# Exploring the intensive and extensive margin of employment in a CGE framework

Christopher King

Centre of Policy Studies

Victoria University

Thesis submitted in fulfilment of the requirements for the degree of Doctorate of Philosophy

November 30, 2020

## 0.1 Abstract

This research incorporates a model of labour demand into a computable general equilibrium model for Australia. It extends the underlying model by disaggregating the price of labour into (1) ordinary wage costs, (2) payments for overtime hours and (3) fixed costs of labour. It also distinguishes the labour input into hours worked per worker and the number of workers.

This research developed a model of labour demand where firms chose between increasing the number of hours worked per worker and paying overtime wage premiums which increase with overtime hours or expanding the size of their workforce and paying fixed costs for each additional worker. A labour services function was adopted to represent the aggregate quantity of labour so that an additional hour per worker had a differing marginal product than an additional worker.

A two-stage least squares model of wage premiums and overtime was estimated to represent Australia's industry-specific overtime regulations based on the set of modern awards and collective bargaining agreements. This provided a relationship between the average wage premium per hour of overtime and the number of overtime hours performed. This was used to calibrate the labour demand model integrated into the CGE framework.

The Australian government has a compulsory retirement savings scheme, the superannuation guarantee, which requires employers to set aside a portion of an employees wage income to provide income support in retirement. The effects of increasing the superannuation guarantee from 9.5% to 12% of employees ordinary wage earnings is simulated using the labour demand model developed within this research. It demonstrates how industries respond to the increase in the superannuation guarantee by substituting away from workers to additional hours per worker. The main findings are that industries with (a) higher fixed costs, (b) higher levels of ordinary hours and (c) flatter wage premium schedules comparatively experience larger increases in overtime.

## 0.2 Doctor of Philosophy Student Declaration

I, Christopher Leigh King, declare that the PhD thesis entitled Exploring the intensive and extensive margin of employment is no more than 100,000 words in length including quotes and exclusive of tables, figures, appendices, bibliography, references and footnotes. This thesis contains no material that has been submitted previously, in whole or in part, for the award of any other academic degree or diploma. Except where otherwise indicated, this thesis is my own work.

Signature Date

## 0.3 Acknowledgements

I would like to thank both of my supervisors Janine Dixon and Philip Adams for their ongoing support over the previous 4 years. I am incredible lucky to have such caring and enthusiastic supervisors. I cannot begin to express my gratitude for all your effort, especially your willingness to read and re-read my many drafts. I am going to miss our weekly Thursday meetings where we discussed my work, among other things. It has been a pleasure to get to know you both over this time.

I would like to thank the entire CoPS family. It has been a pleasure to be surrounded by such an enormously talented and friendly group of people. You have all made me feel very welcome and been of great assistance whenever I have required it.

To my loving partner, Steph. I am incredibly appreciative of the fact that you packed up your life and moved to Melbourne so we could be together so I could finish my PhD.

I would like to thank my parents and brother. You wear my academic achievements as a badge of pride. I have had the ambition to complete this PhD since I was in the first year of university. You have supported me along the entire process, even when it meant sending your "baby" overseas to Canada to complete my Masters. None of this would have been possible without your unwavering support.

# Contents

## 0.4 List of Abbreviations

| Abbreviations | Description |
|---|---|
| ABS | Australian Bureau of Statistics |
| BOT | Balance of Trade |
| CBA | Collective bargaining agreement |
| CGE | Computable General Equilibrium |
| EEH | Employment Earnings and Hours Survey |
| EBA | Enterprise Bargaining Agreement |
| IO | Input Output |
| LHS | Left hand side |
| MA | Modern Award |
| RHS | Right hand side |
| SG | Superannuation guarantee |

## 0.5 List of symbols

The following table provides a list of symbols used within this research.

| Variable | Chapter used | Description |
|---|---|---|
| $(\eta_{WP,OT} - 1)$ | 3,4,5,6 | elasticity of wage premium with respect to overtime |
| A | 2,3,5,6 | Technology parameter |
| a | 2 | unit production requirement |
| A | 2 | Matrix of coefficients |
| AR | 4 | Absent rate |

| Variable | Chapter used | Description |
|---|---|---|
| $\alpha$ | 2 | production function parameter |
| $\alpha$ | 3,4,5 | Returns to hours in labour services function |
| $\alpha_i$ | 4 | regression coefficient |
| alphot | 5,6 | Returns to workers parameter in simulation |
| B | 3 | Vector of exogenous variables |
| $\beta$ | 2 | preference parameter |
| $\beta$ | 3,4,5 | Returns to workers |
| C | 2,3 | Cost of production |
| $d$ | 2 | Depreciation |
| $\epsilon_F$ | 2 | Frisch parameter |
| $\epsilon_M^{HH}$ | 2 | Household expenditure elasticity |
| $\eta_{L0,L}$ | 2 | elasticity of ordinary hours with respect to total hours |
| $\eta_{L0,N}$ | 2 | elasticity of workers with respect to ordinary hours |
| F | 2 | Shift parameter |
| fix | 4 | fixity of labour |
| FC | 3,4,5,6 | Fixed costs |
| H | 3,4,5,6 | Effective hours |
| HR | 5 | Hiring costs |
| I | 2 | Investment |
| K | 2,3 | Capital |
| $\kappa$ | 3,4 | ratio of fixed costs to ordinary |
| L | 3,4,5,6 | Hours worked per worker |

| Variable | Chapter used | Description |
|---|---|---|
| $L_0$ | 3,4,5,6 | Ordinary hours |
| $\mathcal{L}$ | 2,3 | Lagrangian |
| $\lambda$ | 2,3,4,5 | Lagrangian multiplier |
| LC | 3,5,6 | linear costs |
| M | 2 | Expenditure |
| $M_L$ | 2 | Luxury expenditure |
| M | 3,4,5,6 | Person hours |
| $M_S$ | 3 | Subsistence expenditure |
| MC | 4 | Marginal cost |
| MR | 4 | marginal revenue |
| N | 3,4,5,6 | Number of workers |
| $O$ | 2 | big O notation |
| OC | 2 | Other costs |
| $\Omega$ | 2 | Equilibrium rate of return |
| $O_T$ | 3,4,5,6 | Overtime hours per worker |
| P | 2,3 | price |
| $\pi$ | 2,3 | profit |
| prem | 4 | government mandated wage premium |
| $\psi$ | 2 | investment parameter |
| Q | 2,3,4 | quantity |
| R | 2 | rate of return |
| $\rho$ | 2 | CES parameter |

| Variable | Chapter used | Description |
|---|---|---|
| S | 2,5,6 | Share parameter |
| S | 6 | super component of fixed costs |
| S | 4 | ratio of senior to new employees |
| $\sigma$ | 2 | elasticity of substitution |
| T | 2 | Tariff |
| TC | 3 | Labour costs |
| TR | 4 | Training costs |
| U | 2 | Utility |
| CBA | 4 | CBA |
| $\mu_0$ | 3,4 | wage premium parameter |
| $\mu_1$ | 3,4 | wage premium parameter |
| V | 2,4 | Value |
| VA | 2 | Value added |
| VC | 5,6 | variable costs |
| $W_0$ | 4 | ordinary wage rate |
| $W_1$ | 4 | average wage of ordinary and overtime hours |
| $W_P$ | 4 | Wage premium |
| X | 2 | Demand |
| X | 2 | vector of exogenous variables |
| X | 2 | Luxury consumption |
| Y | 2 | Final Demand |
| Y | 2 | Household demand |

| Variable | Chapter used | Description |
|---|---|---|
| y | 2 | vector of endogenous variables |
| Y | 2 | Subsistence consumption |
| Z | 2 | Production possibilities frontier |
| Z | 3 | Fixed costs |

# List of Figures

# List of Tables

# 1 Introduction

The effectiveness of labour-market policies aimed at increasing worker compensation often depend on the associated negative employment effects. For example, the debate about minimum wage policy often focuses on the corresponding decreases in employment for low-skilled workers. What is often missing from this analysis is how such a policy impacts the composition of labour. The composition effects of a policy often determines the effectiveness of that policy. From an inequality perspective, a 10% increase in the minimum wage which causes the hours worked per worker to decrease by 5% is a better policy outcome than a 10% minimum wage increase which causes the number of workers to decrease by 5%. In the first case, the increase in wages offsets the decrease in hours worked per worker. However, in the second case, a subset of the workforce enjoys an increase in its income whilst a subset is now unemployed.

This research incorporates the intensive and extensive margins of employment into a Computable General Equilibrium (CGE) model of the Australian economy. The intensive margin is defined as the hours worked per worker and the extensive margin is defined as the number of workers. The objective of this research is to build upon a pre-existing CGE model to provide a comprehensive analysis of how the intensive and extensive margin of employment responds to policy shocks to the cost of labour.

A worker-hours labour demand model is adopted to understand what determines the level of intensive and extensive margin of employment. In this framework, there are costs associated with increasing the intensive and extensive margin of employment. Furthermore, the labour productivity of an additional hour of work from increasing hours worked per worker may vary from the labour productivity of an additional hour of work from increasing the number of workers working.

Australia specifies the maximum daily and weekly hours of employment for full-time and part-time workers. Any hours of work performed in excess of these specified quantities are considered overtime and require a wage premium to be paid.[1] The variations in intensive margin employment considered in this research are these changes to overtime hours. The corresponding overtime premiums constitute the costs of expanding intensive margin employment.

The costs associated with expanding extensive margin employment are called quasi-fixed costs.[2] These costs are determined by the number of workers employed, and are invariant to the number of hours worked by each employee. For example, the cost associated with occupational health and safety training is an example of a quasi-fixed cost. It usually requires the same amount of time to complete for each worker and is required for all workers, regardless of hours worked.

These competing costs for a firm are what determine the composition of labour employed. Industries which have higher levels of fixed costs tend to employ workers for longer hours whereas industries with higher wage premiums will use more workers. Whilst it is often assumed that overtime is used to address short-run fluctuations in demand, this research illustrates how overtime may be used in the absence of these demand fluctuations. In doing so, it will demonstrate why some industries persistently use overtime more frequently than other industries, as is presented in Figure 1.1.

In addition to different costs for intensive and extensive margin employment, the marginal productivity of an additional hour and worker often vary. For example, 10 workers working 40 hours might yield different output than 20 workers working 20 hours, despite both scenarios having the same total hours. This setup allows for workers to experience fatigue and become less productive the more hours they work or for workers to experience a 'warm up effect' whereby they become more productive the longer they work.

Traditional CGE models offer a very simplistic treatment of labour demand. Usually, labour consists of a single quantity and the composition of labour, i.e 20 person hours comprised of 4

[1] All references to overtime in this thesis are exclusively paid overtime hours.

[2] This term is interchangeably used with fixed costs throughout this research.

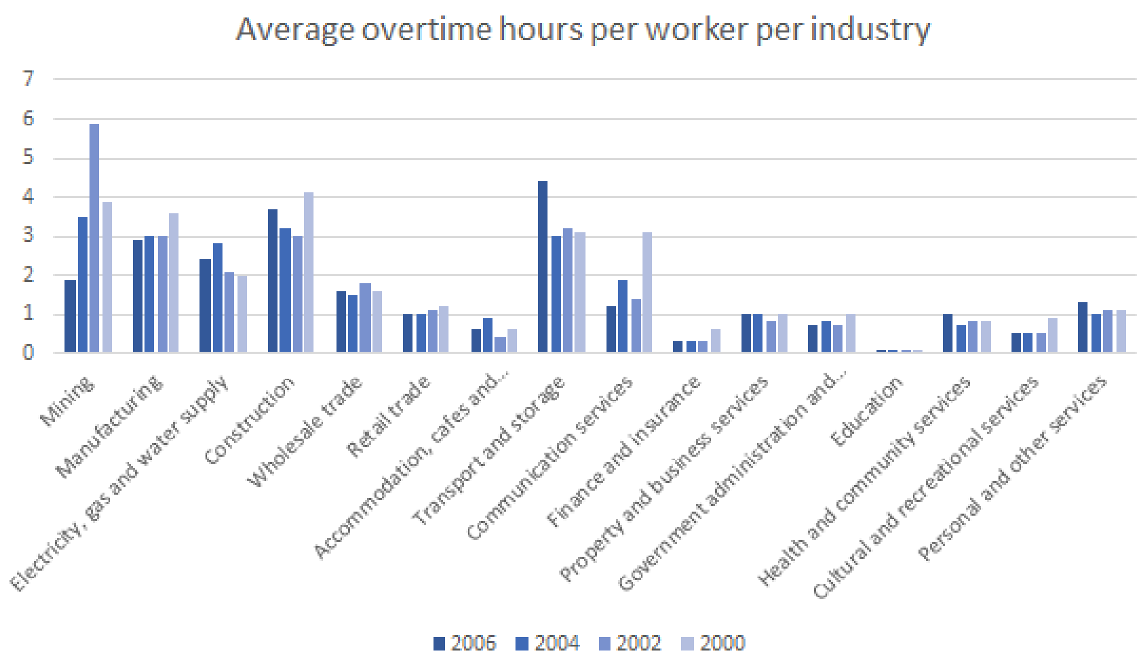


Figure 1.1: Average weekly overtime per worker across years 2000, 2002, 2004 and 2006. (Australian Bureau of Statistics, 2001, 2003, 2005, 2007). Note: In 2006 the industry definitions changed meaning years after 2006 can not be compared to years prior to 2006.

workers performing 5 hours or 2 workers performing 10 hours, has no impact on labour productivity. The demand for labour is determined by cost minimising firms choosing the optimal amount of labour based on a single wage rate. A consequence of this setup is that an increase in a quasi-fixed costs and a proportionate increase in the wage rate would both generate the same employment results.

This approach requires the researcher to make certain assumptions about the quantity of labour. For example, the researcher can assume that the quantity of labour represents total person hours or they can assume that either the extensive or intensive margin of employment is held fixed - usually the intensive margin. The former provides no insight into how either margin is affected and the latter ignores any of the possible endogenous responses that occur.

Consider the research performed by Dixon et al. (2005) where they simulate the effects of a $26.60 increase in award wages per week.[3] This results in real before-tax wages increasing by 4.23% and employment, measured as aggregate person-hours, decreasing by 5.24%. However, it does not distinguish between intensive and extensive margin employment. As illustrated above, The impact to employees depends on whether unemployment is distributed along the intensive or extensive margin. In the extreme circumstance that all unemployment was distributed along the intensive margin, low-skilled workers would only marginally have their wages reduced but would enjoy 5.24% increase in leisure. Conversely, inequality will increase with 5.24% of workers being unemployed. The introduction of the aforementioned labour demand model would demonstrate the impact this policy would have on intensive and extensive margin employment. As such, the introduction of both margins provides an extension of the traditional setup.

The final section of this research simulates the effect of a 26.3% increase in the rate of the Superannuation Guarantee (SG). The superannuation guarantee is a mandated policy which requires

[3] Award wages are industry-specific minimum wages for employees.

firms to deposit 9.5% of a worker's ordinary wages into a retirement saving account on behalf of each of their employees. This exemplifies another shortcoming of the traditional CGE model, as these costs are considered a quasi-fixed cost. If this simulation was performed in the traditional CGE framework, it would generate the same results as a comparable increase in workers ordinary wages. However, since an increase in the SG is likely to make workers more expensive compared to additional hours, this will cause a substitution from workers to hours, which is missed in the standard model.

The remainder of this thesis is organised as follows. Section (1) provides a brief history of CGE modelling and outlines the underlying economic theory for the ORANIG model. The ORANIG model is a static equilibrium CGE model. This chapter outlines the structure of the CGE model before any modifications are made to the labour market. Section (2) outlines competing labour demand models in the presence of fixed costs and overtime. It outlines three workers-hours labour demand models: (1) a model where the wage premium paid for overtime hours is a constant, (2) where the wage premium is a linear function of overtime hours and (3) where the elasticity between wage premiums and overtime is constant. Section 3 reviews the empirical literature on labour demand models. The aim of this section is to provide empirical justification for why the constant elasticity model was chosen. In addition to reviewing the literature, it discusses the approach taken to measuring fixed costs and measures the elasticity between overtime hours and wage premium. Section (4) provides a description of the data used within this research to calibrate the CGE model and explains the process involved with incorporating the theory presented in section 3 into the ORANIG model outlined in section (2). Finally, section (5) uses this model to simulate the effects of a 26.3% increase in the rate of the SG. This section outlines the impact this policy has on the labour market in terms of intensive and extensive margin of employment. In addition, it outlines the effects that each industry experiences as a result of the increase in the SG. To outline the benefits of including a labour demand model, the increase in the rate of the SG is also simulated using the original ORANIG model. The results between the two simulations are compared.

# 2 CGE framework

## 2.1 Leontief's Input Output framework

This chapter provides a brief overview of the history of economy-wide modelling. It starts with an input-output model, transitions to the works of Leif Johansen and then presents the ORANIG model which will be used in this research.

The starting point for any student of CGE modelling is Leontief's Input-Output (IO) model. The IO model is constructed based upon a set of input-output accounts which represents in quantitative terms the key inter-dependencies between different sectors of a national economy (Leontief, 1965). Usually, the input-output data accounts for all economic transactions which occur within an economic region for a given period of time, e.g, a year.

In this section, a two-industry IO model based on the IO data in Table 2.1 is presented. It is defined in terms of an Agriculture and Non-Agriculture sector. The output produced by these two industries consists of either an Agriculture or Non-Agriculture commodity. All final users, such as consumers, investors and government are aggregated and denoted Final Demand (FD). All primary factors provided to firms, such as capital and labour, are denoted as Value Added (VA). Finally, it is assumed the economy is closed.

The table below is referred to as a USE table. It summarises all transactions for Australia in the year 2009-2010 and is drawn from the official IO tables (Australian Bureau of Statistics, 2013b).

| | Agriculture | Non-Agriculture | Final demand | Total use |
|---|---|---|---|---|
| Agriculture | $13 | $32 | $19 | $64 |
| Non-Agriculture | $24 | $1185 | $1,448 | $2,657 |
| Value added | $27 | $1440 | | |
| Total demand | $64 | $2657 | | |

Table 2.1: Australian USE table (Product by Industry) for the year 2009-10 ($B)

Across the first two rows, each element represents how much of each commodity is purchased by a respective agent. Reading across the first row shows that the Agriculture industry purchases $13 billion dollars worth of the Agriculture commodity, the non-Agriculture industry purchases $32 worth of the Agriculture industry and final demand consumes $19 billion. The final column shows the sum of all sales of the agriculture commodity equals $64 billion.

It may seem peculiar that an industry can purchase from itself. This is due, in part, to aggregation. For example, a cattle producing firm who operates in the Agriculture industry may purchase grain from another firm who operates within the Agriculture industry. This would represent a flow from Agriculture to Agriculture.

Down the first two columns, each element describes the value of each commodity that an industry purchases to produce its output. Reading down the Non-Agriculture column, it is observed that Non-Agriculture purchases $32 billion of the Agriculture commodity, $1,185 billion of the Non-Agriculture commodity and $1,440 billion of primary factors denoted as value added.

The third row represents the Value Added (VA) for each industry. It represents the expenditure on primary factors by each industry. For example, reading the Agriculture column shows that the Agriculture industry purchases $27 billion of primary factors.

For each commodity, the total value of output ($Y$) equals the sum of the values of all inputs to the producing industry. In essence, sales equal costs for all industries. This is a fundamental adding-up property of all IO tables.

Thus far it has been assumed that each industry only produces one output commodity. If for example, the Agriculture industry produced some Non-Agriculture commodity (for example, they might use some of their land to produce renewable energy via wind turbines, which would be considered Non-Agriculture) it would result in a situation where the value of total output of the Agriculture commodity does not match the value of total inputs to the Agriculture industry. In this case, there would be a separate matrix, referred to as the MAKE matrix (discussed in Chapter 2.7), which would describe production by each industry. Nonetheless, total sales of a product would still equal the input usage required to produce the respective commodity.

### 2.1.1 Model setup

For industry $i$ where $i =$ Agriculture or Non-Agriculture, it is assumed that the volume of production is the sum of the volume of intermediate and final demand:

$$Q_1 = X_{1,1} + X_{1,2} + Y_1 \tag{2.1a}$$

$$Q_2 = X_{2,1} + X_{2,2} + Y_2 \tag{2.1b}$$

where $Q_i$ represents the total production of commodity $i$, $X_{ij}$ is the intermediate demand of commodity $i$ by industry $j$ and $Y_i$ is the final usage of commodity $i$. In this framework, the possibility of multi-production is ignored, so $Q_i$ represents the production of commodity $i$ and industry $i$.

Output for industry $j$ is produced using fixed proportions in Leontief's IO model, which gives the input demand equations for each commodity as:

$$X_{i,j} = a_{i,j} Q_j \tag{2.2}$$

where $a_{i,j}$ is a technology parameter which describes the unit production requirements of commodity

$i$ used by industry $j$. Substituting the corresponding demand equations from (2.2) into both (2.1a) and (2.1b), yields:

$$Q_1 = a_{1,1}Q_1 + a_{1,2}Q_2 + Y_1 \tag{2.3a}$$

$$Q_2 = a_{2,1}Q_1 + a_{2,2}Q_2 + Y_2 \tag{2.3b}$$

which can be expressed in matrix notation as:

$$Q = AQ + Y \tag{2.4}$$

where:

$$\mathrm{A} = \begin{vmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{vmatrix} \tag{2.5}$$

denotes a square matrix of unit production requirements,

$$\mathrm{Q} = \begin{vmatrix} Q_1 \\ Q_2 \end{vmatrix} \tag{2.6}$$

is a vector of total production and

$$\mathrm{Y} = \begin{vmatrix} Y_1 \\ Y_2 \end{vmatrix} \tag{2.7}$$

is vector of final demands.

### 2.1.2 Solving the IO model

Typically, final demand is treated as exogenous when solving the IO model. Total production can be solved by rearranging and inverting equation (2.4) such that:

$$Q = (I - A)^{-1}Y \tag{2.8}$$

The term $(I - A)^{-1}$ is referred to as the Leontief inverse and can be thought of as a multiplier for output when there is an increase in final demand.

The impact that a shock to final demand has on production can be understood with the following equation:

$$Q + \Delta Q = (1 - A)^{-1}(Y + \Delta Y) \tag{2.9}$$

where $\Delta Y$ is the shock to final demand and $\Delta Q$ is the change in total output. The change in production can be denoted:

$$\Delta Q = (1 - A)^{-1}(\Delta Y) \tag{2.10}$$

Consider the IO model with the following unit requirements for production derived from Table 2.1,

$$\text{A} = \begin{vmatrix} 0.20 & 0.01 \\ 0.38 & 0.45 \end{vmatrix} \tag{2.11}$$

The Leontief inverse $(1 - A)^{-1}$ is calculated as:

$$(1 - A)^{-1} = \begin{vmatrix} 1.27 & 0.02 \\ 0.87 & 1.83 \end{vmatrix} \tag{2.12}$$

The effects of a \$1000 increase in final demand for commodity 1 can be simulated as:

$$\Delta Q = \begin{vmatrix} 1.27 & 0.02 \\ 0.87 & 1.83 \end{vmatrix} \begin{vmatrix} 1000 \\ 0 \end{vmatrix} \tag{2.13}$$

The \$1000 increase in Agriculture final demand leads to a \$1,270 increase in Agriculture production and \$870 worth of Non-Agriculture production. Since \$1,000 worth of the production of Agriculture is for final demand, it follows that \$270 of Agriculture output and \$870 worth of Non-Agriculture is used as an intermediate input.

### 2.1.3 Calibration

Calibration is the process of calculating the input requirement coefficients ($a_{i,j}$) using the data presented in Table 2.1. The coefficients ($a_{i,j}$) are calibrated by rearranging (2.2) such that:

$$a_{i,j} = \frac{X_{i,j}}{Q_j} \tag{2.14}$$

Defining the quantities $X_{i,j}$ and $Q_{i,j}$ is difficult as they tend to represent an aggregation of many commodities. Instead, each commodity is defined in terms of its market value in the base period, which gives the unit input requirements as:

$$a_{i,j} = \frac{P_i X_{i,j}}{P_j Q_j} \tag{2.15}$$

where $P_i X_{i,j}$ is expenditure on commodity $i$ to produce commodity $j$ and $P_j Q_i$ is the total cost of producing commodity $j$. For example, the unit requirements for the Non-Agriculture commodity for the Agriculture industry are calculated:

$$a_{2,1} = \frac{\$24}{\$64} = 0.38 \tag{2.16}$$

Doing this for the remaining commodities yields:

$$\mathrm{A} = \begin{vmatrix} 0.2 & 0.38 \\ 0.01 & 0.45 \end{vmatrix} \tag{2.17}$$

giving the matrix of unit coefficient requirements.

## 2.2 Stylised Johansen

The IO framework presented in (2.1) is a simplified model of economic behaviour where prices play no role in the allocation of resources between economic agents. CGE models were, in part, developed to overcome this lack of price mechanism.

Leif Johansen is credited with developing what is regarded as the first CGE model (Dixon and Rimmer, 2010). His model consisted of 19 industries based on 1950's data from Norway's IO tables Johansen (1960). His economic framework was characterised by utility maximising households and cost minimising firms.

The following section presents a stylised model called Stylised Johansen (SJ) based on the work of Johansen. There are two industries and two primary factors: capital and labour. Usual CGE models require computers to solve as they consist of thousands of equations and variables. The purpose of this model is to introduce the underlying economic concepts of a CGE model in a framework which does not require a computer to solve allowing the economic assumptions and methods employed to be better understood.

## 2.3 Linearisation

In the Leontief IO model, the assumption that inputs are used in fixed proportions is convenient as all economic activity can be modelled as a system of linear equations. In comparison, Johansen's approach uses a Cobb-Douglas functional form for production, resulting in nonlinear intermediate demand equations. Nonlinear equations pose a problem as they are burdensome to solve for large systems of nonlinear equations.

Johansen's solution method is to convert the system of nonlinear equations into a system of first-order linear approximations around an initial equilibrium value. Moreover, the linear approximations are converted to percentage change form so that results can be interpreted as elasticities.

Converting to percentage changes alters the way in which results are interpreted. For example, in (2.13) the results were analysed as a difference, where total production increased by \$2140 in response to a \$1000 increase in final demand. If measured in terms of percentage change, the change in total output would be 68.17% in response to a 78.65% increase final demand.

Nonlinear equations are converted to linear equations by using the Euler method, as discussed in Dixon and Parmenter (1996), which involves taking derivatives around an initial value, such that if given the function is:

$$Y_i = f_i(X_{1,i}, X_{2,i}, ..., X_{n,i}) \quad i = 1, ..., m \tag{2.18}$$

The linearised version of this equation would be:

$$dY_i = \frac{\partial f_i}{\partial X_1, i} dX_{1,i} + \frac{\partial f_i}{\partial X_2, i} dX_{2,i} + ... + \frac{\partial f_i}{\partial X_n, i} dX_{n,i} \quad i = 1, ..., m \tag{2.19}$$

where the partial derivatives are evaluated at some initial condition corresponding to the underlying database. Moreover, all variables are converted into percentage change variables, as such:

$$\frac{dY_i}{Y_i} Y_i = \frac{\partial f_i}{\partial X_{1,i}} \frac{dX_{1,i}}{X_{1,i}} X_{1,i} + \frac{\partial f_i}{\partial X_{2,i}} \frac{dX_{2,i}}{X_{2,i}} X_{2,i} + ... + \frac{\partial f_i}{\partial X_{n,i}} \frac{dX_{n,i}}{X_{n,i}} X_{n,i} \quad i = 1, ..., m \tag{2.20}$$

To reduce notation, percentage change variables are represented with lower case variables, such that:

$$y_i Y_i = \frac{\partial f}{\partial X_{1,i}} x_{1,i} X_{1,i} + \frac{\partial f}{\partial X_{2,i}} x_{2,i} X_{2,i} + ... + \frac{\partial f}{\partial X_{n,i}} x_{n,i} X_{n,i} \quad i = 1, ..., m \tag{2.21}$$

where $x_{i,j} = 100 * \frac{dX_{i,j}}{X_{i,j}}$ and $y_i = 100 * \frac{dY_i}{Y_i}$ both denote percentage change variables.

Consider the following example of linearisation of the following function:

$$Y = X^2 \tag{2.22}$$

The first order derivative is expressed as:

$$dY = 2XdX \tag{2.23}$$

which can be expressed in terms of percentage change variables by multiplying both sides by $\frac{1}{Y}$ and the RHS by $\frac{X}{X}$ such that:

$$\frac{dY}{Y} = 2\frac{X^2}{Y}\frac{dX}{X} \tag{2.24}$$

Using lower case variables for percentage changes, (2.24) can be expressed:

$$y = 2x \tag{2.25}$$

This relationship shows that if the variable $x$ increases by 1% the variable $y$ will increase by 2% in response.

### 2.3.1 Linearisation errors

The linearisation of (2.22) introduces a linearisation error. For example, consider the percentage change in $Y$ when $X$ increases from 1 to 2. When $X = 1, Y = 1$ and when $X = 2, Y = 4$. The true percentage change in $Y$ is calculated:

$$\frac{Y_1 - Y_0}{Y_0} = 100\frac{4-1}{1} = 300\% \tag{2.26}$$

Using the approximation formula in (2.25) yields:

$$y = 2(100\%) = 200\% \tag{2.27}$$

calculating the error as such:

$$error = |Approximation - True| \tag{2.28}$$

gives the error:

$$error = |200\% - 300\%| = 100\% \tag{2.29}$$

This approximation understates the true percentage increase by 100%. The source of this error is due to the fact that the difference of Y, $dY$ is calculated using its first derivative which ignores higher-order terms. For example, letting $f(X) = Y$ the Taylor expansion of this function is:

$$f(X) = f(X_0) + f'(X_0)(X - X_0) + \frac{f''(X_0)}{2}(X - X_0)^2 + O[(X - X_0)^3] \tag{2.30}$$

where $X_0$ is the initial value of $X$ and $O$ represents terms of higher order than 3. Re-arranging and dividing through by $f(X_0)$ gives:

$$\frac{f(X) - f(X_0)}{f(X_0)} = \frac{f'(X_0)}{f(X_0)}(X - X_0) + \frac{f''(X_0)}{2f(X_0)}(X - X_0)^2 + O[(X - X_0)^3] \tag{2.31}$$

which can be expressed as:

$$100 * \frac{f(X) - f(X_0)}{f(X_0)} = 100 * [\frac{f'(X_0)}{f(X_0)} X_0 \frac{(X - X_0)}{X_0} + \frac{f''(X_0)}{2f(X_0)}(X - X_0)^2 + O[(X - X_0)^3]] \quad (2.32)$$

which can be simplified to:

$$y = x[\frac{f'(X_0)}{f(X_0)} X_0] + \frac{f''(X_0)}{2f(X_0)}(X - X_0)^2 + O[(X - X_0)^3] \quad (2.33)$$

where the error is:

$$error = \frac{f''(X_0)}{2f(X_0)}(X - X_0)^2 + O[(X - X_0)^3] \quad (2.34)$$

Thus the error exists because the higher-order terms are ignored. These terms are ignored as the value of the error is usually negligible as long as the percentage change in $X$ is small, whereas in the example with (2.22) the percentage change is $x = 100\%$. Returning to the example (2.22), where $Y = X^2$, the error can be calculated as:

$$\frac{2}{2} * 1^2 = 1 \quad (2.35)$$

which when represented as a percentage is 100%. This is exactly the same as the error calculated above as there are no higher order terms since (2.22) is a quadratic.

It is worthwhile noting that the error term includes the term $(X - X_0)^2$ which means the greater difference between $X$ and $X_0$, the larger the error. For example, consider the Table 2.1. The first column shows the $Y$ value given that $Y = X^2$. The second column shows the $X$ value. The third column, labelled true, shows the actual percentage change in Y assuming that the initial values are $X = 1$ and $Y = 1$. The fourth column shows the approximation using the formula (2.25). The fifth column shows the error as calculated in (2.28). The final column shows the proportion of the error to the size of the true percentage change in $Y$. This column shows that the larger the step size, the greater the error compared to the actual shock size.

| Y | X | true | Approximate | error | $\frac{error}{true}$ |
|---|---|---|---|---|---|
| 1.02 | 1.01 | 2.01% | 2.00% | 0.01% | 0.1% |
| 1.21 | 1.10 | 21.00% | 20.00% | 1.00% | 4.76% |
| 4.00 | 2.00 | 300.00% | 200.00% | 100.00% | 33.33% |

Table 2.2: Approximation error for Y when calculating the percentage change between the X and $X_0 = 1$.

### 2.3.2 Multistep solutions

In the previous section, the approximation formula $y = 2x$ was used to approximate the percentage change in $Y$ when $X$ increased from its initial value of $X_0 = 1$ to $X = 2$. The approximated value of $Y$ is calculated from the initial value of $Y_0$ using the formula:

$$Y = (1 + y)Y_0 = (1 + 2x)Y_0 \tag{2.36}$$

This approach introduced an approximation error which was proportionate to the difference $X - X_0$. This approximation error can be reduced by breaking the steps in-between $X_0$ and $X$ into smaller steps. For example, a 2-step solution could be performed where the percentage change in $Y$ is calculated by calculating how $Y$ changes when $X$ increases from $X_0 = 1$ to $X_1 = 1.5$ then how it changes from $X_1 = 1.5$ to $X = 2$. The first-step approximate of $Y_1$ would be calculated by first using the formula:

$$Y_1 = (1 + y)Y_0 = (1 + 2x)Y_0 \tag{2.37}$$

then using the value for $Y_1$ the value for $Y$ would be calculated as:

$$Y = (1 + y)Y_1 = (1 + 2x)Y_1 \tag{2.38}$$

In step 1, the percentage change in $X$ is $x = 100\frac{1.5-1}{1} = 50\%$ giving:

$$Y_1 = (1 + 2 * 50\%) * 1 = 2 \tag{2.39}$$

| Number of steps | Approximation value |
|---|---|
| 1 | 200% |
| 2 | 233% |
| 3 | 250% |
| 5 | 266% |
| 10 | 282% |
| 50 | 296% |
| 100 | 298% |
| 1000 | 299.80% |

Table 2.3: Approximated values for $Y$ based on different step sizes

In step 2, the percentage change in $X$ is $x = \frac{2-1.5}{1.5} = 33\%$ giving:

$$Y = (1 + 2 * 33\%) * 2 = 3.32 \tag{2.40}$$

By adding a second step, the approximated solution is 232% instead of 200%. The error reduces from 100% down to 68%. This approach can be generalised for $n$ different steps, such that:

$$Y_1 = (1 + y)Y_0 \tag{2.41}$$

$$\vdots$$

$$Y = (1 + y)Y_{n-1}$$

An illustration of this approach and how the solution becomes more accurate is depicted in Figure 2.1. Different step sizes and approximations for the equation (2.22) is presented in Table 2.3.

The more steps that are used the smaller the error becomes in size. Returning to equation (2.34) shows why the error decreases as the number of steps increases. The size of the error is proportionate

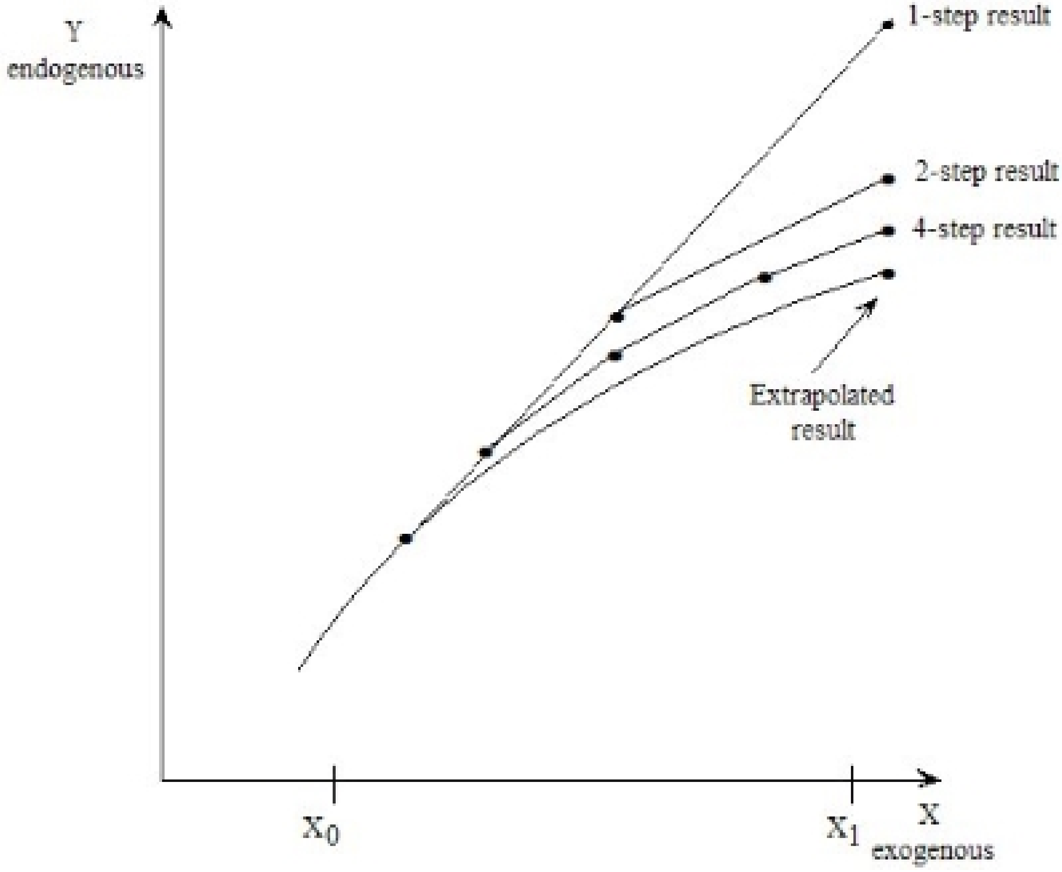


Figure 2.1: Illustration of Euler approximation with different step sizes (Horridge et al., 2018)

to the term $(X - X_0)^2$. Therefore, the size of the error shrinks by a greater proportion than the decrease in the step size.

In addition to these multi-step solutions, the software GEMPACK, which is used to perform the CGE simulations in this thesis, uses numerical methods to increase the accuracy of approximations. An outline of these methods can be found in (Pearson, 1991).

### 2.3.3 Linearisation rules

There are three rules which are often used to convert non-linear relationships in the level form of variables to a linear relationship in the percentage change form of variables.

*Sum Rule*

For the case where a variable is the sum of two or more other variables, such that:

$$X = Y + Z \tag{2.42}$$

The linearised version of this equation is:

$$x = \frac{X}{Y} y + \frac{X}{Z} z \tag{2.43}$$

*Product Rule*

The product rule is used when a variable is the product of two or more variables, such that:

$$X = YZ \tag{2.44}$$

In which case the linearised version of this equation is expressed:

$$x = y + z \tag{2.45}$$

*Power Rule*

The power rule is used for when a relationship involves an exponent, such as:

$$X = Y^{\alpha} \tag{2.46}$$

which in percentage change forms is expressed:

$$x = \alpha y \tag{2.47}$$

Finally, if a function involves a fraction, such that:

$$X = \frac{Y}{Z} \tag{2.48}$$

Noting that this relationship can be expressed as:

$$X = YZ^{-1} \tag{2.49}$$

A combination of the product and power rule can be used to convert this into a percentage change form as:

$$x = y + (-z) \tag{2.50}$$

These rules will be applied in the following section to convert equations from their levels form into their percentage change form.

## 2.4 Household demand

In the Leontief IO model presented in (2.1), there was only one source of final demand and it was treated as exogenous. By contrast, in this Johansen system, final demand is represented by a single representative household with a Cobb-Douglas utility function:

$$U = \prod_{i=1}^{2} Y_i^{\beta_i} \tag{2.51}$$

where $U$ is the household utility function, $Y_i$ is the consumption of commodity $i$ and $\beta_i$ are preference parameters with $0 \geq \beta_i \leq 1$ and $\sum_{i=1}^{2} \beta_i = 1$.

Households maximise their utility subject to an expenditure constraint such that:[4]

$$M = \sum_{i=1}^{2} P_i Y_i \tag{2.52}$$

where $M$ represents household expenditure and $P_i$ is the price of commodity $i$. Solving the utility maximisation problem yields the demand function:

$$Y_i = \frac{\beta_i M}{P_i} \tag{2.53}$$

The linearised version of (2.53) is:

$$y_i = m - p_i \tag{2.54}$$

where, as always, the lower case variables represent percentage changes.

## 2.5 Input demand

In the IO model where each industry uses intermediate commodities in fixed proportions, intermediate demand is proportionate to final demand, and prices play no role in the use of input commodities. In the stylised model, industries produce output according to a Cobb-Douglas production function:

$$Q_j = A_j \prod_{i=1}^{4} X_{i,j}^{\alpha i,j} \tag{2.55}$$

where $Q_j$ is production of commodity $j$; $A_j$ is total factor productivity; $X_{i,j}$ is the usage of intermediate commodities $1, 2$ when $i = 1, 2$; $X_{i,j}$ is the usage of primary factors when $i = 3, 4$ and $\alpha_{i,j}$ is the factor share of input $i$ used to produce commodity $j$. Since production is constant returns to scale $\sum_i^4 \alpha_{i,j} = 1$. The cost of production for industry $i$ is defined:

$$C_j = \sum_{i=1}^{4} P_i X_{i,j} \tag{2.56}$$

[4] Expenditure is used instead of budget to allow for savings.

where $P_i$ is the price of commodity/factor $i$. For a given level of output, the input demand equation for input $i$ used by industry $j$ is:

$$X_{i,j} = \frac{\prod_{n=1}^{N} P_n^{\alpha_{n,j}}}{P_i} \frac{\alpha_{i,j}}{\prod_{m=1}^{M} \alpha_{m,j}^{\alpha_{M,j}}} Q_j \tag{2.57}$$

The linearised version of this equation is expressed:

$$x_{i,j} = q_j - (p_i - \sum_{n=1}^{4} \alpha_{i,j} p_i) \tag{2.58}$$

Given that there are 4 inputs for each industry, there is a total of 8 input demand equations. Equation (2.58) illustrates the CGE model's extension of the IO model. There are two terms on the RHS of the input demand equation. The first term ($q_j$) is a scale effect and the second term ($p_i - \sum_{n=1}^{4} \alpha_{n,j} p_n$) is a relative price effect. In the IO model the relative price effect is ignored and only the scale effect is present. Conversely, the CGE model includes both of these effects thus extending upon the IO framework.

### 2.5.1 Zero-profit condition

It is assumed each industry operates in a perfectly competitive market, meaning pure profits are zero. This implies that revenue equals costs, such that for industry $i$:

$$P_j Q_j = \sum_{i=1}^{4} P_i X_{i,j} \tag{2.59}$$

Substituting the demand equations in (2.57) into (2.59) determines the price for each output commodity, such that:

$$P_j = \frac{\prod_{n=1}^{N} P_{n,j}^{\alpha_{n,j}}}{\prod_{n=1}^{N} \alpha_{n,j}^{\alpha_{n,j}}} \tag{2.60}$$

Linearizing (2.60) gives the following price equation in percentage change form:

$$p_j = \sum_{n=1}^{4} \alpha_{n,j} p_n \tag{2.61}$$

In this framework, the output price is the share-weighted price of all inputs used.

