systems.

2.7 Reliability and Cost Analysis Basics

Australian Energy Market Commission stated that by reducing the reliability standards for NSW state electricity distribution network, the government can save up to A$2.5 billion over 15 years and consumers can benefit by getting cost reduction in their electricity bills. The reliability of a system can be improved by installing additional components. The customer interruption costs in these cases will decrease as the capital and operating costs increase. The main objective is to balance the benefits realized from providing higher reliability and the cost of providing it. A major objective of reliability cost assessment is to determine the optimum level of service reliability. This basic concept is shown in figure 2.2. It is shown in the figure that the utility cost increases while the socio-economic customer interruption cost decreases with increase in the level of service reliability. The total cost is the sum of the two curves. The optimum level of reliability occurs at the point of lowest total cost.

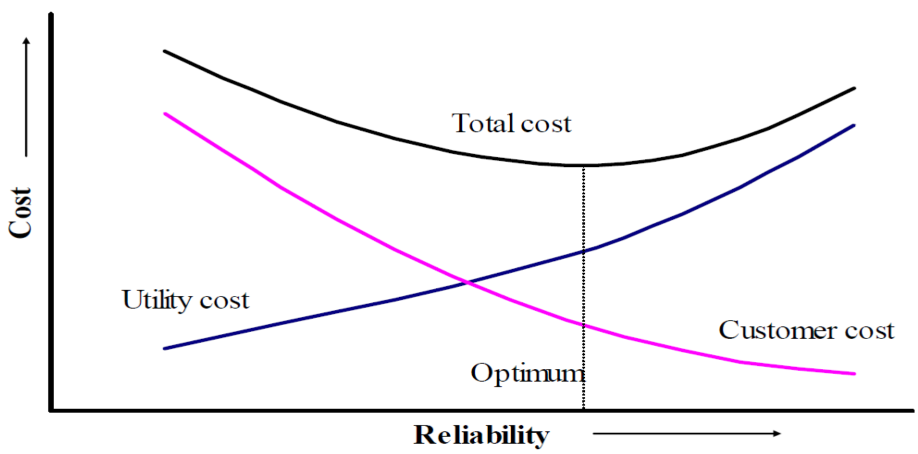

Figure 2.2 Components of Reliability and Cost

2.8 Conclusions

In this chapter the concepts related to the developments in power system reliability have been discussed. Reliability hazard functions, stand by systems, phased mission systems, 'k-out-of-n' systems and proportional hazards model have been evaluated critically, and the appropriateness of these concepts to this research work have been justified.

CHAPTER 3

RELIABILITY OF K-OUT-OF-N COLD STANDBY SYSTEMS WITH RAYLEIGH DISTRIBUTIONS[3]

3.1. Introduction

Cold standby redundancy is used as an effective mechanism for improving system reliability [21]. For example, applications of cold standby redundancy can be found in space explosion and satellite systems [22], electrical power systems [23], telecommunication systems [24], textile manufacturing systems [25], and carbon recovery systems [26]. Cold standby redundancy involves the use of redundant components that are shielded from the operational stresses associated with system operation. Without exposure to those stresses, the likelihood of failure is very low, and assumed to be zero, until the component is required to operate as a substitute for a failed component [21]. When a failure does occur, it is necessary to detect the failure and to activate the redundant component. For a non-repairable system, the failure detection and switching must be accomplished by additional system hardware that would not otherwise be required. When switching mechanisms are perfect, standby redundancy can provide higher system reliability compared to active redundancy with analogous system architecture [21, 27]. However, when switching mechanisms are imperfect, cold standby redundancy may not necessarily provide higher system reliability than the corresponding active redundancy system [8]. Therefore, for analysing the reliability of cold

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[3] Contents of this Chapter have been presented at 18th ISSAT International conference on Reliability and Quality Design. July, 2012, **Boston USA.**

standby systems, it is important to consider switching failures [21, 27-29].

When component failure times follow a non-exponential distribution and the system requires multiple operating components for its success ($k > 1$), then the successive failures of the k-out-of-n cold standby system do not follow any standard stochastic process [7, 9]. This is because, at any given time during the mission, the system can have multiple working components with different operational times (operational ages). Therefore, to calculate the probability of another failure during the mission, the operational ages of all working components must be considered. In other words, at any given time, the next failure in the system occurs with a rate equal to the sum of the hazard rates of all working components. Hence, to calculate the occurrence rate of the next failure, the ages of all working components must be known. Therefore, the direct evaluation of system reliability considering the sequences of component failures involves multiple integral equations. However, efficiently evaluating the multiple integral equations is still a challenging task [30]. It not only involves huge computational times but also is prone to numerical round-off errors. The inherent complexity of this direct method is described in section 4 using an example of a 2-out-of-4 cold standby system.

To avoid the difficulties associated with multiple integral equations and numerical round-off errors, we use a counting process-based method for evaluating system reliability. This method was first proposed in [31] and later generalized in [27] to handle non-identical components, warm standby systems, and switch failures. According to this method, the k operating components are considered to be at k logical locations. The key concept used in this method is that as long as the system is operating, the failure processes in all logical locations are independent. Therefore, we can analyse each logical location independently and combine their state probabilities to find the system state probabilities. Once we find the system state probabilities, we can find the system reliability as the sum of the probabilities of all success states.

In the counting process-based method, we need to find the probability of a given number of failures in a logical location considering that it acts as a 1-out-of-(n-k+1) cold standby system. This calculation involves the computation of convolution integrals. Although this computation is simpler than multiple integrals, it still requires the use of numerical integration methods for general component failure time distributions. To avoid the explicit use of numerical integrations, we consider Rayleigh distributions for component failure times and an approximate formula for computing the cumulative distribution function of sum of Rayleigh distributed random variables. The main advantage of considering the Rayleigh distribution is that it can be used for modeling component lives that exhibits a linearly increasing hazard rate. Most mechanical components, such as rotating shafts, valves, and cams, exhibit linearly increasing hazard rates. Similarly, some electrical components such as relays exhibit linearly increasing hazard rate [32].

In this Chapter, Rayleigh distributions are considered for component lives, counting process-based method for analysing k-out-of-n cold standby systems is proposed and demonstrated. This method also considers the effects of switch failures on system reliability.

3.2. Rayleigh Distribution

The Rayleigh distribution has a linearly increasing hazard rate. Therefore, the hazard rate of the Rayleigh distribution is expressed as [32]:

$$h(t) = \lambda t \tag{3.1}$$

where λ is a constant. The probability density function (pdf), $f(t)$, and cumulative distribution function (cdf), $F(t)$, are obtained as:

$$f(t) = \lambda t \exp\left\{-\frac{\lambda t^2}{2}\right\} \tag{3.2}$$

and

$$F(t) = 1 - \exp\left\{-\frac{\lambda t^2}{2}\right\} \tag{3.3}$$

The reliability function, *R(t)*, is:

$$R(t) = \exp\left\{-\frac{\lambda t^2}{2}\right\} \tag{3.4}$$

The Rayleigh distribution can be expressed in other forms. By substituting $\eta = \sqrt{2/\lambda}$ (or $\lambda = 2/\eta^2$), the reliability function can be expressed as:

$$R(t) = \exp\left\{-\left(\frac{t}{\eta}\right)^2\right\} \tag{3.5}$$

Similarly, substituting $\lambda = 1/\theta$, the reliability function can be expressed as:

$$R(t) = \exp\left\{-\frac{t^2}{2\theta}\right\} \tag{3.6}$$

In this Chapter, the Rayleigh distribution with parameter λ as in (21)-(24) is expressed.

3.2.1 Sum of Rayleigh Random Variables

In the proposed method, we need to calculate the distribution of sum of Rayleigh distributed random variables. The distribution of this sum can be found using the convolution integrals [27]. The distribution of the sum of two Rayleigh distributed random variables exists in closed-form [33]; however, for an arbitrary sum, there is no closed-form solution. As a result, numerical evaluations and approximations must be used [34]. Many different approaches have been proposed to compute the distribution of sum of Rayleigh random variables. They include bounds, infinite series representations, published tables, and cdf curves. A widely used approximation for the cdf of the sum of L independent and identically distributed (i.i.d.) Rayleigh random variables with parameter λ is [34, 35]:

$$F_Z(z) = \Pr\{Z \le z\} \approx 1 - \exp\left(-\alpha t^2\right)\left(\sum_{i=0}^{L-1} \frac{\left(\alpha t^2\right)^i}{i!}\right) \tag{3.7}$$

where

$$
\begin{aligned}
t &= \frac{z}{\sqrt{L}} \\
\alpha = \alpha(L,\lambda) &= \frac{L\lambda}{2\cdot\left[(2L-1)!!\right]^{1/L}} \qquad (3.8) \\
(2L-1)!! &= (2L-1)(2L-3)\cdots 3.1
\end{aligned}
$$

The double factorial in (8) can be expressed in terms of the factorial functions:

$$(2L-1)!! = \frac{(2L-1)!}{2^{L-1}(L-1)!} \qquad (3.9)$$

Note that the approximation in (27) is in the form of Nakagami cumulative distribution function (cdf) with shape parameter $\mu = L$ and scale parameter $\omega = L/\alpha$. Hence,

$$F_Z(z) = \Pr\{Z \le z\} \approx F_N\left(\frac{z}{\sqrt{L}}; L, \frac{L}{\alpha}\right) \qquad (3.10)$$

Further, if X follows Nakagami distribution with parameters μ and ω, then $Y = X^2$ follows gamma distribution with scale parameter $\theta = \omega/\mu$ and shape parameter $k = \mu$. The cdf of gamma distribution can be expressed in terms of the regulated gamma function, which can be evaluated using incomplete gamma functions. Therefore, the approximate cdf of sum of Rayleigh distributed random variables can be obtained as:

$$F_Z(z) = \Pr\{Z \le z\} \approx I\left(L; \frac{\alpha\cdot z^2}{L}\right) \qquad (3.11)$$

where $I(a,x)$ is the regulated gamma function. Because the regulated gamma function (or the gamma distribution itself) is available in several standard statistical or mathematical libraries, we can use (31) to compute the cumulative distribution function of the sum of Rayleigh distributed random variables.

3.3. System Description and Assumptions

For The proposed method is based on the following system description and assumptions:

1. There are a total of *n* identical components in the system.
2. Initially *k* components are operating, and the remaining (*n*-*k*) components are in cold standby.
3. The lifetime (failure time) of a component in operation follows a Rayleigh distribution.
4. Components cannot fail while they are in the standby mode. In other words, the failure rate of a component in the standby mode is zero.
5. Immediately upon the failure of an operating component, the component is replaced by one of the standby components in the queue.
6. Switches are used to replace the failed component with one of the standby components, and the switches themselves can fail to operate on demand.
7. The replacement of the component is successful only if the switching mechanism is successful.
8. The system is operational during the mission when there is *k* operating components.

Although we restrict our focus to the case of identical components and cold standby configurations, the counting process-based method used in this Chapter can be applied to the cases of non-identical components as well as warm standby configurations [27]. In this Chapter, using well known closed-form approximations to Rayleigh distributions, we provide an efficient approximate method for evaluating the reliability of *k*-out-of-*n* cold standby systems. Once the basic concepts of the method are understood, reliability engineers and practitioners can refer to [27] for evaluating the reliability of complex standby configurations.

3.4. Complexity of Direct Method

To demonstrate the complexity of system reliability evaluation using a direct method based on sequence of failure events, we consider a 2-out-of-4 cold standby system with perfect switches. The system has a total of 4 components, and it will be in operation as long as there are two good components. In other words, the system reaches a failed state at the event of third component failure. Initially, components 1 and 2 are in operation, and components 3 and 4 are in cold standby. Upon the first component failure due to the failure of either of the working components (component 1 or component 2), component 3 will be kept in operation. Upon the failure of the next component, component 4 will be kept in operation. Therefore, the system reaches a failed state due to one of the following disjoint sequences of failures:

(1) $x_1 < x_2 < x_3$

(2) $x_1 < x_2 < x_4$

(3) $x_1 < x_3 < x_2$

(4) $x_1 < x_3 < x_4$

(5) $x_2 < x_1 < x_3$

(6) $x_2 < x_1 < x_4$

(7) $x_2 < x_3 < x_1$

(8) $x_2 < x_3 < x_4$

Where x_i is the failure time of component i. It is equivalent to the sum of both operational and standby times of component i at the time of its failure. Because the failure sequences are disjoint, we can find the system unreliability as the sum of probabilities of these failure sequences. However, the method has several disadvantages.

The first disadvantage of this method is that when $k > 2$, the number of sequences with distinct probabilities increases exponentially with (n-k+1) value even when the components are identical. For the 2-out-of-4 system, when the components are identical, the probabilities of sequences (1), (2), (3), and (4) are equivalent to the probabilities of sequences (5), (6), (7), and (8) respectively. However, we still need to find the probabilities for four distinct sequences: (1), (2), (3), and (4). In general, the number of such distinct sequences increases exponentially. For the k-out-of-n system with identical components, the number of distinct sequences is equal to $(s!) \times (k^m)$ where $s = \min\{k, n-k+1\}$ and $m = \max\{0, n-2k+1\}$. Therefore, the computational time for evaluating system reliability increases exponentially with the system size.

The second disadvantage of this method is that the probability calculation of each of these sequences involves multiple integrals that are difficult to solve. The third disadvantage of this method is that, for each sequence, the failure times of components must be tracked down to find valid ranges for the integration limits. These are explained further by developing the equations for each of the failure sequences.

Let t_i be the operational time of component i at the time of its failure. Note that t_i is different from x_i. For example, if component i is kept in the operation at time 100 hours (after the beginning of the mission) and it fails at time 250 hours, then t_i = 150 hours and x_i = 250 hours. For sequence (1), we have: $t_1 = x_1$, $t_2 = x_2$, and $t_3 = x_3 - x_1$. The last event in this sequence occurs at x_3. Hence, the sequence can occur within the mission time t, when $x_3 < t$. The graphical representation of this sequence is shown in Figure 1.

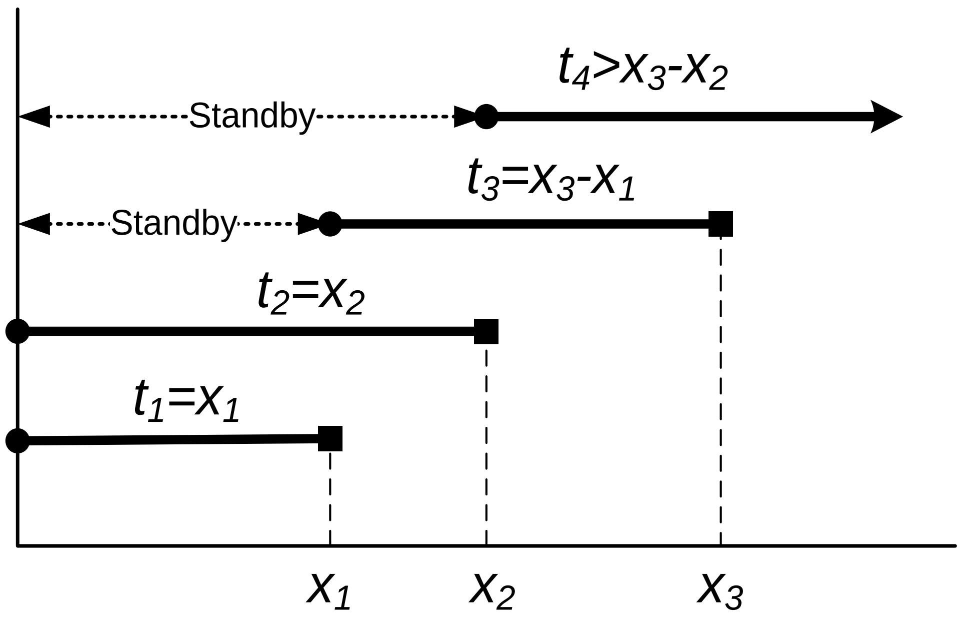

Figure 3.1 – Graphical Representation of Sequence (1)

Let $Q_i(t)$ be the probability of sequence i occurring within the mission time. To calculate this probability, for each sequence, we should determine the valid ranges for the operational times of the components. For sequence (1), we have: $x_1 < x_2 < x_3$. Therefore, valid ranges for the operational times associated with this sequence are:

- $0 < t_1 < t$
- $t_1 < t_2 < t$
- $t_2 - t_1 < t_3 < t - t_1$
- $t_4 > x_3 - x_2 = t_1 + t_3 - t_2$

Hence, the probability of this sequence occurring within the mission time is:

$$Q_1(t) = \int_0^t f_1(t_1) \int_{t1}^t f_2(t_2) \int_{t2-t1}^{t-t1} f_3(t_3) \int_{t1+t3-t2}^{\infty} f_4(t_4) dt_4 dt_3 dt_2 dt_1 \tag{3.12}$$

where $f_i(t)$ is the pdf of failure time of component i. This equation can be simplified as:

$$Q_1(t) = \int_0^t f_1(t_1) \int_{t1}^t f_2(t_2) \int_{t2-t1}^{t-t1} f_3(t_3) R_4(t_1 + t_3 - t_2) dt_3 dt_2 dt_1 \tag{3.13}$$

where $R_i(t)$ is the reliability function of component i. When all components are statistically identical with common pdf, *f(t)*, and reliability function, *R(t)*, we have:

$$Q_1(t) = \int_0^t f(t_1) \int_{t1}^{t} f(t_2) \int_{t2-t1}^{t-t1} f(t_3) R(t_1 + t_3 - t_2) dt_3 dt_2 dt_1 \quad (3.14)$$

Similarly, the graphical representation of sequence (2) is shown in Figure 2. The corresponding formula for $Q_2(t)$ is shown in equation (3.15).

