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EMPIRICAL ESTIMATION OF THE ELASTICITY OF SUBSTITUTION: A REVIEW

by

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C O N T E N T S

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1. INTRODUCTION

Recent development in economic theory have placed increasing emphasis on the importance of measuring the relative ease with which various factors can be substituted for each other in the productive process. Prior to 1961 econometric studies involving the use of production functions dealt almost exclusively with functional forms which assumed specific numerical values for the important elasticity of substitution parameter (σ). The most popular of these, the celebrated Cobb/Douglas function,[1] implies a σ of unity while the alternatives, the Leontief fixed coefficient model and the "straight line isoquant" production function, imply values for σ of zero and infinity respectively. However, in view of the fact that this parameter can conceptually have any value between zero and infinity (in the two factor case), the ad hoc prior restrictions inherent in the above functions deprive the empirical results of much of their interest, as well as introducing the possibility of specification error if the assumption is not substantially correct.

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(1) See Cobb and Douglas (1928).

The Constant Elasticity of Substitution (CES) functional form introduced by Arrow et al. (1961) was intended to overcome this problem to some extent by allowing the value of σ to be determined by the data. Although the CES function represented a substantial generalization beyond the earlier functions the substitution parameter was restricted to be constant over all output and input combinations. Other functional forms have been developed which allow for some systematic variation in σ[(2)] but these have been largely ignored in favour of the CES form by applied workers.

In the ORANI module the specification of the technical relationship between inputs and output, and the firms behavioural characteristics have been used to derive input demand functions for each of the 105 input-output industries.[3] The demand equations for primary inputs in the production of manufactured goods inherit from the production function the elasticity of substitution between capital and labour as one of their parameters. The current specification of the ORANI module defines the substitution possibilities between labour and capital in terms of a CES function. While the CES form is conceptually preferable to the more restrictive Cobb/Douglas function in that it does not assume a priori a value for σ, it does introduce the non-trivial problem of obtaining satisfactory estimates for that parameter.

Despite the considerable intellectual effort that has

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(2) See Revankar (1971) for an outline of some of these functions.

(3) For the specification of the ORANI module and the derivation of demand functions see Dixon (1975) and Caddy (1975).

been directed towards this task the results have, to say the least, been disappointing. The apparent sensitivity of the estimates to the particular data base used, the form of the estimating equation and the estimating technique employed has caused the reported results to show a high degree of instability.

It is not the purpose of this paper to advance any new hypotheses as to why this instability occurs or to present any new results, but rather to highlight the estimation problem that does exist by providing a summary of some of the studies that have been undertaken using industry classifications which have a sufficiently high degree of disaggregation to be of relevance for the ORANI module. If a perusal of the empirical evidence indicates that there is a persistent tendency for the industry production functions to be characterised by unitary elasticity of substitution the generalization afforded by the CES function is unnecessary and therefore the module could revert to the use of the Cobb/Douglas form.

2. THE CES FUNCTION AND ITS ESTIMATION

Although there is evidence of the CES functional form appearing in the literature prior to the joint paper by Arrow, Chenery, Minhas and Solow (1961) it is undoubtedly that article which gave it its current pre-eminent role in production analysis. The derivation of the functional form was based on the empirical observation that a log-linear relationship existed between output per unit of labour and the real wage rate (i.e. $\log V/_L = \log a + b \log W/_P$ where V is value added, L is labour input, W is the wage rate and P is the output price.) It turns out that in a constant elasticity of substitution framework the functional form implied by this relationship is:

$$V = \gamma \{\delta L^{-\rho} + (1 - \delta)K^{-\rho}\}^{-1/\rho}, \qquad (1)$$

where K is capital input and $\sigma = \frac{1}{1 + \rho} = b$.

In practice a number of different approaches have been followed in estimating the parameters of (1). The most obvious is the direct approach in which output is treated as being the dependent variable and capital and labour inputs as the explanatory variables. An obvious disadvantage of this method is that the function is non-linear in parameters and hence the OLS method cannot be used. Kmenta (1964) has suggested using a Taylor series expansion about the point $\rho = 0$ (which implies $\sigma = 1$)to obtain a linear second order approximation of (1). This approximation has the form:

$$\log V = \log \gamma + (1 - \delta) \log K + \delta \log L - \tfrac{1}{2}\rho\delta (1 - \delta)[\log(K/L)]^2. \qquad (2)$$

There are two problems associated with direct estimation of the function. Firstly, the problem of multicollinearity between the inputs leads to imprecise estimates of the parameters.[3a] Secondly, there is the problem of simultaneous equation bias. Marschak and Andrews (1944) pointed out that if the behavioural characteristics of the production unit under consideration are specified, the production function can be seen to be only one in a system of simultaneous relationships. Under the 'traditional' specifying assumptions the inputs will not be independent of the production function error term, and consequently least squares

3a. See Tsang and Persky (1975).

estimates will be biased. Zellner et al. (1966) have rehabilitated the direct estimation approach by providing an alternative specification which does not give rise to the simultaneity problem.

A more popular (and historically the first) way of estimating the parameters of (1) is to use one or other of the necessary first order optimizing conditions which make up the "complete model". If the objective of profit maximization is applied to a competitive firm, these take the form:

$$\log (V/_L) = a_1 + \sigma \log (W/_p), \tag{3}$$

and

$$\log (V/_K) = a_2 + \sigma \log (R/_p). \tag{4}$$

where R is the price of capital. (2) is referred to as the "labour marginal productivity side condition" while (3) is the "capital marginal productivity side condition".

A third condition relating the marginal rate of substitution to the price ratio, can be obtained by subtracting (3) from (4). This gives:

$$\log (K/_L) = a_3 + \sigma \log (W/_R). \tag{5}$$

These forms have the advantage of simplicity, and the fact that σ enters as a first order parameter enhances the possibility of it being estimated with some precision. In the direct approach it is a second order parameter, affecting the curvature of the relations only.