### 2.5.2 Market clearing

All markets are assumed to clear. As each industry only produces one commodity, total supply ($\bar{X}_i$) equals industry output $Q_i$ such that:

$$\bar{X}_i = Q_i \tag{2.62}$$

The extra notation is introduced as it is possible for this assumption to be relaxed in which case total supply would differ from industry output. The market clearing condition can thus be expressed:

$$\bar{X}_i = \sum_{j=1}^{2} X_{i,j} + Y_i \quad \text{for } i = 1, 2 \tag{2.63}$$

where the total supply ($\bar{X}_i$) of commodity $i$ is the sum all of intermediate demand ($\sum_{j=1}^{2} X_{i,j}$) and final demand ($Y_i$) for commodity $i$. The market clearing condition (2.69) can also be expressed in terms of expenditure by multiplying both sides by the output price, such that:

$$P_i\bar{X}_i = P_i(\sum_{j=1}^{2} X_{i,j} + Y_i) \quad \text{for } i = 1, 2 \tag{2.64}$$

This allows the percentage-change form of the market-clearing condition to be expressed as the expenditure-share weighted function such that:

$$\bar{x_i} = S_{i,1}x_{i,1} + S_{i,2}x_{i,2} + S_{i,3}y_{i,3} \quad \text{for } i = 1, 2 \tag{2.65}$$

where $S_{i,j}$ is the expenditure-share on each component of demand such that:

$$S_{i,j} = \frac{P_i X_{i,j}}{P_i \bar{X}_i} \quad \text{for } i = 1, 2 \ \& \ j = 1, 2 \tag{2.66}$$

and

$$S_{i,j} = \frac{P_i Y_i}{P_i \bar{X}_i} \quad \text{for } i = 1, 2 \ \& \ j = 3 \tag{2.67}$$

It is assumed that there is no final demand component for primary factors, so the market clearing conditions for the primary factor market are:

$$\bar{X}_f = \sum_{j=1}^{2} X_{f,j} \quad \text{for } f = 3, 4 \tag{2.68}$$

where: $\bar{X}_f$ is the total supply of primary factor $f$. The linearised version of this becomes:

$$\bar{x}_f = S_{f,1}x_{f,1} + S_{f,2}x_{f,2} \quad \text{for } f = 3, 4 \tag{2.69}$$

where

$$S_{f,j} = \frac{P_f X_{f,j}}{P_f \bar{X}_f} \quad \text{for } j = 3, 4 \tag{2.70}$$

### 2.5.3 Numeraire

A feature of nearly all general equilibrium models is that the level of demand for a commodity or primary factor is only affected by a change in relative prices. A uniform increase in all prices will not impact the demand for any individual commodity. To determine an aggregate price level, a single price must be set exongeously. This price is referred to as the *numeraire.* In the stylised model, the price of commodity 1 is set as the numeraire:

$$P_1 = 1 \tag{2.71}$$

in percentage change form,

$$p_1 = 0 \tag{2.72}$$

Therefore, all changes in prices are interpreted as relative price changes to commodity 1.

### 2.5.4 Summary

The 2 industry stylised model of the Australian economy consists of 17 equations and 19 variables. A Summary of these equations are listed in the Table 2.4. All equations are rearranged such that the RHS is equal to zero.

| Equation List | Linearised version | Description |
|---|---|---|
| EQ1-EQ2 | $y_i - m + p_i = 0 \quad (i = 1, 2)$ | Consumer demand |
| EQ3 | $p_1 = 0$ | Numeraire |
| EQ4-EQ11 | $x_{i,j} + q_j + (p_j - \sum_{i=1}^{4} a_{i,j} p_j) = 0 \quad (i = 1, ..., 4, j = 1, 2)$ | Input demand |
| EQ12-EQ13 | $p_j - \sum_{i=1}^{4} a_{i,j} p_j = 0 \quad (j = 1, 2)$ | Price formation |
| EQ14-EQ15 | $S_{i,1} x_{i,1} + S_{i,2} x_{i,2} + S_{i,3} y_{i,3} - \bar{x}_i = 0 \quad i = (1, 2)$ | Commodity market clearing |
| EQ16-EQ17 | $S_{i,2} x_{i,1} + Si, 2x_{i,2} - \bar{x}_i = 0 \quad (i = 3, 4)$ | Primary factor market clearing |

Table 2.4: List of linearised model equations for SJ model.

### 2.5.5 Calibration

The stylised model is calibrated in a similar manner to the IO model. The data used for calibration is outlined in Table 2.5.

| | Agriculture | Non-Agriculture | Final demand | Total output |
|---|---|---|---|---|
| Agriculture | \$12,993 | \$32,081 | \$18,838 | \$63,912 |
| Non-Agriculture | \$24,062 | \$1,185,272 | \$1,447,819.00 | \$2,657,153 |
| Labour | \$6,446 | \$777,492 | | |
| Capital | \$20,411 | \$662,308 | | |
| VA | \$26,857 | \$1,439,800 | | |
| Total Sales | \$63,912 | \$2,657,153 | | |

Table 2.5: Initial database for stylised model (\$M).

Consider calibrating the share parameter $S_{3,1}$ for the market clearing condition in equation (2.69).

$$
\begin{bmatrix}
-1.00 & 1.00 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.00 & 0 & 0 & 0 \\
-1.00 & 0 & 1.00 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.00 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.00 & 0 & 0 & 0 \\
0 & 0 & 0 & 1.00 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1.00 & 0 & 0 & 0 & 0.80 & -0.38 & -0.10 & -0.90 \\
0 & 0 & 0 & 0 & 1.00 & 0 & 0 & 0 & 0 & 0 & 0 & -1.00 & 0 & 0 & 0 & -0.20 & 0.62 & -0.10 & -0.90 \\
0 & 0 & 0 & 0 & 0 & 1.00 & 0 & 0 & 0 & 0 & 0 & -1.00 & 0 & 0 & 0 & -0.20 & -0.38 & 0.90 & -0.90 \\
0 & 0 & 0 & 0 & 0 & 0 & 1.00 & 0 & 0 & 0 & 0 & -1.00 & 0 & 0 & 0 & -0.20 & -0.38 & -0.10 & 0.10 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.00 & 0 & 0 & 0 & 0 & -1.00 & 0 & 0 & 0.99 & -0.45 & -0.54 & -0.46 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.00 & 0 & 0 & 0 & -1.00 & 0 & 0 & -0.01 & 0.55 & -0.54 & -0.46 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.00 & 0 & 0 & -1.00 & 0 & 0 & -0.01 & -0.45 & 0.46 & -0.46 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.00 & 0 & -1.00 & 0 & 0 & -0.01 & -0.45 & -0.54 & 0.54 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.80 & -0.38 & -0.10 & -0.90 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.01 & 0.55 & -0.54 & -0.46 \\
0 & -0.29 & 0 & -0.20 & 0 & 0 & 0 & -0.50 & 0 & 0 & 0 & 1.00 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -0.54 & 0 & -0.01 & 0 & 0 & 0 & -0.45 & 0 & 0 & 0 & 1.00 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1.00 & 0 & 0 & 0 & 1.00 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -0.04 & 0 & 0 & 0 & -0.96 & 0 & 0 & 0 & 1.00 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
m \\ y_1 \\ y_2 \\ x_{1,1} \\ x_{1,2} \\ x_{1,3} \\ x_{1,4} \\ x_{2,1} \\ x_{2,2} \\ x_{2,3} \\ x_{2,4} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ p_1 \\ p_2 \\ p_3 \\ p_4
\end{bmatrix}
=
\begin{bmatrix}
0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0
\end{bmatrix}
$$

Figure 2.2: Matrix representation of SJ model

Using the data in Table 2.5, the share parameter is calculated as:

$$
S_{3,1} = \frac{\$6,446}{\$6,446 + \$777,492} = 0.0082 \tag{2.73}
$$

Calculating the remaining parameters for the 17 equations from Table 2.4 yields the matrices in Figure 2.2.

### 2.5.6 Closure

In a CGE framework, there are often more equations than there are variables present within the model. Consequently, the system cannot be inverted and the model cannot be solved. Closure is the process of setting variables as a fixed value to ensure that the model can be solved.

This occurs as CGE models often lack economic theory to describe all the variables within a system. The variables in the system are classified as either endogenous or exogenous. The variables which have an equation to explain them are classified as endogenous, hence there is an equation for each endogenous variable. The variables which are determined outside the model are denoted as exogenous and these are the variables which are set as fixed by the researcher.

In this model, the absence of labour supply causes there to be more variables than equations. Usually, the intersection of the labour demand curve and labour supply curve determines the equilibrium wage rate and quantity of labour in the labour market. However, the absence of a labour supply function means that there are two variables for one equation. In this case, either the wage rate or the quantity of labour needs to be set by the researcher.

For example, as there is no labour supply function specified, in the short-run wages are held constant and labour adjusts to clear the labour market. This reflects the idea that wages are rigid in the short run. Alternatively, in a long-run environment, the labour stock is held constant and wages adjust to clear the market. This reflects the idea that employment will return to its natural rate in the long run.

### 2.5.7 Solving the model

In the stylised model presented thus far, there are 19 variables and 17 independent equations requiring that two variables be chosen for closure. Once the two exogenous variables have been determined, it can partitioned such that:

$$Ay + Bx = 0 \tag{2.74}$$

Where $y$ is a vector of the 17 endogenous variables, $A$ is the corresponding square matrix of the equation coefficients, $x$ is the vector of exogenous variables $B$ is a 17x2 matrix of equation coefficients. To solve the model, (2.74) is rearranged such that the endogenous variables are represented in terms of the exogenous, such that:

$$y = -A^{-1}Bx \tag{2.75}$$

The equation (2.75) shows how the vector of endogenous variables ($y$) deviates away from equilibrium when there is a change in the exogenous vector ($x$).

Consider the situation where both the primary factors ($x_3$) and ($x_4$) are set as exogenous. Using equation (2.74) the SJ model is expressed in Figure 2.3. Which when rearranged and inverted, such that $y = -A^{-1}Bx$, can be expressed as the solution depicted in Figure 2.5.7.

$$
\begin{bmatrix}
-1.0 & 0.0 & 1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -1.0 & 0.0 & 0.8 & -0.4 & -0.1 & -0.9 \\
0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -1.0 & 0.0 & -0.2 & 0.6 & -0.1 & -0.9 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -1.0 & 0.0 & -0.2 & -0.4 & 0.9 & -0.9 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 0.0 & 0.0 & 0.0 & 0.0 & -1.0 & 0.0 & -0.2 & -0.4 & -0.1 & 0.1 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 0.0 & 0.0 & 0.0 & 0.0 & -1.0 & 1.0 & -0.4 & -0.5 & -0.5 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 0.0 & 0.0 & 0.0 & -1.0 & 0.0 & 0.6 & -0.5 & -0.5 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 0.0 & 0.0 & -1.0 & 0.0 & -0.4 & 0.5 & -0.5 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 0.0 & -1.0 & 0.0 & -0.4 & -0.5 & 0.5 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.8 & -0.4 & -0.1 & -0.9 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.6 & -0.5 & -0.5 \\
0.0 & -0.3 & 0.0 & -0.2 & 0.0 & 0.0 & 0.0 & -0.5 & 0.0 & 0.0 & 0.0 & 1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & -0.5 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -0.4 & 0.0 & 0.0 & 0.0 & 1.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0
\end{bmatrix}
\begin{bmatrix} m \\ y_1 \\ y_2 \\ x_{1,1} \\ x_{1,2} \\ x_{1,3} \\ x_{1,4} \\ x_{2,1} \\ x_{2,2} \\ x_{2,3} \\ x_{2,4} \\ x_1 \\ x_2 \\ p_1 \\ p_2 \\ p_3 \\ p_4 \end{bmatrix}
+
\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}
\begin{bmatrix} x_3 \\ x_4 \end{bmatrix}
=
\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
$$

Figure 2.3: Partioned matrix $Ay + Bx = 0$ for SJ model

The first column of (2.5.7) represents the elasticity's of the endogenous variables with respect to a 1% increase in the quantity of labour. The second column represents the elasticity's for a 1% increase in the quantity of capital. For example, the second element in column one (0.28) describes the elasticity of household consumption of commodity 1 in response to an increase in the supply of labour. A 1% increase in the labour supply leads to a 0.28% increase in the consumption of commodity 1.

It is observed that the sum across any row for a quantity variable equals unity. For example, summing across the row $x_{1,2} = 0.75 + 0.25 = 1$. This shows that when both primary factors are increased by 1%, all quantity variables increase proportionately. This is a consequence of the production function exhibiting constant returns to scale and the income elasticities for both output commodities being unity, since preferences are modelled using Cobb Douglas preferences. Furthermore, this also implies that price variables will not be impacted, which is observed by summing across a row for any price variable.

When both labour and capital increase, the price $p_1$ does not change. This is due to it being set as a numeraire. When labour increases, commodity 1 becomes more expensive relatively due to the price decrease in $p_2$. Moreover, since the price of commodity 1 is set as the numeraire, the demand equation for it becomes $y_i = m$ which explains why it moves proportionately with income.

$$
\begin{bmatrix} m \\ y_1 \\ y_2 \\ x_{1,1} \\ x_{1,2} \\ x_{1,3} \\ x_{1,4} \\ x_{2,1} \\ x_{2,2} \\ x_{2,3} \\ x_{2,4} \\ \bar{x}_1 \\ \bar{x}_2 \\ p_1 \\ p_2 \\ p_3 \\ p_4 \end{bmatrix} = \begin{bmatrix} 0.28 & 0.72 \\ 0.28 & 0.72 \\ 0.75 & 0.25 \\ 0.28 & 0.72 \\ 0.75 & 0.25 \\ 1.00 & 0.00 \\ 0.00 & 1.00 \\ 0.28 & 0.72 \\ 0.75 & 0.25 \\ 1.00 & 0.00 \\ 0.00 & 1.00 \\ 0.28 & 0.72 \\ 0.75 & 0.25 \\ 0.00 & 0.00 \\ -0.47 & 0.47 \\ -0.72 & 0.72 \\ 0.28 & -0.28 \end{bmatrix} \begin{bmatrix} x_3 \\ x_4 \end{bmatrix}
$$

Figure 2.4: Solution to SJ model with exogenous labour and capital

For both industries to absorb the increase in quantity of labour requires both industries to decrease their capital-labour ratios. For industries to increase their usage of labour, the price of labour must decrease relative to capital, which is true since $p_3 = -0.72\%$ and $p_4 = 0.28\%$. This decrease in the price of labour favours industry 2 which is more labour intensive, resulting in its output price $p_2$ decreasing by $-0.47\%$. This decrease in cost causes the household demand for this commodity to increase by 0.75% which is larger than household demand for commodity 1 which only increases by 0.28%. This same logic can be applied in reverse for an increase in capital.

There are shortcomings to this adaptation of the Johansen model, despite its advancements on the Leontief IO model. This model does not include any margins, meaning there are no costs associated with transporting a commodity from its source of production to its final destination for consumption. The model also includes no taxes on consumption or production and does not have any imports or exports. Finally, the modelling assumptions employed in this framework are restrictive. The Cobb-Douglas utility function imposes income and own-price elasticity values of 1 and cross-price elasticities of 0. The same applies for the production function which also adopts a Cobb-Douglas functional form.

## 2.6 Transitioning to ORANIG

The following section transitions from the SJ model based on Johansens work to ORANIG. This section applies the main theoretical advancements of ORANIG to the recurring 2 industry model that has been presented thus far. The final section outlines the overall theory of the ORANIG framework. Those advancements cover: (a) the introduction of imports (b) margins and taxes (c) multiproduction. A comprehensive treatment of the ORANIG framework can be found in Horridge et al. (2000).

### 2.6.1 Imports

In the following section, imports are incorporated into the SJ model. In Table 2.6 purchases by each industry is decomposed based on the source of purchase, where (D) denotes domestic purchases and (I) denotes imports.

| | Agriculture | Non-Agriculture | Final demand | Total output |
|---|---|---|---|---|
| Agriculture - D | $12,818 | $31,265 | $18,448 | $62,531 |
| Agriculture - I | $175 | $816 | $390 | $1,381 |
| Non-Agriculture - D | $20,614 | $1,050,899 | $1,332,265.00 | $2,403,778 |
| Non-Agriculture - I | $3,448 | $134,373 | $115,554.00 | $253,375 |
| Labour | $6,446 | $777,492 | | |
| Capital | $20,411 | $662,308 | | |
| VA | $26,857 | $1,439,800 | | |
| Total Sales | $63,912 | $2,657,153 | | |

Table 2.6: Australian USE table (Product by Industry), disaggregated by source of production, for the year 2009-10 ($M)

In Table 2.6 the first column shows the Agriculture industry purchases $20,614 worth of the domestic and $3,448 worth of the imported Non-Agriculture commodity, totalling to $24,062, which is the same value that is in Table 2.5 where imports are not distinguished from domestic production.

Each industry purchases a combination of domestic and imported commodities of both commodities. If domestic and imported commodities were perfect substitutes, it would be expected a corner solution where industries only purchased either domestic or imported based on which commodity was cheaper. Given this is not observed, the imported and domestically produced commodities are treated as imperfect substitutes (Armington, 1969).

Each industries production function now is a function of domestic and imported commodities. The demand for domestic and imported commodities are derived from the following cost function:

$$C_j = \sum_{i=1}^{2}\sum_{s=1}^{2} P_{i,s} X_{i,s,j} \quad s.t \quad Q_j = F(X_{1,1,j}, X_{1,2,j}, X_{2,1,j}, X_{2,2,j}, X_{3,j}, X_{4,j}) \tag{2.76}$$

where $X_{i,s,j}$ represents the use of commodity $i$ from source $s$ by indutry $j$. The $s$ subscript represents the source of the purchase: $s = 1$ represents domestic commodities and $s = 2$ represents imported commodities. Thus $X_{i,1,j}$ denotes the purchase of domestically produced commodity $i$ by industry $j$ and $X_{i,2,j}$ denotes the purchase of the imported commodity $i$ by industry $j$. The price of each commodity is also distinguished based on source, such that $P_{i,s}$ is the price for commodity $i$ from source $s$.

The associated cost minimisation problem from (2.76) is often simplified by adopting a separability assumption, where costs and output are defined:

$$Costs = \sum_{i=1}^{2} P_i^{\bar{s}} X_{i,j}^{\bar{s}} + \sum_{i=3}^{4} P_i * X_{i,j} \tag{2.77}$$

and

$$Q_j = F(X_{1,j}^{\bar{s}}, X_{2,j}^{\bar{s}}, X_{3,j}, X_{4,j}) \tag{2.78}$$

where $P_i^{\bar{s}}$ the price of the composite commodity and $X_i^{\bar{s}}$ is a composite of domestic and imported commodities. The composite commodity $X_i^{\bar{s}}$ is defined:

$$X_{i,j}^{\bar{s}} = G_i(X_{i,1,j}, X_{i,2,j}) \tag{2.79}$$

where $G_i()$ describes the ability to substitute between imports and domestic commodities. By defining a composite commodity in (2.78), the optimal combination of imports and domestically produced output can be solved for each composite commodity.

The separation of each decision allows for a different technology structure for each decision. In ORANIG, the decision for imported and domestic commodities is modelled using a CES preference structure:

$$X^{\bar{s}}_{i,j} = [\alpha X^{\rho}_{i,1,j} + (1-\alpha)X^{\rho}_{i,2,j}]^{\frac{1}{\rho}} \tag{2.80}$$

where $\alpha$ is a share parameter and $\rho$ determines the level of elasticity of substitution, which is defined $\sigma = \frac{1}{1-p}$. The decision for composite commodities assumed output is produced according to Leontief production structure, which has inputs used in fixed proportions:

$$Q_j = MIN(X^{\bar{s}}_{1,j}, X^{\bar{s}}_{2,j}, X_{3,j}, X_{4,j}) \tag{2.81}$$

where *MIN* function returns the minimum argument as the output. The determination of demand for commodities based on source and for composite commodities is done in two steps. The first step to determine the demand for imported and domestic commodities is achieved by minimising costs, giving the Lagrangian:

$$\mathcal{L}_{i,j} = P_{i,1}X_{i,1,j} + P_{i,2}X_{i,2,j} + \lambda_i(X^{\bar{s}}_{i,j} - [\alpha X^{-\rho}_{i,1,j} + (1-\alpha)X^{-\rho}_{i,2,j}]^{\frac{-1}{\rho}}) \tag{2.82}$$

The demand function for these commodities are:

$$X_{i,s,j} = X^{\bar{s}}_{i,j}(\alpha^{\frac{1}{\rho+1}}_{i,s,j}(\frac{P_{i,s}}{P^{ave}_{i,j}})^{\frac{\rho}{\rho+1}})^{\frac{1}{p}} \quad \text{for } (i=1,2)(s=1,2)(j=1,2) \tag{2.83}$$

where:

$$P^{ave}_{i,j} = (\alpha^{\frac{1}{\rho+1}}_1 P^{\frac{\rho}{\rho+1}}_{i,1} + \alpha^{\frac{1}{\rho+1}}_2 P^{\frac{\rho}{\rho+1}}_{i,2})^{\frac{\rho+1}{\rho}} \tag{2.84}$$

In essence, these demand equations determine the proportions of domestic and imported commodities that comprise the composite commodity. Using:

$$P_i X^{\bar{s}}_{i,j} = \sum_{s}^{2} P_{i,s}X_{i,s,j} \tag{2.85}$$

and dividing through by $X^{\bar{s}}_i$ gives the price of the composite commodity as:

$$P_i = P_{i,1}[(\alpha^{\frac{1}{\rho+1}}_{i,1,j}(\frac{P_{i,1}}{P^{ave}_{i,j}})^{\frac{\rho}{\rho+1}})^{\frac{1}{p}}] + P_{i,2}[(\alpha^{\frac{1}{\rho+1}}_{i,2,j}(\frac{P_{i,2}}{P^{ave}_{i,j}})^{\frac{\rho}{\rho+1}})^{\frac{1}{p}}] \tag{2.86}$$

Which can be thought of as a weighted average of the price for the domestic and imported commodity. The second step is to determine the optimal bundle of composite commodities demanded by the final users. The top nest uses Leontief preferences, which means all inputs will be proportionate to output, such that:

$$X_{i,j}^{\bar{s}} = A_{i,j} Q_j \tag{2.87}$$

These equations can now be linearised and incorporated into our model. The top level percentage change is:

$$x_{i,j}^{\bar{s}} = a_{i,j} + q_j \tag{2.88}$$

The linearised demand equation of the CES function for domestic and imported commodities can be expressed:

$$x_{i,s,j} = x_{i,j}^{\bar{s}} - \sigma(p_{i,s} - p_{i,j}^{ave}) \tag{2.89}$$

where $\sigma = \frac{1}{1+\rho}$ and the average price term:

$$p_{i,j}^{ave} = S_{i,1,j} P_{i,1} + S_{i,2,j} P_{i,2} \tag{2.90}$$

and

$$S_{i,s,j} = \frac{P_{i,s} X_{i,s,j}}{\sum_s^2 P_{i,s} X_{i,s,j}} \tag{2.91}$$

This approach of using nests is a common technique employed by CGE modellers and is used widely throughout the rest of this chapter. It permits convenient notation and facilitates different nests having different structures of preferences and technology.

### 2.6.2 Margins and Taxes

The price paid by the final consumer of a commodity is often not the same as that received by the producer of the commodity. These differences are due to the presence of margins and taxes on output.

Margins are commodities or services (e.g, transport, wholesale and retail trade) which are used to distribute commodities from their source of production to final users. For example, consider the production of beer. Production typically occurs in a brewery. The brewery product may then be transported to a retailer via road transport. The road transport would be considered a margin. The retailer would then add a markup to the good when they sell the commodity. The markup would be considered a retail margin. In the SJ model it is assumed that all margins commodities are used are used in proportion to final output.

Commodity taxes are those paid on each unit of output sold. In general, taxes on output can be levied ad volerem (i.e as a fraction of sale value) or on a per unit basis. The SJ model only considers the case where taxes are ad volerem. An example of an ad valorem tax for Australia is the Goods and Services (GST).

Taxes on commodities drive a wedge between the price paid by final users and that received by producers. In the SJ model, the price paid by the final user ($u = 3$) can be related to that received by producers as:

$$P_{3,i} = (1 + T_{3,i} + MM_{3,u})P_{0,i} \tag{2.92}$$

where the basic price ($P_{0,i}$), is the price received by producers for commodity $i$; $MM_{3,i}$ is the total value of margins per unit on $i$; $T_{3,i}$ is the taxes levied on $i$ and $P_{3,i}$ is the price paid by the final user.

In the IO data table the data for taxes and margins are listed as a separate row for each commodity. Reading down each column gives the dollar amount of taxes and margins purchased by intermediate and final users. In Table 2.7, GST has been added to our 2 industry data table. Reading down column 3, it is observed that the GST levied on the Agriculture sector is \$356 and \$54,226 on the non Agriculture industry.

Both taxes and margins are treated as an input requirement for each industry producing output.

The GST for example, requires that final users pay a tax of 10% of the dollar value of a purchase to the government. Therefore, the demand for GST can be modelled as a commodity consumed proportionately to the final demand.

| | Agriculture | Non-Agriculture | Final demand |
|---|---|---|---|
| Agriculture | \$12,993 | \$32,081 | \$18,838 |
| Non-Agriculture | \$24,062 | \$1,185,272 | \$1,447,819 |
| GST-Agriculture | \$0 | \$0 | \$356 |
| GST-Non-Agriculture | \$0 | \$0 | \$54,226 |
| Labour | \$6,446 | \$777,492 | |
| Capital | \$20,411 | \$662,308 | |
| VA | \$26,857 | \$1,439,800 | |
| Total Sales | \$63,912 | \$2,657,153 | |

Table 2.7: Australian USE table (Product by Industry), including GST, for the year 2009-10 (\$M)

### 2.6.3 Power of tax

Ignoring the presence of margins, consider the example where an increase in the GST was simulated. The final price is defined:

$$P_{3,i} = (1 + T_i)P_{0,i} \tag{2.93}$$

where $P_{3,i}$ is the price paid by final domestic users, $T_i$ is the GST rate and $P_{0,2}$ is the price that is received by producers. Since in this framework the model is solved as percentage changes from an equilibrium level and taxes can be zero, The relationship above is described using the power of the tax, such that:

$$P_{3,2} = (V_2)P_{0,2} \tag{2.94}$$

Where $V_i = (1 + T_i)$ is referred to as the power of the tax. To illustrate why its important to use the power of the tax, consider the scenario where a tax rate on commodity $i$ is increased from 0 to 0.1. It would not be possible to calculate this as a percentage change, as:

$$t_i = \frac{0.1 - 0}{0} \tag{2.95}$$

.

is undefined. However, when considering the power of the tax, the percentage change can be calculated as:

$$t_i = \frac{1.1 - 1}{1} * 100 = 10\% \tag{2.96}$$

Alternatively, the the relationship could be described as a differential, such that:

$$dP_{3,i} = (1 + T_i)dP_{0,i} + P_{0,i}dT_i \tag{2.97}$$

In which case if the tax was increased from 0 to 0.1, i.e $dT_i = 0.1$, it would be described as an increase of 10 percentage points.

## 2.7 Multiproduction

Up until this point, its been assumed that each industry only produces one commodity. However, this is often not the case, with industries producing a range of products which get classified as different composite commodities. For example, it may be observed that the primary production of a farm consists of farming of cattle. This farm may also install wind turbines which generate electricity. This would result in the Agriculture industry producing both commodities as the cattle would be classified as Agriculture and the electricity generated would be considered non-Agriculture. The data containing the production of commodities by each industry is contained in the MAKE matrix. The MAKE matrix is presented in Table 2.8.

| | Agr | Non-Agr |
|---|---|---|
| Agr | 62,486 | 45 |
| Non-Agr | 3392 | 2,403,778 |

Table 2.8: Australian MAKE matrix (Product by Industry), disaggregated by source of production, forthe year 2009-10 ($M)

The column values describe the value of commodity production by each industry. For example, the first column shows the Agriculture industry produces $62,486 worth of their primary Agriculture commodity and $6,392 worth of the secondary commodity Non-Agriculture.

For each industry, the column value of the use table equals the column value of the MAKE matrix, as the expenditure for an industry must equal the value of industry sales. Moreover, the row of the USE table also equals the row of the MAKE matrix, as the value of production must equal the total value of consumption of the respective commodity. Comparing the MAKE matrix (2.8) with the USE table (2.6) it is observed that the row and column totals are both equal for the USE and MAKE matrix.

To allow for the multiple production of commodities by industries, it is assumed that industries produce output according to a Production Possibilties Frontier (PPF). The PPF describes all the feasible combinations of output that an industry can produce. In ORANIG, the PPF is represented by the Constant Elasticity of Transformation (CET) function, which is defined:

$$Z_i^\rho = \alpha_{1,i} Q_{1,i}^\rho + \alpha_{2,i} Q_{2,i}^\rho \tag{2.98}$$

where $Z_i$ describes the production possibilities frontier, $Q_{1,1}$ is the production of commodity 1 by industry 1 and $Q_{2,1}$ is the production of commodity 2 by industry 1 and $\rho$ and $\alpha$ are both parameters. The CET function is analogous to the CES function, except that it is required that $\rho > 1$.

Industries choose output to maximise profit, given the production possibilities frontier in (2.98).

The Lagrangian is expressed:

$$\mathcal{L}_i = P_{0,1}Q_{1,i} + P_{0,2}Q_{2,i} + \lambda_i[Z_i^\rho - \alpha_{1,i}Q_{1,i}^\rho + \alpha_{2,i}Q_{2,i}^\rho] \tag{2.99}$$

The first-order conditions are:

$$\frac{d\mathcal{L}}{dQ_{1,j}} = P_{0,1} - \lambda_j \alpha_{1,j} \rho Q_{1,j}^\rho = 0 \tag{2.100}$$

$$\frac{d\mathcal{L}}{dQ_{2,j}} = P_{0,2} - \lambda_j \alpha_{2,j} \rho Q_{2,j}^\rho = 0 \tag{2.101}$$

combining (2.100) and (2.101) yields:

$$\frac{\alpha_{2,j}P_{0,1}}{\alpha_{1,j}P_{0,2}} Q_{2,j}^{\rho-1} = Q_{1,j}^{\rho-1} \tag{2.102}$$

setting $Q_{1,i}$ as the subjective yields:

$$Q_{1,i} = [\frac{\alpha_{2,j}P_{0,1}}{\alpha_{1,j}P_{0,2}}]^{\frac{1}{\rho-1}} Q_{2,j} \tag{2.103}$$

Substituting it back into (2.98) yields:

$$Z_j^\rho = \alpha_{1,j}[\frac{\alpha_{2,j}P_{0,1}}{\alpha_{1,j}P_{0,2}}]^{\frac{\rho}{\rho-1}} Q_{2,j}^\rho + \alpha_{2,j}Q_{2,j}^\rho \tag{2.104}$$

which simplifies to:

$$Z_j^\rho = Q_{2,j}^p[\alpha_{1,j}[\frac{\alpha_{2,j}P_{0,1}}{\alpha_{1,j}P_{0,2}}]^{\frac{\rho}{\rho-1}} + \alpha_{2,j}[\frac{\alpha_{2,j}P_{0,2}}{\alpha_{2,j}P_{0,2}}]^{\frac{\rho}{\rho-1}}] \tag{2.105}$$

giving:

$$Z_j^\rho = Q_{2,j}^p(\frac{\alpha_{2,j}}{P_{0,2}})^{\frac{\rho}{\rho-1}}[\alpha_{1,j}[\frac{P_{0,1}}{\alpha_{1,j}}]^{\frac{\rho}{\rho-1}} + \alpha_{2,j}[\frac{P_{0,2}}{\alpha_{2,j}}]^{\frac{\rho}{\rho-1}}] \tag{2.106}$$

which gives the supply response function:

$$Q_{2,j} = Z_j(\frac{P_{0,2}}{\alpha_{2,j}})^{\frac{1}{\rho-1}} \frac{1}{[\alpha_{1,j}^{\frac{1}{\rho-1}} P_{0,1}^{\frac{\rho}{\rho-1}} + \alpha_{2,j}^{\frac{1}{\rho-1}} P_{0,2}^{\frac{p}{\rho-1}}]^{\frac{1}{\rho}}} \tag{2.107}$$

which can be generalised:

$$Q_{i,j} = Z_i(\alpha_{i,j})^{-\sigma}(\frac{P_{0,j}}{P_{AVE}})^\sigma \tag{2.108}$$

where:

$$P_{AVE} = [\alpha_{1,j}^{\frac{1}{\rho-1}} P_{0,1}^{\frac{\rho}{\rho-1}} + \alpha_{2,j}^{\frac{1}{\rho-1}} P_{0,2}^{\frac{\rho}{\rho-1}}]^{\frac{\rho-1}{\rho}} \tag{2.109}$$

and:

$$\sigma = \frac{1}{\rho - 1} \tag{2.110}$$

The linearised version of this equation can be represented as:

$$q_{i,j} = z_j + \sigma(p_i - p_{ave}) \tag{2.111}$$

where $p_{ave}$ is the share weighted price such that:

$$p_{ave} = S_{1,j}p_1 + S_{2,j}p_2 \tag{2.112}$$

and $S_{i,j}$ is the cost share:

$$S_{i,j} = \frac{P_i Q_{i,j}}{P_1 Q_{1,j} + P_2 Q_{2,j}} \tag{2.113}$$

In this setup $Z_j$ determines the feasible combinations of output for $Q_{1,j}$ and $Q_{2,j}$ and is treated as exogenous in the profit maximisation problem (2.99) . The magnitude of the PPF is determined by the intermediate inputs and primary factors available to an industry and provides the link between the resources available to an industry and the output-mix produced by the corresponding industry. The formal explanation of how this is modelled is not provided in this section, but is covered in (2.9.9).

Currently, the theory described for imports, margins and multi-production in the stylised model sets the foundations for the ORANIG framework. The remainder of this chapter will describe the how the model is further disaggregated into more users and their functional forms for those agents.

## 2.8 ORANIG

## 2.9 Final demand

In the stylised model final demand was represented by a single representative household. In the ORANIG framework, final demand is decomposed into 5 different agents: households, investors, exporters, government and inventory. In the stylised model, final demand was represented using the variable $Y_i$. For the remainder of the chapter all users ($u$) of commodities will be represented by the variable $X_{u,c,s,i}$ where the subscript $u = 0$ denotes supply, $u = 1$ intermediate demand, $u = 2$ investment, $u = 3$ for final consumption, $u = 4$ exports, $u = 5$ governent and $u = 6$ inventory. When an index is not applicable to a variable, it will be denoted with $(\cdot)$. For example, household demand is denoted $X_{3,c,s,\cdot}$ since there is no industry ($i$) dimension for households. Finally, a subscript with a bar and raised as an exponent denotes a composite or aggregate of a variable. For example, $X^{\bar{s}}_{3,c,\cdot}$ represents a composite of domestic and imported composite commodity $c$ consumed by households.

### 2.9.1 Household consumption

In Chapter (2.4) household demand is represented with Cobb Douglas preferences. In ORANIG, each commodity is treated as a composite commodity of imported and domestic commodities, similar to how intermediate demand for domestic and imported commodities in section (2.6.1) are treated as a composite commodities. Preferences for composite commodities are represented using Stone-Geary preferences, where household utility ($U$) is defined:

$$U = \prod_{c=1}^{C} (X^{\bar{s}}_{3,c,\cdot} - \bar{Y}_{3,c,\cdot})^{\beta_c} \tag{2.114}$$

where $X^{\bar{s}}_{3,c,\cdot}$ is the luxury consumption, $\bar{Y}_{3,c,\cdot}$ is the subsistence level of consumption and $\beta_c$ is a preference parameter. Similar to Cobb Douglas preferences, $0 \leq \beta_c \leq 1$ and $\sum_{c=0}^{C} \beta_c = 1$. In fact, if the subsistence term $\bar{Y}_{3,c,\cdot} = 0$ for all commodities, (2.114) collapses to Cobb Douglas preferences.

The luxury consumption $X^{\bar{s}}_{3,c,\cdot}$ variable has $\bar{s}$ as an exponent to signify that it is a composite across different sources, e.g, it is a composite of imported and domestic production.

The subsistence term $\bar{Y}_{3,c,\cdot}$ represents the notion that consumers will always consume a fixed proportion of each commodity, regardless of their income.

The expenditure constraint $M$ is defined as:

$$M = \sum_{c=0}^{C} P_{3,c} X^{\bar{s}}_{3,c,\cdot} \tag{2.115}$$

where $P^{\bar{s}}_{3,c}$ is the price of the composite commodity $c$. The Lagrangian function can be constructed by combining the budget constraint (2.115) and the utility function (2.114):

$$\mathcal{L} = \prod_{c=1}^{C} (X^{\bar{s}}_{3,c,\cdot} - \bar{Y}_{3,c,\cdot})^{\beta_c} - \lambda[M - \sum_{c=0}^{C} P_{3,c} X^{\bar{s}}_{3,c,\cdot}] \tag{2.116}$$

The corresponding demand equation for each composite commodity $c$ is expressed:

$$X^{\bar{s}}_{3,c,\cdot} = \bar{Y}_{3,c,\cdot} + \frac{\beta_c}{P_{3,c}} (M - \sum_{c=0}^{C} P_{3,c} \bar{Y}_{3,c,\cdot}) \tag{2.117}$$

These set of demand equations are often referred to as the *Linear Expenditure System (LES)* as demand can be represented as a linear relationship between household expenditure and expenditure on subsistence commodities (Stone, 1954). This can be shown by multiplying both sides of (2.117) by the price $(P^{\bar{s}}_{3,c})$, such that:

$$P^{\bar{s}}_{3,c} X^{\bar{s}}_{3,c,\cdot} = P^{\bar{s}}_{3,c} \bar{Y}_{3,c,\cdot} + \beta_c (M - \sum_{c=0}^{C} P^{\bar{s}}_{3,c} \bar{Y}_{3,c,\cdot}) \tag{2.118}$$

A feature of this demand equation is the preference parameter $\beta_C$ equals the marginal budget share:

$$\frac{d(P_{3,c} X^{\bar{s}}_{3,c,\cdot})}{dM} = \beta_c \tag{2.119}$$

Often expenditure $(M)$ is divided into necessary expenditure $(M_s)$ and luxury expenditure $(M_L)$ such that:

$$M = M_L + M_S \tag{2.120}$$

where the necessary expenditure $M_S$ is the expenditure required to purchase subsistence commodities, giving:

$$M_S = \sum_{c=0}^{C} P_{3,c}\bar{Y}_{3,c,\cdot} \tag{2.121}$$

In addition, luxury demand $X^L_{3,c}$ is defined:

$$X^L_{3,c} = X^{\bar{s}}_{3,c,\cdot} - \bar{Y}_{3,c,\cdot} \tag{2.122}$$

by doing so allows the demand equation in (2.118) to be represented as:

$$P_{3,c}X^L_{3,c} = \beta_c M_L \tag{2.123}$$

The linearised demand equation for luxury consumption is represented as

$$x^L_{3,c} = m_L - p_{3,c} \tag{2.124}$$

which would be the same as the linearised Cobb-Douglas demand equation in (2.54) if there was no subsistence consumption. The total demand for commodity $c$ is expressed as

$$x^{\bar{s}}_{3,c,\cdot} = \beta^{LUX} x^L_{3,c} + (1 - \beta^{LUX})\bar{y}_{3,c,\cdot} \tag{2.125}$$

where the luxury budget share $\beta^{LUX}$ is defined:

$$\beta^{LUX} = \frac{X^{\bar{s}}_{3,c,\cdot} - \bar{Y}_{3,c,\cdot}}{X^{\bar{s}}_{3,c,\cdot}} \tag{2.126}$$

Often there is insufficient data on consumption expenditure to distinguish between the subsistence and luxury expenditure. The ORANIG model relies upon econometric estimates of the Frisch ($\epsilon_F$) and household expenditure elasticity ($\epsilon^{HH}_c$) parameters to infer initial values of $B^{LUX}$.

The household expenditure elasticity $\epsilon^{HH}_c$ is defined:

$$\epsilon^{HH}_c = \frac{dP_{3,c}X^{\bar{s}}_{3,c,\cdot}}{dM} * \frac{M}{P_{3,c}X^{\bar{s}}_{3,c,\cdot}} = \frac{\beta_c M}{P_3, cX^{\bar{s}}_{3,c,\cdot}} \tag{2.127}$$

Rearranging the demand equation (2.117), the marginal budget share can be expressed

$$\beta_c = \frac{(X^{\bar{s}}_{3,c,\cdot} - \bar{Y}_{3,c,\cdot})}{M - M_S} \tag{2.128}$$

Substituting (2.128) into (2.127) gives the following expression for household expenditure:

$$\epsilon^{HH}_c = \frac{X^{\bar{s}}_{3,c,\cdot} - Y^{\bar{s}}_{3,c,\cdot}}{X^{\bar{s}}_{3,c,\cdot}} \frac{M}{M - M_S} \tag{2.129}$$

The Frisch parameter, which can be interpreted as the marginal utility of money with respect to income, is defined:

$$\epsilon_F = \frac{d\lambda}{dM} \frac{M}{\lambda} = \frac{M}{M - M_s} \tag{2.130}$$

where the Frisch parameter equals the inverse ratio of the supernumerary income to total income. Substituting (2.130) into (2.129) gives:

$$\epsilon^{HH}_c = \frac{X_3, c^{\bar{s}} - \bar{Y}_3, c^{\bar{s}}}{X_3, c^{\bar{s}}} \epsilon_F \tag{2.131}$$

Thus, the luxury budget share $B^{LUX}$ is defined as:

$$B^{LUX} = \frac{\epsilon^{HH}_c}{\epsilon_F} \tag{2.132}$$

It is important to note that the luxury budget share $B^{LUX}$ is not a parameter and must be updated as luxury consumption increases.

### 2.9.2 Investment

The following two sections outline the two investment rules adopted in the ORANIG framework.

### 2.9.3 Investment rule 1: rate of return

Investment determines the aggregate level of capital stock for an industry. In ORANIG, there is flexibility in how investment is determined for each industry, with investment being determined via two rules. The first investment rule (Investment rule 1) allows for investment to be determined as a function of the rate of return.

The rate of return $R_{t,j}$ for any period $(t)$ and industry $j$ is defined as:

$$R_{t,j} = \frac{P_{2,j}}{\Pi_j} - d_j \tag{2.133}$$

where $P_{2,j}$ is the rental value, $\Pi_j$ is the cost of a unit of capital and $d_j$ is the depreciation rate of capital, which is assumed to be fixed. In the static model, it is assumed that capital takes one period to be built. In essence, the investor purchases the unit of capital in period $j$ for the cost $\Pi_J$ and then sells the same unit of capital for $P_{2,j}$ the next period, but loses $d_j$ to depreciation.

The evolution of capital is defined:

$$K_{t+1,j} = (1 - d_j)K_{t,j} + X_{2,t,j} \tag{2.134}$$

where $X_{2,t,j}$ is the investment demand for industry j. As it takes one period for investors to realise a return, investment in the current period is determined based upon the expected ROR for the next period. The relationship between expected rate of return $(E[R_{t+1,j}])$ and the expected level of capital $E[K_{t+1,j}]$ is assumed to be negatively related, such that:

$$E[R_{t+1,j}] = R_{t,j}(\frac{E[K_{t+1,j}]}{K_{t,j}})^{\psi_j} \tag{2.135}$$

where $\psi_j > 0$ and represents the sensitivity of the $ROR$ to additional units of capital in the next period. If the expected quantity of capital increases in the next period, the corresponding $ROR$ decreases for that industry. Given that investors only care about maximising profits, the returns will converge across all industries, giving:

$$R_{t,j} = R_{t,i} = \Omega \quad \forall i, j \tag{2.136}$$

The market clearing condition for investment implies that investment expenditure $I$ across all industries equals total capital formation, such that:

$$I = \sum_{j=1}^{J} \Pi_j X_{2,j} \tag{2.137}$$

The equations outlined thus far are used to determine the level of investment per industry and the rate of return within the economy. The linearised version of these equations are outlined below. The $ROR$ of return is expressed:

$$r_{t,j} = RR_j[p_{2,j} - \pi_j] \tag{2.138}$$

where $RR_j = \frac{R_{i,j}+d_j}{R_{t,j}}$ which is the ratio of the gross rate of returns to the net rate of returns. The percentage change form of the capital evolution path is:

$$k_{t+1,j} = (1 - KK_j)k_{t,j} + (KK_j)x_{2,j} \tag{2.139}$$

Where $KK_j = \frac{X_t}{K_{t+1}}$ is the ratio of gross investment to future capital stock. Both $KK_j$ and $RR_j$ are treated as parameters within the ORANIG framework and thus are not calibrated on any underlying data.

