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DEVELOPING A COST OF CAPITAL MODULE FOR COMPUTABLE GENERAL EQUILIBRIUM MODELLING

by

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The Centre of Policy Studies and Impact Project is a research centre at Monash University devoted to quantitative analysis of issues relevant to Australian economic policy.

Developing a Cost of Capital Module for Computable General Equilibrium Modelling

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Abstract

This paper outlines the development of an approach to incorporate business taxation and allowances into a model of a firm to determine the effect of tax policy changes on the firm's behaviour. Following Auerbach, King and Benge, we first develop a model in which the firm maximises the value of its shareholder equity, taking account of: company and personal income taxes; capital-gains taxes (including a treatment of realisation-based capital-gains tax); depreciation allowances; investment allowances; and interest rates on debt linked to financial leverage. This approach takes the revenue streams and income payments generated by the firm as given. Subsequently we derive a function for the user-cost of capital to the firm in an optimising framework in which the expression for the value of the firm is the objective function, and then solve for all of the firm's choice variables. In this way, the model determines the firm's optimal investment policy and the resulting levels of revenues and income streams to shareholders. By embedding this in a dynamic CGE model, we can simulate the effects of tax changes on the user-cost of capital and thus on investment. Our ultimate aim is to enable an analysis of the effects of reforms to business taxation (such as the recent Ralph proposals) using a large-scale dynamic CGE model.

This is a revised version of a paper prepared for the PhD Conference in Economics and Business held at The University of Western Australia, Perth, Australia, November 7-9 2001.

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1. Introduction

A key determinant of the behaviour of producers is the cost of capital. Microeconomic theory tells us that, in the presence of diminishing returns, optimising producers will increase their usage of primary factors until the nominal benefit derived from the marginal unit (its marginal revenue product) falls to equal its price. As such, the growth of the firm, the level of its profit and its value to shareholders are all heavily dependent on the cost of capital.

Modigliani and Miller (1958) showed that in the presence of perfect foresight and in the absence of taxes, the firm would be indifferent between the various sources of capital available to it. Stiglitz (1973) showed that, assuming real economic depreciation is deductible and investment is financed at the margin by debt, the cost of capital is the rate interest. King (1974) developed an optimising framework to analyse the financial incentives of firms in both classical and dividend-imputation company tax system, and showed that different taxation provisions can act to make the costs of various capital sources diverge. Under these circumstances, it is not possible to determine a generic cost of capital function for a firm, because this is then dependent on the firm's financial policy. Auerbach (1979) provides further support for the concept of the relevance of the source of finance for determining the firm's cost of capital. Benge (1997) further developed and refined the framework of King and analysed the financial incentives facing firms under Australian tax conditions, finding (amongst other things) that the introduction of full dividend imputation should have removed any debt–equity biases, except in the presence of inflation with capital gains tax indexing.

The model described in this paper follows the approach of King (1974 and 1977) and Benge (1997 and 1998). We develop an expression for the value of a corporate enterprise to its shareholders under various conditions, including the presence of a realisation-based capital gains tax, and solve a constrained optimisation problem to determine the firm's cost of capital. In a CGE model, the required rate of return constitutes a passive supply side for the market for finance, and the aim of this research is to generate a responsive and more detailed treatment of this important determinant of producer behaviour. We find that the firm's financial policy is sensitive to personal and corporate tax provisions and is critical in determining the cost of capital to the firm.

2. The Value of the Firm

In this section, we develop an expression for the value of the firm to a shareholder. As will soon become apparent, making allowance for the various taxation systems quickly makes the expressions sufficiently unwieldy to obscure their intuitive appeal. In light of this, we proceed by first assuming that there are no taxes in the model, and then add income taxes and capital gains taxes successively.

2.1. *No Taxes*

We begin with the idea that a firm will need to provide some minimum rate of return to attract equity capital. With a broader definition of capital, we can relate this idea to the required rate of return used in CGE models to help determine investment levels. This required rate of return is that needed to at least satisfy the investors marginal rate of time preference or discount rate, and thus to entice them to forego consumption in the current period in return for consumption in a future period.

Further, because firms always have the option of issuing new equity to acquire capital in any period, we need to allow for the dilution of pre-existing owners' equity. An equity issue of one dollar represents a loss to pre-existing shareholders of one dollar's worth of appropriately discounted future earnings from the firm. The total value of equity in the firm at any point is

$$V_t = V_t^O + V_t^N \tag{1}$$

where

V_t is total equity in the firm at the beginning of period t,

V_t^O is pre-existing equity at the beginning of period t, and

V_t^N is the total value of new share issues in period t, which always occur at the beginning of the period, and occur *ex-dividend*.

In the absence of any taxes, shareholder earnings, E_t, at the beginning of period t are denoted by

$$E_t = D_t \tag{2}$$

where

D_t is the dividend payable at the beginning of period t on the previous periods operations.

Earnings are defined as after-tax receipts by shareholders. This does not include un-realised capital gains, but does include capital gains tax liabilities where applicable (although at this stage we choose to abstract from them). We also assume that un-retained profits generated in period t are distributed as dividends at the beginning of period t+1. As we are abstracting from taxes at this stage, equation (2) simply says that shareholder earnings in period t are equal to the dividends distributed at the beginning of that period t from period t-1's operations.

In a one period problem in the absence of taxes, arbitrage behaviour in financial markets ensures that equilibrium is characterised by

$$i_{t+1}V_t = E_{t+1} + \left(V_{t+1}^{O} - V_t\right) \tag{3}$$

where

i_t is the cash rate in period *t* which reflects a riskless required rate of return.

Expression (3) says that, in equilibrium (i.e. after all arbitrage opportunities are exhausted), the one-period rate of return on holding equity (comprising dividends and accrued capital gains) will equal the investor's discount rate.

Extending this, and using (1), (2) and (3), we can set up a discrete time, multi-period problem with perfect foresight and solve for V_0:

$$V_0 = \sum_{t=1}^{T}\left[\frac{D_t - V_t^N}{\prod_{s=1}^{t}(1+i_s)}\right] + \frac{V_T}{\prod_{s=1}^{T}(1+i_s)} \tag{4}$$

where V_0 is the value of the firm today.

The terminal constraint on the firm, V_T, is the value of the firm at the "end" of the time horizon. As we are considering the value of equity in a corporate enterprise, it seems sensible to remove the issue of a terminal value completely and assume an infinite horizon. A corporation, in one sense, is an administrative entity that brings together many sources of capital to finance a succession of investment "projects", and while each project may have a finite horizon, the potential sum of all of the projects a firm can undertake over time is not constrained *ex ante*.

In a typical inter-temporal model of investment, the terminal constraint is assumed away by the application of a transversality condition,

$$\lim_{t\to\infty}\left[\frac{V_t}{\prod_{s=1}^{t}[1+i_s]}\right] = 0 \tag{5}$$

This states that, as long as the present value of the terminal constraint remains bounded as time approaches infinity, the right-most term of (4) approaches zero. This requires that the absolute value of the firm does not grow at a rate faster than the discount factor (in this case, simply *i*). This can probably best be understood by understanding what behaviour it rules out. According to the arbitrage condition (equation (3)), for the firm's value to grow at a rate faster than *i* would require the payment of negative dividends.

i.e. from (3), if $\left(V_{t+1}^{O}-V_{t}\right)>i_{t+1}V_{t}$ then $D_{t+1}<0$

Thus, the transversality condition rules out the possibility of a firm growing faster than i forever while paying negative dividends. While perhaps not immediately intuitive, the concept of negative dividends can be thought of as the case of a firm demanding additional funds from shareholders to cover operating losses. This might happen in the case of contributing shareholders, or for stakeholders in enterprises such as Lloyds of London, but is increasingly uncommon. Efficient capital markets place significant constraints on the ability of firms to behave in this manner for even a single period, let alone forever.

Applying this transversality condition, expression (4) becomes

$$V_0=\sum_{t=1}^{\infty}\left[\frac{D_t-V_t^N}{\prod_{s=1}^{t}\left(1+i_s\right)}\right] \tag{6}$$

Expression (6) says that the present value of equity in the firm is equal (in equilibrium) to the sum of all future dividend streams minus any new share issues.

As we proceed with this derivation, it will become apparent that describing the denominator of this function as the "discount rate" will – strictly speaking – lose some appeal, as various parameters (tax rates and inflation rates, for instance) and dummy variables enter this term. For convenience, however, we will continue to describe it as a discount rate.

2.2. Company and Personal Income Tax

Having developed a basic framework in the form of expression (6) in the previous section, we now add personal and corporate income taxation to the model.

Shareholders face income taxes on income streams they receive from holding assets. These come in the form of company (CIT) and personal (PIT) income taxes on dividends and interest income.

Dividend taxation takes two basic forms:

2.2.1. The Classical CIT System

In this system, dividends are taxed both at the company level and at the personal level, and do not carry tax credits for the shareholder. Therefore, the after-company-tax dividend received is taxed again at the shareholder's full marginal rate of personal income tax. If we denote the firm's before-tax profit generated in period t-1 and distributed in period t as κ_t, then

$$D_t = (1-\tau_t)\kappa_t$$

where

τ is the company tax rate.

Shareholder earnings are now

$$E_t = (1-\theta_t)(1-\tau_t)\kappa_t = (1-\theta_t)D_t \tag{7}$$

where

θ is the shareholder's marginal personal income tax rate.[1]

This highlights the notion of double-taxation of dividends that characterises the classical company tax system.

Following this, (6) becomes

$$V_0 = \sum_{t=1}^{\infty}\left[\frac{(1-\theta_t)D_t - V_t^N}{\prod_{s=1}^{t}\left(1+i_s(1-\theta_s)\right)}\right] \tag{8}$$

Expression (8) states that the value of the firm to shareholders is equal to the present value of all after-income-tax dividend payments minus new share issues. Because interest income is taxable, the discount rate now becomes a function of the after-income-tax rate of interest on liquid deposits.

2.2.2. Dividend Imputation CIT System

Dividend imputation refers to the transfer of tax credits to the shareholder for tax paid at the firm level. This change in the method of taxing income in Australia was a significant move toward a Haig-Simons-Carter style comprehensive income taxation system, and was part of a series of base broadening and rate reduction reforms that were introduced after the Draft White Paper on the Australian taxation system in 1985.

Dividend imputation systems come in three broad categories.

1. Full imputation systems currently operate in Australia, Chile, New Zealand, Singapore, Taiwan, Italy, Malaysia and South Korea. In these tax regimes the company's profits are

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[1] This assumes that all profits are distributed. This is a simplification for illustrative purposes that is not maintained throughout the development of the model.

fully taxed at the corporate income tax rate and are then distributed with a tax credit for the entire amount paid in tax.

2. Countries such as France, the United Kingdom. Norway, Finland, Israel and Germany tax all dividends fully at the company level so that only fully taxed dividends are distributed.

3. Systems such as that in operation in Canada provide a notional credit to shareholders for company tax paid that does not specifically reflect the rate of company tax itself.

A final category that – broadly speaking – belongs in this category comprises countries that exempt distributions from any further taxation at the shareholder level. This group contains countries such as Croatia, Mexico and Brazil.

