

# Integrating a Global Supply Chain Model With a Computable General Equilibrium Model

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# **Integrating a global supply chain model with a computable general equilibrium model**

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## **Abstract**

Global supply chain (GSC) trade results from decisions by firms producing final goods to allocate underlying tasks to dedicated facilities in different countries. These decisions create cross-border flows of products at various stages of completion. We demonstrate a divide-and-conquer approach to integrating GSC and computable general equilibrium (CGE) models: the models are solved separately and information is passed between them. A stylized integrated model suggests that by providing low-skilled jobs in developing countries, GSC trade accelerates the transfer of labour out of low-marginal-productivity agriculture in these countries into higher-marginal-productivity manufacturing. At the same time, GSC trade can leave high-income countries having to transfer considerable fractions of their workforce out of manufacturing and into services. After potentially expensive structural adjustment, high-income countries may be left in the long run with no more than a small equilibrium welfare gain or even a loss.

***JEL: F12; C68; C63***

***Key words: Global supply chain trade; computable general equilibrium; CSC-CGE integration; benefits/costs of GSC***

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## 1. Introduction

GSC trade is a new type of trade that has developed over the last 30 years. It is trade resulting from decisions by firms producing final goods (such as Apple iPhones) to allocate underlying tasks (such as design, component production and assembly) to dedicated facilities in different countries. These decisions create cross-border flows of products at various stages of completion (e.g. iPhone components produced in Thailand and Vietnam sent to China for assembly). GSCs now account for more than half the world's trade in manufactured products (Athukorala and Talgaswatta, 2016).

Economists have responded to the challenge of understanding GSC trade with studies that can be classified as descriptive, input-output, and econometric/theoretical.

An early prominent contribution to the descriptive literature was Stan Shih's (1996) "smiling curve". Shi produced a smile by drawing a curve that relates aspects of value added such as wage rates and profitability to stage in the supply chain. The smile comes about because high value added occurs at the early stages (design and planning) and at the late stages (advertising and sales), while the middle stages (manufacturing and assembly) consist of a large number of separate processes all with low value added characteristics. Firms that undertake activities at the two ends of the supply chain are mainly in developed countries while the middle chain activities are mainly in developing countries. The smiling curve may explain why some countries are pursuing development plans that emphasize early stage activities, especially research, see for example Fang and Walsh's (2018) description of China's MIC2025 plan. Follow up studies using the smiling curve include Hallward-Driemeier & Nayyar (2017), Chen (2004) and Shin *et al.* (2012). The case-study approach is also prominent in descriptive GSC literature. Examples of such studies are Grapper's (2007) description of production sharing arrangements for Boeing's 787 Dreamliner, Dedrik *et al.*'s (2010) study of profit sharing in global production of iPod and notebook PCs, and the studies of the global semiconductor industry by Grunwald & Flamm (1985) and Brown & Linden's (2005). Another strand of the descriptive literature on GSC trade provides statistics on its prevalence, see for example Athukorala (2011), Yeats (2001) and Athukorala & Talgaswatta (2016).

GSCs create situations in which value added generated in one country makes multiple border crossings, including returning to the country from which it originated (e.g. Vietnamese labour embodied in Apple iPhone components exported to China and then returned to Vietnam embodied in assembled iPhones purchased by Vietnamese households). Input-output models can be used to describe the value-added contributions from different countries embedded in each trade flow. Politically significant recalculations of bilateral trade balances can then be made in value-added terms. GSC input-output studies include Amador and Cabral (2017), Dean *et al.* (2011), Koopman, *et al.* (2014), Mattoo *et al.* (2013), Johnson and Noguera (2012) and Productivity Commission (2015).

Theorists and econometricians have measured inter-country differences in participation in GSC trade and the nature of the tasks in GSCs that are allocated to countries at different stages of development. These studies often adopt a gravity framework, see for example, Athukorala (2009) and Athukorala & Yamashita (2006 & 2009). Hanson *et al.* (2005), Golub *et al.* (2007), Baldwin & Taglioni (2011) and Okubo *et al.* (2014) show how econometric equations can be formulated to explain trade in the presence of GSCs. Other authors have investigated, theoretically and econometrically, factors underlying the growth of GSC trade such as changes in the relative costs of *inter*-firm transactions versus *intra*-firm coordination [see, for example, Antràs & Chor (2013) and Grossman & Rossi-Hansberg (2008 & 2013)] and the role of GSC trade in the overall growth in trade [e.g. Yi (2003)].

This paper is in the third category of GSC studies, primarily theoretical but with the theory illustrated by suggestive numerical simulations. Perhaps closest to our paper are Antràs & de Gortari (2017) and Fally & Hillberry (2018). Both these papers rely on simulations with stylized data to illustrate properties of theoretical models.

Antràs & de Gortari (2017, hereafter A&deG) develop an algebraic and numerical model in which there is one final good produced in  $N$  stages. The final good is consumed in  $J$  countries, with the level of consumption in each country set exogenously. The stage-1 good can be produced in any country under constant returns to scale using only labour supplied by residents of that country. The stage- $n$  good,  $n > 1$ , can be produced in any country using a Cobb-Douglas constant-returns-to-scale combination of the country's own labour and an intermediate input consisting of the stage- $(n-1)$  good supplied by any of the  $J$  countries. A&deG assume that markets are purely competitive at each production stage. Thus, prices equal costs. Accordingly, the purchaser's price of a stage- $n$  good supplied to country  $j$  from country  $k$  is a combination of: the stage- $n$  unit labour cost in  $k$  [ $k$ 's wage rate,  $W_k$ , divided by  $k$ 's stage- $n$  productivity variable,  $A_{n,k}$ ]; the purchaser's price in  $k$  of the  $(n-1)$  good; and the trade cost applying to a  $k$ -to- $j$  flow of the stage- $n$  good. On the assumption that demanders of the stage- $n$  good in country  $j$  [that is stage- $(n+1)$  producers if any or final users if  $n = N$ ] always buy at what to them is the lowest price, the A&deG model can be solved by two sequences of calculations: a price sequence and a quantity sequence.

The price sequence starts by determining the purchaser's price in each country and the supplier to that country of the stage-1 good. These can be found for country  $j$  by evaluating  $\min_k (W_k * \tau_{kj} / A_{1,j})$  where  $\tau_{kj}$  is the power of trade costs applying to  $k$ -to- $j$  trade, assumed to be the same for goods at all stages. Once the stage-1 purchaser's price in each country is known, we can combine this with stage-2 unit labour costs and trade costs to determine the supplying country and purchaser's price in each demanding country of the stage-2 good, and so on. The quantity sequence starts by calculating the output of the stage- $N$  good in each country  $k$  by adding up final demands in countries to which  $k$  is the  $N$ -stage supplier. This allows the output of the stage- $(N-1)$  good in each country  $k$  to be calculated taking account of the outputs of the stage- $N$  good in countries to which  $k$  is the  $N-1$  supplier, and so on.

A&deG solve the price sequence 13 million times for a 4-country, 4-stage version of their model: one million assumptions for the vector of 16 unit labour costs,  $W_k/A_{n,k}$ ,  $n, k = 1, \dots, 4$ , *times* 13 assumptions for the vector of powers of trade costs between countries. The million vectors of unit labour costs are obtained by taking random draws from a log normal distribution. In making these draws, A&deG treat countries symmetrically: for a given  $n$ ,  $W_k/A_{n,k}$  is drawn from the same distribution for all  $k$ . The 13 vectors of trade costs are obtained by adopting 13 values of  $s$  from 0 to 50 in the equation

$$\tau_{jk} = 1 + s * (\bar{\tau}_{jk} - 1) = \tau_{kj} \quad (1.1)$$

where  $\bar{\tau}_{jk}$  equals  $\bar{\tau}_{kj}$ ,  $j, k = 1, \dots, 4$ , gives an underlying structure of trade costs that applies in all 13 million solutions. A&deG adopt a structure that makes trade costs: zero within countries; relatively low between countries 1 and 2 and between countries 3 and 4; and high between any country in the 1, 2 group and any country in the 3, 4 group. This is done by setting  $\bar{\tau}_{12} = 1.3$ ,  $\bar{\tau}_{34} = 1.3$ ,  $\bar{\tau}_{13} = 1.8$ ,  $\bar{\tau}_{14} = 1.75$ ,  $\bar{\tau}_{23} = 1.5$ ,  $\bar{\tau}_{24} = 1.8$ . A&deG refer to countries 1 and 2 as one region, and 3 and 4 as another region.

With this setup, A&deG calculate for each value of  $s$  and each country  $j$  the likelihood (proportion of the million solutions) that  $j$  makes a value-added contribution in the supply chain that delivers the final good to a particular country, country 4 in their example. They

use these calculations to build up pictures of how the nature of the global supply chain satisfying the demand for the final good in country 4 depends on trade costs. If trade costs are zero ( $s = 0$ ), then A&deG find in about 2/3rds of their million cases that country 1 is in the supply chain that satisfies 4's final demand. Similarly, in about 2/3rds of the cases, countries 2, 3 and 4 are in the supply chain that satisfies 4's final demand. With zero trade costs and symmetric treatment of unit labour costs, each country is equally well placed to be in the supply chain that terminates in country 4 (referred to as 4's supply chain). As trade costs increase ( $s$  rises from zero) the probability of participation in 4's supply chain by countries outside 4's region falls rapidly. Eventually the probability of participation by 4's regional partner, country 3, also drops away. If trade costs are very high ( $s = 50$ ) then 1, 2 and 3 never contribute to the supply chain that terminates in 4: the supply chain for 4 is always purely domestic.

From other applications of their model, A&deG explain that centralized nations are likely to specialize in products down the supply chain. This is because with trade costs being *ad valorem*, the rate of trade cost applying to value added in downstream stages of production can be very high. Thus, there is an incentive to avoid trade in downstream products between pairs of peripheral nations separated by high *ad valorem* trade costs reflecting long distances.

Another conclusion from A&deG's model is that their specification of supply chains is unlikely to lead to models that produce results for the welfare value of trade that are much different from models without trade in intermediate inputs.

Whereas A&deG's model is partial equilibrium, the model by Fally and Hillberry (2018, hereafter F&H) is general equilibrium with sufficient structure to determine wage rates and final demands in each country endogenously. Rather than consuming a single final good, the household in country  $k$  in F&H's model chooses consumption levels for a continuum of final goods in a set  $\Omega_k$  to maximize a Cobb-Douglas utility function subject to a budget constraint that imposes zero trade balance.

Each final good consumed in country  $k$  is produced in a sequence of stages in which labour is combined with the product of the previous stage in a constant returns to scale production function (Leontief for F&H rather than Cobb-Douglas). This is similar to the production of the single final good in A&deG's model. An important difference between the F&H and A&deG models is F&H's focus on firms. For A&deG, a country or countries undertake the production of the discrete stage- $n$  commodity. By contrast, F&H visualize the stages in the production of a final good for country  $k$  as a continuum of stages or tasks along a line with sub-segments of the line being allocated to firms in country  $j$ . An assumption that simplifies F&H's mathematics is that a firm can contribute to only one supply chain. While the production by a firm of any given good in its sub-segment exhibits constant returns to scale, a firm experiences diminishing returns if it increases its output through an increase in the length of its sub-segment (scope). By assuming that firms in country  $j$  are identical *ex ante*, F&H ensure that the sub-segment of tasks allocated to any particular firm is continuous.

As in A&deG's model, cost-minimizing optimization problems play a key role in F&H's model. For F&H, these determine for each final good consumed in country  $k$  the segments of the task line allocated to each country  $j$  and within a  $j$ -segment, the division into sub-segments to be undertaken by different firms. In the F&H setup, there is always a finite number of segments and sub-segments for each final good delivered to  $k$  and each segment and sub-segment is of non-zero length (scope).

The key innovation in the F&H model is the use of parameters for each country to reflect intra-firm co-ordination costs (costs associated with managing diverse production activities in

a single firm) and inter-firm transaction costs (costs associated with the sale of a product from one firm to another). In combination, these parameters underlie the determination of the size of firms (length of their sub-segments) in country  $j$  in the supply chain that delivers a final good to country  $k$ . If intra-firm co-ordination costs in  $j$  are low relative to inter-firm transaction costs, then firms in  $j$  will tend to be large, and vice versa. As in A&deG's model for F&H the allocation of tasks to countries depends on unit labour costs and trade costs.

F&H experiment numerically with a 10-country version of their model (the U.S. and 9 Asian countries). For each country, they introduce data for aggregate employment. For final goods consumed in country  $k$  they approximate a continuum by 100,000 discrete goods, making 1 million supply chains for their 10-country world economy. For each of these million supply chains, country  $j$  has a labour productivity variable  $A_j(\omega)$  where  $\omega$  identifies a supply chain. The  $A_j(\omega)$ s are determined by a million draws from a probability distribution with a mean calibrated in accordance with country-specific productivity levels. F&H tie down the value of the inter-firm transaction cost parameter in each country by reference to a World Bank index for the costs of doing business. For the power of trade costs, F&H assume the same value for all products and any pair of countries  $k$  and  $j$ ,  $k \neq j$ . This value is obtained by parameterizing so that their model implies a realistic ratio of aggregate world trade to world output.

F&H's method for setting intra-firm co-ordination costs is similar to their method for the labour productivity variables. For every country  $j$  and all million supply chains  $\omega$ , F&H determine a co-ordination cost variable  $\theta_j(\omega)$  by making a draw from a probability distribution. For country  $j$ , they set the mean of the distribution with reference to a measure derived from input-output tables of  $j$ 's average position (from upstream to downstream) in world-wide supply chains. The underlying idea exploits the correlation in F&H's theory between the average position in supply chains of a country and the country's average co-ordination cost across all supply chains. If co-ordination costs are on average high in country  $j$  relative to  $j$ 's inter-firm transaction costs then, on this account,  $j$  will tend to make its value-added contribution through small firms at the upstream end of supply chains. This is because F&H assume inter-firm transaction costs are *ad valorem* which means it is costly to transfer downstream products, that have accumulated considerable value, from one firm to another. Hence, downstream production requires minimization of inter-firm transfers through the use of large firms which have relatively low intra-firm co-ordination costs.

With all the data and parameter values in place, F&H carry out several counterfactual simulations. The first of these demonstrates the role of reduced trade costs in explaining the growth of international supply chains. Consistent with the emergence of GSC trade, F&H show that an across-the-board decline in trade costs increases the import content and reduces direct value-added shares in the gross values of exports from all countries. F&H's second counterfactual simulation is about the effects of China's productivity catch-up. This simulation shows that a uniform percentage increase in labour productivity in China across all supply chains [uniform improvement  $A_{\text{Chn}}(\omega)$  for all  $\omega$ ] on average moves China downstream in international supply chains and other countries upstream. The explanation is that with greater productivity, a more wealthy China consumes a greater proportion of the world's output of final goods. Because exports of downstream goods carry high trade costs relative to value added contributed at downstream stages of their production, China's increased consumption requires relocation of downstream activities towards China and away from other countries. Worldwide welfare gains from China's improved productivity accrue almost entirely to China. In another counterfactual simulation, F&H reduce inter-firm transaction costs in China. This has the effect of lengthening supply chains in China and

elsewhere. With regard to potentially observable statistics, the effects are increased average distances from production to final use (measured by number of firms through which a product passes) and, for China, a higher share of direct value added in its exports reflecting longer country segments on supply-chain task lines. The worldwide welfare benefits of reductions in Chinese transaction costs accrue mainly to China, but the Chinese share is considerably lower than for increases in Chinese labour productivity.

The F&H model and other contributions to the GSC literature have given us valuable insights on: why GSCs have emerged; the position of countries in GSCs; and the distribution between countries of welfare gains from GSC trade. So what is the next step?

