IMPACT OF DEMOGRAPHIC CHANGE ON INDUSTRY STRUCTURE IN AUSTRALIA
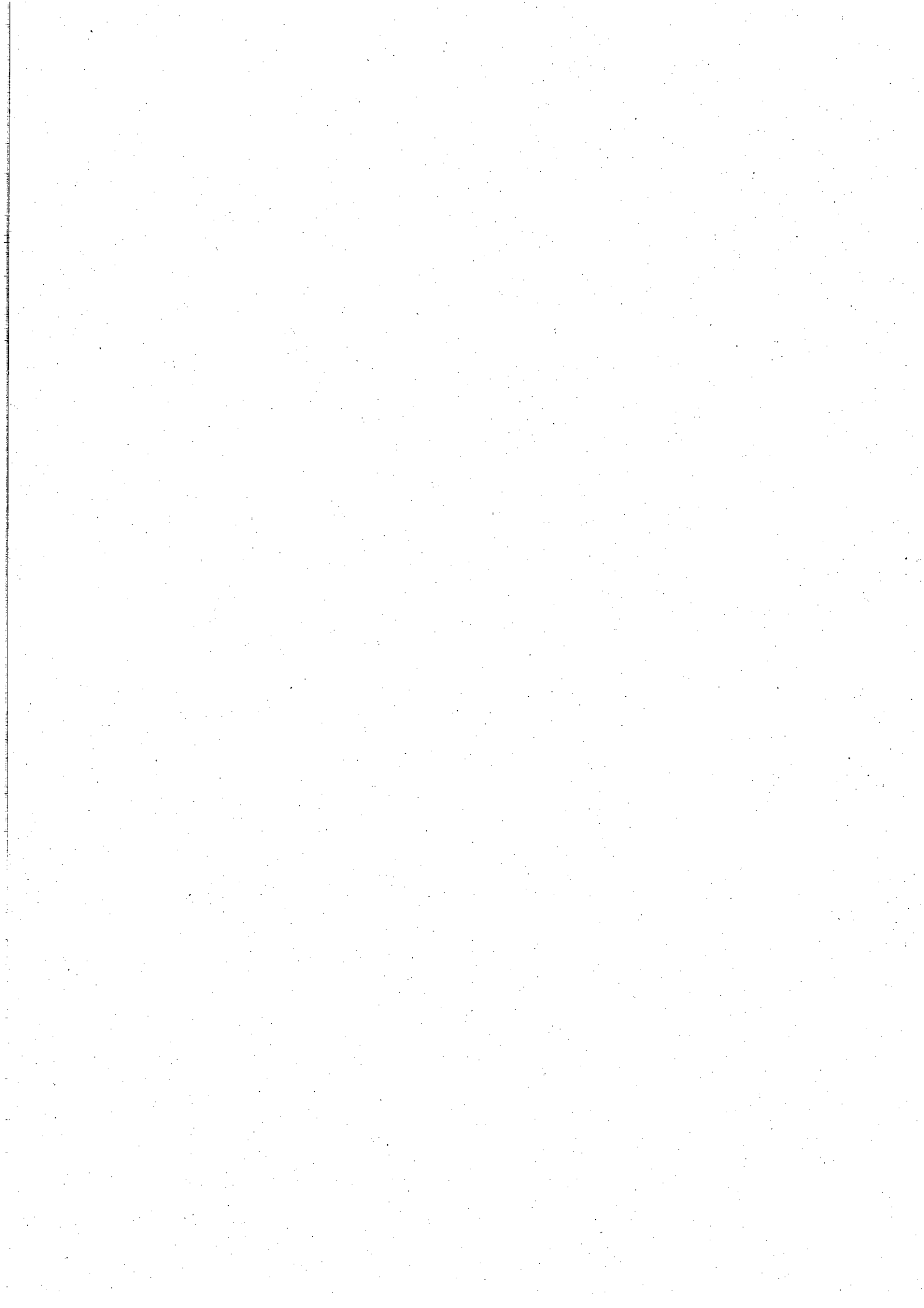
SEPARABILITY AND SUBSTITUTABILITY IN

AUSTRALIAN MANUFACTURING

by

Vern Caddy

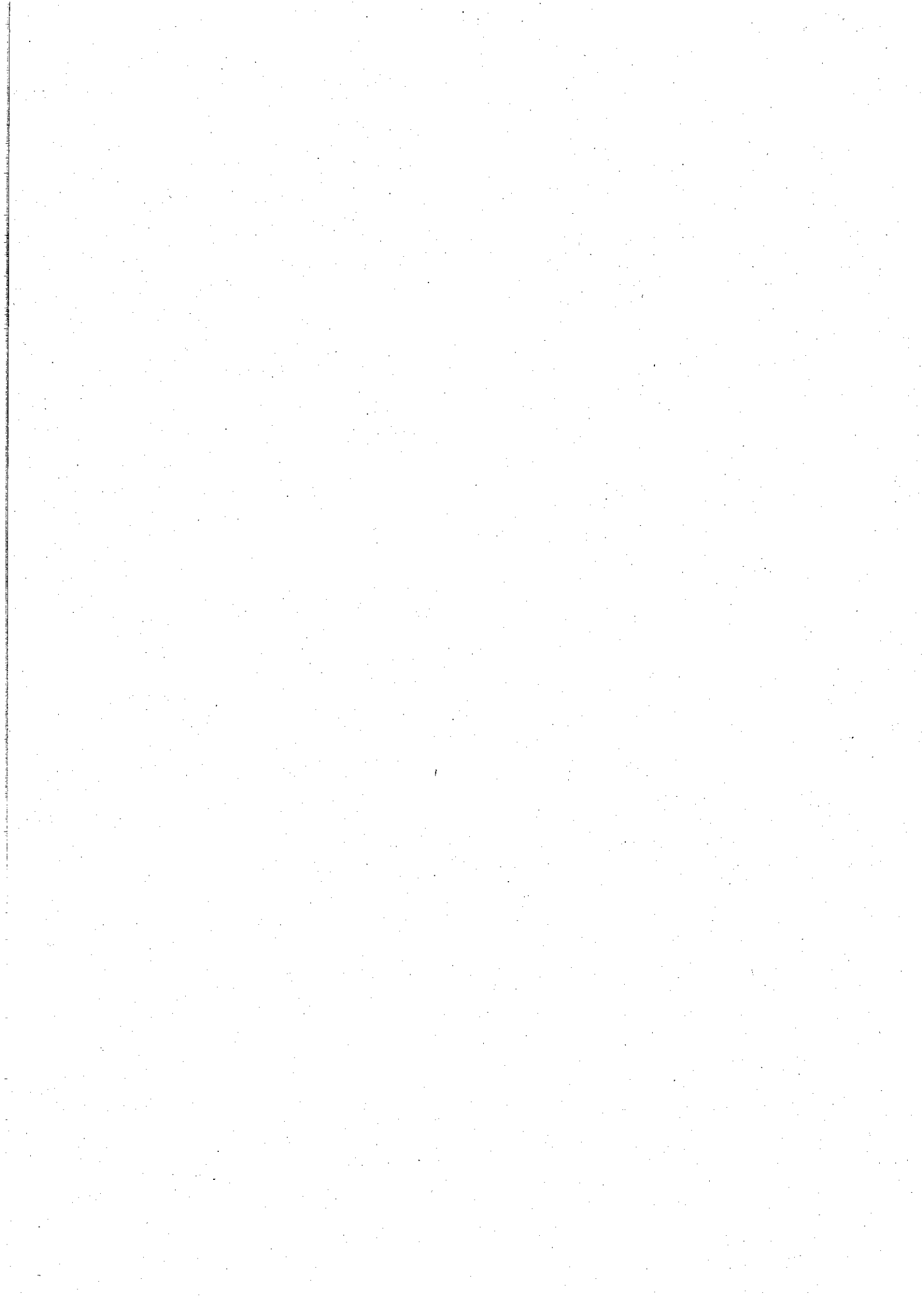Industries Assistance Commission

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CONTENTS

SEPARABILITY AND SUBSTITUTABILITY IN AUSTRALIAN MANUFACTURING

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1.   INTRODUCTION

Over the last fifty years an extensive body of literature

has evolved around the analysis of production. While earlier work

in the area tended to concentrate on a few specific functional forms

for the production relationship, there has recently been an increasing

emphasis on attempts to axiomatically define the production structure

which has led to technology being viewed in a more general and abstract

way. One of the outcomes of this development has been to highlight the

duality that exists between the production function, the cost function

and the input requirement set when certain regularity conditions are

satisfied.[1]

In view of the equivalence between these forms of specifi-

cation the applied researcher is free, without loss of generality or

information, to specify and concentrate attention on any one of the

forms depending on the purpose of the analysis and data availability.