The relationship between the rate of return and capital stock can be expressed:

$$-\psi_j(k_{t+1,j} - k_{t,j}) + r_{t,j} = \omega_j \tag{2.140}$$

Finally the total investment market clearing condition is defined:

$$(\sum_{j=1}^{J} XX_{2,j})i = \sum_{j=1}^{J} XX_{2,j}[\pi_j + x_{2,j}] \tag{2.141}$$

where $XX_{2,j} = \Pi_j X_j$ and $i$ is the percentage change in aggregate investment.

### 2.9.4 Investment rule 2: exogenous

The second investment rule is to assume that investment is exogenous. It is often inappropriate to have investment determined using rates of return, using the rule outlined above. For example, industries such as the health care sector, which are heavily dominated by the government, are unlikely to respond to changes in $ROR$ the same way in which private investors would respond. In these cases, it is often assumed that investment is exogenous or reflects the average level of real investment in the economy, such that:

$$X_n = I_R^{\bar{h}} F_n^{\bar{2}} \tag{2.142}$$

where $I_R^{\bar{h}}$ is the level of real investment, $\bar{h}$ defines whether investment grows at a faster or slower pace than real investment and $F_n^{\bar{2}}$ is a shift variable. the subscript $n$ is used to denote industries which do not respond to the $ROR$ rule above, such that $n \notin J$. The real level of investment $I_R$ is defined as:

$$I_R = \frac{I}{P^{CAP}} \tag{2.143}$$

where $P^{CAP}$ is a capital-goods price index. These two equations can be combined and linearised as follows:

$$x_n = \bar{h}[i + p^{CAP}] + f_n^{\bar{2}} \tag{2.144}$$

This summarises how investors determine their level of investment in each industry.

### 2.9.5 Capital formation

Capital formation is the process by which intermediate commodities purchased by investors are converted into capital goods. The production process for turning intermediate commodities into capital ($K_i$) is similar to that of producing goods for final consumption. Capital ($K_i$) for industry $i$ is produced according to a Leontief production function:

$$K_i = MIN[\frac{1}{A_{1,i}^{\bar{S},CAP}} X_{2,1,i}^{\bar{S}}, \cdots, \frac{1}{A_{C,i}^{\bar{S},CAP}} X_{2,C,i}^{\bar{S}}] \tag{2.145}$$

where $A_{c,i}^{\bar{S},CAP}$ is a technology parameter and $X_{2,c,i}^{\bar{S}}$ is a composite of imported and domestically produce commodity $c$ used by industry $i$. The production of capital does not use any primary factors.

Each composite commodity $X^{\bar{S}}_{2,c,i}$ is produced according to a CES production function such that:

$$X^{2,\bar{S}}_{c,i} = [(\frac{X_{2,c,1,i}}{A^{CAP}_{c,1,i}})^{-\rho^{s}_{2,c,i}} + (\frac{X_{2,c,1,i}}{A^{CAP}_{c,1,i}})^{-\rho^{s}_{2,c,i}}]^{\frac{1}{-\rho^{s}_{2,c,i}}} \tag{2.146}$$

where $A^{CAP}_{c,1,i}$ is a technology parameter. These functional forms give the familiar percentage change demand equations as:

$$k_i = x^{\bar{S}}_{2,c,i} - a^{\bar{S}}_{c,i} \tag{2.147}$$

for the top level of capital production and:

$$x_{2,c,s,i} - a_{2,c,s,i} = x^{\bar{s}}_{2,c,i} + \sigma^{\bar{s}}_{2,c,i}(p_{2,c,s,i} + a_{2,c,s,i} - p^{\bar{s}}_{2,c,i}) \tag{2.148}$$

for the choice between imports and domestic, where $\sigma^{\bar{s}}_{2,c,i} = \frac{1}{1+\rho^{s}_{2,c,i}}$ is the elasticity of substitution parameter. The average price term $p^{\bar{s}}_{2,c,i}$ is defined:

$$p^{\bar{s}}_{2,c,i} = \sum_{s}^{S} S_{2,c,s,i}(p_{2,c,s,i} + a_{2,c,s,i}) \tag{2.149}$$

where $S_{2,c,s,i} = \frac{P_{2,c,s,i}X_{2,c,s,i}}{\sum_{s}^{S} P_{2,c,s,i}X_{2,c,s,i}}$.

### 2.9.6 Exports

Export commodities are segmented into commodities which are traditional exports and those which are non-traditional exports. The traditional export commodities face a downward sloping demand curve which relates the quantity of exports demanded with the price of the respective commodity. Its level form is represented as:

$$X_{4,i} = A^{q}_{4,i} = (\frac{P_{4,i}}{\phi A^{p}_{4,i}})^{\nu_i} \tag{2.150}$$

where $X_{4,i}$ is the demand for the export commodity, $P_{4,i}$ is the price of the export commodity, $\phi$ is the indirect quote of the exchange rate, i.e, how many units of domestic currency for one unit of foreign currency.

The $\nu_i$ represents the constant elasticity of demand parameter which is typically set at -5.0 (Horridge et al., 2000). The terms $A^q_{4,i}$ and $A^p_{4,i}$ are both shift terms with the former used to shift the demand curve out vertically whilst the latter moves the demand curve down horizontally. The linearised version of this equation is:

$$x_{4,i} = a^Q_{4,i} + \nu_i(p_{4,i} - (\phi + a^P_{4,i})) \tag{2.151}$$

The non-traditional export commodities are the collective export group, which are the commodities where demand does not depend the individual's commodity price (e.g, service goods). The export demand for these commodities responds to the collective price (i.e, the average price of all commodities) instead of changes to their individual price. In this case, the linearised relationship is:

$$x_{4,j} = a^{NE}_{4,i} + xx_{4,j} \tag{2.152}$$

where $j$ represents all the non-traditional industries, $a^{NE}_{4,i}$ is a shift variable and $xx_{4,j}$ the collective quantity to be exported. Similar to the individual export demand equation, the collective export demand equation in its linearised form is defined:

$$xx_{4,i} = a^{NQ}_{4,j} + \nu(P^n_{4,j} - (\phi - a^{NP}_{4,i})) \tag{2.153}$$

where $a^{NQ}_{4,j}$ and $a^{NP}_{4,j}$ are both shift parameters, $\nu$ is the constant elasticity of demand parameter and $P^n_{4,j}$ is the average price of all non-traditional commodities. In the ORANI model of Australia, it is assumed all commodities has some market power (Dixon et al., 1982).

### 2.9.7 Government and inventory demand

Government demand and inventory demand comprise the final components of final demand. There is no economic theory within the ORANIG framework which describes government demand. Instead, government demand is either set exogenous or is assumed to be linked to household consumption.

In this case, the government is defined as:

$$X_{5,i} = A_{5,i} X_3^{\bar{c}} \tag{2.154}$$

Where $A_{5,i}$ is a shift variable and $X_3^{\bar{c}}$ represents aggregate real household consumption. Converted to percentage change form, it becomes:

$$x_{5,i} = a_{5,i} + x_3^{\bar{c}} \tag{2.155}$$

Similarly, ORANIG has no underlying economic theory to describe demand for inventories. Instead, it imposes that inventories grow proportionately to the total supply of production, such that:

$$x_{6,c} = x_c^{0,COM} \tag{2.156}$$

where $x_{6,c}$ is the percentage change in inventory for commodity $c$. The inventory demand equation (2.156) is expressed as a differential to allow for an inventory of zero. The relationship for (2.156) is expressed:

$$100 \frac{dX_{6,c}}{X_{6,c}} = x_c^{0,COM} \tag{2.157}$$

$$100(dX_{6,c}) = X_{6,c} x_c^{0,COM} \tag{2.158}$$

Multiplying both sides of this equation by price allows the relationship to be expressed in terms of expenditure:

$$100(P_{6,c} dX_{6,c}) = P_{6,c} X_{6,c} x_c^{0,COM} \tag{2.159}$$

where expenditure is defined as $V_{6,c} = P_{6,c} X_{6,c}$ such that:

$$100(P_{6,c} dX_{6,c}) = V_{6,c} x_c^{0,COM} \tag{2.160}$$

### 2.9.8 Intermediate demand

### 2.9.9 Industry level

In section (2.7) output occurs according to a PPF ($Z_i$). The magnitude of the PPF is defined by the combination of resources available to that industry. The PPF ($Z_i$) is represented by a Leontief production function of composite commodities and factors, such that:

$$Z_i = MIN[\frac{X^{\bar{s}}_{1,1,i}}{A^{1,\bar{s}}_{1,1,i}}, ..., \frac{X^{\bar{s}}_{1,C,i}}{A^{1,\bar{s}}_{C,i}}, \frac{X^{1,PRIM}_i}{A^{1,PRIM}_i}, \frac{OC_i}{A^{OC}_i}] \tag{2.161}$$

where $X^{\bar{s}}_{1,c,i}$ is intermediate demand for the composite of imported and domestically produced commodity $c$, $X^{1,PRIM}_i$ is a composite of primary factors and $OC_i$ is other costs, which covers miscellaneous taxes such as production taxes. $A^{1,\bar{s}}_{C,i}$, $A^{1,PRIM}_i$ and $A^{OC}_i$ are all technology parameters for intermediate ($X^{\bar{s}}_{1,c,i}$), primary factor ($X^{1,PRIM}_i$) and other cost ($OC_i$) demand, respectively.

The corresponding demand equations are all proportionate to the PPF, such that:

$$X^{\bar{s}}_{1,c,i} = A^{1,\bar{s}}_{1,c,i} Z_i, \quad \text{for } c = 1, ..., C \tag{2.162}$$

$$X^{1,PRIM}_i = A^{1,PRIM}_i Z_i \tag{2.163}$$

$$OC_i = A^{oc}_i Z_i \tag{2.164}$$

where (2.162) is the demand for intermediate commodities, (2.163) is the demand for the primary factor composite and (2.164) is the demand for other costs. The percentage change form of these equations may be expressed as follows:

$$x^{\bar{s}}_{1,c,i} = a^{1,\bar{s}}_{1,c,i} + z_i, \quad \text{for } c = 1, ..., C \tag{2.165}$$

$$x^{1,PRIM}_i = a^{1,PRIM}_i + z_i \tag{2.166}$$

$$oc_i = a^{oc}_i + z_i \tag{2.167}$$

where in percentage change form, (2.165) is the demand for intermediate commodities, (2.166) is the demand for the primary factor composite and (2.167) is the demand for other costs.

### 2.9.10 Primary factors

Primary factors consist of labour (a composite of different occupation types), capital and land. Primary factors ($X^{1,PRIM}$) are modelled according to CES production structure, such that:

$$X_i^{1,PRIM} = [\alpha_i^{\bar{o},LAB}(\frac{L_i^{\bar{O}}}{A_i^{\bar{O},LAB}})^{\rho_{prim,i}} + \alpha_i^{CAP}(\frac{K_i}{A_i^{CAP}})^{\rho_{prim,i}} + \alpha_i^{LND}(\frac{N_i}{A_i^{LND}})^{\rho_{i,prim}}]^{\frac{1}{\rho_{prim,i}}} \quad (2.168)$$

where $L_i^{\bar{O}}$ represents the composite commodity of different labour occupations, $K_i$ is capital, $N_i$ is land; $A_i^{\bar{O},LAB}$, $A_i^{CAP}$ and $A_i^{LND}$ are the technology parameters for labour, capital and land, respectively and $\alpha_i^{\bar{O}LAB}$, $\alpha_i^{cap}$ $\alpha_i^{LND}$ are share parameters for labour, capital and land, respectively. Each industry $i$ chooses inputs to minimise costs. The corresponding percentage change demand equations for each primary factor are as follows:

$$l_i^{\bar{O}} - a_i^{\bar{O},LAB} = x_i^{1,PRIM} - \sigma_{prim,i}(p_i^{\bar{O},LAB} + a_i^{\bar{O},LAB} - p_i^{1,PRIM}) \quad (2.169)$$

$$k_i - a_i^{CAP} = x_i^{1,PRIM} - \sigma_{prim,i}(p_i^{CAP} + a_i^{CAP} - p_i^{1,PRIM}) \quad (2.170)$$

$$n_i - a_i^{LND} = x_i^{1,PRIM} - \sigma_{prim,i}(p_i^{LND} + a_i^{LND} - p_i^{1,PRIM}) \quad (2.171)$$

where $P_i^{\bar{O},LAB}$, $P_i^{CAP}$, $P_i^{LND}$ are the prices of composite labour, capital and land respectively. The elasticity of substitution parameter $\sigma_{prim,i}$ is defined:

$$\sigma_{prim,i} = \frac{1}{1 - \rho_{prim,i}} \quad (2.172)$$

$P_i^{1,PRIM}$ is a share-weighted price of all commodities, such that:

$$p_i^{1,PRIM} = S_i^{\bar{O},LAB}(p_i^{\bar{O},LAB} + a_i^{\bar{O},LAB}) + S_i^{CAP}(p_i^{CAP} + a_i^{CAP}) + S_i^{LND}(p_i^{LND} + a_i^{LND}) \quad (2.173)$$

where $S$ is used to denote factor shares of each primary factor, such that:

$$S_i^{LAB} = \frac{P_i^{\bar{O},LAB} X_i^{\bar{O},LAB}}{V^{1,PRIM}} \tag{2.174}$$

$$S_i^{CAP} = \frac{P_i^{CAP} X_i^{CAP}}{V^{1,PRIM}} \tag{2.175}$$

$$S_i^{LND} = \frac{P_i^{LND} X_i^{LND}}{V^{1,PRIM}} \tag{2.176}$$

and

$$V_i^{1,PRIM} = P_i^{\bar{O},LAB} X_i^{\bar{O},LAB} + P_i^{CAP} X_i^{CAP} + P_i^{LND} X_i^{LND} \tag{2.177}$$

which denotes the total expenditure on primary factors by an industry.

### 2.9.11 Labour occupation

In the previous section, the labour primary factor was a composite factor of $O$ different occupation types. The composite ($X_i^{\bar{O},LAB}$) is also defined as a CES function where:

$$X_i^{\bar{O},LAB} = (\sum_o^O \alpha_{i,o}^{lab} (X_{i,o}^{LAB})^{\rho_{LAB,i,o}})^{\frac{1}{\rho_{LAB,i,o}}} \tag{2.178}$$

The demand for each occupation type is derived from minimising the cost function:

$$P_i^{\bar{O},LAB} X_i^{\bar{O},LAB} = \sum_o^O P_{i,o}^{LAB} X_{i,o}^{LAB} \tag{2.179}$$

subject to the production function (2.178). This yields percentage-change demand equations similar to primary factor demand, such that:

$$x_{i,o}^{LAB} = x_i^{\bar{O},LAB} + \sigma_{LAB,i,o}(p_{i,o}^{LAB} - p_i^{\bar{O},LAB}) \tag{2.180}$$

where:

$$p_i^{\bar{O},LAB} = \sum_o^O S_{i,o}^{LAB} p_{i,o}^{LAB} \tag{2.181}$$

and

$$S_{i,o}^{LAB} = \frac{P_{i,o}^{LAB} X_{i,o}^{LAB}}{P_i^{\bar{O},LAB} X_i^{\bar{O},LAB}} \tag{2.182}$$

### 2.9.12 Domestic and imported intermediate demand

The steps followed to determine imported and domestic intermediate demand in ORANIG are the same as those outlined in section (2.6.1) with the exception that ORANIG has the preference parameters ($a_{1,c,s,i}$). The linearised demand equations can thus be described as:

$$x_{1,c,s,i} - a_{1,c,s,i} = x^{\bar{s}}_{1,c,i} + \sigma_{1,c}(p_{1,c,s,i} + a_{1,c,s,i} - p^{\bar{s}}_{1,c,i}) \tag{2.183}$$

## 2.10 Production

In section (2.7), it is outlined how industries can produce multiply commodities and how this production is modelled according to a CET production function. The ORANIG model follows the same approach, giving the linearised supply response function for each commodity $C$ as:

$$q_{c,i} = z_i + -\sigma_{CET,i}[p_{0,c} - p_{0,ave}] \tag{2.184}$$

where $q_{c,i}$ is the quantity of commodity $c$ produced by industry $i$, $z_i$ is the $PPF$, $\sigma_{CET,i}$ is the constant elasticity of transformation parameter, $P_{0,c}$ is the basic price of commodity $c$ and $p_{0,ave}$ is the average price, which is defined:

$$p_{0,ave} = \sum_{c}^{C} S_{1,i}p_{0,1} \tag{2.185}$$

where the $S_{c,i}$ is the share of expenditure on commodity $c$, such that:

$$S_{c,i} = \frac{P_i Q_{c,i}}{\sum_c^C P_{0,c}Q_{c,i}} \tag{2.186}$$

ORANIG often includes a further nest where commodities produced for the domestic market and export market are treated as imperfect substitutes. This has been excluded from this research, as it adds an unnecessary layer of complexity for little benefit to a study of labour markets.

### 2.10.1 Fan decomposition

Domestic production is sold domestically or exported overseas. This relationship can be expressed as:

$$Q = DD * S_{DD} + EXP \tag{2.187}$$

where $DD$ is the total level of domestic demand, $S_{DD}$ is the share of expenditure on domestic demand and $EXP$ are exports sold overseas. A decrease in production can be caused by the following: (1) A decrease in the total level of domestic demand ($DD$), (2) a decrease in the share of expenditure on domestically produced commodities ($S_{DD}$) and (3) a decrease in exports ($EXP$). The Fan decomposition measures how each of these 3 factors influence the change in domestic production. This decomposition is measured by converting (2.187) into percentage-change form:

$$q = DD * S_{DD}[s_{DD} + dd] + EXP * exp \tag{2.188}$$

where $q$ measures the percentage change in production, $s_{DD}$ measures the percentage change in the share of output being domestically consumed and $exp$ measures the percentage change in exports.

### 2.10.2 Sales aggregates

This section outlines the market clearing conditions for the ORANIG framework. The market clearing conditions ensure that the sum of individual demands equal the total production for commodities and primary factors.

### 2.10.3 Aggregate commodity supply

The aggregate supply ($X_{0,c}$) for any given commodity ($c$) is the aggregate of commodity production across industries. In its level form, the aggregate supply equation is:

$$X_{0,c} = \sum_{i=1}^{I} Q_{c,i}. \tag{2.189}$$

where $Q_{c,i}$ is the production of commodity $c$ by industry $i$. In the case that industries only produced their primary commodity, the supply equation collapses to

$$X_{0,c} = \begin{cases} Q_{c,i}, & \text{if } c == i \\ \\ 0, & \text{otherwise.} \end{cases} \tag{2.190}$$

In Chapter 2.5.2, it was observed how it was useful to express market clearing conditions in terms of sale flows (i.e $price * quantity$), given the underlying database is populated with expenditure data. Multiplying both sides of 2.189 by the basic price $P_c^0$ gives the relationship:

$$P_c^0 X_c^0 = \sum_{i=1}^{I} P_c^0 * Q_{c,i} \tag{2.191}$$

which when linearised is:

$$[\sum_{i=1}^{I} MAKE(c,i)] * x_{0,c} = MAKE(c,i) * q_{c,i}, \tag{2.192}$$

where:

$$MAKE(c,i) = P_c^0 * Q_{c,i} \tag{2.193}$$

The total supply of commodity $c$ expressed in percentage change form is:

$$x_{0,c} = \frac{MAKE(c,i)}{\sum_{i=1}^{I} MAKE(c,i)} * q_{c,i}. \tag{2.194}$$

## 2.11 Aggregate commodity demand

Aggregate demand is determined by aggregating demand across intermediate, margin and all final users. Before defining aggregate demand, it is useful to define aggregate domestic margin ($X_m^{\bar{ci},MAR}$) use across all users, industries and sources, as:

$$X_m^{\bar{ci},MAR} = \sum_{c=1}^{C}[X_{c,m}^{4,MAR} + \sum_{s=1}^{S}[X_{c,s,m}^{3,MAR} + X_{c,s,m}^{5,MAR} + \sum_{i=1}^{I}[X_{c,s,i,m}^{1,MAR} + X_{c,s,i,m}^{2,MAR}]]] \tag{2.195}$$

$X^{1,MAR}_{c,s,i,m}$ is the margin commodity $m$ used to facilitate the movement of commodity $c$ from the source $s$ to the industry $i$ which uses the commodity $c$ as an intermediate input. $X^{2,MAR}_{c,s,i,m}$ is the margin commodity $m$ used to facilitate the movement of commodity $c$ from the source $s$ to the industry $i$ which uses commodity $c$ for investment. $X^{3,MAR}_{c,s,m}$ is the margin commodity $m$ used to facilitate the movement of commodity $c$ from source $s$ used for household consumption. $X^{5,MAR}_{c,s,m}$ is the margin commodity $m$ used to facilitate the movement of commodity $c$ from source $s$ which is consumed by the government. $X^{4,MAR}_{c,m}$ is the margin commodity $m$ used to facilitate the export of commodity $c$.

Equating the total domestic supply of commodity $c$ to the sum of demand for all users $u$ and total margins yields:

$$X_{0,c} = \sum_{i}^{I}(X_{1,c,i} + X_{2,c,i}) + X_{3,c,1,\cdot} + X_{4,c,\cdot,\cdot} + X_{5,c,1} + X_{6,c,1} + X_{c}^{\bar{ci},MAR} \tag{2.196}$$

This market-clearing condition can be expressed in terms of expenditures by multiplying both sides of (2.196) by the basic price $P_{0,C}$, such that:

$$P_{0,C}X_{0,c} = \sum_{i}^{I}(P_{0,C}X_{1,c,i}+P_{0,C}X_{2,c,i})+P_{0,C}X_{3,c,1,\cdot}+P_{0,C}X_{4,c,\cdot,\cdot}+P_{0,C}X_{5,c,1}+P_{0,C}X_{6,c,1}+P_{0,C}X_{c}^{\bar{ci},MAR} \tag{2.197}$$

The linearised version of (2.196) can then be expressed as:

$$\begin{aligned} Sales^{\bar{u}}_{c}x_{0,c} = \sum_{i}^{I}(Sales_{1,c,i}x_{1,c,i} + Sales_{2,c,i}x_{2,c,i}) + Sales_{3,c,\cdot} \\ x_{3,c,1,\cdot} + Sales_{4,c,\cdot,\cdot}.x_{4,c,\cdot,\cdot} + Sales_{5,c,1,\cdot}.x_{5,c,1,\cdot} + Sales_{6,c,1,\cdot}.x_{6,c,1} + Sales^{MAR}_{c}x_{c}^{\bar{ci},MAR} \end{aligned} \tag{2.198}$$

where $Sales_{u,c,s,i} = P_{0,C}X_{u,c,s,i}$, $Sales^{\bar{u}}_{c,s,i} = \sum_{u=1}^{6} P_{0,c}X_{u,c,s,i}$ and $Sales^{MAR}_{c} = P_{0,c}X_{c}^{\bar{ci},MAR}$.

## 2.12 Price formation

## 2.13 Margins, Taxes and Tariffs

### 2.13.1 Margins

Margin commodities are used to facilitate trade between the producer of a commodity and the final user of the commodity. Margins commodities $X^{MAR}_{u,c,s,i}$ are used in fixed proportions to output, such that the level forms equation can be expressed:

$$X^{MAR}_{u,c,s,i} = \frac{1}{A^{MAR}_{u,c,s,i}} X_{u,c,s,i} \tag{2.199}$$

where $u$ denotes the user, $c$ the commodity, $s$ the source and $i$ is the industry, $m$ is the margin commodity, $X^{MAR}_{u,c,s,i,m}$ is the margin demand, $A^{MAR}_{u,c,s,i,m}$ is a technology parameter and $X_{u,c,s,i}$ is commodity demand. For example, $X^{MAR}_{2,"Coal",1,"Agriculture","transport"}$ represents the use of a transport commodity (m = transport) associated with the use of a domestic ($s$=1) Coal commodity ($c$ = Coal) by investors ($u$ = 2) in the Agriculture industry ($i$ = Agriculture)

The technology parameter allows for margin use to become more or less efficient. An increase in the value of $A^{MAR}_{u,c,s,i,m}$ implies less of that margin is needed. The percentage change form of (2.199) can be expressed:

$$x^{MAR}_{u,c,s,i,m} = x_{u,c,s,i} - a^{MAR}_{u,c,s,i} \tag{2.200}$$

In the absence of technological change, margins will grow proportionately with the corresponding commodity demand.

### 2.13.2 Tariffs

The treatment of taxes and tariffs are quite simple in the ORANIG framework. In ORANIG, basic prices of imported commodities are defined inclusive of tariffs, such that:

$$P^0_{c,"imp"} = P^{f0,CIF}_c * \Phi * T^{0,imp}_c \tag{2.201}$$

where $P_c^{f0,CIF}$ is the cost, insurance, freight in foreign currency units, $\Phi$ is the nominal exchange rate and $t_c^{0,imp}$ is the power of tariff levied on commodity $c$. In its percentage change form, the price equation becomes:

$$p^0_{c,"imp"} = p_c^{f0,CIF} + \phi + t_c^{0,imp} \tag{2.202}$$

Equation (2.201) can be used to calculate tariff revenue. Since $T_c^{0,IMP}$ is the power of the tariff, the tariff revenue can be calculated:

$$V_c^{0,TAR} = (T^{0,IMP} - 1)[\Phi * P_c^{f0,CIF} * X^0_{c,"imp"}] \tag{2.203}$$

Since it is possible for tariffs to be zero, the tariff revenue is modelled as a differential, such that:

$$100 * \Delta V_c^{0,TAR} = [V_c^{0,BAS}][t_c^{0,imp}] - V_c^{0,TAR}[\phi + p_c^{f0,CIF} + x^0_{c,"imp"}] \tag{2.204}$$

where:

$$V_c^{0,BAS} = \Phi * P_c^{f0,CIF} * X^0_{c,"imp"} * T^{0,IMP} \tag{2.205}$$

### 2.13.3 Indirect Taxes

Indirect taxes are those levied on commodities, not income or profits, and hence are paid indirectly.

Indirect taxes are those taxes which are levied directly on the sale of commodites and thus are paid indirectly by users. They do not include taxes on income or profit. In ORANIG all indirect taxes are modelled as proportionate to commodity sales.

For intermediate users, households, exporters and the government, the tax revenue can be expressed:

$$V^{tax}_{u,c,s,i} = (T_{u,c,s,i} - 1)P^0_{c,s,i}X_{u,c,s,i} \tag{2.206}$$

where $V^{tax}_{u,c,s,i}$ is the tax revenue paid by user $u$, on commodity $c$ from source $s$ produced in industry $i$. $T_{u,c,s,i}$, $P^0_{c,s}$ and $X_{u,c,s,i}$ are the corresponding power of tax, basic price and quantity, respectively.

The tax revenue equation is converted into differentials due to the possibility of tax revenue being zero if there are no indirect taxes applied to a commodity. The differential form for (2.206) is expressed:

$$\Delta V^{tax}_{u,c,s,i} = V^{TAX}_{u,c,s,i}[t_{u,c,s,i} + x_{u,c,s,i} + p^0_{c,s}] - V^{BAS}_{u,c,s,i}[x_{u,c,s,i} + p^0_{c,s}], \tag{2.207}$$

where:

$$V^{TAX}_{u,c,s,i} = T_{u,c,s,i} P^0_{u,c,s} X_{u,c,s,i} \tag{2.208}$$

and:

$$V^{BAS}_{u,c,s,i} = P_{c,s} X_{u,c,s,i} \tag{2.209}$$

### 2.13.4 Price formation

Chapter (2.6.2) demonstrated how taxes and margins can force a wedge between purchase prices and basic prices. In the ORANIG model, both taxes and margins place a wedge between those two prices. The price equation for each user is defined as follows:

$$P^u_{c,s} X^u_{c,s} = T^u_{c,s} X^u_{c,s} + \sum_{m=1}^{M} X^{u,MAR}_{c,s,m} P^0_{m,dom} \tag{2.210}$$

Where $P^u_{c,s}$ is the price paid and quantity consumed, respectively, by user $u$, for commodity $c$, from source $s$. $T^u_{c,s}$ is the power of the tax for user $u$, on commodity $c$, from source $s$. $\sum_{m=1}^{M} P^0_{c,s} X^{u,MAR}_{c,dom,m}$ is the sum of margins used.

Converting the equation to percentage change form gives:

$$V^{1,TOT}_{c,s}(p^u_{c,s} + x^u_{c,s}) = [V^{1,BAS}_{c,s} + V^{1,TAX}_{c,s}](t^u_{c,s} + x^u_{c,s}) + V^{1MAR}_{c,s}(x^{U,MAR\bar{m}}_{c,s} + p^{0\bar{m}}) \tag{2.211}$$

By noting that $x^{uMAR}_{c,s,m} = x_{c,s}$ it can be expressed as follows:

$$V^{1,TOT}_{c,s}(p^u_{c,s} + x^u_{c,s}) = [V^{1,BAS}_{c,s} + V^{1,TAX}_{c,s} + V^{1MAR}_{c,s}]x^u_{c,s} + [V^{1,BAS}_{c,s} + V^{1,TAX}_{c,s}](t^U_{c,s}) + V^{1MAR}_{c,s}(x^{U,MAR\bar{m}}_{c,s} + p^{0\bar{m}}) \tag{2.212}$$

Since $V_{c,s}^{1,TOT} = V_{c,s}^{1,BAS} + V_{c,s}^{1,TAX} + V_{c,s}^{1MAR}$ allows the cancellation of the $x_{c,s}$ terms from both sides, giving:

$$V_{c,s}^{1,TOT}(p_{c,s}^{u}) = [V_{c,s}^{1,BAS} + V_{c,s}^{1,TAX}](t_{c,s}^{u}) + V_{c,s}^{1MAR}(x_{c,s}^{u,MAR\bar{m}} + p^{0\bar{m}}) \tag{2.213}$$

## 2.14 Macroeconomic aggregates

### 2.14.1 Expenditure aggregates

For each final user, an aggregate price index and real output can be calculated using a similar method to what is used when calculating real GDP and the price deflator.

The aggregate earnings for the export industry is defined:

$$V^{4,tot} = P^{4,TOT}X^{4,TOT} \tag{2.214}$$

where

$$P^{4,TOT}X^{4,TOT} = \sum_{c=1}^{C} P_c^4 X_c^4 \tag{2.215}$$

This can be linearised to give:

$$p^{4,TOT} + x^{4,TOT} = \sum_{c=1}^{C} \frac{V_c^{4,PUR}}{V^{4,TOT}}[p_c^4 + x_c^4] \tag{2.216}$$

Nominal GDP can be measured by using either the income or expenditure approach. The income approach involves summing up all payments to factors of production and indirect taxes. In its level forms:

$$V^{0GDP_{inc}} = V^{1,lab\bar{IO}} + V^{1,cap\bar{I}} + V^{1,lnd\bar{I}} + V^{0,TAXC\bar{S}I} \tag{2.217}$$

where $V^{1,lab\bar{IO}}$ are aggregate payments to labour, $V^{1,cap\bar{I}}$ are aggregate payments to capital, $V^{1,lnd\bar{I}}$ are aggregate payments to land and $V^{0,TAXC\bar{S}I}$ are total indirect taxes paid.

As there exists a composite primary factor good, equation (2.217) can be reduced to:

$$V^{0GDP_{inc}} = V^{1,prim\bar{I}} + V^{0,TAXC\bar{S}I} \tag{2.218}$$

where $V^{1,prim\bar{I}} = V^{1,lab\bar{I}O} + V^{1,cap\bar{I}} + V^{1,lnd\bar{I}}$. This is expressed as a combination of percentage changes and differentials, such that:

$$V^{0GDP_{inc}} * v^{0GDP_{inc}} = V^{1,prim\bar{I}}(x^{0,prim\bar{I}} + p^{1,prim\bar{I}}) + 100 * \Delta V^{0,TAXC\bar{S}I} \tag{2.219}$$

The alternate approach to calculating GDP is to use the expenditure approach, which sums across all expenditure for final users. In its level forms,

$$V^{0GDP_{exp}} = V^{2,tot,\bar{I}} + V^{3,tot,\bar{I}} + V^{5,tot,\bar{I}} + V^{6,tot,\bar{I}} + (V^{4,tot,\bar{I}} - V^{CIF,\bar{c}}) \tag{2.220}$$

where $V^{2,tot,\bar{I}}$ is the expenditure by investors, $V^{3,tot,\bar{I}}$ is the expenditure by households, $V^{5,tot,\bar{I}}$ is government expenditure, $V^{6,tot,\bar{I}}$ is the value of inventories and $(V^{4,tot,\bar{I}} - V^{CIF,\bar{c}})$ is net exports.

At this point, it is useful to define an aggregate price and quantity index, such that:

$$V^{0GDP_{exp}} = P^{def} * X^{0GDP_{exp}} \tag{2.221}$$

where $P^{def}$ is the price deflator and $X^{0GDP_{exp}}$ is real GDP. In linearised form (2.222) can be expressed:

$$\begin{aligned} V^{0GDP_{exp}} = P^{def} + X^{0GDP_{exp}} = V^{2,tot,\bar{I}}v^{2,tot,\bar{I}} + V^{3,tot,\bar{I}}v^{3,tot,\bar{I}} + \\ V^{5,tot,\bar{I}}v^{5,tot,\bar{I}} + V^{6,tot,\bar{I}}v^{6,tot,\bar{I}} + (V^{4,tot,\bar{I}}v^{4,tot,\bar{I}} - V^{CIF,\bar{c}}v^{CIF,\bar{c}}) \end{aligned} \tag{2.222}$$

This gives the ability to calculate the percentage-change variables for the price deflator and real GDP as:

$$\begin{aligned} p^{def} = V^{2,tot,\bar{I}}p^{2,tot,\bar{I}} + V^{3,tot,\bar{I}}p^{3,tot,\bar{I}} + V^{5,tot,\bar{I}}p^{5,tot,\bar{I}} + \\ V^{6,tot,\bar{I}}p^{6,tot,\bar{I}} + (V^{4,tot,\bar{I}}p^{4,tot,\bar{I}} - V^{CIF,\bar{c}}p^{CIF,\bar{c}}) \end{aligned} \tag{2.223}$$

and:

$$x^{GDP} = V^{2,tot,\bar{I}}x^{2,tot,\bar{I}} + V^{3,tot,\bar{I}}x^{3,tot,\bar{I}} + V^{5,tot,\bar{I}}x^{5,tot,\bar{I}} + V^{6,tot,\bar{I}}x^{6,tot,\bar{I}} + (V^{4,tot,\bar{I}}x^{4,tot,\bar{I}} - V^{CIF,\bar{c}}x^{CIF,\bar{c}}) \tag{2.224}$$

Although it might appear redundant to use both methods to calculate both variables, it is a useful check to ensure that the model is correct. If there is an imbalance between the two methods, this is typically an indication that a calculation has been performed incorrectly.

## 2.15 Terms of trade

The terms of trade ($P^{0,toft}$) is a measure of relative price of exports to imports, such that:

$$P^{0,toft} = \frac{\text{Price of exports}}{\text{Price of imports}}. \tag{2.225}$$

The terms of trade can be calculated using the import and export price indexes, giving:

$$P^{0,toft} = \frac{P^{4,TOT}}{P^{0,cif\_c}}. \tag{2.226}$$

which when linearised becomes:

$$p^{0,toft} = p^{4,tot} - p^{0,cif\_c}. \tag{2.227}$$

when calculating the terms of trade is required that the prices in the numerator and the denominator are in consistent prices, either foreign prices or domestic prices. If using domestic prices, it is important to measure imports at the Cost, insurance and Freight (C.I.F) inclusive price.

## 2.16 Balance of Trade

The balance of trade is the difference between the value of exports and imports. In levels form, the balance of trade is defined:

$$BOT = P^{4,tot}X^{4,tot} - P^{0CIF\_C}X^{0,imp\_c} \tag{2.228}$$

The balance of trade expressed as a fraction of GDP ($BOT^{GDP}$) is expressed:

$$BOT^{GDP} = \frac{BOT}{V^{0,GDP}} \tag{2.229}$$

Since $BOT^{GDP}$ could feasibly be zero, it is expressed as a differential instead of a percentage change. Using $V^{4,tot}$ and $V^{0,imp\_c}$ to denote the value of exports and imports, respectively, the differential of (2.229) is expressed as:

$$bot^{GDP} + v^{0,GDP} = \frac{1}{BOT} V^{4,tot}(v^{4,tot}) - V^{0,imp\_c}(v^{0,imp\_c}) \tag{2.230}$$

Multiplying both sides by $BOT$ and multiplying the last term on the RHS of equation (2.230) by $\frac{V^{0,GDP}}{V^{0,GDP}}$ gives:

$$\frac{BOT * V^{0,GDP}}{V^{0,GDP}} bot^{GDP} + BOT * v^{0,GDP} = V^{4,tot}(v^{4,tot}) - V^{0,imp\_c}(v^{0,imp\_c}) \tag{2.231}$$

Since $\frac{BOT}{V^{0,GDP}} = BOT^{GDP}$ and $bot^{GDP} = 100 * \frac{\Delta BOT^{GDP}}{BOT^{GDP}}$ the balance of trade formula can be expressed as:

$$100 * BOT * \Delta BOT^{GDP} = V^{4,tot}(v^{4,tot}) + V^{0,imp\_c}(v^{0,imp\_c}) - BOT(v^{0,GDP}) \tag{2.232}$$

### 2.16.1 Summary

This chapter provides a brief overview of the history of CGE modelling and outlines the foundations for the ORANIG framework. The 19 equation model presented at the start of this chapter provides an intuitive insight into the structure of a CGE model. This insight is often difficult to grasp with other CGE models due to the sheer size of the model. Thus, this section is used as a reference in later parts of the thesis to provide clarity for concepts in the ORANIG framework.

This section also outlines the process involved with converting non-linear equations into their linear equivalents. An outline of the approximation error is discussed and processes which can be used to minimise these errors are outlined. Finally, rules for converting non-linear equations into their linear form are outlined. This rules are applied throughout the remainder of this thesis.

The latter half of this chapter focuses on describing the underlying theory for the ORANIG framework. For industries, this entails describing the production function adopted to model how production occurs, deriving the input demand equations and expressing these equations in their linear form. For final users, such as households, this entails describing the model used to represent their preferences, deriving input demand equations and again expressing these equations in their linear form.

Up until now, there has been no discussion about labour demand in the presence of overtime and quasi-fixed costs. The model presented thus far has a single quantity of labour ($X1LAB$) which is purchased a single wage rate ($P1LAB$). The quantity of labour is disaggregated in terms of occupation type, but does not distinguish between hours and workers. The sole focus of this chapter was to provide an overview of the ORANIG framework before any modifications were made to the labour market. The following chapters introduce the theory on labour demand, the empirical support of these models and how to integrate the model into the ORANIG framework presented in this chapter.

# 3 Theoretical contribution

This chapter introduces the theory of labour demand in the presence of overtime and fixed costs. It outlines three competing models of labour demand and discusses implications of each competing model.

## 3.1 Standard Model

How employment responds to a policy outcome often determines the effectiveness of the policy. For example, suppose that a wage increase of 10% causes employment, measured as total person-hours, to decrease by 5%. The potential benefits of this increase in wages depends on how unemployment is distributed. From an inequality perspective, if all workers who experienced the wage increase have their hours cut by 5% then this is a better outcome than if 5% of the workforce lose their job. In the first instance, the increase in wages will compensate workers for the reduction in hours. However, in the second instance, a subset of the workforce loses all their income.

The labour market described in (2.8) does not distinguish between hours worked and number of workers. To analyse such a problem, a theory on what determines demand for hours worked per person and the number of workers is required. The standard description of a firm might have the following cost structure:

$$C = WM + RK \tag{3.1}$$

Subject to the quantity constraint:

$$Q = F(M, K) \tag{3.2}$$

where $C$ are firms costs, $Q$ is the firms output, $W$ is the wage rate, $M$ is labour input, $R$ is the rental rate of capital and $K$ is the quantity of capital. This is often unaccompanied by what defines labour as an input.

### 3.1.1 Effective hours

The labour input is usually measured as the total person hours, such that:

$$M = NL \tag{3.3}$$

where $M$ is the total person hours, $N$ is the number of workers and $L$ is the average hours worked per worker (Dixon and Freebairn, 2009; Freeman, 2008).

Such a setup implies that worker's productivity has no relation to how many hours of work they performed. For example, without a distinction between hours and workers, a firm would be indifferent between hiring 1 worker for 80 hours or 2 workers for 40 hours since both would provide the same amount of person hours.

This is an unrealistic assumption, as it may be the case that the single worker working 80 hours may experience fatigue and a loss of productivity as they work longer hours or it might take time for a worker to 'warm up' causing them to become more productive the longer they work.

In worker-hours models, to account for the notion that different combinations of workers and hours may have different levels of productivity, the labour input is modelled using the labour services function:

$$H = G(L, N) \tag{3.4}$$

where $H$ is the effective hours worked (Dixon and Freebairn, 2009). Usually, $G$ is defined such that the first derivative $G_L > 0$ and the second derivative $G_{LL} < 0$. A property of this functional form is that the productivity of labour does not double if the hours worked per week doubles from 40 hours to 80 hours.

A functional form for the labour services function is:

$$G(L, N) = L^{\alpha} N^{\beta} \tag{3.5}$$

where $0 < \alpha < 1$ and $0 < \beta < 1$ (Hart, 2004; Andrews et al., 2005). If $\alpha = \beta = 1$ then effective hours and person-hours are the same.

### 3.1.2 Cost function

The wage rate ($W$) is the only cost paid for labour in the standard description of a cost function (3.1). In the labour demand literature, costs are segmented into different cost types. Hamermesh (1996) segments the total labor costs into a variable and quasi-fixed cost component. Variable costs are those which vary with the amount of hours worked, such as hourly wages and overtime payments. Quasi-fixed costs are those which do not depend on the amount of hours work but are incurred for each worker a firm hires, e.g, administrative costs such as payroll which are required regardless of how many hours a worker works. Hamermesh (1996) defines labour costs ($TC$) per week in the general form:

$$TC = [W(L)L + Z]N \tag{3.6}$$

where $W(L)$ is the variable component and is a function of the average hours worked per worker and $Z$ is the fixed costs per worker.

Often the cost of labour is segemented into an ordinary wage rate, overtime payments and fixed costs (Andrews et al., 2005; Calmfors and Hoel, 1988; Hart, 2004). The wage rate ($W$) is the price paid for all ordinary hours ($L_0$) of work where the ordinary hours represent the maximum hours per week you can employ a worker without having to pay them an overtime premium.

