A STATISTICAL APPROACH TO AUTOMATIC PROCESS CONTROL (REGULATION SCHEMES)

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DECLARATION

The candidate hereby declares that the work (which is presented in the thesis entitled, 'A Statistical Approach To Automatic Process Control (Regulation Schemes)', for the award of the degree of Doctor of Philosophy submitted to the Department of Computer and Mathematical Sciences, Victoria University of Technology):

(i) is that of the candidate alone and has not been submitted previously, in whole or part, in respect of any other academic award and has not been published in any form by other person except where due reference is given.

(ii) has been carried out during the period from February 1992 to March 1997 under the supervision of Dr.N.Barnett and Dr.P.Cerone.

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Venkatesan Gopalachary

SUMMARY TO THE THESIS A STATISTICAL APPROACH TO AUTOMATIC PROCESS CONTROL (REGULATION SCHEMES)

Automatic process control (APC) techniques have been applied to process variables such as feed rate, temperature, pressure, viscosity, and to product quality variables as well. Conventional practices of engineering control use the potential for step changes to justify an integral term in the controller algorithm to give (long-run) compensation for a shift in the mean of a product quality variable. Application of techniques from the fields of time series analysis and stochastic control to tackle product quality control problems is also common. The focus of this thesis is on the issues of process delay ('dead time') and dynamics ('inertia') which provides opportunity to utilise technologies from both statistical process control (SPC) and APC.

A presentation of the application of techniques from both SPC and APC is made in an approach to control the quality of a product (product variability) at the output. The thesis considers the issues of process control in situations where some form of feedback control is necessary and yet where stability in the feedback control loop cannot be easily attained. 'Disturbances' afflict a process control system which together with issues of dynamics and dead time (time delay), compound the control problem.

An explanation of proportional, integral and derivative (PID) controllers, time series controllers, minimum variance (mean square error) control and MMSE (minimum mean square error) controllers is given after a literature review of stochastic process control and 'dead-time compensation' methods.

The dynamic relationship between (output) controlled and (input) manipulative variables is described by a second-order dynamic model (transfer function) as also is the process dead time. The use of an ARIMA (0,1,1) stochastic time series model characterizes and forecasts the drifting behaviour of process disturbances. A feedback control algorithm is developed which minimizes the variance of the output controlled variable by making an adjustment at every sample point that exactly compensates for the forecasted disturbance. An expression is derived for the input control adjustment required that will exactly cancel the output deviation by imposing feed back control stability conditions. The (dead-time) simulation of the stochastic feedback control algorithm and EWMA process control are critiqued.

The feedback control algorithm is simulated to find the CESTDDVN (control error standard deviation) or control error sigma (product variability) and the adjustment frequency of the time series controller. An analysis of the time series controller performance results and discussion follow the simulation. Time series controller performance is discussed and an outline of a process regulation scheme given. The thesis enhances some of the methodologies that have been recently suggested in the literature on integrating SPC and APC and concludes with details of some suggestions for further research.

Solutions to the problems of statistical process monitoring and feedback control adjustment connected with feedback (closed loop) stability, controller limitations and adequate compensation of dead time in achieving minimum variance control, are found by the application of both process control techniques. By considering the dynamic behaviour of the process and by manipulating the inputs during non-stationary conditions, dynamic optimization is achieved. The IMA parameter, suggested as an on-line tuning parameter to compensate dead time, leads to adaptive (self-tuning) control. It is demonstrated that the performance of the time series controller is superior to that of the EWMA and CUSUM controllers and provides minimum variance control even in the face of dead time and dynamics.

Some articles/papers have appeared in Technometrics, Volume 34, No.3, 1992, in relation to statistical process monitoring and feedback adjustment (251-267), ASPC (286-297), and discourse given on integrating SPC and APC (268-285). By exploiting the time series controller's one-step ahead forecasting feature and considering closed-loop (feedback) stability and dead-time compensation, this thesis adds further to these contributions.

CONTENTS

SYMBOLS AND ABBREVIATIONS

Symbols:

g	-	System or Closed-loop (Feedback) gain
AI	-	Adjustment interval (Sample period)
X	-	Input variable
$\overline{X}$	-	Mean of X
x_t	-	Adjustment in the input variable X
Y	-	Output or controlled variable
Z	-	Disturbance (Noise)
z_t	-	Disturbance variable at time t
a_t	-	Variable representing random shock at time t
$\{a_t\}$	-	Sequence of a_t
Θ	-	Moving average parameter
r	-	Rate of drift of process
R	-	Random number
R_1	-	Independent random number
R_2	-	Independent random number
β	-	Angle between Z_1 axis and radius B
Z_1	-	Standard Normal random variable
Z_2	-	Standard Normal random variable
δ	-	Inertia
σ	-	Standard deviation

$\hat{z}$	-	Estimate of z
$\sigma_{\hat{z}}$	-	Standard deviation of $\hat{z}$
e_t	-	Forecast error at time t
σ_e	-	Forecast error standard deviation
σ_a	-	Standard deviation of the random variable 'a'
B	-	Backward shift operator
	-	Radius
	-	Polynomial representation
∇	-	Backward difference operator
T_d, b	-	Dead time (Time delay)
τ	-	Time constant
e	-	Exponential
δ_1	-	Dynamic parameter
δ_2	-	Dynamic parameter
ω	-	Process response
E, ε	-	Control error
σ_E	-	Control error standard deviation
$L_1(B)$	-	Polynomial in B
$L_2(B)$	-	Polynomial in B
$L_3(B)$	-	Polynomial in B
$L_4(B)$	-	Polynomial in B
t	-	time
ℓ	-	lead time

$\Phi(B)$	-	Autoregressive operator
$\Theta(B)$	-	Moving average operator
I	-	Time constant of (integral) controller
I_u	-	Integral time for zero damping
k	-	Lag
r_k	-	Autocorrelation estimate
L	-	Process chart control limits
T	-	Target
	-	Sampling period/interval
α	-	Units of adjustment in input variable

Abbreviations:

SPC	-	Statistical process control
ASPC	-	Algorithmic statistical process control
APC	-	Automatic process control
EPC	-	Engineering process control
IMA	-	Integrated moving average
ARIMA	-	Autoregressive integrated moving average
EWMA	-	Exponentially weighted moving average
GMA	-	Geometric moving average
CUSUM	-	Cumulative sum
PID	-	Proportional, integral, deriative
PI	-	Proportional, integral
CESTDDVN	-	Control error standard deviation

SE	-	Control error sigma
MSE	-	Mean square error
MMSE	-	Minimum mean square error
CG	-	Controller gain
PG	-	Process gain
LCL	-	Lower control limit
UCL	-	Upper control limit
FREQ	-	Frequency, an indicator variable
MFREQ, MFEQ		Average for FREQ
AF	-	Adjustment frequency
SISO	-	Single input - single output
ARL	-	Average length

CHAPTER 1

INTRODUCTION TO THE THESIS

A STATISTICAL APPROACH TO AUTOMATIC PROCESS CONTROL (REGULATION SCHEMES)

1.1 INTRODUCTION

A manufacturing process is concerned with the conversion of a set of input raw materials into a set of desired outputs. The outputs are generally expected to conform to some measurable *targets*. Exact conformance is not possible due to factors which cause output variables to deviate from their targets. These factors may be variations in the inputs or variations in the conversion process itself. Variations in the inputs may cause instability in the process resulting in a bias in the outputs due to the presence of *assignable* or *special causes* of variation. The presence of non-random effects can be detected as unexpected variations in system measurements. In addition to the inputs, the process may well be afflicted by *disturbances* forcing it to drift off target if no control action is taken to eliminate them.

Satisfactory control of a process requires that its output follows some predetermined command signal and that it remains unaffected by disturbances or variations in the process parameters. *Engineering control systems* continually adjust processes *on-line* in order to counteract the effects of disturbances. *Statistical process control* (SPC) seeks to reduce to an acceptable level the residual variation in the output that can degrade product quality. An objective in automatic process control (APC) is to maintain certain key process variables as near their desired values, called *set points,* for as much of the time as possible. From among the process variables, certain variables are chosen as the key ones. By maintaining these variables at specified values, it is possible to achieve certain production objectives.

1.2 MOTIVATION AND AIMS IN STATISTICAL APPROACH TO AUTOMATIC PROCESS CONTROL

One frequent production objective is to produce material of desired quality by having an acceptable level of variation (product variability) in the measured output characteristics. Based on recent history and/or a particular model, one approach to control is to forecast the deviation from target which would occur if no action were taken and then to act so as to cancel out this deviation. A mathematical model is constructed to describe the behaviour of the production system in a concise way. A structural model is then developed and set up as an hypothesis to account for the sampling observations. The resulting model is used to forecast the behaviour of the time series describing the disturbance. Using the structural model, the process is controlled by generating warning signals of future untoward events.

This thesis considers the application of techniques from both SPC and APC to the control of product quality at the output. In the process industries, close control of a laboratory measured variable to the specified target is frequently necessary in order to meet the desired quality characteristic at the output. It is common that the effect of any process adjustments is delayed beyond when the adjustment is made. This delay is called *dead time* and may be caused by delay in performing laboratory measurements, by processes that take a longer time to deliver material from the point of adjustment to the sample point or by general inertia in the system or in the chemical reaction that is occurring. *The main focus of this thesis is the time series controller and its application to reduce output variance and hence to aid in the control of product quality. This thesis (i) addresses issues related to feedback control (closed-loop) stability without large increases in the system or closed-loop gain by considering a 'critically damped' second-order system, (ii) gives direct indication, through simulation, of when to make an adjustment (change) in the input variable and by how much the input variable should be adjusted so that the mean of the quality variable is at or near target, (iii) the simulation results provide information about the number of adjustment intervals (AIs) (sample periods) after which a sample needs to be taken and the process adjusted, (iv) provides values of the required adjustment (dxt) in the input variable, the adjustment variance (vardxt) and the variance of the control error (varCE) from simulation results and (v) hypothesises use of the IMA parameter, Θ as an on-line tuning parameter of the close-loop time constant for use in Dahlin' algorithm by drawing attention to and making inferences from the work of Harris, MacGregor and Wright [1982].*

The sequential achievements of this thesis are:

(i) the development of a second-order model for a dynamic system with delay (dead time) giving proper justification for doing so giving due reference to the conventional approach of describing a process by a first-order model,

(ii) use made of the Integrated Moving average (IMA)'s Exponentially Weighted Moving Average (EWMA) property for making forecasts of the deviation (error) from target,

(iii) utilization of the time series controller's one-step ahead forecast error variance to predict forecasts over the dead time period,

(iv) derivation of a stochastic feedback control algorithm for the required input adjustment to compensate the output deviation due to disturbance, and a simulation study using the algorithm to obtain the time series controller's performance measures,

(v) showing that the performance of a time series controller is superior to that of the EWMA and CUSUM controllers from information obtained from simulation results,

(vi) achieving optimization under non-stationary conditions and

(vii) the presentation of an outline for a process *regulation* scheme.

1.3 ORGANISATION OF THE THESIS

The thesis is organised into ten Chapters, Chapters 2 to 9 containing the body of the thesis and Chapter 10 providing suggestions for further work at the interface between statistical process control and automatic process control.

1.3.1 Review and Literature Survey of Stochastic Process Control

The second chapter relates to stochastic process control. It gives a review and literature survey spanning the past 30 years. A discussion of statistical and automatic process control is given and the terms *algorithmic process control, feedforward control, feedback control* and *adaptive control* are briefly explained. A comparison of adaptive control with feedback control is made, bringing out the subtle difference between the two. The effect of process dynamics (inertia) on control and the conditions under which adaptation would facilitate feedback regulation are discussed. Some discussion on controllers and in particular, the three term PID controller is given.

1.3.2 Discrete Stochastic Control and Direct Digital Control

The background of discrete stochastic control is reviewed briefly in Chapter 3. The motivation and objectives of the thesis are explained as also its salient features. A description is given of minimum variance control and minimum variance controllers. A review of the design of sampled data controllers is made and the concept of direct digital control explained. The need to identify dead time and its effect on sampled-data (discrete) control systems is also explained. The role and characteristics of inertia are described.

1.3.3 Dead-Time Compensation

The effects of dead time on the performance of a controller are described in Chapter 4. The need for a dead-time compensator is explained as also is the principle of working of some common dead-time compensators.

1.3.4 Stochastic Process Control Algorithm

The salient features of time series controllers are explained in Chapter 5 and the control equation is derived for a feedback control model. An exponent of the criterion for the time series controller algorithm is given and an approximate feedback control equation is derived when there are disturbances and process dynamics (inertia). Justification for considering second-order dynamic models and in particular, restricting attention to the 'critically damped behaviour' of the second-order system are discussed. An expression is also derived for the feedback control adjustment required in the input variable of a time series controller. Time series controller performance measures are explained and an outline of the strategy to be adopted to determine these measures is also detailed. A review of Baxley's (Baxley [1991]) simulation study of statistical process control algorithms for drifting processes is given. The Chapter concludes by detailing how the experimental strategy for simulation studies used in that study can be followed to find the controller (tuning) parameters.

1.3.5 Simulation Study of the Stochastic Process Control Algorithm

Simulation methodology to determine the time series controller performance measures are discussed in Chapter 6 as also is the EWMA control charting procedure. The simulation strategy and the drifts (fast and slow) are explained. A discussion of Baxley's [1991] and Kramer's [1990] results is given before concluding the Chapter.

1.3.6 Time Series Controller Performance Measures - Analysis and Discussion

An analysis and discussion of time series controller performance measures is presented in Chapter 7. The simulation methodology and EWMA process control are explained as too is the feedback control adjustment. The benefits and limitations of integral control are briefly discussed along with details of a constrained variance control scheme. The simulation results are discussed along with some inferences.

1.3.7 Process Regulation Scheme - An Outline

A brief review of process regulation is given in Chapter 8 and an explanation of a feedback control process regulation scheme given. An outline of such a scheme along with a cost model are presented.

1.3.8 Controller Performance and Product Quality Control

The performance, limitations and robustness, in reference to the function of a controller are explained in Chapter 9. The characteristics and requirements of a feedback controller are given along with a brief discussion of the working of a direct digital sampled data (discrete) controller. Examples of controller applications are also provided.

1.4 CONCLUSION

This thesis considers mainly feedback control (closed-loop) stability problems from the automatic or engineering process control point of view by the application of process control techniques at the interface of SPC and APC. The literature on stochastic-dynamic process control is replete with work by (statistical) process control specialists, for example Box [1957], Box and Jenkins [1962, 1963, 1965, 1968, 1970, 1976], Astrom [1970], Box and MacGregor [1974], MacGregor [1987, 1988], Harris [1989], Harris and MacGregor [1987], Harris and MacGregor and Wright [1982]. It is acknowledged that this work covers topics that relate to the analysis of closed-loop dynamic-stochastic systems, assessment of control loop performance, on-line process control, discrete stochastic and linear quadratic controllers etc. Unfortunately, these valuable contributions along with the work done separately by control engineers in automatic process control focus only on particular aspects of process control and are very much disjointed. The main thrust of this thesis is to show that by proper modelling of a dynamic process and the disturbance, with adequate dead-time compensation and (feedback) integral control, it is possible to minimise the control error standard deviation (product variability) at the output by bringing the two methodologies together for more efficient operation.

This research is aimed at finding a solution to the commonly occurring product quality control problem by utilising both SPC and APC techniques. This thesis offers solutions beneficial to both the scientific and technical communities.

CHAPTER 2

STOCHASTIC PROCESS CONTROL: A REVIEW AND LITERATURE SURVEY

2.1 INTRODUCTION

In this chapter, the need for process control is explained and definition given to the terms 'statistical process control' and 'automatic' or 'engineering process control'. Some major contributions to process control by both statisticians and control systems engineers are outlined and an explanation and review given of algorithmic statistical process control. A brief description of proportional integral derivative and time series controllers is also provided.

2.2 PROCESS CONTROL

The manufacturing industries may be broadly classified into two types: (i) the component parts manufacturing industries and (ii) the process industries. Typical examples of the component manufacturing industries are production and assembly lines. The process industries, on the other hand, are typified by the manufacture of bulk chemicals, powders or fluids, plastics, etc. This distinction is a broad one and there is in fact a spectrum of types (Weatherill and Rowlands [1991]). A reality is that some modern processes, such as the production of computer chips, are hybrids, using certain manufacturing aspects of the parts industries and other aspects similar to that of the process industries.

Global production objectives are generally to achieve production targets at an acceptable cost and to manufacture products of a desired quality in a safe manner with the minimum possible harm to the environment. These goals are realised by *monitoring* and controlling production. *Control* tools are used:

(i) to *detect* changes in process performance from a stable state,

(ii) to *identify* the *assignable* or *special causes* of *variation* indicated by these deviations and eliminate the same and/or

(iii) to *adjust* a relevant process variable or variables so as to maintain a performance criterion in some desirable neighbourhood of a target value (Box and Jenkins [1963]).

The first two control actions of process monitoring and control are commonly achieved by traditional statistical process control **(SPC)** techniques.

The objective of statistical process control, (explained subsequently in Section 2.2.2), is to engage in regular process *surveillance* in order to detect the presence of special causes of variation. SPC aims to contain variation in output so that the level of product quality is both predictable and satisfactory. In contrast, one of the main aims of automatic process control **(APC),** (explained in Section 2.2.3), is to provide an instantaneous continuous response, counteracting changes in the balance of a process and to apply self-corrective action to bring the output close to the desired target without, where possible, human intervention (Keats and Hubele [1989]). APC aims also to maintain certain key process variables as near their desired values, called 'set points', for as much of the time as possible in order to satisfy production objectives. One approach is to 'forecast' the output deviation from target which would occur if no control action were taken and then to act so as to cancel out this deviation.

2.2.1 Need for Feedback Control and Feedforward Control

The third process control action, mentioned above, is possible by an appropriate *feedback* or *feedforward* procedure that indicates when and by how much to adjust. This is called *adaptive quality control* (Box and Jenkins [1962]). Feedback control systems are frequently referred to as *closed-loop* systems. When knowledge of the value of some fluctuating measured input variable is used to partially cancel out deviations of the output from the target value, then the action is called *feedforward* control, (Figure 2.1). When it is possible to use the deviation from target or *error signal* of the output characteristic itself to calculate the appropriate compensatory changes that need to be made, it is called *feedback* control (Harrison [1964]), (Figure 2.2). In some situations, *feedforward-feedback* control, a combination of feedforward and feedback control, is used.

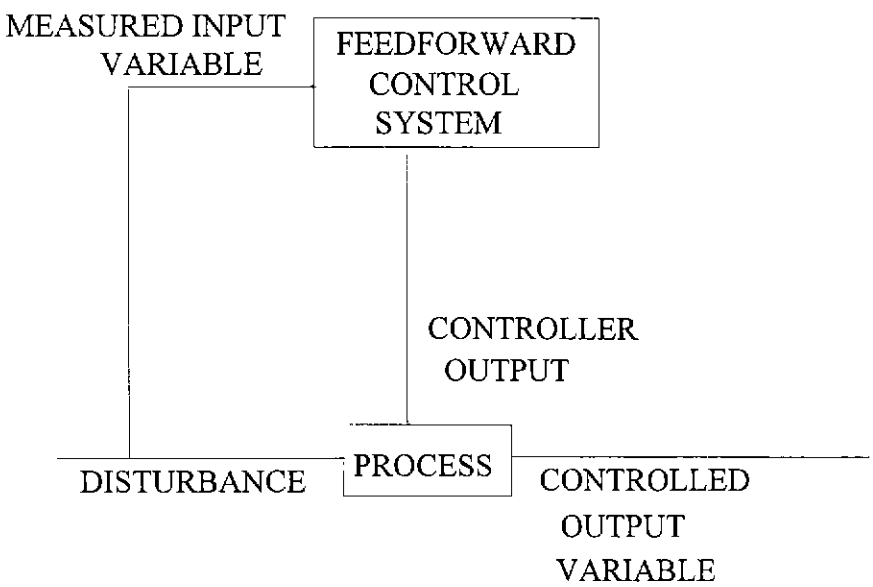


Figure 2.1 FEEDFORWARD CONTROL MODEL

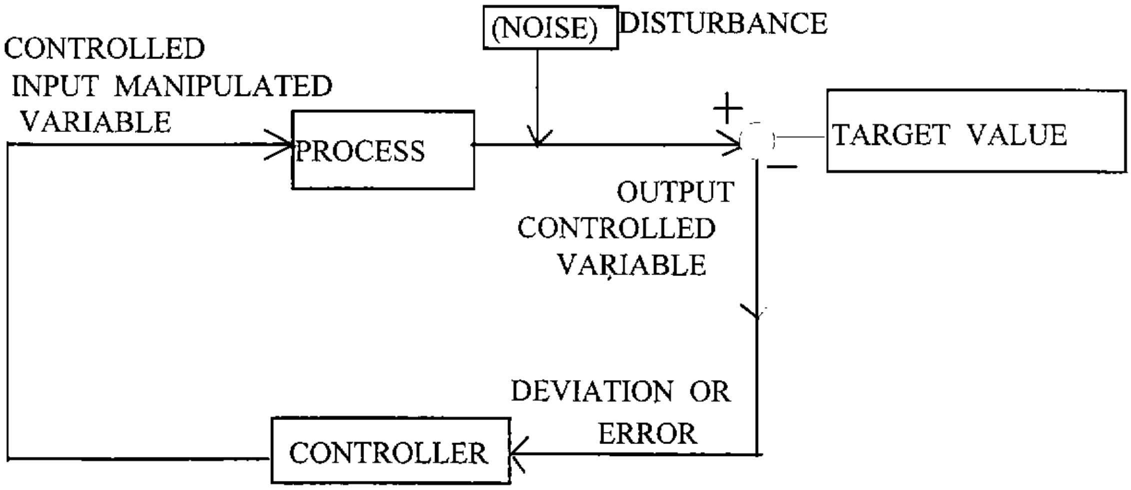


Figure 2.2 FEEDBACK CONTROL MODEL

Feedback that is amenable to computer control, is often termed *empirical* or *technical*. Empirical feedback occurs when the reason for taking action is not very complex in nature and predictable by a simple rule. The rule describes unequivocally the control action that should be taken and the modifications that should be made. Intelligent feedback results from the interaction of the human mind with information obtained from *experimentation* as characterised by, cycle:- *conjecture-design-experiment-analysis* (Box [1957]), leading to new ideas and models. One non-automatic example of technical feedback is the use of quality control charts to indicate when something is wrong and to highlight any abnormal variations in the behaviour of a process (See Figure 2.3).

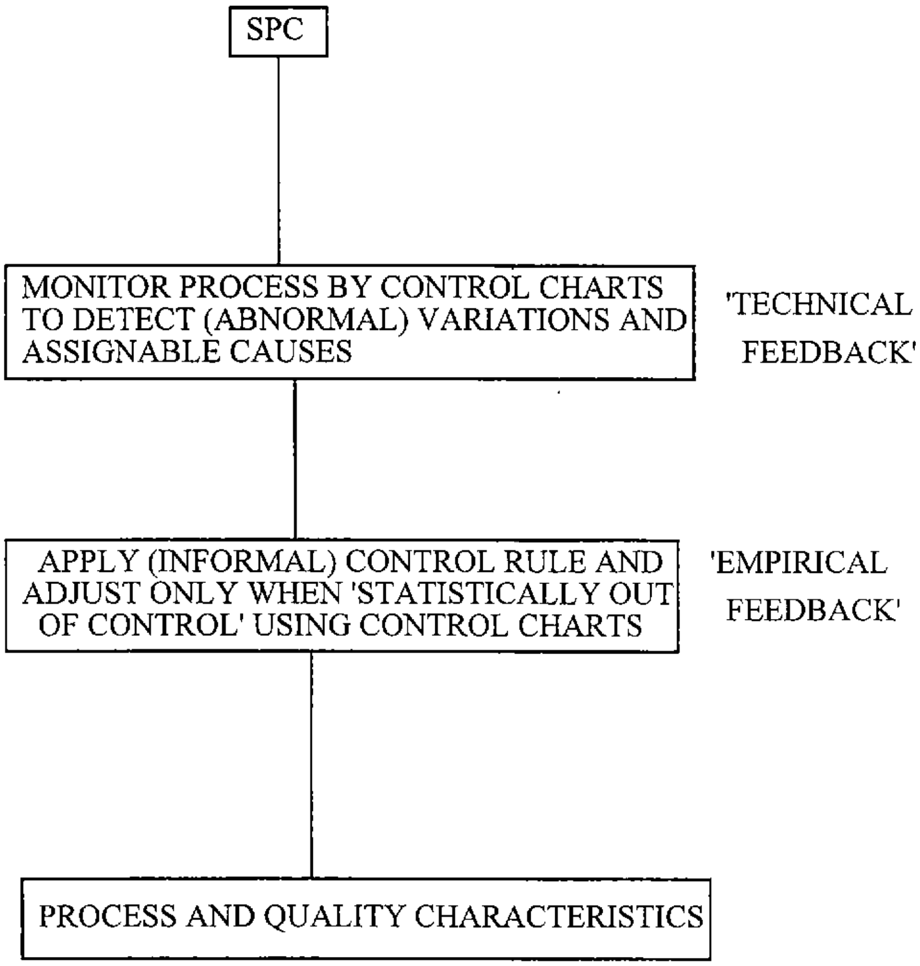


Figure 2.3 STATISTICAL PROCESS CONTROL (SPC)

2.2.2 Control Charts and Statistical Process Control (SPC)

Control charts, the tools of SPC, are used to decide when to adjust the process and when to leave the process alone as a result of either identifying special or assignable causes of variability or concluding that only a *chance cause system* of variation is present. Special causes should be duly accounted for when reacting to control warnings. Control charts are used for detecting apparent departures from a *model.* A model is a theoretical description that adequately accounts for the current behaviour of the system and will allow reasonable prediction of its future behaviour. A model can be constructed by direct analysis of the physical laws governing the system. Due to lack of knowledge of all the factors governing the system's behaviour, it may not be possible to build a complete model of the system. Modelling is combined with experimentation to obtain data from the system. Various techniques are used to determine a model which best fits the measured data (Hunt [1989]). Process monitoring and control is concerned with regularly checking that the system continues to follow the assumed model and as such it parallels hypothesis testing as a statistical procedure. SPC techniques help an analyst in monitoring a process so as to detect and remove special causes of variability that are inconsistent with the working model. SPC attempts to improve the process over the long term by finding and removing these special causes. It may also subsequently prompt action in an attempt to reduce product variability (control error standard deviation). *Single value, X-bar* and *range control charts*, *cumulative sum* **(CUSUM)** charts and *exponentially weighted moving average* **(EWMA)** charts are tools of trade employed in SPC, (Figure 2.3).

2.2.3 Automatic Process Control (APC)

We refer to Section 2.2.1 in which we dealt with feedback and feedforward control. In automatic process control (Figure 2.4), various forms of feedback and feedforward control regulation schemes are used for process adjustment to make the required change in the input level in order to compensate for the output deviation. If no compensatory adjustments are made by taking proper control action, the process drifts off target and the course followed by the output results in 'disturbance' (Box and Kramer [1992]). *Disturbance causes variability in the output or outputs of an otherwise stable process by producing undesirable changes in the (output) mean.* The need for process regulation arises when the system is afflicted with disturbances unless proper control action is taken to compensate them.

APC is a collection of techniques for devising algorithms to manipulate the adjustable variables of a process to achieve the desired process behaviour, namely, output close to a specified target value. Automatic control of a process can be defined as the maintenance of a balanced state by measuring one or more of the conditions representing the balance and providing an automatic counteraction to any change in the condition[1]. Control systems engineers call this 'balanced state' a 'steady state' of the process. The steady state level of the output obtained is the value at which the discrete output from a stable system eventually comes to 'equilibrium' when the input is held at some constant value (page 338, Box and Jenkins 1970, 1976]).

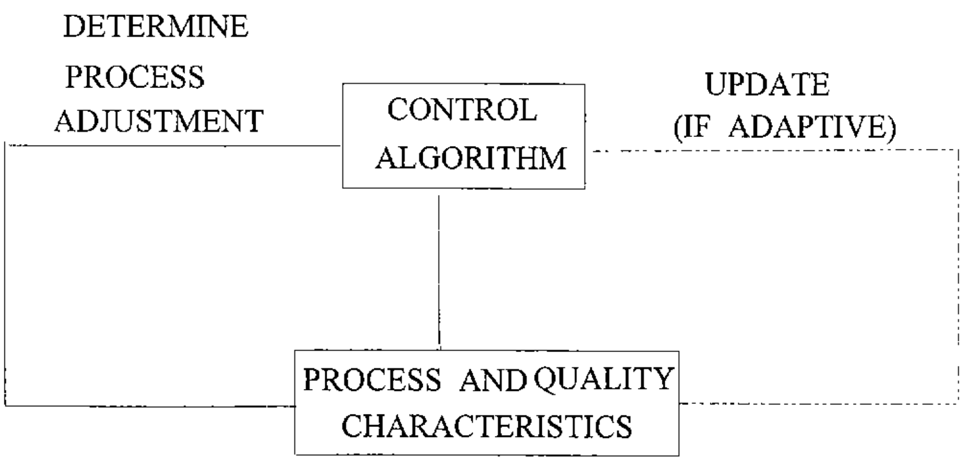


Figure 2.4 AUTOMATIC PROCESS CONTROL (APC)

1 Adapted from the definitions used in the Science of Automatic Control by A.S.M.E. Industrial Instruments and Regulators Division Committee on Technology, U.S.A.

The ISA[2] definition of steady state is, 'a characteristic of a condition such as a value, rate, periodicity, or amplitude exhibiting only negligible change, over an arbitrary long period of time'. The balance in the process may be a balance of any form of energy, for example, heat or pressure (Eckman [1945]). Unstable processes may need frequent control to stabilise them. Consequently, it is worthwhile to concentrate on controlling stable processes (explained subsequently).

Control systems engineers are more concerned with the dynamics (inertial characteristics) and stability of the system. The term 'controlled process' is often used to mean a 'process state' that is (narrowly) interpreted as *stationary* having *iid variation about a target value* (Wiel and Vardeman [1992]). An alternative, has 'a state of control' as *a process state in which future behaviour can be predicted within probability limits determined by the common-cause system* (Box and Kramer [1992]). If a state of statistical control is identified with a process generating independent and identically distributed (iid) random variables, control of such random processes by automatic means invariably leads to an undesirable increase in process variability. Successful application of APC requires that certain conditions are extant such as feedback (closed-loop) control stability without large increases in closed-loop gain.

APC provides a continuous steady dynamic response, ('the behaviour of the output of an automatic control device as a function of the input both with respect to time'), in counteracting changes in the balance of a process and must be properly applied to obtain successful results. Oscillations, (for example, in a feedback control system), of the output controlled variable above and below the set point results in a

[2] ISA stands for Instrument Society of America Standard on Process Instrumentation Technology (ISA-S51.1/1976).

periodic change of the controlled variable from one value to another at a constant amplitude and period, (called 'cycling') and it will take significant time for the process to reach its steady state. The period of oscillation in such a feedback control system depends on the combination of all dynamic elements in it including the controller (control mechanism). During uniform oscillation of the controlled variable, a 'signal' passing through the feedback (path) returns to its starting point with exactly the same amplitude one complete cycle later. If it is attenuated more than it is amplified by the combination of elements through which it passes, the signal will gradually diminish in amplitude and the oscillation is said to be 'damped'. For 'undamped' oscillation to persist, the product of 'gains', ('ratio of output change to the change in input causing it'), for all the elements in the feedback (path) must be unity 'at that period of oscillation'. At this particular period of oscillation, if, for some reason, the gain of the elements in the feedback (path) exceeds the value of 1, each succeeding cycle will exceed the preceding cycle in amplitude until some natural limit is reached, possibly damaging the equipment (controller). A sound principle is to avoid gain products exceeding unity because of the inherent dangers that are present in an 'expanding' cycle. When the gain product is less than 1, the oscillation will dampen and (in a 'linear' feedback control system) eventually disappear. *A gain product of unity is then the limit of stability of feedback control. In feedback control, the conditions for uniform oscillation serve as a convenient reference on which to base rules for controller adjustment. An undamped oscillation may thus indicate the stability of a feedback control system* (page 8, Schinskey [1988]). The aim to increase the feedback (closed-loop) gain of the controller (control mechanism) does not always result in making the control system stable. In fact, such an idea, in particular situations, may make the system totally unstable. Without an increase in initial or a *priori* information, it may not be possible to improve the feedback gain without bringing in a state of instability into the system (Alonefits [1987]). *Higher gains tend to drive the feedback control system into instability and to accentuate any (measurement) noise which is present in the system.* So, a limit exists in the allowable feedback gain which amounts to a trade-off between performance and stable operation of the system.

In situations where the cost of making an adjustment to the process is considerable, APC can result in increased costs. APC, referred to as 'engineering feedback control' is a short-term approach to control that attempts to minimise (output) variation by transferring the predictable component of the output variation to the input manipulated (control) variable [MacGregor] (Box and Kramer [1992]). The appropriate engineering control strategy depends upon (i) the *characteristics of the stochastic (statistical) component of the process modelled by a suitable time series and (ii) the costs associated with making dynamic adjustments to process conditions.*

The purpose of automatic control is to get maximum efficiency of process operation by adjustment of a controller (control mechanism). By using a logical method for selecting controller adjustments and by suitable 'tuning', (which means, to have the freedom of choice to vary the parameters of control), there is the potential for returns in the form of efficient process operation. It is possible to formulate (design) a control mechanism by suitably modelling a probabistically predictable process (Box and Jenkins [1970, 1976]).

2.3 REVIEW OF CONTRIBUTIONS BY BOX AND JENKINS TO STOCHASTIC PROCESS CONTROL

In recent times, several statisticians and control systems engineers have endeavoured to define the contexts in which the various SPC and APC techniques can be applied.

In the early 1960's, Box and Jenkins published a paper entitled *Some statistical aspects of adaptive optimisation and control* [1962]. In this paper, the authors explained what is meant by *adaptive control*. For adaptive control, automatic and continual identification of process dynamics (inertia) is used as a basis for the automatic and continuing structural planning (formulation) of the controller features and characteristics (parameters) (Hunt [1989]), (Figure 2.5).

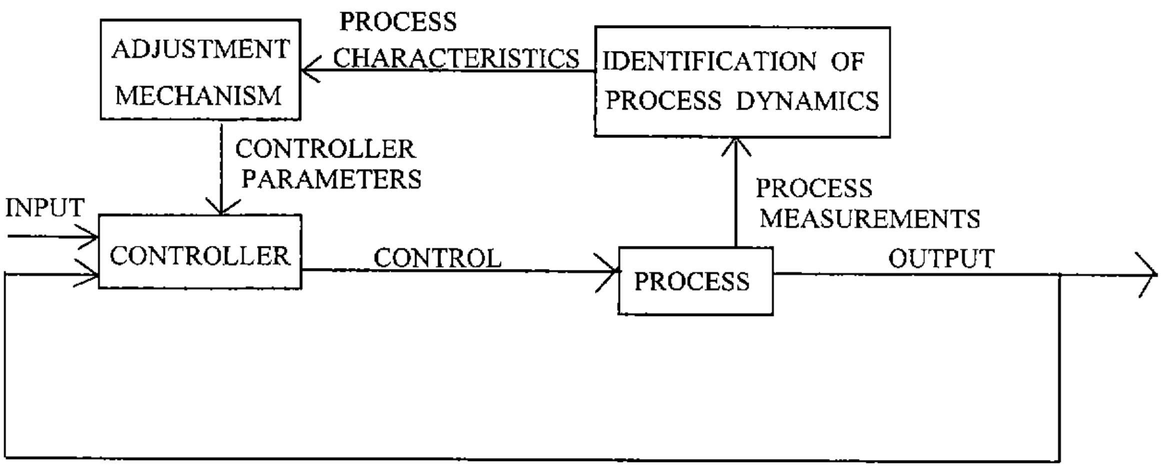


Figure 2.5 ADAPTIVE CONTROL

Variation in process dynamics (inertia) can lead to a deterioration in control performance. By modelling a process and its disturbance, it is possible to formulate a control mechanism leading to the adaptive control situation (Box and Jenkins [1970, 1976]). A similarity exists between the process of adaptation and the complex process of learning by the human mind. The concepts of adaptation are complex and more vague than other concepts in the context of automatic control. A range of definitions link adaptive control to artificial intelligence by trying to simulate the human learning process (Bar-Shalon, Gershwin [1978]). It may be worthwhile to compare adaptive control with feedback control.

One of the main reasons for the use of feedback control is the reduction of the effect of parameter variation; but the degree to which achieving the overall objectives of formulating (designing) a control mechanism depends critically upon the level of available knowledge about the process dynamics (inertia) (Hunt [1989]). This raises the matter of the distinction between adaptive control and feedback control. It also provides a clue that adaptation is closely connected with the lack of a priori information. Putting these ideas in a nut shell, adaptation may be viewed as 'the estimation of information that, if given a priori, would enable feedback regulation (Caines [1988]) in a process control system whose dynamics are uncertain or time-varying and which has the ability to continuously adapt to changing process conditions' (Hunt [1989]).

The remark of Caines and Chen [1985], that *some further level of adaptation must be used if it is desired to relax presently required a priori information,* results in some kind of *hierarchy* of feedback and points to the iterative nature of adaptive control. The distinction between feedback and adaptation is not crystal clear. However, in a certain sense, it may be said that while adaptive control removes ignorance about process dynamics, feedback control copes with ignorance. The term *adaptive automatic control* may be applied to a system that does not require, for its construction and operation, complete initial information regarding the controlled process, yet it ensures a value of the criterion chosen (output) close to its (external) target value. If the initial information regarding the controlled process is incomplete, there is a need to use an *adaptive system,* since such an adaptive system is able to adjust to changing external conditions, change its parameters when these external conditions change, and, in short, adapt itself to the new situation (Andreyev [1969]). It is the lack of knowledge of the system model parameters as opposed to some other quantity that makes the problem an adaptive one. This thesis will generally be dealing with such parameter adaptive control (explained in Section 3.6), (Figure 2.6) problems only when there is lack of knowledge of the system model parameters. **Bayesian** and **Non-Bayesian** classifications (Alonefits [1987]) of adaptive control will not be addressed. Adaptive quality control is explained in Figure 2.7.

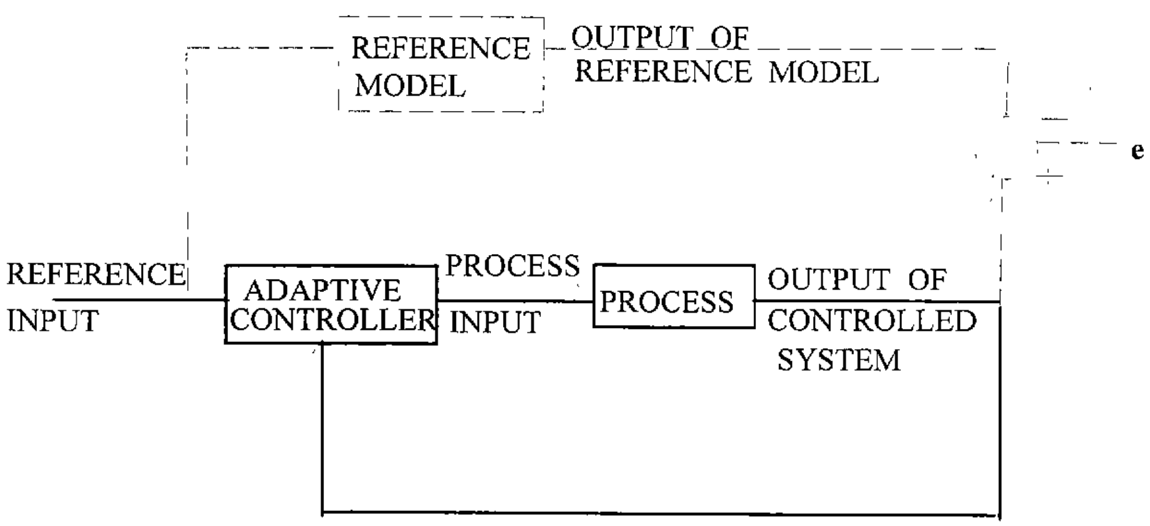


Figure 2.6 PARAMETER - ADAPTIVE SYSTEM *

* Feur and Morse [1978].

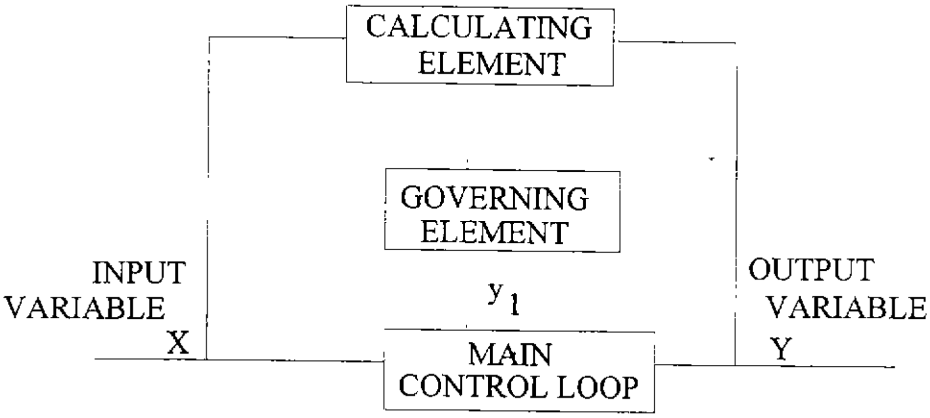


Figure 2.7 ADAPTIVE QUALITY CONTROL *

In the above figure, the input variable 'X' and the output variable 'Y' of the main control loop are applied to the 'calculating element' of an adaptive system.

The calculating element determines the required system characteristic by using the initial and 'working information' ('the set of facts regarding the controlled process that are obtained during the course of operation of the system) which includes information about the deviations of the characteristics of the main control loop as a *system that adapts itself to a standard.* The characteristic of the system thus determined is compared with a standard system and in accordance with the results of this comparison, a 'command' is given to the 'governing element'.

The governing element changes the parameter (y_1) being adjusted in such a way that the actual characteristic of the system approaches that of the standard system and feeds the main control loop.

*Extract from Andreyev [1969], pages 311 and 312. □

Box and Jenkins published another paper, a year later, entitled *On further contributions to adaptive quality control* [1963]. In their paper, Box and Jenkins [1963], developed some approaches to control problems in both the parts and the process industries. In the former, the approach led to a control procedure similar to Shewhart control charting. In the latter, their approach to a specific problem in the chemical manufacturing industry led to control procedures that were similar to those used by control systems engineers. The authors concluded that complete automatic control is not possible without analytical techniques. The authors' view was that the cost of automatic control and the analytical hardware required cannot always be justified. In situations where it is possible to justify the use of automatic control, it may be possible to improve manual adjustment by using the mathematical ideas behind automatic control.

This led further to *Mathematical models for adaptive control and optimisation* by Box and Jenkins [1965]. In building their models, they followed an *evolutionary* (adaptive) approach by submitting mathematical ideas to practical *test*, modifying appropriately and retesting the models. In the paper, the authors set up realistic and flexible stochastic models for disturbances which force the system, unless controlled, away from their optimal operating conditions. They used the process knowledge and took care of the inertia or dynamics of the system which makes the control actions needed to combat these disturbances, more complex in nature. In doing so, they found methods for estimating the unknown parameters in the models from process input-output data. They also used the models, after fitting parameters, to design optimal control schemes.

In subsequent papers, (part I and II), Box and Jenkins [1968] discussed some forecasting and control problems for situations requiring a control scheme which adjusts some variable, whose precise effect on the quality characteristic is known, so as to minimise the variation of this quality characteristic about a target value. In part I ([Box and Jenkins [1968]), the authors described an iterative *model fitting procedure* using a process of *identification, estimation, diagnostic checking, refitting* and *rechecking* to find a satisfactory representation.

In part II of a second paper, (Box, Jenkins and MacGregor [1974]), the authors described how stochastic and dynamic models may be brought together to design feedforward and feedback control schemes. They showed how the parameters in the stochastic and dynamic models may be simultaneously estimated from measurements made on the operating system.

2.4 STOCHASTIC MODELS AND STOCHASTIC DISTURBANCES

In process control, it is common to come across disturbances (noise) that are drifting or non-stationary in nature. The importance of considering disturbances of this type was known to control systems engineers from the early stages of the development of *deterministic control theory*. This theory was developed to provide tools to analyse and synthesise a large variety of feedback control systems. Results from various branches of applied mathematics and control problems were used in developing this theory. The early development focussed on *stability theory* and the *theory* of *analytic functions*. Due to complexity and the stringent performance criteria required of controlled processes, the theory of *optimal control* of *deterministic processes* was developed using the tools of the calculus of variations. In controlling deterministic processes, no significant distinction was made between a feedback control system and a feedforward control system and no dynamics (inertia) were assumed in the feedback.

There were some drawbacks in using deterministic control theory, such as not using realistic models for disturbances and postulating them as a function which is known a priori. In the framework of deterministic control theory, many of the classical methods were capable of dealing with disturbances in an heuristic manner (Hall [1956]). The effects of the disturbances were required to be predicted by suitable models. There existed many situations where there was a need to model the disturbances in a proper and fitting manner. It was possible for statisticians to describe such disturbances in the form of stochastic time series models.

Since analytic functions are limited in their capacity to accurately model, the potential for the use of 'statistical models' became apparent. Barnard [1959] and Bather [1963] linked the control problem and SPC charts. Barnard [1959] suggested that for a 'wandering' industrial process, it is possible to make improvements in process adjustments by means of a model that closely described the disturbance, by using control charts and its signals. He suggested that it may be useful to view the primary function of a control chart as providing an estimate of the current process mean, which in turn, is connected with the control problem. Other statisticians, Box [1970], Jenkins [1970] and Astrom [1970] endeavoured to provide an answer to the problem of how to characterise and model the disturbance.

A 'deterministic model' makes possible exact calculation of the value of some time-dependent quantity at any instant of time. In many process control problems, it might not be possible to write a deterministic model to calculate the future behaviour exactly, because of unknown factors. However, it might be possible to derive a model that could be used to calculate the probability of a future value lying between two specified limits. Such a model is called a 'probabilistic' or a 'stochastic model'.

Box and Jenkins [1970, 1976] adopted this approach and made a major contribution to stochastic control. Since a disturbance causes a process to drift off target, it is necessary to compensate for this by taking proper control action. For process drifts arising from disturbances, the true process level is not even a stationary stochastic process (Box and Kramer [1992]). A process in which the mean is varying in nature with respect to time can be described as a *non-stationary* disturbance. A stationary disturbance represents the situation where there is no drift in the mean and the process is in a perfect state of control.

Disturbances entering at various points of a process are often persistent in nature, such as variations in ambient temperature or a change in the properties of feedstocks to a process input. In many instances, it may not be economically possible or physically feasible to eliminate these. Disturbances envisaged as the result of a sequence of independent random shocks are represented by a first order (linear) differential equation and as dynamic control systems in continuous time. Such a system is referred to as a 'first-order dynamic system'.

Control systems engineers described the system model behaviour in which the response of a system to a given input is certain and well defined (deterministic). The engineers used Laplace transforms to obtain simplified solutions (Deshpande and Ash [1981]). The linearity assumption supplies an approximation for many practical situations. In a similar manner, in dealing with discrete processes, linear difference equations are employed to represent the processes in which the sampling intervals are short enough so that the dynamic or inertial properties of the process cannot be ignored. A first-order process may be represented by the first-order difference equation when sampled at discrete intervals or by the first-order transfer function or 'filter', (the term used in engineering terminology, by control systems engineers), (cf. MacGregor [1987]).

Later, Box and Jenkins compiled their series of papers into a book entitled Time-series analysis: forecasting and control. This book was first published in 1970 and later a second revised version in 1976 (Box and Jenkins [1970, 1976]). In this book, an approach for dealing with drifting processes by a class of *stochastic time series models* called *autoregressive integrated moving average* (ARIMA) models is described in detail. These are used to describe the stochastic disturbances to the system and to provide a means of modelling the process dynamics (inertia). These time series models characterise and forecast the drifting behaviour of the process when no control action is taken and describe the dynamic relationship between the controlled variables (outputs) and the manipulated variables (control inputs). A feedback control algorithm, derived from these models, minimises the variance of the output controlled variable at every sample point that exactly compensates for the forecasted disturbance. Models of this kind are used in inventory control problems, in econometrics and to characterise certain disturbances that regularly occur in industrial processes.

2.5 MODERN TIME SERIES ANALYSIS AND CONTRIBUTIONS OF ASTROM

Modern time series analysis is a subject embracing three closely related fields which have tended to develop somewhat independently. These are:

(i) *Statistical communication and control theory*

(ii) *The probabilistic theory of stochastic processes possessing finite second moments and*

(iii) The statistical theory of regression analysis, correlation analysis and spectral (harmonic) analysis [Parzen] (Box and Jenkins [1962]). Prior to the *publication* of the book by Box and Jenkins, Astrom [1970] published a book, entitled Introduction to Stochastic Control Theory, in which he makes some noteworthy *contribution* to the control of stochastic systems. These books are important milestones in modern time series analysis.

2.6 CONTRIBUTIONS OF CONTROL SYSTEMS ENGINEERS

Control systems engineers, performed important applications in the field of electrical control systems. The engineers were concerned with systems in which the response to a given input is certain and well defined (deterministic). They described the system model behaviour and technical feedback by means of a set of differential equations. These equations, describing the problem in continuous time, are solved by use of Laplace transforms. A large number of books and papers have been published in the control systems engineering area, for example in the design of *direct digital control,* PID controllers and in the design of *discrete data sample systems.*

2.7 CONTRIBUTIONS OF OTHER STATISTICIANS TO STOCHASTIC PROCESS CONTROL

In 1986, Alwan and Roberts published a paper, *Time Series modelling for Statistical Process Control.* In this paper, the authors took a two-fold approach trying to make a possible union of time-series modelling and traditional ideas of process control. Hoadley [1981] and Hunter [1986] perceived similar ideas of integrating time-series modelling and process control.

MacGregor [1987] published a paper, *Interfaces between process control and on-line statistical process control,* in which he outlined areas where the operations of APC and SPC are common and overlapping.

Following this work, two papers were published in 1989, one by Harris on the same subject and one by Van der Weil, Tucker and Faltin [1989]. In the second of these, the authors explained what is meant by Algorithmic Statistical Process Control (ASPC) (Figure 2.8) and gave an appropriate literature review, while discussing its implementation. ASPC is the term used by the authors for an integrated approach to quality improvement, an approach that realises quality gains through appropriate process adjustment and through elimination of special causes of *variability* signalled by process monitors.

Two papers, one on *Statistical process monitoring and feedback adjustment* (Box and Kramer [1992]) and another, *ASPC: Concepts and an application* (Van der Wiel, Tucker, Faltin and Doganaksoy [1992]) have renewed interest and rekindled discussion on the integration of the two process control disciplines.

The second paper gave an insight into obtaining better product quality through an integration of techniques from both the methodologies. Through appropriate process adjustment and by eliminating assignable causes of variability signalled by statistical process monitors, it may be possible to achieve an improvement in quality. It is the contention of the authors that ASPC reduces predictable quality variation using feedback and feedforward techniques and then monitors the complete system to detect and remove the causes of unexpected variation.

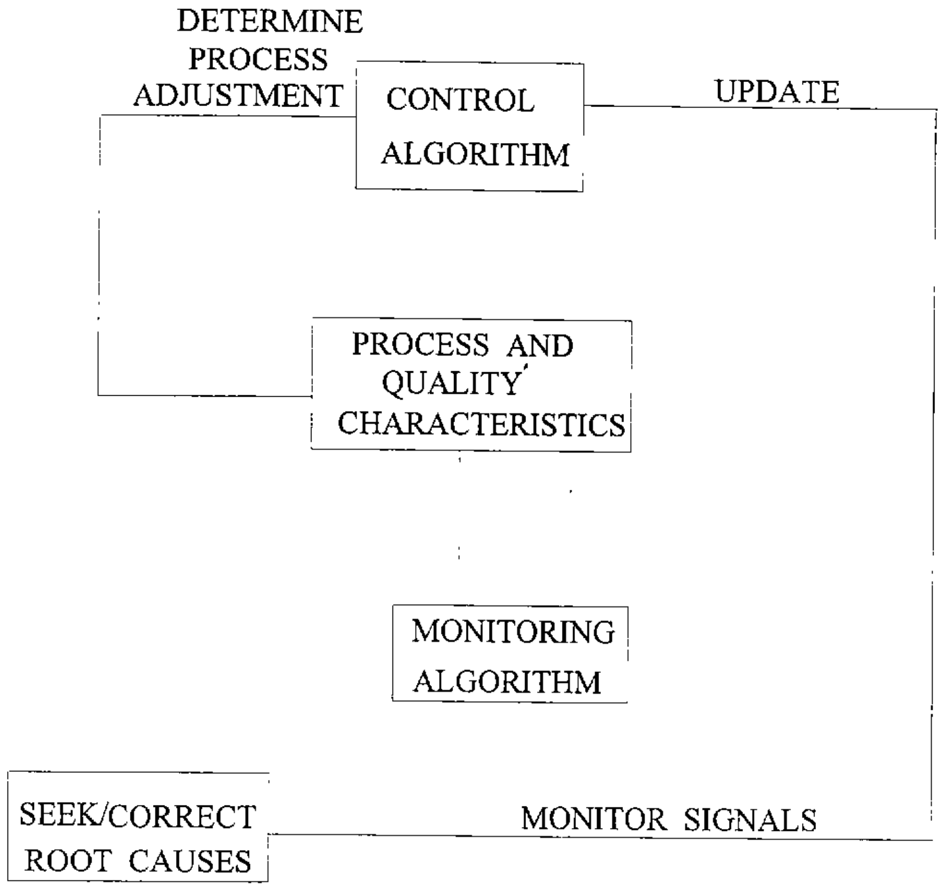


Figure 2.8

ALGORITHMIC STATISTICAL PROCESS CONTROL (ASPC)*

* Taken from Tucker, Faltin and Vander Wiel [1991]

ASPC advocates that compensatory adjustments be applied in *conjunction* with rather than in *competition* with SPC. The authors refer to literature that concerns control of stochastic systems relevant to the algorithmic part of ASPC (Astrom [1970] and Box and Jenkins [1970, 1976]). They refer also to MacGregor's work on *On-line statistical process control* [1988] in which the author suggested that SPC charts be used for statistically monitoring the performance of a feedback (closed-loop) control system. Even before MacGregor's suggestion, Bather [1963] and Barnard [1959] had linked the optimal control problem and SPC charts as mentioned earlier in Section 2.4. MacGregor [1988] reviewed the basic concepts from stochastic control as well as from so called on-line SPC methods, pointed out similarities, overlap and cited reasons for lack of interface between the two fields. Box and Kramer [1992] gave overview descriptions of both methodologies delineating their similarities and differences.