In this paper, we will abstract from the issue of foreign ownership. This assumption allows us to abstract from complications that arise from the tax treatment of foreign sourced income. These issues will be discussed in a forthcoming paper.

Under the dividend imputation system, shareholders can receive franked or unfranked dividends.

Unfranked dividends are distributed with no tax credits, and the shareholder is liable for the full marginal rate of personal income tax on each dollar.

Franked dividends are a little more complicated. To simplify the analysis, let's begin by assuming the shareholder can claim credit for the entire amount of corporate income tax paid on the franked dividend, as would be the case under a full dividend imputation system. Also, for expositional purposes, assume for now that all profits are distributed (i.e. ignore for now the possibility of the firm retaining earnings). Company income tax is paid on the before-tax profit, and the after-company-tax profit is paid as a franked dividend,

$$D_t^f = \kappa_t \left(1-\tau_t\right),$$

and carries tax credits equal to

$$\tau_t \kappa_t \text{, or } \frac{\tau_t}{\left(1-\tau_t\right)} D_t^f .$$

Effectively, then, the shareholder receives κ_t in full:

$$D_t^f + \tau_t \kappa_t = \kappa_t \left(1-\tau_t\right) + \tau_t \kappa_t = \kappa_t .$$

Shareholder after-tax earnings with full imputation are

$$E_t = \kappa_t\left(1-\theta_t\right) = \left[\frac{1-\theta_t}{1-\tau_t}\right] D_t^f \tag{9}$$

While Australia currently allows full dividend imputation, many other countries allow only partial credit for income taxes paid at the company level. To allow us to choose the "degree" of dividend imputation, we introduce a new variable:

γ_t denotes the proportion of total tax paid at the company level in period t that can be claimed as tax credits for personal income taxation purposes.

To make sense of this variable we can run a simple thought experiment, as follows:

With partial imputation, the firm generates κ_t and pays $\tau_t\kappa_t$ tax, but only passes on $\gamma_t\tau_t\kappa_t$ in tax credits. The shareholder then pays tax on the sum of these two amounts, in which case shareholder earnings are

$$E_t = \frac{\left(1-\theta_t\right)\left[1-\left(1-\gamma_t\right)\tau_t\right]}{1-\tau_t}\kappa_t\left(1-\tau_t\right) = \frac{\left(1-\theta_t\right)\left[1-\left(1-\gamma_t\right)\tau_t\right]}{1-\tau_t} D_t^f \tag{10}$$

Therefore,

$\gamma_t = 1$ denotes full dividend imputation,

$0 < \gamma_t < 1$ denotes partial dividend imputation, and

$\gamma_t = 0$ denotes zero imputation, or alternatively, a classical CIT system.

Combining franked and unfranked dividends into a single expression, we have

$$\begin{aligned} E_t &= \left(1-\theta_t\right) S_t^u \kappa_t\left(1-\tau_t\right) + \frac{\left(1-\theta_t\right)\left[1-\left(1-\gamma_t\right)\tau_t\right]}{1-\tau_t} S_t^f \kappa_t\left(1-\tau_t\right) \\ &= \left(1-\theta_t\right) D_t + \frac{\left(1-\theta_t\right)\left[1-\left(1-\gamma_t\right)\tau_t\right]}{1-\tau_t} D_t^f \end{aligned} \tag{11}$$

where $S_t^u + S_t^f = 1$, and denote the share of unfrankable and frankable earnings paid by the firm.

The value of the firm in equilibrium over an infinite horizon is

$$V_0 = \sum_{t=1}^{\infty}\left[\frac{\left(1-\theta_t\right) D_t + \frac{\left(1-\theta_t\right)\left[1-\left(1-\gamma_t\right)\tau_t\right]}{1-\tau_t} D_t^f - V_t^N}{\prod_{s=1}^{t}\left[1+i_s\left(1-\theta_s\right)\right]}\right] \tag{12}$$

2.3. *Capital Gains Taxation*

2.3.1. *Taxing Accrued Capital Gains.*

In this section, we add capital gains taxation to our model. Capital gains taxation can apply to accrued or realised gains, on a real or nominal basis, and sometimes with averaging provisions in place.

Accrual-basis CGT systems require an asset holder to pay CGT at the end of every period on gains accrued during that period. This type of CGT system is usually assumed in theoretical modelling due to its convenience, largely because, in comparison to a realisation-basis system, the accrual-basis system contains no endogenous timing issues. Shareholders have no discretion over the timing of capital gains tax payments, and thus the important realisation-system issue of deferral (delaying the sale of an asset to push the capital gains tax payment into the future and reduce its present value) plays no role.

Taking account of the periodical outflow of funds required to pay the accrued CGT liability, the expression for shareholder earnings becomes

$$E_t = (1-\theta_t) D_t + \frac{(1-\theta_t)\left[1-(1-\gamma_t)\tau_t\right]}{1-\tau_t} D_t^f - c_t^A \left(V_t^o - V_{t-1}\right) \tag{13}$$

where

c_t^A is the effective rate of the capital gains tax under an accrual-basis system.

The effective rate of he capital gains tax will be defined differently for each capital gains system. Under an accrual-basis system, we define this rate as

$$c_t^A = c_t \psi_t \tag{14}$$

where

c_t is the statutory rate of the capital gains tax, and

ψ_t is the proportion of the total capital gain that is taxable.

We include ψ_t because only a proportion of capital gains are taxed under some CGT regimes. Two examples are Australia (50% of the nominal gain) and Canada (75% of the nominal gain). Further, while in some countries we see CGT rates reflect personal income tax rates, this is not universally true and so we denote the CGT rate as a separate parameter of the model.

With this definition of E, the value of the firm over an infinite horizon is

$$V_0=\sum_{t=1}^{\infty}\left[\frac{\left[\frac{1-\theta}{1-c_t^A}\right]D_t+\left[\frac{(1-\theta_t)\left[1-(1-\gamma_t)\tau_t\right]}{(1-\tau_t)\left(1-c_t^A\right)}\right]D_t^f-V_t^N}{\prod_{s=1}^{t}\left[\frac{1+i_s(1-\theta_s)-c_s^A}{1-c_s^A}\right]}\right] \tag{15}$$

The capital gains tax terms enter this expression in two ways. Firstly, the terms in the denominator of the tax coefficient on dividends and in the denominator of the discount factor act to weight the dividend stream to retain the relativities between dividends and new issues. This particular formulation is the result of an algebraic convenience, in which we choose to state the expression without tax coefficients on the new-share-issues variable. The product of these terms in the denominator of the discount factor for all periods prior to period *t* acts to dilute the value of the firm (by increasing the value of the discount factor) to take account of the way in which capital gains tax payments in prior periods dilute the value of the shareholder's equity. Every dollar paid in capital gains tax is equivalent to not receiving one dollar of discounted future earnings. The capital gains tax term in the numerator of the discount rate acts to subtract the base-value of the firm for the capital gain calculation in any period.

2.3.2. Taxing Realised Capital Gains

The essential difference between the accrual- and realisation-basis CGT systems arises from the timing of CGT payments, and therefore their present value. With a realisation-basis CGT, the payment of the CGT liability is delayed until the asset is realised, and thus the liability is discounted at the rate applicable to the period of the sale, which may or may not be the period in which the capital gain was incurred.

From a modelling perspective, the difficulty that arises in incorporating realisation-basis capital gains taxation relates to the determination of investor behaviour. In wealth maximisation frameworks, it becomes difficult to explain why a shareholder would *ever* realise an asset, as doing so only incurs a tax liability that detracts from the assets worth. If the objective is to maximise wealth and not the present value of consumption flows, the shareholder never faces a liquidity constraint that requires asset sales, or indeed never faces a trade-off between present and future consumption because consumption is irrelevant to the maximand pursued by these individuals.

The obvious answer to this problem is to embed the choice in a utility maximisation framework where the maximisation of the net-present-value of consumption flows defines the objective, and the holding of assets becomes part of a lifetime consumption decision. In such a framework, the shareholder would compare the net present value of the income streams generated by a dollar invested in an asset with the marginal utility of the consumption that could otherwise be obtained immediately in an inter-temporal optimisation problem. This would effectively endogenise the realisation behaviour of investors. In this paper, we abstract from these issues and assume that the realisation behaviour of investors is exogenous.

In explaining the intuition behind the modelling of a realisation-basis CGT, we provide two methods below. The first highlights the role of shareholder behaviour but underestimates the value of the firm, while the second corrects this error and dramatically simplifies the resulting expression.

To capture this essential difference we need to account for the timing of CGT payments. In equilibrium, the value of the shareholder's earnings in a single period problem is

$$\begin{aligned}\left(i_1\left(1-\theta_1\right)\right)V_0 = \left(1-\theta_1\right)D_1 + \frac{\left(1-\theta_1\right)\left[1-\left(1-\gamma_1\right)\tau_1\right]}{1-\tau_1}D_1^f + \varepsilon_1\left(V_1^o - V_0\right) - c_1^R\left(V_1^o - V_0\right) \\ + \left(1-\varepsilon_1\right)\left(V_1^o - V_0\right)\end{aligned} \tag{16}$$

where

$$c_t^R = \varepsilon_t \psi_t c_t \tag{17}$$

is the effective rate of capital gains tax under a realisation-basis system, and

ε_t is the proportion of the shareholder's total equity in the firm realised in period t.

Notice how we have divided the value terms into three components:

$\varepsilon_1\left(V_1^o - V_0\right)$ the before-tax realised capital gain,

$\varepsilon_1\psi_1 c_1\left(V_1^o - V_0\right)$ the CGT payable on the realised capital gain, and

$\left(1-\varepsilon_1\right)\left(V_1^o - V_0\right)$ the unrealised capital gain.

Given a value of ε, the realisation system drives a wedge between the value of holding equity and its realisable value. The value of the firm to the holder is the present value of all future income streams minus any tax payments, including CGT payments. The realisable value of the firm is the price that another individual is willing to pay, which is equal to the present value of the (given) income streams and *that* individual's tax liabilities. Assume that the holder purchased the shares in period t, and is looking to realise some proportion of them in period t+1. The value of each share to the holder in period t+1 is the after-tax dividend streams from period t+1 onwards, minus any capital gains tax that must be calculated against period t values. From the potential purchaser's point of view, the shares are worth the present value of after-tax dividend streams from t+1 onward minus any CGT liabilities that are calculated against period t+1 values. Therefore, as long as these shares appreciate, their value to a buyer is always greater than their value to a seller. Left as is, this specification implies that the value of shares to a seller and a buyer are equal, and thus underestimates their value to the original owner in period *0* by overestimating the present value of capital gains tax payments.