We think a policy-relevant direction is the development of computable general equilibrium (CGE) models that embrace GSC specifications for relevant sectors such as motor vehicles and electronics. While F&H and other modellers provide impressive treatments of GSC sectors, their specifications of other aspects of the economy are rudimentary. For example, in the F&H model, there is no investment or capital accumulation, no governments or taxes except those embedded in trade costs, no non-traded services, no land or other natural resources, no balance of payments accounting or treatment of foreign assets and liabilities, and no occupational or regional barriers to labour mobility between different employment activities. All of these phenomena have been included in CGE models. An integrated dynamic GSC-CGE model would provide simulations of adjustment paths recognizing investment-capital links and connections between current account balances and the accumulation of financial assets and liabilities. The effects on GSC trade of shocks outside the GSC sector (such as the imposition of a value-added tax or improved agricultural productivity) could be analysed in an integrated model.

In view of contemporary political discussions of GSC trade, perhaps the most important potential contribution of an integrated GSC-CGE model would be to throw light on the effects of GSC trade taking account of labour markets that work differently in different parts of the world. How does GSC trade affect the occupational and regional composition of employment in each country? How does GSC trade affect wage rates by occupation and by educational level? Within each country, does GSC trade lead to reductions or increases in inequality? Do free trade agreements help or hinder GSC trade? What are the implications of anti-trade policies by the U.S. for participation by China and other developing countries in global production sharing? In broad terms, the aim of an integrated GSC-CGE model would be to help us understand the international allocation of the welfare benefits and adjustment costs generated by GSC trade.

In this paper, we construct a stylized integrated GSC-CGE model. Section 2 describes a 1-sector GSC model and a multi-country CGE model and then describes the theory of an integrated GSC-CGE model. Section 3 sets out a database for a 1-sector, 4-stage, 2-region GSC model and shows how this database is reflected in the database for a 2-region CGE model in which the GSC sector is represented as a single commodity/industry. The GSC sector produces Widgets through four activities (stages): Design; Components; Assembly and Sales distribution. In the databases for both models, one region referred to as the U.S. has much higher productivity in all activities than the other region referred to as Asia. In section 4 we use the U.S.-Asia CGE database from section 3 in generating a standard CGE baseline forecast, and in section 5 we use the GSC database from section 3 in generating a stand-alone baseline forecast for the GSC sector. In section 6, we make our first attempt to integrate the CGE and GSC models in an algorithm that passes economy-wide results from the CGE model to the GSC model, and GSC-sector results from the GSC model to the CGE model. As analysed in section 7, this first attempt reveals a hole in the demand curve for labour in

Asia preventing convergence. In section 8 we introduce surplus labour in Asia. This leads to a converged GSC-CGE baseline forecast which we compare with the standard CGE forecast. Conclusions are in section 9.

## **2. Description of the global supply chain (GSC) and computable general equilibrium (CGE) models**

Subsection 2.1 sets out the theory of a 1-sector GSC model. This model is similar to that of A&deG. However, we allow for economies of scale at each stage of production and introduce a global agent that allocates production activities to different countries and determines trade flows. Subsection 2.2 is an outline of a standard multi-region CGE model focusing on the form of the database. In subsection 2.3 we show how the two models can be integrated. The GSC model is for what we call the Widget sector. Widget variables in both models are denoted with a W subscript. To avoid confusion between which variables belong to which model we attach a gsc superscript to GSC variables and a cge superscript to CGE variables.

### ***2.1. A global supply chain model***

We define a GSC model as a mathematical system describing world-wide output and trade for a particular sector, such as Motor vehicles or Electronic equipment. We see as an essential characteristic of a GSC model optimizing behaviour by one or more agents who take a global perspective in deciding which activities within the sector to locate in different countries and which sectoral products to trade between countries. By activities within a sector we mean Design, production of different Components, Assembly and Sales & distribution of final goods.

We consider a comparatively simple GSC model for the Widget sector in which there is a single global optimizing agent and  $N$  regions.<sup>1</sup> We assume that the Widget sector provides a single final good to the rest of the economy in each region. This single final good is created in a non-traded activity, which we refer to as SalesDist. The input to SalesDist consists of an Assembled product plus labour and possibly other inputs. The Assembled product and constituent Components and Design are readily tradable.

While this model is simple, the required notation for setting it out mathematically is challenging. In what follows we ease the notational burden by providing italic notes beneath the algebraic expressions. Then we give formal definitions and explanations.

In our global supply chain model we assume that for all  $r \in \mathbb{R}$ , the global optimizing agent chooses:

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<sup>1</sup> The GCS Widget model that we describe here is the same as that in Athukorala *et al.* (2018), but we take the analysis and application of it in a different direction. Whereas the earlier paper focused on the properties of the GSC model, this paper is about integrating the GSC model with a CGE model.

$$\begin{aligned}
& X_{jW}^{\text{gsc}}(r), j \in \text{WA}; \\
& \quad \text{output of widget commodity/activity } j \text{ in region } r \\
& A_{ijW}^{\text{gsc}}(r, d), i \in \text{WCI} \ \& \ j \in \text{WA}; \\
& \quad \text{input of widget com } i \text{ from } r \text{ per unit of output of widget com } j \text{ in } d \\
& \text{SCALE}_{jW}^{\text{gsc}}(r), j \in \text{WA}; \\
& \quad \text{scale - economy variable in production of widget com } j \text{ in } r \\
& \text{TTC}_W^{\text{gsc}}; \\
& \quad \text{total cost of tariffs to global agent} \\
& \text{LC}_W^{\text{gsc}}; \\
& \quad \text{total labour costs in the global widget sector}
\end{aligned} \tag{2.1}$$

to minimize

$$\begin{aligned}
& \text{TTC}_W^{\text{gsc}} + \text{LC}_W^{\text{gsc}} \\
& \quad \text{total cost of satisfying world-wide demand for the final widget product}
\end{aligned} \tag{2.2}$$

subject to

$$\begin{aligned}
& X_{iW}^{\text{gsc}}(r) = \sum_{d \in R} \sum_{j \in \text{WA}} A_{ijW}^{\text{gsc}}(r, d) * X_{jW}^{\text{gsc}}(d) \text{ for all } i \in \text{WCI} \text{ and } r \in R \\
& \quad \text{supply/demand balance for intermediate widget com } i \text{ from region } r
\end{aligned} \tag{2.3}$$

$$\begin{aligned}
& X_{FW}^{\text{gsc}}(d) = Y_W^{\text{gsc}}(d) \text{ for all } d \in R \\
& \quad \text{supply/demand balance for final (F) widget com in region } d, F \text{ is not traded}
\end{aligned} \tag{2.4}$$

$$\begin{aligned}
& \bar{A}_{ijW}^{\text{gsc}}(d) = \sum_{r \in R} A_{ijW}^{\text{gsc}}(r, d) \text{ for all } i \in \text{WCI}, j \in \text{WA} \text{ and } d \in R \\
& \quad \text{total requirements of } i \text{ per unit of output of } j \text{ in } d
\end{aligned} \tag{2.5}$$

$$\begin{aligned}
& \text{LC}_W^{\text{gsc}} = \sum_{j \in \text{WA}} \sum_{r \in R} W^{\text{gsc}}(r) * \frac{X_{jW}^{\text{gsc}}(r)}{\text{PROD}_{jW}^{\text{gsc}}(r)} * \text{SCALE}_{jW}^{\text{gsc}}(r) \\
& \quad \text{widget labour costs determined by wage rates, output and productivity modified for scale}
\end{aligned} \tag{2.6}$$

$$\begin{aligned}
& \text{SCALE}_{jW}^{\text{gsc}}(r) = S_{jrW}^{\text{gsc}}(X_{jW}^{\text{gsc}}(r)) \text{ for all } j \in \text{WA} \text{ and } r \in R \\
& \quad \text{scale modification in production of } j \text{ in } r \text{ determined by output}
\end{aligned} \tag{2.7}$$

$$\begin{aligned}
& \text{TTC}_W^{\text{gsc}} = \sum_{i \in \text{WCI}} \sum_{r \in R} \sum_{j \in \text{WA}} \sum_{d \in R} P_{iW}^{\text{gsc}}(r) * [T_{iW}^{\text{gsc}}(r, d) - 1] * A_{ijW}^{\text{gsc}}(r, d) * X_{jW}^{\text{gsc}}(d) \\
& \quad \text{total cost of tariffs to the global agent}
\end{aligned} \tag{2.8}$$

In this optimizing problem:

WCI is the set of Widget commodities used in the Widget sector as intermediate inputs.

These are Design, Components and Assembly. This set excludes the final good, SalesDist. Later, we will denote the set of all Widget commodities, including SalesDist as WC.

WA is the set of Widget activities. Each activity is responsible for production of the correspondingly named Widget commodity,  $\text{WA} = \text{WC}$ .

R is the set of regions.

$X_{iW}^{\text{gsc}}(r)$  is the output of Widget commodity  $i$  in region  $r$ . It can also be viewed as the output of activity  $i$  in region  $r$ . When  $i = F$ , we are referring to the output of the final good, that is the output of the SalesDist activity.

$A_{ijW}^{gsc}(r, d)$  is the quantity of Widget commodity  $i$  from region  $r$  that the global optimizing agent chooses to use per unit of Widget activity  $j$  in region  $d$ .

$\bar{A}_{ijW}^{gsc}(d)$  is the total quantity of Widget commodity  $i$  required per unit of Widget activity  $j$  in region  $d$ . We treat these variables as exogenous or outside the control of the optimizing agent. They reflect Widget technology available in country  $d$ .

$Y_W^{gsc}(d)$  is the total quantity of Widget commodity  $i$  required by final uses in region  $d$ .

These variables are exogenous to the optimizing agent although, as we will see later, they are endogenous in the integrated GSC-CGE model. It is easiest to think of final demands as being demands by public and private consumers and by capital creators.

However, in a detailed empirical model final demands would include intermediate sales of Widget commodities to industries outside the Widget sector.

$LC_W^{gsc}$  is total labour costs incurred in the world-wide Widget sector.

$W^{gsc}(r)$  is the wage rate in region  $r$ , which is exogenous to the optimizing agent but possibly endogenous in the integrated GSC-CGE model.

$PROD_{jW}^{gsc}(r)$  is labour productivity in Widget activity  $j$  in region  $r$  at standard scale for output. This is exogenous to the optimizing agent and remains exogenous in the integrated GSC-CGE model.

$SCALE_{jW}^{gsc}(r)$  allows for variations in labour productivity in Widget activity  $j$  in region  $r$  reflecting economies of scale. If the optimizing agent chooses to produce Widget commodity  $j$  in region  $r$  at a scale greater than standard, then through a suitable specification for the  $S_{jW}^{gsc}$  function on the RHS of (2.7) we can allow output per unit of labour input in activity  $j$  in region  $r$  to be greater than  $PROD_{jW}^{gsc}(r)$ . For example, in the stylized model described in the next section we assume that  $SCALE_{jW}^{gsc}(r) = 0.95$  if region  $r$ 's output is sufficiently large to satisfy world requirements for Widget commodity  $j$ . In that case output per unit of labour in  $r$ 's Widget activity  $j$  is greater than  $PROD_{jW}^{gsc}(r)$ : it is  $PROD_{jW}^{gsc}(r) / 0.95$ .

$T_{iW}^{gsc}(r, d)$  is the power (one plus the rate) of tariffs applying to the flow of Widget commodity  $i$  from region  $r$  to region  $d$ . This is a naturally exogenous variable. We could also include transport costs between  $r$  and  $d$ . But in this simple model we will ignore that complication.

$TTC_W^{gsc}$  is total cost of tariffs to the global agent.

$P_{iW}^{gsc}(r)$  is the price before tariffs of Widget commodity  $i$  produced in region  $r$ . As discussed below, it may seem that  $P_{iW}^{gsc}(r)$  can be controlled by the global agent.

Nevertheless, we treat  $P_{iW}^{gsc}(r)$  as exogenous in the global agent's optimization problem. In the integrated GSC-CGE model, it is endogenous.

Via (2.3) – (2.8) we assume that for given values of the variables  $\bar{A}_{ijW}^{gsc}(d)$ ,  $Y_W^{gsc}(d)$ ,  $W^{gsc}(r)$ ,  $PROD_{jW}^{gsc}(r)$ ,  $T_{iW}^{gsc}(r, d)$  and  $P_{iW}^{gsc}(r)$ , the variables listed in (2.1) are determined by minimizing the total tariff and labour costs, defined by (2.2), of satisfying final demands for Widget commodities, the  $Y_W^{gsc}(d)$ s. In this simple GSC model, we assume that Widget activities use only one primary factor and no intermediate inputs from outside the Widget sector. In a more complete model we would need to include capital costs as well as labour costs and also non-Widget-sector intermediate inputs. Equations (2.3) and (2.4) ensure that the output of Widget

commodity  $i$  in region  $r$ , commodity  $(i,r)$ , satisfies intermediate and final demands for  $(i,r)$ . Equation (2.5) imposes the assumption of perfect substitutability between Widget commodity  $i$  from different sources in satisfying intermediate demands in the Widget sector of region  $d$ . Equations (2.6) to (2.8) define labour costs, the scale variable and total tariff costs.

The only role of prices,  $P_{iW}^{gsc}(r)$ , in the global optimizing problem is in the calculation of *ad valorem* tariff costs see (2.8). We assume that these prices are set to reflect production costs according to

$$P_{iW}^{gsc}(d) = \sum_{j \in WCI} \sum_{r \in R} P_{jW}^{gsc}(r) * T_{jW}^{gsc}(r,d) * A_{jiW}^{gsc}(r,d) + \frac{W^{gsc}(d)}{PROD_{iW}^{gsc}(d)} * SCALE_{iW}^{gsc}(d) \quad (2.9)$$

for all  $i \in WC$  and  $d \in R$

We can think of these prices as being imposed by governments to ensure that the global agent cannot avoid tariffs costs by “clever” setting of intra-Widget prices. The use of (2.9) to determine intra-sectoral prices for the purpose of calculating  $TTC_W^{gsc}$  seems relatively harmless. In the integrated GSC-CGE system we also assume that  $P_{FW}^{gsc}(d)$  determined in (2.9) applies to sales of the final Widget commodity to final users. This seems more problematic. Despite modelling the global agent as a monopolist, we assume that pricing of final goods is competitive. In the background, we are assuming that the global agent is constrained by potential entry of rivals.

## 2.2. A CGE model

Global CGE models are built around input-output databases. Table 2.1 is illustrative for a simple N-region global CGE model in which labour is the only primary factor input and tariffs on intermediate flows are the only wedges between factory door prices and purchasers’ prices.

In Table 2.1,  $V(r,d)$  is a  $C$  by  $C$  matrix where  $C$  is the number of commodities or industries. The  $h,k$  component of  $V(r,d)$  is the pre-tariff value of commodity  $h$  produced in region  $r$  used in industry  $k$  in region  $d$ .  $VTI(r,d)$  is the  $C$  by  $C$  matrix of tariff collections associated with  $V(r,d)$ .  $FD(r,d)$  is a  $C$  by 1 vector in which the  $h$  component is the value of commodity  $h$  from country  $r$  used in final demand in country  $d$ . Because we assume no trade in the final Widget commodity, we do not need to allow for tariffs on final goods to illustrate how the GSC and CGE models can be integrated. Thus, for simplicity we assume no tariffs on *any* final good.  $LAB(r)$  is a 1 by  $C$  vector in which the  $k$  component is the value of labour input to industry  $k$  in region  $r$ .  $Z(r)$  is the  $C$  by 1 vector of sales values of commodities produced in region  $r$ . Its transpose,  $Z'(r)$ , is the 1 by  $C$  vector of total costs incurred by industries in region  $r$ . A fundamental balance condition in CGE models is that the value of sales of each commodity produced in each region is equal to costs in the producing industry. In Table 2.1, these costs are the value of labour plus intermediate goods including tariffs on intermediates.

One interpretation of CGE models is that they are a system of equations that drive the components of an input-output database. The variables in the CGE model that combine to determine the value of each input-output flow are quantities and factory prices. Quantities are determined in cost-minimizing and utility-maximizing problems. Purchasers’ prices reflect factory prices (costs) and tariffs. In this stylized example, we leave out sales taxes and transport costs. Factory prices are determined by wages and by technology (input requirements per unit of output) which is usually treated as exogenous. Wages depend on demand and supply for labour. Demand for labour depends on productivity while supply is

either exogenous or modelled via demographic variables. Total final demand in each region depends on incomes, which depend on wages.

