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# Theoretical specification of a labour-supply module, including HIV/AIDS, for South Africa

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# Theoretical specification of a labour-supply module, including HIV/AIDS, for South Africa.

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**Abstract**

This paper describes the application of a detailed labour-supply module, including HIV/AIDS for South Africa. I begin by developing a model of the South African labour market that embodies two salient features of HIV: (1) incidence and transmission rates follow distinct patterns by age, gender, race and occupation; (2) the disease progresses through a number of distinct stages. To model these features, the labour-market model must be dynamic, and identify large amounts of age, gender, race, labour-market function and HIV-stage detail. Dixon and Rimmer (2003) develop and apply a labour-market model with high levels of occupational detail. They subsequently extend this model to investigate issues related to immigration (Dixon & Rimmer, 2009, 2010, 2011), and of relevance to this paper, health (Dixon *et al.*, 2010).

In developing the labour-market module, we define people between the age of 15 and 65 by labour-market function, age, gender, race and HIV stage. We distinguish between employment, unemployment, new entrant and "permanently departed from the labour force" categories and employment, unemployment and "permanently departed from the labour force" activities. People in each category at the start of year $t$ decide on their labour supply during year $t$. They make this decision by solving an optimisation problem. We assume that people in employment categories supply their labour more strongly to employment activities than people in the unemployment categories. We also assume that supply of labour depends on health status. A person who is HIV negative supplies labour more strongly to employment activities than a person who is HIV positive. Adults grouped in the "permanently departed from the labour force" category do not offer to any labour force activity. As we shall see, under this modelling framework, policies aimed at reducing the transmission of HIV in year $t-1$ increase labour supply in year $t$ by increasing (relative to the basecase) the number of HIV-negative people in year $t$. Although not described in this paper, improved treatment of people already infected with the HIV virus may decrease the rate at which people move through the stages of HIV infection. Improved treatment may increase the effective labour supply of HIV-infected people.

The outline of this paper is as follows. Section1 defines the key concepts necessary for the functioning of the labour-market specification. Section 2 explains the basic labour-supply mechanism. The aim is to introduce the basic mechanism before it is explained in detail later in this chapter. Section 3 describes how categories at the start of year $t$ are determined. Section 4 describes the utility maximisation problem that people face when they decide on their labour offers. This equation shows that labour offers depend on relative wages. Section 5 describes the possible flows between categories and activities. Section 6 describes the equations facilitating these flows. Section 7 explains the wage determination process in the policy simulation. Under this process real wages are sticky in the short term and flexible in the long term, with employment adjusting in the short term and returning to basecase in the long term. Section 8 describes equations that allow for independent HIV forecasts to be introduced to the basecase. Section 9 describes an equation that models the deviation in the number of new HIV cases. This equation can be activated during the policy simulation.

Keywords: Africa, HIV/AIDS
JEL codes: I190, O55

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**LIST OF ABBREVIATIONS**

| | |
|---|---|
| AIDS | Acquired immunodeficiency syndrome |
| HIV | Human immunodeficiency virus |
| PDL | Permanently departed from the labour force |
| RHS | Right hand side |
| SAGE-H | South African General Equilibrium model including the labour-market specification |
| UN | United Nations |
| WAP | Working age population |
| WHO | World Health Organisation |

**LIST OF SETS**

| | |
|---|---|
| $ACT \in$ | OCC, UNEMP and Permanently departed from the labour force (PDL) |
| $AGE \in$ | A15-24, A25-34, A35-44, A45-54, A55-65. |
| $CAT \in$ | OCC, UNEMP, N and Permanently departed from the labour force (PDL). |
| $EUN \in$ | Employment (OCC), Unemployment (UNEMP), New entrant (N). |
| $EUNP \in$ | Employment (OCC), Unemployment (UNEMP), New entrant (N) and Permanently departed from the labour force (PDL). |
| $EUP \in$ | Employment (OCC), Unemployment (UNEMP), Permanently departed from the labour force (PDL). |
| $EU \in$ | Employment (OCC), Unemployment (UNEMP). |
| $GEN \in$ | Female, Male. |
| $H2 \in$ | HIV negative, HIV positive. |
| $H3 \in$ | HIV negative, Stage 1, Stage 2 and Stage 3. |
| $N \in$ | New entrant. |
| $OCC \in$ | Legislators, Professionals, Technicians, Clerks, Service workers, Skilled agricultural workers, Craft workers, Plant and machine operators, Elementary occupations, Domestic workers and Occupations not elsewhere specified. |
| $RACE \in$ | African, Other. |
| $STG \in$ | HIV negative, Stage 1, Stage 2, Stage 3 and Stage 4. |
| $UNEMP \in$ | Short (S) and long-run (L) unemployment. |

# 1. KEY CONCEPTS

## 1.1. Working age population (WAP)

The working age population (WAP) includes all persons between the ages of 15 and 65. The WAP is divided according to age, gender, race, HIV status and labour-market activity. The labour-market activities are employment, unemployment and "permanently departed from the labour force" (PDL).[1]

## 1.2. Health stages

SAGE-H recognises five HIV stages. A person can either be HIV negative or HIV positive. Once a person becomes HIV positive, they cannot return to being HIV negative. They remain HIV positive until they leave the WAP. The HIV-positive group is divided into four stages (Stage 1, 2, 3 and 4). These stages are based on the World Health Organisation's clinical classification of HIV/AIDS for adults and adolescents with confirmed HIV infection. Table 1 describes the clinical stages in simplified terms. Appendix A reports a list of clinical staging events associated with each stage of HIV/AIDS for adults and adolescents with confirmed HIV infection (World Health Organisation, 2007: 15).

**Table 1. WHO clinical staging of established HIV infection**

| *HIV-associated symptoms* | *WHO clinical stage* |
|---|---|
| Asymptomatic | 1 |
| Mild symptoms | 2 |
| Advanced symptoms | 3 |
| Severe symptoms | 4 |

Source: World Health Organisation, 2007: 12. Table 1.

People move sequentially through five HIV stages. For example, a person transitioning from HIV negative to HIV positive starts at Stage 1. From Stage 1 they move sequentially through the stages until they reach Stage 4. People do not move backward through the stages.

[1] People who are "permanently departed from the labour force" are considered not part of any employment or unemployed activity. They are not part of the labour force. PDL is also the only activity that includes people with a Stage 4 HIV status. People with a Stage 4 HIV status are assumed too ill to be part of any employment or unemployment activity. Once a person is PDL, they remain PDL and do not supply their labour to any employment or unemployment activity.

### 1.3. Categories

People are allocated to "categories" at the start of year $t$. These categories are based on common characteristics, namely: age, gender, race, HIV status and labour-market activity during year $t-1$. The five employment statusses are:

- employment in occupation $o$, where $o$ is one of the 11 occupations identified in SAGE;
- short-term unemployed (S), i.e. unemployed in year $t-1$ but employed in year $t-2$;
- long-term unemployed (L), i.e. unemployed in both years $t-1$ and $t-2$;
- "permanently departed from the labour force" (PDL), i.e. not part of any employment or unemployment activity. This category also includes people with an HIV Stage 4 status; and
- new entrants $(N)$ into the labour force. This category is not based on any activity performed in year $t-1$. This category is added exogenously at the start of year $t$. *New entrants* refer to those who were not part of the labour force (employed or unemployed) in year $t-1$. I assume that new entrants fall into the age group 15 to 24.

### 1.4. Activities

Before I define the activities matrix, I wish to point out that the descriptions of the activities and categories matrices are very similar. Both matrices are defined by labour-market function (employment, unemployment and PDL), age, gender and race. There are two main differences between these matrices. Firstly, the matrices relate to different points in time. The activities matrix relates to what a person is doing *during* year $t$, while the categories matrix is defined at the *start* of year $t$ on the basis of the previous year's activities. An adult's activity during the year depends on the category they were grouped into at the start of the year. Secondly, new entrant (N) is an additional category and added exogenously at the start of the year. The activities matrix does not include new entrants as an activity. Rather, new entrants perform a specific activity, such as being employed as a technician, during the year.

The main activities undertaken during year $t$ are:

- employment in occupation $o$, where $o$ is one of the 11 occupations identified in SAGE;

- short-term unemployed[2] (S), i.e. employed in year $t-1$ but unemployed in year $t$;
- long-term unemployed i.e. unemployed in both years $t-1$ and $t$. They are also referred to as discouraged workers; and
- "permanently departed from the labour force" (PDL).

Activities are defined by age, gender, race and HIV status. There are no restrictions imposed on age, gender and race in any activity. The only restriction is regarding Stage 4. We assume that people in Stage 4 are too ill to participate in any productive activity. Hence, they move into the PDL activity. The HIV stages that are allowed in each activity are summarised in Table 2.

**Table 2. HIV stages that are allowed in each activity**

| ***Activity*** | ***HIV status (stage)*** |
|---|---|
| Employment in occupation $o$ | HIV negative, Stages 1, 2, 3 |
| Short and long-term unemployment | HIV negative, Stages 1, 2, 3 |
| Permanently departed from the labour force | HIV negative, Stages 1, 2, 3 and 4 |

### 1.5. Planned labour flows

Once categories have been specified, the people within these categories make plans to offer labour to an activity during year $t$. These plans are based on category-specific solutions to utility maximisation problems. Planned labour supply (offers) is sensitive to changes in both occupation-specific relative wages and personal preferences.

While the theory allows for the possibility of a change in labour market activity, most people will choose to remain in the same activity as the previous year. Only a small proportion of people will move to a different occupation or to the unemployment activity. The strength of these offers, as well as the construction of the planned flows matrix, is explained in detail Section 4.2.

