Centre of Policy Studies Working Paper

No. G-248 September 2014

Revised December 2015

Interfacing a CGE Labour Market Model With the E3ME Multi-Sector Macroeconomic Model

G.A. Meagher,

Centre of Policy Studies, Victoria University

Felicity Pang,

Bank of China, Macau

R.A. Wilson,

Institute for Employment Research, University of Warwick

ISSN 1 031 9034 ISBN 978-1-921654-56-5

The Centre of Policy Studies (CoPS), incorporating the IMPACT project, is a research centre at Victoria University devoted to quantitative analysis of issues relevant to economic policy. Address: Centre of Policy Studies, Victoria University, PO Box 14428, Melbourne, Victoria, 8001 home page: www.vu.edu.au/CoPS/ email: copsinfo@vu.edu.au Telephone +61 3 9919 1877

Abstract

In recent years, a series of European labour market forecasts have been produced on behalf of, and have been published by, the European Centre for the Development of Vocational Training (CEDEFOP). These forecasts were generated using a modular modelling approach containing two major components :

- a multi-sector macroeconomic model (E3ME) for 29 European countries, primarily developed and operated by Cambridge Econometrics, and
- a labour market extension (WLME), primarily developed and operated by the Institute for Employment Research at the University of Warwick.

The countries are treated as an integrated system in E3ME but the extension is applied to each country separately. Forecasts of employment by industry are determined by E3ME; forecasts of employment by occupation and qualification are determined by the extension. The two components rely mainly on time series econometric techniques to generate their forecasts.

This paper describes how WLME can be replaced with an alternative extension (MLME) which incorporates a computable general equilibrium model. The CGE model has been developed primarily at the Centre of Policy Studies at Monash University. Compared to WLME, MLME relies less on time series extrapolation and more on explicitly modelled economic behaviour. This approach introduces a range of behavioural and technical parameters which offer more scope for modelling developments in the labour market which impact on occupations and skills rather industries.

Forecasts produced using the new E3ME-MLME system are reported for the United Kingdom, Greece and the Netherlands, and compared with the corresponding forecasts produced using the existing E3ME-WLME system. The focus of the comparison is on qualitative differences in the way the two sets of forecasts are to be interpreted. In particular, the sense in which explicit specification of technical change and economic behaviour (in the new system) can be substituted for time series extrapolation techniques (in the existing system) is carefully considered. The primary objective of the paper, therefore, is to demonstrate the empirical feasibility of the alternative methodology rather than to produce robust alternative forecasts.

JEL codes: C53, C58, D58, E27, J23, O41

Keywords: Forecasting, CGE models, hybrid models, labour markets

Contents

1. Introduction[1]

In recent years, a series of European labour market forecasts have been produced on behalf of, and have been published by, the European Centre for the Development of Vocational Training (CEDEFOP). These forecasts were generated using a modular modelling approach containing two major components :

- a multi-sector macroeconomic model (E3ME) for 29 European countries, primarily developed and operated by Cambridge Econometrics, and
- a labour market extension, primarily developed and operated by the Institute for Employment Research at the University of Warwick.

The countries are treated as an integrated system in E3ME but the extension is applied to each country separately. Forecasts of employment by industry are determined by E3ME; forecasts of employment by occupation and qualification are determined by the extension.

The labour market extension can be considered to consist of three modules:

- EDMOD which determines the forecasts of employment by occupation,
- QUALMOD which determines provisional forecasts of employment by qualification,
- STOCKMOD which determines labour supply by qualification, and
- BALMOD which revises the provisional qualifications forecasts to conform to the labour supply projections from STOCKMOD.

These modules will be referred to collectively as the Warwick labour market extension (WLME). They rely mainly on time series econometric techniques to generate their forecasts. An overview of the combined E3ME-WLME forecasting system, with references to further documentation, is contained in Wilson et al. (2010).

This paper describes how the WLME can be replaced with an alternative extension in which STOCKMOD is combined with a computable general equilibrium model which will be referred to as CGEMOD. In other words, CGEMOD replaces EDMOD, QUALMOD and BALMOD in the labour market extension. The CGE model has been developed primarily at the Centre of Policy Studies at Monash University and the new extension will be referred to as the Monash labour market extension (MLME). Compared to the WLME, the MLME relies less on time series extrapolation and more on explicitly modelled economic behaviour. This approach introduces a range of behavioural and technical parameters which offer more scope for modelling developments in the labour market which impact on occupations and skills rather industries. Section 2 presents an overview of MLME and its compares them with the corresponding E3ME-WLME forecasts. The focus in the comparison is on

[1] A version of this paper appeared previously as Meagher et al. (2010).

relationship with WLME. Section 3 sets out the specification of CGEMOD in detail. Section 4 presents forecasts for three representative countries prepared with the E3ME-MLME system, and differences in the interpretation of the forecasts generated by the two systems rather than on explaining the source of quantitative differences. Section 4 contains some concluding remarks.

2. The Monash Labour Market Extension: an Overview

The CGE core of MLME describes the operation of 27 occupational labour markets. On the supply side of these markets, the preferences of workers with a particular skill (here represented by qualification) are such that they are indifferent between occupational combinations which lie on the same Constant Elasticity of Transformation (CET) function. Figure 1 presents the idea diagrammatically. The position of the transformation curve is determined by the supply of the skill measured in preference units rather than hours. If the wage rate of occupation 2 increases relative to that of occupation 1, the isorevenue line becomes steeper, and the owners of the skill can increase their income by transforming some of occupation 1 into occupation 2. Hence, they change the occupational mix from E_1 to E_2. In principle, each of the 3 skills identified in WLME can be transformed into any of the 27 occupations. However, if none of a particular skill is used in a particular occupation in the base period, none of it will be used in that occupation in any of the forecasts.

On the demand side, labour of different occupations can be converted into effective units of industry specific labour according to Constant Elasticity Substitution (CES) functions. In Figure 2, the position of the isoquant is determined by the demand for labour in the industry. If the wage rate of occupation 2 decreases relative to that of occupation 1, the isocost line becomes flatter, and the producers in the industry can reduce their costs by substituting some of occupation 2 for occupation 1. Hence they change the occupational mix from E_1 to E_2. In principle, each of the 41 E3ME industries can employ any of 27 occupations but, as before, none of a particular occupation will be used by an industry in a forecast if none of it was used by that industry in the base period.

CGEMOD can accommodate different scenarios concerning the operation of the occupational labour markets. If relative wage rates are fixed, the model determines the skill mismatches (expressed in terms of occupations) which pertain at those wage rates. If relative wage rates are flexible, the model determines the wage rate changes required to clear the labour markets and eliminate any skills mismatches. If relative wage rates are sticky, the model determines the residual mismatches after the partial wage adjustment has occurred.

Figure 1 : Skill Transformations between Occupations

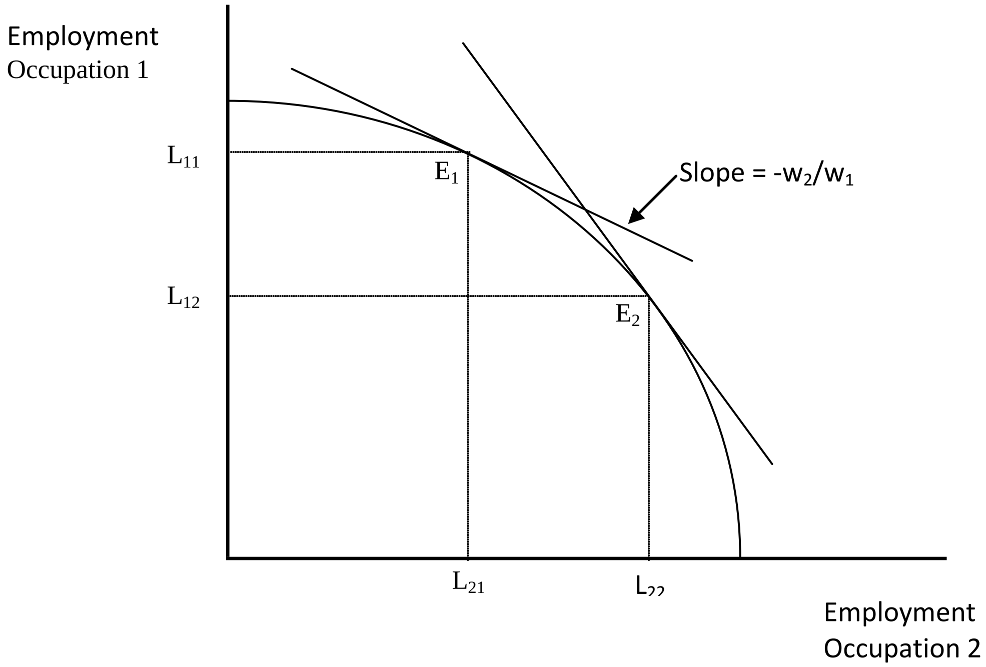
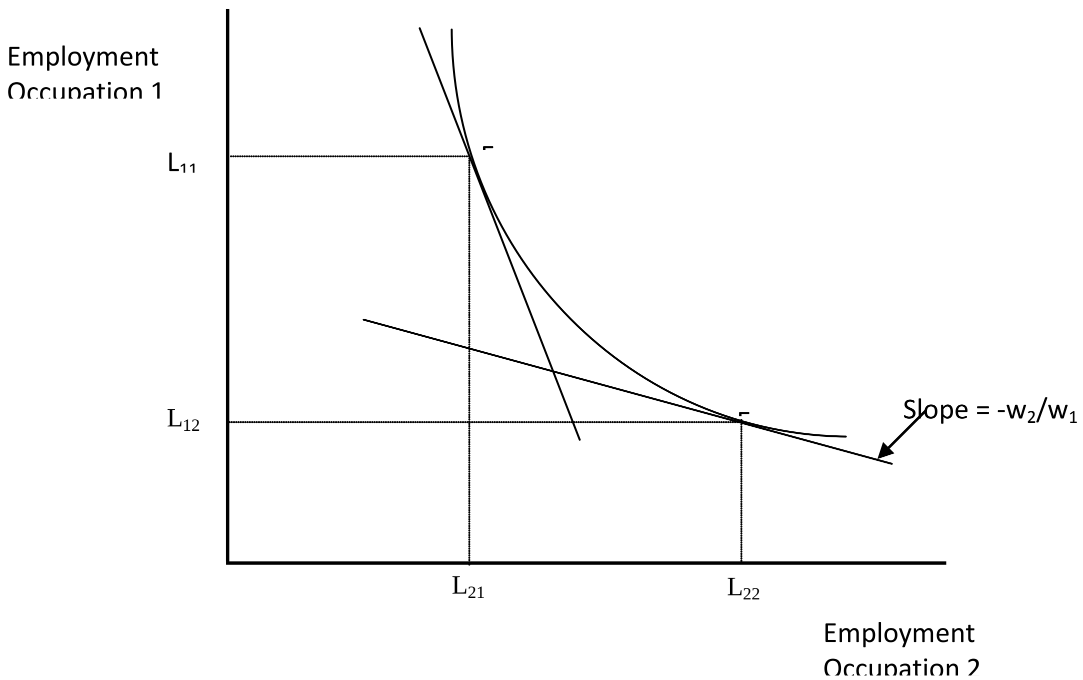


Figure 2: Substitution between Occupations in Industries



To satisfactorily interface the MLME model with the E3ME model, it must be supplied with the following data from the E3ME/WLME forecasts:

- employment measured in persons, and cross classified by industry (41), occupation (27) and qualification (3),
- hours worked per person differentiated by industry (41),
- ware rates differentiated separately by industry (41) and occupation (24),
- labour supply differentiated by qualification (3).