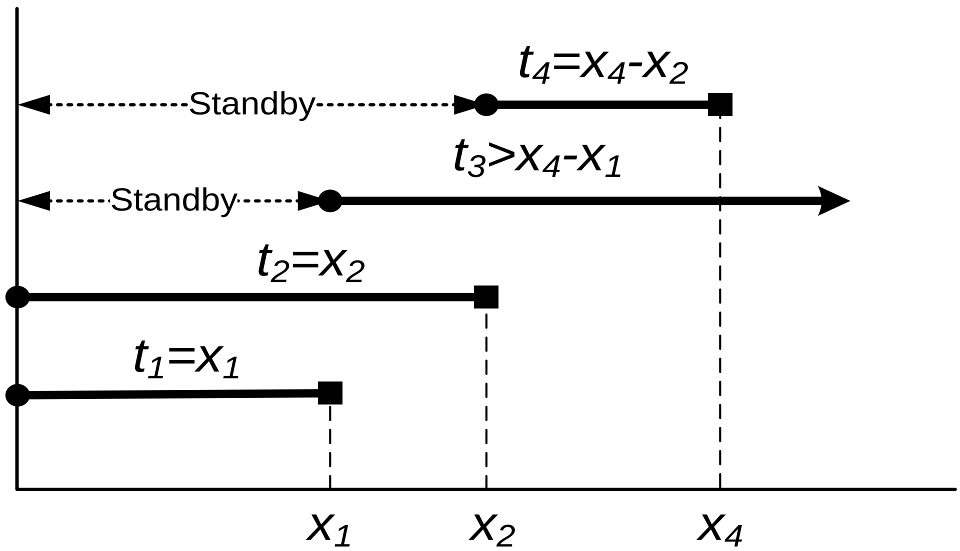

Figure 3.2 – Graphical Representation of Sequence (2)

From Figure 2, the probability of sequence (2) is:

$$Q_2(t) = \int_0^t f(t_1) \int_{t1}^{t} f(t_2) \int_{0}^{t-t1} f(t_4) R(t_4 - t_2) dt_4 dt_2 dt_1 \quad (3.15)$$

Similarly, the formulas for $Q_i(t)$ for sequences (3) and (4) are shown in equations (3.16) and (3.17) respectively.

$$Q_3(t) = \int_0^t f(t_1) \int_0^{t-t1} f(t_3) \int_{t1+t3}^{t} f(t_2) R(t_2 - t_1 - t_3) dt_2 dt_3 dt_1 \quad (3.16)$$

$$Q_4(t) = \int_0^t f(t_1) \int_0^{t-t1} f(t_3) \int_0^{t-t1-t3} f(t_4) R(t_1 + t_2 + t_3) dt_4 dt_3 dt_1 \quad (3.17)$$

Further, when the components are identical, we have: $Q_5(t) = Q_1(t)$, $Q_6(t) = Q_2(t)$, $Q_7(t) = Q_3(t)$, and $Q_8(t) = Q_4(t)$. Once we compute these probabilities, we can find the system reliability as shown in equation (3.18).

$$R_{Sys}(t) = 1 - \sum_{i=1}^{8} Q_i(t) \tag{3.18}$$

The evaluation of $R_{Sys}(t)$ requires computing $Q_i(t)$, which involves multiple integrals as shown in equations (3.14) through (3.17). However, the evaluation of multiple integral equations is still a challenging task [30]. It not only requires huge computational times but also is prone to numerical round-off errors. Therefore, the direct method is not practical for evaluating the reliability of *k*-out-of-*n* cold standby systems.

3.5. Proposed Method

In this section, we describe the basic concepts and theoretical background of the proposed method for evaluating the reliability of *k*-out-of-*n* cold standby systems. The effects of switch failures are considered in subsection 3.5.3.

3.5.1 Counting Process Based Method

The proposed method is based on a counting process. This method was first proposed in [31]. This method is also described in the famous textbook *Mathematical Theory of Reliability* by Barlow and Proschan [16, p. 175]. The method is later generalized in [27] to handle non-identical components, warm standby systems, and switch failures. In this method, we assume that the operating components are kept at *k* logical locations or positions. At any location, after the failure of the operating component, it is replaced by a standby component.

Therefore, the total number of failures in the system is the sum of the failures at all locations. The probability of a given number of failures at each location is calculated assuming that the failure process at each location is independent of the failure processes (number of failures) at other locations. Because of this independence assumption, the computation of these probabilities becomes simple. Using these probabilities, we calculate the probability of a given number of failures in the whole system. When the switches are perfect, the system is operational as long as there are k good components. In other words, the system is considered to be operating if the total number of failures in the system is less than or equal to (n-k). Because we already calculated the probability of a given number of failures (say i failures) in the system, we can calculate system reliability by adding these probabilities for the allowed range of component failures, i.e., i = 0 to (n-k).

The key assumption that simplifies the system reliability evaluation is the independence of the failure processes at different logical locations. Therefore, it is important to understand the validity of this assumption to appreciate and accept the proposed method. Strictly speaking, the failure processes at different logical locations are not independent of each other because they all share the common pool of standby components (spares). However, such a dependency needs to be considered only if there is a shortage of spares. As long as there is no shortage of spares, the failure processes at different logical locations are independent. In the counting process-based method, we consider only those cases where there is no shortage of spares. Hence, the independence assumption used in the proposed method is valid.

To explain this concept further, assume that an infinite number of spares exist (n is infinity). Therefore, irrespective of the number of failures at other locations, after a failure of any operating component in a location, the component is immediately replaced by a spare. If all spares are identical, at each replacement, we use the same type of spare. Because the failed components are always replaced by a spare, the failure process at any location is independent of the number of failures at other locations when all spares are identical. Therefore, in this case, we can analyse each location independently to find the probability of a given number of failures in that location. Using these probabilities, we can find the probability of a given number of total failures in all locations. Using the same concept, we can analyse each location independently as long as the failed components are replaced by a spare, i.e., as long as there is no shortage of spares. The shortage of spares occurs only when the total number of failures is greater than (n-k). Therefore, as long as the total number of failures is less than or equal (n-k), we can analyse each location independently.

3.5.2 System Reliability Analysis

In this Chapter, we considered that all components are statistically identical. Thus, all logical locations are not only independent but also identical. Therefore, effectively we need to analyse only one location. Let Y_i be the failure time of the i^{th} component used in a logical location. Because the components are identical, each Y_i has the same Rayleigh failure time distribution. Let Z_i be the cumulative operational times of all components up to the i^{th} failure. Therefore,

$$Z_i = Y_1 + \cdots + Y_i \tag{3.19}$$

Let $G_i(t) \equiv \Pr\{Z_i < t\}$ be the cdf of Z_i. It is also equivalent to the probability that there are at least i failures in the logical location during the mission time. Note that Z_i is the sum of independent and identically distributed Rayleigh distributions. Hence, $G_i(t)$ can be calculated from equation (27). In addition, $G_i(t)$ can be calculated using regulated gamma function.

$$G_i(t) = I\left(i, \frac{\alpha(i,\lambda)\cdot t^2}{i}\right) \tag{3.20}$$

Here, $\alpha(i,\lambda)$ indicates that α is a function of i and λ. Therefore, while calculating α using equation (3.8), we should use $L = i$. Specifically, we have:

$$\alpha(i,\lambda) = \frac{i\lambda}{2 \cdot \left[(2i-1)!!\right]^{1/i}} \tag{3.21}$$

Note that by definition, we have: $G_0(t) = 1$. Let $p_i \equiv p_i(t)$ be the probability that there are exactly i failures within the mission time t in a logical location. Therefore,

$$p_i = G_i(t) - G_{i+1}(t) = I\left(i, \frac{\alpha(i,\lambda)\cdot t^2}{i}\right) - I\left(i+1, \frac{\alpha(i+1,\lambda)\cdot t^2}{i+1}\right) \tag{3.22}$$

Let $P_i \equiv P_i(t)$ be the probability that there are exactly i component failures in the system during the mission time. Note that the total number of component failures in the system is equal to the sum of the failures at all logical locations. This probability can be calculated using discrete convolution functions. Let $H(m,i)$ be the probability that there are exactly i failures in the first m locations. By definition, we have: $H(1,i) = p_i$. For m=2 to k, we can calculate $H(m,i)$ using the following recursive discrete convolution formula:

$$H(m,i) = \sum_{j=0}^{i} p_j \cdot H(m-1, i-j), \qquad i \le n-k \tag{3.23}$$

According to equation (3.15), we can experience i failures in the first m locations when there are j ($0 \le j \le i$) failures in the m^{th} location and (i-j) failures in the previous (m-1) locations. Further, we have:

$$P_i = H(k,i) \tag{3.24}$$

Finally, system reliability is calculated by summing the probabilities of all success states. Because the system is successful when the number of failures is less than or equal to (n-k), we have:

$$R_{Sys}(t) = \sum_{i=0}^{n-k} P_i \tag{3.25}$$

3.5.3 Switch Failures on Demand

In this section, how to add the effects of switch failures on demand (on request) to the reliability analysis is shown. Consider that, at any time the switch is required, there is a constant probability, p_{sw}, that the switch will be successful. In other words, the switch failure probability is (1-p_{sw}). If there are exactly i failures in the system, the switch needs to perform its operation successfully for all i requests. Hence, the switch probability of success for i requests is $(p_{sw})^i$. In the proposed method, when switches are perfect, system reliability as the sum P_i values is calculated, where P_i is the probability of exactly i failures in the system. When switches are imperfect, then these probabilities need to be multiplied with switch success probabilities. Hence, system reliability is:

$$R_{Sys}(t) = \sum_{i=0}^{n-k} \left(p_{sw}\right)^i P_i \tag{3.26}$$

3.6. Numerical Illustration

To illustrate the proposed method, a cold standby 4-out-of-10 system is studied [27]. The failure distribution of the components is Rayleigh with λ = 2.0E-6 (Weibull with η = 1000 and β = 2.0). Mission time is t = 1000. Switch success probability on demand is 0.95. In this example, k = 4 and n = 10. The steps involved in evaluating system reliability are:

1. Calculate G_i values: $G_0 = 1$ and G_i for i = 1 to (n-k+1) is calculated from (3.20) and (3.21). In this Chapter, G_i in (20) is calculated using MATLAB *gammainc* function.
2. Using G_i values, calculate $p_i = G_i - G_{i+1}$ for i = 0 to (n-k).
3. Set $H(1,i) = p_i$ for i = 0 to (n-k). Then, using equation (3.23), calculate $H(m,i)$ for m = 2 to k and i = 0 to (n-k).
4. Set $P_i = H(k,i)$ for i = 0 to (n-k).
5. Calculate system reliability using equation (26).

All calculations involved in the above procedure are simple. The results obtained at each step of the reliability evaluation are provided in Table 1.

Step	1	2(3.1)	3.2	3.3	4(3.4)	5.1
i	$G_i(t)$	$H(1,i)$ $= p_i$	$H(2,i)$	$H(3,i)$	$H(4,i)$ $= P_i$	$P_i \cdot (p_{sw})^i$
0	1.0	0.368	0.135	0.050	0.018	0.018
1	0.632	0.518	0.381	0.210	0.103	0.098
2	0.115	0.106	0.346	0.339	0.239	0.216
3	0.008	0.008	0.116	0.263	0.295	0.253
4	3.1e-4	3.0e-4	0.020	0.107	0.213	0.174
5	7.1e-6	7.0e-6	0.002	0.026	0.096	0.074
6	1.1e-7	1.1e-7	1.0e-4	0.004	0.029	0.021
7	1.2e-9	1.2e-9	---	---	---	---
Step 5.2: Sum (Reliability)					0.993	0.854

Table 3.1 – Reliability Evaluation Steps

The last row of the table includes the final system reliability values. With perfect switches, system reliability is 0.993. With switch failures on demand, system reliability is reduced to 0.854. These results match with the exact results presented in [27] for up to 3 decimal places accuracy. The CPU time for solving the problem is 0.001 seconds. Refer to [37] for a method to calculate these small CPU times accurately.

3.7. Conclusions

In this Chapter, using the concepts of counting processes, an efficient approximate method to evaluate the reliability of cold standby systems when component lives follow Rayleigh distributions is proposed. This method also considers the effects of switch failures on system reliability. The consideration of Rayleigh distributions allows us to apply this method for analysing cold standby systems with components having linearly increasing hazard rates. The step-by-step procedure of the method is demonstrated using a numerical example. All steps involved in the proposed method are simple and do not include any complex numerical integrations. Therefore, the method can easily be implemented in any reliability software package. The CPU time for the reliability evaluation indicates that the proposed method is extremely fast.

CHAPTER 4

RELIABILITY OF PHASED MISSION SYSTEMS WITH WARM STANDBY SUB-SYSTEMS[4]

4.1 Introduction

Many practical systems are phased-mission systems where the mission consists of multiple, consecutive, non-overlapping phases. For the mission to be a success, the system must operate successfully during each of the phases. In each phase, the system has to accomplish a specific task and may be subject to different stresses. Thus, the system configuration, success criteria, and component failure behavior may change from phase to phase [38]. Systems used in these missions are usually called phased-mission systems (PMS). A typical example of such a system is a Geothermal Power Plant with phases: Phase1- During autumn and spring, when it has to ensure only the manger water, Phase 2- during winter then it has to ensure the heating process and manager water and phase 3-during the summer when it has to ensure the cooling process and the manager water [13]. Another typical example of such a system is an aircraft flight with phases: taxi to runway, take-off, ascend, cruise, descend, land, and taxi back to terminal [39].

Geothermal power plant functioning can be divided into three: the heating (A), the cooling (B) and the ensuring of manager water (C).

[4] Contents of this Chapter have been extracted from my paper published in Annual Proceeding of Reliability and Maintainability Symposium (RAMS) 2013, vol., no., pp.1,5, 28-31 Jan. 2013 doi: 10.1109/RAMS.2013.6517753.

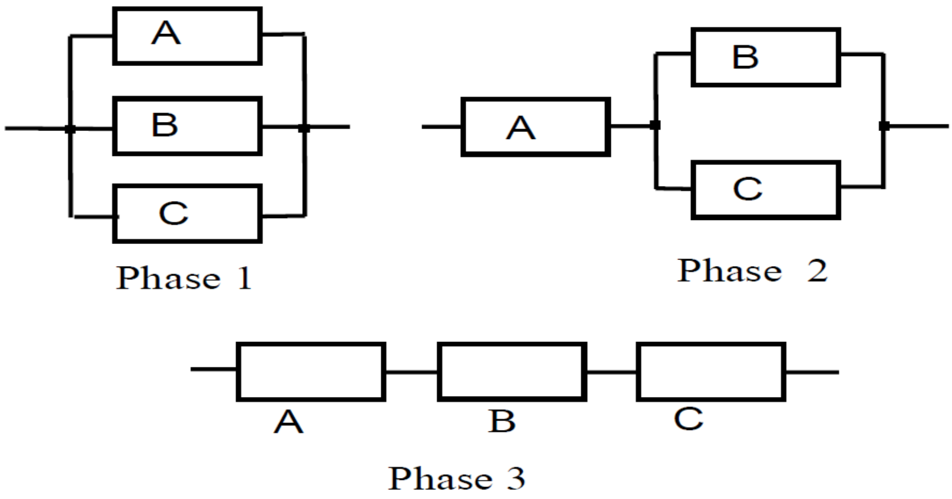

Figure 4.1: Phases: Phase1- during autumn and spring, when it has to ensure only the manger water, Phase 2- during winter then it has to ensure the heating process and manager water and phase 3-during the summer when it has to ensure the cooling process and the manager water [13].

Safety critical systems including the applications of phased-mission systems use either active or standby redundancy to improve the mission reliability. In general, there are three types of standby configurations, i.e., cold, hot and warm standby configurations. Cold standby implies that the inactive redundant components have a zero failure rate and cannot fail while in standby state. Hot standby implies that the redundant component has the same failure rate as active components while in standby state. Warm standby implies that an inactive component has a failure rate between cold standby and hot standby. Warm standby components are partially powered up when they are in standby mode. Therefore, they have a reduced failure rate in the standby mode. However, they are subject to the regular full failure rate when they are kept in operation to replace the faulty primary components. As compared to hot sub-systems, warm sub-systems do not consume much power when they are in standby mode. As compared to a cold standby system, the warm standby system does not need long initialization and recovery time. Warm sub-systems are commonly used in sensor networks and power generation, transmission and distribution systems using wind generators, tidal power generators, geothermal power system, solar power and other backup power generators. Warm standby components are subjected to different failure rates while they are in standby and operational modes. Such a state dependent failure behavior makes the reliability analysis of warm standby system a challenging task.