Overtime hours ($OT$) are defined as all hours which are worked in excess of ordinary hours, such that:

$$O_T = L - L_0 \tag{3.7}$$

Overtime hours earn a wage premium ($W_P$) which is either a constant or an increasing function of overtime. The total overtime payments made to a worker are measured $(1 + W_P)W(L - L_0)$. Thus, the term $W_p$ denotes the premium paid for overtime hours and is usually a value like 0.5. The cost of labour for a working week for all $N$ workers is defined:

$$TC = [WL_0 + (1 + W_p)W(L - L_0) + Z]N \tag{3.8}$$

where wage premium, $W_P$ is assumed to be a constant. This can also be expressed as:

$$TC = [WL + W_pW(L - L_0) + Z]N \tag{3.9}$$

### 3.1.3 A general solution

The Lagrangian for the cost minimisation problem using the cost function (3.9) and the labour services function (3.4) is expressed:

$$\mathcal{L} = [WL + W_PW(L - L_0) + Z]N + \lambda[H - G(L, N)] \tag{3.10}$$

The associated first-order conditions are:

$$\frac{d\mathcal{L}}{dL} = [W + W_PW]N - \lambda G_L = 0 \tag{3.11}$$

$$\frac{d\mathcal{L}}{dN} = WL + W_PW(L - L_0) + Z - \lambda G_N = 0 \tag{3.12}$$

where $G_L$ is the derivative of G with respect to L, $G_N$ is the derivative of G with respect to N and $\lambda$ is the Lagrange multiplier. Combining (3.11) and (3.12) gives:

$$\frac{G_L}{G_N} = \frac{[W + W_PW]N}{WL + W_PW(L - L_0) + Z} \tag{3.13}$$

Depending on the functional form of $G(L, N)$, the equilibrium quantity of $L$ may be a function of the number of workers. According to Hamermesh (1996), there is no evidence to suggest that hours worked per worker is dependent on the number of workers in a firm. Therefore, it is ideal to have a labour services function such that (3.13) is invariant to scale, i.e, $L$ does not depend on $N$. The labour services function defined as:

$$H = N^{\beta}F(L) \tag{3.14}$$

is adopted throughout the literature, in varying forms, as it has the property of being invariant to scale (Calmfors and Hoel, 1988; Bell, 1982; Hart and Moutos, 1995). Using this labour services function, The left-hand side of (3.13) can be expressed:

$$\frac{G_L}{G_N} = \frac{NF'(L)}{\beta F(L)} \tag{3.15}$$

When substituted into (3.13) yields an equilibrium solution:

$$\frac{F'(L)}{\beta F(L)} = \frac{[W + W_P W]}{WL + W_P W(L - L_0) + Z} \tag{3.16}$$

where $L$ is invariant to scale as $N$ does not appear on either the left-hand or right-hand side of the equation.

### 3.1.4 Standard model

The 'standard model', as presented in Hart (2004) and Andrews et al. (2005), uses the labour services function (3.5) with the cost function (3.9) where it is assumed that the wage premium is constant. The Lagrangian function for the corresponding cost minimisation problem is:

$$\mathcal{L} = [WL + W_P W(L - L_0) + Z]N + \lambda[H - L^\alpha N^\beta] \tag{3.17}$$

The associated first-order conditions are:

$$\frac{d\mathcal{L}}{dL} = [W + W_P W]N - \alpha\frac{H}{L} = 0 \tag{3.18}$$

$$\frac{d\mathcal{L}}{dN} = WL + W_P W(L - L_0) + Z - \beta\frac{H}{N} = 0 \tag{3.19}$$

Combining equations (3.18) and (3.19) gives:

$$\frac{\alpha N}{\beta L} = \frac{(W + WW_p)N}{WL + W_p W(L - L_0) + Z} \tag{3.20}$$

Cancelling out the $N$ terms and rearranging yields:

$$\alpha[WL + W_P(L - L_0) + Z] = \beta[WL + W_P WL] \tag{3.21}$$

Solving for $L$ yields the intensive margin demand equation:

$$L = (\frac{\alpha}{\beta - \alpha})\frac{Z - W_p W L_o}{(1 + W_p)W} \tag{3.22}$$

The intensive margin of employment depends on the the wage rate, overtime wage premium, fixed costs and both the parameters from the labour services function. As outlined in the proceeding chapter, it is possible for the equilibrium value of intensive margin employment to be lower than the ordinary hours ($L_0$) if the fixed costs ($Z$) are too low and if the ordinary hours are too high.

### 3.1.5 Second-order condition

For the solution (3.22) to be a minimum, the second-order derivative is required to be larger than 0, which requires that $\beta > \alpha$ (Bell, 1982; Andrews et al., 2005). The second-order derivative test can be applied by converting the cost minimisation problem (3.17) into an unconstrained optimisation problem by eliminating $N$ such that:

$$TC = [WL + W_P W(L - L_0) + Z]\frac{H}{L^\alpha} \tag{3.23}$$

where $N = \frac{H^{1/\beta}}{L^{\frac{\alpha}{\beta}}}$. It is useful for later sections to define the problem as:

$$f(L) = [WL + W_P W(L - L_0) + Z] \tag{3.24}$$

and

$$g(L) = H^{\frac{1}{\beta}} L^{\delta} \tag{3.25}$$

where $\delta = -\frac{\alpha}{\beta}$. The second order condition is:

$$TC_{LL} = f''(L)g(L) + 2f'(L)g'(L) + f(L)g''(L) > 0 \tag{3.26}$$

The labour services function $g(L)$ is homogeneous of degree $\delta$ which means:

$$g'(L) = \frac{\delta}{L}g(L) \tag{3.27}$$

It follows:

$$g'(L) = \frac{L}{\delta - 1} g''(L) \tag{3.28}$$

Substituting out $g(l)$ and $g''(l)$ gives:

$$TC_{LL} = g'(L)[f''(L)\frac{L}{\delta} + 2f'(L) + f(L)\frac{\delta - 1}{L}] > 0 \tag{3.29}$$

In equilibrium, the first-order derivative is equal to 0 such that:

$$TC_L = f'(L)g(L) + g'(L)f(L) = 0 \tag{3.30}$$

re-arranging yields:

$$TC_L = f'(L)g'(L)\frac{L}{\delta} + g'(L)f(L) = 0 \tag{3.31}$$

implying:

$$f(L) = -f'(L)\frac{L}{\delta} \tag{3.32}$$

substituting this into the second-order derivative gives:

$$TC_{LL} = g'(L)[f''(L)\frac{L}{\delta} + 2f'(L) - f'(L)\frac{\delta - 1}{\delta}] > 0 \tag{3.33}$$

allowing it to be expressed as:

$$TC_{LL} = \frac{g'(L)f'(L)}{\delta}[\frac{f''(L)}{f'(L)}L + 2\delta - (\delta - 1)] > 0 \tag{3.34}$$

Since $g'(L) < 0$, $f'(L) > 0$ and $\delta < 0$ this requires:

$$\frac{f''(L)}{f'(L)}L + \delta + 1 > 0 \tag{3.35}$$

as $\delta = -\frac{\alpha}{\beta}$ this can be expressed:

$$\frac{f''(L)}{f'(L)}L + \frac{\beta - \alpha}{\beta} > 0 \tag{3.36}$$

In the standard model, the wage premium is constant, therefore:

$$f'(l) = W + WW_P \tag{3.37}$$

and

$$f''(l) = 0 \tag{3.38}$$

thus, second-order condition requires that:

$$\frac{\beta - \alpha}{\alpha} > 0 \tag{3.39}$$

which gives the condition that $\beta > \alpha$.

### 3.1.6 Properties of the standard model

The effects of an increase in fixed costs, wage premiums and ordinary hours can be understood by evaluating the derivatives of the equilibrium condition (3.22):

$$\frac{dL}{dZ} = \frac{\alpha}{\beta - \alpha} \frac{1}{(1 + W_p)W} > 0 \tag{3.40}$$

$$\frac{dL}{dW_p} = -[\frac{WL_0}{Z - W_pWL_0} + \frac{1}{1 + W_p}]L < 0 \tag{3.41}$$

$$\frac{dL}{dL_0} = -\frac{\alpha}{\beta - \alpha} \frac{W_p}{1 + W_p} < 0 \tag{3.42}$$

Equation (3.40) shows that an increase in the level of fixed costs causes firms to increase their demand for hours worked per worker. The second equation (3.41) shows that an increase in the wage premium, causing the cost of overtime hours to increases, leads to a decrease in demand for intensive margin employment. As standard hours are fixed, this will cause the level of overtime to decrease. Finally, equation (3.42) shows that an increase in ordinary hours causes an increase in hours worked per worker. This means that if ordinary hours are reduced the total hours actually increase, causing overtime to increase by a greater amount than the decrease in ordinary hours.

Whilst the first two equations are as expected, this result is the most controversial as it is not reflected by empirical evidence. The section (4.1) covers the empirical estimation of this effect.

The reason why a decrease in standard hours causes an increase in a workers total hours was outlined in Hunt (1996). Consider the marginal cost an increase in hours per person in the cost equation (3.9):

$$MC_L = (1 + W_p)W \tag{3.43}$$

and the marginal cost of adding another worker:

$$MC_N = [WL_o + W_P W(L - L_0) + Z] \tag{3.44}$$

where $MC_L$ is the marginal cost of another hour and $MC_N$ is the marginal cost of an additional worker. The level of standard hours has no impact on the marginal cost of an additional unit of overtime. The marginal cost of overtime is determined jointly by the wage premium and the wage rate. However, the marginal cost of an additional unit of labour is increased when decreasing the standard workweek as:

$$\frac{dMC_n}{dL_0} = -W_p W \tag{3.45}$$

By decreasing the standard workweek, workers work a greater portion of hours which are overtime hours. As these hours are remunerated at a higher rate, the cost of these workers increases.

### 3.1.7 Fixed costs necessary to generate overtime

It is worthwhile considering what parameters and cost values are necessary to generate positive overtime within this framework. Overtime $(L - L_0)$ can be expressed by subtracting $L_0$ from both sides of equation (3.22), giving:

$$L - L_0 = (\frac{\alpha}{\beta - \alpha})\frac{Z - W_p W L_o}{(1 + W_p)W} - L_0 \tag{3.46}$$

which can be expressed:

$$O_T = (\frac{\alpha}{\beta-\alpha})\frac{Z - W_p W L_o}{(1+W_p)W} - \frac{\beta-\alpha(1+W_P W)}{\beta-\alpha(1+W_P W)} L_0 \tag{3.47}$$

where $O_T = L - L_0$. This can be simplified as:

$$O_T = (\frac{\alpha}{\beta-\alpha})\frac{Z - W_p W L_o - (\frac{\beta-\alpha}{\alpha})(1+W_P)WL_0}{(1+W_p)W} \tag{3.48}$$

For this model to generate a positive amount of overtime, it is required that:

$$Z - W_p W L_o - (\frac{\beta-\alpha}{\alpha})(1+W_P)WL_0 > 0 \tag{3.49}$$

Following the approach of Andrews et al. (2005), the ratio of fixed costs to ordinary costs and fixed costs are defined:

$$\kappa = \frac{Z}{WL_0 + Z} \tag{3.50}$$

This $\kappa$ value measures the fraction of expenditure on labour (excluding overtime) that is represented by fixed costs. It is expected that this value should lay somewhere on the interval of [0,1]. The measurement of $\kappa$ is discussed later in Chapter 4.4.2.

Using (3.50) allows (3.49) to be expressed as:

$$Z > W_p W L_o + (\frac{\beta-\alpha}{\alpha})(1+W_P)WL_0 \tag{3.51}$$

Adding $WL_0$ to both sides yields:

$$Z + WL_0 > W_p W L_o + WL_0 + (\frac{\beta-\alpha}{\alpha})(1+W_P)WL_0 \tag{3.52}$$

Dividing through by $WL_0$ gives:

$$\frac{1}{1-\kappa} > W_p + 1 + (\frac{\beta-\alpha}{\alpha})(1+W_P) \tag{3.53}$$

which can be simplified:

$$\frac{1}{1-\kappa} > (\frac{\beta}{\alpha})(1+W_P) \tag{3.54}$$

such that $\kappa$ can be expressed:

$$\kappa > 1 - \frac{\alpha}{\beta}\frac{1}{1 + W_P} \tag{3.55}$$

Since $\beta > \alpha$ and using the often assumed value of $W_P = 0.5$ Dixon et al. (2005), the lowest value of $\kappa$ is:

$$\lim_{\alpha \rightarrow \beta} \kappa = 1 - \frac{1}{1.5} = 33\% \tag{3.56}$$

implying that the lowest plausible value for fixed costs as a fraction of ordinary and fixed costs is 33%. In section (4.4) the measurement of fixed costs are discussed. Nonetheless, this value seems larger than expected.

## 3.2 Increasing Wage premium

The standard model presented thus far assumes that wage premiums are remunerated at a constant wage premium. However, it is possible that a wage premium function could be an increasing function of overtime hours. This section introduces a model where overtime is a linear function of overtime hours. The increasing wage premium model uses the labour services function (3.5) and the cost function (3.9), except the linear wage premium function used in Hart and Moutos (1995) is adopted, where:

$$W_p = \mu_0 + \frac{\mu_1}{2}(L - L_0) \tag{3.57}$$

where both $\mu_0$ and $\mu_1$ are parameters. If $\mu_0 > 0$ and $\mu_1 = 0$ the wage premium reduces back to a constant. The Lagrangian for the cost minimisation problem is:

$$\mathcal{L} = [WL + W[\mu_0(L - L_0) + \frac{\mu_1}{2}(L - L_0)^2] + Z]N + \lambda[H - L^{\alpha}N^{\beta}] \tag{3.58}$$

The first-order conditions are:

$$\frac{d\mathcal{L}}{dL} = [W + W[\mu_0 + \mu_1(L - L_0)]]N - \alpha\frac{H}{L} = 0 \tag{3.59}$$

$$\frac{d\mathcal{L}}{dN} = WL + W_P W(L - L_0) + Z - \beta\frac{H}{N} = 0 \tag{3.60}$$

The process to derive the equilibrium solution for $L$ is cumbersome and produces an intractable solution. However, the second-order conditions and derivatives can be used to evaluate the model.

### 3.2.1 Second-order condition for increasing overtime

The second-order condition can be evaluated using equation (3.36), such that:

$$\frac{\mu_1 L}{W + W\mu_0 + \mu_1 O_T} + \frac{\beta - \alpha}{\beta} > 0 \tag{3.61}$$

This can be re-arranged such that:

$$(\beta - \alpha)[W + W\mu_0 + \mu_1 O_T] + B\mu_1 L > 0 \tag{3.62}$$

which can be simplified to:

$$(\beta - \alpha)[(1 + \mu_0)W - W\mu_1 L_0 + (\frac{2\beta - \alpha}{\beta - \alpha})W\mu_1 L] > 0 \tag{3.63}$$

It is no longer required that $\beta > \alpha$ to satisfy the second-order condition. The following conditions satisfy the second-order condition (Hart and Moutos, 1995):

(a) $\beta > \alpha$

(b) $\beta = \alpha$

(c) $\beta < \alpha$, $\beta < 2\alpha$ and $1 + \mu_0 < \mu_1 L_0$.

Condition (a) is the same requirement as the constant wage premium where returns to workers is larger than returns to workers. Condition (b) states that it is sufficient for the returns to workers to be the same as the returns to hours. Condition (c) states that the returns to workers can be less than the returns to hours depending on the shape of the overtime function. Overall, the increasing overtime premium permits more flexibility in values for $\alpha$ and $\beta$.

### 3.2.2 Derivative of ordinary hours with respect to total hours with increasing overtime

The standard model was also restrictive in that a decrease in ordinary hours causes overall hours to increase. Whilst no equilibrium value of $L$ is provided, the derivative $\frac{dL}{dL_0}$ can still be evaluated. Firstly, combining the first-order conditions (3.59) and (3.60) yields:

$$\frac{\alpha N}{\beta L} = \frac{N(W + W\mu_0 + W\mu_1(L - L_0))}{WL + W\mu_0(L - L_0) + \frac{W\mu_1}{2}(L - L_0)^2 + Z} \tag{3.64}$$

which can be expressed as:

$$\beta L[W + W\mu_0 + W\mu_1(L - L_0)] - \alpha[WL + W\mu_0(L - L_0) + \frac{W\mu_1}{2}(L - L_0)^2 + Z] = 0 \tag{3.65}$$

Hart and Moutos (1995) define the following function:

$$F = \beta L[W + W\mu_0 + W\mu_1(L - L_0)] - \alpha[WL + W\mu_0(L - L_0) + \frac{\mu_1}{2}(L - L_0)^2 + Z] \tag{3.66}$$

Then take the total differential, such that:

$$dF = F_L dL + F_{L_0} dL_0 \tag{3.67}$$

where $F_{L_0}$ is the derivative of $F$ with respect to $L_0$ and $F_L$ is the derivative of $F$ with respect to $L$. Since the first-order condition must equal 0, this implies $dF = 0$ which gives:

$$\frac{dL}{dL_0} = \frac{-F_{L_0}}{F_L} \tag{3.68}$$

Giving the following derivative for total person hours with respect to ordinary hours:

$$\frac{dL}{dL_0} = \frac{W\mu_1[(\alpha - \beta)L + \alpha L_0] - \alpha W\mu_0}{(\beta - \alpha)[W + W\mu_0 - W\mu_1(L - L_0) + \beta\mu_1 L} \tag{3.69}$$

which permits a range of possible values depending on parameter choices. For example, setting $\beta = \alpha$ simplifies it to:

$$\frac{dL}{dL_0} = \frac{L_0 - \mu_0}{\mu_1 L} \tag{3.70}$$

where if $L_0 > \mu_0$ the derivative is positive. Multiplying both sides by the wage gives $WL_0 > W\mu_0$. It can be determined from this that the derivative will be positive as long as the cost of the first additional hour of overtime is less than the cost of an additional worker.

### 3.2.3 Fixed costs with increasing overtime premium

The standard model, when wage premium is assumed to be constant at 0.5, required the proportion of fixed costs to fixed and ordinary costs be 33% to generate overtime. Following a similar approach in this section, the first costs necessary to generate zero overtime are calculated. Firstly, (3.64) can be rearranged to express fixed costs as:

$$Z = \frac{\beta}{\alpha}[W_p W(L + L_0) - W\mu_0 L] + \frac{\beta - \alpha}{\alpha}[WL_0 + (1 + W_p)W(L - L_0)] \tag{3.71}$$

In the standard model, the fixed costs were calculated as $\alpha \to \beta$. Thus setting $\alpha = \beta$ yields:

$$Z = W_p W(L + L_0) - W\mu_0 L \tag{3.72}$$

Calculating $\kappa$ from (3.50) yields:

$$\kappa = \frac{W_p(L + L_0) - \mu_0 L}{W_p(L + L_0) - \mu_0 L + L_0} \tag{3.73}$$

Setting $L = L_0$ so that overtime is zero, gives:

$$\kappa = \frac{2W_p(L) - \mu_0 L}{2W_p L - \mu_0 L + L} \tag{3.74}$$

Since there is no overtime $W_P = \mu_0$, giving:

$$\kappa = \frac{2\mu_0 L - \mu_0 L}{2\mu_0 L - \mu_0 L + L} \tag{3.75}$$

Assuming that the $\mu_0 = 0.5$, which implies that the wage premium is 50% for the first hours of overtime, yields:

$$\kappa = \frac{0.5L}{1.5L} = 0.33\% \tag{3.76}$$

This model yields the same outcome as the standard model, which is a consequence of assuming that $\mu_0 = 0.5$. In essence, the marginal decision is the same in both cases as both firms face a wage premium of 0.5 for a unit of overtime.

### 3.3 Discrete wage premium function

In the previous section (3.2) the wage premium is a continuous linear function. Sometimes the overtime schedule is a step function, e.g, where the first two hours are remunerated at 50% and then all hours after that are remunerated at 100%. When the wage premium is represented as a step function, it operates the same as the constant wage premium model. For example, following from Hart (2004), suppose that for the first two hours a worker gets remunerated with a wage premium $W_{P,1}$ and for all hours after that are remunerated at $W_{P,2}$ wage premium, where $W_{P,1} < W_{P,2}$. If a firm wanted to employ more than 2 hours of overtime, their cost function would be:

$$TC = [WL + W_{P,1}W2 + W_{P,2}W(L - (2 + L_0) + Z]N \tag{3.77}$$

In this case the demand for hours worked per person would be:

$$L = (\frac{\alpha}{\beta - \alpha})\frac{Z - W_{p,1}WL_o}{(1 + W_{p,1})W} \quad \text{for } L + 2 \leq L > L_0 \tag{3.78}$$

$$L = (\frac{\alpha}{\beta - \alpha})\frac{Z - W_{p,2}WL_o}{(1 + W_{p,2})W} \quad \text{for } L \geq L + 2 \tag{3.79}$$

This case does not overcome any of the shortcomings of the constant wage premium model. It still requires that $\beta > \alpha$ and imposes that the elasticity of ordinary hours with respect to total hours is negative. Yet, it is more complex to solve due to the discontinuity between the two different wage premium rates. Finally, there are significant data issues for such a framework. In an annual model, it would require being able to distinguish how many hours were remunerated at the lower wage premium per week compared to the higher wage premium per week. There is no survey which has such information for Australia.

Yet, it is difficult to solve due to the discontinuity between the two different wage premium rates and because of the data requirements. In terms of data, the model requires knowledge on how overtime is distributed over a year. For example, it might be that a worker initially performs 104 hours of overtime per year evenly spread across all weeks. In this case, the worker would only receive the lower wage premium $W_{P,1}$. However, if the firm wished to increase overtime for this worker they would be required to pay the higher overtime premium $W_{P,2}$ for the overtime performed in this month. Conversely, suppose that the same worker performs 104 hours of overtime in the span of a 30-day month and the overtime is evenly spread over the entire month. This worker would receive both the lower wage premium $W_{P,1}$ and the higher wage premium $W_{P,2}$. If the worker's employer wished to increase overtime by 1 hour they would face two different wage premiums depending on when the employer wanted the employee to perform the overtime. If it was performed in the same month, the worker would receive the wage premium $W_{P,2}$. However, if it was performed in a different month the worker would receive $W_{P,1}$. Thus, an understanding of when the overtime occurred in addition to total overtime is required to implement this model.

The best source for overtime data in Australia is discussed in Chapter 4.3. The data is only collected once per year and only provides information on total overtime hours and remuneration for overtime per individual. The wage premium is derived from these two values and is calculated as the average wage premium for all overtime hours. It does not provide scope for understanding how overtime is distributed across the year.

## 3.4 Constant elasticity model

In (3.1.7), two competing models of labour demand were presented. The first model assumed that the wage premium was constant and the second model assumed that the wage premium was a linear function of overtime hours. In this section, a labour demand model with a constant elasticity between the wage premium and overtime function is presented. This relationship can be expressed:

$$ln(W_P) = ln(A) + (\eta_{WP,OT} - 1)ln(O_T) \tag{3.80}$$

where $O_T = L - L_0$ and $(\eta_{WP,OT} - 1)$ measures the percentage change in the wage premium for a 1% increase in overtime and $A$ is a positive parameter [5]. The labour services function adopted in this model is the same as that presented in (3.5), such that:

$$H = L^{\alpha} N^{\beta} \tag{3.81}$$

The cost function adopted is also the same as that used in (3.9) where:

$$TC = [WL + W_p W(L - L_0) + Z]N \tag{3.82}$$

The equilibrium for hours worked can be determined by equating the ratio of the marginal costs of hours and workers to the ratio of marginal product of hours and workers in the labour services function:

$$\frac{TC_L}{TC_N} = \frac{H_L}{H_N} \tag{3.83}$$

Given the specification of the wage premium in equation (3.80), the first order condition for hours can be expressed:

$$TC_l = [W + \frac{\partial W_P}{\partial O_T} W O_T + W_P W]N = [(1 + (\eta_{WP,OT} - 1)W_P + W_P)W]N \tag{3.84}$$

[5] The -1 in $(\eta_{WP,OT} - 1)$ makes the representation of intensive margin demand more convenient.

Where $(\eta_{WP,OT} - 1) \equiv \frac{\partial W_P}{\partial O_T}\frac{O_T}{W_P}$. Equating the ratio of marginal costs to the marginal product of the labour services function gives:

$$\frac{(1 + \eta_{WP,OT} W_P) WL}{WL + W_P W (L - L_0) + Z} = \frac{\alpha}{\beta} \tag{3.85}$$

which gives the equilibrium condition for the hours worked per worker as:

$$L = \frac{\alpha}{\beta[1 + \eta_{WP,OT} W_p - \frac{\alpha}{\beta}(1 + W_p)]} \frac{Z - W_p W L_0}{W} \tag{3.86}$$

This is not a solution for $L$ since the wage premium is a function of intensive margin employment, i.e, $L$ appears on both the RHS and LHS of the equation as $W_P$ is defined according to (3.80). The lack of solution for $L$ will be overcome by converting the equation into its linear form.

Following the approach of (4.3), which is outlined in Chapter 4.1 it is imposed that the returns to workers are set to unity such that $\beta = 1$ which gives the expression for the marginal product of hours:

$$\alpha = \frac{(1 + \eta_{WP,OT} W_P) WL}{WL + W_P W (L - L_0) + Z} \tag{3.87}$$

where it is possible for $\alpha$ to be larger than, less than or equal to unity. This has no impact on the equilibrium level of intensive margin employment since it was determined by the ratio of $\frac{\alpha}{\beta}$.

### 3.4.1 Second order condition

Similar to the linear increasing wage premium model, this model also provides more flexibility regarding permissible values for $\alpha$. Using equation (3.36), where $f'(l) = W + \eta_{WP,OT} W_P W$ and $f''(L) = \eta_{WP,OT}(\eta_{WP,OT} - 1)\frac{W_P}{O_T}$ yielding:

$$\frac{W \eta_{WP,OT}(\eta_{WP,OT} - 1) W_P}{W[1 + \eta_{WP,OT} W_P]} \frac{L}{O_T} + \frac{\beta - \alpha}{\beta} > 0 \tag{3.88}$$

which can be re-arranged as:

$$\frac{(\eta_{WP,OT} - 1)}{[1 + \frac{1}{W_P \eta_{WP,OT}}]} \frac{L}{O_T} + \frac{\beta - \alpha}{\beta} > 0 \tag{3.89}$$

The following conditions satisfy the second-order conditions:

a) $\beta > \alpha$

b) $\beta = \alpha$

c) $\frac{\alpha-\beta}{\beta} < \frac{(\eta_{WP,OT}-1)}{[1+\frac{1}{W_P \eta_{WP,OT}}]} \frac{L}{O_T}$

Condition (a) states that the second-order condition is satisfied if the returns to workers are higher than the returns to hours. The returns to hours ($\alpha$) and returns to workers ($\beta$) represent the output elasticities for the labour services function (3.5). In essence, they measure the additional output by expanding hours and workers, respectively. Condition (b) states that it is sufficient for the returns to workers and hours to be the same. Condition (c) states that the returns to hours can be higher than the returns to workers depending on the magnitude of the elasticity parameter ($\eta_{WP,OT}$) and ratio of equilibrium hours to overtime.

### 3.4.2 Summary

This section presented three competing labour demand models in the presence of fixed costs and wage premiums. The first model presented was the simplest with a constant wage premium function. However, this was also the most restrictive. It relied on the returns to workers parameter being larger than the returns to hours parameter ($\beta > \alpha$) and imposed that the elasticity of ordinary hours with respect to total hours ($\eta_{L0,L}$) was negative. As will be demonstrated in the next section, this not always consistent with the empirical literature.

The second model presented was the model with a wage premium which is a linear function of overtime hours. This function relaxed the requirements that the returns to workers be larger than the returns to hours ($\beta > \alpha$) and permitted the elasticity $\eta_{L0,L}$ to be positive.

The third model presented was the model with a constant elasticity between the wage premium and overtime. This model also allowed $\beta > \alpha$ and $\eta_{L0,L}$ to be positive. Both these models are superior to the first model presented in that they allow more flexibility in terms of parameter choice. Their downside is it is often difficult to find an exact solution for hours worked ($L$). The inability to find a solution is not a hindrance from a modelling perspective, as both models have a solution once converted to their linear equivalent using the linear approximation techniques discussed in (2.3). Whilst there are other methods which can be adopted to numerically solve these equations, the linear form has the added benefit of being easy to interpret. Ultimately, the model with a constant elasticity will be incorporated into the ORANIG framework. The following section provides the empirical justification for adopting this model.

# 4 Empirical research

In the previous section, three models of labour demand were presented with competing assumptions. The constant wage premium function is the most parsimonious model but also the most restrictive. To choose a model which best fits the empirical evidence and structure of the Australian economy, the following four aspects will be analysed: (a) what are the empirical observations of the elasticity of ordinary hours with respect to total hours (b) what does the empirical literature reveal about the labour services function (c) what is the shape of the wage premium function and (d) the importance of fixed costs. This section will ultimately justify why the constant elasticity model of labour demand is the preferred model to be incorporated into the ORANIG framework.

## 4.1 Work sharing

An empirical focus of the workers-hours literature is whether the quantity of employed workers ($N$) can be increased by decreasing the standard workweek ($L_0$). The hypothesis is that if total hours performed in an economy were fixed, a decrease in ordinary hours would translate into a proportionate increase in the number of workers.

These policies are described as "work-sharing" and their effectiveness is ambiguous (Hunt, 1996). The shortcomings of these policies are that they might cause the cost of labour to increase which would cause a decrease in demand for total hours. In this case, the reduction in employed workers caused by employers scaling back employment might dominate the increase in employed workers caused by a substitution from hours to workers. Moreover, firms may also decide to increase overtime instead of using extra workers.

These studies have evaluated the effectiveness of work-sharing policies by estimating the following elasticities:

$$\eta_{L,L_0} = \frac{dL}{dL_0}\frac{L_0}{L} \tag{4.1}$$

*Standard hours elasticities on actual hours, employment and probability of working overtime: microeconometric evidence*

| Study | Data | $\eta_{H\bar{H}}$ | $\eta_{N\bar{H}}$ | $\frac{\partial \Pr(V>0)}{\partial H}$[a] |
|---|---|---|---|---|
| Hart and Wilson (1988) | 52 UK engineering firms, panel, 1978–1982 | 0.80 | 0.41/ – 0.49[b] | |
| Bell and Hart, (1999) | New Earnings Survey, 24,029 males, 1996; GB | 0.798[c] | | – 0.012 |
| Own regressions | New Earnings Survey, 34,657 manual males, 1978; GB | 0.895[c] | | – 0.0053 |
| Own regressions | New Earnings Survey, 31,360 manual males, 1985; GB | 1.056[c] | | – 0.0005 |
| Kalwij and Gregory (2000) | New Earnings Survey, 1975–1999; GB | 0.98[d] | | |
| Bauer and Zimmermann (1999) | German Socio-Economic Panel, 17,332 individuals, 1984–1997; West Germany | 1.034[c] | | 0.0020 |
| Hübler (1989) | German Socio-Economic Panel, 1031 individuals, 1984; West Germany | 0.924[e] | | ≈ – 0.0075 |
| Hübler and Meyer (1997) | Panel of firms (1024 in 1994; 849 in 1995); West Germany | 0.99 | | |
| Hunt (1999) | German Socio-Economic Panel, 4386 workers, manufacturing/ service sectors, 1984–1994; West Germany | [0.70, 0.85] | | [ – 0.0029, – 0.0017][f] |
| Hunt (1999) | 30 manufacturing industries, biannual pooled, 1982–1993; West Germany | 0.90 | – 0.50[g] | |
| Hernanz *et al.* (1999) | Spanish EESE Panel, 7300 firms, manufacturing, 1990–1997 | 1.09 | – 0.005 | |
| Crépon and Kramarz (2002) | French Labour Force Survey, panel, 1977–1987 | | [0.8, 1.6][h] | |

*Notes:*
[a]Except for Bauer and Zimmermann (1999) and Hunt (1999), estimated as a Probit.
[b]$\eta_{N\bar{H}} = 0.41$ for firms who offer overtime, $\eta_{N\bar{H}} = -0.49$ otherwise.
[c]Is the regression parameter in a Tobit, i.e. estimates $\partial H^*/\partial \bar{H}$, where $H^*$ is unconstrained choice of hours.
[d]ML Fixed Effects Tobit.
[e]As c, but uses Heckman-corrected truncated regression.
[f]Linear probability model with individual fixed-effects.
[g]Insignificant. Changes to 0.71 in a 10-industry panel, 1984–1994.
[h]Effect of reducing $\bar{H}$ from 40 to 39 hours in 1982 on probability of employment to non-employment transition.
GB, Great Britain; EESE, Encuesta de Estrategias Empressariates.

Figure 4.1: Summary of evidence for the worksharing elasticities ($\eta_{L,L_0}$) and ($\eta_{N,L_0}$) (Andrews et al., 2005)

and

$$\eta_{N,L_0} = \frac{dN}{dL_0}\frac{L_0}{N} \tag{4.2}$$

where $\eta_{L,L_0}$ is the elasticity of total hours with respect to ordinary hours and $\eta_{N,L_0}$ is the elasticity of workers with respect to ordinary hours.

These empirical results have implications when deciding which model from (3) to adopt. For example, the standard model in (3.1.7) requires that the elasticity (4.1) be negative per equation (3.42). Comparatively, the increasing wage premium model in Chapter (3.2) is more flexible as it permits a positive or negative elasticity.

Andrews et al. (2005) provide a summary of the microeconometric evidence from a selection of studies conducted using various estimation techniques. The results are presented in Figure 4.1.

The overwhelming evidence is that the elasticity $\eta_{L,L_0}$ is a positive value close to unity. Andrews et al. (2005) state that $\eta_{L,L_0}$ is positive as a stylised fact. This suggests that a decrease in the standard workweek ($L_0$) is accompanied by a decrease in the hours worked ($L$) per worker. This evidence suggests that the standard model in (3.1.7) is not an appropriate model as it imposes a negative elasticity for $\eta_{L,L_0}$.

In terms of choosing a functional form to model labour demand, the empirical results for the elasticity $\eta_{N,L_0}$ is less important as the number of workers are not directly determined by the labour demand models presented in (3.1.4) and (3.2). Nonetheless, Crépon and Kramarz (2002) found that the probability of transitioning from employment to unemployment increased by 2-4% after the French government mandated a decrease in the standard workweek from 39 down to 35 hours. Hunt (1999) finds that a decrease in the standard workweek by 1 hour causes a 3.8% decrease in employment, though the estimate was not statistically significant. Moreover, this estimate is an uncompensated elasticity as it does not control for any wage increases. When controlling for wage increases caused by the reduction in standard hours, Hunt (1999) finds that relationship is practically indistinguishable from zero.

Studies conducted on work-sharing for Australia are limited. Mangan and Steinke (1987) estimated the employment effects of a reduction in standard hours based upon survey data. For the period of 1976-85, they found that the only industries which experienced an increase in employment after a reduction in standard work week were relatively labour intensive and either tertiary or public sector industries. Elasticities were not calculated as the primary source of data was survey data where firms were asked questions such as "did employment increase/decrease" and thus responses were categorical in nature. These results are not promising for worksharing since there appears to be little employment gain from decreasing the standard workweek.

Policy-makers can also implement work-sharing policies by increasing the wage premium. For example, if they were to increase the wage premium (or wage premium schedule in the case of an increasing wage premium), this would cause the marginal cost of an additional hour to increase and cause firms to substitute from hours towards workers, though this would only apply for firms currently utilising overtime. This policy would generate increases in employment as long as the substitution effect from hours to workers outweighed the scale effect caused by an increase in the total cost of labour.

The impact that increasing wage premiums (or decreasing the standard workweek) has on overtime hours depends on the wage-determination process. The models presented thus far have assumed that firms purchase labour for a given wage rate with a corresponding overtime premium. According to Trejo (1991), it is possible that hours and wages are jointly determined. In this case, overtime regulation may have no impact on hours worked. For example, suppose that a worker and firm agree to exchange 10 hours of labour for \$100. In the absence of overtime penalty rates, they would simply agree to set the wage rate at \$10 per hour. However, suppose the government mandates that all hours in excess of 8 incur a penalty rate of 50%. The worker and the firm could mutually agree to lower the standard wage to \$9.09 per hour. In this case, for the first 8 hours the worker would earn \$9.09 per hour and for the remaining 2 hours the worker would earn \$13.64. The standard earnings would be \$72.72 and the overtime earnings would be \$27.27 resulting in a total income of \$100. Trejo (1991) refers to this model of wage determination as the 'fixed job model'.

This hypothesis relies on the wage rate being flexible enough that it can be downward adjusted to compensate for the increase in costs due to the wage premium. If there is a binding minimum wage, this would not be possible.

The evidence supporting the fixed job model is inconclusive. In Trejo (1991) he considers three hypothesis to evaluate the fixed jobs model. Firstly, he compares the wages of identical workers who are covered by overtime regulation to those who are not covered. He finds that wage rates only partially adjust for non-covered workers to compensate for the lack of overtime earnings. While the standard wage rates are higher for the uncovered, they have lower total earnings. Secondly, he compares the level of compliance for overtime regulations between minimum wage and non-minimum wage workers, where non-compliance is when firms illegally avoid paying their workers premiums for their overtime hours. His hypothesis is that if wages are determined by the fixed job model, it would be costless to comply with overtime regulation for non-minimum wage workers. However, it would be costly to comply for minimum wage workers as they would be paid a binding overtime premium. Therefore, there should be a higher incidence of non-compliance with minimum wage workers as there is an economic gain by non-complying. He finds statistical evidence that non-compliance is higher for minimum wage workers compared to non-minimum wage workers. These results are true even when controlling the potential that higher wage workers have a higher level of compliance in general and that smaller firms might disproportionately use minimum wage workers and that they might be more likely to not comply. Finally, he evaluates the observations that hours worked tends to spike at 40 hours which is the point at which firms are required to pay overtime premiums. His hypothesis is that if wage premium regulation is binding, firms will use workers right up until the point of 40 hours but not further to avoid overtime premiums. However, if the fixed job model was applicable, they could employ workers for longer than 40 hours without the premium impacting costs. His findings are that workers covered by wage premium regulation are more likely to work exactly 40 hours than workers who are not covered by wage premium regulation. Overall, his research provides mixed evidence to support the fixed jobs model.

Up until 1980, California had daily overtime pay regulation which stipulated that females were required to be paid a 50% overtime premium on all wages for hours worked in excess of 8 hours per day. In 1980, the daily overtime premium was extended to males. This provided a natural experiment which allowed Hamermesh and Trejo (2000) to test the effects of an increase in wage premium on overtime hours worked. Using difference-in-difference estimation techniques, they found that the increase in coverage extended to males caused the incidence of men working overtime to decrease by 17-20% . This results are also unfavourable to the fixed jobs hypothesis presented by (Trejo, 1991) as there was a reduction in hours in response to an increase in overtime premiums. However, it is also plausible that this reduction was a consequence of minimum wage workers having their hours cut.

Finally, in regards to the Australian economy, Miller and Mulvey (1991) estimated the wage differentials between union and non-union workers. They found that 1% of the wage differentials earned by workers was due to overtime earnings. This differential should not exist if the fixed-jobs model was applicable, unless the workers were on a minimum wage.

Based on the evidence provided in this section, it is concluded that the elasticity of ordinary hours with respect total hours is negative and that wage premiums are binding in that they do impact the overtime decision.

## 4.2 Labour services function

In this section, the returns to hours and workers for the labour services function (3.5) are evaluated. This analysis is often conducted by estimating an aggregate production function with hours per worker ($L$) and number of workers ($N$) as independent variables.

This literature has important implications for how to model labour demand in the presence of overtime and fixed costs. The standard model presented in (3.1.4) requires that the returns to workers be larger than the returns to hours ($\alpha > \beta$) for the second order condition (3.39) to be satisfied. The increasing wage premium (3.2) model is less restrictive; however, it still has limitation based on other parameters.

Variants of the following logarithmic transformation of (3.5) are adopted within the literature to estimate the returns to hours and workers (Hart, 2004):

$$ln(Q_{i,t}) = \alpha ln(L_{i,t}) + \beta ln(N_{i,t}) + -\nu_1 ln(K_{i,t}) + \nu_2 ln(\Phi_{i,t}) + \nu_3 ln(X_{i,t}) + \epsilon_{i,t} \qquad (4.3)$$

where $Q_{i,t}$ is output, $L_{i,t}$ is the average hours worked per person, $N_{i,t}$ is the number of workers, $K_{i,t}$ is capital, $\Phi_{i,t}$ is capital utilisation, $X_{i,t}$ is a vector of controls, $\epsilon_{i,t}$ is an error term, $i$ is an industry dimension and $t$ is a time dimension.

Early studies conducted on this topic, such as those by Feldstein (1967) and Craine (1973), concluded that a) the returns to hours were larger than the returns to workers, i.e, $\alpha > \beta$ and b) that hours exhibited increasing returns to scale.

Feldstein (1967) calculated the equation (4.3) for the years 1954, 1957 and 1960 for 22 British industries. Using gross value added as a measure for $Q_{i,t}$, the average estimates across all 3 years derived by Feldstein were $\alpha = 2.05$ and $\beta = 0.75$.

Craine (1973) estimates an unrestricted version (except he omits capital utilisation such that $\nu_2 = 0$) of (4.3) where he finds that $\alpha = 1.98$ and $\beta = 0.8$. Additionally, again omitting capacity utilisation, he estimates a version where he applies the restriction that $\beta + \nu_1 = 1$, i.e, there is constant returns to scale for workers and capital. In this case he finds that $\alpha = 2.02$ and $\beta = 0.68$. This restriction is adopted as it is expected that doubling both workers and capital, while holding hours constant, should double output.

Feldstein (1967) provided two primary reasons for why returns to hours are larger than returns to workers. Firstly, an increase in hours worked corresponds to an increase in capital utilisation over a given time period. This decreases the unit cost of capital and causes net output to increase. Secondly, there is often a fixed amount of unproductive hours - lunch breaks, setting up time etc. - which do not rise proportionately with the quantity of hours worked. Thus, as hours increase, there is a more than proportionate increase in the quantity of hours actually worked.

Leslie and Wise (1980) offer a third explanation for the magnitude of returns to scale in Feldstein

(1967) and Craine (1973). They suggest that an omitted variable bias, possibly caused by labour hoarding, might be the reason for the estimated elasticity. Labour hoarding is when firms retains workers during downturns who are currently not necessary but are retained due to the cost of firing and re-training. Comparatively, firms which hoard labour are likely to be less productive. They are also unlikely to use overtime less as they are currently hoarding unproductive labour. Therefore, it is likely that there is a negative correlation between hours worked and labour hoarding. It is plausible the coefficient for hours is capturing some of the omitted variable bias for labour hoarding.

Using a pooling of cross section and time-series data, Leslie and Wise (1980) estimated the model used by (Feldstein (1967)) and estimated their own model where they included dummies for industry effects and used the unemployment rate as a proxy for labour hoarding. The model based on Feldstein (1967) generated the values $\alpha = 1.61$ and $\beta = 0.78$. Their own model, however, found that both workers and hours experienced diminishing returns and found that $\alpha = \beta = 0.64$.

Hart and McGregor (1988) reviews the literature covered thus far and finds that inadequate measures of capacity utilisation being the reason for the high values of elasticity of hours with respect to output. They estimated (4.3) with and without a measure of capacity utilisation. They found that omitting capacity utilisation caused the returns to hours be larger than unity. When they included capacity utilisation, they found that both workers and hours experience diminishing marginal products. Yet, they still found the returns to hours being larger than those to workers with $\alpha = 0.82$ and $\beta = 0.31$.