2.8 ALGORITHMIC STATISTICAL PROCESS CONTROL (ASPC)

The aim of ASPC is to use SPC and APC methodologies to perform the separate functions of process monitoring and control. ASPC is the term given by Scott A. Vander Wiel, Tucker, Faltin and Doganaksoy [1992] to represent the situation in which techniques from both the methodologies are employed in a *synergistic* way to make quality gains. In ASPC, the role of statistical monitoring is to detect and give (warning) signals when the operation of the closed-loop process is not consistent with the estimated model and the control algorithm. ASPC represents a *proactive* approach to quality improvement. Variations in product quality are reduced in two ways, one through algorithmic compensation for predictable deviations and the second through elimination of special causes of variability as signalled by statistical monitoring charts.

2.9 REVIEW AND DEVELOPMENT OF ALGORITHMIC STATISTICAL PROCESS CONTROL

2.9.1 Feedforward And Feedback Control Models

A feedforward control model is proposed when the major disturbances to a production system can be measured. Feedback control may be applied, when the primary sources of disturbance are either not known or cannot be measured. Making use of the available knowledge of the production process and the serially occurring industrial data (which are very likely correlated), it is often possible to build stochastic models to represent and model the disturbances. Box and Jenkins [1970] expressed the process inputs and outputs in terms of time series and described the disturbances by time series models in order to manipulate the system for control purposes.

Feedforward or open-loop control is used to eliminate the effect of some fluctuating measured input by making an adjustment from direct calculation of its effect on the output. For a specified target value of the output, the feedforward control model gives an estimate of the required change to be made in the compensating variable to minimise the *mean square error (sum of the squared deviations between an output value and the target value)*. When the time series model predicts an out-of-control signal for shifts in the mean of the quality deviations from target, changes are made to the compensating variable to offset the effects of the predicted situation (Keats and Hubele [1989]).

Feedback or closed-loop control uses past output deviations from target to determine a process adjustment. The approach makes use of the error (difference between the output and the target values) as the mechanism of identifying changes to the input. Using time series analysis, the effect of the disturbance in the absence of a control action is estimated and a dynamic model is developed linking the input and the process output.

Automatic feedback control is an *adaptive process* since, (i) the feedback from the output changes the process variables to maintain the output close to some desired target and (ii) there is no initial information regarding the disturbances. The intention of feedback control action is to minimise the system variation, which in the process industries includes drifts along with measurement error.

2.10 PROPORTIONAL INTEGRAL DERIVATIVE (PID) CONTROLLERS

In feedback control systems, the process adjustments (control actions) are performed either manually or by automatic means through the use of 'controllers'. A digital computer connected directly to the process accomplishes the execution of the control action by observing the system so that the available data appears in discrete-time.

Slow changes are encountered in many chemical processes. Under such circumstances, it may be adequate to monitor and take whatever control action is necessary at convenient time intervals. For many automatic controllers, as soon as the measurements are made, the control actions are initiated immediately. By means of the discrete data, the adjustments are made to bring the process into a state of control. With the process data available, it is possible to control the mean square error, about the target by **proportional-integral feedback control** schemes (Box and Kramer [1992]).

The proportional plus integral (PI) controller makes a compensation (correction) (which lags behind the trend, if any, in the disturbance) proportional to a (linear) combination of terms involving the deviation and the integral of all the previous errors. A PI controller is a 'standard linear controller'. A special case of such a controller is, regulation based on the control-modified EWMA statistic.

The proportional integral derivative (PID) controller is a modified form of the PI controller in which an additional term involving the first derivative with respect to the time of the error is included. This type of automatic control action makes a correction which is proportional to a (linear) combination of *(i) the first derivative of the current deviation ('the difference between value of the output controlled variable and position of the final controller set point'), (ii) the deviation itself and (iii) the integral of the* *deviations over all past history* (Box and Kramer [1992]). PID controllers, also known as *three-term* controllers, are automatic, continuous time controllers. These controllers (i) are not capable of providing tight control over processes in which the effect of an adjustment is delayed until the following sample due to time taken to deliver material from the point of adjustment to the sample point (called, the 'dead time'), (ii) tend to perform poorly unless 'detuned' in the face of dead times in order to take necessary action at each sampling instant (page 428, Harris, MacGregor & Wright [1982]), and (iii) are not suited to direct-digital (discrete) control. PID controllers are also not capable of producing control actions that might be called for by a minimum variance feedback controller (page 437, Box and Jenkins [1970, 1976]). Time series controllers employ stochastic characteristics to regulate production processes. With time series controllers, it is possible to provide tight control of processes with dead time and to provide minimum variance at the same time. PID controllers are compared with time series controllers by Palmor and Shinnar [1979]. A description of time series controllers, their statistical control algorithms and performance measures are given in chapter 5.

2.11 CONCLUSION

In this chapter, we have discussed statistical and automatic process control. A review of the contributions of statisticians and control systems engineers has been given. Explanation of the terms feedforward control, feedback control, adaptive control and algorithmic statistical process control have been provided. A comparison of adaptive control was made with feedback control. Various kinds of controllers have been discussed, in particular, PID controllers.

In the following chapters, attention is focussed on time series controllers which take the hypothesis setting approach to decide whether or not the process mean is on target. Process monitoring parallels hypothesis testing and concerns continuous checking of the assumed process model. Checking for a state of control is regarded as a test of a null hypothesis. An hypothesis that is alternative to this has a 'state of control' as a process state in which future behaviour can be predicted within probability limits determined by the common-cause system. It may be possible that sudden and substantial shocks impinge on the constant cause system of a process that was in a state of control. Against such an alternate hypothesis, the *three-sigma control limits* provide enough control to detect shocks without sounding false alarms that may be present when the shocks are not there in the system (Alwan and Roberts [1986]). In this context, a review is given of a method suggested in *Simulation study of statistical process control algorithms for drifting processes* by Baxley (Baxley [1991]) in Chapter 5. Later, use is made of some of the principles developed in the simulation to formulate a time series control algorithm which takes into due consideration drifts and dead time (delay) in the process and its dynamic characteristics.

CHAPTER 3

DISCRETE STOCHASTIC CONTROL AND DIRECT DIGITAL CONTROL

3.1 INTRODUCTION

3.1.1 Background of Discrete Stochastic Control

SPC and APC techniques overlap (explained subsequently), in the areas of on-line quality control and the control logic based on knowledge about the nature of the process and the disturbances. Stochastic control theory provides a means to explore this area of overlap between SPC and APC. Process knowledge is necessary to make meaningful decisions in responding to the upsets occurring in a process due to disturbances. Knowledge about the nature of the disturbances (noise) is necessary to detect out-of-control state situations and to estimate the true levels of the output deviations from the required targets. Coping with delay or dead time (defined in Section 3.6.1), the inability of a process to adapt to an adjustment, is important part to effective operations. The need to identify the dead time and the effects of system dynamics (inertia) (explained in Section 3.8) while controlling a process are discussed in this Chapter. The design of sampled data controllers (Palmor and Shinnar [1979]) is reviewed and a brief description given of direct digital control (DDC) with justification for its use. Some methods to compensate for the dead time are discussed as also is the role of the IMA parameter, r which measures the rate of drift of the process.

MacGregor [1988] suggested the use of statistical process monitoring on closed-loop (feedback) control systems and pointed out the overlap between SPC and APC when (i) control actions have their full effect on the process outputs in the immediately succeeding periods, (ii) process noise (disturbance) is modelled as a first-order integrated moving average process (IMA), (iii) a fixed cost is associated with taking any non-zero control action and (iv) additional costs are assessed in proportion to the *squared deviation* of the outputs from the required target. Unfortunately, there are situations, especially in the process industries, when the control actions ('adjustments') have little or no effect on the process outputs in the immediately succeeding periods due to dead time and dynamics (inertia).

3.2 THESIS OUTLINE AND SALIENT FEATURES

3.2.1 Thesis Outline

The traditional Shewhart control charting procedures, when there are drifting disturbances, have a relatively high 'control error variability' compared with time series controllers (described in Section 5.2) under a similar situation (Baxley [1991]). This is one of the reasons for studying the performance of time series stochastic feedback control algorithms for drifting processes. Another reason is that Baxley [1991] contended that the performance of time series controllers for some particular cases is well known. A more general approach to the control of product quality by simulation of statistical time series process control algorithms for drifting (dynamic) processes with the existence of dead time (time delay) is made in this thesis. Baxley [1991], in his 'simulation study of statistical process control algorithms for drifting processes' (Baxley [1991]), (reviewed in Section 5.10), provided results for only some 6 values of the time series controller for the IMA parameter, Θ ranging from 0.25 to 0.75 for the dead time, $b = 0$ and 1 only and for the dynamics (inertia) $\delta = 0$. Baxley [1991] derived the statistical algorithm considering the drifting nature of the process and not the process dynamics and closed-loop (feedback control) stability. Kramer [1990] considered values of Θ ranging from 0.1 to 0.9 and b = 0 only.

This thesis studies cases where dead time values, b range from 1 to 2. Baxley [1991] considered a (first-order) system with delay described by a first-order dynamic model. The mathematical derivation of the feedback control algorithm from first principles given in this thesis proposes a novel process 'control' approach which requires only an increase in the order of the system (second-order) and leads to a dynamic model requiring less computer storage and less computing time for simulation of the algorithm. Some of the features of good (feedback) control considered : (i) Permissible gain of the feedback (closed) loop, (ii) Stability of the feedback control loop and (iii) Precise regulation of loops containing dead time. The control engineer faces a challenge in the control of production processes involving time delays (dead time) because of their non-linear nature. The significant amounts of lag(s) introduced (by the dead time) into the system response frequently makes use of conventional control algorithms a poor prospect. The innovative feedback control algorithm obtained gives minimum variance control even in the presence of dead time and it is shown that it has both integral action and dead-time compensation (explained in Chapter 4). Integral control is extensively used in the continuous process industries for control of noisy, drifting processes. The use of the ARIMA (0,1,1) stochastic model to describe non-stationary disturbances implies necessarily integral action in the controller. There need not be any constraint placed on the input variable in order to obtain minimum variance at the output. Process control loops containing pure dead time are difficult to stabilise with conventional three-mode or three-term (PID) controllers (explained in Section 2.10). In the (control engineering) literature, (Smith [1957]), it is suggested to use an integral or 'floating' controller (Buckley [1960]) for control of a (dynamic) process with dead time. The integral controller is simple in operation and easy to adjust. The time series control algorithm derived in this thesis gives information about the adjustment intervals (AIs) (sample periods), obtained via simulation. Sampling and adjustment done at these AIs result only in a slight increase in control error variability and does not call for an adjustment every time a sample is taken.

Shewhart charts lack the capacity to detect small *shifts* in the process mean unlike the EWMA and the CUSUM charting methods which detect such shifts more rapidly. So, an intermediate charting method, such as the MTA {*machine tool adjustment*} chart, developed by Kramer [1990] is required which can detect such shifts at a pace that would enable proper control action to be taken economically at the appropriate time. Box and Jenkins [1963] developed minimal cost procedures and mentioned their similarity with Shewhart charts. These procedures have been developed and extended by Kramer [1990] to include monitoring and adjustment costs when the system has no dead time (time delay). A time series controller with suitable design parameters (Chapter 7) and an outline of a process regulation scheme is proposed (Chapter 8). The EWMA controller is not explored in detail for the reason that the feedback control equation does not adequately provide a term for compensating dead time, which is possible in a time series controller algorithm; and moreover, the EWMA controller requires a 'controller gain' (explained in Section 6.6) below one in order to avoid over-control (Baxley [1991]). The CUSUM controller with accompanying CUSUM chart and V-mask is complex for analysis, particularly when the process dynamics and the dead time are to be considered.

3.2.2 Salient Features

The salient features of the thesis are

(i) Drifting processes due to disturbances are modelled as ARIMA (0,1,1) non-stationary time series for values of $0 < \Theta < 1$ and categorised as 'fast' or 'slow' drifts (explained in Section 6.7.2).

(ii) The presence of dead time in the process requires that forecasts are made of the disturbance and these are provided using the EWMA made over the delay period as explained in Chapter 6.

(iii) The property of the time series controller is used to obtain the necessary one step ahead forecasts.

(iv) The feedback control algorithm (Chapter 5) gives information about when to make an adjustment and by how much.

(v) The IMA parameter, Θ is used as an on-line tuning parameter for dead-time compensation.

(vi) Complex processes are represented by a second-order dynamic model.

(vii) Dead-time simulation (explained in Chapter 5) is used for comparing variances at the sampling points.

(viii) Extensive simulations, approximations and use of special triple-entry tables are not required (as in the regulation schemes of Box and Kramer [1992]).

(ix) Closed-loop (feedback) control stability is taken care of by considering a 'critically damped' system by keeping the system gain (g) less than 1.0 and

(x) The stochastic feedback control algorithm provides minimum variance control even in the face of dead time and process dynamics.

3.3 MINIMUM VARIANCE CONTROLLERS

Control error variance (also called *mean square error* or *mean square deviation*, explained in Section 3.3.1) is a(n arbitrary) benchmark to measure the theoretical best achievable control performance and to evaluate the performance of the feedback control strategy employed. Suppose that this theoretical best achievable control represents a significant improvement over current performance, then, this feedback control strategy can possibly be considered as standard if this improved performance is required and warranted. However, if the best achievable performance itself is inadequate, alternate approaches such as feedforward control may be used to achieve such a reduction in variability. The controllers employing this principle of *minimum variance* for process control are termed *minimum variance controllers*. Box, Jenkins and MacGregor [1974] discussed in their paper, 'Some recent advances in forecasting and control, part II', the relationship between minimum variance controllers and methods used in statistical process control (SPC). They showed that the feedback control algorithms of minimum variance controllers are similar to most feedback control algorithms derived by using statistical process control (SPC) methods.

3.3.1 Minimum Variance Control

A feedback control strategy to minimise the mean square of the output deviation (error) from the target is by minimum mean square error ('sum of the squared deviations between an output value and the target value') or variance control. Minimum variance control is the best possible control in the mean square error sense for processes described by linear functions with disturbances which can be added together and treated as a single disturbance for purposes of mathematical analysis and convenience. Its implementation may demand aggressive control much in excess of what is (normally) required and so may not be practically desirable. However, minimum variance control provides a convenient bound on achievable performance against which the performance of other controllers may be compared. Such a basis is important in the context of deciding corrective control actions. Harris [1989] described a technique to ascertain the best theoretically achievable feedback control performance as measured by the output mean square error. The performance of an optimal stochastic minimum variance controller depends upon the form and parameters of the dynamic stochastic model used for the disturbance (noise). The form of the stochastic model considered here is the autoregressive integrated moving average (ARIMA) time series model.

An inherently continuous system, when sampled periodically, followed by a sequence of 'discrete control' actions, is called a 'sampled-data' system (explained in Section 3.5.3). Examples of sampled-data systems are found (i) in power electronics, (ii) in signal transmission, in 'pulse' form in biological nervous systems and (iii) in the generation of a 'torque pulse' at each ignition that synchronises the operation of an internal combustion engine. Systems where the signals are sufficient to describe the system's behaviour at the sampling instants at discrete times are called 'discrete-time systems'. In discrete control systems, time delays are integral multiples of the 'sampling period' (known as the 'clock period').

Univariate stochastic control theory based on discrete time series dynamic (transfer function) models for the process and its disturbances, leads to digital control algorithms. These algorithms include the classical PID (proportional, integral and derivative) and dead-time compensation, (explained in Chapter 4), terms. Palmor and Shinnar [1979] gave a set of rules to choose the parameters of discrete controllers with dead-time compensation and stability properties. For this purpose, they used the structure of a particular form of stochastic controller proposed by Box and Jenkins [1970, 1976] (Harris, MacGregor and Wright [1982]). The results of Palmor and Shinnar [1979] are discussed and efforts made to follow some of their rules to determine the process parameters of a time series controller with suitable compensation for dead time.

3.4 DESIGN OF SAMPLED DATA CONTROLLERS - A REVIEW

Palmor and Shinnar [1979] unified some of the ideas of stochastic control theory and provided certain control algorithms. In their paper, they gave an overview of the design of sampled data controllers. The design was based on the connections between these discrete controllers, which have the required dead-time compensation and stability properties, and other control techniques aimed at solving specific problems. The authors introduced the basic ideas of the design of minimum variance and constrained minimum variance controllers and considered the properties of the resulting control algorithms. In particular, they showed the relationship between the dead-time compensation terms in these stochastic controllers and the dead-time compensation via the predictors which employ the Smith [1957] prediction techniques. Palmor and Shinnar [1979] presented an analysis of the stability of these stochastic controllers and their properties. They discussed also the situation when there is substantial error in the estimated process dead time. For this, they recommended reducing the gain of the optimal controller in order to make amends for the improper compensation of the 'phase (change in the slope of the input-output curve) lag' due to the dead time by providing a 'phase lead' term in the controller.

3.5 DIRECT DIGITAL CONTROL (DDC)

3.5.1 Digital Computers Application to Process Control

Use of automation techniques in industry made it possible to apply digital computer capabilities for the solution of process control problems. The development of minicomputer technology combined with the knowledge gained about computer process control has led to an increase in the application of digital computers to process control.

3.5.2 Direct Digital Control (DDC) - CONCEPT

Digital computers are used for on-line automatic process control (APC) to monitor processes, where the output is measured and the input is changed only at discrete intervals of time. Computer based controllers incorporating tuning tools make complex algorithms practically possible.

Direct digital control (DDC) is the term used for controlling processes directly by computer. DDC highlights some basic control functions such as problems relating to choice of sampling period, ('the time interval between observations in a periodic sampling control system'), control algorithms and reliability of processors.

3.5.3 Direct Digital Control (DDC) or Sampled-Data Systems

A measuring system senses the value of the output controlled variable in the traditional (conventional) feedback control system and transmits a message dependent on it to its controller. The controller compares this value with the value of the chosen controlled variable, called the desired value - or an input variable which sets the desired value of the controlled variable, called the set point, so as to generate a deviation called the error. The controller acts on this error to produce a control signal. This signal is then fed to a final control element, which is an automatic positioning valve to reduce the error. In sampled-data (discrete) digital control systems, in contrast to conventional control systems, the discrete (digital) signals represent information by a set of discrete values in accordance with a prescribed law.

The electrical signal, in a basic sampled-data (discrete) (digital) control system, represents the output controlled variable and is fed to a device called an analogue*-to-digital converter, where it is sampled. The sampling period, a constant in process control applications, is called the 'clock period' in digital computer terminology. The value of the discrete (digital) signal is compared with the discrete form of the set point in the digital computer to produce an error. A control algorithm is executed yielding a discrete controller output. This discrete (digital) signal is then converted to an electrical signal by a digital-to-analogue converter and then fed to a final control element. The control strategy is repeated so as to achieve closed-loop (feedback) computer control of the process and this type of sampled-data (discrete) control technique is referred to as 'direct-digital computer control' (*An analogue system is one in which the data are everywhere known or specified at all instants of time and the (input, output) variables are continuous functions of time).

In a sampled-data (discrete) control system, the analogue controller in a conventional control system is replaced by a digital computer and the control action produced by the controller in the feedback (closed) loop is initiated by the computer programme. The feedback controller is a special-purpose analogue computer used in the direct digital (discrete) (sampled-data) control of production processes. Digital computers automatically collect data about a process and its operating conditions and provide details about the product produced by the plant, its reliability and specifications.

It is possible to achieve the objectives for sampled-data (discrete) digital control (DDC) by using digital techniques in the process control of some time-delay models.

3.5.4 Justification for the Use of DDC

The use of digital controllers offers advantages such as (i) making available a wider selection of process control algorithms than in analogue controllers, (ii) faster calculations, (iii) logic capabilities, both at the controller input and the output, (iv) on-line restructuring of (control) loops and (v) adaptive-control features. Factors in justifying computer control for a given application include the number of conventional controllers that are to be replaced by digital computers and whether or not there is better process control performance. The computer needs to be used to automate functions and operations that could not be automatically accomplished earlier. Feedforward control, dead-time compensation and optimal control techniques can be implemented by exploiting the capabilities of the computer and by the use of DDC hardware systems.

Control strategies can be implemented that are otherwise impractical or impossible with conventional analogue hardware. The availability of control computers makes possible a type of hybrid approach to process control involving both the digital capabilities and conventional analogue capabilities. The implementation of control strategies is achieved by leaving those (feedback) loops with conventional analogue control systems where feedback control is envisaged and employing direct digital control (DDC) only for those process loops in which there can be significant improvements made in control performance.

3.5.5 Self-tuning (Adaptive) Control

Marshall [1979] showed that process control is possible if a system is sufficiently understood to be modelled. Time-delay control systems benefit from improved process modelling. Good modelling makes possible the application of the principles of prediction to the stochastic disturbances in order to improve control. Identification methods for systems afflicted by disturbance on-line give estimates recursively for use in adaptive controllers. Parameter estimation (identification) methods result in adaptive control. Control systems combining recursive methods of estimation and use of minimum variance controllers are called 'self-tuning' (adaptive) controllers.

Digital computation facilitates simultaneous estimation of parameters and on-line control and provides the required computer solutions for adaptive control. Process regulation schemes that involve recursive techniques are programmed using micro-processors. Modelling and formulation (design) of self-tuning regulators is by

(i) determining suitable model structures, (ii) estimating model parameters recursively and (iii) using estimates to calculate the control. A regulator (controller) with facilities for tuning its own parameters is called a 'self-tuning regulator'. The self-tuning regulator controls processes by suitably altering algorithms to track process parameters which change with time.

3.6 DEAD TIME OR TIME DELAY

3.6.1 The Need to Identify Dead time

Identifying and minimising dead time in production processes is one of the measures that can be adopted to achieve a reduction in product variability (control error standard deviation), if the best achievable performance is not adequate enough to provide minimum variance control. Dead time is the property of a production system by which the response to a control adjustment is delayed in its effect. It is 'the interval of time between initiation of an input change and the start of the resulting observable response'. There may be a finite delay before any effect is observed in the output when changes are made in a process input. Dead time occurs when process materials move from one processing stage to another without any change taking place in the properties or characteristics of the processed materials. Such delays are caused by flows of liquids or gases through pipes. Time delays are also known by various other names such as transportation lag or velocity-distance lag, or pure delay. Dead time may be a problem of transportation and is present in (process) control systems. Time delays occur also in human biological, political and economic systems. The effect of dead time in these systems is discussed in the works of Bateman [1945], Justin [1953], Howarth and Parks [1972], York [1972], Howarth [1973] and Smith [1974] (Marshall [1979]).

Dead time causes difficulties in satisfactory control of processes by sluggish response to control actions and so, where possible, efforts must be made to reduce it. Time delays are often created by sampling systems. So, it may be necessary to decrease the frequency of taking samples from a process. Sampling at periods that are shorter than the delay period may not be useful when delays occur in a process. An effective manner of improving process control is to reduce or eliminate the (feedback) dead time since a feedback control strategy alone by itself cannot return the process output to its target value until the process dead time or time delay has elapsed.

A feedback controller applies corrective action to the input of a process based on the present observation of its output. In this way, control action is moderated by its effect on a process. A process containing dead time does not produce any immediate effect and thereby delays control action. Dead time is one of the difficult dynamic elements that occurs in many production systems. The delay produces a change* in slope of the input-output curve and this property becomes an essential consideration in feedback loops characterised by the behaviour of the critical quality variable (during transition between two steady states). In view of this fact, feedback control-system design techniques must be capable of identifying and dealing with dead time (*called 'phase shift' or 'span shift' in control theory terminology).

There is a time delay when an adjustment is made to the flow rate of a liquid travelling a significant distance between two receptacles and in measuring the thickness of insulation while coating a wire. A time delay is significant over long distances in remote control systems and in processes which involve complex chemical reactions. There will be cases of delay in control because of remote operations, which may be several time periods in duration. Many industrial processes, particularly thermal processes and distillation processes may be best represented by including an element of time delay in the system model.

3.6.2 Sampled-data (Discrete) Control Systems and Dead time

Sampled-data techniques involve the use of storing or holding and releasing information when required, which is a delay process. In this manner, a connection exists between sampled-data (discrete) control systems and delays. Systems involving the use of digital computers in process control rely on the use of stores of memory. Reliable storage or holding of data is the delay between the input or the calculation and the output at some multiple of the clock period later. Sampled-data techniques enable algorithms to be used in numerical analysis (digital computing methods). Formulation of a process control problem by sampled-data enables a solution to be found whereas it is difficult to analyse the corresponding control problem in continuous time. The control problem in the sampled-data (discrete) case is solved by modelling the disturbance by time series techniques. The characterisation of disturbance in continuous time is difficult to treat in a rigorous fashion. Smith [1957] prediction techniques and its extensions are capable of using digital computing (numerical) methods.

In production systems, where there are lags as in production involving chemical reactions, it is often convenient, from a modelling point of view, to replace the accumulation effects of the lags by a single time delay. There are many complex processes in which this assumption is helpful.

3.7 IDENTIFICATION OF DEAD TIME

For satisfactory operation, it is necessary to ensure that a process containing an element of time delay should not be affected by parametric variations or extraneous noise (disturbance). Suitable (feedback) control strategies may be employed to minimise the effects of external disturbance and variations in the process parameters. A control strategy may be defined as a set of rules by which a control action is determined when an output deviates from a desired set point. It is an algorithm or a control equation that determines the controller output as a function of the present and past measured errors (Deshpande and Ash [1981]). An appropriate (feedback) control strategy for a process containing an element of time delay, is to assume a dynamic model which adequately represents the process that it is required to control. This model should be capable of tracking any variations in the parameters ,of the process. Thus a process must be identified continually and the parameters of the model adapted accordingly.

Identification of a process consists of deriving a suitable form for the model and fitting it with the required parameters. The form of the model and the initial values for it are determined beforehand and as the process operates, it is usual in practice to determine the changes in the process parameters. For this purpose, an ARIMA (0,1,1) time series model is assumed for the process, a brief description of which is given in Section 5.6. In order to reduce the effects of disturbance on the system output, an estimate of the disturbance is required (Mitchell [1987]).

Having discussed the dead time and its characteristics, focus is turned to the other dynamic parameters of the process, namely, the dynamics (inertia) and r, the rate of process drift.

3.8 THE ROLE AND CHARACTERISTICS OF INERTIA

The concept of inertia is explained by the term 'capacity'. In automatic control, capacity is a location where mass or energy can be stored and acts as a buffer between inflowing and outflowing streams, determining how fast the level of mass or energy can change. The mechanical measure of the property 'capacitance' is 'inertia', which determines the amount of energy that can be stored in a stationary or flowing liquid, fluid, gas or fine granular material. The inertia is an important determinant of an optimal process control system. A control action applied to a process at time zero may not be fully effective until an elapse of some significant time due to the system dynamics (inertia). This is particularly true in the process industries, where attempts to compensate for the disturbances ignoring the dynamics may lead to inappropriate control actions. The need to allow for dynamics is less common in the parts industries and in view of this fact, controllers specifically built for the purpose of dealing with the dynamics and not tuned properly may be ineffective in such situations.

Excessive changes in the input variable may be required when a minimum variance feedback control scheme is applied to a monitored process. This may be due to (i) the parameter governing the dynamics (inertia) of the system, δ, being large in relation to the monitoring interval and (ii) there may not be any penalty associated with large adjustments. Kramer [1990] showed a method to evaluate the expected variance of the control actions (adjustment variance) by using the fact that minimum variance control generates deviations from target that are equivalent to the uncorrelated random shocks. The adjustment variance becomes larger as δ becomes closer to the value one. Since the dynamic parameter δ is a function of the monitoring (sampling) interval, it is possible to reduce its inertial effects by lengthening the (monitoring) interval. However, as δ gets larger, the adjustment variance also can be reduced by suitably lengthening the monitoring interval. This fact was substantiated by Kramer [1990] with arguments which led to the conclusion that altering the monitoring interval changes also the variance. Abraham and Box [1979] showed that changes in δ have effects on the optimal control adjustment and also affect the resulting variance of the optimum control adjustment (vardxt). The parameter, δ, plays a minor role in determining the monitoring interval corresponding to an increase in the mean square error (variance) deviation, whereas the rate of drift of the process, r, plays a dominant role. This is true when the value of δ is not near one as a result of the small bias resulting from the dynamic nature of the input-output relationship (Baxley [1991]). In view of this, the role of the parameter r in making changes in the variance is discussed.

3.9 THE RATE OF DRIFT OF THE PROCESS (r)

For the ARIMA (0,1,1) disturbance model for processes with drifting behaviour from a given fixed-target value, the disturbance process z_t is

$$z_t = \hat{z}_t + a_t, \tag{3.1}$$

where $\hat{z}_t$ is an estimate of z_t which is independent of a_t and is an EWMA of the past data defined by

$$\hat{z}_t = r(z_{t-1} + \Theta z_{t-2} + \Theta^2 z_{t-3} + \ldots). \qquad 0 \le \Theta < 1 \tag{3.2}$$

where, the rate of drift of the process, $r = 1 - \Theta$.

The coefficients r, $r\Theta$, $r\Theta^2$, ... in equation (3.2) form a convergent sequence that sums to unity. Algebraic manipulation of equation (3.1) and equation (3.2) gives

$$\hat{z}_{t+1} = rz_t + \Theta\hat{z}_t, \hat{z}_{t+1} - \hat{z}_t = ra_t.$$

For the first-order ARIMA(0,1,1) disturbance model

$$z_t - z_{t-1} = a_t - \Theta a_{t-1}.$$

Summing this equation and using equation (3.1) with t = 1 leads to

$$z_t = \hat{z}_1 + a_t + r\sum_{i=1}^{t-1} a_i. \qquad 0 < r \le 1 \tag{3.3}$$

In particular, if the process mean is set on target at time t = 1 by adjusting its level so that $\hat{z}_1 = 0$ then, the subsequent course of the deviations from the target is represented by

$$z_t = a_t + r\sum_{i=1}^{t-1} a_i, \quad 0 < r \le 1. \tag{3.4}$$

Equation (3.4) is an interpolation between the sequence of uncorrelated random shocks, $N(0,\sigma_a^2)$, of the stationary disturbance equation, $z_t = a_t$ for a process in a perfect state of statistical control with no drift obtained as r approaches the value 0 and the highly nonstationary random-walk model,

$$z_t = \sum_{i=1}^{t} a_t \,. \tag{3.5}$$

Equation (3.5) is obtained when r = 1 in equation (3.4).

The purpose of this discussion is to show that for intermediate values of r, the process can represent slight, moderate, or severe degrees of non-stationarity (drifts). When the process drift, r = 0, the disturbance is a sequence of random shocks and the process is known to be in a perfect state of control requiring no control action to be taken. When the drift, r = 1, this degree of non-stationarity is so extreme that it can hardly be regarded as describing any control situation likely to be met in real life, although it has been shown in the literature that the variance doubles after only two monitoring intervals.

3.10 CONCLUSION

In this chapter, a description of the background of stochastic control theory has been given and an explanation provided of direct digital control (DDC) and justification for its use. The need to identify dead time has been explained as also has been the inertia and the parameter r.

In Section 2.2.3, mention was made that statistical process control charts can be considered an appropriate engineering control strategy under certain specific conditions. One of these is specifying a loss function that quantifies the cost of being away from the desired or target value and the cost of making an adjustment. In light of optimal control theory and by using the quadratic criterion function, it is possible to derive minimum variance controllers (discussed in Section 3.3.1). The principle employed in the quadratic loss function is that the penalty or loss associated with being off target depends only on the squared magnitude of the mean square error (variance). The quadratic loss function so derived depends only on the absolute value of the standard deviation from target. It will be shown in Chapter 5 that the control adjustment equation of the MMSE (minimum mean square error) controller is the discrete equivalent of a properly tuned integral controller. This form of the minimum variance controller would minimise the mean overall adjustment cost when it is possible to neglect other variable costs. Apart from the process adjustment costs, if there are other costs in monitoring and controlling a process and in taking observations, then the resulting minimum-cost feedback adjustment schemes have to be formulated on the basis of different configurations. In this context, these aspects are considered in the subsequent chapters of the thesis for the stochastic control algorithm of the time series controller derived in Chapter 5.

CHAPTER 4

DEAD - TIME COMPENSATION

4.1 INTRODUCTION

The term dead time and the need to eliminate it if possible was explained in Section 3.6. The effects of the adjustment and the dead time on the performance of a controller are explained in this current Chapter. The need for a dead-time compensator is explained as also is the principle of working of a variety of the dead-time compensators that are useful in practical situations.

4.2 SAMPLED-DATA (DISCRETE) CONTROL AND DEAD TIME

Process control schemes incorporate information regarding the process into the controller, by having a process model (delay) built into the controller mechanism. The dead-time element required for building the controller mechanism is not usually physically realisable and even if approximated, results in increased costs and inaccuracies in process modelling.

So, it is usual to assume the value of the time delay of the process in discrete (sampled-data) control systems, is *a priori* information. The sampled-data (discrete) control is used to provide the plant operator with information about control actions (adjustments) that should be taken to account for the plant dynamics (inertia) and the nature of the stochastic (random) disturbances.

4.3 SAMPLING AND FEEDBACK CONTROL PERFORMANCE

Sampling at periods that are much shorter than the time delay (dead time) is likely to result in poor control. The sampling rate in sampled-data control has its influence on the closed-loop (feedback) behaviour. A rational choice of sampling rate is based on this influence and also on the recommendations for its selection. Sampling is economically advantageous where high production rates combine with relatively expensive or time-consuming measurements of individual items. Output-sampling is a practical necessity in the control of a large variety of continuous processes such as paper and sheet plastic. It is observed that as the sampling interval is decreased, the feedback control loop performance improves, but at the same time, the effort necessary to accomplish this also increases. MacGregor [1976] introduced a variance constraint on the input manipulated variable since the control error variance often increases for a decreasing sampling period, (relative to the time response of the process). Box and Jenkins [1970, 1976] and Astrom [1970] showed that under minimum variance control, the error in the process output is the forecast error of the effective disturbance at the output. It is interesting to evaluate the changes in the variance of this output error at the sampling instants by increasing the sampling interval rate. According to Abraham and Box [1979], the effect of lengthening the sampling interval is (i) to increase the mean square error slightly and (ii) to reduce the cost of the feedback control scheme. The control performance is affected by too large sampling periods and long time delays tend to reduce the controller gain (CG). So, there is a need for an optimal choice of the sampling interval.

The control achieved using a sampling interval larger than the time delay may be 'tight' requiring less-frequent sampling and there may be little economic incentive for such tighter control. Tighter control can reduce the stable operation of processes. However, some processes, such as polymerisation, sheet forming and fiber and other 'no blend' type processes require tighter control and efforts are constantly made to control the quality variables as tightly as possible by minimising the variance of the output deviation about given set points (Kelly, MacGregor and Hoffman [1987]). A controller gives different levels of performance for the same process depending upon how tightly it is tuned. In some other situations, there can be definite economic incentives for moving process set points closer to the process or quality constraints. It is usual to minimise the product variability (control error sigma) for the required adjustment interval in order to achieve this objective (Harris and MacGregor [1987]). This can be done by simulating the feedback control algorithm (derived in Chapter 5 under the effects of the dead time) for values of the IMA parameter Θ ranging from 0 to 1.0.

4.4 THE NEED FOR A DEAD-TIME COMPENSATOR

A time lag (time delay or dead time) in limiting the permissible process gain (PG) reduces the ability to control the process. So, a controller mechanism is necessary to reduce this limitation. This mechanism is called the 'dead-time compensator'. The principle of working of the dead-time compensator is explained below.

Assume that a small adjustment is made in the input variable at the 'n'th sample. The adjustment made will not have any effect on the next sample if the sampling interval is smaller than the dead time (made up of the process delay and the measurement delay). If there is no appreciable effect of the adjustment in the process, the same control error deviation from the desired target will be measured at the output and there is a tendency to overcorrect the control error deviation if another adjustment is made. In this scenario, there exists an option either (i) to reduce the controller gain (CG) and to apportion a part of the adjustment to each of the samples occurring during the dead time (time delay), or (ii) to reduce the control action by accounting for all the control actions already taken during the time delay, the effects of which are not yet perceived. The first option is achieved if the CG is chosen by a proper stability analysis. Long time delays reduce the CG (Palmor and Shinnar [1979]). Baxley [1991] found different values for the CG in his simulation study by the 'Central Composite Design' method and the corresponding standard deviation of the control error along with the mean adjustment interval (AI). The maximum value of the controller gain for the stable operation of a time-delay process is 1.0 (Chandra Prasad and Krishnaswamy [1974]). So, the first option is chosen by setting the value of the time series controller gain to be 1.0 and minimising the control actions by accounting for all the control actions taken during the time delay period. Plausible reasons for setting the value of the controller gain to be 1.0 are given in Chapter 6. This is in contrast to the EWMA controller for which a controller gain below 1.0 is required in order to avoid overcompensation of the control error and over control of the process.

The second option results in deriving an equation for the dead-time compensator from optimal control algorithms. This type of compensator is advantageous in that the problem of over-correction reduces during the time delay and it may be possible to choose the value of the controller gain without the aid of the dead-time compensator. The dead-time compensator, though it may not be able to eliminate (completely) the dead time in real systems, has a stabilising effect on the process. The response of a dead-time compensator is faster and smoother than an analogue (continuous) conventional controller in spite of sampling infrequently. The Smith predictor, (or the

Smith dead-time compensator) is a result of the minimal variance strategy and that minimal variance control for processes having dead time includes this type of dead-time compensation. A description of the Smith predictor is given in the next Section.

4.5 THE SMITH'S PREDICTOR AND THE DAHLIN'S CONTROLLER

In this Section, the Smith's predictor and the principle of operation of Dahlin's controller are described briefly. Smith's [1957] principle provides a criterion for selecting a control strategy for time delay processes and dead-time compensation techniques. The technique is an approach to control of systems with long dead times. This principle, known as the Smith predictor, states that the response of a process with a time delay should be the same as that for the same process without the delay, but delayed by a time equal to that of the delay. Smith [1957] proposed a discrete version of a dead-time compensator based on this principle. This (linear) predictor consists of a conventional PID controller in combination with a process model, which is used as a predictor of the output over the interval of the dead time, in a feedback loop around it. Figure 4.1 gives a block diagram of the Smith predictor.

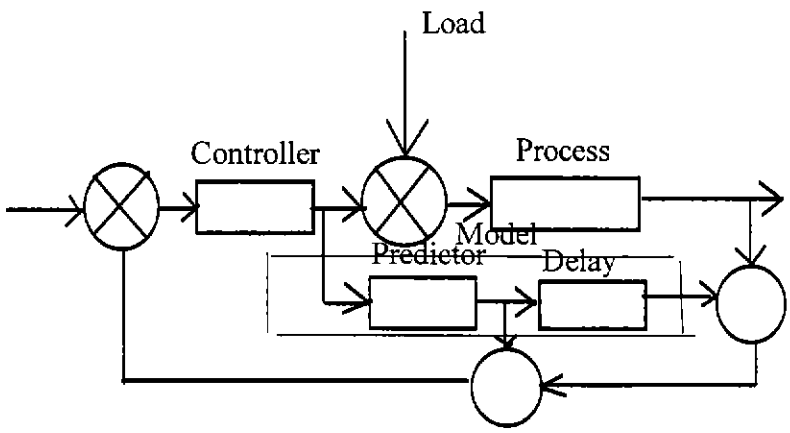


Figure 4.1 Dead-time compensation with Smith predictor

The Smith predictor contains two feedback loops; a positive loop containing the dead time and a negative loop without it. The positive feedback loop cancels out the effect of the negative feedback loop through the process, leaving the negative feedback loop in the predictor with only the lag and gain of the model in it. This arrangement makes the predictor input identical to that which would exist if there were no dead time in the process resulting in better control. The compensation technique involves the prediction of the process output through the use of a process model which does not contain the dead time. The output of this predictor element is also delayed with a time-delay element which constitutes a separate model of the process dead time. With model dead time, lag and the controller gain matched to the process, the Smith predictor reproduces a step change exactly one dead time later. A Smith Predictor achieves some form of derivative action required for compensating dead time in first-order processes by a lag in its feedback path. By matching the lag in the Smith Predictor to the lag (inertia) in a dead-time process, the input manipulated variable follows the process lag exactly but delayed by the dead time. The delayed predictor output is compared to the measured process output and the resulting model error quantity is added to the current predictor output to correct for predictor deficiencies, provided that the model is a true representation of the process and there are no further disturbances to the process during the dead-time period. It is observed that the optimal predictor part of the controller algorithm changes also with the time delay. The Smith predictor is an optimal dead-time compensator for only those systems having disturbances for which the optimal prediction is a constant over the period of the dead time.

In brief, the Dahlin's controller works on the principle proposed by Dahlin (1968) that digital controllers be designed to yield a desired first order plus dead-time response to a set point or load change. Dahlin's algorithm specifies that the sampled-data (discrete) closed-loop (feedback) control system behaves as though it were a continuous first-order process with dead time. For designing sampled-data controllers, Dahlin considered a tuning parameter ranging from 0 to 1 whereas in the original formulation, the parameter could take values from -1 to 1. Thus a dead-time compensator allows the use of a large process gain. In order to select a suitable value for the time constant of the closed-loop response, namely, the Dahlin's tuning parameter, an (trial or) initial value is assumed and the control system is simulated on a computer. A proper selection of this parameter can be made by repeatedly varying this parameter and examining the closed-loop response. The Dahlin computer-control algorithm is designed for a specific input, for example, a step change in set point. If an input (load) change occurs in a process for which the control algorithm is based on a change in set point, the response may not be equally good. The usual procedure, therefore, is to design for the worst possible change in either set point or load that is likely to occur.

Dead-time compensators are usually complex to deal with in real systems. Nevertheless, they have a stabilising effect on the process in a manner which is similar to that of a controller working on an unconstrained optimal control algorithm. The precaution to be taken against instability for large gains in the real systems by having a dead-time compensator, is by ensuring that there is no deviation between the assumed dynamic (transfer function) model and the real system (Palmor and Shinnar [1979]). It is explained later in Section 5.6 how the benefit of having built in the Dahlin's dead-time compensator in the stochastic feedback control algorithm Equation (5.16) helps to achieve the required dead-time compensation and minimum variance.

4.6 CONCLUSION

In this chapter, an exposition of the influence of the sampling interval on feedback control performance was discussed. A brief explanation of the need to compensate dead time and a description of the working of some of the dead-time compensators was also given. Dead-time simulation is a method to control sampled-data processes with time delay. In the next chapter, the derivation of the stochastic time series controller algorithm is given and its simulation discussed in Chapter 6.

CHAPTER 5

STOCHASTIC PROCESS CONTROL ALGORITHM

5.1 INTRODUCTION

Time series controllers are described and a feedback control model is given in this Chapter. The derivation of an approximate feedback control difference equation together with an algorithm for calculating the control adjustment required in the input variable when there are dynamics and delay in a process, are also presented. The *statistical control algorithm* developed for the time series controller and the controller *performance* measures (explained in Section 5.9) are discussed in detail. A brief review of a method suggested by Baxley in controlling drifting processes (Baxley [1991]) is presented.

5.2 TIME SERIES CONTROLLERS

5.2.1 Characteristics and Features

Time series controllers are used in the chemical and process industries for regulating quality variables measured at discrete time intervals. Their 'stochastic feedback control algorithms'[1] are used to calculate a series of adjustments which compensate the disturbances. Recourse to ARIMA models is often made in order to forecast the drifting behaviour. The stochastic feedback control algorithm or equation derived from these ARIMA models is computerised. Thus, 'time series control algorithms', by calculating a series of adjustments, compensate for the disturbances by

[1]The terms 'stochastic feedback control algorithm' and 'statistical time series control algorithm' are synonymous and used alternatively in this thesis.

making an adjustment at every sample point. It will be shown in Section 5.6 that the stochastic feedback control algorithm derived for the time series controller has 'reset'-or 'integral' action and a controller having integral action can eliminate (steady-state) 'offset' (deviation from set point) (Shinskey [1988]).

Time series controllers require an adjustment for every sample and provide a *performance benchmark* by giving a minimum control error variance so long as the underlying process model remains correct. It is used to test the performance of feedback control strategies formulated (designed) using various available techniques employing methods used in statistical process control (SPC) or more elaborate strategies such as 'pole placement designs' and 'linear quadratic controllers'.

Time series controllers are capable of giving one-step ahead forecast error variance over the time delay (dead time) period in a process. It may be possible to restrict sampling and adjusting a process until an acceptable control error variance is achieved by making use of the time series controller's forecast error variance property (explained in Section 5.7) and to minimise monitoring and adjustment costs.

5.2.2 Statistical Control Algorithm - Criterion

The modified Shewhart approach of splitting the control chart into six zones and employing run rules takes a *hypothesis testing* approach to decide whether or not the process mean is on target. Since this hypothesis testing approach is ruled out, (since we require the mean to be exactly on target or as close as possible to the desired target), it is necessary to have an alternate method, to calculate the process adjustment from statistics calculated from historical data. If the null hypothesis (on target) is rejected, there is no statistic built into the control logic which provides an estimate of the new process level and hence no adjustment can be calculated. The statistical control algorithm employed by a time series controller makes an adjustment to compensate exactly for the 'forecasted deviation' from target. There are also the (i) EWMA controller and (ii) the CUSUM controller which meet the same criterion; but these controllers are different from time series controllers in that they use the EWMA and the CUSUM statistics respectively to derive their own individual 'stochastic feedback control algorithms' to calculate the adjustment that may exactly compensate for the disturbance (Baxley [1991]).

5.3 A COMPARISON OF TIME SERIES AND PID CONTROLLER PERFORMANCES

The proportional integral derivative (PID) controller, though simple in performance, does not possess the capability of providing tight control over processes with dead time. Consider a discrete PID controller taking a control action on an output deviation from target occurring at time t. This control action will not affect the output until the lapse of dead time. At time t+1, the PID controller makes another correction for the same output error if there are no (new) disturbances to the process. The effect of the first correction will come through to compensate for the original error over the next adjustment interval, but then, overcompensation of the output error is the result of the second correction coming through a period later. A controller that is tuned tightly compensates the disturbance and not the real changes in the process and leads to overcompensation. This problem may be overcome by tuning the controller correctly and doing away with overcompensation (Wardrop and Garcia [1992]). PID controllers tend to give unstable (oscillatory) performance in the face of dead times unless they are detuned so that only part of the necessary action is taken at each instant. There is a tendency to compound the problem of tight control with more periods of dead time (time delay) introduced by sampling. On the other hand, the optimal time series controller in which the prediction is made at time t, of the disturbance b+1 (where b is the dead time), periods into the future, over the period of the process dead time, will not compensate for the same error a second time (Harris, MacGregor and Wright [1982]). Dead-time compensators are built based on a similar principle. A time series controller is better in performance with respect to dead-time compensation than the EWMA controller which requires the controller gain to be less than 1.0 and has no dead-time compensation term in its control algorithm (page 286, Baxley [1991]).

The effect of the choice of the sampling interval on the controller performance can be considered by comparing the minimum output error variance obtainable at the sampling instants for various sampling intervals. The sampling intervals are different from one another and the same error variances are not compared at these sampling instants. Simulation is a mechanism to evaluate these variances at intermediate times. The technique of dead-time simulation and control used for sampled-data control of processes with time delay (dead time), is explained in Chapter 6.

Thus, by building a dynamic-stochastic model of the process based upon data collected at a single interval, the time series controller performance is predicted at longer sampling intervals and thereby a reasonable choice of the sampling interval is arrived at which is used for sampling, adjustment and process control.

5.4 DEVELOPMENT OF TIME SERIES MODELS

5.4.1 Feedback Control Difference Equation

The 'stochastic difference equation' for the feedback control model is derived with the help of a block diagram shown in Figure 5.1.

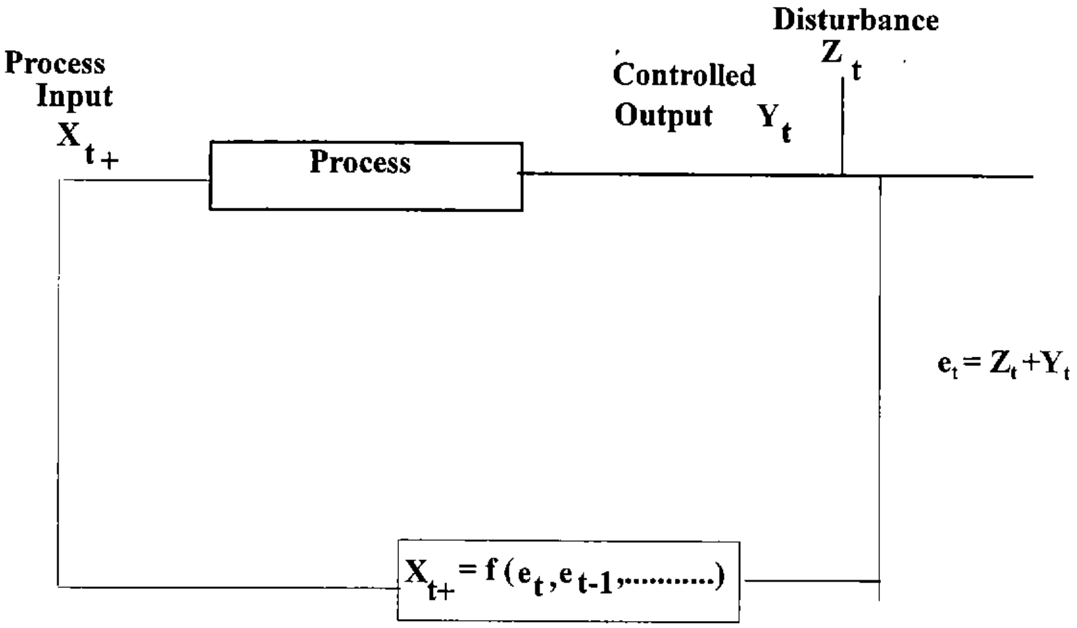


Figure 5.1 Block Diagram for the Feedback Control Model

In the feedback control scheme shown in Figure 5.1, the process is regulated by manipulating the input variable X_t which in turn affects the controlled output Y_t. X_{t+} is the setting of the input variable (the plus sign on the subscript of X_{t+} implies that the adjustment is made at time t during the interval between t and t+1). A definite deterministic relationship exists between the process input X_t and its output Y_t which does not exhibit stochastic characteristics. Z_t, the non-stationary disturbance, is the output of the (linear) system, when subjected to a sequence of uncorrelated random shocks $\{a_t\}$ where $a_t \sim N(0,\sigma_a^2)$.

5.4.2 Symbols used in the Feedback Control Model Block Diagram of Figure 5.1.

a_t Random shocks $N(0, \sigma_a^2)$,

Z_t Disturbance,[2]

e_t Forecast error,

X_{t+} Input Manipulative Variable (Linear function of e_t and of integral over time of past errors),

Y_t Output or Controlled Variable,

$e_t = Z_t + Y_t$.

Box and Jenkins [1970, 1976] described some dynamic models of the order (r,s) by

$\delta_r(B)Y_t = \omega_s(B)B^bX_t$, (Table 10.1, page 350, Box and Jenkins [1970, 1976]),

'b' being the number of whole periods of dead time, where $\delta_r(B)$ and $\omega_s(B)$ are polynomials in B and $BX_t = X_{t-1}$, $B^bX_t = X_{t-b}$; B is the backward shift operator.

A first-order *discrete* (sampled data) *single input single output* (SISO) dynamic system is parsimoniously represented by the general (linear) difference equation

$$(1+\xi\nabla)Y_t = gX_{t-b}, \qquad 0 \le \delta < 1 \qquad (5.1)$$

where $\xi = \delta/(1-\delta)$ and ∇ is the backward difference operator, $\nabla = 1-B$.

The terms g(ain) and δ are explained subsequently.