[2] Statistics South Africa uses two definitions of unemployment: the official (narrow) and the expanded definition. According to the official definition, the unemployed are those people within the economically active population who (a) did not work during the seven days prior to the interview (b) want to work and are available to start work within a week of the interview and (c) have taken active steps to look for work or to start some form of self-employment in the four weeks prior to the interview. The expanded unemployment rate excludes criterion (c). Those who are included in the expanded but not the official definition of unemployment are discouraged job seekers, that is, those who said they were unemployed but had not taken active steps to find work in the four weeks prior to the interview (Statistics South Africa, 2003: 103). In SAGE-H we use the expanded definition where we model both short-term and long-term unemployment.

### 1.6. Actual flows

Actual flows link categories at the start of year $t$ to activities undertaken during year $t$. Determining who secures employment, that is, the actual flow from category $c$ to employment activity $o$, is dependent on (1) the number of occupation-specific vacancies and (2) the total supply to that specific activity from all categories $c$.

### 1.7. Vacancies

Occupation-specific vacancies are defined as employment less the number of jobs filled by incumbents. Vacancies are filled by flows from (1) other occupations where occupation, (2) short and long-term unemployed and (3) new entrants. In any given year it is possible that the number of people seeking employment exceeds the number of vacancies. People who do not secure employment move to the short-term unemployment activity.

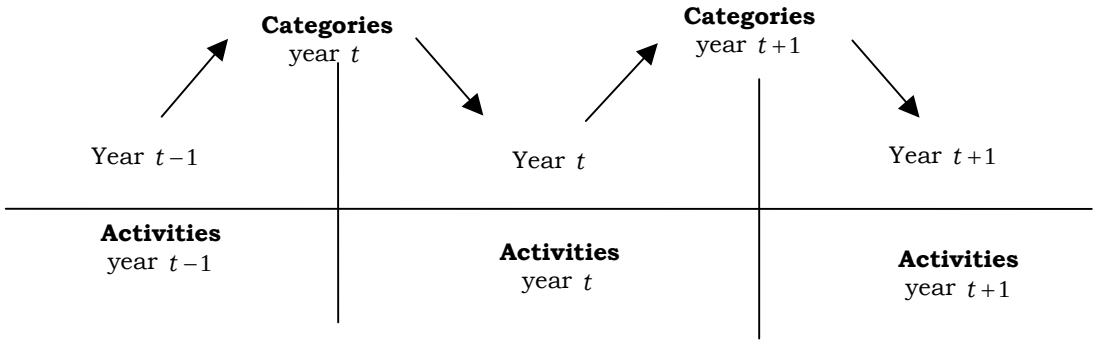
## 2. THE FUNCTIONING OF THE LABOUR-MARKET SPECIFICATION

The labour-market extension of the SAGE-H model draws heavily on the key ingredients specified in the MONASH-Health model (Dixon *et al.*, 2007; Dixon & Rimmer, 2010, 2011). The following key ingredients are specified:

- assign the WAP into categories at the start of the year. I define an equation that links the number of people in activity $a$ in year $t-1$ to the number of people in categories $c$ at the start of year $t$;
- identification of workforce activities, i.e. what people do during the year;
- determine the supply of labour from each category $c$ to each activity $a$;
- determine the demand for labour for each employment activity $a$. Equations determining the demand for labour are contained in the core model of SAGE-H (Dixon and Rimmer, 2002);
- specify the wage adjustment process reflecting the gap between labour demand and supply; and
- activity determination, i.e. determining who gets the jobs and what happens to those who don't secure employment.

At the start of year $t$, people are divided into categories based on common characteristics. These characteristics are age, gender, race, health status and labour-market activity in year $t-1$. People in categories offer their labour services to activities. In Figure 1, this flow is illustrated by the downward-sloping arrow between categories and activities. At the end of year $t$, people still part of the WAP progress one year in age and may change their health status. Some people leave the WAP due to retirement or death. This transition is illustrated by the upward-sloping arrow in Figure 1.

**Figure 1. Movement of labour supply from year t-1 to year t**




Source: Adapted from Dixon and Rimmer, 2008.

After this transition, people are again grouped into categories, based on common characteristics. The process of labour supply from a category to an activity is then repeated. Figure 1 abstracts from a large number of labour market flows. This detail is expanded upon in Figure 2. In Sections 3 to 6 below we explain the theory underlying the flows described in Figure 2. A summary of the dimensions used in the labour-market specification is provided in Appendix B.

## 3. DETERMINING CATEGORIES AT THE START OF YEAR $t$ (Figure 5.2; flow o)

At the beginning of each year, we allocate people in the WAP into categories according to their recent labour-market activity, age, gender, race and HIV stage. We allow for 5 age groups, 2 genders, 2 races and 5 HIV stages.

Every year we allow a small proportion of people to leave the WAP through death or retirement. This is implicitly shown in flow (o) in Figure 2. Those who survive are assumed to progress one year in age. In our framework, the majority of people survive and remain part of the WAP. We also allow people to change their HIV stage. For example, a person may change their status from HIV negative in year $t-1$ to Stage 1

in year $t$. The alternative to this change is to remain HIV negative in year $t$. An HIV-positive person moves sequentially through Stage 1 to Stage 4 where they remain until death. Once a person is HIV positive, they cannot change their HIV status back to HIV negative.

A person's recent labour-market activity refers to what a person did as an activity in year $t-1$. The activities are specified in Section 1.4 and illustrated in Figure 2. In Figure 2, categories are determined via flow (o).

Categories at the start of the year are specified in (E.1):

$$CAT_{(o,a,g,r,h)} = \sum_{aa \in AGE} \sum_{hh \in STG} ACT\_L_{(o,aa,g,r,hh)} * T_{(o,aa,g,r,hh,a,h)} \qquad \text{(E.1)}$$

$o \in EUP$;[3] $a \in AGE$; $g \in GEN$; $r \in RACE$ and $h \in STG$.[4]

New entrants, described in Figure 2 by flow (p), are determined exogenously:

$$CAT_{(N,a,g,r,h)} = exogenous \qquad \text{(E.2)}$$

$a \in AGE$; $g \in GEN$; $r \in RACE$ and $h \in STG$.

where

- $CAT_{(o,a,g,r,h)}$ is the level of the number of people in category $o$ of gender $g$, race $r$, allocated to age $a$ and HIV stage $h$ at the start of year $t$;
- $ACT\_L_{(o,aa,g,r,hh)}$ is the level of the number of people who performed activity $o$ in year $t-1$, given their age $aa$, gender $g$, race $r$ and HIV stage $hh$;
- $T_{(o,aa,g,r,hh,a,h)}$ is the proportion of people, given their gender and race, who were in activity $o$ and in age $aa$ and HIV stage $hh$ in year $t-1$ who are allocated to category $o$, age $a$ and HIV stage $h$ at the start of year $t$. $T$ is referred to as the transition matrix. We do not allow people to change their activity, gender and race from the previous year. Only age and HIV stage may change; and
- $CAT_{(N,a,g,r,h)}$ is the level of the number of new entrants at the start of year $t$ by age $a$, gender $g$, race $r$, HIV stage $h$. They are exogenously

[3] $EUP$ is the set of all employment, short and long-term unemployment and "permanently departed from the labour force" employment statuses.
[4] $STG$ is a set containing all HIV stages: HIV negative, Stage 1, Stage 2, Stage 3 and Stage 4.

added every year.

The percentage-change form of Equation (E.1) is:

$$CAT_{(o,a,g,r,h)} * cat_{(o,a,g,r,h)} = \sum_{aa \in AGE} \sum_{hh \in STG} \left\{ ACT\_L_{(o,aa,g,r,hh)} * \left( T_{(o,aa,g,r,hh,a,h)} * act\_l_{(o,aa,g,r,hh)} + 100 * d\_transit_{(o,aa,g,r,hh,a,h)} \right) \right\} \quad \text{(E.3)}$$

$o \in EUP$ ; $a \in AGE$ ; $g \in GEN$ ; $r \in RACE$ and $h \in STG$ .

where
- $cat_{(o,a,g,r,h)}$ is the percentage change in the number of people in each category $o$ for all age $a$ , gender $g$ , race $r$ and HIV stage $h$ ;
- $act\_l_{(o,aa,g,r,hh)}$ is the percentage change in the number of people in activities $o$ performed in year $t-1$, for all age $aa$ , gender $g$ , race $r$ and HIV stage $hh$ ; and
- $d\_transit_{(o,aa,g,r,hh,a,h)}$ is the ordinary change in the transition rate allowing people by gender $g$ and race $r$ and in activity $o$ , to change their age from $aa$ to $a$ and HIV stage from $hh$ to $h$ .

## 4. PLANNED LABOUR SUPPLY FROM CATEGORIES TO ACTIVITIES

### 4.1. Utility maximisation problem

Following Dixon and Rimmer (2008), we assume that people in category $c$ choose their labour offers across activities $a$ by solving a utility maximisation problem. The utility function takes a CES form. The general maximisation problem is defined as an adult choosing $L_t(c,a)$ to:

$$\text{Max } U_{(c)} = \left[ \sum_a \left( B_{(c,a)} * ATW_{(a)} * L_{(c,a)} \right)^{\frac{\eta}{1+\eta}} \right]^{\frac{1+\eta}{\eta}} \text{ for all } a \in EU \text{ [5]}, \quad \text{(E.4)}$$

$$\text{subject to } \sum_{a \in ACT} L_{(c,a)} = CAT_{(c)} , \quad \text{(E.5)}$$

where
- $U_{(c)}$ is a category-specific utility function;
- $B_{(c,a)}$ captures exogenous non-wage factors, such as preferences, that may motivate people from category $c$ to offer their labour to employment in activity $a$;
- $ATW_{(a)}$ is the real after-tax wage rate in activity $a$;
- $L_{(c,a)}$ is the number of offers from category $c$ to activity $a$; and
- $CAT_{(c)}$ is the number of people allocated to category $c$ at the start of year $t$.