DeBeaumont and Singell Jr (1999) expand on the idea presented in Leslie and Wise (1980) that different firms may have different levels of productivity. They relax the assumption of a common production structure across industries which was assumed by Feldstein (1967), Craine (1973) and Hart and McGregor (1988). They suggest that not disaggreating by industry will cause a an aggregation bias as industries with higher levels of productivity will utilise labour for longer hours. The

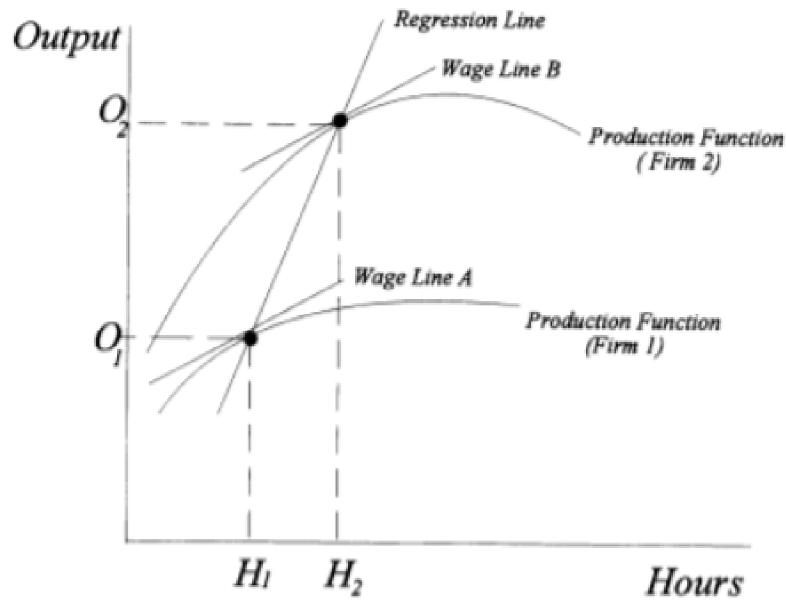


**Figure 1.** Illustration of Aggregation Bias for Return to Hour

Figure 4.2: Aggregation bias caused by assuming common production structure across different firms (DeBeaumont and Singell Jr, 1999)

likely bias is depicted in Figure 4.2.

In this case, Firm 2 has a higher level of productivity than Firm 1. For a given wage rate, Firm 2 will wish to hire workers for longer than firm 1. If they do not account for varying productivity across firms, the regression output will estimate a coefficient with the slope labelled ‘Regression line’ instead of the correct slope denoted by “Wage Line B” and “Wage Line “A”.

Using a fixed effects model based on U.S 2 digit industry data, they find the results presented in Figure 4.3. For all industries except stone, there are diminishing marginal products for both hours and workers. Moreover, they find that for a majority of industries that the returns to workers are larger than the returns to hours. Given that CGE modellers have long recognised that production structures vary across industries, these results are promising. Unfortunately, estimation based on

**Table 2.** Fixed-Effects Estimates by Industry Using Two-Digit State Data for 1972–1978[ab]

| Industry (SE) | Return to Hours Coefficient | Return to Workers Coefficient | Return to Capital Coefficient | Unemployment Rate Coefficient | Number of Observations |
|---|---|---|---|---|---|
| Food | 0.30** | 0.29* | −0.16 | −0.01 | 301 |
| | (0.18) | (0.10) | (0.25) | (0.05) | |
| Textiles | −0.06 | 0.70* | −0.02 | −0.07 | 154 |
| | (0.06) | (0.12) | (0.25) | (0.10) | |
| Apparel | 0.62* | 0.97* | −0.08 | 0.14* | 168 |
| | (0.25) | (0.10) | (0.17) | (0.08) | |
| Lumber | 0.76* | 0.75* | −0.21 | 0.01 | 119 |
| | (0.31) | (0.1) | (0.31) | (0.06) | |
| Furniture | 0.97* | 0.61* | 0.08** | −0.01 | 175 |
| | (0.16) | (0.08) | (0.05) | (0.06) | |
| Paper | 0.84* | 0.57* | −0.01 | −0.02 | 224 |
| | (0.23) | (0.09) | (0.20) | (0.25) | |
| Printing | 0.26* | 0.32* | 0.14 | −0.05 | 336 |
| | (0.13) | (0.08) | (0.16) | (0.05) | |
| Chemical | 0.31** | 0.65* | −0.01 | −0.08 | 245 |
| | (0.19) | (0.07) | (0.16) | (0.06) | |
| Petroleum | 0.28 | 0.67* | −0.65 | 0.07 | 112 |
| | (0.20) | (0.18) | (0.82) | (0.23) | |
| Rubber | 0.90* | 0.80* | −0.37 | −0.03 | 189 |
| | (0.17) | (0.05) | (0.28) | (0.05) | |
| Leather | −0.70 | 0.61* | −0.13 | 0.12 | 70 |
| | (0.55) | (0.2) | (0.24) | (0.13) | |
| Stone | 1.34* | 0.89* | 0.16 | 0.16* | 189 |
| | (0.16) | (0.05) | (0.34) | (0.08) | |
| Precious Metals | 0.19 | 0.89* | −0.39 | −0.18* | 210 |
| | (0.36) | (0.08) | (0.32) | (0.08) | |
| Fine Metals | 0.63* | 0.73* | −0.10 | −0.06 | 245 |
| | (0.09) | (0.06) | (0.19) | (0.05) | |
| Machinery | 0.21 | 0.87* | 0.15 | −0.07** | 308 |
| | (0.19) | (0.04) | (0.13) | (0.04) | |
| Electric | 0.72* | 0.73* | 0.58* | −0.12* | 217 |
| | (0.19) | (0.06) | (0.17) | (0.04) | |
| Transportation | 0.06 | 0.70* | 0.21** | −0.13* | 189 |
| | (0.05) | (0.07) | (0.13) | (0.06) | |
| Instruments | −0.05 | 0.75* | 0.77* | 0.08 | 168 |
| | (0.26) | (0.06) | (0.36) | (0.08) | |
| Miscellaneous | 0.41** | 0.90* | −0.04 | 0.08 | 175 |
| | (0.30) | (0.08) | (0.23) | (0.12) | |

[a] The estimates for each industry are corrected for first-order autocorrelation except for those from the petroleum, rubber, machinery, transportation, and miscellaneous industries, where the Durbin–Watson statistic does not indicate possible autocorrelation.
[b] *, Significant at 5% level; **, significant at 10% level.

Figure 4.3: Estimates of returns to hours ($\alpha$) and returns to workers ($\beta$) by industry (DeBeaumont and Singell Jr, 1999)

Australian data is relatively scarce.

Similar to the methods adopted to calibrate a CGE model, Dixon and Freebairn (2009) calibrate the returns for hours for the labour services function:

$$H = L^{\alpha} N \tag{4.4}$$

where $\beta = 1$ is imposed. They use the cost function:

$$TC = [WL + Z]N \tag{4.5}$$

Equating the ratio of marginal costs to marginal products gives the following:

$$\frac{WN}{WL + Z} = \frac{\alpha L^{\alpha-1} N}{L^{\alpha}} \tag{4.6}$$

Which yields the following result:

$$\alpha = \frac{WL}{WL + Z} \tag{4.7}$$

Following this approach they find that $\alpha = 0.83$. This approach does not rely on econometric evidence; instead, relying on estimates for quasi-fixed costs. Whilst the authors treat these estimates of fixed costs with skepticism, this approach provides a method which is consistent with the approach adopted by CGE modellers (Horridge et al., 2000).

The evidence provided in this section does not appear to definitively conclude whether the returns to workers are larger than the returns to hours ($\alpha > \beta$) or whether the returns to hours ($\alpha$) is larger than or less than unity. As such, no restrictions will be placed on the $\alpha$ parameter. Its value will be determined by calibration, similar to the process outline above in (4.7).

## 4.3 Wage premium

The relationship between wage premium and overtime needs to be specified given that it has been concluded that the elasticity of ordinary hours with respect to total hours is positive, ruling out a constant wage premium model.

Hart and Ruffell (1993) estimate the relationship between wage premium and overtime hours to see whether the wage premium is a constant or increasing function of overtime. They show that firms may face wage premium schedule which is an increasing function of overtime even if the underlying wage premium schedule for an individual is a step function. Production bottlenecks and other organisation constraints might ensure that overtime hours may vary across the workforce. This may result in some workers earning a wage premium $W_{p,2}$ whilst there is still a pool of workers who could still be remunerated at $W_{p,1}$.

Assuming that workers face the same wage premium schedule, Hart and Ruffell (1993) calculate some potential wage premium schedules based upon the composition of overtime hours observed in the English economy for different mean overtime hours. The data is presented in the Figure 4.4.

The first row states that when the average overtime hours per industry is 1 then 84.4% of the workforce does not work overtime, 4.2% work between 0 and 2 hours, so on, until it is observed that 1.9% of the workforce works 20+ hours of overtime. When mean overtime hours increases to 2 hours, the fraction working 0 hours decreases from 84.4 to 72.4.

The authors in Hart and Ruffell (1993) assumed 3 different wage premium schedules for individual workers (a) that overtime hours are remunerated at 1.25 (e.g, $W_P = 0.25$) for the first 2 hours and then 1.5 for all remaining, (b) until 4 hours workers receive a 1.25, between 4 and 8 hours they receive a wage premium of 1.5 and after 8 hours they receive a wage premium of 2 and (c) up until 20 hours they receive 1.25 and for all hours after that they receive 1.5 hours.

Considering case (a) the wage premium for the first hour of overtime is:

$$\bar{W_p} = (.884 + 0.042)1.25 + (1 - .884 + 0.042)1.5 = 1.28 \tag{4.8}$$

Doing this for the remaining mean overtime hours and cases yields the wage premium function in Figure 4.5.

In case (a), the wage premium has initial upward spike before it tapers off and becomes flat. Case

| Mean overtime hours | Interval (hours)[a] | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 0 | 0–2 | 2–4 | 4–6 | 6–8 | 8–14 | 14–20 | 20+ |
| 1 | 84·4 | 4·2 | 3·9 | 3·5 | 0·5 | 0·3 | 1·4 | 1·9 |
| 2 | 72·4 | 5·4 | 6·2 | 5·3 | 3·1 | 4·1 | 2·0 | 1·5 |
| 3 | 62·0 | 6·3 | 7·9 | 6·6 | 5·2 | 7·4 | 2·9 | 1·6 |
| 4 | 53·3 | 6·8 | 9·0 | 7·6 | 6·7 | 10·2 | 4·1 | 2·3 |
| 5 | 46·1 | 7·0 | 9·5 | 8·2 | 7·7 | 12·5 | 5·4 | 3·6 |
| 6 | 40·4 | 6·7 | 9·5 | 8·4 | 8·1 | 14·4 | 7·1 | 5·4 |
| 7 | 36·4 | 6·1 | 8·8 | 8·2 | 8·0 | 15·8 | 9·0 | 7·8 |
| 8 | 34·0 | 5·0 | 7·5 | 7·6 | 7·3 | 16·7 | 11·1 | 10·8 |
| 9 | 33·1 | 3·6 | 5·6 | 6·6 | 6·1 | 17·2 | 13·5 | 14·3 |
| 10 | 33·9 | 1·8 | 3·2 | 5·3 | 4·4 | 17·2 | 16·1 | 18·3 |

[a] Over the lower figure but not more than the higher figure; the top class was assumed to be 20–44 when drawing Figure 1. The figures are predictions from OLS regressions of actual percentage frequencies in each class on mean overtime and mean overtime squared.

*Data*: 2-digit industries; obtained from NES distributions of total hours assuming a standard work-week of 40 hours.

Figure 4.4: Proportions of workforce in working different levels of overtime based on the mean level of overtime (Hart and Ruffell, 1993)

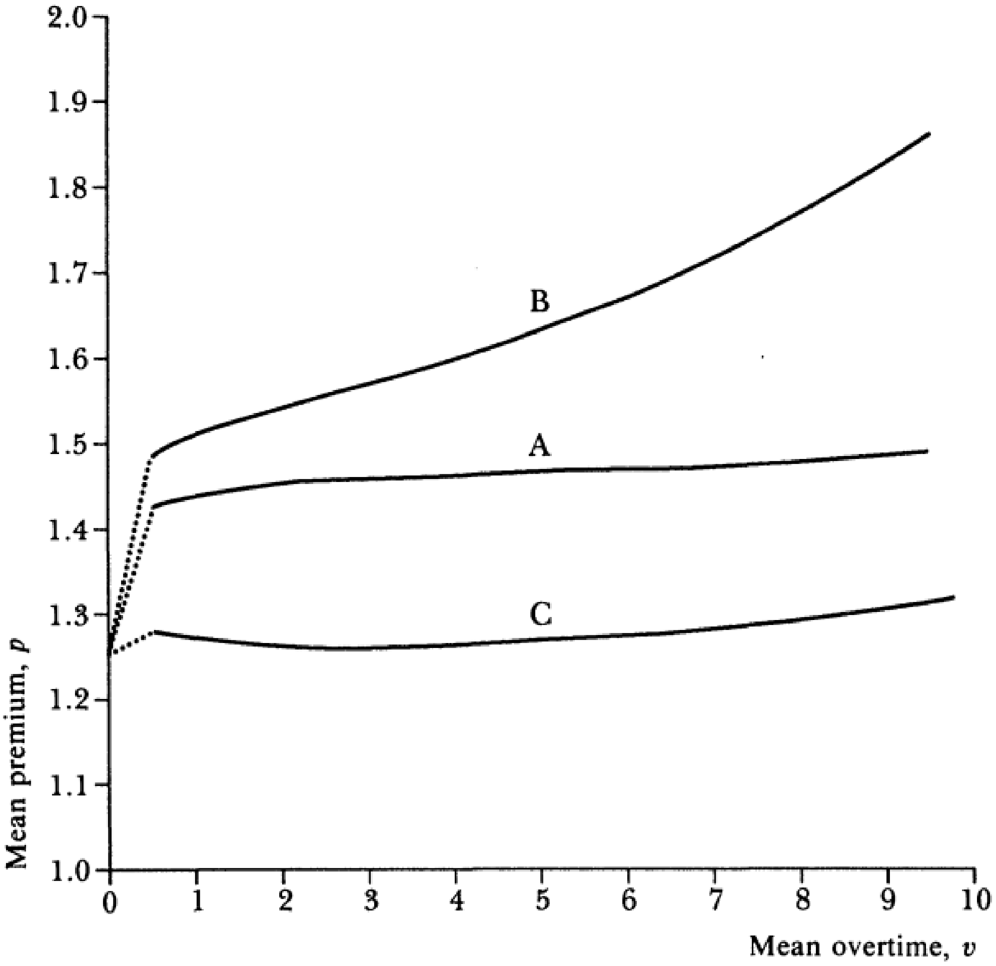


Figure 4.5: Relationship between mean wage premium and mean overtime (Hart and Ruffell, 1993)

(b) has a pronounced upward slope over the entire interval. Case (c) has a significant downward slope for a significant interval before increasing towards the end. This illustrates the plausible shapes of a mean overtime function depending on the underlying wage premium schedule faced by an individual.

Hart and Ruffell (1993) empirically measure the shape of the wage premium function. To capture all the different plausible shapes of the wage premium schedule, they use a third-degree polynomial, where:

$$W_P = \mu_0 + \mu_1 O_T + \mu_2 O_T^2 + \mu_3 O_T^3 + \epsilon \tag{4.9}$$

where $W_P$ is the average wage premium paid by either a firm or an industry and $O_T$ is the average amount of overtime hours worked by all individuals within that same economic unit. A function where $\mu_0 = 1$ and $\mu_1 = \mu_2 = \mu_3 = 0$ would support the hypothesis that the mean wage premium function is a constant whereas a function where any of the coefficients are greater than zero would support the conclusion that the wage premium function is an increasing function.

Hart et al. (1996) have estimated (4.9) for Japan, U.K and U.S.A. These countries are interesting for comparison as they all have different overtime regulations. The U.K sits at one extreme with no mandated wage premium policies for overtime hours. At the other extreme, the USA mandates that all overtime hours be paid at 1.5 times the base wage rate. Japan also mandates that overtime hours are remunerated at a higher rate, but only 1.25 times the base wage rate. The results from their finding are presented in the Figure 4.6.

For the U.K they find that the estimated values for $\mu_1$, $\mu_2$, $\mu_3$, are not statistically significant. They suggest that this supports the conclusion that the wage premium function is a constant of 1.3 in the U.K. The results for the U.S are the same except that it appears the regulation dominates with the wage premium being 1.5. Conversely, Japan which has a mandated wage premium, albeit lower, does show the presence of an increasing wage premium function.

There is no simple formula to explain wage premiums for the Australian economy. Wage pre-

| Japan | | United Kingdom | | United States | |
|---|---|---|---|---|---|
| Period 1958–90<br>No. of obs. per ind. 33<br>No. of industries 16<br>FIML estimates | | Period 1981-88<br>No. of obs. per ind. 3<br>No. of industries 23<br>OLS: Pooled 1981, 1984, 1988 | | Period 1960–90<br>No. of obs. per ind. 31<br>No. of industries 16<br>FIML estimates | |
| **Parameter** | **Estimate** | **Parameter** | **Estimate** | **Parameter** | **Estimate** |
| $\kappa$ | 1.25<br>(n/a) | $\kappa$ | 1.30<br>(0.11) | $\kappa$ | 1.50<br>(n/a) |
| $\mu_1$ | 0.153<br>0.00e-3) | $\mu_1$ | −0.050<br>(0.10) | $\mu_1$ | 0.0008<br>(0.38e-4) |
| $\mu_2$ | −0.082<br>(0.75e-4) | $\mu_2$ | 0.009<br>(0.027) | $\mu_2$ | −0.0006<br>(0.21e-4) |
| $\mu_3$ | 0.0077<br>(0.81e-5) | $\mu_3$ | −0.001<br>(0.002) | $\mu_3$ | 0.0001<br>(0.32e-5) |

*Note*: The figures in parentheses are standard errors.

Figure 4.6: Estimates for mean wage premium schedule for Japan, U.K and USA (Hart et al., 1996)

miums can vary between 50% up to 200% depending on how many hours of overtime a worker has already performed, the time of the day in which it is performed and the day of the week that overtime is performed.

In Australia wage premium and overtime regulation is either outlined in the Modern Awards (MA) or via a Collective Bargaining Agreement (CBA). In addition to outlining the maximum ordinary hours for an occupation, the MA stipulates the minimum entitlements afforded to a worker based on the occupation and industry in which they work. It outlines the minimum wage that a worker can be paid, the maximum hours that can be performed and the premiums that must be paid for overtime, among other things. The MA applies to all employees in Australia and the entitlements described within it cannot be undercut by any other form of arrangement, e.g, if a worker is on an CBA they must receive at least the same wage rate as a worker who is on an award contract.

CBA's which are also known as Enterprise Bargaining Agreements (EBA) apply to workers from occupations and industry where their union has negotiated a better set of basic entitlements than those described in the award. These EBA's usually also include specification for maximum hours and overtime payments similar to the MA's.

Usually, overtime hours are remunerated at an extra 50% of the ordinary wage for the first two hours of overtime and then 100% for all additional hours. This rule is not universal in application, though. For example, a hairdresser must work 3 hours before their premium increases to 100%. Overtime that is performed on Sundays and Public holidays is often remunerated at an even higher rate. For example, nurses who work overtime on a Sunday receive an overtime premium of 200% and they earn 250% if they perform overtime on a public holiday, regardless on the quantity of overtime hours performed.

A peculiar feature of the Australian economy is that overtime regulation does not specify a premium for overtime hours, but instead specifies an overtime wage rate ($W_1$) for overtime hours. For example, the award for nurses [MA000034] specifies that for a Registered nurse with a 4 year degree must earn at least $25.82 for an ordinary hour of work and should earn $38.73 for their first two hours of overtime conditional upon it being performed on a Monday to Friday. Therefore, the implied wage premium is 50%. However, if the overtime hour is performed on a Sunday, the rate of pay for each hour of overtime, regardless of how many hours overtime have been worked, is $51.64 implying an overtime premium of 100%.

A consequence of this policy legislation is that workers who receive above award wages and are not on a collective bargaining agreement actually have an effective wage premium that is lower than their award counterpart as firms are only required to pay the overtime wage rate. For example, using the award rates specified above, if the registered nurse received a wage rate of $30 per hour, their employer would still only be required to pay them $38.73 for their overtime hours, implying a wage premium of 29.1%.

The overtime wage rate ($W_1$) for a worker is defined as:

$$W_1 = (1 + prem)W_{award} \tag{4.10}$$

where $prem$ is the legislated premium and $W_{award}$ is the award wage rate. Subtracting and dividing by the ordinary wage rate ($W$) yields the wage premium function:

$$\frac{W_1 - W_0}{W_0} = (1 + prem)\frac{W_{award}}{W_0} - 1 \tag{4.11}$$

If the overtime hours are zero, it is expected that $prem = 0.5$. This relationship also implies that the greater the differential between award and those workers on above award wages, the lower the wage premium.

There are other reasons why a wage premium may be less than the legislated amount in addition to wage premiums being specified in absolute terms. Employees are often able take Time Off in Lieu (TOIL) instead of receiving an overtime payment. In this case, they receive time off in proportion to the corresponding wage premium. For example, if the wage premium was 100% the worker would receive two hours off in lieu of receiving payment for the additional hours.

In addition, if loadings comprise a significant component of an employees wage bill, then the measured wage premium will appear to be lower than the legislated amount. For example, hair dressers often receive an additional 100% of their ordinary wage for working on Sunday, increasing their average ordinary time earnings.

Consider the example for a hairdresser who works on Monday to Thursday at the ordinary wage rate of \$10 per hour and also works on Sunday, receiving 100% loading, earning \$20 per hour. Assume that they work 8 hours for each day. The average ordinary hourly wage for this worker is: $\frac{4*8*10+1*8*20}{40} = \$12$. If their overtime was remunerated at \$15 per hour, an implied 50% higher than the ordinary wage rate, their measured wage premium would be calculated as:

$$W_P = \frac{15 - 12}{12} = 0.25 \tag{4.12}$$

Thus, despite having a wage premium of 50% of their ordinary wage, the effective wage premium is only 25% of their average ordinary time earnings.

Given the complex nature of wage premium legislation for Australia it would be too difficult to specify an individual wage premium policy for each worker type in the underlying CGE model. Moreover, industries will be comprised of composites of different worker types which may face different overtime regimes. To be able to specify wage premiums for each worker, the model would require knowledge of the time of day worked, day of the week, and whether any loadings etc. applied to the underlying wage rate which is not feasible for this project. Considering these complexities, the wage premium will be calculated based on the observed empirical relationship instead of based on the legislated award.

The data source used to measure ordinary and overtime hours and ordinary and overtime earnings are contained in the Employment Earnings and Hours (EEH) series from the ABS. The restricted access version of the EEH series contains unit record data on a subset of Australian employees. To calculate wage rates, the EEH survey distinguished hours worked based on ordinary and overtime hours and provides detail on ordinary time earnings and overtime earnings. In addition, it provides information based on the sex, age, industry and occupation of employment, full-time/part-time employment status and method of setting pay (e.g, award wage rates or collective bargaining agreement). The industry data used is based on the Australian and New Zealand Industry Classification (ANZSIC) first division. The first division of the ANZSIC classification classifies employees according to 19 different industries (Australian Bureau of Statistics, 2008). The occupation data used is based on the Australian and New Zealand Occupation Classification (ANZSCO) third division. This classifies employees into 88 different occupation types (Australian Bureau of Statistics, 2013a).

The benefit of this data is that it distinguishes ordinary hours and overtime hours of work. This allows the analysis of how earnings vary for ordinary and overtime hours. Unfortunately, it still does not provide any details on the corresponding wage premiums or the days in which hours of work are performed, which makes it difficult to ascertain the wage premium which a worker would have received for performing overtime work.

The ordinary and average overtime wage rate can be derived from the EEH survey. The ordinary wage rate is calculated:

$$W_0 = \frac{WORTCE}{WORHPF} \tag{4.13}$$

where $WORTCE$ is the Weekly Ordinary Time Cash Earnings and $WORHPF$ is the Weekly Ordinary Hours Paid For. The average overtime wage rate is calculated:

$$W_1 = \frac{WOVTCE}{WOVHPF} \tag{4.14}$$

where $WOVTCE$ is the Weekly Overtime Cash Earnings and $WOVHPF$ is the Weekly Overtime Hours Paid For. $W_1$ is referred to as the 'average overtime wage rate' as the underlying wage premium ($prem$) may vary based on the amount of overtime hours worked. For example, a worker might have worked two hours overtime where they received 50% for the first hour and then 100% for the second hour. In this case $W_1$ would equal $1.75 * W_0$. The average wage premium $W_p$ can be calculated:

$$W_P = \frac{W_1 - W_0}{W_1} \tag{4.15}$$

where following the logic above the wage premium is the average wage premium for all overtime hours worked. In Table 4.1 the summary statistics for 19 industries are provided for overtime hours and wage premiums. Unfortunately, there was no wage premium and overtime data available for the Agriculture, forestry and fishing industry. Due to this missing data, it is assumed that the industry averages for wage premiums and overtime apply to this industry.

Thus far the relationship between mean wage premium and mean overtime has been discussed. In summary, it is hypothesised that because the composition of overtime might change as the average level of overtime hours increases, the mean wage premium for an industry might have a different shape to the wage premium of individual workers. The results were mixed with most studies finding

| Industry | Wage premium | Overtime Hours |
|---|---|---|
| Agriculture, forestry and fishing | 0.49 | 1.11 |
| Mining | 0.28 | 1.70 |
| Manufacturing | 0.50 | 2.02 |
| Electricity, gas, water and waste services | 0.64 | 3.03 |
| Construction | 0.44 | 3.40 |
| Wholesale trade | 0.52 | 1.22 |
| Retail trade | 0.49 | 0.41 |
| Accommodation and food services | 0.46 | 0.31 |
| Transport, postal and warehousing | 0.49 | 2.61 |
| Information media and telecommunications | 0.52 | 0.28 |
| Financial and insurance services | 0.49 | 0.24 |
| Rental, hiring and real estate services | 0.50 | 0.80 |
| Professional, scientific and technical services | 0.36 | 0.28 |
| Administrative and support services | 0.54 | 1.39 |
| Public administration and safety | 0.51 | 0.68 |
| Education and training | 0.49 | 0.16 |
| Health care and social assistance | 0.48 | 0.39 |
| Arts and recreation services | 0.56 | 0.31 |
| Other services | 0.46 | 0.80 |

Table 4.1: Average wage premium and overtime hours per worker by industry, Australia, 2010 (Australian Bureau of Statistics, 2011)

a flat mean wage premium. There were significant difficulties when attempting to reproduce these studies using Australian data. Attempts to estimate the equation:

$$W_P = \mu_0 + \mu_1 O_T + \mu_2 O_T^2 + \epsilon \tag{4.16}$$

were met with significant challenges. To calculate this relationship, it is required that the wage premium be averaged across each industry. However, given that the data only included 18 industries this reduced the sample size to 18. This made it infeasible to control for other factors which might influence the average wage premium (e.g, union status). Alternatively, the data could be averaged across occupations, however, data limitations meant that a significant subset of occupations had only one worker in that occupation, defeating the purpose of averaging across occupations.

There are further reasons to be suspicious of the mean overtime hypothesis. It relies upon the assumption that composition of overtime for an industry is representative of the composition of overtime for an individual firm within that industry which given the lack of data, this cannot be tested. Furthermore, as will be demonstrated later, it is likely that overtime is not an exogenous variable and as such the estimate suffers from endogeneity bias. Since the demand for overtime is a function of the wage premium the error term will be correlated with the wage premium.

From a modelling standpoint there are further complications using mean overtime. CGE models are based upon microfoundations. The ORANIG framework to be used for this simulation assumes that all workers in an industry are homogenous. Thus there is no underlying mechanism which allows for varying degrees of overtime per worker. Moreover, the ordinary hours worked per worker and the ordinary wage rate also vary across an industry. Therefore, it is possible that the average wage for ordinary hours is not a constant if the composition of high wage and low wage workers changes as hours increases. Thus the same problem presented for overtime hours could exist for ordinary hours.

Given the limitations of the mean wage premium hypothesis, it will be assumed that the individual average wage premium represents the wage premium paid by firms for this data set. Despite the regulation often having wage premiums as a step function, given that wages above award can cause wage premiums to be effectively lower, it will be assumed that wage premiums are a continuous. A 2 Staged Least Squares (2SLS) model which accounts for labour demand will be estimated to measure the relationship between overtime and wage premium.

It is assumed that the equation describing the wage premium function for an individual worker can be specified as:

$$log(W_P) = \alpha_0 + \alpha_1 log(O_T) + \alpha_2 MOSP + \alpha_3 log(W) + \alpha_4 OCC + \epsilon \tag{4.17}$$

where $log(O_T)$ represents the logarithm of overtime hours; $MOSP$ represents the regulation which specifies how wage premiums are determined, i.e, via an award or CBA; $log(W)$ represents the ordinary wage rate and $OCC$ describes the occupation in which an employee is employed.

There are no preference parameters included in this equation as it is assumed that the government regulation determines the wage premiums and that firms can utilise as many workers as desired for a given wage premium.

In this equation, the $O_T$ variable is likely to be endogenous since the demand for overtime is a function of the wage premium. An approximation of the overtime demand function, based on the specified models in Chapter 3.1.4 and 3.4, is specified as follows:

$$log(O_T) = \beta_0 + \beta_1 log(W_P) + \beta_2 log(L_0) + \beta_3 log(W) + \beta_4 IND + \beta_5 FTPT + \beta_6 SEX + \beta_7 SIZE + \epsilon \tag{4.18}$$

where $L_0$ is the ordinary hours worked per worker; $IND$ represents industry; $FPTP$ defines whether the worker is a full time or part time worker; SEX describes the sex of a worker and $SIZE$ represents the size of the firm in which an employee is employed. Whilst it cannot be tested, the terms $IND$, $FTPT$, $SEX$ and $SIZE$ are all used as proxies for fixed costs since there is no reliable measure of fixed costs for this data set. It is assumed that full time workers are likely to have higher fixed costs than part time workers because firms would be less likely to hire a worker part time if they had high fixed costs. The industry variable is used to capture fact that fixed costs vary across industries, so using an industry dummy should hopefully capture some of this variation. The size and sex variables have the most tenuous link to fixed costs, they were simply used in case there was variations across sexes or firm sizes.

Both a logarithmic and second-order polynomial was estimated for these equations. As the explanatory power of the logarithmic function was higher, it was the preferred model.

The data used to estimate the above equations is based on the 2010 EEH survey data (Australian Bureau of Statistics, 2011). The data set contains unit record information for 60,271 individuals. The system of equations specified above are based on the available variables contained within this data set. It is not possible to add other variables to this data set due to confidentiality issues. There are approximately 10,000 workers out of 60,000 who perform overtime duties. From this sample of workers, all workers who did not receive a wage premium for working overtime (i.e $W_1 \leq W_0$); all workers who are managers or self-employed, whom are often not bound by overtime regulation and all outliers where either the wage premium was larger than 300% or the overtime hours were in excess of 15 hours were removed from the sample. There were 6363 data points remaining after removing all these values. Unfortunately, it was not possible to use more up-to-date data than 2010 as later series from the EEH survey did not have adequate data on industry of employment for individuals.

The 2SLS estimates of equation (4.17) and (4.18) are presented in the Table 4.2. The wage premium function is expressed in row (1) and the overtime demand function is in row (2). The OCC, IND, MOSP, FTPT, SEX and size are not reported. However, they all provide explanatory power for the model.

The output shows that a 1% increase in overtime hours results in a 0.7% increase in the wage premium. For the overtime demand function, the signs on the coefficients are mostly as expected.

| Table : 2SLS regression output | | |
|---|---|---|
| | (1) | (2) |
| | log(Wage Premium) | log(Overtime) |
| log(overtime) | 0.7*** | - |
| | (0.03) | |
| log(Wage Premium) | - | -0.10* |
| | | (0.04) |
| Ordinary hours | - | -0.06 |
| | | (0.04) |
| log(Wage Rate) | -0.81*** | 0.08* |
| | (0.053) | (0.036) |
| Observations | 6300 | 6300 |
| t statistics in parantheses | | |
| * p <0.05, ** p <0.01, *** p <0.001 | | |

Table 4.2: 2SLS: regression of wage premium and overtime hours

An increase in the wage premium causes the level of overtime to decrease. Although not significant, the coefficient for the relationship between ordinary hours and overtime is similar to those results found in section (4.1) and is the correct sign. This point estimate, despite being insignificant, is consistent with the evidence that supports the conclusion that the wage premium is an increasing function since $-1 < \frac{dO_T}{dL_0} < 0$ which implies $\frac{dL}{dL_0} > 0$ whereas the constant wage premium model requires $\frac{dL}{dL_0} < 0$.

The positive coefficient $\beta_3$ suggests that workers with higher wages work more overtime. This is consistent with Oi (1962) who suggests that lower skill workers tend to have lower fixed costs. Thus wages appear to be acting as a proxy for fixed costs with high wages leading to more overtime.

Finally, as was suspected a test for endogeneity suggests that an OLS estimate of equation (4.17) is not consistent. This implies that the covariance between overtime and the error in the equation (4.17) was not zero when estimating (4.17) using OLS. As such, overtime is not an exogenous variable. 2SLS overcomes this problem by estimating the relationship (4.18) first then using these estimated values, which are not correlated with the error term in (4.17), as an instrumental variable in the equation (4.17).

There are significant potential shortcomings to this approach. This model assumes that the supply of workers is determined solely by regulation. There is possible for bias in the estimates if the sample suffers from omitted variable bias. This may occur if highly motivated workers tend to perform more overtime than non-highly motivated workers. If these workers are also compensated with wage premiums higher than the regulated amount this may lead to an upward bias for the regression coefficient between wage premiums and overtime.

Heckman correction was not used as it is expected that the wage premium for a worker who was not selected would be zero. Compared to the case of labour supply, assuming that the wage rate for a non-worker is zero is not true since it is possible that they would be willing to receive a wage rate which is not feasible due to a minimum wage or some other variable.

Furthermore, the specification of this model suffers from the inability to apply bootstrapping to the data due to confidentiality issues.

Overall, this estimate suggests that the elasticity between wage premium and overtime hours is 0.7. This value can be used to calibrate the wage premium parameter ($\eta_{WP,OT}-1$) specified in (3.80) for the constant elasticity model of labour demand. This estimation and the empirical literature on the labour services model supports using the constant elasticity of labour demand model.

## 4.4 Fixed costs

Fixed costs are the final component of the labour demand model yet to be discussed. Similar to how the labour services function is simply assumed to be the product of workers and hours, i.e, $H = NL$, labour is often assumed to be a variable factor. Neoclassical theory often assumes that firms can hire as much labour as they desire for a fixed wage rate.

Quasi-fixed costs represent the costs of hiring which are invariant to the quantity of hours worked and only apply to the quantity of workers a firm has employed. Oi (1962) analyses the effects that quasi-fixed costs have on the level of employment. He considers a short-run theory of employment where hiring and training costs constitute fixed labour costs. Adopting a simplified model presented in Norris (1996), the total cost ($C$) of a worker working for ($T$) years is defined:

$$C = \sum_{t}^{T} \frac{W_t}{(1+r_t)^t} + HR + TR \tag{4.19}$$

where $W_t$ is the wage at time $t$, $(1 + r_t)$ is the interest rate at time $t$, $HR$ is the cost of hiring and $TR$ is the training cost. The value $V$ of an additional worker is the sum of its marginal revenue, such that:

$$V = \sum_{t}^{T} \frac{MR_t}{(1+r_t)^t} \tag{4.20}$$

where $MR_t$ is the marginal revenue of a worker at time $t$. In equilibrium, costs equal the value of an additional worker, such that:

$$\sum_t^T \frac{W_t}{(1+r_t)^t} + HR + TR = \sum_t^T \frac{MR_t}{(1+r_t)^t} \tag{4.21}$$

Allowing for expenditure on training and hiring costs can be amortised gives the expressions:

$$HR = \sum_t^T \frac{hr^*}{(1+r_t)^t} \tag{4.22}$$

and

$$TR = \sum_t^T \frac{tr^*}{(1+r_t)^t} \tag{4.23}$$

For example, if $T = 5, r = 0.05$ and $HR = 50$ the total expenditure could be spread evenly across all 5 periods as $hr = 11.55$. Further simplifying the analysis by assuming that wages and marginal revenue are constant across time yields the following expression:

$$W\sum_t^T \frac{1}{(1+r_t)^t} + hr\sum_t^T \frac{1}{(1+r_t)^t} + tr\sum_t^T \frac{1}{(1+r_t)^t} = MR\sum_t^T \frac{1}{(1+r_t)^t} \tag{4.24}$$

where $W$ is the constant wage and $MR$ is the constant marginal revenue. By defining $Z = hr + tr$, (4.24) simplifies to:

$$W + Z = MR \tag{4.25}$$

To measure what share of employees costs are fixed (i.e cannot be changed by varying hours), a measure of flexibility called fixity ($fix$) is defined:

$$fix = \frac{Z}{W + Z} \tag{4.26}$$

where if $fix = 0$ labour is a completely variable factor and if $fix = 1$ it is completely fixed. In the latter case, there would be no difference in costs for hiring someone for 20 hours per week compared to 40 hours. Thus, there would be no incentive to vary hours in response to an economic shock. Conversely, when $fix = 0$ there are no costs of firing or hiring an additional worker.

This model implies that for any positive amount of fixed costs, the wage rate will be less than the marginal revenue. The greater the level of fixity, the greater the difference between the wage rate and marginal product.

The implications of this framework are that the greater the fixity of labour, the lower the expected variation in employment. In the short run, previous period's hiring and training costs are sunk. Therefore, if there is a negative demand shock such that the marginal product of a worker decreases, the decision of whether to retain an employee depends on whether $W \leq MR$, i.e, the fixed cost component is ignored. The greater the fixed cost component, the larger the decrease in the marginal product required to have the firm terminate the employment of the worker. Oi (1962) tested this hypothesis by comparing the variations in employment for those of low-skilled compared with high-skilled workers. Unfortunately, data for the level of fixity of a worker was not available, so the wage rate of the worker was used as a proxy. The findings in his research support this hypothesis, as workers with higher wage rates tend to experience lower turnover. These results are supported by the research conducted by Rosen (1968) who analyses the labour turnover for railroad workers and finds that those with higher fixed costs also experience lower job turnover.

Ehrenberg (1971) Estimate the effects that fixed costs have on overtime. Using linear estimation techniques, the following equation is estimated:

$$O_T = a_0 + a_1 \frac{Z}{W_p} + a_2 L_0 + a_3 Q^T + a_4 S^{NS} + a_5 AR + a_6 CBA \tag{4.27}$$

where $O_T$ is the annual amount of overtime hours per worker, $Z$ is the fixed costs, $W_P$ is the wage premium, $L_0$ is the number of hours per week after which an overtime premium must be paid, $Q^T$ is the quit rate, $S^{NS}$ is the ratio of new to senior workers $AR$ is the establishment absentee rate and $CBA$ is a dummy representing the presence of collective bargaining agreement.

This analysis is conducted for 24 different industries and the coefficient $a_1$ is statistically significant and positive for 18 of these 24 industries. The positive coefficient for $\alpha_1$ supports the conclusion that an increase in fixed costs causes a substitution to workers performing longer hours away from hiring more workers.

### 4.4.1 Measurement issues for fixed costs

There is significant evidence to suggest that fixed costs play a pivotal role in determining the level of overtime and employment used within an economy. Unfortunately, there is difficulty in measuring fixed costs. Firstly, statistical agencies do not provide measures of the quasi fixed costs in the same way they provide measures on the wage rate. Secondly, it is often difficult to distinguish between wage payments and quasi-fixed costs.

(Garbarino, 1964, p.435) states that "Fringe costs and overtime costs are both a direct function of pay rates. It is obvious that overtime cost moves with the base rate (and this is almost the entire explanation for the upward movement over the years), but so do the costs of many fringes, such as social insurance, vacations, and holidays" without formalizing the idea. It is worth expanding upon this idea to see how it affects the calculation of fixed costs. For example, suppose that fixed costs $(Z)$ are such that $Z = sWL_0$. It follows that the cost function can be expressed as:

$$TC = [WL + sL_0 + W_P W(L - L_0)]N \tag{4.28}$$

Adding and subtracting $s(L - L_0)$ yields the expression

$$TC = [(1 + s)WL + (W_P - s)W(L - L_0)]N \tag{4.29}$$

Given the labour services function $H = L^{\alpha} N^{\beta}$, setting the ratio of marginal products of the labour services function to the marginal costs of hours and workers yields:

$$\frac{W(1 + s)L + (W_p - s)W(L - L_0)}{[W(1 + s) + (W_p - s)W]N} = \frac{\frac{\beta H}{N}}{\frac{\alpha H}{L}} \tag{4.30}$$

which yields:

$$\frac{\frac{\alpha H}{L}}{W+(W_p)W}=\frac{\frac{\beta H}{N}}{W(1+s)L+(W_p-s)W(L-L_0)} \tag{4.31}$$

giving:

$$\alpha[W(1+s)L+(W_p-s)W(L-L_0)]=\beta[W(1+s)L+(W_p-s)WL] \tag{4.32}$$

Further giving:

$$-\alpha(W_p-s)WL_0=(\alpha+\beta)[W(1+s)L+(W_p-s)WL] \tag{4.33}$$

removing the $WsL$ from the RHS gives:

$$-\alpha(W_p-s)WL_0=(\beta-\alpha)[WL+W_pWL] \tag{4.34}$$

Finally giving an expression for hours per worker as:

$$L=(\frac{\alpha}{\beta-\alpha})\frac{sWL_0-W_pWL_0}{[WL+W_pWL]} \tag{4.35}$$

And since $sWL_0=Z$ substituting $Z$ into the above equation yields the same result derived earlier in Chapter 3.1.4. Therefore, as long as any payment to labour does not incur the overtime wage premium, it will have the effect of a fixed costs. For example, since superannuation is not paid on working overtime it has the impact of acting like a fixed cost, despite it typically being calculated as 9% of the worker's ordinary wage rate. Thus an increase in the superannuation guarantee would have the same impact as an increase in fixed costs, causing a movement away from workers towards overtime.

### 4.4.2 Calculating fixed costs

In regards to calculations for fixed costs, Table 4.7 provides a summary of labour costs for the U.S for the year 2010.

Ehrenberg and Smith (2012) state that the fixed costs components are: Unemployment insurance and other; Employment costs based on benefits formulas (defined benefits plans), Insurance (medical,

**Table 5.3**

**Employee Benefits as a Percentage of Total Compensation, 2010 (Average Hourly Cost in Parentheses)**

| | | |
|---|---|---|
| **Legally required payments** | **7.7** | **($2.30)** |
| Social Security | 5.6 | ($1.68) |
| Workers' compensation | 1.5 | ($0.44) |
| [a]Unemployment insurance and other | 0.6 | ($0.18) |
| **Retirement** | **4.5** | **($1.32)** |
| [a]Employment costs based on benefit formulas (defined benefit plans) | 2.7 | ($0.81) |
| Employer costs proportional to earnings (defined contribution plans) | 1.7 | ($0.51) |
| [a]**Insurance** (medical, life) | **8.8** | **($2.62)** |
| [a]**Paid vacations, holidays, sick leave** | **6.9** | **($2.06)** |
| **Other** | **2.5** | **($0.73)** |
| **Total** | **30.4** | **($9.04)** |

[a]Category of costs believed by authors to be largely quasi-fixed (see discussion in the text).

*Source:* U.S. Labor Department, Bureau of Labor Statistics, "Employer Costs for Employee Compensation—March 2010," Table 1, news release USDL: 10-0774 (June 9, 2010).