Each is capable of completely summarizing the characteristics of the

production technology under consideration.

Regardless of which relationship is used to describe the

technology, the empirical specification requires the use of a specific

functional form. The choice of such a function is, of necessity,

somewhat arbitrary, as the regularity conditions necessary to ensure

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1.   The duality of cost and production functions was first developed
     by Shephard (1953). A proof of the regularity conditions can be
     found also in Diewert (1971).

that it is "well behaved" impose only relatively mild restrictions on its algebraic form.

While it is desirable to use a function which is capable of representing a wide range of technologies, it is usually considered necessary to impose additional prior restrictions in order to reduce the number of parameters to be estimated to a manageable level. These constraints usually take the form of assumptions about the separability of the function[1] and determine the nature and extent of the aggregation procedures used in data handling, as well as the restrictions placed on the substitution matrix. This paper attempts to test the legitimacy of two propositions related to the separability of the estimated function that are commonly assumed to be valid in applied production studies. These assumptions, which imply the existence firstly of a value added function and secondly of a consistent capital index, are tested using cost data for Australian Manufacturing Industries.

2. SEPARABILITY AND SUBSTITUTABILITY

We consider the separability of a production function $Y = G(X_1, \ldots, X_m)$ in which the m inputs have been partitioned into r mutually exclusive and exhaustive subsets $[N_1, \ldots, N_r]$. The function $G(X)$ is said to be weakly separable with respect to the above partition if the marginal rate of substitution between any

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1. See Goldman and Uzawa (1964) for an exposition of the various forms of separability.

two inputs X_i and X_j from any subset N_s, $s=1,\ldots,r$, is independent of the level of inputs outside the subset N_s. That is,

$$\frac{\partial(G_i/G_j)}{\partial X_q} = 0 \quad , \tag{1}$$

where $G_i = \frac{\partial G}{\partial X_i}$, etc. and $i,j \in N_s$, $q \notin N_s$.

This condition implies the following relationship between the first and second order partial derivatives of G :

$$\frac{\partial(G_i/G_j)}{\partial X_q} = \frac{1}{G_j} G_{iq} + G_i \frac{\partial(1/G_j)}{\partial X_q} \quad ,$$

$$= \frac{G_{iq}}{G_j} - \frac{G_i \, G_{jq}}{G_j^2} \quad ,$$

$$= 0 \text{ (from (1) above) } ,$$

where $G_{iq} = \frac{\partial^2 G}{\partial X_i \, \partial X_q}$, $G_j^2 = \left[\frac{\partial G}{\partial X_j}\right]^2$.

If $G_j \neq 0$ then we have

$$G_{iq} \, G_j = G_{jq} \, G_i \quad . \tag{2}$$

Goldman and Uzama (1964) have also proved that weak separability of the function with respect to the above partition is a necessary and sufficient condition for the function to be written in the form :

$$Y = G(X^1, X^2, \ldots, X^r) \tag{3}$$

where X^s is a function of the elements of N_s only.

In view of the earlier comments on duality between production and cost functions it would be expected that any inferences that can be drawn about the separability of one could also be drawn from the other. This is in fact the case since, by "Lau's Lemma", separability of the cost function with respect to prices has exactly the same implications as separability of the production function with respect to the corresponding inputs.[1] Hence an equivalent condition to (2) is :

$$C_{iq} C_j = C_{jq} C_i \quad , \tag{4}$$

where $C(P)$ is the cost function $C(P_1, \ldots, P_m, Y)$, P_s is the price of X_s, Y is output and $C_{iq} = \dfrac{\partial C}{\partial P_i \, \partial P_q}$ etc.

Berndt and Christensen (1973) have shown that for homothetic functions the concept of functional separability has direct implications for the relationship between the various Allen Partial Elasticities of Substitution (AES).[2] In general the weak separability of a function at a point in its domain is a necessary and sufficient condition for all AES σ_{iq}, $\sigma_{jq} (i, j \epsilon N_s, q \notin N_s)$ to be equal at that point.

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1. Lau (1969) has proved this proposition for direct and indirect utility functions. An analogous relationship exists between production and cost functions.

2. See Allen (1938).

3. SEPARABILITY IN PRACTICE

In several areas of applied production analysis separability is assumed (usually implicitly) as part of the maintained hypothesis. As the validity of the procedures used often depend on such assumptions (for which there is no substantial economic justification) it would appear that some empirical testing is required. The two propositions that will be examined in this paper are the assumed existence of a "value added function" and a consistent aggregate capital input index. Both these concepts are inherent to the bulk of applied production studies and impose certain separability conditions on the underlying technology. In this paper the Australian Manufacturing sector will be studied using a production function of the general form :

$$Y \;=\; G(B,P,L,M) \qquad , \tag{5}$$

where Y is real gross output, B and P represent the capital inputs "buildings and structures" and "plant and equipment" respectively, L is a measure of labour inputs and M represents intermediate inputs.

i) Value Added

An assumption commonly made by economists is that a functional relationship exists between the primary inputs which provides a consistent measure of the real contribution of these factors to the productive process (called "value added"). In the absence of equi-proportional changes in the prices of the various inputs (in which case Hicks' (1946) composite commodity theorem would apply), Leontief (1947) has shown that a consistent index of this nature will only exist if the production function exhibits weak separability between the primary and other inputs.

That is, if the function is of the form :

$$Y = G^*(H(B,P,L),M) \quad , \tag{6}$$

where H(B,P,L) is the value added function.

Probably the most familiar measure of value added is that given in the Australian National Accounts. The implicit assumptions underlying any attempt to use this measure to reflect real production changes are more stringent than those given in (6). Since value added is taken to be the difference between gross output and material inputs the implied production technology is of the form :

$$Y = H(P,B,L) + M \quad . \tag{7}$$

This function is additively separable and imposes an AES of infinity between value added and intermediate inputs, i.e., they are perfect substitutes $(\sigma_{HM} = 0)$.

The other method sometimes employed in the literature to incorporate the concept of a value added function into the underlying production technology   is to use a Leontief type specification.[1] That is :

$$Y = \min\{H(B,P,L),aM\} \quad , \tag{8}$$

where a is a fixed coefficient. In this case value added and intermediate inputs are perfect complements and consequently have an AES of zero $(\sigma_{HM} = 0)$.[2]

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1. See for example Dixon (1975).

2. To make this statement fully rigorous, it is necessary to replace (8) by a differentiable function, such as $Y = \gamma \left[\alpha\{H(B,P,L)\}^{-\rho} + (1-\alpha)M^{-\rho} \right]^{-\frac{1}{1+\rho}}$, and to note that for σ_{HM} small enough, the difference between (8) and this function can be made arbitrarily small.