In specifying utility from labour allocation according to (E.4), we implicitly assume that people in category $c$ treat South African Rand earned in different activities as imperfect substitutes. By specifying a separate utility function for each category, we ensure that each category supplies labour to activities that are compatible with that category's age, gender, race, health status and occupational characteristics. Within the SAGE-H framework, we allow for limited movement between occupations and between occupations and short-term unemployment. Hence, the majority of people continue to offer their labour supply to the same activity they performed during the previous year.

Based on the optimisation problem specified by Equations (E.4) and (E.5), the utility-maximising planned labour-supply equations take the form:

$$L_{(c,a)} = CAT_{(c)} * \left[ \frac{\left( B_{(c,a)} * ATW_{(a)} \right)^{\eta}}{\sum_{q} \left( B_{(c,q)} * ATW_{(q)} \right)^{\eta}} \right] \qquad \text{(E.6)}$$

The percentage-change form of (E5.6) is:

$$l_{(oo,a,g,r,h,o)} = cat_{(oo,a,g,r,h)} + \eta \left[ atw_{(o)} - atw^{ave}_{(oo,a,g,r,h)} + b\_pref_{(oo,a,g,r,h,o)} - b\_pref^{ave}_{(oo,a,g,r,h)} \right] \qquad \text{(E.7)}$$

$oo \in EUN$;[6] $a \in AGE$; $g \in GEN$; $r \in RACE$; $h \in H3$[7] and $o \in EU$.

[5] $EU$ is the set for all employment and unemployment activities.
[6] $EUN$ is a set of all employment, unemployment and new entrant categories.
[7] $H3$ is a set containing four elements: HIV negative, Stage1, Stage 2 and Stage 3.

where

- $l_{(oo,a,g,r,h,o)}$ is the percentage change in the number of persons of age $a$, gender $g$, race $r$ and HIV stage $h$, moving from category $oo$ to employment activity $o$;
- $cat_{(oo,a,g,r,h)}$ is the percentage change in the number of people in each $oo$ category by age $a$, gender $g$, race $r$ and HIV stage $h$;
- $atw_{(o)}$ is the percentage change in the occupation-specific real after-tax wage earned;
- $b\_pref_{(oo,a,g,r,h,o)}$ is the preference of a person of age $a$, gender $g$, race $r$ and HIV stage $h$, offering their labour supply from category $oo$ to activity $o$;
- $\eta$ reflects the ease with which people shift between activities; and
- $atw^{ave}_{(oo,a,g,r,h)}$ and $b\_pref^{ave}_{(oo,a,g,r,h)}$ are the weighted percentage change in the average after-tax wage and preference variable for adults in category $oo$. The weights reflect the share of activity $q$ in the offers made by people from category $c$. These variables are determined via Equations (E.9) and (E.10).

The above discussion focuses on adults who are employed, unemployed and new entrants to the labour force. What remains is to define the movement of people who are permanently departed from the labour force (PDL). We assume that no one in the PDL category offers to employment or unemployment activities, they simply move to the PDL activity. In level form, the equation is:

$$L_{(PDL,a,g,r,h,PDL)} = CAT_{(PDL,a,g,r,h,)} \qquad \text{(E.8)}$$

$a \in AGE$; $g \in GEN$; $r \in RACE$ and $h \in STG$.

The percentage-change form of (E5.8) is:

$$l_{(PDL,a,g,r,h,PDL)} = cat_{(PDL,a,g,r,h)} \qquad \text{(E.9)}$$

where

- $L_{(PDL,a,g,r,h,PDL)}$ is the number of people of age $a$, gender $g$, race $r$ and HIV stage $h$, moving from the PDL category to the PDL activity. $l_{(PDL,a,g,r,h,PDL)}$ is the corresponding percentage-change variable; and

- $CAT_{(PDL,a,g,r,h,)}$ is the number of people of age $a$, gender $g$, race $r$ and HIV stage $h$, grouped into the PDL category. $cat_{(PDL,a,g,r,h)}$ is the corresponding percentage-change variable.

In interpreting Equation (E.7), begin by assuming that there are no changes in the after-tax real wage and preference variables. Then, the percentage change in the offers from $oo$ to activity $o$ will follow the percentage change in the supply of labour in general from category $oo$.

In the absence of changes in preferences, people in category $oo$ will shift their labour offers towards activity $o$ when the real after-tax wage in activity $o$ rises relative to the average wage rates across all activities in which category $oo$ people could participate.[8] Generally, an increase in the ATW will not have much effect on labour supply from category $o$ to activity $o$. This is because a large part of labour supplies from category $o$ to employment activity $o$ is from incumbents, reflecting the assumption that the majority of people perform the same occupation in year $t$ as in year t-1. Hence, $L_{(a,a)}$ is a large fraction of $L_{(a)}$, hence $atw_{(o)} - atw^{ave}_{(oo)}$ will typically be close to zero.

People in category $oo$ will shift their offers towards activity $o$ if their preference to work in activity $o$ increases relative to the average preference. The derivation of the labour-supply equations from the optimisation problem specified by (E5.3) – (E5.4) is reported in Appendix 5B.

Equations (E.10) and (E.11) determine the average wage rate and average preference shift respectively as appropriate share-weighted averages:

$$CAT_{(oo,a,g,r,h)} * atw^{ave}_{(oo,a,g,r,h)} = \sum_{o \in EU} L_{(oo,a,g,r,h,o)} * atw_{(o)} \qquad \text{(E.10)}$$

$$CAT_{(oo,a,g,r,h)} * b\_pref^{ave}_{(oo,a,g,r,h)} = \sum_{o \in EU} L_{(oo,a,g,r,h,o)} * b\_pref_{(oo,a,g,r,h,o)} \qquad \text{(E.11)}$$

$oo \in EUN$; $a \in AGE$; $g \in GEN$; $r \in RACE$; $h \in H3$ and $o \in EU$.[9]

[8] We restrict the supply of labour between the different occupations included in SAGE-H. For example, we allow people employed in unskilled occupations to offer to occupations with a similar skill profile. They can not offer to occupations requiring skilled labour. Hence, we ensure that people can only offer to occupations with a similar skills profile.

[9] $EU$ is a set for all employment and unemployment employment statusses.

Once the offer from each category of labour supply to each employment activity is determined by Equations (E.7), the planned labour supply to each employment activity is the sum of the offers to the activity $o$ by people from all categories of labour supply. The levels form of this equation is presented in (E.12) and the percentage-change form in (E.13):

$$LS_{(o)} = \sum_{oo \in EUN} \sum_{a \in AGE} \sum_{g \in GEN} \sum_{r \in RACE} \sum_{h \in H3} L_{(oo,a,g,r,h,o)} \qquad \text{(E.12)}$$

$oo \in EUN$ , $a \in AGE$ ; $g \in GEN$ ; $r \in RACE$ ; $h \in H3$ ; $o \in OCC$ .

$$LS_{(o)} * ls_{(o)} = \sum_{oo \in EUN} \sum_{a \in AGE} \sum_{g \in GEN} \sum_{r \in RACE} \sum_{h \in H3} L_{(oo,a,g,r,h,o)} * l_{(oo,a,g,r,h,o)} \qquad \text{(E.13)}$$

where

- $L_{(oo,a,g,r,h,o)}$ is the number of people of age $a$, gender $g$, race $r$ and HIV stage $h$, moving from the category $oo$ to employment activity $o$. $l_{(oo,a,g,r,h,o)}$ is the corresponding percentage-change variable; and
- $LS_{(o)}$ is the total number of people moving to employment activity $o$. $ls_{(o)}$ is the corresponding percentage-change variable.

### 4.2. The role of the preference variable

The offers matrix shows how people, given their age, gender, race and HIV stage characteristics, offer their labour from category $c$ to activity $a$. The strength of these offers depends on preferences. The preference variable fulfils a dual role. The first is via their initial setting in the database. When the offers matrix is constructed, we assign values reflecting people's preference to move to a different occupation.

- By setting $B(c,a)$ at relatively high values, we ensure that people who worked in occupation $o$ in year $t-1$ strongly offer their labour supply to the same occupation $o$ in year $t$. This reflects the empirical regularity that people tend to remain in the same occupation from year to year.
- By setting $B(c,a)$ at suitably positive values we ensure that people offer their labour only to occupations compatible with their skills.

- By setting $B(c,a)$ at moderately large values, where the category is short-term unemployment and the activity is long-term unemployment, we introduce a mild discouraged worker effect for those who are short-term unemployed.
- By setting $B(c,a)$ at large values when the characteristics of the category $c$ and activity $a$ are the same and when both category and activity are long-term unemployed, we introduce a stronger discouraged worker effect.
- By setting $B(c,a)$ at zero when the category is either short or long-term unemployment and the activity is short-term unemployment, we ensure that no one can stay in the short-term unemployment activity for more than one year or move from long-term unemployment to short-term unemployment.
- By setting $B(c,a)$ at zero if the $a,g,r,h$ characteristics of the category differ from the $a,g,r,h$ characteristics of the activities, we ensure that people in categories with $a,g,r,h$ characteristics only offer labour to activities with similar $a,g,r,h$ characteristics. For example, I do not allow people allocated to the "male" gender category to offer to a "female" gender activity.
- By setting $B(c,a)$ at zero when the category includes an HIV status of Stage 4, we ensure that no one is employed or unemployed with this HIV status. They offer only to the $PDL$ activity.

A full description of the preference variables is included in Roos (2013) Section 4. Specifically, we set out there assumptions specifying (1) the proportion of people who would like to offer their labour to a different occupation; (2) the proportion of people voluntarily moving from an employment category to the short-term unemployment activity; and (3) the strength of offers given a person's HIV status.