In a typical global CGE model we can think of Widgets as a single commodity and industry in each region. The Widget row of the input-output data for region  $r$  shows Widget sales to the Widget industry and other parts of region  $r$ 's economy as well as Widget sales to export. The Widget column for region  $r$  shows inputs to Widget production in region  $r$  and associated tariffs.

Given the specification of the global Widget sector in subsection 2.1, what we would see in the Widget row for source region  $r$  in the input-output data in Table 2.1 is zero entries except (possibly) in the Widget-Widget or WW position of the  $V(r,d)$  matrices and the W position of the  $FD(r,r)$  vector. This is because Widgets are traded only as intermediate inputs to Widget industries. What we would see in the Widget column for destination region  $d$  is zero entries except for the Widget entry in  $LAB(d)$  and (possibly) the WW entries in  $V(r,d)$  for all  $r$  and  $VIT(r,d)$  for all  $r \neq d$ .

What we would not see in the input-output data is the underlying nature of the Widget flows. With the GSC Widget specification in subsection 2.1, the intermediate inputs to region  $d$ 's Widget industry and Widget exports from region  $d$  would be aggregations of commodities in WCI. Region  $d$ 's domestic Widget sales outside the Widget sector would consist entirely of the final good. But these features are not revealed by the input-output data: we simply see undifferentiated flows of Widgets.

In essence, our approach to integrating GSC and CGE models is to devise a method for driving the undifferentiated Widget flows and primary factor inputs to the Widget industry in each region in the CGE model in a way that takes account of the underlying changes in activities in the GSC model. To do this, we must work out how output, input and price variables for the undifferentiated Widget commodity/industry in each region in the CGE model should react to changes in output, input and price variables for the underlying Widget activities in the GSC model.

### ***2.3. Integrating GSC and CGE: sharing roles between the two models in an integrated system***

Looked at through a CGE lens, the channels through which a GSC Widget sector affects the broader economy in each region include:

- (a) the price of the final Widget good;
- (b) demands for domestic and imported inputs per unit of output in the Widget sector;
- (c) demands for primary-factor inputs per unit of output in the Widget sector; and
- (d) average tariff rates in each region on imports of Widget goods.

If developments (e.g. productivity improvements) in the Widget sector of region  $d$  lead to lower prices for the final Widget good, then imposed on a CGE model this will produce a series of repercussions. These may include increased consumption of Widgets in region  $d$ , increased or reduced consumption of other goods depending on trade-offs between substitution and income effects, and increased real wages in region  $d$ . If there is substitution against domestically produced Widget inputs to the Widget industry in region  $d$  in favour of imported inputs from region  $r$ , then imposed on a CGE model this may cause a temporary trade deficit for region  $d$  generating real devaluation in  $d$  and eventual correction of the trade deficit through stimulation of  $d$ 's exports and reduction in  $d$ 's imports. With real devaluation in region  $d$ , there will be flow-on effects to other regions through their trade relations with  $d$ . Equilibrium will be re-established with a new set of input-output flows satisfying the balance conditions between sales values and input costs.

*Table 2.1. Input-output database for a simple global CGE model*

V(1,1)	...	V(1,N)	FD(1,1)	...	FD(1,N)	Z(1)
V(2,1)	...	...	...	...	...	...
...						
V(N,1)	...	V(N,N)	FD(N,1)	...	FD(N,N)	Z(N)
0	...	VTI(1,N)				
VTI(2,1)	0					
...	...	0				
VTI(N,1)	...	0				
LAB(1)	...	LAB(N)				
Z'(1)		Z'(N)				

Looked at through a GSC lens, the channels through which the broader multi-region economy affects the Widget sector in each region include:

- (e) wage rates; and
- (f) Widget demand by final users.

To integrate a CGE and a GSC model we impose on the CGE model outcomes from the GSC model for (a) to (d) and impose on the GSC model outcomes from the CGE model for (e) and (f). In this way, we ensure that Widget variables in the CGE model are driven by the GSC model and economy-wide variables in the GSC model are driven by the CGE model.

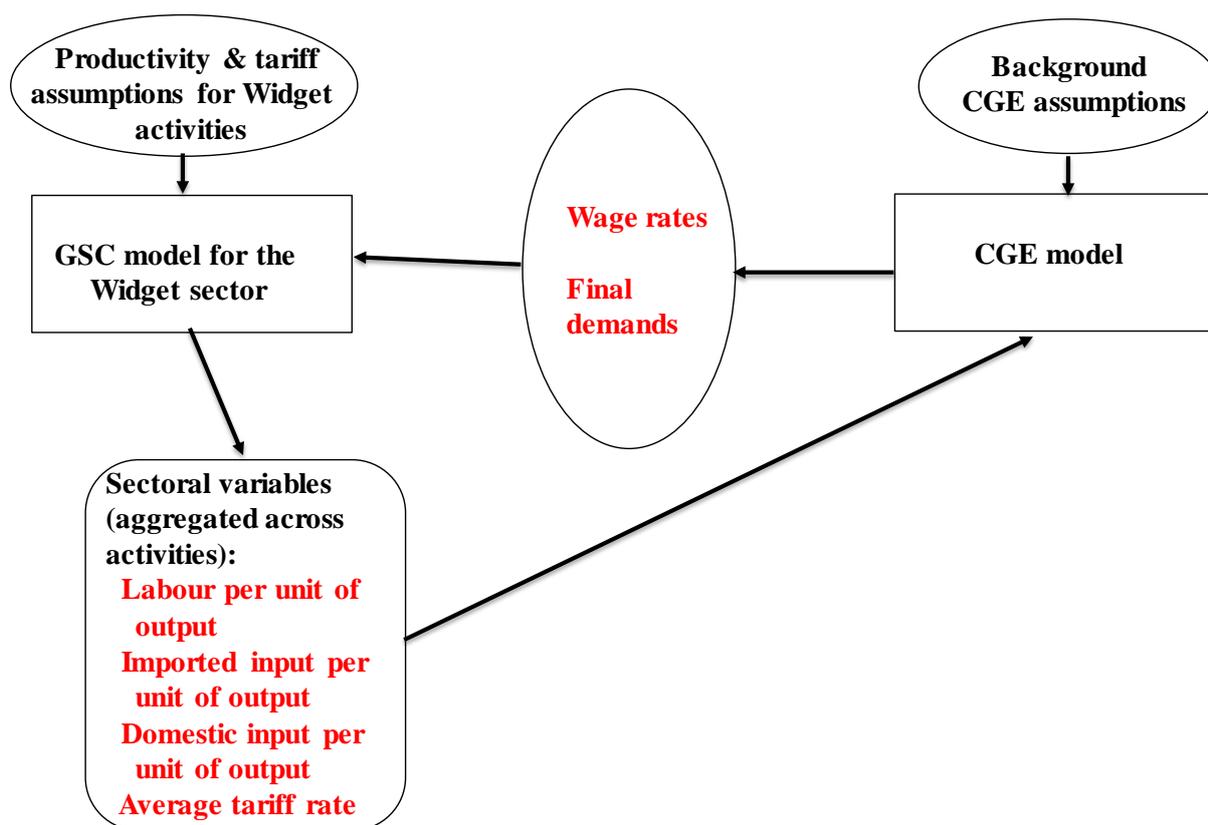
In the case of the GSC and CGE models described in sections 2.1 and 2.2, we say that they are fully integrated if and only if

- (i) wage rates and final demand for Widgets used in determining the GSC solution are the same as those determined in the CGE model;
- (ii) the power of the tariff, and the value of the flow and related tariff collection for the single Widget commodity applying to any pair of regions  $r$  and  $d$  in the CGE model is the value of these variables determined in the GSC model as appropriate aggregations over Widget commodities of powers, values and tariff collections;
- (iii) the technology (intermediate and primary-factor input per unit of output) in each region's Widget industry in the CGE model is the same as that for the Widget sector in the GSC model derived by appropriate aggregation over the four Widget activities;
- (iv) the price of region  $r$ 's single Widget good in the CGE model is the price of  $r$ 's final Widget good determined in the GSC model; and
- (v) employment in each region's Widget industry in the CGE model is the same as that for the Widget sector in the GSC model derived by adding over Widget activities.

Through conditions (i) to (v) we are saying that two models are fully integrated if there are no inconsistencies between them in their solutions for variables of relevance to welfare and resource allocation.

Figure 2.1 illustrates a process for implementing full integration of the GSC model for Widgets and the CGE model. The GSC model receives shocks describing changes in productivity and tariff variables applying to production and trade in the Widget sector's activities in each region. Then on the basis of assumed values for wage rates and final

*Figure 2.1. An algorithm for integrating GSC and CGE models*



demands for Widgets, the GSC model produces results for Widget activities. From these, movements for each region in inputs per unit of output and average tariff rates at the aggregated sectoral level for Widgets can be calculated. As shown in the figure, these can be passed to the CGE model. The CGE model can also receive shocks to the myriad of technology, tariff and other exogenous CGE variables outside the Widget sector. The CGE model then produces results for wage rates and final demands for Widgets in each region. Starting with a CGE solution incorporating initial guesses for Widget sectoral variables or with a GSC solution incorporating initial guesses for wage rates and final demands for Widgets, we can follow the arrows around Figure 2.1 looking for a converged solution. A converged solution occurs when wage rates and final demands being passed to the GSC model are unchanged between successive iterations or equivalently when the Widget sectoral variables being passed to the CGE model are unchanged between successive iterations.

As demonstrated in the Appendix, full integration between the GSC Widget model and a CGE model is achieved by computing fully converged solutions. By computing such solutions we can reveal (1) the economy-wide (CGE) effects of shocks to productivity and tariffs applying to GSC activities and (2) the intra-widget (GSC) effects of shocks to exogenous CGE variables such as changes in productivity in sectors apart from Widgets.

Will the algorithm converge? We were confident that this algorithm or a variant of it (e.g. partial adjustment between steps) would work because the GSC sector is usually quite a small fraction of the total economy. Consequently, we expected wages and final demands determined in the broader economy to be relatively insensitive to developments in the GSC sector. However, as explained in section 6, in a stylized example there was sufficient

sensitivity of wages to GSC developments that convergence could not be achieved without an important modification to the standard CGE model.

### 3. A GSC sector in a CGE database: an illustrative numerical example

In this section, we provide a numerical example showing how a GSC sector is represented in a world input-output database of the type used in CGE modelling. We assume that there are two regions, R1 and R2, which we will think of as the U.S. and Asia, and two industries, Ind1 and Ind2. Ind1 is a potential GSC industry while Ind2 is the rest of the economy consisting mainly of services but also including tradable goods such as agriculture and mining.

For Ind1 we adopt a special case of the Widget industry in the GSC model described in subsection 2.1. Within this Widget industry, there are four activities: Design; Components; Assembly; and SalesDist. Data for these four activities are given in Table 3.1, part A. For concreteness, we will think of these data as referring to 1990.

As shown in the top panel of Table 3.1A: production of a unit of Components requires 1 unit of Design; production of a unit of Assembly requires 1 unit of Components; and production of a unit of SalesDist requires 1 unit of Assembly. In terms of the model in subsection 2.1, this defines  $\bar{A}$  on the LHS of (2.5). Panel (b) in Table 3.1A gives the values for labour productivity in each Widget activity at standard scale [PROD in (2.6)]. As explained in subsection 2.1, we assume that labour productivity in activity  $j$  in region  $r$  is 5 per cent greater than PROD if  $r$  undertakes all of the output of  $j$  required for both regions (see the definition of SCALE). Panel (c) gives the powers of the tariffs [T in (2.8)] and panel (e) gives wage rates [W in (2.6)]. SalesDist is the non-traded final Widget commodity. Quantities of this commodity [Y in (2.4)] used in each region are given in panel (d).

Panel (f) in Table 3.1A shows wage rates divided by PROD, that is labour costs per unit of output at standard scale for each Widget activity in the two regions. Even though wage rates are much lower in R2 than in R1, wage costs per unit of output in all traded Widget activities are higher in R2 than in R1. This reflects the very low productivity levels assumed for R2 in panel (b) relative to those for R1.

Part B of Table 3.1 shows the GSC Widget solution generated under the assumptions in Part A. In view of the high labour costs per unit of output applying to Design, Components and Assembly in R2, it is not surprising that R1 is dominant in world production of these three traded Widget commodities. The only non-zero Widget activity in R2 is production of the non-traded commodity SalesDist. Despite a tariff of 20 per cent, R2 satisfies all of its requirement for Assembly by importing from R1. Imported Assembly goes to R2's SalesDist activity. Because R2 does not produce either Components or Assembly, it does not import either Design or Components.

The shaded parts of Table 3.2A depict the Widget industry of Tables 3.1A&B in a world input-output database of the form used in CGE modelling. In this database, Widgets is industry 1 (Ind1). It is represented as producing a single composite commodity (C1). The underlying details of Design, Components, Assembly and SalesDist are suppressed.

Exports of Widgets (C1) from R1 to R2 are shown in Table 3.2A with a cif value of 1.425. This consists of 0.5 units of Assembly priced at 2.850 per unit (see Table 3.1B).

The flow of C1 to Ind1 in R1 in Table 3.2A is 7.125. This is the value of output of Design, Components and Assembly produced in R1 less exports of Assembly, that is:  $0.95 \cdot 1.5 + 1.9 \cdot 1.5 + 2.85 \cdot 1.5 - 1.425 = 7.125$  (see Table 3.1B).

**Table 3.1. World Widget sector in 1990**

**A. Technology assumptions and data for 1990**

	Design	Components	Assembly	SalesDist
<i>(a) Intermediate inputs for producing 1 unit of output of each activity in both regions (<math>\bar{A}</math>)</i>				
Design	0	1	0	0
Components	0	0	1	0
Assembly	0	0	0	1
SalesDist	0	0	0	0
<i>(b) Output per unit of labour input, standard scale (Productivity, PROD)</i>				
R1(US)	1	1	1	1
R2 (Asia)	0.0833	0.1667	0.125	0.25
<i>(c) Powers of tariffs (T) on imports by importing region</i>				
R1 (US)	1.1	1.2	1.2	1
R2 (Asia)	1.1	1.2	1.2	1
<i>(d) Demand for final product:</i>				
R1 (US)				Y(R1) = 1
R2 (Asia)				Y(R2) = 0.5
<i>(e) Wage rate (W)</i>				
R1 (US)		1.0		
R2 (Asia)		0.25		
<i>(f) Labour costs per unit of output at standard scale (SLC), that is W/PROD</i>				
R1 (US)	1	1	1	1
R2 (Asia)	3	1.5	2	1

**B. Output, employment, trade and prices in 1990: solution from GSC model**

	Price	Output	Employment	Exports, qty	Exports, value
<b>R1</b>					
Design	0.950	1.5	1.425	0.0	0.0
Components	1.900	1.5	1.425	0.0	0.0
Assembly	2.850	1.5	1.425	0.5	1.425
SalesDist	3.850	1.0	1.000	0.0	0.0
<b>Total</b>			5.275		1.425
			VA=5.275		
<b>R2</b>					
Design	3.000	0.0	0.0	0.0	0.0
Components	2.545	0.0	0.0	0.0	0.0
Assembly	4.280	0.0	0.0	0.0	0.0
SalesDist	4.420	0.5	2.0	0.0	0.0
<b>Total</b>			2.0		0.0
			VA=0.5		

The values in Table 3.2A of labour input to Ind1 in the two regions (5.275 and 0.5) are simply the value added (VA) numbers in Table 3.1B. These numbers are the wage rates in the two regions (1.0 and 0.25, Table 3.1A) multiplied by the Widget employment levels (5.275 and 2.0, Table 3.1B).