Assume for now that the value of a parcel of shares to seller and purchaser are identical. At the beginning of period 1, the shareholder holds a proportion (1-ε_t) of the shares held at the beginning of period *0*. From this point on (assuming no more share realisations), the shareholder will receive a proportion (1-ε_t) of the distributions (dividends and capital gains) paid each period relative to the original entitlement. However, the price the seller receives for the proportion ε_t that was realised is, by definition, equal to the present value (at the beginning of period 1) of a proportion ε_t of all future distributions. Thus, although ε_t of the original holding has been sold, the shareholder has a claim over (1-ε_t) of all future distributions *plus* cash from the share sale equal in value to ε_t of all future distributions. The shareholder has transformed part of the original total asset holdings (originally comprised entirely of claims over future income flows) into cash equal in present value to the proportion of the claim sold. Most importantly, the value of the shareholder's total asset holdings has not changed (i.e. $(1-\varepsilon_1)+\varepsilon_1=1$).

This point is important when we move to a multi-period problem. The shareholder will receive only (1-ε_t) of next periods distributions, but is already holding cash to the value of the residual ε_t. Therefore, if we pre-multiply the next periods distribution by (1-ε_t) we also need to add back in ε_t of these distributions to take account of the value of the cash generated by selling ε_t the previous period. In the accrual case, we noted that the capital gains tax has the effect of diluting the shareholders future earnings, because the cash due on the CGT liability has an equivalent present value in terms of future distributions. The same is true in this case, only now the dilution is due to a tax calculated on the realised capital gain (1-$\varepsilon_t\psi_t c_t$) and not an accrued gain (1-$\psi_t c_t$). In light of this and the discussion of ε_t above, it is clear that it is the capital gains tax itself that dilutes the value of the shareholders equity at time *0* and not the sale of shares themselves.

Solving across an infinite horizon we obtain

$$V_0=\Gamma\cdot\sum_{t=1}^{\infty}\left[\frac{\left[\frac{1-\theta_t}{1-\varepsilon_t\psi_t c_t}\right]D_t+\left[\frac{(1-\theta_t)\left[1-(1-\gamma_t)\tau_t\right]}{(1-\tau_t)(1-\varepsilon_t\psi_t c_t)}\right]D_t^f-V_t^N}{\prod_{s=1}^{t}\left[\frac{1+i_s(1-\theta_s)}{1-\varepsilon_s\psi_s c_s}\right]}\right] \tag{18}$$

where

$$
\Gamma = \left[1 - \sum_{t=1}^{\infty} \left[\frac{\left[\dfrac{\varepsilon_t \psi_t c_t}{1 - \varepsilon_t \psi_t c_t} \right]}{\prod_{s=1}^{t} \left[\dfrac{1 + i_s \left(1 - \theta_s\right)}{1 - \varepsilon_s \psi_s c_s} \right]} \right] \right]^{-1}
$$

This expression contains an approximation and is unnecessarily cumbersome. The nature of the approximation and its result has already been pointed out above, and the solution to both issues is discussed hereafter.[2]

If we take a slightly different view, we can ask ourselves a different question. From the perspective of an individual in period t, what is the difference between the value of shares held at time t but purchased in t-1 and those held and purchased at time t? The answer is the additional capital gains payments that will be due on the shares acquired in period t-1 (assuming that the shares appreciated in value). Taking (16) we can add an additional term to take account of this difference:

$$
\begin{aligned}
\left(i_t \left(1 - \theta_t\right)\right) V_{t-1} = \left(1 - \theta_t\right) D_t + \frac{\left(1 - \theta_t\right)\left[1 - \left(1 - \gamma_t\right)\tau_t\right]}{1 - \tau_t} D_t^f \\
+ \varepsilon_t \left(V_t^o - V_{t-1}\right) - c_t^R \left(V_t^o - V_{t-1}\right) + \left(1 - \varepsilon_t\right)\left(V_t^o - V_{t-1}\right) - Z_t
\end{aligned}
\tag{19}
$$

where

$$
Z_t = \left(V_t^o - V_{t-1}\right) \left[\frac{\varepsilon_{t+1} \left(1 - \varepsilon_t\right) \psi_{t+1} c_{t+1}}{1 + i_{t+1} \left(1 - \theta_{t+1}\right)} + \frac{\varepsilon_{t+2} \prod_{s=t}^{t+1} \left(1 - \varepsilon_s\right) \psi_{t+2} c_{t+2}}{\prod_{s=t+1}^{t+2} \left(1 + i_s \left(1 - \theta_s\right)\right)} + \ldots \right]
\tag{20}
$$

is the present value of the additional future capital gains taxes payable on the capital gain received in period t-1.

In this specification, the price received by the seller for realising a proportion ε of shares is defined as the value of these shares to the purchaser.

i.e. V_t^o is now the *realisable* value of the original shareholder's equity, which is given by their value to a new shareholder.

[2] Following an excellent suggestion by Matt Benge, School of Economics, The Australian National University.

This makes sense, because in determining the value of ε to be exogenous, we make the supply curve for shares perfectly inelastic and, thus, the market price is determined by the purchaser. This defines the value of these shares to the seller as

Value to seller $= V_t^o - Z_t$.

If we define an accrual-equivalent rate of capital gains tax under the realisation system[3] as

$$c_t^R = \varepsilon_t \psi_t c_t + \frac{\varepsilon_{t+1}\left(1-\varepsilon_t\right)\psi_{t+1}c_{t+1}}{1+i_{t+1}\left(1-\theta_{t+1}\right)} + \frac{\varepsilon_{t+2}\prod_{s=t}^{t+1}\left(1-\varepsilon_s\right)\psi_{t+2}c_{t+2}}{\prod_{s=t+1}^{t+2}\left(1+i_s\left(1-\theta_s\right)\right)} + \ldots$$

or, simplifying,

$$c_t^R = \varepsilon_t \psi_t c_t + \sum_{z=t+1}^{\infty}\left[\frac{\varepsilon_z \prod_{k=t}^{z-1}\left(1-\varepsilon_k\right)\psi_z c_z}{\prod_{s=t+1}^{z}\left(1+i_s\left(1-\theta_s\right)\right)}\right] \tag{21}$$

then we can write (19) as

$$\left(i_t\left(1-\theta_t\right)\right)V_{t-1} = \left(1-\theta_t\right)D_t + \frac{\left(1-\theta_t\right)\left[1-\left(1-\gamma_t\right)\tau_t\right]}{1-\tau_t}D_t^f + \left(1-c_t^R\right)\left(V_t^o - V_{t-1}\right) \tag{22}$$

When taken out to an infinite horizon, this provides

$$V_0 = \sum_{t=1}^{\infty}\left[\frac{\left[\frac{1-\theta}{1-c_t^R}\right]D_t + \left[\frac{\left(1-\theta_t\right)\left[1-\left(1-\gamma_t\right)\tau_t\right]}{\left(1-\tau_t\right)\left(1-c_t^R\right)}\right]D_t^f - V_t^N}{\prod_{s=1}^{t}\left[\frac{1+i_s\left(1-\theta_s\right)-c_s^R}{1-c_s^R}\right]}\right] \tag{23}$$

Taking (23) we can see that switching between the accrual- and realisation-basis capital gains tax systems can be accomplished by setting the value of ε. If we set ε at 1, all of the terms in (21) relating to subsequent years become zero fall out of the expression, thus reducing (23) to equivalency with (15) and allowing us, from hereon in, to simply use c as the symbol for the capital gains tax rate regardless of the capital gains tax system we assume. Thus,

$\varepsilon_t = 1$ imposes accrual-basis capital gains taxation, and

[3] Following King (1977).

$0 \leq \varepsilon_t < 1$ imposes realisation-basis capital gains taxation.

One final point to note is that this formulation implicitly allows the immediate deduction of capital losses from taxable income at the rate of the capital gains tax. The ability to offset capital losses against capital gains is not universally applied to capital taxation, and it is not uncommon for there to be no provision at all for capital loss offset. When offset is allowed, it can be for all or part of the capital loss or it can be against only current capital gains, future capital gains, or both. Therefore, as specified, expression (23) will overestimate the value of the firm in the presence of capital losses because it will (a) allow an immediate deduction when it is not allowed in practise or (b) not appropriately discount the value of a capital loss offset if it is carried forward. A further extension to be developed in future work is to allow for different capital loss treatments in this model.

2.3.3. Real vs. Nominal Capital Gains.

Allowing for inflation indexing in this model is quite simple, and we use a method that follows Benge (1997 and 1998). The intuition behind this adjustment can be found in recognising that we need to index the base value of shares for the capital gains tax calculation to take account of changes in the value of the unit of measurement – money. We modify shareholder earnings to be partly a function of real or nominal capital gains, depending on the value of a dummy variable that allows us to activate a price index. The arbitrage condition becomes

$$\left(i_t\left(1-\theta_t\right)\right)V_{t-1} = \left(1-\theta_t\right)D_t + \frac{\left(1-\theta_t\right)\left[1-\left(1-\gamma_t\right)\tau_t\right]}{1-\tau_t}D_t^f + \left(1-c_t\right)\left(V_t^o - \left[1+\alpha\pi_t\right]V_{t-1}\right) \tag{24}$$

Solving for the value of the firm over an infinite horizon we find

$$V_0 = \sum_{t=1}^{\infty}\left[\frac{\left[\frac{1-\theta_t}{1-c_t}\right]D_t + \left[\frac{\left(1-\theta_t\right)\left[1-\left(1-\gamma_t\right)\tau_t\right]}{\left(1-\tau_t\right)\left(1-c_t\right)}\right]D_t^f - V_t^N}{\prod_{s=1}^{t}\left[\frac{1+i_s\left(1-\theta_s\right)-c_s\left[1+\alpha\pi_s\right]}{1-c_s}\right]}\right] \tag{25}$$

where

π_t is the proportionate growth in a general price index, such as a GDP deflator, and

α is a dummy variable which the takes the value 1 for real capital gains taxation and 0 for nominal capital gains taxation.

With α set at zero, π_t falls out of the expression and the nominal change in the value of equity is taxed. When α is set at one, the value of the firm's equity in the base period is inflated to take account of the price level.

Expression (25) defines the role that corporate and personal tax provisions play in determining the value of the firm to its shareholders. This asset price will represent the base value against which a rate of return on equity can be calculated. This expression could be used as-is to inform an analysis of the affect of tax changes on investors' willingness to contribute equity capital to the enterprise. While this has some appeal, it suffers from the obvious problem that the firm's behaviour is exogenous – its investment decisions and profit streams are assumed rather than calculated. Thus, at this point we have a formulation that can provide answers to questions of the form; *given* the firm's revenues and dividend payout policy, what affect does taxation play in determining the value of these flows to shareholders. A more thorough and rigorous approach would be to see how the firm's choices regarding investment plans, input choice and dividend payout policy are effected by tax provisions, and then derive an explicit cost of capital expression from an optimising framework where (25) is the objective function. Such an approach is described below.

3. Constraining the Firm's Behaviour

Having derived an objective function for the firm (the maximisation of shareholder value) we now need to define those things that constrain the firm's ability to pursue this objective.