The second role of the preference variable is to model a change in preferences to work in a specific occupation. In policy simulations, this is achieved by introducing a shock to $b\_pref$ (see (E.7) above).

## 5. DESCRIPTION OF EVERYONE'S ACTIVITY: FLOWS FROM CATEGORIES TO ACTIVITIES

Figure 2 is an illustration of all model flows from categories to activities. In this section, I describe the flows in Figure 2 as a starting point to introducing the equations that explain these flows in Section 6. Recall that Section 4 addressed the theory explaining planned labour market flows. However, the model allows for the

possibility of short-run wage stickiness.[10] This implies that gaps between labour supply and demand in activity $o$ are allowed in this model. I therefore need to specify which offers to employment activity $o$ are accepted and which activities are undertaken by those whose offers to employment activities are not accepted. The following labour-supply flows (see Figure 2) are modelled:

$CAT_{EMP}^{1,a,g,r,h}$ to $CAT_{EMP}^{z,a,g,r,h}$

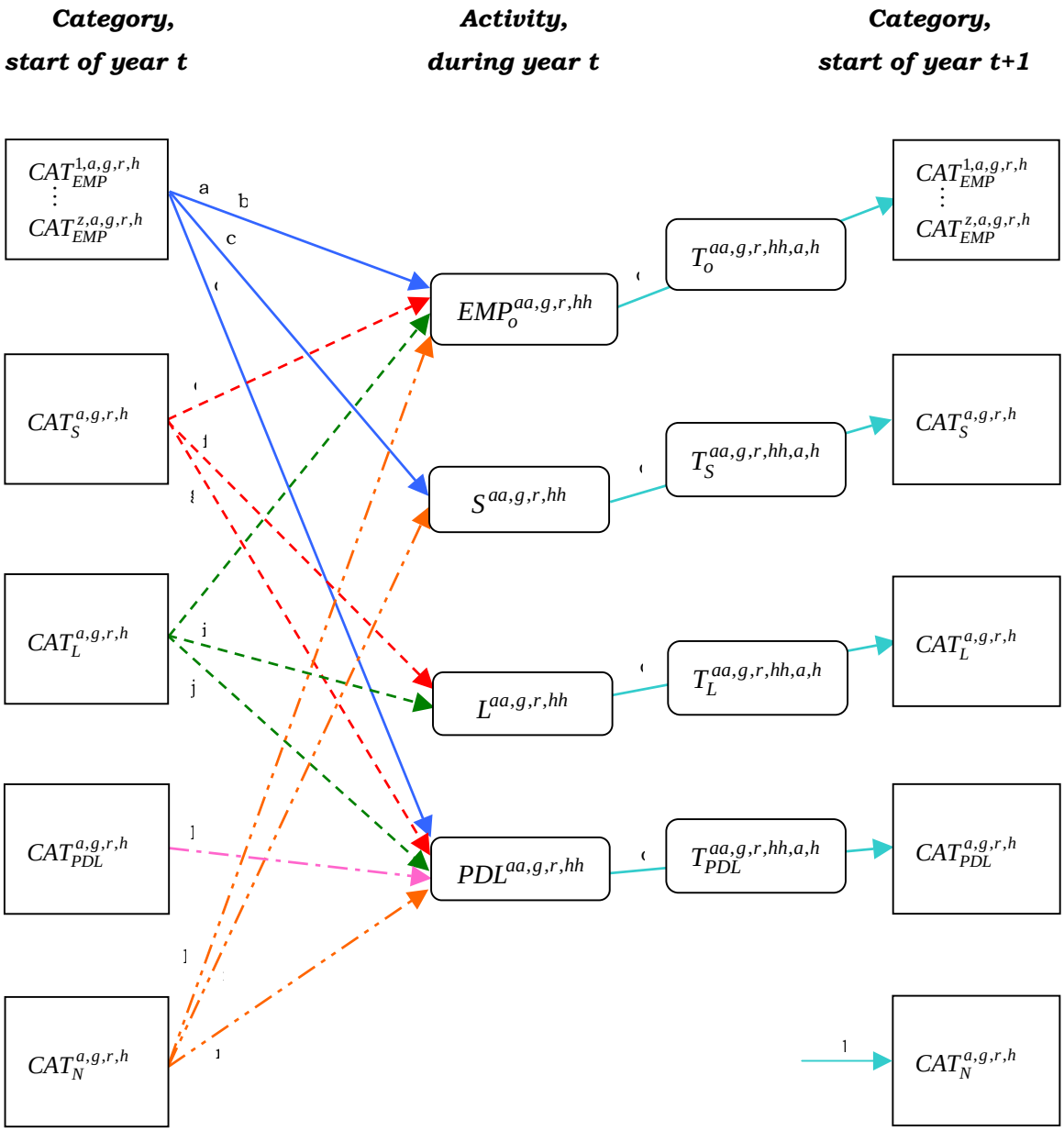
This category describes people employed in year $t-1$ for a given age $a$, gender $g$, race $r$ and HIV stage $h$. In year $t$, people in this category can return to the same occupation $o$ (flow a); move to a different occupation (flow b); move to short-term unemployment (flow c); or move to the PDL activity (flow d). Flow (d) only applies to a proportion of people who were employed and in HIV Stage 3 during year $t-1$. A proportion of people in HIV Stage 3 will move to HIV Stage 4 at the beginning of year $t$. They are grouped in an employment category with an HIV stage of Stage 4. We do not allow anyone with a Stage 4 status to be employed or unemployed. Instead, they will move to the PDL activity in year $t$.

$CAT_{S}^{a,g,r,h}$

This category describes people of a given age $a$, gender $g$, race $r$ and HIV stage $h$, who were not employed in year $t-1$, but employed in year $t-2$. In year $t$, people in this category can move to employment activity $o$ (flow e); move into the long-term unemployment activity (flow f); or move to the "permanently departed from the labour force" activity (flow g). Flow (g) only refers to those who were unemployed and in HIV Stage 3 during year $t-1$. A proportion of those in HIV Stage 3 change their health status to HIV Stage 4 at the beginning of year $t$. They are grouped into the short-term unemployment category, given their $a,g,r$ characteristics, with a Stage 4 status. Again, adults with a Stage 4 status move to the PDL activity in year $t$.

[10] See Section 7.

**Figure 2. Flows between categories at the start of year $t$ and activities during year $t$**




$CAT_L^{a,g,r,h}$ This category describes people of a given age $a$, gender $g$, race $r$ and HIV stage $h$, who were unemployed in years $t-1$ and $t-2$. In year $t$, people in this category can move to employment activity $o$ (flow h); remain in the long-term unemployment activity (flow i); or move to the "permanently departed from the labour force" activity (flow j). Flow (j) only refers to those who were long-term unemployed in year $t-1$ with an HIV Stage 3 status. A proportion of those in HIV Stage 3 will move to HIV Stage 4 at the beginning of year $t$. Given their $a,g,r$ characteristics, they are grouped into the long-term unemployment

category with a Stage 4 status. Adults with a Stage 4 status move to the $PDL$ activity in year $t$.

$CAT_{PDL}^{a,g,r,h}$ This category describes people of a given age $a$, gender $g$, race $r$ and HIV stage $h$, who were "permanently departed from the labour force" in year $t-1$. They are not considered part of the labour force and do not move to any employment or unemployment activities. PDL also includes all persons with a Stage 4 status. In year $t$, people in this category only move into the PDL activity (flow k).

$CAT_{N}^{a,g,r,h}$ The final describes people of a given age $a$, gender $g$, race $r$ and HIV stage $h$, who enter the labour force for the first time. In year $t$, people in this category can move to employment activity $o$ (flow l); move to the short-term unemployment activity (flow m); or move to the "permanently departed from the labour force" activity (flow n). Flows (l) and (m) describe people who are HIV negative or HIV positive and in Stages 1, 2 or 3. New entrants in Stage 4 move to the PDL activity (flow n).

Flows (a) to (n) in Figure 2 are derived from equations explained in Section 6. Flow (o) is derived via Equation (E.3) explained in Section 3.

## 6. DETERMINING EVERYONE'S ACTIVITY: WHO SECURES EMPLOYMENT AND WHAT HAPPENS TO THOSE WHO DON'T

In linking the categories at the start of year $t$ to the activities in year $t$, we specify a flow from each category $c$ to each activity $a$, $H_{(c,a)}$ in year $t$. Notice that $H_{(c,a)}$ includes flows from all categories (employed, unemployed, new entrant and $PDL$) to all activities (employed, unemployed and $PDL$).

### 6.1. Non-diagonal flows from the employment, unemployment and new entrant categories, to employment across all occupations (Figure 2; flows a, e, h, l)

In deriving the non-diagonal flows from categories $oo$ to employment activity $o$, we

begin by defining occupation-specific vacancies $o$.[11] Occupation-specific vacancies are defined as employment in activity $o$ minus jobs filled by incumbents. In levels form the equation is:

$$VAC_{(o)} = LD_{(o)} - H_{(o,o)} \text{ for } o \in OCC \qquad \text{(E.14)}$$

The percentage-change form of (E.14) is:

$$VAC_{(o)} * vac_{(o)} = LD_{(o)} * x1lab\_i_{(o)} - H_{(o,o)} * h_{(o,o)} \qquad \text{(E.15)}$$

where
- $VAC_{(o)}$ is the level of the number of occupation-specific vacancies and $vac_{(o)}$ is the corresponding percentage-change variable;
- $LD_{(o)}$ is the level of the number of people in employment activity $o$ and $x1lab\_i_{(o)}$ is the corresponding percentage-change variable; and
- $H_{(o,o)}$ is actual flows from occupation $o$ to occupation $o$, where $o = o$, summed across age, gender, race and HIV stage. These are the "diagonal flows". $h_{(o,o)}$ is the percentage change in the number of people staying in the same occupation, i.e. the percentage change in $H_{(o,o)}$.