Other estimating equations could be used (e.g. factor demand or cost functions) but as these have not been used in any applied studies of the type which are relevant to this survey, they will not be discussed here.[4]

3. CROSS-SECTIONAL STUDIES

The estimates of σ reported in the ACMS article were obtained from the regression of log $(PV/_L)$ on log W. The log W coefficient was interpreted as the elasticity of substitution of a CES function;

$$\text{i.e.,} \quad \log (PV/_L) = a_1' + \sigma \log W. \qquad (3a)$$

It should be noted that this regression equation differs from the labour side condition implied by the CES function as it measures output and wages in money rather than real terms. McKinnon (1963) has pointed out that if prices vary across units of observation, the use of the above equation (3a) instead of the relationship (3) leads to a specification error due to an omitted variable. This can be readily seen if we attempt to derive (3a) from (3) by adding log P. This gives

$$\log (PV/_L) = a_1 + \sigma \log W/_P + \log P,$$

$$= a_1 + \sigma \log W + (1 - \sigma) \log P; \qquad (3b)$$

i.e., equation (3a) omits the log P term. If P is constant over all observations then the omitted term can be regarded as being absorbed into the constant a_1. Since the "efficiency" and "distribution"

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4. For a survey of the alternative approaches see Woodland (1976).

parameters which normally make up the constant term cannot in any case be identified, the characteristics of the estimates will not be adversely effected if (3a) is used under these circumstances. If, however, P does vary then the estimates will be inefficient and will also be biased if P and W are correlated. ACMS recognise this problem but decide, on the basis of the rather inconclusive evidence that is available, that the quantitative significance of the error will be small and hence can be ignored.[5a]

The data used consisted of observations on 24 industries from a cross-section of 19 countries. The results are summarized in Column 1 of Table 1.[5b] Of the 24 values for σ that were estimated, 14 proved to be insignificantly different fron one at the 95% level of confidence.

ACMS point out however that variations in efficiency from one country to another could, like variations in output price, cause the estimates to be biased. If γ is in fact variable it cannot be regarded as being part of the constant term and must be included explicitly as an explanatory variable if a specification error is to be avoided. In this case the correct specification of the labour side condition is

$$\log (V/_L) = a + \sigma \log (W/_P) + (1 - \sigma) \log \gamma. \qquad (3c)$$

If the actual σ is less than one then the coefficient on

5a. It will be seen below that this premise is implicitly accepted in most cross-sectional studies.

5b. In order to make the various results more comparable Table 1 attempts to present them using a common industry classification (USSIC).

the log γ term will be positive. If in addition there is a positive correlation between efficiency and money wage (as postulated by ACMS)[6], the exclusion of the last term will bias the estimate of σ upwards.

ACMS suggest that if capital as well as labour data is available at two points on a production function, an unbiased estimate of σ can be obtained by using the relationship between factor and price ratio expressed by (5). The reason for this is that in deriving (5) from (3) and (4) the efficiency parameter γ is eliminated and the term a_3 contains only the distribution parameter δ. Thus even if γ changes from one country to another, so long as δ remains constant (i.e. the efficiency changes are neutral) the following relationship will hold[7]:

$$\frac{(K/L)_1}{(K/L)_2} = \left\{\frac{(W/R)_1}{(W/R)_2}\right\}^{\sigma} , \qquad (6)$$

where the subscripts refer to countries.

They used this relationship, which is independent of the efficiency parameter, with data for Japan and the U.S. to get an alternative set of estimates. The two sets of results differed substantially. The latter estimates exceeded the former in 67% of the industries which appeared in both sets of data and had a median value of .93 compared with .87 obtained in the earlier analysis. As the measure of capital used in the estimation did not include working capital this apparently perverse result was taken to indicate that the

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6. Arrow et al. (1961, p.236).

7. This equation is obtained by dividing the "expansion path" (i.e. eqn (5)) for country one by the "expansion path" for country two.

elasticity of substitution between labour and working capital was "much less" than unity. Although not intended by ACMS this conclusion draws attention to one of the undesirably restrictive aspects of the CES function. If the elasticity of substitution between working capital and labour is different to that between fixed capital and labour then this implies that the two types of capital inputs are not separable and hence cannot be aggregated into a consistent index. A correct specification of the production function would be one that recognizes the three inputs but which does not make any prior assumptions about equality between the various substitution elasticities[8a]. The multi-factor generalization of the CES does not satisfy this requirement.

Fuchs (1963) attempted to account for the expected differences in efficiency by dividing the countries used by ACMS into two groups according to differences in development. He then re-estimated the labour side condition with a shift variable included to allow the constant term (which contains the efficiency parameter) to differ between the two groups.

The estimates that were obtained[8b] were, in all but three cases, higher than those obtained by ACMS and only in two cases did they differ significantly from one. Again the relationship between efficiency and the wage rate had an effect contrary to that expected. In most cases the constant term was higher for the under-developed group than for the developed group. This implied that for any given wage rate, output per unit of labour would be

8a. See Berndt and Christensen (1973) for an elaboration of this point.

8b. See Column 2 Table 1.

higher in the less developed countries. Fuchs attempts to rationalize this result by postulating that observed wages in these countries do not reflect labour costs as fully as they do in developed countries.

Estimation of the CES labour side condition using international cross-sectional data has also been carried out by Arrow and Murata (1965). Using U.N. and I.L.O. data for a total of 23 countries estimates were made for two periods, 1953-6, and 1957-9. These results appear in columns 3 and 4 of Table 1. The estimated values for σ for the two periods agree reasonably well and, in general, tend to be of slightly smaller magnitude than those estimated by ACMS and Fuchs[9]. Probably the most significant difference between the Murata-Arrow results and those cited earlier is the much wider range of industries over which the hypothesis that $\sigma = 1$ can be rejected.

A large number of cross-sectional studies have been carried out using data collected in the U.S. "Survey of Manufacturers" conducted during 1957 and 1958 (e.g. Minsian (1961), Solow (1964), Dhrymes (1965, 1970), Bell (1965), Hildebrand and Liu (1965)). A perusal of the results of these studies highlights the extreme instability exhibited by the σ estimates when slightly different data concepts, estimating equations or estimating techniques are used.

In attempting to estimate the elasticity of demand for labour with output held constant Minsian (1961), independently of ACMS, derived an estimating equation of the form.

$$\log \left[\frac{WL}{PV}\right] = C + (1 + \eta) \log W \quad . \tag{3d}$$

9. In making such comparisons it should be noted that a higher level of industry aggregation has been used by Murata and Arrow.

ACMS were able to prove that under the assumptions adopted by Minsian the estimated demand elasticity (η) was equal to the elasticity of substitution between labour and capital and hence established the equivalence of (3d) and (3a). These results should therefore be comparable to those of Solow (1964) who, like Minsian, uses U.S. Survey of Manufacturers data to estimate (3a).[10] They do however, use different years and different geographical areas of aggregation.

Minsian uses State data for 1957 while Solow employs aggregates over Census regions for 1956. In several cases (e.g. industries 21, 29, 30, 33, 36 and 37) there are substantial differences between the two sets of estimates which can only be accounted for in terms of the differences in regions of aggregation and the years used. Nerlove (1967, p.70) suggests that the different areas of aggregation will effect the results because they will lead to different product mixes being compared, and that different years would have an effect due to the higher level of capacity utilization in 1956 than in 1957 (which was a recession year). It does not appear however, that the observed pattern of differences can be adequately explained by reference to these factors.

