IMPACT OF DEMOGRAPHIC CHANGE ON INDUSTRY STRUCTURE IN AUSTRALIA

A joint study by the Australian Bureau of Statistics, the Department of Employment and Industrial Relations, the Department of Environment, Housing and Community Development, the Department of Industry and Commerce and the Industries Assistance Commission

FACTOR DEMAND AND PRODUCT SUPPLY RELATIONS IN AUSTRALIAN AGRICULTURE : THE CRESH/CRETH PRODUCTION SYSTEM

by

Peter B. Dixon, David P. Vincent & Alan A. Powell

Industries Assistance Commission

Preliminary Working Paper No. OP-08 Melbourne November 1976

Reprinted September 1977

The views expressed in this paper do not necessarily reflect the opinions of the participating agencies, nor of the Australian government.

IMPACT Project, Industries Assistance Commission, 606 St. Kilda Road, Melbourne, Victoria 3004 Australia
Area Code 03 Telephone 51 8611

CONTENTS

FACTOR DEMAND AND PRODUCT SUPPLY RELATIONS IN AUSTRALIAN AGRICULTURE : THE CRESH/CRETH PRODUCTION SYSTEM

by

Peter B. Dixon, David P. Vincent &
Alan A. Powell
Industries Assistance Commission

1. PURPOSE OF THIS PAPER

This paper is part of a larger project designed to model the Australian economy at a level of disaggregation which distinguishes about 100 different industry groups and about a dozen different occupational categories of labour.[1] In the initial design of the economy-wide model, no provision was made for joint production, so that for every product recognized in the model, the production function could be written as[2]

(1) Gross output of product produced by industry i = function of inputs into industry i ;

i.e.,

$$Y_i = f_i(X_{1i}, X_{2i}, \ldots, X_{Mi}) \quad . \tag{1'}$$

Whilst characterizations of the type (1') may be satisfactory for modelling production relations in the secondary, tertiary and extractive sectors, they are less satisfactory for the Australian agricultural sector which is to a large extent dominated by joint production. The purpose of this paper is to develop a more plausible specification of the production technology to accommodate this case.

---

1. For details of the overall project design, see Alan A. Powell and Tony Lawson, "IMPACT: An Economic-Demographic Model of Australian Industry Structure - Preliminary Outline," Impact of Demographic Change on Industry Structure in Australia, Working Paper No. I-01, Industries Assistance Commission, Melbourne, September, 1975.

2. For details of the specification of the production technology in the initial design, see Peter B. Dixon, "The Theoretical Structure of the ORANI Module," Impact of Demographic Change on Industry Structure in Australia, Working Paper No. O-01, Industries Assistance Commission, Melbourne, October, 1975.

Various types of separability have been postulated in the literature of production relations.[1] One possible type of separability is reflected in the fact that multi-product production functions do not necessarily involve jointness. Such a lack of jointness in (say) a two product production process with three factors would mean that the single product procuction functions would be well defined and could be written

$$(2.1) \qquad Y_1 = f_1(\alpha_1 X_1, \alpha_2 X_2, \alpha_3 X_3) \quad ,$$

$$(2.2) \qquad Y_2 = f_2\,((1 - \alpha_1)X_1, (1 - \alpha_2)X_2, (1 - \alpha_3)X_3) \quad ,$$

in which α_j is the share of total usage of factor j which is allocated to the first product. Systems of the type (2.1) - (2.2), however, involve a degree of separability which is incompatible with the degree of jointness known to exist between important agricultural products in Australia (e.g., between beef and wool, or between wool and wheat).[2] Indeed, from a technical (and even from an accounting) viewpoint, the shares $\{\alpha_j\}$ are often impossible to identify in any meaningful way.[3] It is more attractive to think of inputs of a non-specific nature (e.g., of fertilizer, as distinct from shearing) as

---

1. Ragnar Frisch (Theory of Production (Dordrecht & Chicago: D. Reidel & Rand McNally, 1965), Chapter 14) devotes a chapter to "multi-ware production" in which various separability concepts are put forward. For a recent example which takes the separability idea from the consumer literature and applies it to factor demands in the context of a one-product production function, see Henri Theil, "The Independent Inputs of Production," University of Chicago, Center for Mathematical Studies in Business and Economics, Report 7535, September, 1975, pp. 80 (mimeo).

2. See, e.g., I. R. Wills and A. G. Lloyd, "Economic Theory and Sheep Cattle Combinations," Australian Journal of Agricultural Economics, Vol. 17, No. 1 (April, 1973), pp. 58-67.

3. Consider, for example, a farmer who sows a grain crop in autumn, grazes it lightly during the winter, and harvests grain from it in the early summer. What fraction of fertilizer, ploughing, weed killing, sowing and similar costs, should be attributed to the production of wool, and what fraction to grain?

determining the location of the short-run product transformation curve. If all inputs are non-specific, then the simplest option is to define a production function whose left hand variable is a scalar index Z defining generalized capacity to produce

$$Z = f(X_1, \ldots, X_M) \quad , \tag{3.1}$$

where the $\{X_j\}$ are the total levels of factors (including both primary factors and intermediate inputs) which are input into the multi-product production activity. This capacity index then serves to locate the product transformation schedule; namely,

$$g(Y_1, \ldots, Y_N) = Z \ . \tag{3.2}$$

In this paper we postulate functional forms for f and g (namely CRESH[1] and CRETH[2,3]). On the assumption that certain factors are fixed (e.g., land), the optimal behaviour of a representative producer is modelled within a one period setting. This generates factor demand and product supply relations which recognize the essential jointness of production relations in Australian agriculture. All of this is done in Section 2. In Section 3, the data base available for carrying out the estimation is discussed in some detail. After that in Section 4 an econometric specification is made in the light of the model and the salient features of the data.

---

1. Giora Hanoch, "CRESH Production Functions," Econometrica, Vol. 39, No. 5 (September, 1971), pp. 695-712.

2. CRETH (to be discussed below) is in relation to CET as CRESH is to CES. The CRETH formulation was suggested by Peter B. Dixon in, "The Costs of Protection : The Old and New Arguments," Impact of Demographic Change on Industry Structure in Australia, Preliminary Working Paper No. IP-02, Industries Assistance Commission, Melbourne, June, 1976.

3. In the development below we do not require either f or g to have explicit functional forms; indeed, CRETH and CRESH do not possess functional forms which are explicit in terms of (3.1) and (3.2).

Proposed estimation procedures are then discussed. Section 5 contains brief concluding remarks.

2. SPECIFICATION OF THE MODEL

The approach embodied in (3.1) and (3.2) is that suggested by Jorgenson, Christensen and Lau[1], and recently taken up by Hasenkamp[2]. Among the alternatives considered by these authors was the case in which f took the CES form[3] and g took the CET form[4]. For situations in which $N = M = 2$ (two factors and two products), such a specification is sufficiently flexible to accommodate most empirically interesting cases. When both the number of factors and the number of products exceeds two, the CES/CET specification suffers from the very serious weakness that each of the $[\frac{1}{2}(M - 1)M]$ pairwise partial substitution elasticities are constrained to equality, whilst the analogous restriction applies also to partial transformation elasticities.

---

1. Dale W. Jorgenson, Laurits R. Christensen, and Lawrence J. Lau, "Transcendental Logarithmic Production Frontiers," Review of Economics and Statistics, Vol. 55, No. 1 (February, 1973), pp. 28-45.

2. Georg Hasenkamp, "A Study of Multiple-Output Production Functions : Klein's Railroad Study Revisited," Journal of Econometrics, Vol. 4, No. 3 (August, 1976), pp. 253-262.

3. K. J. Arrow, H. B. Chenery, B. S. Minhas and R. M. Solow, "Capital-Labour Substitution and Economic Efficiency," Review of Economics and Statistics, Vol. 43, No. 3 (August, 1961), pp. 225-250.

4. Alan A. Powell and F. H. Gruen, "The Constant Elasticity of Transformation Production Frontier and Linear Supply System," International Economic Review, Vol. 9, No. 3 (October, 1968), pp. 315-328.