Considerable research efforts have been expended in the reliability analysis of PMS over the past four decades. However, even with the many advances in computing technology, only small-scale PMS problems can be solved accurately due to the high computational complexity of existing methods [40]. A state-of-the-art review of PMS reliability modeling and analysis techniques is provided in [39]. A major source of computational complexity in PMS reliability evaluation is due to its inherent dynamic dependencies. Specifically, it is important to consider the dynamics associated with variable system configurations at different phases subject to different stresses [38-40]. This dynamic behavior usually requires a distinct model for each phase of the mission in the reliability analysis [40]. Further complicating the analysis are statistical dependencies across the phases for a given component. For example, the state of a component at the beginning of a new phase is identical to the state at the end of the previous phase. The consideration of these dynamic dependencies poses unique challenges to existing reliability analysis methods [38-40].

To overcome these difficulties, reference [38] proposed a special structure for PMS models that are applicable for a wide range of practical systems. Further, most of the published examples of PMS models belong to this special structure [41-43]. The rationale behind this special structure is that, even though the configurations of the systems are varying with phases, it is also unrealistic to assume that the configurations at different phases are totally unrelated. This is because we are considering the same system at different phases, there must be a relationship between the system configurations at different phases. Therefore, it is valid to consider that the phase-dependent reliability requirements of the sub-systems change in a certain restricted fashion. Specifically, reference [38] has assumed that the changes in the system configuration can be described in terms of the changes in the sub-systems configurations, including their active or inactive status at different phases. Using this special structure and modularization techniques, reference [38] proposed an efficient method for evaluating the reliability of PMS models.

This section extends the scope of the model presented in [38] by considering warm standby sub-systems in phased mission analysis. This Chapter also presents a new method for reliability analysis of PMS with warm standby sub-systems. In the analysis, multiple sub-systems are considered in the system where each sub-system uses warm standby redundancy. The working and reserve failure rates of parts can change with the phases. Furthermore, the composition of individual sub-system can differ with the phases. The recommended algorithm is cultivated based on: (1) modularization methods, (2) closed-form equation for conditional reliability of warm standby sub-systems and (3) a recursive formula for gauging the dependencies of sub-systems over the phases. The proposed method is also applicable for analysing the phased mission system with cold and hot standby redundancies, since cold and hot standby configurations are special cases of warm standby configuration. The reliability assessment algorithm is depicted using an example of fault tolerant computing system consisting of multiple warm standby sub-systems.

4.2 System Description and Assumptions

The proposed method is based on the following system description and assumptions:

- The system mission consists of multiple, consecutive, non-overlapping phases.
- The system has several statistically independent and non-identical sub-systems.
- Each sub-system has several identical components.
- The components can have phase-dependent operational and standby failure rates.
- Each sub-system uses a k-out-of-n active/standby redundancy structure. The type of redundancy and the minimum number of good components required (k value) can vary with the phases.
- Some sub-systems are required only in certain phases; in other phases, they are kept idle or are switched off.
- When a sub-system is kept idle during a phase, all components within the sub-system are considered to be in a cold/warm standby mode. The components are still subject to fail, even when they are kept idle (warm standby). However, irrespective of the number of component failures, the sub-system is not considered to be failed during their idle phases.
- If any one of the required sub-systems is failed in a phase, the system is considered to be failed in that phase.

The overall mission is considered to be failed, if the system fails in any one of the phases.

4.3 Redundancy and Failure Criteria

1. Each sub-system uses a k-out-of-n warm standby redundancy. The minimum number of good components required (k value) can vary with the phases. Specifically, the configuration of sub-system l in phase j is k_{lj}-out-of-n_l warm standby redundancy. Hence, the sub-system is considered to be failed in phase j, when the number of working components is less than k_{lj}.
2. There is no repair.
3. Sensing and switching are perfect, e.g., instantaneous, error free, and failure free.

4.4 Modularization Method

In this Chapter, the modularization method proposed in [38] for evaluating the mission reliability is utilized. The modularization method is based on the following assumptions:

- If any one of the required sub-systems is failed in a phase, the system is considered to be failed in that phase.
- The overall mission is considered to be failed if the system fails in any one of the phases. Hence, the system reliability evaluation can be simplified as shown in equation (4.1). In other words, each sub-system can be analysed independently.

$$R_{PMS} = \prod_{l=1}^{N} R_l \tag{4.1}$$

Because reliabilities of sub-systems are calculated independently, the modularization method drastically reduces the computational complexity of the PMS reliability evaluation. The modularization method does not make any assumptions on the sub-system configuration. Therefore, this method can be used when different sub-systems use different redundancy types. However, in this Chapter, the primary focus is on warm standby redundancy. Refer to [38,44] for more details on the modularization method.

4.5 Reliability of Warm Standby Sub-Systems in a Specific Phase

This section describes a method proposed in [45] for computing the reliability of warm standby sub-system with exponential failure distributions with failure rate parameter λ_0 in operational state and failure rate parameter λ_d in the dormant state. We first considered a special case where k' $=k/d$ $= k\lambda_o/\lambda_d$ is an integer. In this case as shown in [45] the Markov Chain for warm standby case is equal to the Markov chain for active redundancy case where the model parameters *k, n,* λ and $p \equiv exp(-\lambda t)$ are modified to $k', n' = n - k + k'$, $\lambda' = \lambda_d$, and $p' \equiv \exp(-\lambda' t)$. The state probability of the system with active redundancy is calculated using binomial distribution with $p = exp(-\lambda t)$.

$$P_i = \binom{n}{i}(p)^{n-i}(1-p)^i \tag{4.2}$$

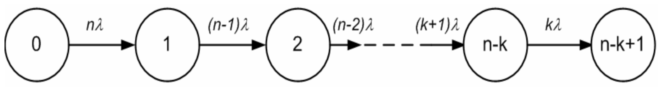

Figure 4.2: Markov Chain for *k-out-of-n* Systems with Active Redundancy

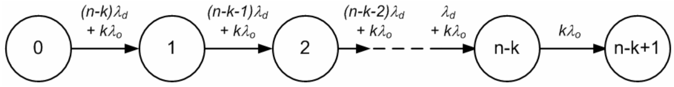

Figure 4.3: Markov Chain for *k-out-of-n* System with Warm Standby Redundancy

The state probabilities of the system with warm standby redundancy computed using binomial distribution with $p' \equiv \exp(\lambda' t)$.

$$P_i = \binom{n'}{i}\left(p^i\right)^{n'-i}(1-p')^i \tag{4.3}$$

Using the closed-form expressions for sum of exponential random variables in [46], reference [47] has shown that the equation (4.3) is also applicable even when where $k' = k\lambda_d/\lambda_o$ is a real number. Therefore, we use equation (4.3) to compute the state probabilities of warm standby sub-system in different phases.

4.6 Reliability of warm standby sub-system over all phases

In this Chapter we are evaluating the system reliability and state probabilities using recursive method proposed in [46]. Assume that there are n components in a given sub-system. The sub-system requires at least k_j working components in phase j. Hence, the sub-system is considered to be failed if there are at least $m_j = (n-k_j+1)$ failed components during phase j. In addition, the sub-system is considered to be failed if it fails in any one of the phases. Let x_j be the number of components that have failed before the completion of phase j, where $j=1,2,\ldots,M$. Hence, the sub-system is considered to successful if $x_j < m_j$ for all values *of j* (all phases). The sub-system reliability can be calculated by summing the probabilities of all combinations *of* x_j *values:* $(x1,x2,\ldots,x_M)$ *where* $x_j < m_j$ for all values of j.

Let $Z_{j,i}$ be the probability of the sub-system state such that $x_j = i$ and $x_l < m_l$ for all $l < j$. Hence,

$$Z_{j,i} = Pr\{x_j = i;\ x_{j-1} < m_{j-1}; \ldots; x_1 < m_1\} \tag{4.4}$$

Using the Markov property of the x_j sequence [48], the conditional probability term in equation (4.7) can be simplified.

$$Z_{j,i} = \sum_{a=0}^{m_{j-1}-1} Z_{(j-1),a} \cdot \Pr\{x_j = i|x_{j-1} = a\} \tag{4.5}$$

Where $Z_{1,i} = \Pr\{x_1 = i\} = \binom{n'}{i}\left(1 - p'_j\right)^{i}(p'_j)^{n'-i}$ (4.6)

$$\Pr\{x_j = i | x_{j-1} = a\} = \begin{cases} 0 & : i < a \\ \binom{n'-a}{i-a}\left(1 - p'_j\right)^{i-a}\left(p'_j\right)^{n'-i} & : i \geq a \end{cases} \quad (4.7)$$

Where p'_i is defined in (4.2). The equation (4.3) forms the basic recursion for sub-system reliability calculations. To improve the efficiency of the calculations and reduce the storage requirements, we use the following recursive relationships:

$$\Pr\{x_j = i | x_{j-1} = 0\} = \frac{n'-i+1}{i} \cdot \frac{1-p'_j}{p'_j} \cdot \Pr\{x_j = i-1 | x_{j-1} = 0\}$$

$$\Pr\{x_j = i | x_{j-1} = a\} = \frac{i-a+1}{n'-a+1} . \frac{1}{1-p'_j} . \Pr\{x_j = i | x_{j-1} = a-1\}$$

Where, $\Pr\{x_j = 0 | x_{j-1} = 0\} = \left(p'_j\right)^{n'}$ (4.8)

Once we calculate $Z_{M,i}$ values using the recursive formulas, we can calculate the sub-system reliability, R_l (where the suffix l is for the l^{th} sub-system).

$$R_l = \sum_{i=0}^{m_M - 1} Z_{M,i} \quad (4.9)$$

4.7 Algorithm For Sub-System Reliability

This section provides a detailed algorithm to compute the sub-system reliability.

Inputs: n, M, $\boldsymbol{k} = [k_1, k_2, \ldots, k_M]$,

$\lambda_0 = [\lambda_{01,}\lambda_{02\,,}, \ldots \lambda_{0M}\ ], \lambda_d = [\lambda_{d1}, \lambda_{d2} \ldots \lambda_{dM}]$

Output: Sub-system reliability: R_l

1. Calculate the vector $\boldsymbol{m}=[m_1, m_2, \ldots, m_M]$

   // for i =1 to M: $m_i = n\text{-}k_i+1$

2. **for** j = 1 to M **do**
3. $p'_j = \exp(\lambda'_j . t)$.
4. $d_{ij} = \lambda_{diM}./\lambda_{oiM}$
5. $p_G \Leftarrow power(p_{j,} d_{ij}); p_F \Leftarrow 1 - p_G$ // where: $p_F = q_j$
6. **if** $(p_F == 0)$ **continue** // skip the iteration j
7. $\boldsymbol{pZ} \Leftarrow [1,0, \ldots 0]; m_0 \Leftarrow 1$ // means: $pZ_0 = 1$

   // $\boldsymbol{pZ}$ means: previous $\boldsymbol{Z}$ vector

8. $Pr0 \Leftarrow (p_G)^{n'}$; $n' \equiv n - k_j + k_j/d_j$
9. **for** i = *0* to n ***do***
10. $Pr \Leftarrow Pr0; Z_i \Leftarrow 0$
11. **for** a = *0* **to** min{i, m_{j-1} -1} **do**
12. $Z_i \Leftarrow Z_i + pZ_a \times Pr$
13. $Pr \Leftarrow \frac{Pr}{pF} \times \frac{i-a}{n'-a+1}$ // for next a
14. **end for**
15. $Pr0 \Leftarrow Pr0 \times \frac{n'-i+1}{i} \times \frac{p_G}{p_F}$ // for next i
16. **end for**
17. $\boldsymbol{pZ} \Leftarrow \boldsymbol{Z}$ // set $\boldsymbol{pZ}$ to $\boldsymbol{Z} = [Z_0, \ldots, Z_n]$
18. **end for** Sub-system Reliability:$R_l \Leftarrow \sum_{i=0}^{m_M-1} Z_i$

4.8 Illustrative example

In this section, we illustrate the proposed method based on a hypothetical example of a fault tolerant computing system discussed in [48]. The number of components in each sub-system is shown in Table 4.1. The system has 4 sub-systems and 4 phases. The redundancy configurations of each sub-system is specified in table 4.2.

Sub-system		#Comp.
ID	*Name*	*N*
A	*PE1*	3
B	*PE2*	4
C	*PE3*	3
D	*ME*	2

Table 4.1 – Sub-system Parameters

The duration of phases and the phase-dependent sub-system parameters (*k*, λ_o *and* λ_d *values*) are shown in Table 4.2.

Phase		Phase 1	Phase 2	Phase 3	Phase 4
Duration		10	30	40	20
SS NAME	Phase-Dependent Sub-system Parameters				
PE1	k	2	2	0(Idle)	2
	λ_o	0.001	0.0002	0.002	0.0012
	λ_d	0.00083	0.00067	0.00083	0.001
PE2	k	0 (Idle)	2	0 (Idle)	2
	λ_o	0.00083	0.001	0(Idle)	0.000125
	λ_d	0.00025	0.0005	0(Idle)	0.0000625
PE3	k	0(Idle)	3	0(Idle)	2
	λ_o	0.0002	0.0008	0.0006	0.0004
	λ_d	0.000067	0.0004	0.0002	0.0002
ME	k	0(Idle)	1	1	1
	λ_o	0.001	0.0002	0.002	0.0012
	λ_d	0.00083	0.00067	0.00083	0.001

Table 4.2 – Phase-Dependent Requirements and Parameters

SS NAME	Phase	Phase 1	Phase 2	Phase 3	Phase 4
PE1	n'_{1j}	5	5	3	4
	p'_{1j}	0.9900	0.9940	0.9231	0.9763
PE2	n'_{2j}	4	7	4	10
	p'_{2j}	0.9917	0.9704	0.608	0.9976
PE3	n'_{3j}	3	12	3	5
	p'_{3j}	0.9993	0.9881	0.9920	0.9960
MEM	n'_{4j}	2	2	4	3
	p'_{4j}	0.9917	0.9801	0.9673	0.9802

Table 4.3 – Phase-Dependent intermediate parameters

Sub-system-A, sub-system-B, sub-system-C and sub-system-D are kept idle during phase-3, phases-1&3, phsases-1&3 and phase-1 respectively. Hence, the required number of working components for these sub-systems in these phases is zero. In the proposed method, we first calculate the mission reliability for each sub-system. For example, for sub-system-*A*, we calculate the conditional reliabilities of components using equation $p'_j \equiv exp(-\lambda_{dj}.t)$, $\boldsymbol{p'_{1j}}$ = $[p'_1, p'_2, p'_3, p'_4]$ = [0.9900, 0.9940, 0.9231, 0.9763]. From Table 2, we have: $\boldsymbol{k} = [k_1, k_2, k_3, k_4]$ = [2, 2, 0, 2] for sub-system-*A*. From Table 1 and Table 2, we have: n = 3 and $\boldsymbol{M} = \mathbf{4}$. Hence, using the algorithm in section 6, we calculate the mission reliability of sub-system-A: R_A = 0.9846. Similarly, we obtain the mission reliabilities for other sub-systems: R_B = 0.9996, R_C = 0.9645 and R_D = 09888. The overall mission reliability of the entire system is calculated as the product of mission reliabilities of individual sub-system as shown in (1). Hence, R_{PMS} = 0.9387. The CPU time for solving this problem is 7.125 e-4 seconds. Refer to [49] for a method to calculate these small CPU times accurately.

4.8 Conclusions

In this Chapter, a new method for reliability evaluation of phased mission systems with warm standby systems is presented. The proposed algorithm is established based on: (1) a modularization approach, (2) an efficient closed form equation for conditional reliability of warm standby sub-systems and (3) a recursive expression for enumerating the reliance of sub-system over the phases. The CPU time for solving the numerical example demonstrates that the method is computationally powerful. The reliability assessment algorithm is depicted using an example of a *fault tolerant system.*

CHAPTER 5

RELIABILITY OF LOAD-SHARING SYSTEMS SUBJECT TO PROPORTIONAL HAZARDS MODEL[5]

5.1 Introduction

The biggest problem with electrical power systems is aging. The failure due to aging is a non-repairable failure. The aging failure of power system components such as Power plants, Transformers, Power Transmission cables, breakers, Capacitors and reactors etc, have been major concern and a major factor in system planning of many utilities since more and more power system components not only in Victoria and but also all over the world approaching end of their life period. Excluding aging failure in the reliability analysis of power system will lead to underestimation of the risks associated with power system failures. If some key components in any system are aged, then the aging failure could become a major factor of system unreliability. Low reliability due to aging not only reduces a competitive advantage on valuation in the power utilities market, but also requires greater operation and repair costs. There may be a trade off between reliability and cost suggestions, system performance optimization based on cost reliability analysis but research shows that recent blackout and bush fires in Victoria and NSW are due to aging power systems. There is no comparison between the human loss and economic loss due to this problem compared to replacing the aging power system infrastructure. There

[5] The contents of this Chapter have been extracted from my paper published at annual proceeding *Reliability and Maintainability Symposium (RAMS), 2013,*, vol., no., pp.1,5, 28-31 Jan. 2013, doi: 10.1109/RAMS.2013.6517708

are several research papers separately considering aging and separately considering load-sharing system. In this Chapter, author is presenting the research findings, when Load-Sharing and aging is taken together to evaluate the reliability of power systems.