3.1. Cash Flow

The firm pursues what is effectively a distributed-profit maximisation objective. The firm's ability to distribute profits is constrained (from one point of view) by a cash flow constraint. Simply put, the ability of the firm to distribute dividends is a function its sources and uses of funds. The firm's sources and uses of funds are summarised in the following expression:

$$\begin{aligned} D_t + D_t^f = p_t A_t \left[a_1 L_t^{-\rho} + a_2 K_{t-1}^{-\rho} + a_3 Q_t^{-\rho} \right]^{-\frac{1}{\rho}} + B_t + V_t^N - w_t L_t \left(1 + \upsilon_t\right) \\ - P_t Q_t - \left(1 + r_t\right) B_{t-1} - q_t I_t - T_t \end{aligned} \tag{26}$$

where

$Y_t = A_t \left[a_1 L_t^{-\rho} + a_2 K_{t-1}^{-\rho} + a_3 Q_t^{-\rho} \right]^{-\frac{1}{\rho}}$ is the firm's CES production function, where

A_t is an efficiency parameter (always positive) that says something about the state of technology,

a_i is a distributional factor (positive) denoting input shares, so $\sum_{i=1}^{3} a_i = 1$,

ρ is a parameter taking a value greater than or equal to –1 but not equal to zero[4],embodying a constant elasticity of substitution $\sigma = \frac{1}{1+\rho}$,

L_t is the firms total employment of labour in period *t*,

w_t is the price of labour,

K_t is the firm's effective capital stock in period *t*,

Q_t is the firm's effective total intermediate usage of goods,

P_t is the purchaser's price of intermediate goods,

p_t is the producer or basic price of the firm's output in period *t*,

B_t is the total size of issues of one period bonds in period *t*,

r_t is the interest rate on the firm's debt (to be discussed below),

I_t is the level of real investment in new capital goods,

q_t is the asset price of capital goods,

υ_t is the rate of payroll tax in period *t*, and

T_t is corporate income tax payable in year *t*.

Expression (26) says that the firm's sources of funds are operating revenue and funds received from issuing claims over fixed (debt) and variable (new equity) proportions of future cash flows, while its uses of funds are related to its wage bill, intermediate goods purchases, debt servicing, capital expenditures and company income tax. This serves to define the relationship between the variables in the maximand and the firm's activities.

Compared to using (25) as a stand alone analytical tool, we can see here that – amongst other things – dividend payout policy becomes a part of a wider optimisation problem that raises the possibility of the firm choosing to retain earnings to finance investment expenditures in order to maximise the value of shareholder wealth.

3.2. *Taxation Liabilities*

Now we define the company tax liability that a firm faces at any point in time. Tax payable in period t is

$$T_t = \tau_t \left[p_t A_t \left[a_1 L_t^{-\rho} + a_2 K_{t-1}^{-\rho} + a_3 Q_t^{-\rho} \right]^{-\frac{1}{\rho}} - w_t L_t \left(1+\upsilon_t\right) - P_t Q_t - r_t B_{t-1} - \sum_{s=-\infty}^{t} \Delta_{s,t-s} q_s I_s \right] \quad (27)$$

where

$\Delta_{t,t-s}$ is the deductible capital allowance (investment expenditure and depreciation allowances) on a dollar of capital purchased in period s, payable t-s periods later.

Expression (27) defines the firm's taxable income for company income tax purposes. Taxable income equals operating revenue minus labour costs, intermediate goods costs, interest payments and capital allowances (both capital expenditure allowances and depreciation allowances). Defining this separately from (26) helps to keep the two concepts distinct and allows a little more flexibility in process of finding a solution.

3.3. *Capital Stock*

Next, we define the accumulation relationship for physical capital in the model. The firm's capital stock in any period t will increase in size by the level of real investment, I_t, and depreciate at a constant geometric rate δ, and so

$$K_t = I_t + \left(1-\delta\right) K_{t-1} \quad (28)$$

Depreciation in this context is real economic depreciation, and differs from the accounting principle defined in (27) above. The firm's capital stock at the end of period t is equal to the depreciated value of the previous period's capital stock plus real investment. With the firm using the capital stock available to it at the end of period t-1 in production in period t, this specification implies that there are time or "gestation" costs involved in installing capital. Investment decisions in period t are based on an analysis of the revenue streams produced by the marginal unit of capital starting from period t+1.

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[4] As ρ approaches zero, the CES production function approaches the form of a Cobb- Douglas production function with an elasticity of substitution equal to 1.

3.4. Financial Leverage

A standard assumption in finance and financial economics is that the cost of debt increases with a firm's financial leverage. Simply put, as long as the net rate of return on the firm's capital is greater than the net cost of debt, increasing the proportion of debt in the firm's financial structure will increase the profitability of the firm and, therefore, the rate of return to shareholders. However, because debt service obligations do not vary with the firm's profitability, an increase in indebtedness commits the firm to larger fixed funds outflows. Thus, from the point of view of investors, higher leverage tends to be associated with greater risk in the presence of uncertainty.

In applying this concept to our model, we therefore effectively incorporate a very basic type of uncertainty. It is certainly true that the cost of debt is related to many more things in a much more complicated way than a simple appeal to risk associated with financial structure might imply[5], but as a component of a practically-focussed CGE model it seems an attractive proposition to make some allowance for this in a relatively simplified form. Further, in seeking to analyse the optimal financial policy of the firm, assuming such a relationship will tend to promote interior solutions that better reflect the financing choices we observe in real-world firm behaviour.

Firstly, we define a ratio to capture the firm's financial leverage:

$$\text{Leverage} = \frac{B_t}{q_t K_{t-1}},$$

where we define leverage to be the firm's total debt liabilities as a proportion of the nominal value of its capital stock.

In effect, what we are attempting to do here is to formulate an active supply side to the market for debt. We formulate the equation as a non-linear function with an intercept term given by *i*, the required riskless rate of return, and so

$$r_t = i_t + s_t \left[\frac{B_{t-1}}{q_t K_{t-2}} \right]^{b_t} \tag{29}$$

The exponent, b_t, determines the shape of the function. Increasing the size of b_t acts to "focus" investor reactions into relatively large values of r_t. The intercept term also insures against the unrealistic situation of the firm being offered a zero interest rate if it is seeking debt finance but has

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[5] A few examples are the investor's confidence in the firm's management, the state of the economy in general, the state of the firm's product or factor markets, and the likelihood of unfavourable policy changes in the future.

no outstanding debt liabilities. The parameter s_t enables us to scale the impact of changes in the firm's leverage on the cost of debt.

Notice that we have defined the cost of debt in period t as a function of the firm's financial leverage in the previous period. This means that the firm can issue as much debt as it desires in the current period at a constant cost, but as it increases the size of this issue, it makes borrowing in the following period more expensive.

Making appropriate substitutions in (26) and (27) provides

$$\begin{aligned} D_t + D_t^f = {} & p_t A_t \left[a_1 L_t^{-\rho} + a_2 K_{t-1}^{-\rho} + a_3 Q_t^{-\rho} \right]^{-\frac{1}{\rho}} - w_t L_t \left(1 + \upsilon_t\right) - P_t Q_t + B_t + V_t^N \\ & - \left(1 + i_t\right) B_{t-1} - \frac{s_t B_{t-1}^{1+b_t}}{q_{t-1} K_{t-2}^{b_t}} - q_t I_t - T_t \end{aligned} \tag{30}$$

$$T_t = \tau_t \left[\begin{aligned} & p_t A_t \left[a_1 L_t^{-\rho} + a_2 K_{t-1}^{-\rho} + a_3 Q_t^{-\rho} \right]^{-\frac{1}{\rho}} - w_t L_t \left(1 + \upsilon_t\right) \\ & - P_t Q_t - i_t B_{t-1} - \frac{s_t B_{t-1}^{1+b_t}}{q_{t-1} K_{t-2}^{b_t}} - \sum_{s=-\infty}^{t} \Delta_{s,t-s} q_s I_s \end{aligned} \right] \tag{31}$$

3.5. Product Demand Conditions

So far, we have assumed that the price of the firm's product is exogenous, implying that it is a perfect competitor. We now incorporate a modification that allows us to set the "competitiveness" of the firm by determining the own-price elasticity of demand for its product. This also has the added benefit of aiding the upward sloping supply curve for debt defined above in constraining the firm to a finite level of investment. Let's now define the price of the firm's output as

$$p_t = Y_t^{-\eta} \tag{32}$$

where η is a parameter determining the own-price elasticity of demand as follows:

First linearise[6] (32),

$$\hat{p}_t = -\eta \cdot y_t$$

(where $\hat{p}_t$ and y_t are the percentage changes in the p_t and Y_t respectively), and solve for y

$$y_t = -\frac{\hat{p}_t}{\eta}.$$

Thus the own-price elasticity of demand is

$$ED_t = -\frac{1}{\eta}.$$

We now redefine the revenue functions in (30) and (31) to take account of (32):

i.e. Knowing that

$$Y_t = A_t \left[a_1 L_t^{-\rho} + a_2 K_{t-1}^{-\rho} + a_3 Q_t^{-\rho} \right]^{-\frac{1}{\rho}}$$

and

$$p_t = Y_t^{-\eta}$$

it follows that

$$p_t Y_t = \left[A_t \left[a_1 L_t^{-\rho} + a_2 K_{t-1}^{-\rho} + a_3 Q_t^{-\rho} \right]^{-\frac{1}{\rho}} \right]^{1-\eta} \tag{33}$$

Expression (33) is a total revenue function for the firm with factor inputs explicit. Therefore, taking the derivative of (33) with respect to *L*, *K*, or *Q* provides the marginal revenue product of that factor.

Substituting this into (30) and (31) we have

$$\begin{aligned} D_t + D_t^f = {} & \left[A_t \left[a_1 L_t^{-\rho} + a_2 K_{t-1}^{-\rho} + a_3 Q_t^{-\rho} \right]^{-\frac{1}{\rho}} \right]^{1-\eta} - w_t L_t \left(1+\upsilon_t\right) - P_t Q_t + B_t + V_t^N \\ & - \left(1+i_t\right) B_{t-1} - \frac{s_t B_{t-1}^{1+b_t}}{q_{t-1} K_{t-2}^{b_t}} - q_t I_t - T_t \end{aligned} \tag{34}$$

and

$$T_t = \tau_t \left[\begin{aligned} & \left[A_t \left[a_1 L_t^{-\rho} + a_2 K_{t-1}^{-\rho} + a_3 Q_t^{-\rho} \right]^{-\frac{1}{\rho}} \right]^{1-\eta} - w_t L_t \left(1+\upsilon_t\right) \\ & - P_t Q_t - i_t B_{t-1} - \frac{s_t B_{t-1}^{1+b_t}}{q_{t-1} K_{t-2}^{b_t}} - \sum_{s=-\infty}^{t} \Delta_{s,t-s} q_s I_s \end{aligned} \right] \tag{35}$$

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[6] Linearisation involves taking the natural log form of the expression. For small changes in the equations variables or parameters, this provides a good approximation of a percentage change form.

3.6. *Inequality Constraints*

Finally, we had a few more constraints on the firm's behaviour. Firstly, we assume that investment, unfranked dividends, franked dividends, outstanding debt and new equity issues must be non-negative,

$$I_t \geq 0 \tag{36}$$

$$D_t \geq 0 \tag{37}$$

$$D_t^f \geq 0 \tag{38}$$

$$B_t \geq 0 \tag{39}$$

$$V_t^N \geq 0 \tag{40}$$

Constraining investment in this way stops the firm from liquidating its productive capital in order to finance any use of funds (paying dividends, servicing debt or funding new capital). This sign restriction also limits the rate at which the firm's capital stock can shrink to the rate of depreciation, δ .