Hanoch has suggested a generalization of CES (namely, CRESH : constant ratio of elasticities of substitution, homethetic) which permits substitution elasticities to differ among different pairs of factors, without, however, introducing a large number of additional parameters[1]. The latter feature makes CRESH attractive for empirical work; a slight drawback is that the functional form of the CRESH production function, f, cannot be obtained explicitly. An implicit representation for the constant returns to scale case is given by[2]

$$(4.1) \qquad \sum_{\substack{j \\ h_j \neq 0}} \frac{t_j}{h_j} \left(\frac{X_j}{Z}\right)^{h_j} + \sum_{\substack{j \\ h_j = 0}} t_j \, \ell n \left(\frac{X_j}{Z}\right) = \kappa_1 \quad ,$$

in which t_j, h_j and κ_1 are parameters with $t_j \geqslant 0$ and $h_j < 1$ for all j. We can also assume that t_j and κ_1 are normalized so that

$$\sum_{j=1}^{M} t_j \equiv 1 \quad .$$

---

1. Hanoch, op. cit..

2. Hanoch's (op. cit.) representation of CRESH differs slightly from ours in that he adopted an alternative normalization of the parameters. Also, he allowed for non-constant returns to scale by replacing Z in (4.1) by a general function $\psi(Z)$. In this paper, we are assuming constant returns to scale, and therefore only the special case $Z = \psi(Z)$ is required.

From (4.1) we find that in writing the marginal products, $\partial Z/\partial X_j$, the need for the partition vanishes, as the same algebraic expression, namely

$$\text{(4.2)} \qquad \frac{\partial Z}{\partial X_j} = t_j \frac{X_j}{Z} \left(\frac{X_j}{Z}\right)^{h_j - 1} \Bigg/ \sum_{j=1}^{M} t_j \left(\frac{X_j}{Z}\right)^{h_j}$$

accommodates both the case $h_j = 0$ and $h_j \neq 0$. (Indeed, it is this feature which justifies the form of the second term on the left of (4.1)[1]).

Finally, we note that the marginal rate of substitution between any pair of factors ℓ and k is given by

$$\text{(4.3)} \qquad MRS_{k\ell} \equiv \left.\frac{\partial X_\ell}{\partial X_k}\right|_{Z, X_j (j \neq k, \ell) \text{ constant}} \equiv -\frac{\partial Z/\partial X_k}{\partial Z/\partial X_\ell}$$

$$= -t_k \left(\frac{X_k}{Z}\right)^{h_k - 1} \Bigg/ t_\ell \left(\frac{X_\ell}{Z}\right)^{h_\ell - 1}$$

and that the restriction on the h_j's ensures that the (absolute value of the) $MRS_{k\ell}$ falls as we increase X_k/X_ℓ, holding all other inputs and Z constant.[2]

---

1. The addition of the logarithmic term on the left of (4.1) to cover the case of a zero exponent is in the same spirit as the suggestion made by Johansen in the context of his utility function. See Leif Johansen, "On the Relationships Between Some Systems of the Demand Functions," University of Oslo, Institute of Economics, Reprint Series No. 47, Oslo, 1969; reprinted from Liiketaloudellinen Aikakauskirja, pp. 41.

2. The restriction that $h_j < 1$ for all j is necessary and sufficient to ensure that CRESH is globally strictly-quasi-concave. On the other hand, if we are content with local strict quasi-concavity, then one (but only one) of the h_j may exceed 1. However, if $h_1 > 1$ and $h_i < 1$, $i = 2, \ldots, M$, then it can be shown that factor 1 is a substitute for all other factors while the factors 2, ..., M form a group of complements, with negative elasticity of substitution for any pair in the group -- see Hanoch, op. cit. p. 700. It seems to us that in a study which deals with "general factors," as this one will, such as land, labour, capital, etc., little is lost by ruling out the sort of local complementarity which is potentially allowable in CRESH.

2.1 The CRESH/CRETH Production System

Turning now to the product-product space, Dixon has recently suggested extending the CET formulation in an analogous way; the resulting product transformation frontier (CRETH -- constant ratio of elasticities of transformation, homothetic) is[1]

$$(5.1) \qquad \sum_{i=1}^{N} \frac{r_i}{k_i} \left(\frac{Y_i}{Z} \right)^{k_i} = \kappa_2 \quad ,$$

where Z, as before, has the interpretation of a scalar measure of total capacity which may be thought of as either an input index or an output index.[2] The r_i and κ_2 are normalized so that

$$(5.2) \qquad \sum_{i=1}^{N} r_i \equiv 1 \quad .$$

The restriction on the k_i's ensures that for any pair of outputs w and v, the marginal rate of transformation,

$$(5.3) \qquad MRT_{vw} \equiv \left. \frac{\partial Y_w}{\partial Y_v} \right|_{Z, Y_i (i \neq v, w) \, constant} \equiv - \frac{\partial Z / \partial Y_v}{\partial Z / \partial Y_w}$$

$$= - r_v \left(\frac{Y_v}{Z} \right)^{k_v - 1} \bigg/ \; r_w \left(\frac{Y_w}{Z} \right)^{k_w - 1}$$

---

1. Peter B. Dixon, "The Costs of Protection," op. cit..

2. As with CRESH, it is possible to introduce non-constant returns to scale into CRETH by replacing Z with a function Θ(Z). In this paper we are concerned only with the constant returns case.

increases (in absolute value) as we increase Y_v/Y_w, holding all other outputs and Z constant.[1]

The CRESH/CRETH production system is then defined by

$$(6)\quad \begin{cases} \sum\limits_{\substack{j \\ h_j \neq 0}} \dfrac{t_j}{h_j}\left(\dfrac{X_j}{Z}\right)^{h_j} + \sum\limits_{\substack{j \\ h_j = 0}} t_j \,\ln\left(\dfrac{X_j}{Z}\right) = \kappa_1 & j \varepsilon [1, \ldots, M] \\ \sum\limits_{i=1}^{N} \dfrac{r_i}{k_i}\left(\dfrac{Y_i}{Z}\right)^{k_i} = \kappa_2 \,. & \end{cases}$$

In the case where the h_j share a common value h,(h < 1) for all M factors and the k_i share a common value k, (k > 1) for all products, (6) becomes the CES/CET system, with substitution elasticity 1/(1 - h) and with elasticity of transformation equal to 1/(1 - k). In the case where the common values of these elasticities are plus and minus one respectively, the system collapses further to a Cobb-Douglas/Ellipse pair of functional forms.

2.2 Specific versus Non-specific Inputs

Both the CES-CET approach and ours as specified in (3.1) - (3.2) and the special case, (6), provide a severe simplification of the relationship between inputs and outputs. In the general case where we write the multi-product multi-factor production function as

---

1. More formally, $k_i > 1$ for all i is necessary and sufficient to ensure that CRETH is globally strictly-quasi-convex. It will also be noticed that since zero is not in the domain of the k_i's, the need for a logarithmic term, such as that in (4.1), does not arise in (5.1).

$$H(X_1, \ldots, X_M ; \; Y_1, \ldots, Y_N) = 0 \; ,$$

there are, at every point in the input-output space, MN "free" elasticities E_{ij}, where E_{ij} is the elasticity of output of product i with respect to factor j, i.e., E_{ij} measures the effect on i of increasing j while holding all other outputs and inputs constant. Under (3.1) - (3.2), the number of free elasticities is reduced to M + N. We notice that the E_{ij} may be written as

$$(7) \qquad E_{ij} = A_i B_j \qquad (i = 1, \ldots, N \; ; \; j = 1, \ldots, M) \; ,$$

where A_i and B_j are respectively the elasticity of Y_i with respect to Z and the elasticity of Z with respect to X_j . A_i is computed from (3.2) by allowing Z and Y_i to vary while fixing all other Y's, whilst B_j is obtained from (3.1) by computing the effect of a change in X_j on the level of Z, with all other X's unchanged.[1]

The simplification (7) implies that if the elasticity of wool production with respect to labour inputs is twice that of wheat production with respect to labour inputs, then the elasticity of wool production with respect to capital inputs is also twice that of wheat production with respect to capital inputs. This means that factors are completely <u>non-specific</u>. If it is relatively easy to expand wool output

---

1. In the case of CRESH/CRETH

$$A_i = \sum_{\ell=1}^{N} r_\ell \left(\frac{Y_\ell}{Z}\right)^{k_\ell} \Big/ \; r_i \left(\frac{Y_i}{Z}\right)^{k_i}$$

and

$$B_j = t_j \left(\frac{X_j}{Z}\right)^{h_j} \Big/ \; \sum_{\ell=1}^{M} t_\ell \left(\frac{X_\ell}{Z}\right)^{h_\ell}$$

by applying more of factor 1, then it is relatively easy to expand wool output by applying more of factor 2. In summary, no factor has comparative advantage in increasing the output of any particular product.