Apart from the wage rates, each of these data items is required for the base period and for every year of the forecast. The wage rates are only required for the base period. The labour supply projections are provided by STOCKMOD.

3. Specification of the CGE Core of MLME

The equations and notation for CGEMOD are listed in Tables 1 to 4. The computations are performed with a system of equations that is linear in percentage changes of the variables. That is, the system computes the percentage changes in the endogenous variables in some period t arising from changes ("shocks") to the exogenous variables. The coefficients in the system are shares. Sets, coefficients and parameters are denoted by upper-case or Greek symbols. The convention is adopted that lower-case symbols denote percentage changes in the levels of the variables represented by the corresponding upper case symbols, that is, the notation assumes y=100 (dY/Y). The levels variables Y do not appear in the equations but they will be used in the discussion of the equations which follows. Variables denoting amounts of labour or wage rates carry three subscripts which refer in strict order to industry, occupation and skill. If one of these subscripts is inoperative for a particular variable, it is replaced with an asterisk.

Consider the equations T1 in Table 1. This equation represents the generalisation of the economic theory presented in Figure 1 and can be derived from cost minimising behaviour by producers. The derivation is set out in Appendix 1, the equation which corresponds to TI being (A1.15). To obtain T1 from (A1.15), two additional assumptions are required. First, it is assumed that the wage rate for labour of occupation *o* is the same regardless of the industry in which it is employed, i.e.,

$$p_{io*} = p_{*o*}$$

for all industries i. That is, for the version of CGEMOD considered in this paper, markets exist only for labour of different occupations. Second, it is assumed that the rate of occupation-specific technical change is also the same regardless of the industry in which it is employed, i.e.,

$$a_{io*} = a^{D}_{*o*}$$

for all industries i. These assumptions are adopted for the sake of simplicity and could be relaxed in future versions of the model.

To interpret equations T1, consider first the case in which there is no technical change, i.e., a^{D}_{*o*} is zero for all occupations. Then equation maintains that, if there are no changes in the relative occupational wage rates P_{*o*}, i.e., if

$$p_{*o*} = 0 ,$$

a one per cent increase in the demand D^{E}_{i**} for effective units of labour in industry i leads to a one per cent increase in the demand D_{io*} for labour of each occupation by the industry. Here, the number of "effective" units is obtained by aggregating the occupational demands measured in hours according to a constant elasticity of substitution function[2]. If, however, the wage rate P_{*o*} for occupation o rises relative to the average wage rate for the industry, i.e., if

$$p_{*o*} > \sum_{k=1}^{OCC} SH^{W}_{ik*} \, p_{*k*} ,$$

the demand D_{io*} for occupation o will increase less rapidly than D^{E}_{i**}. Producers will substitute against occupation o in favour of other occupations. If it is difficult to substitute other occupations for occupation o, i.e., if the elasticity of substitution σ^{S}_{i} is small, the amount by which d^{E}_{i**} exceeds d_{io*} will also tend to be small. Note that the superscript W attached to the SH^{W}_{ik*} indicates that wage cost shares are to be used in computing the average wage rate for industry i, i.e.,

$$SH^{W}_{ik*} = P_{*k*} \, D_{ik*} \; / \; \sum_{o=1}^{OCC} P_{*o*} \, D_{io*} \; .$$

Now consider the case in which the wage rates P_{*k*} and the effective demand D^{E}_{i**} are constant but technical change is taking place.

[2] See equations (A1.1) and (A1.2) in Appendix 1 for a detailed account. Note that the notation in the text differs from the notation in the Appendix in order to facilitate the exposition in the latter.

Table 1. Equations of CGEMOD

Equation T1: Demand for labour of occupation *o* by industry *i*, hours

$$d_{io*} = d^{E}_{i**} - \sigma^{S}_{i} [\, p_{*o*} - \sum_{k=1}^{OCC} SH^{W}_{ik*}\, p_{*k*} \,] + a^{D}_{*o*} - \sigma^{S}_{i} [\, a^{D}_{*o*} - \sum_{k=1}^{OCC} SH^{W}_{ik*}\, a^{D}_{*k*} \,]$$

(all $i \in IND$, $o \in OCC$)

Equation T2: Average demand for labour of all occupations by industry *i*, hours

$$d^{H}_{i**} = \sum_{o=1}^{OCC} SH^{DI}_{io*}\, d_{io*}$$ (all $i \in IND$)

Equation T3: Average demand for labour of occupation *o* by all industries, hours

$$d^{H}_{*o*} = \sum_{i=1}^{IND} SH^{DO}_{io*}\, d_{io*}$$ (all $o \in OCC$)

Equation T4: Supply of labour by skill *s* to occupation *o*, hours

$$s_{*os} = s^{E}_{**s} + \sigma^{T}_{s} [\, p_{*o*} - \sum_{k=1}^{OCC} SH^{W}_{*ks}\, p_{*k*} \,] - a^{S}_{*o*} - \sigma^{T}_{s} [\, a^{S}_{*o*} - \sum_{k=1}^{OCC} SH^{W}_{*ks}\, a^{S}_{*k*} \,]$$

(all $o \in OCC$, $s \in SKL$)

Equation T5: Supply of labour by skill *s* to all occupations, hours

$$s^{H}_{**s} = \sum_{o=1}^{OCC} SH^{SS}_{*os}\, s_{*os}$$ (all $s \in SKL$)

Equation T6: Supply of labour by all skills to occupation *o*, hours

$$s^{H}_{*o*} = \sum_{s=1}^{SKL} SH^{SO}_{*os}\, s_{*os}$$ (all $o \in OCC$)

Equation T7: Market clearing for labour of occupation *o*, hours

$$d^{H}_{*o*} = s^{H}_{*o*}$$ (all $o \in OCC$)

Equation T8: Average hourly wage rate

$$p_{***} = \sum_{o=1}^{OCC} SH^{DI}_{*o*}\, p_{*o*}$$

Equation T9: Flexible handling of labour supply by workers with skill *s*, hours

$$s^{H}_{**s} = \overline{s}^{H}_{**s} + f\_s^{H}_{***}$$ (all $s \in SKL$)

Table 2. Variables of CGEMOD

Name	Description	
d_{io*}	Demand for labour of occupation *o* by industry *i*, hours	(all $i \in IND$, $o \in OCC$)
d^{E}_{i**}	Demand for labour of all occupations by industry *i*, effective units	(all $i \in IND$)
d^{H}_{i**}	Demand for labour of all occupations by industry *i*, hours	(all $i \in IND$)
d^{H}_{*o*}	Demand for labour of occupation *o* by all industries, hours	(all $o \in OCC$)
s_{*os}	Supply of labour to occupation *o* by all skills, hours	(all $o \in OCC$, $s \in SKL$)
s^{E}_{**s}	Supply of labour to all occupations by all skill *s*, effective units	(all $s \in SKL$)
s^{H}_{**s}	Supply of labour to all occupations by skill *s*, hours	(all $s \in SKL$)
s^{H}_{*o*}	Supply of labour to occupation *o* by all skills, hours	(all $o \in OCC$)
p_{*o*}	Hourly wage rate for occupation *o*	(all $o \in OCC$)
p_{***}	Average hourly wage rate	
$\overline{s}^{H}_{**s}$	Exogenous supply of labour to occupation *o* by all skills, hours	(all $o \in OCC$)
$f\_s^{H}_{***}$	Wage shift variable	
a^{D}_{*o*}	Occupation-*o*-augmenting technical change in production	(all $o \in OCC$)
a^{S}_{*o*}	Occupation-o-increasing technical change in labour supply	(all $o \in OCC$)

Table 3. Sets in CGEMOD

Name	Description	Number of Elements
IND	Industries	41
OCC	Occupations	27
SKL	Skills	3

Table 4. Coefficients and parameters of CGEMOD

Name	Description	
σ_i^S	Elasticity of substitution between occupations in industry *i*	(all $i \in IND$)
σ_s^T	Elasticity of transformation between occupations for skill *s*	(all $s \in SKL$)
SH_{io*}^{W}	Share of occupation *o* in cost of labour in industry *i*	(all $i \in IND$, $o \in OCC$)
SH_{io*}^{DI}	Share of occupation *o* in demand by industry *i*	(all $i \in IND$, $o \in OCC$)
SH_{io*}^{DO}	Share of industry *i* in demand for occupation *o*	(all $i \in IND$, $o \in OCC$)
SH_{*os}^{W}	Share of occupation *o* in income from skill *s*	(all $o \in$ OCC, $s \in SKL$)
SH_{*os}^{SS}	Share of occupation *o* in supply of skill *s*	(all $o \in$ OCC, $s \in SKL$)
SH_{*os}^{SO}	Share of skill *s* in supply of occupation *o*	(all $o \in$ OCC, $s \in SKL$)
SH_{*o*}^{DI}	Share of occupation *o* in total demand	($o \in OCC$)

If the change is *o*-augmenting at the rate of one per cent, i.e.,

$$a^{D}_{*o*} = -1$$

and

$$a^{D}_{*k*} = 0$$

for $k \neq o$, then industry *i* 's demand for labour of occupation *o* falls by

$$(1 - \sigma^{S}_{i}(1 - SH^{W}_{io*}))$$

per cent, i.e. by less than one per cent. Thus the *o*-augmenting technical progress induces some substitution in favour of occupation *o* and away from occupation *k* , $k \neq o$. Note that industry *i* 's demand for labour of occupation *k* , $k \neq o$, falls by

$$\sigma^{S}_{i} \, SH^{W}_{io*}$$

per cent.

Equations T2 defines the percentage change in the average demand for labour by industry *i* measured in hours (rather than effective units). The superscript *D* attached to the SH^{DI}_{io*} indicates that demand shares are to be used in computing the average, i.e.,

$$SH^{DI}_{io*} = D_{io*} / \sum_{k=1}^{OCC} D_{ik*} \; .$$

That is, SH^{DI}_{io*} is the share of occupation *o* in the total demand for labour (measured in hours) by industry *i*. Similarly equations T3 defines the percentage change in the average demand for labour of occupation *o* by all industries measured in hours. In this case, the demand shares to be used in computing the average are given by

$$SH^{DO}_{io*} = D_{io*} / \sum_{k=1}^{IND} D_{ko*} \; .$$

That is, SH^{DO}_{io*} is the share of industry *i* in the total demand by all industries for labour of occupation *o* (measured in hours).

Equations T4 in Table 1 represents the generalisation of the economic theory presented in Figure 2. It can be derived from income maximising behaviour by workers as set out in Appendix 2. Equations T4 can be obtained from (A2.5) by assuming that the wage rate for labour of occupation *o* and the rate of occupation-specific technical change are the same regardless of the skill from which the occupation was derived, i.e.,

$$p_{*os} = p_{*o*}$$

and

$$a_{*os} = a^{S}_{*o*}$$

for all skill groups *s*. As before, these assumptions are adopted for the sake of simplicity and could be relaxed in future versions of the model.