A further comparison can be made between these and the inter-national cross-sectional results summarized above. If, as has been suggested earlier, differences in efficiency between the observational units causes σ to be biased upwards, it is to be anticipated that the intra-national studies would yield lower estimates

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10. These estimates appear in Columns 5 and 6 of Table 1.

than those made on an international basis. This is so because efficiency differences would be expected to be substantially lower in the former than the latter. However, again the empirical results do not appear to conform to this prior reasoning.

Although all the above studies have used the labour marginal productivity side condition as the estimating equation there are no a priori theoretical grounds for using this in preference to the capital side condition. If the specified functional form is valid then either side condition should yield substantially the same results. Dhrymes (1965) tests this proposition by estimating both equations using U.S. data on individual States for 1957. In his 1965 article Dhrymes claimed that his estimates were based on the regressions (3a) and the corresponding "money version" of (4)

$$\text{i.e.} \quad \log \frac{PV}{K} = a_2' + \sigma \log R \quad . \tag{4a}$$

Subsequently however Dhrymes and Zarembka (1970) discovered that the results were in fact based on the reciprocal regression.

$$\log W = b_1 + \left(1/_\sigma\right) \log \left(PV/_L\right) \tag{7}$$

$$\log R = b_2 + \left(1/_\sigma\right) \log \left(PV/_K\right) \tag{8}$$

Consequently the Dhrymes (1965) results are estimates of $(1/_\sigma)$ not σ as reported. Dhrymes and Zarembka (1970) presented a "corrected" set of results derived from the equations originally thought to have been used.[11]

Which version does in fact constitute the appropriate regression equation depends on the assumptions on which the endogenous/exogenous classification of variables is made. The implicit assumption underlying the Dhrymes-Zarembka desire to "correct" the estimates is that factor prices are exogenous while quantities are endogenous and hence the original (Dhrymes (1965)) estimates will suffer from simultaneous equation bias. Since it may be more appropriate to also treat prices as being, at least to some extent, endogenous to the production model, a comparison of the small sample properties of the estimates based on (3) and (4) and their reciprocal regressions (7) and (8) is of interest.

Unfortunately, the magnitude and direction of the simultaneous equation bias cannot in general be determined. Maddala and Kadane (1966) have however conducted Monte Carlo studies in which they compare the small sample properties of the σ estimates from (3) and (7) (call these estimates σ_3 and σ_7 respectively). They found that σ_3 estimates are biased downwards, that the bias is fairly large except when the true values of σ are close to one, and that σ_7 estimates are more robust in the face of simultaneous equations mis-specification. Also Nerlove (1967, p.89) claims that "it can be argued that the two estimates of the elasticity of substitution (i.e. estimates from the "original" equation and its reciprocal) tend to bracket the true value as the sample size increases".

11. Berndt (1976) has pointed out that the R^2 figures reported by Dhrymes (1965) do not equal the R^2 figures for the "corrected" reciprocal regression as would be expected. He concludes that the correction must therefore be incomplete or additional errors must have been contained in the original estimates.

The estimates from both the original reciprocal regressions (Dhrymes 1965) and the "corrected" results (Dhrymes/Zarembka 1970) have been reported in Table 1 (ref. Col. 7 to 10). In general the point estimates from the labour side condition were higher than for the capital side condition. Since both relations are equally admissable under the standard ACMS model, Dhrymes (1965) attempts to avoid the apparent mis-specification by developing a more general model which does not assume a priori that homogeneity of degree one and perfectly competitive factor markets exist as does the ACMS model. The estimates of σ arising from this model appear in Column 11 of Table 1.

A conflicting result was obtained by Bell (1965) who found that the use of capital data generally resulted in higher estimates of σ. His estimates were made using the labour side condition (eqn.3a) and a variant of the reciprocal regression of (5), namely

$$\log \left[\frac{WL}{PV-WL} \right] = c + \rho \log \frac{K}{L} . \qquad (9)$$

These results appear in columns 12 and 13. All but one of the estimates obtained using capital data and half those obtained from labour data alone exceeded unity. Bell (1965, p.330) claims that "there is general agreement amongst many industries in the estimation of the elasticity using the two equations" but finds "the disagreement between the two methods for some industries difficult to explain". Although he suggests lack of first degree homogeneity in the production function, non-competitive factor markets and variations in capacity utilization as possible reasons for the inconsistencies, he does not pursue the matter.

Moroney (1972) has also used the marginal rate of substitution condition (eqn.(5)) as an estimating equation. He suggests that in view of the specification problems which arise if output prices and the efficiency parameter are not constant over observations, the "expansion path" equation (5) should be preferred for estimation. Since neither the product price nor the efficiency parameter appear in this equation the problems which would be otherwise introduced by their variability are eliminated.

The results (ref. columns 14) which are based on US Census of Manufacturers data for 1963, show a substantially different distribution to those reported above. In only one case is the point estimate of σ greater than one and the hypothesis that σ equals one can be accepted in only 4 of the 18 cases.

By way of contrast Moroney has also used the labour side condition (eqn.(3a)) for estimation. There is a considerable difference between the two sets of estimates with the latter (ref. Col.15) being in almost all cases higher than the former. Such a result could be explained in terms of bias due to output price variations. If the true σ is less than one then the coefficient on the log P term in the true relationship (10) will be positive. If in addition there is a positive correlation between prices and money wages (as might reasonably be expected) the exclusion of this term will bias the estimate of σ upwards.

Probably the most disturbing aspect of a comparison of the results of all the above studies is the lack of any consistent relationship between the various industry estimates. As they all use basically the same data and production specification (and in several cases the same estimating equation), the magnitude of the discrepancies is unacceptable.

Hildebrand and Liu (1965) have used a similar data set (i.e. U.S. data by States for 1957) with an estimating equation which is derived from a variable elasticity of substitution function (which includes the CES as a special case)[12]. The labour marginal productivity condition for the function has the form

$$\log\ (PV/_L) = \log \alpha_1 + \alpha_2 \log W + \alpha_3 \log\ (K/_L) \quad , \tag{11}$$

and the elasticity of substitution is given by

$$\sigma = \frac{1}{\frac{1}{\alpha} - \frac{\alpha_3}{\alpha_2 S_k}} \quad , \tag{12}$$

where S_k is capital's share in output.

Although it is intuitively appealing to have the capital/labour ratio serving as an explanator for output per unit of labour[13], the behaviour of σ implied by the function is not easily comprehensible. The values for σ have been calculated from the Hildebrand and Liu results[14] and appear in column 16 of Table 1. In terms of the data used these results should be comparable with those discussed earlier.

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12. This function was also suggested by Bruno (1962).