This *discrete dynamic* model is of the order (1,0) and has the form

$$(1-\delta B)Y_t = \omega_0 B^bX_t. \qquad 0 \le \delta < 1$$

With s = 0, the *impulse response* tails off exponentially (geometrically) from the initial starting value

[2] 'z' denotes the *stochastic variable* and 'Z' represents the **stochastic disturbance**. The same logic holds good for $\mathbf{a}_t$ which denotes the *variable* and $\{\mathbf{a}_t\}$ represents the **sequence** of *random variables*.

$$\omega_0 = g/(1 + \xi) = g/(1 + (\delta/(1 - \delta)) = g(1 - \delta)$$

(page 352, Box and Jenkins [1970, 1976])

where g, the (steady-state) gain denotes the ratio of change in the steady-state process output to the change in the input which caused it (Deshpande and Ash [1981], (Shinskey [1988]). δ represents the inertial capacity or the dynamics of a process to recover back to its equilibrium conditions after an adjustment is made where the adjustments do not have an immediate effect on the process. It is connected to the sampling interval and the time constant by means of the relation $\delta = e^{-T/\tau}$ where T is the sampling interval of the discrete process and τ, the process time constant. The time constant* is the time required for the process output to complete 63.2% (τ = 1-1/e = 1-1/2.718 = 0.632) of its final steady-state value after a (step) change is made in the input (*Time constant is the ratio of change in the output controlled variable to the product of the process (static) gain and the input step change (Shinskey [1988])).

So, the *recursive feedback control (linear) difference equation* for the discrete dynamic model with b units of delay (dead time) can be written in the form

$$(1-\delta B)Y_t = \omega_0 X_{t-b} = g(1-\delta)X_{t-b} = g(1-\delta)B^b X_t. \qquad 0 \le \delta < 1 \qquad (5.2)$$

For this feedback control first-order difference equation, the output change asymptotically approaches 'g' for a unit change in the input. 'g' is also called the 'system gain 'or 'pure gain' (Box and Kramer [1992]).

5.5 JUSTIFICATION FOR SECOND-ORDER DYNAMIC MODELS

For feedback control (closed-loop) stability, the parameter δ must satisfy the condition that $0 < \delta < 1$ in the discrete dynamic model of the process and the gain should be less than or equal to 1.0 (Shinskey [1988]).

The first-order dynamic model that can approximate the behaviour of a number of processes which is characterised by the linear difference equation (5.2) can be written as

$$Y_t = \delta Y_{t-1} + g(1 - \delta)X_{t-b}. \qquad 0 \leq \delta < 1 \qquad (5.3)$$

The views of Box and Kramer [1992] in regard to the effect on control action of larger values of δ are briefly reviewed. The MMSE (minimum mean square error) or minimum variance control schemes based on the first-order dynamic model and the ARIMA (0,1,1) disturbance model produce the minimum mean square error (MMSE) at the output for the particular form of adjustment that may require excessive control action in the following situations. (i) if the values of δ are not small; (ii) as δ becomes larger and in particular, as it approaches unity and (iii) when the time constant of the process is large with respect to the sampling interval (Kramer [1990]). As δ becomes larger, the minimum variance control exhibits large 'alternating' character in the required adjustments (control actions) to give minimum output variance (Box and Kramer [1992]).

For larger values of δ, the general recourse is to go in for constrained variance control schemes. In such control schemes, reduced control action may be achieved at a cost of small increases in the mean squared error at the output by placing a constraint on the input manipulated variable. Kramer [1990] developed a constrained variance control scheme in which he showed the effect on both adjustment variance and the specified output variance in order to evaluate the trade-offs between the two variances.

Processes found in practice are often complex because of their dynamic characteristics which change with time. Approximating such processes by first-order dynamic models does not always seem to be satisfactory. It can be shown from the simulation study results of the time series controller for a first-order (plus dead time) dynamic model and the ARIMA (0,1,1) model that for drifting processes, for values of δ from 0 to 1, the required adjustments are of alternating character and sometimes with huge increases in control error standard deviation and its adjustment variance (vardxt). A control algorithm based on ARIMA (0,1,1) disturbance model and the first-order dynamic model of equation (5.3) is of 'practical use' only for fairly small values of δ (page 256, Box and Kramer [1992]). It is likely that some processes may have more than one dynamic element and the exact mathematical model relating the output and the input could be greater than the first-order. The aim is to develop a stochastic process control algorithm based on a process model that would be of practical utility in manufacturing industry. Many complicated dynamic systems can be fairly closely approximated by second-order systems with delay (dead time). The mathematical justification and suggestions of process control practitioners in regard to the use of a second-order dynamic model are discussed subsequently. Detailed analysis and identification of the dynamic models and their suitability can be found in Box and Jenkins [1970, 1976].

Some commonly used identification techniques are maximum likelihood or prediction error methods and the recursive least squares method. The recursive least squares method and its extensions require that the value of the dead time (time delay) of the process must be known in order to account for the delay. Palmor and Shinnar [1979], though, of the view that the first-order model with dead time is sufficient for representing some of the most commonly occurring simple processes, cautioned that complex (process) model that helps to 'identify', ('Identification' is the term used by engineers for process modelling), processes is (also) not (very) accurate. They contended that there had been several claims in the process industries to second-order models coupled with delays as sufficient for most purposes. In particular, the dynamic models (transfer function) of the form

$$\delta_r(B)Y_t = \omega_s(B)B^{b+1}X_t$$

are sufficient for most practical cases of interest in sampled-data (discrete) control, based on laboratory measurements (Palmor and Shinnar [1979]).

It can be shown that for a given stable (transfer function) dynamic model, the higher terms in the polynomials decrease exponentially with increasing sampling interval (T). In practice, therefore, a second-order polynomial for $\omega(B)$ and $\delta(B)$ described by

$$Y_t = [(\omega_0 - \omega_1 B)/ (1- \delta_1 B - \delta_2 B^2)]B^{b+1} X_t$$

is probably the highest order that can be justified.

The transfer function, $[(\omega_0 - \omega_1 B)/ (1- \delta_1 B - \delta_2 B^2)]$ depends on the sampling interval (T) and also on the tuning of the process controller and therefore it can be adjusted and can approximate any (linear) response at fixed time intervals. However, the polynomial in the numerator can be extended further if there is a large variation in the assumed 'a priori' dead time value. The disadvantage of this is that the matrices used in calculating the process parameters would increase and the procedure would take a longer time than usual to identify the process.

It is possible to approximate the behaviour of high-order processes by a system having one or two time constants and a dead time. When one or two time constants dominate, the smaller ones ('work together') add up to produce a lag that almost resembles (pure) dead time. It is possible also to approximate the actual input-output mathematical model of a high-order, complex dynamic process with a simplified model consisting of a second-order process combined with a dead-time element. The second-order model will reduce to the first-order model if one of the two time constants of the former model is smaller than the other (pages 12-15, Deshpande and Ash [1981]).

Box and Jenkins [1963] suggested that many dynamic models could be adequately represented by, at most, two exponential stages with variable gains and delay governed by parameters δ_1, δ_2, g and dead time b. Box and Jenkins [1970, 1976] mentioned also in their monograph that complex processes with dead time (delay) can reputedly be closely approximated by second-order systems having more than one time constant, (usually, two) (Page 345, Box and Jenkins [1970, 1976]). This view is shared also by MacGregor [1988], he commented:- 'To adequately characterise the dynamic behaviour of more complex processes, it may sometimes be necessary to use higher-order dynamic models' (page 25, MacGregor [1988]). A second-order process with dead time is a useful model for some complex processes which are fairly common (Shinskey [1988]).

It is conjectured, therefore, that the dynamic system will be better described by a second-order system that is represented by a dynamic model of the order (2,1), ('a discretely coincident' continuous system, page 358, Box and Jenkins [1970, 1976]).

Some methodologies were suggested by MacGregor [1988], Box and Kramer [1992] to use statistical process control charts to monitor the performance of closed-loop controlled systems. Such methodologies, though taking care of on-line process control and monitoring, do not always seem to deal with stability problems of the feedback control loop in an explicit and simple manner. It is acknowledged that the literature contains derivations for general formulae for minimum variance control. However, it is worth considering a concrete example of controlling a second-order dynamic system by a second-order dynamic model of the form:-

$$Y_t (1- \delta_1 B - \delta_2 B^2) = (\omega_0 - \omega_1 B) B^{b+1} X_t. \qquad (5.4)$$

For stability reasons, interest is focused on the 'critically damped' behaviour of the second-order dynamic system (for which the time constants τ_1 and τ_2 are real and equal), as a special case, and not on the behaviour of the system when it is either 'overdamped' or 'underdamped' for which τ_1 and τ_2 may be complex. Some justification for restricting attention to a (critically) damped second-order system with dead time is given in Sections 7.2.3, 7.2.5, 9.5.3 and 9.6.

The second-order dynamic system can then be thought of as equivalent to two discrete first-order systems arranged in series.

The second-order model will be

(i) underdamped, when the roots are complex, that is, when

$\delta_1^2 + 4\,\delta_2 < 0$;

(ii) overdamped, when the roots are real (and not equal), that is, when

$\delta_1^2 + 4\,\delta_2 > 0$, and

(iii) critically damped, when the roots are real and equal, that is when

$\delta_1^2 + 4\,\delta_2 = 0$.

Stability is achieved when the point (δ_1, δ_2), lies in a triangular region defined by the conditions, $\delta_2 - \delta_1 = 1$, $\delta_1 + \delta_2 = 1$ and $\delta_2 < 1$. This is shown in Figure 5.2.

It can be seen that the process delay, b is distributed in equation (5.4) between the terms B^{b-1} and the $\omega(B)$ polynomial.

Using $\omega_1 \approx \omega_1 B$, equation (5.4) is approximated to

$$(1- \delta_1 B - \delta_2 B^2) Y_t = \omega\, B^{b+1} X_t\ , \qquad (5.5)$$

where $(\omega_0 - \omega_1) = \omega$, for mathematical convenience in dealing with a single term in ω. ω is the magnitude of the process response to a unit step change in the first period following the dead time carrying over into additional sample periods. Only whole periods of dead time (delay) are being considered.

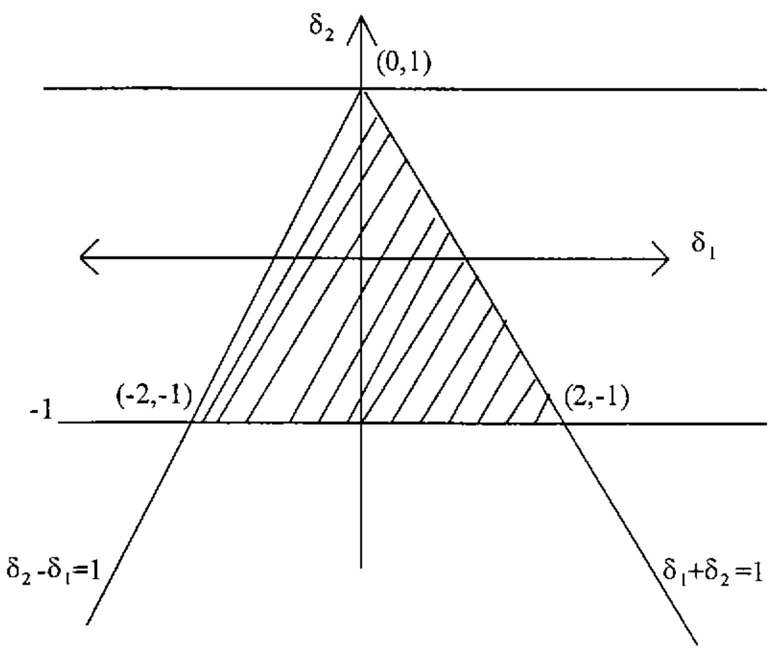


Figure 5.2 Triangular region defined by the inequality conditions for achieving stability

For dead time b = 1, equation (5.5) becomes

$$Y_t = \delta_1 Y_{t-1} + \delta_2 Y_{t-2} + \omega B^{1+1} X_t = \delta_1 Y_{t-1} + \delta_2 Y_{t-2} + \omega X_{t-2}.$$

For dead time b =2, equation (5.5) becomes

$$Y_t = \delta_1 Y_{t-1} + \delta_2 Y_{t-2} + \omega B^{2+1} X_t = \delta_1 Y_{t-1} + \delta_2 Y_{t-2} + \omega X_{t-3}.$$

These equations are directly built into and used in the subsequent simulations.

The following points in regard to the feedback control equation (5.5) should be noted:

(1) A (linear) difference equation is employed to represent the *discrete* (sampled-data) second-order dynamic system. This is similar to representing continuous dynamic systems by linear differential equations. The (linear) model implies that the response to

a set of impulses of an input series can be added to provide the output and suggests an approximation for practical situations. In dealing with discrete processes, linear difference equations representing the processes in which the sampling intervals are short, take care of the dynamic or inertial properties of the process. A second-order process is represented by the second-order difference equation when sampled at discrete intervals or by the second-order transfer function or 'filter', (the term used in engineering terminology, by control system engineers), (cf. MacGregor [1987]).

(2) The initial parameter errors are assumed to be small in order to reduce the sensitivity of the difference equation, describing the discrete dynamic control system, to computer round off errors when conducting simulation experiments.

(3) The second-order model is capable of representing some dynamic systems with dead time for some reasonable value ranges of X_t and Y_t.

Moreover, equation (5.5) reduces to that of Baxley's [1991] first-order dynamic model, namely,

$$Y_t = \delta Y_{t-1} + \omega B^{b+1} X_t$$

which describes the first-order system with dead time (delay) when $\delta_2 = 0$ and $\delta_1 = \delta$.

The steady-state gain, 'g', of such a second-order discrete dynamic model is given by

$$g = (\omega_0 - \omega_1)/(1-\delta_1-\delta_2).$$

(Equation 10.2.5, page 346, Box and Jenkins [1970, 1976])

$$(1-\delta_1 B-\delta_2 B^2)Y_t = (\omega_0-\omega_1 B)B^b X_t$$

$$= (1-S_1 B)(1-S_2 B)Y_t$$

$$= (\omega_0 - \omega_1 B)\, B^b\, X_t$$

where

$$\omega_0 = [PG/(\tau_1-\tau_2)]\{(\tau_1(1-S_1)-\tau_2(1-S_2)\},$$

$$\omega_1=[PG/(\tau_1-\tau_2)]\{(S_1+S_2)(\tau_1-\tau_2)+\tau_2S_2(1+S_1)-\tau_1S_1(1+S_2)\},$$

(Palmor and Shinnar [1979]),

$$S_1 = e^{-1/\tau_1},$$

$$S_2 = e^{-1/\tau_2},$$

$$\delta_1 = S_1 + S_2 = e^{-1/\tau_1} + e^{-1/\tau_2},$$

$$\delta_2 = - S_1 \times S_2 = - e^{-(1/\tau_1 + 1/\tau_2)} \text{ and}$$

PG represents the process gain, realised by the total effect in output caused by a unit change in the input variable after the completion of the dynamic response (Baxley [1991]).

Now,

$$\omega = (\omega_0 - \omega_1) = [[PG/(\tau_1 - \tau_2)]\{(\tau_1(1 - S_1) -\tau_2(1 - S_2)\}] -$$

$$[[PG/(\tau_1 - \tau_2)]\{(S_1 + S_2)(\tau_1 - \tau_2) +\tau_2S_2(1 + S_1) -\tau_1S_1(1+S_2)\}]$$

which on simplification, gives, for a critically damped system,

$$\omega = PG[1 - S_1 - S_2 + S_1S_2]$$

$$= PG[1-(S_1+S_2)+S_1S_2]$$

$$= PG[1- (e^{-1/\tau_1} + e^{-1/\tau_2})+e^{-(1/\tau_1+1/\tau_2)}]$$

$$= PG[1 - \delta_1 - \delta_2].$$

Therefore, the steady-state or system gain

$$g = (\omega_0 - \omega_1)/(1 - \delta_1 - \delta_2) = PG[1- \delta_1 - \delta_2]/[1- \delta_1 - \delta_2] = PG, \text{ the process gain.}$$

Baxley [1991] used $PG = 1/1-\delta$ and made $PG = 1.0$ by setting $\delta = 0$, meaning that there are no carry-over effects (inertia) into the next observation and seemed to have tackled the problem of feedback control stability in a convincing manner in his simulation studies for drifting processes. Kramer [1990], derived expressions for the disturbance and output effect of control actions as functions of random shocks, *independent* of the *control scheme.* Moreover, Kramer [1990] considered approaches for reducing adjustment variability. Since interest here is in reducing the product variability (control error standard deviation, CESTDDVN), at the output, it is worthwhile to consider the critically damped behaviour of the second-order dynamic system for which the time constants are real and equal thus ensuring closed-loop stability. Furthermore, the steady-state gain of such a critically damped second-order system is shown to be PG, the process gain itself.

An additional term in the parameter δ, (δ_2) of the second-order dynamic model makes it possible to account for more of the process dynamics for both small and large values of δ and to represent the dynamic nature of the process more adequately. It is easier to control a dead-time process having an additional dynamic lag rather than a (pure) delay process (Chandra Prasad and Krishnaswamy [1975]). The additional term Y_{t-2}, defines the input-output relationship in a better manner than the first-order dynamic model.

For stability of the second-order dynamic model, the parameters δ_1 and δ_2 must satisfy the following inequality conditions given by

$$\delta_1 + \delta_2 < 1,$$

$$\delta_2 - \delta_1 < 1,$$

$$-1 < \delta_2 < 1.$$

The 'characteristic equation' for the second-order dynamic system is

$$(1-\delta_1 B-\delta_2 B^2) = 0. \qquad (5.6)$$

When the roots of this equation are real, that is, when $\delta_1^2 + 4\,\delta_2 \geq 0$, the solution will be the sum of two exponentials.

The roots of the characteristic equation (5.6) determine the stability of the second-order dynamic system. When these roots are real and positive, the step response, which is the sum of two exponential terms, approaches its asymptote g, the steady-state gain, without crossing it. When the roots are complex, as can be seen from the Figure 5.3, (Reproduced from Figure 10.5, page 344, Box and Jenkins [1970]), (Attachment 5.I), the step response of the output variable above the target value, which is a problem in APC due to complex roots, overshoots the value g. From Figure 5.3, it can also be seen that the system has no overshoot when the characteristic equation has real positive roots. This explains the interest in the critically damped second-order discrete dynamic model which ensures closed-loop (feedback control) stability. The focus on the critically damped second-order dynamic model is one of many justifications for restricting attention to such a special case. It is shown later in Chapter 9, how the advantages of having an integral term in the stochastic feedback control algorithm (5.16) help determine the damping of the feedback control loop and guards it and the controller from the occurrence of over and under damped oscillations leading to an unstable feedback loop.

It is known that for the critically damped second-order dynamic system, $\tau_1 = \tau_2 = \tau$. So,

$$\delta_1 = 2e^{-1/\tau} \text{ and}$$

$$\delta_2 = -e^{-2/\tau} .$$

As per Figures 5.2 and 5.3 (Attachment 5.I), the values of δ_1 and δ_2 should satisfy the following equation (5.7) given by

$$-2 < \delta_1 < 2,$$

$$-1 < \delta_2 < 1. \tag{5.7}$$

These are built into subsequent simulations.

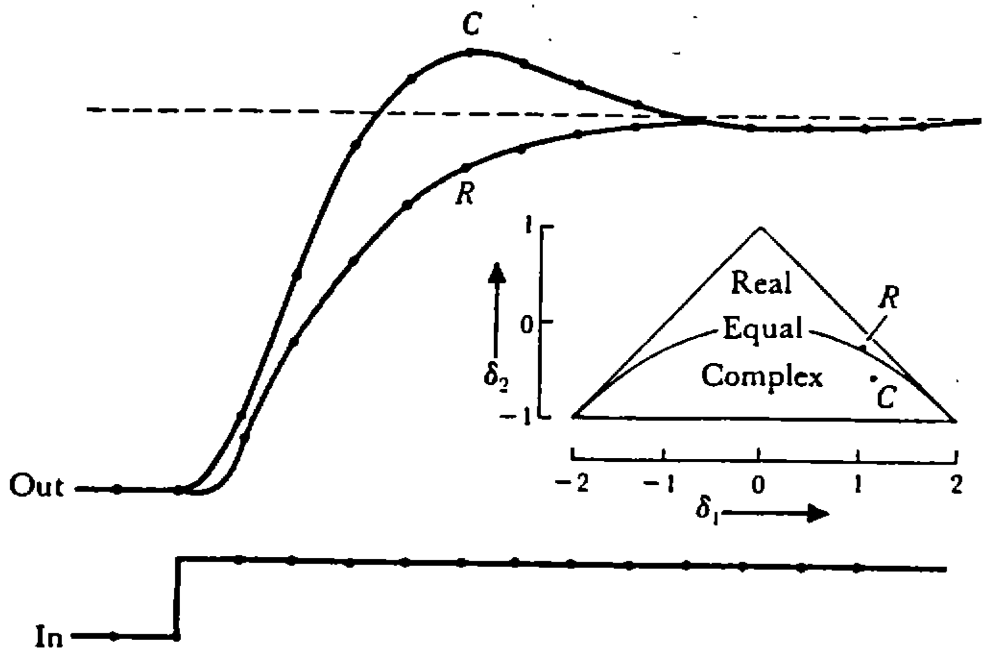


FIG. 5.3 Step responses of coincident, discrete and continuous second-order systems having characteristic equations with real roots (Curve R) and complex roots (Curve C)

5.6 EXPRESSION FOR THE CONTROL ADJUSTMENT IN THE INPUT VARIABLE OF A TIME SERIES CONTROLLER

An expression is derived for the *feedback control adjustment* required in the input manipulated variable of a time series controller for a dynamic process with dead time (delay). This expression is different from equation (5.5) which explains the feedback control model. Figure 5.4 shows the feedback control scheme to compensate a disturbance Z_t by means of a time series controller. Baxley [1991] considered the dead time equal to one period when deriving the feedback control equation. In this Section, the feedback control (adjustment) algorithm is derived considering b periods of dead time. It conforms to the minimum variance (mean square) control equation derived by Kramer [1990] for a system in which adjustments to the input variable are made after the process is observed and so their effects are first seen at the next observation (b = 0).

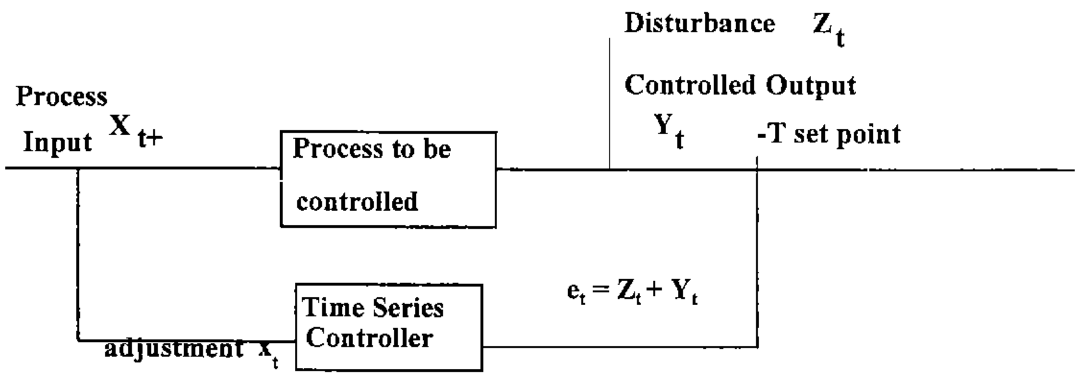


Figure 5.4 Feedback Control scheme to compensate disturbance Z_t by a Time Series Controller in the existence of Dynamics and Dead time

Using the same symbols and notation of Section 5.5,

$$(1 - \delta_1 B - \delta_2 B^2) Y_t = \omega B^{b+1} X_t. \qquad (5.5)$$

Changes are made in the input X at times t,t-1,t-2,---, immediately after observing the disturbances z_t, z_{t-1}, z_{t-2},---. Because of this, a pulsed input results and the level of X in the interval t to t+1 is denoted by X_{t+}. For this pulsed input, assume that the dynamic model which connects the input manipulated variable X_{t+} and the controlled output Y_t is

$$Y_t = L_1^{-1}(B)L_2(B)B^{b+1}X_{t+}, \tag{5.8}$$

where

$L_1(B)$ is a polynomial in B of degree r,

$L_2(B)$ is a polynomial in B of degree s and

b is the number of complete intervals of delay before an adjustment in the input X_{t+} begins to affect the output Y_t.

The non-stationary disturbance is represented by the ARIMA (0,1,1) model

$$\nabla Z_t = (1-\Theta B)a_t.$$

That is,

$$z_t = z_{t-1} + a_t - \Theta a_{t-1}. \tag{5.9}$$

Z_t measures the effect at the output of an unobserved disturbance, that is, an uncompensated non-stationary disturbance that reaches the output before it is possible for the compensating control action to become effective. This causes the process to wander off target and is defined as the deviation from the target that would occur if no control action were taken. The effect of the disturbance would be cancelled if it were possible to set

$$X_{t+} = -L_1(B)L_2^{-1}(B)Z_{t+b+1}, \qquad b > -1.$$

This control action is not realisable since (b+1) is positive; but, the minimum mean square error of the deviation of the output from its target value can be obtained by replacing Z_{t+b+1} by its forecast estimate $\hat{z}_t(b+1)$ made at time t.

That is, by taking the minimum variance control action

$$X_{t+} = -L_1(B)L_2^{-1}(B)\ \hat{z}_t(b+1)\ . \tag{5.10}$$

The change or adjustment to be made in the input manipulated variable is then

$$x_t = -L_1(B)L_2^{-1}(B)\{\hat{z}_t(b+1) - \hat{z}_{t-1}(b+1)\}. \tag{5.11}$$

The error at the output or deviation from the target at time (t+b+1) is the forecast error $e_t(b+1)$ at lead time b+1 for the Z_t disturbance.

That is,

$$e_t(b+1) = Z_{t+b-1} - \hat{z}_t(b+1)$$

made (b+1) steps ahead at time t.

The error observed at time t is

$$\varepsilon_t = e_{t-b-1}(b+1)$$

$$= Z_t - \hat{z}_{t-b-1}(b+1)\ .$$

$\hat{z}_t(b+1) - \hat{z}_{t-1}(b+1)$ can be deduced from the observed error sequence

$\varepsilon_t, \varepsilon_{t-1}, \varepsilon_{t-2}$,---.

Here $e_t(b+1)$ and $\hat{z}_t(b+1)$ are linear functions of the $\{a_t\}$'s.

So,

$$Z_{t+b+1} = L_4(B)a_{t+b+1} + L_3(B)a_t, \text{ where}$$

$L_3(B)$ and $L_4(B)$ are operators in B which can be deduced from the relations

$$e_{t-b-1}(b+1) = L_4(B)a_t \text{ and}$$

$$\hat{z}_t(b+1) = L_3(B)a_t.$$

From these,

$$\hat{z}_t(b+1) = L_3(B)/L_4(B)e_{t-b-1}(b+1) = \{L_3(B)/L_4(B)\}\varepsilon_t$$

and

$$\hat{z}_t(b+1) = \{(1-\Theta)/(1-B)\}a_t = L_3(B)a_t.$$

Similarly, $L_4(B)$ is found by expressing the forecast errors as a linear function of future shocks (Box and Jenkins [page 128, 1970, 1976]).

Then,

$$L_1(B) = (1-\delta_1 B - \delta_2 B^2),$$

$$L_2(B) = PG(1-\delta_1-\delta_2),$$

$$L_3(B) = (1-\Theta)/(1-B)$$

$$\text{and } L_4(B) = 1 + (1-\Theta)B.$$

So, for a time series controller, when the disturbance is described by the ARIMA (0,1,1) model and there are <u>definite</u> carry over effects, the adjustment (x_t) in the input manipulated variable required to make the control and forecast error variances equal, is given by

$$X_{t+} = -\{L_1(B)L_3(B)/(L_2(B)L_4(B))\}\varepsilon_t.$$

(Box and Jenkins [1970, 1976])

The control action in terms of the adjustment $x_t = x_{t+} - x_{t-1+}$, to be made at time t is,

$$x_t = -\{(L_1(B)L_3(B)(1-B))/(L_2(B)L_4(B))\}\varepsilon_t.$$

(Equation 12.2.8 page 435 Box and Jenkins [1970, 1976]).

This 'feedback control equation defines the adjustment to be made to the process at time t which would produce the feedback control action yielding the smallest possible mean square error since it exactly compensates the predicted deviation from target' (page 213, Box and Jenkins [1968]).

The above equation, on substituting in the expressions for $L_1(B)$, $L_2(B)$, $L_3(B)$ and $L_4(B)$ gives:-

$$x_t = -\frac{(1-\delta_1 B - \delta_2 B^2)(1-\Theta)}{PG(1-\delta_1-\delta_2)(1+(1-\Theta)B)}\varepsilon_t \tag{5.12}$$

where Θ is the *moving average* (operator) parameter.

The control (forecast) errors which turn out to be the one-step ahead forecast errors are measured in practice.

It is known that the (forecast) error ε_t at the output at time t is the forecast error at lead time b+1 for the Z_t disturbance.

So,

$$\varepsilon_t = e_{t-b-1}(b+1) = \psi_0 a_t + \psi_1 a_{t-1}. \tag{5.13}$$

For the ARIMA (0, 1, 1) model, the weights are $\psi_0 = 1$ and $\psi_1=1-\Theta$, so

$$\begin{aligned}\varepsilon_t &= a_t+(1-\Theta)a_{t-1}\\ &= (1+(1-\Theta)B)a_t\end{aligned}$$

and further from eqution (5.12),

$$x_t = -\frac{(1-\delta_1 B - \delta_2 B^2)(1-\Theta)}{PG(1-\delta_1-\delta_2)(1+(1-\Theta)B)}(1+(1-\Theta)B)a_t\ . \tag{5.14}$$

Since $(1-\Theta)\times 100$ per cent of the control error will affect the future process behaviour as per the disturbance model, for a dead time b,

$$\begin{aligned}e_t &= a_t + (1-\Theta)a_{t-b}\\ &= a_t[1 + (1-\Theta)B^b]\end{aligned}$$

and so

$$a_t = e_t/[1+(1-\Theta)B^b]. \tag{5.15}$$

Therefore, the control adjustment equation for b periods of dead time is

$$x_t = -\frac{(1-\delta_1 B - \delta_2 B^2)(1-\Theta)}{PG(1-\delta_1-\delta_2)} \times \frac{e_t}{(1+(1-\Theta)B^b)}\ .$$

That is,

$$x_t + (1-\Theta)\,x_{t-b} = -\frac{(1-\delta_1 B - \delta_2 B^2)(1-\Theta)}{PG(1-\delta_1-\delta_2)}\,e_t$$

giving

$$x_t = -\frac{(e_t - \delta_1 e_{t-1} - \delta_2 e_{t-2})(1-\Theta)}{PG(1-\delta_1-\delta_2)} - (1-\Theta)\,x_{t-b}\,. \tag{5.16}$$

The control adjustment action given by equation (5.16) minimises the variance of the output controlled variable.

The equation (5.16) is in conformance with the feedback control action adjustment equation of Kramer [1990] when the output variance is made equal to the variance (σ_a^2) of the random shocks, the a_t's, for achieving minimum variance or mean square control when b = 0. The control adjustment action is made up of the current deviation (e_t) and the past adjustment action x_{t-b} (Kramer [1990]). It is observed also that this is similar to the feedback control action adjustment equation for one period of dead time derived by Baxley [1991] on taking a value 1 for b, the dead time and when there are no carry-over effects for a 'standard' time series controller. On comparison with the equation of Baxley [1991], it is found that the first term in equation (5.16) gives the integral action and the second term, the dead time-compensator, developed by Smith [1959] (Baxley [1991]).

Some simulation results of equation (5.16) for the control error standard deviation (CESTDDVN), obtained when $\delta_1 = \delta_2 = 0$, PG = g = 1 and b = 1 are shown in Table 5.1. These results match closely with that of Baxley's [1991] values for the time series controller.

Table 5.1 Simulation Results of equation (5.16)

$\delta_2 = \delta_1 = \delta = 0$ (no carry-over effects), Dead Time, b = 1.0,

Controller Gain, CG = 1.0, Process Gain, PG = $1/(1-\delta) = 1.0$

Θ (theta)	CESTDDVN	Control error sigma(SE) (Baxley)
0.25	1.260	1.250
0.50	1.112	1.118
0.75	1.010	1.031

The control adjustment action given by equation (5.16) minimises the variance of the output controlled variable.

5.7 TIME SERIES CONTROLLERS-FORECAST ERROR VARIANCE FEATURE

The time series controller has the characteristic that its control error variance is the (b+1) step-ahead *forecast error variance*. This is explained as follows.

The ARIMA (0,1,1) (Box and Jenkins [1970, 1976]) disturbance model is represented by

$$\Phi(B)Z_t = \Theta(B)a_t$$

where

$\Phi(B)$ is the *stationary autoregressive operator* and

$\Theta(B)$ is the *Moving average operator*.

The one step-ahead forecast error for this model can be shown as

$$e_t(\ell) = z_{t+\ell} - \hat{z}_t(\ell) \tag{5.17}$$

where t represents the time at which the forecast is being made and ℓ, the *leadtime*, (the time forecast in terms of the sample periods). That is, the forecast is made at origin t for lead time ℓ. In this equation (5.17), z_t is the effect of the disturbance at the origin t and $\hat{z}_t$ is an estimate of the *expected* value of the disturbance (Z) for any future time, conditional upon the realisation of Z up to time t.

The forecast errors help determine the appropriate adjustment in the input manipulated variable for returning the process to target by making the forecast and control errors equal. The derivation of the expression for the control adjustment in the input variable was shown in Section 5.6.

Although the forecasts are the same for all future sample points (values of ℓ), the forecast error variance increases with ℓ. This can be seen by expressing the forecast errors as a linear combination of future shocks

$$e_t(\ell) = \psi_0 a_{t+\ell} + \psi_1 a_{t+\ell-1} + \ldots + \psi_{\ell-1} a_{t+1}. \tag{5.18}$$

The forecast error variance is,

$$\mathrm{Var}[e_t(\ell)] = [1+(\ell-1)(1-\Theta)^2]\sigma_a^2 \tag{5.19}$$

of the random component of the disturbance where ℓ is the same time forecast, (defined above), in terms of the number of sample intervals into the future and σ_a^2, the variance of the random shocks.

So, the control error variance for the ARIMA (0,1,1) time series disturbance model is

$$\mathrm{Var}[e_t(b+1)] = [1+b(1-\Theta)^2]\sigma_a^2. \tag{5.20}$$

This is the (b+1)-step-ahead forecast error variance. In this, the effect of dead time (b) is to increase the control error variance by an amount which depends on Θ, the *moving* *average operator*, also called the *smoothing* or *time series constant*. For slow drifts, (values of Θ closer to 1), the dead time causes only a small increase in the *control error variance*, while for fast drifts, (values of Θ closer to 0), the increase is large (Baxley [1991]). For processes with fast drifts (explained in Section 6.7.2), since the effect of dead time is more pronounced, it is important to reduce dead time achieved by employing a dead-time compensator as shown in Section 4.5.

This particular characteristic regarding control error variance enables a time series controller to provide an important baseline that can be used for studying the particular class of controllers which require occasional adjustments. As mentioned earlier, the controller's objective is to minimise the mean squared deviation from target of the quality characteristic. This is accomplished by positioning the process in order to exactly compensate for the forecasted deviation from target at the time when the current adjustment will take effect, b+1 periods into the future. If this is done, then the deviation from target or control error ε_t is just the error from a forecast originating from b+1 periods in the past. As the level of the input manipulated variable at time (t), X_{t+} is placed to compensate for the forecast, the adjustment or change x_t (that is, $X_{t+}-X_{t-1}$) in the input manipulated variable is calculated to compensate for the change in the forecast from the previous sample period. This feature of a time series controller helps to know the control error variance (b+1) step-ahead of the forecast error variance of a process with b periods of dead time (time delay).

5.8 ASSUMPTIONS IN THE FORMULATION (DESIGN) OF TIME SERIES CONTROLLER PARAMETERS.

Before proceeding with further discussions, the following assumptions are made in order to simplify the formulation (design) of the tuning parameter combinations for the time series controller.

(1) There are only b full time periods of dead time (delay) and no fractional periods of delay in the system. Any fractional periods of dead time will be rounded off to the nearest integer.

(2) There is no effect of additional noise on the input manipulated variable.

(3) There are no large observational errors in the measurement of the manipulated variable and these uncorrelated errors, even if present, are assumed to be negligible compared with the errors in forecasting or prediction. The measurement errors are independent of the (conditional maximal) setting of the controller.

(4) Continuous plant process production records are available so that it is possible to obtain an approximate knowledge of the process and system behaviour under different operating conditions.

(5) The present study is not for new or start-up processes or initial pilot-run production schemes.

(6) There is no model error in the assumed process model.

5.9 TIME SERIES CONTROLLER PERFORMANCE MEASURES

5.9.1 Time Series Controller Tuning Parameter Combinations

The performance measures for a time.series controller are

(1) CESTDDVN, *control error standard deviation* σ_E (expressed as multiples of σ_a, the standard deviation of the random noise component of process disturbance $\{a_t\}$), CESTDDVN or $\sigma_E = \sigma_e/\sigma_a$, where σ_e is the forecast error standard deviation and σ_a, the standard deviation of the random shocks.

(2) The average number of sample periods between adjustments denoted by 'AI', the sampling interval.

An optimal controller is one which for any adjustment interval (AI), the value of r(=1-Θ), a measure of the rate of drift of the process and transfer function gives the lowest process variability (CESTDDVN or σ_E) (Baxley [1991]). The controller gain (CG) is set equal to 1.0 in order to reduce the size of adjustment that will exactly compensate for the forecast deviation from target (Baxley [1991]).

The parameters which determine the process simulation are

(i) b, the full number of periods of dead time

(ii) $\omega = PG(1-\delta_1-\delta_2)$, a measure of the amount of process response carrying over into additional sample periods and

(iii) r, a measure of the rate of drift of the process, called the IMA parameter (page 248, Baxley [1991]). Though, Baxley [1991] did not show the explicit relationship between r and Θ, it is shown in Section 6.7.3 that the values of the parameter r are similar to the EWMA weights used for statistical process control.

It is proposed to determine these tuning parameter combinations, CESTDDVN (σ_E) and AI, so as to eliminate over-control, characterised by more variable control errors (Baxley [1991]).

5.9.2 Need for Simulation Study of Statistical Process Control Algorithms for Drifting Processes

In practice, it has been found that for many feedback control loops, the performance improves significantly with a decrease in the sampling interval time. Moreover, when there are delays, sampling at time periods which are much shorter than the dead time, may result in little improvement in the performance of a time series controller. The sampling time should be such that there will not be too many adjustments in the process during the sampling time period. The sampling interval is related to the type of process loop to be controlled and the process parameters, namely, the time constant and dead time. Some feedback control loops respond faster to adjustments than some other feedback loops. Due to this, the samples are required to be taken at regular and faster intervals. A method to find the effects of sampling interval on controller performance is to compare the various minimum output error variances at the sampling instants for different intervals. In this method, the corresponding error variances are not compared for different sampling intervals. Simulation is a method to evaluate these variances at the intermediate times. In this context, a review of the *Simulation study of statistical process control algorithms for processes with drifts* (Baxley [1991]) is given and some of the principles of this simulation methodology are used to predict the performance of the time series controller (Chapter 6) at longer sampling interval time periods.

5.10 SIMULATION STUDY OF STATISTICAL PROCESS CONTROL ALGORITHMS FOR PROCESSES WITH DRIFTS - A REVIEW

The study has been limited to six sets of the three parameters, (i) the dead time (b), (ii) the inertia constant (δ) and (iii) the moving average parameter Θ (which is set to match the IMA disturbance) for reasons (such as cost factors and dead time) explained in the paper. *Response surface experiments* were run for the feedback control algorithms on the six sets in order to determine *optimal tuning parameter combinations*. The two *responses of interest* were the *control error sigma* (SE or σ_E) and the *adjustment frequency* (AF), the reciprocal of which is the average adjustment interval (AI). The *tuning parameters* served as experimental factors. Each experiment consisted of a simulation run of 2000 sample intervals. The *experimental design* used was *Central Composite Design* with the relative spacing of star and factorial points set to give uniform precision. Empirical models of the form given in the study (Baxley [1991]) were fitted to the data in each of the 12 experiments using least squares regression analysis. Baxley [1991], used an optimisation procedure based on the Nelder-Mead Simplex Search Algorithm (Nash [1979]) to find tuning parameter combinations giving minimum control error sigma subject to an upper constraint on the adjustment frequency and generated, by varying these constraints, a series of optimal parameter combinations covering a range of adjustment intervals from 3 to 20. Then, an additional simulation run of 10,000 sample periods was made for each optimal set of tuning parameters in order to estimate more precisely the controller performance.

Baxley [1991] made a stepwise regression analysis on the simulated data. From the analysis of variance (ANOVA) tables for (SE or σ_E) and AF, Baxley [1991] found the variability among the simulation runs to detect evidence for lack of fit. Contour plots of AI = 1/AF and SE versus, the control parameter L (the number of multiples of σ_a used for the control limits) and CG (Control Gain) were drawn. A scatterplot of SE versus AI for the extended runs at optimal settings (CG = 1.0) and 'experimental data' for other values of CG.

Baxley [1991] determined the 'form' of the model by observing that for any adjustment interval, the slope of the relationship between SE and AI varies with Θ in the same manner as the fractional increase in SE caused by one period of dead time. As a check for the model, Baxley [1991] compared the performance model predictions with the work of Box, Jenkins and MacGregor [1974]. Baxley [1991] discussed also in detail the simulation results for both the EWMA and the CUSUM controllers.

5.11 CONCLUSION

In this chapter, the characteristics, features and the criterion for a statistical control algorithm for time series controllers were presented. An approximate feedback control equation was derived and the time series controller performance measures discussed in light of the statistical control algorithm developed for this controller. The justification for considering a higher (second-order) dynamic model under critically damped conditions was also given. The time series controller performance measures were explained and also the tuning parameter combinations. A review of a simulation study of statistical process control algorithms for drifting processes was given. The (dead-time) simulation of the stochastic feedback control algorithm and EWMA process control are explained in Chapter 6 and the simulation results discussed in Chapter 7.

CHAPTER 6

SIMULATION AND PROCESS CONTROL

6.1 INTRODUCTION

The feedback control difference Equation (5.2) was derived in Chapter 5 and the expression (Equation 5.16) for the control adjustment in the input variable of a time series controller in the existence of dynamics (inertia) and dead time (delay) given. A brief review of the simulation study of statistical process control algorithms for drifting processes by Baxley [1991] was also given. The principles of simulation methodology to determine the performance measures of the time series controller, namely, control error standard deviation (CESTDDVN) and the adjustment interval (AI) and EWMA process control are discussed in this Chapter.

6.2 SIMULATION METHODOLOGY

The drifting behaviour of the process is simulated using a first order ARIMA (0,1,1) model fed by standard normal shocks N(0,1). The shocks are obtained from a random number generator with the seed based on the clock time in the computer. In this method, two standard normal random variables, Z_1 and Z_2, are plotted as a point in the plane as shown in Figure 6.1 and represented in polar coordinates as

$$Z_1 = B \cos \beta \text{ and}$$

$$Z_2 = B \sin \beta$$

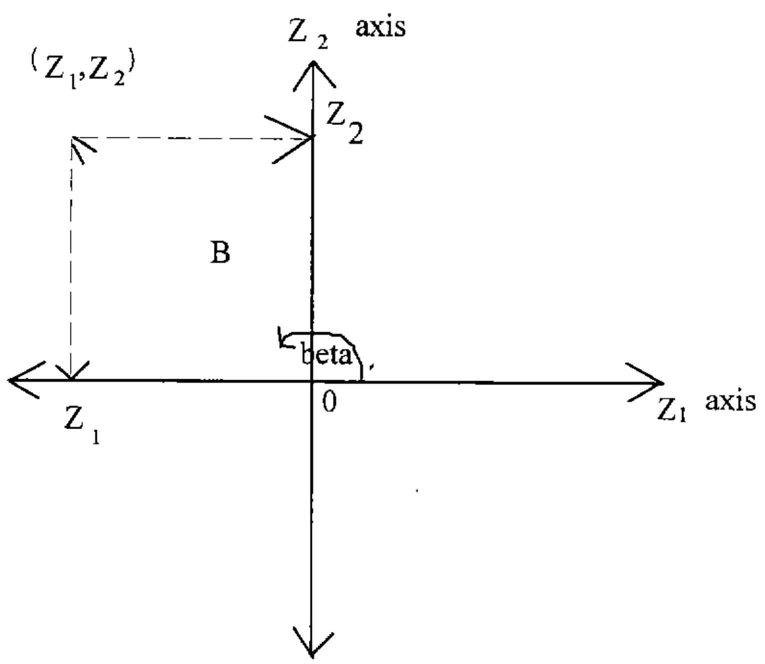


Figure 6.1 Polar representation of a pair of standard normal variables.

It is known that $B^2 = Z_1^2 + Z_2^2$ has a chi-square distribution with 2 degrees of freedom. Thus, the radius, B, can be generated by use of the equation

$$B = (-2 \ln R)^{1/2},$$

where R is a random number.

By the symmetry of the normal distribution, it seems reasonable to suppose that the angle β is uniformly distributed between 0 and 2π radians. In addition, the radius B, and the angle, β are mutually independent. Combining the above three equations gives a direct method for generating two independent standard normal variates, Z_1 and Z_2, from two independent random numbers R1 and R2.

$$Z_1 = (-2 \ln R1)^{1/2} \cos (2\pi\, R2),$$

$$Z_2 = (-2 \ln R1)^{1/2} \sin (2\pi\, R2).$$

This method is used in the Fortran simulation programme to generate the random shocks. The seed is based on the clock time in the computer to ensure complete randomisation of the simulation runs. The rate of drift, r, of the process, from target is varied by giving different values to Θ ($0 < \Theta < 1$). The property of the IMA (0,1,1) model, that the *forecasts for all future time is an exponentially weighted moving average* (EWMA) *of current and past values of the disturbance* z's (pages 106 and 145, Box and Jenkins [1970, 1976]) is made use of to predict the future GMA, the *geometric moving average*. An ARIMA (0,1,1) time series model is fitted to the variable quality data by superimposing the one-step-ahead forecasts along with the control limits. The forecast originating at any time t is a weighted average of the previous forecast (at time t-1) and the current data. Box and Jenkins (page 128, [1970, 1976]) showed that, for a lead time ℓ, these forecasts estimate the process deviation from target without bias and the forecast errors,

$$e_t(\ell) = z_{t+\ell} - \hat{z}_{t+\ell}$$

have a lower variance than those for any other statistic calculated from historical data. The forecasts also help determine the appropriate adjustment for returning the process to target. By making an adjustment at every sample point which exactly compensates for the forecasted disturbance, the variance of the output controlled variable can be minimised. The time series controller feedback control algorithm fits into this criterion since it requires an adjustment for every sample and gives the minimum control error variance as long as the dynamic model describing the process and the stochastic model describing the disturbance are correct.

At first, the general control charting procedure is briefly explained which provides the means of plotting the EWMA forecasts in a geometric moving average (gma) or EWMA chart. The forecasts help in monitoring and regulating the process by comparing the quality deviations from target, this is discussed in Section 6.5 as also is the test of hypothesis aimed at ensuring that the mean is on target.

6.3 CONTROL CHARTS AND STATISTICAL CONTROL

6.3.1 State of Statistical Control

The terms state of control and stable process were discussed in chapter 2. Statistical control in relation to process control charts is explained in detail in this Section.

A process is said to be in a state of statistical control when assignable or special causes of variation have been detected, identified and eliminated. It is of subsequent interest to determine any change in the process, either in the mean level or in the variation about the mean. Statistical control relates to the distribution of the observations of the process and also to future observations which are expected to possess the same statistical properties as past observations. When the process has been operating with random fluctuations about a fixed mean, in a state of statistical control, the general aim is to keep the process at this target level or to improve it by reducing the overall variation about the required target. Control charts are used to detect changes from a target level or changes in the variability of the process.

6.3.2 X Bar Control Chart and the Control Charting Procedure

Control charts are tools for process monitoring in industrial applications. One of the standard tools in the process control environment is the Shewhart $\bar{X}$ chart. The $\bar{X}$ chart uses the data to detect general or abrupt shifts in the mean level of the process. In the $\bar{X}$ chart, small sample means of successive observations of the process quality data, (that are assumed to be serially independent and approximately normally distributed), are plotted in time order. Chart construction generally presumes that the same number of samples will be taken at each sampling in order to evaluate the mean and to assess the state of the process. Previous process data are used to estimate the variance from which the control chart limits (lines) are drawn $\pm 3\sigma$ (standard deviation) about the target T. The performance characteristics of a product are the primary quality characteristics that determine the product's performance in satisfying the customer's requirements. The variation of a performance characteristic about its target value is referred to as 'performance variation'. The smaller the performance variation generally, the better is the quality of the product. The central (target) line is a measure of the general level of the process. The control limits help decide whether the process is operating in a state of statistical control or not.

A statistically controlled process will oscillate fairly evenly about the mean with a concentration of points in a one standard deviation band either side of the mean. So, the process should be left alone unless a disturbance arising from a special cause is detected by a hypothesis-testing type of procedure, making it also necessary to specify the probability level to be used for such hypothesis testing. The response is then to look for an assignable cause and to correct the process back to target by removing this. The modus operandi to be followed if the assumption is made that the true process level is not a constant and the common-cause variation and the process state of statistical control follow only a stable or stationary model, is discussed in Chapter 8.

Consider a situation where the data is not normally distributed. In practice, there are situations, in which non-normality is a characteristic of the observed process data. $\bar{X}$ charts are generally robust with respect to deviation from normality, although not with respect to departures from the independent or uncorrelated data assumption. Hoerl and Palm (Hoerl and Palm [1992]) contended that all processes are autocorrelated, depending on the sampling scheme used.

Lack of independence between successive observations in the form of autocorrelation is an intrinsic part of the data structure in the continuous process industries. The data is often similar to that arising from a first or second order autoregressive process. The availability of autocorrelated data helps in predicting the future observations accurately. If the autocorrelation estimates r_k for lags $k > 0$ are not significantly different from 0 or are small, (and damp out rapidly), then, the true level of the process variable at any time t is assumed to be a constant.

Process data taken sequentially in time are likely to be serially correlated, (that is, the data are not random; high values tend to follow high values, and low values follow low values (Berthouex and Hunter [1983]). Generally, the effect of serial correlation is to cause many false 'out-of-control signals', (explained shortly).

To cope with serial correlation, the original process data may be modelled by an appropriate stochastic model. The residuals, (the difference between the predicted value and the actual value of the next observation), from such a model are then uncorrelated. The control charting methods of SPC can then be applied to the residuals (Montgomery and Friedman, Keats and Hubele [1989]).

The Shewhart chart highlights special non-random causes affecting the process mean by establishing limits on the sample averages. These causes may be identified by the subgroup means falling outside the control limits or by runs (explained subsequently) tests, (to detect early small shifts in the mean), applied to sequences of plotted points. The control limits are defined by a probabilistic statement of the form

$$UCL = K_{\alpha}\sigma \text{ and } LCL = K_{1-\alpha}\,\sigma$$

where K_α and $K_{1-\alpha}$ are the α and $1-\alpha$ points of the distribution of the mean $\bar{x}$. This same principle is used to plot the EWMA forecasts from the data against two parallel action lines in a geometric moving average control chart. In a feedback control scheme, employing such a principle, the position of these limit lines is determined on (i) the relative costs of adjustment and of being off-target, and (ii) by the degree of non-stationarity of the process and not by questions of statistical significance. The relative value of these costs is an important factor in deciding the choice of a feedback control scheme, which is discussed in Chapter 8.

6.3.3 The EWMA Chart

In the simulation study, the geometric moving average Θ is used for monitoring by the EWMA chart and an appropriate alarm criteria based on the GMA (EWMA) statistic for sounding the 'out-of-control alarm signal'. The EWMA control limits give an indication of how the forecast is significantly different from the target. When an EWMA signal is obtained, appropriate corrective control action based on the forecast is devised. This is explained in detail in Section 6.5. An alarm signal that indicates that the process may be out of control is the appearance of a single last plotted point falling beyond the 3σ control limits. One of the purposes of the process control chart is to monitor a stable operation and to reveal special or assignable causes. It will then be sensible to react to process changes only when some monitoring criteria is established as statistically significant. In practice, the alarm control signal that the process needs immediate attention is aided by the use of 'runs' (short sequences of observations).

There are two possibilities for dealing with data that are serially correlated; one, using original observations and suitably modifying the control limits and rules to account for the correct process variance and another possibility is to model the observations as a time series and plot the resulting residuals on a control chart. If the observations have positive serial correlation, the variance of the process will be underestimated. If the observations are negatively correlated, the resulting estimate of the variation will be overestimated. It can be shown that for data from a first-order autoregressive process, a negative lag-one correlation will decrease the variance of the time series and also the average run length (explained in Chapter 8). A positive lag-one correlation, while reducing the ARL as compared to a Normal process, would increase the estimated variance over the true value. If this is ignored, such an action would result in a control chart that has limits set too far from the mean and may fail to indicate problems when they do truly exist. To analyse such a situation is to 'correct' the estimated variance to account for autocorrelation and to use the correct value to compute the modified control limits (Berthouex [1989]). This is discussed in Section 8.4.