Using the Berndt and Christensen result referred to above, the specifications (5) - (8) can be interpreted in terms of the restrictions they impose on the various AES. These restrictions are given in Table 1 below.

TABLE 1
RESTRICTIONS IMPLICIT IN VALUE ADDED FUNCTIONS

Equation	Restrictions on AES				
(5)	Nil				
(6)	σ_{BM}	$= \sigma_{PM}$	$= \sigma_{LM}$		
(7)	σ_{BM}	$= \sigma_{PM}$	$= \sigma_{LM}$	σ_{HM}	$= \infty$
(8)	σ_{BM}	$= \sigma_{PM}$	$= \sigma_{LM}$	σ_{HM}	$= 0$

ii) Capital Aggregation

The other proposition to be tested in this paper is related to the aggregation procedures used in measuring the input of primary factors into the productive process. In most applied studies the degree of aggregation used is extremely high with, in most cases, only two forms of primary inputs - namely capital and labour - being identified as arguments of the function. While analytically it is convenient to aggregate the many forms of these inputs which must enter any productive process, it is nevertheless necessary to recognise that this also implicitly assumes the conditions necessary for the existence of consistent capital and labour indices are satisfied. Again (if we ignore the Hicks composite commodity case) it can be shown that this

requires the production function to be weakly separable with respect to the partition which differentiates between the various forms of capital and labour inputs.

While the range of capital and labour inputs that could conceptually be included in the production function is almost limitless, it is necessary to severely restrict the number if the model is to be given any empirical content. For this study it has been decided (as can be seen from equation (5)), to distinguish between the two capital inputs Buildings and Structures and Plant and Equipment. The necessary conditions for the separability of these components from the other inputs (which is required for a consistent capital index to exist) imply that equation (5) can be written as :

$$Y = G'(F(B,P),L,M) \quad . \tag{9}$$

In practice the aggregator function is usually defined as a fixed weight arithmetic sum. The assumption implicit in such aggregation is that the AES between the two capital inputs is infinity (i.e., they are perfect substitutes). In this case we have :

$$Y = G'((B + P),L,M) \quad . \tag{10}$$

The restrictions on the AES inherent in (9) and (10) are given in Table 2.

TABLE 2

RESTRICTIONS IMPLICIT IN CAPITAL AGGREGATION

Equation	Restrictions on AES
(9)	$\sigma_{BL} = \sigma_{PL}$
	$\sigma_{BM} = \sigma_{PM}$
(10)	$\sigma_{BL} = \sigma_{PL}$
	$\sigma_{BM} = \sigma_{PM}$
	$\sigma_{BP} = \infty$

4. NON-SEPARABLE FUNCTIONAL FORMS

In order to test the above propositions it is obviously necessary to work with a function in which the inherent restrictions in terms of separability (and substitutability) are minimal. A group of models, the so-called "flexible functional forms", which have recently gained a measure of prominence in the literature would appear to be suitable for the task. These functional forms - the "Generalized Leontief" and "Generalized Cobb/Douglas" proposed by Diewert (1971, 1973) and the "Transcendental Logarithmic (translog) Function" of Christensen et al. (1971) - all provide second order approximations to any twice differentiable function in an arbitrary number of variables. If for example we have an arbitrary cost function $C^*(P,Y)$ with first and second order partial derivatives $C_i^*(P^*,Y)$ and $C_{ij}^*(P^*,Y^*)$ at a point (P^*,Y^*), then there exists a cost function $C(P,Y)$ in each of the above forms such that the first and second order partial derivatives at (P^*,Y^*) coincide

with those of C^* . Since the AES can be expressed in terms of first

and second order partial derivatives of the cost function, it follows

that there exists a set of parameters for each of the above functional

forms which will reproduce any matrix of AES at the point of approximation.

In view of the extreme generality of these functions they are suitable

for testing the separability assumptions inherent in the production

functions discussed above in Section 3. In this paper the translog

functional form is used to specify a cost function rather than

production function.[1] There are several advantages in doing this.

Firstly linear homogeneity of the production function does not have to

be assumed to derive the estimating equations. Cost functions are

homogeneous of degree one in prices regardless of the degree of

homogeneity of the production function. Secondly the elasticity of

substitution can be calculated as a linear transformation of the

estimated parameters.[2] As a consequence the econometric properties of

the calculated substitution parameters can readily be related to those

of the estimated parameters. Thirdly it is hoped that the problem of

multicollinearity, which plagues attempts at direct estimation of the

production function, may be ameliorated if prices rather than input

quantities represent the explanatory variables.[3] Finally it can be

argued that the problem of simultaneous equation bias will be eliminated

if prices rather than input quantities (which are unambiguously

endogenous in the model under consideration) are used as explanators.

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1. Berndt and Christensen (1973a, 1973b) have used translog production
    functions in two recent applied studies of the type undertaken in
    this paper.

2. The form of this transformation is derived below.

3. See Tsang and Persky (1975) for an analysis of this problem in
    relation to the CES function.

5. DERIVATION OF THE TRANSLOG COST FUNCTION

If we have an arbitrary cost function (corresponding to the production function (5)) in logarithmic form :

$$\ell nC = J(\ell nP_b, \ell nP_p, \ell nP_\ell, \ell nP_m, \ell nY) \tag{11}$$

where P_i is the price of input $i(i=b,p,\ell,m)$ and Y is the level of output, then J can be written as a Taylor series expansion in the ℓnP_i's about the fixed point $(\ell n\overline{P}_b,\ldots,\ell n\overline{P}_m, \ell n\overline{Y})$.

$$\ell nC = J(\ell n\overline{P}_b,\ldots,\ell n\overline{P}_m, \ell n\overline{Y}) + \sum_{i=1} \frac{\partial J}{\partial \ell nP_i} (\ell nP_i - \ell n\overline{P}_i)$$

$$+ \frac{1}{2} \sum_i^m \sum_j^m \frac{\partial^2 J}{\partial \ell nP_i \partial \ell nP_j} (\ell nP_i - \ell n\overline{P}_i)(\ell nP_j - \ell n\overline{P}_j)$$

$$+ \sum_i^m \frac{\partial^2 J}{\partial \ell nP_i \partial \ell nY} (\ell nP_i - \ell n\overline{P}_i)(\ell nY - \ell n\overline{Y}) + \frac{\partial J}{\partial \ell nY} (\ell nY - \ell n\overline{Y})$$

$$+ \frac{1}{2} \frac{\partial^2 J}{\partial \ell nY \partial \ell nY} (\ell nY - \ell n\overline{Y})^2 + \text{higher order terms} , \tag{12}$$

where the partial derivatives are evaluated at the point $(\ell n\overline{P}_b,\ldots,\ell n\overline{P}_m, \ell n\overline{Y})$.