Figure 4.7: Measurements of different labour costs, including fixed costs, per worker in U.S.A (Ehrenberg and Smith, 2012)

life); Paid vacations, holidays, sick leave; and other. The data suggests that about 19% of the workers compensation is comprised of quasi-fixed costs. For Australia, Dixon and Freebairn (2009) provide a breakdown of fixed costs, presented in Table 4.3.

| Category of quasi-fixed labour costs | Indicative importance of items in Australian total labour costs |
|---|---|
| Hiring | Not Available |
| Training | 2.5% of gross wages |
| Firing and retention | 6.6% of gross wages |
| Fringe benefits | 3.4% of gross wages |
| Regulatory and administrative requirements | Not Available |
| Staff payrolls | |
| OHS | About 1% |
| Taxation | About 6% |
| Negotiations new agreements | Not Available |

Table 4.3: Note: OHS denotes occupational health and safety Indicative estimates of the relative importance of Quasi-fixed labour costs in Australian Total labour costs (Dixon and Freebairn, 2009)

The data for Australia suggests that 19.5% of costs are fixed costs. However, Dixon and Freebairn (2009) note that they are uncertain about their estimates, as they are unable to quantify all fixed costs.

An alternative approach has been followed in Delmez and Vandenberghe (2018). They approximate a cost function and output equation using a translog function. From this they recover estimates for fixed costs and find that for Belgium fixed costs constitute around 20-23% of wage costs. This approach does not rely on measurements of fixed costs; instead, calculates fixed costs based on an econometric model of costs and output. The results presented in this research are similar to the measured values presented in (Dixon and Freebairn, 2009).

### 4.4.3 Summary

The objective of this chapter was to provide an empirical justification for which model to choose from Chapter 3. It did this by reviewing the empirical literature on (a) the elasticity of ordinary hours with respect to total hours ($\eta_{L,L0}$), (b) the returns to hours ($\alpha$) and workers parameter ($\beta$) for the labour services function and (c) the shape of the wage premium schedule.

Given there was overwhelming evidence to suggest that the elasticity of ordinary hours with respect to total hours ($\eta_{L_0,L}$) is negative, that the returns to hours are often larger than the returns to workers ($\alpha > \beta$) and that wage premiums in Australia are an increasing function of overtime, this ruled out choosing the standard model with the constant wage premium. Ultimately, the constant elasticity model was preferred to the linear wage premium model as it produced a model with better explanatory power and the regression for the linear wage premium model generated a negative value for ($\eta_{WP,OT} - 1$).

The measurement of fixed costs and associated issues with measuring fixed costs were also discussed in this section. This review provided estimates for fixed costs overseas and outlined some of the key economic variables which comprise fixed costs. Most importantly, it was shown that even costs which are proportionate to ordinary earnings can be considered fixed costs as long as firms are not required to pay these costs for overtime hours. This means that the superannuation guarantee can be treated as fixed costs. These concepts were used when measuring fixed costs in the following chapter.

# 5 Model Integration and Data

## 5.1 Model Linearisation

To incorporate this theory into a CGE-style model, the theory specified in (3.4) needs to be converted into its linearised form. The rules for linearisation were presented in Chapter 2.3.3.

The notational convention is to represent all level equations as uppercase variables and to denote percentage change variables as lower case. Following this approach, the labour services function (3.5) is represented as:

$$h = \alpha l + \beta n \tag{5.1}$$

where $h$ is the percentage change in effective labour, $l$ is the percentage change in hours per worker and $n$ is the percentage change in workers. The $\alpha$ and $\beta$ represents the returns to workers and hours, respectively. It is expected that these values will vary across industries depending on each industries characteristics.

In the simulation the returns to workers parameter will be set to 1, such that $\beta = 1$. Unfortunately, in the absence of an appropriate econometric estimate for Australia, this was required since effective labour is not observed. Whilst this is not ideal, it has no impact on the level of intensive margin employment. Furthermore, it has no impact on any other variable as employment does not appear anywhere else in the ORANIG code.

Earlier in this research it was discussed how it was expected that the marginal product of an additional hour of work may be increasing due to a 'warming up effect' or decreasing due to a 'fatigue effect'. A value of $\alpha > 1$ would support the former conclusion meanwhile a conclusion of $\alpha < 1$ would support the latter conclusion. The calibrated values for $\alpha$ are discussed in Chapter 6.10

The total cost function (3.9) can be expressed as:

$$tc = S_{LC}[w + l] + S_{VC}[w_p + w + o_t] + S_Z[z] + n \tag{5.2}$$

where $o_t$ is the percentage change in overtime, $S_{LC}$ is the share of linear costs, $S_{VC}$ is the share of variable costs and $S_Z$ is the share of fixed costs. The formulas for each of these shares are:

$$S_{LC} = \frac{WLN}{TC} \tag{5.3}$$

$$S_{VC} = \frac{W_P W(L - L_0)N}{TC} \tag{5.4}$$

$$S_Z = \frac{ZN}{TC} \tag{5.5}$$

The expression for the wage premium (3.80) is expressed:

$$w_p = (1 - \eta_{WP,OT}) * o_t \tag{5.6}$$

The overtime function is defined as:

$$o_t = \frac{L}{L - L_0} * l + \frac{L_0}{L - L_0} * l_0 \tag{5.7}$$

where $L_0$ is ordinary hours and is exogenous in this model. However, the $l_0$ could be shocked to simulate the effects of a work-sharing policy.

Finally, the expression for the equilibrium condition can be derived by linearising equation (3.86) which gives the following expression:

$$w + l + \frac{W_p}{1 + (\eta_{WP,OT})W_p} w_p = S_{LC}[w + l] + S_{VC}[w_p + w + o_t] + S_Z[z] \tag{5.8}$$

In Chapter 5.3 these linearised equations will be converted into the TABLO language and incorporated into the ORANIG framework.

To integrate this into the ORANIG framework requires two modifications to the baseline code. In the underlying ORANIG framework the variable $x1lab$ represents the quantity of labour. In this simulation $x1lab \equiv h$ such that:

$$x1lab = \alpha l + \beta n \tag{5.9}$$

The second modification occurs to the price of labour $p1lab$. In the underlying ORANIG model, the only cost of labour is the wage rate. In this simulation, the cost of labour is segmented into three different components. Therefore, the price of labour is redefined as:

$$p1lab + l + n = S_{LC}[w + l] + S_{VC}[w_p + w + o_t] + S_Z[z] + n \tag{5.10}$$

These two modifications incorporate the theory of labour demand into the underlying ORANIG framework which was outlined in Chapter 2.8. This theory introduced 9 new variables and 7 equations into the model. For closure, this will require that two of the variables be set as exogenous.

## 5.2 Data requirements

Data to calibrate the model is required in addition to incorporating the theory into the ORANIG framework. The process of calibration is explained in Chapter 2.5.5. The data for share parameters (5.3), (5.4), (5.5) along with $\alpha$, $\beta$, $W_P$, $L$ and $L_0$ are required to calibrate the equations (5.1), (5.2), (5.7) and (5.8).

The underlying database used for this simulation is based on 2016 Input-Output tables for the Australian economy and is sourced from Dixon and Nassios (2016). This database only has data on total expenditure on labour and is denoted by the header $V1LAB$. To account for the three components of labour, the total expenditure on labour, $V1LAB$ is defined as:

$$V1LAB = [LC + VC + FC]N \tag{5.11}$$

where $LC$ are the total linear costs, $VC$ are the total variable costs and $FC$ are the total fixed cost components for all $N$ workers. The $LC$ accounts for all ordinary term earnings and is defined:

$$LC = WL \tag{5.12}$$

The $VC$ are all the overtime payments defined as:

$$VC = W_P W(L - L_0) \tag{5.13}$$

The $FC$ are defined:

$$FC = Z \tag{5.14}$$

### 5.2.1 Ordinary and overtime costs

The most reliable data for ordinary and overtime hours for Australia is sourced from the EEH survey. It measures earnings and hours worked for both ordinary and overtime hours for individual workers. Unfortunately, the data from the $EEH$ survey measures based on earnings per worker whereas the data used for $V1LAB$ is based on aggregate expenditure on labour by industry. Therefore, they are not consistent data sources.

To overcome this problem, the ratio between ordinary costs ($LC$) and overtime costs ($VC$) are calculated using the EEH data, such that:

$$\gamma_{VC} = \frac{VC}{LC} \tag{5.15}$$

Using the relationship between fixed costs and $V1LAB$ (which is outlined in Chapter 5.2.4) allows fixed costs to be expressed as :

$$FC = \gamma_{FC} V1LAB \tag{5.16}$$

Substituting (5.15) and (5.16) into (5.11) yields:

$$LC = \frac{(1 + \gamma_{FC})}{\gamma_{VC}} V1LAB \tag{5.17}$$

This approach avoids the need to find an appropriate value for the number of workers. This will have no impact on the model simulation as the number of workers are not required in any equation to calibrate the model.

### 5.2.2 Ordinary and overtime hours

The equation (5.7) requires an initial value for total ($L$) and ordinary hours ($L_0$) to calculate the percentage change in overtime. Data on ordinary and overtime hours are presented in Table 5.1.

| Industry | Ordinary income | OT income | Ordinary hours | OT hours |
|---|---|---|---|---|
| Mining | 2036.60 | 24.73 | 39.49 | 1.70 |
| Manufacturing | 1085.18 | 32.06 | 33.97 | 2.02 |
| Electricity, gas, water and waste services | 1321.78 | 75.29 | 34.19 | 3.03 |
| Construction | 1289.12 | 57.77 | 33.47 | 3.40 |
| Wholesale trade | 1072.49 | 21.67 | 31.74 | 1.22 |
| Retail trade | 562.24 | 4.75 | 24.20 | 0.41 |
| Accommodation and food services | 474.58 | 3.13 | 21.57 | 0.31 |
| Transport, postal and warehousing | 1119.82 | 44.17 | 32.68 | 2.61 |
| Information media and telecommunications | 1217.24 | 5.99 | 29.33 | 0.28 |
| Financial and insurance services | 1409.32 | 5.23 | 31.23 | 0.24 |
| Rental, hiring and real estate services | 912.74 | 12.59 | 28.77 | 0.80 |
| Professional, scientific and technical services | 1373.02 | 4.69 | 29.34 | 0.28 |
| Administrative and support services | 877.96 | 22.56 | 29.39 | 1.39 |
| Public administration and safety | 1209.01 | 13.87 | 30.08 | 0.68 |
| Education and training | 1006.56 | 3.01 | 25.77 | 0.16 |
| Health care and social assistance | 908.26 | 6.13 | 27.51 | 0.39 |
| Arts and recreation services | 725.68 | 5.58 | 22.45 | 0.31 |
| Other services | 792.82 | 10.92 | 26.77 | 0.80 |

Table 5.1: Average ordinary time earnings, overtime earnings, ordinary hours and overtime from (Australian Bureau of Statistics, 2011)

This data is sourced from the EEH survey which measures "ordinary hours paid for" and "overtime hours paid for" for individual workers.

Whilst this measure is the most reliable, it is also likely "ordinary hours paid for" overstates the true amount of ordinary hours worked since it includes hours for which workers are paid that were not actually worked, e.g, paid sick leave.

The labour demand models presented in Chapter 3 require that the ordinary hours ($L_{0,i}$) be exogenously determined for each industry ($i$), otherwise, firms would simply ensure that the ordinary hours is set as total hours ($L_0 = L$) and not pay the wage premium.

The ordinary hours ($L_0$) worked by an individual worker are determined by government regulation for most workers in Australia. The usual rule of thumb is that ordinary hours of work are 8-hours per day or 40-hours per week, with industry specific legislation governing the ordinary hours for each industry.

The model is calibrated based on the observed ordinary hours (presented in table 5.1) instead of based on legislated levels of ordinary hours. This is done for two reasons. Firstly, each industry is comprised of full-time and part-time employees who have different ordinary hours. Secondly, due to the level of aggregation, there are different levels of ordinary hours within an industry. Thus, the observed levels of ordinary hours reflects the different compositions of workers than the legislated ordinary hours.

### 5.2.3 Wage premium data

The linearised equilibrium condition (5.8) requires wage premium data to calibrate the equation. Data on wage premiums are also collected from the EEH survey and are outlined in Table 5.2.

| Industry | Wage Premium |
|---|---|
| Mining | 0.28 |
| Manufacturing | 0.50 |
| Electricity, gas, water and waste services | 0.64 |
| Construction | 0.44 |
| Wholesale trade | 0.52 |
| Retail trade | 0.49 |
| Accommodation and food services | 0.46 |
| Transport, postal and warehousing | 0.49 |
| Information media and telecommunications | 0.52 |
| Financial and insurance services | 0.49 |
| Rental, hiring and real estate services | 0.50 |
| Professional, scientific and technical services | 0.36 |
| Administrative and support services | 0.54 |
| Public Administration and safety | 0.51 |
| Education and training | 0.49 |
| Health care and social assistance | 0.48 |
| Arts and recreation services | 0.56 |
| Other services | 0.46 |

Table 5.2: Average wage premium per worker for each industry, 2010, Australia (Australian Bureau of Statistics, 2011)

### 5.2.4 Fixed costs

There is no direct data for fixed costs of employment for workers in Australia; instead, fixed costs can be estimated using ABS data. This data is sourced from the Australian Industry (AI), Labour Account (LA) and Other Labour Costs (OLC) series. All these data sets contain aggregate expenditure data for each industry.

For the Australian economy, the main sources of fixed costs of employment are likely to be:

- Superannuation
- Administration
- Recruitment
- Termination
- Training
- Sick leave
- Holiday leave

The labour account series contains a variable defined "Other Labour costs" (OC) which is the sum of training, recruitment and payroll tax paid. Note, this variable is different from the series OLC mentioned above. The training and recruitment costs can be calculated by subtracting Payroll Tax (PT) from this variable. The data on payroll tax is contained within the AI series.

The Leave related Fixed Costs (LFC) are derived from the labour account series. They consist of the costs such as sick and holiday pay. They are calculated by measuring the fraction of hours which are paid but not worked by employees. The formula for leave-related fixed costs are defined:

$$\text{LFC} = \frac{(\text{hours paid for} - \text{hours actually worked})}{\text{hours actually worked}}(\text{compensation of employees}) \tag{5.18}$$

The data for superannuation costs (SC) are contained within the OLC series. It measures the contributions made by employers to their employees for each industry. Due to differences in scope and coverage, the OLC and AI series have differing values for total labour costs. As a consequence, there are significantly varying values for superannuation as a fraction of total labour costs depending on whether total labour costs are sourced from the OLC or AI series. To overcome this problem, it is assumed that the correct ratio of superannuation to total labour costs is determined using the data in the OLC series. However, the absolute values of superannuation used to calculate fixed costs are determined by using the AI dataset. To calculate this value, the fraction determined initially using the OLC values are multiplied by the AI values.

There is unfortunately no reliable data for Administration or Termination costs for the Australian economy. They are assumed to be zero in this simulation. However, this will cause the estimates for fixed costs to understate the true values of fixed costs. Hopefully, both those components entail a negligible amount of fixed costs. The shares of total labours costs for each component of fixed costs outlined above are outlined in Table 5.3.

As there was no labour account data for Other Services its value represents the average of all other industries. Therefore, the average fixed costs comprise approximately 16% of all labour costs, which is consistent with the findings of Dixon and Freebairn (2009); Hamermesh (1996); Ehrenberg and Smith (2012).

Another shortcoming of this approach is that the data used to calculate the fixed costs values is from a different period to our underlying ORANIG data. This is why the data is represented as percentages of total employee compensation. The fixed costs will be calculated as a fraction of total employee compensation such that:

$$FC = \gamma_{FC} * V1LAB \tag{5.19}$$

where $\gamma_{FC}$ for each industry are in Table 5.3.

| Industries | OC | SC | PT | LFC | Fixed costs |
|---|---|---|---|---|---|
| Agriculture, forestry and fishing | 0.02 | 0.03 | 0.00 | 0.02 | 0.07 |
| Mining | 0.08 | 0.06 | 0.04 | 0.06 | 0.16 |
| Manufacturing | 0.06 | 0.07 | 0.03 | 0.06 | 0.15 |
| Electricity, gas, water and waste services | 0.08 | 0.07 | 0.04 | 0.08 | 0.20 |
| Construction | 0.04 | 0.05 | 0.02 | 0.06 | 0.13 |
| Wholesale trade | 0.04 | 0.07 | 0.03 | 0.08 | 0.16 |
| Retail trade | 0.03 | 0.07 | 0.03 | 0.08 | 0.16 |
| Accommodation and food services | 0.04 | 0.07 | 0.02 | 0.11 | 0.20 |
| Transport, postal and warehousing | 0.06 | 0.06 | 0.03 | 0.06 | 0.15 |
| Information media and telecommunications | 0.06 | 0.07 | 0.04 | 0.08 | 0.17 |
| Financial and insurance services | 0.13 | 0.05 | 0.02 | 0.07 | 0.23 |
| Rental, hiring and real estate services | 0.07 | 0.07 | 0.02 | 0.09 | 0.20 |
| Professional, scientific and technical services | 0.06 | 0.06 | 0.02 | 0.09 | 0.19 |
| Administrative and support services | 0.06 | 0.06 | 0.03 | 0.08 | 0.17 |
| Public administration and safety | 0.03 | 0.09 | 0.00 | 0.07 | 0.10 |
| Education and training | 0.07 | 0.09 | 0.00 | 0.08 | 0.17 |
| Health care and social assistance | 0.02 | 0.08 | 0.01 | 0.09 | 0.14 |
| Arts and recreation services | 0.07 | 0.06 | 0.02 | 0.11 | 0.21 |
| Other services | 0.06 | 0.05 | 0.02 | 0.08 | 0.16 |

Table 5.3: Quasi-fixed costs as a fraction of total employee compensation

| Description | Coefficient name | TABLO name |
| --- | --- | --- |
| The share of linear costs | $S_{LC,i}$ | $\frac{V1LC_i}{V1LAB_i}$ |
| The share of variable costs | $S_{VC,i}$ | $\frac{V1VC_i}{V1LAB_i}$ |
| The share of fixed costs | $S_{FC,i}$ | $\frac{V1FC_i}{V1LAB_i}$ |
| Standard hours | $L_{0,i}$ | $L_i$ |
| Total hours | $L_i$ | $LEQ_i$ |
| Overtime hours | $OT_i$ | $OT_i$ |
| Returns to hours | $\alpha_i$ | $alphot_i$ |
| Wage premium | $W_{p,i}$ | $WAGEPREM_i$ |

Table 5.4: Table of model coefficients and their corresponding TABLO names

## 5.3 Model code

So far the linearisation and data requirements have been covered. The following section discusses the process involved with incorporating the theoretical model into the TABLO language.

Table 5.4 lists the model coefficients, the corresponding TABLO name and a description for the labour demand model. Coefficients are the values which populate the $A$ matrix, such as $a_{i,j}$ in (2.5). For example, in equation (5.9) the $\alpha$ is a coefficient.

In Table 5.5, the list of variables and their corresponding TABLO names are outlined. The variable $wage\_io$ is introduced to represent the average ordinary time hourly wage.

The following section of outlines how to declare the model variables.

```
VARIABLE
(all,i,IND)(all,o,OCC) x1labi(i,o)   #Intensive margin labour demand#;
(all,i,IND)(all,o,OCC) x1labx(i,o)   #extensive margin labour demand#;
(all,i,IND)(all,o,OCC) vc(i,o)          #Variable non-wage cost#;
(all,i,IND)(all,o,OCC) tc(i,o)          #total non-wage cost#;
```

| Variable name | Variable | Tablo variable |
|---|---|---|
| Labour services | $h_i$ | $xlab_i$ |
| Intensive margin employment | $n_i$ | $xlabx_i$ |
| Extensive margin employment | $l_i$ | $xlabi_i$ |
| Total cost of labour | $tc_i$ | $p1lab_i + x1lab_i$ |
| Fixed cost of labour | $z_i$ | $z_i$ |
| ordinary wage | $w_i$ | $wage_i$ |
| Wage premium | $wp_i$ | $wp_i$ |
| Overtime | $ot_i$ | $ot_i$ |

Table 5.5: Table of model variables and their corresponding TABLO names

```
(all,i,IND)(all,o,OCC) wp(i,o)          #wage premium#;
(all,i,IND)(all,o,OCC) wage(i,o)        #Standard wage rate#;
(all,i,IND)(all,o,OCC) z(i,o)          #Fixed costs#;
wage_io                                 #Weighted average of wage across industries#;
```

The following section of code declares the model coefficients that have been introduced.

```
COEFFICIENT
(all,i,IND)(all,o,OCC) alphot(i,o) #Returns to hours#;
(all,i,IND)(all,o,OCC) LS(i,o) #labour supply hours#;
(all,i,IND)(all,o,OCC) LEQ(i,o) #Labour supply (initial) equilibirum)#;
(all,i,IND)(all,o,OCC) V1LVC(i,o) #Variable cost#;
(all,i,IND)(all,o,OCC) V1LFC(i,o) #Fixed cost#;
(all,i,IND)(all,o,OCC) V1LC(i,o) #Linear wage component#;
(all,i,IND)(all,o,OCC) V1SUP(i,o) #Superannuation's component of fixed costs#;
(all,i,IND)(all,o,OCC) WAGEPREM(i,o);
 SHAREW_IO #Total value of share of wages#;
```

In the above section, $V1SUP$ was defined as superannuation's component of fixed costs. It is required that the fixed cost variable be shocked to simulate the effects of an increase in the SG. This data is used to ensure the shock size is proportionate to the size of superannuation.

The following section reads the coefficient values from the previous section from the model database.

```
READ
V1LVC from file BASEDATA header "V1VC";
V1LFC from file BASEDATA header "V1FC";
LEQ from file BASEDATA header "LABE";
LS from file BASEDATA header "LABI";

FORMULA
(all,i,IND)(all,o,OCC) V1LC(i,o) = (V1LAB(i,o) - V1VC(i,o) -V1FC(i,o));
SHAREW_IO = sum{i,IND, sum{o,OCC, V1LC(i,o)}};

UPDATE
(all,i,IND)(all,o,OCC) V1LVC(i,o) = vc(i,o);
(all,i,IND)(all,o,OCC) V1LFC(i,o) = x1labx(i,o);
(all,i,IND)(all,o,OCC) XLABS(i,o) = x1labi(i,o);
```

The following section outlines the equations of the model. They represent the TABLO form of the linearised equations from Chapter 5.1.

```
EQUATION
E_labourservices #Labour services function# (all,i,IND)(all,o,OCC) x1lab(i,o) =
    alphot(i,o)*x1labi(i,o) + x1labx(i,o);
```

```
E_personhours #Hours worked per person# (all,i,IND)(all,o,OCC) person_hours(i,o) =
    x1labi(i,o) + x1labx(i,o);

E_overtime #Overtime hours per worker# (all,i,ind)(all,o,OCC) ot(i,o) = ([LS(i,o)]/[
    LS(i,o)-LEQ(i,o)])* x1labi(i,o);

E_variablecost #Variable labour costs# (all,i,ind)(all,o,OCC) vc(i,o) = wage(i,o) +
    wp(i,o) + ot(i,o);

E_totallabourcost #Total labour costs# (all,i,IND)(all,o,OCC) tc(i,o) + x1labx(i,o) =
     p1lab(i,o) + x1labi(i,o) + x1labx(i,o);

E_wagepremium #Wage premium# (all,i,IND)(all,o,OCC) wp(i,o) = OT_ELAS*ot(i,o);

E_totcost #Costs decomposed# (all,i,IND)(all,o,OCC)  V1LAB(i,o)*tc(i,o) = V1LC(i,o)*[
    wage(i,o)+x1labi(i,o)] + V1NWVC(i,o)*[vc(i,o)] + V1NWFC(i,o)*[fc(i,o)];

E_int_hours #Equilibrium condition for hours per worker#  (all,i,IND)(all,o,OCC)
    x1labi(i,o) = tc(i,o) - wage(i,o) - ([(1+OT_ELAS)*WAGEPREM(i,o)]/[ 1 + (OT_ELAS
    +1)*WAGEPREM(i,o)])*wp(i,o);

E_wage_io #average wage across sectors and industries# SHAREW_IO*wage_io = sum{i,IND,
     sum{o,OCC, V1LC(i,o)*wage(i,o)}};

!Flexible setting of money wages using ordinary wage rate!

 E_p1lab   # Flexible setting of money wages #
  (all,i,IND)(all,o,OCC)
    wage(i,o)= p3tot + f1lab_io + f1lab_o(i) + f1lab_i(o) + f1lab(i,o);
! Real wage equation using ordinary wage rate!
```

```
E_realwage realwage = wage_io - p3tot;
```

In addition to describing the linearisation of the labour demand model, this section also redefines some pre-existing variables. For example, instead of having the real wage defined the real ordinary wage is defined.

## 5.4 Summary

This initial component of this chapter involved incorporating the theory presented in Chapter 3.4 into the ORANIG model presented in Chapter 2.8. This involved linking the labour services function to the quantity of labour in the ORANIG framework and relating the labour cost function to the price of labour in ORANIG.

In addition, the theory in Chapter 3.4 was converted into its linear form. To calibrate the linear form of this model required data on hours worked and a decomposition of labour costs into ordinary, overtime and fixed costs. This chapter the process involved with incorporating this data into the underlying database.

Finally, the process involved with converting the model from its linear form into a the TABLO language was outlined. The final section of this thesis can now use this model to simulate the effects of an increase in the rate of the superannuation guarantee.

# 6 Superannuation Simulation

## 6.1 Superannuation

The Superannuation Guarantee ($SG$) is a government scheme which requires employers to make contributions on behalf of employees who meet a minimum income threshold. Its purpose is to provide savings for workers for when they retire. Since federation all Australian residents have been entitled to an age pension, provided and paid for by the government, when they are of retirement age. In 1986, as a means to supplement the aged pensions, superannuation payments of 3% of ordinary time earnings were awarded to award workers as a result of a bargaining agreement between the unions and the government. By 1990, 64% of all workers had superannuation coverage (Nielson et al., 2010). In 1992, the government legislated the SG which legally entitled all workers earning over $450 per month to 3% of all earnings to be paid into a superannuation account (Warren et al., 2008). By 2003, 90% percent of workers were covered by the SG (Nielson et al., 2010). In 2002, the coverage for superannuation payments was reduced back to ordinary time earnings, which meant that superannuation was not required to be paid on overtime hours. Currently, the SG is 9.5% and is planned to increase to 12% by 2025 (Australian Taxation Office, 2020).

The effect this increase in the SG has on the economy is simulated in this chapter. In the original ORANIG model without the labour demand component, this is simulated by simply increasing the wage rate by the corresponding increase in the rate of superannuation. As the original model does not distinguish between intensive and extensive margin employment, this framework will only show how total person hours change in response to a shock.

Using the labour demand model allows for a more specific shock to the cost of labour and will generate employment responses along both the intensive and extensive margin. The removal of superannuation payments for overtime hours causes superannuation to be viewed as a fixed cost and not part of the ordinary wage rate by firms, despite the superannuation payments being proportionate to ordinary time earnings. This can be understood by considering the decision being made by a firm when deciding between an additional worker or more overtime hours. If they hire an additional worker, they will be required to pay ordinary time earnings plus superannuation. Conversely, if they use overtime hours, they are required to pay an overtime wage premium but no additional superannuation. Therefore, in the labour demand model, the shock will be simulated by increasing the fixed cost of labour. This idea is also outlined in Chapter 4.4.1.

The remainder of the chapter is as follows: the closure environment for the simulation is outlined, followed by a description of how the shock will be simulated and an explanation of the results for the labour market and each industry is presented.

## 6.2 Closure assumptions

To simulate the effects of an increase in SG from 9.5% to 12%, the CGE model used requires the closure environment to be set. The process of closure is explained in Chapter 2.5.6. The ORANIG framework presented in Chapter 2.8 augmented with the labour demand theory presented in Chapter 5.3 is used to perform this simulation. It has 21,174 equations describing the economy and 34,350 variables describing the model with 20 different industries. This requires 13,176 variables to be set exogenously for closure.

Similar to Chapter 2.5.6, the choice of closure determines whether the simulation operates within a short-run or a long-run environment. However, the ORANIG framework also has variables (e.g, tax rates) which have no economic theory to explain them and are naturally exogenous regardless of whether it is a long or short-run simulation. The remainder of this section will provide justification for the choice of closure for each variable which is set as exogenous.

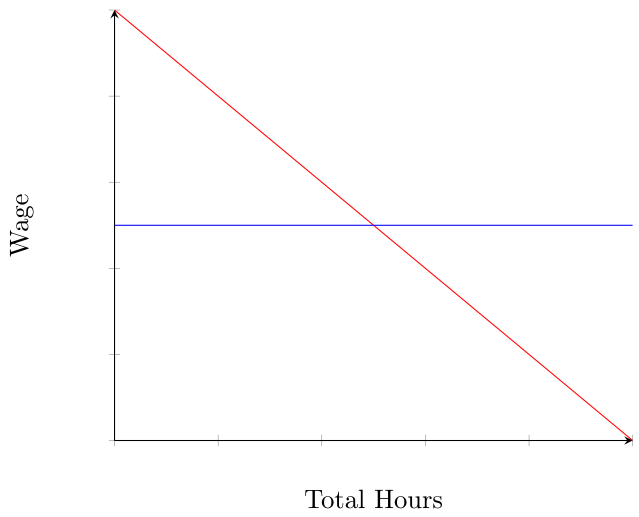


Figure 6.1: Labour market closure for Short-run simulation with red line representing demand and blue line supply.

### 6.2.1 Labour supply

There are no behavioural equations which determine labour supply in the standard ORANIG framework, requiring that the researcher specify a labour supply function. Usually, the labour supply is specified as either a) perfectly elastic or b) perfectly inelastic.

The first case corresponds to a short-run closure environment where the wage rate is assumed to be fixed and firms can adjust their labour demand accordingly. This closure environment is depicted in the Figure 6.1.

The second case corresponds to a long-run closure where it is assumed that employment is fixed at its natural rate of employment. In this case, the wage rate adjusts to ensure that the market clears. This situation is depicted in Figure 6.2.

When the model is extended to incorporate the labour demand theory presented in Chapter 3.4, the quantity of labour is comprised of an extensive and an intensive margin. Moreover, the price of

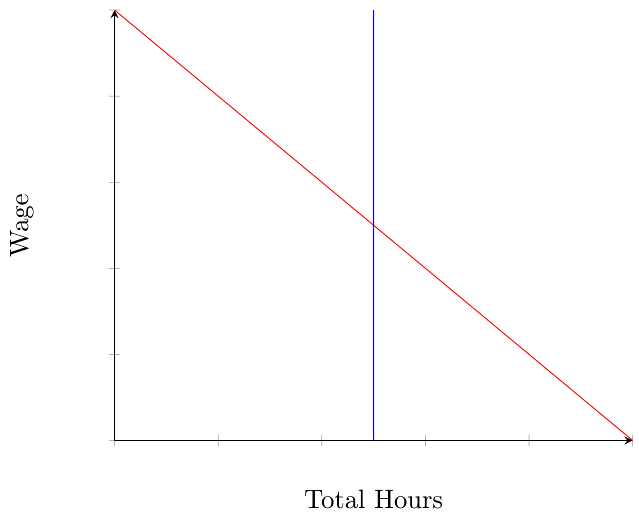


Figure 6.2: Labour market closure for long-run simulation with red line representing demand and blue line supply

labour consists of an ordinary wage rate, an overtime wage premium and a fixed cost component. These modification alter how closure is determined.

The ordinary hours of work ($L_0$) are assumed to be exogenous as they are determined by institutional factors. The wage premium ($W_P$) is a function of the level of overtime ($O_T$) so is determined based on the equilibrium level of hours worked ($L$). The hours worked per person are determined by the wage rate ($W$) and the fixed costs ($Z$). There is no economic theory which determines the wage rate ($W$), fixed costs ($Z$) or the number of workers ($N$) in this framework, requiring them to be specified by the researcher.

If the researcher wishes to adopt a short-run closure, the two price variables ($W$) and ($Z$) can be set exogenous and the number of workers can adjust to ensure the labour market clears.

If the researcher wished to perform a simulation in a long-run environment, they could set the number of workers ($N$) as exogenous and the ratio of fixed costs to wage $\frac{Z}{W}$ as constant. This reflects an environment where employment is at its natural level and the intensive margin of employment remains constant, as the intensive margin is determined by the ratio of fixed costs to wage[6].

This simulation will employ a short-run closure where the wage rate and fixed costs are set exogenously. This simulation will thus demonstrate the short-run impact that an increase in superannuation will have on the level of employment. For the long-run effects of an increase in the superannuation rate, Henry (2009) and Coates et al. (2020) report that the increase in the rate of superannuation leads to corresponding decrease in the wage rate.

### 6.2.2 Capital and land

The capital stock is assumed to be fixed and immobile between industries as this simulation assumes a short-run closure environment. The stock of capital is held constant since it is assumed that it takes time for new capital to be produced or re-purposed for other industries.

The quantity of land is immobile between sectors and exogenous in this simulation. Similar to the capital stock, it takes time to clear land for productive purposes or to re-purpose it for uses.

### 6.2.3 Consumption

The aggregate level of consumption expenditure is usually assumed to be determined via one of the three following rules 1) set as exogenously, 2) consumption function or 3) balance of trade constraint. The first rule is to assume that consumption remains fixed and typically corresponds to a short-run simulation. The second rule has a permanent income flavour and has domestic consumption increase as a function of GDP. Finally, the third rule says that consumption must adjust to ensure that the balance of trade remains at its initial level, whether that be surplus, balanced or deficit.

[6] This can be seen by rearranging (3.86) such that:

$$\frac{Z}{W} = \frac{\beta[1 + ELASW_p - \frac{\alpha}{\beta}(1 + W_p)}{\alpha} L + W_p L_0 \tag{6.1}$$

Since $W$ or $Z$ does not appear on the RHS, as long as the ratio $\frac{Z}{W}$ does not change, there is no impact the equilibrium $L$.

This assumption is based on the idea that foreign countries, over the longer run, will not allow a country to continue to build up a negative or positive trade balance out of the fear that they will not be repaid.

In this policy simulation the level of aggregate consumption is held constant. It is assumed that for the majority of households, the increase in the rate of superannuation will not initially increase disposable income as workers are required to wait until retirement until they can access their superannuation savings. However, it is possible that households could offset the superannuation savings by reducing other forms of savings or by borrowing money with the intention of repaying it when workers reach the age of retirement. Gruen et al. (2011) states that for each $1 in superannuation households decrease their savings by 30 cents from other savings. However, it is expected that such adjustments will take time and therefore in the short-run it seems reasonable to assume that consumption is fixed.

### 6.2.4 Investment

Similar to aggregate consumption expenditure, there is little economic theory within the ORANIG framework to determine the level of aggregate investment expenditure. Despite the intention of the SG to increase the level of savings within the economy (Connolly et al., 2004), there is no theory within the model to suggest whether any increases in savings are invested domestically or overseas. Furthermore, it is possible that any direct increase in savings caused by the SG might be offset by a reduction in other savings which are undertaken by households.

Ultimately, given that Australia is a small open economy it is likely that the aggregate investment is largely determined by external factors and by central bank policy. Therefore, by assumption, the level of aggregate investment remains fixed for this simulation.

At the industry level, investment is determined by the rules set out in Chapter 2.9.2. The following industries, which tend to be dominated by the government sector, have their investment set exogenous: Electricity, Gas and Water; Construction; Finance and Insurance; Owner Dwellings; Property and Business Services; Government Administration and Defence; Education; and Health and Community. The remaining industries investment is a positive function of the expected rate of return in the next period, as discussed in Chapter 2.9.3.

### 6.2.5 Government

Government demand in this simulation is treated as exogenous. It is assumed that governments plan their expenditure in advance and in the short run would not adjust their spending. Given the preferable tax treatment of superannuation savings, government revenue might be affected in the long run if wages decrease in response to the rate of superannuation.

### 6.2.6 Inventories

The level of inventory demand is assumed to be unaffected and thus treated as exogenous.

### 6.2.7 Net exports

The level of exports are determined by the export demand curve for each commodity. There is no reason to suspect a change in domestic policy would impact the rest of the worlds demand for Australian commodities, therefore the quantity shifter $A^{Q}_{4,c}$ and price shifter $A^{P}_{4,c}$ are both set exogenously. In this case, the quantity of exports supplied will adjust to reflect any changes to the nominal exchange rate ($\phi$) and the commodity price ($P_{4,S}$). The foreign currency import prices are treated as exogenous in this simulation based on the assumption that Australia is a small open economy. The quantity of imports will adjust to reflect the relative price changes in domestic and imported commodities. Moreover, as all other components of aggregate demand are treated as exogenous, the trade balance will adjust to ensure that aggregate demand equals aggregate supply.

### 6.2.8 Other Variables

The remaining variables are the technological change, household preferences, number of households and the tax rates. All of these variables were treated as exogenous for this simulation.

It is plausible that the utilisation rate of capital might be impacted by changes to the amount of hours worked per worker, which would be represented by a change in the technology parameter $\alpha_K$. For example, an increase in the hours worked per worker might result in capital spending less time idle which may cause an increase in technical efficiency of capital. However, this would rely upon capital being idle for the hours in which workers do not work, which may not be the case. For example, a single truck could be shared by 3 workers working an 8-hour shift. If one of those workers increased their hours to 12 hours, there would be no idle capital for the extra 4 hours. Unfortunately, endogenising capital's technology would require further economic theory which is beyond the scope of this research.

Finally, it is likely that tax rates might be affected by policy changes to the rate of superannuation. However, given the simple treatment of households taxes in the ORANIG framework it is unlikely that taxes would be that significantly altered such that it warrants attempting to endogenise taxes.

## 6.3 Back of the Envelope

Ensuring that solutions are accurate can be difficult due to the size of the system and the interrelated nation of the model. Back Of The Envelope (BOTE) calculations are a reduced form version of the model which aids researchers in understanding results by providing a simplified explanation and exposing the main determinants of the final outcome.

For example, consider the effect that a policy which causes the quantity of labour to decrease by 1% will have on GDP. Ignoring the presence of land, GDP in percentage change form can be approximated using the formula:

$$y = S_h h + S_K k \tag{6.2}$$

where $S_H$ is the share of labour, $h$ is the percentage change in labour, $S_K$ is the share of capital and $k$ is the percentage change of capital. If the simulation is a short-run simulation, the change in capital will be 0, therefore, the percentage decrease in GDP should approximately be the share of labour $S_h$. BOTE calculations will be used throughout this chapter as a way to gain insights into our results and to ensure the accuracy of the results.

## 6.4 Shock simulation

In Chapter 4.4.1 it was shown that costs which are not paid for overtime but are proportional to the ordinary time earnings are considered as fixed costs by firms. This means that since 2002 firms have considered the SG as a fixed cost, since the SG was removed from overtime earnings. This means that the increase in the SG from 9.5% to 12% represents a 26.31% increase in fixed costs. Given that fixed costs are not uniform across industries, the increase from 9.5% to 12% will be represented by different shock sizes in percentage change terms for each industry.

To calculate the size of the percentage change in fixed costs, it is useful to redefine fixed costs as:

$$Z_i = SUPER_i + FC_i \tag{6.3}$$

where $Z_i$ is total fixed costs, $SUPER_i = 0.095 * WL_0$ (representing the component of fixed costs which are superannuation) and $FC_i$ is the remaining part of fixed costs for each industry. The percentage change of $Z_i$ is defined:

$$z_i = S_i^{SUP} super_i + S_i^{FC} fc_i \tag{6.4}$$

where $S_{Si}$ is superannuation's share in fixed costs and $S_{FCi}$ is the share of the remaining component.

Therefore, the percentage change in fixed costs can be calculated:

$$z_i = \frac{0.095 * W * L_{0,i}}{0.095 * W * L_{0,i} + FC_i} super_i \tag{6.5}$$

Therefore, industries which have larger ratio of fixed costs to ordinary earnings will experience a smaller shock in percentage change terms. The shock applied to each industry is presented in Table 6.1.

## 6.5 Simulation Results

The effects of an increase from a 9.5% to 12% increase in the SG for the Macroeconomic, labour-market and industry-specific results are presented in the remainder of this chapter. There are two simulations are performed in this section. The first simulation simulates the effects of an increase in the rate of superannuation with the original ORANIG model excluding the labour demand component. In this case, the price of labour $P1LAB$ is increased proportionately to the size of the shock to superannuation.[7] This model will be denoted the Basic Labour (B-L) market model.

The second simulation simulates the effect of an increase in the SG for the ORANIG model including the labour demand module. In this case, the fixed cost of labour is increased proportionately to the size of the shock of superannuation. This will be denoted the Labour Demand (L-D) market model.

The B-L simulation is based upon the work of Corden and Dixon (1980) which simulates the economy-wide impact of a decrease in the real wage rate. The purpose of including this simulation is to demonstrate how the addition of a labour demand module can build upon previously established findings to provide a more detailed description of the labour market.

[7] The shock for the baseline is applied to the shifter term $f1lab$ for each industry to ensure the relative size of the shock for each industry is the same. A second shock is applied to the real wage to ensure the economy-wide shock is the same in both simulations.

| Percentage change shock to fixed costs | |
|---|---|
| Agriculture, forestry and fishing | 0.11 |
| Mining | 0.08 |
| Manufacturing | 0.09 |
| Electricity, gas, water and waste services | 0.06 |
| Construction | 0.09 |
| Wholesale trade | 0.09 |
| Retail trade | 0.09 |
| Accommodation and Food Services | 0.08 |
| Transport, postal, warehousing | 0.09 |
| Information Media and telecommunications | 0.08 |
| Finance and insurance services | 0.06 |
| Rental, hiring and real estate services | 0.07 |
| Professional, scientific and technical services | 0.08 |
| Administrative and support services | 0.08 |
| Public administration and safety | 0.07 |
| Education and training | 0.06 |
| Health care and social assistance | 0.08 |
| Arts and recreation services | 0.07 |
| Other services | 0.08 |

Table 6.1: Shock to fixed costs for each industry

In addition, the B-L is used to highlight the implicit assumptions made about the labour market. In the B-L simulation, there is only one quantity of labour [8] ($X1LAB$) and it is left to the researcher to specify how this quantity is defined, e.g, they might define it as person hours or number of workers. Usually, it is assumed that intensive margin employment is held fixed. In the B-L simulation, a shock to fixed costs of labour is conducted by increasing the price of labour. As there is no distinction between intensive and extensive, there are no endogenous changes to intensive or extensive margin of employment, only a change to the quantity of effective labour. However, the increase in fixed costs will cause a substitution from hours to workers. This research will highlight how the assumptions made about this change in effective labour often causes the true level of the decrease extensive margin employment to be understated, as the B-L does not account for these substitution effects.

### 6.5.1 Comparability of results

The introduction of different labour costs, such as overtime and fixed costs, has implications for how certain variables are interpreted. In the L-D simulation, the price of labour ($P1LAB$) is defined:

$$P1LAB = W + W_P W \frac{L - L_0}{L} + \frac{Z}{L} \tag{6.6}$$

This represents the average price of labour as overtime hours are remunerated at a higher rate than ordinary time hours but the fixed cost per hour of work decreases. Thus, the price of labour varies based on the quantity of hours worked per worker. In the B-L simulation, all hours of work are remunerated at the constant rate of $P1LAB$ and there are no overtime or fixed costs allowing it to be interpreted as the wage rate. In this case, the average hourly price of labour and the wage rate are equivalent.

[8] When discussing the B-L simulation employment, person hours and effective labour all represent the single quantity of labour ($X1LAB$)

For the L-D and B-L simulation, the *real wage* is defined:

$$\text{real wage} = p1lab\_io - p3tot \tag{6.7}$$

where $p1lab\_io$ is the average nominal price of labour across all industries and occupations and $p3tot$ is the consumer price index. In the L-D simulation, the real wage represents the average remuneration per hour per worker in real terms. As fixed costs are based on entitlements such as sick leave and superannuation payments, they are considered part of the remuneration package for workers. Similar to $P1LAB$, in the L-D simulation the real wage varies based on the level of intensive margin employment.