13. Wise and Yeh (1965) have compared production functions for several countries and have found that σ is strongly correlated to the capital labour ratio.

14. See Nerlove (1967, pp.84, 85) for these calculations.

Again the most striking characteristic of the estimates is their diversity. The lack of standard errors on the Hildebrand/Liu results precludes the possibility of making tests of significances on σ. However it has been suggested that the large standard errors which are associated with the coefficients of the estimating equation (11) are consistent with the results that would be expected if the true production function was in fact Cobb/Douglas. This would imply a log-linear relationship between $(K/_L)$ and W which would in turn lead to a high level of multi-collinearity between the variables, and hence large standard errors on the regression coefficients.

More recent estimates are available from the work of Zarembka and Chernicoff (1975) and Griliches and Ringstad (1971). The former study, which uses the U.S. Census of Manufacturers data for 1963, estimates two sets of results using different levels of aggregation. The results are presented to support the claim made by Zarembka in an earlier paper that "for most empirical purposes the elasticity should be assumed equal to unity and the Cobb/Douglas function employed rather than the CES function" (1970, p.53). The equation used was of the general form (3a) but four regional dummies were included to absorb any inter-regional variations in product price or labour quality. At the higher level of aggregation (table 1, Col.17) 6 of the 19 estimated parameters show a significant departure from one at the 95% confidence level, while the more dis-aggregated results show σ differing from one in only 15 of the 79 cases (table 1,Col.18). Although both sets of estimates have the characteristic of centering about unity the point estimates show a considerable amount of variation. In some cases the σ value for an industry group as a whole falls outside the values posited for each of the activities that go to make up that industry group. This further adds to σ's image of a highly unstable parameter.

As the Zarembka-Chernicoff study provides estimates at two different levels of aggregation a comparison should provide some indication of the validity of Solow's (1964, p.118) assertion that "elasticities of substitution should be smaller the more narrowly defined the industrial classification, and larger the higher the degree of aggregation". It is not apparent from a perusal of the results that such a relationship exists.

The Griliches-Ringstad (1971) estimates have been made from the data obtained in the 1963 Norwegian Census of Manufacturing Establishments. They have obtained σ values from direct estimation of the production function (using both non-linear estimation and OLS on the Kmenta approximation) as well as the ever-popular labour side condition. These results appear in columns 19, 20 and 21 of Table 1.

In view of the recent work by Tsang and Persky (1975) it is not surprising that the direct estimation procedures provide results of low statistical significance. When the Kmenta approximation is used the hypothesis that $\sigma = 1$ can be tested by checking to see whether or not the coefficient on the squared term differs significantly from one. In 75% of the industries examined by Griliches and Ringstad the null hypothesis could not be rejected. Furthermore, of the cases in which it is rejected, half the estimates imply the wrong curvature for the isquants ($\hat{\sigma} < o$) and hence cannot be taken seriously.

The non-linear estimates (obtained by searching over a grid of σ values) are similarly unable to reject the Cobb-Douglas specification. While the point estimates obtained from the search technique fluctuate considerably about one, in only one case was the residual sum of squares (adjusted for degrees of freedom) significantly lower than for the Cobb-Douglas form. As an indication of the extreme flatness of the likelihood function in the σ direction Griliches and Ringstad reported that the residual sum of squares for total manufacturing differed by less than one per cent for values of σ between 0.6 and 1.5.

The use of the labour marginal productivity condition again gave estimates clustered around unity. In only 6 of the 27 industries could the estimated values of σ be regarded as significantly different from one. A comparison between these and the direct estimates does not indicate any systematic relationship between the alternative sets of results. Griliches and Ringstad suggest that "since they are all imprecisely estimated, perhaps this is not so surprising" (1971, p.85).

4. SUMMARY OF CROSS-SECTIONAL ESTIMATES

The above survey of cross-sectional estimates of σ appears to yield little in the way of substantive results. Perhaps the most obvious question to ask is whether or not the generalization of the Cobb-Douglas afforded by the more complex CES form is in fact necessary. On the basis of the results cited above there is no strong evidence to refute the maintained hypothesis that the production function for the various industries is of the Cobb-Douglas form. In a great majority of the cases the estimated value for σ did not show any significant deviation from unity. However, in many cases the power of the tests is very low and these may equally be taken to indicate the poor quality of the data rather than the validity of the Cobb-Douglas form.

5. TIME SERIES STUDIES

The main difference between the specification of the regression model for use with time series data and the cross-sectional models considered above lies in the need to allow for technological change and other effects that may cause the production function to shift over time. In most of the studies in which σ rather than the rate of technological progress *per se* is of prime interest, the technological change is specified as being Hicks neutral and constant (i.e. increases in the marginal productivity of both labour and capital are equi-proportional and advance at a constant rate of time). This feature is incorporated into the CES function by making the efficiency parameter (γ) a function of time. For example let $\gamma_t = \gamma_o \ell^{ct}$ where t measures units of time and c is the rate of constant neutral technical progress. Under these circumstances the time series version of the labour marginal productivity condition (3) is

$$\log \left[V/_L\right] = a + \sigma \log \left[W/_P\right] + ct \quad . \tag{13}$$

McKinnon (1962) uses an estimating equation of the general form of (13) but allows for a distributed lag in the relationship.[15] The data used was taken from a number of secondary sources. The most apparent difference between these long-run results (ref. Col.1 table 2) and the cross-sectional ones summarized above is the tendency for the estimates to be consistently less than one. As will be seen below this is a characteristic common to most time series studies.

An exception to this are the estimates made by Ferguson (1965). He used the labour side condition with time series data

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15. McKinnon's derivation of the relationship is not based on the use of a CES function. In his interpretation of (13) the log W coefficient must take a value between 0 and 1. However since this restriction is not enforced in the estimation, the results are comparable to those derived using a CES function.

covering 19 U.S. manufacturing industries over the period 1949-61. The initial estimates for seven of the industries yielded a technological change coefficient which was negative and insignificantly different from zero. For these industries the trend term was eliminated and the parameters were re-estimated. The final results for all industries are listed in column 2 of table 2. Although the period covered was roughly the same as that used by McKinnon there are substantial differences between the two sets of estimates. Ferguson's estimates are distributed about one with, in the majority of cases, the deviation being insignificant. However as his data was in current dollar values and output prices would certainly have risen over the period the estimates could, according to the argument advanced earlier, be expected to have an upwards bias.

Lucas (1969) provides some support for the lower estimates of McKinnon, based on a larger sample size. Lucas uses U.S. data for the period 1931-58 to compute two sets of estimates for σ using (13).