If we neglect the higher order terms and let

$$J(\ell n\overline{P}_b,\ldots,\ell n\overline{P}_m, \ell n\overline{Y}) = \gamma_{oo} \; ; \; \frac{\partial J}{\partial \ell nP_i} = \gamma_{oi} \; ;$$

$$\frac{\partial^2 J}{\partial \ell nP_i \partial \ell nP_j} = \gamma_{ij} \; ; \; \frac{\partial^2 J}{\partial \ell nP_i \partial \ell nY} = \gamma_{iy} \; ; \; \frac{\partial J}{\partial \ell nY} = \gamma_{oy} \; ; \tag{13}$$

$$\frac{\partial^2 J}{\partial \ell nY \partial \ell nY} = \gamma_{yy} \quad \text{(in which all of the derivatives are}$$

understood to be evaluated at the fixed point $(\ell n\overline{P}_b,\ldots,\ell n\overline{Y}))$;

and expand about the point at which all the variables have a value of unity,[1] (i.e., $\ln \overline{P}_i = 0$, $\ln \overline{Y} = 0$), then, at the point in question, (12) can be written as :

$$\ln C = \gamma_{oo} + \sum_i \gamma_{oi} \ln P_i + \tfrac{1}{2} \sum_i \sum_j \gamma_{ij} \ln P_i \ln P_j$$

$$+ \sum_i \gamma_{iy} \ln P_i \ln Y + \gamma_{oy} \ln Y + \tfrac{1}{2}\gamma_{yy} (\ln Y)^2 \quad . \tag{14}$$

Note that the equality of the cross derivatives in (13) implies the symmetry condition

$$\gamma_{ij} = \gamma_{ji} \quad . \tag{15}$$

If we assume that all first and second order derivatives are approximately constant in the vicinity of the chosen point then (14) can be used as a regression equation to provide estimates of the parameters of the true (but unknown) cost function (11). Alternatively (14) may be taken as a functional form in its own right and assumed to represent accurately the cost function associated with the existing technology.

Whichever interpretation is used it is necessary to ensure that certain regularity conditions are satisfied if the function is to be considered "well behaved". These conditions are :

(i)   concavity in prices,

(ii)   monotonicity in output and prices, and

(iii)   homogeneity of degree one in prices.

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1. The data can always be scaled to include this point.

The translog function will not, in general, satisfy these conditions globally. However while global regularity may be desirable it is sufficient for applied work that the restrictions are not violated in the empirically relevant region of the price space. The usual practice is simply to check the regularity of the estimated function at each of the sample points.

(i) <u>Concavity.</u> The estimated function will be strictly concave if corresponding bordered Hessian matrix is negative definite.

(ii) <u>Monotonicity.</u> The necessary conditions for monotonicity of the cost function C in output and prices are :

$$\frac{\partial C}{\partial P_i} > 0 \quad , \tag{16a}$$

$$\frac{\partial C}{\partial Y} > 0 \quad . \tag{16b}$$

Since costs and prices will always be positive, (16a) can be written as

$$\frac{P_i}{C} \frac{\partial C}{\partial P_i} = \frac{\partial \ell nC}{\partial \ell nP_i} > 0 \quad .$$

Relating this to the specified function (14) we get;

$$\frac{\partial \ell nC}{\partial \ell nP_i} = \gamma_{oi} + \sum_j \gamma_{ij} \ell nP_j + \gamma_{iy} \ell nY > 0 \quad . \tag{17a}$$

This condition can be readily interpreted in terms of cost shares. Shephard (1965) has shown that if C is differentiable in P

then the factor demand functions will coincide with the partial

derivatives of C with respect to factor prices. That is,

$$\frac{\partial C}{\partial P_i} = X_i \quad,$$

where X_i is the demand for factor i .

The condition (16a) can therefore be written as

$$S_i > 0 \quad,$$

where S_i is the share of the ith factor in total costs.

Similarly, since costs and output will always be positive,
(16b) is equivalent to

$$\frac{Y}{C} \frac{\partial C}{\partial Y} = \frac{\partial \ell n C}{\partial \ell n Y} \quad,$$

$$= \sum_i \gamma_{iy} \ell n P_i + \gamma_{oy} + \gamma_{yy} \ell n Y > 0 \quad. \tag{17b}$$

(iii) <u>Homogeneity</u>. The requirement that the function be linearly

homogeneous in prices can be shown to imply the following restrictions

on the parameters[1] :

$$\sum_i \gamma_{oi} = 1 \tag{18a}$$

$$\sum_i \gamma_{ij} = 0 \tag{18b}$$

$$\sum_i \gamma_{ji} = 0 \tag{18c}$$

$$\sum_i \gamma_{yi} = 0 \tag{18d}$$

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1. See Appendix 1.

If the function C is assumed to be homothetic[1] then (14) can be written as :

$$\ell nC = \gamma_{00} + \sum_i^m \gamma_{0i} \ell nP_i + \tfrac{1}{2} \sum_i^m \sum_j^m \gamma_{ij} \ell nP_i P_j$$

$$+ \gamma_{oy} \ell nY + \gamma_{yy} (\ell nY)^2 \qquad . \qquad (19)$$

With the inclusion of an appropriately specified error term to account for errors in cost minimization by producers, equation (19) could be used for estimation. There is however some gain in estimating the share equations rather than (19). Since

$$S_i = \frac{P_i X_i}{C} = \frac{P_i}{C} \frac{\partial C}{\partial P_i} = \frac{\partial \ell nC}{\partial \ell nP_i} \quad ;$$

we have

$$S_b = \gamma_{01} + \gamma_{11} \ell nP_b + \gamma_{12} \ell nP_p + \gamma_{13} \ell nP_\ell + \gamma_{14} \ell nP_m + \mu_1 \; ; \quad (20a)$$

$$S_p = \gamma_{02} + \gamma_{21} \ell nP_b + \gamma_{22} \ell nP_p + \gamma_{23} \ell nP_\ell + \gamma_{24} \ell nP_m + \mu_2 \; ; \quad (20b)$$

$$S_\ell = \gamma_{03} + \gamma_{31} \ell nP_b + \gamma_{32} \ell nP_p + \gamma_{33} \ell nP_\ell + \gamma_{34} \ell nP_m + \mu_3 \; ; \quad (20c)$$

$$S_m = \gamma_{04} + \gamma_{41} \ell nP_b + \gamma_{42} \ell nP_p + \gamma_{43} \ell nP_\ell + \gamma_{44} \ell nP_m + \mu_4 \; . \quad (20d)$$

---

1. Since homotheticity means that factor shares (S_i) are independent of output (Y) it can be seen from (17a) that this implies $\gamma_{iy} = 0 \; \forall \, i$. The more restrictive assumption of homogeneity could be adopted by setting $\gamma_{yy} = 0$. Finally if $\gamma_{oy} = 1$ then linear homogeneity is imposed (refer Appendix 1).

where the μ_i are "classically well behaved" random distrubances which reflect the failures of producers to achieve their optimum position. The advantage in using (20) arises from the fact that the assumed homotheticity of the cost function[1] makes S_i independent of Y. Although this means that scale effects cannot be estimated it does allow estimation of the substitution possibilities to proceed without having to obtain observations on real output. Furthermore, the assumption that the distrubance terms are homoskedastic is less likely to be violated in (20) than in (19) since scale effects are excluded.

As shown above the regularity conditions for a "well behaved" cost function impose certain restrictions on the coefficients. In particular the conditions necessary for linear homogeneity in prices can be used to restrict the number of free parameters in the share equations (20).

These conditions can be expressed as in (18) or alternatively as[2]:

$$\sum_{i}^{m} \gamma_{oi} = 1 \quad , \tag{21a}$$

$$\sum_{i}^{m} \gamma_{ij} = 0 \qquad j = b,p,\ell,m \quad , \tag{21b}$$

$$\gamma_{ij} = \gamma_{ji} \qquad i,j = b,p,\ell,m \quad . \tag{21c}$$

---

1. Technological change could be handled in an analogous manner. If some (perhaps unobservable) variable Z which causes technology to change at a constant logarithmic rate is included in the cost function (14) then the coefficient γ_{oz} would measure the rate of technical change while γ_{zi} would measure the rate of bias. If we are prepared to assume that technical change is unbiased ($\gamma_{zi} = 0$) then the share equations (20) can be used to estimate the other parameters.

2. Note that zero row and column totals (18b and 18c) imply symmetry (21c). Conversely zero column totals (21b) and symmetry (21c) imply zero row totals. Note also that the condition (18d) is now satisfied by assumption since the assumption of homotheticity made above sets all γ_{iy} equal to zero.