It is clear that in applications of models based on (3.1) - (3.2), the inputs $X_1, \ldots, X_M$ should be broadly defined (e.g., labour, capital, land, intermediate inputs). Simplification (7) would not be appropriate if the factor list included, for example, "contract shearing." Obviously, this input is quite specific to the particular output, wool. Fortunately, however, fully specific inputs, such as contract shearing, often have virtually zero substitution elasticity with respect to every other factor involved in the production of their specific output, in this case wool. For the firm's decision making, the costs of such factors can be deducted from the product price (i.e., treated as an excise tax). Other specific factors in this category are commission and freight charges involved in getting products to market. Likewise, these factors do not possess identifiable substitutes. Thus in adopting (3.1) - (3.2) we are implicitly making the assumption that any factors which are product-specific do not have substitutes (or at least that product-specific factors with substitutes can be ignored).

2.3 Fixed versus Variable Inputs

We assume that the representative farmer is efficient and seeks to minimize the cost of producing any given multi-product output bundle chosen as optimal in the light of relative product prices. The common resource base consists of fixed and variable inputs. All products are assumed to have the same production period (one year) with common starting date.

The model assumes constant returns to scale to all inputs. This implies diminishing returns to scale to the variable inputs. Hence the scale of output is determinate and the model has a solution. At the commencement of each production period, the representative farmer chooses his combination of products and variable inputs given the supply of his fixed inputs (land, capital and his own labour and management)[1].

2.4 The Objective Function

Let F be the set of subscripts identifying fixed factors. The constrained maximization problem facing the representative farm firm may now be stated as :

> Choose the bundle of outputs (Y_i), inputs (X_j), and an overall level of farm activity (Z) to maximize total farm gross margin -- that is,

---

1. An alternative approach to factor fixity would proceed along general equilibrium lines. Resources fixed in the short run from the viewpoint of agriculture as a whole (land, capital) would be taken as variable by individual farmers (in the light of their ability to buy and sell land and equipment among themselves). Factor demand and product supply schedules would be generated for a representative agent, α, on the assumption of a given level for his overall activity, Z_α. Then because of our assumption of constant returns to scale, these factor demand and product supply schedules could be turned into industry schedules simply by replacing Z_α by Z, the industry overall activity level. The model would then be closed as follows : product prices and prices of variable factors would be given exogenously on the basis of the small country and small industry assumptions respectively. Prices of fixed factors would be determined by industry demand and exogenously given supply and finally, the overall level of activity Z would be such that the fixed factor prices or rentals would leave the industry with zero profits. The net result of this paradigm, we believe, would be indistinguishable from the story which we tell for the market on the basis of a representative farmer optimising subject to factor fixity.

(8) $$\text{Max} \quad P'Y - \sum_{j \notin F} Q_j X_j$$

subject to

(9.1) $$\sum_{i=1}^{N} \left(\frac{Y_i}{Z}\right)^{k_i} \frac{r_i}{k_i} - \kappa_2 = 0$$ (product transformation constraint) ;

(9.2) $$\sum_{\substack{j \\ h_j \neq 0}} \left(\frac{X_j}{Z}\right)^{h_j} \frac{t_j}{h_j} + \sum_{\substack{j \\ h_j = 0}} \left[\ell n \left(\frac{X_j}{Z}\right)\right] t_j - \kappa_1 = 0$$ (input substitution constraint) ;

(9.3) $$X_j = \bar{X}_j \quad (j \varepsilon F)$$ (fixed factor constraint) ;

where the $\bar{X}_j$ represent the exogenously given supplies of the fixed factors. P is the vector of net prices for products, i.e., the price after allowance for the costs of product specific inputs, and for $j \notin F$, Q_j is the price of the j^{th} variable input.[1]

---

1. From the viewpoint of the model described in this paper, the exogeneity of variable input prices is based on the assumption that agriculture is a 'small industry.' Within the ORANI module of IMPACT, however, input prices (in the economy at large) are endogenous, due to the general equilibrium nature of ORANI.

2.5 First Order Conditions

The solution can be found by forming the Lagrangean (L) and solving the set of equations from the first order conditions for a maximum. The Lagrangean function is :

$$(10)\qquad L = P'Y - \sum_{j\notin F} Q_j X_j - \sum_{j\varepsilon F} Q_j (X_j - \bar{X}_j) - \Lambda \left[\sum_{i=1}^{N} \left(\frac{Y_i}{Z}\right)^{k_i} \frac{r_i}{k_i} - \kappa_2 \right]$$

$$+ \Gamma \left[\sum_{\substack{j \\ h_j \neq 0}} \left(\frac{X_j}{Z}\right)^{h_j} \frac{t_j}{h_j} + \sum_{\substack{j \\ h_j = 0}} t_j \,\ell n \left(\frac{X_j}{Z}\right) - \kappa_1 \right] ,$$

where Λ, Γ and Q_j, $j\varepsilon F$ are Lagrangean multipliers. The Q_j, for $j\varepsilon F$, may be interpreted as the endogenously determined rental prices of the fixed inputs. The signs appearing in front of each of the Q_i, Λ and Γ have been chosen to enforce the convention that each Lagrange multiplier (viz., shadow price) is expected to be positive.

Differentiating (10) with respect to the $(M + M^* + N + 3)$ variables (X_j, Q_j for $j\varepsilon F$, Y_i, Z, Λ, Γ), we obtain[1] :

---

1. Our assumptions that $h_j < 1$, $k_i > 1$ for all k, j ensure that there is no danger of corner solutions. Provided t_j and h_j have the same sign (a sensible restriction - - see Hanoch, <u>op. cit</u>., p. 697), the CRESH isoquants either fail to reach the axes or meet them tangentially with slopes of (minus) zero and (minus) infinity at the tangency points. Also, with the r_i constrained to positive values, for any given Z the CRETH transformation frontiers meet the axes at right angles. Hence even when the substitution and transformation elasticities are very high (i.e., h_j, $k_j \to 1$ - - see (24.2) and (25.2)), the first order conditions will hold as equalities. It follows that an appropriate choice of parameters enables CRESH/CRETH to approximate to an arbitrarily high degree of accuracy the corner solutions involved in perfect substitutability and/or transformability.

$$(11.1) \qquad \frac{\partial L}{\partial Y_i} = P_i - \Lambda \left(\frac{Y_i}{Z}\right)^{k_i} \frac{r_i}{Y_i} = 0 \qquad \text{(N equations) ;}$$

$$(11.2) \qquad \frac{\partial L}{\partial X_j} = -Q_j + \Gamma \left(\frac{X_j}{Z}\right)^{h_j} \frac{t_j}{X_j} = 0 \qquad \text{(M equations) ;}$$

$$(11.3) \qquad \frac{\partial L}{\partial \Lambda} = \sum_{i=1}^{N} \left(\frac{Y_i}{Z}\right)^{k_i} \frac{r_i}{k_i} - \kappa_2 = 0 \qquad \text{(1 equation) ;}$$

$$(11.4) \qquad \frac{\partial L}{\partial \Gamma} = \sum_{\substack{j \\ h_j \neq 0}} \left(\frac{X_j}{Z}\right)^{h_j} \frac{t_j}{h_j} + \sum_{\substack{j \\ h_j = 0}} t_j \, \ell n \left(\frac{X_j}{Z}\right) - \kappa_1 = 0 \qquad \text{(1 equation) ;}$$

$$(11.5) \qquad \frac{\partial L}{\partial Q_j} = X_j - \bar{X}_j = 0 \quad (j \varepsilon F) \qquad \text{(M* equations) ;}$$

$$(11.6) \qquad \frac{\partial L}{\partial Z} = \Lambda \sum_{i=1}^{N} \left(\frac{Y_i}{Z}\right)^{k_i} \frac{r_i}{Z} - \Gamma \sum_{j=1}^{M} \left(\frac{X_j}{Z}\right)^{h_j} \frac{t_j}{Z} = 0 \qquad \text{(1 equation) .}$$

In the equation count above, M* is the number of fixed inputs.

Using (11.1) and (11.2), (11.6) can be restated as

$$(11.6') \qquad \sum_{i=1}^{N} P_i Y_i = \sum_{j=1}^{M} Q_j X_j = \sum_{j \varepsilon F} Q_j X_j + \sum_{j \notin F} Q_j X_j \quad ;$$

that is, the value of output is equal to the cost of inputs, where the cost of the fixed inputs has been reckoned at their shadow rental values.

2.6 The Linearization of the First Order Conditions and the Elimination of Non-observable Variables

From a practical viewpoint, the system of first order equations (11.1) - (11.6) is awkward in two respects : (i) it is non-linear, and (ii) it contains the non-observable variables Λ, Γ, Z and Q_j $(j \varepsilon F)$.