Equations T4 can be interpreted in a similar way to equations T1. If there is no technical change and there are no changes in the relative occupational wage rates, a one per cent increase in the supply S^{E}_{**s} of composite units of labour by workers with skill s leads to a one per cent increase in the supply S_{*os} of labour of each occupation by those workers. The number of composite units is obtained by aggregating the occupational supplies measured in hours according to a constant elasticity of transformation function[3]. If, however, the wage rate P_{*o*} for occupation *o* rises relative to the average wage rate for the skill group, i.e., if

$$p_{*o*} > \sum_{k=1}^{OCC} SH^{W}_{*ks}\ p_{*k*}\ ,$$

the supply S_{*os} of occupation *o* will increase more rapidly than S^{E}_{**s}. Workers will transform their composite labour in favour of occupation *o* and against other occupations. If it is difficult to transform other occupations into occupation *o*, i.e., if the elasticity of transformation σ^{T}_{s} is small, the amount by which s^{E}_{**s} exceeds s_{*os} will also tend to be small. The superscript W attached to the SH^{W}_{*ks} again indicates that wage cost shares are to be used in computing the average wage rate for skill *s*, i.e.,

$$SH^{W}_{*ks} = P_{*k*}\ S_{*ks}\ /\ \sum_{o=1}^{OCC} P_{*o*}\ \mathrm{S}_{*os}\ .$$

Now consider the case in which the wage rates P_{*k*} and the supply S^{E}_{**s} are constant but technical change is taking place. If the change is *o*-expanding at the rate of one per cent, i.e.,

$$a^{S}_{*o*} = -1$$

and

$$a^{S}_{*k*} = 0$$

for $k \neq o$, then supply of labour of occupation *o* by workers with skill *s* increases by

$$(1 + \sigma^{T}_{s}\ (1 - SH^{W}_{*os}))$$

[3] See equations (A2.1) and (A2.2) in Appendix 2 for a detailed account.

per cent, i.e. by more than one per cent. Thus the *o*-increasing technical progress induces some transformation of composite labour in favour of occupation *o* and away from occupation k , $k \neq o$. Note that the supply labour of occupation k , $k \neq o$, by workers with skill *s* falls by

$$\sigma_s^T \; SH_{*os}^W$$

per cent.

Equations T5 defines the percentage change in the average supply of labour by workers with skill *s* measured in hours (rather than effective units). The first superscript *S* attached to the SH_{*os}^{SS} indicates that supply shares are to be used in computing the average, i.e.,

$$SH_{*os}^{SS} \;=\; S_{*os} \;/\; \sum_{k=1}^{OCC} \; S_{*ks} \;\; .$$

That is, SH_{*os}^{SS} is the share of occupation *o* in the total supply of labour (measured in hours) by workers with skill *s*. Similarly equations T6 defines the percentage change in the average supply of labour to occupation *o* by all workers measured in hours. In this case, the supply shares to be used in computing the average are given by

$$SH_{*os}^{SO} \;=\; S_{*os} \;/\; \sum_{k=1}^{SKL} \; S_{*ok} \;\; .$$

That is, SH_{*os}^{SO} is the share of skill *s* in the total supply by all skill types of labour to occupation *o* (measured in hours).

Equations T7 require that the occupational labour markets clear. If slack variables are introduced into T7, the model can be used to compute skills gaps at given relative wage rates rather than equilibrium levels of employment.

Equation T8 defines the average hourly wage rate using demand shares, i.e.,

$$SH_{*o*}^{DI} \;=\; D_{*o*} \,/\, \sum_{k=1}^{OCC} \; D_{*k*} \;\; .$$

In CGE models, agents respond to changes in relative, rather than absolute, prices. Hence a uniform increase in all prices does not affect any quantity variables, and the price of one quantity variable must be set exogenously to determine the absolute price level. The quantity variable chosen for this purpose is referred to as the *numeraire*. E3ME includes among its variables the average wage income (measured in euros per person per year) for each industry. Hence it implicitly includes the average wage rate (euros per person per hour) where the average is calculated across industries. If the variable (in levels) P_{***} in equation T8 is set equal to this average wage rate, the changes in the absolute price level in CGEMOD will reflect the amount of wage inflation determined in E3ME.

Since the average wage rate in E3ME is calculated across industries, it is appropriate to use demand shares (rather than supply shares) in defining p_{***}.

To solve the model, the system must contain the same number of equations as endogenous variables. As the number of variables exceeds the number of equations, the values of some variables must be set exogenously. In a standard closure of CGEMOD (i.e., a standard division of its variables in exogenous and endogenous groups), the variables

$$d^H_{i**},\ \bar{s}^H_{**s},\ a^D_{*o*},\ a^S_{*o*}\ \text{and}\ p_{***}$$

comprise the exogenous set. Note that, if the market clearing conditions T7 are binding, the exogenous specification of the d^H_{i**} determines the change in aggregate employment. Hence the s^H_{**s} cannot also be specified exogenously without overdetermining the model. Equations T9 are included to avoid this eventuality and the $\bar{s}^H_{**s}$, rather than the s^H_{**s}, appear on the exogenous list. In other words, if the absolute changes in the demand for labour by industry are exogenous, and if labour markets clear, only the relative changes in the supply of labour by skill can be exogenous.

The model is solved recursively. In the first period of the forecast, the coefficients in Table 4 are evaluated from historical (base period) data. In subsequent periods, the CGEMOD solutions are used to update the coefficients. Linearisation errors are eliminated by introducing the shocks to the exogenous variables in small steps and updating the coefficients at each step. The method is described in some detail in Appendix 3.

Much of the data necessary to determine the CGEMOD shares listed in Table 4 can be obtained from the E3ME-WLME database. However four additional matrices are required for the base year 2009. They are

- employment by industry and occupation measured in hours,
- wage costs by industry and occupation,
- employment by occupation and skill measured in hours, and
- wage incomes by occupation and skill.

The matrices are derived from the E3ME-WLME database, from estimates of relative occupational wage rates prepared by Stehrer and Ward (2011), and from a variety of plausible assumptions concerning the range of applicability of the available data. The method is set out in Appendix 4. In years subsequent to the 2009, the shares can be updated using the CGEMOD solutions.

4. Interfacing CGEMOD and E3ME

In this section, forecasts of employment by occupation computed using the E3ME-MLME system are reported and compared with the corresponding forecasts computed with the E3ME-WLME system. Forecasts are reported for three countries - the United Kingdom, the Netherlands and Greece – and cover the period 2009 to 2020. For the exogenous variables for CGEMOD, the d_{i**}^{H} and p_{***} are taken from E3ME, the $\overline{s}_{**_s}^{H}$ are taken from STOCKMOD, and all the technical change variables $a_{*_o*}^{D}$ and $a_{*_o*}^{S}$ are set to zero[4]. The elasticities of substitution σ_i^S are all set to 0.35 and the elasticities of transformation σ_s^T are all set to 0.50. Hence the E3ME-MLME forecasts are to be regarded as illustrative only. Their purpose is to establish the operational viability of the E3ME-MLME system and to elucidate the relationship between that system and E3ME-WLME.

The E3ME-WLME system determines industry-by-occupation-by-skill forecasts of employment measured in persons. It also determines forecasts of the number of hours worked per person per year by industry. On the assumption the number of hours worked depends only on the industry in which a person works, industry-by-occupation-by-skill forecasts of employment measured in hours are also determined.[5] In particular, E3ME-WLME determines an industry-by-occupation matrix of hours worked (A^D, say) and an occupation-by-skill matrix of hours worked (A^S, say). These matrices allow the E3ME forecasts of employment by industry and the STOCKMOD forecasts of employment by skill to be converted from persons to hours, and hence to provide projections for the exogenous variables d_{i**}^{H} and $\overline{s}_{**_s}^{H}$ in CGEMOD.

CGEMOD separately determines industry-by-occupation and occupation-by-skill employment forecasts measured in hours. It remains to explain how these forecasts are to be converted from hours to persons for comparison with the E3ME-WLME forecasts. By construction, employment by industry (measured in hours) is the same in both systems. But the mix of occupations within industries is different. Similarly, employment by skill (measured in hours) is the same in both systems but the mix of occupations within skill groups is different. It follows that, if the matrices A^D and A^S are used to convert the CGEMOD forecasts from hours to persons, the demand for labour of a

[4] Note that the E3ME and STOCKMOD employment forecasts must first be converted from persons to hours in the manner described below. Then the choice of the d_{i**}^{H} and the $\overline{s}_{**_s}^{H}$ implies that the wage shift variable $f\_s_{***}^{H}$ will be endogenously determined as zero by CGEMOD.

[5] For purposes of the E3ME-MLME system this assumption is severe, especially as the forecasts of hours worked are available only for a restricted number of the 41 industries identified. In future work, it would be desirable to collect more comprehensive data on hours worked by both industry and occupation.

particular occupation (measured in persons) will not be equal to the supply. This is notwithstanding the fact that demand is equal to supply when both are measured in hours. Evidently, if the occupational labour markets are to clear when labour is measured in either persons or hours, the hours worked matrices A^D and A^S must change (to B^D and B^S, say).

It turns out that, as currently formulated, the E3ME-MLME system does not contain enough theory to determine B^D and B^S uniquely. If B^D is set equal to A^D, the demand for labour by occupation (measured in persons) will be determined. The elements of A^S must then be adjusted to bring the supply of labour by occupation into conformity with demand. That is, B^s will differ from A^S and all the adjustment will occur on the supply side of the labour market. Alternatively, if B^s is set equally to A^S, the elements of A^D must be adjusted to bring the demand for labour by occupation into conformity with supply. That is, all the adjustment will occur on the demand side of the labour market. Any distribution of the adjustment between these two extremes is also possible. Table 5 shows how the different modes of adjustment affect the forecasts of employment by occupation (measured in persons) for the United Kingdom in 2020. The effect is not large but nor is it negligible. In the forecasts reported in this paper, the adjustment is distributed equally between the demand and supply sides of the labour market. For this purpose, an additional module HTOPMOD is included in the E3ME-MLME system to convert the hours forecasts into persons forecasts. It is described in Appendix 5.

Table 6 compares the forecasts of employment by occupation measured in persons produced by the two systems. The differences are significant. However, a rationalisation of these differences is a complicated task and must be left for a future paper. Here, it will simply be noted that significant differences are to be expected when the technical change variables in CGEMOD have all been set at zero. The foreshadowed rationalisation will begin, then, with a determination using E3ME-MLME of the type and amount of the technical change implicit in the E3ME-WLME database and forecasts. Based on an analysis of the results of that determination, appropriate non-zero values will be assigned to the CGEMOD variables.

The relationship between the two systems can be usefully understood in a different way if WLME and MLME are regarded as alternative ways of modelling the future state of the markets for the 27 types of occupational labour identified in the two systems. This represents a departure of the usual interpretation of E3ME-WLME in which BALMOD is regarded as adjusting the demand for labour by qualification to match the supply by qualification. Thus, in the usual interpretation, E3ME and EDMOD together determine the demand by occupation. QUALMOD determines an occupation-by-qualification matrix which can be used to convert demand by occupation into demand by qualification. STOCKMOD determines supply by qualification. BALMOD adjusts the occupation-by-qualification matrix using the RAS method so that it conforms to both the demand by occupation determined by EDMOD and the supply by qualification determined by STOCKMOD. The occupational mix within each qualification category is regarded as having been adjusted by BALMOD to bring the demand for labour by qualification into conformity ("balance") with the supply of labour by qualification.