The test of hypothesis is discussed in the next Section before reverting to EWMA forecasting in Section 6.5.

6.4 THE NULL HYPOTHESIS

The IMA property of the EWMA forecast is to (i) compare the quality deviations from target, that is, the mean of the quality data GMA with the control limits (LCL and UCL) and (ii) to adjust only when the GMA is beyond its limits. These limits are set at

$$LCL = -L\,\sigma_{\hat{z}} \tag{6.1}$$

and

$$UCL = L\,\sigma_{\hat{z}} \tag{6.2}$$

where $\sigma_{\hat{z}}$ is the standard deviation of an estimate of the expected value of the disturbance z for any future time, conditional upon the realisations of z up to time t.

The null hypothesis, denoted by H_0 is that the (process) mean of the quality variable is on target. The EWMA forecasts estimate the process deviation from target and help determine the approximate adjustment for returning the process to target. The test statistic, (which is a function of the observed random sample), is calculated from the quality deviations from target and compared with the control limits for that statistic. Under the null hypothesis of drifting behaviour and special causes, there is a probability of the test statistic falling outside the critical region of the control limits. The rejection of H_0 would mean that there is evidence to suggest that the product mean is not on target and lead to the acceptance of the *alternative hypothesis,* denoted by H_1(mean not on target). By not taking action when the process is out of control, leads to *Type II error* by judging the process to be in control when it is actually not.

The risk of committing a type II error is minimised in the following manner. When a limit violation occurs, it is assumed that there is a need to correct a special cause which is present in the process. The adjustment is then calculated to compensate for the change in the forecast from the previous sample period. This approach seems to be better than the hypothesis-testing Shewhart approach which is aimed at minimising the risk of taking action when the process is in control, (explained below) (Baxley [1991]). Since the test statistic falls inside the critical region, this hypothesis-testing approach does not aid in providing an estimate of the new process level to calculate the required adjustment.

For a time series controller, a sample is taken and corrective action is initiated at once, following an out-of-control alarm signal for level shifts in the mean of the quality data away from the target, in such a manner that returns the quality index back to target. This is achieved for values of Θ closer to 0 and 1.0 for drifting processes and the EWMA control limits L fixed by Equations (6.1) and (6.2) for controlling the average number of sample periods. The term control is used here in the sense of the stationary or iid variation about a target value (page 266, Box and Kramer [1992]) and embraces Shewhart's [1931] definition of control to predict within limits, the future behaviour of a process with the possibility of bringing the process into a state of statistical control by *adjustment* (page 257, Box and Kramer [1992]). The adjustment interval (AI), the control error standard deviation (CESTDDVN) σ_E (control error sigma) are obtained from simulation results (see Attachment 6.I for the Fortran F77L computer programme). These are important to determine the time series controller performance measures. The programme simulates a time series controller with a second-order plus dead time dynamic model and ARIMA (0,1,1) disturbance.

6.5 EWMA FORECASTING AND FEEDBACK CONTROL

The Equation (5.9) for the non-stationary disturbance represented by the ARIMA (0,1,1) model is rewritten to give the following recursive formula for updating a forecast of the process level

$$\hat{z}_t(\ell) = \Theta\, \hat{z}_{t-1}(\ell) + (1-\Theta)\, z_t . \qquad (6.3)$$

The current forecast $\hat{z}_t(\ell)$ of Equation (6.3) for lead time ℓ is re-expressed by making successive substitutions for the previous forecasts to obtain

$$\hat{z}_t(\ell) = (1-\Theta)(z_t + \Theta\, z_{t-1} + \Theta^2 + \ldots). \qquad (6.4)$$

The EWMA forecasts are a weighted average of the current and historical data, where the weights are decreasing exponentially for the data further back in time and the weights are, $(1 - \Theta)$,$(1 - \Theta)\Theta$, $(1 - \Theta)\Theta^2$ --- etc. which sum to unity since $0 < \Theta < 1$, and because of the fact that

$$\sum_0^{\infty} x^i = 1/(1-x),\ 0 < x < 1.$$

These weights are relatively heavy on recent data for fast drifts (explained in Section 6.7.2) but spread back in time for slow drifts as per Figure 6.2 (reproduced from Figure 4, page 256, Baxley [1991]) (Attachment 6.II). The control limits are set at multiples of (usually 3 times) the standard deviation of $\hat{Z}_{t+1}(\ell)$.

It is known that

$$\sigma_{\hat{z}} = ([1-\Theta]^{2t}[(1-\Theta)/(1+\Theta)])^{1/2}\sigma_a \qquad \text{(page 262, Baxley [1991])}$$

asymptotically converges to

$$[(1-\Theta)/(1+\Theta)]^{1/2}\sigma_a$$

after a small number of sample periods, t, since the last out-of-control signal (page 262, Baxley [1991]). At this stage, it is assumed that the value of $\sigma_{\hat{z}}$ in the situation when there are no drifts will be the same when there are process drifts.

The control parameter L denotes the number of multiples of σ_a (notation SA in the computer simulation) used for the control limits. The upper control limit and the lower control limits are set at the values given by Equations (6.1) and (6.2).

The one step ahead forecasts are plotted about a target value T and refer to the control lines drawn at a distance of $\pm L\sigma_{\hat{z}}$, that is, $\pm L\sqrt{(1-\Theta)/(1+\Theta)}\,\sigma_a$ above and below the target. Such a plotting procedure provides timely warning of a deviation from target and of the possible need for corrective feedback control action and may also provide clues as to possible assignable causes of variation which may subsequently be eliminated or compensated for in order to improve the process. As long as the predicted forecast falls within these control limits (and hence is considered close to the target), no change is made in the process. Appropriate adjustment is made when a forecast crosses the control limits. This is similar to keeping a Shewhart chart on the predicted deviation from target one step ahead when the series of uncorrelated random shocks is a sequence of highly dependent random deviates about a fixed mean and has a tendency to drift. These control limits are related to the cost of making a change relative to that of being off-target and to the parameters of the non-stationary stochastic model. The minimum mean square error forecast of the non-stationary stochastic model is the geometrically (exponentially) weighted moving average of the previous observations. The geometric moving average is the IMA parameter, Θ, of the ARIMA (0,1,1) process. For a dynamic system, where a cost is associated with making a change, we obtain the standard Shewhart chart with different control limits which are not related to any tests of significance and probabilities of being outside control limits. This control action is the discrete (analogue of) integral control action which is accumulating the deviations from target when action is being taken at every sampling or adjustment interval. This action is equivalent to taking an exponentially weighted moving average of past disturbances Z_t, Z_{t-1}, Z_{t-2}--- (Box, Jenkins and MacGregor [1974]).

In the computer simulation, the quality deviations from target, that is, the mean of the quality data gma (notation used in the computer simulation) are compared with the control limits, UCL and LCL. Then, the required adjustment (dxt) in the input manipulated variable X is calculated by means of the feedback control algorithm and the necessary required corrective action is applied to compensate for the change in the forecast from the previous sample period to return the quality index to target. From the simulation results, it is possible to know (i) *when* to make an adjustment to the process and (ii) by *how much* to change the input variable so that the output quality variable is at or near target.

The output results of the simulation programme given in Tables 6.I and 6.II, titled 'Time Series Controller Performance' (Attachments 6.III and 6.IV), show the standard deviation of the control errors (CESTDDVN) and an average (MFREQ) for an indicator variable (FREQ), which takes the value 1 for sample periods with an adjustment and zero otherwise. The results are shown for values of Θ of 0.05, 0.25 (fast drifts) and 0.70, 0.75 and 0.95 (slow drifts) only. The tables can be used as ready reckoner which give an idea of CESTDDVN for (some) known dynamics of a process control system and a particular (regularly) occurring disturbance.

The performance of the time series controller measured by the control error standard deviation (CESTDDVN) and the average adjustment interval, the mean AI being equal to 1/MFREQ, (the mean adjustment frequency AF), are obtained from the simulation results of the feedback control algorithm. Control error standard deviation (CESTDDVN) = SE/SA (standard deviation of the forecast (control) error/standard deviation of the random shocks σ_e/σ_a).

There is no prior work that needs to be done which estimates the control error sigma CESTDDVN) or the adjustment interval (AI) when the disturbances follow an ARIMA (0,1,1) process, (Baxley [1991]). Graphical plots (Figures 7.3 and 7.4) (Attachments 7.VIII and 7.IX to Chapter 7), of the values of CESTDDVN versus AI and Θ show the variation in CESTDDVN due to AI and the IMA parameter Θ.

6.6 A NOTE ON CONTROLLER GAIN (CG)

The reasons for setting the controller gain CG to 1.0, is discussed in brief in this Section. The maximum value of the controller gain for stable operation of a (pure) delay process is one (Chandra Prasad and Krishnaswamy [1975]). At this stage, it is intuitively assumed that the tuning parameter for the time series controller is Θ only, which depends on the rate of drift (r) of the process, (r being equal to 1-Θ). For making such an assumption, inferences are drawn from Baxley's [1991] simulation study on the behaviour of the EWMA and the CUSUM controllers for drifting processes. Baxley [1991], from the contour plots of AI(=1/AF), and SE versus CG for the EWMA controller, showed that for the AI contours, the control error sigma (SE) is lowest when the controller gain is about 1.0. Baxley [1991] used the optimisation procedure based on the Nelder-Mead Search algorithm (Nash [1979]) to find tuning parameter combinations that gave a minimum SE subject to a constraint on the adjustment frequency. He found from the sample results of these optimisations along with the results of additional simulation runs of 10,000 sample periods that the optimum controller gain is near 1.0. The adjustment control action exactly compensating for the forecasted deviation had a strong appeal to Baxley [1991] to set CG to 1.0 for the zero dead time case. By drawing a scatter plot of SE versus AI for these runs at optimal settings (CG = 1.0), Baxley [1991] showed that the optimal controllers lie along the lower edge of the scatter plot. For these reasons, it is assumed that this property of optimal controllers will also hold good for a time series controller, being a MMSE controller. It is thus sufficient to consider Θ only as the tuning parameter for other cases of dead time (b = 1, 2). This is also in view of the use of the EWMA forecasts which exhibit the IMA property.

6.7 EXPERIMENTAL STRATEGY FOR SIMULATION

6.7.1 Review of Kramer and Baxley's Results

The steps suggested by Baxley [1991] are followed for the experimental strategy for conducting the simulation study. Without loss of generality, the first period response to a unit step change (l) and the standard deviation of the random shocks (σ_a) are set equal to unity. The emphasis of this simulation study is on the control of quality indices which are evaluated in the laboratory from the production samples. A sample interval which is large in relation to the dynamic response $PG(1-\delta_1-\delta_2)/(1-\delta_1 B-\delta_2 B^2)$ of the process to an adjustment, contributes to the costs of sampling and measurement. There are also situations that result in delays when performing the laboratory measurement or in which the effect of the control adjustment takes time in reaching the sample point. With these facts in mind, the respective observations made by Kramer [1990] and Baxley [1991] in regard to the control of dynamic processes (in which there is no dead time), and processes in which only dead time is present (and there are no dynamics) are reviewed briefly.

Kramer [1990] studied the effects of dynamics (inertia), δ on the adjustment variance for a first order dynamic system in which the adjustments to the input variable are made after the process is observed and so their effects are seen at the next observation (meaning that the dead time is zero). Baxley [1991] in his simulation study, gave some values of Θ and SE and $\delta = 0$ for the time series controller performance with one dead time period and $\sigma_a = 1.0$. The aim is to show that even with the combined effects of both the dynamics (inertia) and the dead time, the time series controller still gives MMSE control with (minimum) adjustments for slight increases in control error sigma (CESTDDVN).

Baxley [1991] observed that for a time series controller, the presence of dead time means that the process disturbances must be forecasted farther ahead in order to determine the best adjustment which results in a larger forecast error sigma and hence a larger control error sigma. The penalty for having one period of dead time, for example, in the feedback loop is seen to be more severe for processes with fast drifts, that is, for processes for which the values of Θ tend to zero. Baxley [1991] observed also that for any adjustment interval, the slope of the relationship between SE and AI varies with Θ in the same manner as the fractional increase in SE caused by one period of dead time. Both increase as the rate of drift of the process, r, increases, that is, as Θ tends to zero. Since the joint effects of both the dynamics, and the dead time, b, on the control error standard deviation (CESTDDVN) for the time series controller are required, the simulation study is conducted for values of Θ ranging from 0 to 1 and these values of Θ are repeated for b = 2.0. The reasons for limiting b to the values of 1 and 2 are given in Chapter 7.

An advantage of conducting the simulation study is that the values of the output variances (varCE) and control adjustment action variances (SDdxt) are directly obtained from the simulation results. It obviates the need for developing complex expressions as shown by Kramer [1990] for (constrained) variance control schemes.

Kramer [1990] and Baxley [1991] used the terms 'monitoring interval' and 'adjustment interval' in reference to their respective process regulation schemes and simulation studies and so had specific intent. Kramer [1990] defined the *monitoring* ('sampling') *interval* as the multiple of the initial short *base* (unit) interval at which the process is experimentally monitored. Baxley [1991] defined the average number of sample periods (intervals) between adjustments as adjustment interval when the EWMA statistic violated the control limits for that statistic requiring an adjustment to be made to the process. So, as per Baxley's [1991] definition, an adjustment interval is made up of a number of sample intervals (periods). This means that the process is under constant surveillance, the gma and the value of the adjustment calculated at each instant and necessary adjustment made, if required, to the process when the gma crosses the control limits. This is an advantage for monitoring the process closely but may become costly if the adjustments cannot be automated thus increasing the total cost of process regulation. Another advantage of using the concept of the adjustment interval is that for the time series controller, the sampling and control adjustment actions need not be done separately or independent of each other, the adjustment being made as soon as an out-of-control signal appears on the control chart, the required adjustment being known from computing the feedback control algorithm. The use of AI presents an opportunity to sample the process at every instant of time, (at each iteration of the simulation run), if a knowledge of the state of the process at every instant of time is required; but this may not be necessary as the process may require to be sampled only when the process is out-of-control in order to know the exact reason for such an out-of-control signal. This may help in reducing the sampling cost and eventually the overall cost of regulation of the process control scheme. While Kramer [1990] explained the effects of altering the monitoring (sampling) interval on δ, Baxley [1991] showed the effect of Θ on the adjustment interval. Baxley's [1991] notation and meaning of AI are followed in order to discuss the simulation results. The value of AI (the reciprocal of the mean frequency MFREQ in the simulation programme) corresponds to the total number of sample periods (intervals) with an adjustment.

6.7.2 Fast and Slow Drifts

It is known that ARIMA models can be used to characterise and forecast the drifting behaviour of process disturbances and to describe the dynamic relationship between the output controlled variable and the input manipulated variable. A process with fast drifts is one where the true process level can move rapidly over the range equal to the magnitude of the variance of the random shocks, $N(0, \sigma_a^2)$ because of a relatively large variance of the disturbance (Z_t). In such processes with fast drifts, the factors/causes driving the process away from target are large relative to the sampling and measurement errors (Baxley [1991]). One of the assumptions mentioned in Section 5.8 is that there are no large errors in measurement and in sampling errors that exist are negligible when compared with the forecasting errors. The IMA parameter Θ with values from zero to 1, approaches zero for such processes. As Θ approaches zero, the process becomes less and less stable and closer and closer to a random walk. The random walk is an IMA (0,1,1) process with $\Theta = 0$. On the other hand, processes with slow drifts, that is, those processes with values of Θ approaching 1, have a small variance of the shocks (and larger sampling and measurement errors), which can be modelled as IMA processes with Θ approaching 1 (Baxley [1991]). As Θ approaches unity, the time series model representing the drifting disturbance (Equation 5.9), behaves more and more like the stationary model. When $\Theta = 1$, (Equation 5.9) becomes the stationary model where the errors are iid about a fixed mean. The rate of drift of the process, r, away from the target for drifting processes is determined by the IMA parameter Θ. Baxley [1991] described the rate of drift for the IMA (0,1,1) disturbance model with the values of Θ ranging from those approaching 0 and up to 0.25 as fast drifts and for values of Θ above 0.75 and approaching 1.0 as slow drifts (See Figure 6.2, reproduced from Figure 4, Page 256, Baxley [1991]) (Attachment 6.II). The same principle is followed in our discussions. r is taken as the forecast weight (explained in Section 6.5) of the gma plot in the simulation. The justification for use of these weights in the simulation study are discussed by reviewing some of the earlier results available in the literature in connection with EWMA control charting procedures.

6.7.3 Identification of the IMA Parameter, r =(1 - Θ), as EWMA Forecast Weight

It was mentioned in Section 6.5, that the one-step ahead EWMA forecast weights are $(1-\Theta), \Theta(1-\Theta), \Theta(1-\Theta)^2, \ldots$ etc which sum to unity. Baxley [1991] showed that these weights are relatively heavy on recent data for fast drifts but spread further back in time for slow drifts. These weights are identical to the rate of drift of the process, r, which can be used to calculate the gma plots. In this context, it may be appropriate to mention the following results available in relation to the EWMA forecasts.

(i) The EWMA weighting constant determines the memory of the EWMA (gma) statistic. That is, the EWMA constant determines the rate of decay of the weights. When the process is under control, points plotted on the EWMA chart are equal in their capability of detecting signals of departures from assumptions, (that the process is under control), to points plotted on the Shewhart chart. Hunter [1986] suggested values of 0.2 ± 0.1 for the weights based on similar experiences with econometric data.

(ii) Roberts [1959] and Crowder [1987] assumed that there were no drifts present in the system and recommended the choice of Θ (=1-r) (and also the control limits L) for controlling the average number of sample periods until an out-of-control signal for specified level shifts in the mean of quality data away from target. Crowder [1987] showed that setting $\Theta = 0.95$ (and L = 2.5) resulted in an in-control average run length of 379 and an ARL of 10.8 following a level shift of $\pm 1\ \sigma_a$ (Baxley [1991]).

(iii) Berthouex [1989] suggested that the values of the weight (in the EWMA calculation) in the range of 0.1 to 0.3 are useful in effectively smoothing the time series while being responsive to the change.

(iv) Montgomery and Friedman (Keats and Hubele [1989]) recommended small values of r (which they called a 'discount factor'), say, $0.05 \le r \le 0.20$) to plot the geometric moving average and the width of the control limits (L) in chart-sigma units as either 2.5 or 3.0.

(v) Harris and Ross [1991] quoted, (from the Tables for the EWMA given in Lucas and Saccucci [1990]), that the value of the weight (which they called the 'smoothing parameter'), required to implement the EWMA equal to 0.18, and the other parameter L, (which they called the critical value for EWMA), equal to 3, closely match the average run lengths for the cumulative sum procedures used to calculate the ARLs via Markov chain considerations or via a Monte Carlo simulation.

(vi) Baxley [1991] gave a graphical representation of forecasting an IMA process and showed that the forecast weights, (the rate of drift, r), increases for slow drifts and decreases for fast drifts.

In view of the above with regard to the choice of the value for r, the simulation run of 2000 sample periods (intervals) was conducted for:-

(i) values of Θ for 0.25, 0.5, and 0.75, $\delta = 0$ and values of b = 1.0 and shown in Table 5.1 confirms the agreement of our results with that of Baxley [1991].

(ii) fast drifts (for values of Θ = 0.05, 0.1, 0.15, 0.20 and 0.25), for moderate drifts (for values of Θ = 0.50, 0.55, 0.60, 0.65, 0.70) and for slow drifts (for values of Θ =

0.75, 0.80, 0.85, 0.90, 0.95) for 10,000 sample periods (intervals) for values of dead time b = 1.0 and b = 2.0 in order to show that the time series controller still gives minimum variance control under drifting process conditions with dead time.

6.7.4 Experimental Strategy - Continuation

The standard deviation of random shocks (σ_a) is set to be unity. Vander Wiel and Vardeman [1992] opined that a process disturbance, if different from an IMA, will result in poor performance of a feedback control algorithm formulated for an IMA disturbance. It is likely that its performance may be worse with no feedback control if the disturbance is really a moving average. The feedback control based on the IMA disturbance model works whenever the variance of the error of the EWMA forecast of the disturbance is (substantially) lower than the variance of the original disturbance (Box and Kramer [1992]). So, the IMA parameter Θ is set to match the disturbance.

Baxley [1991] used experimental design to find the control limits L and the controller gain (CG) in his simulation studies. It is not required to run 'response surface experiments' on the sets of values of Θ, δ and b for the time series controller to determine its optimal tuning parameter combinations. This is due to the facts that (i) the controller gain (CG) is set to be 1.0, and (ii) the control limits given by Equations (6.1) and (6.2) depend only in turn on Θ. Yet, the control error sigma (CESTDDVN) and the adjustment frequency (FREQ in the computer simulation) are maintained as the responses of interest since, as noted earlier, in Section 5.9, CESTDDVN and AI, the average number of sample periods (intervals) between adjustments, are the performance measures for the time series controller. It was mentioned in Section 6.7.1 that the reciprocal of the average adjustment frequency (MFREQ in the computer simulation programme) is the adjustment interval (AI). A controller, to be optimal, should give the lowest process variability (CESTDDVN) for (i) an average adjustment interval, (ii) the value of r, the rate of the process drift and (iii) the process dynamic model (transfer function). Instead of Baxley's [1991] analytical approach of using the Nelder-Mead Simplex Search Algorithm (Nash [1979]) to find tuning parameter combinations that give minimal control error sigma, the observation of Box and Jenkins ([1970, 1976]), made in Section 5.6, that the values of the CESTDDVN's, given by the simulation runs are the minimum variance of the output variable since the feedback control adjustment action x_t (dxt in the computer simulation) given by Equation (5.16) exactly compensates for the forecasted disturbance, is recalled. The minimal CESTDDVN and the AI are found directly from simulation results. Since these minimum variance controllers are derived via simulation, it is possible to find adjustment intervals to minimise the sum of the adjustment, (which includes the sampling cost) and off-target costs (explained in Chapter 8 along with an outline of description of the regulation procedure).

6.8 CONCLUSION

The simulation methodology has been explained in this Chapter as also has the control charting procedure and EWMA forecasting procedure. The simulation strategy and the drifts (fast and slow drifts) have also been discussed with a review of Baxley's and Kramer's results. The results of the simulation are discussed in Chapter 7.

```
C                                                    Attachment 6.I
C              FORTRAN F77L COMPUTER PROGRAMME
C #include <f77_floatingpoint.h>

C   FORTRAN PROGRAMME FOR SIMULATION OF A TIME SERIES
C   CONTROLLER WITH A SECOND-ORDER PLUS DEAD TIME DYNAMIC
C   MODEL (TRANSFER FUNCTION) AND ARIMA (0,1,1) DISTURBANCE
C   THE GMA THETA IS CONSTRAINED TO BE THE SAME AS THETA FOR
C    THE DISTURBANCE
C
C   NOTATION
C
C   R1     -  INDEPENDENT RANDOM NUMBER
C
C   R2     -  ANOTHER INDEPENDENT RANDOM NUMBER
C
C   rr     - SEED BASED ON THE CLOCK TIME IN COMPUTER
C
C   rrand -  RANDOM NUMBER GENERATOR
C
C   Z1     -  INDEPENDENT STANDARD NORMAL RANDOM VARIABLE
C
C   Z2     - ANOTHER INDEPENDENT STANDARD NORMAL VARIABLE
C
C   j      - ITERATION VARIABLE
C
C SPROC -  STANDARD DEVIATION SET EQUAL TO 1.0
C
C  ATAN -  ARCTAN
C
C   iter   - NUMBER OF COMPUTER ITERATIONS
C
C   iteron2 - iter/2
C
C  x       - VALUE OF INPUT MANIPULATIVE VARIABLE AT TIME t
C
C  xtm1  -  VALUE OF x AT TIME t-1
C
C  dxt    - CONTROL ADJUSTMENT' REQUIRED IN THE INPUT
C            MANIPULATED VARIABLE AT TIME t
C
C  dxtmb - VALUE OF CONTROL ADJUSTMENT AT TIME t WITH DEAD TIME
C            b
C
C  Mdxt  - MEAN OF dxt
C
C  SDdxt - STANDARD DEVIATION OF dxt
C
```

```
C  b       - THE FULL (WHOLE) NUMBER OF PERIODS OF DEAD TIME
C
C  PG      - PROCESS GAIN FOR THE CRITICALLY DAMPED CONDITION,
C            THE EVENTUAL EFFECT OF A UNIT CHANGE IN THE INPUT
C            VARIABLE AFTER THE COMPLETION OF THE DYNAMIC
C            RESPONSE
C
C  delta1  -INERTIA CONSTANT GE -2 LT 2
C
C  delta2  -INERTIA CONSTANT GE -1 LT 1
C
C  w       -PG*(1-delta1-delta2), MAGNITUDE OF THE DYNAMIC RESPONSE
C            TO A UNIT CHANGE IN THE FIRST PERIOD FOLLOWING DEAD
C            TIME SET EQUAL TO 1.0
C
C  w/(1-delta1*B-delta2*B*B)
C          -DYNAMIC RESPONSE OF THE PROCESS
C
C  CG      -CONTROLLER GAIN SET EQUAL TO 1.0
C
C  r       - = (1-theta), A MEASURE OF THE RATE OF DRIFT OF THE
C              PROCESS
C
C  theta   - INTEGRATED MOVING AVERAGE(IMA)PARAMETER 'theta'
C
C  e       - FORECAST ERROR
C
C  ME      - MEAN OF FORECAST ERROR
C
C  SE      - STANDARD DEVIATION OF FORECAST ERROR
C
C  A       - RANDOM SHOCK NID(0, SA)
C
C  atm1    - VALUE OF RANDOM SHOCK {a} AT TIME t-1
C
C  AVMA- MEAN OF {a}
C
C  SA      - STANDARD DEVIATION OF RANDOM SHOCK
C
C  CESTDDVN
C          - CONTROL ERROR STANDARD DEVIATION (EQUALS SE
C            OVER SA)
C
C  varCE - CONTROL ERROR VARIANCE
C
C  vardxt - VARIANCE OF THE CONTROL ADJUSTMENT (SDdxt**2)
C
C  gma     - GEOMETRIC MOVING AVERAGE theta AT TIME t
```

```
C
C   gmatm1
C          - gma at time t-1
C
C   z      - DISTURBANCE AT TIME t
C
C   ztm1  - VALUE OF DISTURBANCE AT TIME t-1
C
C   y      - VALUE OF OUTPUT OR CONTROLLED VARIABLE AT TIME t
C
C   ytm1  - VALUE OF y AT TIME t-1
C
C   theta1 - IMA PARAMETER 'theta' AT TIME t-1
C
C   FREQ - ADJUSTMENT FREQUENCY
C
C   MFREQ
C          - MEAN OF FREQ
C
C   AL     - CONTROL PARAMETER = 3*SA

    integer accrued, ieeer, under,inv,inx,over,under
    character*16 out

              DIMENSION A(-10001:10001),e(-10001:10001),
   &          dxt(-10001:10001),
   &          x(-10001:10001),y(-10001:10001),
   &          z(-10001:10001),FREQ(10000),gma(-10001:10001),
   &          atm1(-10001:10001),ztm1(-10001:10001),
   &          gmatm1(-10001:10001),xtm3(-10001:10001),
   &          xtm2(-10001:10001),
   &          xtm1(-10001:10001),
   &          ytm1(-10001:10001),ytm2(-10001:10001),
   &          etm1(-10001:10001),
   &          etm2(-10001:10001),dxtmb(-10001:10001)

              INTEGER j,iter,iteron2,b,recno, currec

    REAL      MFREQ,Mdxt,ME,SDE,SE,SDdxt,SDFREQ,FREQ,e,r,AVMA,SA,q,
   &          PG,w,ATAN,rr,vardxt,varCE,CESTDDVN

    DATA      SPROC/1/,CG/1/,
   &          atm1(0)/0/,ztm1(0)/0/,xtm1(0)/0/,
   &          x(0)/0/,ytm1(0)/0/,etm1(0)/0/,etm2(0)/0/,
   &          y(0)/0/,e(0)/0/,gma(0)/0/,z(0)/0/,gmatm1(0)/0/
   &          dxt(0)/0/,ytm2(0)/0/,xtm2(0)/0/,xtm3(0)/0/,dxtmb(0)/0/

    write(*,'(" this is unit 66")')
```

```
      open(1,file='file11.txt',access='sequential',status='unknown',
     &form='formatted')
      open(2,file='file22.txt',access='sequential',status='unknown',
     &form='formatted')
      open(3,file='file33.txt',access='sequential',status='unknown',
     &form='formatted')
      open(4,file='file44.txt',access='sequential',status='unknown',
     &form='formatted')
      open(5,file='file55.txt',access='sequential',status='unknown',
     &form='formatted')
      open(6,file='file66.txt',access='sequential',status='unknown',
     &form='formatted')
      open(7,file='file77.txt',access='sequential',status='unknown',
     &form='formatted')
      open(8,file='file88.txt',access='sequential',status='unknown',
     &form='formatted')

c     Number of iterations
      iter=10000
c     program to find the values of delta1 and delta2
c LK
      recno=0
      do 22 delta2= -0.99,1.01,0.01
      do 21 delta1= -1.99,2.01,0.01
      C = delta1**2+4*delta2
      if ((delta1+delta2).lt. 1.0.and.(delta2-delta1).lt. 1.0) then
      if (C .gt. -0.01 .and. C .lt. 0.01)then
      write(1,10) delta1,delta2
      recno=recno+1
      PG=1/(1-delta1-delta2)
      if (PG  .gt. 1.0 ) go to 22
      endif
      endif
10    FORMAT(2F30.15)
21    continue
22    continue
      close(1)
      open(1,file='file11.txt',access='sequential',status='old',
     &        form='formatted' )

c     Limit of steady-state gain for closed-loop stability, that is, g=1.0
c     PG=g, for a critically damped second-order system
c     g = w/(1-delta1-delta2)
      w = 1

      do 20 theta=0.05,1.0,0.05
      do 25 currec= 1, recno
```

```
      read(1,fmt=10,iostat=n) delta1,delta2
      if (n .lt. 0) go to 13
      PG=1/(1-delta1-delta2)
      if (PG .gt. 1.0) go to 25
      if (theta .gt. 0.25 .and. theta .lt. 0.50) go to 20

c     set value of q
      r = 1 - theta
      q = sqrt(r/(1+theta))

c     Initialise arrays
      Do 40 j=1,iter,1
      atm1(j)=0
      ztm1(j)=0
      gmatm1(j)=0
      xtm3(j)=0
      xtm2(j)=0
      xtm1(j)=0
      ytm1(j)=0
      ytm2(j)=0
      etm1(j)=0
      etm2(j)=0
      dxt(j) =0
      dxtmb(j)=0
40      continue

C     generate random normal numbers
      iteron2 =iter/2
      rr =rand(0)
         DO 50 j=1,iteron2
            R1=RAND(0)
            R2=RAND(0)
            PI=4*ATAN(1.)
            Z1=SQRT(-2*ALOG(R1))*COS(2*PI*R2)
            Z2=SQRT(-2*ALOG(R1))*SIN(2*PI*R2)
      A(2*j-1)=Z1*SPROC
      A(2*j)=Z2*SPROC
50      CONTINUE

C     CALC MEAN AND STANDARD DEVIATION OF A(SA)
C     SUM OF ITER
            MA=0
         DO 60 j=6,ITER
            MA=MA+A(j)
60    CONTINUE

C     DIVIDE BY (ITER-5)
```

```
        AVMA=MA/(ITER-5)

C    SUM OF SQUARED DEVIATIONS FROM MEAN
        SDA=0
      DO 70 j=6,ITER
        SDA=SDA+(A(j)-AVMA)**2
70   CONTINUE

C    STANDARD DEVIATION...S`QR DIVDED BY ITER
          SA=SQRT(SDA/(ITER-6))
C    set value of the control parameter, AL=3*SA
        AL=3*SA
        LCL=-AL*q*SA
        UCL= AL*q*SA

         b=1
C    compare quality deviations from target,that is,mean of quality
C    data gma(j) with control limits
C    calculate dxt(j) to compensate for change in forecast from previous
C    sample period

        z(1)=a(1)
c      xtm1(1)=0
        y(1)=delta1*ytm1(1)+delta2*ytm2(1)+w*xtm2(1)
        e(1)=z(1)+y(1)
      dxt(1)=-(1/w)*(r*e(1)-r*delta1*etm1(1)
   &          -r*delta2*etm2(1))-r*dxtmb(1)
        x(1)=dxt(1)
      gma(1)=r*e(1)

        z(2)=ztm1(2)+a(2)-theta*atm1(2)
        y(2)=delta1*ytm1(2)+delta2*ytm2(2)+w*xtm2(2)
        e(2)=z(2)+y(2)
      dxt(2)=-(1/w)*(r*e(2)-r*delta1*etm1(2)
   &          -r*delta2*etm2(2))-r*dxtmb(2)
        x(2)=xtm1(2)+dxt(2)
      gma(2)=r*e(2)+theta*gmatm1(2)

      do 80 j=3,iter
        z(j)=ztm1(j)+a(j)-theta*atm1(j)
      gma(j)=r*e(j)+theta*gmatm1(j)
80   continue

      do 90 j=3,iter
   if (xtm2(j) .le. 1.E-37) then
      xtm2(j)=0
   endif
```

```
 if (ytm1(j) .le. 1.E-37) then
    ytm1(j)=0
 endif

 if (ytm2(j) .le. 1.E-37) then
    ytm2(j)=0
 endif

 if (y(j) .le. 1.E-37) then
      y(j)=0
 endif

 if (etm1(j) .le. 1.E-37) then
    etm1(j)=0
 endif

 if (etm2(j) .le. 1.E-37) then
    etm2(j)=0
 endif

 if (dxtmb(j) .le. 1.E-37) then
   dxtmb(j)=0
 endif

      y(j)=delta1*ytm1(j)+delta2*ytm2(j)+w*xtm2(j)
      e(j)=z(j)+y(j)

 if (gma(j) .lt. LCL .or. gma(j) .gt. UCL) then
     dxt(j)=(-1/w)*(r*e(j)-r*delta1*etm1(j)
&           -r*delta2*etm2(j))-r*dxtmb(j)
      x(j)=xtm1(j)+dxt(j)
     gma(j)=0
    freq(j)=1
 else
     dxt(j)=0
    freq(j)=0
 endif

      x(j)=xtm1(j)+dxt(j)
  atm1(j+1)=a(j)
 gmatm1(j+1)=gma(j)
  ztm1(j+1)=z(j)
  xtm2(j+1)=xtm1(j)
  xtm1(j+1)=x(j)
  ytm1(j+1)=y(j)
  ytm2(j+1)=ytm1(j)
  etm1(j+1)=e(j)
  etm2(j+1)=etm1(j)
```

```
      if (ytm1(j+1) .le. 1.E-37) then
        ytm1(j+1)=0
      endif

      if (ytm2(j+1) .le. 1.E-37) then
        ytm2(j+1)=0
      endif

      if (xtm1(j+1) .le. 1.E-37) then
        xtm1(j+1)=0
      endif

      if (xtm2(j+1) .le. 1.E-37) then
        xtm2(j+1)=0
      endif

      if(x(j) .le. 1.E-37) then
          x(j)=0
      endif

      if (j .le. 25) then
      WRITE(2,110) j,x(j),xtm1(j),dxt(j),e(j),y(j)
110   FORMAT(I3,1X,F8.3,1X,F8.3,1X,F8.3,1X,F8.3,1X,F8.3)
      endif
90    continue

C     CALC MEAN AND STANDARD DEVIATION (CESTDDVN)
C     SUM OF ITER
        ME=0
         Mdxt=0
        MFREQ=0

       DO 200 j=6,ITER
          ME=ME+E(j)
         Mdxt=Mdxt+dxt(j)
        MFREQ=MFREQ+FREQ(j)
200   CONTINUE

c     DIVIDE BY (ITER-5)
          ME=ME/(ITER-5)
         Mdxt=Mdxt/(ITER-5)
        MFREQ=MFREQ/(ITER-5)

C     SUM OF SQUARED DEVIATIONS FROM MEAN
         SDE=0
        SDFREQ=0
         SDdxt=0
```

```
      vardxt=0

      DO 210 j=6,ITER
       SDFREQ=SDFREQ+(FREQ(j)-MFREQ)**2
          SDE=SDE+(E(j)-ME)**2
         SDdxt=SDdxt+(dxt(j)-Mdxt)**2
210   CONTINUE
        SE=0
       varE=0
      varCE=0
      CESTDDVN=0

       varE=(SDE/(ITER-6))
        SE=SQRT(SDE/(ITER-6))
         SDdxt=SQRT(SDdxt/(ITER-6))
        SDFREQ=SQRT(SDFREQ/(ITER-6))
      CESTDDVN=SE/SA
       varCE=(CESTDDVN)**2
       vardxt=(SDdxt)**2

      WRITE(3,230) theta,delta1,delta2,CESTDDVN,MFREQ
230     FORMAT(1X,F8.2,1X,F8.2,1X,F8.2,1X,F8.3,1X,F8.3)

      if (r_infinity(se)) then
       ieeer=ieee_flags('clear','exception','all',out)
      endif
      if (r_quiet_nan(se)) then
      print *, 'se bfore ', se
          se=r_min_normal(se)
      print *, 'se after ', se
       ieeer=ieee_flags('clear','exception','all',out)
      endif
      accrued=ieee_flags('get','exception', '', out)
      if (accrued.ne.0) then
      print *, ' accrued', accrued, ' SE ', se
c           inx=and(rshift(accrued, fp_inexact), 1)
c           div=and(rshift(accrued, fp_division), 1)
c         under=and(rshift(accrued, fp_underflow), 1)
c          over=and(rshift(accrued, fp_overflow), 1)
c           inv=and(rshift(accrued, fp_invalid), 1)
      print *, "Highest priority exception is: ", out
      print *, 'invalid divide overflo underflo inexact'
      print '(5i8)', inv, div, over, under, inx
       ieeer=ieee_flags('clear','exception','all',out)
      endif

      WRITE(4,250) theta,delta1,delta2,b,AL,PG,CG,SA
250   FORMAT('theta=',F8.2,1X,'delta1=',F8.2,1X,'delta2=',F8.2,1X,
```

```
   &'b=',I3,1X,'AL=',F8.2,1X,'PG=',F8.2,4X,'CG=',F8.1,1X,'SA=',F8.2)

    WRITE(4,*)'VARIABLE   j   MEAN  STD.DVN  VARIANCE'
    WRITE(4,*)'  E     10000'
    WRITE(4,260)            ME,  CESTDDVN,  varCE
260  FORMAT(15X,F8.3,5X,F8.3,5X,F8.3)

    WRITE(4,*)'  dxt   10000'
    WRITE(4,270)            Mdxt, SDdxt,     vardxt
270  FORMAT(15X,F8.3,5X,F8.3,5X,F8.3)

    WRITE(4,*)'  FREQ   10000'
    WRITE(4,280)                 MFREQ,     SDFREQ
280  FORMAT (15X,F8.3,5X,F8.3)
         b=2
C    compare quality deviations from target,that is,mean of quality
C    data gma(j) with control limits
C    calculate dxt(j) to compensate for change in forecast from previous
c    sample period

        z(1)=a(1)
c      xtm1(1)=0
c      xtm2(1)=0
        y(1)=delta1*ytm1(1)+delta2*ytm2(1)+w*xtm3(1)
        e(1)=z(1)+y(1)
      dxt(1)=-(1/w)*(r*e(1)-r*delta1*etm1(1)
   &          -r*delta2*etm2(1))-r*dxtmb(1)
          x(1)=dxt(1)
      gma(1)=r*e(1)

        z(2)=ztm1(2)+a(2)-theta*atm1(2)
c      xtm1(2)=0
c      xtm2(2)=0
        y(2)=delta1*ytm1(2)+delta2*ytm2(2)+w*xtm3(2)
        e(2)=z(2)+y(2)
      dxt(2)=-(1/w)*(r*e(2)-r*delta1*etm1(2)
   &          -r*delta2*etm2(2))-r*dxtmb(2)
        x(2)=xtm1(2)+dxt(2)
      gma(2)=r*e(2)+theta*gmatm1(2)

     do 290 j=3,iter
        z(j)=ztm1(j)+a(j)-theta*atm1(j)
          gma(j)=r*e(j)+theta*gmatm1(j)
290  continue

        do 300 j=3,iter
        if (xtm3(j) .le. 1.E-37) then
     xtm3(j)=0
```

```
  endif
     if (ytm1(j) .le. 1.E-20) then
   ytm1(j)=0
  endif
     if (ytm2(j) .le. 1.E-20) then
      ytm2(j)=0
     endif
     if (y(j) .le. 1.E-20) then
        y(j)=0
  endif
  if (dxtmb(j) .le. 1.E-37) then
  dxtmb(j)=0
  endif
  if (etm1(j) .le. 1.E-37) then
   etm1(j)=0
  endif
  if (etm2(j) .le. 1.E-37) then
   etm2(j)=0
  endif

     y(j)=delta1*ytm1(j)+delta2*ytm2(j)+w*xtm3(j)
     e(j)=z(j)+y(j)

    if (gma(j) .lt. LCL .or. gma(j) .gt. UCL) then
    dxt(j)=-(1/w)*(r*e(j)-delta1*r*etm1(j)
&          -r*delta2*etm2(j))-r*dxtmb(j)
       x(j)=xtm1(j)+dxt(j)
      gma(j)=0
     freq(j)=1
    else
      dxt(j)=0
     freq(j)=0
    endif

       x(j)=xtm1(j)+dxt(j)
    atm1(j+1)=a(j)
    gmatm1(j+1)=gma(j)
    ztm1(j+1)=z(j)
 xtm3(j+1)=xtm2(j)
 xtm2(j+1)=xtm1(j)
    xtm1(j+1)=x(j)
 ytm1(j+1)=y(j)
 ytm2(j+1)=ytm1(j)
 etm1(j+1)=e(j)
 etm2(j+1)=etm1(j)

 if (ytm1(j+1) .le. 1.E-20) then
 ytm1(j+1)=0
```

```
      endif
      if (ytm2(j+1) .le. 1.E-20) then
      ytm2(j+1)=0
      endif
      if (xtm1(j+1) .le. 1.E-37) then
      xtm1(j+1)=0
      endif
      if (xtm2(j+1) .le. 1.E-37) then
      xtm2(j+1)=0
      endif
      if (xtm3(j+1) .le. 1.E-37) then
      xtm3(j+1)=0
      endif
      if (x(j) .le. 1.E-37) then
         x(j)=0
      endif

      if (j .le. 25) then
      WRITE(5,221) j,x(j),xtm1(j),dxt(j),e(j),y(j)
221   FORMAT(I3,1X,F8.2,1X,F8.2,1X,F8.2,1X,F8.2,1X,F8.2)
      endif
300   continue

C     CALCULATE MEAN AND STANDARD DEVIATION (CESTDDVN)
C     SUM OF ITER
           ME=0
         Mdxt=0
        MFREQ=0
      DO 310 j=6,ITER
           ME=ME+E(j)
         Mdxt= Mdxt+dxt(j)
        MFREQ=MFREQ+FREQ(j)
310    continue
C     DIVIDE BY (ITER-5)
           ME=ME/(ITER-5)
         Mdxt=Mdxt/(ITER-5)
        MFREQ=MFREQ/(ITER-5)
C     SUM OF SQUARED DEVIATIONS FROM MEAN
          SDE=0
        SDFREQ=0
         SDdxt=0
      DO 320  j=6,ITER
        SDFREQ=SDFREQ+(FREQ(J)-MFREQ)**2
          SDE=SDE+(E(j)-ME)**2
         SDdxt=Sddxt+(dxt(j)-Mdxt)**2
320   CONTINUE

C    STANDARD DEVIATION...S'QR DIVIDED BY ITER
```

```
        SE=0
       varE=0
      varCE=0
    CESTDDVN=0
     vardxt=0
       varE=(SDE/(ITER-6))
        SE=SQRT(SDE/(ITER-6))
      SDdxt=SQRT(SDdxt/(ITER-6))
     SDFREQ=SQRT(SDFREQ/(ITER-6))
    CESTDDVN=SE/SA
      varCE=(CESTDDVN)**2
     vardxt=(SDdxt)**2

    WRITE(7,231) theta,delta1,delta2,CESTDDVN,MFREQ
231 FORMAT(1X,F8.2,1X,F8.2,1X,F8.2,1X,F8.3,1X,F8.3)

    WRITE(8,251) theta,delta1,delta2,b,AL,PG,CG,SA
251 FORMAT('theta=',F8.2,1X,'delta1=',F8.2,1X,'delta2=',1X,F8.2,
   &1X,'b=',I3,1X,'AL=',F8.2,1X,'PG=',F8.2,4X,'CG=',F8.1,1X,'SA=',
   &F8.2)
    WRITE(8,*)'VARIABLE  j  MEAN  STD.DVN  VARIANCE'

    WRITE(8,*) '  E   10000'
    WRITE(8,261)          ME,  CESTDDVN,varCE
261 FORMAT(15X,F8.3,5X,F8.3,5X,F8.3)
    WRITE(8,*) '  dxt  10000'
    WRITE(8,271)          Mdxt, SDdxt,  vardxt
271 FORMAT(15X,F8.3,5X,F8.3,5X,F8.3)

    WRITE(8,*) '  FREQ 10000'
    WRITE(8,281)             MFREQ,  SDFREQ
281 FORMAT(15X,F8.3,5X,F8.3)
25  CONTINUE
13  CLOSE(1)
    open(1,file='file11.txt',access='sequential',status='old',
   &form='formatted')
    continue
20  continue
    end
```

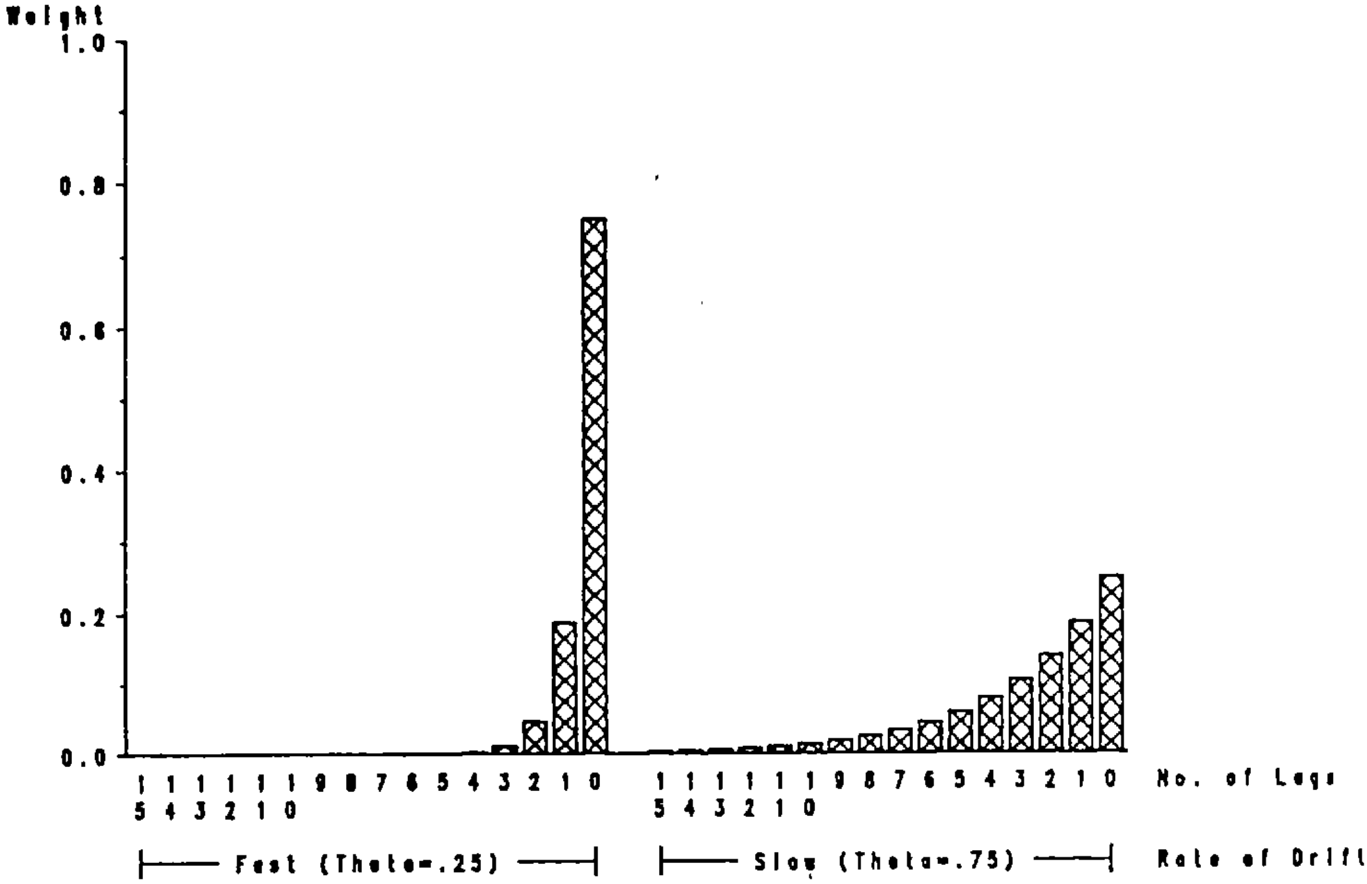


Figure 6.2 Forecast Weights for IMA(0,1,1) Model

(Figure 4, page 256, Baxley [1991])