The "payment" of negative dividends is constrained by corporations law. For example, incorporation and limited liability for public companies caps the liability of shareholders for company debts to, at most, their paid-up capital[7]. Further, market pressures make it unlikely that firms would ask shareholders to contribute more capital to cover operating losses on a regular basis. On the matter of new equity issues, in the context of our analysis it seems sensible to assume that firms are not interested in share buy-backs, quite apart from the fact that a firm's ability to re-purchase its own equity is usually restricted by corporations law.

Under a dividend imputation regime, it is illegal for a firm to issue franked dividends over and above the balance in its franking account. This constraint is effectively brought to bear when tax deductions cause a dichotomy between the statutory rate of CIT and the actual amount paid. Therefore, we impose an inequality constraint, following Benge (1997 and 1998):

$$\sum_{s=-\infty}^{t} \left[\left(\frac{1-\tau}{\tau} \right) T_s - D_s^f \right] \geq 0 \tag{41}$$

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[7] Excluding the instance of contributing shares, which represent a small part of equity markets.

This requires that the firm have a dollar in its franking account for every dollar of franked dividends it issues. It is here that the effect of tax deductions on the firm's ability to pay franked dividends is brought to bear, and also, therefore, on its cost of capital.

The firm's optimisation problem is thus defined as the maximisation of (25) subject to (28) and (34) through (41).

4. The Firm's Optimal Financial Policy

The firm's optimal financial policy is crucial to the solution. Because of different tax treatments, the firm's potential sources of financial capital have different costs. Further, because of the way we have endogenised the interest cost of debt, the level of output also has a role and, thus, so too does investment. As will be discussed in more detail below, the firm's level of investment is a decreasing function of the cost of finance, and thus an analysis of the cost of the firm's alternative sources is required.

4.1. The Cost of Retaining Unfrankable Earnings

For those variables that appear directly in the maximand (dividends and new equity issues), we simply need to find the partial derivative of the maximand with respect to each variable. When the firm retains a dollar of unfrankable earnings in period t, the cost to shareholders is

$$\frac{\partial V_0}{\partial D_t} = \frac{\left[\dfrac{1-\theta_t}{1-c_t}\right]}{\prod_{s=1}^{t}\left[\dfrac{1+i_s(1-\theta_s)-c_s[1+\alpha\pi_s]}{1-c_s}\right]} \tag{42}$$

the present value of the after-tax unfranked dividend that they forego in that period.

4.2. The Cost of Retaining Frankable Earnings

The cost to shareholders of retaining frankable earnings is

$$\frac{\partial V_0}{\partial D_t^f} = \frac{\dfrac{(1-\theta_t)\left[1-(1-\gamma_t)\tau_t\right]}{(1-\tau_t)(1-c_t)}}{\prod_{s=1}^{t}\left[\dfrac{1+i_s(1-\theta_s)-c_s[1+\alpha\pi_s]}{1-c_s}\right]} \tag{43}$$

the present value of the after-tax franked dividend forgone.

4.3. *The Cost of Issuing Equity*

The cost to existing shareholders of a new share issue is

$$\frac{\partial V_0}{\partial V_t^N} = \frac{1}{\prod_{s=1}^{t}\left[\frac{1+i_s(1-\theta_s)-c_s[1+\alpha\pi_s]}{1-c_s}\right]} \tag{44}$$

the after-capital-gains-tax, present value of this issue. The logic is straightforward; to convince a potential new equity investor to part with a dollar, the firm must offer in return a stream of future earnings equal in present value terms (in equilibrium) to one dollar. This before-tax loss of one dollar to current shareholders has a net-of-CGT value equal to (44).

4.4. *The Cost of Issuing Debt*

This case is a little more difficult to analyse, because bond issues do not appear explicitly in the expression for the value of the firm. Because of this, we need to make some assumptions about how a firm will behave. The repayment of interest and principal one period later on a one-period bond, by (34) and (35), reduces the firm's ability to pay dividends. In a classical company taxation system, this has a cost equal to

$$\frac{\partial V_0}{\partial B_t} = \frac{\left(1+i(1-\tau_{t+1})\right)\left[\frac{(1-\theta_{t+1})}{(1-c_{t+1})}\right]}{\prod_{s=1}^{t+1}\left[\frac{1+i_s(1-\theta_s)-c_s[1+\alpha\pi_s]}{1-c_s}\right]} \tag{45}$$

the after-tax present value of the unfranked dividends foregone in servicing the after-tax liability on the debt issue.

In a dividend imputation system, we will assume that the firm will always retain unfranked dividends over franked dividends, and thus the cost of a debt issue is

$$\frac{\partial V_0}{\partial B_t} = \frac{\left(1+i(1-\tau_{t+1})\right)\left[\frac{(1-\theta_{t+1})\left[1-(1-\gamma_{t+1})\tau_{t+1}\right]}{(1-\tau_{t+1})(1-c_{t+1})}\right]}{\prod_{s=1}^{t+1}\left[\frac{1+i_s(1-\theta_s)-c_s[1+\alpha\pi_s]}{1-c_s}\right]} \tag{46}$$

the after-tax present value of the franked dividends foregone.

If we assume that the firm issues a perpetuity, on the other hand, the cost is

$$\frac{\partial V_0}{\partial B_t}=\sum_{z=t+1}^{\infty}\left[\frac{i\left(1-\tau_z\right)\left[\frac{\left(1-\theta_z\right)}{\left(1-c_z\right)}\right]}{\prod_{s=t+1}^{z}\left[\frac{1+i_s\left(1-\theta_s\right)-c_s\left[1+\alpha\pi_s\right]}{1-c_s}\right]}\right] \tag{47}$$

for the classical company tax system, and

$$\frac{\partial V_0}{\partial B_t}=\sum_{z=t+1}^{\infty}\left[\frac{i_z\left[\frac{\left(1-\theta_z\right)\left[1-\left(1-\gamma_z\right)\tau_z\right]}{\left(1-c_z\right)}\right]}{\prod_{s=t+1}^{z}\left[\frac{1+i_s\left(1-\theta_s\right)-c_s\left[1+\alpha\pi_s\right]}{1-c_s}\right]}\right] \tag{48}$$

under a dividend imputation regime. Notice that if we assume full dividend imputation, (48) becomes

$$\frac{\partial V_0}{\partial B_t}=\sum_{z=t+1}^{\infty}\left[\frac{i_z\left[\frac{\left(1-\theta_z\right)}{\left(1-c_z\right)}\right]}{\prod_{s=t+1}^{z}\left[\frac{1+i_s\left(1-\theta_s\right)-c_s\left[1+\alpha\pi_s\right]}{1-c_s}\right]}\right] \tag{49}$$

In the absence of indexing or (equivalently) no inflation, this expression collapses to

$$\frac{\partial V_0}{\partial B_t}=\sum_{z=t+1}^{\infty}\left[\frac{i_z\left[\frac{\left(1-\theta_z\right)}{\left(1-c_z\right)}\right]}{\prod_{s=t+1}^{z}\left[1+\frac{i_s\left(1-\theta_s\right)}{1-c_s}\right]}\right] \tag{50}$$

An infinite stream of payments of this size has a present value of one dollar (because the size of the payment is exactly equal to the discount factor that is applied to it). This will cause a fall in the value of the firm by one dollar, and the after capital gains tax value of this dollar is the net cost to shareholders of this debt policy. This makes debt and equity equivalent under these conditions, a point noted by a number of authors including Benge (1997).

Given some knowledge of the tax system and the rates of the taxes within it, we can determine an ordering of preferred financing options based on their relative costs to shareholders. Within any ordering there is also a range of possibilities. Suppose that the firm’s preferred financing options in descending order are unfrankable earnings, new equity, debt, and frankable earnings. Depending on the prior profitability of the firm and the size of planned investment expenditure, the firm could have one of two financing policies: if retained unfrankable earnings are sufficient to finance investment, the firm might not require any other forms of finance; the firm might not have sufficient unfrankable earnings to entirely finance investment and so might issue equity. Frankable are – almost by definition – always a more expensive source of finance than unfrankable earnings. Further, as the model now stands, the costs of debt, equity and retentions are constant, given a set of parameter values. Although the cost of debt is affected by a change in the firm's financial leverage, this does not have an impact until the following period. Therefore, if equity is cheaper at the start of period *t*, it is always cheaper during period *t* regardless of how the firm's financial structure changes. Making the cost of equity issues and retentions endogenous is left as a task for future research.

4.5. Financial Policy Summary

The firm faces a number of potential financial policies depending on the tax system and tax rates it faces.

In a classical company taxation system, the firm faces four possibilities:

1. Issue debt only,
2. Issue equity only,
3. Retain (unfranked) earnings only,
4. Retain earnings and issue debt,
5. Retain earnings and issue equity.

Alternative 3 arises when the firm's retained earnings are sufficient to finance planned investment expenditures. Alternatives 4 and 5 arise when retained earnings are insufficient to finance investment and a source of external finance must be sought.

Notice that we have not allowed for the firm issuing debt and equity at the same time. Although the firm faces an upward-sloping supply curve for debt, this is an inter-temporal relationship – as it increases debt issues in period *t*, the cost of debt in period *t*+1 increases. As such, the firm faces a given cost of debt in any single period. Thus, the relative costs of debt and equity for any single period are also given, and the firm will not issue both other than in the unlikely instance of the tax system, tax rates and interest rates combining to ender them equal – a possibility that we will assume does not occur.

Under a dividend imputation regime, we can start by ruling out a few possibilities. We can rule out the possibility of the firm ever retaining frankable earnings over unfrankable earnings. As long as the firm generates unfrankable earnings (that is, as long as it can claim investment allowances and depreciation allowances), unfrankable earnings are always a cheaper source of finance (in fact, by definition). If there is any transfer of imputation credits at all, franked dividends are tax preferred and, therefore, a more expensive form of finance. If no imputation credits are available, then all dividends are unfrankable and the comparison loses any relevance. In light of this, the imputation system provides some extra possibilities for a firms financial policy:

6. Issue debt and equity,

7. Retain unfrankable and frankable earnings,

8. Retain unfrankable and frankable earnings and issue debt,

9. Retain unfrankable and frankable earnings and issue equity.

There are therefore 9 potential ways of financing investment.

We have now fully defined the firm's objectives and constraints. The firm chooses a production plan – including an investment policy – and a financial policy (which determines the cheapest means to finance the investment policy) to maximise the present value of income streams to its residual claimants.

5. The Cost of Capital

5.1. Methodology

The firm's cost of capital is now attainable from the solution to a constrained optimisation problem in which the firm seeks to maximise V_0 subject to the constraints discussed above.

To simplify matters, assume

Φ_t is the appropriate discount rate from period *0* to *t*, and

ϕ_t is the appropriate discount rate between periods *t*-1 to *t*.