In the new interpretation, E3ME and EDMOD together determine the demand by occupation as before. STOCKMOD determines supply by qualification as before. QUALMOD determines an occupation-by-qualification matrix as before, but this time its role is to convert supply by qualification into supply by occupation. As before, BALMOD adjusts the occupation-by-qualification matrix using the RAS method so that it conforms to both the demand by occupation determined by EDMOD and the supply by qualification determined by STOCKMOD. This time the qualification mix within each occupation is regarded as having been adjusted by BALMOD to bring the demand for labour by occupation into conformity with the supply of labour by occupation.

The difference is only in the way the operation of the E3ME-WLME system is interpreted. There is no change in the set of forecasts produced by the system. The point of adopting the new interpretation is that it allows for a more transparent presentation of the differences between the E3ME-WLME forecasts and the E3ME-MLME forecasts.

In the new interpretation, then, WLME contains three processes which can be considered to affect the change in demand for labour (measured in hours):

(1) The *industry shift effect* (ISE) which describes how the demand for occupations (measured in persons) would change if there were no change in the mix of occupations within each of the 41 industries. It is determined by the E3ME model.

(2) The *occupational share effect* (OSE) which describes how the mix of occupations (measured in persons) within each industry changes over time. It is determined by extrapolating past trends in occupational shares. The ISE and OSE together determine the demand for labour by occupation when labour is measured in persons.

(3) The *average hours effect* (AHE) which describes how the average hours worked per employed person in each occupation changes over time. The AHE, insofar as it applies to occupations, does not usually form part of WLME but, as it is required to interface MLME with E3ME, it will be counted as part of WLME for purposes of the present discussion. It allows the occupational forecasts produced by WLME and MLME to be compared in either persons or hours.

Table 5. Alternative Employment Forecasts, MLME, 2020, Persons, United Kingdom

Seq. No	Occupation	(1)	(2)	(3)
		Type of Average Hours Adjustment		
		Supply Only	Demand Only	Equal Contributions
1	01 Armed Forces	61778	61464	61621
2	11 Legislators and senior officials	60725	61225	60975
3	12 Corporate managers	3893747	3893223	3893485
4	13 Managers of small enterprises	1157664	1171099	1164381
5	21 Physical mathematical and engineering science profs	1290689	1293005	1291847
6	22 Life science and health professionals	474760	471508	473134
7	23 Teaching professionals	1399572	1388542	1394057
8	24 Other professionals	1786497	1785760	1786129
9	31 Physical and engineering science associate professionals	774197	776511	775354
10	32 Life science and health associate professionals	1003992	997741	1000866
11	33 Teaching associate professionals	211046	211861	211453
12	34 Other associate professionals	2437162	2448104	2442633
13	41 Office clerks	3295336	3289378	3292357
14	42 Customer services clerks	974222	971278	972750
15	51 Personal and protective services workers	3672106	3661368	3666737
16	52 Models salespersons and demonstrators	1810189	1808950	1809569
17	61 Skilled agricultural and fishery workers	379826	380856	380341
18	71 Extraction and building trades workers	1395406	1401699	1398553
19	72 Metal machinery and related trades workers	870123	873500	871811
20	73 Precision handicraft craft printing and related trades wkrs	124114	124168	124141
21	74 Other craft and related trades workers	149245	148803	149024
22	81 Stationary plant and related operators	146852	145958	146405
23	82 Machine operators and assemblers	504686	505622	505154
24	83 Drivers and mobile plant operators	1130773	1130190	1130482
25	91 Sales and services elementary occupations	2322588	2324144	2323366
26	92 Agricultural fishery and related labourers	87803	90484	89144
27	93 Mining construction manufacturing transport labourers	1049746	1048407	1049077
28	99 All occupations	32464844	32464844	32464844

Table 6. Deviations in Employment Forecasts, WLME and MLME, 2020, United Kingdom

Seq. No	Occupation	(1) WLME (persons)	(2) MLME (persons)	(3) Deviations (per cent)
1	01 Armed Forces	31343	61621	96.60
2	11 Legislators and senior officials	35127	60975	73.58
3	12 Corporate managers	3912781	3893485	-0.49
4	13 Managers of small enterprises	1162210	1164381	0.19
5	21 Physical mathematical and engineering science profs	1144780	1291847	12.85
6	22 Life science and health professionals	566434	473134	-16.47
7	23 Teaching professionals	1103303	1394057	26.35
8	24 Other professionals	1918894	1786129	-6.92
9	31 Physical and engineering science associate professionals	762539	775354	1.68
10	32 Life science and health associate professionals	895199	1000866	11.80
11	33 Teaching associate professionals	282439	211453	-25.13
12	34 Other associate professionals	3217418	2442633	-24.08
13	41 Office clerks	2634253	3292357	24.98
14	42 Customer services clerks	935288	972750	4.01
15	51 Personal and protective services workers	3718364	3666737	-1.39
16	52 Models salespersons and demonstrators	1955543	1809569	-7.46
17	61 Skilled agricultural and fishery workers	365800	380341	3.98
18	71 Extraction and building trades workers	1504690	1398553	-7.05
19	72 Metal machinery and related trades workers	630823	871811	38.20
20	73 Precision handicraft craft printing and related trades wkrs	70840	124141	75.24
21	74 Other craft and related trades workers	117991	149024	26.30
22	81 Stationary plant and related operators	154054	146405	-4.96
23	82 Machine operators and assemblers	520789	505154	-3.00
24	83 Drivers and mobile plant operators	1271056	1130482	-11.06
25	91 Sales and services elementary occupations	2225239	2323366	4.41
26	92 Agricultural fishery and related labourers	53600	89144	66.31
27	93 Mining construction manufacturing transport labourers	1274047	1049077	-17.66
28	99 All occupations	32464844	32464844	0.00

Similarly, three processes can be considered to operate on changes in the supply of labour by occupation (measured in hours):

(1) The skill shift effect (SSE) which describes how the supply of labour (measured in persons) would change if there were no change in the mix of occupations within each of the three skill groups. It is determined by STOCKMOD and QUALMOD. STOCKMOD determines the supply of labour by qualification by extrapolating past trends in various variables affecting labour supply such as population growth and labour force participation rates. QUALMOD determines the provisional occupation-by-qualification matrix for converting supply by qualification into supply by occupation.

(2) The *occupational share effect* (OSE) which describes how the mix of occupations (measured in persons) within each skill group changes over time. Thus, the OSE must be interpreted differently on the demand and supply sides of a labour market but the interpretations are quite analogous. In WLME, the OSE on the supply side is determined by the requirement that all occupational labour markets must clear (or be in "balance"). Balance is achieved by application of the mechanical RAS method. The SSE and OSE together determine the supply for labour by occupation when labour is measured in persons.

(3) The *average hours effect* (AHE) which again describes how the average hours worked per employed person in each occupation changes over time. There is no distinction between average hours worked on the demand and supply sides of the (postulated) occupational labour markets in WLME. It follows that, if a market is in balance (if the market clears) when labour is measured in persons, it is also in balance when labour is measured in hours.

In this interpretation, the demand side in WLME is determined without reference to the supply side. In other words the adjustment required to achieve balance falls entirely on the supply side of the labour markets in the sense that only the supply-side OSE values are forced to diverge from their trend values. It would be possible to impose supply-side OSE values which do reflect trend values in the same way that trend values are currently imposed on the demand-side OSE. In that case, WLME would determine the excess demands for, or supplies of, labour implied by the trend OSE's. That is, the model would determine the "skills gaps" that would develop in the labour markets if existing trends were to persist. Note also that there is no necessity for balance to be achieved entirely by supply-side adjustment within the WLME framework. In principle, it would be possible to achieve balance at any set of occupational employment levels intermediate between the two sets implied by the trend OSE's. One would simply apply the RAS method on both the demand and supply sides rather than just the supply side.

An analogous set of processes operates in MLME. Changes in the demand for labour by occupation (measured in persons) are affected by:

(1) The *industry shift effect* (ISE) which describes how the demand for occupations (measured in hours) would change if there were no change in the mix of occupations within each of the 41 industries. It is determined by the E3ME model supplemented with an industry-by-occupation hours worked matrix (the A^D matrix in terms of the above discussion).

(2) The *occupational share effect* (OSE) which describes how the mix of occupations (measured in hours) within each industry changes over time. For a given set of occupational wage rates, the demand-side OSE values are determined by technology (as embodied in the CES production function) and by the cost-minimising behaviour of producers. These technological and behavioural assumptions perform a similar function in MLME to the trend assumption for the demand-side OSE in WLME (i.e., the assumption that the occupational mix within industries will continue to change in the future in the way as it has on average in the past). The ISE and OSE together determine the demand for labour by occupation when labour is measured in hours.

(3) The *average hours effect* (AHE) which describes how the average hours worked per employed person in each occupation changes over time. It is determined by the module HTOPMOD.

As before, three processes can be considered to operate on changes in the supply of labour by occupation (measured in persons):

(1) The skill shift effect (SSE) which describes how the supply of labour (measured in hours) would change if there were no change in the mix of occupations within each of the three skill groups. It is determined by the STOCKMOD module supplemented with an occupation-by-skill hours worked matrix (the A^S matrix in terms of the preceding discussion). That is, if an occupation-by-skill matrix were to be added to STOCKMOD and if there are no changes in occupational mix, the augmented version would determine the SSE.

(2) The *occupational share effect* (OSE) which describes how the mix of occupations (measured in hours) within each skill group changes over time. On the supply-side, the OSE values are determined by technology (as embodied in the CET transformation function) and by the income-maximising behaviour of workers. The SSE and OSE together determine the supply of labour by occupation when labour is measured in hours.

(3) The *average hours effect* (AHE) which describes how the average hours worked per employed person in each occupation changes over time. It is determined by the module HTOPMOD.

When the same set of occupational wage rates is assumed for both the demand and supply sides of the labour markets, MLME determines the skills gaps which will pertain at those wage rates. For practical purposes, only one such set of wage rates will clear all the markets (i.e., eliminate all the skills gaps). Correspondingly, there will be only one set of equilibrium (or balanced) occupational employment levels. In an equilibrium solution, it is the relative wage rates, rather than the skills gaps, which reflect structural pressures in the economy and provide an indicator for, say, training policy.

For each year of the forecast, CGEMOD determines an industry-by-occupation employment matrix and an occupation-by-skill employment matrix, where employment is measured in hours. HTOPMOD determines industry-by-occupation and occupation-by-skill matrices for hours worked (i.e., the matrices B^D and B^S in terms of the above discussion) which enable the employment forecasts to be converted from hours to persons. Once the employment matrices have been so converted, it is possible to compute the demand side ISE and OSE, and the supply side SSE and OSE, measured in persons rather than in hours. These effects computed from the E3ME-MLME system can then be compared more directly with the corresponding effects computed from the E3ME-WLME system. In particular, when computed in this way, the ISE and SSE are identical for the two systems and any differences between their employment forecasts are entirely accounted for by differences in the OSE. Forecasts of these effects for changes in employment in the United Kingdom between 2009 and 2020, as computed from the E3ME-MLME system, are reported in Tables 7 and 8.