The system may be linearized by expressing each of the six equations in proportional change form to give :

$$(12.1) \qquad p_i = \lambda + k_i(y_i - z) - y_i \quad ;$$

$$(12.2) \qquad q_j = \gamma + h_j(x_j - z) - x_j \quad ;$$

$$(12.3) \qquad \sum_{i=1}^{N} (y_i - z) \; r_i \left(\frac{Y_i}{Z}\right)^{k_i} = 0 \quad ;$$

$$(12.4) \qquad \sum_{j=1}^{M} (x_j - z) \; t_j \left(\frac{X_j}{Z}\right)^{h_j} = 0 \quad ;$$

$$(12.5) \qquad x_j = \bar{x}_j \quad (j \varepsilon F) \quad ;$$

$$(12.6) \qquad \sum_{i=1}^{N} (p_i + y_i) \; S_i = \sum_{j=1}^{M} (q_j + x_j) \; W_j \quad ,$$

where lower case symbols p_i, q_j, x_j, y_i, z, λ, γ, represent small changes in the logarithms of the corresponding upper case variables P_i, Q_j, X_j, Y_i, Z, Λ, Γ ; and where

$$(13.1) \qquad S_i = P_i Y_i \Big/ \sum_{\ell=1}^{N} P_\ell Y_\ell \quad ,$$

the share of product Y_i in total farm revenue, whilst

$$(13.2) \qquad W_j = Q_j X_j \Big/ \sum_{\ell=1}^{M} Q_\ell X_\ell \quad ,$$

the cost share of input j in total costs. For fixed inputs the W_j represent estimated annual rentals as a percentage of total costs.

Equations (12.3) and (12.4) may be further simplified, via (11.1) and (11.2), to give :

$$(14.1) \qquad \sum_{i=1}^{N} y_i \, S_i = z \quad ,$$

and

$$(14.2) \qquad \sum_{j=1}^{M} x_j \, W_j = z \quad .$$

Hence it follows that (12.6) reduces to :

$$(15) \qquad \sum_{i=1}^{N} p_i \, S_i = \sum_{j=1}^{M} q_j \, W_j \quad .$$

We now work with the $(N + M + M* + 3)$ equation system (12.1), (12.2), (12.5), (14.1), (14.2) and (15). This system has overcome the first of our difficulties. It is linear in the endogenous variables y_i, x_j, q_j for $j \varepsilon F$, z, λ and γ.[1] The second difficulty remains, viz., q_j for $j \varepsilon F$, z, λ and γ are non-observable. Also, it might be argued that the fixed cost shares W_j, $j \varepsilon F$ are non-observable. However, in this paper we will assume that all the shares S_j and W_j can be observed or estimated. In the case of the fixed factors, we are assuming that it is not possible to observe, with any reliability, the time series on changes in the rental value of farmer-owned capital and land. On the other hand, we are assuming that it is possible to estimate an average long-run share for various types of fixed factors in total costs.[2]

The elimination of the non-observables will be performed in several steps. We commence with λ and γ.

Equation (12.1) may be rewritten as :

(16) $$p_i = \lambda - k_i z + (k_i - 1)y_i \; .$$

After multiplying through by $[S_i/(k_i - 1)]$, summing over products, and recalling from (14.1) that $\Sigma_i S_i y_i = z$, we see equation (16) implies :

---

1. The shares, S_i, W_j, can be regarded as "approximately" predetermined. If changes in exogenous variables, x_j for $j \varepsilon F$, p and q_j for $j \notin F$, are not very large, then the system (12.1), (12.2), (12.5), (14.1), (14.2) and (15) will be approximately valid for time t where the shares refer to time $t - 1$.

2. This issue is further discussed in section 3.9.

$$(17.1) \qquad \sum_{i=1}^{N} \hat{S}_i p_i = \lambda \sum_{i=1}^{N} \hat{S}_i + z(1 - \sum_{i=1}^{N} \hat{S}_i k_i) ,$$

where

$$(17.2) \qquad \hat{S}_i = S_i \Big/ (k_i - 1) .$$

But

$$(1 - \sum_{i=1}^{N} \hat{S}_i k_i) = - \sum_{i=1}^{N} \hat{S}_i ;$$

hence

$$(18) \qquad \lambda = \sum_{i=1}^{N} p_i S_i^* + z ,$$

where

$$(19) \qquad S_i^* = \frac{\hat{S}_i}{\sum_{i'=1}^{N} \hat{S}_{i'}} = \frac{S_i \big/ (k_i - 1)}{\sum_{i'=1}^{N} S_{i'} \Big/ (k_{i'} - 1)} .$$

Using similar reasoning, equation (12.2) can be solved for γ, yielding

$$(20) \qquad \gamma = \sum_{j=1}^{M} q_j W_j^* + z ,$$

where

$$(21) \qquad W_j^* = \frac{\hat{W}_j}{\sum_{j'} \hat{W}_{j'}} = \frac{W_j \Big/ (h_j - 1)}{\sum_{j'=1}^{M} W_{j'} \Big/ (h_{j'} - 1)} .$$

2.6.1 Derivation of CRETH Supply and CRESH Demand Functions

Substituting (18) and (20) into (12.1) and (12.2), we obtain :

$$(22) \qquad y_i = z + \frac{1}{k_i - 1}\left(p_i - \sum_{i'=1}^{N} p_{i'} S^*_{i'}\right) \qquad \text{(CRETH Supply Functions -- Structural Form)}$$

and

$$(23) \qquad x_j = z + \frac{1}{h_j - 1}\left(q_j - \sum_{j'=1}^{M} q_{j'} W^*_{j'}\right) \qquad \text{(CRESH Demand Functions -- Structural Form)}$$

which together with (12.5) and (15) constitute the revised system of $(N + M + M^* + 1)$ equations, linear in the endogenous variables y_i, x_j, q_j for $j \varepsilon F$, and z. It remains to eliminate the non-observables q_j for $j \varepsilon F$ and z. However, at this stage we will pause to interpret (22) and (23).

2.6.2 Interpretation of CRETH Supply and CRESH Demand Functions

The interpretation of (22), the product supply equation, is as follows : the percentage change in output of product i is linear in the percentage change z in overall farm activity, and in the percentage change in the relative price of product i. In determining the latter relative price movement, the percentage change in the absolute price of product i has subtracted from it a weighted average of the percentage changes in the prices of all products, where the weights used depend on the relative shares of the different products in the gross value of production at the farm gate, and on the parameters, $k_1, \ldots, k_N$.

By setting $p_\ell = 1$, $z = 0$ and $p_{i'} = 0$ for all $i' \neq \ell$, we see that (22) implies that

$$\xi_{i\ell} = -\frac{1}{k_i - 1} S^*_\ell , \qquad i \neq \ell , \tag{24.1}$$

where $\xi_{i\ell}$ is the cross price elasticity of supply of product i with respect to changes in the price of product ℓ. Under our restriction that $k_i > 1$ for all i, it is apparent that all product-product cross price elasticities are negative. With the overall level of activity fixed, an increase in the price of product ℓ will lead to a reduction in the output of product i, $i \neq \ell$. Under CRETH it also follows that all partial transformation elasticities between products are negative. If $\tau_{i\ell}$ is the pair-wise product transformation elasticity between i and ℓ, then as a matter of definition,

$$\tau_{i\ell} \equiv \frac{\xi_{i\ell}}{S_\ell} , \tag{24.2}$$

$$= -\frac{1}{k_i - 1}\,\frac{1}{k_\ell - 1}\,\frac{1}{\sum_{i'=1}^{N} \hat{S}_{i'}} \qquad (i \neq \ell) .$$

On the input side, we find from (23) that under CRESH,

$$\eta_{jm} = -\frac{1}{h_j - 1} W^*_m \qquad (j \neq m) , \tag{25.1}$$

and that

$$\sigma_{jm} = -\frac{1}{h_j - 1}\,\frac{1}{h_m - 1}\,\frac{1}{\sum_{j'=1}^{M} \hat{W}_{j'}} \qquad (j \neq m) , \tag{25.2}$$

where η_{jm} and σ_{jm} are respectively the cross-price elasticity of demand for factor j with respect to changes in the price of factor m, and σ_{jm} is the (Allen-Uzawa) partial elasticity of substitution between j and m. With all $h_j < 1$, since the elasticities η_{jm} and σ_{jm} are positive for all pairs of factors, it follows that the possibility of complements is excluded.