Since we require the various cost shares to add to unity
it is necessary to drop one equation in order to overcome the problem
of singular contemporaneous covariance matrices of disturbances.
As the estimates will be invariant to the equation the following
application will use the plant, labour and materials share equations
(20b, c and d).

While the dropping of one equation ensures that the "adding
up" conditions (21a and b) will be satisfied, it is also necessary to
ensure that the estimating procedure enforces the across equation
symmetry conditions (21c). These restrictions reduce the number of
free parameters to 9. By substituting from (21b) into (21c) these
restrictions can be expressed in terms of the parameters appearing in
the equations to be estimated.

TABLE 3

SYMMETRY/HOMOGENEITY RESTRICTIONS

$$\gamma_{21} = - (\gamma_{22} + \gamma_{23} + \gamma_{24})$$

$$\gamma_{31} = - (\gamma_{32} + \gamma_{33} + \gamma_{34})$$

$$\gamma_{41} = - (\gamma_{42} + \gamma_{43} + \gamma_{44})$$

$$\gamma_{32} = \gamma_{23}$$

$$\gamma_{43} = \gamma_{34}$$

$$\gamma_{42} = \gamma_{24}$$

Free Parameters $\gamma_{02}, \gamma_{03}, \gamma_{04}, \gamma_{22}, \gamma_{23}, \gamma_{33}, \gamma_{34}, \gamma_{24}, \gamma_{44}$.

6. SUBSTITUTION ELASTICITIES IN THE TRANSLOG FUNCTION

Uzawa (1962) proved that the AES could be calculated from the cost function as[1] :

$$\sigma_{ij} = \frac{C}{C_i C_j} C_{ij} \quad , \tag{22}$$

where $C_i = \frac{\partial C}{\partial P_i}$, etc., as before.

Binswanger (1974) has shown that this reduces to a relatively simple relationship between cost shares and parameters when the cost function is of the translog form. In this case we have :

$$\frac{\partial^2 \ell nC}{\partial \ell nP_i \, \partial \ell nP_j} = \gamma_{ij} \tag{23}$$

and

$$\frac{\partial^2 \ell nC}{\partial \ell nP_i \, \partial \ell nP_j} = P_j \frac{\partial (C_i P_i / C)}{\partial P_j} \quad ,$$

$$= P_j \left(C_{ij} \frac{P_i}{C} - C_j C_i \frac{P_i}{C^2} + \frac{C_i}{C} \frac{\partial P_i}{\partial P_j} \right) \tag{24}$$

When $i \neq j$ then $\frac{\partial P_i}{\partial P_j} = 0$. In this case using Shephard's Lemma (i.e., $C_i = X_i$) and combining (23) and (24) we get :

$$\gamma_{ij} = \frac{P_i P_j}{C} C_{ij} - \frac{P_i P_j}{C^2} X_i X_j \quad ,$$

or

$$C_{ij} = \frac{C}{P_i P_j} \left(\gamma_{ij} + \frac{P_i X_i}{C} \frac{P_j X_j}{C} \right) \tag{25}$$

---

1. This result was originally proved for a homogeneous production function. Binswanger (1974) has provided a proof which does not rely on homogeneity.

Substituting (25) into (22) gives :

$$\sigma_{ij} = \left[\frac{C}{C_i C_j} \right] \frac{C}{P_i P_j} \left(\gamma_{ij} + \frac{P_i X_i}{C} \frac{P_j X_j}{C} \right) ,$$

$$= \frac{\gamma_{ij}}{S_i S_j} + 1 \qquad\qquad i \neq j . \qquad\qquad (26a)$$

In the case of $i=j$ we have $\frac{\partial P_i}{\partial P_i} = 1$ in equation (24)
which, when similar manipulations to those outlined above are used, gives ;

$$\sigma_{ii} = \frac{\gamma_{ii}}{S_i^2} - S_i + 1 . \qquad\qquad (26b)$$

Once the parameters γ_{ij} have been estimated from the share
equations the substitution elasticities between the various inputs can
be readily calculated from (26). As this a linear transformation of
the $\hat\gamma_{ij}$ the econometric properties of these estimates will carry over
to the $\hat\sigma_{ij}$.

When more than two factors are involved Allen termed X_i
and X_j to be substitutes or complements depending on whether σ_{ij}
is positive or negative.

In addition the factor demand elasticities will also be
calculated. From (22) we have

$$\sigma_{ij} = \frac{C}{X_i X_j} \frac{\partial X_i}{\partial P_j} , \qquad\qquad (27)$$

since $C_k = \frac{\partial C}{\partial P_k} = X_k$ and $C_{ij} = \frac{\partial \left(\frac{\partial C}{\partial P_i} \right)}{\partial P_i} = \frac{\partial X_i}{\partial P_j}$.

Multiplying both sides of (27) by $\dfrac{P_j}{X_i}$, and rearranging, gives;

$$\sigma_{ij} = n_{ij}/S_j \quad ,$$

OR (using (26a) and (26b))

$$n_{ij} = \frac{\gamma_{ij}}{S_i} + S_j \qquad i \neq j \quad , \qquad \qquad \text{(28a)}$$

and

$$n_{ii} = \frac{\gamma_{ii}}{S_i} + S_i - 1 \quad . \qquad \qquad \text{(28b)}$$

7. ESTIMATING AND TESTING PROCEDURE

The two equations from which the cost function parameters will be estimated are the Equipment, Labour and Materials share equations (20b), (20c) and (20d). The estimation will be carried out using the Wymer RESIMUL Package.[1] This program calculates full information maximum likelihood estimates for a system of linear equations in which there may be non-linear restrictions on the coefficients within or across equations. The ability of this program to compute the value and asymtotic standard error of any combination of the estimated parameters is exploited in the hypothesis testing.

The estimating and testing will be carried out in two stages. The initial tests will be for the symmetry/homogeneity of the function[2]

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1. This program has been made available to the IMPACT Project by Dr C.R. Wymer, I.M.F. (formerly of the London School of Economics).

2. Strictly speaking these are only partial tests. In the data construction the cost share have been assumed to sum to unity - this implies linear homogeneity.

(refer Table 3). This characteristic will be tested in two ways. Firstly the model will be estimated without any restrictions being imposed on the coefficients. Linear combinations of the estimates so obtained will be used to test the restrictions given in Table 3 both individually and collectively, i.e., the following hypotheses will be tested :

TABLE 4

HYPOTHESES TO BE TESTED FOR SYMMETRY/HOMOGENEITY

$$\beta_1 = \hat{\gamma}_{21} + \hat{\gamma}_{22} + \hat{\gamma}_{23} + \hat{\gamma}_{24} = 0 \quad ,$$

$$\beta_2 = \hat{\gamma}_{31} + \hat{\gamma}_{32} + \hat{\gamma}_{33} + \hat{\gamma}_{34} = 0 \quad ,$$

$$\beta_3 = \hat{\gamma}_{41} + \hat{\gamma}_{42} + \hat{\gamma}_{43} + \hat{\gamma}_{44} = 0 \quad ,$$

$$\beta_4 = \hat{\gamma}_{23} - \hat{\gamma}_{32} = 0 \quad ,$$

$$\beta_5 = \hat{\gamma}_{43} - \hat{\gamma}_{34} = 0 \quad ,$$

$$\beta_6 = \hat{\gamma}_{42} - \hat{\gamma}_{24} = 0 \quad ,$$

$$\beta_1 = \beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_5 + \beta_6 = 0 \quad .$$

An alternative method of testing these restrictions is to use a likelihood ratio test. This test is carried out by confining the parameters to a subset of the total parameter space (in this case the subset that satisfies the symmetry/homogeneity conditions) and comparing the resultant maximum value of the likelihood function with the value that is achieved when no restrictions are imposed. Whether or not the reduction that occurs indicates that the restrictions are "over-riding" the information being yielded by the data to any significant extent can be tested using the statistic :

$$L = -2 \log \left\{ \frac{\text{max. value of likelihood fn. with constraints}}{\text{max. value of likelihood fn. without constraints}} \right\}$$

It can be shown that L is distributed as Chi-Squared with the number of degrees of freedom equal to the number of restrictions.[1]

If the above tests indicate that the data is consistent with a cost function that is linearly homogenous in prices, the model will be re-estimated with the restrictions of Table 3 imposed as part of the maintained hypothesis. The other regularity conditions will then be checked by seeing if all factor shares are positive (monotonicity) and whether or not the bordered Hessian matrix is negative definite at each data point (concavity).