The discussion in this section makes it clear that, notwithstanding the universally accepted CGE nomenclature, computable general models do not always (or even usually) assume that all markets clear. In the context of labour market forecasting, an assumption of sticky wage rates (whereby wage rates only partially adapt to excess demands and supplies in any period) is an attractive option that will be pursued in future work.

The final two tables presented in the paper compare E3ME-WLME and E3ME-MLME employment forecasts for Greece (Table 9) and the Netherlands (Table 10). They are computed in the same way as the forecasts in Table 6 for the United Kingdom. It can be seen that the differences between the forecasts for particular occupations vary considerably from one country to another. In other words, differences in the forecasts derived from the two systems do not reflect any simple kind of occupational bias. Rather, they originate in a complicated way from differences (explicit and implicit) assumed for technical change and economic behaviour in the competing forecasts. An elaboration of these differences, insofar as they can be identified, will also be pursued in future work.

Table 7. Contributions to Growth in Demand, 2009-20, Thousands of persons, MLME, United Kingdom

Seq. No.	Occupation	(1) Employ-ment 2009	(2) Industry Shift Effect	(3) Occupation Share Effect	(4) Employ-ment 2020
1	01 Armed Forces	61	-2	3	62
2	11 Legislators and senior officials	56	2	3	61
3	12 Corporate managers	3591	207	96	3893
4	13 Managers of small enterprises	1090	76	-2	1164
5	21 Physical mathematical and engineering science profs	1169	80	43	1292
6	22 Life science and health professionals	436	13	24	473
7	23 Teaching professionals	1346	-23	71	1394
8	24 Other professionals	1555	219	13	1786
9	31 Physical and engineering science associate professionals	721	35	19	775
10	32 Life science and health associate professionals	939	13	49	1001
11	33 Teaching associate professionals	203	4	5	211
12	34 Other associate professionals	2220	201	22	2443
13	41 Office clerks	3139	197	-44	3292
14	42 Customer services clerks	941	48	-17	973
15	51 Personal and protective services workers	3612	63	-8	3667
16	52 Models salespersons and demonstrators	1704	169	-63	1810
17	61 Skilled agricultural and fishery workers	384	1	-5	380
18	71 Extraction and building trades workers	1362	25	11	1399
19	72 Metal machinery and related trades workers	867	-17	22	872
20	73 Precision handicraft craft printing and related trades wkrs	124	1	0	124
21	74 Other craft and related trades workers	152	-6	3	149
22	81 Stationary plant and related operators	154	-7	-1	146
23	82 Machine operators and assemblers	556	-51	0	505
24	83 Drivers and mobile plant operators	1117	30	-16	1130
25	91 Sales and services elementary occupations	2310	201	-188	2323
26	92 Agricultural fishery and related labourers	92	-4	1	89
27	93 Mining construction manufacturing transport labourers	1071	18	-40	1049
28	Total	30972	1493	0	32465

Table 8. Contributions to Growth in Supply, MLME, 2009-20, Thousands of Persons, United Kingdom

Seq. No.	Occupation	(1) Employ-ment 2009	(2) Skill Shift Effect	(3) Occupation Share Effect	(4) Employ-ment 2020
1	01 Armed Forces	61	4	-4	62
2	11 Legislators and senior officials	56	7	-2	61
3	12 Corporate managers	3591	386	-83	3893
4	13 Managers of small enterprises	1090	41	34	1164
5	21 Physical mathematical and engineering science profs	1169	161	-39	1292
6	22 Life science and health professionals	436	68	-31	473
7	23 Teaching professionals	1346	204	-156	1394
8	24 Other professionals	1555	213	18	1786
9	31 Physical and engineering science associate professionals	721	74	-20	775
10	32 Life science and health associate professionals	939	132	-70	1001
11	33 Teaching associate professionals	203	17	-9	211
12	34 Other associate professionals	2220	212	11	2443
13	41 Office clerks	3139	91	62	3292
14	42 Customer services clerks	941	10	21	973
15	51 Personal and protective services workers	3612	188	-133	3667
16	52 Models salespersons and demonstrators	1704	-49	155	1810
17	61 Skilled agricultural and fishery workers	384	-14	10	380
18	71 Extraction and building trades workers	1362	47	-10	1399
19	72 Metal machinery and related trades workers	867	43	-38	872
20	73 Precision handicraft craft printing and related trades wkrs	124	-1	1	124
21	74 Other craft and related trades workers	152	-1	-2	149
22	81 Stationary plant and related operators	154	-6	-1	146
23	82 Machine operators and assemblers	556	-43	-8	505
24	83 Drivers and mobile plant operators	1117	-19	32	1130
25	91 Sales and services elementary occupations	2310	-193	206	2323
26	92 Agricultural fishery and related labourers	92	-5	2	89
27	93 Mining construction manufacturing transport labourers	1071	-75	53	1049
28	Total	30972	1493	0	32465

Table 9. Deviations in Employment Forecasts, WLME and MLME, 2020, Greece

Seq. No	Occupation	(1) WLME (persons)	(2) MLME (persons)	(3) Deviations (per cent)
1	01 Armed Forces	56020	69761	24.53
2	11 Legislators and senior officials	1064	2170	103.93
3	12 Corporate managers	101528	85416	-15.87
4	13 Managers of small enterprises	350675	426966	21.76
5	21 Physical mathematical and engineering science profs	109679	122570	11.75
6	22 Life science and health professionals	97287	117469	20.74
7	23 Teaching professionals	266087	279765	5.14
8	24 Other professionals	168745	208129	23.34
9	31 Physical and engineering science associate professionals	126712	113504	-10.42
10	32 Life science and health associate professionals	95911	87536	-8.73
11	33 Teaching associate professionals	22250	15173	-31.81
12	34 Other associate professionals	349341	251387	-28.04
13	41 Office clerks	360008	420630	16.84
14	42 Customer services clerks	167326	131313	-21.52
15	51 Personal and protective services workers	395141	401389	1.58
16	52 Models salespersons and demonstrators	360357	348941	-3.17
17	61 Skilled agricultural and fishery workers	446027	442644	-0.76
18	71 Extraction and building trades workers	329940	312611	-5.25
19	72 Metal machinery and related trades workers	167822	189841	13.12
20	73 Precision handicraft craft printing and related trades wkrs	13552	20374	50.34
21	74 Other craft and related trades workers	94193	93944	-0.26
22	81 Stationary plant and related operators	22738	16597	-27.01
23	82 Machine operators and assemblers	91361	79331	-13.17
24	83 Drivers and mobile plant operators	229342	217123	-5.33
25	91 Sales and services elementary occupations	260500	233660	-10.30
26	92 Agricultural fishery and related labourers	10922	15028	37.59
27	93 Mining construction manufacturing transport labourers	83545	74804	-10.46
28	99 All occupations	4778073	4778073	0.00

Table 10. Deviations in Employment Forecasts, WLME and MLME, 2020, Netherlands

Seq. No	Occupation	(1) WLME (persons)	(2) MLME (persons)	(3) Deviations (per cent)
1	01 Armed Forces	19499	28768	47.53
2	11 Legislators and senior officials	24527	20017	-18.39
3	12 Corporate managers	406827	443416	8.99
4	13 Managers of small enterprises	562626	506769	-9.93
5	21 Physical mathematical and engineering science profs	394893	466925	18.24
6	22 Life science and health professionals	163296	169947	4.07
7	23 Teaching professionals	259417	344420	32.77
8	24 Other professionals	1091184	932331	-14.56
9	31 Physical and engineering science associate professionals	346575	341103	-1.58
10	32 Life science and health associate professionals	398990	413696	3.69
11	33 Teaching associate professionals	11334	8584	-24.26
12	34 Other associate professionals	847415	866518	2.25
13	41 Office clerks	698204	785507	12.50
14	42 Customer services clerks	313151	261536	-16.48
15	51 Personal and protective services workers	785701	732892	-6.72
16	52 Models salespersons and demonstrators	440087	477527	8.51
17	61 Skilled agricultural and fishery workers	92112	99371	7.88
18	71 Extraction and building trades workers	346436	349479	0.88
19	72 Metal machinery and related trades workers	255472	288885	13.08
20	73 Precision handicraft craft printing and related trades wkrs	25233	27693	9.75
21	74 Other craft and related trades workers	56401	64208	13.84
22	81 Stationary plant and related operators	62004	55797	-10.01
23	82 Machine operators and assemblers	109389	127485	16.54
24	83 Drivers and mobile plant operators	231216	256169	10.79
25	91 Sales and services elementary occupations	485369	476536	-1.82
26	92 Agricultural fishery and related labourers	9665	10326	6.84
27	93 Mining construction manufacturing transport labourers	467840	348959	-25.41
28	99 All occupations	8904864	8904864	0.00

5. Concluding Remarks

In this paper, a methodology has been established for interfacing a CGE labour market model with the E3ME multi-sector macroeconomic model for purposes of producing labour market forecasts for the European Union. Forecasts produced using the new E3ME-MLME system are reported for the United Kingdom, Greece and the Netherlands, and compared with the corresponding forecasts produced using the existing E3ME-WLME system. The focus of the comparison has been on qualitative differences in the way the two sets of forecasts are to be interpreted. In particular, the sense in which explicit specification of technical change and economic behaviour (in the new system) can be substituted for time series extrapolation techniques (in the existing system) has been carefully identified. The primary objective of the paper, therefore, has been to demonstrate the empirical feasibility of the alternative methodology rather than to produce robust alternative forecasts.

To facilitate more robust employment forecasts in future, the most important task is to provide the new system with adequate projections of technical change. Preliminary work indicates that suitable projections can indeed be derived from the E3ME-WLME database. However, it seems likely that more than one specification will be available which adequately accounts for the data. A method for deciding which specification is the most appropriate is not obvious. On the other hand, it is precisely because the new system provides an opportunity to explore this kind of issue that it was conceived in the first place.

Appendices

A1. The Producers' Cost Minimisation Problem[6]

In CGEMOD, the CGE core of the Monash Labour Market Extension (MLME), the production technology available to producers in industry j is assumed to be such that units X_{ij} of labour belonging to various occupations i can be combined to provide a unit X_j of effective labour input according to the equation

$$X_j = CES_i\left(\frac{X_{ij}}{A_{ij}};\rho_j, b_{ij}\right). \qquad \text{(A1.1)}$$

The notation CES_s (f_s; ρ, b_s) means that the variables f_s are to be aggregated according to a constant elasticity of substitution function with parameters ρ and b_s, i.e.,

$$CES_s\,(f_s;\rho, b_s) \equiv \left(\sum\nolimits_s f_s^{-\rho} b_s\right)^{-(1/\rho)} \qquad \text{(A1.2)}$$

with the b_s being non-negative and ρ being greater than -1 but not equal to zero. In what follows the parameter list will generally be omitted and the left hand side of (A1.2) written simply as $CES_s(f_s)$. The A_{ij}'s are positive coefficients which allow for technical change.