Equation (E.16) specifies that jobs in employment activity $o$ given to non-incumbents are proportional to the vacancies in employment activity $o$ and the share of category $c$ in the supply of labour to activity $o$ from people outside category $o$. For example, if people in category $c$ account for 10 per cent of the people outside category $o$ who want jobs in employment activity $o$, then people in category $c$ fill 10 per cent of the vacancies in $o$.

In levels form, the equation for non-diagonal flows is:

$$H_{(c,a,g,r,h,o)} = VAC_{(o)} * \left[ \frac{L_{(c,a,g,r,h,o)}}{OfferTo_{(o)}} \right] \qquad \text{(E.16)}$$

$c \in EUN$; $a \in AGE$; $g \in GEN$; $r \in RACE$; $h \in H3$ and $o \in OCC$.

where
- $H_{(c,a,g,r,h,o)}$ is the level of the number of non-diagonal flows of people of

[11] An example of non-diagonal flows is all offer of people to occupation 1 from all unemployment, new entrant and all other employment (except occupation 1) categories.

age $a$, gender $g$, race $r$ and HIV stage $h$, moving from category $c$ to activity $o$ where $c \neq o$;

- $VAC_{(o)}$ is the level of the number of occupation-specific vacancies; and
- $\frac{L_{(c,a,g,r,h,o)}}{OfferTo_{(o)}}$ is the proportion of people moving from category $c$, age $a$, gender $g$, race $r$ and HIV stage $h$ to occupation $o$ from people outside category $c$. $L_{(c,a,g,r,h,o)}$ is the level of the number of labour flows from categories $c$ to occupation $o$. $OfferTo_{(o)}$ is the level of the number of labour flows to occupation $o$ from outside $o$, summed across all age, gender, race and HIV status.

The percentage-change form of Equation (E.16) is:

$$h_{(c,a,g,r,h,o)} = vac_{(o)} + l_{(c,a,g,r,h,o)} - offerto_{(o)} \qquad \text{(E.17)}$$

where

- $h_{(c,a,g,r,h,o)}$ is the percentage change in the number of non-diagonal flows from categories $c$ to employment activity $o$, for all age $a$, gender $g$, race $r$ and HIV stage $h$;
- $vac_{(o)}$ is the percentage change in occupation-specific vacancies as determined in Equation (E.18);
- $l_{(c,a,g,r,h,o)}$ is the percentage change in offers from category $c$ to employment activity $o$, for all age $a$, gender $g$, race $r$ and HIV stage $h$; and
- $offerto_{(o)}$ is the percentage change in offers to employment activity $o$ from outside $o$.

I further assume that:

$$h_{(c,a,g,r,Stage4,o)} = 0 \qquad \text{(E.18)}$$

Equation (E.18) states that people in category $c$, age $a$, gender $g$, race $r$ with an HIV Stage 4 status, do not move into employment activity $o$. As mentioned before, people in Stage 4 do not form part of the labour force and are considered too ill to offer their labour to employment activities (see Section 1.2). They move into the PDL activity, which is determined via (E.32).

### 6.2. Diagonal flows from occupation $o$ to occupation $o$ (Figure 2; flow b)

Incumbents are defined as the number of people in employment category $o$ who remain in employment activity $o$. They are determined as the number of people in employment category $o$ less the number who move *out* of employment category $o$. People can move *out* of employment category $o$ to a different occupation or to short-term unemployment. In levels form, the equation is:

$$H_{(o,a,g,r,h,o)} = CAT_{(o,a,g,r,h)} - \sum_{o \neq oo} H_{(o,a,g,r,h,oo)} \qquad \text{(E.19)}$$

$o \in OCC$; $a \in AGE$; $g \in GEN$; $r \in RACE$; $h \in H3$ and $oo \in EU$.

The percentage-change form of Equation (E5.19) yields:

$$H_{(o,a,g,r,h,o)} * h_{(o,a,g,r,h,o)} = CAT_{(o,a,g,r,h)} * cat_{(o,a,g,r,h)} - \mathrm{H}_{(o,a,g,r,h,oo)} * h_{(o,a,g,r,h,oo)} \qquad \text{(E.20)}$$

where
- $H_{(o,a,g,r,h,o)}$ is the level of the flow from employment categories $o$ to employment activities $o$, for all age $a$, gender $g$, race $r$ and HIV stage $h$. $h_{(o,a,g,r,h,o)}$ is the corresponding percentage-change variable;
- $CAT_{(o,a,g,r,h)}$ is the level of the number of people in employment category $o$, gender $g$ and race $r$, allocated to age $a$ and HIV stage $h$ at the start of year $t$; and
- $H_{(o,a,g,r,h,oo)}$ is the flow of people of age $a$, gender $g$, race $r$ and HIV stage $h$, moving from employment category $o$ into activity $oo$, where $oo$ is another occupation or short-term unemployment $S$. $h_{(o,a,g,r,h,oo)}$ is the corresponding percentage-change variable.

Again, it is assumed that:

$$h_{(o,a,g,r,Stage4,o)} = 0 \qquad \text{(E.21)}$$

### 6.3. Flows from all employment categories $o$ to short-term unemployment activity $S$ (Figure 2; flow c)

Equation (E.22) determines the number of people moving from employment category $o$

to short-term unemployment activity $S$. The equation in levels form is:

$$H_{(o,a,g,r,h,S)} = L_{(o,a,g,r,h,S)} + \mu * CAT_{(o,a,g,r,h)} \quad \text{(E.22)}$$

$o \in OCC$; $a \in AGE$; $g \in GEN$; $r \in RACE$ and $h \in H3$.

here
- $H_{(o,a,g,r,h,S)}$ is the flow of people of age $a$, gender $g$, race $r$ and HIV stage $h$, moving from employment category $o$ into short-term unemployment activity $S$;
- $L_{(o,a,g,r,h,S)}$ is the level of the number of people of age $a$, gender $g$, race $r$ and HIV stage $h$, voluntarily moving from employment category $o$ into unemployment activity $S$; and
- $\mu * CAT_{(o,a,g,r,h)}$ is the level of the number of people of age $a$, gender $g$, race $r$ and HIV stage $h$, involuntarily moving from employment category $o$ to unemployment activity $S$. $\mu$ is the percentage of people in each category $o$ who are fired each year.

The percentage-change form of Equation (E.22) is:

$$H_{(o,a,g,r,h,S)} * h_{(o,a,g,r,h,S)} = L_{(o,a,g,r,h,S)} * l_{(o,a,g,r,h,S)} + \mu * CAT_{(o,a,g,r,h)} * cat_{(o,a,g,r,h)} \quad \text{(E.23)}$$

where
- $h_{(o,a,g,r,h,S)}$ is the percentage change in the number of people of age $a$, gender $g$, race $r$ and HIV stage $h$, moving from employment category $o$ to unemployment activity $S$;
- $l_{(o,a,g,r,h,S)}$ is the percentage change in the number of people of age $a$, gender $g$, race $r$ and HIV stage $h$, voluntarily moving from employment category $o$ to unemployment activity $S$; and
- $cat_{(o,a,g,r,h)}$ is the percentage-change variable in the number of people in each category $o$, age $a$, gender $g$, race $r$ and HIV stage $h$.

Note that in linearising (E.23), we treat $\mu$ as a parameter.

Again, it is assumed that:

$$h_{(o,a,g,r,Stage4,S)} = 0 \quad \text{(E.24)}$$

### 6.4. Flows from short and long-term unemployment categories $uu$ to long-term unemployment activities $L$ (Figure .2; flows f and i)

Equation (E.24) determines the flow from all unemployment categories $uu$ to unemployment activity $L$. In levels form this flow is:

$$H_{(uu,a,g,r,h,L)} = CAT_{(uu,a,g,r,h)} - \sum_{o \in OCC} H_{(uu,a,g,r,h,o)} \qquad \text{(E.25)}$$

$uu \in UNEMP$;[12] $a \in AGE$; $g \in GEN$; $r \in RACE$; $h \in H3$ and $o \in OCC$.

The percentage-change form of Equation (E.25) is:

$$H_{(uu,a,g,r,h,L)} * h_{(uu,a,g,r,h,L)} = \left[ CAT_{(uu,a,g,r,h)} * cat_{(uu,a,g,r,h)} - \sum_{o \in OCC} H_{(uu,a,g,r,h,o)} * h_{(uu,a,g,r,h,o)} \right] \qquad \text{(E.26)}$$

where

- $H_{(uu,a,g,r,h,L)}$ is the flow of people of age $a$, gender $g$, race $r$ and HIV stage $h$, moving from unemployment category $uu$ into unemployment activity $L$. $h_{(uu,a,g,r,h,u)}$ is the corresponding percentage-change variable;
- $CAT_{(uu,a,g,r,h)}$ is the level of the number of people in unemployment category $uu$, gender $g$ and race $r$, allocated to age $a$ and HIV stage $h$ at the start of the year. $cat_{(uu,a,g,r,h)}$ is the corresponding percentage-change variable; and
- $\sum_{o \in OCC} H_{(uu,a,g,r,h,o)}$ is the flow of people of age $a$, gender $g$, race $r$ and HIV stage $h$, moving from unemployment category $uu$ into occupation $o$. $h_{(uu,a,g,r,h,o)}$ is the corresponding percentage-change variable. This flow is determined in (E.20).

In determining flows from the unemployment category $uu$ to the unemployment activity $L$, we ensure that:

- those who were short-term unemployed and fail to secure employment flow to the long-term unemployed activity; and

[12] *UNEMP* is a set including short-run (S) and long-term (L) unemployment categories.

- those who were long-term unemployed and fail to secure employment remain in the long-term unemployment activity.