For the L-D simulation the term real ordinary wage rate is introduced to represent the hourly wage rate, exclusive of overtime or fixed costs, adjusted for inflation. It is defined:

$$\text{real ordinary wage} = wage\_io - p3tot \tag{6.8}$$

where $wage\_io$ is the average ordinary wage rate across industries and occupations. In the L-D simulation, the real ordinary wage rate does vary with the level of intensive margin employment.

In the L-D simulation the labour variable ($X1LAB$) is interpreted as the effective quantity of labour[9] as there are different returns to hours ($\alpha$) and workers ($\beta$). This variable should be used when comparing the employment effects between the two simulations. However, when measuring the total person hours in the economy, the sum of the intensive and extensive margin variation should be used. The effective labour variable ($X1LAB$) should be used when comparing the L-D and B-L simulation.

[9]The term effective labour is used to describe the labour input $X1LAB$ so that it is not mistakenly interpreted as person hours, as assumptions are required to interpret $X1LAB$ as person hours. In the B-L simulation effective labour is represented by an undefined labour services function. In the L-D simulation the term effective labour represents the labour services function represented by $H = L^{\alpha}N^{\beta}$ .

### 6.5.2 Similarities between shocks

The magnitude of the shock to the real wage was the same for both simulations so that any differences in outcomes between the L-D and B-L simulations were due to differences in modelling and not because one simulation experienced a larger shock than the other simulation. As such, both simulations experienced a 2.09% increase in the real wage. However, despite the increase in real wage being the same across simulations, the approach to simulate this increase varied across simulations.

In the labour demand simulation, the real ordinary wage rate was held constant while the fixed cost was shocked. This caused the nominal price of labour[10] to increase by 4.04% and translated into an increase of 2.09% in the real wage.

In the B-L simulation the real wage is treated as exogenous and was increased by 2.09% to ensure that the B-L simulation experienced the same magnitude shock to the overall economy as it did in the L-D simulation. To reflect the different proportions of superannuation in labour costs across industries, the relative price of labour was shocked according to the magnitude of superannuation payments. For example, in the following line of code, the variable $super(i,o)$ is shocked by 25% for each industry:

```
E_SG #SG impact on labour costs#
(all,i,IND)(all,o,OCC) V1LAB(i,o)*f1lab(i,o) = V1SUP(i,o)*super(i,o);
```

where $V1SUP(i,o)$ is the expenditure on superannuation contributions for each industry. As $\frac{V1SUP(i,o)}{V1LAB(i,o)}$ varies across industries, this will cause prices to change by varying degrees across industries. The variable $f1lab(i,o)$ is used to adjust the price variable $p1lab(i,o)$ in the following equation:

```
(all,i,IND)(all,o,OCC)
p1lab(i,o)=  p3tot + f1lab_io + f1lab_o(i) + f1lab_i(o) + f1lab(i,o);
```

[10] The numeraire in this simulation is the nominal exchange rate

| Macroeconomic variable | B-L | L-D |
|---|---|---|
| Real GDP | -0.78 | -0.77 |
| Nominal price of labour | 3.78 | 3.75 |
| Real price of labour | 2.09 | 2.09 |
| Total hours | -1.39 | -1.77 |
| Effective labour ($X1LAB$) | -1.39 | -1.38 |
| Employed workers | - | -2.00 |
| Hours per worker | - | 0.23 |
| Terms of Trade (TOT) | 0.77 | 0.76 |
| CPI | 1.69 | 1.66 |
| Export volume index | -3.83 | -3.81 |
| Import volume index, duty-paid weights | 0.24 | 0.23 |
| Exports price index, local currency | 0.77 | 0.76 |

Table 6.2: Macroeconomics effects of a 26.3% shock to superannuation

Consequently, the price of labour varies across industries proportionately to the size of superannuation expenditure in that industry whilst also having real wages increase by the same as the L-D simulation.

## 6.6 Macroeconomic results

A summary of the macroeconomic impact of the 26.3% increase in the rate of superannuation is presented in Table 6.2 for both the L-D and B-L simulation.

In the B-L simulation, the shock to real wages causes a decrease in demand for labour, resulting in a decrease of 1.39% in effective labour. As the simulation is conducted in a short-run environment, the capital and land stock for each industry remains constant as industries are unable to substitute between primary factors.

The nominal price of labour ($p1lab_{io}$) in this simulation increases by 3.78% as a consequence of the increase in real wages. The increase in the price of labour causes output prices to increase, causing the consumer price index to increase by 1.69%.

The decrease in effective labour causes real GDP to decrease by 0.78%. As all components of domestic absorption are held fixed in this simulation, all variations in output are due to changes in the trade balance. The increase in the cost of labour causes output prices to increase, which has two effects. Firstly, exports become less attractive to the rest of the world, decreasing their demand. And secondly, imports become relatively cheaper, causing domestic consumers to substitute towards imported goods away from domestically produced commodities. There is an improvement in the terms of trade of 0.77% resulting in a reduction in export demand leading to a 3.83% decrease in the volume of exports. Whilst the foreign import price is not impacted by this policy, as Australia is a small open economy, the local currency price of imports compared to domestic commodities decreases, causing an increase of 0.24% in the quantity of imports.

The macroeconomic impacts of the shock in the L-D simulation are similar to the B-L simulation due to the real hourly labour cost per worker being shocked by the same amount in both simulations.

### 6.6.1 Effective and person-hours

In the L-D simulation, the 26.3% increase in the SG is simulated by increasing the fixed costs of labour holding the real ordinary wage rate constant. This causes the real hourly labour cost per worker to increase by 2.09%. The nominal price of labour increases by 3.75%. The increase in the nominal price of labour also causes output prices to increase, causing the CPI to increase by 1.66%.

The increase in the real hourly cost of labour causes a decrease in demand for the effective labour input resulting in a 1.38% decrease in effective labour. The decrease in effective labour leads to an even larger decrease in person hours of 1.77%.

The decrease in person hours (-1.77%) is larger than the decrease in effective labour (-1.38%). The person hours and effective hours can be decomposed to show why this is the case. The person hours can be decomposed as:

$$m = n + l = -2 + 0.23 = -1.77 \tag{6.9}$$

where $m$ is the percentage in person hours. The effective labour can be expressed:

$$h = n + \alpha l = -2 + \alpha 0.23 = -1.38 \tag{6.10}$$

This is because $\alpha > 1$ on average across industries, meaning the aggregate returns to hours parameter is larger than unity. As such, since intensive margin employment increases effective labour experiences less of a decline than person hours. This also implies that, on average, workers experience increasing returns to hours worked within this model.

The ability to substitute hours for workers in the L-D simulation means that the decline in effective labour is marginally smaller than the decline in effective labour in the B-L simulation. In response to the increase of fixed costs, industries substitute towards overtime hours.

It is observed that the effective labour decreases by slightly more in the B-L simulation compared to the L-D simulation, despite both simulations experiencing the same size shock to real hourly labour costs per worker. This is likely due to varying changes in effective labour at the industry level. The composition of industry effects will be impacted by individuals industries ability to substitute hours for workers.

In this setup, the elasticity of effective hours with respect to the real price of labour is -0.66 which is relatively consistent with other CGE models such as Dixon et al. (2010) who found an elasticity of -0.63 when they simulated the effects of an award wage increase. However, the elasticity of person hours is much higher at -0.84. This implies that depending on the nature of the shock, the B-L model may understate the elasticity of person hours with respect to real hourly labour costs.

### 6.6.2 Aggregate intensive and extensive margin

In the L-D simulation the increase in fixed costs causes adjustments along the intensive and extensive margin of employment. There is a substitution from workers to hours as the increase in fixed costs causes workers to become comparatively more expensive than additional hours. In aggregate the increase in hours worked per worker increase by 0.23%. In absolute terms, this results in an additional 4 extra minutes work per week for an average worker. Effectively, this may result in one in every 15 workers working an extra hour.

On average across all industries, the increase in overtime was 6.71%. The 26.3% increase in superannuation translated to an average of 13.26% increase in fixed costs across all industries. This implies that the elasticity of overtime with respect to fixed costs is 0.51.

The number of workers decreases by 2.00% which is more pronounced than the decline in effective hours. The percentage of workers decreased due to a substitution effect from workers to hours and due to a scale effect caused by a decline in demand for effective labour. The implied elasticity of workers with respect to the real hourly labour cost per person is -0.96. This significantly higher than the -0.66 for effective labour and highlights how assuming that the change in effective labour was represented by a change in workers would understate the true extensive margin employment effects.

### 6.6.3 GDP

GDP decreases for both simulations as a consequence of the cost of labour increasing. The impact that the decrease in effective labour has on GDP can be understood using the linearised version of the income approach to measuring GDP. Ignoring the presence of land, the percentage change in

GDP is defined:

$$gdp = S_h h + S_k k \tag{6.11}$$

where $S_h$ is the share of total effective labour, $S_k$ is the share of capital, $h$ is the percentage change in hours and $k$ is the percentage change in capital. The share of effective labour is approximately 52%. As the percentage change in effective labour is -1.38, this implies the expected decrease in GDP is -0.71%. This is relatively close to the simulated value of 0.77%. This BOTE analysis is useful to ensure that results are accurate.

### 6.6.4 Trade balance

The variation in GDP occurs to the trade balance on the expenditure side of the economy, since all components of domestic absorption are fixed. The increase in the cost of labour flows onto the cost of output causing the price of exports to increase lowering export demand. Consequently, the terms of trade appreciate in both simulations with the B-L experiencing an 0.77% and the L-D a 0.76% increase in the TOT. This relative price increase causes domestic consumers to substitute towards imports from domestically produced commodities and causes foreigners to demand less of domestic commodities resulting in a deterioration in the trade balance.

## 6.7 Labour market

Table 6.3 shows the employment effects for each industry for the L-D and B-L simulation. The first 4 columns show the effective hours, person hours, intensive margin and extensive margin employment changes for the L-D simulation in that order. The last column shows change in employment for the B-L simulation.

| | L-D | | | | B-L |
|---|---|---|---|---|---|
| Industry | H | M = | L | N | H |
| Agriculture, forestry and fishing | -4.04 | -4.19 | 0.26 | -4.45 | -4.03 |
| Mining | -2.61 | -2.68 | 0.37 | -3.05 | -2.64 |
| Manufacturing | -3.26 | -3.45 | 0.36 | -3.81 | -3.23 |
| Electricity, gas, water and waste services | -1.96 | -2.29 | 0.57 | -2.86 | -1.91 |
| Construction | -0.12 | -0.41 | 0.66 | -1.07 | -0.12 |
| Wholesale trade | -1.16 | -1.3 | 0.24 | -1.54 | -1.15 |
| Retail trade | 0.11 | 0.05 | 0.1 | -0.05 | 0.12 |
| Accommodation and food services | -2.15 | -2.19 | 0.09 | -2.28 | -2.19 |
| Transport, postal and warehousing | -2.96 | -3.21 | 0.48 | -3.69 | -2.91 |
| Information media and telecommunications | -2.28 | -2.31 | 0.06 | -2.37 | -2.32 |
| Financial and insurance services | -1.44 | -1.46 | 0.04 | -1.5 | -1.48 |
| Rental, hiring and real estate services | -2.32 | -2.4 | 0.16 | -2.56 | -2.35 |
| Professional, scientific and technical services | -1.43 | -1.45 | 0.07 | -1.52 | -1.45 |
| Administrative and support services | -1.37 | -1.52 | 0.27 | -1.79 | -1.39 |
| Public administration and safety | -0.19 | -0.27 | 0.17 | -0.44 | -0.19 |
| Education and training | -1.86 | -1.88 | 0.04 | -1.92 | -1.92 |
| Health care and social assistance | -0.41 | -0.45 | 0.09 | -0.54 | -0.42 |
| Arts and recreation services | -1.23 | -1.27 | 0.08 | -1.35 | -1.26 |
| Other services | -0.94 | -1.03 | 0.2 | -1.23 | -0.94 |

Table 6.3: Labour market response to an increase in SG for each industry. The first 4 columns are effective labour, person hours, hours worked and number of workers for L-D simulation. The last column is effective labour for B-L simulation.

### 6.7.1 Effective labour hours

In the short run, the increase in the price of labour leads to a decrease in demand for domestic commodities. As capital and land are fixed, this causes a decrease in the use of effective labour. The magnitude of the change in effective labour often depends on the capital-labour ratio. Ignoring industry subscripts, in the short-run the production function can be expressed as:

$$q = S_h h \tag{6.12}$$

where $q$ is industry output, $S_h$ is the share of effective labour in production and $h$ measures effective labour. Noting that $S_H = 1 - S_K$, it can be seen that the relationship between output and effective labour is more elastic the greater the share of capital, as:

$$h = \frac{q}{1 - S_K} \tag{6.13}$$

In Figure 6.3, the relationship between capitals share in production, measured as expenditure on capital divided by expenditure on labour and capital, is plotted with respect to the percentage change in effective labour. This demonstrates a negative relationship between the two variables. The R squared for this plot is 36.5%, which suggests that 36.5% percent of variation can be explained by the capital-labour ratio.

For example, consider the Arts and the Information Technology industries. The Arts has a capital labour ratio of 0.4 whereas the Information Technology industry has a ratio close to 1.06. The Arts industry experiences a much smaller decline in effective employment only experiencing a decline of 1.23% compared with a decline of 2.28% for the Information Technology industry.

This relationship is not so compelling when comparing similar industries which have different levels of trade exposure. For example, consider the Healthcare industry compared with the Education sector. Both industries have similar capital-labour ratios of about 0.15 yet the education industry experiences a decline 4 times larger of -1.86% compared with the healthcare which only experiences

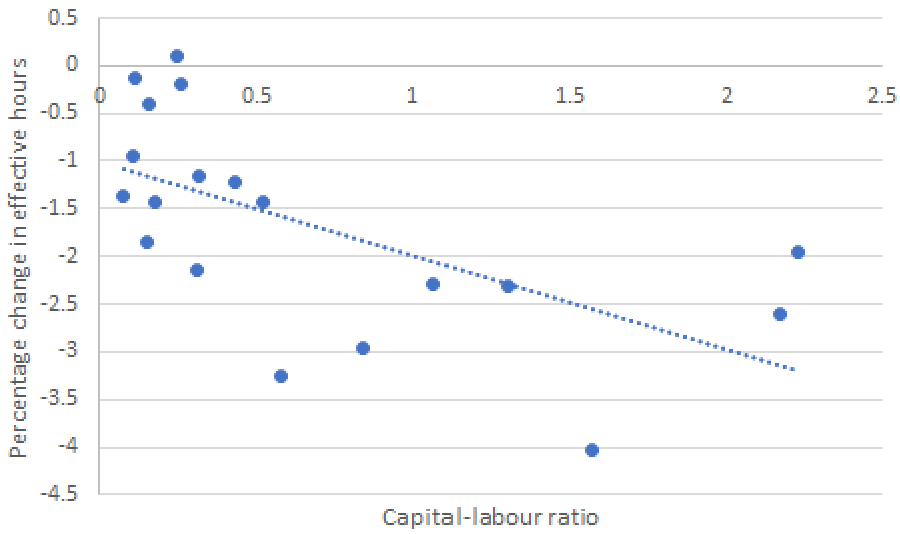


Figure 6.3: Relationship between the capital-labour ratio and percentage change in effective labour

-0.41%. This is a consequence of the Education industry being significantly more trade exposed than the Health Care industry, where most of its demand is fixed as its determined by government demand. Conversely, the Education industry has a significant proportion of its demand as exports to foreign students. As they are unable to increase their output prices due to overseas competition, they experience a larger decline in employment.

In the graph there are a group of 4 industries which sit above the trend line that perform reasonably well. These industries (Retail, Construction, Public admin and Healthcare) all share the traits of being labour-intensive industries and are sheltered from import competition. As such they are able to pass on price increases to consumers without experiencing any significant declines in employment activity. This outcome is largely a consequence of the choice for closure. As all components of domestic absorption are assumed to be fixed in the short run all variations in output are due to adjustments to the trade balance.

There is a strong link between an industry's output price for a commodity and the variation in effective labour for that industry. The variation in effective employment across industries can be understood using BOTE analysis. Firstly, ignoring industry subscripts, consider the first-order condition from the CES function in linear form, where:

$$h - k = -\sigma(p_h - p_k) \tag{6.14}$$

where $h$ is effective hours, $k$ is capital, $p_h$ is the price of labour and $p_k$ is the rental rate on capital. Secondly, consider the price setting equation, ignoring taxes and other costs, where:

$$p_q = S_h p_h + S_k p_k \tag{6.15}$$

where $p_q$ is the output price, $S_h$ is the share of effective labour in output and $S_k$ is the share of capital. Combining these two equations and using the fact $k = 0$ yields:

$$h = -\frac{\sigma}{S_k}(p_h - p_q) \tag{6.16}$$

It can be seen from (6.16) that if the price rises by the same amount as the cost of labour, there would be no decline in effective labour. Thus industries with inelastic demand, who are able to increase their prices in response to the increase in the cost of labour, experience smaller declines in effective labour. The relationship between output prices and effective labour are illustrated in Figure 6.4.

For example, consider the construction industry. Its labour costs increase by 4.14%. Given that it has inelastic demand, it can pass the increase in costs onto consumers with the price of output increasing by 2.95%. As a consequence, the industry only reduces its effective labour by -0.13%.

Figure 6.4 further illustrates how the level of trade exposure impacts the usage of effective labour. Consider the Education and Health care industries again. Education is a trade exposed industry. Its output prices increase by 3.31% and its effective labour decreases by 2.11%. Conversely, the Healthcare industry is not trade exposed. It experiences a similar price increase of 3.21%. However,

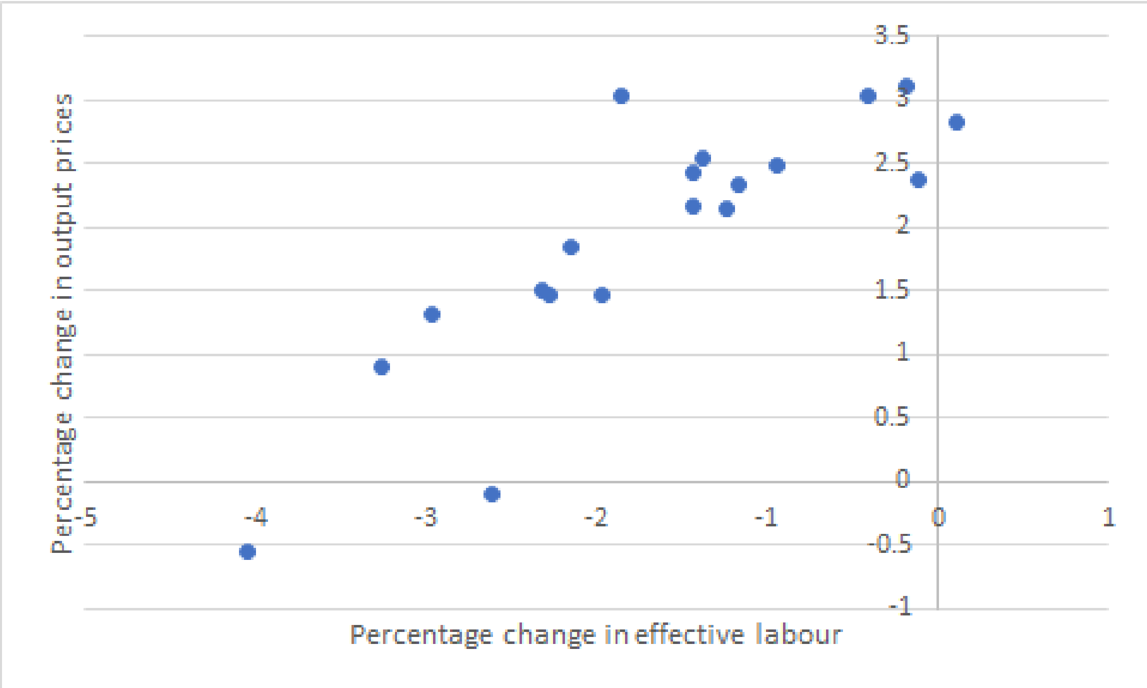


Figure 6.4: The relationship between output prices and effective labour for L-D simulation

its effective labour only decreases by 0.46%. Thus, despite having similar capital-labour ratios and similar increases in price, there is a significant variation between industries which are trade exposed and those which are not trade exposed.

### 6.7.2 Differences between the B-L and L-D simulation

When comparing the B-L and the L-D simulation, there are often differences in the price of labour ($p1lab$) across the two simulations and this corresponds to a difference in the use of effective labour ($x1lab$) across industries. For the Mining industry, the percentage change in the price of labour ($p1lab$) was 3.75% in the L-D simulation. In the B-L simulation it was 3.69% with the difference in price between the B-L and L-D simulation being 0.06%. The percentage change in effective labour ($x1lab$) was -2.61% for the L-D simulation. For the B-L simulation, the percentage change in $x1lab$ was -2.67% with the difference between the L-D simulation and the B-L simulation being 0.05%. Since the increase in $p1lab$ was larger in the L-D compared to the B-L simulation the percentage decrease in $x1lab$ was larger in the L-D than the B-L. Thus there is a positive relationship between the difference in $p1lab$ between the L-D and B-L simulation and the difference in $x1lab$ between the L-D and B-L simulation. This relationship for all industries is illustrated in Figure 6.5.

The construction and retail industry are both notable outliers in Figure 6.5. Both of these industries use comparatively more intensive margin employment than the remaining industries. The L-D and B-L models lead to different results for the price of labour. In the L-D simulation, industries which use comparatively more intensive margin employment and thus overtime experience higher increases in the price of labour as they are paying wage premiums for more hours per worker. Conversely, in some exceptional circumstances where demand is inelastic, there impact on effective employment of an increase in price is always small. Thus, there is little variation in employment regardless of the price change in effective labour.

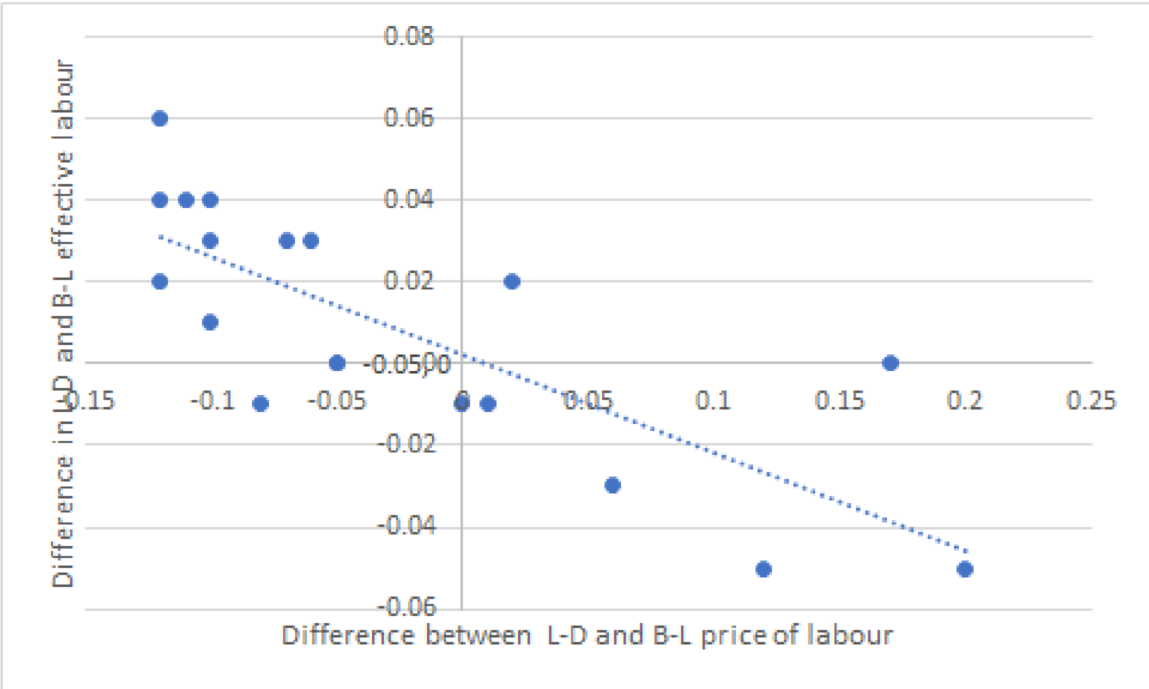


Figure 6.5: Relationship between difference in price and effective labour between the L-D and B-L simulation

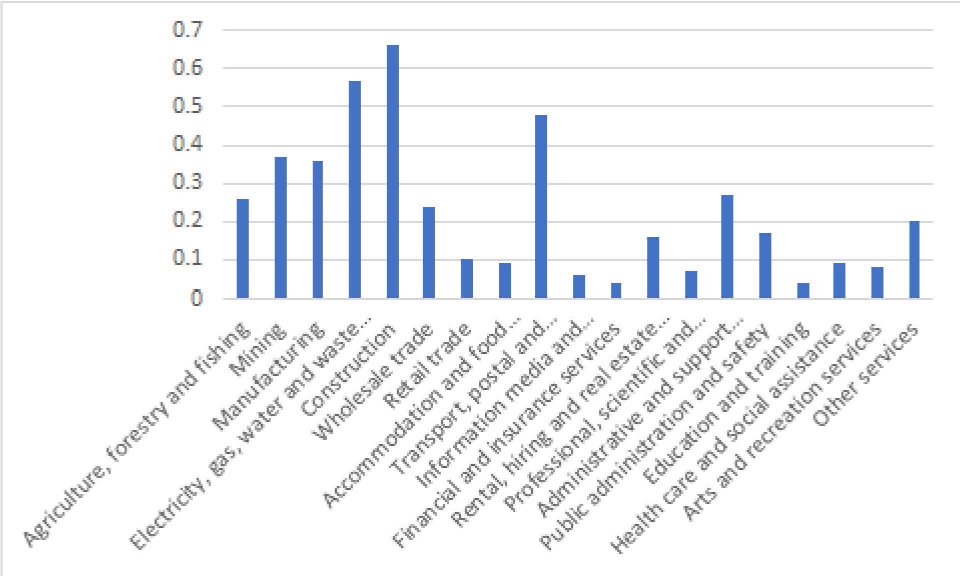


Figure 6.6: Percentage change in hours worked per worker by industry

### 6.7.3 Hours worked per person

The intensive margin adjustment of employment for each industry is presented in Figure 6.6. It shows how each industry chooses to substitute hours for workers in response to an increase in fixed costs resulting from SG being increased. As the level of ordinary hours are fixed, the adjustment to intensive margin employment occurs to the level of overtime for each industry.

The corresponding change in overtime is depicted in Figure 6.7. The industries with the greatest increase in overtime are: Mining (8.94%); Professional, scientific and technical services (7.57%) and Public administration and safety (8.05%) whilst those with the least adjustment in overtime are: Arts and recreational services (5.74%); Finance and insurance (5.8%) and Administration and support services (5.91%).

The relationship between overtime and increases in intensive margin employment is not a 1-to-1 relationship. For example, consider the Education and Arts industries. The Education industry experiences a 7.18% increase in overtime compared with the Arts industry which experiences 5.74%.

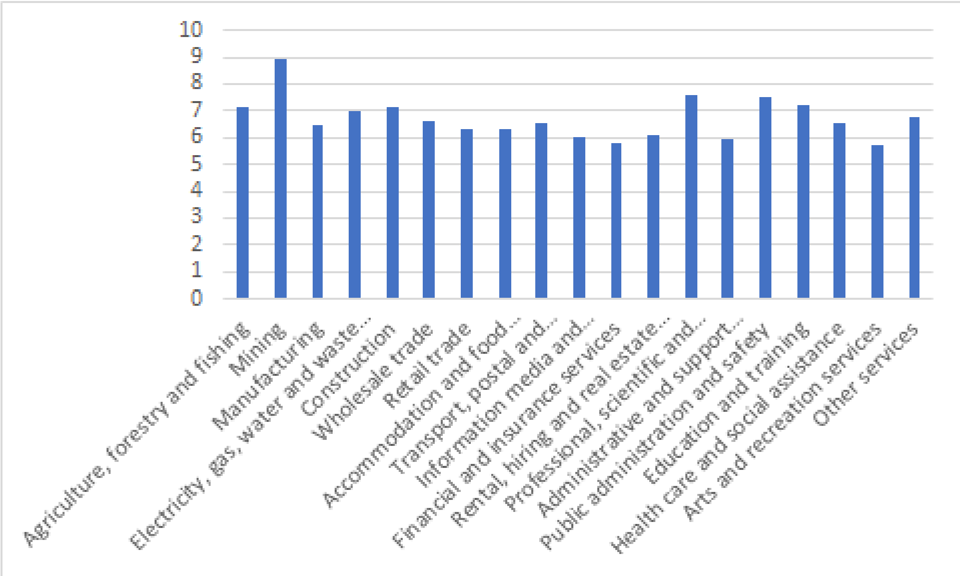


Figure 6.7: Percentage change in overtime per worker by industry

However, the Arts industry experiences a 0.08% increase in intensive margin employment compared to the Education industry which experiences 0.04% increase in intensive margin employment. This is because the education industry initially uses a lower quantity of overtime hours compared to the Arts industry.

The variations in intensive margin employment can mostly be attributed to the following three factors: (1) variations in the percentage change in fixed costs (2) different wage premium schedules across industries and (3) different levels of ordinary hours worked across industries.

### 6.7.4 Relationship between the percentage change overtime and fixed costs

Despite a uniform 25% increase in superannuation, the percentage change in fixed costs varies across industries as the share of superannuation in fixed costs varies across industries. For example, the share of superannuation in fixed costs in the Finance industry is only 28.5% compared to the Construction industry where it is 52.5% of fixed costs. Therefore, the 25% increase in superannuation

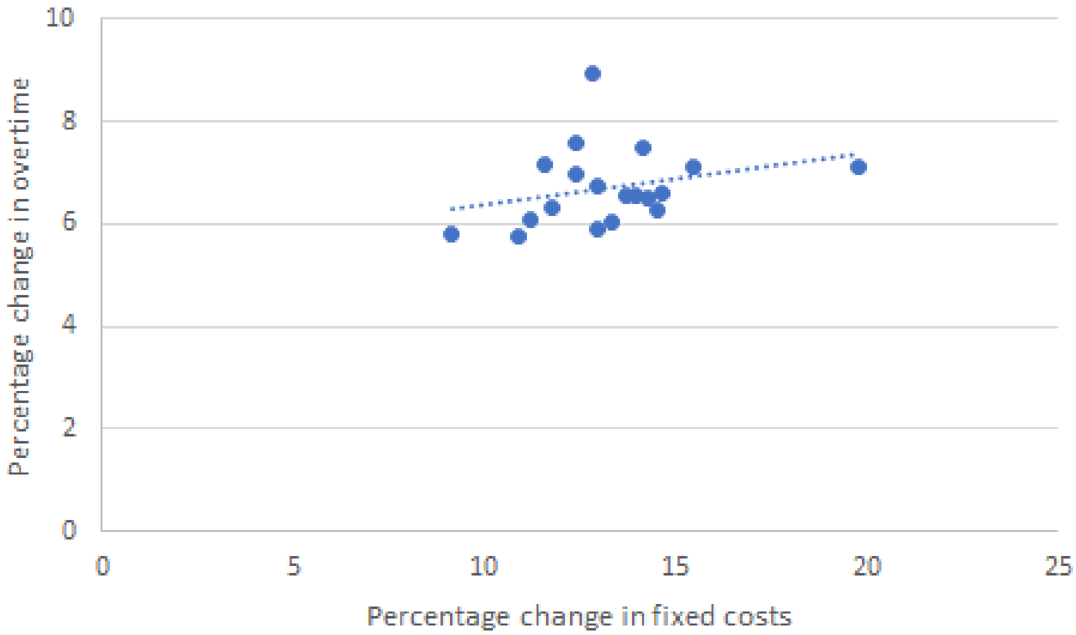


Figure 6.8: Relationship between the percentage change in overtime and percentage change in fixed costs

will have a more significant impact on the Construction industry than the Finance industry. Ceteris paribus, it is expected that the Construction industry will be more impacted than Finance. The relationship between the size of the shock to fixed costs and the percentage change in overtime is depicted in Figure 6.8. As expected, the industries with larger increases in fixed costs tend to experience larger percentage increases in the level of overtime used.

### 6.7.5 Relationship between the percentage change in overtime and wage premium schedule

In Figure 6.8, there are industries which experience similar percentage increases in fixed costs that have varying increases in overtime. For example, the mining industry experiences a 12.81% increase

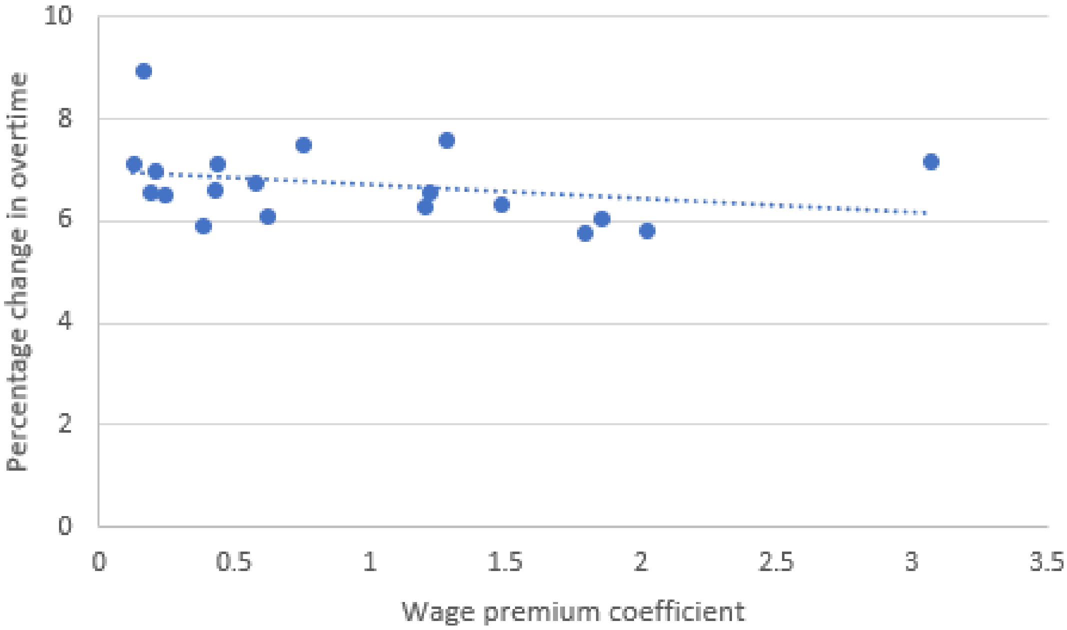


Figure 6.9: Relationship between wage premium schedule and overtime

in fixed costs and the Admin industry experiences a 12.94% increase in fixed costs, yet the mining industry increases overtime by 8.94% compared to the Admin industry increasing overtime by 5.91%. The contrast between these two industries can be explained by differing wage premium schedules.

The elasticity between wage premium and overtime is constant, giving the relationship:

$$W_P = A_{OT} OT^{\eta_{WP,O_T}-1} \tag{6.17}$$

where $(\eta_{WP,OT} - 1)$ is the elasticity of wage premium with respect to overtime hours. The term $A_{OT}$ represents the different wage premium regimes across industries. The larger the value for $A_{OT}$ the more expensive it is to use overtime. The relationship between overtime and this $A_{OT}$ for each industry is depicted in the Figure 6.9.

There is a negative relationship between the two variables. For the example above, the $A_{OT}$ for the Admin industry is 0.39 compared to the Mining industry which has an $A_{OT}$ value of 0.17. This

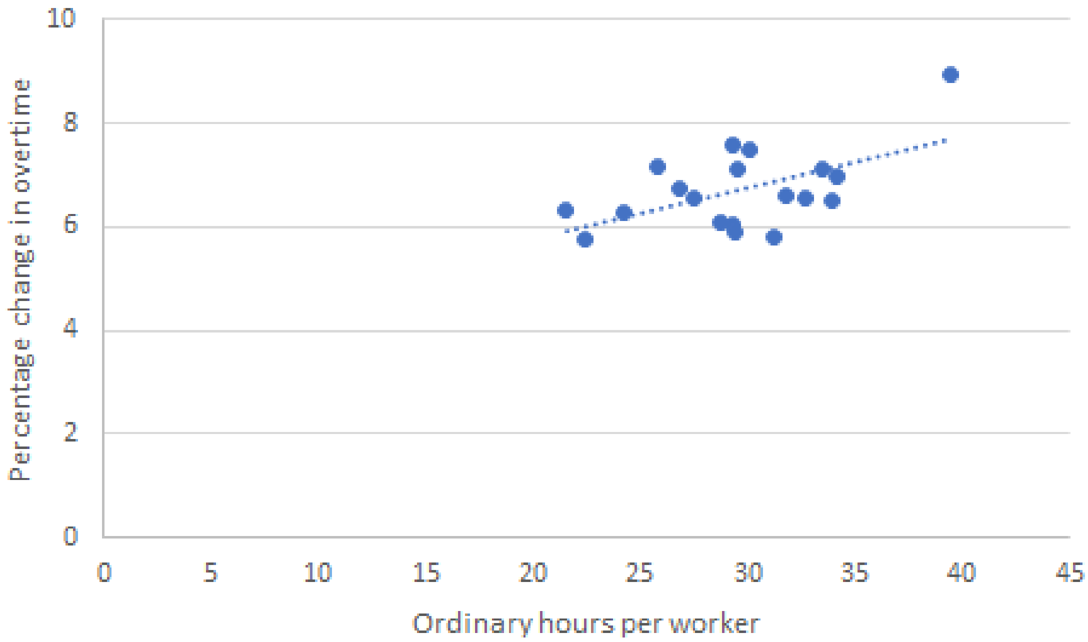


Figure 6.10: Relationship between ordinary hours and overtime

helps to explain why the Mining industry (8.94%) increased its overtime by almost twice the amount of the Admin industry (5.91%).

### 6.7.6 Relationship between percentage change overtime and initial hours per worker

The final determinant of the variation in overtime is the initial level of ordinary hours worked within an industry. The larger the ordinary hours for an industry, the more likely that overtime will be employed. Since ordinary hours are fixed, it is more expensive to hire an additional worker if they have a higher level of ordinary hours, as they are required to be paid for those fixed hours. For example, if industry 1 has ordinary hours of 20 hours, industry 2 has ordinary hours of 40 hours and the wage rate for both industries is $10, the minimum cost of an additional worker in industry 1 is $200 whereas it is $400 in industry 2. The relationship is shown in the Figure 6.10.

Consider the Electricity industry compared to the Arts industry. The Electricity industry has ordinary hours of 34.19 and experiences a 6.99% increase in overtime whereas the Arts Industry only has 22.45 ordinary hours and only experiences a 5.74% increase in overtime hours.

### 6.7.7 Proportion of variation in person hours

Table 6.4 disaggregates the variation in person hours into what proportion is due to intensive margin adjustments and extensive margin adjustments.

The Proportion of Variation (PV) attributed to the intensive margin is calculated:

$$PV_l = \frac{|l|}{|l| + |n|} \tag{6.18}$$

where $PV_l$ represents the proportion of variation attributed to the adjustment in intensive margin employment and $|l|$ and $|n|$ represents the absolute value of $l$ and $n$, respectively. Absolute values are used to ensure the negative decreases in extensive margin employment do not cancel out positive increase in intensive margin employment.

The $PV$ for extensive margin is defined:

$$PV_n = \frac{|n|}{|l| \; + \; |n|} \tag{6.19}$$

The extensive margin accounts for most variation in employment, with it accounting for 86% of total variation in employment across all industries. The only industry in which intensive margin adjustments are larger than extensive margin adjustments is the Construction industry.

These results demonstrate the potential for systematic bias if it is assumed that all variation in person hours occurs along the extensive margin. Given that the increase in the SG causes the intensive margin to increase, the decrease in extensive margin is larger than the decrease in person hours. This assumption would lead to the false conclusion that the decrease in the number of workers are not as large as the true decrease. In particular, if this assumption was applied to the construction

| Industry | $PV_l$ | $PV_n$ |
|---|---|---|
| Agriculture, forestry and fishing | 0.06 | 0.94 |
| Mining | 0.11 | 0.89 |
| Manufacturing | 0.09 | 0.91 |
| Electricity, gas, water and waste services | 0.17 | 0.83 |
| Construction | 0.38 | 0.62 |
| Wholesale trade | 0.13 | 0.87 |
| Retail trade | 0.67 | 0.33 |
| Accommodation and food services | 0.04 | 0.96 |
| Transport, postal and warehousing | 0.12 | 0.88 |
| Information media and telecommunications | 0.02 | 0.98 |
| Financial and insurance services | 0.03 | 0.97 |
| Rental, hiring and real estate services | 0.06 | 0.94 |
| Professional, scientific and technical services | 0.04 | 0.96 |
| Administrative and support services | 0.13 | 0.87 |
| Public administration and safety | 0.28 | 0.72 |
| Education and training | 0.02 | 0.98 |
| Health care and social assistance | 0.14 | 0.86 |
| Arts and recreation services | 0.06 | 0.94 |
| Other services | 0.14 | 0.86 |

Table 6.4: The proportion of variation due to intensive margin ($PV_l$) and extensive margin employment ($PV_n$).

industry, it would be falsely concluded that employment decreased by 0.12% despite the number of workers actually decreasing by 1.07%. Thus, the industries with a greater share of intensive margin in total variation will have more misleading results.

### 6.7.8 The cost of labour

The results for the cost of labour $P1LAB$ and the overtime premium are relatively straightforward to interpret. For the L-D simulation, the cost of labour, $P1LAB$ measures the average hourly labour cost per worker. Since the increase in fixed costs causes there to be a substitution to overtime hours, this causes the wage premium to increase. This means that the cost of labour increases for all industries as the overtime and fixed costs increase. The percentage change in the price of labour and the wage premium are presented in the Table 6.5.

The specification of the wage premium function means that the wage premium increases proportionately to overtime. As such, the mining industry experiences the largest increase in its wage premium since it experiences the largest increase in overtime.

### 6.7.9 Extensive margin employment

In this setup, the level of effective labour is determined by the hourly labour cost per worker and intensive margin is determined by fixed costs, ordinary wage rates and wage premiums. Thus, extensive margin employment is residually determined, as it does not appear anywhere in the model outside the labour services function. Ignoring the industry subscripts, extensive margin employment can be determined by rearranging the labour services function, such that:

$$n = x1lab - \alpha l \tag{6.20}$$

The pattern of extensive margin employment mostly follows that of the effective labour for industries where the variation in intensive margin employment is relatively small. However, for industries such

| Industry | p1lab | wp |
|---|---|---|
| Agriculture, forestry and fishing | 3.92 | 4.98 |
| Mining | 3.69 | 6.25 |
| Manufacturing | 3.75 | 4.55 |
| Electricity, gas, water and waste services | 4.22 | 4.89 |
| Construction | 3.8 | 4.98 |
| Wholesale trade | 3.84 | 4.62 |
| Retail trade | 3.68 | 4.4 |
| Accommodation and food services | 3.6 | 4.41 |
| Transport, postal and warehousing | 3.76 | 4.58 |
| Information media and telecommunications | 3.65 | 4.23 |
| Financial and insurance services | 3.5 | 4.06 |
| Rental, hiring and real estate services | 3.61 | 4.26 |
| Professional, scientific and technical services | 3.67 | 5.3 |
| Administrative and support services | 3.65 | 4.14 |
| Public administration and safety | 4.1 | 5.23 |
| Education and training | 3.95 | 5.03 |
| Health care and social assistance | 3.72 | 4.59 |
| Arts and recreation services | 3.62 | 4.02 |
| Other services | 3.74 | 4.71 |

Table 6.5: Percentage change in average hourly labour cost ($P1LAB$) and wage premium ($WP$)

as Construction and Retail where intensive margin adjustments account for a large share in the variation in person hours, this relationship breaks down and the change in number of workers is significantly larger.