In reliability engineering, it is a widespread practice to use redundancy techniques to enhance system reliability. A standard form of redundancy is a *k*-out-of-*n* arrangement in which at least *k*-out-of-*n* components must work for the triumphant operation of the system. The *k*-out-of-*n* configuration redundancy finds capacious purpose in both industrial and military systems. Examples include the **generators in power systems**, cables in transmission lines and the multi-engine system in an airplane. Several examples of *k*-out-of-*n* systems are available in [46].

In numerous cases, when investigating redundancy, autonomy is ascertained across the components within the system. In other words, it is assumed that the failure of a component does not alter the failure properties (failure rates) of the remaining components. In the real-world, however, numerous systems are load-sharing, where the conjecturing of independence is no longer accurate. In a load-sharing system, if a component breaks down, the same workload has to be shared by the remaining components, resulting in an increased load shared by each surviving component. In most circumstances, an aggrandized load induces a colossal component failure rate [46]. Many empirical studies of mechanical systems [50] and computer systems [51] have showed that the workload strongly impinges the component failure rate. Applications of load-sharing systems include electric generators sharing an electrical load in a power plant, CPUs in a multiprocessor computer system, cables in a suspension bridge, and valves or pumps in a hydraulic system [53].

5.2 Related Work

While there are many Chapters on reliability modelling for *k*-out-of-*n* systems, not much attention has been paid to load-sharing *k*-out-of-*n* systems [53], [54]. In most of the existing literature on load-sharing systems, the solutions are provided only for:

- System with independent and identical distributed (i.i.d). exponential failure times [50],[52].
- 1-out-of-2 or 1-out-of-3 systems with non-identical components following exponential distributions.
- 1-out-of-2 systems with general distributions.

Scheuer [55] studied the reliability of a *k*-out-of-*n* system where component failure induces higher failure rates in the survivors and assumed that the components are i.i.d. with constant failure rates. Astonishingly, it came into sight that until 1997 [46], there is no closed form solution for all cases of load-sharing *k*-out-of-*n* systems, ever for the i.i.d. exponential failure times.

Although a generalized accelerated failure-time model (AFTM) for load-sharing *k*-out-of-*n* systems with arbitrary load-dependent component lifetime distributions is presented in [54], the solution provided in [54] is complex. Therefore, as mentioned in [53], [54], it can only be applied for simple systems where $n \leq 6$. Therefore, more efficient methods for handling arbitrary load-dependent component lifetime distributions are needed. In this Chapter, we provide a closed-form analytical solution for the reliability of PHM load-sharing *k*-out-of-*n* systems with identical components where all surviving components share the load equally.

5.3 Load-sharing systems

In order to analyse the reliability of load-sharing systems, we should consider the relationship between the load and the failure behaviour of a component over a time period.

5.4 Load Distribution

In a load-sharing system, upon a component failure, the load on the failed component is redistributed among the surviving components. In a majority of cases, the load is equally distributed over all surviving components. If the total load is *L*, and there are m good components, then the load on each component is $z = L/m$. The equal distribution of load is appropriate when all components are of the same type. Hence, when the load is distributed equally, it is also reasonable to assume i.i.d. components.

Let *n* be the total number of components in the system and z_i be the load on each of the surviving components when *i* components are failed. Hence,

$$z_0 = \frac{L}{n}; \quad z_i = \frac{L}{n-i} = z_0 \frac{n}{n-i} \tag{5.1}$$

5.5 Load-Life Relationship

To analyse the reliability of a load-sharing system, the effect of the rise of load levels on the remaining lifetime of the products needs to be explained. An accelerated step-stress model that depicts the progress of the acceleration of failure or degradation of the products under high stresses can also be applied for the load-sharing systems. In this Chapter, we consider the PHM.

5.6 THE PHM

In this model the acceleration of failure when the stress is raised from a lower level to a higher level is reflected in the hazard rate function. Consider a component that is subjected to an ordered sequence of loads, where load z_i ($i = 0, 1, \ldots, n - k$) is applied during the time interval $[\tau_i, \tau_{i+1}]$ where $\tau_0 = 0$. In other words, the load changes at times $\tau_1, \tau_2, \ldots, \tau_{n-k}$.

According to PHM, the hazard rate of the component at time t is:

$$h(t) = h_i(t) = \delta_i . h_0(t) \quad \text{for } \tau_{i-1} \le t < \tau_i \qquad (5.2)$$

where $\delta_0 = 1$, $h_0(t)$ is the hazard rate at the lower load z_0 and δ_i is the failure rate adjustment factor at load level z_i. The failure rate adjustment factor is a function of the applied stress. Hence, the PHM can be express as:

$$h(t) = \delta(z). h_0(t) \qquad (5.3)$$

Where z is the load at the time t. Considering these load dependent time varying failure rates makes the reliability analysis a challenging task.

5.7 Reliability analysis

Assumptions

In this Chapter, we have taken the following assumptions:

- There are *n* components in the system.
- The system functions successfully if and only if there are at least k good components.
- After a component failure, the load is equally distributed among all surviving components.
- The failure rate of a component varies as per the PHM.
- The baseline failure rate of the PHM can follow an arbitrary distribution such as Normal, Weibull, Lognormal, and Gamma.
- The redistribution and reconfiguration mechanisms can be imperfect.

The system and its components are non-repairable.

5.8 k-out-of-n Identical Components

First consider a *k*-out-of-*n* system with identical components where the failure rate of each component is $\lambda_i(t)$. When the system is put into operation at time zero, all components are working, and they are equally sharing the total load that the system is supposed to carry. In this case, the failure rate of every component is denoted by $\lambda_0(t)$. Because there are *n* working components in the system, the first failure occurs at rate:

$$\alpha_1(t) = n.\lambda_0(t) = n.\delta_0.\lambda(t) \tag{5.4}$$

Where the system experiences the first failure, the remaining *n-1* working components must carry the same load on the system. As a result, the failure rate of each working component becomes:

$$\lambda_1(t) = \delta_1.\lambda(t) \tag{5.5}$$

which is typically higher than $\lambda_0(t)$. The second failure occurs at rate:

$$\alpha_2(t) = (n-1).\lambda_1(t) = (n-1)\delta_1\lambda(t) \tag{5.6}$$

When *i* components are failed, the failure rate of each of the (*n*-*i*) working components is represented by $\lambda_i(t)(0 \le i \le n-k)$. The next failure that is $(i+1)^{th}$ failure occurs at rate:

$$\alpha_{i+1}(t) = (n-i).\lambda_i(t) = (n-i).\delta_i.\lambda(t) \tag{5.7}$$

The system fails when more than (*n* - *k*) components are failed. Therefore the failure process can be represented by non-homogenous Markov process as shown in Figure 5.1

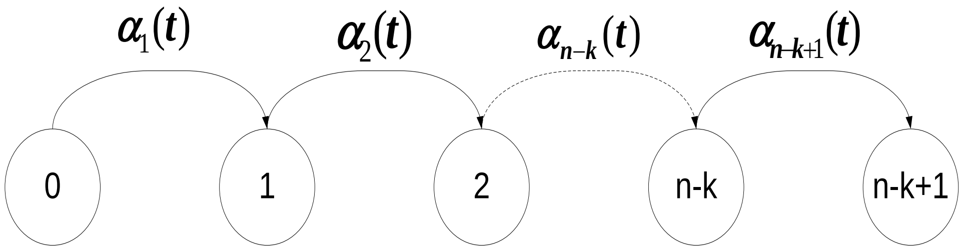

Figure 5.1: Non-homogeneous Markov process for identical components case.

Then the solution to this can be obtained by the non-homogeneous Markov model. In Figure 1, state *i* represent the system with *i* failures. State 0 is initial state, state (*n*-*k*+1) is failed state and states 0 to *n*-*k* are all working states. After substituting α_i values the Figure1 can be expressed as in figure2.

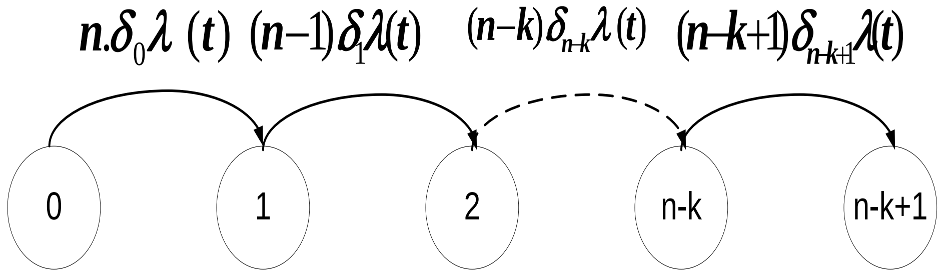

Figure 5.2: *Non-homogeneous Markov process after substituting α_i*

$$p'(t) = p_0(t).\Lambda(t).p(t) \quad (5.8)$$

where p is a vector of probability of system states. $\Lambda(t)$ is transition rate matrix and $p_0(t)$ is initial probability sate vector. However in identical component case the system reliability can be found in a closed form expression using transformation in [56] the system reliability can be obtained as in equation (46). Using the transformations in [50] the reliability of the system described in Figure 5.2 can be obtained as shown below.

$$R(t) = \sum_{i=0}^{n-k+1} A_i.\exp((n-i-1)\delta_{i-1}\Lambda(t)) \quad (5.9)$$

Where $\Lambda(t) = \int_0^t \lambda(x).dx$ (5.10)

$$A_i \equiv \prod_{\substack{j=1 \\ j\neq i}}^{n-k+1} \frac{(n-j+1).\delta_{j-1}}{(n-j+1)\delta_{j-1}-(n-i+1).\delta_{i-1}}; i = 1,2, \ldots . n-k+1 \tag{5.11}$$

Example 1: Consider a 5-out-of-8 system with weibull as the baseline distribution, where η = 1000, β = 1.5. The failure rate multiplication factor is: $\delta_i = \left(\frac{n}{n-i}\right)^2$.

Model: k = 5, n = 8, t = 100

Base line parameters η = 1000, β = 1.5

Solution: The base line failure rate is $\lambda(t) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1}$

Hence $\Lambda(t) = \int_0^t \lambda(x).dx$

$$= \int_0^t \frac{\beta}{\eta}\left(\frac{x}{\eta}\right)^{\beta-1}.dx$$

$$= \left(\frac{t}{\beta}\right)^{\beta} \tag{5.12}$$

Now substituting numerical inputs

$$\Lambda(t) = \left(\frac{100}{1000}\right)^{1.5} = 0.0316227766$$

The multiplication factors are: δ(1) = 8, δ (2) = 9.1429 and δ (3) = 10.667, δ (4) =128. Further substituting these values in equation (5.9) the load sharing system reliability is 0.9996.

5.9 Load Sharing Systems with Switch Failures

In the previous section it is assumed that load distribution is perfect. However the load systems can fail due to imperfect load redistribution. Let c_i be the success probability of the load redistribution (switch success probability on demand) at the time of i^{th} failure. Hence the system failure process can be described using non-homogeneous Markov chain as shown in

Figure 5.3.

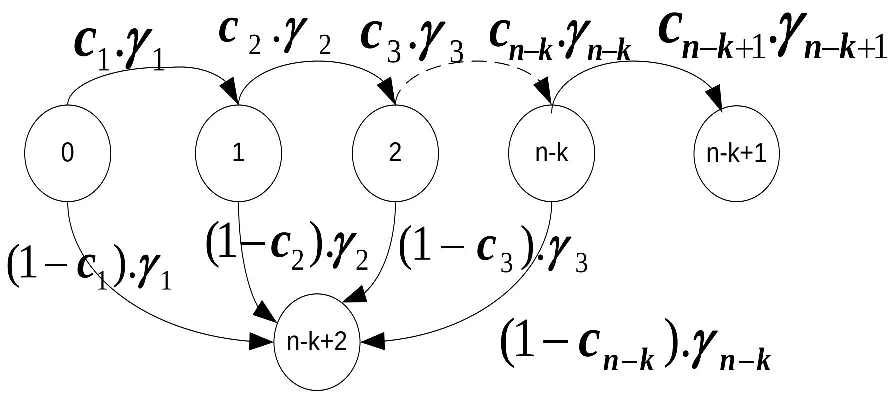

Figure 5.3: System failure process using non-homogeneous Markov chain.

In Figure 5.3 the state (*n*-*k*+2) represent the system failure due to load redistribution. Now by re-writing the transition rates in Figure 3 and substituting $\alpha_i(t) = \gamma_i.\lambda(t)$. The α_i in Figure 5.3 is

$\alpha_i(t) = (n - i + 1) = (n - i + 1)\delta_{i-1}.\lambda(t)$.

Now define $\gamma_i = (n - i + 1).\delta_{i-1}$.

Hence $\alpha_i(t) = \gamma_i.\lambda(t)$.

Now using these values, the Figure 3 can be expressed as in Figure 5.4.

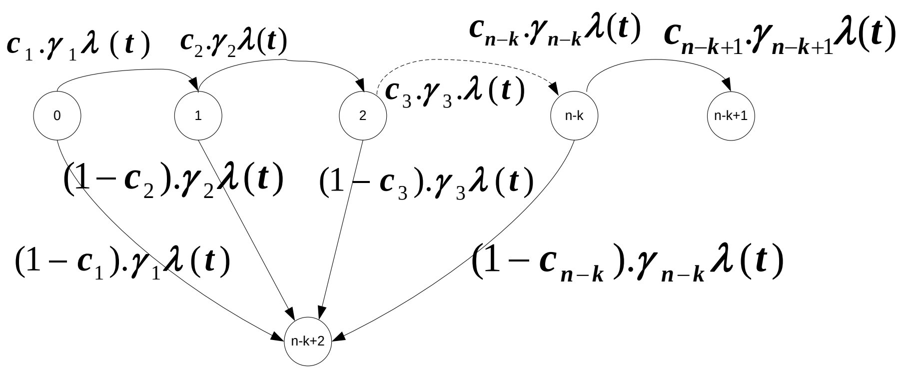

Figure 5.4:Non-Homogeneous Markov Chain

Using the transformation in [56] the reliability of the system described in Figure 5.4 (to the non-homogeneous Markov chain in Figure 5.4) can be obtained as shown below.

$$R(t) = \sum_{i=0}^{n-k} p_i(t) \tag{5.13}$$

where $p_i(t)$ is the state probability of non-homogeneous Markov chain shown in Figure 4. $p_i(t)$ is calculated by applying the transformations in [56] then solving underlying homogeneous Markov chain. Under the transformed scale y = Λ(t) defined in equation (5.12) the failure process becomes a homogeneous Markov chain as shown in figure 5.5

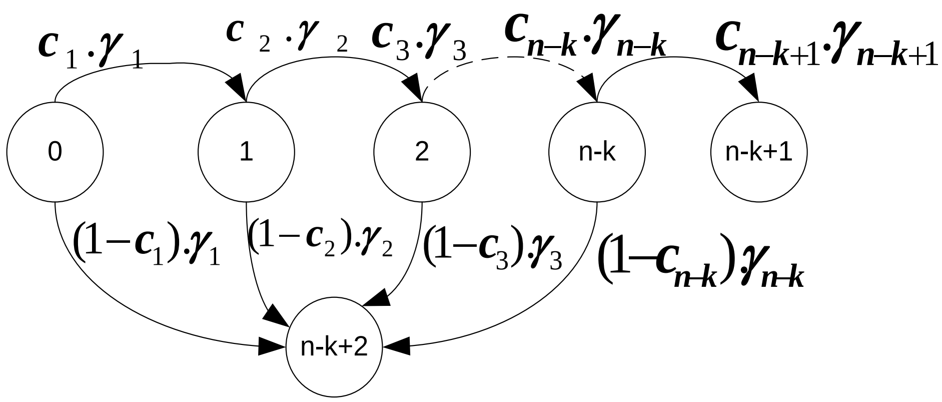

Figure 5.5: Homogeneous Markov Chain

The Laplace Transforms from [46]

$$p_0 = \frac{1}{s+\gamma_1} \tag{5.14}$$

$$p_1(s) = \frac{c_1\gamma_1}{(s+\gamma_1)(s+\gamma_2)} \tag{5.15}$$

$$p_i(s) = \frac{(c_1\gamma_1)(c_2\gamma_2)\ldots\ldots(c_i\gamma_i)}{(s+\gamma_1)(s+\gamma_2)\ldots.(s+\gamma_n)} \tag{5.16}$$

By definition $c_0 = 1, R_0 = 1$

By taking the inverse Laplace Transforms author have:

$$p_0(t) = e^{\gamma_1 t} \tag{5.17}$$

$$p_1(t) = A_{11}e^{-\gamma_1 t} + A_{12}e^{-\gamma_2 t} \tag{5.18}$$

Where

$$A_{11} = \left[\frac{c_1\gamma_1}{(s+\gamma_2)}\right]_{s=-\gamma_1} = \left[\frac{c_1\gamma_1}{(\gamma_2-\gamma_1)}\right]$$

$$A_{12} = \left[\frac{c_1\gamma_1}{(s+\gamma_1)}\right]_{s=-\gamma_1} = \left[\frac{c_1\gamma_1}{(\gamma_1-\gamma_2)}\right]$$

$$p_2(s) = \frac{(c_1\gamma_1)(c_2\gamma_2)}{(s+\gamma_1)(s+\gamma_2)(s+\gamma_3)} \tag{5.19}$$