Producers are assumed to be competitive and efficient. They are competitive in that they treat all wage rates as exogenously given. They are efficient in that, for any given amount of effective labour input X_j, producers in industry j select the combination of occupational inputs X_{ij} which minimises their costs. That is, producers choose the input levels X_{ij} to minimise $\sum_i P_{ij} X_{ij}$ subject to (A.1.1), where P_{ij} is the wage rate for occupation i in industry j. The problem can be conveniently rewritten as

choose $\overline{X}_{ij}$ to

minimise $\sum_i \overline{P}_{ij} \overline{X}_{ij}$

subject to $X_j = CES_i(\overline{X}_{ij})$,

[6] This appendix is adapted from Dixon et al. (1982), pp.68-74, 76-90.

where

$$\bar{P}_{ij} = A_{ij} P_{ij} \ , \qquad \text{(A1.3)}$$

and

$$\bar{X}_{ij} = X_{ij} / A_{ij} \ . \qquad \text{(A1.4)}$$

The first-order conditions for a solution to this problem are

$$\bar{P}_{ij} - \Lambda \frac{\partial CES_i(\bar{X}_{ij})}{\partial \bar{X}_{ij}} = 0 \qquad \text{(A1.5)}$$

and

$$X_j - CES_i(\bar{X}_{ij}) = 0 \ , \qquad \text{(A1.6)}$$

where Λ is the Lagrangian multiplier. Equation (A1.5) can be rewritten as

$$\bar{P}_{ij} - \Lambda b_{ij} \left(\frac{\bar{X}_{ij}}{X_j} \right)^{-\rho_j - 1} = 0,$$

which implies

$$\bar{p}_{ij} - \lambda + (\rho_j + 1)(\bar{x}_{ij} - x_j) = 0, \qquad \text{(A1.7)}$$

where the lower-case symbols denote percentage changes in the variables represented by the corresponding upper-case symbols.

Now, totally differentiating (A1.6), and using (A1.5), gives

$$dX_j = \sum_i \left(\frac{\partial CES_i(\bar{X}_{ij})}{\partial \bar{X}_{ij}} \right) d\,\bar{X}_{ij} \ ,$$

i.e.,

$$dX_j = \sum_i \left(\frac{\bar{P}_{ij} d\bar{X}_{ij}}{\Lambda} \right) \ ,$$

and

$$x_j = \sum_i \left(\frac{\bar{P}_{ij} \bar{X}_{ij}}{\Lambda X_j} \right) \bar{x}_{ij} \ .$$

Hence

$$x_j = \sum_i S_{ij} \bar{x}_{ij} \ , \qquad \text{(A1.8)}$$

where

$$S_{ij} = \frac{\bar{P}_{ij} \bar{X}_{ij}}{\Lambda X_j} \ . \qquad \text{(A1.9)}$$

It turns out that

$$S_{ij} = P_{ij} X_{ij} / \sum_k P_{kj} X_{kj} \ , \qquad \text{(A1.10)}$$

i.e., S_{ij} is the share of occupation i in the total cost of labour to industry j. To show this, equation (A1.5) is multiplied through by $\bar{X}_{ij}$ and aggregated over i, giving

$$\sum_i \bar{P}_{ij} \bar{X}_{ij} - \Lambda \sum_i \left(\frac{\partial CES_i(\bar{X}_{ij})}{\partial \bar{X}_{ij}} \right) \bar{X}_{ij} = 0. \qquad \text{(A1.11)}$$

But, according to Euler's theorem for linearly homogeneous functions,

$$X_j = \sum_i \left(\frac{\partial CES_i(\bar{X}_{ij})}{\partial \bar{X}_{ij}} \right) \bar{X}_{ij} \ .$$

Hence equation (A1.11) reduces to

$$\sum_i \bar{P}_{ij} \bar{X}_{ij} - \Lambda X_j \ . \qquad \text{(A1.12)}$$

Equations (A1.12), (A1.9), (A1.4) and (A1.5) together imply (A1.10).

Returning now to equation (A1.7), multiplying through by S_{ij}, aggregating over i and using (A1.8), gives

$$\lambda = \sum_i S_{ij} \bar{P}_{ij} \ .$$

Substituting λ into (A1.7) yields the percentage change form of the demand functions for labour belonging to occupation i:

$$\bar{x}_{ij} = x_j - \sigma_j (\bar{p}_{ij} - \sum_i S_{ij} \bar{p}_{ij}) \ , \qquad \text{(A1.13)}$$

where

$$\sigma_j = \frac{1}{(1 + \rho_j)}$$

is the elasticity of substitution between different occupations in industry j.

Finally, given that

$$\overline{x}_{ij} = x_{ij} - a_{ij} \,, \qquad \text{(A1.14)}$$

and

$$\overline{p}_{ij} = p_{ij} + a_{ij}$$

from equations (A1.3) and (A1.4), (A1.13) can be written

$$x_{ij} = x_j - \sigma_j (p_{ij} - \sum_k S_{kj} p_{kj}) + a_{ij} - \sigma_j (a_{ij} - \sum_k S_{kj} a_{kj}) \,. \qquad \text{(A1.15)}$$

Now let the variable P_j be defined by the equation

$$P_j X_j = \sum_i P_{ij} X_{ij}$$

In percentage change form, this equation becomes

$$p_j + x_j = \sum_i S_{ij} p_{ij} + \sum_i S_{ij} x_{ij} \,.$$

But, from (A1.8) and (A1.14)

$$x_j = \sum_i S_{ij} (x_{ij} - a_{ij}) \,.$$

Hence,

$$p_j = \sum_i S_{ij} p_{ij} + \sum_i S_{ij} a_{ij} \,. \qquad \text{(A1.16)}$$

In MLME, it is appropriate to set

$$\sum_i S_{ij} a_{ij} = 0. \qquad \text{(A1.17)}$$

Then the a_{ij}'s simulate the effects of occupation-i-augmenting technical change in industry j. Other types of technical change have already been included, at least in principle, in the industry employment forecasts provided by E3ME. For example, general labour-augmenting technical change, which tends to shift the capital-labour ratio in favour of labour, cannot be accommodated in MLME as MLME does not include any information about the relative use of capital and labour. That is, MLME includes only a partial description of the production technology in each industry.

If the convention of equation (A1.17) is adopted, equation (A1.16) implies that the percentage change in P_j is a weighted average of the percentage changes in the costs to industry j of units of labour from different occupations, the weights being the shares of each occupation in industry j's total labour costs.

A2. The Workers' Income Maximisiation Problem[7]

In CGEMOD, the preferences of workers with skill j are assumed to be such that they are indifferent between combinations of hours worked X_{ij} in various occupations i which satisfy the equation

$$X_j = \text{CET}_i\,(X_{ij} A_{ij}\,;\, \rho_j,\, b_{ij})\,. \qquad \text{(A2.1)}$$

Here, X_j can be regarded as the amount of composite labour supplied where labour is measured in preference units rather than hours. The notation $\text{CET}_s\,(f_s;\, \rho,\, b_s)$ means that the variables f_s are to be aggregated according to a constant elasticity of transformation function with parameters ρ and b_s, i.e.,

$$CET_s\,(f_s;\, \rho,\, b_s) \equiv \left(\sum_s f_s^{\,-\rho} b_s\right)^{-(1/\rho)} \quad , \qquad \text{(A2.2)}$$

where the b_s are positive and sum to 1, and ρ is less than or equal to -1. The A_{ij}'s are positive coefficients which allow for preference change. Thus, apart from the restrictions on the parameters, the CET function is identical to the CES function. The parameter restrictions for CES ensure isoquants which are convex to the origin whereas the parameter restrictions on CET ensure transformation surfaces which are concave to the origin.

Workers are assumed to be competitive in that they treat all wage rates as exogenously given. For any given amount of composite labour supply X_j, workers with skill j select the combination of occupations X_{ij} which maximises their incomes. That is, workers choose the labour supply levels X_{ij} to maximise $\sum_i P_{ij} X_{ij}$ subject to (A.2.1), where P_{ij} is the wage rate for labour of occupation i and skill j. The problem can be rewritten as

choose $\overline{X}_{ij}$ to

[7] This appendix is adapted from Dixon et al. (1982), pp.74-76, 90-94. Note that, in order to maintain symmetry in the notation, the interpretation of the variables here is different from their interpretation in the Appendix A1.

maximise $\sum_i \bar{P}_{ij} \bar{X}_{ij}$

subject to $X_j = CET_i(\bar{X}_{ij})$,

where

$$\bar{P}_{ij} = P_{ij} / A_{ij}, \tag{A2.3}$$

and

$$\bar{X}_{ij} = X_{ij} A_{ij} \,. \tag{A2.4}$$

The solution to the workers' maximisation problem in percentage change form is given by

$$x_{ij} = x_j + \sigma_j (p_{ij} - \sum_k S_{kj} p_{kj}) - a_{ij} - \sigma_j (a_{ij} - \sum_k S_{kj} a_{kj}), \tag{A2.5}$$

where

$$\sigma_j = \frac{1}{(1+\rho_j)}$$

is the elasticity of transformation between different occupations for skill j. The weights S_{ij} used to compute the average of the percentage changes in the occupational wage rates for skill j are income shares defined by

$$S_{ij} = P_{ij} X_{ij} / \sum_k P_{kj} X_{kj} \,. \tag{A2.6}$$

If the variable P_j be defined by the equation

$$P_j X_j = \sum_i P_{ij} X_{ij} \,,$$

it follows by analogy with equation (A1.16), that

$$p_j = \sum_i S_{ij} p_{ij} + \sum_i S_{ij} a_{ij} \,. \tag{A2.7}$$

In MLME, it is again appropriate to set

$$\sum_i S_{ij} a_{ij} = 0. \qquad \text{(A2.8)}$$

Then the a_{ij}'s simulate the effects of occupation-i-expanding technical change for skill j. Other types of technical change which affect the mix of skills (rather than the mix of occupations within a skill) have already been included, at least in principle, in the labour supply forecasts provided by STOCKMOD.

If the convention of equation (A2.8) is adopted, equation (A2.7) implies that the percentage change in P_j is a weighted average of the percentage changes in the incomes of workers with skill j from the supply of units of labour to different occupations, the weights being the shares of each occupation in skill j's total labour income.

A3. The Percentage-Change Approach to Model Solution[8]

Many of the equations of CGEMOD are non-linear—demands depend on price ratios, for example. However, following Johansen (1960), CGEMOD is solved by representing it as a series of linear equations relating percentage changes in model variables. This appendix explains how the linearised form can be used to generate exact solutions of the underlying, non-linear, equations, as well as to compute linear approximations to those solutions.

A typical CGE model can be represented in the levels as:

$$\mathbf{F}(\mathbf{Y},\mathbf{X}) = \mathbf{0}, \qquad \text{(A3.1)}$$

where **Y** is a vector of endogenous variables, **X** is a vector of exogenous variables and **F** is a system of non-linear functions. The problem is to compute **Y**, given **X**. Normally **Y** cannot be written as an explicit function of **X**.

Several techniques have been devised for computing **Y**. The linearised approach starts by assuming that some solution to the system, $\{\mathbf{Y}^0,\mathbf{X}^0\}$ already exists, i.e.,

$$\mathbf{F}(\mathbf{Y}^0,\mathbf{X}^0) = \mathbf{0}. \qquad \text{(A3.2)}$$

[8] This appendix is adapted from Horridge (2000), pp.3-7.