The only tax collection on Widgets is the tariff on R2's imports of Widgets. These imports consist of Assembly with a tariff rate of 20 per cent (Table 3.1A). With the cif value of imports being 1.425, the tariff collection is, as shown in Table 3.2A, 0.285 (=0.2\*1.425).

**Table 3.2A. World input-output table for 1990 (\$)**

		R1	R1	R2	R2	R1	R2	Totals
		Ind1	Ind2	Ind1	Ind2	Consumption		
R1	C1	7.125		1.425		3.85		12.400
R1	C2					23.525	2.85	26.375
R2	C1						2.210	2.210
R2	C2					4.275	5.725	10.000
Labour	C1	5.275	26.375					31.650
Labour	C2			0.5	10			10.500
Taxes	C1	0	0	0.285	0	0	0	
Taxes	C2	0	0	0	0	0	0	
Totals		12.400	26.375	2.210	10.000	31.650	10.785	

The numbers in the shaded rows and columns are for flows of commodity 1 (C1) and inputs to industry 1 (Ind1) that produces C1. These numbers are obtained from the Widget data in Table 3.1. The numbers for C2 and Ind2 were set so that the Widget Industry (manufacturing) contributes approximately 17 per cent of GDP in region 1 (R1) and 5 per cent of GDP in region 2 (R2).

**Table 3.2B. World input-output table for 1990 (\$) modified for use in CGE model**

		R1	R1	R2	R2	R1	R2	Totals
		Ind1	Ind2	Ind1	Ind2	Consumption		
R1	C1	7.125		1.425		3.85		12.400
R1	C2					23.525	2.85	26.375
R2	C1	0.01		0.01			2.200	2.220
R2	C2					4.265	5.735	10.000
Labour	C1	5.265	26.375					31.640
Labour	C2			0.5	10			10.500
Taxes	C1	0	0	0.285	0	0	0	
Taxes	C2	0	0	0	0	0	0	
Totals		12.400	26.375	2.220	10.000	31.640	10.785	

We can calculate the value of Widget consumption shown in Table 3.2A for each region as the value of output *plus* imports *less* exports *less* intermediate use. For R1 this gives consumption of Widgets at 3.85 ( $= 12.4 + 0 - 1.425 - 7.125$ ). For R2, consumption of Widgets is 2.21 ( $= 2.21 + 1.425 - 0 - 1.425$ ). These consumption values can be checked from Table 3.1. They are the consumption (or output) quantities of SalesDist (1 and 0.5, Table 3.1A) times the prices of SalesDist (3.850 and 4.420, Table 3.1B).

For simplicity we assume that Ind2 in each region uses only labour as an input and sells only to final demand. We also assume there are no tariffs on trade in Ind2's product (C2). In both regions, Ind2 is much larger than Ind1. As shown in Table 3.2A, Ind2 accounts for 83.33 per cent of employment in R1 (26.375 out of 31.650) and 95.24 per cent in R2 (10 out of 10.5).

We assume that trade in 1990 is balanced. Reflecting its specialization in Widgets, R1 has a Widget trade surplus of 1.425 while R2 has a surplus of 1.425 in C2 trade ( $= 4.275 - 2.85$ )

Before it can be used in a CGE model the database in Table 3.2A needs to be slightly modified. The problem is zero flows. In CGE modelling it is difficult to project non-zero flows from a zero starting point. In Table 3.2A there are zero flows of C1 from R2 to Ind1 in both countries. In section 5, we simulate situations in which these flows are non-zero. To make this possible, we adjust the database in Table 3.2A by adding tiny amounts (0.01) to the flows of C1 from R2 to Ind1 in R1 and to Ind1 in R2. We then rebalance the table, by

reducing labour input to Ind1 in R1 by 0.01 and reducing consumption of C1 in R2 by 0.01, to arrive at Table 3.2B. In rebalancing, we preserve the original trade balances, zero for each region.

#### 4. Baseline CGE forecast

Imagine that we are standing in 1990 trying to project forward to 2000. We have the input-output database set out in Table 3.2B and decide to build a standard CGE model calibrated to this database. In the model, we assume that: production functions for the two industries in each of the two regions are Leontief in intermediate inputs of C1 and C2 and the single primary factor labour; household preferences are Cobb-Douglas between C1 and C2; and Armington elasticities set at 3.8 determine substitution by industries and households between imported and domestic varieties of the same commodity. In modelling each labour and intermediate input, we allow for technical change by introducing an exogenous variable that affects the use of the input per unit of output.

##### *Shocks*

In applying this CGE model to the task of projecting from 1990 to 2000, we introduce three ideas. First, R2 (Asia) is rapidly catching up to R1 (U.S.) in terms of productivity and wages. Second, productivity growth is rapid in Ind1 (think manufacturing) relative to Ind2 (dominated by services). Third, tariffs are being dismantled. Looking at these ideas through CGE eyes, we project from 1990 to 2000 by applying the following shocks:

- (1) labour-saving technical progress in Ind1, R1 = 15%
- (2) labour-saving technical progress in Ind2, R1 = 0%
- (3) labour-saving technical progress in Ind1, R2 = 27.75%
- (4) labour-saving technical progress in Ind2, R2 = 15%
- (5) reduction in the power of the tariff on R2's imports of C1 = 12.5%

Shocks (1) and (2) give R1 a background rate of labour-saving technical change of 0% with 15% extra for Ind1. Shocks (3) and (4) give R2 a background rate of labour-saving technical change of 15% with 15% extra for Ind1. The 15% extra means that instead of falling from 1 to 0.85, the index of labour requirements per unit of output in R2's Widget industry falls from 1 to 0.7225 ( $= 0.85 \times 0.85$ ). Shock (5) introduces a reduction in the rate of the tariff imposed by R2 on imports of C1 from 20% to 5% [ $-12.5 = 100 \times (1.05/1.20 - 1)$ ].

In this section, and later in sections 7 and 8, we assume no change in both regions in employment measured in people. In generating standard CGE forecasts, we also assume no growth in aggregate labour input (row 22, Table 4.1), implying that labour input is adequately measured by employment (number of people employed). In sections 7 and 8, describing results from the integrated GSC-CGE model, we allow for changes in labour input associated with movement of surplus but employed labour from Ind2 to Ind1 in R2.

We could add other shocks to (1) - (5).<sup>2</sup> For example, we could include shocks to the number of people employed in each region reflecting demographic developments and to the trade balance reflecting capital flows. These variables are exogenous in our projections. Correspondingly, real wage rates and real exchange rates are endogenous. However, including shocks to employment and the trade balance is unnecessary for our current illustrative purposes.

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<sup>2</sup> We did in fact include a sixth shock to deal with a minor problem caused by inclusion in the 1990 database of the artificial flow (0.01) of C1 from R2 to R1. We didn't include a tariff collection on this flow which means that the 1990 database implies a zero rate of tariff on the flow. In 2000, where there may be genuine flows, we want the tariff rate to be 5%. The sixth shock was an increase in the power of the tariff on the flow of C1 from R2 to R1 by 5% (from 1 to 1.05).

## Results

Table 4.1 shows the projections for R1 and R2 derived by applying shocks (1) to (5) to our stylized CGE model. With no capital in this simple model and with no growth in labour input, the projected increases in real GDP (row 1) can be explained purely from our technology assumptions and items in the input-output data in Table 3.2B. R1 is projected to have labour-saving technical progress of 15 per cent [shock (1)] in 16.67 per cent of its economy (Ind1's share of R1's labour input) giving it a GDP increase of about 2.6 per cent, close to the number shown in Table 4.1 (2.72, row 1, col 1). R2 is projected to have labour-saving technical progress of 27.75 per cent [shock (3)] in about 4.76 per cent (Ind1) of its economy and 15 per cent [shock (4)] in about 95.24 per cent (Ind2) of its economy. For a given level of output this technical progress frees up about 15.6 per cent of the labour force. Re-employing this labour enables R2 to increase its GDP by about 18.5 per cent [=  $100 \times 0.156 / (1 - 0.156)$ ], which is close to the result in Table 4.1 (18.76, row 1, col 2).

Rapid technical progress in R2 relative to R1 gives workers in R2 a wage increase of 13.82 per cent relative to workers in R1 (row 3 in Table 4.1).<sup>3</sup> In real terms the wage increase in R2 is nearly 20 percentage points greater than that in R1 (row 4). The wage differential is accentuated in real terms by the improvement in R2's terms of trade (discussed below) and by the cut in its tariffs on its imports of C1 [shock (5)].<sup>4</sup>

Reflecting the 1990 situation of balanced trade and the assumption of no-change in the trade balance, real consumption (row 2, Table 4.1) in the two regions increases broadly in line with GDP. Small GDP-consumption discrepancies arise from terms-of-trade movements: a slightly smaller percentage increase in consumption than in GDP in R1 (2.41 compared with 2.72) and a slightly larger increase in R2 (19.76 compared with 18.76). R2 benefits from a terms-of-trade improvement (row 15) which increases the amount of consumption that it can undertake per unit of output (or GDP). The reverse is true for R1. R2 experiences a terms-of-trade improvement because it is a net exporter of C2 and it imports C1: prices for C1 fall relative to those for C2 (rows 5 to 8) reflecting extra technical progress in the production of C1 relative to C2.

With balanced trade and strong growth in R2 relative to R1, R2's trade falls as a share of GDP [real export growth of 12.84 per cent (row 13) compared with GDP growth of 18.76 per cent]. The explanation is that the export market for R2 (namely R1) is shrinking relative to the size of R2's domestic market. The opposite is true for R1.

Rows 16 to 21 show developments at the industry/commodity level. In both regions, labour input in industry 1 declines relative to that in industry 2 (rows 16 and 17). For R1, the decline is about 1.80 per cent (=  $1.51 + 0.30$ ) and for R2 it is about 4.5 per cent (=  $4.29 + 0.21$ ). These declines are brought about by rapid technical progress in Ind1 relative to Ind2. They can be accommodated by small switches between industries in labour input. In R1, labour input in Ind1, which accounts for 16.6 per cent of R1's total labour input in 1990 (Table 3.2B), falls by 1.51 per cent. With no change in aggregate labour input this implies a reallocation between 1990 and 2000 of 0.25 per cent of R1's workforce from Ind1 to Ind2 (=  $1.51 \times 0.166$ ). The workforce-switch percentage is even smaller for R2, 0.20 per cent.

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<sup>3</sup> The wage rate in R1 is the numeraire. Consequently, it is shown in Table 4.1 with zero change.

<sup>4</sup> A cut in tariffs, as with a cut in any indirect tax, allows a given level of employment to be maintained with a higher pre-tax real wage rate. In post-tax terms workers may not be better off if lost tariff revenue is replaced by a direct income tax.

**Table 4.1. Projection from 1990 to 2000 using a standard CGE model**

		R1	R2
1	Real GDP (% change from 1990 to 2000)	2.72	18.76
2	Real consumption, welfare (% change)	2.41	19.76
3	Wage rate	0.00	13.82
4	Real wage rate, CPI delated (% change)	2.41	22.19
5	Factory price of C1 (% change)	-15.01	-23.85
6	Factory price of C2 (% change)	0.00	-3.25
7	Purchasers' price to consumers of C1 (% change)	-15.01	-23.85
8	Purchasers' price to consumers of C2 (% change)	-0.53	-2.22
9	Consumption of C1, quantity (% change)	9.77	32.46
10	Consumption of C2, quantity (% change)	1.47	16.89
11	Value of exports (% change)	9.11	9.11
12	Value of imports (% change)	9.11	9.11
13	Quantity of exports (% change)	15.28	12.84
14	Quantity of imports (% change)	12.84	15.28
15	Terms of trade (% change)	-2.12	2.17
16	Labour input in Ind1	-1.51	-4.49
17	Labour input in Ind2	0.30	0.21
18	Exports C1 (100 times change in value at initial prices as share of initial GDP) *	1.47	0.04
19	Exports C2 (100 times change in value at initial prices as share of initial GDP) *	0.66	5.05
20	Imports C1 (100 times change in value at initial prices as share of initial GDP) *	0.01	4.30
21	Imports C2 (100 times change in value at initial prices as share of initial GDP) *	1.72	1.94
22	Aggregate labour input (no. of employed persons in the standard CGE projection)	0	0

\*  $100 * (\text{Quantity in final year times price in initial year} - \text{Value in initial year}) / \text{GDP in initial year}$

Rows 18 to 21 show changes in the commodity composition of each region's trade. In these rows, it is convenient to report changes in volume flows as percentage-point changes in shares of initial GDP. For example, the entry in the R1 column of row 18 means that between 1990 and 2000 R1's exports of C1 valued at 1990 prices increased as a share of initial GDP by 1.47 percentage points, that is, from 4.5038 per cent of GDP in the 1990 database in Table 3.2B to 5.9693 per cent. In this case, the volume increase is finite and interpretable, 32.54 per cent  $[=100 * (5.9693/4.5038 - 1)]$ . However, as we will see in section 8, our integrated GSC-CGE model can generate substantial trade flows for 2000 from an arbitrarily small starting point in 1990, making percentage change results uninformative. By reporting percentage-point share changes in GDP, we not only avoid this problem but we also highlight changes in the commodity structure of trade.

Viewed this way, the trade projections in Table 4.1 can be described as "business as usual". In 1990, R2 specialized in the export of C2 (4.265 out of total exports 4.275, see Table 3.2B). This specialization continues in 2000 with the expansion of R2's exports accounted for almost entirely by C2 (5.05 in row 19 compared with 0.04 in row 18). In 1990, R1 exported both commodities (1.425 for C1 and 2.85 for C2, see Table 3.2B). This mixed pattern continues for R1 with substantial increases in exports of both commodities (rows 18 & 19, col 1). Technical progress is 15 per cent faster in each industry in R2 than in R1. Thus, R1's

comparative advantage in the production of C1 is preserved. Given the relative weakness of R2 in the production of C1, R2's consumers draw strongly on R1 to satisfy their rapidly growing demand for C1. This explains the growth in R1's exports of C1 (row 18) relative to its exports of C2 (row 19).

The import results in rows 20 and 21 of Table 4.1 follow in a mechanical way from the export results. Consequently, no further explanation is required.

## 5. World Widget industry in 2000: technology and tariff assumptions, and GSC solution

Now imagine that we are specialists on the Widget industry, wishing to project the industry's prospects from 1990 to 2000 using the GSC model described earlier. Our views on exogenous variables for the Widget industry in 2000 are shown in Table 5.1 part A and our assumptions concerning movements in these variables from their 1990 values can be deduced by comparing Table 5.1A with Table 3.1A.

As in 1990, we assume for 2000 that: one unit of Design is required per unit of Components; one unit of Components is required per unit of Assembly; and one unit of Assembly is required per unit of SalesDist.

Consistent with the assumptions we made as CGE modellers, we assume as GSC modellers that between 1990 and 2000 there will be labour-saving technical change of 15 per cent in R1's Widget industry, and that this applies to the four Widget activities. For 1990 output per unit of labour in the four activities in R1 at standard scale was one (Table 3.1A). Thus, as shown in panel (b) of Table 5.1A, R1's output per unit of labour at standard scale in the four activities in 2000 is assumed to be  $1.1765 [= 1/(1-0.15)]$ .

As CGE modellers in section 4 we assumed labour-saving technical progress between 1990 and 2000 in R2's Widget industry of 27.75 per cent, made up of 15 per cent background labour-saving technical progress applying generally in R2 plus an extra 15 per cent in Ind1. In 1990 the only Widget activity in R2 was SalesDist (Table 3.1B). Now as GSC modellers we assume that the 27.75 per cent labour-saving technical progress applies in R2 to this activity. This assumption is reflected in panels (b) of Tables 3.1A and 5.1A which show an increase in PROD for SalesDist in R2 from 0.25 in 1990 to 0.3460 in 2000.<sup>5</sup> In each of the other three Widget activities we assume that PROD in R2 more than doubles from its very low levels in 1990. Despite this, as can be seen from Table 5.1A panel (b), we assume that R2's productivity levels in traded Widget activities in 2000 remain well below those in R1.