2.6.3 Matrix Representation of the CRESH/CRETH System

Rather complicated manipulations are necessary to eliminate the remaining unobservables, q_j for $j \epsilon F$ and z, from our condensed system (22), (23), (12.5) and (15). Unfortunately, the introduction of matrix notation seems unavoidable. A matrix representation of our system follows in which it is assumed that the variable factor inputs are numbered 1, 2, ..., M - M*, and the fixed factor inputs are M - M* + 1, ..., M :

$$(26.1) \qquad \underline{y} = z\,\underline{1} + \hat{\underline{\underline{K}}}\,(\underline{\underline{I}} - \underline{\underline{S}}^*)\,\underline{p} \quad ;$$

$$(26.2) \qquad \underline{x}_1 = z\,\underline{1} + \hat{\underline{\underline{H}}}_1\,(\underline{\underline{I}} - \underline{\underline{W}}^*_{11})\,\underline{q}_1 - \hat{\underline{\underline{H}}}_1\,\underline{\underline{W}}^*_{12}\,\underline{q}_2 \quad ;$$

$$(26.3) \qquad \bar{\underline{x}}_2 = z\,\underline{1} + \hat{\underline{\underline{H}}}_2\,(-\underline{\underline{W}}^*_{21})\,\underline{q}_1 + \hat{\underline{\underline{H}}}_2\,(\underline{\underline{I}} - \underline{\underline{W}}^*_{22})\,\underline{q}_2 \quad ;$$

$$(26.4) \qquad \underline{\underline{S}}'\underline{p} = \underline{\underline{W}}'_1\,\underline{q}_1 + \underline{\underline{W}}'_2\,\underline{q}_2 \quad ,$$

Equation (26.1) restates equation (22) whilst (26.2) and (26.3) restate (23) for variable and fixed inputs respectively. Equation (12.5) has been used to eliminate $\underline{x}_2$, i.e., $\underline{x}_2$ is replaced by $\bar{\underline{x}}_2$. and (26.4) is a restatement of (15). The notational definitions are :

$\underline{\underline{y}}$: the N-vector of proportional rates of change in product outputs, with typical element y_i .

$\underline{\underline{p}}$: the N-vector of proportional rates of change in product prices, with typical element p_i .

z : the proportional rate of change in the representative farm's overall activity level, viz., $d\ell nZ$ (as previously defined; a scalar) .

$\underline{\underline{x}}_1$: the (M - M*)-vector collecting the proportional rates of change x_j for $j \notin F$ (i.e., variable inputs) .

$\underline{\underline{q}}_1$: the (M - M*)-vector of proportional rates of change in prices of the variable inputs .

$\underline{\underline{x}}_2$: the M*-vector collecting the proportional rates of change x_j for $j \varepsilon F$ (i.e., fixed inputs) .

$\bar{\underline{\underline{x}}}_2$: the exogenously set values of $\underline{\underline{x}}_2$.

$\underline{\underline{q}}_2$: the M*-vector of proportional rates of change in shadow rental prices of the fixed inputs .

$\hat{\underline{\underline{K}}}$: the N × N diagonal matrix with typical element $1/(k_i - 1)$.

$\hat{\underline{\underline{H}}}_1$: the (M - M*) × (M - M*) diagonal matrix with typical element $1/(h_j - 1)$ for $j \notin F$.

$\hat{\underline{\underline{H}}}_2$: the M* × M* diagonal matrix with typical element $1/(h_j - 1)$ for $j \varepsilon F$.

$\underline{\underline{S}}^*$: the N × N matrix with every row the same and equal to $(S_1^*, \ldots, S_N^*)$.

$\underline{\underline{W}}^*$: the M × M matrix with every row the same and equal to $(W_1^*, \ldots, W_M^*)$.

$\underline{\underline{W}}^*_{11}$: the (M - M*) × (M - M*) north-west block in a partition of $\underline{\underline{W}}^*$;

$\underline{\underline{W}}^*_{11}$ has every row the same and containing the W^*_j values for $j \notin F$.

$\underline{\underline{W}}^*_{22}$: the M* × M* south-east block in the above partition having every row identical and containing W^*_j values for $j \varepsilon F$.

$\underline{\underline{W}}^*_{12}$, $\underline{\underline{W}}^*_{21}$: respectively the north-east and south-west blocks in the partition of $\underline{\underline{W}}^*$; apart from the number of rows involved -- (M - M*) and M* -- these matrices are respectively identical to $\underline{\underline{W}}^*_{22}$ and $\underline{\underline{W}}^*_{11}$.

$\underline{\underline{S}}$: the N-vector of product shares S_i in gross value of the representative farm's output net of product-specific costs .

$\underline{\underline{W}}_1$: the (M - M*)-vector of shares of non-product specific variable inputs in total input cost excluding product specific charges..

$\underline{\underline{W}}_2$: the M*-vector of shares of imputed returns of fixed factors in total input cost excluding product specific charges .

$\underline{\underline{I}}$: the identity matrix (with order defined by the context) .

$\underline{\underline{1}}$: column vector of ones (with order defined by the context) .

2.7 Completion of the Elimination of Non-observables

Rearrangement of (26.3) gives

$$q_2 = (I - W^*_{22})^{-1} \hat{H}_2^{-1} \left[\tilde{x}_2 - z\, 1 + \hat{H}_2 W^*_{21} q_1\right] \quad (27)$$ [1]

q_2 is then eliminated by substituting (27) into (26.2) and (26.4) to obtain

$$\begin{aligned} x_1 = {} & z\,[I^{\dagger} + \hat{H}_1 W^*_{12} (I - W^*_{22})^{-1} \hat{H}_2^{-1}]\, 1 \\ & + \hat{H}_1 [(I - W^*_{11}) - W^*_{12} (I - W^*_{22})^{-1} W^*_{21}]\, q_1 \\ & - \hat{H}_1 W^*_{12} (I - W^*_{22})^{-1} \hat{H}_2^{-1} \tilde{x}_2 \, , \end{aligned} \quad (28)$$ [2]

and

$$\begin{aligned} S' p - W_1' q_1 = {} & W_2' (I - W^*_{22})^{-1} \hat{H}_2^{-1} (\tilde{x}_2 + \hat{H}_2 W^*_{21} q_1) \\ & - z\, W_2' (I - W^*_{22})^{-1} \hat{H}_2^{-1}\, 1 \, . \end{aligned} \quad (29)$$

---

1. Equation (27) has two roles. First it is the vehicle by which we eliminate the unobservable q_2 on our way to setting up an estimable system of input demand and output supply equations. Second, it could, following estimation of the relevant parameters, be used in the calculation of a series describing changes in implicit rentals of fixed factor inputs.

2. The matrix $I^{\dagger}$ is a rectangular matrix of dimension $(M - M^*) \times M$, and contains exactly one unit element in each row (the positioning is arbitrary), and zeros elsewhere.

We now have a system, (28), (29) and (26.1) which contains only one non-observable, z. The elimination of z proceeds as follows. From (29) we find that

$$(30.1) \qquad z = \frac{\underline{W}_1' \underline{q}_1 - \underline{S}' \underline{p} + \underline{W}_2' (\underline{I} - \underline{w}^*_{22})^{-1} \hat{\underline{H}}_2^{-1} (\bar{\underline{x}}_2 + \hat{\underline{H}}_2 \underline{w}^*_{21} \underline{q}_1)}{\underline{W}_2' (I - \underline{w}^*_{22})^{-1} \hat{\underline{H}}_2^{-1} \underline{1}}$$

$$(30.2) \qquad = \mu(\underline{A}' \underline{q}_1 + \underline{B}' \bar{\underline{x}}_2 - \underline{S}' \underline{p}) \quad ,$$

where

$$(30.3) \qquad \mu = 1 \Big/ [\underline{W}_2' (I - \underline{w}^*_{22})^{-1} \hat{\underline{H}}_2^{-1} \underline{1}] \quad ;$$

$$(30.4) \qquad \underline{A}' = \underline{W}_1' + \underline{W}_2' (\underline{I} - \underline{w}^*_{22})^{-1} \underline{w}^*_{21} \quad ;$$

and

$$(30.5) \qquad \underline{B}' = \underline{W}_2' (\underline{I} - \underline{w}^*_{22})^{-1} \hat{\underline{H}}_2^{-1} \; .$$