Table 6.I Time Series Controller Performance for σ_a = 1.0 and dead time b = 1.0

theta	delta1	delta2	STDDVN	MFREQ
0.05	-1.82	-0.83	1.000	0.000
0.05	-1.81	-0.82	1.035	0.090
0.05	-1.80	-0.81	1.036	0.101
0.05	-1.79	-0.80	1.033	0.100
0.05	-1.78	-0.79	1.037	0.087
0.05	-1.73	-0.75	1.038	0.101
0.05	-1.72	-0.74	1.040	0.092
0.05	-1.71	-0.73	1.034	0.102
0.05	-1.70	-0.72	1.037	0.096
0.05	-1.66	-0.69	1.029	0.102
0.05	-1.65	-0.68	1.025	0.092
0.05	-1.64	-0.67	1.034	0.099
0.05	-1.61	-0.65	1.037	0.093
0.05	-1.60	-0.64	1.038	0.094
0.05	-1.59	-0.63	1.041	0.092
0.05	-1.56	-0.61	1.032	0.102
0.05	-1.55	-0.60	1.029	0.094
0.05	-1.52	-0.58	1.033	0.098
0.05	-1.51	-0.57	1.029	0.092
0.05	-1.50	-0.56	1.032	0.095
0.05	-1.48	-0.55	1.027	0.097
0.05	-1.47	-0.54	1.029	0.092
0.05	-1.44	-0.52	1.037	0.092
0.05	-1.43	-0.51	1.026	0.088
0.05	-1.40	-0.49	1.033	0.093
0.05	-1.37	-0.47	1.033	0.117
0.05	-1.36	-0.46	1.031	0.100
0.05	-1.34	-0.45	1.034	0.097
0.05	-1.33	-0.44	1.032	0.099
0.05	-1.31	-0.43	1.033	0.101
0.05	-1.30	-0.42	1.037	0.102
0.05	-1.28	-0.41	1.036	0.106
0.05	-1.25	-0.39	1.030	0.092
0.05	-1.23	-0.38	1.028	0.107
0.05	-1.22	-0.37	1.029	0.101
0.05	-1.20	-0.36	1.038	0.097
0.05	-1.18	-0.35	1.039	0.107
0.05	-1.17	-0.34	1.036	0.117
0.05	-1.15	-0.33	1.044	0.110
0.05	-1.13	-0.32	1.039	0.104
0.05	-1.11	-0.31	1.044	0.101
0.05	-1.10	-0.30	1.050	0.104
0.05	-1.08	-0.29	1.041	0.109

theta	delta1	delta2	STDDVN	MFREQ
0.05	-1.06	-0.28	1.043	0.122
0.05	-1.04	-0.27	1.049	0.105
0.05	-1.02	-0.26	1.042	0.128
0.05	-1.00	-0.25	1.039	0.102
0.05	-0.98	-0.24	1.041	0.111
0.05	-0.96	-0.23	1.050	0.126
0.05	-0.94	-0.22	1.049	0.124
0.05	-0.92	-0.21	1.047	0.124
0.05	-0.90	-0.20	1.061	0.110
0.05	-0.89	-0.20	1.049	0.130
0.05	-0.87	-0.19	1.052	0.115
0.05	-0.85	-0.18	1.052	0.110
0.05	-0.83	-0.17	1.057	0.127
0.05	-0.82	-0.17	1.056	0.133
0.05	-0.80	-0.16	1.058	0.111
0.05	-0.78	-0.15	1.060	0.152
0.05	-0.77	-0.15	1.058	0.124
0.05	-0.75	-0.14	1.039	0.129
0.05	-0.72	-0.13	1.056	0.144
0.05	-0.70	-0.12	1.057	0.129
0.05	-0.69	-0.12	1.061	0.127
0.05	-0.67	-0.11	1.074	0.162
0.05	-0.66	-0.11	1.067	0.119
0.05	-0.64	-0.10	1.074	0.107
0.05	-0.63	-0.10	1.072	0.165
0.05	-0.60	-0.09	1.082	0.160
0.05	-0.57	-0.08	1.056	0.114
0.05	-0.56	-0.08	1.084	0.155
0.05	-0.53	-0.07	1.082	0.132
0.05	-0.52	-0.07	1.082	0.135
0.05	-0.50	-0.06	1.095	0.178
0.05	-0.49	-0.06	1.097	0.150
0.05	-0.48	-0.06	1.094	0.141
0.05	-0.45	-0.05	1.067	0.159
0.05	-0.44	-0.05	1.089	0.140
0.05	-0.41	-0.04	1.105	0.172
0.05	-0.40	-0.04	1.086	0.118
0.05	-0.39	-0.04	1.097	0.163
0.05	-0.36	-0.03	1.097	0.171
0.05	-0.35	-0.03	1.111	0.199
0.05	-0.34	-0.03	1.106	0.159
0.05	-0.30	-0.02	1.108	0.166
0.05	-0.29	-0.02	1.111	0.165
0.05	-0.28	-0.02	1.113	0.191
0.05	-0.27	-0.02	1.157	0.190
0.05	-0.22	-0.01	1.127	0.168
0.05	-0.21	-0.01	1.118	0.176

theta	delta1	delta2	STDDVN	MFREQ
0.05	-0.20	-0.01	1.127	0.196
0.05	-0.19	-0.01	1.137	0.181
0.05	-0.18	-0.01	1.153	0.152
0.05	-0.10	0.00	1.139	0.199
0.05	-0.09	0.00	1.160	0.209
0.05	-0.08	0.00	1.158	0.199
0.05	-0.07	0.00	1.168	0.185
0.05	-0.06	0.00	1.179	0.195
0.05	-0.05	0.00	1.175	0.202
0.05	-0.04	0.00	1.174	0.180
0.05	-0.03	0.00	1.172	0.190
0.05	-0.02	0.00	1.158	0.221
0.05	-0.01	0.00	1.186	0.176
0.05	0.00	0.00	1.217	0.224
0.25	-1.82	-0.83	1.018	0.086
0.25	-1.81	-0.82	1.012	0.026
0.25	-1.80	-0.81	1.010	0.030
0.25	-1.79	-0.80	1.010	0.031
0.25	-1.78	-0.79	1.011	0.031
0.25	-1.73	-0.75	1.013	0.032
0.25	-1.72	-0.74	1.010	0.024
0.25	-1.71	-0.73	1.012	0.039
0.25	-1.70	-0.72	1.012	0.028
0.25	-1.66	-0.69	1.008	0.028
0.25	-1.65	-0.68	1.014	0.026
0.25	-1.64	-0.67	1.006	0.033
0.25	-1.61	-0.65	1.006	0.028
0.25	-1.60	-0.64	1.019	0.050
0.25	-1.59	-0.63	1.011	0.025
0.25	-1.56	-0.61	1.018	0.030
0.25	-1.55	-0.60	1.015	0.034
0.25	-1.52	-0.58	1.008	0.030
0.25	-1.51	-0.57	1.012	0.034
0.25	-1.50	-0.56	1.010	0.037
0.25	-1.48	-0.55	1.017	0.034
0.25	-1.47	-0.54	1.011	0.035
0.25	-1.44	-0.52	1.016	0.039
0.25	-1.43	-0.51	1.011	0.028
0.25	-1.40	-0.49	1.012	0.042
0.25	-1.37	-0.47	1.016	0.031
0.25	-1.36	-0.46	1.014	0.028
0.25	-1.34	-0.45	1.017	0.033
0.25	-1.33	-0.44	1.020	0.038
0.25	-1.31	-0.43	1.013	0.037
0.25	-1.30	-0.42	1.022	0.032
0.25	-1.28	-0.41	1.017	0.039

theta	delta1	delta2	STDDVN	MFREQ
0.25	-1.25	-0.39	1.018	0.038
0.25	-1.23	-0.38	1.011	0.029
0.25	-1.22	-0.37	1.019	0.054
0.25	-1.20	-0.36	1.022	0.032
0.25	-1.18	-0.35	1.018	0.045
0.25	-1.17	-0.34	1.021	0.034
0.25	-1.15	-0.33	1.015	0.033
0.25	-1.13	-0.32	1.012	0.042
0.25	-1.11	-0.31	1.015	0.037
0.25	-1.10	-0.30	1.016	0.039
0.25	-1.08	-0.29	1.012	0.048
0.25	-1.06	-0.28	1.020	0.053
0.25	-1.04	-0.27	1.021	0.046
0.25	-1.02	-0.26	1.023	0.050
0.25	-1.00	-0.25	1.030	0.050
0.25	-0.98	-0.24	1.029	0.037
0.25	-0.96	-0.23	1.021	0.051
0.25	-0.94	-0.22	1.026	0.038
0.25	-0.92	-0.21	1.017	0.041
0.25	-0.90	-0.20	1.038	0.041
0.25	-0.89	-0.20	1.024	0.037
0.25	-0.87	-0.19	1.023	0.061
0.25	-0.85	-0.18	1.041	0.065
0.25	-0.83	-0.17	1.021	0.030
0.25	-0.82	-0.17	1.031	0.060
0.25	-0.80	-0.16	1.030	0.037
0.25	-0.78	-0.15	1.037	0.032
0.25	-0.77	-0.15	1.026	0.041
0.25	-0.75	-0.14	1.034	0.046
0.25	-0.72	-0.13	1.027	0.056
0.25	-0.70	-0.12	1.027	0.062
0.25	-0.69	-0.12	1.048	0.034
0.25	-0.67	-0.11	1.039	0.062
0.25	-0.66	-0.11	1.035	0.048
0.25	-0.64	-0.10	1.042	0.065
0.25	-0.63	-0.10	1.047	0.051
0.25	-0.60	-0.09	1.032	0.064
0.25	-0.57	-0.08	1.030	0.048
0.25	-0.56	-0.08	1.039	0.066
0.25	-0.53	-0.07	1.046	0.061
0.25	-0.52	-0.07	1.059	0.056
0.25	-0.50	-0.06	1.049	0.038
0.25	-0.49	-0.06	1.045	0.052
0.25	-0.48	-0.06	1.045	0.066
0.25	-0.45	-0.05	1.060	0.059
0.25	-0.44	-0.05	1.049	0.068
0.25	-0.41	-0.04	1.062	0.060

theta	delta1	delta2	STDDVN	MFREQ
0.25	-0.40	-0.04	1.059	0.056
0.25	-0.39	-0.04	1.046	0.081
0.25	-0.36	-0.03	1.065	0.039
0.25	-0.35	-0.03	1.076	0.074
0.25	-0.34	-0.03	1.055	0.066
0.25	-0.30	-0.02	1.069	0.072
0.25	-0.29	-0.02	1.078	0.073
0.25	-0.28	-0.02	1.056	0.056
0.25	-0.27	-0.02	1.073	0.075
0.25	-0.22	-0.01	1.080	0.102
0.25	-0.21	-0.01	1.061	0.078
0.25	-0.20	-0.01	1.070	0.072
0.25	-0.19	-0.01	1.088	0.085
0.25	-0.18	-0.01	1.082	0.080
0.25	-0.10	0.00	1.109	0.061
0.25	-0.09	0.00	1.075	0.074
0.25	-0.08	0.00	1.081	0.090
0.25	-0.07	0.00	1.108	0.108
0.25	-0.06	0.00	1.102	0.086
0.25	-0.05	0.00	1.097	0.100
0.25	-0.04	0.00	1.129	0.111
0.25	-0.03	0.00	1.114	0.074
0.25	-0.02	0.00	1.135	0.105
0.25	-0.01	0.00	1.100	0.111
0.25	0.00	0.00	1.099	0.085
0.70	-1.82	-0.83	1.001	0.017
0.70	-1.81	-0.82	1.000	0.001
0.70	-1.80	-0.81	1.000	0.000
0.70	-1.79	-0.80	1.000	0.000
0.70	-1.78	-0.79	1.001	0.000
0.70	-1.73	-0.75	1.002	0.001
0.70	-1.72	-0.74	1.000	0.001
0.70	-1.71	-0.73	1.001	0.001
0.70	-1.70	-0.72	1.000	0.000
0.70	-1.66	-0.69	1.000	0.000
0.70	-1.65	-0.68	1.000	0.000
0.70	-1.64	-0.67	1.003	0.000
0.70	-1.61	-0.65	1.004	0.001
0.70	-1.60	-0.64	1.000	0.001
0.70	-1.59	-0.63	1.001	0.001
0.70	-1.56	-0.61	1.001	0.000
0.70	-1.55	-0.60	1.000	0.000
0.70	-1.52	-0.58	1.001	0.001
0.70	-1.51	-0.57	1.000	0.000
0.70	-1.50	-0.56	1.000	0.000
0.70	-1.48	-0.55	1.000	0.001

theta	delta1	delta2	STDDVN	MFREQ
0.70	-1.47	-0.54	1.001	0.001
0.70	-1.44	-0.52	1.000	0.001
0.70	-1.43	-0.51	1.005	0.001
0.70	-1.40	-0.49	1.001	0.001
0.70	-1.37	-0.47	1.001	0.001
0.70	-1.36	-0.46	1.000	0.001
0.70	-1.34	-0.45	1.001	0.000
0.70	-1.33	-0.44	1.000	0.001
0.70	-1.31	-0.43	1.001	0.000
0.70	-1.30	-0.42	1.002	0.000
0.70	-1.28	-0.41	1.003	0.001
0.70	-1.25	-0.39	1.002	0.001
0.70	-1.23	-0.38	1.000	0.000
0.70	-1.22	-0.37	1.001	0.000
0.70	-1.20	-0.36	1.004	0.001
0.70	-1.18	-0.35	1.003	0.001
0.70	-1.17	-0.34	1.009	0.000
0.70	-1.15	-0.33	1.003	0.001
0.70	-1.13	-0.32	1.002	0.001
0.70	-1.11	-0.31	1.002	0.000
0.70	-1.10	-0.30	1.007	0.001
0.70	-1.08	-0.29	1.000	0.000
0.70	-1.06	-0.28	1.014	0.001
0.70	-1.04	-0.27	1.000	0.001
0.70	-1.02	-0.26	1.006	0.001
0.70	-1.00	-0.25	1.002	0.001
0.70	-0.98	-0.24	1.001	0.001
0.70	-0.96	-0.23	1.001	0.001
0.70	-0.94	-0.22	1.005	0.000
0.70	-0.92	-0.21	1.007	0.001
0.70	-0.90	-0.20	1.000	0.001
0.70	-0.89	-0.20	1.004	0.001
0.70	-0.87	-0.19	1.002	0.001
0.70	-0.85	-0.18	1.004	0.002
0.70	-0.83	-0.17	1.000	0.001
0.70	-0.82	-0.17	1.000	0.001
0.70	-0.80	-0.16	1.000	0.000
0.70	-0.78	-0.15	1.002	0.001
0.70	-0.77	-0.15	1.010	0.001
0.70	-0.75	-0.14	1.003	0.000
0.70	-0.72	-0.13	1.012	0.001
0.70	-0.70	-0.12	1.000	0.001
0.70	-0.69	-0.12	1.000	0.001
0.70	-0.67	-0.11	1.004	0.002
0.70	-0.66	-0.11	1.011	0.001
0.70	-0.64	-0.10	1.000	0.001
0.70	-0.63	-0.10	1.005	0.001

theta	delta1	delta2	STDDVN	MFREQ
0.70	-0.60	-0.09	1.002	0.001
0.70	-0.57	-0.08	1.004	0.001
0.70	-0.56	-0.08	1.013	0.000
0.70	-0.53	-0.07	1.016	0.003
0.70	-0.52	-0.07	1.005	0.001
0.70	-0.50	-0.06	1.004	0.002
0.70	-0.49	-0.06	1.001	0.001
0.70	-0.48	-0.06	1.024	0.001
0.70	-0.45	-0.05	1.000	0.000
0.70	-0.44	-0.05	1.004	0.002
0.70	-0.41	-0.04	1.005	0.003
0.70	-0.40	-0.04	1.000	0.001
0.70	-0.39	-0.04	1.005	0.006
0.70	-0.36	-0.03	1.004	0.001
0.70	-0.35	-0.03	1.005	0.002
0.70	-0.34	-0.03	1.028	0.001
0.70	-0.30	-0.02	1.041	0.001
0.70	-0.29	-0.02	1.002	0.001
0.70	-0.28	-0.02	1.010	0.003
0.70	-0.27	-0.02	1.033	0.002
0.70	-0.22	-0.01	1.001	0.001
0.70	-0.21	-0.01	1.022	0.001
0.70	-0.20	-0.01	1.014	0.014
0.70	-0.19	-0.01	1.001	0.002
0.70	-0.18	-0.01	1.007	0.005
0.70	-0.10	0.00	1.040	0.002
0.70	-0.09	0.00	1.004	0.002
0.70	-0.08	0.00	1.014	0.009
0.70	-0.07	0.00	1.012	0.014
0.70	-0.06	0.00	1.014	0.013
0.70	-0.05	0.00	1.007	0.005
0.70	-0.04	0.00	1.038	0.001
0.70	-0.03	0.00	1.026	0.021
0.70	-0.02	0.00	1.003	0.001
0.70	-0.01	0.00	1.023	0.036
0.70	0.00	0.00	1.010	0.001
0.75	-1.82	-0.83	1.002	0.019
0.75	-1.81	-0.82	1.000	0.000
0.75	-1.80	-0.81	1.000	0.000
0.75	-1.79	-0.80	1.000	0.000
0.75	-1.78	-0.79	1.000	0.000
0.75	-1.73	-0.75	1.000	0.000
0.75	-1.72	-0.74	1.000	0.000
0.75	-1.71	-0.73	1.000	0.000
0.75	-1.70	-0.72	1.000	0.000
0.75	-1.66	-0.69	1.000	0.000

theta	delta1	delta2	STDDVN	MFREQ
0.75	-1.65	-0.68	1.000	0.000
0.75	-1.64	-0.67	1.000	0.000
0.75	-1.61	-0.65	1.000	0.000
0.75	-1.60	-0.64	1.000	0.000
0.75	-1.59	-0.63	1.000	0.000
0.75	-1.56	-0.61	1.000	0.000
0.75	-1.55	-0.60	1.000	0.000
0.75	-1.52	-0.58	1.000	0.000
0.75	-1.51	-0.57	1.000	0.000
0.75	-1.50	-0.56	1.000	0.000
0.75	-1.48	-0.55	1.000	0.000
0.75	-1.47	-0.54	1.000	0.000
0.75	-1.44	-0.52	1.000	0.000
0.75	-1.43	-0.51	1.000	0.000
0.75	-1.40	-0.49	1.000	0.000
0.75	-1.37	-0.47	1.000	0.000
0.75	-1.36	-0.46	1.000	0.000
0.75	-1.34	-0.45	1.000	0.000
0.75	-1.33	-0.44	1.002	0.000
0.75	-1.31	-0.43	1.000	0.000
0.75	-1.30	-0.42	1.000	0.000
0.75	-1.28	-0.41	1.000	0.000
0.75	-1.25	-0.39	1.000	0.000
0.75	-1.23	-0.38	1.000	0.000
0.75	-1.22	-0.37	1.000	0.000
0.75	-1.20	-0.36	1.000	0.000
0.75	-1.18	-0.35	1.000	0.000
0.75	-1.17	-0.34	1.000	0.000
0.75	-1.15	-0.33	1.000	0.000
0.75	-1.13	-0.32	1.000	0.000
0.75	-1.11	-0.31	1.000	0.000
0.75	-1.10	-0.30	1.000	0.000
0.75	-1.08	-0.29	1.000	0.000
0.75	-1.06	-0.28	1.001	0.000
0.75	-1.04	-0.27	1.000	0.000
0.75	-1.02	-0.26	1.000	0.000
0.75	-1.00	-0.25	1.000	0.000
0.75	-0.98	-0.24	1.000	0.000
0.75	-0.96	-0.23	1.000	0.000
0.75	-0.94	-0.22	1.000	0.000
0.75	-0.92	-0.21	1.000	0.000
0.75	-0.90	-0.20	1.000	0.000
0.75	-0.89	-0.20	1.001	0.000
0.75	-0.87	-0.19	1.000	0.000
0.75	-0.85	-0.18	1.004	0.002
0.75	-0.83	-0.17	1.000	0.000
0.75	-0.82	-0.17	1.000	0.000

theta	delta1	delta2	STDDVN	MFREQ
0.75	-0.80	-0.16	1.000	0.000
0.75	-0.78	-0.15	1.000	0.000
0.75	-0.77	-0.15	1.000	0.000
0.75	-0.75	-0.14	1.000	0.000
0.75	-0.72	-0.13	1.000	0.000
0.75	-0.70	-0.12	1.002	0.000
0.75	-0.69	-0.12	1.000	0.000
0.75	-0.67	-0.11	1.000	0.000
0.75	-0.66	-0.11	1.000	0.000
0.75	-0.64	-0.10	1.002	0.000
0.75	-0.63	-0.10	1.000	0.000
0.75	-0.60	-0.09	1.000	0.000
0.75	-0.57	-0.08	1.000	0.000
0.75	-0.56	-0.08	1.000	0.000
0.75	-0.53	-0.07	1.003	0.001
0.75	-0.52	-0.07	1.006	0.000
0.75	-0.50	-0.06	1.000	0.000
0.75	-0.49	-0.06	1.000	0.000
0.75	-0.48	-0.06	1.001	0.000
0.75	-0.45	-0.05	1.000	0.000
0.75	-0.44	-0.05	1.000	0.000
0.75	-0.41	-0.04	1.000	0.000
0.75	-0.40	-0.04	1.000	0.000
0.75	-0.39	-0.04	1.000	0.000
0.75	-0.36	-0.03	1.008	0.000
0.75	-0.35	-0.03	1.000	0.000
0.75	-0.34	-0.03	1.000	0.000
0.75	-0.30	-0.02	1.000	0.000
0.75	-0.29	-0.02	1.000	0.000
0.75	-0.28	-0.02	1.000	0.004
0.75	-0.27	-0.02	1.000	0.000
0.75	-0.22	-0.01	1.002	0.001
0.75	-0.21	-0.01	1.000	0.000
0.75	-0.20	-0.01	1.000	0.001
0.75	-0.19	-0.01	1.000	0.000
0.75	-0.18	-0.01	1.000	0.000
0.75	-0.10	0.00	1.011	0.002
0.75	-0.09	0.00	1.000	0.000
0.75	-0.08	0.00	1.019	0.001
0.75	-0.07	0.00	1.002	0.000
0.75	-0.06	0.00	1.011	0.003
0.75	-0.05	0.00	1.000	0.000
0.75	-0.04	0.00	1.014	0.000
0.75	-0.03	0.00	1.013	0.001
0.75	-0.02	0.00	1.000	0.000
0.75	-0.01	0.00	1.008	0.002
0.75	0.00	0.00	1.000	0.000

theta	delta1	delta2	STDDVN	MFREQ
0.95	-1.82	-0.83	1.000	0.116
0.95	-1.81	-0.82	1.001	0.472
0.95	-1.80	-0.81	1.001	0.488
0.95	-1.79	-0.80	1.001	0.484
0.95	-1.78	-0.79	1.001	0.490
0.95	-1.73	-0.75	1.000	0.488
0.95	-1.72	-0.74	1.000	0.479
0.95	-1.71	-0.73	1.001	0.478
0.95	-1.70	-0.72	1.001	0.480
0.95	-1.66	-0.69	1.001	0.483
0.95	-1.65	-0.68	1.000	0.481
0.95	-1.64	-0.67	1.001	0.477
0.95	-1.61	-0.65	1.000	0.476
0.95	-1.60	-0.64	1.000	0.473
0.95	-1.59	-0.63	1.001	0.476
0.95	-1.56	-0.61	1.000	0.483
0.95	-1.55	-0.60	1.000	0.469
0.95	-1.52	-0.58	1.000	0.468
0.95	-1.51	-0.57	1.000	0.469
0.95	-1.50	-0.56	1.000	0.477
0.95	-1.48	-0.55	1.001	0.474
0.95	-1.47	-0.54	1.000	0.464
0.95	-1.44	-0.52	1.000	0.472
0.95	-1.43	-0.51	1.001	0.465
0.95	-1.40	-0.49	1.000	0.467
0.95	-1.37	-0.47	1.001	0.461
0.95	-1.36	-0.46	1.001	0.457
0.95	-1.34	-0.45	1.001	0.453
0.95	-1.33	-0.44	1.000	0.457
0.95	-1.31	-0.43	1.000	0.444
0.95	-1.30	-0.42	1.001	0.455
0.95	-1.28	-0.41	1.000	0.450
0.95	-1.25	-0.39	1.000	0.443
0.95	-1.23	-0.38	1.000	0.444
0.95	-1.22	-0.37	1.000	0.434
0.95	-1.20	-0.36	1.001	0.431
0.95	-1.18	-0.35	1.000	0.438
0.95	-1.17	-0.34	1.000	0.431
0.95	-1.15	-0.33	1.001	0.429
0.95	-1.13	-0.32	1.001	0.419
0.95	-1.11	-0.31	1.000	0.417
0.95	-1.10	-0.30	1.001	0.414
0.95	-1.08	-0.29	1.000	0.411
0.95	-1.06	-0.28	1.001	0.408
0.95	-1.04	-0.27	1.000	0.409
0.95	-1.02	-0.26	1.000	0.397
0.95	-1.00	-0.25	1.001	0.389

theta	delta1	delta2	STDDVN	MFREQ
0.95	-0.98	-0.24	1.001	0.399
0.95	-0.96	-0.23	1.001	0.393
0.95	-0.94	-0.22	1.000	0.392
0.95	-0.92	-0.21	1.000	0.390
0.95	-0.90	-0.20	1.001	0.377
0.95	-0.89	-0.20	1.000	0.381
0.95	-0.87	-0.19	1.001	0.371
0.95	-0.85	-0.18	1.001	0.365
0.95	-0.83	-0.17	1.000	0.366
0.95	-0.82	-0.17	1.001	0.358
0.95	-0.80	-0.16	1.000	0.357
0.95	-0.78	-0.15	1.001	0.362
0.95	-0.77	-0.15	1.001	0.347
0.95	-0.75	-0.14	1.001	0.347
0.95	-0.72	-0.13	1.000	0.336
0.95	-0.70	-0.12	1.001	0.334
0.95	-0.69	-0.12	1.001	0.331
0.95	-0.67	-0.11	1.000	0.330
0.95	-0.66	-0.11	1.001	0.333
0.95	-0.64	-0.10	1.001	0.324
0.95	-0.63	-0.10	1.001	0.315
0.95	-0.60	-0.09	1.001	0.319
0.95	-0.57	-0.08	1.000	0.310
0.95	-0.56	-0.08	1.001	0.303
0.95	-0.53	-0.07	1.002	0.297
0.95	-0.52	-0.07	1.001	0.289
0.95	-0.50	-0.06	1.001	0.281
0.95	-0.49	-0.06	1.002	0.283
0.95	-0.48	-0.06	1.001	0.283
0.95	-0.45	-0.05	1.001	0.271
0.95	-0.44	-0.05	1.002	0.270
0.95	-0.41	-0.04	1.003	0.262
0.95	-0.40	-0.04	1.001	0.245
0.95	-0.39	-0.04	1.001	0.243
0.95	-0.36	-0.03	1.001	0.249
0.95	-0.35	-0.03	1.003	0.237
0.95	-0.34	-0.03	1.002	0.233
0.95	-0.30	-0.02	1.002	0.227
0.95	-0.29	-0.02	1.003	0.221
0.95	-0.28	-0.02	1.003	0.218
0.95	-0.27	-0.02	1.002	0.213
0.95	-0.22	-0.01	1.003	0.207
0.95	-0.21	-0.01	1.002	0.196
0.95	-0.20	-0.01	1.002	0.191
0.95	-0.19	-0.01	1.002	0.191
0.95	-0.18	-0.01	1.004	0.186
0.95	-0.10	0.00	1.004	0.187

theta	delta1	delta2	STDDVN	MFREQ
0.95	-0.09	0.00	1.003	0.154
0.95	-0.08	0.00	1.003	0.154
0.95	-0.07	0.00	1.006	0.149
0.95	-0.06	0.00	1.004	0.145
0.95	-0.05	0.00	1.002	0.145
0.95	-0.04	0.00	1.005	0.138
0.95	-0.03	0.00	1.005	0.132
0.95	-0.02	0.00	1.005	0.131
0.95	-0.01	0.00	1.004	0.129
0.95	0.00	0.00	1.005	0.125

Table 6.II Time Series Controller Performance for σ_a =1.0 and dead time b = 2.0

theta	delta1	delta2	STDDVN	MFREQ
0.05	-1.82	-0.83	1.514	0.019
0.05	-1.81	-0.82	1.549	0.021
0.05	-1.80	-0.81	1.543	0.022
0.05	-1.79	-0.80	1.517	0.020
0.05	-1.78	-0.79	1.552	0.020
0.05	-1.73	-0.75	1.518	0.023
0.05	-1.72	-0.74	1.555	0.023
0.05	-1.71	-0.73	1.547	0.020
0.05	-1.70	-0.72	1.539	0.021
0.05	-1.66	-0.69	1.556	0.018
0.05	-1.65	-0.68	1.545	0.020
0.05	-1.64	-0.67	1.536	0.019
0.05	-1.61	-0.65	1.541	0.021
0.05	-1.60	-0.64	1.533	0.021
0.05	-1.59	-0.63	1.561	0.020
0.05	-1.56	-0.61	1.532	0.018
0.05	-1.55	-0.60	1.540	0.020
0.05	-1.52	-0.58	1.519	0.019
0.05	-1.51	-0.57	1.524	0.018
0.05	-1.50	-0.56	1.538	0.020
0.05	-1.48	-0.55	1.516	0.020
0.05	-1.47	-0.54	1.507	0.017
0.05	-1.44	-0.52	1.523	0.017
0.05	-1.43	-0.51	1.523	0.020
0.05	-1.40	-0.49	1.618	0.019
0.05	-1.37	-0.47	1.537	0.018
0.05	-1.36	-0.46	1.552	0.020
0.05	-1.34	-0.45	1.553	0.019
0.05	-1.33	-0.44	1.551	0.018
0.05	-1.31	-0.43	1.565	0.020
0.05	-1.30	-0.42	1.574	0.020
0.05	-1.28	-0.41	1.542	0.019
0.05	-1.25	-0.39	1.588	0.017
0.05	-1.23	-0.38	1.525	0.018
0.05	-1.22	-0.37	1.523	0.018
0.05	-1.20	-0.36	1.580	0.019
0.05	-1.18	-0.35	1.557	0.017
0.05	-1.17	-0.34	1.577	0.018
0.05	-1.15	-0.33	1.549	0.020
0.05	-1.13	-0.32	1.540	0.018
0.05	-1.11	-0.31	1.531	0.019
0.05	-1.10	-0.30	1.592	0.020
0.05	-1.08	-0.29	1.601	0.018

theta	delta1	delta2	STDDVN	MFREQ
0.05	-1.06	-0.28	1.560	0.018
0.05	-1.04	-0.27	1.647	0.019
0.05	-1.02	-0.26	1.568	0.021
0.05	-1.00	-0.25	1.567	0.018
0.05	-0.98	-0.24	1.600	0.017
0.05	-0.96	-0.23	1.650	0.020
0.05	-0.94	-0.22	1.600	0.021
0.05	-0.92	-0.21	1.597	0.020
0.05	-0.90	-0.20	1.602	0.018
0.05	-0.89	-0.20	1.595	0.017
0.05	-0.87	-0.19	1.546	0.016
0.05	-0.85	-0.18	1.644	0.017
0.05	-0.83	-0.17	1.612	0.019
0.05	-0.82	-0.17	1.591	0.018
0.05	-0.80	-0.16	1.677	0.019
0.05	-0.78	-0.15	1.644	0.018
0.05	-0.77	-0.15	1.614	0.020
0.05	-0.75	-0.14	1.685	0.016
0.05	-0.72	-0.13	1.621	0.020
0.05	-0.70	-0.12	1.639	0.017
0.05	-0.69	-0.12	1.709	0.019
0.05	-0.67	-0.11	1.616	0.021
0.05	-0.66	-0.11	1.601	0.017
0.05	-0.64	-0.10	1.714	0.021
0.05	-0.63	-0.10	1.695	0.019
0.05	-0.60	-0.09	1.591	0.021
0.05	-0.57	-0.08	1.730	0.018
0.05	-0.56	-0.08	1.646	0.017
0.05	-0.53	-0.07	1.718	0.021
0.05	-0.52	-0.07	1.744	0.021
0.05	-0.50	-0.06	1.676	0.023
0.05	-0.49	-0.06	1.735	0.022
0.05	-0.48	-0.06	1.754	0.021
0.05	-0.45	-0.05	1.635	0.019
0.05	-0.44	-0.05	1.834	0.019
0.05	-0.41	-0.04	1.622	0.022
0.05	-0.40	-0.04	1.685	0.022
0.05	-0.39	-0.04	1.760	0.021
0.05	-0.36	-0.03	1.774	0.022
0.05	-0.35	-0.03	1.798	0.022
0.05	-0.34	-0.03	1.740	0.022
0.05	-0.30	-0.02	1.746	0.024
0.05	-0.29	-0.02	1.872	0.023
0.05	-0.28	-0.02	1.808	0.022
0.05	-0.27	-0.02	1.795	0.033
0.05	-0.22	-0.01	1.815	0.024
0.05	-0.21	-0.01	1.788	0.021

theta	delta1	delta2	STDDVN	MFREQ
0.05	-0.20	-0.01	1.928	0.023
0.05	-0.19	-0.01	1.750	0.027
0.05	-0.18	-0.01	1.853	0.031
0.05	-0.10	0.00	2.081	0.025
0.05	-0.09	0.00	1.895	0.030
0.05	-0.08	0.00	1.840	0.030
0.05	-0.07	0.00	1.948	0.030
0.05	-0.06	0.00	1.968	0.031
0.05	-0.05	0.00	1.871	0.031
0.05	-0.04	0.00	1.901	0.032
0.05	-0.03	0.00	1.940	0.031
0.05	-0.02	0.00	1.818	0.030
0.05	-0.01	0.00	2.021	0.034
0.05	0.00	0.00	1.859	0.038
0.25	-1.82	-0.83	1.306	0.007
0.25	-1.81	-0.82	1.335	0.006
0.25	-1.80	-0.81	1.328	0.007
0.25	-1.79	-0.80	1.322	0.007
0.25	-1.78	-0.79	1.321	0.007
0.25	-1.73	-0.75	1.313	0.007
0.25	-1.72	-0.74	1.352	0.007
0.25	-1.71	-0.73	1.327	0.006
0.25	-1.70	-0.72	1.308	0.006
0.25	-1.66	-0.69	1.303	0.007
0.25	-1.65	-0.68	1.325	0.007
0.25	-1.64	-0.67	1.300	0.007
0.25	-1.61	-0.65	1.397	0.005
0.25	-1.60	-0.64	1.297	0.008
0.25	-1.59	-0.63	1.315	0.007
0.25	-1.56	-0.61	1.353	0.007
0.25	-1.55	-0.60	1.326	0.006
0.25	-1.52	-0.58	1.342	0.006
0.25	-1.51	-0.57	1.370	0.006
0.25	-1.50	-0.56	1.346	0.007
0.25	-1.48	-0.55	1.344	0.007
0.25	-1.47	-0.54	1.390	0.006
0.25	-1.44	-0.52	1.299	0.007
0.25	-1.43	-0.51	1.393	0.007
0.25	-1.40	-0.49	1.296	0.007
0.25	-1.37	-0.47	1.335	0.005
0.25	-1.36	-0.46	1.370	0.006
0.25	-1.34	-0.45	1.360	0.007
0.25	-1.33	-0.44	1.356	0.007
0.25	-1.31	-0.43	1.329	0.006
0.25	-1.30	-0.42	1.369	0.008
0.25	-1.28	-0.41	1.366	0.007

theta	delta1	delta2	STDDVN	MFREQ
0.25	-1.25	-0.39	1.336	0.006
0.25	-1.23	-0.38	1.463	0.007
0.25	-1.22	-0.37	1.340	0.007
0.25	-1.20	-0.36	1.384	0.006
0.25	-1.18	-0.35	1.339	0.008
0.25	-1.17	-0.34	1.332	0.008
0.25	-1.15	-0.33	1.377	0.006
0.25	-1.13	-0.32	1.371	0.007
0.25	-1.11	-0.31	1.375	0.006
0.25	-1.10	-0.30	1.397	0.007
0.25	-1.08	-0.29	1.456	0.005
0.25	-1.06	-0.28	1.414	0.008
0.25	-1.04	-0.27	1.410	0.006
0.25	-1.02	-0.26	1.420	0.008
0.25	-1.00	-0.25	1.388	0.007
0.25	-0.98	-0.24	1.426	0.008
0.25	-0.96	-0.23	1.384	0.008
0.25	-0.94	-0.22	1.407	0.007
0.25	-0.92	-0.21	1.351	0.006
0.25	-0.90	-0.20	1.396	0.007
0.25	-0.89	-0.20	1.448	0.009
0.25	-0.87	-0.19	1.482	0.007
0.25	-0.85	-0.18	1.336	0.009
0.25	-0.83	-0.17	1.414	0.006
0.25	-0.82	-0.17	1.365	0.009
0.25	-0.80	-0.16	1.351	0.008
0.25	-0.78	-0.15	1.374	0.008
0.25	-0.77	-0.15	1.408	0.007
0.25	-0.75	-0.14	1.479	0.008
0.25	-0.72	-0.13	1.473	0.006
0.25	-0.70	-0.12	1.372	0.007
0.25	-0.69	-0.12	1.454	0.008
0.25	-0.67	-0.11	1.435	0.008
0.25	-0.66	-0.11	1.456	0.009
0.25	-0.64	-0.10	1.436	0.008
0.25	-0.63	-0.10	1.502	0.009
0.25	-0.60	-0.09	1.447	0.009
0.25	-0.57	-0.08	1.443	0.007
0.25	-0.56	-0.08	1.474	0.008
0.25	-0.53	-0.07	1.476	0.010
0.25	-0.52	-0.07	1.391	0.010
0.25	-0.50	-0.06	1.467	0.009
0.25	-0.49	-0.06	1.477	0.009
0.25	-0.48	-0.06	1.483	0.010
0.25	-0.45	-0.05	1.503	0.010
0.25	-0.44	-0.05	1.473	0.011
0.25	-0.41	-0.04	1.492	0.009

theta	delta1	delta2	STDDVN	MFREQ
0.25	-0.40	-0.04	1.518	0.010
0.25	-0.39	-0.04	1.414	0.010
0.25	-0.36	-0.03	1.548	0.010
0.25	-0.35	-0.03	1.523	0.013
0.25	-0.34	-0.03	1.519	0.011
0.25	-0.30	-0.02	1.587	0.011
0.25	-0.29	-0.02	1.490	0.013
0.25	-0.28	-0.02	1.552	0.008
0.25	-0.27	-0.02	1.635	0.011
0.25	-0.22	-0.01	1.550	0.013
0.25	-0.21	-0.01	1.559	0.012
0.25	-0.20	-0.01	1.611	0.011
0.25	-0.19	-0.01	1.593	0.012
0.25	-0.18	-0.01	1.488	0.010
0.25	-0.10	0.00	1.555	0.015
0.25	-0.09	0.00	1.649	0.011
0.25	-0.08	0.00	1.658	0.012
0.25	-0.07	0.00	1.610	0.015
0.25	-0.06	0.00	1.696	0.014
0.25	-0.05	0.00	1.694	0.014
0.25	-0.04	0.00	1.562	0.019
0.25	-0.03	0.00	1.672	0.016
0.25	-0.02	0.00	1.674	0.019
0.25	-0.01	0.00	1.625	0.012
0.25	0.00	0.00	1.570	0.014
0.70	-1.82	-0.83	1.055	0.006
0.70	-1.81	-0.82	1.061	0.003
0.70	-1.80	-0.81	1.054	0.003
0.70	-1.79	-0.80	1.068	0.003
0.70	-1.78	-0.79	1.066	0.003
0.70	-1.73	-0.75	1.065	0.003
0.70	-1.72	-0.74	1.077	0.003
0.70	-1.71	-0.73	1.079	0.003
0.70	-1.70	-0.72	1.076	0.003
0.70	-1.66	-0.69	1.067	0.003
0.70	-1.65	-0.68	1.065	0.003
0.70	-1.64	-0.67	1.072	0.003
0.70	-1.61	-0.65	1.081	0.002
0.70	-1.60	-0.64	1.057	0.003
0.70	-1.59	-0.63	1.109	0.003
0.70	-1.56	-0.61	1.042	0.003
0.70	-1.55	-0.60	1.106	0.004
0.70	-1.52	-0.58	1.071	0.003
0.70	-1.51	-0.57	1.064	0.003
0.70	-1.50	-0.56	1.098	0.003
0.70	-1.48	-0.55	1.061	0.002

theta	delta1	delta2	STDDVN	MFREQ
0.70	-1.47	-0.54	1.103	0.002
0.70	-1.44	-0.52	1.072	0.004
0.70	-1.43	-0.51	1.071	0.002
0.70	-1.40	-0.49	1.076	0.003
0.70	-1.37	-0.47	1.059	0.003
0.70	-1.36	-0.46	1.059	0.003
0.70	-1.34	-0.45	1.063	0.003
0.70	-1.33	-0.44	1.073	0.003
0.70	-1.31	-0.43	1.081	0.003
0.70	-1.30	-0.42	1.058	0.002
0.70	-1.28	-0.41	1.101	0.003
0.70	-1.25	-0.39	1.050	0.004
0.70	-1.23	-0.38	1.101	0.003
0.70	-1.22	-0.37	1.065	0.003
0.70	-1.20	-0.36	1.084	0.004
0.70	-1.18	-0.35	1.093	0.002
0.70	-1.17	-0.34	1.120	0.004
0.70	-1.15	-0.33	1.065	0.003
0.70	-1.13	-0.32	1.081	0.003
0.70	-1.11	-0.31	1.070	0.002
0.70	-1.10	-0.30	1.079	0.002
0.70	-1.08	-0.29	1.079	0.003
0.70	-1.06	-0.28	1.132	0.003
0.70	-1.04	-0.27	1.101	0.003
0.70	-1.02	-0.26	1.086	0.002
0.70	-1.00	-0.25	1.092	0.002
0.70	-0.98	-0.24	1.098	0.003
0.70	-0.96	-0.23	1.092	0.003
0.70	-0.94	-0.22	1.132	0.003
0.70	-0.92	-0.21	1.102	0.003
0.70	-0.90	-0.20	1.100	0.003
0.70	-0.89	-0.20	1.096	0.004
0.70	-0.87	-0.19	1.194	0.003
0.70	-0.85	-0.18	1.083	0.006
0.70	-0.83	-0.17	1.146	0.003
0.70	-0.82	-0.17	1.123	0.005
0.70	-0.80	-0.16	1.100	0.002
0.70	-0.78	-0.15	1.124	0.003
0.70	-0.77	-0.15	1.148	0.004
0.70	-0.75	-0.14	1.178	0.004
0.70	-0.72	-0.13	1.146	0.003
0.70	-0.70	-0.12	1.147	0.003
0.70	-0.69	-0.12	1.201	0.003
0.70	-0.67	-0.11	1.151	0.005
0.70	-0.66	-0.11	1.090	0.003
0.70	-0.64	-0.10	1.138	0.003
0.70	-0.63	-0.10	1.114	0.004

theta	delta1	delta2	STDDVN	MFREQ
0.70	-0.60	-0.09	1.059	0.002
0.70	-0.57	-0.08	1.117	0.002
0.70	-0.56	-0.08	1.217	0.004
0.70	-0.53	-0.07	1.093	0.005
0.70	-0.52	-0.07	1.172	0.004
0.70	-0.50	-0.06	1.167	0.003
0.70	-0.49	-0.06	1.155	0.005
0.70	-0.48	-0.06	1.127	0.004
0.70	-0.45	-0.05	1.191	0.002
0.70	-0.44	-0.05	1.263	0.004
0.70	-0.41	-0.04	1.210	0.005
0.70	-0.40	-0.04	1.290	0.003
0.70	-0.39	-0.04	1.178	0.005
0.70	-0.36	-0.03	1.186	0.004
0.70	-0.35	-0.03	1.128	0.005
0.70	-0.34	-0.03	1.207	0.004
0.70	-0.30	-0.02	1.058	0.005
0.70	-0.29	-0.02	1.185	0.003
0.70	-0.28	-0.02	1.170	0.005
0.70	-0.27	-0.02	1.181	0.003
0.70	-0.22	-0.01	1.203	0.003
0.70	-0.21	-0.01	1.412	0.005
0.70	-0.20	-0.01	1.157	0.010
0.70	-0.19	-0.01	1.291	0.004
0.70	-0.18	-0.01	1.201	0.007
0.70	-0.10	0.00	1.144	0.005
0.70	-0.09	0.00	1.352	0.003
0.70	-0.08	0.00	1.440	0.008
0.70	-0.07	0.00	1.446	0.008
0.70	-0.06	0.00	1.303	0.009
0.70	-0.05	0.00	1.065	0.006
0.70	-0.04	0.00	1.236	0.003
0.70	-0.03	0.00	1.047	0.018
0.70	-0.02	0.00	1.298	0.004
0.70	-0.01	0.00	1.089	0.018
0.70	0.00	0.00	1.196	0.004
0.75	-1.82	-0.83	1.035	0.009
0.75	-1.81	-0.82	1.034	0.001
0.75	-1.80	-0.81	1.029	0.001
0.75	-1.79	-0.80	1.044	0.001
0.75	-1.78	-0.79	1.033	0.001
0.75	-1.73	-0.75	1.059	0.001
0.75	-1.72	-0.74	1.029	0.001
0.75	-1.71	-0.73	1.045	0.001
0.75	-1.70	-0.72	1.045	0.001
0.75	-1.66	-0.69	1.041	0.001

theta	delta1	delta2	STDDVN	MFREQ
0.75	-1.65	-0.68	1.058	0.001
0.75	-1.64	-0.67	1.036	0.001
0.75	-1.61	-0.65	1.033	0.001
0.75	-1.60	-0.64	1.049	0.001
0.75	-1.59	-0.63	1.036	0.001
0.75	-1.56	-0.61	1.053	0.001
0.75	-1.55	-0.60	1.030	0.000
0.75	-1.52	-0.58	1.041	0.001
0.75	-1.51	-0.57	1.045	0.001
0.75	-1.50	-0.56	1.035	0.001
0.75	-1.48	-0.55	1.059	0.001
0.75	-1.47	-0.54	1.036	0.001
0.75	-1.44	-0.52	1.048	0.001
0.75	-1.43	-0.51	1.036	0.000
0.75	-1.40	-0.49	1.043	0.001
0.75	-1.37	-0.47	1.069	0.001
0.75	-1.36	-0.46	1.044	0.001
0.75	-1.34	-0.45	1.045	0.001
0.75	-1.33	-0.44	1.058	0.001
0.75	-1.31	-0.43	1.033	0.001
0.75	-1.30	-0.42	1.050	0.001
0.75	-1.28	-0.41	1.042	0.001
0.75	-1.25	-0.39	1.039	0.001
0.75	-1.23	-0.38	1.053	0.001
0.75	-1.22	-0.37	1.054	0.001
0.75	-1.20	-0.36	1.037	0.001
0.75	-1.18	-0.35	1.078	0.001
0.75	-1.17	-0.34	1.080	0.002
0.75	-1.15	-0.33	1.047	0.001
0.75	-1.13	-0.32	1.040	0.001
0.75	-1.11	-0.31	1.052	0.001
0.75	-1.10	-0.30	1.039	0.001
0.75	-1.08	-0.29	1.052	0.001
0.75	-1.06	-0.28	1.048	0.001
0.75	-1.04	-0.27	1.057	0.001
0.75	-1.02	-0.26	1.086	0.001
0.75	-1.00	-0.25	1.054	0.001
0.75	-0.98	-0.24	1.127	0.001
0.75	-0.96	-0.23	1.055	0.001
0.75	-0.94	-0.22	1.057	0.001
0.75	-0.92	-0.21	1.065	0.001
0.75	-0.90	-0.20	1.103	0.001
0.75	-0.89	-0.20	1.050	0.001
0.75	-0.87	-0.19	1.143	0.001
0.75	-0.85	-0.18	1.029	0.003
0.75	-0.83	-0.17	1.151	0.001
0.75	-0.82	-0.17	1.081	0.001

theta	delta1	delta2	STDDVN	MFREQ
0.75	-0.80	-0.16	1.064	0.001
0.75	-0.78	-0.15	1.042	0.001
0.75	-0.77	-0.15	1.041	0.001
0.75	-0.75	-0.14	1.085	0.001
0.75	-0.72	-0.13	1.070	0.001
0.75	-0.70	-0.12	1.057	0.001
0.75	-0.69	-0.12	1.084	0.001
0.75	-0.67	-0.11	1.046	0.001
0.75	-0.66	-0.11	1.062	0.001
0.75	-0.64	-0.10	1.091	0.001
0.75	-0.63	-0.10	1.088	0.001
0.75	-0.60	-0.09	1.032	0.001
0.75	-0.57	-0.08	1.148	0.001
0.75	-0.56	-0.08	1.227	0.001
0.75	-0.53	-0.07	1.080	0.002
0.75	-0.52	-0.07	1.042	0.001
0.75	-0.50	-0.06	1.135	0.001
0.75	-0.49	-0.06	1.093	0.001
0.75	-0.48	-0.06	1.063	0.001
0.75	-0.45	-0.05	1.086	0.000
0.75	-0.44	-0.05	1.081	0.002
0.75	-0.41	-0.04	1.065	0.000
0.75	-0.40	-0.04	1.072	0.001
0.75	-0.39	-0.04	1.087	0.001
0.75	-0.36	-0.03	1.052	0.002
0.75	-0.35	-0.03	1.068	0.001
0.75	-0.34	-0.03	1.071	0.001
0.75	-0.30	-0.02	1.037	0.001
0.75	-0.29	-0.02	1.140	0.001
0.75	-0.28	-0.02	1.031	0.005
0.75	-0.27	-0.02	1.162	0.001
0.75	-0.22	-0.01	1.099	0.002
0.75	-0.21	-0.01	1.197	0.001
0.75	-0.20	-0.01	1.141	0.001
0.75	-0.19	-0.01	1.105	0.001
0.75	-0.18	-0.01	1.279	0.001
0.75	-0.10	0.00	1.052	0.004
0.75	-0.09	0.00	1.118	0.001
0.75	-0.08	0.00	1.031	0.004
0.75	-0.07	0.00	1.310	0.001
0.75	-0.06	0.00	1.059	0.006
0.75	-0.05	0.00	1.116	0.001
0.75	-0.04	0.00	1.265	0.001
0.75	-0.03	0.00	1.104	0.003
0.75	-0.02	0.00	1.232	0.001
0.75	-0.01	0.00	1.119	0.004
0.75	0.00	0.00	1.274	0.001

theta	delta1	delta2	STDDVN	MFREQ
0.95	-1.82	-0.83	1.005	0.188
0.95	-1.81	-0.82	1.004	0.368
0.95	-1.80	-0.81	1.003	0.369
0.95	-1.79	-0.80	1.005	0.377
0.95	-1.78	-0.79	1.003	0.375
0.95	-1.73	-0.75	1.004	0.372
0.95	-1.72	-0.74	1.005	0.370
0.95	-1.71	-0.73	1.003	0.369
0.95	-1.70	-0.72	1.004	0.374
0.95	-1.66	-0.69	1.003	0.370
0.95	-1.65	-0.68	1.004	0.372
0.95	-1.64	-0.67	1.005	0.365
0.95	-1.61	-0.65	1.005	0.360
0.95	-1.60	-0.64	1.003	0.366
0.95	-1.59	-0.63	1.004	0.374
0.95	-1.56	-0.61	1.005	0.367
0.95	-1.55	-0.60	1.004	0.367
0.95	-1.52	-0.58	1.004	0.362
0.95	-1.51	-0.57	1.005	0.373
0.95	-1.50	-0.56	1.004	0.374
0.95	-1.48	-0.55	1.005	0.366
0.95	-1.47	-0.54	1.005	0.373
0.95	-1.44	-0.52	1.005	0.358
0.95	-1.43	-0.51	1.005	0.371
0.95	-1.40	-0.49	1.005	0.364
0.95	-1.37	-0.47	1.004	0.364
0.95	-1.36	-0.46	1.006	0.359
0.95	-1.34	-0.45	1.005	0.358
0.95	-1.33	-0.44	1.005	0.358
0.95	-1.31	-0.43	1.004	0.361
0.95	-1.30	-0.42	1.004	0.365
0.95	-1.28	-0.41	1.005	0.357
0.95	-1.25	-0.39	1.004	0.358
0.95	-1.23	-0.38	1.006	0.353
0.95	-1.22	-0.37	1.006	0.346
0.95	-1.20	-0.36	1.005	0.352
0.95	-1.18	-0.35	1.005	0.354
0.95	-1.17	-0.34	1.006	0.353
0.95	-1.15	-0.33	1.007	0.348
0.95	-1.13	-0.32	1.006	0.341
0.95	-1.11	-0.31	1.006	0.338
0.95	-1.10	-0.30	1.003	0.336
0.95	-1.08	-0.29	1.006	0.338
0.95	-1.06	-0.28	1.006	0.339
0.95	-1.04	-0.27	1.006	0.337
0.95	-1.02	-0.26	1.005	0.325
0.95	-1.00	-0.25	1.005	0.330

theta	delta1	delta2	STDDVN	MFREQ
0.95	-0.98	-0.24	1.004	0.335
0.95	-0.96	-0.23	1.004	0.337
0.95	-0.94	-0.22	1.005	0.334
0.95	-0.92	-0.21	1.004	0.332
0.95	-0.90	-0.20	1.005	0.321
0.95	-0.89	-0.20	1.005	0.323
0.95	-0.87	-0.19	1.004	0.317
0.95	-0.85	-0.18	1.006	0.326
0.95	-0.83	-0.17	1.004	0.316
0.95	-0.82	-0.17	1.006	0.306
0.95	-0.80	-0.16	1.005	0.314
0.95	-0.78	-0.15	1.006	0.320
0.95	-0.77	-0.15	1.005	0.316
0.95	-0.75	-0.14	1.004	0.310
0.95	-0.72	-0.13	1.006	0.304
0.95	-0.70	-0.12	1.006	0.305
0.95	-0.69	-0.12	1.005	0.310
0.95	-0.67	-0.11	1.006	0.295
0.95	-0.66	-0.11	1.005	0.303
0.95	-0.64	-0.10	1.005	0.299
0.95	-0.63	-0.10	1.004	0.303
0.95	-0.60	-0.09	1.005	0.294
0.95	-0.57	-0.08	1.005	0.298
0.95	-0.56	-0.08	1.006	0.288
0.95	-0.53	-0.07	1.007	0.286
0.95	-0.52	-0.07	1.008	0.277
0.95	-0.50	-0.06	1.006	0.278
0.95	-0.49	-0.06	1.007	0.278
0.95	-0.48	-0.06	1.004	0.281
0.95	-0.45	-0.05	1.007	0.274
0.95	-0.44	-0.05	1.007	0.270
0.95	-0.41	-0.04	1.008	0.256
0.95	-0.40	-0.04	1.009	0.251
0.95	-0.39	-0.04	1.006	0.257
0.95	-0.36	-0.03	1.005	0.250
0.95	-0.35	-0.03	1.008	0.250
0.95	-0.34	-0.03	1.005	0.243
0.95	-0.30	-0.02	1.006	0.240
0.95	-0.29	-0.02	1.008	0.238
0.95	-0.28	-0.02	1.007	0.238
0.95	-0.27	-0.02	1.004	0.229
0.95	-0.22	-0.01	1.008	0.232
0.95	-0.21	-0.01	1.008	0.220
0.95	-0.20	-0.01	1.008	0.217
0.95	-0.19	-0.01	1.007	0.216
0.95	-0.18	-0.01	1.008	0.224
0.95	-0.10	0.00	1.009	0.202

theta	delta1	delta2	STDDVN	MFREQ
0.95	-0.09	0.00	1.009	0.201
0.95	-0.08	0.00	1.008	0.197
0.95	-0.07	0.00	1.012	0.184
0.95	-0.06	0.00	1.009	0.191
0.95	-0.05	0.00	1.009	0.187
0.95	-0.04	0.00	1.010	0.178
0.95	-0.03	0.00	1.011	0.178
0.95	-0.02	0.00	1.008	0.171
0.95	-0.01	0.00	1.009	0.179
0.95	0.00	0.00	1.008	0.169

CHAPTER 7

TIME SERIES CONTROLLER PERFORMANCE RESULTS - ANALYSIS AND DISCUSSION

7.1 INTRODUCTION

In this Chapter, the simulation results are used to analyse performance measures of the time series controller. The effects of the rate of process drift, r, on the control error standard deviation (CESTDDVN) are discussed for the Time Series Controller presented in Chapter 6 which were obtained directly from simulation of the feedback control algorithm. The results, with the adjustment intervals (AIs), are given in Tables 7.II and 7.V and in Tables 7.III and 7.VI, the model and controller parameters for dead time b = 1 and b = 2 respectively. The feedback (closed-loop) control stability, adjustment and benefits and problems of using integral control, are briefly discussed in Sections 7.2.3, 7.2.4 and 7.2.5. The effect of control limits on product variability and the dependence of control action variance on dynamic parameters and process drift are explained in Sections 7.3 and 7.4. Details on the constrained variance control scheme are given in Table 7.IV and in Section 7.5. The effect of increase in dead time from b = 1 to b = 2 on CESTDDVN and AI is discussed in Section 7.6. It is shown that the EWMA has fairly good control of the process for values of Θ in the interval, 0.75 ± 0.05. An approximate probabilistic model for the situations in which the mean of the product quality variable is on target and the mean not on target is given in Section 7.7.