Now

$$p_2(t) = A_{21}e^{-\gamma_1 t} + A_{22}e^{-\gamma_2 t} + A_{23}e^{-\gamma_3 t} \tag{5.20}$$

Where

$$A_{21} = \left[\frac{(c_1\gamma_1)(c_2\gamma_2)}{(s+\gamma_2)(s+\gamma_3)}\right]_{s=-\gamma_1} = \left[\frac{(c_1\gamma_1)(c_2\gamma_2)}{(\gamma_2-\gamma_1)(\gamma_3-\gamma_1)}\right] \tag{5.21}$$

$$A_{22} = \left[\frac{(c_1\gamma_1)(c_2\gamma_2)}{(s+\gamma_1)(s+\gamma_3)}\right]_{s=-\gamma_2} = \left[\frac{(c_1\gamma_1)(c_2\gamma_2)}{(\gamma_1-\gamma_2)(\gamma_3-\gamma_1)}\right] \tag{5.22}$$

$$A_{23} = \left[\frac{(c_1\gamma_1)(c_2\gamma_2)}{(s+\gamma_1)(s+\gamma_3)}\right]_{s=-\gamma_3} = \left[\frac{(c_1\gamma_1)(c_2\gamma_2)}{(\gamma_1-\gamma_3)(\gamma_2-\gamma_3)}\right] \tag{5.23}$$

$$p_i(t) = A_{i,1}e^{-\gamma_1 .t} + A_{i,2}e^{-\gamma_2 .t} + \ldots\ldots + A_{i,i+1}e^{-\gamma_{i+1}.t} \tag{5.24}$$

$$p_i(t) = \sum_{j=1}^{i+1} A_{i,j}e^{-\gamma_j .t} \tag{5.25}$$

Where

$$A_{i,j} = \left[p_i(s).(s+\gamma_j)\right]_{s=-\gamma_j} \tag{5.26}$$

$$p_i(s) = \frac{\prod_{m=1}^{i} c_m\gamma_m}{\prod_{m=1}^{i+1}(s+\gamma_m)} \tag{5.27}$$

Now

$$A_{i,j} = \left[\left[\frac{\prod_{m=1}^{i} c_m\gamma_m}{\prod_{m=1}^{i+1}(s+\gamma_m)}\right](s+\gamma_m)\right]_{s=-\gamma_m} \tag{5.28}$$

$$A_{i,j} = \left[\frac{\prod_{m=1}^{i} c_m\gamma_m}{\prod_{m=1}^{i+1}(s+\gamma_m)}\right]_{s=-\gamma_m} \tag{5.29}$$

$$A_{i,j} = \frac{\prod_{m=1}^{i} c_m\gamma_m}{\prod_{\substack{m=1\\ m\neq j}}^{i+1}(\gamma_m-\gamma_j)} \tag{5.30}$$

5.10 ILLUSTRATIVE EXAMPLE

In this section, the proposed method is illustrated using two numerical examples.

Example 2: Same as example 1, except the switchover mechanism is at load redistribution is imperfect where c_i=0.99 for all *i*.

Solution: Using equation (25) the state probabilities are:

$p_0(t) = 0.768717, p_1 = 0.18909, p_2(t) = 0.026473$ and $p_3(t) = 0.002866$.

Hence, system reliability is:

$$R(t) = p_0(t) + p_1(t) + p_2(t) + p_3(t) = 0.987146$$

Example 3: Same as example 1. Except the switch mechanism is at load redistribution is imperfect, where $c_i = 1 - 0.001.i$.

Solution: $R_0 = 1$, $c_1 = 0.99$, $c_2 = 0.98$, $c_3 = 0.97$,

$c_4 = 0.96$, $R_1 = c_1$, $R_2 = c_1.c_2$, $R_3 = c_1.c_2.c_3$

$R_4 = c_1.c_2.c_3.c_4$.

$R(t) = p_0.R_o + p_1R_1 + p_2R_2 + p_3R_3 = 0.984268$.

5.11 Conclusions

In this Chapter, a new method for reliability evaluation of Load Sharing systems using k-out-of-n structure subject to proportional hazards model is presented. The solution is based on a time transformation using cumulative hazard function and equivalent problem formulation based on continuous Markov chains. The analysis also considers the effects of imperfect switches at the time of load redistribution. Numerical results obtained using closed form expressions are also compared with Monte Carlo simulation.

The method can also be extended to non-identical components case. Author is planning to extend this method for analysing Phased Mission Systems with load sharing components subject to imperfect switches.

CHAPTER 6

RELIABILITY EVALUATION OF PHASED MISSION SYSTEMS WITH LOAD-SHARING COMPONENTS[6]

(The content of this Chapter has been presented at Reliability And Maintainability Symposium (RAMS) 2012 conference USA)

6.1 Introduction

Many practical systems such as Nuclear power [86] and Geothermal Power [13] systems are phased-mission systems where the mission consists of multiple, consecutive, non-overlapping phases [38, 39, 61-63]. For the mission to be a success, the system must operate successfully during each of the phases. In each phase, the system has to accomplish a specific task and may be subject to different stresses. Thus, the system configuration, success criteria, and component failure behavior may change from phase to phase [38]. Systems used in these missions are usually called phased-mission systems (PMS). A typical example of such a system is an aircraft flight with phases: taxi to runway, take-off, ascend, cruise, descend, land, and taxi back to terminal [40, 61, 62,74].

Considerable research efforts have been expended in the reliability analysis of PMS over the past four decades [65-73]. However, even with the many advances in computing technology, only small-scale PMS problems can be solved accurately due to the high computational complexity of existing methods [74]. A state-of-the-art review of PMS reliability modeling and analysis techniques is provided in [74]. A major source of computational complexity in PMS reliability evaluation is due to its inherent dynamic

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dependencies. Specifically, it is important to consider the dynamics associated with variable system configurations at different phases subject to different stresses [38,74,75]. This dynamic behavior usually requires a distinct model for each phase of the mission in the reliability analysis [75]. Further complicating the analysis are statistical dependencies across the phases for a given component. For example, the state of a component at the beginning of a new phase is identical to the state at the end of the previous phase. The consideration of these dynamic dependencies poses unique challenges to existing reliability analysis methods [38, 74, 75].

To overcome these difficulties, reference [38] proposed a special structure for PMS models that are applicable for a wide range of practical systems. Further, most of the published examples of PMS models belong to this special structure [41-43]. The rationale behind this special structure is that, even though the configurations of the systems are varying with phases, it is also unrealistic to assume that the configurations at different phases are totally unrelated. This is because the same system is considered at different phases, there must be a relationship between the system configurations at different phases. Therefore, it is valid to consider that the phase-dependent reliability requirements of the sub-systems change in a certain restricted fashion. Specifically, reference [38] has assumed that the changes in the system configuration can be described in terms of the changes in the sub-systems configurations, including their active or inactive status at different phases. Using this special structure and modularization techniques, reference [38] proposed an efficient method for evaluating the reliability of PMS models.

Although the special structure considered in [38] has several applications, it also has some limitations. Specifically, it assumes that all components within a sub-system are statistically independent of each other during a phase. In other words, it is assumed that the failure of a component does not affect the failure properties (failure rates) of the remaining components.

In the real world, however, many systems are load-sharing, which makes the assumption of independence invalid [56,57]. In a load-sharing system (LSS), if a component fails, the same workload is shared by the remaining components, resulting in an increased load on each surviving component. In most circumstances, an increased load induces a higher component failure rate. Many empirical studies of mechanical systems [58] and computer systems [59] have proved that the workload strongly affects the component failure rate. This introduces the dependency among the components even within a phase. Therefore, to extend the applicability of PMS models, the statistical dependencies between the components due to the variations in the applied loads across the phases as well as within the phases should be considered.

In such cases, the load on a component depends on its operational phase as well as the number of working components within the sub-system that share the load along with the component. Further, the number of working components in a sub-system depends on the cumulative hazard rates of its components. The cumulative hazard rate of a component depends on the durations of previous phases, the phase dependent total load on the sub-system, and the number of working components at different durations in the past. This introduces complex dynamic dependencies among the load-sharing components. The reliability evaluation methods that can handle these complex dependencies are very limited [74]. The only available method for analysing these complex dependencies are the Monte Carlo simulation and state-space oriented methods. Simulation typically offers greater generality in system representation, but it is often more expensive in computational requirements [74, 75]. This is particularly a concern with the crude Monte Carlo simulation for analysing safety-critical systems, especially those with ultra-high reliabilities often found in aerospace and nuclear industries. State space-oriented approaches, which are based on Markov chains and/or Petri nets, are flexible and powerful in modeling complex dependencies among system components. However, they suffer from state explosion when modeling large-scale systems [38, 64, 76]. Therefore, all existing methods for PMS reliability analysis with load-sharing components are limited to either small-scale problems (state-space methods) or non-critical systems with moderate reliability requirements (crude Monte Carlo simulation).

In this Chapter, an efficient recursive algorithm for reliability evaluation of phased mission systems with load-sharing components is proposed. In the analysis, multiple sub-systems where each sub-system can have multiple load-sharing components is considered. Due to the complex nature of the problem, the analysis is restricted to exponential failure time distributions for the components. The proposed algorithm is developed based on: (1) a modularization technique, (2) an easily computable closed-form expression for conditional reliability of load-sharing sub-systems, and (3) a recursive formula for the reliabilities of sub-systems across the phases. The algorithm proposed in this Chapter helps reliability engineers to accurately evaluate the reliability of phased mission systems with load-sharing components in an efficient way.

6.2 System Description and Assumptions

The proposed method is based on the following system description and assumptions. *

6.2.1. Phases

1) The overall mission of the system can be divided into M consecutive and non-overlapping phases.

2) The duration of phase j is tj. Hence, the duration of the entire mission is: $T = t_1 + t_2 + \cdots + t_M$.

6.2.2. System Elements and Failure Rates

The following lists all the required input parameters for solving the problem.

1) The system has N statistically independent and non-identical sub-systems.
2) Each sub-system is subjected to a certain load that can vary with the phases. Specifically, the sub-system l is subjected to load $L_T(j,l)$ during the phase j.
3) Each sub-system has several identical components arranged according to a load-sharing configuration. The phase-dependent load on the sub-system is equally shared by all its surviving components.
4) Failure rate of the components vary with the load. Therefore, failure rate of a component is a function of the load. Components from different sub-systems can have different failure rate functions.
5) At the beginning of the mission, all components are in good conditions (working condition).
6) The components and the sub-systems are non-repairable during the mission.

6.2.3. Redundancy and Failure Criteria

1) Each sub-system uses a k-out-of-n load-sharing redundancy. The minimum number of good components required (k value) can vary with the phases. Specifically, the configuration of sub-system l in phase j is k_{lj}-out-of-n_l load-sharing redundancy. Hence, the sub-system is considered to be failed in phase j, when the number of working components is less than k_{lj}.
2) Some sub-systems are required only in certain phases; in other phases, they are kept

idle or are switched off.

3) If any one of the required sub-systems is failed in a phase, the system is considered to be failed in that phase.
4) The overall mission is considered to be failed, if the system fails in any one of the phases.

6.3 Modularization

In this Chapter, author utilizes the modularization method proposed in [38] for evaluating the mission reliability. The modularization method is based on the following assumptions:

- If any one of the required sub-systems is failed in a phase, the system is considered to be failed in that phase.
- The overall mission is considered to be failed if the system fails in any one of the phases.

Under these assumptions, the overall mission reliability of the system can be calculated as the product of mission reliabilities of individual sub-systems. The rationale behind this method is that both the active sub-systems and the phases are logically in series. Let s_{lj} be the Boolean variable that represents the success status of sub-system *l* in phase *j*. Hence, s_{lj} is TRUE, when sub-system *l* is successful in phase *j*. Therefore, the system structure function can be represented as:

$$\Phi = \cap_{j=1}^{M}\left[\cap_{l=1}^{N} s_{lj}\right] \qquad (6.1)$$

When the sub-system *l* is idle in phase *j*, it cannot fail in that phase. In such cases, s_{lj} is always TRUE. Note that the s_{lj} variables in equation (1) are not independent. Therefore, the system reliability cannot be calculated simply as the product of probabilities of s_{lj}. However, equation (1) can be rearranged:

When the sub-system l is idle in phase j, it cannot fail in that phase. In such cases, s_{lj} is always TRUE. Note that the s_{lj} variables in equation (1) are not independent. Therefore, the system reliability cannot be calculated simply as the product of probabilities of s_{lj}. However, equation (1) can be rearranged:

$$\Phi = \cap_{l=1}^{N}\left[\cap_{j=1}^{M} s_{lj}\right] \tag{6.2}$$

Because the sub-systems are statistically independent, the system reliability can be calculated as the product over all sub-systems.

$$R_{PMS} = \cap_{l=1}^{N} \Pr\left[\cap_{j=1}^{M} s_{lj}\right] \tag{6.3}$$

The probability term in equation (6.3) is nothing but the mission reliability (R_l) of the sub-system l. Therefore, according the modularization method, the mission reliability of the overall system is:

$$R_{PMS} = \prod_{l=1}^{N} R_l \tag{6.4}$$

Because reliabilities of sub-systems are calculated independently, the modularization method drastically reduces the computational complexity of the PMS reliability evaluation. The modularization method does not make any assumptions on the sub-system configuration. Therefore, this method is also applicable for sub-systems with load-sharing redundancy. Further, the same modularization can be used even when different sub-systems use different redundancy types. However, in this Chapter, the primarily focus is on load-sharing redundancy. Refer to [38] for PMS reliability analysis with active and standby redundancies.

6.4 Load-Sharing Sub-systems

In this Chapter, it is considered that the sub-systems in each phase use a load-sharing redundancy. To analyse the sub-system reliability in a particular phase, there is a need to find the loads on each component and their load-dependent failure rates.

6.4.1. Failure Rate versus Load

To analyse the reliability of load-sharing systems, the relationship between the load and the failure rate of a component should be considered. In general, failure rate of a component increases with the applied load. The following models are commonly used to describe the failure rate of components subjected to load-sharing redundancy [56, 57].

- Power Law: $\lambda(L) = C.L^{\alpha}$
- Exponential Law: $\lambda(L) = C.e^{L\alpha}$

Where C and α are the model parameters and L is load on the component. The model parameters can be obtained from the accelerated life testing analysis [60].

6.4.2. Failure Rate versus Number of Failures

Let L_T be the total load on the sub-system in a given phase. If there are i $(0 \le i \le n\text{-}k)$ component failures in the sub-system, the load is shared by the remaining (n-i) components. Hence, the load on each component is

$$L_i = \frac{L_T}{n-i} \tag{6.5}$$

Let λ_i be the failure rate of each of the surviving components when there are i failures. The value of λ_i can be found using load-life relationship [69]. If the failure rate follows the power-law, then:

$$\lambda_i = C.\left(\frac{L_T}{n-i}\right)^{\alpha} \quad (6.6)$$

If failure rate follows the exponential-law, then: $\lambda_i = C.exp\left(\frac{\alpha.L_T}{n-i}\right)$ (6.5)

6.4.3. Conditional State Probabilities in a Single Phase

In the proposed method, the conditional state probabilities for each sub-system in each phase are calculated. Let $P_{a,i}$ be the conditional probability that the sub-system is in state-i (S_i) at the end of the phase given that it was in state-a (S_a) at the beginning of the phase. Because the sub-systems are non-repairable, the number of failures in the sub-system increases with time. Hence,

$$P_{a,i} = 0; \quad \text{for} \quad a > i \quad (6.8)$$

Therefore, to calculate $P_{a,i}$ only for $a \leq$ i *is needed*. Let X_m be the time spent in state-m (S_m). When the system is in S_m, there will be (n-m) surviving components. Because any one of the surviving (n-m) components can fail with rate λ_m, the next failure in the system occurs with rate γ_m.

Therefore, to calculate $P_{a,i}$ only for $a \leq i$ is needed Let X_m be the time spent in state-m (S_m). When the system is in S_m, there will be (n-m) surviving components. Because any one of the surviving (n-m) components can fail with rate λ_m, the next failure in the system occurs with rate γ_m.

$$\gamma_m = (n-m).\lambda_m; \quad for \quad m \leq n-k \quad (6.9)$$

Therefore, X_m follows exponential distribution with rate parameter γ_m. Let $T_{a,i}$ be the total time spent in S_a to S_i. Hence, $T_{a,i} = X_a + X_{a+1} + \cdots + X_i = \sum_{m=a}^{i} X_m$

$$(6.10)$$

Where $T_{a,i}$ is equal to the sum of *(i-a+1)* independent random variables following exponential distributions with possibly different parameters (rates). Closed-form expressions for the cumulative distribution function and survival function of $T_{a,i}$ are available in [29]. Let $G_{a,i}(t)$ be the survival function of $T_{a,i}$, where t is the phase duration. It is defined as:

$$G_{a,i}(t) = \Pr\{T_{a,i} > t\} = \Pr\{X_a + X_{a+1} + \cdots X_i > t\} \qquad (6.11)$$

To find the closed-form expressions for $G_{a,i}(t)$, the following two cases should be considered separately:

Case-1: All γ_i*'s* are equal (say γ) [13].

$$G_{a,i}(t) = \sum_{m=0}^{i-a} \frac{(\gamma t)^m \exp(-\gamma t)}{m!} \qquad (6.12)$$

In this case, $T_{a,i}$ follows the gamma (Erlang) distribution. This case arises when the failure rate of each surviving component is linearly increases with the load. If the power-law model is used for failure rate, this case occurs when $\alpha = 1$.