On using (30.2) in (26.1) and (28) we obtain our final system :

$$(31.1) \qquad \underline{y} = (\mu \underline{A}' \underline{q}_1) \underline{1} + (\mu \underline{B}' \bar{\underline{x}}_2) \underline{1} + \underline{C} \underline{p} \; ,$$

where

$$(31.2) \qquad \underline{C} = \hat{\underline{K}} (\underline{I} - \underline{S}^*) - \mu(\underline{S}' \theta \underline{1}) \quad ,$$

and

$$\mathbf{x}_1 = (\mu\, \mathbf{A}'\mathbf{q}_1\, \mathbf{D}\, \mathbf{1} + \mathbf{G}\, \mathbf{q}_1) + (\mu\, \mathbf{B}'\bar{\mathbf{x}}_2\, \mathbf{D}\, \mathbf{1} + \mathbf{J}\, \bar{\mathbf{x}}_2) - \mu\, \mathbf{S}'\, \mathbf{p}\, \mathbf{D}\, \mathbf{1} \quad , \tag{32.1}$$

where

$$\mathbf{D} = \mathbf{I}^{\dagger} + \hat{\mathbf{H}}_1\, \mathbf{w}^*_{12}\, (\mathbf{I} - \mathbf{w}^*_{22})^{-1}\, \hat{\mathbf{H}}_2^{-1} \quad , \tag{32.2}$$

$$\mathbf{G} = \hat{\mathbf{H}}_1\, [(\mathbf{I} - \mathbf{w}^*_{11}) - \mathbf{w}^*_{12}\, (\mathbf{I} - \mathbf{w}^*_{22})^{-1}\, \mathbf{w}^*_{21}] \quad , \tag{32.3}$$

and

$$\mathbf{J} = -\, \hat{\mathbf{H}}_1\, \mathbf{w}^*_{12}\, (\mathbf{I} - \mathbf{w}^*_{22})^{-1}\, \hat{\mathbf{H}}_2^{-1} \quad . \tag{32.4}$$

Equations (31.1) and (32.1) are our final CRESH/CRETH product supply and input demand system. They are linear in the observable variables y, q_1, x_1 and p and contain no entities which are inherently unobservable. In the next section we will describe the data base which we intend to apply to our system and in the following section we will discuss our proposed econometric methods. However, to conclude this section, it is worth highlighting one of the system's more interesting features.

The form of the first two terms on the right of (31.1) indicates that the impact of any given change in variable factor prices and/or fixed factor supplies is neutral as between products : that is, the elasticity of supply of every product with respect to any given variable input's price or with respect to the stock of any fixed factor, is the same for every product. This is as would be expected. If reflects the nonspecificity of factors (see subsection 2.2). On the other hand, the form of the last term on the right of (32.1) indicates that variable input demand elasticities with respect to the price of any given product will, except in exceptional circumstances, differ between inputs. This difference is due, however, to the presence of fixed factors. The reason that a given product price rise, having led to a higher level of overall activity, doesn't lead to equiproportional changes in the demand for all variable inputs is clear : in terms of the variable factors alone, the CRESH function (4.1) is not homothetic (except when every h_j for $j \notin F$ is the same).[1]

3. THE DATA BASE

The model will be fitted to Bureau of Agricultural Economics (BAE) representative farm data from the Australian Sheep Industry Survey (ASIS).[2] The survey data covers the period 1952/53 to 1974/75. Within the ASIS framework, agricultural and pastoral enterprises are classified into three distinct geographic zones (termed Pastoral, Wheat-sheep and High Rainfall) according to rainfall. This zonal approach is particularly important

---

1. If $\hat{\underline{\underline{H}}}_1 = \alpha \underline{\underline{I}}$ where α is any scalar, it can be shown that $D \underline{\underline{1}}$ is a scalar multiple of $\underline{\underline{1}}$, and in that case changes in product prices lead to equiproportional changes in the demand for all variable inputs.

2. The ASIS is drawn from a population of about 90,000 farms. For a property to be eligible for inclusion in the ASIS it must run at least 200 sheep, provide full-time occupation for at least one person and not be principally a stud, part of a multiple holding or used mainly for dealing. The survey population encompasses nearly all commercial output from the grazing-livestock complex.

in the Australian environment where because of climatic and biological influences, the same product may be produced using quite different technologies in different regions.

Initially, the model will be fitted to input-output information from each of the three zones. It is anticipated that at a later stage of the project, each zone will be further subdivided on a State basis in order to provide regional detail from the model.

The treatment of the ASIS data base will be as follows :

3.1 Products Y_i $(i = 1, \ldots, 5)$

At least 5 types of product groups (wool, sheep, cattle enterprise, cereals enterprise, other) will be recognized. The (1973/74) relative shares of each product's revenue in total farm revenue for each zone is shown in Table 1.

TABLE 1 : PRODUCT SHARES BY ZONE : 1973/74
(per cent)

	Pastoral	Wheat-sheep	High Rainfall
Sheep enterprise - wool	53.5	31.6	45.6
- sheep	19.0	17.6	22.9
Cattle enterprise	15.4	12.6	22.6
Cereals enterprise			
- wheat	8.0	23.4	0.6
- oats	-	1.5	0.6
- barley	0.4	4.9	0.6
'Other' enterprise	3.7	8.4	7.1

Source : Bureau of Agricultural Economics, The Australian Sheep Industry Survey : 1970/71 to 1972/73 (Canberra : Australian Government Publishing Service, 1976).

The product supply data to be used in our model should represent changes in output as planned by the representative producer. Adjustment for weather conditions and for autonomous technological improvements will be made to the ASIS information in forming our series $\{Y_{it}, \ i = 1, \ldots, N \ ; \ \ t = 1, \ldots, T\}$.

3.2 Fixed Inputs X_j $(j = 6, 7, \ \text{i.e.,} \ j \varepsilon F)$

The farm firm's land area, capital input and the labour supply of the owner-operator are regarded as fixed factors in the short run.[1] In this analysis, the onwer-operator's labour is treated as part of the firm's capital input. That is, payments to capital are assumed to consist, in part, of the reward to the owner-operator for his management and labour inputs.

Land : Units are total area of farm in hectares.[2]

Capital : Capital inputs are notoriously difficult to measure in studies of this type. The main difficulty lies in the interpretation of the concept of depreciation. There are two general approaches (both based on perpetual inventory formulations but adopting different treatments of depreciation) which can be followed to derive a suitable capital input series.

(i) Assume that the capital service flow is proportional to the capital stock and construct a stock series of the form $\kappa_{t+1} = \kappa_t + I_t - d\kappa_t$, where κ_t is capital stock

1. Australian agriculture is characterised by a family farming structure. Labour inputs of the owner-operator, particularly in the Wheat-sheep and High Rainfall zones, represent a very high percentage of the total labour inputs of the firm.

2. In the case of the Pastoral zone, both the concept and measurement of land inputs into the production process is less clear cut than is the case with the other zones because of the very large areas and low outputs/area of Pastoral zone properties.

(constant prices), I_t is gross investment net of sales (constant prices), and d is the annual rate of depreciation. This series would then be used as an estimate of the real capital stock embodied in structures and plant and machinery.

The choosing of an appropriate value for d is an unresolved problem. If market data on second-hand values is used, then the resulting d does not refer solely to the capacity of an asset to produce current services. It includes, in addition to a deterioration factor (which is a legitimate determinant of the service flow) an obsolescence factor which is in the nature of a penalty the market attaches to existing equipment in view of better equipment becoming available. The obsolescence factor need not affect the service flow. Hence the depreciated capital stock series is likely to 'over depreciate' the service flow.

(ii) Follow Griliches[1] and Yotopoulos[2] and formulate a measure of capital service flow under a 'one-hoss shay' specification, the assumption being that there is no deterioration with age and that the service flow is

---

1. Z. Griliches, "Measuring Inputs in Agriculture : A Critical Survey," Journal of Farm Economics, Vol. 42, 1960, pp. 1411-1433.

2. P. A. Yotopoulos, "From Stock to Flow Capital Inputs for Agricultural Production Functions : A Microanalytic Approach," Journal of Farm Economics, Vol. 49, 1967, pp. 476-491.

constant over the life span of the capital item. The service flow is then computed from the stock as the annuity associated with an n year moving sum (n = working life) of cumulated gross investment in structures, plant and machinery.

The annuity can be thought of as representing the sum of interest and depreciation, with the interest charges declining and the depreciation charges rising as the capital item ages. The assumption in this approach that the service flow does not decline with age is likely to result in an overstatement of the actual service flow.

Our judgment is that there is no compelling reason for preferring either one of the foregoing approaches to capital input measurement. A final decision will be made at a later stage of our research.

3.3 Variable Inputs X_j $(j = 1, \ldots, 5,\ \text{i.e.},\ j \notin F)$

All labour hired[1] by the owner-operator (including family labour) together with material and service inputs constitute the variable inputs.

Labour : The ASIS contains details of payments to the following categories of labour -

hired ,
contract ,
shearing and crutching ,
family .

1. The different treatment of hired and owner-operator labour is consistent with the labour fixity phenomenon frequently observed in the rural sector.