The first of these (Col.3) uses current period's price as the explanatory variable while the other (Col.4) uses previous period's price (i.e. a lag of one period is assumed). In some industries (e.g. 28, 33, 35 & 37) there are considerable differences between the two estimates of the elasticity of substitution. Although there appears to be no consistent relationship between these and McKinnon's estimates at the individual industry level, both sets of results give the same overall picture of being substantially lower than the cross-sectional estimates.

Maddala (1965) presents estimates of the elasticity which are in one respect similar to those (unintentionally) provided by

Dhrymes. He uses two different types of information on the rate of return to capital[16] to estimate equation (5) and it's reciprocal,

$$\log \left(W/_R\right) = a_3' + \left(1/_\sigma\right) \log \left(K/_L\right) \tag{14}$$

The data used covers the period 1947-58. The results (ref. Col.5 - 8 of Table 2) again tend to be less than one but otherwise show little similarity to those summarized above. The degree of variability in the estimates (which refer to roughly the same time period) indicates that like the cross-sectional estimates the time series estimates of σ are extremely sensitive to the form of the fitted relationship and the data definitions used.

In view of the crucial role played by the data in these estimates it is surprising that Australian statistics - claimed by P.H. Douglas to be "the best statistics in this field in the world" - have not been more fully exploited. The only comprehensive set of Australian estimates that are available are those of Sampson (1969). He uses data on a selection of 22 industries over the period 1949-50 to 1964-65 to estimate equation (13)[17]. The results follow the pattern established in overseas studies in so far as all the estimated values were less than one. Although this pattern of behaviour appears to be established the considerable disparity between the estimates at the individual industry level is difficult to rationalize.

Mayor (1975) has used a generalization of the Hall and Jorgenson (1967) investment model to obtain estimates of the elasticity

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16. The first rate of return series used was taken from Stigler (1963) while the other was determined by "residual computation".

17. Refer Column 9 of Table 2 for these results.

of substitution. The investment equation that is estimated is based on a distributed lag model which uses the capital side condition of the CES function (eqn. 4) to determine the desired capital stock. Unlike the models that have been referred to above, the estimating equation in this study has σ entering in a non-linear fashion. Since the equation was linear in all other parameters estimation was carried out by scanning over a grid of σ values and using the results which yielded the lowest residual sum of squares. The standard errors which accompany these estimates (ref. Col.10) were calculated using a method suggested by Hartley and Booker (1965). The estimates cover a much wider range of values than has been observed in the other time series studies.

Non-linear least squares estimating techniques have also been employed by Tsurumi (1970). He presents estimates based on both a direct and two stage least squares application of Marquardt's (1959) maximum neighbourhood method. For comparison he also computes estimates using ordinary least squares on the labour side-condition (13). None of these estimates (ref. Col. 11-13) exceed unity. However although there is some measure of agreement between the alternative estimates for the various industries it is again difficult to detect any systematic relationship between these and the other time series estimates.

6. SUMMARY OF TIME SERIES RESULTS

The time series estimates by themselves appear to provide little on which to base any strong conclusions about the value of the elasticity of substitution parameter for various industries. In general the estimates are lower than those obtained from cross-sectional data and fall in the range 0 to 1.

7. CONCLUSION

This review of time series and cross-sectional studies has indicated that there is little agreement as to the "true" value of the elasticity of substitution. There are significant differences between the estimates and there appears to be no clear-cut explanation for their diversity. Even slight changes in the period or concepts produce drastic changes in the estimates. Suggested causes of the instability in the estimates cover such factors as variations in the efficiency parameter, output price or quality of inputs, lagged adjustment, cyclical changes in utilization, serial correlation, simultaneous equation bias and random measurement errors. The overall impression is that there are a large number of biases, the magnitude and direction of whose effects are uncertain, operating simultaneously to produce very inconsistent and untrustworthy results.

Not only do the various studies attribute widely different absolute levels to the elasticity of substitution in the industries considered,but there is also no apparent consistency between the ordinal rankings of the industries within each study. There is however a general pattern of differences between the cross-sectional and time series estimates - namely the former are usually larger than the latter.

A more basic criticism of the general model underlying the above estimates has been advanced by Harcourt (1966) and others. He has shown that serious biases in the estimates may stem from a failure to identify clearly the differences between the <u>ex post</u> and <u>ex ante</u> substitution possibilities. As factor proportions of plant cannot be easily varied after it has been installed it is only the factor proportions of the newest plant that will be adjusted to current price conditions. The practice of regressing aggregate factor proportions against recent costs will lead to a hybrid substitution parameter which actually reflects neither the ex post nor ex ante production possibilities and whose value depends on the relation of average-practice and best-practice factor proportions.

Johansen (1972, p. 1) points out that when there is a difference between the ex ante and ex post substitution possibilities "it may well be that we should get different estimates by different types of data, these different estimates reflecting different aspects of a more complex technological structure than that described by traditional production functions." To overcome this problem he develops a model which carefully distinguishes between the short and long run, ex ante and ex post production functions. It is my view that the Johansen model would provide a more realistic production framework for future estimating attempts.

8. RECOMMENDATION

In spite of the dubious record of attempts to estimate neo-classical production functions econometrically, in a forthcoming paper I show that recent cross-sectional evidence on Australian manufacturing industries will, when fitted under a flexible econometric specification, yield a capital-labour substitution elasticity which has a relatively narrow confidence band. Such an estimate may temporarily fill a vacuum until more ambitious work recognizing the vintage composition of capital can be attempted. But in any event, prudence requires that ORANI results be subjected to intensive analysis of their sensitivity to the values of the capital-labour substitution elasticities.