Case-2: All γ_i*'s* are distinct [13].

$$G_{a,i}(t) = \sum_{m=i}^{a} A_m \exp(-\gamma_m t) \qquad (6.13)$$

where

$$A_m = \prod_{j=i;j\neq m}^{a} \frac{\gamma_j}{\gamma_j - \gamma_m} \qquad (6.14)$$

In this case, $T_{a,i}$ follows the Hypo-exponential distribution. This case arises more frequently. If the power-law model is used, this case occurs when $\alpha \neq 1$. Once $G_{a,i}(t)$ is calculated, the conditional probability, $P_{a,i}$, can easily be calculated:

$$P_{a,i} = \begin{cases} 0; & a > i \\ G_{a,i+1(t)} - G_{a,i}(t) & a \le i \le n-k \\ 1 - G_{a,i}(t); & a \le n-k;\, i = n-k+1 \\ 1; & a = n-k+1;\, i = n-k+1 \end{cases} \tag{6.15}$$

6.5 Sub-system Reliabilities

Assume that there are n components in a given sub-system. The sub-system requires at least k_j working components in phase j. In other words, the sub-system is considered to be failed if there are at least $d_j = (n\text{-}k_j\text{+}1)$ failed components during phase j. Further, the sub-system is considered to be failed if it fails in any one of the phases. Let $\boldsymbol{x_j}$ be the number of components that have failed before the completion of phase j, where j=1,2,…,M. Hence, the sub-system is considered to be successful if $\boldsymbol{x_j < d_j}$ for all values of j (all phases). Let $\boldsymbol{Z_{j,i}}$ be the probability of the sub-system state such that $\boldsymbol{x_j = i}$ and $\boldsymbol{x_l < d_l}$ for all $\boldsymbol{l < j}$. Hence,

$$Z_{j,i} = Pr\{x_j = i;\ x_{j-1} < d_{j-1}; \dots ; x_1 < d_1\} \tag{6.16}$$

Using the Markov property of the x_j sequence [29], the conditional probability term in equation (6) can be simplified.

$$Z_{j,i} = \sum_{a=0}^{d_{j-1}-1} Z_{(j-1),a} \cdot Pr\{x_j = i | x_{j-1} = a\} \tag{6.17}$$

Where,

$$Z_{1,i} = Pr\{x_1 = i\} = P_{0,i} \tag{6.18}$$

$$Pr\{x_j = i | x_{j-1} = a\} = P_{a,i} \tag{6.19}$$

The equation for $P_{a,i}$ is provided in equation (14). Once $Z_{M,i}$ values are calculated using the recursive formulas, the sub-system reliability, R_l (where the suffix l is for the l^{th} sub-system) can be calculated.

$$R_l = \sum_{i=0}^{d_M - 1} Z_{M,i} \tag{6.20}$$

Finally, the mission reliability of the entire system is calculated using equation (6.4).

6.6 Illustrative Example

In this section, the proposed method is illustrated using a numerical example. The system has 3 sub-systems and the mission is divided into 4 phases. The number of components in each sub-system is shown in Table 1. The parameter values for the load-dependent failure rate model depend on the sub-system.

They are shown in Table 6.1.

Sub-system	**#Comp.**	**Failure Rate Model**	**Failure Rate Model Parameters**	
ID	***n***	***$\lambda(L)$***	***C***	α
A	**4**	**Power Law**	**1.0E-4**	**1.5**
B	**3**	**Power Law**	**1.2E-6**	**2.0**
C	**5**	**Exponential**	**5.0E-5**	**1.25**

Table 6.1 – Sub-system Parameters

The duration of phases and the phase-dependent sub-system parameters (k and L_T values) are shown in Table 6.2.

Phase		Phase 1	Phase 2	Phase 3	Phase 4
Duration		20	30	50	25
Phase-Dependent Sub-system Parameters					
Sub-system *A*	K	2	1	3	1
	L_T	10	20	5	15
Sub-system *B*	K	1	2	2	1
	L_T	50	30	40	25
Sub-system *C*	K	3	5	4	2
	L_T	2	5	6	3

Table 6.2 – Phase-Dependent Requirements and Parameters

In the proposed method, the mission reliability for each sub-system is calculated. The step-by-step procedure for the sub-system-A is demonstrated. The same procedure can be used for other sub-systems. However, for the sub-system-C, instead of power-law, exponential-law should be used for calculating the failure rates of components.

For each sub-system, the state probabilities ($Z_{j,i}$) associated with phase-1 are calculated first. Using these probabilities, the state probabilities for the next phase (phase 2) are calculated, and so on. To calculate the state probabilities, the loads on each of the surviving components when there are i ($0 \leq i \leq n\text{-}k$) failures in the sub-system is determined. Then the load-dependent failure rates for the components is calculated. Using component failure rates, the state transition rates are determined. From Table 1 and Table 2, we have: $n = 4$, $k = 2$, and L_T =10 for sub-system-A in phase-1. Therefore, from equations (6.5) and (6.8), λ_i and γ_i as in Table 6.3 are calculated.

I (*# Failures)*	λ_i (Failure Rates)	γ_i (Transition Rates)
0	0.004	0.0016
1	0.006	0.0018
2 = (n-k)	0.0011	0.0022

Table 6.3 – Failure Rates and Transition Rates for Sub-system-A in Phase 1

From Table 2, the duration of phase-1 is 20. Now using the transition rates in Table 6.3, $P_{0,i}$ are calculated using equation (6.14) and set to $Z_{1,i}$ as in equation (6.17). These values are shown in Table 6. 4.

i *(# Failures)*	$P_{0,i}$	$Z_{1,i}$
0	0.9689	0.9689
1	0.0306	0.0306
2 = (n-k)	0.0006	0.0006

Table 6. 4 –State Probabilities for Sub-system-A in Phase 1

Repeating the above procedure for phase 2, 3, and 4, the $Z_{j,i}$ values are calculated. For these subsequent calculations, the equations (6.14) and (6.18) are used. The results are summarized in Table 6.5.

i *(# Failures)*	$Z_{2,i}$ (Phase 2)	$Z_{3,i}$ (Phase 3)	$Z_{4,i}$ (Phase 4)
0	0.8472	0.8239	0.7662
1	0.1387	0.1572	0.1999
2	0.0130	0.0178	0.0143
3	0.0010	0.0188	0.0007
4	0.001	0.0188	0
Sum(Reliability)			0.9811

Table 6. 5 –State Probabilities for Sub-system-A in Phase 2,3 and 4

Finally, the mission reliability of sub-system-A is calculated from equation (6.19). From Table 2, *k* value for the phase-4 is 1. Therefore, $d_M = n\text{-}k_M+1$ = 4-1+1 = 4. Therefore, the sub-system reliability is calculated by summing the first four values in the last column of Table 5. Hence, the mission reliability of sub-system-A is 0.9811. If the switch success probability on demand is considered as 0.95, then the mission reliability of sub-system-A is 0.9696.

Similarly, the mission reliabilities for other sub-systems: R_B = 0.9855 and R_C = 0.9532 are calculated. Finally, the overall mission reliability of the entire system is calculated as the product of mission reliabilities of individual sub-system as in (4). Hence, $R_{PMS(sw)} = 0.9108$. With perfect switches, system reliability is 0.9436. With switch failures on demand, system reliability is reduced to 0.9108. The CPU time for solving this problem is 0.0018 seconds. Refer to [17] for a method to calculate these small CPU times accurately.

6.6 Conclusions

In this Chapter, a new method for reliability evaluation of phased mission systems with load-sharing components subjected to switch failures is presented. The proposed algorithm is developed based on: (1) a modularization technique, (2) an easily computable closed-form expression for conditional reliability of load-sharing sub-systems, and (3) a recursive formula for the reliabilities of sub-systems across the phases. The CPU time for solving the numerical example demonstrates that the proposed method is computationally efficient. As in reference [1], the proposed method can easily be extended for analysing phased mission systems with random phase durations. Further, as discussed in section 3, it is straightforward to apply the proposed method to the cases where different sub-systems use different types of redundancies, such as active, standby, and load-sharing redundancies. Time-varying hazard rates and non-identical components will be considered as the future research work.

CHAPTER 7[7]

COST-EFFECTIVE EARLY WARNING SYSTEM

This research Chapters takes into consideration the development of a unique system in association with the aspects of power industry, to enhance the Reliability of power systems. Abundance of wind, solar, wave and other natural, other domestic resources in Australia develop creativity in technologies of cleaner varieties. Requirements that are regulatory innovative and new or the ones which could be generated with the renewable energy sources and an increment in carbon emission pricing, there has been an increase in the demand of consistent, renewable power systems. Reliable power systems are of significant value in terms of environment conservation procedures, global warming phenomenon and the economic condition of a country. In an era where carbon pricing is excessive, a cost effective yet simpler way to enhance the reliability of power systems has to be approached to achieve societies of greener properties. Through this Chapter, it is evident that the functions of remote fault identification and detection mechanism for Generation, Transmission and Distribution (GTD) system for both kinds of renewable and non-renewable resources are to reduce catastrophes and their outcome by implementing a unique hardware and software solution integrated with the help of M2M technology and cloud computing mechanism. The early warning system developed for the improvement of power system Reliability assures the authors to alleviate the possible negative circumstances on power system functionalities. The equipment developed for this objective has multiple benefits because of its low cost, low consumption of power and feasible installation and regularities.

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In this Chapter, the authors have depicted their success stories and conclusions that they have derived in the application of their system to work on the reliability of electrical power systems in multiple industries involving telecommunication, electrical, transport and agricultural ones. A particular instance of installation, and testing of SMS Alert System to for the monitoring of change in maintenance of circuit breaker and auto-enclosure status created within extra high voltage (EHV) switchyard materials in an electrical sub-station will be deliberated from Reliability point of view. A comparative study of the system developed will also be elaborated between different locations demographically distinguished among non-renewable and renewable power mechanisms. Finally authors show that there is a significant improvement in reliability indices of power systems as a result of this solution.

7.1 Introduction

Because of the reason of bush fires, floods as well as other climatic conditions in major parts of the Australia like Victoria and Queensland have suffered from the recent blackouts and have directed almost everyone's major thoughtfulness on the examination and the determination of the Victorian Electrical Power Systems. Since the main functionality of any electrical power system is to spring their customers the satisfaction of having the sort of energy of not just an electrical kind having the unsurpassed economical rates but also of a constant continuation as well as distinct quality. Since last ten years the blackouts that have been recurrent in Victoria are being confronted on a regular basis every twelve months. For instance in February 2005 40,000 Victorians were devoid of the electrical power supply due to the poor climatic and storm conditions when storms and temperature changes reached their peak limits during January 2006 the results were seen in the form of 618,000 supply interventions and that is not it, in year 2007 due to a bush fire the rolling power shut down across the whole Victoria was forced. When a storm took over in April 2008, 420,000 Victorians had to suffer through the off supply for many days. In the year January 2009 because of the labour-intensive problems and break downs in the admirations in the Bas link interconnect caused a huge power supply cut off to more than 500,000 Victorians. [5].

As far as the areas of natural calamities and the power failure because of it is concerned, there has not been much work and research done on it along with their initial warnings for the Australian energy providing sectors. In references [79-81] the guidance that is produced due to the natural disasters on the power systems is presented and all the early warnings regarding the usual catastrophes and the power systems were proposed. In reference [82] which deals with the risk assessments, monitoring of early warnings and their related mechanism are delivered. The fore most shortcomings of this arrangement are:

1. It will be very expensive to build, install and maintain.
2. It cannot detect the fault location in the major power systems.

Power industry all over the world looking for the ways to improve the reliability of supply to its customers by investing hugely on its infrastructure. The most important part of reliability improvement is monitoring. Monitoring includes recording root causes and assets affected during outages and how well the process of restoring supply was managed. Reliability indices such as System Average Interruption Duration Index (SAIDI), System Average Interruption Frequency Index (SAIFI), and Average Availability Index (ASAI) are measures of supply wide reliability, operation and maintenance efficiency.

The given and planned hardware elucidations are very much effective in terms of cost and also operative in the detection of the faults plus relaying of the guidelines. The critical components of a power systems and its status of working and conditions are monitored by the system and when the estimated data has been collected the failure chances of the components in all normal and extreme weather conditions are anticipated including the time duration of the natural disaster. By an effective application of this technology, this analysis is aggravating to access the power system reliability and sustainability by giving added information to the operator of the approaching failures in the system. This concept is advised to be an application of the Smart-Grid philosophy. This concept can be used in Victorian power systems to improve the Reliability of Victorian power systems.

7.2 Electrical Substations

Problems associated with in Electrical Power Utilities within these times are confronted with accretion accident of burnouts and black outs. This in association with uncalled conservation costs and anytime accretion appeal for ceaseless power accumulation to the customer, it becomes majorly essential to instantaneously acquaint the outage status in the substation to the staff so that the fault rectification can be done as soon as it can be accessible and reduce power failures..

The planned explanation is that the outage consistency of transmission manual aspect is dejected through SMS to the individuals concerned. The above elements which are monitored are Circuit Breakers, Protection Relays, Auto reclose relays. Early Warning System with up to 32 Digital inputs (expandable up to 64) and up to 16 Analog input channels, finer takes use of the absolute GSM Network for communication. As an aftereffect of their proposed outcome, several Early Warning Systems (EWSs) accommodated in assorted substations beyond India viz., Warangal, Nellore,Vizag, Hyderabad (Andhra Pradesh), Trichy, Hosur (Tamil Nadu), Wardha (Maharashtra), Baroda/Jambuva (Gujarat), etc. Power grid 400/220kV substations and 220kV manual association substations are auspiciously active and allowance the aliment agents in befitting themselves abreast on the failing of assorted substation essentials and appropriately augmenting their aliment costs.

Figure 7.1: Typical 400KV substation, the control room and EWS in action. An SMS sent from the device can also be seen in the picture above.

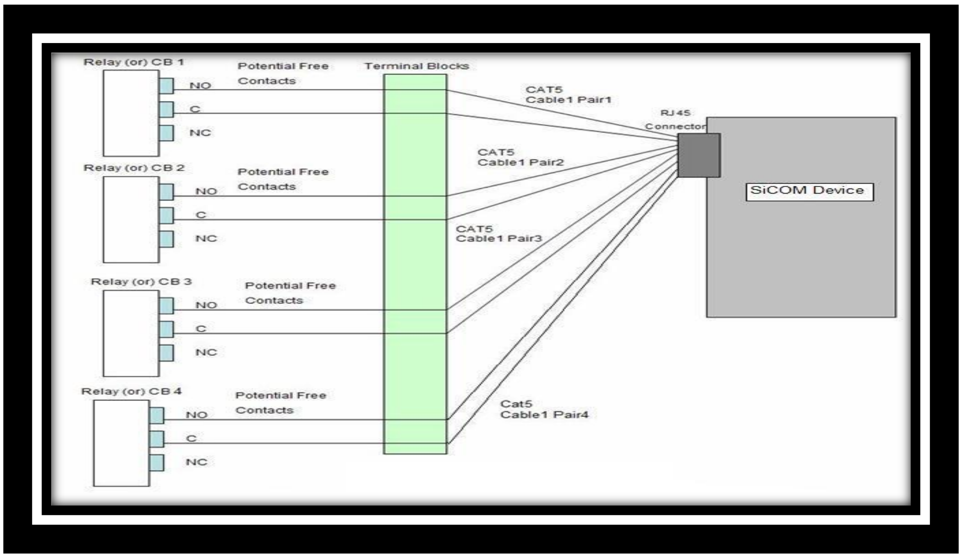

Figure 7.2: Wiring diagram for digital inputs

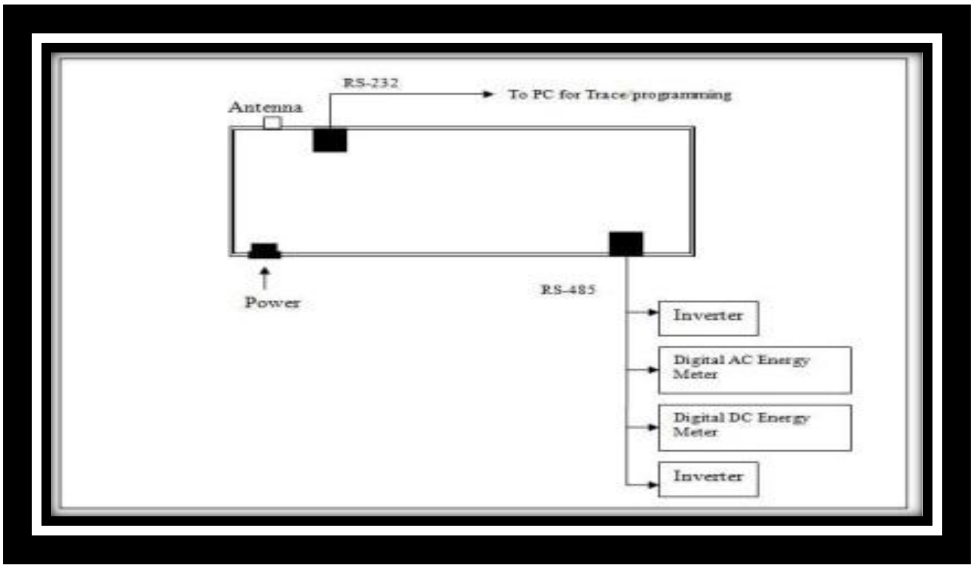

Figure 7.3: EWS Typical Layout showing RS-232 AND RS-485.