The problem the firm attempts to solve is a profit maximisation problem. The objective function defines the present value of the firm's profit streams, in this case labelled "dividends" (which is simply a label we attach to flows distributed to the firm's residual claimants). With the sign constraints on some variables, this becomes a Kuhn-Tucker problem.

The Lagrangian function is

$$L=\sum_{t=1}^{\infty}\left[\Phi_t^{-1}\left[\left[\frac{1-\theta_t}{1-c_t}\right]D_t+\left[\frac{(1-\theta_t)\left[1-(1-\gamma_t)\tau_t\right]}{(1-\tau_t)(1-c_t)}\right]D_t^f-V_t^N\right]\right]$$

$$+\sum_{t=1}^{\infty}\left[\lambda_t^1\cdot\Phi_t^{-1}\left[\begin{array}{l}\left[A_t\left[a_1L_t^{-\rho}+a_2K_{t-1}{}^{-\rho}+a_3Q_t^{-\rho}\right]^{-\frac{1}{\rho}}\right]^{1-\eta}-w_tL_t\left(1+\upsilon_t\right)-P_tQ_t\\+B_t+V_t^N-\left(1+i_t\right)B_{t-1}-\dfrac{s_tB_{t-1}^{1+b_t}}{q_{t-1}K_{t-2}^{b_t}}-q_tI_t-T_t-D_t-D_t^f\end{array}\right]\right]$$

$$+\sum_{t=1}^{\infty}\left[\lambda_t^2\cdot\Phi_t^{-1}\cdot\left[T_t-\tau_t\left[\begin{array}{l}\left[A_t\left[a_1L_t^{-\rho}+a_2K_{t-1}{}^{-\rho}+a_3Q_t^{-\rho}\right]^{-\frac{1}{\rho}}\right]^{1-\eta}-w_tL_t\left(1+\upsilon_t\right)-P_tQ_t\\-i_tB_{t-1}-\dfrac{s_tB_{t-1}^{1+b_t}}{q_{t-1}K_{t-2}^{b_t}}+\sum_{s=-\infty}^{t}\Delta_{s,t-s}q_sI_s\end{array}\right]\right]\right]$$

$$+\sum_{t=1}^{\infty}\left[\lambda_t^3\cdot\Phi_t^{-1}\left[I_t+(1-\delta)K_{t-1}-K_t\right]\right]$$

$$+\sum_{t=1}^{\infty}\Phi_t^{-1}\left[\lambda_t^4I_t+\lambda_t^5D_t+\lambda_t^6D_t^f+\lambda_t^7B_t+\lambda_t^8V_t^N\right]$$

$$+\sum_{t=1}^{\infty}\left[\lambda_t^9\cdot\Phi_t^{-1}\left[\sum_{s=-\infty}^{t}\left[\left(\frac{1-\tau}{\tau}\right)T_s-D_s^f\right]\right]\right] \tag{51}$$

The solution requires

(a) a set of first order conditions,

$$\frac{\partial L}{\partial L_t}=\frac{\left(\lambda_t^1-\lambda_t^2\tau_t\right)\left[\left[\dfrac{(1-\eta)a_1}{A_t^{\rho}}\right]^{-\eta}\left[\dfrac{Y_t}{L_t}\right]^{-\eta(1+\rho)}-w_t\left(1+\upsilon_t\right)\right]}{\Phi_t}=0 \tag{52}$$

$$\frac{\partial L}{\partial K_t}=$$

$$\frac{\left(\lambda_{t+1}^1-\lambda_{t+1}^2\tau_{t+1}\right)\left[\dfrac{(1-\eta)a_2}{A_{t+1}^{\rho}}\right]^{-\eta}\left[\dfrac{Y_{t+1}}{K_t}\right]^{-\eta(1+\rho)}-\left(\lambda_{t+2}^1-\lambda_{t+2}^2\tau_{t+2}\right)\dfrac{b_{t+2}s_{t+2}}{\phi_{t+2}}\left[\dfrac{B_{t+1}}{q_{t+1}K_t}\right]^{1+b_{t+2}}+\lambda_{t+1}^3(1-\delta)-\lambda_t^3\phi_{t+1}}{\Phi_{t+1}}=0 \tag{53}$$

$$\frac{\partial L}{\partial Q_t}=\frac{\left(\lambda_t^1-\lambda_t^2\tau_t\right)\left[\left[\dfrac{(1-\eta)a_3}{A_t^{\rho}}\right]^{-\eta}\left[\dfrac{Y_t}{Q_t}\right]^{-\eta(1+\rho)}-P_t\right]}{\Phi_t}=0 \tag{54}$$

$$\frac{\partial L}{\partial T_t} = \frac{-\lambda_t^1 + \lambda_t^2}{\Phi_t} + \sum_{s=t}^{\infty} \frac{\lambda_s^9 \left[\frac{1-\tau_s}{\tau_s}\right]}{\Phi_s} = 0 \tag{55}$$

$$\frac{\partial L}{\partial I_t} = \frac{q_t \left[-\lambda_t^1 + \lambda_t^2 \tau_t \Delta_{t,0} + \sum_{h=1}^{t+r} \frac{\lambda_h^2 \tau_h \Delta_{t,h}}{\prod_{s=t}^{t+r} \phi_s}\right] + \lambda_t^3 + \lambda_t^4}{\Phi_t} = 0 \tag{56}$$

$$\frac{\partial L}{\partial D_t} = \frac{\left[\frac{1-\theta_t}{1-c_t}\right] - \lambda_t^1 + \lambda_t^5}{\Phi_t} = 0 \tag{57}$$

$$\frac{\partial L}{\partial D_t^f} = \frac{\left[\frac{(1-\theta_t)\left[1-(1-\gamma_t)\tau_t\right]}{(1-\tau_t)(1-c_t)}\right] - \lambda_t^1 + \lambda_t^6}{\Phi_t} - \sum_{s=t}^{\infty} \frac{\lambda_s^9}{\Phi_s} = 0 \tag{58}$$

$$\frac{\partial L}{\partial B_t} = \frac{\left[\lambda_t^1 + \lambda_t^7\right]\phi_{t+1} - \left(\lambda_{t+1}^1 - \lambda_{t+1}^2 \tau_{t+1}\right)\left[1 + i_{t+1} + (1+b_{t+1}) s_{t+1} \left[\frac{B_t}{q_t K_{t-1}}\right]^{b_{t+1}}\right]}{\Phi_{t+1}} = 0 \tag{59}$$

$$\frac{\partial L}{\partial V_t^N} = \frac{-1 + \lambda_t^1 + \lambda_t^8}{\Phi_t} = 0 \tag{60}$$

$$\frac{\partial L}{\partial \lambda_t^1} = \frac{\left[\begin{array}{l} \left[A_t\left[a_1 L_t^{-\rho} + a_2 K_{t-1}^{-\rho} + a_3 Q_t^{-\rho}\right]^{-\frac{1}{\rho}}\right]^{1-\eta} - w_t L_t (1+\upsilon_t) - P_t Q_t + B_t + V_t^N \\ -(1+i_t) B_{t-1} - \frac{s_t B_{t-1}^{1+b_t}}{q_{t-1} K_{t-2}^{b_t}} - q_t I_t + \sum_{s=-\infty}^{t} \Delta_{s,t-s} q_s I_s - T_t - D_t - D_t^f \end{array}\right]}{\Phi_t} = 0 \tag{61}$$

$$\frac{\partial L}{\partial \lambda_t^2} = \frac{\left[T_t - \tau_t \left[\begin{array}{l} \left[A_t\left[a_1 L_t^{-\rho} + a_2 K_{t-1}^{-\rho} + a_3 Q_t^{-\rho}\right]^{-\frac{1}{\rho}}\right]^{1-\eta} - w_t L_t (1+\upsilon_t) - P_t Q_t \\ -i_t B_{t-1} - \frac{s_t B_{t-1}^{1+b_t}}{q_{t-1} K_{t-2}^{b_t}} + \sum_{s=-\infty}^{t} \Delta_{s,t-s} q_s I_s \end{array}\right]\right]}{\Phi_t} = 0 \tag{62}$$

$$\frac{\partial L}{\partial \lambda_t^3} = \frac{I_t + (1-\delta) K_{t-1} - K_t}{\Phi_t} = 0 \tag{63}$$

(b) a set of non-negativity restrictions,

$$\lambda_t^i \geq 0, \ \forall i \tag{64}$$

$$I_t \geq 0 \tag{65}$$

$$D_t \geq 0 \tag{66}$$

$$D_t^f \geq 0 \tag{67}$$

$$B_t \geq 0 \tag{68}$$

$$V_t^N \geq 0 \tag{69}$$

$$\sum_{s=-\infty}^{t} \left[\left(\frac{1-\tau}{\tau} \right) T_s - D_s^f \right] \geq 0 \tag{70}$$

and, (c) a set of complimentary slackness conditions,

$$\lambda_t^4 \geq 0, \text{ and } \lambda_t^4 = 0 \text{ if } I_t > 0 \tag{71}$$

$$\lambda_t^5 \geq 0, \text{ and } \lambda_t^5 = 0 \text{ if } D_t > 0 \tag{72}$$

$$\lambda_t^6 \geq 0, \text{ and } \lambda_t^6 = 0 \text{ if } D_t^f > 0 \tag{73}$$

$$\lambda_t^7 \geq 0, \text{ and } \lambda_t^7 = 0 \text{ if } B_t > 0 \tag{74}$$

$$\lambda_t^8 \geq 0, \text{ and } \lambda_t^8 = 0 \text{ if } V_t^N > 0 \tag{75}$$

$$\lambda_t^9 \geq 0, \text{ and } \lambda_t^9 = 0 \text{ if } \sum_{s=\infty}^{t} \left[\left(\frac{1-\tau}{\tau} \right) T_s - D_s^f \right] > 0 \tag{76}$$

We associate the Lagrangian multipliers $\lambda_t^4, \lambda_t^5, \lambda_t^6, \lambda_t^7, \lambda_t^8$ and λ_t^9 with the non-negativity constraints to act as slack variables. For each variable, the value of this multiplier is set at 0 if the constraint is slack. The Kuhn-Tucker conditions include a set of complimentary slackness conditions that state:

- for any variable x subject to a non-negativity constraint $x \geq 0$, and $x \frac{\partial L}{\partial x} = 0$.

This effectively means that the solution involves a non-negative stationary value of x **or**, as the alternative is a negative stationary value, the value of x must be zero. The addition of the slack variables means that

$$\frac{\partial L}{\partial x} = \lambda ,$$

and thus $\lambda = 0$ when $x \geq 0$. Conversely, when $\lambda > 0$, $x = 0$.

These complimentary slackness conditions allow for the possibility that the stationary value of *x* might be negative, and so at the boundary (where *x*=0) the first derivative of *L* with respect to *x* might not be zero. The slack variables allow us to set-up all of the first-order conditions as equalities.