## 6.8 Industry

The results for activity in each industry are presented in the Table 6.6.

The difference between the B-L and the L-D model can be attributed to differences in the percentage change in the price of labour across industries. Industries where substitutions towards overtime mitigated some of the effects of the increase in fixed costs had lower increases in the total cost of labour compared the B-L simulation. As a consequence, the decline in activity for these industries was more muted. The remainder of this section discusses why, with the exception of retail, that industry activity declined by varying amounts across industries. There will be no distinction between the L-D and B-L simulation since the difference between the two simulations is relatively small compared to the magnitude of the variation in output and this variation in output can be explained more succinctly by other economic fundamentals. However, any figures on industry activity used will be based on the L-D simulation. Throughout this analysis, reference will be given to the labour-capital ratio. It is defined as:

$$\frac{P_H M}{P_k K} \tag{6.21}$$

Where $P_H$ and $P_K$ are the initial price of person-hours and capital, respectively. As land comprises an insignificant component of primary factors for most industries, it is ignored. The term $M$ appears instead of $H$ as workers are remunerated based on the hours worked and not on their effective hours.

| | x1tot | | p0com | |
|---|---|---|---|---|
| Industry | L-D | B-L | L-D | B-L |
| Agriculture, forestry and fishing | -1.07 | -1.07 | -0.54 | -0.55 |
| Mining | -0.58 | -0.57 | -0.10 | -0.10 |
| Manufacturing | -2.04 | -2.06 | 0.89 | 0.90 |
| Electricity, gas, water and waste services | -0.59 | -0.61 | 1.39 | 1.47 |
| Construction | -0.10 | -0.10 | 2.32 | 2.38 |
| Wholesale trade | -0.88 | -0.88 | 2.34 | 2.33 |
| Retail trade | 0.10 | 0.09 | 2.88 | 2.82 |
| Accommodation and food services | -1.67 | -1.64 | 1.88 | 1.84 |
| Transport, postal and warehousing | -1.58 | -1.61 | 1.28 | 1.31 |
| Information media and telecommunications | -1.13 | -1.10 | 1.51 | 1.47 |
| Financial and insurance services | -0.80 | -0.78 | 2.24 | 2.16 |
| Rental, hiring and real estate services | -1.02 | -1.01 | 1.53 | 1.50 |
| Professional, scientific and technical services | -1.18 | -1.16 | 2.49 | 2.42 |
| Administrative and support services | -1.29 | -1.28 | 2.55 | 2.54 |
| Public administration and safety | -0.15 | -0.15 | 3.14 | 3.10 |
| Education and training | -1.67 | -1.62 | 3.12 | 3.04 |
| Health care and social assistance | -0.36 | -0.35 | 3.10 | 3.03 |
| Arts and recreation services | -0.88 | -0.86 | 2.20 | 2.15 |
| Other services | -0.83 | -0.83 | 2.52 | 2.49 |

Table 6.6: Percentage change in industry activity ($x1tot$) and percentage change in general output price of locally produced commodity ($p0dom$) for each industry.

### 6.8.1 Sales decomposition

Table 6.7 decomposes the change in output based on the source of its destination. It shows how the change in demand by each purchase of a commodity affects the output of each industry. This is useful for understanding the main causes for a decline in an industry's economic activity and is used throughout the remainder of this chapter.

### 6.8.2 Agriculture

The results for the Agriculture industry must be interpreted with caution given that certain data, such as overtime and wage premiums, could not be estimated for the Agriculture industry; instead, industry averages of the remaining industries were used for overtime and wage premium data. This assumption has little impact on the results for industry output which is evident from the fact that there is little difference in output between the B-L and L-D simulation. These results are driven by what happens to the average price of labour $P1LAB$ and how each industry responds to this price increase. The introduction of wage premiums and overtime only have modest impacts on how prices respond to changes in fixed costs. The increase in the average price of labour is mostly driven by the size of the increase in the fixed costs of labour.

The Agriculture industry experienced an above average decrease in industry activity of 1.07%, despite the price of the Agriculture commodity decreasing by 0.55%. The reason why the price for the Agriculture industry decreases can be understood by rearranging the BOTE equation (6.16), such that:

$$p_q = \frac{S_k}{\sigma} h + p_h \tag{6.22}$$

Noting that agriculture's share of capital in production is 0.61, the elasticity of substitution $\sigma = 0.5$, the percentage change in price 3.92 and effective labour decreases by 4.04, which gives:

$$p_q = \frac{0.61}{0.5}(-4.04) + 3.92 = -1 \tag{6.23}$$

| Industry | Interm | Invest | HouseH | Export | Margins | Total |
|---|---|---|---|---|---|---|
| Agriculture, forestry and fishing | -1.14 | -0.08 | 0.05 | 0.10 | 0.00 | -1.07 |
| Mining | -0.44 | -0.01 | 0.01 | -0.13 | 0.00 | -0.57 |
| Manufacturing | -0.82 | -0.06 | -0.03 | -1.15 | 0.00 | -2.06 |
| Electricity, gas, water and waste services | -0.37 | 0.00 | 0.01 | 0.00 | -0.25 | -0.61 |
| Construction | -0.12 | 0.02 | 0.00 | 0.00 | 0.00 | -0.10 |
| Wholesale trade | -0.03 | 0.00 | 0.00 | -0.04 | -0.81 | -0.88 |
| Retail trade | -0.01 | 0.00 | 0.00 | 0.00 | 0.10 | 0.09 |
| Accommodation and food services | -0.31 | 0.00 | -0.27 | -1.05 | 0.00 | -1.64 |
| Transport, postal and warehousing | -0.52 | 0.00 | -0.06 | -0.72 | -0.30 | -1.61 |
| Information media and telecommunications | -0.72 | 0.02 | -0.02 | -0.38 | 0.00 | -1.10 |
| Financial and insurance services | -0.43 | 0.00 | -0.11 | -0.25 | 0.00 | -0.78 |
| Rental, hiring and real estate services | -0.86 | -0.01 | 0.00 | -0.14 | 0.00 | -1.01 |
| Professional, scientific and technical services | -0.76 | 0.04 | -0.01 | -0.43 | 0.00 | -1.16 |
| Administrative and support services | -0.98 | 0.00 | -0.02 | -0.28 | 0.00 | -1.28 |
| Public administration and safety | -0.12 | 0.00 | -0.01 | -0.02 | 0.00 | -0.15 |
| Education and training | -0.11 | 0.00 | -0.38 | -1.13 | 0.00 | -1.62 |
| Health care and social assistance | -0.01 | 0.00 | -0.30 | -0.04 | 0.00 | -0.35 |
| Arts and recreation services | -0.26 | 0.01 | -0.26 | -0.35 | 0.00 | -0.86 |
| Other services | -0.59 | 0.00 | -0.20 | -0.04 | 0.00 | -0.83 |

Table 6.7: Sales decomposition for each industry by each user.

which is larger than the observed value of -0.52. This is a consequence of ignoring intermediate inputs role in price formation, as the price of intermediate inputs for agriculture increase, the price also increases. However, this still shows that if the percentage decline in effective labour and the share of capital are large, the price can decrease in response to an increase in the price of labour.

Despite being export-exposed, the decrease in price caused exports share in sales to increase for the Agriculture by 0.1%. In addition household consumption's share in sales increased by 0.05% while investment decreased by 0.08%.

The main determinant of why industry activity declined overall is due to decline in intermediate usage. The decline in intermediate usage accounts for 1.14% of its decrease in sales. As intermediate inputs are used in fixed proportions, the decrease in price of the Agriculture does not impact how other industries use Agriculture as an intermediate input. Since output in all other industries (except Retail trade) decrease, the demand for Agriculture as an intermediate commodity also decreases.

### 6.8.3 Mining

The mining industry only experienced about half the decline in activity as the Agriculture industry. Despite being export-oriented, the mining industry is relatively sheltered from the increase in price of effective hours as it is capital intensive. The major contributor to the mining industry's decrease in output is due to a decrease in use as an intermediate input. This decline accounts for 0.44% of its decline in output.

Similar to the Agriculture industry, the mining industry also experiences a 0.1% decrease in the general price ($p0com$) of output. However, the export price ($p4$) for the mining industry increases by 0.05%, which is a consequence of the increase in the price of margins used by the mining industry. Therefore, the mining industry experience a decline in exports of 0.24% which accounts for 0.13% of the decline in total output, as exports comprise approximately 54% of Mining sales.

Even though the Mining and Agriculture commodities are trade exposed, they experience a smaller decline in economic activity since they benefit from a decrease in the price of land. This allows them to pass on these savings in the form of lower prices to their purchasers.

### 6.8.4 Manufacturing

The manufacturing industry experiences the greatest decline in economic activity (-2.06%) of all industries since it is both trade-exposed and labour intensive. The decline in manufacturing can mostly be attributed to a decline in intermediate usage and exports of the manufacturing commodity with little variation in investment or household consumption. Whilst both consumption and investment is exogenous, consumers are still able to substitute domestic for imported commodities, which can result in consumption decreasing.

The decline in exports is due to a 1.05% increase in export prices which leads to a decrease in the quantity of exports by 5.25%. As exports account for approximately 22% of all manufacturing output, accounting for a decline of 1.15% in output.

Typically, the increase in the domestic price of a commodity should lead to a substitution towards imported commodities. In this case, household consumption of domestically produced goods only declined by -0.17% and imported commodities only increased by 0.69%. This can be attributed to the fact that the purchase price of Manufactured commodities increased for both the domestic and imported commodity with the purchase price for domestic consumers increasing by 1.56% and by 1.13% for imported commodities. Whilst the import price is held constant in this simulation, the increase in the price of margins causes the local currency import price to increase.

As a consequence of the decline in overall economic activity, each industry's intermediate demand for the manufacturing commodity decreases. This decline in economic activity causes the intermediate usage of the manufacturing commodity to decline by 0.82%.

### 6.8.5 Electricity, gas, water, waste services

The Electricity industry experiences a similar decline in economic activity as the Mining industry, experiencing an 0.61% decrease in economic activity, despite being neither trade exposed nor labour intensive. Given it has a similar capital-labour ratio, it was expected that the decline in economic activity would be lesser in the Electricity industry than the Mining industry.

The decline in the electricity commodity can be attributed to its decrease in use as a margin and intermediate goods. Whilst Electricity is immune to a decrease in export demand as it is not trade exposed, it is often used as an intermediate good for commodities which are trade exposed. As such, 0.25% of its decline in activity can be attributed to a reduction in margin demand.

The use of Electricity as an intermediate commodity causes the other 0.37% decrease in the Electricity commodity. As intermediate inputs are used according to a Leontief production structure, it will decrease proportionately to the output commodities for which it is used as an input. The Leontief production structure means that there will be no substitution either away from this commodity or towards it in response to a relative price change of margin commodities.

### 6.8.6 Construction

The Construction industry only experiences a 0.1% decrease in output despite a 2.38% increase in basic and output prices. There is little decline in economic activity for the construction industry due to the assumptions employed within this simulation. There is almost no household consumption or export of the Construction commodity. It is primarily used as an intermediate and investment commodity. It accounts for close to 60% of all investment spending. Given that aggregate investment is held constant in this simulation, the demand remains relatively constant. The decrease in activity is due to the -0.12% decrease in its use as a intermediate commodity.

### 6.8.7 Wholesale Trade

The Wholesale industry experiences a modest decline in economic activity with output reducing by -0.88%. Similar to the Electricity industry, Wholesale is used as a margin commodity by other industries. Therefore, 0.81% of the decrease in output for the wholesale industry can be attributed to a decrease in its use as a margin. The remaining decrease in Wholesale is evenly split between exports and intermediate usage.

### 6.8.8 Retail Trade

The Retail Trade industry is the only industry which performs better as a consequence of the increase in superannuation, with economic activity increasing by 0.09%.

A significant component of the demand for the Retail commodity is derived from being used as a margin. As aggregate household consumption is fixed and retail margins are used in fixed proportion to household consumption, there is little scope for variation in retail output. However, the price increase in service commodities (such as Healthcare and Education) causes a substitution towards commodities which use retail as margins, such as manufactured Imported goods. As such, the increase in margins on these commodities causes Retail's use as a margin to increase overall, accounting for a 10% increase in retail activity.

### 6.8.9 Accommodation and Food services

The Accommodation and food services industry is both labour-intensive and an export-oriented industry. Consequently, it is one of the more significantly affected industries, experiencing a decline in activity by 1.64%.

| Fan decomposition | |
|---|---|
| Local Market | -0.18 |
| Domestic Share | -0.40 |
| Export | -1.05 |
| Total | -1.64 |

Table 6.8: Fan decomposition for the Accommodation industry

The Accommodation industry is impacted by both import substitution and a decline in exports. Domestic consumers substitute domestic for overseas accommodation commodities in response to the increase in its domestic price as a consequence of the increase in the cost of labour. That is, consumers are more likely to take an overseas holiday as a consequence of the increase in the SG. Table 6.8 shows the Fan decomposition for the accommodation industry, which decomposes the change in output into a local-market effect, export effect and domestic-share effect, as is discussed in Chapter 2.10.1. The reduction in the domestic share of output, caused by a substitution from domestic production to imports, accounts for 0.4% of the decrease in demand. The increase in price causes a movement up the export demand curve which causes the quantity of exports to decrease, accounting for 1.05% of the decline in activity. In addition, the price increase causes the local market to shrink, accounting for the 0.18% decrease in economic activity.

The decrease in domestic sales is attributed to the decline in Accommodation's use as an intermediate and household commodity. The decline in activity in the household sector accounts for

0.27% of the decline in activity while the decline in use as an intermediate input accounts for 0.31% of the decline.

The Accommodation commodity is used as a margin by other industries. Despite this, it does not experience a decline in activity due to its use as margin commodity. This is because it is not used as a margin for exported commodities and its domestic use comprises a small share of its total output.

#### 6.8.10 Transport, postal and warehousing

The Transport, postal and warehousing industry is very reliant on trade. Consequently, it is one of the largest impacted industries with its economic activity declining by -1.61%. Its demand decreases due to a decline in export demand, intermediate and margin use.

The increase in price of Transport caused by the increase in labour costs causes exports to become less favourable to the rest of the world. For example, overseas consumers are less likely to consume flights to Australia when the price of a flight increases. The decrease in demand for exports causes activity to decline by 0.72% and is a main reason why transport performs so poorly compared to margins like retail and Wholesale.

The remaining reduction in activity is predominantly a result of a reduction in use as an intermediate input. The demand for Transport as an intermediate commodity decreases because of the decline in economic activity overall, accounting for a 0.52% decrease in activity for the Transport industry.

#### 6.8.11 Information media and telecommunications

The Information industry is neither trade exposed or labour intensive, yet it experiences a decline in economic activity of 1.1%. The Information industry is responsible for producing commodities related to computer equipment and internet services which are often used as intermediates in production. As such the decline in intermediate demand declines by 0.72% which accounts for the largest component of the variation in economic activity. The remaining decline in economic activity is due to the reduction in exports. The increase in output price causes export demand to decrease, which accounts for 0.38% of its decline in activity.

### 6.8.12 Financial and insurance services

The Finance industry has the lowest share of superannuation as a proportion of fixed costs out of all industries, resulting in it having the smallest increase in fixed costs in percentage terms. Coupled with it not being an export-oriented industry meant that it only experienced a modest decrease in activity of 0.78%, despite it being a relatively labour-intensive industry.

The main contributor to the decline in activity was its use as an intermediate input with intermediate demand accounting for 0.78% of its decline in activity. The Finance industry is used as an intermediate input widely throughout the economy and thus it declines proportionately with overall economic output.

The remaining variation in output is due to a reduction in export demand. This reduction accounts for 0.25% of the decline in economic activity.

### 6.8.13 Rental, hiring and real estate services

The Rental, hiring and real estate services industry experiences a 1.01% decrease in economic activity. The use of the Rental commodity as an intermediate input contributes most to this decline with its intermediate demand accounting for 0.86% of the decline in activity. With the exception of Rental using itself as an intermediate input, the main users of the Rental commodity as an intermediate input are the manufacturing and professional services industry. As the manufacturing experiences the largest decline in economic activity, this flows onto the Rental industry, as the manufacturing industry decreases its intermediate demand for the Rental commodity. Moreover, the Professional services industry's output declines by 1.16%, causing a further reduction in input demand.

A decrease in export demand accounts for another 0.14% of the decline in economic activity. The Rental industry only experiences a modest decline in exports as it is not an export-oriented industry.

#### 6.8.14 Professional, scientific and technical services

The Professional services industry experiences an above average decrease in activity of 1.16% which is due to its use as an intermediate input and export demand.

The Professional services industry consists of scientific research, architecture and engineering. These services are used by most industries as an intermediate input. Therefore, as there is an overall decline in economic activity, it experiences a decline in use as an intermediate input. The decline in economic activity accounts for 0.76% of the decline in the professional services industry's activity.

The decline in exports across non-export-oriented industries decreases by a uniform amount in percentage terms. However, exports share in sales determines the magnitude of the effect that a decrease in exports has on total sales of a commodity. The Professional services commodity has the largest share of exports in total sales of all the non-export-oriented commodities and thus experiences the most significant decline.

#### 6.8.15 Administrative and support services

The Administrative and support services industry experiences the second largest decline in economic activity of 1.28%. Whilst the Administrative industry is the most labour-intensive industry, with labour accounting for 93% of expenditure on primary factors, the decline in intermediate usage is the main contributor to its economic decline. It experiences the largest decline in intermediate usage with its decline in intermediate usage accounting for 0.98% of its decline. Similarly to the professional services industry, its use as an intermediate input is widespread throughout the economy. Therefore, it experiences a decline in intermediate usage as a result of the overall decrease in economic activity.

The industry industry is not a trade exposed industry. However, it still experiences a moderate decline in export activity with exports declining by 0.28%.

#### 6.8.16 Public administration and safety

Despite being a labour-intensive industry, the Public industry only experiences a 0.15% decrease in output. The industry only suffers a small decline in economic activity as the government is the largest user of the Public commodity and its demand is held constant in this simulation. The industry is also not very trade exposed so does not suffer from import-substitution or a decline in exports with its export demand only accounting for 0.02% decrease in economic activity.

The decline in economic activity for the Public Industry can be attributed to a reduction in intermediate demand which accounts for 0.12% of its reduction in sales.

#### 6.8.17 Education and training

The Education industry is an export-oriented industry which is characterised by a labour-intensive production process. As a result, the education industry experiences a larger than average decline in economic activity of 1.62%. Historically, Education typically was consumed domestically by households and the government sector. This usually makes it sheltered from price increases as the demand is relatively stable. However, the boom in foreign students means that Education is now an export commodity. In basic prices, exports now account for almost 20% of all sales of the Education commodity. As a consequence, the increase in price of labour causes it to become less competitive internationally and causes output to decrease. The decrease in export demand contributes to 1.13% of the reduction in demand, which was the second largest decline in exports.

The Education industry experiences the greatest decline in household consumption of any industry with it contributing to -0.38% of economic activity. This is a consequence of the price of education increasing by 3.12% due to its large share of labour in production, which accounts for 87% of all expenditure on primary factors. This is compounded by the fact that the Education industry is unable to substitute to hours from workers. The Education industry has a steep wage premium schedule which makes it expensive for the education industry to use overtime.

#### 6.8.18 Health care and social assistance

The Health care industry experiences a decline in economic activity of 0.31%. Similar to the Education industry, the healthcare industry is labour intensive with labours share in production accounting for 87% of expenditure on primary factors.

The Healthcare industry also experiences a significant decline in household demand (-0.3%) as did the Education industry, due to the significant share of labour in production.

Unlike the Education industry, the Healthcare industry does not export its commodity and thus does not experience a significant decline in output due to a decline in exports. Combined with the fact that Healthcare has limited use as an intermediate commodity, this leads to healthcare being largely unaffected.

The small decline in economic activity is also a consequence of the closure assumptions employed within this simulation. As the governments demand are treated as exogenous and considering government demand comprises a significant portion of the Healthcare commodity, there is little scope for a decline in output. If the governments do adjust their demand as consequence of the increase in fixed costs, there might be a larger decline in economic activity.

#### 6.8.19 Arts and recreation services

The Arts industry experiences a decline in economic activity of 0.873% which is rather uniformly spread across intermediate, households and export users.

Household demand for the Arts commodity accounts for 0.26% of the decline in economic activity. The arts industry consists of sports and entertainment; museums, cultural activities and gambling activities. These industries are predominantly used by households which account for 73% of total sales. Despite aggregate consumption being held constant, the Arts industry still suffers from import competition with households substituting to imports instead of domestically produced commodities. In addition, the Arts industry experiences an above average increase in its general output price ($p0dom$) of 2.2%. Whilst aggregate consumption is fixed, the relatively high increase in its output price causes a substitution towards other commodities.

Exports account for the largest component of the decrease in sales with export demand decreasing by 0.35%. The last significant component of decrease in sales can be attributed to a reduction in intermediate demand which accounts for 0.26% decline in activity. Overall, the industry is relatively unaffected by the increase in the cost of labour as most of its demand remains fixed in this simulation.

#### 6.8.20 Other services

The final industry is the Other services industry which is a very labour-intensive industry with labour accounting for 90% of expenditure on primary factors. Its output declines by 0.895% which is larger than the decrease in GDP of 0.77%.

The majority of this decline in economic activity can be attributed to its use as an intermediate input. The intermediate demand for other services declines by 0.59%. Household demand accounts for the majority of the remaining variation in economic activity, explaining 0.2% of the decrease in economic activity. As the industry does not export a significant component of its output, there is almost no variation in output due to a reduction in exports.

#### 6.8.21 Owner dwelling

The Owner dwelling industry consists entirely of capital stock and does not use any labour. As the capital stock is fixed, there is no scope for a variation in employment, so its output remains fixed.

## 6.9 Long Run impacts

This research focused on the short-run implications of an increase in the SG, despite the policy being permanent. This was largely done due to the limited impacts that would occur in the long run. In the long run, the labour market would return to equilibrium, which would require the ordinary wage rate to decrease accompanied by a proportionate decrease in fixed costs. Therefore, there would be no changes to employment.

There would be a long-run increase in the use of overtime and intensive margin employment. However, these results would be the same as they are in the short-run simulation, as the ratio of ordinary wages to fixed cost do evolve over time in this framework. Given that there would only be modest difference in a short-run and long-run simulation, it was deemed little insight would be gained from performing a long-run simulation.

## 6.10 Return to hours parameters

In Chapter 4.2 the literature on the labour services function was reviewed, in particular, the returns to hours parameter $\alpha$. The literature presented conflicting results with some studies suggesting that the labour services function experienced increasing returns to scale for hours whereas some suggesting decreasing returns to scale. The same applied for whether the returns to hours were larger than the returns to workers. In this model $\beta = 1$ was imposed and the $\alpha$ value was calculated from the first order conditions using equation (3.87). These estimated values are presented in the Table 6.9.

| Industry | $\alpha$ |
|---|---|
| Agriculture, forestry and fishing | 1.59 |
| Mining | 1.21 |
| Manufacturing | 1.53 |
| Electricity, gas, water and waste services | 1.58 |
| Construction | 1.46 |
| Wholesale trade | 1.56 |
| Retail trade | 1.55 |
| Accommodation and food services | 1.44 |
| Transport, postal and warehousing | 1.51 |
| Information media and telecommunications | 1.56 |
| Financial and insurance services | 1.38 |
| Rental, hiring and real estate services | 1.46 |
| Professional, scientific and technical services | 1.31 |
| Administrative and support services | 1.57 |
| Public administration and safety | 1.50 |
| Education and training | 1.41 |
| Health care and social assistance | 1.50 |
| Arts and recreation services | 1.53 |
| Other services | 1.45 |

Table 6.9: Returns to hours parameter for labour services function for each industry

These results suggest that all industries experience increasing returns to scale and that the returns to hours are larger than the returns to workers.

Counter-intuitively, the returns to hours is lowest for the mining industry despite the mining industry being one of the industries which expands overtime most. It is expected that industries with higher productivity's of hours would be more inclined to use more hours, comparatively.

There appears to be three immediate reasons as to why this may have been the case. Firstly, the cost of expanding overtime in the mining industry is relatively cheap. The initial wage premium observed in the mining industry is only 28.13%. Thus despite hours being relatively unproductive it is outweighed by the relatively low cost of overtime. Secondly, the $\alpha$ value is calibrated based on the observed wage premium value. If there was measurement error with the data used to calculate wage premiums for mining workers, then this could cause the estimate for $\alpha$ to be biased for the mining industry.

Finally, this may be a consequence of the underlying model used to calculate the returns to hours parameter being mis-specified. In this case, the estimate for $\alpha$ across all industries would be biased. The Cobb-Douglas style labour services function does not allow for the productivity of hours $\alpha$ to vary based on hours worked per week. However, it seems plausible that as hours increase, that $\alpha$ may decrease, representing a decrease in the productivity of hours. This would justify mining having such a low value for $\alpha$ since workers in the mining industry work an average of 39.5 hours of ordinary time per week compared with the economy average of 29.6 ordinary hours per week. In which case, the estimate for $\alpha$ provides an approximate for productivity in the neighbourhood of ordinary hours worked.

## 6.11 Summary

Overall, the increase in the rate of superannuation in the short-run causes a decline in GDP and the number of workers employed within the economy. Extensive margin employment decreases by 2% and GDP decreases by 0.77% compared to the scenario where there is no increase in the SG.

In terms of the labour market, the main takeaway from this analysis is how the decline in extensive margin employment (-2.00%) is larger than the decrease in effective labour (-1.77%). Therefore, a simulation performed without this labour demand component will understate the true effects on employment, as it will not capture the effects of firms substituting hours for workers. For example, in the B-L simulation the decrease in effective labour was 1.39%. Even if the researcher assumed all this variation in employment was caused by a reduction in extensive margin demand, it would still understate the decrease in extensive margin employment by 0.61%.

This research highlighted three contributing factors which impacted industries abilities to substitute hours for workers. Firstly, the larger the variation in fixed costs, the more that an industry will increase its intensive margin employment. Secondly, the steeper the slope of the wage premium schedule, the less likely that industries will use overtime. Thirdly, the higher the initial level of ordinary hours, the more likely industries would use overtime.

In addition, this policy did not have a uniform impact on all industries. As domestic absorption is held constant in this simulation, there tends to be four sources of decline in activity: (1) decrease in the level of exports, (2) decrease in margin use, (3) decrease in intermediate usage and (4) import substitution. Overall, industries which are trade exposed tend to suffer the largest decline in activity, as they are unable to pass the increase in costs of labour onto consumers. Industries which supply their commodities as intermediates or margins experience decline in activity since both intermediate commodities and margins are used in fixed proportions by industries. Thus, when other industries decrease their output it causes them to use less intermediate and margin commodities.

# 7 Final Remarks

There are three main contributions of this research: (1) The development of a labour demand model in the presence of overtime and fixed costs which is incorporated into a CGE model (2) The review of wage premium policy in Australia and the estimation of the relationship between wage premiums and overtime hours (3) The simulation of an increase in the SG from 9.5% to 12%.

This research addresses some the shortcomings of CGE models which are often ill-equipped to explain what causes variations in intensive margin and extensive margin employment. A labour demand model is incorporated into the ORANIG framework to allow it to simulate the effects on the intensive and extensive margins of employment. The effective labour input was modified so that the marginal productivity an additional hour of work could vary across industries. In addition, industry's costs were segmented into ordinary, overtime and fixed costs, with these costs determining the composition of hours and workers within an industry.

The benefits of this labour demand model are that changes to intensive and extensive margin employment are endogenously determined by the model. This avoids researchers having to specify how hours and workers will respond to such a policy change, and avoids the possibility for a bias to be introduced based on assumptions employed.

The standard model of labour demand in the presence of fixed costs and overtime with a constant wage premium was extended to a model with a wage premium function which had a constant elasticity between wage premium and overtime hours. This model overcame some of the shortcomings of the current literature by allowing the returns to hours to be larger than the returns to workers ($\alpha > \beta$) and allowing the elasticity of ordinary hours per worker and total hours per worker to have a positive elasticity ($\eta_{L_0,L}$).

This model also provides a framework to extend upon the empirical research in Chapter 4 conducted on labour demand using workers-hours models. The current literature has been able to explain how intensive margin employment responds to policy changes such as changes in wage premiums, ordinary hours and fixed costs. However, it has not always been able to explain how such policies impact the number of workers employed. The adoption of a CGE model is useful in this instance since CGE models include theory on the entire economy. As such, they can be used to simulate the how industries change their demand for effective labour in response to labour market policy shocks. Thus, a CGE framework can provide insight into how both intensive and extensive margin employment responds. In particular, this framework would simulate the effect of a reduction in standard hours, as it would provide both an intensive and extensive margin response. Especially given there are limited studies on this topic for Australia.

This research provides insight into the complex nature of the system of Modern Awards in Australia and their relation to overtime hours. In particular, how overtime wages depend on the ratio between workers wage rate and the corresponding award wage rate. This research overcame the difficulty of trying to incorporate wage premium data from Australia's 122 modern awards into a CGE model by instead estimating a relationship between wage premiums and overtime econometrically.

The estimation was conducted using a 2SLS model for wage premiums and overtime. This overcame some of the shortcomings of the previous literature where it is likely that overtime was an endogenous regressor. It was found that the relationship between wage premium and overtime was best modelled using a logarithmic function and that the elasticity between wage premiums and overtime was estimated to be 0.7. This implies that a 1% increase in overtime causes a 0.7% increase in wage premiums.

Finally, the ORANIG framework augmented with the labour demand component was used to simulate the effects of an increase in SG from 9.5% to 12%. In the short run, this causes the real labour cost per hour to increase by 2.09% causing effect labour to decrease by 1.38%. The real GDP in this simulation decreased by 0.77%.

The increase in the SG caused the total hours to decline by 1.77%. As this increased fixed costs, it caused a substitution from workers to hours of 0.23% resulting in a decrease in workers by 2%. Overall, this policy had the biggest impact in terms of workers with 2% of workers being without a job compared to the situation where superannuation is not increased. It is possible that the simulation overstates the decrease in extensive margin employment if ordinary wages do decrease in response to an increase in superannuation, as it was assumed that ordinary wages remained fixed in this simulation. Given that the simulation was conducted in a short-run environment, it was assumed that it would take time for this adjustment to wages to be implemented.

The SG simulation from 9.5% to 12% highlighted how different industries with different cost structures responded to a change in fixed costs. This simulation shows that the main determinants of overtime usage are (a) level of fixed costs (b) wage premium schedule and (c) the initial level of ordinary hours. For example, the Mining industry experienced a significant increase in overtime (8.94) despite having the highest ordinary hours (39.49) as it had the cheapest overtime hours. Conversely, the Finance industry experienced a much smaller increase in overtime (5.8%) as it had a steeper wage premium schedule.

The simulation shows the importance about assumptions on the quantity of labour. Consistent with the findings in Chapter 4.2, the calibration of the CGE model suggests that the marginal product of an additional hour per person is different to the marginal product of an additional worker. On average, the returns to hours parameter ($\alpha$) was 1.48 which suggests increasing returns to hours worked. This implies the composition of hours and workers impacts the productivity labour as an input in the production process.

In addition, this simulation illustrates how assumptions about the labour input might lead to researchers erroneously understating the effects of a labour market policy. In this instance, the increase in the superannuation caused there to be a decline in the demand for effective labour and a substitution to intensive margin employment from extensive margin employment. A simulation which does not account for the fixed costs and overtime will miss this as it will not capture the substitution effect. Even if it assumes all employment variation occurs to extensive margin employment, it will still understate the variation in extensive margin employment.

## 7.1 Shortcomings

It is important to recognise that there were certain restrictive assumptions employed regard parameters used within this simulation. In particular, the returns to workers and the elasticity of overtime with respect to the wage premium are likely to have influenced the results.

### 7.1.1 Returns to workers

It was imposed that $\beta = 1$ with little economic justification. This was done for two reasons: (1) to ensure that the production function exhibited constant returns to scale for all primary factors while holding hours worked constant and (2) as there was a lack of Australian econometric evidence to determine the parameter.

Unfortunately, it is possible that the calibration of this value caused the employment effects, in terms of the number of workers, to be either under-stated or over-stated. However, since extensive margin employment only enters into the model through the labour services function, it did not impact any other element of the simulation. Furthermore, it had no impact on the level of intensive margin employment, which depends on the ratio $\frac{\alpha}{\beta}$.

### 7.1.2 Overtime elasticity

There are possible shortcomings with the elasticity parameter $(\eta_{WP,OT} - 1)$ adopted for the relationship between wage premium and overtime hours. Firstly, it is assumed that the elasticity parameter is constant across all industries. This was done due to data limitations when estimating the parameter. There is no underlying economic logic for this assumption. It is plausible that this value varies across industries. Secondly, the econometric estimation techniques imposes a restrictive structure onto the overtime demand function. Unfortunately, confidentiality issues surrounding the EEH dataset limited the statistical analysis which could be applied. This precluded any form of non-parametric techniques being applied to the data. As such, the estimated value could be sensitive to the statistical modelling assumptions employed.

## 7.2 Future research

The research performed in this thesis is a novel approach at modelling labour demand in a CGE model. It provides further scope for future research. In addition, there are current shortcomings of this research which could be addressed in future research.

### 7.2.1 Disaggregation

Ideally, the underlying database adopted in this simulation would be extended upon the current 19 industries. Further disaggregation would better reflect the underlying industry structure. For example, the current database assumes that the Agriculture, Forestry and Fishing industries all use the same production technology. While the output commodities are similar, it seems unlikely that a commercial fishing boat uses the same technology as a cotton farmer.

It would also be useful to disaggregate occupations as different occupations within an industry may work different levels of overtime. Furthermore, some occupations, such as managers, do not tend to receive overtime premiums if they receive a salary.

### 7.2.2 Simulate the effects of change in ordinary hours

Chapter 4.1 analysed the economic impacts of decreasing the ordinary hours of work ($L_0$). The CGE model built in this research provides a framework to further contribute to this area of research. There has been little research conducted on this topic in Australia. Moreover, a CGE model provides a useful framework for analysing how both the intensive and extensive margin of employment is impacted by such a policy.

### 7.2.3 Ordinary hours worked

In addition to variations in overtime across industries, the ordinary hours worked per worker varies across industries. This is another avenue for future research. It is likely that varying levels of ordinary hours per worker across industries reflects the different levels of fixed costs and marginal productivity of labour across industries.

Another contributing factor to the difference in the ordinary hours per worker ($L_0$) was the composition of full and part-time workers within an industry. The data on overtime hours worked per worker showed that part-time workers tended to work different levels of overtime compared with their corresponding full-time workers (Australian Bureau of Statistics, 2011). Moreover, there is also scope to analyse how the level of fixed costs determines the quantities of part-time and full time workers within an industry. This would hopefully provide further insights into the Australian labour market.

### 7.2.4 Dynamic models

Finally, an avenue for future research would be converting this model into a dynamic model. This would provide insight into how unemployment, in terms of the number of workers without a job, evolves over time in response to such policy changes. Moreover, the benefits of such an approach is that in sample forecasts could be performed to evaluate the accuracy of the labour demand model in the presence of fixed costs.

# References


Andrews, M. J., Schank, T., and Simmons, R. (2005). Does worksharing work? some empirical evidence from the IAB-establishment panel. *Scottish Journal of Political Economy*, 52(2):141–176.

Armington, P. S. (1969). A theory of demand for products distinguished by place of production. *Staff Papers*, 16(1):159–178.

Australian Bureau of Statistics (2001). Employee earnings and hours, Australia, 2000, cat no 6306.0.

Australian Bureau of Statistics (2003). Employee earnings and hours, Australia, 2002, cat no 6306.0.

Australian Bureau of Statistics (2005). Employee earnings and hours, Australia, 2004, cat no 6306.0.

Australian Bureau of Statistics (2007). Employee earnings and hours, Australia, 2006, cat no 6306.0.

Australian Bureau of Statistics (2008). Australian and new zealand standard industrial classification (anzsic). *Canberra: Australian Bureau of Statistics.*

Australian Bureau of Statistics (2011). Microdata: Employee earnings and hours, Australia, 2010, cat no 6306.0.55.001.

Australian Bureau of Statistics (2013a). 1220.0-anzsco–australian and new zealand standard classification of occupations, 2013, version 1.2.

Australian Bureau of Statistics (2013b). *Australian National Accounts: Input-Output Tables: Table 2. Use table - input by industry and final use category and supply by product group, 2009-2010, cat. no. 5209.0.55.001_2013.* ABS.

Australian Taxation Office (2020). Key superannuation rates and thresholds.

Bell, D. (1982). Labour utilization and statutory non-wage costs. *Economica*, 49(195):335–343.

Calmfors, L. and Hoel, M. (1988). Work sharing and overtime. *The Scandinavian Journal of Economics*, pages 45–62.

Coates, B., Mackey, W., and Cowgill, M. (2020). No free lunch: Higher superannuation means lower wages.

Connolly, E., Kohler, M., et al. (2004). *The impact of superannuation on household saving.*

Corden, W. and Dixon, P. (1980). A tax-wage bargain in Australia: Is a free lunch possible? *Economic Record*, 56(154):209–221.

Craine, R. (1973). On the service flow from labour. *The Review of Economic Studies*, 40(1):39–46.

Crépon, B. and Kramarz, F. (2002). Employed 40 hours or not employed 39: Lessons from the 1982 mandatory reduction of the workweek. *Journal of Political Economy*, 110(6):1355–1389.

DeBeaumont, R. and Singell Jr, L. D. (1999). The return to hours and workers in US manufacturing: evidence on aggregation bias. *Southern Economic Journal*, 66(2):336.

Delmez, F. and Vandenberghe, V. (2018). Long working hours make us less productive but also less costly. *Labour*, 32(4):259–287.

Dixon, J. and Nassios, J. (2016). *Modelling the impacts of a cut to company tax in Australia.* Centre of Policy Studies (CoPS), Victoria University.

Dixon, P. B., Madden, J. R., and Rimmer, M. T. (2010). Linking national and multi-regional Computable General Equilibrium (CGE) Models: The Effects of an Increase in Award Wage Rates in Australia. *Regional Studies*, 44(10):1369–1385.

Dixon, P. B. and Parmenter, B. R. (1996). Computable general equilibrium modelling for policy analysis and forecasting. *Handbook of computational economics*, 1:3–85.

Dixon, P. B., Parmenter, B. R., Sutton, J., and Vincent, D. (1982). *ORANI, a multisectoral model of the Australian economy.* North-Holland Pub. Co. Sole distributors for the U.S.A. and Canada, Elsevier Science Pub. Co, Amsterdam New York New York, N.Y.

Dixon, P. B. and Rimmer, M. T. (2010). Johansen's contribution to CGE modelling: originator and guiding light for 50 years. Centre of Policy Studies/IMPACT Centre Working Papers g-203, Victoria University, Centre of Policy Studies/IMPACT Centre.

Dixon, R. and Freebairn, J. (2009). Models of labour services and estimates of Australian productivity. *Australian Economic Review*, 42(2):131–142.

Dixon, R., Freebairn, J., and Lim, G. C. (2005). An employment equation for Australia. *Economic Record*, 81(254):204–214.

Ehrenberg, R. G. (1971). The impact of the overtime premium on employment and hours in US industry. *Economic Inquiry*, 9(2):199–207.

Ehrenberg, R. G. and Smith, R. S. (2012). *Modern labor economics theory and public policy.* Upper Saddle River, N.J Prentice Hall, eleventh ed edition.

Feldstein, M. S. (1967). Specification of the labour input in the aggregate production function. *The Review of Economic Studies*, 34(4):375–386.

Freeman, R. (2008). Labour productivity indicators. *Comparison of two OECD databases productivity differentials & the Balassa-Samuelson effect: OECD Statistics Directorate Division of Structural Economic Statistics*, 76.

Garbarino, J. W. (1964). Fringe benefits and overtime as barriers to expanding employment. *ILR Review*, 17(3):426–442.

Gruen, D., Soding, L., et al. (2011). Compulsory superannuation and national saving. *Economic Round-up*, (3):45.

Hamermesh, D. S. (1996). *Labor demand*. princeton University press.

Hamermesh, D. S. and Trejo, S. J. (2000). The demand for hours of labor: Direct evidence from California. *Review of economics and statistics*, 82(1):38–47.

Hart, R. A. (2004). *The economics of overtime working*. Cambridge University Press.

Hart, R. A., Malley, J. R., and Ruffell, R. J. (1996). What shapes are overtime premium schedules? some evidence from Japan, the UK, and the US. *Economics letters*, 53(1):97–102.

Hart, R. A. and McGregor, P. G. (1988). The returns to labour services in West German manufacturing industry. *European Economic Review*, 32(4):947–963.

Hart, R. A. and Moutos, T. (1995). *Human capital, employment and bargaining*. Cambridge University Press.

Hart, R. A. and Ruffell, R. J. (1993). The cost of overtime hours in British production industries. *Economica*, pages 183–201.

Henry, K. (2009). *Australia's future tax system : the retirement income system : report on strategic issues*. Commonwealth of Australia, Dept. of Treasury, Barton, ACT.

Horridge, J., Jerie, M., Mustakinov, D., and Schiffmann, F. (2018). Gempack manual. *GEMPACK software*.

Horridge, M. et al. (2000). Orani-g: A general equilibrium model of the Australian Economy. Technical report, Victoria University, Centre of Policy Studies/IMPACT Centre.

Hunt, J. (1996). The response of wages and actual hours worked to the reductions of standard hours. Technical report, National Bureau of Economic Research.

Hunt, J. (1999). Has work-sharing worked in Germany? *The Quarterly Journal of Economics*, 114(1):117–148.

Johansen, L. (1960). Multi-sectoral study of economic growth.

Leontief, W. W. (1965). The structure of the US economy. *Scientific American*, 212(4):25–35.

Leslie, D. and Wise, J. (1980). The productivity of hours in UK manufacturing and production industries. *The Economic Journal*, 90(357):74–84.

Mangan, J. and Steinke, J. (1987). Working-time reductions, employment and wage costs. *Asia Pacific Journal of Human Resources*, 25(2):34–39.

Miller, P. and Mulvey, C. (1991). Trade unions and the distribution of paid overtime. *Journal of Industrial Relations*, 33(2):220–233.

Nielson, L., Harris, B., et al. (2010). *Chronology of superannuation and retirement income in Australia*. Department of Parliamentary Services.

Norris, K. (1996). Economics of Australian labour markets.

Oi, W. Y. (1962). Labor as a quasi-fixed factor. *Journal of political economy*, 70(6):538–555.

Pearson, K. (1991). Solving nonlinear economic models accurately via a linear representation.

Rosen, S. (1968). Short-run employment variation on class-I railroads in the US, 1947-1963. *Econometrica: Journal of the Econometric Society*, pages 511–529.

Stone, R. (1954). Linear expenditure systems and demand analysis: an application to the pattern of british demand. *The Economic Journal*, 64(255):511–527.

Trejo, S. J. (1991). The effects of overtime pay regulation on worker compensation. *The American Economic Review*, 81(4):719–740.

Warren, D. et al. (2008). Australia's retirement income system: Historical development and effects of recent reforms. Technical report, Melbourne Institute of Applied Economic and Social Research.