7.3 Illustration of Data from EWS

- **HVDC-VIZAG: ICT-1 GR-A/B Operated on 2012/07/11, 21:00:03:45.**
- **HVDC-VIZAG: ICT-1 GR-A/B Restored on 2012/07/11, 2100:15:25.**
- **HVDC-VIZAG: Kalapaka-1 GR-A/B Operated on 2012/07/11, 15:40:57:50.**
- **HVDC-VIZAG: Kalapaka-1 GR-A/B Restored on 2012/07/11, 15:41:15:40.**
- **HVDC-VIZAG: BUSBAR-2 Protection Operated on 2012/07/11, 15:40:58:95.**
- **HVDC-VIZAG: BUSBAR-2 Protection Restored on 2012/07/11, 16:56:58:55. After 1 hrs 16 minutes.**
- **HVDC-VIZAG: 400 KV BPL-2 TIE CB Operated on 2012/07/11, 15:40:58:10.**
- **HVDC-VIZAG: 400 KV BPL-2 TIE CB Restored on 2012/07/11, 15:41:25:30.**
- **HVDC-VIZAG: 220 KV DUR-2 LINE CB Operated on 2012/07/11, 15:40:58:95.**
- **HVDC-VIZAG: 220 KV DUR-2 LINE CB Restored on 2012/07/11, 15:40:58:10.**

Device Configurations:

Set Time:

Command: admin123 setdt 11/06/01, 16:02:00

Response: SUCCESS setdt 11/06/01, 16:02:00

Set the Phonebook numbers

Command: admin123 setpb, 9885608760, 9000200120

Response: SUCCESS setpb, 9885608760, 9000200120

To 9885608760

SMS: Your No is added in the Phonebook of HVDC-VIZAG Sicom

Set Massages:

Command: admin123 setport 02, 1, 1, 220 KV NGM-2 LINE CB Operated, 220 KV NGM-2 LINE CB Restored

Response: SUCCESS setport 02, 1, 1, 220 KV NGM-2 LINE CB Operated, 220 KV NGM-2 LINE CB Restored.

7.4 Railways signalling and Telecommunications

The key issues in Railway Signalling and Telecom (S&T) Installations are generally installed forth the clue at assorted limited locations area accessibility is low, positioning an individual for connected ecology is approved to be expensive and the information of issues itself requires time for accountability rectification, which will consequence could cause losses of consistencies besides the losses due to faulty equipment or materials implied (S&T element) and the adventitious conservation. Though there is a substantial availability of monitoring equipment such as data loggers, they are generally concentrated on cataloguing the information rather than mistaken allusion and abbreviation the interruption. The EWS advised for S&T is an Short Messaging Service (SMS) based accountability monitoring system. It is a microcontroller based equipment with congenital accountability argumentation engine for notifying assorted Signal faults like AC Main Fails, Direct Current Distribution Panel (DCDP), Inverters Fail, Signal Machine Replication (SMR) Fail, Battery Low, Surge Protection Device (SPD) AC/DC issues for Integrated Power Supplies (IPS) and Optical Fiber Communication (OFC) charger can be alongside arrangement to data logger for day to day failure administration and ability enhancement. Embedded Intelligence engine for the acceptance of assorted issues highlighting assorted accountability alarm situations in S&T Power supplied for Optical Fibre Communication (OFC) Huts and Integrated Power Supply (IPS) for the purpose of supporting the signalling gear.

- OFC AC MAINS FAIL
- OFC RECTIFIER DC OUTPUT FAIL
- OFC DC OUTPUT SPD FAIL
- OFC AC INPUT SPD FAIL

- OFC DC UNDER VOLTAGE
- IPS AC MAINS FAIL
- IPS INVERTER 1 FAIL
- IPS INVERTER 2 FAIL
- IPS SMR FAIL
- IPS DCDP FAIL
- IPS BATTERY LOW (109V)

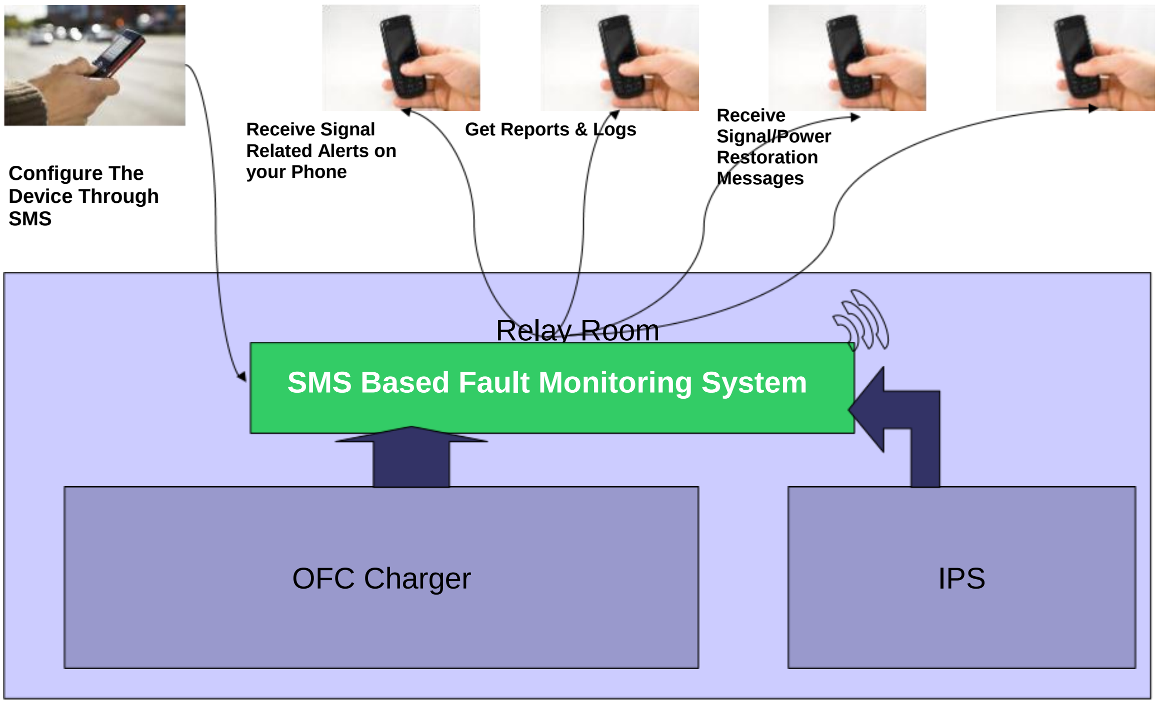

Figure 7.4: A solution to improve the reliability of Railway Signaling and Telecommunications. This system has been installed at a test site and working reliabily

The system shown in figure 7.4, Railway installation is helping the railway authorities in:

• Instantaneous failure communication directly from the field enabling the key personnel informed all the time.

• Better fault handling in terms of man power deployment or machinery so that the line is up and running.

• Fewer punctuality losses and improved over all system efficiency.

Optimized manpower and resource utilization enabling cost effective maintenance.

7.5 Power generation stations

In this section, the authors points out the essential and basic most issues in power generation stations and resolutions given with EWS to improve the reliability and maintainability of the absolute power station equipment. The only method which is cooperating enough is the one optimizing the absolute substructure and processes. So that the plant and mechanical arrangements irregularity is diminished and the operations are consistent and cheap.

Proposed solution: Suggested EWS fabricated process for power generating stations absolutely bridges the unusual capacities between the SCADA and the Operating personnel/Maintenance engineers. The authors activated the proposed initial warning arrangement by positioned in assorted APGENCO Power plants viz, NTTPS (Vijayawada), RTPP (Kadapa), Kakatiya Thermal Power Project (Warangal) assures the engineers and managers to access several configurationally options.

7.6 Power Generation System at Victoria University Melbourne

Conventional activity technologies are not ecology friendly, not renewable and also the amount of deposit fuels traveling college (anecdotal affirmation reflect that consumers will be spending three times more money for their accepted bill in 5 years' time). Hydrogen Activity and Fuel Cells appliance may be a key basic to play a basic role in electrical administration system, back the alone by-product is calefaction and H2O and is safer than gasoline, agent and accustomed gas. So Hydrogen Activity and Fuel Cells appliance on electrical administration arrangement are acceptable awful adorable in the limited Remote Area Power System (RAPS) due to top manual and administration losses, top amount to carriage of accepted fuels to limited locations or top amount of filigree extensions [83]. For the application of RAPS, Hydrogen Energy and Fuel Cells appliance arrangement comprising of hydrogen accumulator tank (50L) having a pressure of 2000PSI, T-1000 1.2kW PEMFC, and (4×12) Volts bank of the battery, 3 circuit breakers, a voltmeter and an ammeter are acclimated for analysis in Power Systems Research Lab at Victoria University to accomplish generation of the electrical power in an effectual manner.

If Hydrogen Energy and Fuel Cells appliance arrangement is appropriately managed according to their corresponding capacities, such synchronized acceptance prevents boundless amount and ensures all the sub-systems and isolated constituents in every sub-system are at the aforementioned phase of their activity span. Outcomes from sudden issues in an arrangement can be secured with the help of effective planning and maintenance of thoughts [78].

7.7 Illustrative Example

In this particular section, the authors depict information collected for the improvisation of the reliable power systems implementing accessible and suitable warning systems operating in Victoria University Power Lab. Early Warning System operated from Fuel cell is outlined below.

- From SiCOM Device at Mar 4, 2012 23:25 : VU : Fault restored in Fuel Cell on 2000/03/06,00:17:58:30From SiCOM Device at Mar 4, 2012 23:25 : VU : Fault in Fuel Cell on 2000/03/06,00:17:59:20
- From SiCOM Device at Mar 4, 2012 23:25 : VU : Fault restored in Fuel Cell on 2000/03/06,00:17:59:30
- From SiCOM Device at Mar 4, 2012 23:25 : VU : Fault in Fuel Cell on 2000/03/06,00:18:00:65
- From SiCOM Device at Mar 4, 2012 23:25 : VU : Fault restored in Fuel Cell on 2000/03/06,00:18:00:10
- From SiCOM Device at Mar 4, 2012 23:28 : VU : SiCOM Started on 2000/03/06,00:21:29 Hrs
- From SiCOM Device at Mar 4, 2012 23:29 : VU : Fault in Fuel Cell on 2000/03/06,00:22:31:15
- From SiCOM Device at Mar 4, 2012 23:29 : VU : Fault restored in Fuel Cell on 2000/03/06,00:22:31:40
- Send To SiCOM Device at Mar 4, 2012 23:34 : admin123 setconf 28,1,1,Fault in Roof Top Solar Plant, Fault Restored in Roof Top Solar Panels
- From SiCOM Device at Mar 4, 2012 23:34 : SUCCESS setconf 28,1,1,Fault in Roof Top Solar Plant, Fault Restored in Roof Top Solar Panels
- From SiCOM Device at Mar 4, 2012 23:35 : VU : Fault in Roof Top Solar Plant on 2000/03/06,00:27:56:65
- From SiCOM Device at Mar 4, 2012 23:35:VU: Fault Restored in Roof Top Solar Panels on 2000/03/06, 00:27:56:85.

7.8 COST

Cost effectiveness of EWS in comparison to any other instance and example given by the industry partners of the Power Industry is highly commendable. Systems of similar assortments in power industry are usually the ones that are founded on a large scale SCADA systems with RTUs (Remote Terminal Units) on which an ample amount of restricted copper wiring has to be implied, larger structural elements along with the software regulations are included. Given EWS is a miniature RTU, fulfilled with communication procedure, a simple plug and play sort of a gadget, therefore, works efficiently being powerful enough, portable and not as expensive.

7.9 Reliability Issues and Results

The author's Initial installation had some issue related to system malfunction during some conditions like network non-availability and fault occurrence. After this, all the EWSs installed are running successfully for over a year at several power grid and GENCO substations. However, for S&T, the requirements were different. The authors had to monitor 24 digital points and 8 analog voltages, evaluate failure conditions based on logics among those digital input states and keep track of voltage levels of each analog channel High Normal and low but not just ON/OFF alone.

These requirements made the authors to bring out second generation modular Early Warning System, which has pluggable hardware blocks instead of fixed number of DI /DO and AI points. The Hardware has a base board or the mother board on which the following provisions are given.

1) Power supply card with rechargeable battery and charger provision.
2) Mobile Communication Module.
3) Serial Communication Module.
4) Optically Isolated Digital Input modules.
5) Analogue input Modules.
6) Digital output Modules (Dry Contacts).

Depending on the field requirement, the hardware configuration can be done. This System was highly appreciated by customer like Power Grid and GENCO as they have the flexibility of asking for required number of points to be monitored along with all the other features that this small box does.

7.10 Renewable Power Systems.

When the authors first started their research into monitoring of solar installations, the requirements were slightly different; there is a necessity to interact with several energy meters, Inverters at the solar installations over communication ports on serial lines.

This led the authors to develop the 3rd Generation Early Warning System which the authors call it EWS3.The overall system is classified into two types of modules,

1) EWS3 - **G2** : GSM/GPRS Gateway Module
2) EWS3 - **ION**: Input/output Nodes.

This section of research Chapter discussed about the development of third generation EWS using an innovative technology in collaboration with the industry to improve the reliability of power systems.

Devices and people collaborate over cloud based collaboration platform. Many machines located in many places, these remote machines could be equipment, power supplies, inverters since these equipment typically have to monitor KWH, Voltage, Current, liquid level, temperature, pressure, on/off status, GPS location and other vital parameters. And control on/off equipment and alarm condition. These parameters are technically Digital inputs, ADC inputs, RS232/485 and LAN. These are connected to a Remote Terminal Unit (RTU), Micro Controller plus GSM/GPS Modem. RTU connects to Network Operating Centre (NOC) server through GPRS. NOC server has TCP server to communicate with RTUs database to store configurations and history, application for configuration and ticket management.

M2M enables thousands of devices, located in anywhere in this world to be simultaneously monitored and managed to provide real-time information for any business to analyse and act upon to improve the reliability. M2M allows key information to be exchanged without human intervention, making it possible to reduce costs, improves efficiency, reliability service to customers.

This research focus is cellular M2M, the figure below represents a typical cellular M2M setup. This setup is Smart-Grid integration capable solution. Smart grid is always aware of current power supply and demand. It uses intelligent power networks which use M2M solutions to immediately react to critical event such as unexpected spikes or drops in the availability of renewable resources. The automated interaction of intelligent load sharing and smart metering generates a massive amount of information to streamline grids, lower costs and create more reliable energy supply.

Figure 7.5: Existing M2M setup

Cellular M2M setup is divided into four stages.

1. Sensor and equipment interface
2. Remote device and m2m server communication
3. Configuration of remote devices
4. Content delivery mechanism.

Each stage has certain challenges. There is no standardization in interconnection, therefore chance of field staff doing mistakes in interconnecting input/outputs (IOs) and interfaces to Remote Terminal Unit (RTU) are not reliable. In remote devices and M2M server communication stage, most of the existing M2M players using proprietary protocols to communicate between RTU and M2M server, consistency and robustness in data are compromised, apart from security.

The proposed solution is shown in Figure 7.5 below.

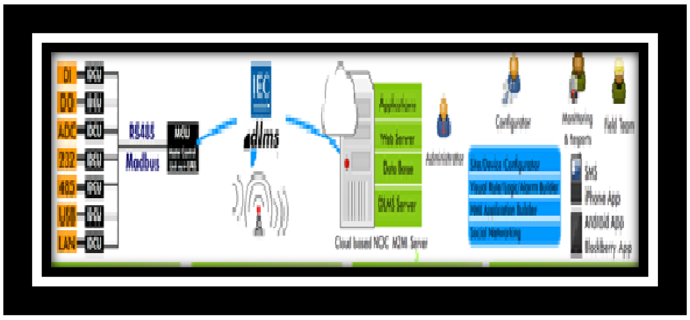

Figure 7.6: Proposed M2M solution

In server and equipment interface, Device Control Unit (DCU) is a simple IO gateway or protocol gateway to interface with equipment with priority protocol and physical interface. All DCUs data would be available to Master Control Unit (MCU) on industry standard RS485/Modbus. This makes over all solutions highly scalable and maintainable, and even the MCU can be replaced with any future technology beyond GSM/GPRS, without touching the DCU infrastructure.

In Remote devices and M2M server communication, our proposed solution, adapts standard protocols(IEC specs, DLMS/COSEM) to communicate between MCU and M2M server to bring robustness, security and reliability in data transmission.