Again, I assume that:

$$h_{(uu,a,g,r,Stage4,L)} = 0 \qquad \text{(E.27)}$$

### 6.5. Flows from the new entrant category $N$ to short-term unemployment activity $S$ (Figure 2; flow m)

New entrants who fail to secure employment in employment activity $o$ move to short-term unemployment activity $S$. This flow is specified as:

$$N_{(N,a,g,r,h,S)} = CAT_{(N,a,g,r,h)} - \sum_{o \in OCC} H_{(N,a,g,r,h,o)} \qquad \text{(E.28)}$$

$a \in AGE$; $g \in GEN$; $r \in RACE$; $h \in H3$ and $o \in OCC$.

The percentage-change form of Equation (E.28) is:

$$H_{(N,a,g,r,h,S)} * h_{(N,a,g,r,h,S)} = CAT_{(N,a,g,r,h)} * cat_{(N,a,g,r,h)} - \sum_{o \in OCC} H_{(N,a,g,r,h,o)} * h_{(N,a,g,r,h,o)} \qquad \text{(E.29)}$$

where
- $H_{(N,a,g,r,h,S)}$ is the flow of people of age $a$, gender $g$, race $r$ and HIV stage $h$, moving from new entrant category $N$ into unemployment activity $S$. $h_{(N,a,g,r,h,S)}$ is the corresponding percentage-change variable;
- $CAT_{(N,a,g,r,h)}$ is the level of the number of people in new entrant category $N$ of age $a$, gender $g$, race $r$ and HIV stage $h$ at the start of the year. $cat_{(N,a,g,r,h)}$ is the corresponding percentage-change variable; and
- $\sum_{o \in OCC} H_{(N,a,g,r,h,o)}$ is the level of the number of new entrants of age $a$, gender $g$, race $r$ and HIV stage $h$, who secure employment in occupation $o$. $h_{(N,a,g,r,h,o)}$ is the corresponding percentage-change variable, determined via (E.20).

Again I assume that:

$$h_{(N,a,g,r,Stage4,S)} = 0 \qquad \text{(E.30)}$$

### 6.6. Flows from all employment, unemployment, new entrant and *PDL* categories to *PDL* activities (Figure 2; flows d, g, j, k, n)

Equation (E.31) determines the number of people from all categories moving into the "permanently departed from the labour force" (PDL) activity. There are two flows into the PDL activity. The first flow is determined via the first variable on the RHS of (E.34). This variable shows the number of people in the PDL category at the start of year $t$. As mentioned in Section 1.4 and footnote 7, we assume that people in the *PDL* category are not part of the labour force and do not move into any employment or unemployment activity. Once a person is PDL they remain PDL, that is, they simply move from the PDL category into the PDL activity. In Figure 2 this is flow (n). The second variable on the RHS of (E.31) is the number of people, in any employment, unemployment or new entrant (*EUN*) category $o$, with a Stage 4 HIV status. Equations (E.18), (E.21), (E.24), (E.27) and (E.30) prohibit the flow from any *EUN* category to any employment and unemployment (*EU*) activity if the person has an HIV status of Stage 4. People in Stage 4 are too ill to participate in any productive activity, and thus move into the PDL activity (see Section 1.2). These are flows (d), (g), (j) and (n) in Figure 2.

$$H_{(oo,a,g,r,h,PDL)} = CAT_{(PDL,a,g,r,h)} + CAT_{(oo,a,g,r,Stage4)} \qquad \text{(E.31)}$$

$oo \in EUNP$;[13] $a \in AGE$; $g \in GEN$; $r \in RACE$; $h \in STG$ and $o \in EUN$.

The percentage-change form of (E.31) is:

$$H_{(oo,a,g,r,h,PDL)} * h_{(oo,a,g,r,h,PDL)} = CAT_{(PDL,a,g,r,h)} * cat_{(PDL,a,g,r,h)} + CAT_{(oo,a,g,r,Stage4)} * cat_{(oo,a,g,r,Stage4)} \qquad \text{(E.32)}$$

where
- $H_{(oo,a,g,r,h,PDL)}$ is the flow of people of age $a$, gender $g$, race $r$ and HIV stage $h$, moving from all *EUNP* category *oo* into the PDL activity.

[13] *EUNP* is a set of all employment, unemployment, new entrant and "permanently departed from the labour force" (PDL) categories.

$h_{(oo,a,g,r,h,PDL)}$ is the corresponding percentage-change variable; and

- $CAT_{(PDL,a,g,r,h)}$ is the level of the number of people in the PDL category by gender $g$ and race $r$, allocated to age $a$ and HIV stage $h$ at the start of the year. $cat_{(PDL,a,g,r,h)}$ is the corresponding percentage-change variable; and
- $CAT_{(oo,a,g,r,Stage4)}$ is the level of the number of people in category $oo$, gender $g$, race $r$ and HIV status of Stage 4, allocated to age $a$ at the start of the year. $cat_{(oo,a,g,r,Stage4)}$ is the corresponding percentage-change variable.

## 7. WAGE ADJUSTMENT EQUATIONS REFLECTING LABOUR DEMAND AND SUPPLY

In year-on-year simulations we might begin by envisaging two possible extreme representations of how the labour market adjusts to a shock:

- employment adjusts while the real wage remains unchanged from basecase level; or
- real wages adjust while employment remains unchanged from its basecase level.

The former, sticky-wage, environment might be a reasonable representation of labour-market adjustment immediately following some policy shock, while the latter might be a reasonable representation of the long-term post-shock labour market. In SAGE-H we recognise a mechanism operational during policy simulations, which moves between these two short-term and long-term representations of labour-market functioning.

In SAGE-H we adopt the wage adjustment process described in Dixon and Rimmer (2008). In policy simulations, we assume that wage rates adjust according to Equation (E.33):

$$\frac{ATW^{policy}_{t(o)}}{ATW^{base}_{t(o)}} - \frac{ATW^{policy}_{t-1(o)}}{ATW^{base}_{t-1(o)}} = \alpha\left[\frac{LD^{policy}_{t(o)}}{LD^{base}_{t(o)}} - \frac{LS^{policy}_{t(o)}}{LS^{base}_{t(o)}}\right] \text{ for } o \in OCC \qquad \text{(E.33)}$$

where

- $ATW^{base}_{t(o)}$ and $ATW^{policy}_{t(o)}$ are the wage in the baseline and policy

simulation in year $t$;

- $ATW^{base}_{t-1(o)}$ and $ATW^{policy}_{t-1(o)}$ are the wage in the baseline and policy simulation in year $t-1$;
- $LD^{base}_{t(o)}$ and $LD^{policy}_{t(o)}$ are the demand for labour in the baseline and policy simulation in year $t$; and
- $LS^{base}_{t(o)}$ and $LS^{policy}_{t(o)}$ are the labour supply in the baseline and policy simulation in year $t$.

In the policy simulation, $ATW^{base}_{t(o,r)}$, $LD^{base}_{t(o,r)}$ and $LS^{base}_{t(o,r)}$ are exogenously determined and equal to their baseline forecast values. $ATW^{policy}_{t(o,r)}$, $LD^{policy}_{t(o,r)}$ and $LS^{policy}_{t(o,r)}$ evolve endogenously. Equation (E5.33) states that if, for example, a policy causes a larger percentage deviation in labour supply than labour demand, then there will be a decrease between years $t-1$ and $t$ in the deviation in occupation $o's$ real after-tax wage rate. In other words, in periods in which a policy elevates labour supply relative to labour demand, real wages fall relative to their basecase values. However, short-term wage rates are sticky, so the initial effect of the increase in labour supply is to increase unemployment while only slowly decreasing the wage. The positive parameter reflects the speed of adjustment in the labour market, that is, it controls the response of after-tax-real wage to gaps between labour supply and demand.

In SAGE-H, (E.34) determines the after-tax real wage rate for all occupations $o$. This wage is important in determining labour supply. For determining labour demand, before-tax wage is important. The real after-tax wage is linked to the real before-tax wage via Equation (E.34).

$$ATW_{(o)} = BTW_{(o)} * (1 - TAX\_LAB) \text{ for } o \in OCC \qquad \text{(E.34)}$$

where
- $ATW_{(o)}$ is the occupation-specific after-tax wage and $BTW_{(o)}$ is the occupation-specific before-tax wage;
- $TAX\_LAB$ is the rate of tax on labour income.

The percentage-change form of Equations (E.35) is:

$$atw_{(o)} = btw_{(o)} - \frac{TAX\_LAB}{(1 - TAX\_LAB)} * tax_{(lab)} \text{ for } o \in OCC \qquad \text{(E.35)}$$

where $atw_{(o)}$, $btw_{(o)}$ and $atw_{(u)}$ are the percentage-change variables for the corresponding levels variables in (E.34).

## 8. ADDITIONAL EQUATIONS ACCOMMODATING HIV DATA DURING THE BASECASE FORECAST SIMULATION

I specify additional equations, not part of the core labour-market theory, to calculate the percentage change in the number of new HIV cases at the start of year $t$. At the start of year $t$, the number of people of age $a$, gender $g$ and race $r$ who become newly infected with HIV is determined via (E.36):

$$New\_HIV_{(a,g,r)} = \sum_{o \in EUP} \sum_{aa \in AGE} ACT\_L_{(o,aa,g,r,HIVn)} * T_{(o,aa,g,r,HIVn,a,Stage1)} \qquad \text{(E.36)}$$

$o \in EUP$; $aa \in AGE$; $g \in GEN$; $r \in RACE$ and $a \in AGE$.

where
- $New\_HIV_{(a,g,r)}$ is the level of the number of new HIV cases by age $a$, gender $g$ and race $r$;
- $ACT\_L_{(o,aa,g,r,HIVn)}$ is the level of the number of people in labour-market activity $o$ of gender $g$ and race $r$, who were in age $aa$ and HIV negative in year $t-1$; and
- $T_{(o,aa,g,r,HIVn,a,Stage1)}$ is the transition from age $aa$ to age $a$ and from HIV status HIV negative to Stage 1, for all labour-market activities $o$, gender $g$ and race $r$.