TABLE 1 — CROSS-SECTIONAL STUDIES

SOURCES OF ESTIMATES APPEARING IN TABLE

COLUMN	AUTHOR	DATA	ESTIMATING EQUATION
1	Arrow, Chenery, Minhas and Solow (1961)	International (1950-5)	$\log PV/_L = a_1 + \sigma \log W$
2	Fuchs (1963)	International (1950-5)	$\log PV/_L = a_1 + \sigma \log W + \beta \log D$ where $D = 10$ for developed countries $= 1$ for underdeveloped countries
3	Arrow and Murata (1965)	International (1953-6)	$\log PV/_L = a_1 + \sigma \log W$
4	Arrow and Murata (1965)	International (1957-9)	$\log PV/_L = a_1 + \sigma \log W$
5	Minsian (1961)	U.S. (1957)	$\log LW/_{PV} = c + \sigma \log W$
6	Solow (1964)	U.S. (1956)	$\log PV/_L = a_1 + \sigma \log W$
7	Dhrymes (1965)	U.S. (1957)	$\log W = a_1' + 1/_\sigma \log PV/_L$
8	Dhrymes (1965)	U.S. (1957)	$\log R = a_2' + 1/_\sigma \log PV/_k$
9	Dhrymes and Zarembka (1970)	U.S. (1957)	$\log PV/_L = a_1 + \sigma \log W$
10	Dhrymes and Zarembka (1970)	U.S. (1957)	$\log PV/_k = a_2 + \sigma \log R$
11	Dhrymes (1965)	U.S. (1957)	$\log L = G - \frac{1}{\gamma} \log W + \frac{\beta}{\gamma} \log V$ where $\sigma = -1/_\gamma$
12	Bell (1965)	U.S. (1958)	$\log [WL/(PV - WL)] = C_2 + \rho \log K/_L$
13	Bell (1965)	U.S. (1958)	$\log PV/_L = a_1 + \sigma \log W$
14	Moroney (1972)	U.S. (1963)	$\log K/_L = a_3 + \sigma \log W/_R$
15	Moroney (1972)	U.S. (1963)	$\log PV/_L = a_1 + \sigma \log W$
16	Hildebrand and Liu (1965)	U.S. (1957)	$\log V/_L = \log \alpha_1 + \alpha_2 \log W + \alpha_3 \log K/_L$ where $\sigma = \left(\frac{1}{\alpha_2} - \frac{\alpha_3}{\alpha_2 S_k}\right)^{-1}$
17	Zarembka and Chernicoff (1975)	U.S. (1963)	$\log PV/_L = a_1 + \log c W$) plus regional dummies
18	Zarembka and Chernicoff (1975)	U.S. (1963)	$\log PV/_L = a_1 + \log \sigma W$)
19	Griliches and Ringstad (1971)	Norwegian (1963)	$\log V/_L = b_0 + b_1 \log L + b_2 \log K/_L + b_3 \log (K/_L)^2$
20	Griliches and Ringstad (1971)	Norwegian (1963)	$\log V = \gamma (\delta K^{-\rho} + (1 - \delta) L^{-\rho})^{-\mu/\rho}$
21	Griliches and Ringstad (1971)	Norwegian (1963)	$\log V/_L = a_1 + \sigma \log W$

USSIC INDUSTRY	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
FOOD AND KINDRED PRODUCTS																					
201			.722 (.054)	.725 (.054)	0.58 (.16)	0.69 (.22)	1.786	1.028	.822 (.164)	.7683 (.070)	.469 (.142)	.908	.648 (.117)	.538 (.123)	.873	2.15	.692 (.104)	.558 (.113)	1.242	0.5	1.022 (0.144)
202	.721 (.073)	.902 (.080)																.714 (.106)	*	1.3	0.660 (0.138)
203	.855 (.075)	1.086 (.098)																1.259 (.340)	1.912	1.5	1.134 (.144)
204	.909 (.096)	1.324 (.167)																.867 (.231)	.154	0.5	1.560 (0.403)
205	.900 (.065)	1.065 (.105)																.949 (.135	.703	.70	.964 (.075)
206	.781 (.115)	.895 (.183)																			
207																		1.081 (.259)			
208																		1.309 (.139)	*	1.3	1.074 (.242)
209																		1.779 (.244)			
TOBACCO MANUFACTURERS																					
211-4	.753 (.151)	1.215 (.208)			3.46 (.52)	1.96 (.30)															
TEXTILE MILL PRODUCTS																					
221-4	.809 (.068)	.976 (.104)	.793 (.049)	.827 (.069)	1.58 (.35)	1.27 (.15)	1.479	.968	.968 (.154)	.813 (.095)	.936 (.167)	1.161	.975 (.190)	.609 (.168)	.890	1.65	1.236 (.157)		1.126	1.5	1.028 (.120)
225	.785 (.064)	.948 (.083)																.927 (.387)			
226-7																					
228																		.082 (.714)			
229																		1.386 .429			

APPAREL	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
231																		1.540 (.363)			
232																		1.959 (.431)			
233																		1.476 (.219)			
234			.660 (.067)	.804 (.043)		1.01 (.13)	1.859	.972	1.214 (.197)	0.768 (.076)	1.169 (.207)	1.535	1.373 (.355)	.759 (.122)	1.065	1.43	1.371 (.184)	2.363 (.457)	1.121	.9	1.101 (0.097)
235																					
236																		1.869 (.497)			
237																					
238																		1.207 (.231)	9.1	1.5	1.343 (.242)
239																		1.068 (.102)			

LUMBER AND WOOD PRODUCTS	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
241																		.990 (.178)	1.075	1.1	0.736 ((.093)
242	.860 (.066)	1.083 (.141)	.818 (.068)	.919 (.074)	.94 (.11)	.99 (.09)	1.284	.908	.875 (.069)	.635 (.077)	1.109 (.117)	1.010	.942 (.053)	.825 (.074)	1.035	1.00	.926 (.087)	.469 (.281)			
243																		.835 (.123)	2.008	1.3	.949 (.129)
244																		.964 (.138)			
249																		.959 (.168)			

FURNITURE AND FIXTURES	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
251-2																		1.134 (.150)	167	1.3	1.002 (.085)
253	.894 (.042)	.1043 (.090)	.818 (.068)	.919 (.074)	1.09 (.23)	1.12 (.11)	1.437	.717	1.173 (.125)	.7427 (.031)	1.001 (.159)	1.149	1.032 (.106)	.621 (.064)	.921	.92	1.046 (.121)	.747 (.206)			
254																		.953 (.176)	1.359	1.5	0.985 (.071)
259																		.562 (.262)			

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
PAPER AND ALLIED PRODUCTS																					
261-2																		2.308 (.521)			
263	.965 (.101)	.912 (.175)	.904 (.050)	.788 (.061)	1.60 (.35)	1.77 (1.01)	4.926	1.567	1.432 (.454)	1.156 (.109)	1.470 (.488)	1.557	1.529 (.307)	1.276 (.186)	1.308	1.06	1.592 (.140)	.883 (.629)	.784	.7	1.111 (.169)
264																		1.172 (.458)			
265																		.727 (.225)			
PRINTING AND PUBLISHING																					
271																		.868 (.101)			
272																		1.108 (.102)			
273	.868 (.056)	1.021 (.085)	.836 (.075)	.926 (.063)		1.02 (.21)	1.468	.904	.998 (.206)	.826 (.059)	.562 (.199)	1.113	1.115 (.216)	.737 (.080)	.889		.818 (.087)	1.059 (.188)	1.222	.9	.593
274																		.998 (.171)			
275																		.818 (.095)			
276																		.884 (.437)			
278																		.832 (.197)			
279																		1.066 (.100)			
CHEMICALS AND ALLIED PRODUCTS																					
281	.831 (.070)	1.113 (.104)																.939 (.311)	∞	1.5	2.171 (.672)
283																		1.796 (.497)			
284	.839 (.090)	1.058 (.181)	.838 (.050)	.834 (.087)		.14 (.95)	3.235	.971	.869 (.272)	.906 (.048)	.506 (.267)	1.050	.694 (.252)	.764 (.124)	.889	1.24	1.230 (.177)	2.231 (.414)			
285-6																		2.053 (.291)	∞	1.5	1.203 (.279)
287																		1.406 (.258)			
289	.895 (.059)	1.060 (.088)																1.170 (.306)			