On tariffs, we note the trend towards free trade. As can be seen from panels (c) in Tables 5.1A and 3.1A, we assume that this trend applies to the Widget industry, and that tariffs on Design will fall from 10 per cent in 1990 in both regions to 5 per cent in 2000. For Components we assume a fall from 20 per cent to 5 per cent and for Assembly a fall from 20 per cent to 10 per cent.

In our role as Widget specialists wishing to apply the GSC model, we need to make assumptions about economy-wide wage rates and final demands for Widgets (demands for the product SalesDist). These assumptions must be guided by movements in productivity and income outside the Widget industry. Comparing panels (d) and (e) in Tables 5.1A with the corresponding panels in 3.1A shows our wage and demand assumptions: 20 per cent wage and 50 per cent demand growth in R2, and zero growth in these variables in R1. This is consistent with rapid catch-up by R2.

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<sup>5</sup> Labour-saving technical progress of 27.75 per cent means that a given level of output can be produced with 27.75 per cent less workers. Hence output per worker increases by 38.4 per cent  $[= 100*(1/(1-0.2775)-1)]$ . The movement in PROD from 0.25 to 0.3460 is a 38.4 per cent increase.

**Table 5.1. World Widget sector in 2000**

**A. Technology assumptions and data for 2000**

	Design	Components	Assembly	SalesDist
<i>(a) Intermediate inputs for producing 1 unit of output of each commodity in both regions (<math>\bar{A}</math>)</i>				
Design	0	1	0	0
Components	0	0	1	0
Assembly	0	0	0	1
SalesDist	0	0	0	0
<i>(b) Output per unit of labour input, standard scale (Productivity, <math>PROD</math>)</i>				
R1 (US)	1.1765	1.1765	1.1765	1.1765
R2 (Asia)	0.1765	0.3922	0.3069	0.3460
<i>(c) Powers of tariffs (<math>T</math>) on imports by importing region</i>				
R1 (US)	1.05	1.05	1.1	1
R2 (Asia)	1.05	1.05	1.1	1
<i>(d) Demand for final product:</i>				
R1 (US)				Y(R1) = 1
R2 (Asia)				Y(R2) = 0.75
<i>(e) Wage rate (<math>W</math>)</i>				
R1 (US)		1.0		
R2 (Asia)		0.3		
<i>(f) Labour costs per unit of output at standard scale (SLC), that is <math>W/PROD</math></i>				
R1 (US)	0.85	0.85	0.85	0.85
R2 (Asia)	1.70	0.76	0.98	0.87

**B. Output, employment, trade and prices in 2000: solution from GSC model**

	Price	Output	Employment	Exports, qty	Exports, value
<b>R1</b>					
Design	0.808	1.75	1.413	1.75	1.413
Components	1.657	0.00	0.000	0.00	0.000
Assembly	2.503	1.00	0.850	0.00	0.000
SalesDist	3.353	1.00	0.850	0.00	0.000
<b>Total</b>			3.113		1.413
VA=3.113					
<b>R2</b>					
Design	1.700	0.00	0.000	0.00	0.000
Components	1.575	1.75	4.239	1.00	1.575
Assembly	2.552	0.75	2.444	0.00	0.000
SalesDist	3.419	0.75	2.168	0.00	0.000
<b>Total</b>			8.851		1.575
VA=2.655					

Given the assumptions in Table 5.1A, our GSC model produces the solution shown in Table 5.1B. Comparing this 2000 solution with the 1990 solution (Table 3.1B), we see that production of Components has switched entirely from R1 to R2. Although R2's productivity in Components in 2000 is low relative to that in R1, R2's wage rate remains sufficiently low relative to that in R1 to give R2 a competitive edge in Component production. As can be seen from panel (f) in Table 5.1A, labour cost at standard scale per unit of output in Components in R2 in 2000 is less than that in R1 (0.76 compared with 0.85). For Assembly production, R2's labor cost per unit of output remains above that in R1 in 2000 (0.98 compared with 0.85). Despite this, R2 undertakes Assembly production to satisfy its own

needs. R1 continues to produce Assembly but no longer exports. Why shouldn't R1 continue to produce all of the Assembly required by both regions?

Given that Components are entirely produced in R2, splitting Assembly production not only saves trade costs (tariff payments) on Assembly but also on Components. It turns out that the saving of trade costs more than offsets the now relatively small reduction in world Assembly costs that would follow from leaving R1 as the sole Assembler.

With the complete switch of world Components productions and the partial switch of Assembly production from R1 to R2, together with rapid productivity growth, Widget employment in R1 declines sharply, from 5.275 in 1990 to 3.113 in 2000. By contrast, Widget employment in R2 increases sharply, from 2 in 1990 to 8.851 in 2000. R1's 1990 trade surplus in Widgets of 1.425 (Table 3.1B) turns into a 2000 trade deficit of 0.162 (= 1.575-1.413, Table 5.1B).

## 6. Iterating to impose the GSC solution on the CGE model: a non-converging case

In the GSC Widget solution for 2000 we assumed wage and Widget demand increases between 1990 and 2000 in R1 of zero and in R2 of 20% and 50%. If Widget productivity and tariff movements for 1990 to 2000 were as assumed in our GSC model, what would a CGE model tell us about wage rates and Widget demands?

In section 2 (see Figure 2.1) we described an algorithm that aims to ensure consistency: (a) between wage and Widget demand assumptions in GSC solutions and outcomes for these variables in CGE solutions; and (b) between tariff and Widget technology assumptions in CGE solutions and outcomes for these variables in GSC solutions. Achievement of these consistencies is what we call integration of the models.

Table 6.1 shows our first attempt to implement the consistency algorithm. We started with CGE iter1. This is the baseline CGE solution described in section 4. It incorporates the shocks listed in section 4 together with the assumption of no change in labour input in either region. As shown in the CGE iter1 column of Table 6.1, this solution implies a wage increase in R2 of 13.8192 per cent (zero in R1 by the numeraire assumption) and Widget demand increases in R1 and R2 of 9.7714 and 32.4588 per cent. These results can also be seen (with less decimal places) in rows 3 and 9 of Table 4.1.

In GSC iter1 we solve the GSC model with the Widget assumptions given in Table 5.1A except that the wage ( $W$ ) and final demand assumptions ( $Y$ ) are replaced by results from CGE iter1.<sup>6</sup> This replacement is indicated by the arrow from the CGE iter1 column in Table 6.1 to the GSC iter1 column. After the GSC model is solved with these new  $W$  and  $Y$  values, selected results are passed to the CGE model.

The block of selected GSC results from GSC iter1 that are passed to the CGE model are indicated by the arrow from the GSC iter1 column to the CGE iter2 column. These GSC results show percentage changes between 1990 and 2000 in labour and intermediate inputs per unit of output in each region's Widget industry and also average powers of tariffs. Inputs, output and powers of tariffs are derived for each region's Widget industry as a whole by aggregating results for Widget activities in the GSC model. The aggregation formulas are in the Appendix.

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<sup>6</sup> Instead of  $W(R2) = 0.3$  as in Table 5.1A, in GSC iter1 we use  $W(R2) = 0.2845$  (i.e. the 1990 value, 0.25, times 1.138192). Instead of  $Y(R1) = 1$  as in Table 5.1A, in GSC iter1 we use  $Y(R1) = 1.097714$  (i.e.  $1 * 1.097714$ ). Instead of  $Y(R2) = 0.75$  as in Table 5.1A, in GSC iter1 we use  $Y(R2) = 0.6623$  (i.e.  $0.5 * 1.324588$ ).

**Table 6.1 Iterating between the CGE and GSC models: non-converging  
(percentage differences between values of variables in 1990 and 2000)**

	CGE iter1	GSC iter1	CGE iter2	GSC iter2	CGE iter3	GSC iter3	CGE iter4
Baseline shocks (1) to (5)	YES						
Baseline shocks (2) and (4) (excludes Widget shocks, these come from GSC model)			YES		YES		YES
GSC productivity and tariff assumptions for Widget activities		YES		YES		YES	
<b>Widget results from GSC used as shocks in CGE</b>							
labour per unit of output, Ind1, R1		-14.4245		-14.0802		-14.5368	
labour per unit of output, Ind1, R2		5.4564		-27.7126		5.5539	
C1 from R1 per unit output in Ind1, R1		-39.3689		-0.8059		-39.3070	
C1 from R1 per unit output in Ind1, R2		-70.3976	→	0.0457	→	-70.2490	→
C1 from R2 per unit output in Ind1, R1		31301.7		8.5895		31326.4	
C1 from R2 per unit output in Ind1, R2		8950.6		-10.1346		8917.7	
power of tariff on R1's imports of C1		5*		0*		5*	
power of tariff on R2's imports of C1		-12.5		-8.33		-12.5	
<b>Results from CGE used as shocks in GSC</b>							
quantity of final demand for C1 in R1	9.7714		7.0179		9.7609		7.0208
quantity of final demand of C1 in R2	32.4588	→	37.8331	→	30.5935	→	37.8380
wage rate in R2	13.8192		25.2489		13.8918		25.2412

\* As shown in Table 3.2B, in 1990 R1 collected zero tariff revenue on negligible but non-zero imports of C1. These data imply (artificially) that the power of the tariff on R1's imports of C1 was one. In the GSC iter1 and iter3 solutions, R1 imports Components with a tariff of 5 per cent. Thus from the point of view of the CGE model, the power of the tariff on R1's imports of C1 has risen from 1 to 1.05, an increase of 5%. In the GSC iter 2 solution R1 imports no Widget products. We interpret this as implying no change in the average tariff rate.

**Table 6.2. Output, employment, trade and prices in 2000: solution from GSC iter1\***

	Price	Output	Employment	Exports, qty	Exports, value
<b>R1</b>					
Design	0.808	1.76	1.421	1.76	1.421
Components	1.657	0.00	0.000	0.00	0.000
Assembly	2.464	1.10	0.933	0.00	0.000
SalesDist	3.314	1.10	0.933	0.00	0.000
<b>Total</b>			3.287		1.421
			VA=3.287		
<b>R2</b>					
Design	1.612	0.00	0.000	0.00	0.000
Components	1.537	1.76	4.264	1.10	1.687
Assembly	2.464	0.66	2.158	0.00	0.000
SalesDist	3.287	0.66	1.914	0.00	0.000
<b>Total</b>			8.338		1.687
			VA=2.372		

\* This GSC solution was generated with  $W(R1) = 1$ ,  $W(R2) = 0.2845$ ,  $Y(R1) = 1.097714$  and  $Y(R2) = 0.6623$ . In the solution in Table 5.1B,  $W(R1) = 1$ ,  $W(R2) = 0.3$ ,  $Y(R1) = 1$  and  $Y(R2) = 0.75$ .

After the CGE model is solved in CGE iter2 with these selected inputs from GSC iter1, wage and final demand results from CGE iter2 are passed to the GSC model as indicated by the arrow from the CGE iter2 column to the GSC iter2 column. The GSC model is resolved in GSC iter2 and passes revised Widget tariff and technology results to the CGE model to be used in CGE iter3 and so on.

Before discussing convergence or lack of it, we pause to look at the GSC iter1 results that are passed to CGE iter2.

### **Results from GSC iter1**

The GSC iter1 solution is in Table 6.2. Although the wage and final demand assumptions in GSC iter1 are different from those in the stand-alone GSC solution for 2000 in Table 5.1B, the solution is quite similar. In both these GSC solutions R2 takes over the entire world production of Components, the production of Assembly is split between the two regions and R1 continues to be responsible for world production of Designs.

With R2 taking over Components production and Assembly shared, R2 supplies Components to Assembly in both regions. As shown in the GSC iter1 column of Table 6.1, this generates huge percentage increases (31301.7 & 8950.6) in the use of C1 from R2 as an intermediate input per unit of output in Widget production (Ind1) in R1 and R2. The percentage increases are huge because they are calculated from the negligible 1990 values (0.01) adopted in the 1990 input-output database (see Table 3.2B).<sup>7</sup> Corresponding to the huge percentage increases in the use of C1 from R2 in the production of C1 in both regions, GSC iter1 implies sharp decreases in C1 from R1 used as an intermediate input in Widget production per unit of output (39.3689% in R1 and 70.3976% in R2).

With regard to labour input per unit of output in Widgets, GSC iter1 shows an increase of 5.4564 per cent in R2 between 1990 and 2000 and a decrease of 14.4245 per cent in R1. The increase in R2 is the outcome of two factors, one positive and one negative. The positive

<sup>7</sup> It might be objected that because the negligible starting points, the 0.01s, are arbitrary then so are the percentage increases. But this doesn't matter. The important point is that the percentage increases will take us to the correct 2000 values.

factor reflects the changing composition of R2's Widget production from being purely SalesDist in 1990 to also including Components and Assembly in 2000. Because the value of output in SalesDist is dominated by the cost of the intermediate input (Assembly), labour input per unit of output in SalesDist is low. Thus, the move towards Components and Assembly production in R2 has a positive effect on the overall use of labour per unit of output for the sector. The negative factor is increased labour productivity occurring in all of R2's Widget activities. For R2 the positive effect of the composition change on labor input per unit of output outweighs the negative effect of productivity improvement. The 14.4245 per cent decrease in R1's use of labour per unit of output in Widget production is mainly a reflection of the 15 per cent productivity increases assumed for all of R1's Widget activities. R1's employment per unit of output in the Widget sector is also affected by compositional changes (loss of Component production) and by loss of scale in Assembly production.

The tariff results in GSC iter1 reflect the composition of each region's Widget imports and the powers of the tariffs applying to the separate Widget commodities. In 1990, R2's Widget imports consisted entirely of Assembly with a tariff power of 1.2. In the GSC iter1 solution for 2000, R2's Widget imports consist entirely of Design with a tariff power of 1.05. Thus for R2 the average power falls by 12.5 per cent. In 1990, R1 had no Widget imports. As explained in the note below Table 6.1, the 5 per cent increase in the power of the tariff on R1's Widget imports shown in the GSC iter1 column is an artefact of the emergence of Widget imports from a negligible base.

#### ***Why didn't the algorithm work?***

It is clear from Table 6.1 that our first attempt to implement the GSC-CGE integrating algorithm failed. The differences between the solutions in CGE iter4 and CGE iter3 are no smaller than the differences between CGE iter3 and CGE iter2. Rather than converging, our results are cycling between two solutions: one given by CGE iter2 & 4 and the other given by CGE iter1 & 3. From the point of view of the GSC model we can also see two distinct solutions: one given by GSC iter1 & 3 and the other given by GSC iter2.

The behaviour of the wage rate in R2 is the key to the cycling that is apparent in Table 6.1. In CGE iter1 there are no shocks from the GSC model. Given the CGE baseline productivity and tariff shocks listed in section 4, the CGE model implies a moderate increase in the wage rate for R2, 13.8192 per cent. On the basis of this moderate wage increase and the productivity and tariff shocks assumed at the Widget activity level, GSC iter1 produced the radical location changes in Widget activities that we can see in the comparison of Table 6.2 with Table 3.1B (or in the earlier comparison of Table 5.1B with Tables 3.1B). Given the changes in inputs per unit of Widget output implied by GSC iter1, CGE iter2 gives a large wage increase for R2, 25.2489 per cent.

With this large wage increase, the GSC model tells us that Widget activity will not be transferred from R1 to R2. Consequently, GSC iter2 shows relatively mild percentage changes in inputs per unit of Widget output, reflecting mainly the background productivity changes assumed for the four Widget activities in R1 and for SalesDist in R2. Given these relatively small Widget shocks, CGE iter3 implies a moderate wage increase for R2, close to that in CGE iter1 in which there were no shocks from the GSC model.

When the moderate wage increase for R2 from CGE iter3 is introduced to the GSC model, the GSC model once more generates the radical location shift in Widget activity that we saw in Table 6.2. Thus, when used in CGE iter4, the shocks coming out of GSC iter3 again support a large wage increase in R2, 25.2412 per cent. And hence the cycling continues.