7.2 ANALYSIS OF SIMULATION RESULTS

7.2.1 Background of the Simulation

The main focus of the simulation is on studying the effect of the rate of drift on control of a dynamic process with dead time. It was mentioned in Section 6.7, that what is critical is the combined effects of the inertia and the dead time on the output control error sigma (CESTDDVN). Although Baxley's [1991] feedback control strategies for drifting processes, provided closer control to target with control error sigmas lower than that obtained by Shewhart control charting procedures, they resulted in slight increases in control error sigmas (product variability) obtained through time series controllers but required less frequent adjustments. The variance of the output variable obtained from simulation of the feedback control algorithm (Equation (5.16)) is a minimum because of the following observations made earlier in Section 5.6.

(i) The feedback control Equation (5.16) defines the adjustment to be made to the process at time t which would produce the feedback control action compensating for the forecasted disturbance and yielding the smallest possible mean square error' at the output. In other words, the control adjustment action given by Equation (5.16) minimises the variance of the output controlled variable.

(ii) The Equation (5.16) reduces to that of the control adjustment equation of the time series controller algorithm derived by Baxley [1991] when there are no carryover effects of the process response into succeeding sample periods (the dynamics or inertia $\delta = 0$). Baxley [1991] identified that the first term in his algorithm represented integral action and the second term, the dead-time compensator developed by Smith [1959]. Palmor and Shinnar [page 15, 1979] observed that 'the Smith predictor is a direct result of minimal variance strategy and that minimal variance control for processes having dead times includes this type of dead-time compensation'. At this stage, an intuitive conjecture, using this principle, is made that the inclusion of the dead-time compensation term of either the Smith predictor type and/or the Dahlin's (on-line) tuning parameter, (whose values range from 0 to 1), in a feedback control algorithm will also result in a minimum variance strategy for processes with dead time. This draws on the comparison made by Harris, MacGregor and Wright [1982] to the minimum variance controller they derived for the process with dead time (for which the number of whole periods of delay was equal to 2) and the Dahlin controller given in their paper. The authors showed that the two controllers were identical upon setting the value of Dahlin's parameter, (the discrete time constant of the closed-loop process), equal to Θ, the IMA parameter in the stochastic disturbance model. They reconciled the different approaches by noting that the 'IMA parameter Θ provides information about the magnitude of the closed-loop time constant'. Equation (5.16) is identical to Baxley's [1991] algorithm and includes both integral action and dead-time compensation terms and the IMA parameter Θ, (whose values range from 0 to 1), is set to take care of the drift ($r = 1 - \Theta$) as well as Dahlin's parameter to compensate for the dead time. So, it follows that *the variance of the output product variable achieved by using Equation (5.16) with integral action and dead-time compensation terms is a minimum. The dead-time compensation term (seemingly) removes the delay from stability considerations and definitely provides a stabilising effect on the feedback control system. These principles are used for designing (formulating) the discrete (sampled data) time series controller.* Such a controller will maintain the mean of the process quality variable at or near target and will allow for a (rapid) response to process disturbances without much overcompensation or overcorrection.

The CESTDDVN's (control error standard deviations) of the output variable achieved are a minimum by appeal to an observation made by Box and Jenkins [1970, 1976] mentioned above in (i) and in Chapter 6. Moreover, it has been pointed out that Equation (5.16) contains integral action. The control obtained through the first term in Equation (5.16) is the discrete analogue of 'integral control', a form of adjustment employed in engineering or automatic process control. So, a minimum variance controller built on this algorithm will be able to maintain the desired output variable at or near a given set point (Palmor and Shinnor [1979]), thereby fulfilling one of the eight* criteria specified as the requirements of a sampled data (discrete) controller algorithm by Palmor and Shinnar [1979] in their paper (*Details of the criteria are given in Chapter 9).

It is possible to show that the response of the feedback control algorithm (Equation 5.16) to a change in set point will be fast and smooth and will have no overshoot by taking care of and considering the stability of the feedback control (closed-loop) under critically damped conditions and by satisfying the stability conditions (Equation 5.7) in the simulation of the feedback control algorithm. The feedback (closed-loop) control stability is further discussed in Section 7.2.3. It will also give a fairly good response even if there is a slight perturbation in the process parameters. See Palmor and Shinnar (page 20, [1979]) for further information regarding characteristics of good integral control.

The following facts also support the claim that the feedback control Equation (5.16) provides minimum variance control and the necessary feedback for process control adjustment.

(iii) According to Astrom, [1970], Astrom and Wittenmark [1973], a *moving average process of order f has the property that its Autocorrelation Function (ACF) is zero beyond the lag f.* The autocorrelation function of the output process variable Y_t, (f = 1 for the ARIMA (0,1,1) model considered in this thesis), was checked and it was found that minimum variance control is obtained at the output for the control strategy adopted by using Equation (5.16).

(iv) Moreover, the effect of the feedback control provided by Equation (5.16) is that the *error* in forecasting the deviation from target is the deviation itself and that the EWMA Equation (5.17) produces a minimum mean squared error forecast since the disturbance is represented by the (integrated moving average) IMA (0,1,1) time series model Equation (5.9) (Muth [1960]).

(v) In feedback control, where the input is calculated (completely) from the output, the 'cross-correlation' technique is used for testing the presence of feedback. A substantial and significant sample 'cross-correlation' $r_{x'e'}(k)$ (between the 'prewhitened' input x'_t and the transformed output e'_t) at lag k = 0 would (actually) indicate the presence of a feedback loop, where $\{x'_t\}$ is a 'white noise' sequence with mean zero and variance $\sigma_{x'}^2$. A large (negative) cross-correlation at lag k = 0, calculated for the feedback control algorithm and found to be significant gave the clue to the presence of feedback. For more information on the cross-correlation technique, see Box and MacGregor [1974].

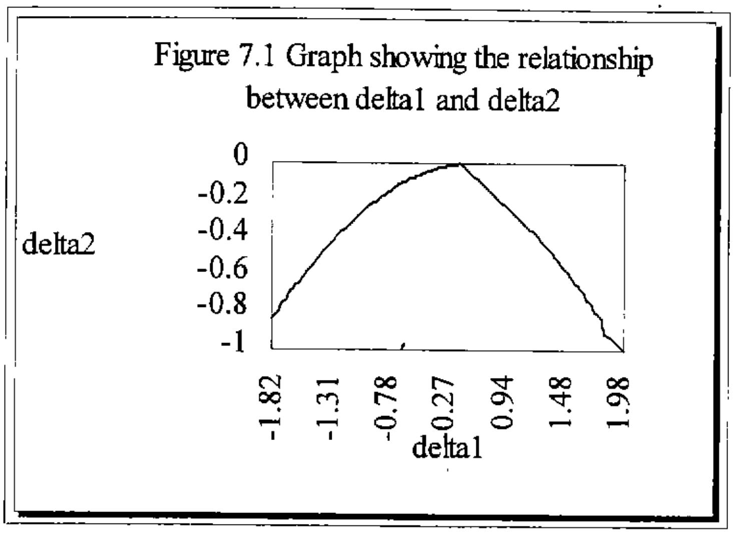
7.2.2 Simulation Methodology and EWMA Process Control

The satisfactory control of a process with time delay was made possible by (i) the process output following the command input (signal) closely and remaining unaffected by variations in the process parameters, (ii) the second-order dynamic model (Equation 5.5) with time delay matched the process and was an adaptive model of the process and (iii) as the feedback control system operated, the model tracked the variations in the parameters of the process. Under steady state conditions, due to drifting of the (sensitive) feedback control adjustment equation, a small mismatch between the process and model did not significantly affect the operation of the feedback control stability. This is important since closed-loop stability cannot be guaranteed when there is process/model mismatch (page 1486, Harris and MacGregor [1987]). Feedback control (closed-loop) stability is discussed in the next Section.

7.2.3 Feedback (Closed - loop) Control Stability

Stability is the first and foremost problem to be faced. The values of δ_1,δ_2 generated by the computer through the FORTRAN programme, shown in Table 7.I (Attachment 7.I) satisfy the (closed-loop) stability conditions given in Section 5.5 (Equation 5.7). It is possible to generate other sets of values of δ_1,δ_2 that also satisfy the stability conditions. The Figure 7.1, (an inverted parabola ($\delta_1^2 = -4\delta_2$)), shows the relationship between δ_1, δ_2 satisfying the inequality conditions for achieving stability.

The roots of the characteristic Equation (5.6) determine stability of the feedback control system (described by Equation 5.5). It is interesting to note that the region for values of δ_1 from -2 to 0 and δ_2 from -1 to 0 (in Figures 5.2 and 5.3) satisfying Equation



(5.7) is the stable operation region without oscillation in the feedback control loop. This is similar to the stability condition used by control engineers, that all the roots of the characteristic equation of the closed loop transfer function* (dynamic) model of the process must lie to the left of the imaginary axis on the *s* plane, as shown in Figure 7.2 (*Control engineers represent the process P(s) in terms of the Laplace operator 's' by a second-order model of the form $P(s) = \frac{g e^{-sT_d}}{(1+sT_1)(1+sT_2)}$ where g is the (steady-state) gain of the process, T_1, T_2 are the time constants of the exponential (inertial) lags and T_d is the time delay).

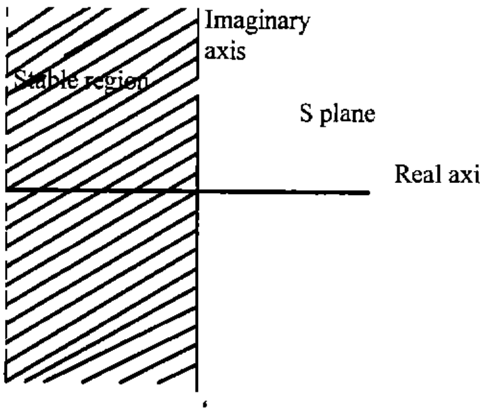


Figure 7.2 Stable Region in the S Plane

There are some advantages and/or benefits that can be derived from using Figures 5.2 and 5.3, such as employing the dynamic parameters δ_1, δ_2 in the stable region and without overshoot of the output variable, as explained in Section 5.5. The 'Routh test' is used by engineers in APC to determine if a control system is stable. Routh's stability criterion provides information about the number of positive roots without the need to actually solve a polynomial, it applies to polynomials with only a finite number of terms. When this criterion is applied to a control system, information about stability can be obtained (directly) from the coefficients only of the characteristic equation which should be positive and real. However, Routh's criterion does not give the roots of the characteristic equation nor the degree of stability (that is, how far the roots are from the imaginary axis), of the control system. Moreover, Routh's test cannot be applied to systems containing dead time and in these instances, control engineers often make recourse to 'frequency response analysis'.

The points on the parabolic curve ($\delta_1^2 = -4\delta_2$) are real and equal. The portion of this curve where the value of system/process gain (PG) ranges for values of (δ_1, δ_2) from (-2, -1), (for which PG = 1/4 = 0.25) and for values of (δ_1,δ_2), (0,0) (for which PG = 1) is the stable region where the (feedback) control system can be operated without oscillations in the feedback loop. In the portion to the right of this stable region, the roots are still positive and equal but the process gain (PG) of the system for values of (δ_1,δ_2) from (0, 0) to (2, -1) range from 1 to ∞. Interest is only in the positive values of the process gain, PG less than or equal to 1 for satisfying the closed-loop stability conditions. Values of PG, given by PG = $1/1-\delta_1-\delta_2$, greater than 1 are in the unstable region as shown in Figures 5.2 and 5.3. Higher gains tend to drive the feedback control system unstable and cause oscillations in the feedback loop. For this reason, the values of (δ_1,δ_2) are considered in the region (-2, -1) and (0, 0) and lying on the parabola $(\delta_1^2 = -4\delta_2)$ in order to satisfy the stability conditions. Thus, the upper stability limit of the feedback (closed-loop) gain, g, (shown equal to the process gain, PG), cannot exceed the value 1 without leading to undamped oscillations. This upper limit has been explored in order to determine how stable and effective the time series controller built on feedback control algorithm (Equation 5.16) will be when it is applied to control a process.

This discussion, incidentally, provides justification, in a way, for restricting attention, as a special case, to a 'critically damped' second-order system.

The issues and control problems arising out of 'gain margin' and 'phase margin' present in damped loops containing integrating or capacitive (that is, dynamic or inertial) elements are not addressed in this thesis since the concept of gain and phase margins are not particularly relevant to the process control practitioner. This is especially so in the current situation where the thrust is directed towards controlling the product quality of the output variable not withstanding other allied or connected issues that may arise in **synergising** the techniques from both SPC and APC disciplines without being unduly concerned about other problems of (automatic) process control that may also arise in applying process control techniques from both areas simultaneously.

7.2.4 Feedback Control Adjustment

The programme calculated the value of the required adjustment, given a small increment in the input variable x_t, (notation dxt in the simulation), for each sample interval, whenever the geometric moving average (gma) was less than LCL or greater than UCL. During simulation runs, the values of x_t (x at time t) (obtained from adding the adjustment, the increment in x_t and x_{t-1}(xtm1 in the computer simulation)) and the gma were obtained at various sample intervals (instants of time). It was found that whenever gma had a value lying between the control limits, no adjustment was needed, (and hence no sampling was necessary), and so the increment in x_t (the adjustment dxt) was zero as also was the mean, though a time series controller calls for an adjustment for every sample interval. Thus, by using the gma statistic in EWMA control for monitoring, unnecessary adjustments are avoided and so too are the associated costs of sampling and adjustment. As long as dxt was zero, then x_t was equal to x_{t-1} On the other hand, when the gma crossed either of the control limits, dxt had some significant value and so a control adjustment action was required to be made for that particular sampling interval. It was found also that the value of x_t was zero even though e_t had some significant value. This was due to the fact that the values for the term x_{t-b-1} for b = 1, 2, namely, x_{t-2} and x_{t-3} were made equal to zero whenever their values were sufficiently small. So, the value of x_t becomes zero and so also dxt. The process then requires no adjustment, the gma plot falls within the control limits and thus the process is in control.

Table 7.II, titled, 'Time series controller performance measures' (Attachment 7.II), shows the values of the CESTDDVN and the adjustment interval (AI) (1/MFREQ) for values of Θ ranging from 0.05 (fast drifts) to 0.95 (slow drifts). Values of Θ from 0.30 to 0.65 were of less interest since only fast and slow process drifts are considered for the ARIMA (0,1,1) disturbance model. Disturbances with Θ closer to 0, may be termed less noisy, while a value of Θ =' 0.7 denotes a fairly noisy non-stationary disturbance. The simulation results indicate that the feedback control algorithm,

Equation (5.16) holds potential in reducing product variability (control error sigma, CESTDDVN). If the input to the system (SISO, single input - single output, considered in this thesis), is zero, then the process described by Equation (5.5) will return to the desired final state $Y_t = 0$, due to the iterative nature of the feedback control algorithm (5.16) even if no control is applied. There are,some benefits in applying this algorithm for controlling product variability as will be shown subsequently.

7.2.5 Benefits and Limitations of Integral Control

Automatic process control (APC) techniques have been applied to process variables such as feed rate, temperature, pressure, viscosity, etc. APC or engineering process control techniques have been applied to product quality variables as well. Conventional practices of engineering control use the potential for step changes to justify an integral term in the controller algorithm to give (long-run) compensation for a shift in the mean of the product quality variable.

Process regulation is an important function of a controller intended to keep the output controlled variable at the desired set point by changing it as often as necessary. Every process is subject to load variations. In a (well)-regulated feedback control loop, the input manipulated variable will be driven to balance the load, as a consequence of which, the load is usually measured by engineers in terms of the corresponding value of the output controlled quality variable.

The block diagram for the feedback control illustrated in Figure 5.1 (Section 5.4, Chapter 5) assumed that the adjustment $x_t = f\ (e_t, e_{t-1}, \ldots)$ can be written as a (linear) function of past deviations from target which is appropriate if the costs of adjustment and of sampling are negligible. This is likely to be the situation in process industries where the adjustments are automated and so the assumption of a linear function holds good for the form of feedback control considered. This is evident from the feedback control adjustment equation (Equation 5.16) for the adjustment x_t which shows x_t as a (linear) function of the past deviations $e_t, e_{t-1}, \ldots$.

A controller with integral action changes the output variable as long as a deviation from target or set point exists (page 15, Shinskey [1988]) and produces slightly greater mean square error (MSE) at the output than actually required. The rate of change of the output (variable) with respect to time is proportional to the deviation. As mentioned earlier in Section 5.5, the closed-loop gain must be 1 in order to sustain oscillations in the feedback control (closed) loop.

A process variable that has a uniform 'cycle' and sustained oscillations does not threaten the stability of the feedback control system. Under integral control, the (feedback) closed-loop oscillates with uniform amplitude. The feedback loop tends to oscillate at the period where the system gain is unity, the integral (also known as reset) time, that is, the time constant (I) of the controller, then, affects only the period of oscillation which increases with damping for a controller with integral action in a dead-time loop. It can be shown that the 'integral time' (I_u) for zero damping (which requires a loop gain of 1.0), is 0.64PG T_d where PG is the process gain and T_d, the dead time (page 16, Shinskey [1988]). A process with a dead time of 1 minute would cycle with a period of 4 minutes under integral control with oscillations sustained by an integral time constant of about 0.64PG minutes. Control engineers endeavour constantly to have this integral time as nearly equal to the process dead time as possible so that the process variable can take the same path as the dead time. It is possible to achieve damping by reducing the closed-loop (feedback) gain and by increasing the integral time as will be shown in Section 9.5.3.

Sampling actually improves control of a dead-time process that allows a controller with integral action to approach best-possible performance (explained in Section 9.7.1), if its sample interval*(or 'scan, period') is set equal to the process dead time. This loop is even robust (also explained in Section 9.7.2), for increases that occur in dead time as well, by setting the sample interval at the maximum expected dead time. For this, the integral time should be set at the product value of PG and AI (sample period). *Sample interval or scan period of a digital controller, (in process control terminology), is the interval between executions of a digital controller operating intermittently at regular intervals.

The integrating control action is successful in eliminating 'offset' (deviation obtained with proportional control) at the expense of reduced speed of response and increasing the period of the feedback control loop with its phase lag. When integral time is too long, the feedback loop is overdamped, leading to unstable conditions. Moreover, integral mode slows and destabilises a (feedback control) loop (Shinskey [1988]).

Another limitation is that there may be a maximum integral (reset) rate which cannot be exceeded without encountering stability difficulties and which saturates its integral mode when the input exceeds the range of the input manipulated variable. This condition is called 'integral windup' by engineers and results in overshoot before control is restored. Overshoot can be avoided by setting the integral time higher than that required for (load) regulation and can also be minimized by limiting the rate of set-point changes (Shinskey [1988]).

During a step change (change from a steady level of zero by an instantaneous change to a steady level of unity) of the input process variable, the closed-loop output is serially independent when pure one-step minimum mean square control is used which is often the practice in process control operations. A time delay of two time periods was taken care of by considering dead time, b = 2.0, in the feedback control algorithm.

A dynamic element, such as integral, within the domain of (linear) controllers, has both beneficial and undesirable properties. The selection of the control mode requires a prior understanding of the benefits and drawbacks of the control mode. It may once again be emphasised that since there are some criticisms and drawbacks in using (pure) integral control such as integral wind-up and overshoot etc., the objective is in reducing the CESTDDVN (product variability), of the outgoing product quality and so, such criticisms can be put to rest.

7.2.6 Feedback Control Adjustment - Methodology

In practice, the mean square error (MSE) control adjustment algorithm is applied for a trial period to the process whose product variability is required to be reduced. EWMA charts are installed after observing changes in the process that could cause the adjustment algorithm to underestimate (or overestimate) the required control adjustment to make the product quality variable to be exactly on-target and making it possible to realise a reduction in the output control error standard deviation (CESTDDVN). The EWMA process monitoring system notifies, by means of out-of-control signals, the shifts in the quality variable needed to maintain on-target performance. Further reductions in product variability are possible by re-programming the closed-loop control. When the cost of adjustment and cost of sampling are significant, and when the controller tuning appears/(happens) to be excessive or tight, the continuous feedback can be temporarily removed or disconnected from the feedback loop and connected again after a (fairly) short period of time. This course of action may be necessary to prevent overcompensation of the output product variable which is characterised by more variable control errors and more frequent adjustments. A minimum variance feedback control algorithm will bring the process back to set point with less oscillatory behaviour than usually experienced under manual control. It will help also in accomplishing set point changes in a smooth and rapid manner (Shinskey [1988]). It is expected that the time series controller designed and built on the principles enunciated in Sections 7.2.1 and 7.2.4 with dead-time compensation and integral action terms in Equation (5.16) will also possess these characteristics.

7.2.7 Analysis of Simulation Results for Dead Time b = 1

The mean (ME), and standard deviation of the control error, namely, the control error standard deviation (CESTDDVN), and that of the adjustment, (Mdxt) and SDdxt and the values of the process gain (PG) are shown in Table 7.III (Attachment 7.III). In monitoring a closed-loop process operating under a known control algorithm, the 'information' about underlying changes in the process are reflected in the sequences of control actions and the process output. This was found from the values of adjustment (dxt) and adjustment variance (vardxt) from simulation results. The effect of control actions needs to be taken into account by an effective process monitoring scheme.

This information is used to detect process changes by means of EWMA forecasts and gma theta falling outside the control limits. By virtue of the observations made earlier in Section 7.2, a range of minimal control error sigmas (CESTDDVN) are available for values of Θ and hence the rate of drift, r = (1-Θ) in Table 7.III. As the process drift, r, decreases from fast drift, [that is, as the IMA parameter Θ increases from a value of zero, (random walk)], to slow drift, [that is, as Θ becomes closer to the value of 0.70, a non-stationary disturbance], the CESTDDVNs also decrease to a value which is close to 1.0 *when* $\Theta = 0.75 \pm 0.05$*, and the EWMA seems to have good control of the process.* Around this value of Θ, EWMA forecasts are effective in controlling a process. **This inference is made possible because of the fact that the control error sigma achieved in controlling a process with no dead time (and no carry over effects) is 1.0 (page 286, Baxley [1991]).** *Since the CESTDDVN values, (close to the value of 1.0), are obtained for the second-order dynamic process with dead time b = 1, 2, it is possible to achieve good (feedback) control possessing features such as*

(i) permissible gain of the feedback (closed) loop, (ii) stability of the feedback control loop and (iii) precise regulation of loops containing dead time, mentioned in Section 3.2. The range of control error sigmas (CESTDDVN) for corresponding values of Θ and the process drift, r can be used to formulate process regulation schemes. Whole periods of dead time (b = 1, 2) are considered in this thesis for illustration and to avoid complex controller algorithm whose dead-time compensator/Smith predictor changes with the time delay. Hence, it is possible to estimate the best achievable performance when measured by the variance of the output (mean square error), knowing the ratio of the integer portion, (being b = 1 and 2), of the process time delay divided by the control interval, (the AI values), obtained from the simulation results (Harris [1989]). These values of Θ and AI are used to formulate process regulation schemes in Chapter 8.

The control error sigma (SE) and adjustment frequency (AF) obtained by Baxley [1991] for extended simulation runs with IMA parameter Θ = 0.75 for

(i) an EWMA controller are 1.15 and 0.035 for dead time b = 1.0 and controller gain CG = 1.0 with no carry-over effects (for dynamics or inertia $\delta = 0$), and control limits L=3.15 (page 278, Baxley [1991] and

(ii) for a CUSUM controller, the corresponding SE and AF values are 1.13 and 0.064 (page 280, Baxley [1991]) respectively for dead time b = 1.0 and CG = 1.04 again with no carry-over effects and 'h' (the cusum controller tuning parameter which is analogous to the spacing between control limits), equal to 3.98.

The simulation results obtained for the time series controller with IMA parameter $\Theta = 0.75$ for dead time b = 1.0, $\delta_1 = \delta_2 = 0$ are CESTDDVN = 1.0 and AF = 0 (Refer Table 7.II, Attachment 7.II).

The SE and AF obtained by Baxley [1991] with IMA parameter $\Theta = 0.25$ and $\Theta = 0.50$ are (1.77, 0.065) (L = 3.63) and (1.37, 0.059) (L = 3.12) for the EWMA controller and (1.73, 0.105) (h = 3.64) and (1.48, 0.053) (h = 4.50) for the CUSUM controller (extended) simulation runs respectively. The controller gain (CG) in these situations were 0.83 and 0.85 for the EWMA controller and 0.77 and 0.85 obtained with the CUSUM controller. The EWMA controller has no dead-time compensation term and requires a controller gain below one in order to avoid over-control or overcompensation of the output controlled variable. Again, this is one of the issues raised by Kramer (Box and Kramer [1992]) which is taken care of, it is argued, by the time series controller feedback control algorithm with integral and dead-time compensation terms (Equation 5.16).

The corresponding CESTDDVN and AF given by the time series controller are 1.099 and 0.085 for $\Theta = 0.25$, and 1.068 and 0.06 respectively for $\Theta = 0.50$ for inertia

$\delta = 0$, (dead time, $b = 1.0$). In fact, these CESTDDVN values are far below the SE values (1.25 for $\Theta = 0.25$ and 1.118 for $\Theta = 0.50$) presented by Baxley (page 286, [1991]) as indicated earlier in Table 5.I (Chapter 5).

Hence, it can be concluded that the feedback control provided by the time series controller adjustment Equation (5.16) is superior in performance to the performance of the EWMA and CUSUM controllers. The model and controller parameters are given in Table 7.III (Attachment 7.III).

7.3 THE EFFECT OF CONTROL LIMITS ON PRODUCT VARIABILITY

In order to know the effect of the control limits L on varCE and vardxt due to different monitoring intervals (AI's), the increase in varCE for different AI and L are found from Table 7.IV (Attachment 7.IV). The choice of L is based on statistical considerations which have been set to $L = 3\sigma_a$, (3 times the standard deviation of the random shocks). Lower CESTDDVN values for L = 3, is the result of more frequent interventions with smaller adjustment intervals in order to return the process with fast drifts to target. If the adjustments are made automatically, as in some chemical processes involving small adjustment costs, and if the specification limits are narrow relative to process variability, then it may be proper to set a low value for L to minimise control error variability so that the *production of off-quality material will be a minimal.* In this way, increase in product variability due to overcontrol can be avoided by widening the control limits (increasing L). This is substantiated by Box, Jenkins and MacGregor [1974] who showed that the motivation for increasing L is to reduce costs when process adjustments are expensive. On the contrary, if automatic adjustments are not possible and if specification limits are wide, then it may be proper to set L = 3 to minimise adjustment costs. Since the values of L in the simulation are around 3, which in turn depends upon the nature of the drifts, the performance for values of L around 3 are similar to that of using a statistical process control strategy with an ARL of 400 under the assumption that the process is on target with no drifts (Baxley [1991]).

For a process with dynamic parameters δ_1, δ_2 and process gain, PG, (defined in Section 5.5, as 'the eventual effect of a unit change in the input manipulative variable after the dynamic response has been completed'), the value of ω, ('the magnitude of the response to a unit step change in the first period following the dead time'), is equal to

$$PG(1-\delta_1-\delta_2) = [1/(1-\delta_1-\delta_2)]\,[(1-\delta_1-\delta_2)] = 1.$$

This shows that the response to a unit step change is total (100%) and completely reflected in the process due to the dynamic parameters, δ_1,δ_2 and b, the dead time. The dynamic parameters measure the carry-over of the exponential process response into succeeding sample periods. If there are no dynamics in the system (δ is very nearly 0), there are no carry over effects and so PG = ω.

7.4 DEPENDENCE OF ADJUSTMENT VARIANCE ON DYNAMIC PARAMETERS AND PROCESS DRIFT

In Section 5.5, the expressions for δ_1 and δ_2 were shown for a 'critically damped' second-order system to be $\delta_1 = 2e^{-1/\tau}$, $\delta_2 = -e^{-2/\tau}$ where τ is the process time constant. Since $\delta = e^{-T/\tau}$, δ_1 and δ_2 are functions of sampling interval T, the control adjustment action variance (vardxt) depends on the rate of drift, r, and also on the inertial process lags, δ_1 and δ_2.

The expected variance of control adjustment action (vardxt), (from the expression for x_t, Equation (5.16)) is evaluated by using the fact that minimum variance control generates deviations, e_t, from target that are equivalent to the random shocks $\{a_t\}$.

That is,

$$x_t = \frac{(1-\Theta)}{PG(1-\delta_1-\delta_2)}(1-\delta_1 B-\delta_2 B^2)a_t$$

The second term on the right is in the form of a general ARIMA(2,0,1) process.

It can be shown that the variance of this process is

$$= \frac{(1-\delta_2)\,\sigma_a^2}{(1-\delta_1-\delta_2)(1+\delta_1-\delta_2)(1+\delta_2)}.$$

[Refer to Equation (3.2.28), page 62, Box and Jenkins [1970, 1976] which gives the expression for the variance of the second-order Autoregressive process described by Equation (3.2.17), page 58 in the above monograph].

So,

$$\mathrm{Var}(x_t) = \mathrm{Var}[\frac{(1-\Theta)(1-\delta_1 B-\delta_2 B^2)}{PG(1-\delta_1-\delta_2)}a_t]$$

$$= \frac{(1-\delta_2)(1-\Theta)^2\,\sigma_a^2}{PG^2(1-\delta 1-\delta_2)^2(1-\delta_1-\delta_2)(1+\delta_1-\delta_2)(1+\delta_2)}$$

$$= \frac{(1-\delta_2)r^2}{(1-\delta_1-\delta_2)(1+\delta_1-\delta_2)(1+\delta_2)\,\omega^2}$$

$$= \frac{(1-\delta_2)r^2}{(1-\delta_1-\delta_2)(1+\delta_1-\delta_2)(1+\delta_2)},$$

(7.1)

because of the fact that $\sigma_a = 1.0$, $r = 1-\Theta$ and $PG(1-\delta_1-\delta_2) = \omega = 1$.

Table 7.IV (Attachment 7.IV) shows the adjustment variance (vardxt) values for values of dead time, b = 1 and for values of δ_1 and δ_2 that satisfy stability conditions. It

can be shown that the values of vardxt obtained from simulation of Equation (5.16) compare reasonably well with the numerical values calculated from Equation (7.1) above. Since δ_1 and δ_2 are functions of the monitoring (sampling) interval, the effect of inertia may be reduced by lengthening the monitoring interval. The adjustment variance is a minimum (0.003) for large monitoring intervals (52.6) such as for example, for $\Theta = 0.75$, $\delta_1 = -1.82$ and $\delta_2 = -0.83$ and for dead time, b = 1.0.

From Table 7.IV, it is observed also that for an increase in varCE and a particular value of Θ, large values of δ_1 and δ_2 yield considerable reductions in the adjustment control action variance (vardxt). The dependence on past control actions increases as δ_1 and δ_2 get larger which is in agreement with the observation of Kramer [page 157, 1990].

For varCE equal to 1.002 and $\Theta = 0.70$, the (longest) monitoring interval (58.8) occurs when $\delta_1 = -1.82$ and $\delta_2 = -0.83$ *When δ_1 and δ_2 are both equal to 0, the control adjustment action (dxt) leads to an immediate adjustment in the controller set point and there is no bias due to the process dynamics. Longer monitoring intervals are possible as the bias due to the process dynamics is reduced.* Plots of CESTDDVN and AI = 1/MFREQ against Θ for various values of the parameters $\delta_1$and δ_2 and dead time b = 1 are shown in Figures 7.3 (Attachment 7.VIII) and 7.4 (Attachment 7.IX). From Figure 7.3 for plots of Θ versus CESTDDVN, it can be seen that as the values of δ_1, δ_2 approach 0, the plot becomes flatter and almost straight for δ_1, δ_2 values equal to 0. The peaks in Figure 7.4 for plots of Θ versus AI are due to the process requiring adjustments, (in fact, zero adjustments, dxt = 0), after long sampling periods, say, 500 or even 1000 adjustment intervals for $\Theta = 0.70$ and 0.75. This is because the EWMA has a fairly good control of the process around these Θ values. It is shown also that the control adjustment action variance (vardxt) is a fraction of the adjustment variance obtained when the control is minimum variance for large δ_1 and δ_2 and small values of r (large Θ values). When r is small and the drifts are slow, long monitoring intervals can be considered for increases in varCE leading to reductions in the control adjustment action variance (vardxt). The amount of increase in CESTDDVN by extending AI's is the same irrespective of dead time as this does not contribute to increase in control error sigma, this can be seen from the results.

Minimum variance control requires large alternating control actions to give minimum output variance when δ_1 and δ_2 are large. The alternating character is eliminated by allowing slight increases in output variance (varCE). Substantial reductions in control action variance can be achieved for minor increases in output variance by constraining gma theta. By means of this constraint, minimum variance (or MMSE) control for minimising control actions was achieved independent of the monitoring interval.

7.5 CONSTRAINED VARIANCE CONTROL

7.5.1 Constrained Minimum Variance Control and Control Action Variance

The objective in schemes that employ minimum variance control, is to find a control scheme such that $E(e_t^2)$ is minimised. The aim in constrained variance control schemes is to find a control strategy that minimises gma theta subject to a constraint that the calculated individual gmas are less than some limit, say L where L denotes the number of multiples of σ_a, ($L = 3\sigma_a$, 3 times the standard deviation of the random component of the disturbance) used for EWMA control limits. These control limits were set as in Equations (6.1) and (6.2). As the control limits increase, more emphasis is placed on reducing the variance of control adjustment actions and less on the variance of deviations from target or output variances, (also termed by Kramer as MSD, mean squared deviation from target) (page 155, Kramer [1990]). Following the principle of Baxley [1991] in the EWMA simulation programme (page 293, Baxley [1991]), the gma theta is constrained to be the same as the IMA parameter theta, (the process drift, r, being equal to 1-Θ), for the disturbance in the simulation of the stochastic feedback control Equation (5.16), (Fortran 77L computer programme, Attachmant 6.I). Table 7.IV (Attachment 7.IV) shows that the constrained variance control scheme generates a smaller control adjustment action variance (vardxt) than that would have been possible by minimum variance control at the expense of a larger output variance (varCE). The position of the control limits which determined the control scheme changed also with Θ as can be seen from Equations (6.1) and (6.2). Table 7.IV is used to identify a range of L that will yield an increase in varCE and the corresponding increase in control action adjustment variance for a combination of Θ, δ_1 and δ_2. The constrained variance control scheme is determined once a value of L is chosen, usually corresponding to a particular increase in varCE. From Table 7.IV, it is found that *the constrained control adjustment action variance can be reduced by 0.7 % of the minimal variance control action variance (namely 0), when a 0.2 % increase in varCE can be allowed in the final product (*for $\Theta = 0.70$, $\delta_1 = -1.82$ and $\delta_2 = -0.83$ for dead time, $b = 1.0$*)*. The value of L corresponding to this scheme is between 3.04 and 2.97. It can be shown that substantial reductions in vardxt could be achieved for, in some instances, with minor increases in varCE by using a constrained control scheme. Tables 7.IV (Attachment 7.IV) and 7.VI (Attachment 7.VI) give information on AIs (adjustment intervals) for a certain reduction in vardxt which will result in an increase in varCE. In some situations, there may be substantial increases in varCE due to the control scheme being based only on the deviations associated with the AI's. *With higher AI's, it is possible to have larger L and hence the process is allowed to drift farther from target before an adjustment is made, causing an increase in varCE.*

7.6 EFFECT OF INCREASE IN DEAD TIME ON CONTROL ERROR STANDARD DEVIATION (PRODUCT VARIABILITY) AND ADJUSTMENT INTERVAL

Dead time and inertia have a bearing and influence on determining the adjustment interval (AI) when a process is drifting. Tables 7.II (Attachment 7.II) and 7.V (Attachment 7.V) are used for discussing the effect of increase in dead time from $b = 1$ to $b = 2$. For a set of values of $\delta_1 = -1$, $\delta_2 = -0.25$ and $\Theta = 0.05$, the CESTDDVN and AI from Table 7.II are 1.039 and 9.8. The corresponding values for dead time, $b = 2$ from Table 7.V are 1.567 and 55.56. On comparing these sets of values, it is found that for a process with the same parameters, $\delta_1 = -1$, $\delta_2 = -0.25$ and Θ, the CESTDDVN has increased from 1.039 to 1.567. However, the process gain is the same in both cases showing that dead time does not contribute to the steady-state feedback control (closed-loop) gain or process gain. This means that *dead time offers no gain contribution.* However, it has increased CESTDDVN from 1.039 for $b = 1$ to a value of 1.567 for $b = 2$. The corresponding adjustment intervals (AIs) being, 9.8 for $b = 1$ and 55.56 for $b = 2$.

It can also be shown that the penalty for a process with dead time is more severe when the process has fast drifts than when the drifts are slow. This can be seen by considering the decrease in the rate of drift from $r = 0.95(\Theta = 0.05)$ to $r = 0.30$

(Θ = 0.70), the CESTDDVNs achieved are near 1.0 and comparable to a process with no dead time. As the drifts further decrease and become slow, the EWMA weights have less effect on the forecasts and there is a slight increase in CESTDDVN.

The situation is similar with the adjustment intervals. In these situations, if the control scheme is based only on the deviations associated with the adjustment intervals (monitoring periods), there is an (substantial) increase in the output variance (varCE). For a dead time, b = 1, the adjustment interval (AI) is 0 for δ_1 = -1.82, δ_2 = -0.83 and Θ = 0.05. On comparing these sets of values, it is found that for a process with the same parameters, δ_1 = -1.82 , δ_2 = -0.83 and Θ, the AI has increased from 0 to 52.63 for b = 2.0. Again, as the drifts decrease and reach about (0.30) (Θ = 0.70), the EWMA has effective control of the process and the process requires no adjustment (AI = 0). As the drifts decrease still further and the process becomes almost stationary, larger adjustment intervals are required to bring the already stationary process back to control. The AI's decrease from large values for a process with fast drift to a process with slow drift, as also do the CESTDDVN's. It is observed from Table 7.IV (Attachment 7.IV) that for longer adjustment (monitoring) intervals, both the output variance (varCE) and the control action (adjustment) variance (vardxt) are large. *It is required to know how sensitive the simulation results are to the assumptions made in deriving the feedback control algorithm. It may be a matter of surprise to note that small values of CESTDDVN could be associated with large AI's.*

It can be seen that the output variance (varCE) depends on the IMA parameter Θ and in turn, the rate of drift of the process, r, whereas the control action (adjustment) variance (vardxt) depends on the process dynamics parameters as well as the process drift, r, as shown in Section 7.4. It is shown that as Θ gets larger and the parameters (δ_1,

δ_2,b) remain constant, changes in the monitoring interval (AI) have a smaller effect on the output variance when Θ is near the value of 1.0 (slow drifts), (for example, 0.95, that is, when the process is tending to become stationary), than when it takes smaller values such as 0.05 (fast drifts) (random walk). From Table 7.II (Attachment 7.II), it can be seen that for values of r = 0.95, 0.25, the AI's are 0 and 1.62. As r decreases from 0.95 to 0.05, the AI increases from 0 to 8.62 showing that larger adjustment intervals are required for controlling slow drifts and smaller AI's are required for fast drifts and it is comparatively easier to control fast drifts than slow drifts. This might be expected since when the process drifts are slow, the disturbance is almost stationary and it may be practically possible and sufficient to adjust a process which is already nearly under control after a long adjustment interval.

7.7 PROBABILITY MODEL FOR FEEDBACK CONTROL ADJUSTMENT

A probability model for feedback control adjustment (x_t) in the input manipulated variable is given in this Section. After making an adjustment, the CESTDDVN of the output quality variable is checked for any off-quality product and if the product is within specification limits, the process is continued. Otherwise, a sample is taken based on the number of AI's from the simulation results and the process adjusted.

The test for feedback control is that an adjustment is either made or not following the gma theta falling outside the control limits L given by Equations 6.1 and 6.2. The input control adjustment is assumed to be say 'α' units of adjustment made at a certain instant in a process under observation and control. Also, depending on the amount of feedback, the adjustment quantity changes and differs from the previous one.

Here, the feedback is characterised by two probabilities. Firstly, the probability that an adjustment will make the mean of the output variable on target conditional upon it being not currently on target and secondly, the probability that there will not be an adjustment that will make the output variable move away from target conditional upon it already being on target.

Let p = Pr(Adjustment /Mean Not On Target), i.e., Pr (A/MNOT), where A denotes an adjustment requirement.

Let q = Pr(No Adjustment/Mean On Target), i.e., Pr(NA/MOT), where NA denotes the stage of the process requiring no adjustment.

Clearly, p+q = 1

The units of adjustment is characterised by α, where

α = Pr(MNOT), that is, Pr(Mean Not On Target).

The ideal situation will be when the adjustment units, $\alpha = 0$.

p and q are the properties of adjustment applied to the feedback, mean not on target (MNOT) and mean on target (MOT) respectively and α units of adjustment is an adjustment measure.

The larger the deviation (the error), the greater is the opportunity for feedback control to adjust the process and so, the feedback will be associated with a larger α level.

The basic feedback control adjustment is shown in Figure 7.5.

If the adjustment is not complete, there will be out of control signals again appearing on the EWMA chart.

Interest is focused on:-

(i) The required adjustment, that is α, to avoid out of control signals and further adjustment;

(ii) The values of p and q for the required feedback control adjustment.

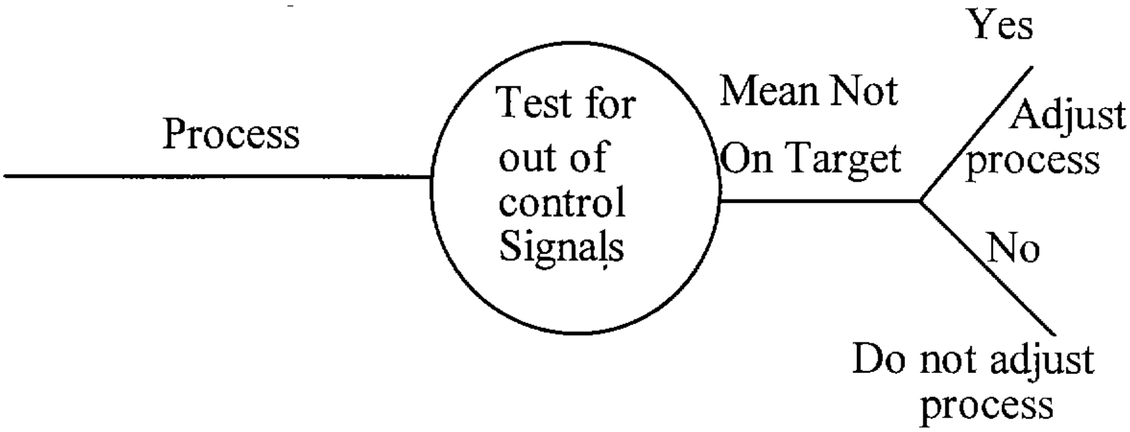


Figure 7.5 Basic probability test for feedback adjustment

A basic probability model is developed in which either an adjustment is not made or made depending upon the process conditions, that is, either the process is in control or out of control, identified by out of control signals. If an adjustment is required, the process is not in control and the mean of the product quality variable is not at or near target.

Then, the probability that a feedback control with α units of adjustment will bring the mean closer to target is given by

$$\Pr(\text{No Adjustment}) = \Pr(NA/MOT)\Pr(MOT) + \Pr(NA/MNOT)\Pr(MNOT)$$

$$= q(1-\alpha) + (1-p)\alpha. \tag{7.2}$$

$$\Pr(\text{Adjustment}) = \Pr(A/MOT)\Pr(MOT) + \Pr(A/MNOT)\Pr(MNOT)$$

$$= (1-q)(1-\alpha) + p\alpha. \tag{7.3}$$

It is obvious that Pr(No Adjustment) + Pr(Adjustment) = 1

Having adjusted or not adjusted the process, a new feedback control is imminent depending on the position of the product mean from the target. In the case of no adjustment, the appropriate α value, is given by Pr(MNOT/No Adjustment); that is from Equation (7.2),

$$\Pr(\text{MNOT/NA}) = \frac{[\Pr(\text{NA / MNOT})][\Pr(\text{MNOT})]}{\Pr(\text{NA})}$$

$$= \frac{(1-p)\alpha}{q(1-\alpha)+(1-p)\alpha}. \tag{7.4}$$

Using Equation (7.4), the approximate value of α for the situations when the mean is not on target and hence requires adjustment is given by

$$\frac{\Pr(\text{MNOT})}{\Pr(\text{Adjustment})} = \frac{\Pr(\text{Adjustment / MNOT})\Pr(\text{MNOT})}{\Pr(\text{Adjustment})}$$

$$= \frac{p\alpha}{(1-q)(1-\alpha)+p\alpha}. \tag{7.5}$$

Equations (7.4) and (7.5) provide the appropriate measures of the probability of the mean not on target situations which would apply to further adjustments when the mean is either on target or not on target. Equations (7.3), (7.4) and (7.5) provide the basis for probabilistic expressions of interest in an adjustment situation.

An expression for a likelihood estimate of the parameters α, p and q is not warranted or required to be developed at this stage since the feedback control adjustment requires that the adjustment α units exactly compensates for the deviation from target and the disturbance bringing the mean closer to or on target and the process under control. Alternatively, the parameters α, p and q can be approximately estimated for a process based upon a batch of test data collected on a sample test of the process, noting the number of times the process has to be adjusted for 'mean not on target' situations. A mathematical expression for the likelihood of the observations can then be maximised with respect to the parameters α, p and q to provide a maximum likelihood estimate of these parameters. If the value of α is found to be high, meaning that the probability of the mean of the product quality variable is not on target consistently for a considerably long time, requiring frequent adjustments, then, the production process would likely be investigated to identify and correct the underlying problem.

Knowing the required adjustment (x_t) from the algorithm (or α in the probability model), in the input manipulated variable, the probability of the mean not on target and hence requiring adjustment can be calculated by applying Equation (7.5).

7.8 REGRESSION ANALYSIS

A simple regression analysis was performed on the simulation results for control error standard deviation (CESTDDVN) and its dependence on the parameters Θ, δ_1 and δ_2. Table 7.VII (Attachment 7.VII) shows the coefficients of the fitted model for CESTDDVN and Table 7.VIII, the analysis of variance. The linear regression model is appropriate for the values of the control error standard deviation (CESTDDVN) obtained via simulation. The coefficients of the dynamic process parameters Θ, δ_1 and δ_2 are significantly different from 0. A value of 71.1% for the residual sum of squares (adjusted) suggests that 71% of the total variability in the observed response (CESTDDVN) is explained by the model thus indicating a good fit. It can be seen that there is no strong evidence of lack of fit for the model for CESTDDVN.

The adjustment interval changes also depending on the values of the dynamic parameters δ_1 and δ_2 and on the value of the IMA parameter Θ for fast and slow process drifts. The actual regression analysis for AI shows a varying non-linear relationship with the process parameters and indicates only a value of 35% for the residual sum of squares. For these reasons, the regression analysis results are not pursued for the adjustment interval (AI).

7.9 DISCUSSION OF SIMULATION RESULTS

The characteristics of the simulation change when the rate of drift either increases or decreases or when there is a change in dead time or in the dynamic properties of the process. From simulation of the stochastic feedback control algorithm (Equation 5.16), the amount of control adjustment action, the frequency of adjustment (MFREQ) and AI, the adjustment interval (AI = 1/MFREQ), can be found as also can the output control error sigma for a particular rate of drift (r).

A dead-time compensation scheme which provides a process gain (PG) in the feedback path whose value depends on both the process output and model has been devised. This scheme is suited to use in situations where the process dead time results from a measurement device in a laboratory and is a known quantity. A process control approach to product quality based on discrete laboratory data has the potential for improvements (in product quality). A practical control strategy would then be (i) based on the use of quality control laboratory analyses and (ii) based on a time series analysis of plant data collected from a designed closed-loop experiment and using the laboratory data to update the set point of the minimum variance time series controller to verify the quality of outgoing product.

The intelligent use of any control algorithm is an iterative process which depends upon the skill and judgement of the designer. Most controller designs involve compromises and intelligent choices. For successful use of complex algorithms, it is imperative that results are presented to the designer in such a way that they are assisted in making compromises intelligently. This allows for more efficient use of the designer's judgement and experience. Competent use of modern control theory involves the use of principles of interactive trial and error that can be successfully utilised to improve practical controller design. Algorithms derived from stochastic optimal control theory have the potential for efficient control, and modern control methods are useful tools in an iterative design method. Stochastic (optimal) design algorithms provide valuable clues as to the controller structure and an understanding of the controller design. These clues lead to controller designs which appear to be better than those obtained by conventional approaches. If proper account is taken of stability (as done in this formulation), the resulting design will lead to (efficient) controllers that are not sensitive to the dynamic model (exact form of the transfer function) of the process or to small perturbations in its parameters. Practical control strategies result from employing an appropriate model for the process dynamics (such as the second-order model considered) and disturbances (ARIMA (0,1,1)).

It has been reported in the literature that the performance of (optimal) algorithms for sampled data controllers appear to be sensitive to the structure of the disturbance but do not seem to be sensitive to the parameters of the (noise) model. The dynamics may be known from earlier experimental and theoretical work. In these cases, the suggested second-order model can be used with appropriate dynamic parameters set to their pre-determined values since (optimal) algorithms appear also to be sensitive to small deviations between the real process model and the model used for controller design. Process dead time can be determined from the process step response under manual control. The control algorithm thus derived has not only desirable properties (integral control and adequate dead-time compensation) but is also practically 'robust' in nature to the basic assumptions made in developing the control algorithm.

It may be difficult to obtain satisfactory control due to process characteristics and the control system's inability to operate in a proper manner. Some of these difficulties may be overcome by (i) a thorough analysis of the process and periodical inspection of the output controlled product variable, (ii) a tabulation of various rates and magnitudes of load changes, (change in process conditions requiring a change in the average value of the input manipulated variable to maintain the output controlled variable at the desired value or target), (iii) a study of the degree of the process lags, (retardation or delay in response of output controlled variable at point of measurement to a change in value of input manipulated variable), and (iv) an observation of the relation among these factors. It may be also difficult at times to assign a proper relationship between the output controlled variable and the state of balance of a process which may not be satisfactory even though there is satisfactory control of that (output) controlled variable. Under these circumstances, it is important to control directly from the final output product in order to eliminate the possibility of any variance between the controlled variable and the control conditions of a process. It may be difficult to maintain the balance of a process when the load or any of the uncontrolled variables associated with the process are subject to frequent and fast changes. The deviation of the controlled variable is in direct proportion to the rate of the changes. In practice, it is found that it is often necessary to add a controller to the changing variable in order to eliminate its effect on the control of the main process variable. For example, in large (ceramic) kilns and furnaces, it is necessary to control the pressure in the furnace in order that the furnace temperature may be sufficiently stabilised.

Maximum efforts should be made to eliminate the effect of supply variations and disturbances. It is possible to improve the control of a process by making minor alterations or by redesigning to obtain smaller lag due to the inertial characteristics of the process or shorter dead time. The process improvement may be handicapped unless these steps are undertaken with careful consideration of the effects of dynamic process reactions. By a re-arrangement of the supply medium, it may sometimes be possible to reduce transfer lag, caused by large temperature or pressure differences.

The most serious lag in automatic control, namely, the dead time, must be kept to a minimum in the controlled system. This type of lag should be investigated to determine if it can be reduced or eliminated.