Equation (E.36) states that at the start of year $t$, the age, gender and race-specific number of new HIV cases are those who were HIV negative in year $t-1$ and survive to year $t$ and become HIV positive. The linearised form of (E.36) is:

$$New\_HIV_{(a,g,r)} * new\_hiv_{(a,g,r)} = \sum_{o \in EUP} \sum_{aa \in AGE} \left\{ ACT\_L_{(o,aa,g,r,HIVn)} * \left( T_{(o,aa,g,r,HIVn,a,Stage1)} * act\_l_{(o,aa,g,r,HIVn)} + 100 * d\_transit_{(o,aa,g,r,HIVn,a,Stage1)} \right) \right\} \qquad \text{(E.37)}$$

$o \in EUP$; $aa \in AGE$; $g \in GEN$; $r \in RACE$ and $a \in AGE$.

where
- $new\_hiv_{(a,g,r)}$ is the percentage change in the number of new HIV cases by age $a$, gender $g$ and race $r$;
- $act\_l_{(o,aa,g,r,HIVn)}$ is the percentage change in the number of people in labour-market activity $o$ of gender $g$ and race $r$, who were in age $aa$ and HIV negative in year $t-1$; and
- $d\_transit_{(o,aa,g,r,HIVn,a,Stage1)}$ is the ordinary change in the transition from age $aa$ to age $a$ and from HIV status HIV negative to Stage 1, for all labour-market activities $o$, gender $g$ and race $r$.

In the baseline simulation, I wish to impose on the model an independent forecast for the annual number of new HIV cases.[14] To exogenously determine the percentage change in the number of new HIV cases in the baseline simulation, we endogenise the ordinary change in the transition from HIV negative to Stage 1. In (E.38) we set the transition rate equal to a shift variable. The levels form of the transition rate is:

$$T_{(o,aa,g,r,HIVn,a,Stage1)} = FF\_T_{(aa,g,r,HIVn,a,Stage1)} * FFF\_T_{(o,aa,g,r,HIVn,a,Stage1)} \qquad \text{(E.38)}$$

$o \in EUP$; $aa \in AGE$; $g \in GEN$; $r \in RACE$ and $a \in AGE$.

where
- $T_{(o,aa,g,r,HIVn,a,Stage1)}$ is the transition rate from age $aa$ to age $a$ and from HIV negative to Stage 1, for all labour-market activities $o$, gender $g$ and race $r$; and
- $FF\_T_{(aa,g,r,HIVn,a,Stage1)}$ is a shift variable that allows a person to change their age from $aa$ to $a$ and HIV status from HIV negative to Stage 1, for all gender $g$ and race $r$. $FFF\_T_{(o,aa,g,r,HIVn,a,Stage1)}$ is a shift variable that allows for the same change as $FF\_T$ and is specific to all categories $o$.

The ordinary-change form of (E.38) is:

$$d\_transit_{(o,aa,g,r,hh,a,h)} = \left[\frac{T_{(o,aa,g,r,hh,a,h)}}{100}\right] * \left[ff\_d\_transit_{(aa,g,r,hh,a,h)} + fff\_d\_transit_{(o,aa,g,r,hh,a,h)}\right] \qquad \text{(E.39)}$$

$o \in EUP$; $aa \in AGE$; $g \in GEN$; $r \in RACE$, $hh \in STG$, $a \in AGE$ and $h \in STG$.

[14] See Chapter 8, Section 8.2 for a description of the exogenous variables in the baseline simulation.

where
- $d\_transit_{(o,aa,g,r,hh,a,h)}$ is the ordinary change in the transition from age $aa$ to age $a$ and HIV stage from $hh$ to $h$, for all labour-market activities $o$, gender $g$ and race $r$; and
- $ff\_d\_transit_{(aa,g,r,hh,a,h)}$ is a change in the shift variable that allows a person to change their age from $aa$ to $a$ and HIV stage from $hh$ to $h$, for all gender $g$ and race $r$. $fff\_d\_transit_{(o,aa,g,r,hh,a,h)}$ allows for the same change but is category specific.

Three factors regarding the ordinary change variables included in (E5.39) are noted. Firstly, in terms of age, we allow people to move from age $aa$ in year $t-1$ to age $a$ at the start of year $t$. A small proportion of people will not survive, that is, they will not move from age $aa$ to age $a$. Secondly, to accommodate the percentage change in new HIV cases we only consider changes in HIV status from HIV negative to Stage 1. All other changes in health status remain unchanged.[15] Thirdly, if the transition from HIV negative to Stage 1 changes, we must also adjust the transition rate for those who remain HIV negative.[16] If the probability of remaining HIV negative does not adjust in light of a change in the transition from HIV negative to Stage 1, the sum of the transitions across age $aa$ and HIV status $hh$ will not equal their initial setting.[17] We assume that the change in transition allowing people to remain HIV negative is offset by the change in the transition of HIV negative to Stage 1. This is achieved via (E.40):

$$d\_transit_{(o,aa,g,r,HIVn,a,HIVn)} = - d\_transit_{(o,aa,g,r,HIVn,a,Stage1)} \quad \text{(E.40)}$$

where
- $d\_transit_{(o,aa,g,r,HIVn,a,HIVn)}$ is an ordinary-change variable that allows people, given their labour-market activity $o$, gender $g$ and race $r$, to change their age from $aa$ to $a$ and remain HIV negative; and
- $d\_transit_{(o,aa,g,r,HIVn,a,Stage1)}$ is an ordinary-change variable that allows people, given their labour-market activity $o$, gender $g$ and race $r$, to change

[15] We keep the transitions from Stage 1 to Stage 2, Stage 2 to Stage 3 and from Stage 3 to Stage 4 as well as the transitions of staying in Stage 1, Stage 2, Stage 3 and Stage 4 at their initial level.
[16] We assume that there is no change in age and gender-specific death rates. If the transition of moving from HIV negative to Stage 1 falls, it means that fewer people become HIV positive and more people remain HIV negative. We assume that the change in the transition from HIV negative to Stage 1 is completely offset in the transition from HIV negative to HIV negative. For example, if the transition rate in moving from HIV negative to Stage 1 falls by 1 percentage point then the transition of remaining HIV negative increases by 1 percentage point.
[17] Assuming no change in age and gender-specific death rates, $o,g,r$ -specific transitions summed across age $aa$ and HIV status $hh$ should always sum to 1 minus the death rate. The construction of the transition matrix is described in Roos (2013).

their age from $aa$ to $a$ and HIV status from HIV negative to Stage 1. This ordinary change is determined via (E.39).

If the variables on the RHS of (E.39) remain exogenous and unshocked, $d\_transit$ remains at its initial setting in the base year. However, when exogenously determined age, gender and race-specific data on the number of new HIV cases are introduced, the age, gender and race-specific shift variable allowing people to change their age from $aa$ to $a$ and HIV status from HIV negative to Stage 1 $\left(ff\_d\_transit_{(o,aa,g,r,HIVn,a,Stage1)}\right)$ is endogenised. This allows the ordinary change in transition of $o,g,r$-specific adults to change their age from $aa$ to $a$ and HIV status from HIV negative to Stage 1 $\left(d\_transit_{(o,aa,g,r,HIVn,a,Stage1)}\right)$. This in turn will adjust the ordinary change in the transition of $o,g,r$-specific people moving from age $aa$ to $a$ and HIV status from HIV negative to HIV negative $\left(d\_transit_{(o,aa,g,r,HIVn,a,HIVn)}\right)$ in (E.40).

## 9. ADDITIONAL EQUATIONS ACTIVATED DURING THE POLICY SIMULATION

In some policy simulations it can be useful to impose exogenously determined deviations in the number of new HIV cases from its basecase level. For example, Equation (E5.37) is the levels form of the age, gender and race-specific number of new HIV cases. The percentage deviation in the number of new HIV cases by age, gender and race can be written as:

$$DEV\_NEW\_HIV_{(a,g,r)} = \left(\frac{NEW\_HIV_{(a,g,r)}^{policy}}{NEW\_HIV_{(a,g,r)}^{base}} - 1\right) * 100 \qquad \text{(E.41)}$$

$a \in AGE$ ; $g \in GEN$ ; $r \in RACE$ .

The ordinary change of Equation (E5.41) is:

$$d\_dev\_new\_hiv_{(a,g,r)} = \frac{NEW\_HIV_{(a,g,r)}^{policy}}{NEW\_HIV_{(a,g,r)}^{base}} * \left[new\_hiv_{(a,g,r)} - new\_hiv_{(a,g,r)}^{base}\right] \qquad \text{(E.42)}$$

$a \in AGE$ ; $g \in GEN$ ; $r \in RACE$ .

where
- $DEV\_NEW\_HIV_{(a,g,r)}$ is the deviation in the number of age, gender and race-specific new HIV cases from its basecase level and $d\_dev\_new\_hiv_{(a,g,r)}$ is the ordinary change in the deviation of total new HIV cases;
- $NEW\_HIV^{base}_{(a,g,r)}$ and $NEW\_HIV^{policy}_{(a,g,r)}$ are the level of the number of new HIV cases by age, gender and race in the base forecast and policy simulation; and
- $new\_hiv_{(a,g,r)}$ and $new\_hiv^{base}_{(a,g,r)}$ are the percentage change in the age, gender and race-specific number of new HIV cases in the policy and base forecast.