Figure 6.1 is a picture of what is happening in Table 6.1. The figure shows three CGE demand curves for labour in R2: the demand curve in 1990; the demand curve in 2000 on the assumption that R2 is not receiving any GSC shocks; and the demand curve in 2000 under the assumption that R2 is receiving GSC shocks such as those going into CGE iter2 and CGE iter4. The figure also shows the labour-supply curve for 1990 and 2000 assumed in the CGE simulations in Table 6.1: no-change in employment. The demand and supply curves in Figure 6.1 are drawn to be consistent with our CGE results. They imply wage increases in R2 of about 25.2 per cent when R2 benefits from participation in Widget GSC activities and about 13.8 per cent in the no-GSC case. As we move across the CGE solutions in Table 6.1, we cycle between the solution on the low demand curve for 2000 and the solution on high demand curve.

The figure makes it clear that no amount of fine-tuning of our algorithm (e.g. partial adjustment between iterations) is going to lead to a converged solution. The truth is that there is no converged solution under the assumptions we are making. If R2 is participating in the Widget GSC then the CGE model implies that it can't participate – its wage rate is too high. If R2 is not participating in the Widget GSC then the CGE model implies that it will participate – its wage rate is sufficiently low.

## 7. Obtaining GSC-CGE convergence: giving R2 surplus labour

As illustrated in Figure 7.1, the opportunity to participate in GSC trade in effect gives R2 a demand curve for labour that contains a hole. By experiment, we found that if the wage rate in R2 is greater than about 0.301, then the GSC model indicates that R2 will be on the *without*-GSC demand curve in 2000, otherwise it will be on the *with*-GSC curve. The vertical labour supply curve that we assumed in our initial formulation of the GSC-CGE integration problem happens to pass through the hole between the two segments of the demand curve. In theory, we could determine the exact value of  $W(R2)$  at which the global agent controlling the Widget industry is indifferent between radical location switches, of the type we saw in Tables 6.2 and 5.1B, and leaving all traded Widget activities entirely in R1. But, with a vertical labour supply curve, this still wouldn't give us a solution to the combined GSC-CGE model. If, at the indifference-level of  $W(R2)$  the global agent chose to switch, then demand for labour in R2 would exceed supply. If, on the other hand, the global agent chose not to switch, then supply of labour in R2 would exceed demand.

Figure 7.1 not only helps us understand why there is no solution to the GSC-CGE problem with inelastic labour supply in R2, but it also suggests how we should reformulate the GSC-CGE problem. We need to recognize that the opportunity to participate in GSC activity can affect R2's labour supply, not the supply of people but the supply of labour input.

Assume that R2 has surplus labour. We are thinking of a developing country, e.g. China in 1990, with large numbers of agriculture workers whose marginal product is close to zero. As explained by Lewis (1954), these workers can survive in agriculture because they are paid the average product of their family group rather than their individual marginal product. Now assume that a global supply chain opportunity becomes available that offers jobs compatible with the skills of R2's surplus workers at wages that are considerably higher than average farm product. At these higher wages, we assume that there is an unlimited supply of labour to GSC activities in R2 of workers whose marginal product in their previous employment was close to zero. We represent this situation in Figure 7.1 by a perfectly elastic supply curve of labour input. Provided labour-input supply is perfectly elastic at a wage below the critical 0.301, Figure 7.1 indicates that a converged GSC-CGE solution with R2 participation in the Widget GSC exists. This is confirmed in Table 7.1.

Figure 6.1. Demand and supply curves for labour in R2: non-converging CGE solutions

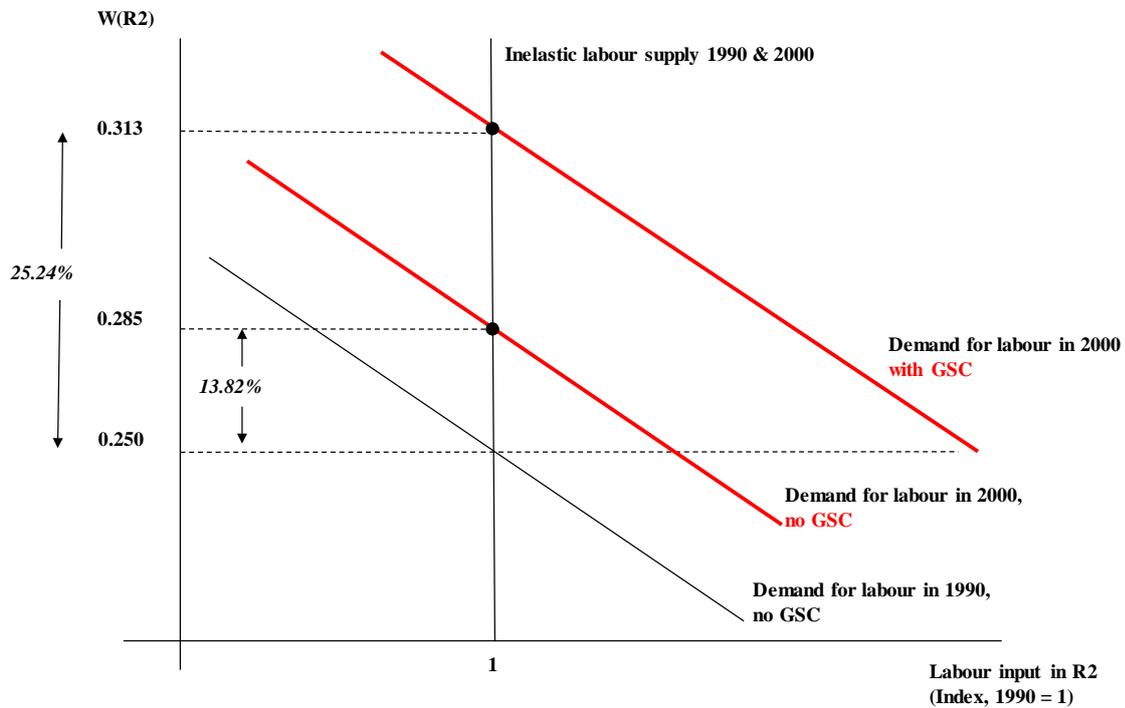
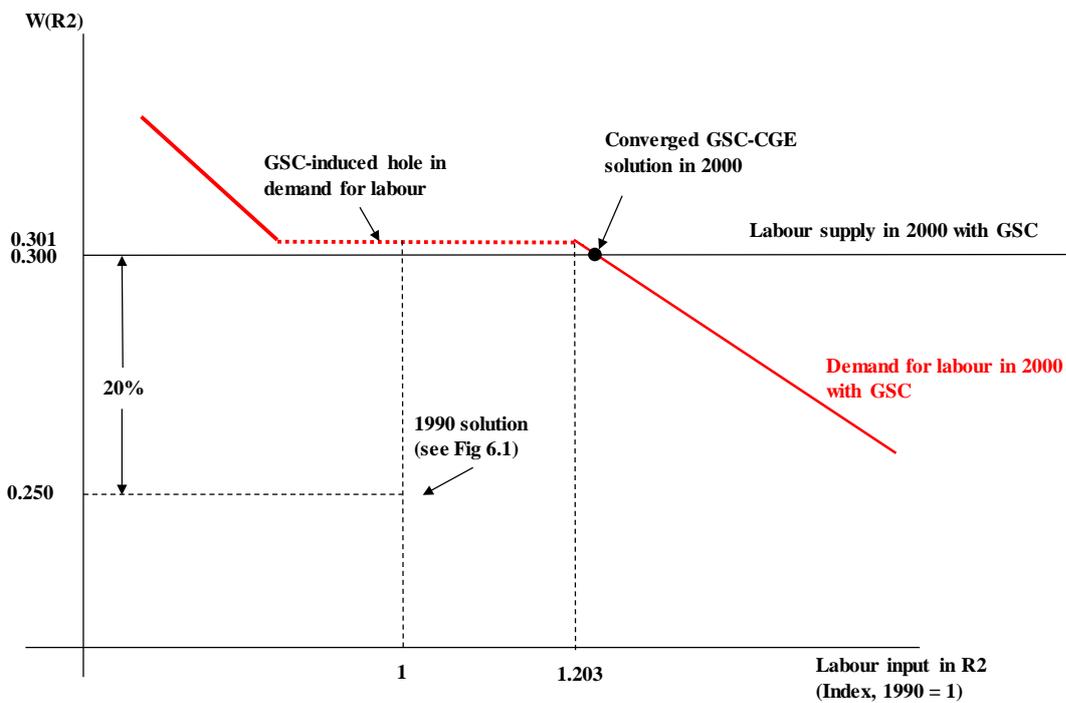


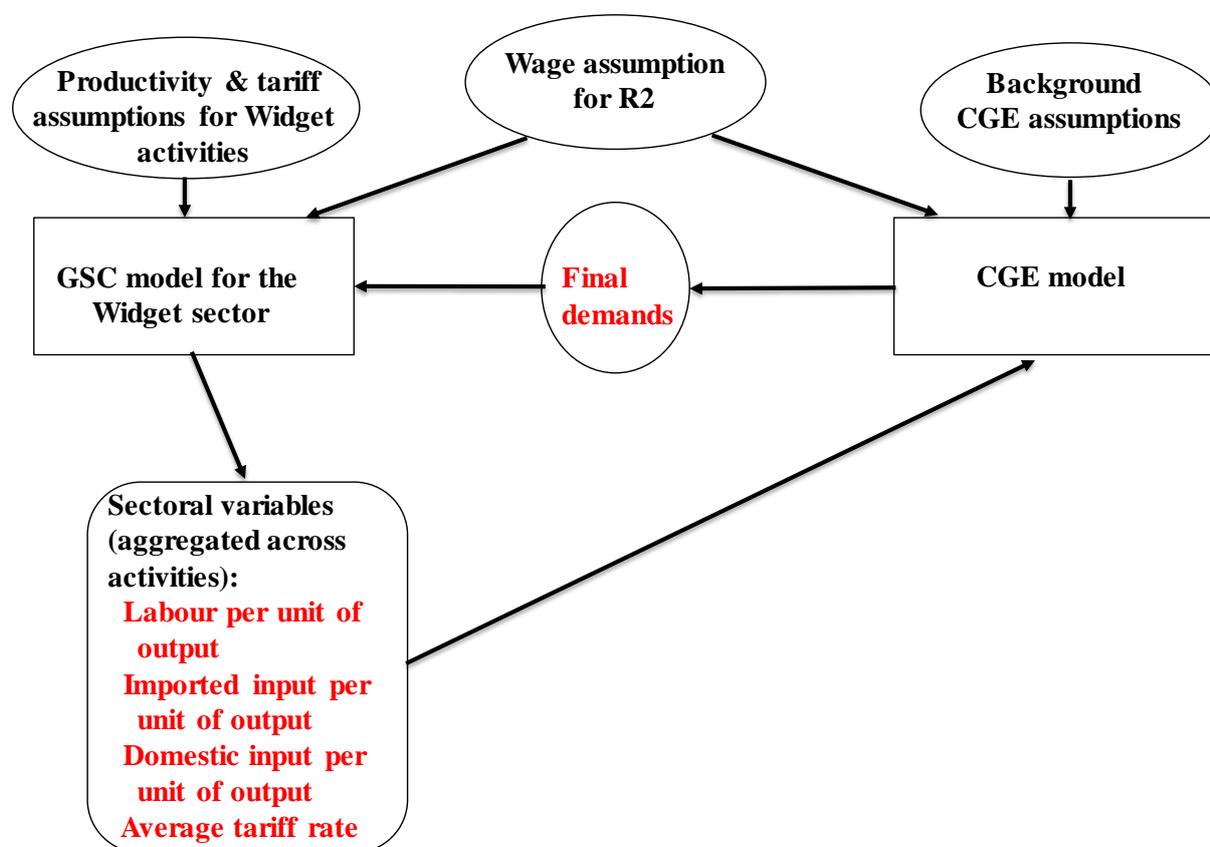
Figure 7.1. Demand and supply curves for labour in R2: converging CGE solution



**Table 7.1. Iterating between the CGE and GSC models: converging with elastic labour supply in R2  
(percentage differences between values of variables in 1990 and 2000)**

	GSC iter1	CGE iter2b	GSC iter2b	CGE iter3b	GSC iter3b	CGE iter4b
Baseline shocks (1) to (5)						
Baseline shocks (2) and (4) (excludes Widget shocks, these come from GSC model)		YES		YES		YES
GSC productivity and tariff assumptions for Widget activities	YES		YES		YES	
<b>Widget results from GSC used as shocks in CGE</b>						
labour per unit of output, Ind1, R1	-14.4245		-12.3505		-12.3401	
labour per unit of output, Ind1, R2	5.4564		4.8818		4.8759	
C1 from R1 per unit output in Ind1, R1	-39.3689	→	-40.1062	→	-40.1114	→
C1 from R1 per unit output in Ind1, R2	-70.3976		-72.7764		-72.7861	
C1 from R2 per unit output in Ind1, R1	31301.7		30320.1		30317.3	
C1 from R2 per unit output in Ind1, R2	8950.6		9427.66		9429.8	
power of tariff on R1's imports of C1	5		5		5*	
power of tariff on R2's imports of C1	-12.5		-12.5		-12.5	
<b>Results from CGE used as shocks in GSC</b>						
quantity of final demand for C1 in R1		7.9829	→	8.0679	→	8.0678
quantity of final demand of C1 in R2		63.1089		63.4067		63.4083
wage rate in R2		20	20	20	20	20
employment in R2		19.9		20.3		20.3

*Figure 7.2 An algorithm for integrating GSC & CGE models: elastic labour supply in R2*



In generating Table 7.1 we used the algorithm in Figure 7.2. This is a modified version of the initial algorithm in Figure 2.1. In the initial algorithm, the wage rate in R2 is determined endogenously in the CGE model and passed as a shock to the GSC model. Now, under the assumption that labour supply is perfectly elastic in R2, we make an assumption about the wage rate in R2 and feed it exogenously into both the GSC and CGE models.

In applying the modified algorithm we started by generating the CGE solution shown in the CGE iter2b column of Table 7.1. In this solution, we used the GSC-related shocks from GSC iter1 in Table 6.1 (reproduced in the first column of Table 7.1). However, instead of allowing the CGE model to determine the wage rate in R2, we set its increase between 1990 and 2000 exogenously at 20 per cent and determined employment endogenously.

As indicated by the arrow out of the CGE iter2b column of Table 7.1, we transferred the final demand results from CGE iter2b into GSC iter2b. In GSC iter2b we adopted the same wage increase for R2 (20 per cent) that was assumed in CGE iter2b. Continuing as in Table 6.1, we transferred GSC Widget results from GSC iter2b into CGE iter3b. Convergence was achieved quickly.

The R2 wage assumption we chose for our stylized example, an increase between 1990 and 2000 of 20 per cent, takes R2's wage to 0.300 ( $= 0.25 \times 1.2$ ). This wage is close to the

maximum level compatible with R2's participation in the Widget GSC. By choosing this wage increase we illustrate a situation in which R2 receives close to the maximum wage benefit available from GSC participation and close to the minimum labour-input benefit. Other combinations of wage and labour-input benefit are obviously available. But even the minimum labour-input benefit is substantial. In the converged GSC-CGE solution in Table 7.1, labour-input in R2 grows by 20.3 per cent.

## **8. Comparison of the standard CGE and the integrated GSC-CGE projections for 1990 to 2000**

Table 8.1 compares the projections discussed in section 4 (Table 4.1) made using the standard CGE model with those implied by the converged GSC-CGE solution in Table 7.1.