Since shearing and crutching labour is specific to the sheep enterprise and has no clearly identifiable substitute (see earlier discussion) it will be treated as a 'mark-up' expense. That is, payments to shearing and crutching will be deducted from the gross revenue of the sheep enterprise.

Payments to contractors, other than shearing and crutching, are of minor significance and will be combined with hired labour payments.. Furthermore, we assume that hired labour and family labour inputs behave as perfect substitutes in production. Hence hired, family and contract labour inputs will be combined into one labour variable input termed hired farm labour.

Non-labour inputs : Two broad types of non-labour inputs are recognized : (a) materials whose main components are fuels, fertilizer and maintenance expenditure on capital items, and (b) services whose main components are product marketing charges and rates and taxes. Since marketing charges are product specific they will be treated as mark-ups to be deducted from the appropriate product's gross revenue. Rates and taxes will be regarded as a cost of holding land. Since the "rental" on land has been substituted out of our CRESH/CRETH system, rates and taxes will not have a role in our econometric estimation of the parameters h_j, k_i, $(j = 1, \ldots, M ;$ $i = 1, \ldots, N)$. Series on rates and taxes may be useful, however, at a subsequent stage in interpreting the endogenously generated implicit rental series for land (see the footnote attached to equation (27), and subsection (3.5)).

Amongst the materials inputs, the maintenance item warrants further comment since its interpretation is somewhat ambiguous. One approach would be to regard annual maintenance expenditure as a fixed percentage of the value of the capital stock. An alternative (followed here) is to treat it as determinable by the farmer. The latter approach is probably the more realistic one in the Australian farm environment where the farmer appears able to exercise a good deal of choice over his level of maintenance expenditure.

In summary, the five variable inputs that will be distinguished in the CRESH/CRETH system are : hired farm labour, fuels, fertilizer, maintenance, and an 'other' category which includes all remaining intermediate inputs.

3.4 Product prices P_i $(i = 1, \ldots, 5)$

Not all farm enterprises distinguished earlier are unique with respect to products sold by the firm. For example, the cereals enterprise yields wheat, barley and oats (though predominantly wheat). In the case of the wheat-sheep zone, the three cereals components are sufficiently important to be treated as separate commodities. However, for the other zones, a finer disaggregation of cereals into individual products is considered unwarranted in view of the minor importance of the individual cereal products. In these instances, weighted average prices per unit of enterprise output will be computed.

The product price units will be consistent with the units in which product outputs are expressed. For example, if planned output of wheat is expressed as dollars/area sown, then the corresponding wheat price is expressed as dollars/area sown. This measurement consistency is necessitated by the presence of the value shares (S_i) of products in gross income, in the estimating model.

It will be necessary to modify actual product prices to reflect production expectations. Some formal expectational model will be required, perhaps along classic Koyck/Nerlove lines,[1] or along the variant explored by Powell and Gruen.[2] However, whatever expectational model is chosen, we will follow the convenient convention of locating all lags on the expectational side. No data set is likely to be rich enough to partition accurately lags in observed output response to product prices into frictions on the adjustment side and on the updating of expectations.[3]

3.5 Variable input prices Q_j ($j = 1, \ldots, 5$, i.e., $j \notin F$)

We assume that prices of variable inputs are anticipated accurately. Therefore we will be able to use BAE indexes of prices paid for inputs without modification.

3.6 Fixed input prices Q_j ($j = 6, 7$, i.e., $j \in F$)

The prices of the fixed factors are not part of the data base. They are endogenously determined in the system. However, we make reference to them at this stage because they are of interest in the development of a theory of agricultural investment. Using econometric estimates based on the observable variables, back substitution (see equation (27)) will allow

---

1. L. M. Koyck, Distributed Lags and Investment Analysis (Amsterdam : North-Holland Publishing Company, 1954).

   Marc Nerlove, The Dynamics of Supply : Estimation of Farmers' Response to Price (Baltimore : The Johns Hopkins Press, 1958).

2. Alan A. Powell and F. H. Gruen, "The Estimation of Production Frontiers: The Australian Livestock/Cereals Complex," Australian Journal of Agricultural Economics, Vol. 11, No. 1 (June, 1967), pp. 63-81.

3. Nerlove (op. cit., Ch. II) demonstrates the theoretical possibility of separately identifying coefficients of expectations and of adjustment; in practice this is not likely to be very successful. See Alan A. Powell and F. H. Gruen, "Problems in Aggregate Agricultural Supply Analysis," Review of Marketing and Agricultural Economics, Vol. 34, Nos. 3 and 4 (September and December, 1966), pp. 112-135; and pp. 186-201.

the estimation of implicit shadow prices on the fixed factors. In the ORANI module, investment takes place in response to the gap between the shadow price and the supply price of investment capital in agriculture. It therefore seems likely that the CRESH/CRETH estimates of the shadow prices on capital and land could form an important part of the specification of an aggregate investment function for the various agricultural regions of Australia.

3.7 Product shares of income S_i $(i = 1, \ldots, 5)$

These are readily computed from information on gross revenue (net of product-specific costs) of individual enterprises and from total gross revenue. The series will be corrected for the influences of weather.

3.8 Variable input shares of costs W_j $(j = 1, \ldots, 5, \text{ i.e., } j \notin F)$

As with product shares, these are readily computed from information on the total value of output and expenditure on the various input categories.

3.9 Fixed input shares of costs W_j $(j = 6, 7, \text{ i.e., } j \varepsilon F)$

There is no difficulty in making estimates of the aggregate fixed input share, $W_7 + W_8$. One can simply subtract the variable input share from unity. The difficult problem is to make the split into a capital share and a land share.

Initial estimates can be generated via a residual imputation of farm value added. For example, implied rental payments to land can be estimated from the market price (capitalised rent) of land. The market price reflects the discounted present value of expected returns from the land. Under the assumption that land yields an inexhaustible flow of services over time, land rental payments represent the perpetuity associated with the market value. Payments to the remaining factor, owner-operator labour and capital, may then be computed as a residual. However, splits based on the above method are known to be unreliable : alternative equally plausible methods can give very different results. Therefore, we plan to conduct some sensitivity analysis, by varying the split and noting the effect on parameter estimates. The more ambitious approach of making the ratio W_6/W_7 a parameter to be estimated may also be explored.

4. ESTIMATION FRAMEWORK

Our approach to estimation is via the system (31.1) and (32.1). We consider this system to be a reduced form, i.e., each of the endogenous variables $\underline{y}$ and $\underline{\underline{x}}_1$ appears on the left while only exogenous variables appear on the right. The system is linear in the variables $\underline{\underline{y}}$, $\underline{\underline{x}}_1$, $\underline{\underline{q}}_1$, $\bar{\underline{\underline{x}}}_2$ and $\underline{\underline{p}}$, and does not involve unobservables. There remains, however, an important decision to be made regarding the handling of the shares and modified shares $\underline{\underline{S}}$, $\underline{\underline{S}}^*$, $\underline{\underline{W}}$ and $\underline{\underline{W}}^*$.

The simplest method, and the one we plan to adopt, is to treat the shares $\underline{\underline{S}}$ and $\underline{\underline{W}}$ as constants. Although the shares do in fact vary over time, our judgement is that system (31.1) and (32.1) can be reasonably well approximated by fixing the shares at central values. In effect, we are interpreting (31.1) and (32.1) as a system which is precisely CRESH/CRETH at co-ordinates corresponding to the chosen central values of the shares S_i and W_j, but which elsewhere is to be regarded as an approximation to the CRESH/CRETH system. Under the assumption of constant shares, (31.1) and (32.1) can be rewritten in stochastic form as :

$$(33.1)\qquad \underline{\underline{y}}_t = (\mu\, \underline{\underline{A}}'\otimes \underline{\underline{1}})\underline{\underline{q}}_{1t} + (\mu\, \underline{\underline{B}}'\otimes \underline{\underline{1}})\bar{\underline{\underline{x}}}_{2t} + \underline{\underline{C}}\, \underline{\underline{p}}_t + \underline{\underline{u}}_t\ ,$$

or

$$(33.1')\qquad \underline{\underline{y}}_t = \underline{\underline{L}}\, \underline{\underline{q}}_{1t} + \underline{\underline{M}}\, \bar{\underline{\underline{x}}}_{2t} + \underline{\underline{C}}\, \underline{\underline{p}}_t + \underline{\underline{u}}_t\ ;$$

and

$$(33.2)\qquad \underline{\underline{x}}_{1t} = [\mu\, \underline{\underline{D}}(\underline{\underline{A}}'\otimes \underline{\underline{1}}) + \underline{\underline{G}}]\underline{\underline{q}}_{1t} + [\mu\, \underline{\underline{D}}(\underline{\underline{B}}'\otimes \underline{\underline{1}}) + \underline{\underline{J}}]\bar{\underline{\underline{x}}}_{2t} - [\mu\, \underline{\underline{D}}(\underline{\underline{S}}'\otimes \underline{\underline{1}})]\, \underline{\underline{p}}_t + \underline{\underline{v}}_t\ ,$$

or

$$(33.2')\qquad \underline{\underline{x}}_{1t} = \underline{\underline{N}}\, \underline{\underline{q}}_{1t} + \underline{\underline{\Omega}}\, \bar{\underline{\underline{x}}}_{2t} + \underline{\underline{\Pi}}\, \underline{\underline{p}}_t + \underline{\underline{v}}_t\ ,$$

where no time subscripts are required on the matrices $\underline{\underline{L}}$, $\underline{\underline{M}}$, $\underline{\underline{C}}$, $\underline{\underline{N}}$, $\underline{\underline{\Omega}}$ and $\underline{\underline{\Pi}}$. The appended vectors $\underline{\underline{u}}_t$, $\underline{\underline{v}}_t$ are the zero mean additive disturbances.