Solving for the nominal wage rate,

$$w_t = \left[\frac{1}{1+\upsilon_t}\right]\left[\frac{(1-\eta)a_1}{A_t^\rho}\right]^{-\eta}\left[\frac{Y_t}{L_t}\right]^{-\eta(1+\rho)} \tag{77}$$

This simply says that the wage rate will equal the marginal revenue product of labour in equilibrium.

Repeating for Q

$$P_t = \left[\left[\frac{(1-\eta)a_3}{A_t^\rho}\right]^{-\eta}\left[\frac{Y_t}{Q_t}\right]^{-\eta(1+\rho)}\right] \tag{78}$$

and for K,

$$\frac{\lambda_t^3\phi_{t+1} - \lambda_{t+1}^3(1-\delta) + \left(\lambda_{t+2}^1 - \lambda_{t+2}^2\tau_{t+2}\right)\frac{b_{t+2}s_{t+2}}{\phi_{t+2}}\left[\frac{B_{t+1}}{q_{t+1}K_t}\right]^{1+b_{t+2}}}{\left(\lambda_{t+1}^1 - \lambda_{t+1}^2\tau_{t+1}\right)} = \left[\frac{(1-\eta)a_2}{A_{t+1}^\rho}\right]^{-\eta}\left[\frac{Y_{t+1}}{K_t}\right]^{-\eta(1+\rho)} \tag{79}$$

The cost of capital expression is based on these manipulations of the first order conditions. By assuming that this firm exhibits optimising behaviour (with diminishing short-run returns to factors of production, a negatively sloped product demand curve and the increasing cost of finance implicit in the model) we can infer that it will continue to invest up to the point at which the value of the marginal product of capital is equal to the value of the marginal cost of capital (i.e. we will obtain an interior solution). In its "raw" form, the first order condition for capital provides some useful insights.

$$\frac{\left(\lambda_{t+1}^1 - \lambda_{t+1}^2\tau_{t+1}\right)\left[\frac{(1-\eta)a_2}{A_{t+1}^\rho}\right]^{-\eta}\left[\frac{Y_{t+1}}{K_t}\right]^{-\eta(1+\rho)} - \left(\lambda_{t+2}^1 - \lambda_{t+2}^2\tau_{t+2}\right)\frac{b_{t+2}s_{t+2}}{\phi_{t+2}}\left[\frac{B_{t+1}}{q_{t+1}K_t}\right]^{1+b_{t+2}} + \lambda_{t+1}^3(1-\delta) - \lambda_t^3\phi_{t+1}}{\Phi_{t+1}} = 0 \tag{80}$$

This expression tells us about the costs and benefits of increasing the firm's capital stock in period t by one unit, and embodies the assumption that the firm should continue to increase its usage of capital until these costs and benefits are equal at the margin. The benefits of an extra unit of capital are

$$\left(\lambda_{t+1}^{1}-\lambda_{t+1}^{2}\tau_{t+1}\right)\left[\frac{(1-\eta)a_{2}}{A_{t+1}^{\rho}}\right]^{-\eta}\left[\frac{Y_{t+1}}{K_{t}}\right]^{-\eta(1+\rho)},$$

the net-of-tax value of the revenue generated by the marginal unit of capital,

$$\left(\lambda_{t+2}^{1}-\lambda_{t+2}^{2}\tau_{t+2}\right)\frac{b_{t+2}s_{t+2}}{\phi_{t+2}}\left[\frac{B_{t+1}}{q_{t+1}K_{t}}\right]^{1+b_{t+2}},$$

the reduction in the (tax deductible) interest rate the firm must pay on debt due to a reduction financial leverage caused by the increase in the capital stock by one unit, and

$$\lambda_{t+1}^{3}(1-\delta),$$

the residual value in period t+1 of the marginal unit of capital installed in period t. This captures the present value of the product of this residual, measured in period t+1.

The cost of this marginal unit of capital is given by

$$\lambda_{t}^{3}\phi_{t+1}$$

the net monetary cost of purchasing a unit of capital.

The Lagrangian multipliers λ_t^1, λ_t^2 and λ_t^3 denote the sensitivity of the solution to an extra unit of capital via its effect on the firm's cash flow, tax liability and capital stock constraint respectively. In the expression for the firm's cost of capital, solving for these multiplier terms will net-out the taxes and allowances applicable to each component of (79) so that we can determine the after-tax cost of capital. As we will see below, these tax terms will be unique to a particular financial policy – that is, as mentioned above, the different tax treatments attached to each finance source cause their costs to differ, and so the firm's choice of financing sources will determine the tax treatment of its cost of capital.

Expression (79) equates the marginal revenue product of capital with its marginal cost. Therefore, the cost of a marginal unit of capital in period t– which we'll denote from now on as COC_t - is

$$COC_t = \frac{\lambda_t^3\phi_{t+1} - \lambda_{t+1}^3(1-\delta) + \left(\lambda_{t+2}^1 - \lambda_{t+2}^2\tau_{t+2}\right)\frac{b_{t+2}s_{t+2}}{\phi_{t+2}}\left[\frac{B_{t+1}}{q_{t+1}K_t}\right]^{1+b_{t+2}}}{\left(\lambda_{t+1}^1 - \lambda_{t+1}^2\tau_{t+1}\right)} \tag{81}$$

Simply put, if the firm increases its capital stock by a unit when behaving optimally, its ability to pay dividends is reduced by the net-of-tax cost of purchasing that unit of capital, but increases its ability to pay dividends via: the extra net-of-tax revenue this capital generates, the impact of a lower net-of-tax interest rate on debt, and by the residual value of the unit of capital available for production in subsequent periods.

The right hand side of (79) is effectively the firm's investment demand curve. Solving this expression for each period generates a time path for investment. In the current paper, we have no exogenously imposed structural changes in the firm's economic environment, and thus the firm does not have any reason to grow or shrink. This firm, under the conditions assumed in this paper, will find an optimal, profit maximising capital stock, which it will then seek to attain in every period. Knowing this, and using expression (28), we can deduce that

$$K_t = K_{t-1} = \bar{K}$$

and therefore

$$I_t = \bar{K} - (1-\delta)\bar{K} = \delta\bar{K} .$$

This says that investment in new capital in every period will be that amount required to restore that portion of the capital stock lost through depreciation – that is, we have defined a static steady state. Growth is easily imposed on this firm, most obviously via exogenous technical change (via the parameters A and a in the CES production function) or by imposing a change in the prices of the firm's product and/or inputs.

The exact nature of a firm's cost of capital function will be determined by the financing and investment decisions that it makes, which in turn will be determined by the legal system (represented by some of the inequality constraints), tax system and rates of taxation that it faces, *ceteris paribus*. There will exist a unique cost of capital function for each variant of the firm's financial and investment policies. This is clear from (81) – the underlying structure of all of these cost of capital expressions is the same, and it is the different tax coefficients, determined by the nature of relevant legal and tax systems, and the firms' financial policies, that makes them case-specific.

5.2. An Illustrative Example: Different Financial Policies

Assume that these firms are identical except for their methods of financing investment. This situation could easily arise when firms are owned by different cohorts of shareholders with different tax positions. We'll assume that they all operate within a dividend imputation system.

For this first firm, we'll assume that it will retain unfrankable earnings and pay the full amount of franked dividends possible. Therefore, looking at the complimentary slackness conditions, we need to set

$$\lambda_t^6 = 0 \tag{82}$$

Further, with the firm also issuing equity, we can set

$$\lambda_t^8 = 0 \tag{83}$$

We will also assume that this firm is a going concern, in the sense that it intends to undertake a non-zero level of investment in this period. This does not rule out the possibility of the firm shrinking, because its level of planned investment can still be less than that amount required to replace capital lost through physical depreciation in the previous period. Not assuming a positive level of investment, when we are considering the cost of capital that is purchased in undertaking this investment, seems unreasonable. If the level of investment was zero, we could assume that the cost of capital was also zero, and this process of analysis would be redundant. Assuming a non-zero level of investment allows us to set

$$\lambda_t^4 = 0 \tag{84}$$

Given these parameter values, equation (60) implies

$$\lambda_t^1 = 1 \tag{85}$$

From (58),

$$\frac{\left[\dfrac{(1-\theta_t)\left[1-(1-\gamma_t)\tau_t\right]}{(1-\tau_t)(1-c_t)}\right]-1}{\Phi_t} = \sum_{s=t}^{\infty} \frac{\lambda_s^9}{\Phi_s} \tag{86}$$

The term on the right hand side of this expression takes account of the ability of the firm to pay franked dividends due to its franking account constraint. As it appears above, this term allows for the possibility that the firm might not have paid the maximum possible amount of franked dividends in previous periods, and thus has franking credits available over and above those generated in the current period. Assuming that this firm will always pay franked earnings when it is in a position to do so (which, in reality, is a quite weak assumption), we need only take account of the current period, and we can alter this expression to obtain

$$\left[\frac{(1-\theta_t)\left[1-(1-\gamma_t)\tau_t\right]}{(1-\tau_t)(1-c_t)}\right]-1=\lambda_t^9 \tag{87}$$

Assuming that full dividend imputation is available to shareholders receiving franked dividends, we can set γ_t at 1, providing

$$\lambda_t^9=\frac{\tau_t+(1-\tau_t)c_t-\theta_t}{(1-\tau_t)(1-c_t)} \tag{88}$$

Again, under the assumption that firms always pay franked dividends if they have the franking credits available, substituting (88) for λ_t^9 in (55) provides

$$\lambda_t^2=\frac{\theta_t-c_t}{(1-c_t)\tau_t} \tag{89}$$

If we assume that shareholders will never have more than 1 periods forewarning of changes in tax rates (which does not appear unreasonable), and if, following Benge (1997), we denote the present value of all tax allowances arising from I_t as

$$\Xi_t=\Delta_{t,0}+\sum_{h=1}^{t+r}\frac{\Delta_{t,h}}{\prod_{s=t}^{t+r}\phi_s} \tag{90}$$

from (56) we obtain

$$\lambda_t^3=q_t\left[1-\left[\frac{\theta_t-c_t}{1-c_t}\right]\Xi_t\right]=q_t\left[\frac{1-\theta_t\Xi_t+c_t(1-\Xi_t)}{1-c_t}\right] \tag{91}$$

Taking (81) and making the appropriate substitutions with (85), (89) and (91), we have

$$COC_t=\left[\frac{1-c_{t+1}}{1-\theta_{t+1}}\right]\left[\begin{array}{l} q_t\left[\frac{1-\theta_t\Xi_t+c_t(1-\Xi_t)}{1-c_t}\right]\phi_{t+1} \\ -q_{t+1}\left[\frac{1-\theta_{t+1}\Xi_{t+1}+c_{t+1}(1-\Xi_{t+1})}{1-c_{t+1}}\right](1-\delta) \\ +\left[\frac{1-\theta_{t+2}}{1-c_{t+2}}\right]\frac{b_{t+2}s_{t+2}}{\phi_{t+2}}\left[\frac{B_{t+1}}{q_{t+1}K_t}\right]^{1+b_{t+2}} \end{array}\right] \tag{92}$$

If the various tax rates do not change, or at least are not expected to change from the point of view of financiers in period t, then this becomes

$$COC_t = q_t \phi_{t+1} \left[\frac{1-\theta \Xi_t - c\left(1-\Xi_t\right)}{1-\theta} \right] - q_{t+1}\left(1-\delta\right) \left[\frac{1-\theta \Xi_{t+1} - c\left(1-\Xi_{t+1}\right)}{1-\theta} \right] + \frac{b_{t+2} s_{t+2}}{\phi_{t+2}} \left[\frac{B_{t+1}}{q_{t+1} K_t} \right]^{1+b_{t+2}} \tag{93}$$

Now consider a firm that retains unfrankable and frankable earnings. The analysis of section 4 shows that the firm will always retain unfrankable earnings before it turns to frankable earnings, and so we need to make an assumption about whether the firm retains all frankable earnings, or indeed whether there are sufficient funds available in the firm's internal sources of finance to entirely fund investment at all. For this example, we'll analyse both cases – in the first case, the firm has sufficient unfrankable and frankable earnings to finance investment *and* pay some franked dividends, and in the second case, the firm retains all available unfrankable and frankable earnings *and* issues equity.