Tests for the types of separability discussed in Section 3 above will be made next. Berndt and Christensen (1973) suggested that separability of a translog function can be tested through the imposition of parameter restrictions on the function. Blackorby <u>et al</u>.

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1. See Theil (1971).

(1977) have however shown that the "separability inflexibility" of the translog (and other) functional forms makes these tests considerably less general than originally thought. The argument is that although the function (14) has sufficient parameters to provide a second order approximation to any function, when restrictions of the type suggested by Berndt and Christensen are imposed, the model loses its flexibility and can only approximate a limited range of separable functions. In fact testing for weak separability using parameter restrictions on the translog function is "equivalent to testing for a hybrid of strong (additive) separability and homothetic weak separability (in fact, with Cobb-Douglas aggregator functions)". (Blackorby et al., 1977, P. 196).

To overcome these problems the separability tests in this paper will be carried out using the various AES calculated from the parameters estimated above.[1] The equality restrictions given in Tables 1 and 2 will be tested in the sequence shown in Diagram 1.

Should either form of separability be accepted as consistent with the data, other functional forms, which incorporate the observed form of separability in their structure, will be tested. The Sato (1967) two level CES function requires all inter-group AES to be equal and also the various intra-group AES to be equal. Uzawa (1962) restricts the various groups to interact in a Cobb-Douglas fashion, i.e., the

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1. The values will be calculated using the average factor shares for the period.

SEQUENCE OF TESTING

$$Y = G(B,P,L,M)$$

(No restrictions on AES)

Test for
Symmetry/Homogeneity.

NO   YES

Search for new

class of models

$$Y = G^*(H(B,P,L),M)$$

$$\sigma_{BM} = \sigma_{PM} = \sigma_{LM}$$

YES   NO

$$Y = G^*(H^*(F(B,P),L),M)$$

$$\sigma_{BL} = \sigma_{PL}$$

$$Y = G'(F(B,P),L,M)$$

$$\sigma_{BL} = \sigma_{PL}$$

$$\sigma_{BM} = \sigma_{PM}$$

YES   NO   YES   NO

Consistent indices
exist for both value
added and capital

A consistent
value added
index exists

A consistent
capital index
exists

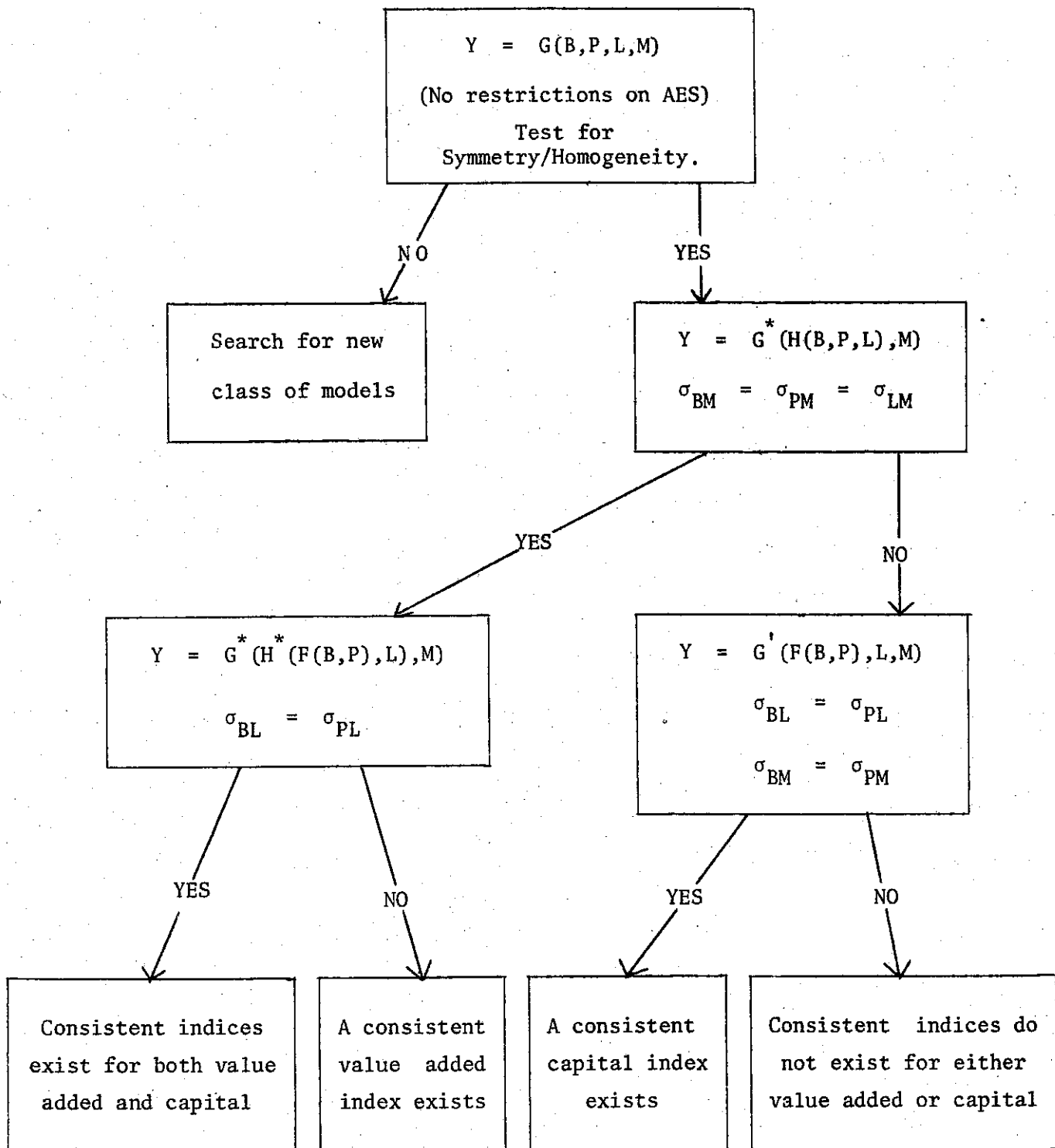Consistent indices do
not exist for either
value added or capital

Diagram 1

inter-group AES are unity. In addition to strong separability, the CES function of Arrow et al. (1961) has each factor forming its own composite. In this case we have all AES being equal. Finally, the Cobb-Douglas function not only assumes strong separability with each factor forming its own composite, but it also requires that all the AES be unity.

8. RESULTS

The unconstrained parameter estimates are reported in column 1 of Table 5, while column 2 contains the estimates obtained when the symmetry/homogeneity restrictions are imposed. As the γ_{ij}'s have little economic meaning in their own right it is not possible to comment on their magnitude or sign. The failure of some of the estimates to differ significantly from zero is of little consequence as this simply implies an AES of one.

Imposing symmetry and homogeneity on the function during estimation resulted in significant changes in some of the parameters. There were however, no cases in which sign reversals occurred.