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
PETROLEUM REFINING																					
291			.838 (.050)	.834 (.087)	-0.54 (1.06)	1.45 (.71)	8.889	.763	.891 (.549)	.770 (.037)	.334 (.870)	1.707	1.316 (.426)	.445 (.130)	1.265		1.361 (.424)	1.053 (.437)			
295																		.879 (.459)			
RUBBER AND MISCELLANEOUS PLASTIC PRODUCTS																					
301-3			.829 (.058)	.768 (.106)	.82 (.29)	1.48 (.88)	2.498	.964	1.563 (.349)	.808 (.097)	1.984 (.290)	1.352	1.347 (.308)	.772 (.185)	1.117	1.44	1.041 (.135)				
306																		.649 (.277)			
307																		.643 (.278)			
LEATHER AND LEATHER PRODUCTS																					
311-3	.857 (.062)	.975 (.100)	.711 (.059)	.699 (.050)	0.96 (.29)	0.89 (.27)	1.969	.888	.857 (.261)	.817 (.0825)	.853 (.262)	1.089	.900 (.247)	.775 (.107)	.961	0.79	1.015 (.149)				
314																		.991 (.407)			
315-9																					
STONE CLAY AND GLASS PRODUCTS																					
321-3	.999 (.084)	1.269 (.096)	.847 (.046)	.859 (.051)	.59 (.25)	.32 (.46)	2.080	1.127	1.027 (.192)	.968 (.065)	1.063 (.232)	1.235	.858 (.182)	.982 (.105)	1.036	1.28	1.204 (.122)	1.235 (.344)	*	1.3	1.029 (.199)
324	.920 (.149)	1.308 (.217)																.366 (.699)			
325	.919 (.098)	.658 (.197)																1.424 (.311)			
326	.901 (.044)	1.078 (.125)																			
327																		1.207 (.120)	1.976	1.5	1.376 (.130)
328																		.912 (.183)			
329																		1.107 (.318)			

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
PRIMARY METAL INDUSTRIES																					
331																		.999 (.217)			
332	.811 (.051)	.756 (.112)																1.151 (.154)	-	1.1	1.173 (.296)
333																					
334	1.011 (.120)	.935 (.197)	.856 (.066)	.873 (.063)	.92 (.24)	1.87 (1.25)	10.50	1.033	.765 (.397)	.836 (.096)	.945 (.600)	1.277	1.069 (.247)	.601 (.148)	1.221	.99	1.306 (.139)				
335																		1.297 (.337)			
336																		1.164 (.149)			
339																		.680 (.286)			
FABRICATED METAL PRODUCTS																					
341																					
342																		1.248 (.209)			
343																		1.323 (.291)			
344	.902 (.088)	1.006 (.166)	.917 (.052)	.922 (.069)		.80 (.29)	2.497	1.05	.557 (.198)	.717 (.096)	.401 (.206)	1.344	.803 (.211)	.593 (.127)	.640	.70	.887 (.110)	0.692 (.120)	1.946	1.5	.901 (.076)
345																		.811 (.191)			
346																		.786 (.158)			
347																		.659 (.209)			
348																		.907 (.363)			
349																		.680 (.156)			

MACHINERY (EXCEPT ELECTRICAL)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
351																					
352																		.717 (.205)			
353																		.786 (.317)			
354					.31 (.21)	.64 (.45)	8.292	4.090	.747 (.466)	.364 (.122)	.050 (.358)	1.571	.535 (.269)	.150 (.157)	.555	.60	.681 (.146)	.882 (.122)	.282	0.5	.858 (.134)
355																		1.119 (.149)			
356																		.853 (.138)			
357																					
358																		.907 (.314)			
359																		.820 (.125)			

ELECTRICAL MACHINERY AND EQUIPMENT

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
361																		1.015 (.230)			
362																		.923 (.293)			
363	.870 (.118)	1.026 (.214)			1.26 .33	0.37 (.54)	5.163	1.613	.592 (.363)	.729 (.060)	.195 (.391)	1.341	.778 (.229)	.552 (.092)	.540	.79	.563 (.280)	1.098 (.600)	.404	.5	.827 (.188)
364																		.816 (.482)			
365-6																		.763 (.292)			
366-7																		.875 (.225)			
369																		1.027 (.249)			

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
TRANSPORT EQUIPMENT																					
371																		1.706 (.362)			
372																					
373					2.04 (.49)	0.06 (.82)	7.686	1.095	1.243 (.637)	.685 (.131)	1.775 (.887)	1.623	1.891 (.498)	.692 (.139)	.893	2.01	.242 (.105)	1.556 (.263)	1.698	.7	.445
374-5																					
379																		.854 (.281)			
INSTRUMENTS AND PHOTOGRAPHIC GOODS																					
381																		.543 (.203)			
382						1.59 (.15)						1.504	.735 (.333)	.554 (.164)	.999	1.24	1.143 (.213)	1.006 (.272)			
383-4																		1.213 (.250)			
385-8																					
MISCELLANEOUS MANUFACTURING																					
391																		-0.253 (.512)			
394																	1.082 (.134)	1.259 (.226)			
395																		.805 (.312)			
396																		.787 (.353)			
399																		1.215 (.132)			