In the first case, knowing that the firm will pay some franked dividends, we set

$$\lambda_t^6 = 0 \tag{94}$$

However, because it does not pay the *maximum possible* amount of franked dividends, we know that the inequality constraint on the franking account-franked dividends relationship will not be binding, and so

$$\lambda_t^9 = 0 \tag{95}$$

We also again set the slack variable on investment to zero.

Expression (58) implies

$$\lambda_t^1 = \left[\frac{1-\theta_t}{\left(1-\tau_t\right)\left(1-c_t\right)} \right] \tag{96}$$

and therefore, expression (55) implies

$$\lambda_t^2 = \left[\frac{1-\theta_t}{\left(1-\tau_t\right)\left(1-c_t\right)} \right] \tag{97}$$

Substituting (96) and (97) into (56) provides

$$\lambda_t^3 = q_t \left[\frac{\left(1-\theta_t\right)\left(1-\tau_t \Xi_t\right)}{\left(1-\tau_t\right)\left(1-c_t\right)} \right] \tag{98}$$

Making the appropriate substitutions in (81) provides

$$COC_t = \left[\frac{1-c_{t+1}}{1-\theta_{t+1}}\right]\left[\begin{array}{l} q_t\left[\frac{(1-\theta_t)(1-\tau_t\Xi_t)}{(1-\tau_t)(1-c_t)}\right]\phi_{t+1} \\ -q_{t+1}\left[\frac{(1-\theta_{t+1})(1-\tau_{t+1}\Xi_{t+1})}{(1-\tau_{t+1})(1-c_{t+1})}\right](1-\delta) \\ +\left[\frac{1-\theta_{t+2}}{1-c_{t+2}}\right]\left[\frac{b_{t+2}s_{t+2}}{\phi_{t+2}}\right]\left[\frac{B_{t+1}}{q_{t+1}K_t}\right]^{1+b_{t+2}} \end{array}\right] \tag{99}$$

or, if tax rates are constant from the potential investor's point of view, we obtain

$$COC_t = q_t\left[\frac{1-\tau\Xi_t}{1-\tau}\right]\phi_{t+1} - q_{t+1}\left[\frac{1-\tau\Xi_{t+1}}{1-\tau}\right](1-\delta) + \frac{b_{t+2}s_{t+2}}{\phi_{t+2}}\left[\frac{B_{t+1}}{q_{t+1}K_t}\right]^{1+b_{t+2}} \tag{100}$$

For the second case, we know that no dividends will be paid, the inequality constraints on the franking account-franked dividends relationship and new equity issues will not be binding and investment will be non-zero, therefore

$$\lambda_t^4 = \lambda_t^8 = \lambda_t^9 = 0 \tag{101}$$

Therefore, expression (60) implies

$$\lambda_t^1 = 1 \tag{102}$$

which, along with (55) provides

$$\lambda_t^2 = 1 \tag{103}$$

Substituting (102) and (103) into (56), we obtain

$$\lambda_t^3 = q_t\left[1-\tau_t\Xi_t\right] \tag{104}$$

Making the appropriate substitutions in (81) this time provides

$$COC_t = q_t\left[\frac{1-\tau_t\Xi_t}{1-\tau_{t+1}}\right]\phi_{t+1} - q_{t+1}\left[\frac{1-\tau_{t+1}\Xi_{t+1}}{1-\tau_{t+1}}\right](1-\delta) + \left[\frac{1-\tau_{t+2}}{1-\tau_{t+1}}\right]\frac{b_{t+2}s_{t+2}}{\phi_{t+2}}\left[\frac{B_{t+1}}{q_{t+1}K_t}\right]^{1+b_{t+2}} \tag{105}$$

or, if tax rates are constant from the potential investor's point of view,

$$COC_t = q_t\left[\frac{1-\tau\Xi_t}{1-\tau}\right]\phi_{t+1} - q_{t+1}\left[\frac{1-\tau\Xi_{t+1}}{1-\tau}\right](1-\delta) + \frac{b_{t+2}s_{t+2}}{\phi_{t+2}}\left[\frac{B_{t+1}}{q_{t+1}K_t}\right]^{1+b_{t+2}} \tag{106}$$

The intuition behind these expressions is more easily understood by assuming that tax rates are not expected to change and, secondly, starting with the tax coefficients on the first asset price terms as a means for comparison. We have

$$q_t\left[\frac{1-\theta\Xi_t-c\left(1-\Xi_t\right)}{1-\theta}\right]\phi_{t+1} \tag{107}$$

$$q_t\left[\frac{1-\tau\Xi_t}{1-\tau}\right]\phi_{t+1} \tag{108}$$

$$q_t\left[\frac{1-\tau\Xi_t}{1-\tau}\right]\phi_{t+1} \tag{109}$$

respectively for each of the three cases under consideration.

Starting with the last of the three, (109) reflects the fact that, because *all* dividends are retained, the shareholder's earnings are effectively taxed at the rate of company tax, adjusted for asset allowances. Even if all earnings were frankable (not possible if Ξ is non-zero), imputation credits cannot be applied when capital gains tax is paid (which is how these earnings are effectively distributed).

For (107) and (108), some franked dividends are paid – in the first case the maximum allowable (therefore the franking account - franked dividend constraint is binding), and in the second case some amount less than this, depending on the relationship between the funds required and the frankable earnings available. In the case of (109), no franked dividends are paid.

For (107), because the maximum level of frankable earnings is distributed, the various investment-related tax allowances reduce the firm's ability to pay franked dividends. At the margin, therefore, these allowances and the effect of the binding nature of the franking account – franked dividend constraint causes an adjustment in the composition of distributions. The weighting of unfranked dividends and new equity issues in the firm's financial structure for this particular period is effectively determined by the size of the allowances – if no allowances were available, the firm would have no unfrankable earnings and the entire project would be financed by equity. As the size of the allowable deductions increases, the firm retains more unfranked earnings, pays less franked dividends, and needs to issue less equity to finance its given investment expenditure needs. In fact, the firm will retain *d*Ξ more franked dividends and issue *d*(1-Ξ) less equity. In the numerator of (107) we can see the weighting of each source is Ξ retained unfrankable earnings and (1-Ξ) new equity issues. An increase in the personal income tax rate causes a reduction in the cost of purchasing this unit of capital, because the after-tax value of the foregone dividend falls. Likewise, an increase on the capital gains tax rate causes a fall in the cost of the purchase, because it reduces the net present value of the income streams that the existing shareholder loses due to the new equity issue.

Notice also that the rate of company income tax, τ, does not affect this firm's cost of capital. This is because it is paying the maximum possible level of franked dividends. With the franking account constraint binding, an increase in the company tax rate increases tax payments at the company level and reduces them at the personal level by equal and offsetting amounts.

In the second case (expression (108)), the cost to shareholders of purchasing this unit of capital *is* dependent on the company tax rate. In this case, with the franking account constraint *not* binding, a change in the company tax rate affects the amount of tax paid at the company level but does not affect the amount of franked dividends that can potentially be paid. If there were no investment or depreciation allowances the after-tax cost of the purchase would simply be the grossed-up value of the foregone distributions, which would have been entirely frankable. If the entire value of the asset could be expensed, the tax coefficient disappears – this highlights the important role of the deductions in causing a dichotomy between the tax treatment of company profits and shareholder distributions.

We can also see that (109) is identical to (108). This is due to our assumption that tax rates are constant, as otherwise (as can be seen in comparing (99) and (105)), they differ significantly.

A more thorough discussion of the full range of possible cost-of-capital expressions is left to a forthcoming paper.

6. Concluding Remarks

In this paper, we have outlined the theoretical underpinnings and methodology of an approach to incorporating business taxation into an investment problem for a corporate enterprise.

The first task was to analyse the role of taxes and allowances on the value of the firm, assuming that its investment decisions and before-tax profit streams are given. We saw that the interplay between taxes and the value of the firm can be quite complex, especially when we apply a dividend imputation system and a realisation-basis CGT system. Understanding the impact of a policy shift on the firm enables us to infer something about the required rate of return on investment and, thus, the level of investment undertaken.

In the second part of this process we set up a constrained optimisation problem and solved for the optimal level of investment and its associated cost, as well as the levels of all of the firm's choice variables (that is, a complete solution to the so-called "producer problem"). In this way, we determine the optimal level of investment to maximise the firm's value to its shareholders, and generate an expression that tells us how the tax system affects the cost of funds to the firm in equilibrium. This allows us to actually maximise the value of the expression that we discussed in the first section and then generate a time path for investment. In this framework, we can see how taxation can influence both the choices the firm makes and the outcomes it can expect *given* those choices.

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Appendix 1: List of Symbols

V	Total value of the firm.
V^O	Value of pre-existing equity.
V^N	Value of new share issues.
θ	Personal income tax rate.
τ	Company tax rate.
γ	Tax credits available on dividends for tax paid at company level (rate of imputation).
υ	Payroll tax rate.
c	Effective capital gains tax rate.
ψ	Proportion of capital gains that are taxable.
ε	Rate of share realisations.
α	Dummy variable on inflation index for capital gains tax.
π	Inflation rate.
δ	Rate of real economic depreciation.
Δ	Capital allowances – depreciation and capital expenditure.
B	Debt issues.
r	Interest cost of debt issues.
b	Parameter determining the shape of the upward sloping supply curve for debt – higher values act to focus investor reactions into relatively large values of r_t.
s	Parameter that scales the impact of changes in the firm's leverage on the cost of debt.
i	Cash rate.
p	Price of firm's output (producer's price).

η Parameter for setting (constant) own-price elasticity of firm's product demand curve.

K Firm's effective total usage of capital.

q Asset price of capital.

Q Firm's intermediate good usage.

P Price of intermediate goods (purchaser's price).

L Firm's labour usage.

w Nominal wage rate.

Y Level of firm's output.

A Efficiency parameter (all primary factor technical change) in firm's CES production function.

ρ Parameter for setting (constant) elasticity of substitution in firm's CES production function, where $ES = \dfrac{1}{1+\rho}$.

a Distributional factor denoting input shares in firm's CES production function.