7.10 CONCLUSION

The effects of the rate of process drift, r, on the control error standard deviation (CESTDDVN) have been discussed in this Chapter. The simulation results were analysed and the performance measures of the time series controller discussed. The combined effects of the process dynamics and dead time on CESTDDVN and AI were explained for $b = 1$ and the effect of increase in dead time from $b = 1$ to $b = 2$ on CESTDDVN and AI. The benefits and limitations of integral control were briefly discussed along with details of a constrained variance control scheme. It was shown that the EWMA has fairly good control of the process for values of Θ in the interval 0.75 ± 0.05. An outline of a process regulation scheme is given in Chapter 8

Table 7.1 Values of δ_1 and δ_2 for a second-order dynamic system satisfying feedback control stability under critically damped conditions

δ_1	δ_2	δ_1	δ_2	δ_1	δ_2	δ_1	δ_2
2.00	-1.00	1.55	-0.61	1.16	-0.34	-0.72	-0.13
1.99	-1.00	-1.55	-0.60	-1.15	-0.33	0.71	-0.13
1.98	-0.99	1.54	-0.60	1.14	-0.33	-0.70	-0.12
1.97	-0.98	-1.52	-0.58	-1.13	-0.32	-0.69	-0.12
1.96	-0.97	1.51	-0.58	1.12	-0.32	0.68	-0.12
1.95	-0.96	-1.51	-0.57	-1.11	-0.31	-0.67	-0.11
1.94	-0.95	1.50	-0.57	1.10	-0.31	-0.66	-0.11
1.93	-0.94	-1.50	-0.56	-1.10	-0.30	0.65	-0.11
1.92	-0.93	1.49	-0.56	1.09	-0.30	-0.64	-0.10
1.91	-0.92	-1.48	-0.55	-1.08	-0.29	-0.63	-0.10
1.90	-0.91	1.47	-0.55	1.07	-0.29	0.62	-0.10
1.89	-0.90	-1.47	-0.54	-1.06	-0.28	-0.60	-0.09
-1.82	-0.83	1.46	-0.54	1.05	-0.28	0.59	-0.09
1.81	-0.83	-1.44	-0.52	-1.04	-0.27	-0.57	-0.08
-1.81	-0.82	1.43	-0.52	1.03	-0.27	-0.56	-0.08
1.80	-0.82	-1.43	-0.51	-1.02	-0.26	0.55	-0.08
-1.80	-0.81	1.42	-0.51	1.01	-0.26	-0.53	-0.07
1.79	-0.81	-1.40	-0.49	-1.00	-0.25	-0.52	-0.07
-1.79	-0.80	1.39	-0.49	0.99	-0.25	0.51	-0.07
1.78	-0.80	-1.37	-0.47	-0.98	-0.24	-0.50	-0.06
-1.78	-0.79	1.36	-0.47	0.97	-0.24	-0.49	-0.06
1.77	-0.79	-1.36	-0.46	-0.96	-0.23	-0.48	-0.06
-1.73	-0.75	1.35	-0.46	0.95	-0.23	0.47	-0.06
1.72	-0.75	-1.34	-0.45	-0.94	-0.22	-0.45	-0.05
-1.72	-0.74	1.33	-0.45	0.93	-0.22	-0.44	-0.05
1.71	-0.74	-1.33	-0.44	-0.92	-0.21	0.43	-0.05
-1.71	-0.73	1.32	-0.44	0.91	-0.21	-0.41	-0.04
1.70	-0.73	-1.31	-0.43	-0.90	-0.20	-0.40	-0.04
-1.70	-0.72	1.30	-0.43	-0.89	-0.20	-0.39	-0.04
1.69	-0.72	-1.30	-0.42	0.88	-0.20	0.38	-0.04
-1.66	-0.69	1.29	-0.42	-0.87	-0.19	-0.36	-0.03
1.65	-0.69	-1.28	-0.41	0.86	-0.19	-0.35	-0.03
-1.65	-0.68	1.27	-0.41	-0.85	-0.18	-0.34	-0.03
1.64	-0.68	-1.25	-0.39	0.84	-0.18	0.33	-0.03
-1.64	-0.67	1.24	-0.39	-0.83	-0.17	-0.30	-0.02
1.63	-0.67	-1.23	-0.38	-0.82	-0.17	-0.29	-0.02
-1.61	-0.65	1.22	-0.38	0.81	-0.17	-0.28	-0.02
1.60	-0.65	-1.22	-0.37	-0.80	-0.16	-0.27	-0.02
-1.60	-0.64	1.21	-0.37	0.79	-0.16	0.26	-0.02

δ_1	δ_2	δ_1	δ_2	δ_1	δ_2	δ_1	δ_2
1.59	-0.64	-1.20	-0.36	-0.78	-0.15	-0.22	-0.01
-1.59	-0.63	1.19	-0.36	-0.77	-0.15	-0.21	-0.01
1.58	-0.63	-1.18	-0.35	0.76	-0.15	-0.20	-0.01
-1.56	-0.61	1.17	-0.35	-0.75	-0.14	-0.19	-0.01

δ_1	δ_2
-0.18	-0.01
0.17	-0.01
-0.10	0.00
-0.09	0.00
-0.08	0.00
-0.07	0.00
-0.06	0.00
-0.05	0.00
-0.04	0.00
-0.03	0.00
-0.02	0.00
-0.01	0.00
0.00	0.00

Table 7.II Time Series Controller Performance Measures for Θ, δ_1, δ_2 and dead time b = 1.0

Θ	r = 1-Θ	δ_1	δ_2	δ_2-δ_1	MFREQ	AI = 1/MFQ	CESTDDVN
0.05	0.95	-1.82	-0.83	0.99	0.000	0.000	1.000
0.10	0.90	-1.82	-0.83	0.99	0.186	5.370	1.046
0.15	0.85	-1.82	-0.83	0.99	0.181	5.520	1.040
0.20	0.80	-1.82	-0.83	0.99	0.172	5.810	1.035
0.25	0.75	-1.82	-0.83	0.99	0.086	11.62	1.018
0.70	0.30	-1.82	-0.83	0.99	0.017	58.82	1.001
0.75	0.25	-1.82	-0.83	0.99	0.019	52.63	1.002
0.80	0.20	-1.82	-0.83	0.99	0.001	1000	1.000
0.95	0.05	-1.82	-0.83	0.99	0.116	8.620	1.000
0.05	0.95	-1.00	-0.25	0.75	0.102	9.800	1.039
0.10	0.90	-1.00	-0.25	0.75	0.094	10.63	1.030
0.15	0.85	-1.00	-0.25	0.75	0.066	15.15	1.033
0.20	0.80	-1.00	-0.25	0.75	0.066	15.15	1.026
0.25	0.75	-1.00	-0.25	0.75	0.050	20.00	1.030
0.70	0.30	-1.00	-0.25	0.75	0.001	1000	1.002
0.75	0.25	-1.00	-0.25	0.75	0.000	0.000	1.000
0.80	0.20	-1.00	-0.25	0.75	0.500	2.000	1.007
0.95	0.05	-1.00	-0.25	0.75	0.389	2.570	1.001
0.05	0.95	-0.27	-0.02	0.25	0.190	5.260	1.157
0.10	0.90	-0.27	-0.02	0.25	0.164	6.090	1.083
0.15	0.85	-0.27	-0.02	0.25	0.105	9.520	1.089
0.20	0.80	-0.27	-0.02	0.25	0.086	11.62	1.082
0.25	0.75	-0.27	-0.02	0.25	0.075	13.33	1.073
0.70	0.30	-0.27	-0.02	0.25	0.002	500.0	1.033
0.75	0.25	-0.27	-0.02	0.25	0.000	0.000	1.000
0.80	0.20	-0.27	-0.02	0.25	0.480	2.080	1.012
0.95	0.05	-0.27	-0.02	0.25	0.213	4.690	1.002
0.05	0.95	-0.10	0.00	0.10	0.199	5.020	1.139
0.10	0.90	-0.10	0.00	0.10	0.138	7.240	1.134
0.15	0.85	-0.10	0.00	0.10	0.120	8.330	1.115
0.20	0.80	-0.10	0.00	0.10	0.089	11.23	1.146
0.25	0.75	-0.10	0.00	0.10	0.061	16.39	1.109
0.70	0.30	-0.10	0.00	0.10	0.002	500.0	1.040
0.75	0.25	-0.10	0.00	0.10	0.002	500.0	1.011
0.80	0.20	-0.10	0.00	0.10	0.355	2.810	1.022
0.95	0.05	-0.10	0.00	0.10	0.187	5.340	1.004

Θ	$r = 1-\Theta$	δ_1	δ_2	$\delta_2-\delta_1$	MFREQ	AI = 1/MFQ	CESTDDVN
0.05	0.95	0.00	0.00	0.00	0.224	4.460	1.217
0.10	0.90	0.00	0.00	0.00	0.127	7.870	1.202
0.15	0.85	0.00	0.00	0.00	0.162	6.170	1.165
0.20	0.80	0.00	0.00	0.00	0.118	8.470	1.161
0.25	0.75	0.00	0.00	0.00	0.085	11.76	1.099
0.70	0.30	0.00	0.00	0.00	0.001	1000	1.010
0.75	0.25	0.00	0.00	0.00	0.000	0.000	1.000
0.80	0.20	0.00	0.00	0.00	0.000	0.000	1.000
0.90	0.10	0.00	0.00	0.00	0.121	8.260	1.010
0.95	0.05	0.00	0.00	0.00	0.125	8.000	1.005

Attachment 7.III

Table 7.III Model and Controller Parameters

Model parameters — Controller parameters

Second order model with dead time b = 1.0

theta = 0.05 delta1 = -1.82 delta2 = -0.83 — PG = 0.27 CG = 1.0

VARIABLE	j(No.of runs)	MEAN	STD.DVN
Control error (E)	10000	-0.012	1.000
Adjustment (dxt)	10000	0.000	0.000
FREQ	10000	0.000	0.000

theta = 0.05 delta1 = -1.00 delta2 = -0.25 — PG = 0.44 CG = 1.0

VARIABLE	j (No.of runs)	MEAN	STD.DVN
Control error (E)	10000	0.138	1.039
Adjustment (dxt)	10000	-0.072	0.450
FREQ	10000	0.102	0.303

theta = 0.05 delta1 = -0.27 delta2 = -0.02 — PG = 0.78 CG = 1.0

VARIABLE	j (No.of runs)	MEAN	STD.DVN
Control error (E)	10000	0.311	1.157
Adjustment (dxt)	10000	-0.096	0.558
FREQ	10000	0.190	0.393

theta = 0.05 delta1 = -0.10 delta2 = 0.00 — PG = 0.91 CG = 1.0

VARIABLE	j (No.of runs)	MEAN	STD.DVN
Control error (E)	10000	0.371	1.139
Adjustment (dxt)	10000	-0.088	0.522
FREQ	10000	0.199	0.399

theta = 0.05 delta1 = 0.00 delta2 = 0.00 — PG = 1.00 CG = 1.0

VARIABLE	j (No.of runs)	MEAN	STD.DVN
Control error (E)	10000	0.447	1.217
Adjustment (dxt)	10000	-0.100	0.568
FREQ	10000	0.224	0.417

theta = 0.25 delta1 = -1.82 delta2 = -0.83 — PG = 0.27 CG = 1.0

VARIABLE	j (No.of runs)	MEAN	STD.DVN
Control error (E)	10000	0.033	1.018
Adjustment (dxt)	10000	-0.074	0.404
FREQ	10000	0.086	0.280

theta = 0.25 delta1 = -1.00 delta2 = -0.25 — PG = 0.44 CG = 1.0

VARIABLE	j(No.of runs)	MEAN	STD.DVN
Control error (E)	10000	0.104	1.030
Adjustment (dxt)	10000	-0.027	0.248
FREQ	10000	0.050	0.217

theta = 0.25 delta1 = -0.27 delta2 = -0.02 PG = 0.78 CG = 1.0

VARIABLE	j (No.of runs)	MEAN	STD.DVN
Control error (E)	10000	0.272	1.073
Adjustment (dxt)	10000	-0.025	0.256
FREQ	10000	0.075	0.264

---

theta = 0.25 delta1 = -0.10 delta2 = 0.00 PG = 0.91 CG = 1.0

VARIABLE	j (No.of runs)	MEAN	STD.DVN
Control error (E)	10000	0.361	1.109
Adjustment (dxt)	10000	-0.020	0.226
FREQ	10000	0.061	0.240

---

theta = 0.25 delta1 = 0.00 delta2 = 0.00 PG = 1.00 CG = 1.0

VARIABLE	j (No.of runs)	MEAN	STD.DVN
Control error (E)	10000	0.332	1.099
Adjustment (dxt)	10000	-0.022	0.242
FREQ	10000	0.085	0.279

---

theta = 0.70 delta1 = -1.82 delta2 = -0.83 PG = 0.27 CG = 1.0

VARIABLE	j (No.of runs)	MEAN	STD.DVN
Control error (E)	10000	-0.003	1.001
Adjustment (dxt)	10000	-0.006	0.082
FREQ	10000	0.017	0.128

---

theta = 0.70 delta1 = -1.00 delta2 = -0.25 PG = 0.44 CG = 1.0

VARIABLE	j (No.of runs)	MEAN	STD.DVN
Control error (E)	10000	0.073	1.002
Adjustment (dxt)	10000	0.000	0.008
FREQ	0.001	0.024	

---

theta = 0.70 delta1 = -0.27 delta2 = -0.02 PG = 0.78 CG = 1.0

VARIABLE	j (No.of runs)	MEAN	STD.DVN
Control error (E)	10000	0.230	1.033
Adjustment (dxt)	10000	0.000	0.014
FREQ	10000	0.002	0.045

---

theta = 0.70 delta1 = -0.10 delta2 = 0.00 PG = 0.91 CG = 1.0

VARIABLE	j (No.of runs)	MEAN	STD.DVN
Control error (E)	10000	0.200	1.040
Adjustment (dxt)	10000	0.000	0.014
FREQ	0.002	0.042	

---

theta = 0.70 delta1 = 0.00 delta2 = 0.00 PG = 1.00 CG = 1.0

VARIABLE	j (No.of runs)	MEAN	STD.DVN
Control error (E)	10000	0.081	1.010
Adjustment (dxt)	10000	0.000	0.011
FREQ	10000	0.001	0.033

---

theta = 0.75 delta1 = -1.82 delta2 = -0.83 PG = 0.27 CG = 1.0

VARIABLE	j (No.of runs)	MEAN	STD.DVN
Control error (E)	10000	0.029	1.002
Adjustment (dxt)	10000	-0.004	0.058
FREQ	10000	0.019	0.135

---

theta = 0.75 delta1 = -1.00 delta2 = -0.25 PG = 0.44 CG = 1.0

VARIABLE	j (No.of runs)	MEAN	STD.DVN
Control error (E)	10000	-0.002	1.000
Adjustment (dxt)	10000	0.000	0.000
FREQ	10000	0.000	0.000

---

theta = 0.75 delta1 = -0.27 delta2 = -0.02 PG = 0.78 CG = 1.0

VARIABLE	j (No.of runs)	MEAN	STD.DVN
Control error (E)	10000	0.002	1.000
Adjustment (dxt)	10000	0.000	0.004
FREQ	10000	0.000	0.010

---

theta = 0.75 delta1 = -0.10 delta2 = 0.00 PG = .91 CG = 1.0

VARIABLE	j (No.of runs)	MEAN	STD.DVN
Control error (E)	10000	0.093	1.011
Adjustment (dxt)	10000	0.000	0.009
FREQ	10000	0.002	0.040

---

theta = 0.75 delta1 = 0.00 delta2 = 0.00 PG = 1.00 CG = 1.0

VARIABLE	j (No.of runs)	MEAN	STD.DVN
Control error (E)	10000	0.014	1.000
Adjustment (dxt)	10000	0.000	0.000
FREQ	10000	0.000	0.000

---

theta = 0.95 delta1 = -1.82 delta2 = -0.83 PG = 0.27 CG = 1.0

VARIABLE	j (No.of runs)	MEAN	STD.DVN
Control error (E)	10000	-0.009	1.000
Adjustment (dxt)	10000	-0.006	0.031
FREQ	10000	0.116	0.320

---

theta = 0.95 delta1 = -1.00 delta2 = -0.25 PG = 0.44 CG = 1.0

VARIABLE	j (No.of runs)	MEAN	STD.DVN
Control error (E)	10000	0.021	1.001
Adjustment (dxt)	-0.011	0.039	
FREQ	10000	0.389	0.488

---

theta = 0.95 delta1 = -0.27 delta2 = -0.02 PG = 0.78 CG = 1.0

VARIABLE	j (No.of runs)	MEAN	STD.DVN
Control error (E)	10000	0.086	1.002
Adjustment (dxt)	10000	-0.002	0.024
FREQ	10000	0.213	0.410

Attachment 7.IV

Table 7.IV Constrained Variance Control Scheme

theta = 0.05 delta1 = -1.82 delta2 = -0.83 b = 1.0 AL = 3.02 SA = 1.01

VARIABLE	VARIANCE		
Control error (E)	1.000		
Adjustment (dxt)	0.000		
FREQ	0(MFREQ)	AI = 0	

---

theta = 0.05 delta1= -1.00 delta2 = -0.25 b = 1.0 AL = 3.02 SA = 1.01

VARIABLE	VARIANCE	Increase in varCE	Increase in vardxt
Control error (E)	1.079	7.90%	
Adjustment (dxt)	0.203		20.30%
FREQ	0.102(MFREQ)	AI = 1/MFREQ = 9.80	

---

theta = 0.05 delta1 = -0.27 delta2 = -0.02 b = 1.0 AL = 3.00 SA = 1.00

VARIABLE	VARIANCE	Increase in varCE	Increase in vardxt
Control error (E)	1.338	24.00%	
Adjustment (dxt)	0.311		53.2%
FREQ	0.19(MFREQ)	AI = 1/MFREQ = 5.26	

---

theta = 0.05 delta1 = -0.10 delta2 = 0.00 b = 1.0 AL = 2.97 SA = 0.99

VARIABLE	VARIANCE	Decrease in varCE	Decrease in vardxt
Control error (E)	1.298	2.98%	
Adjustment (dxt)	0.273		12.22%
FREQ	0.199(MFREQ)	AI = 1/MFREQ = 5.03	

---

theta = 0.05 delta1 = 0.00 delta2 = 0.00 b = 1.0 AL = 3.00 SA = 1.00

VARIABLE	VARIANCE	Increase in varCE	Increase in vardxt
Control error (E)	1.480	14.02%	
Adjustment (dxt)	0.323		18.32%
FREQ	0.224(MFREQ)	AI = 1/MFREQ = 4.46	

---

theta = 0.25 delta1 = -1.82 delta2 = -0.83 b = 1.0 AL = 3.00 SA = 1.00

VARIABLE	VARIANCE	Increase in varCE	Increase in vardxt
Control error (E)	1.036	14.02%	
Adjustment (dxt)	0.163		18.32%
FREQ	0.086(MFREQ)	AI = 1/MFREQ = 11.63	

---

theta = 0.25 delta1= -1.00 delta2 = -0.25 b = 1.0 AL = 2.98 SA = 0.99

VARIABLE	VARIANCE	Increase in varCE	Increase in vardxt
Control error (E)	1.061	2.41%	
Adjustment (dxt)	0.062		61.96%
FREQ	0.05(MFREQ)	AI = 1/MFREQ = 20.00	

---

theta = 0.25 delta1 = -0.27 delta2 = -0.02 b = 1.0 AL = 3.03 SA = 1.01

VARIABLE	VARIANCE	Increase in varCE	Increase in vardxt
Control error (E)	1.152	8.57%	
Adjustment (dxt)	0.066		6.45%
FREQ	0.075(MFREQ)	AI = 1/MFREQ = 13.33	

---

theta = 0.25 delta1 = -0.10 delta2 = 0.00 b = 1.0 AL = 2.98 SA = 0.99

VARIABLE	VARIANCE	Increase in varCE	Decrease in vardxt
Control error (E)	1.229	6.68%	
Adjustment (dxt)	0.051		22.72%
FREQ	0.061(MFREQ)	AI = 1/MFREQ = 16.39	

---

theta = 0.25 delta1 = 0.00 delta2 = 0.00 b = 1.0 AL = 3.02 SA = 1.01

VARIABLE	VARIANCE	Decrease in varCE	Increase in vardxt
Control error (E)	1.208	1.71%	
Adjustment (dxt)	0.059		15.69%
FREQ	0.085(MFREQ)	AI = 1/MFREQ = 11.76	

---

theta=0.70 delta1 = -1.82 delta2 = -0.83 b = 1.0 AL = 3.00 SA = 1.00

VARIABLE	VARIANCE	Decrease in varCE	Decrease in vardxt
Control error (E)	1.002	17.05%	
Adjustment (dxt)	0.007		88.14%
FREQ	0.017(MFREQ)	AI = 1/MFREQ = 58.82	

---

theta = 0.70 delta1 = -1.00 delta2 = -0.25 b = 1.0 AL = 3.00 SA = 1.00

VARIABLE	VARIANCE	Increase in varCE	Decrease in vardxt
Control error (E)	1.004	0.19%	
Adjustment (dxt)	0.000		100%
FREQ	0.001(MFREQ)	AI = 1000.00	

---

theta = 0.70 delta1 = -0.27 delta2 = -0.02 b = 1.0 AL = 2.99 SA = 1.00

VARIABLE	VARIANCE	Increase in varCE	Change in vardxt
Control error (E)	1.067	6.27%	
Adjustment (dxt)	0.000		Nil
FREQ	0.002(MFREQ)	AI = 500.00	

---

theta = 0.70 delta1 = -0.10 delta2 = 0.00 b = 1.0 AL = 3.02 SA = 1.01

VARIABLE	VARIANCE	Increase in varCE	Change in vardxt
Control error (E)	1.081	1.31%	
Adjustment (dxt)	0.000		Nil
FREQ	0.002(MFREQ)	AI = 500.00	

---

theta = 0.70 delta1 = 0.00 delta2 = 0.00 b = 1.0 AL = 3.00 SA = 1.00

VARIABLE	VARIANCE	Decrease in varCE	Change in vardxt
Control error (E)	1.020	5.64%	
Adjustment (dxt)	0.000		Nil
FREQ	0.001 (MFREQ)	AI = 1000.00	

---

theta = 0.75 delta1 = -1.82 delta2 = -0.83 b = 1.0   AL = 3.01   SA = 1.00

VARIABLE	VARIANCE	Decrease in varCE	Increase in vardxt
Control error (E)	1.005	1.47%	
Adjustment (dxt)	0.003		0.3%
FREQ	0.019(MFREQ)	AI = 1/0.019 = 52.63	

---

theta = 0.75 delta1 = -1.00 delta2 = -0.25 b = 1.0   AL = 2.98   SA = 0.99

VARIABLE	VARIANCE	Decrease in varCE	Decrease in vardxt
Control error (E)	1.000	0.49%	
Adjustment (dxt)	0.000		0.30%
FREQ	0.000(MFREQ)	AI = 0.00	

---

theta = 0.75 delta1 = -0.27 delta2 = -0.02 b = 1.0   AL = 3.01   SA = 1.00

VARIABLE	VARIANCE	Change in varCE	Change in vardxt
Control error (E)	1.000	Nil	
Adjustment (dxt)	0.000		Nil
FREQ	0.000(MFREQ)	AI = 0.00	

---

theta = 0.75 delta1 = -0.10 delta2 = 0.00 b = 1.0   AL = 2.98   SA = 0.99

VARIABLE	VARIANCE	Increase in varCE	Change in vardxt
Control error (E)	1.023	2.30%	
Adjustment (dxt)	0.000		NIl
FREQ	0.002 (MFREQ)	AI = 500.00	

---

theta = 0.75 delta1 = 0.00 delta2 = 0.00 b = 1.0   AL = 2.98   SA = 0.99

VARIABLE	VARIANCE	Decrease in varCE	Change in vardxt
Control error (E)	1.000	2.24%	
Adjustment (dxt)	0.000		Nil
FREQ	0.000 (MFREQ)	AI = 0.00	

---

theta = 0.95 delta1 = -1.82 delta2 = -0.83 b = 1.0   AL = 2.99   SA = 1.00

VARIABLE	VARIANCE	Decrease in varCE	Change in vardxt
Control error (E)	1.000	Nil	
Adjustment (dxt)	0.001		0.10%
FREQ	0.116 (MFREQ)	AI = 1/0.116 = 8.62	

---

theta = 0.95 delta1 = -1.00 delta2 = -0.25 b = 1.0   AL = 2.99   SA = 1.00

VARIABLE	VARIANCE	Increase in varCE	Increase in vardxt
Control error (E)	1.001	0.10%	
Adjustment (dxt)	0.002		100.00%
FREQ	0.389 (MFREQ)	AI = 1/0.389 =2.57	

---

theta = 0.95 delta1 = -0.27 delta2 = -0.02 b = 1.0   AL = 3.04   SA = 1.01

VARIABLE	VARIANCE	Increase in varCE	Decrease in vardxt
Control error (E)	1.003	0.19%	
Adjustment (dxt)	0.001		50.00%
FREQ	0.213 (MFREQ)	AI = 1/0.213 = 4.69	

---

theta = 0.95 delta1 = -0.10 delta2 = 0.00 b = 1.0 AL = 2.98 SA = 0.99

VARIABLE	VARIANCE	Increase in varCE	Decrease in vardxt
Control error (E)	1.007	0.39%	
Adjustment (dxt)	0.000		100.00%
FREQ	0.187 (MFREQ)	AI = 1/0.187 = 5.34	

---

theta = 0.95 delta1 = 0.00 delta2 = 0.00 b = 1.0 AL = 2.98 SA = 0.99

VARIABLE	VARIANCE	Increase in varCE	Change in vardxt
Control error (E)	1.009	0.19%	
Adjustment (dxt)	0.000		Nil
FREQ	0.125 (MFREQ)	AI = 1/0.125 = 8.00	

Attachment 7.V

Table 7.V Time Series Controller Performance Measures for Θ, δ_1, δ_2 and dead time b = 2.0

Θ	r= 1-Θ	δ_1	δ_2	δ_2-δ_1	MFREQ	AI= 1/MFQ	CESTDDVN
0.05	0.95	-1.82	-0.83	0.99	0.019	52.63	1.514
0.10	0.90	-1.82	-0.83	0.99	0.016	62.50	1.440
0.15	0.85	-1.82	-0.83	0.99	0.013	76.92	1.433
0.20	0.80	-1.82	-0.83	0.99	0.009	111.1	1.352
0.25	0.75	-1.82	-0.83	0.99	0.007	142.8	1.306
0.70	0.30	-1.82	-0.83	0.99	0.006	166.6	1.055
0.75	0.25	-1.82	-0.83	0.99	0.009	111.1	1.035
0.80	0.20	-1.82	-0.83	0.99	0.001	1000	1.023
0.95	0.05	-1.82	-0.83	0.99	0.188	5.319	1.005
0.05	0.95	-1.00	-0.25	0.75	0.018	55.55	1.567
0.10	0.90	-1.00	-0.25	0.75	0.014	71.42	1.532
0.15	0.85	-1.00	-0.25	0.75	0.010	100.0	1.425
0.20	0.80	-1.00	-0.25	0.75	0.009	111.1	1.479
0.25	0.75	-1.00	-0.25	0.75	0.007	142.8	1.388
0.70	0.30	-1.00	-0.25	0.75	0.002	500.0	1.092
0.75	0.25	-1.00	-0.25	0.75	0.001	1000	1.054
0.80	0.20	-1.00	-0.25	0.75	0.375	2.666	1.040
0.95	0.05	-1.00	-0.25	0.75	0.330	3.030	1.005
0.05	0.95	-0.27	-0.02	0.25	0.033	30.30	1.795
0.10	0.90	-0.27	-0.02	0.25	0.017	58.82	1.712
0.15	0.85	-0.27	-0.02	0.25	0.016	62.50	1.675
0.20	0.80	-0.27	-0.02	0.25	0.014	71.42	1.543
0.25	0.75	-0.27	-0.02	0.25	0.011	90.90	1.635
0.70	0.30	-0.27	-0.02	0.25	0.003	333.3	1.181
0.75	0.25	-0.27	-0.02	0.25	0.001	1000	1.162
0.80	0.20	-0.27	-0.02	0.25	0.331	3.021	1.049
0.95	0.05	-0.27	-0.02	0.25	0.229	4.366	1.004
0.05	0.95	-0.10	0.00	0.10	0.025	40.00	2.081
0.10	0.90	-0.10	0.00	0.10	0.024	41.66	1.857
0.15	0.85	-0.10	0.00	0.10	0.021	47.61	1.923
0.20	0.80	-0.10	0.00	0.10	0.026	38.46	1.686
0.25	0.75	-0.10	0.00	0.10	0.015	66.66	1.555
0.70	0.30	-0.10	0.00	0.10	0.005	200.0	1.144
0.75	0.25	-0.10	0.00	0.10	0.004	250.0	1.052
0.80	0.20	-0.10	0.00	0.10	0.289	3.460	1.053
0.95	0.05	-0.10	0.00	0.10	0.202	4.950	1.009

Θ	$r = 1 - \Theta$	δ_1	δ_2	$\delta_2 - \delta_1$	MFREQ	AI= 1/MFQ	CESTDDVN
0.05	0.95	0.00	0.00	0.00	0.038	26.30	1.859
0.10	0.90	0.00	0.00	0.00	0.033	30.30	1.916
0.15	0.85	0.00	0.00	0.00	0.025	40.00	1.836
0.20	0.80	0.00	0.00	0.00	0.023	43.47	1.630
0.25	0.75	0.00	0.00	0.00	0.014	71.42	1.570
0.70	0.30	0.00	0.00	0.00	0.004	250.0	1.196
0.75	0.25	0.00	0.00	0.00	0.001	1000	1.274
0.80	0.20	0.00	0.00	0.00	0.000	0.000	1.064
0.95	0.05	0.00	0.00	0.00	0.169	5.916	1.008

Attachment 7.VI

Table7.VI Model and Controller parameters

Model parameters	Controller parameters

Second order model with dead time b = 2.0

theta = 0.05 delta1 = -1.82 delta2 = -0.83 AL = 3.02 PG = 0.27 CG = 1.0 SA = 1.01

VARIABLE	j(No.of runs)	MEAN	STD.DVN	VARIANCE
Control error (E)	10000	0.454	1.514	2.292
Adjustment (dxt)	10000	-0.013	0.438	0.191
FREQ	10000	0.019	0.137	AI = 1/0.019 = 52.63

theta = 0.05 delta1 = -1.00 delta2 = -0.25 AL = 3.02 PG = 0.44 CG = 1.0 SA = 1.01

VARIABLE	j(No.of runs)	MEAN	STD.DVN	VARIANCE
Control error (E)	10000	0.829	1.567	2.457
Adjustment (dxt)	10000	-0.012	0.382	0.146
FREQ	10000	0.018	0.132	AI = 1/0.018 = 55.55

theta = 0.05 delta1 = -0.27 delta2 = -0.02 AL = 3.00 PG = 0.78 CG = 1.0 SA = 1.00

VARIABLE	j(No.of runs)	MEAN	STD.DVN	VARIANCE
Control error (E)	10000	1.069	1.795	3.222
Adjustment (dxt)	10000	-0.033	0.509	0.259
FREQ	10000	0.031	0.177	AI = 1/0.031 = 30.30

theta = 0.05 delta1 = -0.10delta2 = 0.00 AL = 2.97 PG = 0.91CG = 1.0 SA = 0.99

VARIABLE	j(No.of runs)	MEAN	STD.DVN	VARIANCE
Control error (E)	10000	1.222	2.081	4.329
Adjustment (dxt)	10000	-0.028	0.453	0.206
FREQ	10000	0.025	0.155	AI = 1/0.025 = 40

theta = 0.05 delta1 = 0.00 delta2 = 0.00 AL = 3.00 PG = 1.00 CG = 1.0 SA = 1.00

VARIABLE	j(No.of runs)	MEAN	STD.DVN	VARIANCE
Control error (E)	10000	1.002	1.859	3.457
Adjustment (dxt)	10000	-0.041	0.460	0.211
FREQ	10000	0.038	0.191	AI = 1/0.038 = 26.31

theta = 0.25 delta1 = -1.82 delta2 = -0.83 AL = 3.00 PG = 0.27 CG = 1.0 SA =1.00

VARIABLE	j(No.of runs)	MEAN	STD.DVN	VARIANCE
Control error (E)	10000	0.281	1.306	1.705
Adjustment (dxt)	10000	-0.006	0.202	0.041
FREQ	10000	0.007	0.084	AI = 1/0.007 = 142.86

theta = 0.25 delta1 = -1.00 delta2 = -0.25 AL = 2.98 PG = 0.44 CG = 1.0 SA = 0.99

VARIABLE	j(No.of runs)	MEAN	STD.DVN	VARIANCE
Control error (E)	10000	0.449	1.388	1.926
Adjustment (dxt)	10000	-0.008	0.241	0.058
FREQ	10000	0.007	0.083	AI = 1/0.007 = 142.86

theta = 0.25 delta1 = -0.27 delta2 = -0.02 AL = 3.03 PG = 0.78 CG = 1.0 SA = 1.01

VARIABLE	j(No.of runs)	MEAN	STD.DVN	VARIANCE
Control error (E)	10000	0.810	1.635	2.672
Adjustment (dxt)	10000	-0.011	0.240	0.058
FREQ	10000	0.011	0.104	AI = 1/0.011 = 90.90

---

theta = 0.25 delta1 = -0.10delta2 = 0.00 AL = 2.98 PG = 0.91CG = 1.0 SA = 0.99

VARIABLE	j(No.of runs)	MEAN	STD.DVN	VARIANCE
Control error (E)	10000	0.489	1.555	2.419
Adjustment (dxt)	10000	-0.019	0.250	0.062
FREQ	10000	0.015	0.123	AI = 1/0.015 = 66.66

---

theta = 0.25 delta1 = 0.00 delta2 = 0.00 AL = 3.02 PG = 1.00 CG = 1.0 SA = 1.01

VARIABLE	j(No.of runs)	MEAN	STD.DVN	VARIANCE
Control error (E)	10000	0.450	1.570	2.465
Adjustment (dxt)	10000	-0.017	0.239	0.057
FREQ	10000	0.014	0.118	AI = 1/0.014 = 71.42

---

theta = 0.70 delta1 = -1.82 delta2 = -0.83 AL = 3.00 PG = 0.27 CG = 1.0 SA = 1.00

VARIABLE	j(No.of runs)	MEAN	STD.DVN	VARIANCE
Control error (E)	10000	0.027	1.055	1.114
Adjustment (dxt)	10000	-0.005	0.087	0.008
FREQ	10000	0.006	0.079	AI = 1/0.006 = 166.66

---

theta = 0.70 delta1 = -1.00 delta2 = -0.25 AL = 3.00PG = 0.44 CG = 1.0 SA = 1.00

VARIABLE	j(No.of runs)	MEAN	STD.DVN	VARIANCE
Control error (E)	10000	0.311	1.092	1.192
Adjustment (dxt)	10000	0.000	0.041	0.002
FREQ	10000	0.002	0.047	AI = 1/0.002 = 500

---

theta = 0.70 delta1 = -0.27 delta2 = -0.02 AL = 2.99 PG = 0.78 CG = 1.0 SA = 1.00

VARIABLE	j(No.of runs)	MEAN	STD.DVN	VARIANCE
Control error (E)	10000	0.598	1.181	1.394
Adjustment (dxt)	10000	-0.001	0.046	0.002
FREQ	10000	0.001	0.056	AI = 1/0.001 = 333.33

---

theta = 0.70 delta1 = -0.10 delta2 = 0.00 AL = 3.02 PG = 0.91CG = 1.0 SA = 1.01

VARIABLE	j(No.of runs)	MEAN	STD.DVN	VARIANCE
Control error (E)	10000	0.308	1.144	1.309
Adjustment (dxt)	10000	-0.002	0.052	0.003
FREQ	10000	0.005	0.068	AI = 1/0.005 = 200

---

theta = 0.70 delta1 = 0.00 delta2 = 0.00 AL = 3.00 PG = 1.00 CG = 1.0 SA = 1.00

VARIABLE	j(No.of runs)	MEAN	STD.DVN	VARIANCE
Control error (E)	10000	2.144	1.196	1.429
Adjustment (dxt)	10000	0.000	0.036	0.001
FREQ	10000	0.004	0.060	AI = 250

---

theta = 0.75 delta1 = -1.82 delta2 = -0.83 AL = 3.01 PG = 0.27 CG = 1.0 SA = 1.00

VARIABLE	j(No.of runs)	MEAN	STD.DVN	VARIANCE
Control error (E)	10000	-0.010	1.035	1.071
Adjustment (dxt)	10000	-0.007	0.081	0.007
FREQ	10000	0.009	0.095	AI = 1/0.009 = 111.11

---

theta = 0.75 delta1 = -1.00 delta2 = -0.25 AL = 2.98 PG = 0.44 CG = 1.0 SA = 0.99

VARIABLE	j(No.of runs)	MEAN	STD.DVN	VARIANCE
Control error (E)	10000	0.165	1.054	1.110
Adjustment (dxt)	10000	0.000	0.021	0.000
FREQ	10000	0.001	0.030	AI = 1000

---

theta = 0.75 delta1 = -0.27 delta2 = -0.02 AL = 3.01 PG = 0.78 CG = 1.0 SA = 1.00

VARIABLE	j(No.of runs)	MEAN	STD.DVN	VARIANCE
Control error (E)	10000	0.841	1.162	1.349
Adjustment (dxt)	10000	0.000	0.024	0.001
FREQ	10000	0.001	0.037	AI = 1000

---

theta = 0.75 delta1 = -0.10 delta2 = 0.00 AL = 2.98 PG = 0.91CG = 1.0 SA = 0.99

VARIABLE	j(No.of runs)	MEAN	STD.DVN	VARIANCE
Control error (E)	10000	0.131	1.052	1.106
Adjustment (dxt)	10000	-0.002	0.040	0.002
FREQ	10000	0.004	0.064	AI = 1/004 = 250

---

theta = 0.75 delta1 = 0.00 delta2 = 0.00 AL = 2.98 PG = 1.00 CG = 1.0 SA = 0.99

VARIABLE	j(No.of runs)	MEAN	STD.DVN	VARIANCE
Control error (E)	10000	0.910	1.274	1.622
Adjustment (dxt)	10000	0.000	0.025	0.001
FREQ	10000	0.001	0.036	AI = 1000

---

theta = 0.95 delta1 = -1.82 delta2 = -0.83 AL = 2.99 PG = 0.27 CG = 1.0 SA = 1.00

VARIABLE	j(No.of runs)	MEAN	STD.DVN	VARIANCE
Control error (E)	10000	0.066	1.005	1.011
Adjustment (dxt)	10000	-0.001	0.031	0.001
FREQ	10000	0.188	0.391	AI = 1/0.188 = 5.319

---

theta = 0.95 delta1 = -1.00 delta2 = -0.25 AL = 2.99 PG = 0.44 CG = 1.0 SA = 1.00

VARIABLE	j(No.of runs)	MEAN	STD.DVN	VARIANCE
Control error (E)	10000	0.265	1.005	1.010
Adjustment (dxt)	10000	0.000	0.028	0.001
FREQ	10000	0.330	0.470	AI = 1/0.33 = 3.03

---

theta = 0.95 delta1 = -0.27 delta2 = -0.02 AL = 3.04 PG = 0.78 CG = 1.0 SA = 1.01

VARIABLE	j(No.of runs)	MEAN	STD.DVN	VARIANCE
Control error (E)	10000	0.830	1.004	1.008
Adjustment (dxt)	10000	0.000	0.017	0.000
FREQ	10000	0.229	0.420	AI = 1/0.229 = 4.366

---

theta = 0.95 delta1 = -0.10 delta2 = 0.00 AL = 2.98 PG = 0.91CG = 1.0 SA = 0.99

VARIABLE	j(No.of runs)	MEAN	STD.DVN	VARIANCE
Control error (E)	10000	1.007	1.009	1.018
Adjustment (dxt)	10000	0.000	0.015	0.000
FREQ	10000	0.202	0.402	AI = 5

---

theta = 0.95 delta1 = 0.00 delta2 = 0.00 AL = 2.98 PG = 1.00 CG = 1.0 SA = 0.99

VARIABLE	j(No.of runs)	MEAN	STD.DVN	VARIANCE
Control error (E)	10000	1.193	1.008	1.017
Adjustment (dxt)	10000	0.000	0.011	0.000
FREQ	10000	0.169	0.375	AI = 59

Table 7.VII Regression Coefficients for CESTDDVN Attachment 7.VII

Predictor	Coefficients	Std.deviation	t-ratio	p	VIF
Constant	1.10440	0.00150	738.26	0.000	
C1(theta)	-0.074723	0.001536	-48.64	0.000	1.0
C2(delta1)	0.081039	0.003233	25.07	0.000	14.8
C3(delta2)	-0.115836	0.006953	-16.66	0.000	14.8

Standard error in CESTDDVN (s) = 0.01809 R-square = 71.2%
R-square(adjusted) = 71.1%

The regression equation is
C4 = 1.10 - 0.0747 C1 + 0.0810 C2 - 0.116 C3

Table 7.VIII Analysis of Variance for CESTDDVN

Analysis of Variance (ANOVA) Table for Control error standard deviation

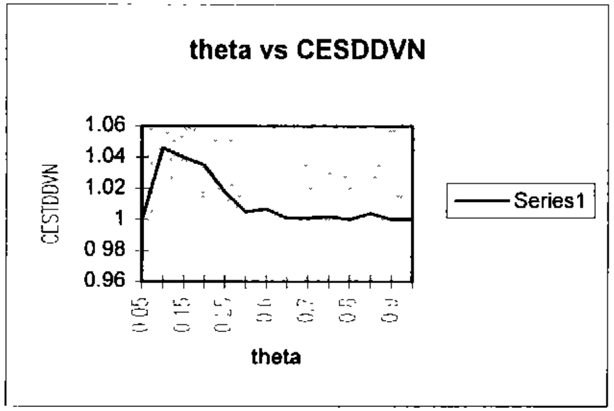
SOURCE	DF	Sum Sq.	Mean Sq	F-ratio	p
Regression	3	1.25505	0.41835	1278.51	0.000
Error	1552	0.50784	0.00033		
Total	1555	1.76289			

SOURCE	DF	SEQ SS
C1(theta)	1	0.77445
C2(delta1)	1	0.38978
C3(delta2)	1	0.09082

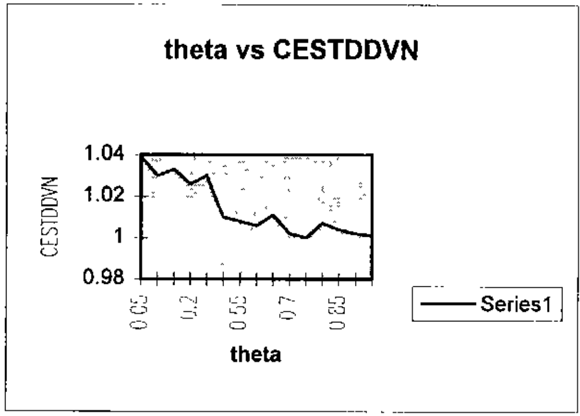
Lack of fit test
Overall lack of fit test is significant at P = 0.000

Figure 7.3 Graph of theta versus CESTDDVN for various delta values and dead time b = 1.0

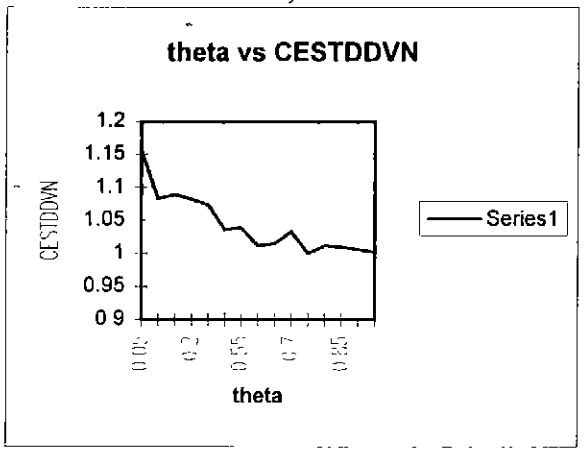
delta1 = -1.82, delta2 = -0.83



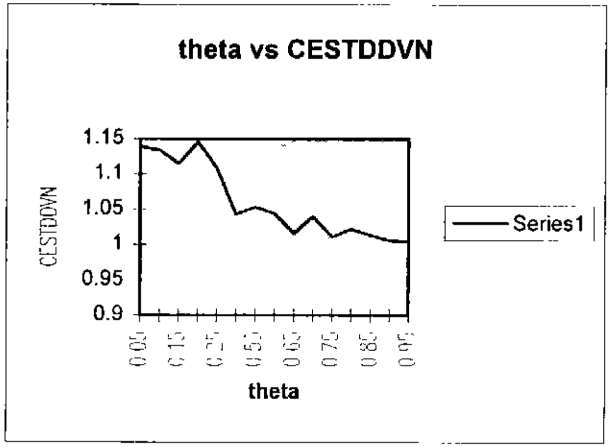
delta1 = -1.0, delta2 = -0.25



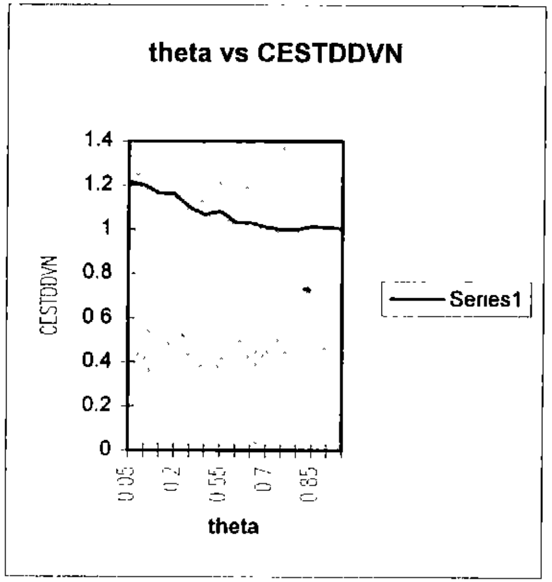
delta1 = -0.27, delta2 = -0.02



delta1 = -0.10, delta2 = 0.00



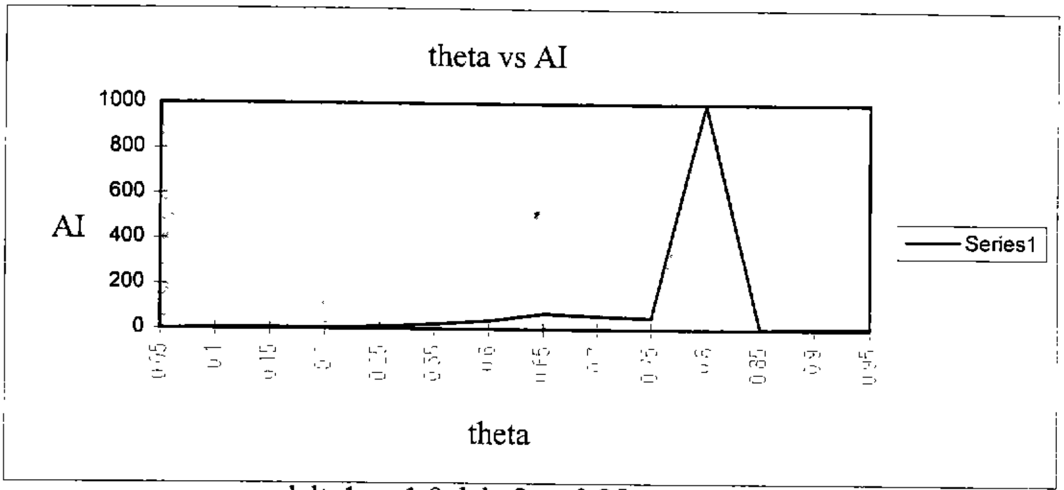
delta1 = 0, delta2 = 0



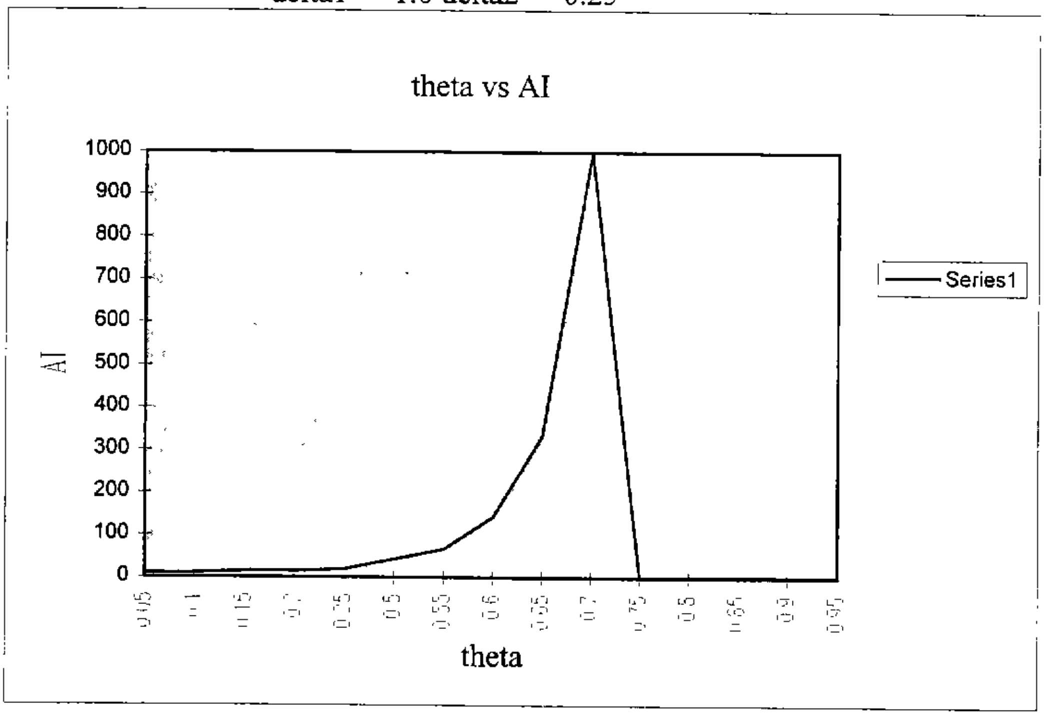
Attachment 7.IX

Figure 7.4 Graph of theta versus AI for various delta values for dead time b = 1.0

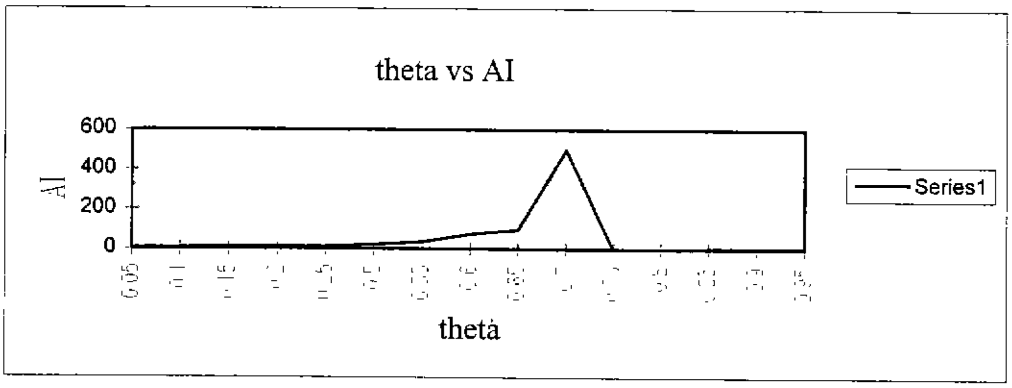
delta1 = -1.82, delta2 = -0.83



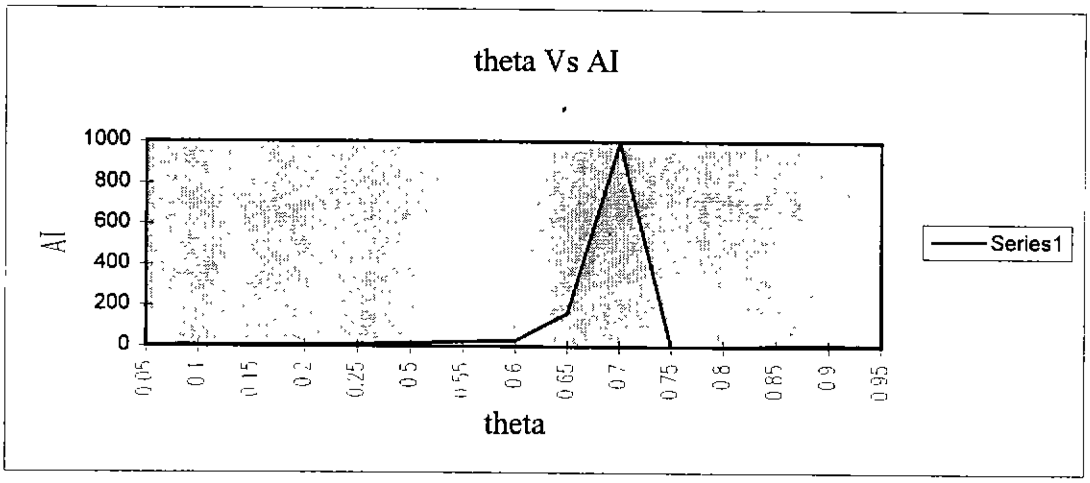
delta1 = -1.0 delta2 = -0.25



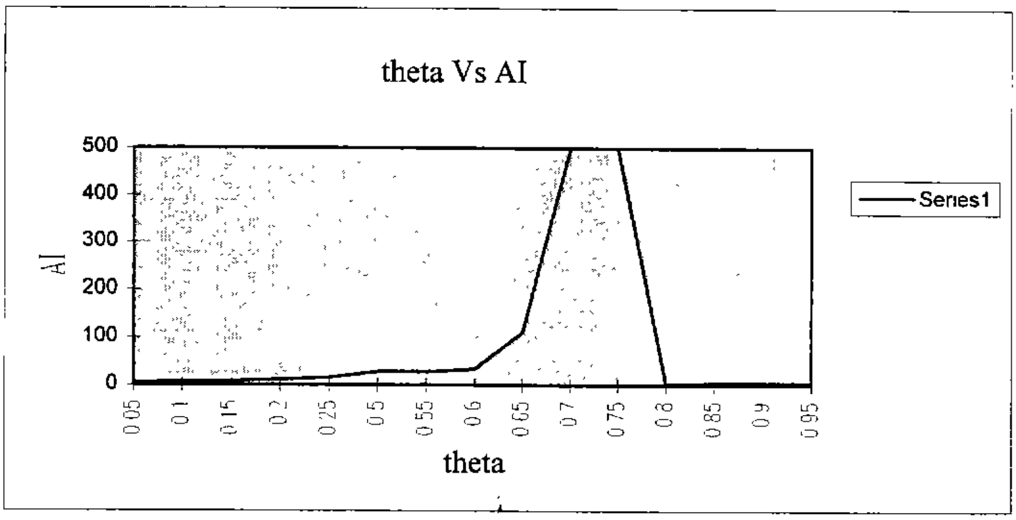
delta1 = -0.27 delta2 = -0.02



delta1= 0 delta2 = 0



delta1=-0.10 delta2=0



CHAPTER 8

PROCESS REGULATION SCHEME - AN OUTLINE

8.1 INTRODUCTION

In Chapter 5, it was mentioned that the time series controller given by the feedback control adjustment Equation (5.16) is the discrete equivalent of a properly tuned integral controller. This equation defines the adjustment to be made to the process at time t which would produce the feedback control action compensating for the forecasted disturbance yielding the smallest possible mean square error (variance). In other words, the control adjustment action given by Equation (5.16) minimizes the variance of the output controlled variable. A feedback control scheme employing a MMSE (or minimum variance) controller would minimize the overall cost if it is assumed that (i) the off-target cost is the *only* cost and (ii) the cost is a quadratic function of the output deviation from the target (Box and Kramer [1992]) So, the time series controller based on Equation (5.16) would minimize the mean overall cost of the feedback control scheme if the cost of being off-target is assumed to be proportional to the square of the deviation from target and that other variable costs, (given below), are negligible.