TABLE 2

TIME SERIES STUDIES

Column	Author	Data	Estimating Equation
1	McKinnon (1962)	U.S. 1947-58	$\log (V/_L) = a_1 + a_2 \log (W/_P)_t + a_3 \log (V/_L)_{t-1} + a_4 t$
2	Ferguson (1965)	U.S. 1949-61	$\log (V/_L)_t = A + \sigma (W/_P)_t + b_t$
3	Lucas (1969)	U.S. 1931-58	$\log (V/_L)_t = A + \sigma (W/P)_t + bt$
4.	Lucas (1969)	U.S. 1931-58	$\log (V/_L)_t = A + \sigma (W/P)_{t-1} + bt$
5	Maddala (1965)	U.S. 1947-58	$\log (K/_L)_t = a_3 + \sigma \log (W/_R)_t$ (Stigler Capital Price Data)
6	Maddala (1965)	U.S. 1947-58	$\log (K/_L)_t = a_3 + \sigma \log (W/_R)_t$ (Capital Price by Residual Comp.)
7	Maddala (1965)	U.S. 1947-58	$\log (W/_R)_t = a_3' + 1/_\sigma \log (K/_L)_t$ (Stigler Data)
8	Maddala (1965)	U.S. 1947-58	$\log (W/_R)_t = a_3' + 1/_\sigma \log (K/_L)_t$ (Residual Comp.)
9	Sampson (1969)	Aust.1949-65	$\log (V/_L) = a + \sigma \log W + ct$
10	Mayor (1975)	U.S. 1947-67	$I_t = \alpha\left\{\left(\frac{V}{R^\sigma}\right)_t - \left(\frac{V}{R^\sigma}\right)_{t-1}\right\} + \beta\left\{\left(\frac{V}{R^\sigma}\right)_{t-1} - \left(\frac{V}{R^\sigma}\right)_{t-2}\right\} + \lambda I_t$
11	Tsurumi (1970)	Canadian (1926-67)	$V = A\ell^{\lambda t} \{\alpha K_t^{-\rho} + (1 - \alpha) L_t^{-\rho}\}^{-\upsilon/\rho}$
12	Tsurumi (1970)	Canadian (1926-67)	"
13	Tsurumi (1970)	Canadian (1926-67)	$\log V/_L + a + \sigma \log W + ct$

	1	2	3	4	5	6	7	8	9	10	11	12	13
FOOD AND KINDRED PRODUCTS													
201										.37 (.11)			
202									.616	.32 (.15)			
203										1.16 (.09)			
204										2.10 (.20)			
205	.373	.24 (.20)	.397 (.056)	.235 (.085)	.033	.142	.088	.423	.938	2.10 (.25)	.496	.483	.58
206										.35 (.07)			
207										.49 (.20)			
208									.270	1.02 (.16)			
209										.0 (.12)			
TOBACCO MANUFACTURERS													
211-4	.921	1.18 (.46)	.152 (.050)	.089 (.055)	.089	.463	-.142	-.525	.551	.0 (.27)	1.000	1.000	.89
TEXTILE MILL PRODUCTS													
221-4									.732				
225	.162	1.10 (.44)	.131 (.063)	.070 (.061)	.058	.099	.138	.216	.667	.49 (.05)	.840	.840	.84
226-7													
228													
229									.686	.61 (.24)			

	1	2	3	4	5	6	7	8	9	10	11	12	13
APPAREL													
231-9	.694	1.08 (.16)			-.045	-.134	-.024	-1.030		2.10 (.04)	.423	.517	.53
LUMBER AND WOOD PRODUCTS													
241-3										.44 (.33)			
244	.802	.91 (.07)	.480 (.068)	.418 (.093)	.171	.262	.251	.309		2.10 (.14)	.202	.206	.23
249													
FURNITURE & FIXTURES													
251										.99 (.17)			
252	1.021	1.12 (.05)			.109	.206	.184	.442		.75 (.19)			
253-9													
PAPER AND ALLIED PRODUCTS													
261-4	.094	1.02 (.06)	.505 (.098)	.599 (.076)	.170	.225	.260	.389	.574	.0 (.17)	.393	.314	.56
265										.0 (.15)			
PRINTING AND PUBLISHING													
271-9	.844	1.15 (.31)	.488 (.069)	.477 (.065)	-.037	-.102	-.079	-.400		.26 (.21)			

	1	2	3	4	5	6	7	8	9	10	11	12	13
CHEMICALS AND ALLIED PRODUCTS													
281										.55 (.11)			
282										.72 (.09)			
283-4	-1.109	1.25 (.07)	.678 (.089)	.368 (.150)	.101	.221	.106	1.139	.609	.53 (.22)	.169	.168	.41
285										.88 (.20)			
286-9									.597				
PETROLEUM REFINING													
291-9		1.30 (.15)	.375 (.068)	.154 (.076)	.273	.374	.359	.486		.61 (.13)	1.000	1.000	.90
RUBBER AND MISC. PLASTIC PRODUCTS													
301-6	.354	.76 (.56)	.323 (.062)	.286 (.064)	.186	.339	.041	.224		.0 (.16)	1.000	1.000	.89
307													
LEATHER AND LEATHER PRODUCTS													
311										.97 (.07)			
313-4	.251	.87 (.14)	.407 (.095)	.320 (.088)	-.010	-1.318	-.052	-.307	.689	.33 (.24)	1.000	1.000	.89

	1	2	3	4	5	6	7	8	9	10	11	12	13
STONE CLAY AND GLASS PRODUCTS													
321-3										1.84 (.14)			
324	-1.124	.67 (.47)	-.205 (.107)	-.273 (.101)	.266	.400	.539	1.418	.155		1.000	1.000	.90
325-9										2.10 (.09)			
329									.524				
PRIMARY METAL INDUSTRIES													
331-2									.587	.0 (.20)	.317	.265	.63
333-6	.033	1.20 (.11)	.641 (.193)	.422 (.215)	.215	.266	.327	.463	.581	1.18 (.12)	.252	.254	.27
339													
FABRICATED METAL PRODUCTS													
341										.41 (.25)			
342										1.60 (.10)			
343	.328	.93 (.26)			.038	.405	.062	.713					
344										.14 (.73)			
345-6										.89 (.21)			
347-9									.879				

	1	2	3	4	5	6	7	8	9	10	11	12	13
MACHINERY (EXCEPT ELECTRICAL)													
351										.0 (.40)			
352										.16 (.65)			
353										.24 (.32)			
354	.754	1.04 (.04)	.476 (.152)	.175 (.173)	.147	.247	.334	.671		1.05 (.13)			
355										.05 (1.48)			
356										.62 (.16)			
357										.35 (.31)			
358										.0 (.20)			
359													
ELECTRICAL MACHINERY AND EQUIP.													
361-2										.02 (7.51)			
363										1.08 (.21)			
364	.432	.64 (.36)			.108	.224	-.026	-.4.305		1.08 (.21)	.252	.254	.27
365-6										2.09 (.15)			
367										.0 (.27)			
369										.96 (.12)			

	1	2	3	4	5	6	7	8	9	10	11	12	13
TRANSPORT EQUIPMENT													
371			.730 (.094)	.486 (.144)						1.03 (.25)			
372	.182	.24 (.56)			.052	.460	-.008	-2.270		.0 (.51)	.391	.350	.71
373-5										.0 (.26)			
379													
OPTICAL INSTRUMENTS AND LENSES													
381-2	.379	.76 (.29)			.416	.583	.577	1.048		.0 (.55)			
383										2.10 (.08)			
MISC. MANUFACTURING													
399										2.10 (.17)			

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