The inclusion of GSC trade has a generally negative effect on the projected macro prospects of R1. This confirms the impression of a negative outcome for the advanced region from the emergence of GSC trade that we obtained from the GSC-only analysis in Athukorala *et al.* (2018). With R2's participation in the Widget GSC, the projected increase in R1's GDP falls from 2.72 per cent to 1.83 per cent (row 1, Table 8.1). While we assume that the GSC makes no difference to aggregate labour-input in R1 (row 22), the recognition of GSC trade reduces R1's GDP growth by transferring resources from Ind1 in which productivity growth is rapid to Ind2 in which productivity growth is slow. Reflecting both the reductions in productivity growth and GDP growth, real wage and consumption growth is lower in R1 in the GSC-CGE integrated solution than in the standard CGE solution (1.36 per cent compared with 2.41 per cent and 1.63 per cent compared with 2.41 per cent, rows 4 and 2). A slight positive for R1 from GSC trade is a less negative movement in its terms of trade (-1.42 per cent compared with -2.12 per cent, row 15). With technical change reducing the price of C1 relative to that of C2, the less-negative terms-of-trade outlook for R1 is explained by a GSC-induced reduction in the C1-share in its exports and increase in the C1-share of its imports.

By contrast, for R2 the inclusion of GSC trade has a major positive effect on the region's projected macro prospects. Whereas the standard CGE projection for GDP growth in R2 was 18.76 per cent, this is raised to 45.93 per cent in the converged GSC-CGE solution. Correspondingly, the converged GSC-CGE solution shows much larger increase in R2's real consumption than standard CGE solution (46.82 per cent compared with 19.76 per cent, row 2).

The extra GDP growth for R2 in the GSC-CGE solution comes mainly from extra labour input, 20.34 per cent (row 22). This extra labour input is not more people in work. Rather, it is more people employed in activities in which their marginal product is non-negligible, in line with their wage. GSC trade has allowed some people to contribute to labour input who, although previously employed, were not adding positively to labour input. Fast technical change in Ind1 relative to Ind2 also contributes to R2's extra GDP growth in the GSC-CGE solution relative to the standard CGE solution.

In section 4 we explained that the standard CGE model produced conservative business-as-usual projections at the industry/commodity level. By contrast, the integrated GSC-CGE model projects major changes in the industrial composition of employment in each region and the commodity composition of trade.

**Table 8.1. Projection from 1990 to 2000 using standard CGE and integrated GSC-CGE models**

		Standard CGE		Integrated GSC-CGE	
		R1	R2	R1	R2
1	Real GDP (% change from 1990 to 2000)	2.72	18.76	1.83	45.93
2	Real consumption, welfare (% change)	2.41	19.76	1.63	46.82
3	Wage rate	0.00	13.82	0.00	20.00
4	Real wage rate, CPI delated (% change)	2.41	22.19	1.36	24.68
5	Factory price of C1 (% change)	-15.01	-23.85	-12.74	-22.30
6	Factory price of C2 (% change)	0.00	-3.25	0.00	2.00
7	Purchasers' price to consumers of C1 (% change)	-15.01	-23.85	-12.74	-22.30
8	Purchasers' price to consumers of C2 (% change)	-0.53	-2.22	0.30	1.32
9	Consumption of C1, quantity (% change)	9.77	32.46	8.07	63.41
10	Consumption of C2, quantity (% change)	1.47	16.89	0.80	43.10
11	Value of exports (% change)	9.11	9.11	36.62	36.62
12	Value of imports (% change)	9.11	9.11	36.62	36.62
13	Quantity of exports (% change)	15.28	12.84	43.26	41.23
14	Quantity of imports (% change)	12.84	15.28	41.23	43.26
15	Terms of trade (% change)	-2.12	2.17	-1.42	1.44
16	Labour input in Ind1	-1.51	-4.29	-35.96	381.48
17	Labour input in Ind2	0.30	0.21	7.18	2.28
18	Exports C1 (100 times change in value at initial prices as share of initial GDP)*	1.47	0.04	1.12	20.52
19	Exports C2 (100 times change in value at initial prices as share of initial GDP)	0.66	5.05	4.54	-2.15
20	Imports C1 (100 times change in value at initial prices as share of initial GDP)	0.01	4.30	6.99	3.30
21	Imports C2 (100 times change in value at initial prices as share of initial GDP)	1.72	1.94	-0.73	13.32
22	Aggregate labour input	0	0	0	20.34

\* 100\*(Quantity in final year times price in initial year – Value in initial year)/GDP in initial year

In rows 16 and 17 of the GSC-CGE projections, R1's labour input (and employment) in Ind1 falls by 35.96 per cent while in Ind2 it increases by 7.81 per cent. With no change in aggregate labour input (row 22), which is the same as employment in R1, this compositional change means that 6 per cent of R1's workforce must be reassigned from Ind1 to Ind2.<sup>8</sup> This can be compared with the standard CGE projection which showed a shift of only 0.25 per cent of the workforce between the two industries.

For R2, the integrated GSC-CGE model projects labour-input increases in both industries: by 381.48 per cent in Ind1 and 2.28 per cent in Ind2, giving an overall increase of 20.34 per cent (row 22). This 20.34 per cent increase in labour input is a transfer of unproductive Ind2 workers into productive work either in Ind1 or Ind2. Enough of these unproductive workers find employment in Ind1 to boost the economy's total labour input by 18.16 per cent [ $=381.48*0.5/(0.5+10)$ ]. A further 2.17 per cent [ $=2.28*10/(0.5+10)$ ] boost comes from transfer of unproductive Ind2 workers into productive Ind2 work. Most of R2's extra demand for C2 associated with extra consumption (row 2) is satisfied by extra imports. Nevertheless, under the Armington assumption, extra output of the domestic product is required. This generates the increased demand for labour input in R2's Ind2.

While GSC trade imposes large changes in the industrial composition of employment in both regions, the nature of the structural adjustments in the two regions is quite different. R1 is faced with a situation in which workers in one industry must move to another industry or suffer unemployment. Consequently, R1 faces a potential structural adjustment problem. By contrast, GSC trade gives unproductive workers in R2's Ind2 an opportunity to move voluntarily to higher wage more productive employment in Ind1 or to contribute fully to output in Ind2. Rather than a structural problem, GSC trade gives R2 the potential for a structural improvement.

As illustrated by rows 11 to 14 of Table 8.1, the integrated GSC-CGE model projects much more growth in trade values and volumes than the standard CGE model. Trade growth is increased for both regions by the dramatic increase in R2's GDP made possible by GSC trade. Although R2 is the major beneficiary of GSC trade, perhaps counter-intuitively, R1's trade share in GDP is increased much more strongly by GSC trade than R2's trade share. As explained in section 4 for the standard CGE model, from R1's point of view, the size of export markets increases strongly relative to domestic markets. This effect is accentuated in the GSC-CGE model in which the gap widens between GDP growth in R2 (a size indicator of R1's export market) and GDP growth in R1 (a size indicator for R1's domestic market).

Relative to the conservative projections generated by the standard CGE model, the integrated GSC-CGE model shows sharp changes in the commodity composition of trade, particularly R2's exports. Whereas the standard CGE model projected continuation of the status quo from 1990 in which R2's exports are dominated by C2, the integrated model projects an increase in R2's exports of C1 worth (at 1990 prices) 20.52 per cent of R2's 1990 GDP. This boosts the share of C1 in R2's exports from close to zero in 1990 to 30 per cent in 2000.<sup>9</sup>

## 9. Concluding remarks

We have demonstrated a method for building and solving an integrated GSC-CGE model in which the two models are solved separately with information being passed from one to the other. This is a divide-and-conquer approach similar to that pioneered by Balistreri and

<sup>8</sup> This can be worked out from the percentage change results in Table 8.1 and the labour-input data in Table 3.2B as either  $35.96*5.265/(5.265+26.375)$  or  $7.18*26.375/(5.265+26.375)$ .

<sup>9</sup> The values of R2's exports of C1 and C2 in 2000 can be computed from the results in Table 8.1 and the data in Table 3.2B using the formula;  $VAL_{2000} = VAL_{1990} + \Delta P*Q + P*\Delta Q + \Delta P*\Delta Q$ .

Rutherford (2013) for solving CGE models with embedded Melitz sectors<sup>10</sup> (heterogeneous firms, imperfect competition and scale economies). Balistreri and Rutherford compute solutions for each Melitz sector separately using guesses for sectoral demands and wage rates. Parts of these solutions (technology and preference variables) are fed into a standard general equilibrium model to generate revised guesses for demands and wage rates. A solution for the Melitz-enhanced CGE model is achieved when wage rates and demands coming out of the CGE model are the same as the guesses going into the previous round of Melitz sectoral computations.

The divide-and-conquer approach adopted here allows separate development of GSC sectoral models without the clutter of the CGE detail. This is efficient from computational and research management points of view. In this paper, it had the additional bonus of alerting us to the possibility of a hole in the labour demand curve for the developing country. Coping with the hole led us to a surplus-labour specification for the developing country and the striking results showing that low-income countries with surplus labour may be the major beneficiaries from the emergence of GSC trade. This contrasts with analyses based on Shih's (1996) smiling curve which is sometimes used to argue that high-income countries are the major beneficiaries because they undertake the high value-added activities (Design and Sales/distribution) at the two ends of supply chains. What our integrated GSC-CGE model suggests is that by providing low-skilled jobs, GSC trade can accelerate the transfer of labour out of agriculture, where workers have low marginal productivity, into manufacturing where their marginal productivity is much higher. At the same time, our integrated model suggests that GSC trade can leave high-income countries having to transfer considerable fractions of their workforce out of manufacturing and into services. Even when they have achieved this potentially expensive structural adjustment, they may be left in the long run with no more than a small equilibrium welfare gain or even a loss.

### **Appendix. The mathematics of integrating the GSC Widget model and a CGE model**

This appendix has four parts. In the first part we set out the conditions for full integration of the GSC and CGE models described in subsections 2.1 and 2.2. The second part defines aggregate Widget variables from the GSC model. These variables must be reconciled with variables from the Widget sector in the CGE model. The third part states and proves a proposition that justifies the algorithm described in Figures 2.1 and 7.2. The final part contains a note on the algorithmic approach versus simultaneous solution of the two models.

#### ***Conditions for full integration***

For the case of the GSC and CGE models described in subsections 2.1 and 2.2, we listed in subsection 2.3 five conditions for full integration. Algebraically, these conditions can be expressed as:

$$(i) \quad W^{\text{gsc}}(d) = W^{\text{cge}}(d) \quad \text{and} \quad Y_{\text{w}}^{\text{gsc}}(d) = Y_{\text{w}}^{\text{cge}}(d) \quad (\text{A1})$$

$$(ii) \quad T_{\text{w}}^{\text{gsc}}(r, d) = T_{\text{w}}^{\text{cge}}(r, d), \quad V_{\text{w}}^{\text{gsc}}(r, d) = V_{\text{w}}^{\text{cge}}(r, d), \quad VT_{\text{w}}^{\text{gsc}}(r, d) = VT_{\text{w}}^{\text{cge}}(r, d) \quad (\text{A2})$$

$$(iii) \quad A_{\text{ww}}^{\text{gsc}}(r, d) = A_{\text{ww}}^{\text{cge}}(r, d) \quad \text{and} \quad ALAB_{\text{w}}^{\text{gsc}}(d) = ALAB_{\text{w}}^{\text{cge}}(d) \quad (\text{A3})$$

$$(iv) \quad P_{\text{fw}}^{\text{gsc}}(d) = P_{\text{w}}^{\text{cge}}(d) \quad (\text{A4})$$

$$(v) \quad EMP_{\text{w}}^{\text{gsc}}(d) = EMP_{\text{w}}^{\text{cge}}(d) \quad (\text{A5})$$

In (A1) – (A5), superscript gsc indicates a variable of the GSC model and superscript cge indicates a variable of the CGE model. The first condition for full integration is that wage

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<sup>10</sup> See Melitz (2003).

rates [  $W^{\text{gsc}}(d)$  and  $W^{\text{cge}}(d)$  ] and final demands for Widgets [  $Y_W^{\text{gsc}}(d)$  and  $Y_W^{\text{cge}}(d)$  ] are the same in the two models. The second condition is that powers of tariffs [  $T_W^{\text{gsc}}(r,d)$  and  $T_W^{\text{cge}}(r,d)$  ], the values of Widget trade flows [  $V_W^{\text{gsc}}(r,d)$  and  $V_W^{\text{cge}}(r,d)$  ] and associated tariff collections [  $VT_W^{\text{gsc}}(r,d)$  and  $VT_W^{\text{cge}}(r,d)$  ] are the same in the two models. The third condition is that Widget inputs from region  $r$  per unit of Widget output in region  $d$  [  $A_{\text{WW}}^{\text{gsc}}(r,d)$  and  $A_{\text{WW}}^{\text{cge}}(r,d)$  ] and labour input per unit of Widget output in region  $d$  [  $ALAB_W^{\text{gsc}}(d)$  and  $ALAB_W^{\text{cge}}(d)$  ] are same in the two models. The fourth condition is that the price in region  $d$  of the Widget good in the CGE model [  $P_W^{\text{cge}}(d)$  ] is the same as the price in region  $d$  of the final Widget good in the GSC model [  $P_{\text{FW}}^{\text{gsc}}(d)$  ]. The final condition is that Widget employment in both models is the same in all regions.

### ***Defining the sectoral or aggregate Widget variables for the GSC model***

The CGE variables mentioned in (A1) to (A5) are naturally occurring or core variables of the CGE model. They don't need any further elaboration. By contrast, the sectoral variables in (A1) to (A5) from the GSC model are add-ons or reporting variables requiring explicit definitions. The definitions we adopt for GSC sectoral variables are as follows:

$$X_W^{\text{gsc}}(r) = \frac{\sum_{i \in \text{WC}} P_{iW}^{\text{gsc}}(r) * X_{iW}^{\text{gsc}}(r)}{P_{\text{FW}}^{\text{gsc}}(r)} \quad \text{for all } r \in R \quad (\text{A6})$$

$$A_{\text{WW}}^{\text{gsc}}(r,d) = \frac{\sum_{i \in \text{WCI}} \sum_{j \in \text{WA}} P_{iW}^{\text{gsc}}(r) * A_{ijW}^{\text{gsc}}(r,d) * X_{jW}^{\text{gsc}}(d)}{P_{\text{FW}}^{\text{gsc}}(r)} * \frac{1}{X_W^{\text{gsc}}(d)} \quad \text{for all } r, d \quad (\text{A7})$$

$$\text{EMP}_W^{\text{gsc}}(r) = \sum_{j \in \text{WA}} X_{jW}^{\text{gsc}}(r) * \frac{\text{SCALE}_{jW}^{\text{gsc}}(r)}{\text{PROD}_{jW}^{\text{gsc}}(r)} \quad \text{for all } r \in R \quad (\text{A8})$$

$$ALAB_W^{\text{gsc}}(d) = \frac{\text{EMP}_W^{\text{gsc}}(r)}{X_W^{\text{gsc}}(r)} \quad \text{for all } d \quad (\text{A9})$$

$$V_W^{\text{gsc}}(r,d) = \sum_{i \in \text{WCI}} \sum_{j \in \text{WA}} P_{iW}^{\text{gsc}}(r) * A_{ijW}^{\text{gsc}}(r,d) * X_{jW}^{\text{gsc}}(d) \quad \text{for all } r, d \in R \quad (\text{A10})$$

$$VT_W^{\text{gsc}}(r,d) = \sum_{i \in \text{WCI}} \sum_{j \in \text{WA}} [T_{iW}^{\text{gsc}}(r,d) - 1] * P_{iW}^{\text{gsc}}(r) * A_{ijW}^{\text{gsc}}(r,d) * X_{jW}^{\text{gsc}}(d) \quad \text{for all } r, d \in R \quad (\text{A11})$$

$$T_W^{\text{gsc}}(r,d) = 1 + \frac{VT_W^{\text{gsc}}(r,d)}{V_W^{\text{gsc}}(r,d)} \quad \text{for all } r, d \in R \quad (\text{A12})$$

(A6) gives the definition for the GSC model of output [  $X_W^{\text{gsc}}(r)$  ] by the Widget sector in region  $r$ . It adds over outputs of Widget commodities using price-ratio weights [  $P_{iW}^{\text{gsc}}(r)/P_{\text{FW}}^{\text{gsc}}(r)$  ].  $P_{iW}^{\text{gsc}}(r)$  is the value of inputs embedded in the production of a unit of Widget commodity  $i$ . Consequently,  $P_{iW}^{\text{gsc}}(r)/P_{\text{FW}}^{\text{gsc}}(r)$  is the fraction of a unit of the final good completed by the production of a unit of Widget commodity  $i$ . In (A6), production of the final good gets a weight of one in the calculation of sectoral output whereas production of a

Widget commodity  $i$  half way along the production process to the final good gets a weight of 0.5.