Normally the initial solution $\{\mathbf{Y}^0,\mathbf{X}^0\}$ is drawn from historical data—the equation system is assumed to have been true for some point in the past. With conventional assumptions about the form of the **F** function it will be true that for small changes **dY** and **dX**:

$$\mathbf{F}_Y(\mathbf{Y},\mathbf{X})\mathbf{dY} + \mathbf{F}_X(\mathbf{Y},\mathbf{X})\mathbf{dX} = \mathbf{0}, \qquad \text{(A3.3)}$$

where $\mathbf{F}_Y$ and $\mathbf{F}_X$ are matrices of the derivatives of **F** with respect to **Y** and **X**, evaluated at $\{\mathbf{Y}^0,\mathbf{X}^0\}$. For reasons explained below, it is more convenient to express **dY** and **dX** as small percentage changes **y** and **x**. Thus y and x, some typical elements of **y** and **x**, are given by:

$$y = 100dY/Y \quad \text{and} \quad x = 100dX/X. \qquad \text{(A3.4)}$$

Correspondingly,

$$G_Y(\mathbf{Y},\mathbf{X}) = F_Y(\mathbf{Y},\mathbf{X})\hat{\mathbf{Y}} \quad \text{and} \quad G_X(\mathbf{Y},\mathbf{X}) = F_X(\mathbf{Y},\mathbf{X})\hat{\mathbf{X}}, \qquad \text{(A3.5)}$$

where $\hat{\mathbf{Y}}$ and $\hat{\mathbf{X}}$ are diagonal matrices. Hence the linearised system becomes:

$$\mathbf{G}_Y(\mathbf{Y},\mathbf{X})\mathbf{y} + \mathbf{G}_X(\mathbf{Y},\mathbf{X})\mathbf{x} = \mathbf{0}. \qquad \text{(A3.6)}$$

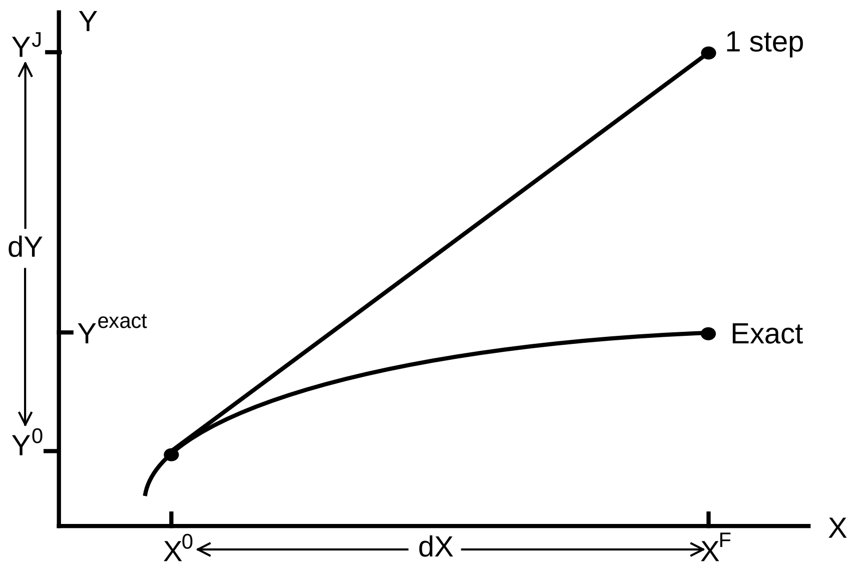


Figure A3.1. Linearisation error

Such systems are easy for computers to solve, using standard techniques of linear algebra. But they are accurate only for small changes in **Y** and **X**. Otherwise, linearisation error may occur. The error is illustrated by Figure A3.1, which shows how some endogenous variable Y changes as an exogenous variable X moves from X^0 to X^F. The true, non-linear relation between X and Y is shown as a curve. The linear, or first-order, approximation:

$$y = -\mathbf{G}_Y(\mathbf{Y},\mathbf{X})^{-1}\mathbf{G}_X(\mathbf{Y},\mathbf{X})x \qquad \text{(A3.7)}$$

leads to the Johansen estimate Y^J—an approximation to the true answer, Y^{exact}.

Figure A3.1 suggests that, the larger is x, the greater is the proportional error in y. This observation leads to the idea of breaking large changes in X into a number of steps, as shown in Figure A3.2. For each sub-change in X, the linear approximation is used to derive the consequent sub-change in Y. Then, using the new values of X and Y, the coefficient matrices $\mathbf{G}_Y$ and $\mathbf{G}_X$ are recomputed. The process is repeated for each step. If 3 steps are used (see Figure A3.2), the final value of Y, Y^3, is closer to Y^{exact} than was the Johansen estimate Y^J. It can be shown, in fact, that given sensible restrictions on the derivatives of **F**(**Y**,**X**), a solution as accurate as desired can be obtained by dividing the process into sufficiently many steps.

The technique illustrated in Figure A3.2, known as the Euler method, is the simplest of several related techniques of numerical integration—the process of using differential equations (change formulae) to move from one solution to another. The GEMPACK software package offers the choice of several such techniques. Each requires the user to supply an initial solution $\{\mathbf{Y}^0,\mathbf{X}^0\}$, formulae for the derivative matrices $\mathbf{G}_Y$ and $\mathbf{G}_X$, and the total percentage change in the exogenous variables, **x**. The levels functional form, **F**(**Y**,**X**), need not be specified, although it underlies $\mathbf{G}_Y$ and $\mathbf{G}_X$.

The accuracy of multistep solution techniques can be improved by extrapolation. Suppose the same experiment were repeated using 4-step, 8-step and 16-step Euler computations, yielding the following estimates for the total percentage change in some endogenous variable Y:

y(4-step) = 4.5%,
y(8-step) = 4.3% (0.2% less), and
y(16-step) = 4.2% (0.1% less).

Extrapolation suggests that the 32-step solution would be:

y(32-step) = 4.15% (0.05% less),

and that the exact solution would be:

y(∞-step) = 4.1%.

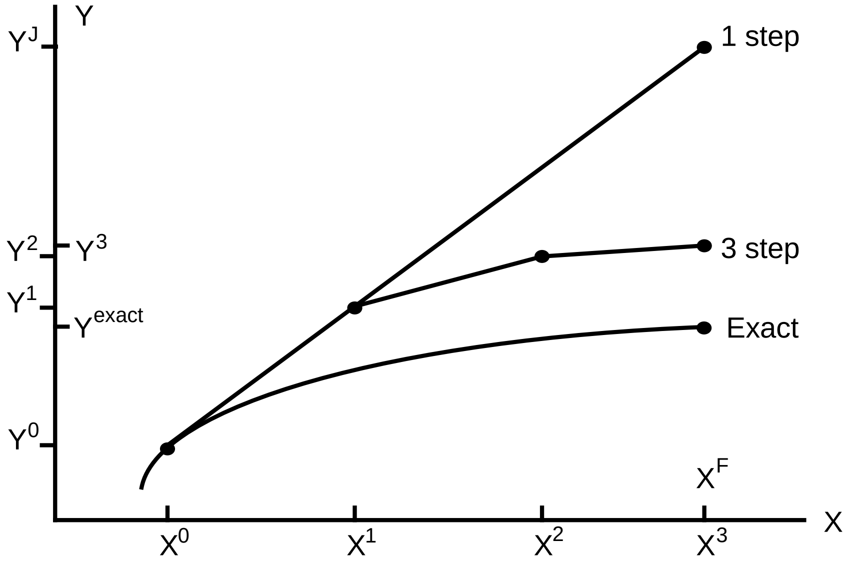


Figure A3.2. Multistep process to reduce linearisation error

The extrapolated result requires 28 (= 4+8+16) steps to compute but would normally be more accurate than that given by a single 28-step computation. Alternatively, extrapolation given accuracy to be obtained with fewer steps. As noted above, each step of a multi-step solution requires: computation from data of the percentage-change derivative matrices $\mathbf{G}_Y$ and $\mathbf{G}_X$; solution of the linear system (6); and use of that solution to update the data (**X**,**Y**).

In practice, for typical CGE models, it is unnecessary, during a multistep computation, to record values for every element in **X** and **Y**. Instead, a set of *data coefficients* **V** can be defined which are functions of **X** and **Y**, i.e., **V** = **H**(**X**,**Y**). Most elements of **V** are simple cost or expenditure flows such as appear in input-output tables. $\mathbf{G}_Y$ and $\mathbf{G}_X$ turn out to be simple functions of **V**; indeed they are often identical to elements of **V**. After each small change, **V** is updated using the formula

$$\mathbf{v} = \mathbf{H}_Y(\mathbf{X},\mathbf{Y})\mathbf{y} + \mathbf{H}_X(\mathbf{X},\mathbf{Y})\mathbf{x}.$$

The advantages of storing **V**, rather than **X** and **Y**, are twofold:

- the expressions for $\mathbf{G}_{\mathrm{Y}}$ and $\mathbf{G}_{\mathrm{X}}$ in terms of **V** tend to be simple, often far simpler than the original **F** functions; and
- there are fewer elements in **V** than in **X** and **Y** (e.g., instead of storing prices and quantities separately, only their products (the values of commodity or factor flows) are stored

Apart from its simplicity, the linearised approach has three further advantages.

- It allows free choice of which variables are to be exogenous or endogenous. Many levels algorithms do not allow this flexibility.
- To reduce CGE models to manageable size, it is often necessary to use model equations to substitute out matrix variables of large dimensions. In a linear system, any variable can always be made the subject of any equation in which it appears. Hence, substitution is a simple mechanical process. In fact, because GEMPACK performs this routine algebra for the user, the model can be specified in terms of its original behavioural equations, rather than in a reduced form. This reduces the potential for error and makes model equations easier to check.
- Perhaps most importantly, the linearized equations contribute to an understanding simulation results. In particular, the contribution of (the change in) each RHS variable to the LHS of each equation can be easily identified.

The discussion of the solution procedure has so far assumed that an initial solution of the model—$\{\mathbf{Y}^0,\mathbf{X}^0\}$ or the equivalent $\mathbf{V}^0$—exists, and that results show percentage deviations from this initial state. In CGEMOD, historical data for the base period is used for the first period of the forecast, that is, the deviations represent the changes that occur during the first period. These deviations are then used to update the base period dataset to period 1. The updated dataset provides the initial solution for period 2 and so on.

A4. Extending the Base Year Database for CGEMOD

From the E3ME database, the following matrices are known for the base year 2009:

A_{ijk}^{01} - employment measured in persons (i=1,..,41; j=1,..,27; k=1,..,3),

A_{i}^{02} - hours worked per person per year (i=1,..,41),

A_{i}^{03} - wage rates per person per year (i=1,..,41).

From Stehrer and Ward (2011), the following vector is also known:

A_j^{04} - relative wage rates expressed as percentages of the average wage rate (i=1,..,41).

Here the indices *i*, *j* and *k* refer to industries, occupations and skill groups, respectively. This appendix sets out a sequence of computational steps whereby the following additional matrices can be derived:

- employment by industry and occupation measured in hours,
- wage costs by industry and occupation,
- employment by occupation and skill measured in hours, and
- wage incomes by occupation and skill.