Minimum variance control that can be achieved with feedback control schemes will be minimum cost schemes if based on the assumption that an off-target cost is an associated cost for the deviation of the output quality characteristic being away from the desired target. This assumption is possible since the minimization of the mean square error at the output is equivalent to minimization of the quadratic off-target cost (Kramer [1990]). However, there are other variable costs such as (i) the cost of adjustment and

(ii) the cost associated with the frequency of sampling to examine the process, called the monitoring or 'observation cost'. The resulting minimum-cost feedback adjustment schemes have then to be formulated in a different manner from the minimum cost schemes based on the minimization of the mean square error at the output. This is due to the fact that the cost of the feedback control scheme changes as the objectives of the scheme are changed to minimize the deviation and also to adjust the process (Box and Kramer [1992]). With this as our objective, an outline of a process regulation scheme is given when there are (i) off-target costs, and (ii) adjustment costs (which include sampling costs) in a stochastically controlled process employing feedback control.

8.2 REVIEW AND BACKGROUND OF THE PROCESS REGULATION SCHEME FORMULATION

Box and Jenkins [1963] evaluated the performance of an EWMA controller using the mass production of ball bearings as an example. The diameter of the bearings was chosen by them as a quality index and they considered the fact that an adjustment to the machine requiring an interruption of production resulted in an adjustment cost. They assumed that the diameters deviated from target according to an IMA (0,1,1) process and that there was no delay (dead time) in realising the adjustments. The cost of being off-target was also assumed, by them, to be proportional to the square of the deviation. The IMA parameter, Θ, was chosen to generate the forecast errors. Box and Jenkins [1963] showed that the sum of the adjustment and off-target costs could be minimized with an appropriate choice of EWMA control limits (L). For certain values of Θ and L, they calculated the expected run length between adjustments and the mean square deviation from target. From these run lengths, the values of L and the control error standard deviation were calculated for values of Θ ranging from zero to one. Box and Jenkins [1970, 1976] further described an approach to the formulation of feedback control schemes assuming that the sampling interval had been decided earlier before designing the scheme.

Box, Jenkins and MacGregor [1974] showed that a control strategy comparing the EWMA to a set of fixed limits is optimal when the adjustment cost is significant when compared with off-target cost. They calculated the effect, of changing the EWMA, on average adjustment interval and the control error variance when dead time is not present. Kramer [1990], considering only quadratic off-target cost for adjustments made without delay, developed minimal cost process regulation schemes for the case of fixed monitoring and adjustment costs. For a dynamic system without delay (dead time), the adjustment cost can be proportional to the square of the required adjustment, which Kramer [1990] also considered. An outline of a process regulation scheme using the feed back control algorithm derived in Chapter 5 is formulated in this Chapter.

In their monograph, Box and Jenkins [1970, 1976] developed the MMSE controller with fixed sampling interval. However, in practice, it is often possible and desirable to change the sampling interval. Abraham and Box [1979] suggested a method to carry out pilot runs from a process to determine its dynamic and stochastic characteristics with a short sampling interval. They used the information obtained to develop an optimal feedback control scheme. Some of these principles are followed in developing an approximate feedback control cost model in this thesis.

8.3 DEAD TIME AND FEEDBACK CONTROL SCHEME

It is generally assumed, in controlling a process by applying SPC techniques, that (i) the true process level is a constant and (ii) the common-cause variation and the process state of statistical control follow only a (stable or) stationary model. If this assumption proves to be incorrect, then, there may be slight amounts of autocorrelations in the process level affecting the run-length and the control chart limits (Harrison and Ross [1991]). This is frequently the scenario in the continuous process industries, where the true process level is not constant due to process drifts. Feedback control can be an approach to compensate for the drifts in these circumstances. *If the autocorrelations are large and persistent, then, a feedback control approach may be more appropriate than a SPC approach to control the process.* Feedback control can compensate only for the predictable component of the uncompensated process output (MacGregor [1992]). So, the effectiveness of feedback control will depend on how much it will be possible to compensate the output. Thus a situation arises when a decision must be taken as when to use SPC and APC. This depends upon the process level remaining constant or where there are changes in the process due to some significance in the autocorrelation estimates of the process data. In the final analysis, a suggestion is made that integrates both the SPC and APC procedures by judicial use of techniques from both the disciplines as may be considered necessary by current process control conditions.

The EWMA forecasts of the (simulated) data have been plotted against two parallel action lines in a geometric moving average (gma) control chart. In a feedback control scheme, the position of these control limits is determined on the basis of:- (i) the relative costs of adjustment and of being off-target, and (ii) by the degree of non-stationarity of the process. The relative value of these costs is an important factor in deciding the optimal choice of a feedback control scheme. A procedure, to reduce the cost of the scheme, is to lengthen the sampling (monitoring) interval. This procedure may be less satisfactory in that it may increase slightly the mean square error due to dynamics (inertia) of the system (page 261, Box and Kramer [1992]). Since an assumption was made to represent a complex dynamic process by a second-order dynamic model, the focus is currently on the influence and effects of dead time on a feedback control scheme.

For ARIMA (transfer function) models with b > 0 periods of delay, minimum mean square action yields a process output that becomes a moving average of $a_t, a_{t-1}, \ldots a_{t-b}$ (page 279, Vander Wiel and Vardeman [1992]). Due to delay, the process deviation is a moving average time series model of order b-1. For b>2, the adjacent values of the process output will be autocorrelated. When the delay exceeds one period, this autocorrelation will still be present, regardless of the feedback control scheme. There will not be any significant autocorrelations beyond lag 2 (MacGregor [1992]). The geometric moving average (gma) Θ was used for monitoring and sounding the out-of-control signal based on this gma statistic. The process with dead time will still be in statistical control though the observations may be serially independent (Koty and Johnson [1985]). The Shewhart chart helps to monitor stable operation due to common causes and to reveal special causes. So, only when it is possible to establish some statistically significant monitoring criterion, will it be proper to react to process changes. A suitable feedback control scheme should then be specifically designed and used to regulate (adjust) the process (Box and Kramer [1992]). This is to avoid a (pure) feedback control adjustment scheme only from obtaining a large mean square error. However, it is possible to obtain the residual sequence $\hat{a}_t$, which can be used for process monitoring even with dead time (in the feedback control loop). Having taken care of both the system dynamics and dead time, the discussion is focused on developing an approximate feedback control scheme.

8.4 FEEDBACK CONTROL PROCESS REGULATION SCHEME

8.4.1 Average Run Length and Control Procedure

The 'average run length' (ARL) measures the performance of a control procedure. A percentage point of the run length distribution can also be an appropriate measure in some applications. The average run length is the mean number of points plotted on a control chart before signalling. control action. This number should be large for a stable process and the average quality of the process output should be acceptable to both the manufacturer and the consumer. The ARL should be small if there is a shift in the mean.

For a (frequently sampled) continuous industrial process, ARL is the average number of sample intervals from the time a shift in the mean occurs until the control chart signals it. Disturbances interrupt stable periods of operation of a manufacturing process and result in drifting behaviour of the process and shifts the output mean from target. Due to this, the product quality data does not fit in with this characterisation for measuring the performance of a control procedure using ARL (Baxley [1991]).

The ARIMA (0,1,1) model characterised the disturbance. The objective in process regulation schemes is to regulate a process and not to discover the cause of disturbance. So, stochastic process control by the ARIMA model approach is preferable to the ARL and Shewhart process control approach. Also, a first order autoregressive integrated moving average model is an appropriate choice since the disturbance model exhibits (charaterises) the correlation structure of the data. A process control EWMA chart scheme was formulated based on this modelling procedure. Box and Jenkins [1970, 1976] showed that, the residuals for a correctly identified and fitted model, form an independent identically distributed (i.i.d.) sequence. Special causes were identified and associated with temporary deviations from the modelled process as leading to departures from the stochastic time series model. This was indicated by the outliers (independent observations) in the error sequence $\{e_t\}$ of the deviations from target. An adjustment control action is triggered when some function of the e_t's exceeds certain boundary values. Special causes were highlighted by applying Shewhart control charting techniques to the residual series for a properly tuned feedback control system. For information on identifying the model parameters by an examination of the correlation structure of the residuals from a fitted ARIMA model, reference may be made to Box and Jenkins [1970, 1976].

In relation to performance measurement, it is required that a control procedure must have a large ARL when the process is in control and a small ARL otherwise. The run length of a control procedure is the mean of the number of samples required before a process gives an out-of-control signal. An out-of-control signal indicates that a shift in the mean is likely to have occurred and that control action should be taken to find and correct the assignable or special cause of this shift in the mean (Woodall [1985]).

Attempting to adjust a process for slight shift in the mean leads to over-correction and introduces additional variability into the process. The average run length should be large if the process is stable and if the average outgoing product quality level (AOQL) is acceptable. The ARL should be small if the mean has shifted to a particular quality level (Devore [1982]) and remains (steady) for a certain time. It is considered disadvantageous for a control procedure to have low ARL values for shifts in the mean that are not of much significance.

A method to construct 'modified' control limits is described in Western Electric Company's handbook [1956] when the product specification limits are wide compared to the process standard deviation. The modified control limits obtained according to this method are wider than Shewhart chart limits. However, the use of modified control limits are acceptable only when some shift in the mean can occur without a significant increase in the percentage of products that are non-conforming to product specifications. Hill [1956] pointed out that the calculations of the modified control limits for a Shewhart chart may contain all values of the shift in the mean for which the percentage non-conforming product is between two specified (probability) values, measured in units of the standard error of the sample mean. These calculations depend on the probability distribution of the quality characteristic and may result in a control region too large and contain shifts in the mean that have to be detected quickly and corrected.

The underlying principle in quality control is that not only should the product meet specifications but that the quality characteristic is concentrated as closely as possible about the desired target value. The basic idea is that the process should be so controlled that if the process mean shifts to within 3σ of specification limits, then this shift should be detected immediately. This means that the shifted mean should be three standard errors beyond the control action limits (Wetherill and Rowlands [1991]).

In relation to the modified control limits, if the time series model has been correctly identified, then, the sample means will appear to be independent identically distributed variables. This once again emphasises the need for identifying correctly the time series model.

8.4.2 Sample Size

The choice of a particular control procedure out of a number of feedback control procedures depends on the selection of in-control and out-of-control regions. The effect of an in-control region increases as the sample size increases. So, it is better to have a large sample size. A typical value for the component parts manufacturing industries is for the samples to be taken in groups of size 4 and 1 in the process industries (Wetherill and Rowlands [1991]). The ability of a process control chart to signal trouble depends on the sampling plan used. In general, it is preferable to sample less frequently in order to eliminate extreme autocorreleration and thus provide a tool for assessing the process behaviour over a long time period. By careful selection of sampling and subgrouping, it may be possible to control the sources of variation that may show as special causes based on 'the control limits or the in control length' (ARL) (Hoerl and Palm [1992]). Using this principle, Wheeler [1991] generalised the concept of rational subgrouping as 'rational sampling'. A sampling scheme is rational when all of the ranges (moving or sub-group) are generated by the same common cause system and so have a consistent interpretation. The connection between sample size and the in control length is illustrated by the following example.

The in-control run length may be the statistic used to describe the performance of an $\bar{X}$ chart. By considering the problem of model identification described in Box and Jenkins [1970, 1976], it can be shown for an AR(1) process $\rho_k = \alpha^{|k|}$, given estimates r_k of the theoretical autocorrelations ρ_k, that $|r_k| = 2/\sqrt{N}$. It can also be shown that a large sample $N \geq 100$ observations is required to identify a small lag 1 autocorrelation of $\rho_1 = 0.2$ that has a significant effect on the in-control run length.

8.5 COST MODEL

Abraham and Box [1979] modified the approach to the design of feedback control scheme of Box and Jenkins [1970, 1976] in which the monitoring interval had been decided prior to the design of the scheme. In their paper, Abraham and Box [1979] derived general results for the ('new' sampled) process and the parameters of the new process, (the term referred by the authors to the resulting process after sampling), when the disturbance (process)

$$\nabla^d Z_t = (1 - \Theta_1 B - \ldots \Theta_q B^q) a_t,$$

is sampled at sampling (integer) interval (h) subject to conditions (i) $h > q-d$ and (ii) $q \geq d$ without considering delay in the system. The authors showed also, by imposing an additional restriction ($h > b$, the delay (dead time) in the system), that the sampling interval obtained is optimum even when there is whole or fractional periods of delay in effecting the adjustments after sampling. The sampling interval was obtained by assuming a certain class of stochastic disturbance models and a specified cost function. They considered a second order moving average process as a special case and illustrated it with a numerical example.

The method of Abraham and Box [1979] is modified to show that the use of sampling periods (adjustment intervals, AIs) obtained directly from simulation of stochastic feedback control algorithm (5.16), still lead to minimum cost control schemes even with time delay (dead time) in the system. This is due to the fact that these adjustment intervals (AIs) are for minimum mean square error (variance) (MMSE) control for the second-order dynamic model with delay, considered in this thesis. By assuming an adequate 'transfer function (dynamic second-order) noise model' to describe the process and the disturbance (noise), the effect of 'model misspecification' (on optimization, a problem encountered by the authors), is minimized. The disturbance model considered is the ARIMA (0,1,1) and it is shown that the derived cost model leads to a minimum cost regulation scheme.

Notation

AI > b, dead time in whole periods of delay,

C_t is the cost associated with being off-target (off-target cost),

C_a is the cost of sampling and adjustment,

$C = C_t/C_a$ is assumed to be known,

σ_ε^2 is the variance of the observed error from the target corresponding to the sampling interval (AI).

Consider a cost function of the form

$$F^*(AI) = C_t(\sigma_\varepsilon^2/\sigma_a^2) + C_a(1/AI). \qquad (8.1)$$

An economical sampling interval will be the value of AI that makes the above cost function a minimum.

It is assumed that the system has no dead time, (b = 0), and that the disturbance Z_t is an integrated moving average process of order 1 given by

$$\nabla Z_t = (1 - \Theta B)a_t.$$

[d = q = 1 for first-order ARIMA(0,1,1) disturbance process.]

Sampling and adjustment, (because the time series controller requires an adjustment for every sample), is done with an adjustment (sampling) interval such that AI > 0 and q = d = 1.

Then, the resulting process M_t and $a_{AI,t}$, the random shocks which are assumed to be $N(0,\sigma^2_{AI})$ with variance σ^2_{AI} is given by

$$\nabla_{AI} M_t = (1-\Theta_{AI} B_{AI})\, a_{AI,t} \tag{8.2}$$

where ∇_{AI} = (1-B_{AI}) = (1-B^{AI}), ∇_{AI} being a differencing operator associated with the adjustment (sampling) interval AI and Θ_{AI}, the IMA parameter for the new process M_t.

Let γ_0(AI) be the variance of $\nabla_{AI} M_t$ and

γ_1 (AI) be the first lag autocovariance of $\nabla_{AI} M_t$.

Abraham and Box [1979] used the following Lemma from the monograph, 'The Statistical Analysis of Time Series', of Anderson [1971] to obtain (a) the result given in equation (8.2) and (b) to obtain also expressions for the variance, γ_0 (AI) and γ_1 (AI), the first lag autocovariance of $\nabla_{AI} M_t$.

Lemma. "Given any arbitrary covariance or correlation sequence with only a finite number of non-zero elements, there is a finite moving average process corresponding to the sequence" (Anderson [1971] - Abraham and Box [1979]).

(a) <u>Proof of Equation (8.2):</u>

$$\nabla_{AI} M_t = \left(\frac{1-B^{AI}}{1-B}\right)(1-B)Z_t$$

$$= [(1+B+\ldots+B^{AI-1})].[(1-\Theta B)\, a_t]. \tag{8.3}$$

This is written as

$$\nabla_{AI} M_t = a_t + \psi_1 a_{t-1} + \ldots + \psi_{1+(AI-1)} a_{t-1-(AI-1)}.$$

Let γ_k(AI) denote kth lag autocovariance of $\nabla_{AI} M_t$.

Then it is enough to show that (i) $\gamma_1(AI) \neq 0$ and (ii) $\gamma_{1+k}(AI) = 0$ for all k > 0.

Now,

$$\nabla_{AI} M_{t-(1+k)AI} = a_{t-(1+k)AI} + \psi_1 a_{t-(1+k)(AI-1)} + \ldots + \psi_{1+AI-1} a_{t-(1+k)AI-AI} \, . \qquad (8.4)$$

Since q=d=1, using k=0 in equation (8.4), it can be seen that $\gamma_1(\text{AI}) = E(\nabla_{AI} M_t \nabla_{AI} M_{t-AI}) \neq 0$, where E denotes the expected value.

It is obvious that (1+k)AI > AI for all values of k ≥ 1 in order to show that $\gamma_{1+k}(\text{AI}) = 0$ for all k ≥ 1.

It is known that since sampling and adjustment is done with an adjustment (sampling) interval AI > (q-d = 1-1=) 0 and so (AI)×(d) + (q-d) > (AI)×(d) + AI > (AI)×(d) + k(AI) for all k ≥ 1 (page 7, Abraham and Box [1979]).

Now, using the fact that $\gamma_1(\text{AI}) \neq 0$ and $\gamma_{1+k}(\text{AI}) = 0$ for all k ≥ 1 and the Lemma, the result of equation (8.2) follows.

(b) <u>Expressions for the variance,</u> γ_0 (AI) and γ_1 (AI), <u>the first lag autocovariance of</u>

$\nabla_{AI} M_t$.

With d = 1 in equation (8.2) and using equation (8.3), it can be shown that

$$\nabla_{AI} M_t = a_t + (1-\Theta)a_{t-1} + \ldots + (1-\Theta)a_t + (1-\Theta)(a_{t-1} + \ldots + a_{t-AI+1}) + (-\Theta)a_{t-AI} \, .$$

Hence,

$$\gamma_0(\text{AI}) = [1 + (1-\Theta)^2 + \ldots + .(1-\Theta)^2 + (\text{AI}-1) \quad (1-\Theta)^2 + (\Theta)^2]\sigma_a^2 \, .$$

Now

$$\gamma_1(\text{AI}) = E((\nabla_{AI} M_t)(\nabla_{AI} M_{t-AI})) = -[(\Theta)]\sigma_a^2 = -\Theta\sigma_a^2$$

and after some tedious algebraic manipulations,

$$\frac{\gamma_0(\text{AI}) + 2\gamma_1(\text{AI})}{\gamma_1(\text{AI})} = -\frac{\text{AI}(1-\Theta)^2}{\Theta} \, . \qquad (8.5)$$

For a first order moving average process with parameters Θ_{AI} and σ^2_{AI}, it can also be shown that

$$\gamma_1(AI) = -\Theta_{AI}\sigma^2_{AI} ,$$

and

$$\frac{\gamma_0(AI)+2\gamma_1(AI)}{\gamma_1(AI)} = -\frac{(1-\Theta_{AI})^2}{\Theta_{AI}} . \qquad (8.6)$$

Hence from equations (8.5) and (8.6), the following relations connecting the parameters of the processes, Z_t and M_t are obtained:

$$AI\ (1-\Theta)^2/\Theta = (1-\Theta_{AI})^2/\Theta_{AI}$$

or

$$[(1-\Theta_{AI})^2/AI] = [(1-\Theta)^2/\Theta][\Theta_{AI}], \qquad (8.7)$$

and

$$\sigma_{AI}^{\ 2} = (\sigma_a^{\ 2})(\Theta/\Theta_{AI})$$

or

$$[\Theta_{AI}/\Theta] = [\sigma_a^{\ 2}/\sigma_{AI}^{\ 2}]. \qquad (8.8)$$

The variance of the error ε_t in the output at time t is given by

$$\sigma_{\varepsilon}{}^2 = \sigma^2_{AI}\ \ [1+(b/AI)(1-\Theta_{AI})^2] \cdot \qquad (8.9)$$

When b = 0, $\varepsilon_{AI,t}$, the observed error from target at time t is the one-step ahead forecast error $a_{AI,t+1}$, and hence $\sigma_{\varepsilon}^{\ 2} = \sigma_{AI}^{\ 2}$.

Using this, Equation (8.1) is simplified and a cost function considered,

$$F(AI) = C(\sigma_{AI}^{\ 2}/\sigma_a^{\ 2}) + (1/AI), \qquad (8.10)$$

where $C = C_t/C_a$.

The equation for the cost function that includes fractional periods of delay, (equation 5.4, page 6, Abraham and Box [1979]), is modified for whole periods of delay to give the cost function corresponding to Equation (8.7).

$$G(AI) = C(\sigma_{AI}^2/\sigma_a^2)[1 + (1-\Theta_{AI})^2(b/AI)] + (1/AI). \tag{8.11}$$

So, equation (8.11) becomes, on using equations (8.7) and (8.8),

$$G(AI) = C(\sigma_{AI}^2/\sigma_a^2) + (1/AI) + C(\sigma_{AI}^2/\sigma_a^2)[b(1-\Theta)^2(\sigma_a^2/\sigma_{AI}^2)].$$

That is,

$$G(AI) = C(\sigma_{AI}^2/\sigma_a^2) + (1/AI)+C(b(1-\Theta)^2) \tag{8.12}$$

$$= F(AI) + k,$$

where $k = C(b)(1- \Theta)^2$, a known constant, since the dead time, b, the rate of drift $r = (1-\Theta)$ and information about C, which will always be available, are known quantities. The minimization of G(AI) and F(AI) are equivalent and hence the sampling period (adjustment interval) obtained will definitely provide minimum cost control schemes even when there is a delay in the system.

A similar result is obtained by Abraham and Box [1979] for a general case and a second order disturbance process.

The function F(AI) given by Equation (8.9) when written as a function of Θ_{AI} becomes

$$F_1(\Theta_{AI}) = C(\Theta/\Theta_{AI}) + [(1-\Theta)^2/(1-\Theta_{AI})^2](\Theta_{AI}/\Theta). \tag{8.13}$$

After locating Θ_{AI} which minimizes Equation (8.13), a corresponding control scheme can be obtained using Equations (8.7) and (8.8) and the control adjustment equation (5.16) derived in Chapter 5. In this case, since the disturbance is described by the ARIMA(0,1,1) model and the IMA parameter Θ is the tuning parameter for the time series controller, Θ and Θ_{AI} are the same. Hence the resulting cost regulation scheme depends only on the ratio of C, that is on C_t, the off-target cost and C_a, the sampling and adjustment cost and the process drift r = (1-Θ) as shown subsequently.

As mentioned earlier, the value of the adjustment interval obtained from the simulation is used to sample the process, and immediately an adjustment is made. The geometric moving average (gma) theta determines the instant at which compensatory action is to be taken depending on the EWMA control limit lines L given by Equations (6.1) and (6.2), L = $\pm 3\sigma_a\sqrt{(1-\Theta)/(1+\Theta)}$ where Θ is the IMA parameter of the stochastic disturbance and σ_a, the standard deviation of the random shocks $\{a_t\}N(0,\sigma_a^2)$. Adjustments, when needed, are made to bring the mean of the output quality variable close to the target value.

The control limits are determined by the relative costs R_a and by the degree of non-stationarity of the process,

$$R_a = (C_a/C_t)/(1-\Theta)^2$$

where C_a is the cost per unit adjustment interval (AI),

$$C_t = k_t(\sigma_a)^2$$

where

k_t = reprocessing or processing cost in terms of material cost per hour (C_o)/(deviation from target, $\Delta)^2$, since as the deviation (Δ) from target increases, it will reach a point at which the manufactured material must be discarded or reprocessed at a cost C_o. For compensating a non-stationary disturbance, the only cost is that of being off-target (Box and Kramer [1992]).

The process is sampled and adjusted regularly at the AIs (sample periods given by simulation), resulting in substantial cost C_a for adjusting the process.

Overall cost C per unit time,

$$C = (AI)\$a + k_1(\sigma_a)^2$$

where

AI is the number of sample periods required to make an adjustment,

\$a, the cost per unit AI.

When a digital computer is used to execute the single integral mode control equations, an (approximate rule) for the lower limit on the adjustment interval (sampling period, T) for integral control is

$$T > \tau_1,$$

where τ_1, is the integral time constant of the controller (Deshpande and Ash [1981]).

It is sometimes difficult to judge the costs directly since different kinds of cost models may be appropriate for various circumstances. One approach is to draw a list of options from which the choice among minimum-cost schemes should be made empirically by balancing the advantage of longer ARL's against the consequential increase in the mean squared error about the target. Such a table is provided by Box in his paper published in 1991 (Box [1991b]).

8.6 PROCESS REGULATION SCHEMES

The choice of feedback control process regulation schemes depends on 'how capable' the 'controlled process' was of providing quality products within manufacturing specifications. If the process capability index was high, then, a moderate increase in the control error deviation (product variability) might be tolerated if this action resulted in savings in sampling and adjustment costs. Table 8.1 shows the adjustment interval (AI) and the corresponding CESTDDVN for some alternative schemes using combinations of L = 2.98, 3.0 and 3.04. These schemes are denoted by A, B, ...0.

Table 8.1 Alternative Process Regulation Schemes

Control Limits $L(= 3\ \sigma_a)$											
2.98				3.0				3.04			
Scheme	*AI*	*CESTDDVN*	*ISTD.*	*Scheme*	*AI*	*CESTDDVN.*	*ISTD*	*Scheme*	*AI*	*CESTDDVN*	*ISTD.*
A	10.99	1.022	-	F	10.0	1.033	-	K	20	1.025	-
($\Theta = 0.10, \delta_1 = -1.51, \delta_2 = -0.57$)				($\Theta = 0.05, \delta_1 = -1.79, \delta_2 = -0.80$)				($\Theta = 0.15, \delta_1 = -1.44, \delta_2 = -0.52$)			
B	10.52	1.032	0.98	G	9.0	1.058	2.42	L	12.20	1.032	0.68
($\Theta = 0.05, \delta_1 = -1.50, \delta_2 = -0.56$)				($\Theta = 0.05, \delta_1 = -0.80, \delta_2 = -0.16$)				($\Theta = 0.10, \delta_1 = -1.22, \delta_2 = -0.37$)			
C	8.20	1.043	1.066	H	5.26	1.157	9.36	M	12.35	1.037	0.48
($\Theta = 0.05, \delta_1 = -1.06, \delta_2 = -0.28$)				($\Theta = 0.05, \delta_1 = -0.27, \delta_2 = -0.02$)				($\Theta = 0.10, \delta_1 = -1.06, \delta_2 = -0.28$)			
D	5.10	1.127	8.05	I	5.02	1.158	0.09	N	9.52	1.089	5.01
($\Theta = 0.05, \delta_1 = -0.20, \delta_2 = -0.01$)				($\Theta = 0.05, \delta_1 = -0.08, \delta_2 = -0.00$)				($\Theta = 0.15, \delta_1 = -0.27, \delta_2 = -0.02$)			
E	5.68	1.186	5.24	J	4.46	1.217	5.09	O	6.94	1.119	2.75
($\Theta = 0.05, \delta_1 = -0.01, \delta_2 = 0.00$)				($\Theta = 0.05, \delta_1 = 0.00, \delta_2 = 0.00$)				($\Theta = 0.10, \delta_1 = -1.06, \delta_2 = -0.28$)			
L/σ_a	3.010			3.000				3.009			

The alternative schemes are: (i) Scheme B: To set L = 2.98 and adjust process at 10.5 sample periods, with an increase in CESTDDVN (ISTD.) of 0.98 or (ii) Scheme E: Adjust process at 5.7 sample periods and ISTD of 5.24 or (iii) Scheme J: by setting L = 3.0 and adjusting the process at 4.46 sample periods, the same ISTD. (5.09) could be achieved with an AI of 4.5.

These schemes are based on how much the CESTDDVN would need to increase to achieve the advantage of taking samples and making adjustments less frequently. This approach avoids the direct assignment of values to costs C_a (cost of adjustment and sampling) and C_t (cost of being off-target). The table shows for various values of the standardised action limit, L/σ_a = 3.010, 3.000, 3.009 and the adjustment interval (AI), the percent of increase in CESTDDVN (ISTD.) with respect to σ_a and AI. The IMA parameter value Θ which determines the process drift and the dynamic parameters δ_1, δ_2 are given below the scheme.

8.7 COMPARISON OF CONTROL SCHEMES

Table 8.2 Control Schemes for Fast and Slow Drifts

IMA Θ	*Rate of drift* $r=1-\Theta$	*Adj. Interval* *AI*	*Control Adj.* *dxt*	*Var. of adj.* *vardxt*	*Cost* $F = C[(\sigma_{AI}^2/\sigma_a^2) + b(1-\Theta)^2] + 1/AI$ $= C[1+ b(1-\Theta)^2] + 1/AI$
0.05	0.95(Fast)	10.52	-0.092	0.266	0.59
0.95	0.05(Slow)	8.62	-0.006	0.001	0.616

$C(\sigma_{AI}^2/\sigma_a^2)$ is taken as C itself since from Equation (8.8), $\sigma_{AI}^2/\sigma_a^2 = 1$ and as mentioned in Section 8.5, the tuning parameter Θ is the same as Θ_{AI}.

8.8 CONCLUSION

A brief review and background of process regulation were given in this Chapter. The effect of dead time on feedback control and feedback control process regulation schemes was explained. It was shown that the cost regulation scheme is a minimum for a process with delay and an approximate cost model was presented. Some process regulation schemes and comparisons were also given. Controller performance and application to product quality control are discussed in Chapter 9.

CHAPTER 9

CONTROLLER PERFORMANCE AND PRODUCT QUALITY CONTROL

9.1 INTRODUCTION

In this Chapter, we explain briefly the terms performance and robustness as applied to a process controller. The characteristics and requirements of a controller are also given as too is a brief description of the working of a direct digital (discrete) feedback controller along with the means to obtain damping in the feedback control loop. These explanations are given to complete the discussion on controllers and the economic benefits of process control application to product quality.

9.2 CHARACTERISTICS REQUIREMENTS OF CONTROLLERS

9.2.1 Dynamic Optimization

Various management objectives are described by profit maximization, loss minimization or by optimization or minimization of other functions. Process dynamics are ignored in steady state optimization of industrial processes. In these processes, a reference value (set point) is set in the various control systems to hold the process variables within the desired limits. Dynamic optimization considers the dynamic behaviour of the process by manipulating the inputs during non-stationary conditions. *By considering a second-order dynamic model and simulation of the feedback control algorithm that minimized the variance at the output, dynamic optimization under non-stationary conditions has been achieved in this thesis*

9.2.2 Controllers Requirements

An industrial process control system, regardless of the control structure or algorithm used, must be robust; that is, it must function reasonably despite process and/or modelling errors over the full range of process operations.

The controller required for a particular application depends upon the objectives the process control system and the dynamic process model (transfer function). The preferred controller for a specific task depends on the type of process to be controlled and the relative importance of 'performance' and 'robustness' (explained subsequently). The response of the controller and the system depend upon the process disturbances. A detailed model of any industrial process is likely to be complicated, therefore, the control system engineer faces a situation in which a control system must be formulated (designed) on the basis of a simplified description (model). In this regard, it is often better to work with an 'acceptable' solution than to be unable to find the 'perfect solution'.

9.3 FEEDBACK CONTROL SYSTEMS

The performance of a feedback control system employing mechanical, hydraulic or pneumatic elements is measured quantitatively by the root mean square (rms) error. It is the ratio of the rms 'error' to the rms input (signal). In many control applications, it is essential that the desired output be obtained instantaneously in time. In a control problem, a satisfactory (linear) phase shift, (explained in Section 3.6.1), may be the cause of excessive error. The purpose of feedback compensation is to alter the performance of a system so that the resulting error will fall within specifications. The desired output (signal) is frequently instantaneously equal to the input (signal) and imposes a severe specification on the control system design.

9.4 SAMPLED-DATA FEEDBACK CONTROL SYSTEMS AND DIGITAL CONTROLLERS

9.4.1 Sampled-Data Feedback Control Systems

A discrete ('sampled-data') system was defined in Section 3.5.3 as one of those 'purely digital systems where the input and the output are described only at the sampling instants' (Marshall [1979]). A sampled-data (discrete) system is a control system in which the data appear at one or more points as a sequence of numbers. An analogue system is one in which the data are everywhere known or specified at all instants of time and the variables are continuous functions of time. If the system includes elements which feed the output (dependent variables) back to the input (independent variables) and if a sampling operation is included, the system is referred to as a 'sampled-data (discrete) feedback control system'.

9.4.2 Digital Controllers

Some of the salient features of a sampled-data system are discussed in this Section. The digital controller is a computer that accepts a sequence of numbers at its input, processes it in accordance with some logical programme, (usually linearly) to produce an output. The output (number) sequence is reconstructed into a command signal and the resultant sequence is applied to the controlled element. By properly designing the (linear) computer programme of the digital controller, the system can be stabilized and its dynamic performance made to conform to rigid specifications. It is possible to implement it by means of digital computer techniques or its equivalent as a mixture of analogue and digital components or wholly by an analogue computer. If the process programmed in the computer is linear, it can be expressed in terms of a recursion formula (or equation) which is transformed into a generating function. The sampling periods are equal for all the samplers in a (linear) sampled-data dynamical system.

Sampled-data (discrete) systems are often subjected to random disturbances. Any compensating device preceded by (synchronous) samplers is referred to as a digital controller. A process control system requires the storage of only a finite number of input and output samples. A pure regulator (controller) system has a fixed reference or set point and the only dynamic effect is the result of disturbances. In regulator system-design, the input in the form of disturbances has an influence in the controller design. In a stable digital system, the reference input is assumed to be a constant and if the system is linear, only that component of the output caused by the disturbance need be considered since it can be superimposed on any other outputs produced by other sources.

One of the advantages of digital controllers is that they can be applied to systems with large time constants. A control programme from the digital computer could be used to compute commands to the plant at sampling points. The exact form of the digital controller depends upon the required application.

9.4.3 Principle of Operation of Discrete Digital Feedback Controllers

With a basic idea of sampled data (discrete) systems and digital controllers as background, the principle of operation of discrete digital feedback controllers is explained briefly in this Section.

The characteristics of the discrete (sampled data) digital feedback controller loop determine how well the process can be controlled and what controller settings are required to produce minimum output variance of the product. The conditions of uniform oscillation of a feedback control loop serve as a convenient reference on which rules for controller adjustment can be based. It is known that the tendency towards oscillation is one of the characteristics of a feedback control loop. So, the feedback control loop as well as the feedback controller should be guarded from the occurrence of over and under damped oscillations which may lead to an unstable feedback control loop.

9.5 MEANS TO ACHIEVE DAMPING IN A FEEDBACK CONTROL LOOP

The integral action term in the control adjustment equation (5.16), determines the damping of the feedback control loop. The period and the integral time required for damping are established by the process characteristics. Minimizing the integral error ensures damping of the feedback control loop. The minimum integral (absolute) error, (IAE), for (integral) control of dead time is achieved by setting the integral time constant of the feedback control loop, $I = 1.6PGT_d$, where T_d is the time delay (dead time) (Shinskey [1988]). Under integral (floating) control, the feedback control loop tends to oscillate with uniform amplitude and at the period where, the (pure) steady-state gain is unity. For an (integral) time constant equal to the dead time, the period of oscillation changes to about half of the original period. Increases in the (integral) time constant contribute more (phase) lag to the feedback control loop and extends the period of oscillation of the feedback control loop. There must be a rise in integral time constant so that the controller can then contribute less (phase) lag. Since the controller algorithm also plays a part in the stable functioning of a feedback control loop, some of the requirements of a sampled data controller algorithm are discussed in the next Section.

9.6 REQUIREMENTS OF A SAMPLED - DATA CONTROLLER ALGORITHM

(i) *The first and foremost requirement of a controller is that it must be able to maintain the desired output variable at a given set point.*

(ii) *The set point changes should be fast and smooth.*

(iii) *The algorithm must lead to stable overall control and converge fast on the desired steady state, thus ensuring asymptotic stability and satisfactory performance for different types of disturbances that could arise.*

(iv) *The controller should be designable with a minimum of information with respect to the nature of the inputs and the structure of the system. This is because of the fact that optimal algorithms, by their nature, tend to be (very) sensitive to both the structure and the exact values of the parameters of the model describing the process.*

(v) *The controller must be reasonably insensitive to changes in system limits. This means that it must be stable and perform well over a reasonable range of system parameters.*

(vi) *It is preferable to avoid excessive control actions.*

(Palmor and Shinnar [1979])

The performance of the computer-controlled algorithm depends not only on the tuning constants but also on the sampling period (adjustment interval). It is reported in control engineering literature that although a second-order (conventional) control system is stable for all values of the (controller) gain, it is likely that computer control of the same system can give unstable response for some specific combinations of the gain and the sampling period.. This is one of the main reasons to consider the critically damped behaviour of the second-order model in Section 5.5, Chapter5.

9.7 CONTROLLER PERFORMANCE AND LIMITATIONS

9.7.1 Performance

Performance, an important characteristic of a controller, refers to how closely the unit holds the output controlled variable to set point in the face of disturbances. The nature of the process, particularly its dead time, determines the best performance achievable by a feedback controller. One way of improving control performance is to reduce dead time in the closed-loop. Performance depends not only on the controller, but on the tuning as well. Tuning rules are affected by the process being controlled. When an input change is encountered, the output controlled variable moves away from the set point along a trajectory (path) determined by the disturbance and the lags in the path between the input and the output controlled variable. Concurrently, the controller output changes according to its algorithm and tuning parameters. However, the controlled variable cannot respond to the controller until the dead time in the feedback control loop lapses. Once a controller is tuned for a given performance, a change in the process gain or dead time could bring the loop to the limit of its stability (that is in the meaning of an undamped oscillation). The smallest change in any process parameter capable of bringing the loop to this point is the 'robustness index'. The robustness of the Dahlin's controller, can be improved by setting its dead time relative to machine speed. Sampling, it is believed, produces a phase lag equal to half of that produced by the same dead time in digital controllers.

The performance of a controller is usually referred to by a performance index. Desborough and Harris [1992] introduced a 'normalized performance index' to characterize the performance of feedback control schemes. A method is outlined in their paper to estimate the index from routine closed-loop process data using linear regression methods.

9.7.2 Robustness

Robustness of a process control loop is that quality that keeps its closed-loop (feedback) stable following variations in process parameters. A 'robust' control loop is one that performs well even in the presence of moderate changes in process parameters. Changes in dead time and gain make a controller reach instability. It is known that sustained undamped oscillations in a control loop represents the limit of stability. This can be brought about by increasing the steady-state gain of a process that reduces the damping of the control loop. The increase in gain required to bring the loop to the limit of stability is a measure of robustness.

The performance of a feedback controller in responding to load changes is at the cost of reduced robustness. Processes having lags (like inertia) form more robust feedback control loops with PID controllers. A process lag, however, requires derivative control action to attain a level of performance that returns the feedback control loop to essentially the same level of robustness as a (pure) dead-time process. There is a trade-off between robustness and performance. The process dead time determines the best performance achievable by a feedback controller. For a feedback controller that is not robust enough but provides an acceptable performance, the recourse is to go in for self-tuning by adapting the controller settings to follow variations in the process parameters in order to keep high performance. An advantage of discrete (sampled-data) model-based controllers is increased robustness that results from sampling the process at slow intervals, (slower sampling).

Robustness can be improved by detuning a controller, but its performance is decreased at the same time. Performance and robustness are inversely related. Lowering the controller gain and slower sampling, improve robustness at the cost of performance. The highest performance also brings with it the lowest robustness. So, high-performance controllers should be capable of on-line self-tuning; i.e. they should be adaptive in nature.

9.7.3 Performance Limitations

An (optimal) controller design depends (i) in defining the input and disturbance to the system in statistical terms and (ii) on a performance criteria such as the mean square error. The mean square error for stochastic (signals) is an example of a quality measure for the performance of a process control system. The mean square error is identically equal to the value of the autocorrelation function of the error at zero shift, (a parallel change in slope of the input-output curve). An attempt to predict the future value of a stochastic (signal) by means of a (linear) system leads to a definite performance limitation in the sense that the mean square error cannot be made (indefinitely) small. Even when the system is adjusted for minimum mean square error, there is a lower limit below which this error cannot be reduced. Imposing a requirement for prediction on a linear system that operates only on present and past information, limits the performance that can be achieved. Disturbance (noise) makes it impossible for a linear system to establish equality between the ideal output and the actual output. A time delay (dead time) must be regarded as a fundamental factor in limiting the performance that may be achieved with a linear system. In many industrial processes, it is difficult to properly tune standard regulators (controllers). It becomes necessary to use sophisticated controllers when long delays or time constants are present, especially in complex systems and when the minimum output variance conditions are imposed.

9.8 ECONOMIC BENEFITS OF PROCESS CONTROL APPLICATION

9.8.1 Engineering Control Application to Product Quality

To control the quality of a product at the output, the set point of a product-quality controller is adjusted so that the product remains within its specification limits following expected load changes or disturbances. In product quality control, the product-quality set point is adjusted away from the specification limit in proportion to the peak deviation expected to be yielded by the controller. Again, the adjustment is in a direction that increases operating costs. Deviation in the 'safe' direction increase operating costs in proportion to the deviation. The quality-controller's set point is positioned relative to the specification limit so that the limit will not be violated for most upsets (load changes). Since the average output product quality will be equal to the set point, the product will be more expensive to make than if the set point were positioned exactly at the specification limit. Excess manufacturing cost is proportional to the difference between the set point and the specification limit and so, proportional to the peak deviation expected. By limiting the peak deviation, excess manufacturing cost and product quality are controlled in engineering control. Peak deviation of the controlled variable from set point is significant when excessive deviation will cause an incident such as rejecting a product due to failure to meet the specification.

Process control provides operating conditions under which a process will function safely, productively and profitably. Ineffective control can be costly in causing, amongst other things, plant shutdown, in consuming resources excessively, in allowing off-specification product to be made, and in unnecessarily reducing the produvtion rate. For a particular control loop, it is often possible to relate operating cost to deviation of the output controlled variable. In product-quality control loop, it is not surprising that the cost function is (usually) found to be different on opposite sides of set point; i.e. it is possible to have both both possitive and negative cost functions.

Process operators frequently place a large margin between the measured quality of a product and its specification to counteract the changes in economic performance when a product specification is violated. It will cost more to produce higher-quality product. Maximum profit can be realized when product quality meets the specifications exactly; but variations in product quality are not equally acceptable on both sides of the specification. As a consequence, the quality set-point must be positioned far enough, without excessive operating costs, on the most acceptable side of the specification. The operating cost can be reduced by better control and smaller variation in quality, allowing the set point to be moved to closer the specification. The deviation between the output controlled variable and its set point can be related (linearly) to operating cost. Overshoot of the output controlled variable can be minimised by limiting the rate of the set-point changes that are likely to be introduced by the operator during the course of plant operation and process control.

Integration of deviation (error) over time could be equated to accumulated (excess) operating cost. Under such circumstances, the control objective would be to minimize integrated error*. This criterion could be applied to control the quality of a product flowing into a storage tank, for example. This can be achieved by keeping the integrated error as low as possible and the quality of the product closer to the set point. (*Integrated error can be estimated from the feedback control algorithm equation (5.16), being equal to (e_t-$\delta_1 e_{t-1}$-$\delta_2 e_{t-2}$)). It is a function of the change required in the input manipulated variable and the setting of the integral mode of the (time series) controller. Integrated error can be significant in product-quality loops, where it may represent excessive operating cost such as product giveaway. Lag-dominant dynamics, (similar to the second-order dynamic model with two exponential terms), characterize, most of the important plant loops such as product quality. For these processes, the integral error, (explained in Section 7.2.5) varies linearly with time.

9.8.2 Statistical Application to Product Quality Control

For the sample values of a product variable whose measurements are normally distributed, its mean will equal the set point if the integral of the error approaches zero over a period of time. In minimizing the deviation of the output controlled variable, the standard deviation is a transformation of that deviation over a statistically significant number of samples or time of operation. The economic incentive behind the standard - deviation criterion is that this criterion estimates the percentage of time the controlled variable violates the specification based on a normal distribution. If samples of an output controlled variable that is 'cycling' uniformly are averaged over a complete cycle to form a subgroup, then the mean of subgroups will lie on the set point and their standard deviation will approach zero (if there are no disturbances in the feedback control system).

The assignment of subgroup size should reflect the capacity of a process to absorb variations in product quality. The method used to average samples also needs to be selected to match the characteristics of the process. If the product is segregated into lots, then, the samples should be segregated into the same lots and averaged equally.

9.9 COMPUTER CONTROL IMPLEMENTATION ON A PROCESS LOOP

A brief description of implementing computer control on a a typical process loop is given in this Section. Consider the implementation of computer control on a single-loop process control system. The single-loop system is a quality control loop currently working on (conventional) control mode. The controller of this loop is the Integral type. It is desired to place this loop under computer control, utilizing the discrete equivalent of the integral controller. A suitable measuring device, strategically placed in the process control system, measures some required characteristic of the output quality process variable and converts it into an (electrical) signal. The integral controller compares this signal with the desired value, set point. If an error exists, the controller outputs a feedback signal which manipulates an input adjustment to eliminate the error.

In computer control, the electrical signal is transmitted to the control computer terminals which represent one of the analogue-to digital (A/D) converter channels. The computer hardware design is such that it can access the discrete output of the A/D converter. The discrete output from the computer is converted to a continuous signal, on demand, by one of the digital-to-analogue (D/A) converter channels. The D/A output is available at the analogue output terminals of the computer. The control computer is instructed to sample the A/D channel every T seconds, T being the sampling period. The computer programme operates on this measurement, representing the value of the measured variable at the sampling instant. For this purpose, the computer uses the discrete equivalent of the controller adjustment equation and computes the desired control-algorithm output. The computer is then instructed to forward this output to the D/A converter and to the compensating (correcting) device. This procedure is repeated every T seconds to achieve closed-loop computer control.

The benefit of computer application to process control is the facility to implement control strategies that might not be practical with analogue hardware. Development of such strategies requires the analysis of computer-control loops to determine their stability characteristics.

9.10 MODEL BASED CONTROLLERS

The question arises whether integral control is the best control mode in application to product quality control. Easily controllable processes can justify the use of integral control. It is essential, in practice, to have integral action to control difficult processes. Integral action control mode is, in actuality, time constant just like the time constants in a process, but they bear no resemblance to dead time that exists in the process or plant.

A feedback control system that is modelled after the process, called a 'model-based' controller has a feature that critical damping can be achieved with a loop gain of 1.0. This is accomplished by the simultaneous feedback of controller output through the process and controller. If both signals arrive at the integral (summing) junction downstream (of the integral block or mode) with the same amplitude at the same time, they will cancel, avoiding further changes in output. This avoids oscillation even in spite of achieving a loop gain of 1.0. The controller output follows the load with open-loop response. Both set point and load responses are ideal for a (pure) dead-time process. A model-based controller provides good feedback control of a dead-time process, called 'dead-beat' response (a loop gain of 1.0). Model-based controllers are superior to PID controllers in performance for processes in which dead time has a dominating effect on the process characteristics and behaviour when a process is subject to different loads and disturbances. Model-based controllers perform well on a dead time process, but do not possess much robustness. A practical application of the model-based controller principle is to control a sugarcane crushing mill. Details of the model-based control of sugarcane crushing mill can be found in the guest editorial by Bob Bitmedd, Process & Control Engineering Journal (1994), Volume 47, No.10, 65-66 and that of an integral controller application for control of dye concentration in Buckley [1960].

9.11 CONCLUSION

The performance, limitations and robustness and the function of a controller were explained in this Chapter. The characteristics and requirements of a feedback controller were also given. The working of a direct digital sampled data (discrete) feedback controller and a model-based controller was described in brief.

CHAPTER 10

CONCLUSION

10.1 BACKGROUND AND METHODOLOGY

This thesis has considered feedback ,control (closed-loop) stability problems from the automatic process control point of view and application of process control techniques at the interface of SPC and APC. A detailed review of the literature on stochastic-dynamic process control has been given. The thesis has focused attention on bringing together contributions made in the analysis of closed-loop dynamic-stochastic systems, assessment of control loop performance, on-line process control together with the work done separately by control engineers on automatic process control.

The issues connected with statistical process monitoring and feedback control adjustment, namely, feedback (closed-loop) stability, controller limitations, dead-time compensation and process dynamics in achieving minimum variance control have been discussed.

A second-order dynamic model was considered to account for process dynamics and an expression was derived for the input adjustment required that will exactly compensate the output deviation. The feedback control algorithm was simulated and the simulation results were discussed. It was shown that it is possible to achieve integral control and dead-time compensation in the presence of dead time in a dynamic process. The performance of the time series controller was shown to be better than that of the EWMA and CUSUM controllers. The results demonstrate that minimum mean square error (variance) control can be achieved in practice and that the product variability can be minimised at the output. An outline of a process regulation scheme was given and also an application of model-based control to product quality.

This thesis has shown that an approachable solution to the commonly occurring product quality control problem is possible by properly making use of SPC techniques such as EWMA forecasting, time series modelling and the APC techniques of dead-time compensation and process regulation. The results of this research show that by knowing the input feedback control adjustment, its variance and adjustment interval (AI), it is possible to devise suitable process regulation schemes. By means of the methods proposed, it is possible to take a different approach to solve the commonly occurring product quality control problem. It has widely made use of the operating principles of controllers, basic principles of feedback control stability and stochastic time series ARIMA models. The application of techniques such as control charting and process regulation have been shown to mix and blend together at the interface of the two process control methodologies. It is hoped that the issues raised and the solutions offered will further kindle interest and offer scope to enhance the work that is currently under way by the scientific and technical communities to integrate SPC and APC.

Optimization under non-stationary conditions has been achieved by considering the dynamic behaviour of the process and by manipulating the input during non-stationary conditions. The solution to the problems of statistical monitoring and feedback adjustment connected with feedback (closed loop) stability, controller performance and robustness and adequate dead-time compensation to achieve minimum variance control, were found by the application of both the process control techniques. The IMA parameter, Θ, set to match the disturbance, was suggested also as an on-line tuning parameter to compensate for the dead time. As the magnitude of the random shocks through the system change, so also does the nature of the disturbance and the process drifts. As the IMA parameter Θ changes, the process adjusts itself, and (in a way), adapts to the new (random shocks) conditions. So, in a way, adaptive (self-tuning) control has been achieved in an indirect manner. The performance of the time series controller was shown to be superior to that of the EWMA and CUSUM controllers and provides minimum variance control even in the face of dead time and dynamics and provides adequate dead-time compensation with the controller gain (CG) set to 1. By exploiting a time series controller's one-step ahead forecasting feature and considering closed-loop (feedback) stability and dead-time compensation, this thesis takes one step further from the articles/papers that have appeared recently in relation to statistical process monitoring and feedback adjustment, ASPC and discussion on integrating SPC and APC.

Demonstration of an application to an actual control design problem has not been done because the objective of this thesis has been to find solutions to issues connected with statistical and feedback control adjustment at the interface of SPC and APC.

10.2 FURTHER SCOPE FOR RESEARCH

Dead time was considered as whole periods. It is possible to consider fractional dead time periods also and processes in which dead time is more dominant (called, dead-time dominant processes) than the inertial (called lag-dominant) characteristic processes. The thesis considered jumps in the mean (step shifts) taken care of by the EWMA control limits. It is possible to consider also ramping shifts in the mean in which there is a slow and gradual increase and decrease in the mean value and square impulses in which there is a sudden upward step increase to a new value for a time and then a decrease in the mean value. Another area of interest is the disturbance level and its cyclical variations. A single-input, single-output (SISO) system was considered in this thesis. It is possible also to consider more than one input and the effect of the change in one of the input variables on the output. The effect of reducing, if not possible to eliminate completely, the dead time, by an appropriate control strategy that gives good performance and at the same time takes care of the controller robustness can also be considered with a view to an improvement in performance. Apart from imposing feedback control stability conditions, the conditions for achieving (minimal) controller robustness without sacrificing performance can also be considered. The dead-beat response of a model-based controller under critically damped conditions can also be exploited to the advantage of the process control industry.

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