In the proposed solution, configuration of remote devices partial for MCU and DCUs configuration enhances the remote configuration and diagnostics capabilities and Visual rule/logic/alarm builder to reduce the dependency of firmware programmers. This would also automate the firmware up gradation for logic change in MCU.

In the proposed solution content delivery is through Cloud based M2M server. Once built, it will deliver the content as Rich Internet Application (RIA) web application; phone App, Android App or Blackberry App., aggregated data delivery to social media-Facebook, Twitter etc. Integration of social media for reporting and escalation of aggregated data.

Priority wise, Energy, Utilities, Supply Chain management and manufacturing is shown in the figure 7.6 below.

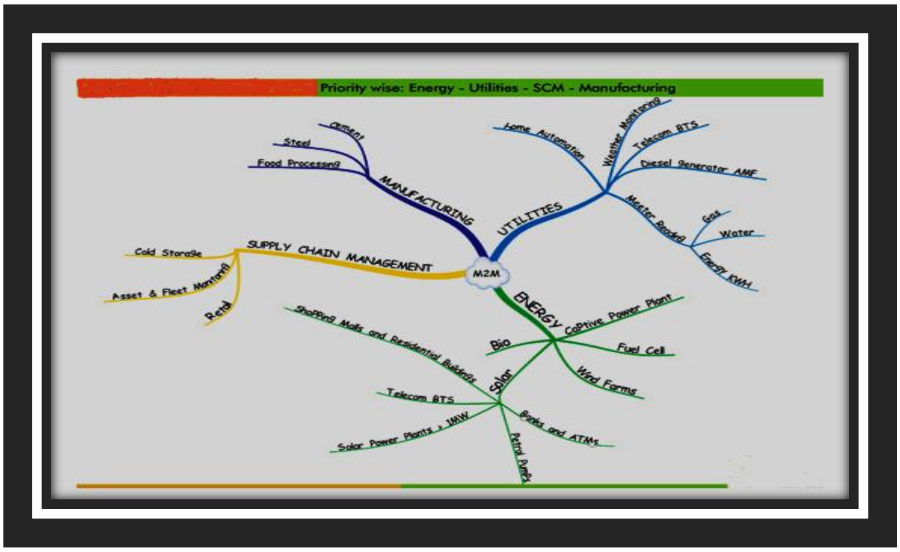

Figure 7.7: Energy, Utilities, Supply Chain management and manufacturing

This proposed solution has distributed control architecture for remote devices, uses robust protocols for over the air data, uses visual approach for application development, content development and content delivery. Similar systems are under evaluation in US energy sector; however this research project focuses on Cloud Infrastructure, software applications/platforms, and low cost highly maintainable devices for remote monitoring to improve the reliability of power systems.

During this project author has developed and installed third generation early warning system in 300 remote locations in collaboration with our industry partner, where formers are using solar powered water pumps and author is able to monitor, maintain and improve the reliability of this huge system. The following date collected from those 300 remote locations 24 hours 7days 365 days, for the analysis. The hardware, software tool and data base developed for this project collects data for use in reporting PV generating unit reliability, availability and productivity. For this project the following data is captured on test database:

- Operating frequency(current frequency)
- Input voltage (DC detection)
- Cumulative operation time
- Output voltage
- Compensated frequency
- Estimated speed
- Torque
- Integral input power to the solar powered pump from solar panels
- Integral output power
- Frequency command value
- Output current
- Torque current
- PID feedback value
- Motor load factor
- Inverter load factor
- Input Power
- Output power

From the collected data, author was able to calculate generation unit reliability, availability and productive indices. According to IEEE standard definition for use in reporting electric Generating unit Reliability, Availability and Productivity ANSI/IEEE std 762-2006 are shown below :

Availability factor (AF): The fraction of a given operating period in which a generating unit is available without any outages. AF = Availability hours/Period hours *100.

Unavailability Factor (UF): The fraction of a given operating period in which a generating unit is not available due to outages. UF = Unavailable hours/period hours*100.

Service Factor (SF) = Service hours/period hours*100

Starting Reliability (SR) = number of starting successes/(number of starting successes + number of starting failures)*100.

Application of proposed solution to solar powered water pump is providing valuable information to improve the system reliability, to choose better system design options and to realize maximum benefit of solar power to poor farmers in India.

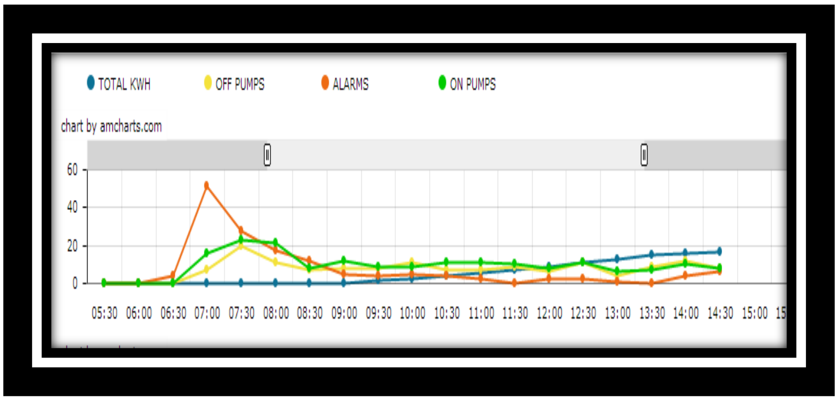

Figure 7.8: Total amount of energy produced, number of solar pumps in operation and number of alarm messages at particular time of the day.

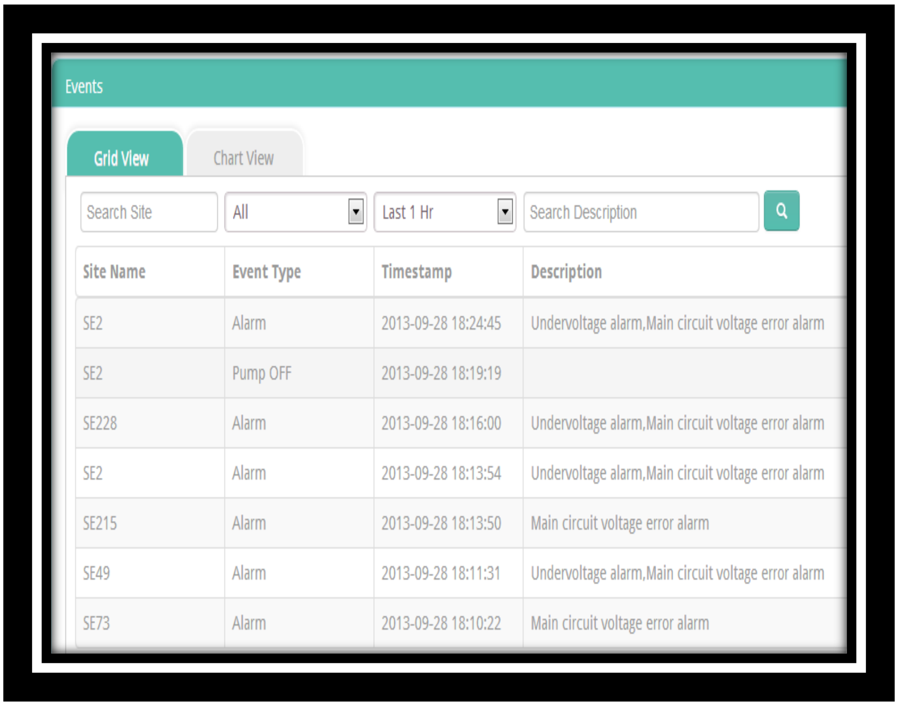

Figure 7.9: Shows site name, solar pump location, even type, time stamp, and description of the event.

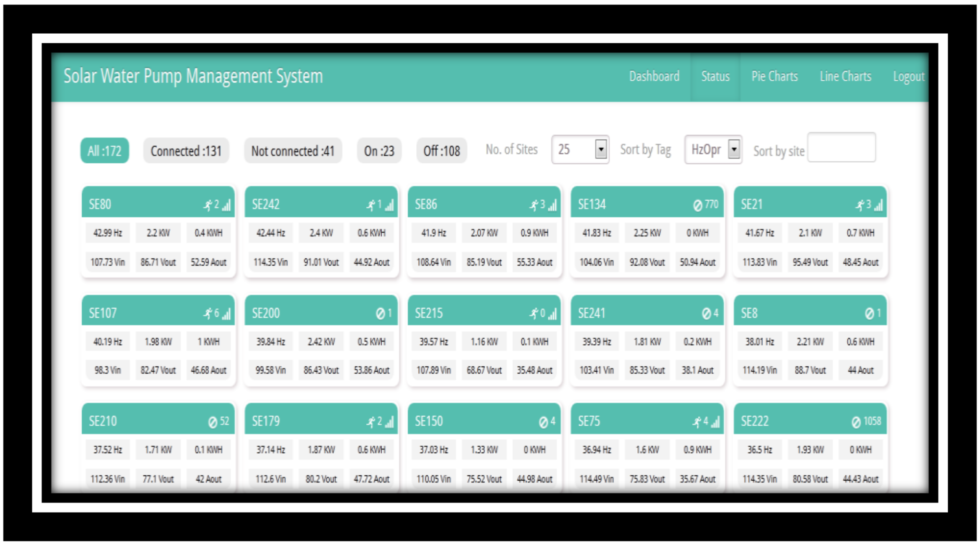

Figure 7.9: Sample data with more clarity

7.11 Conclusions

In this Chapter, a considerable and balanced cost managingearly warning arrangement for enhancing the reliability of power systems is approved using assorted case studies. Early Warning Systems action abundant advantage in effectualsuitability and accountability administration in power systems, Though this analysis plan sheds light on the progresses that has happened so far, the future timegrasps abundant prospective for this arrangement in agreement of, affiliation with ecology management materials, Digital Fault Recording, Failure anticipation and Preventive maintenance.

CHAPTER 8

SUMMARY AND FUTURE WORK

8.1 Summary

This thesis presented a number of research suggestions related to the concept of improving the reliability of power systems by finding new methods for reliability evaluation and enhancement of power systems more efficiently and quickly. It began with an introduction of power systems reliability issues; solutions presented in the past and need for newer methods to solve the reliability problems associated with Smart-Grid connected power systems.

By using the concepts of counting processes, an efficient approximate method to evaluate the reliability of cold standby systems when component lives follow Rayleigh distributions was presented. The proposed method also considers the effects of switch failures on system reliability. The consideration of Rayleigh distributions allowed this research to apply this method for analysing cold standby systems with components having linearly increasing hazard rates. The step-by-step procedure of the method is demonstrated using a numerical example. All steps involved in the proposed method are simple and do not include any complex numerical integrations. Therefore, the method can easily be implemented in any reliability software package.

This method is new because there have been no previous attempts to find a solution to the reliability of “k-out-of-n” cold standby system with **Rayleigh distributions**. The results indicate that the CPU time for the reliability evaluation of the proposed method is extremely fast.

In Chapters 4 to 6 another three new methods were proposed. In Chapter 6 a new method for reliability evaluation of phased mission systems with warm standby systems was presented. In the past several researchers have worked on solving the problem of finding the reliability of phased mission systems with different situations, but no solution has been found to the reliability of "Phased Mission Systems with Warm Standby Sub-Systems".

This new method proposed an algorithm to solve this problem in a fast and computationally efficient way. The proposed algorithm is established based on: (1) a modularization approach, (2) an efficient closed form equation for conditional reliability of warm standby sub-systems and (3) a recursive expression for enumerating the reliance of sub-system over the phases. Results show that time to solve the problem is fast and the method is computationally powerful.

In Chapter 5, another new method for reliability evaluation of Load Sharing systems using "k-out-of-n" structure subject to proportional hazards model was presented. The solution is based on a time transformation using cumulative hazard function and equivalent problem formulation based on continuous Markov chains. The analysis also considers the effects of imperfect switches at the time of load redistribution. Numerical results obtained using closed form expressions are compared with Monte Carlo simulation result.

There are several papers on reliability evaluation of Load-sharing systems using "k-out-of-n structures" but no solutions to this problem have been offered when components in power systems are ageing. This new method solves the problem of finding reliability of Load Sharing systems using "k-out-of-n" structure subject to proportional hazards model.

In Chapter 6, another new method for reliability evaluation of phased mission systems with load-sharing components is presented. There are several methods individually for reliability evaluation of phased missions and some other methods for reliability evaluation of load sharing systems. But this method is a combination of both problems as it solves the problem of ‘reliability evaluation of phased mission system with load-sharing components’. No previous attempts have been made in this regard. The proposed method consists of an algorithm, which is developed based on: (1) a modularization technique, (2) an easily computable closed-form expression for conditional reliability of load-sharing sub-systems, and (3) a recursive formula for the reliabilities of sub-systems across the phases. The CPU time for solving the numerical example demonstrates that the proposed method is computationally efficient. The proposed method can easily be extended for analysing phased mission systems with random phase durations. Further, it is straightforward to apply the proposed method to the cases where different sub-systems use different types of redundancies, such as active, standby, and load-sharing redundancies.

This research took the consideration of the development of a unique system in association with the aspects of power industry, to enhance the Reliability of power systems. Abundance of wind, solar, wave and other natural, other domestic resources in Australia develop creativity in technologies of cleaner varieties. Requirements that are regulatory innovative and new or the ones which could be generated with the renewable energy sources and an increment in carbon emission pricing, there has been an increase in the demand of consistent, renewable power systems. Reliable power systems are of significant value in terms of environment conservation procedures, global warming phenomenon and the economic condition of a country. In an era where carbon pricing is excessive, a cost effective yet simpler way to enhance the reliability of power systems has to be approached to achieve societies of greener properties. Through this research, it is evident that the functions of remote fault identification and detection mechanism for Generation, Transmission and Distribution (GTD) system for both kinds of renewable and non-renewable resources are to reduce catastrophes and their outcome by implementing a unique hardware and software solution integrated with the help of M2M technology and cloud computing mechanism. The early warning system developed for the improvement of power system Reliability assures the research to alleviate the possible negative circumstances on power system functionalities. The equipment developed for this objective has multiple benefits because of its low cost, low consumption of power and feasible installation and regularities.

In this thesis, this research has depicted the success stories and conclusions have derived in the application of the system to work on the reliability of electrical power systems in multiple industries involving telecommunication, electrical, transport and agricultural ones. A particular instance of installation, and testing of SMS Alert System to for the monitoring of change in maintenance of circuit breaker and auto-enclosure status created within extra high voltage (EHV) switchyard materials in an electrical sub-station was deliberated from Reliability point of view. A comparative study of the system developed was also elaborated between different locations demographically distinguished among non-renewable and renewable power mechanisms. Finally, research show that there is a significant improvement in reliability indices of power systems as a result of this solution.

8.2 Future work

The work reported in this thesis involves the development of new methods for Reliability evaluation and enhancement of Power Systems. The following aspects relating to future research are outlined below:

- Reliability Evaluation of Load-Sharing systems using “k-out-of-n” structure subject to proportional hazards model can be extended to solve the problem when components are non-identical and subject to imperfect switches.
- This model can be applied to do the reliability analysis of aging power systems all over the world to avoid blackouts and subsequent economic loss to nations.
- This research can be extended to solve the same problem when components are non-identical in the case of Smart-Grid-enabled power systems.
- Reliability Evaluation of Phased Mission Systems with load-sharing components can be extended to solve the systems with random phase durations, with different types of redundancies, such as active, standby and load-sharing redundancies, for time-varying hazard rates and non-identical components.
- These methods can easily be implemented in any reliability software package or can develop a general purpose software to quantitatively access the reliability of various parts of the power system or sub-system or entire system configurations by using techniques shown in this thesis and evaluate the reliability in terms of probability of failure, frequency of failure, failure due to aging, failure due to load-sharing, cold standby redundancy, warm standby redundancy, k-out-of-n configuration, different structures and different component failure distributions.

- The developed software can be used for research in power industry, to make cost effective decision about maintenance, equipment replacement decisions, train power engineers and train graduate electrical engineers on power system reliability assessment. It can provide power engineers with an efficient and effective tool for estimating the performance of power systems. Powerful calculation techniques allow engineers to choose the depth of system design and the associated results. The key features of reliability assessment software could be: System reliability, Customer oriented indices, Energy (cost) indices, Sensitivity analysis, Single & double contingency.
- Propagated failures are one type of common-cause failures (CCF) that involve simultaneous failure of multiple power systems elements due to a failure originating from some internal power system element. Common-cause failure can also be caused by external factors such a sudden change in environment, power supply disturbances and power system design mistakes. Many studies have shown that both types of common-cause failures tend to increase the joint system failure unreliability. Therefore in future research needs to done for the accurate reliability analysis of power system with common-cause failures especially bush fires, storms, earthquakes and tsunamis.
- The system developed in Chapter 7 – "Smart-Grid Integration capable, Cost-Effective, Cloud Service based Early Warning System for Improving the Reliability of Power Systems using M2M Technology" can be further improved in future research to integrate in the design of electrical substation automation. It can be improved to meet IEC61850 standard and generate Generic Objective Oriented Substation Events (GOOSE) messages in a more user friendly manner.

- ‘System Configuration descriptive Language (SCL)’ and representation format specified by IEC61850 for configuration of electric substation devices can be modified in a more user friendly and configure electric substations remotely using cloud technology.

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