In the policy simulation, $new\_hiv^{base}_{(a,g,r)}$ is set to the baseline projections and $d\_dev\_new\_hiv_{(a.g.r)}$ is exogenous and adopts the deviation in new HIV cases. Equation (E.42) therefore determines $new\_hiv_{(a,g,r)}$. Via (E.37) the change in the transition rate from HIV negative to Stage 1 is then determined.

## 10. CONCLUDING REMARKS

There are six key components in the labour-market specification. They are:

1. Equations linking the number of people in activity $o$ in year $t-1$ to the number of people in categories $c$ at the start of year $t$.
2. The identification of workforce activities, that is, what people do during the year.
3. Supply of labour by each category $c$ to each activity $a$.
4. Demand for labour for each employment activity $o$. This block of equations are contained in the core SAGE-H model (Dixon and Rimmer, 2002).
5. Wage adjustment in the policy simulation reflecting the gap between labour demand and supply.
6. Activity determination, that is, determination of who gets the jobs and what happens to those who don't secure employment.

In this chapter I described all the key components, except the demand for labour, necessary for the functioning of the labour-market specification. The discussion starts by allocating adults into categories at the start of year $t$ based on common characteristics. These characteristics are age, gender, race, HIV status and labour-

market activity in year $t-1$. At the beginning of year $t$, people decide on which activity to perform during year $t$. They base their decision on occupation-specific relative wages and preferences.

Additional equations were added to the model. The first set of equations is activated during the forecast simulation and accommodates exogenously determined data on the number of new HIV cases. The second set of equations is activated during the policy simulation. During the policy simulation we impose the deviation in the number of new HIV cases from it basecase level. In both the forecast and policy simulations, we allow the transition from HIV negative to Stage 1 and from HIV negative to HIV negative to adjust to accommodate the imposed data.

To implement the theory described in this paper, I need to create a database that represents an initial solution to the levels form of the labour-market specification. The data requirements for the labour-market specification are extensive. A number of detailed matrices describing activities performed in the base year, actual and planned flows as well as the initial setting of the transition matrix are required. See Roos (2013) for a detailed description of the construction of such a database.

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**Appendix A. WHO clinical staging of HIV/AIDS for adults and adolescents with confirmed HIV infection**

The information in this table summarises the clinical staging events as defined by the World Health Organisation (WHO, 2007: 15).

| **Clinical Stage 1** |
|---|
| Asymptomatic<br>Persistent generalised lymphadenopathy<br>Performance scale 1: asymtomatic, normal activity |
| **Clinical Stage 2** |
| Moderate unexplained weight loss[18]<br>(<10% of presumed or measured body weight)I<br>Recurrent respiratory tract infections, sinusitis, tonsillitis, otitis media and pharyngitis)<br>Herpes zoster<br>Angular cheilitis<br>Recurrent oral ulceration<br>Papular pruritic eruptions<br>Seborrhoeic dermatitis<br>Fungal nail infections<br>And/or performance scale 2: symptomatic, normal activity |
| **Clinical Stage 3** |
| Unexplained severe weight loss (>10% of presumed or measured body weight)<br>Unexplained chronic diarrhoea for longer than one month<br>Unexplained persistent fever (above 37.6°C intermittent or constant, for longer than one month)<br>Persistent oral candidiasis<br>Oral hairy leukoplakia<br>Pulmonary tuberculosis (current)<br>Severe bacterial infections (such as pneumonia, empyema, pyomyositis, bone or joint infection, meningitis or bacteraemia)<br>Acute necrotising ulcerative stomatitis, gingivitis or periodontitis<br>Unexplained anaemia (<8 g/dl), neutropaenia (<0.5 × 109 per litre) or chronic thrombocytopaenia (<50 × 109 per litre)<br>And/or performance scale 3: bedridden < 50% of the day during the last month |
| **Clinical Stage 4** |
| HIV wasting syndrome<br>Pneumocystis pneumonia<br>Recurrent severe bacterial pneumonia<br>Chronic herpes simplex infection (orolabial, genital or anorectal of more than one month's duration or visceral at any site)<br>Oesophageal candidiasis (or candidiasis of trachea, bronchi or lungs)<br>Extrapulmonary tuberculosis<br>Kaposi's sarcoma<br>Cytomegalovirus infection (retinitis or infection of other organs)<br>Central nervous system toxoplasmosis<br>HIV encephalopathy<br>Extrapulmonary cryptococcosis including meningitis<br>Disseminated non-tuberculous mycobacterial infection<br>Progressive multifocal leukoencephalopathy<br>Chronic cryptosporidiosis (with diarrhoea)<br>Chronic isosporiasis |

[18] Unexplained refers to where the condition is not explained by other causes.

| |
|---|
| Disseminated mycosis (coccidiomycosis or histoplasmosis)<br>Recurrent non-typhoidal Salmonella bacteraemia<br>Lymphoma (cerebral or B-cell non-Hodgkin) or other solid HIV-associated tumours<br>Invasive cervical carcinoma<br>Atypical disseminated leishmaniasis<br>Symptomatic HIV-associated nephropathy or symptomatic HIV-associated ardiomyopathy<br>And/or performance scale 4: bedridden > 50% of the day during the last month |

## Appendix B. Sets and elements modelled in the labour-market specification

| | **All categories at the beginning of year *t*<br>Set EUNP** | |
|---|---|---|
| | **Description** | **Short name** |
| 1. | Legislators | Legislator |
| 2. | Professionals | Profes |
| 3. | Technicians | Technical |
| 4. | Clerks | Clerks |
| 5. | Service workers | Service |
| 6. | Skilled agricultural workers | SkilledAgr |
| 7. | Craft workers | Craft |
| 8. | Plant and machine operators | PlantMach |
| 9. | Elementary occupations | Elementary |
| 10. | Domestic workers | Domestic |
| 11. | Occupation unspecified | Unspecified |
| 12. | Short-term unemployed | S |
| 13. | Long-term unemployed | L |
| 14. | New entrants | N |
| 15. | Permanently departed from the labour force | PDL |

| | **All activities undertaken during year *t*<br>Set EUP** | |
|---|---|---|
| | **Description** | **Short name** |
| 1. | Legislators | Legislator |
| 2. | Professionals | Profes |
| 3. | Technicians | Technical |
| 4. | Clerks | Clerks |
| 5. | Service workers | Service |
| 6. | Skilled agricultural workers | SkilledAgr |
| 7. | Craft workers | Craft |
| 8. | Plant and machine operators | PlantMach |
| 9. | Elementary occupations | Elementary |
| 10. | Domestic workers | Domestic |
| 11. | Occupation unspecified | Unspecified |
| 12. | Short-term unemployed | S |
| 13. | Long-term unemployed | L |
| 14. | Permanently departed from the labour force | PDL |

| | **Set EU** | |
|---|---|---|
| | **Description** | **Short name** |
| 1. | Legislators | Legislator |
| 2. | Professionals | Profes |
| 3. | Technicians | Technical |
| 4. | Clerks | Clerks |
| 5. | Service workers | Service |
| 6. | Skilled agricultural workers | SkilledAgr |
| 7. | Craft workers | Craft |
| 8. | Plant and machine operators | PlantMach |
| 9. | Elementary occupations | Elementary |
| 10. | Domestic workers | Domestic |
| 11. | Occupation unspecified | Unspecified |
| 12. | Short-term unemployed | S |
| 13. | Long-term unemployed | L |

**Appendix B (continue).**

| | Set EUN | |
|---|---|---|
| | **Description** | **Short name** |
| 1. | Legislators | Legislator |
| 2. | Professionals | Profes |
| 3. | Technicians | Technical |
| 4. | Clerks | Clerks |
| 5. | Service workers | Service |
| 6. | Skilled agricultural workers | SkilledAgr |
| 7. | Craft workers | Craft |
| 8. | Plant and machine operators | PlantMach |
| 9. | Elementary occupations | Elementary |
| 10. | Domestic workers | Domestic |
| 11. | Occupation unspecified | Unspecified |
| 12. | Short-term unemployed | S |
| 13. | Long-term unemployed | L |
| 14. | New entrants | N |

| | Set UN | |
|---|---|---|
| | **Description** | **Short name** |
| 1. | Short-term unemployed | S |
| 2. | Long-term unemployed | L |
| 3. | New entrants | N |

| | Set UP | |
|---|---|---|
| | **Description** | **Short name** |
| 1. | Short-term unemployed | S |
| 2. | Long-term unemployed | L |
| 3. | Permanently departed from the labour force | PDL |

| | Age groups<br>Set AGE | |
|---|---|---|
| | **Age groups** | **Short name** |
| 1. | A15–19 | A15–24 |
| 2. | A20–24 | |
| 3. | A25–29 | A25–34 |
| 4. | A30–34 | |
| 5. | A35–39 | A35–44 |
| 6. | A40–44 | |
| 7. | A45–49 | A45–54 |
| 8. | A50–54 | |
| 9. | A55–59 | A55–65 |
| 10. | A60–65 | |

| | Race<br>Set RACE | |
|---|---|---|
| | **Description** | **Short name** |
| 1. | Africa | A |
| 2. | Coloured | |
| 3. | Indian | |
| 4. | White | Other |

**Appendix B (continue).**

| | **Gender Set GEN** | |
|---|---|---|
| | **Description** | **Short name** |
| 1. | Female | F |
| 2. | Male | M |

| | **HIV Stage Set STG** | |
|---|---|---|
| | **Description** | **Short name** |
| 1. | HIV negative | HIVn |
| | HIV positive – Stages[19] | |
| 2. | Stage 1 | Stage1 |
| 3. | Stage 2 | Stage2 |
| 4. | Stage 3 | Stage3 |
| 5. | Stage 4 | Stage4 |

[19] An HIV-positive person will fall into one of these stages. These stages are based on the WHO classification of HIV progression (WHO, 2007).