The values of the statistics, calculated from these unconstrained parameter estimates, that are used to test the symmetry/ homogeneity of the cost function (refer Table 4) are given in Table 6. Of the seven restrictions tested only one could be rejected as not being satisfied at the 95 per cent confidence level.

TABLE 5

TRANSLOG PARAMETER ESTIMATES

(Asymptotic t-values in parentheses)

Parameter	(1) Unconstrained	(2) Constrained
$\hat{\gamma}_{21}$	0.039 (4.40)	0.001* (0.84)
$\hat{\gamma}_{22}$	-0.037 (3.73)	-0.033 (2.60)
$\hat{\gamma}_{23}$	-0.043* (1.66)	-0.002* (0.15)
$\hat{\gamma}_{24}$	0.034* (0.89)	0.030 (4.68)
$\hat{\gamma}_{31}$	-0.028 (2.70)	-0.004 (2.06)
$\hat{\gamma}_{32}$	-0.011* (0.93)	0.002* (0.15)
$\hat{\gamma}_{33}$	0.112 (3.78)	0.027 (2.07)
$\hat{\gamma}_{34}$	-0.097 (2.22)	-0.026 (3.60)
$\hat{\gamma}_{41}$	-0.019 (2.25)	-0.008 (8.49)
$\hat{\gamma}_{42}$	0.047 (5.18)	0.030 (4.68)
$\hat{\gamma}_{43}$	-0.073 (3.05)	-0.026 (3.60)
$\hat{\gamma}_{44}$	0.074 (2.12)	0.004* (0.68)
Implicit Estimates		
$\hat{\gamma}_{11}$	0.008	-0.011
$\hat{\gamma}_{12}$	0.001	-0.001
$\hat{\gamma}_{13}$	0.004	-0.004
$\hat{\gamma}_{14}$	-0.011	-0.008
log-likelihood Value	37.23	36.20

* Insignificantly different from zero at 95 per cent level.

TABLE 6

TEST STATISTICS (refer Table 4)
(Asymptotic t-values in parentheses)

β_1 = -0.006 (0.041)		β_5 = -0.024 (0.67)
β_2 = -0.023 (1.29)		β_6 = 0.013 (0.33)
β_3 = 0.030* (2.08)		β_7 = -0.043 (0.46)
β_4 = -0.032 (1.35)		

* Hypothesis that β_3 = 0 rejected at 95 per cent level

The other test used to check the validity of these regularity conditions (the likelihood ratio test) produced a negative result. The value of this likelihood ratio is 24.66 while the critical Chi-squared value at the 95 per cent significance level (with 6 degrees of freedom) is 12.59. This rejection of the hypothesis that the data was generated in accordance with a cost function that is linearly homogeneous in prices although disturbing, is not surprising. A perusal of the large number of papers that have appeared over the last four or five years which use likelihood ratio tests on the economic theoretic restrictions of demand and production systems, indicate an extremely high rejection rate of the restrictions. Although no consensus has emerged as to the reason for this rejection, it is generally felt that there is sufficient uncertainty about the power of these tests and their implications to warrant an agnostic attitude being taken towards the result. At this stage it would appear to be rather too extreme to suggest that the failure of such tests

provides conclusive evidence of the empirical relevance of the production (or demand) theory on which the models are based.

The model specification was further tested by computing the monotonicity and convexity conditions using the (constrained) parameter estimates. The estimated cost shares calculated from the fitted equations all proved to be strictly positive as required. Furthermore, the conditions necessary for strict convexity of the isoquants were found to be satisfied for each period. It was therefore concluded that the estimated cost function was "well behaved" in the region of the observations.

We next test for separability of the function in the manner shown in Diagram 1. The estimated Allen Elasticities of Substitution and the factor demand elasticities, calculated using average factor shares, are reported in Table 7 together with the linear combinations of those parameters necessary to test for separability.

TABLE 7

ESTIMATES OF ALLEN ELASTICITIES OF SUBSTITUTION

(Asymptotic std. errors in parentheses)

$\hat{\sigma}_{BP} = 1.085$ (0.103)	$\hat{\sigma}_{PM} = 1.471$ (0.101)	$\hat{\sigma}_{BM} - \hat{\sigma}_{LM} = -0.794$ (0.088)
$\hat{\sigma}_{BL} = -0.009$ (0.493)	$\hat{\sigma}_{LM} = 0.841$ (0.044)	$\hat{\sigma}_{PM} - \hat{\sigma}_{LM} = 0.630$ (0.133)
$\hat{\sigma}_{BM} = 0.047$ (0.112)	$\hat{\sigma}_{BL} - \hat{\sigma}_{PL} = -1.076$ (0.861)	
$\hat{\sigma}_{PL} = 1.066$ (0.442)	$\hat{\sigma}_{BM} - \hat{\sigma}_{PM} = -1.424$ (0.186)	

TABLE 8

ESTIMATES OF PRICE ELASTICITIES $(\hat{n}_{ij})$

(Asymptotic std. errors in parentheses)

i \ j	B	P	L	M
B	-0.213 (0.042)	-2.317 (0.930)	-0.002 (0.128)	0.030 (0.070)
P	0.025 (0.014)	-0.776 (0.125)	0.277 (0.115)	0.922 (0.063)
L	-0.000 (0.007)	0.107 (0.044)	-0.634 (0.051)	0.527 (0.027)
M	0.001 (0.002)	0.147 (0.010)	0.218 (0.011)	-0.366 (0.010)

These results indicate that there may be a measure of complementary between "buildings and structures" and "labour". However, the relatively large t-value on the AES does not allow any firm conclusions to be drawn. All other factors appear to substitute for each other with the substitution possibilities being greatest between "materials" and "plant and equipment" and least between "buildings and structures" and "materials". Although not reported here the AES were computed for various years between 1948-9 and 1972-3. These did not indicate that there was any consistent movement in these parameters over the period. Finally the equality between the various AES, necessary for the types of separability discussed earlier to exist, does not appear to exist.

9. CONCLUSION

A translog production function has been used to quantify the substitution possibilities among buildings, plant, labour and materials inputs. The advantage of using the translog function for this purpose is that it makes no a priori assumptions about the separability of the various factors and consequently places no restrictions on the AES. The function proved to be "well behaved" in the empirically relevant region and yielded coefficient estimates which were, in general, significantly different from zero.

The significant differences that existed in the absolute magnitudes of the various AES indicated that the functional separability necessary for the existence of consistent value added and/or capital indices did not exist. It must therefore be concluded that studies which are based on this general class of models and which assume a priori that such indices exist contain a mis-specification.

APPENDIX 1

THE TRANSLOG FUNCTION

$$\ell n \; C(P,Y) = \gamma_{oo} + \sum_i \gamma_{oi} \, \ell n P_i + \tfrac{1}{2} \sum_i \sum_j \gamma_{ij} \, \ell n P_i \, \ell n P_j + \sum_i \gamma_{yi} \, \ell n Y \, \ell n P_i$$

$$+ \; \gamma_{oy} \, \ell n Y + \tfrac{1}{2} \gamma_{yy} \, \ell n Y \, \ell n Y \qquad\qquad (1.1)$$

$$\gamma_{ij} = \gamma_{ji}$$

(i)   Homogeneity of degree one in prices

In general this means :

$$C(\lambda P_1, \lambda P_2, \ldots, \lambda P_n, Y) = \lambda \; C(P_1, P_2, \ldots, P_n, Y)$$

OR

$$\ell n \; \{C(\lambda P_1, \lambda P_2, \ldots, \lambda P_n, Y)\} = \ell n \lambda + \ell n C(P_1, P_2, \ldots, P_n, Y)$$