Each of the elements of the matrices $\underline{\underline{L}}$, $\underline{\underline{M}}$, $\underline{\underline{C}}$, $\underline{\underline{N}}$, $\underline{\underline{\Omega}}$ and $\underline{\underline{\Pi}}$ is a highly non-linear function of the parameters h_j, $j = 1, \ldots, M$, k_i, $i = 1, \ldots, N$. Having decided to fix the values of the S_i and W_j for all i and j, we can know the explicit numerical form of these functions. However, without further simplifications, the chances for successful estimation of the h_j and k_i would seem to be small. We propose, therefore, at each round of our estimation procedure, to treat not only the S_i and W_j as known constants but also the S_i^* and W_j^* . We will make a provisional guess of the values of h_j, k_i . This will enable us to make a provisional calculation of the S_i^*, W_j^* (see (19) and (21)).[1]

---

1. After obtaining estimates for the h_j, k_i based on our provisional values for the S_i^*, W_j^*, we will update our estimates of S_i^* and W_j^* and re-estimate iteratively till convergence (hopefully!) is achieved.

While it is true that even with the S_j^*, W_i^* treated as known constants, the elements of $\underline{\underline{L}}$, $\underline{\underline{M}}$, $\underline{\underline{C}}$, $\underline{\underline{N}}$, $\underline{\underline{\Omega}}$ and $\underline{\underline{\Pi}}$ are still non-linear functions of the parameters h_j and k_i, we do not expect these remaining non-linearities to present major computational difficulties.

We propose to use Wymer's RESIMUL package for the estimation of (33.1) and (33.2) by full information maximum likelihood (FIML).[1] RESIMUL is well suited to handle constraints, linear and non-linear, among the elements of the reduced form coefficients. In our proposed application, the reduced form coefficient matrices $\underline{\underline{L}}$, $\underline{\underline{M}}$, $\underline{\underline{C}}$ of (33.1') and $\underline{\underline{N}}$, $\underline{\underline{\Omega}}$ and $\underline{\underline{\Pi}}$ of (33.2') contain respective totals of $N(M - M^*)$, NM^*. N^2, $(M - M^*)^2$, $(M - M^*)M^*$ and $(M - M^*)N$ coefficients. If $M = 7$, $M^* = 2$ and $N = 4$, the total number of reduced form coefficients is 99. Each of the 99 coefficients is determined by the values of the $N + M$ (= 11) parameters h_j, k_i. The 88 constraints which will be imposed on the estimates of the 99 values of the reduced form coefficients are implied by equations (30.3), (30.4), (30.5) (31.2), (32.2), (32.3), (32.4) and the definitions of $\underline{\underline{L}}$, $\underline{\underline{M}}$, $\underline{\underline{C}}$, $\underline{\underline{N}}$, $\underline{\underline{\Omega}}$ and $\underline{\underline{\Pi}}$ implicit in (33.1') and (33.2').[2]

4.1 Serial Properties

In the reduced form (33.1'), (33.2'), the variables are all differentials of logarithms. Operationally we propose to use the simplest discrete analogue; i.e., $d \log (a_t)$ is replaced by $[(a_t - a_{t-1})/a_t]$.

---

1. C. R. Wymer, Computer Programs : Resimul Manual, London School of Economics, 1973 (mimeo), pp. 25.

2. Wymer's RESIMUL package requires each coefficient in the linear reduced form to be explicitly written as a differentiable function of the structural form parameters. This constitutes the major task in coding a problem for RESIMUL.

It is difficult to specify, a priori, to which version of the CRESH/CRETH equation set it would be most reasonable to append white noise errors. In any case, the mapping of the serial properties of the CRESH/CRETH system as we move from levels to differential changes to discrete annual percentage changes is obviously very complicated. Therefore we opt, provisionally, for the simplest alternative. If we define

$$(34) \qquad \varepsilon_t' = (\tilde{\underline{\underline{u}}}_t', \tilde{\underline{\underline{v}}}_t') \quad ,$$

where $\tilde{\underline{\underline{u}}}_t'$, $\tilde{\underline{\underline{v}}}_t'$ are the disturbance terms in the discrete analogues of (33.1') and (33.2'), we assume that

$$(35.1) \qquad \underline{\underline{\varepsilon}}_t \sim N(\underline{\underline{0}}, \underline{\underline{\Sigma}}) \quad , \qquad \text{for all } t \;;$$

and

$$(35.2) \qquad E(\underline{\underline{\varepsilon}}_t \underline{\underline{\varepsilon}}_{t'}) = \underline{\underline{0}} \;; \qquad \text{for } t \neq t' \quad .$$

Failure, ex post, of this assumption may require transformation of (33.1') and (33.2') prior to estimation.

5. CONCLUSION

Since the work done by Powell and Gruen in the mid-1960's,[1] the only multi-product model of Australian agriculture recognising joint production to emerge has been the University of New England's highly disaggregated (and therefore very ambitious) Aggregative Programming Model of Australian Agriculture (APMAA).[2] Our CRESH/CRETH framework constitutes an improvement over the earlier work of Powell and Gruen in that input demands are treated simultaneously with product supplies and in the degree of consistency in the theoretical specification of the model. In particular, whereas under the CET specification adopted by Powell and Gruen, transformation elasticities τ_{ij} would under a strict interpretation of the CET framework be constrained to equality for all pairs of products, under CRETH this difficulty is avoided. Also, the basis for disaggregation and the quality of the data contemplated for use with CRESH/CRETH hopefully are both superior to the national aggregate data used in the earlier study.

The APMAA study has the advantage of a great deal of regional detail, consisting of a set of approximately 500 regional representative farms for each of which a complete activity analysis linear programming framework is developed. In terms of regional and microeconomic detail, the APMAA study covers territory not to be attempted in our CRESH/CRETH study. Because at the level of microeconomic detail modelled in APMAA

---

1. Powell and Gruen, "The Estimation of Production Frontiers," op. cit.

2. John R. Monypenny, "APMAA '74 : Model, Algorithm, Testing and Application," APMAA Report No. 7, Department of Agricultural Economics and Business Management, University of New England, November, 1975.

it is possible to distinguish several inputs which are fixed in the short-run, a more detailed picture of factor fixity and bottlenecks can be built up than is possible in the context of our treatment involving only two fixed factors. Moreover, the APMAA framework has the advantage of generating shadow prices on each of the fixed factors distinguished.

On the debit side, the APMAA model does not yet have linkages with the economy at large, whilst its linear structure gives a much less convincing story for input substitution and product transformation than does CRESH/CRETH. Further, because the analysis of aggregate supply in APMAA involves simultaneous solution of all 500 representative farm programmes, its use for aggregate supply analysis is likely to be extremely costly and cumbersome. Thus it is likely that APMAA will be used mainly for analysis of regional issues.

Unresolved in the Powell and Gruen study was the question of specifying and estimating an investment function in agriculture.[1] This issue has also been side-stepped in the current paper, although estimates of shadow prices $\underset{\sim}{Q}_2$ on capital and land would fit naturally into a theory of investment and may in fact have an important role to play in the econometric estimation of an aggregate investment function for Australian agriculture. Certainly, in the ORANI module investment takes place in response to the gap between the shadow prices $\underset{\sim}{Q}_2$ and the supply price of investment capital in agriculture. The econometrics of this relationship will be taken up in later work.

---

1. Powell and Gruen, "Problems in Aggregate Agricultural Supply Analysis," op. cit., p. 199.