(A7) gives the definition for the GSC model of Widget input from region  $r$  per unit of Widget output in region  $d$ ,  $[A_{ww}^{gsc}(r, d)]$ . The quantity of Widget input from region  $r$  is defined in accordance with (A6) as a price-ratio-weighted  $[P_{iw}^{gsc}(r)/P_{fw}^{gsc}(r)]$  sum of the quantities of individual Widget goods ( $i \in WCI$ ) flowing from  $r$  to Widget activities ( $j \in WA$ ) in  $d$ ,  $[A_{ijw}^{gsc}(r, d) * X_{jw}^{gsc}(d); i \in WCI, j \in WA]$ .

(A8) gives the definition for the GSC model of employment  $[EMP_w^{gsc}(r)]$  in region  $r$ 's Widget sector. This is the sum of labour inputs over individual Widget activities ( $j \in WA$ ) in region  $r$ . Labour input in each activity is calculated as output  $[X_{jw}^{gsc}(d)]$  times labour requirement per unit of output,  $[SCALE_{jw}^{gsc}(r)/PROD_{jw}^{gsc}(r)]$ .

(A9) gives the definition for the GSC model of labour input per unit of Widget sector output in region  $d$ ,  $[ALAB_w^{gsc}(d)]$ . This is the ratio of sectoral employment to sectoral output.

(A10) gives the definition for the GSC model of the total factory value (same as CIF in our simple GSC model) of Widget flows from  $r$  to  $d$ ,  $[V_w^{gsc}(r, d)]$ . (A11) defines total tariff collection by  $d$  on Widget imports from  $r$ ,  $[VT_w^{gsc}(r, d)]$ .<sup>11</sup> (A12) defines the average power of the tariff  $[T_w^{gsc}(r, d)]$  in the GSC model on the Widget flow from  $r$  to  $d$  as one plus the ratio of the tariff collection on this flow to its factory value.

***Proving that a converged GSC-CGE solution is a full integration solution***

Now we are ready to state and prove a proposition that justifies the algorithm described in Figures 2.1 and 7.2. We prove that if

- (a) Widget sectoral values derived from the GSC model for labour and intermediate-input coefficients and for the average tariff rates are the same as those used in the CGE model, and
- (b) wage rates and Widget final demands assumed in the GSC model are the same as those in the CGE model

then the two models are fully integrated.

***Proposition 1:***

If

$$\begin{aligned} W^{gsc}(d) = W^{cge}(d), \quad Y_w^{gsc}(d) = Y_w^{cge}(d), \quad T_w^{gsc}(r, d) = T_w^{cge}(r, d), \\ A_{ww}^{gsc}(r, d) = A_{ww}^{cge}(r, d) \quad \text{and} \quad ALAB_w^{gsc}(d) = ALAB_w^{cge}(d) \end{aligned} \tag{A13}$$

then the remaining conditions for full integration of the Widget GSC model described in subsection 2.1 and the CGE model described in subsection 2.2 are satisfied, that is

$$V_w^{gsc}(r, d) = V_w^{cge}(r, d), \quad VT_w^{gsc}(r, d) = VT_w^{cge}(r, d), \quad P_{fw}^{gsc}(r) = P_{fw}^{cge}(r), \quad EMP_w^{gsc}(r) = EMP_w^{cge}(r). \tag{A14}$$

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<sup>11</sup> To avoid having to identify trade flows separately from domestic flows, we assume that  $T_{iw}^{gsc}(r, r) - 1 = 0$  for all  $i$  and  $r$ .

In proving this proposition, we draw on: the equations of the GSC model set out in subsection 2.1; the GSC sectoral definitions given in (A6) to (A12); the conditions set out in (A13); and CGE equations relating Widget sectoral variables to each other. These CGE equations are:

$$X_W^{cge}(r) = \sum_d A_{ww}^{cge}(r,d) * X_W^{cge}(d) + Y_W^{cge}(r) \quad \text{for all } r \in R \quad (A15)$$

$$P_W^{cge}(d) = \sum_{r \in R} P_W^{cge}(r) * T_{ww}^{cge}(r,d) * A_{ww}^{cge}(r,d) + ALAB_W^{cge}(d) * W^{cge}(d) \quad \text{for all } d \in R \quad (A16)$$

$$V_W^{cge}(r,d) = P_W^{cge}(r) * A_{ww}^{cge}(r,d) * X_W^{cge}(d) \quad \text{for all } r,d \in R \quad (A17)$$

$$VT_W^{cge}(r,d) = P_W^{cge}(r) * [T_{ww}^{cge}(r,d) - 1] * A_{ww}^{cge}(r,d) * X_W^{cge}(d) \quad \text{for all } r,d \in R \quad (A18)$$

$$EMP_W^{cge}(r) = ALAB_W^{cge}(r) * X_W^{cge}(r) \quad \text{for all } r \in R \quad (A19)$$

In these equations

$X_W^{cge}(r)$  is the output of Widgets in region  $r$ ;

$A_{ww}^{cge}(r,d)$  is the quantity of Widget input from region  $r$  per unit of output of Widgets in region  $d$ ;

$Y_W^{cge}(r)$  is the final demand for Widgets in region  $r$ ;

$P_W^{cge}(d)$  is the factory-door price of Widgets in region  $d$ ;

$T_{ww}^{cge}(r,d)$  is the power of the tariff applying to Widget flows from region  $r$  for use in the Widget industry in region  $d$ ;

$ALAB_W^{cge}(d)$  is the labour input per unit of Widget output in region  $d$ ;

$W^{cge}(d)$  is the wage rate in region  $d$ ;

$V_W^{cge}(r,d)$  is the pre-tariff value of Widgets sent from region  $r$  to be used as input to the Widget industry in region  $d$ ; and

$VT_W^{cge}(r,d)$  is the tariff revenue from the  $(r,d)$  Widget flow.

(A15) is the CGE condition equating supply of Widgets from region  $r$  to demand. This equation reflects the special assumptions in our Widget sector: Widgets from region  $r$  can be used only as intermediate inputs to Widget production in all regions and in final demand in region  $r$ . (A16) imposes the CGE condition that the factory-door price of Widgets from region  $d$  equals unit costs consisting in the Widget sector of region  $d$  of just labour and intermediate inputs of Widgets. (A17) defines the factory value of the Widget flow from  $r$  to  $d$ . (A18) defines the tariff collection on this flow. We assume that there are no transport costs or other margins separating the factory value of the  $r,d$  Widget flow from the CIF value. (A19) computes employment in the CGE Widget industry as the product of output and labour input per unit of output

*Proof of Proposition 1:*

The first step in the proof is to deduce

$$X_W^{gsc}(r) = \sum_d A_{ww}^{gsc}(r,d) * X_W^{gsc}(d) + Y_{FW}^{gsc}(r) \quad \text{for all } r \in R \quad (A20)$$

and

$$P_{FW}^{gsc}(d) = \sum_{r \in R} P_{FW}^{gsc}(r) * T_W^{gsc}(r,d) * A_{ww}^{gsc}(r,d) + ALAB_W^{gsc}(d) * W^{gsc}(d) \quad \text{for all } d \in R \quad (A21)$$

To make these deductions we start by splitting the RHS of (A6) into intermediate and final and use (2.4) to give

$$X_W^{\text{gsc}}(r) = \frac{\sum_{i \in \text{WCI}} P_{iW}^{\text{gsc}}(r) * X_{iW}^{\text{gsc}}(r)}{P_{FW}^{\text{gsc}}(r)} + Y_W^{\text{gsc}}(r) \quad \text{for all } r \in R \quad (\text{A22})$$

By substituting from (2.3) we obtain

$$X_W^{\text{gsc}}(r) = \frac{\sum_{i \in \text{WCI}} \sum_{d \in R} \sum_{j \in \text{WA}} P_{iW}^{\text{gsc}}(r) * A_{ijW}^{\text{gsc}}(r, d) * X_{jW}^{\text{gsc}}(d)}{P_{FW}^{\text{gsc}}(r)} + Y_W^{\text{gsc}}(r) \quad \text{for all } r \in R \quad (\text{A23})$$

and using (A7) we arrive at (A20).

To obtain (A21) we start by substituting (2.9), with arguments  $d$  and  $r$  interchanged, into (A6):

$$X_W^{\text{gsc}}(r) = \frac{\sum_{i \in \text{WC}} \sum_{j \in \text{WCI}} \sum_{d \in R} P_{jW}^{\text{gsc}}(d) * T_{jW}^{\text{gsc}}(d, r) * A_{jiW}^{\text{gsc}}(d, r) * X_{iW}^{\text{gsc}}(r)}{P_{FW}^{\text{gsc}}(r)} \quad \text{for all } r \in R \quad (\text{A24})$$

$$+ \frac{\sum_{i \in \text{WC}} \frac{W^{\text{gsc}}(r)}{\text{PROD}_{iW}^{\text{gsc}}(r)} * \text{SCALE}_{iW}^{\text{gsc}}(r) * X_{iW}^{\text{gsc}}(r)}{P_{FW}^{\text{gsc}}(r)}$$

Substituting (A10) and (A11) into (A12) and interchanging  $i$  and  $j$  arguments and  $r$  and  $d$  arguments we obtain

$$T_W^{\text{gsc}}(d, r) = \frac{\sum_{j \in \text{WCI}} \sum_{i \in \text{WA}} [T_{jW}^{\text{gsc}}(d, r)] * P_{jW}^{\text{gsc}}(d) * A_{jiW}^{\text{gsc}}(d, r) * X_{iW}^{\text{gsc}}(r)}{\sum_{j \in \text{WCI}} \sum_{i \in \text{WA}} P_{jW}^{\text{gsc}}(d) * A_{jiW}^{\text{gsc}}(d, r) * X_{iW}^{\text{gsc}}(r)} \quad \text{for all } r, d \in R \quad (\text{A25})$$

Then using (A25) in (A24) and recalling that  $\text{WA} = \text{WC}$ , we find that

$$X_W^{\text{gsc}}(r) = \sum_d \frac{T_W^{\text{gsc}}(d, r)}{P_{FW}^{\text{gsc}}(r)} * \sum_{i \in \text{WC}} \sum_{j \in \text{WCI}} P_{jW}^{\text{gsc}}(d) * A_{jiW}^{\text{gsc}}(d, r) * X_{iW}^{\text{gsc}}(r)$$

$$+ \frac{\sum_{i \in \text{WC}} \frac{W^{\text{gsc}}(r)}{\text{PROD}_{iW}^{\text{gsc}}(r)} * \text{SCALE}_{iW}^{\text{gsc}}(r) * X_{iW}^{\text{gsc}}(r)}{P_{FW}^{\text{gsc}}(r)} \quad \text{for all } r \in R \quad (\text{A26})$$

Multiplying through by  $P_{FW}^{\text{gsc}}(r) / X_W^{\text{gsc}}(r)$  and using (A7), (A8) and (A9) leads to (A21).

Having established (A20) and (A21), we compare them with (A15) and (A16). Invoking (A13) we can conclude that:

$$X_W^{\text{gsc}}(r) = X_W^{\text{cge}}(r) \quad \text{for all } r \in R \quad (\text{A27})$$

and

$$P_{FW}^{\text{gsc}}(r) = P_W^{\text{cge}}(r) \quad \text{for all } r \in R \quad (\text{A28})$$

(A28) partially establishes (A14).

Now we consider values of trade flows and tariff collections in the GSC and CGE models.

From (A7) and (A10) we see that

$$V_W^{gsc}(r, d) = P_{FW}^{gsc}(r) * A_{WW}^{gsc}(r, d) * X_W^{gsc}(d) \quad \text{for all } r, d \in R \quad (A29)$$

and from (A7), (A25) and (A11) we see that

$$VT_W^{gsc}(r, d) = P_{FW}^{gsc}(r) * [T_W^{gsc}(r, d) - 1] * A_{WW}^{gsc}(r, d) * X_W^{gsc}(d) \quad \text{for all } r, d \in R \quad (A30)$$

Comparing (A29) with (A17) and invoking (A13), (A27) and (A28) we see that

$$V_W^{gsc}(r, d) = V_W^{cge}(r, d). \quad (A31)$$

Comparing (A30) with (A18) and invoking (A13), (A27) and (A28) we see that

$$VT_W^{gsc}(r, d) = VT_W^{cge}(r, d). \quad (A31)$$

Finally, we consider Widget employment in the two models. Comparing (A9) and (A19) and invoking (A13) and (A27) we see that

$$EMP_W^{gsc}(r) = EMP_W^{cge}(r) \quad \text{for all } r \in R \quad (A32)$$

This completes the proof.

***Final note on the algorithm for generating a fully integrated GSC-CGE solution***

Let's assume to start with that  $Y_W^{gsc}(r)$  and  $W^{gsc}(r)$  are exogenous variables in the GSC model and  $A_{WW}^{cge}(r, d)$ ,  $ALAB_W^{cge}(d)$  and  $T_W^{cge}(r, d)$  are exogenous variables in the CGE model. Then in theory, we could write all of the equations of both models in a single system and simply add the equations

$$\begin{aligned} W^{gsc}(d) &= W^{cge}(d), \quad Y_W^{gsc}(d) = Y_W^{cge}(d), \quad T_W^{gsc}(r, d) = T_W^{cge}(r, d), \\ A_{WW}^{gsc}(r, d) &= A_{WW}^{cge}(r, d) \quad \text{and} \quad ALAB_W^{gsc}(d) = ALAB_W^{cge}(d) \end{aligned} \quad (A33)$$

to allow endogenous determination of  $W^{gsc}(r)$ ,  $Y_W^{gsc}(r)$ ,  $T_W^{cge}(r, d)$ ,  $A_{WW}^{cge}(r, d)$  and  $ALAB_W^{cge}(d)$ . The remaining conditions for full integration listed in (A1) to (A5) are

$$V_W^{gsc}(r, d) = V_W^{cge}(r, d), \quad VT_W^{gsc}(r, d) = VT_W^{cge}(r, d), \quad P_{FW}^{gsc}(d) = P_W^{cge}(d), \quad EMP_W^{gsc}(d) = EMP_W^{cge}(d). \quad (A34)$$

These refer to variables that are endogenous in both models. Via Proposition 1, we know that with (A33) in place, the conditions in (A34) are satisfied.

Rather than adding (A33) to a combined GSC-CGE system, we used an algorithmic approach in which the two models are solved separately and information on economy-wide variables is passed from CGE solutions to the GSC model and information on sectoral widget variables is passed from GSC solutions to the CGE model.

What do we do if an endogenous variable in one of the models is to be informed by movements in the corresponding variable in the other model? This problem does not arise with  $Y_W^{gsc}(r)$  and  $W^{gsc}(r)$  which are naturally exogenous in the GSC model or with  $T_W^{cge}(r, d)$  which is naturally exogenous in the CGE model. However, it is easy to imagine situations in which  $A_{WW}^{cge}(r, d)$  and  $ALAB_W^{cge}(d)$  are endogenous in the CGE model reflecting price-sensitive substitution between inputs to region d's Widget industry. In this case we must change the CGE closure so that  $A_{WW}^{cge}(r, d)$  and  $ALAB_W^{cge}(d)$  are exogenous in the CGE

model. This can normally be done by endogenizing input-saving or -using technology variables. In doing this we let the GSC model tell the CGE model about developments in inputs per unit of output in the Widget industries of each region while reconciling the GSC story with the CGE theory through endogenous changes in variables that cause both substitution between inputs and overall changes in productivity.

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