In terms of the translog function :

$$C(\lambda P_1, \lambda P_2, \ldots, \lambda P_n, Y)$$

$$= \gamma_{oo} + \sum_i \gamma_{oi} \ell n (\lambda P_i) + \tfrac{1}{2} \sum_i \sum_j \gamma_{ij} \ell n (\lambda P_i) \, \ell n (\lambda P_j)$$

$$+ \sum_i \gamma_{yi} \ell n Y \, \ell n (\lambda P_i) + \gamma_{oy} \, \ell n Y + \tfrac{1}{2} \gamma_{yy} \, \ell n Y \, \ell n Y \quad .$$

$$= \gamma_{oo} + \sum_i \gamma_{oi} \ln\lambda + \sum_i \gamma_{oi} \ln P_i + \frac{1}{2} \sum_i \sum_j \gamma_{ij} \ln P_i \ln P_j$$

$$+ \frac{1}{2} \sum_i \sum_j \gamma_{ij} (\ln \lambda)^2 + \frac{1}{2} \sum_i \sum_j \gamma_{ij} \ln P_i \ln\lambda$$

$$+ \frac{1}{2} \sum_i \sum_j \gamma_{ij} \ln P_j \ln\lambda + \sum \gamma_{yi} \ln Y \ln\lambda + \sum \gamma_{yi} \ln P_i \ln Y$$

$$+ \gamma_{oy} \ln Y + \frac{1}{2}\gamma_{yy} \ln Y \ln Y \quad ; \tag{1.2}$$

AND

$$\ln\lambda + \ln\{C(P_1,P_2,\ldots,P_n,Y)\} = \ln\lambda + \gamma_{oo} + \sum_i \gamma_{oi} \ln P_i + \frac{1}{2} \sum_i \sum_j \gamma_{ij} \ln P_i \ln P_j$$

$$+ \sum_i \gamma_{yi} \ln P_i \ln Y + \gamma_{oy} \ln Y + \frac{1}{2}\gamma_{yy} \ln Y \ln Y \quad . \tag{1.3}$$

Equating coefficients in (1.2) and (1.3) we get the following parameter restrictions :

$$\sum_i \gamma_{oi} = 1 \tag{1.4a}$$

$$\sum_i \gamma_{ij} = 0 \tag{1.4b}$$

$$\sum_i \gamma_{ij} = 0 \tag{1.4c}$$

$$\sum_i \gamma_{yi} = 0 \tag{1.4d}$$

(ii) Homogeneity of degree one in output

This means :

$$\ln\{C(P_1,P_2,\ldots,P_n,\lambda Y)\} = \ln\lambda + C(P_1,P_2,\ldots,P_n,Y)$$

Using the translog function we have :

$$\ln\{C(P_1,P_2,\ldots,P_n,\lambda Y)\} = \gamma_{oo} + \sum_i \gamma_{oi} \ln P_i + \tfrac{1}{2} \sum_i \sum_j \gamma_{ij} \ln P_i \ln P_j$$

$$+ \sum_i \gamma_{yi} \ln Y \ln P_i + \sum_i \gamma_{yi} \ln\lambda \ln P_i + \gamma_{oy} \ln Y$$

$$+ \gamma_{oy} \ln\lambda + \tfrac{1}{2}\gamma_{yy} \ln Y \ln Y + 2 \gamma_{yy} \ln Y \ln\lambda$$

$$+ \tfrac{1}{2}\gamma_{yy} (\ln\lambda)^2 \quad ; \tag{1.5}$$

AND

$$\ln\lambda + \ln\{C(P_1,P_2,\ldots,P_n,Y)\} = \ln\lambda + \gamma_{oo} + \sum_i \gamma_{oi} \ln P_i + \tfrac{1}{2} \sum_i \sum_j \gamma_{ij} \ln P_i \ln P_j$$

$$+ \sum_i \gamma_{yi} \ln P_i \ln Y + \gamma_{oy} \ln Y + \tfrac{1}{2}\gamma_{yy} \ln Y \ln Y \tag{1.6}$$

Again equating coefficients we get :

$$\gamma_{oy} = 1 \tag{1.7a}$$

$$\gamma_{yi} = 0 \qquad i = 1,\ldots,n \quad . \tag{1.7b}$$

$$\gamma_{yy} = 0 \tag{1.7c}$$

N.B.   If the restriction (1.7a) is not imposed then the function will be homogeneous of degree γ_{oy} in output.   If in addition (1.7c) is relaxed the function will be homothetic but not homogeneous.   If none of the restrictions (1.7) are imposed the function is non-homothetic.

APPENDIX 2

The data used covered the period 1948/9 to 1972/3 for

Australian manufacturing industries as defined in the Australian Bureau

of Statistics publication Manufacturing Establishments : Details of

Operations.[1]

(a) Capital Inputs

The level of capital inputs is assumed to be proportional

to the stocks existing in each year. Capital stock series for

"Buildings and Structures" and "Plant and Equipment" have been developed

using a perpetual inventory procedure. The stock estimates of Edwards

and Drane (1963) for 1947/8, adjusted to exclude "Heat, Light and Power",

have been used as the base year values.[2] These values have been

incremented using the gross investment figures given in Manufacturing

Establishments, The Australian National Accounts deflators for

"Other Building and Construction" and "All Other" were used to give

constant value estimates for the years after 1958/9. For earlier years

it has been necessary to supplement these with the National Accounts

"Dwellings" deflator and the Plant and Equipment price index constructed

by Edwards and Drane (1963).

The depreciation rates used are those calculated from the

depreciation allowances and book value of capital stock given in

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1. Some earlier issues of this publication carried different titles.

2. The book values for 1947/8 given in Manufacturing Establishments
    were used to determine the proportion of total stock in each category
    that is attributable to the Head, Light and Power group.

Manufacturing Industry Bulletins. These rates ranged between 9 per cent and 11 per cent for Plant and Equipment and between 1.5 per cent and 1.8 per cent for Buildings and Structures over the period.

(b) Price of Capital Inputs

In constructing a price series for capital inputs it is necessary to recognise that it is the cost to the user rather than the acquisition cost of new capital that is of relevance. In this paper the service price (P_t) is calculated as :

$$P_t = q_t (r_t + \delta_t - g_t) + FM_t/X_t \quad ,$$

where X_t is the quantity of the capital good in use, q_t is the acquisition price of the asset, r_t is the rate of interest payable on borrowed funds, δ_t is the annual depreciation rate, g_t measures the rate of capital gains or losses during the period and FM_t represents the maintenance and running costs.

The price indices and the depreciation rates used in the construction of the capital stock series above have been used for q and δ in the construction of P. The interest rate (r) that has been used is the price bank overdraft rate.

While the above components of cost could be reasonably assumed to be known with some certainty at the time the production decision is made the same could not be claimed for the level of capital gains (g). It is the anticipated rather than realized gain that is of consequence in attempting to establish the price relevant to the choice of production techniques. For this exercise it is assumed that the

anticipated capital gain was constant over the period and equal to the average annual percentage rate of growth of the price indices over the period of observation.

The maintenance and running costs for Plant and Equipment are taken from Manufacturing Establishments. As equivalent figures are not available for Buildings and Structures it has been necessary to make an arbitrary allowance of 1 per cent of the current value of stock for this component.

(c) Labour Input and Price

The labour input measure used is the average number of people employed in manufacturing in each year as given in Manufacturing Establishments. Total wages and salaries paid (from the same publication) have been divided by the employment figure to give the unit cost of labour.

(d) Intermediate Inputs and their Price

The value of "Materials, Containers and Packages" used in production (as given in Manufacturing Establishments) has been used to measure this input. The National Accounts "Non-Farm Product" deflator has been used as an indicator of the price of intermediate inputs for years after 1958/9. For earlier years the general G.N.P. deflator has been used.

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