1. Employment levels measured in hours are computed according to

$$A_i^{05} = A_i^{02} \sum_{j=1}^{27} \sum_{k=1}^{3} A_{ijk}^{01} \qquad \text{(i=1,..,41).}$$

This step assumes that the number of hours worked per person per year depends only on the industry in which the person works.

2. Wage incomes per person per year are computed according to

$$A_i^{06} = A_i^{03} \sum_{j=1}^{27} \sum_{k=1}^{3} A_{ijk}^{01} \qquad \text{(i=1,..,41).}$$

This step provisionally assumes that the wage rate per person per year depends only on the industry in which the person works.

3. Wage rates per person per hour by industry are computed according to

$$A_i^{07} = A_i^{06} / A_i^{05} \qquad \text{(i=1,..,41).}$$

4. The average wage rate per person per hour is computed according to

$$A^{08} = \sum_{i=1}^{41} A_i^{06} / \sum_{i=1}^{41} A_i^{05} .$$

5. Wage rates per person per hour by occupation are computed according to

$$A_j^{09} = A^{08} A_j^{04} / 100 \qquad \text{(j=1,..,27).}$$

6. An industry-by-occupation employment matrix measured in hours is computed according to

$$A_{ij}^{10} = A_i^{02} \sum_{k=1}^{3} A_{ijk}^{01} \qquad \text{(i=1,..,41; j=1,..,27).}$$

As for Step 1, this step assumes that the number of hours worked per person per year depends only on the industry in which the person works.

7. An industry-by-occupation matrix A_{ij}^{11} (i=1,..,41; j=1,..,27) of wage costs is computed using the RAS method.

   The starting matrix is A_{ij}^{10} (i=1,..,41; j=1,..,27).

   The industry (row) targets are $A_i^{07} \sum_{j=1}^{27} A_{ij}^{10}$ (i=1,..,41).

   The occupation (column) targets are $A_j^{09} \sum_{i=1}^{41} A_{ij}^{10}$ (j=1,..,27).

8. The number of hours worked per person per year by occupation is computed according to

$$A_j^{12} = \sum_{i=1}^{41} A_{ij}^{10} \Big/ \sum_{i=1}^{41} \sum_{k=1}^{3} A_{ijk}^{01} \qquad \text{(j=1,..,27).}$$

9. A provisional occupation-by-skill employment matrix measured in hours is computed according to

$$A_{jk}^{13} = A_j^{12} \sum_{i=1}^{41} A_{ijk}^{01} \qquad \text{(j=1,..,27; k=1,..,3).}$$

10. A revised occupation-by-skill employment matrix is computed according to

$$A_{jk}^{14} = A_{jk}^{13} \sum_{i=1}^{41} A_{ij}^{11} \Big/ \sum_{k=1}^{3} A_{jk}^{13} \qquad \text{(j=1,..,27; k=1,..,3).}$$

    This revision brings the occupation-by-skill employment matrix into conformity with the industry-by-occupation matrix A^{11}.

11. Wage rates per person per hour by occupation are computed according to

$$A_j^{15} = \sum_{i=1}^{41} A_{ij}^{11} \Big/ \sum_{i=1}^{41} A_{ij}^{10} \qquad \text{(j=1,..,27).}$$

    This step assumes that occupational wage rates are independent of skill.

12. A provisional occupation-by-skill matrix of wage incomes is computed according to

$$A_{jk}^{16} = A_j^{15}\, A_{jk}^{14} \qquad \text{(j=1,..,27; k=1,..,3).}$$

13. The provisional wage incomes matrix is revised according to

$$A_{jk}^{17} = A_{jk}^{16} \sum_{i=1}^{41} A_{ij}^{11} \Big/ \sum_{k=1}^{3} A_{jk}^{16} \qquad \text{(j=1,..,27; k=1,..,3).}$$

This revision brings the occupation-by-skill income matrix into conformity with the industry-by-occupation cost matrix A^{11}.

The required additional matrices are given by A^{10}, A^{11}, A^{14} and A^{17}, respectively.

A5. HTOPMOD – A Module for Converting CGEMOD Employment Forecasts from Hours to Persons

From the E3ME forecasts, the following matrices are known for each year

A_{ijk}^{01} - employment measured in persons (i=1,..,41; j=1,..,27; k=1,..,3),

A_{i}^{02} - hours worked per person per year (i=1,..,41),

As before, the indices *i*, *j* and *k* refer to industries, occupations and skill groups, respectively. On the assumption that the number of hours worked per person per year depends only on the industry in which a person works, a three dimensional employment matrix (measured in hours) can be computed from

$$A_{ijk}^{03} = A_{i}^{02} A_{ijk}^{01} \qquad \text{(i=1,..,41; j=1,..,27; k=1,..,3).}$$

Hence an industry-by-occupation matrix of hours worked per person per year can be computed from

$$A_{ij}^{04} = \sum_{k=1}^{3} A_{ijk}^{03} \Big/ \sum_{k=1}^{3} A_{ijk}^{01} \qquad \text{(i=1,..,41; j=1,..,27),}$$

and an occupation-by-skill matrix of hours worked per person per year can be computed from

$$A_{jk}^{05} = \sum_{i=1}^{41} A_{ijk}^{03} \Big/ \sum_{i=1}^{41} A_{ijk}^{01} \qquad \text{(j=1,..,27; k=1,..,3).}$$

The matrices A^{04} and A^{05} are just the matrices A^{D} and A^{S}, respectively, referred to in Section 4.

From the CGEMOD forecasts, the following matrices are known for each year:

A_{ij}^{06} - employment measured in hours (i=1,..,41; j=1,..,27),

A_{jk}^{07} - employment measured in hours (j=1,..,27; k=1,..,3).

This appendix sets out a sequence of computational steps whereby the CGEMOD forecasts can be converted from hours to persons.

1. A provisional industry-by-occupation employment matrix measured in persons is computed according to

$$A_{ij}^{08} = A_{ij}^{06} / A_{ij}^{04} \qquad \text{(i=1,..,41, j=1,..,27).}$$

2. A provisional occupation-by-skill employment matrix measured in persons is computed according to

$$A_{jk}^{09} = A_{jk}^{07} / A_{jk}^{05} \qquad \text{(j=1,..,27;k=1,..,3).}$$

3. A revised occupation-by-skill employment matrix A_{jk}^{10} (j=1,..,27; k=1,..,3) is computed using the RAS method.

   The starting matrix is $\sum_{i=1}^{41} A_{ijk}^{01}$ (j=1,..,27; k=1,..,3).

   The occupation (row) targets are $\sum_{i=1}^{41} A_{ij}^{08}$ (j=1,..,27).

   The skill (column) targets are $\sum_{j=1}^{27} A_{jk}^{09}$ (k=1,..,3).

   The two matrices A^{08} and A^{10} together comprise the E3ME-MLME forecast of employment measured in persons if it is assumed that the entire adjustment to the number of hours worked per person occurs on the supply side of the labour market. This choice is represented by the first column in Table 5. In the notation of Section 4, the revised hours worked matrices are given by

$$B_{ij}^{D} = A_{ij}^{06} / A_{ij}^{08} \qquad \text{(i=1,..,41, j=1,..,27),}$$

$$B_{jk}^{S} = A_{jk}^{07} / A_{jk}^{10} \qquad \text{(j=1,..,27; k=1,..,3).}$$

4. A revised industry-by-occupation employment matrix A_{ij}^{11} (i=1,..,41;j=1,..,27) is computed using the RAS method.

   The starting matrix is $\sum_{k=1}^{3} A_{ijk}^{01}$ (i=1,..,41;j=1,..,27).

   The industry (row) targets are $\sum_{j=1}^{27} A_{ij}^{08}$ (i=1,..,41).

The occupation (column) targets are $\sum_{k=1}^{3} A_{jk}^{09}$ (j=1,27).

The two matrices A^{11} and A^{09} together comprise the E3ME-MLME forecast of employment measured in persons if it is assumed that the entire adjustment to the number of hours worked per person occurs on the demand side of the labour market. This choice is represented by the second column in Table 5. In the notation of Section 4, the revised hours worked matrices are given by

$$B_{ij}^{D} = A_{ij}^{06} / A_{ij}^{11} \quad \text{(i=1,..,41, j=1,..,27),}$$

$$B_{jk}^{S} = A_{jk}^{07} / A_{jk}^{09} \quad \text{(j=1,..,27; k=1,..,3).}$$

5. A revised industry-by-occupation employment matrix A_{ij}^{12} (i=1,..,41;j=1,..,27) is computed using the RAS method.

   The starting matrix is $\sum_{k=1}^{3} A_{ijk}^{01}$ (i=1,..,41;j=1,..,27).

   The industry (row) targets are $\sum_{j=1}^{27} A_{ij}^{08}$ (i=1,..,41).

   The occupation (column) targets are $0.5\left(\sum_{i=1}^{41} A_{ij}^{08} + \sum_{k=1}^{3} A_{jk}^{09}\right)$ (j=1,..,27).

6. A revised occupation-by-skill employment matrix A_{jk}^{13} (j=1,..,27; k=1,..,3) is computed using the RAS method.

   The starting matrix is $\sum_{i=1}^{41} A_{ijk}^{01}$ (j=1,..,27; k=1,3).

   The occupation (row) targets are $0.5\left(\sum_{i=1}^{41} A_{ij}^{08} + \sum_{k=1}^{3} A_{jk}^{09}\right)$ (j=1,..,27).

   The skill (column) targets are $\sum_{j=1}^{27} A_{jk}^{09}$ (k=1,..,3).

   The two matrices A^{12} and A^{13} together comprise the E3ME-MLME forecast of employment measured in persons if it is assumed that the adjustment to the number of hours worked per person is distributed equally between the demand and supply sides of the labour market. This choice is represented by the third column in Table 5. It is also the choice underlying Tables 6 to 10. In the notation of Section 4, the revised hours worked matrices are given by

$$B_{ij}^{D} = A_{ij}^{06} / A_{ij}^{12} \qquad (i=1,..,41, j=1,..,27),$$

$$B_{jk}^{S} = A_{jk}^{07} / A_{jk}^{13} \qquad (j=1,..,27; k=1,..,3).$$

References

Dixon, P.B., B.R.Parmenter, John Sutton and D.P.Vincent (1982). *ORANI: A Multisectoral Model of the Australian Economy*, North-Holland, Amsterdam.

Horridge, J.M. (2000), "ORANI-G: A General Equilibrium Model of the Australian Economy", CoPS/IMPACT Working Paper Number OP-93, Centre of Policy Studies, Monash University, downloadable from http://www.monash.edu.au/policy/elecpapr/op-93.htm.

Johansen, L. (1960). *A Multisectoral Study of Economic Growth*, North-Holland, Amsterdam.

Meagher, G.A., Pang, F., Wilson, R.A. (2012), " Interfacing a CGE Labour Market Model with the E3ME Multi-sector macroeconomic Model" , Technical Paper No. 13, CEDEFOP Project on Forecasting Skill Supply and Demand in Europe 2020. Cedefop, Thessaloniki.

Wilson, R.A. et al. (2010), "Forecasting skill supply and demand in Europe: Overview and Synthesis", report prepared for the European Centre for the Development of Vocational Training (CEDEFOP), tender No. AO/RPA/AZU-TODUN/European-skills-forecasts/009/08, Thessloniki.